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Personalized Pricing with Imperfect Customer Recognition

Abstract

We consider a duopoly model where firms can identify only a share of consumers, which is positively correlated with the consumer' preferences. Firms charge personalized prices to the consumers they can recognize and a uniform price to the rest of consumers. The firms' available information is given by the combination of two factors: the intensive margin, which determines the share of consumers the firms can recognize in each single location, and the extensive margin, which determines how many locations the firms can identify. Different market configurations emerge according to the size of these margins. We characterize the values of the intensive and extensive margins that maximize firms' profits, and we show that profits are non-monotonic. We also show that the composition, in addition to the size, of the available information – i.e., the mix of these margins – affects firms' profits significantly. Implications for regulatory policies concerning the protection of consumers' information are finally discussed.

JEL-Codes: D800, D430, L100.

Keywords: personalized pricing, price discrimination, privacy, margins of information.

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1. Introduction

The technological progress in the last decades has enabled firms to acquire and store a large amount of information about their consumers. This information is then used to tightly target pricing, advertising and products according to their preferences with important effects on consumer surplus and social welfare. On the one hand, more information makes firm's pricing strategies more aggressive at the benefit of consumers (Thisse and Vives 1988, Corts 1998) and improves the quality of recommendations and other services offered to the consumers (Ichihashi 2020). On the other hand, better information on consumer preferences allows firms to infer consumer's willingness to pay and to extract more consumer surplus by charging personalized prices, so that some consumers are made worse off and some are made better off (Taylor and Wagman 2014).[1]

This issue is particularly relevant for regulators and policy makers worried about the potential unfair and anti-competitive use of consumer data when formulating the privacy policy especially for digital markets.[2] Indeed, competition between firms is influenced by the quantity and quality of information they are able to collect (Campbell *et al.* 2015, de Corniere and Taylor 2021). The current regulation on data protection in the digital markets (see e.g., the European GDPR and the FTC 2012 recommendations on protecting consumer privacy[3]) imposes some restrictions on gathering information

[1] Shiller (2016) showed that tailoring prices based on web browsing histories could increase Netflix profits by 14.55%, with some consumers paying nearly double the price others do for the same product. He also found that using only demographics to personalize prices raises profits by only 0.30%, which suggests that the percent profit gain from personalized pricing through the use of big data has increased 48-fold.

[2] Experts and policymakers have recognized the relevance of price targeting (see e.g., OFT, 2013, and OECD, 2018) at least from a theoretical point of view. However, they also noted a relative shortage of studies about the extent to which personalized pricing is widespread in the marketplace. Only in the recent years, Antitrust Authorities are experiencing an increasing number of high-profile cases of personalized prices. Describing the potential harms of algorithmic systems for consumers, the CMA (2021) mentioned the case of a British home improvement firm, called B&Q, which used computerized price tags in its stores to implement dynamic pricing. These price tags used data from customers' phones to modify the price that was shown in accordance with the customer's spending patterns and loyalty card information. In 2018, the Brazilian Consumer Defense Office of the Department of Justice fined 7.5 $ million the online travel agency Decolar.com for having adopted practices known as geopricing and geotagging – i.e., products were offered at very different prices to consumers depending on their location. Finally, in 2017, Thomson Reuters documented the use of consumers' personal information by the international airline AirAsia to personalize baggage pricing.

[3] See the FTC Issues Final Commission Report for businesses and policymakers on Protecting Consumer Privacy in an Era of Rapid Change.

about past purchases, search history and personal data and improves transparency about their use. For example, the GDPR gives the consumers more control over their data and it allows them to block a website from collecting personal data by simply denying the consent. In the same vein, the FTC promoted the implementation of an easy-to use, persistent, and effective Do Not Track system.

In addition to the limits imposed by the regulators, the amount of information firms are able to collect is influenced by the consumer' behavior. When consumers search online for a product, firms can track their search pattern obtaining some useful information on their preferences. Given the tracking technology, a larger number of visits by the same consumer to the online store provides more information than a single visit. Thus, each firm is more likely to track the behavior of consumers whose preferences are more geared to its product, because a consumer with strong preferences for that particular product leaves more traces than a consumer with a smaller interest.[4] As a result, the amount of information available to each firm is likely to be less than perfect and positively related to the consumer' preference for its brand.

Motivated by these observations, we investigate the effect of more information in a model where firms compete à la Hotelling having only partial information on consumer' preferences. Specifically, we assume that for each point of the Hotelling line the firm knows only the preferences of a fraction of consumers and this fraction decreases as we move away from the firm's location. Thus, each firm's ability to detect consumers' preferences is higher for consumers with strong preference for its brand.

We model the available information to the firms by using two parameters, representing the *extensive margin* (how many locations the firm knows) and the *intensive margin* (how many customers the firm can identify in each single location). Doing this,

[4] This is verified not only in the online stores but also in physical markets. Let us consider, for example, the pre-purchase tests and free trials, which are very common for high-tech products, cars, and food and beverages. These experimentations allow consumers to know better the products' characteristics, but they also allow firms to get some information about the prospective customers. However, these tests are usually time-consuming, thus an individual is more likely to take a test only for a preferred product (see Colombo and Pignataro, 2022).

we can investigate the unintended effects of the privacy regulation that impacts not only the amount of information gathered by the firms, but also the extension of the power of consumer geolocation. In other words, the setup of our theoretical model allows us to analyze the effects of changes in both the size and the composition of the firms' information set.

Firms charge personalized prices (first-degree price discrimination) to the identified consumers, and a uniform price to the rest of consumers. In this information structure, the set of identified consumers of one firm never coincides with the set of the identified consumers of the rival, and different market configurations emerge according to the degree of the overlapping between these sets. Accordingly, we characterize three distinct regions of parameters in which personalized prices are charged by both firms, only one firm, and neither firm. These regions give rise to different competitive outcomes according to their relative size. The aggressive pricing strategies used when both firms can price discriminate are restricted to the region where the sets of identified consumers overlap. By contrast, if consumers are unidentified by both firms, competition is softened by the use of a uniform price. In between, there is a group of consumers that is recognized only by one firm, which acts as a sort of residual monopolist, with the rival's uniform price as the unique competitive constraint. We show that the existence of areas where firms have different information has interesting implications for the firms' pricing strategies, equilibrium outcomes, and privacy regulation. Moreover, we contrast the results obtained in two alternative information regimes, one in which each firm can collect information on consumers located also in the rival's turf, which is denoted as the *unconstrained regime*, and one in which each firm can collect information only on consumers in its own turf, which is denoted as the *constrained regime*.

Our main findings are as follows. We find that a change in either the intensive or the extensive margin has a non-monotonic impact on firms' profits. In the *unconstrained regime*, we show that if the intensive (extensive) margin is sufficiently small (large), an increase in one of the two dimensions of the information set – e.g., it may coincide with loosening privacy rules – can affect firms' profits positively. Otherwise, any increase in the margins of information decreases firms' profits. Hence, according to the starting level of information, enlarging the set of consumers to charge personalized (rather than uniform) prices can be profit increasing. This result holds also in the *constrained regime* but with a significant difference. In the *unconstrained regime* an increase in the available information can lead to a local maximum, but the global maximum corresponds to a corner solution with no information and firms competing in uniform prices. Conversely, adopting personalized prices is a profit maximizing strategy in the *constrained regime.* Indeed, we show that, in this case, personalized and uniform prices shall coexist to maximize firms' profits, which implies that the level of information shall be less than perfect.

We also characterize the region of parameters in which firms' profits are minimized (coinciding the case where the consumer surplus is maximum), by managing in turn one of the two margins of information. Specifically, we show that personalized prices make competition tough and therefore making the market as transparent as possible – i.e., maximizing the set of available information – protects consumers from their surplus extraction. This is verified in any regime with one exemption. In the *constrained regime*, when the intensive margin is sufficiently small, such that an increase in the extensive margin does not allow firms to recognize the entire set of consumers, an intermediate value of the extensive margin is optimal from the consumers' point of view. In this case, there is an interior value of the extensive margin that leads firms to adopt both personalized and uniform prices. Indeed, managing the extensive margin does not allow to make customer recognition perfect and the *constrained regime* prevents firms from competing harshly through personalized prices. This suggests that the information gathering process has important effects on the firms' desired level of information and on consumer surplus. Thus, the optimality of the policy interventions should be evaluated by taking into account how firms collect information and if they have any restriction in doing it.

Finally, we argue that both the size and the composition (in terms of consumer locations) of information matter, so that sets of recognized consumers with the same size but with a different composition are not equivalent. Indeed, modeling information as a two-dimension variable enables us to evaluate changes in the two margins that modify the composition of the sets of recognized consumers while leaving unchanged their size. Each firm is eager to trade a higher intensive margin with a lower extensive margin, keeping the size of the information set constant, as the customers who are closer to its location have a higher willingness to pay. Therefore, our model provides a useful theoretical approach to evaluate any regulatory intervention that may have conflicting effects on the two margins of information. For example, a regulation like the California Privacy Rights Act (CPRA), that limits the use of geolocation tracking tools but does not prevent firms to collect information about consumers, guarantees an intensive margin but restricts the extensive margin of information.

Our paper is related to the stream of literature on consumer privacy that considers how the availability of consumer data affects the firms' pricing strategies and the competitive environment in which they operate.[5] A common finding in this literature is that firms are hurt by the possibility of perfect price discrimination (Thisse and Vives 1988, Corts 1998, Taylor and Wagman 2014). A more nuanced effect of information on firms' profits emerges in a series of recent papers that underline how the presence of heterogeneities at the firm level can soften competition. The heterogeneity may arise either because of asymmetries in costs (Houba *et al*. 2023, Matsumura and Matsushima 2015), or asymmetries in the available information (Chen *et al*. 2020, Shy and Stenbacka 2016), or in market shares (Colombo *et al.* 2021, Gehrig *et al*. 2012).[6] Shy and Stenbacka (2016) compare three regimes characterized by diverse degrees of privacy protection. They show that firms are better off in the intermediate regime where they can gather consumer information but cannot share it with rivals. Chen *et al*. (2020) analyze a duopoly model where consumers can prevent firms from tracking their behaviors

[5] For a recent survey see Acquisti et al. (2016). For an analysis of the monopolistic market see Acquisti and Varian (2005), Taylor (2004) [NON CITATO], Conitzer et al. (2012), and Ichihashi (2020).

[6] Matsumura and Matsushima (2015) find that, when there are cost asymmetries, the low-cost firm benefits from price discrimination. Houba et al. (2023) confirm this finding with vertically differentiated products and cost asymmetries. Asymmetries in the market shares may affect the profitability of price discrimination (Colombo et al., 2021) or let the firm enjoy the advantage provided by switching costs (Gehrig et al., 2012).

through identity management. They find that consumers actively engaged in identity management to prevent price discrimination may lower consumer surplus and benefit firm profits. Hence, they suggest that privacy regulation should be carefully studied because stricter privacy rules may harm consumers. Similarly, Clavorà Braulin (2023) considers a two-dimensional model of product differentiation where information on both or only one dimension of consumer preferences can be acquired by two competing firms. The main finding is that firms' profits are higher when firms have information only on one consumers' dimension because the lack of information on one dimension attenuates competition. Different information structures are analyzed also in the dynamic model of Bimpikis *et al.* (2021) where two competing firms, one multiproduct and one single-product, can leverage the information generated by earlier transactions in one product market to infer consumer preferences about the other product. They find that the impact of information acquisition on consumer surplus depends on the degree of competition: consumer surplus may be higher with data tracking technologies when markets are competitive.

Our paper departs from the above-mentioned literature in two aspects: *i*) firms' information is modeled along two margins (extensive and intensive) rather than one; *ii*) firms are not assumed being able to identify all consumers or none of them (that is, partial information is allowed). A paper closer to ours is Chen and Iyer (2002). They analyze the investment in information (addressability) by two competing firms that are able to identify only a portion of the segment where consumers are located. The authors show that firms choose low investment levels to reduce price competition. In their model, the equilibrium level of addressability – i.e., the proportion of consumers at each point on the line – is uniform. However, the authors also suggest that exploring a location-specific addressability would be an interesting extension of the model, because it would allow firms to decrease their investment as the distance from them increases. The present paper meets this challenge. By doing this, it explores a larger variety of competitive situations resulting from the possible degree of overlapping between the set of identified consumers of the two firms.[7,8]

Our paper is finally related to a fast-growing literature that focuses on how much information should a data broker sell to firms competing in the same market (Montes *et al.*, 2019, Bounie *et al.*, 2021, Abrardi *et al.*, 2022). A few papers in this strand of literature focus on how to partition the consumer segment to maximize the data broker revenue. Bounie *et al.* (2021) study how much information should a data broker sell to two competing firms, by using the consumer partition proposed by Liu and Serfes (2004). The authors find that the broker maximizes its profit by selling information about the high-willingness-to-pay consumers and withholding information on the low-willingness-to-pay consumers. Hence, firms may end up with a partial knowledge of the consumers because the data broker strategically manipulates information to soften competition between firms. Abrardi *et al.* (2022) find a qualitatively similar result in a circular model à la Salop where the data broker sells information to alternating firms in such a way to create local monopolies. The present paper contributes to this stream of literature by showing which kind of information the data broker should collect and then sell to firms. Indeed, we show that a good amount of information on a relatively small set of consumers (high-willingness-to-pay customers) is in general more valuable than some information on all the consumers. In other words, focused information (high intensive margin and low extensive margin) is more valuable than dispersed information (low intensive margin and high extensive margin).

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[7] Another paper that assumes that firms can identify only a fraction of consumers is Belleflamme et al. (2020) where two firms selling a homogeneous product can profile consumers with a probability smaller than one. Our model shares with theirs the assumption that firms know the preferences of some, but not all, consumers. However, it differs in several features, the most important of which is the fact that they consider a homogeneous product and therefore consumers preferences do not play any role, contrary to what happens in our model.

[8] There are other papers that consider that firms may have less than perfect information. For example, Chen et al. (2001) and Esteves (2014) consider the case in which firms receive some private information about their customers. The information is noisy, and therefore it may result in misclassification of consumers. Our approach is different in the sense that information is always correct though it may be partial.

The paper is organized in the following way. The next section describes the model. Section 3 presents some preliminary results useful for the subsequent analysis. Section 4 illustrates the equilibrium analysis for the unconstrained information model. Section 5 describes the equilibrium in the constrained information model. In Section 6 we turn to the policy implications of the theoretical models. Finally, Section 7 concludes. All proofs are relegated to the Appendix.

2. The model

Players. Consider a duopoly in which firms produce differentiated products at a constant marginal cost, which is normalized to zero without loss of generality. Firm A is located at the left endpoint of a Hotelling line of length 1, while firm B is located at the right endpoint. We assume there is a unit mass of consumers, who are distributed uniformly along this segment. The location of each consumer in the product space is denoted by $x \in [0,1]$ which represents the preferred variety of a given good and the relative preference for the products on sale: the more the consumer is close to 0 (1), the higher the willingness to pay for the product sold by firm A (B). Consumers, whose outside option is equal to zero, are willing to purchase at most one of the products on sale. Thus, the utility function of a consumer i located in x, if she buys a product from firm j (j = A, B) is:

$$u_i^j(x) = v - p_j - t|x - l_j|$$

where v is the intrinsic value of the product, which is assumed to be large enough so that each consumer always buys one product in equilibrium, p_j is the price charged by firm j, t represents the standard transportation or distaste cost – i.e., the disutility of consuming a variety of product which is different from the preferred one – and l_j is the location of each firm along the Hotelling line – i.e., $l_A = 0$ for firm A and $l_B = 1$ for firm B.

Information technology. There is an information technology which allows firms to collect data about the preference (location) of each single consumer. We consider an exogenously given and symmetric information technology which might be imperfect and reveal the preferences only of a share of consumers. Specifically, each firm perfectly knows the preferences of those consumers belonging to a defined subset, one for each firm, whose size depends on the degree of accuracy of the information technology.

We assume that the subset of consumers whose preference is known by a firm gets smaller as the distance from the firm's location becomes larger. In other words, each firm is more likely to track the behavior of those consumers whose preferences are closer to the firm's location in the product characteristic space: indeed, these consumers are expected to leave more traces than those consumers with a milder interest for the firm's product (see the Introduction).

In particular, we define the subset of consumers whose preferences are known by firm A and firm B as the areas underlying the following functions:

$$\text{Firm A:} \quad g(x) = a - \frac{1}{\gamma}x$$

$$\text{Firm B:} \quad f(x) = a - \frac{1}{\gamma}(1 - x)$$

where $a \geq 0$ and $\gamma \geq 0$ represent the *intensive* and *extensive* margin of information, respectively.[9] More specifically, a denotes the fraction of consumers with the highest willingness to pay (i.e., with the closest location to one of the two firms) each firm knows ("intensive margin"). The parameter $\frac{1}{\gamma}$, instead, represents the rate at which the accuracy of information declines with the distance from the firm's location. Therefore, for any given value of a, the larger is γ, the flatter the above functions, which implies that firms have *more* information about the consumers far away from their locations ("extensive margin"). Thus, when the value of the two functions is less than one,[10] they can represent

[9] The intensive margin of information represents the maximum amount of information that firms can collect in compliance with the rules on privacy and processing of personal data. Instead, the extensive margin of information represents the firms' ability to collect information about even more distant consumers in terms of location and/or preferred variety. A Public Authority can affect both margins of information by restricting or relaxing the privacy rules and facilitating or hindering the use of tracking tools. The intensive margin of information can be affected, for instance, by the prohibition of acquiring consumers' data from third parties, so that firms are less likely to know the preferred variety of each consumer. The extensive margin of information, instead, can be affected by a reduction in the search costs for consumers, that may induce them to gather information even on products that are further from their preferences, leaving traces of their research (see e.g., the digital footprints traced by the current available technologies such as cookies, mobile platforms, apps, and geolocation device, etc.).

[10] If this is not the case, the firms recognize all the consumers located in x.

the probability with which the firm identifies all consumers located in x, with such probability decreasing as we move far away from the firm location.

Based on this information, there are two groups of consumers for each firm: the "identified" consumers and the "unidentified" consumers. The former refers to the consumers whose preferences are perfectly known, while the latter refers to the consumers whose single preferences are unknown. A larger value of a implies an increase in the set of identified consumers at any location. Conversely, an increase in γ results in a larger proportion of distant consumers which are identified. This way of modeling firm's information allows us to analyze separately the effects of a change in the size of the firm information set and a change in its composition, in terms of the distance of the identified consumers. Indeed, both the size and the shape of the area below the curves matter.

Figure 1 illustrates the effect of a variation of a and γ on the g-curve (obviously, similar effects arise when the f-curve is considered).

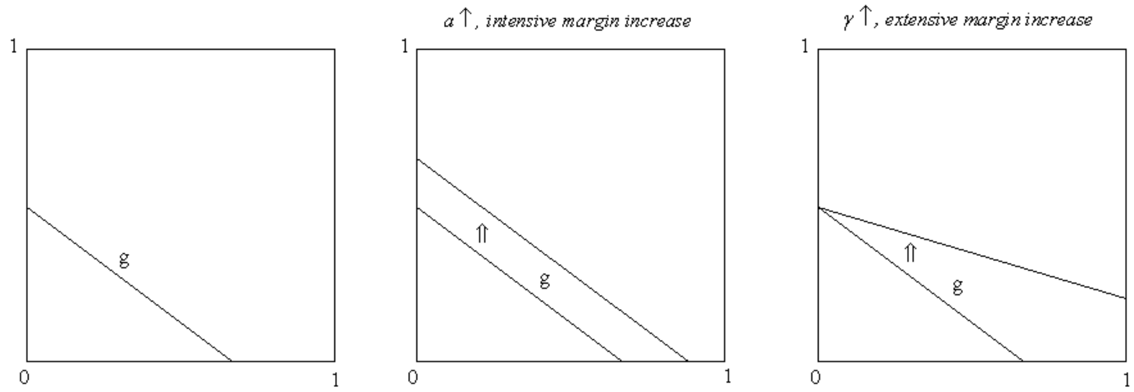

Figure 1: variations in the intensive and extensive margins

Firms can charge a personalized price for each consumer belonging to its identified group of consumers, while the rest of consumers is offered a uniform price. Therefore, each firm might make personalized offers to a fraction of consumers, and this set becomes smaller as we move toward the location of the rival. Hence, according to the size of the subsets of "identified" consumers, there are regions where none, one, or both firms can charge personalized prices.

Information regime. We consider two alternative information regimes: the first in which firms can collect information also about consumers belonging to the rival's turf (denoted with UR – i.e., unconstrained regime); the second in which firms have some restrictions in gathering consumers' information and, specifically, they cannot identify consumers belonging to the rival's turf (denoted with CR – i.e., constrained regime). In particular, we assume that the turf of firm A (B) is composed of those consumers located at the left (right) of ½.

Notice that, under an unconstrained regime each firm can enforce a tough competition on a share of consumers much closer to its rival, by charging personalized prices. Instead, under a constrained regime, each firm posts a uniform price to the group of "unidentified" consumers, which includes the entire rival's turf and a quota of consumers, whose size depends on the parameters a and γ, belonging to its own turf.

In practice, while in the unconstrained regime firms can enforce tracking tools that provide information even on remotely located consumers (or with preferences closer to the product sold by the rival), in the constrained regime firms can acquire information only about the closest consumers. This might be due to the fact that firms, which shared the market equally in previous periods, might collect information only about past consumers.

We will solve the model first under the UR assumption, and then under the CR assumption.

Market structure. According to the size of the group of identified consumers, which depends on the parameters a and γ and the information regime (UR or CR), there might be several different market structures. Basically, the two curves, g and f, might intersect within the market "square", above the market, or below the market. Furthermore, the vertical intercept of g and f might be in the market square (when a is lower than 1) or above (when a is greater than 1). Hence, in the former case, there always exists a group of unidentified consumers for any value of γ, while in the latter case there exists a value of γ above which the group of unidentified consumers disappears. This gives raise to several market structures, which are fully depicted in Figure 2. For the sake of an easy but complete representation, we distinguish between the market structures resulting in both information regimes, only in UR, and only in CR.

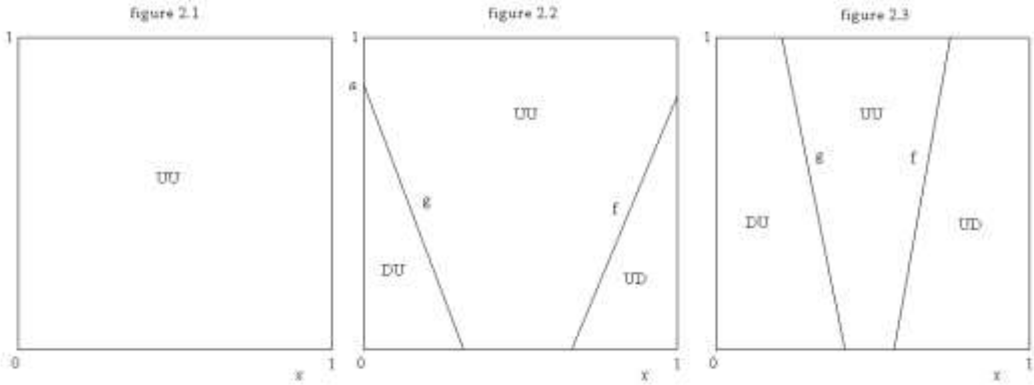

Figure 2.1-2.3: market structures both in UR and CR

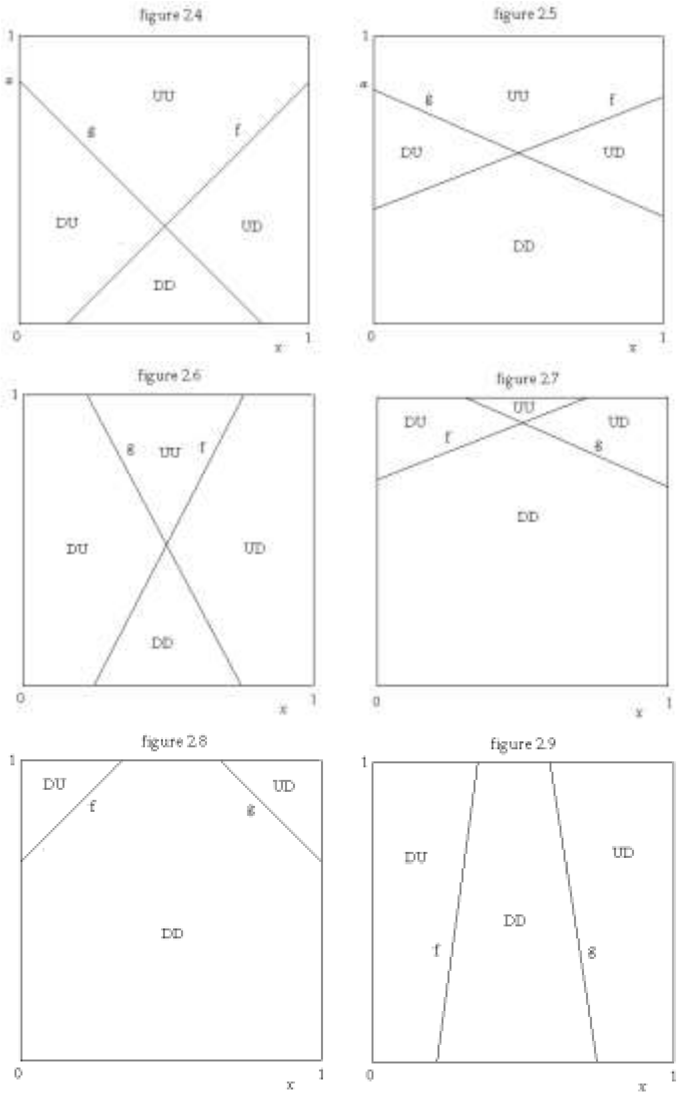

Figure 2.4-2.9: market structures only in UR

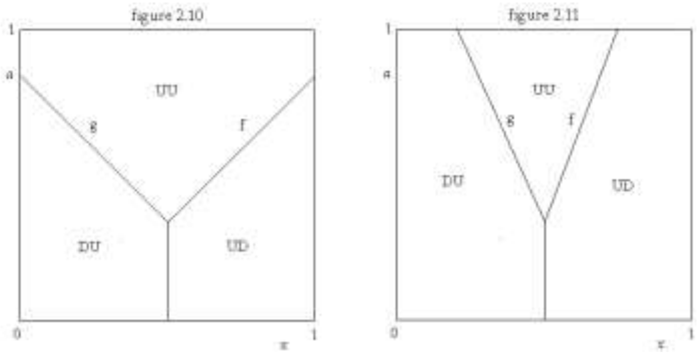

Figure 2.10-2.11: market structures only in CR

Figure 2: Market structures for any information regime

3. Preliminaries

To begin with, we first analyze how to tackle any possible area emerging from the market structures illustrated in Section 2. It is worth noting that, whatever is the level of a and γ, we might observe the following areas within the market square (depending on a and γ, some or all of them might co-exist):

1) a UU area, which is composed of those consumers who are unidentified by both firms, that have no choice but to charge a uniform price;
2) a DD area, which is composed of those consumers who are identified by both firms, charging discriminatory prices;
3) a DU (UD) area, which is composed of those consumers who are identified only by firm A (B), that charges discriminatory prices, while the rival, having no information about this group of consumers, posts a uniform price.

In particular, the following limit cases can be observed. When $a = 0$ and/or $\gamma = 0$, there is only area UU, so we are back to the traditional Hotelling model. By contrast, in the unconstrained (constrained) regime, when $a \geq 1$ and $\gamma \geq \frac{1}{a-1}$ ($\gamma \geq \frac{1}{2(a-1)}$) there is only area DD (area DU/UD).

Area UU. Let us indicate by k the position of those consumers which are indifferent between the two firms given the uniform prices p_A and p_B. Notice that k comes from the solution of $v - p_A - tk = v - p_B - t(1-k)$. Therefore, $k = \frac{p_B - p_A + t}{2t}$.[11] Given k, the profits of the two firms are $\pi_A^{UU} = \int_{UU_A} p_A dx$ and $\pi_B^{UU} = \int_{UU_B} p_B dx$, where UU_j is the subset of area UU served by firm *j*=A,B when the prices are p_A and p_B. The maximization of π_A^{UU} and π_B^{UU} yields the equilibrium uniform prices p_A^{UU*} and p_B^{UU*}.

Area DU and UD. It is well-known (see Thisse and Vives, 1988, for instance) that, in the asymmetric case where one firm can set personalized prices whereas the other sets a uniform price, an equilibrium in pure strategies exists only if the former moves after observing the choice of the latter.[12] Consider area DU (the analysis for area UD is symmetric). The optimal personalized price - $p_{A,x}^{DU*}$ - is the highest value that allows the price discriminating firm to capture the consumer given the equilibrium uniform price, or $u_x\left(p_{A,x}^{DU*}\right) = u_x(p_B^{UU*})$. That is, the optimal personalized price is equal to the uniform price charged by the rival firm plus the difference in the transportation costs borne by the consumer when purchasing from the most distant firm – firm B – rather than from the closest firm – firm A. It follows that the equilibrium profits of firm A and B in area DU are $\pi_A^{DU*} = \int_{DU_A} p_{A,x}^{DU*} dx$ and $\pi_B^{DU*} = \int_{DU_B} p_B^{UU*} dx$.

With regard to the area DU and UD, we can state the following lemma:

Lemma 1. In a symmetric equilibrium, $\pi_B^{DU*} = 0$ and $\pi_A^{UD*} = 0$.

Lemma 1 clarifies that, in a symmetric equilibrium, a firm is not active in the area where only the rival is able to set personalized prices. However, it should be observed that its potential competition sets a ceiling to the price that the price discriminating firm can charge, thus preventing it from fully exploiting its monopolistic position.

Area DD. In this area, both firms set personalized prices. In equilibrium, a consumer located at x buys from the closest firm, and the personalized price is equal to the difference between the consumer's transportation costs when buying from the furthest

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[11] For any location along the horizontal axis of the market square, there is a set of consumers identified along the vertical axis, which ranges from 0 to 1 (see Figure 2). Therefore, we have a set of indifferent consumers, rather than a unique indifferent consumer.

[12] Indeed, the price discriminating firm enjoys more flexibility in its pricing strategy than the uniform pricing firm.

firm and those sustained when buying from the closest firm (see Lederer and Hurter, 1986, for instance). In other words, focusing on symmetric situations, the equilibrium personalized price at $x \leq \frac{1}{2}$ is $p_{A,x}^{DD*} = t(1-x) - tx$ and that at $x \geq \frac{1}{2}$ is $p_{B,x}^{DD*} = tx - t(1-x)$. The equilibrium profits are therefore $\pi_A^{DD*} = \int_{DD_A} p_{A,x}^{DD*} dx$ and $\pi_B^{DD*} = \int_{DD_B} p_{B,x}^{DD*} dx$.

4. Unconstrained Regime: Equilibrium Analysis

In what follows we characterize the equilibrium outcomes in the unconstrained regime (UR). In particular, we distinguish the following three different cases according to the role played by the indifferent consumers:

1) the set of indifferent consumers, k, belongs entirely to the UU area, that is, all the indifferent consumers are unidentified by both firms.
2) the set of indifferent consumers, k, belongs partially to the UU area and partially to the DD area, that is some of the indifferent consumers are unidentified by both firms, and some are identified by both firms.
3) the set of indifferent consumers, k, belongs entirely to the DD area, that is, all the indifferent consumers are identified by both firms.

4.1 Case 1: unidentified indifferent consumers

Let us look first to the case where the indifferent consumers are all unidentified. This implies that the sets of consumers identified by the two firms are disjoint and there doesn't exist a DD region. In other words, the last consumer whose location is known by firm A is $x_A = a\gamma \leq \frac{1}{2}$, while the last consumer identified by firm B is $x_B = 1 - a\gamma \geq \frac{1}{2}$. By inspecting the relevant horizontal and vertical intercepts of line *g* and line *f* with the boundaries of the square, we observe that this case occurs for any a and for $\gamma \in \left[0, \frac{1}{2a}\right]$ or, alternatively, for any γ and for $a \in \left[0, \frac{1}{2\gamma}\right]$. These situations are illustrated in Figure 2.1, 2.2, and 2.3.

In this market configuration three areas might emerge: the UU area that lies at the centre of the market square,[13] and the DU and UD areas that are located in the left and right portion of the square, respectively. This implies that all consumers located at $x = \frac{1}{2}$, i.e., the indifferent consumers, are unidentified. Given the symmetry in the model, we restrict our attention to firm A and we focus on the left half of the market square.

Consider the UU area where both firms are active and charge the uniform prices p_A^{UU1} and p_B^{UU1}. As illustrated in the Preliminaries, the optimal uniform price for firm A is the one that maximizes the following profit function:[14]

$$\pi_A^{UU1} = \int_{UU_A} p_A^{UU1}\, dx = \begin{cases} \int_0^{a\gamma} p_A^{UU1}[1 - g(x)]\, dx + \int_{a\gamma}^{k} p_A^{UU1}\, dx & if \;\; a \leq 1 \\ \int_{(a-1)\gamma}^{a\gamma} p_A^{UU1}[1 - g(x)]\, dx + \int_{a\gamma}^{k} p_A^{UU1}\, dx & if \; a > 1 \end{cases}$$

By the first-order conditions and imposing symmetry – i.e., $p_A^{UU1} = p_B^{UU1}$ –, the equilibrium (uniform) price is:

$$p_A^{UU1*} = p_B^{UU1*} = \begin{cases} t(1 - a^2\gamma) & if \;\; a \leq 1 \\ t(1 - 2a\gamma + \gamma) & if \; a > 1 \end{cases},$$

Note that the uniform price is always decreasing in both margins implying that as the set of unidentified consumers shrinks, competition for the remaining consumers become tougher. The profit for firm A in the UU region is equal to:

$$\pi_A^{UU1*} = \begin{cases} \frac{t}{2}(a^2\gamma - 1)^2 & if \;\; a \leq 1 \\ \frac{t}{2}(\gamma - 2a\gamma + 1)^2 & if \; a > 1 \end{cases}$$

It is not surprising that also the profit π_A^{UU1*} is decreasing in the intensive and the extensive margin as well.

Consider now the other two regions. Firm A, as argued above, does not sell to any consumer in the UD area where consumers are identified by firm B only. Hence, we can restrict our attention to the DU area where the whole demand is for product A. We know from the Preliminaries that the equilibrium personalized price for firm A, $p_{A,x}^{DU*}$, is obtained by adding to the uniform price charged by the rival the difference in the

[13] As shown in figure 2.1, this is the unique area in the limit case in which $a = 0$ and/or $\gamma = 0$.
[14] The same holds for firm B.

transportation cost borne by the consumer. As a result, the impact of the intensive and extensive margins, a and γ, on the personalized price $p_{A,x}^{DU*}$ is the same as the one on the uniform price.

The profit in this asymmetric area is given by the following function:

$$\pi_A^{DU1*} = \begin{cases} \int_0^{a\gamma} p_{A,x}^{DU1*}\left(a - \frac{1}{\gamma}x\right)dx & if \quad a \leq 1 \\ \int_0^{(a-1)\gamma} p_{A,x}^{DU1*}dx + \int_{(a-1)\gamma}^{a\gamma} p_{A,x}^{DU1*}\left(a - \frac{1}{\gamma}x\right)dx & if \ a > 1 \end{cases}$$

which takes into account both the share of identified consumers that hinges on the size of the extensive margin and the quota of consumers that, when $a > 1$, firm A can identify regardless of the size of the extensive margin.

The profit in the DU region increases both with the intensive margin a and with the extensive margin γ, as the positive effect on the size of the area offsets the negative impact on the personalized prices. The reason is fairly intuitive: in the asymmetric area DU, firm A is more informed than firm B, which is forced to charge a uniform price, and an increase in the size of the group of identified consumers enhances the firm A's information advantage.

Summing up, the equilibrium profits when the indifferent consumers are totally unidentified are equal to:

$$\pi_A^1 = \pi_B^1 = \begin{cases} \frac{t(3-2a^3\gamma^2)}{6} & if \ \ a \leq 1 \\ \frac{t[3-\gamma^2(2-6a+6a^2)]}{6} & if \ \ a > 1 \end{cases} \tag{4}$$

Studying the effect of a and γ on (4) leads to the following Lemma.

Lemma 2. If the indifferent consumers are totally unidentified, firms' profits are decreasing in the intensive and extensive margin of information.

The above Lemma shows that a marginal increase in the firms' available information, such that it does not change the quota of unidentified indifferent consumers, decreases the aggregate profits. In other words, the negative impact on the profits from the group of unidentified consumers (area UU) overwhelms the positive effect on the gains from the group of identified consumers (area DU/UD). This implies that firms' profits are negatively affected by an increase in their ability to recognize customers.

4.2 Case 2: partially identified indifferent consumers

We now consider the case where some, but not all, indifferent consumers are identified, as shown in Figures 2.4-2.7. With the same logic adopted in the previous Section, we need to inspect the horizontal and vertical intercepts of line g and line f with the boundaries of the square to determine the relevant parameter values. Specifically, this case occurs for $a \in [0,1]$ and $\gamma \geq \frac{1}{2a}$, or for $a \geq 1$ and $\gamma \in \left[\frac{1}{2a}, \frac{1}{2a-2}\right]$; or alternatively, for any γ when $a \in \left[\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}\right]$. These values of the parameters are such that the information sets of the two firms overlap (i.e., $x_A > \frac{1}{2}$ and $x_B < \frac{1}{2}$) giving rise to a positive DD area. Hence, in this case we have four market regions: UU, DU, DD and UD.

Consider first the case with $a \in [0,1]$, which is depicted in Figure 2.4 and 2.5.[15] In the UU region, firm A maximizes the following profit function:

$$\pi_A^{UU21} = \int_{UU_A} p_A^{UU21}\, dx = \int_0^k p_A^{UU21}[1 - g(x)]\, dx$$

where k indicates the unidentified indifferent consumers. Differentiating with respect to the price and imposing symmetry yield the following optimal uniform price:

$$p_A^{UU21*} = p_B^{UU21*} = \frac{t\big(1 + 4\gamma(1-a)\big)}{2 + 4\gamma(1-a)}.$$

Interestingly, in contrast to our findings in the previous case, the above uniform price is increasing in γ implying that an improved ability to identify the more distant consumers (extensive margin) leads to a higher uniform price despite the reduction of the UU region.

This is due to two factors, the combination of which dampens the effect of γ on the uniform price. First, when $a < 1$, there is a share of consumers that firms cannot identify independently of the value of γ. Eventually, for γ sufficiently large, the g curve becomes almost horizontal and the share of unidentified consumers is determined mainly by the

[15] In this case, we disentangle the market structures with $a \leq 1$ and with $a > 1$ because, as we show and discuss later, the impacts of the intensive and extensive margin on firms' profits vary between the two.

intensive margin a. This means that any change in γ has a small impact on the size of this area, reducing the price impact on the demand of unidentified consumers. Second, an increase in γ reduces the size of indifferent consumers that one firm is able to attract by lowering its (uniform) price. Hence, a price cut has a smaller (positive) demand effect than in the previous case, when the indifferent consumers are totally unidentified. More technically, an increase in γ reduces the demand elasticity in the UU area.

By contrast, the impact of a rise in the intensive margin a remains negative, as it simply coincides with a decrease in demand, lowering the share of consumers that firms cannot identify.

Consider now the case with $a > 1$ as depicted in Figure 2.6 and Figure 2.7. In the UU region, firm A maximizes the following profit function:

$$\pi_A^{UU22} = \int_{UU_A} p_A^{UU22}\, dx = \int_{(a-1)\gamma}^{k} p_A^{UU22}\, [1 - g(x)] dx$$

which yields, after imposing symmetry, the optimal uniform price:

$$p_A^{UU22*} = p_B^{UU22*} = t\left(\frac{1}{2} + \gamma - a\gamma\right)$$

which is decreasing both in γ and a. Let us focus on the extensive margin.[16] The intuition for the negative impact of the extensive margin on the uniform price relies on the relatively small size of the UU area, which now crucially hinges on γ. Indeed, in contrast with the case in which $a < 1$, any change in γ has a strong impact on the demand of the unidentified consumers and, in the limit case, for γ going to infinity, the UU area disappears. Furthermore, if γ increases, the region of unidentified consumers is more concentrated in the centre of the Hotelling line, where consumers pay the largest transportation costs. These factors lead firms to decrease their uniform price as γ increases in order to attract the indifferent consumers.

Consider now the DU area where consumers are served only by firm A, as argued in Section 3. By using the optimal price defined in the Preliminaries, we characterize the following profit functions, distinguishing for $a \leq 1$

[16] The intuition is the same for the intensive margin of information.

$$\pi_A^{DU21*} = \begin{cases} \int_0^{1-a\gamma} p_{A,x}^{DU2*} g(x)dx + \int_{1-a\gamma}^{k} p_{A,x}^{DU2*}[g(x)-f(x)]dx & if \quad \frac{1}{2a} \leq \gamma \leq \frac{1}{a} \\ \int_0^{k} p_{A,x}^{DU2*}[g(x)-f(x)]dx & if \quad \gamma \geq \frac{1}{a} \end{cases}$$

and $a > 1$

$$\pi_A^{DU22*} \begin{cases} \int_0^{(a-1)\gamma} p_{A,x}^{DU2*} dx + \int_{(a-1)\gamma}^{k} p_{A,x}^{DU2*}[g(x)-f(x)]dx & if \quad \frac{1}{2a} \leq \gamma \leq \frac{1}{a} \\ \int_0^{(a-1)\gamma} p_{A,x}^{DU2*}[1-f(x)]dx + \int_{(a-1)\gamma}^{k} p_{A,x}^{DU2*}[g(x)-f(x)]dx & if \quad \gamma \geq \frac{1}{a} \end{cases}$$

Finally, let us consider the DD area where both firms can identify consumers so that competition takes place as in the models where firms have perfect information. Given the pricing strategies in the DD area described in the Preliminaries, the firm A's profit function is:

$$\pi_A^{DD2*} = \begin{cases} \int_{1-a\gamma}^{k} p_{A,x}^{DD2*} f(x)dx & if \quad \frac{1}{2a} \leq \gamma \leq \frac{1}{a} \\ \int_0^{k} p_{A,x}^{DD2*} f(x)dx & if \quad \gamma \geq \frac{1}{a} \end{cases}$$

By taking the profits in each single area, we can evaluate the firm A's aggregate profit when $a \leq 1$

$$\pi_A^{21} = \begin{cases} \frac{t[(20a-4)\gamma - 5 + 12\gamma^2(4+2a-7a^2) - 48(1-a)^2\gamma^3]}{48\gamma[1-2\gamma(1-a)]} & if \quad \frac{1}{2a} \leq \gamma \leq \frac{1}{a} \\ \frac{t[(44a-32)\gamma + 7 + 24\gamma^2(2-3a+a^2)]}{48\gamma[1-2\gamma(1-a)]} & if \quad \gamma \geq \frac{1}{a} \end{cases}$$

and when $a > 1$

$$\pi_A^{22} \begin{cases} \frac{t[6(1+5a)\gamma - 5 + 12a\gamma^2(4-5a) - 8(2-6a+9a^2-5a^3)\gamma^3]}{48\gamma} & if \quad \frac{1}{2a} \leq \gamma \leq \frac{1}{a} \\ \frac{t[6(5-3a)\gamma + 7 - 16\gamma^3(1-a^3)]}{48\gamma} & if \quad \gamma \geq \frac{1}{a} \end{cases}$$

The firm A's aggregate profit behaviour is summarized in the following lemma.

Lemma 3. If the indifferent consumers are partially identified, firms' profits are inverse U-shape in the intensive margin of information if a is sufficiently small and in the extensive margin of information if γ is sufficiently small. Otherwise, they are decreasing in both margins.

The impact of a marginal increase in the extensive margin on profits is a combined result of the effect on prices and on the set of identified consumers, both in terms of size and shape. When $a \leq 1$ and for intermediate values of the extensive margin – i.e., $\gamma \in [\frac{1}{2a}, \frac{1}{a}]$ – the uniform price is increasing in γ. Since the personalized price hinges on the uniform price, an increase in γ has a positive effect on the personalized price as well. Furthermore, an increase in γ enlarges all the regions in which consumers are identified at least by one firm. Therefore, these market regions increase at the expenses of the UU area. In this region of parameters, the positive effect on the DU area more than compensates the increase in the less profitable DD and UD areas and profit increases. However, if γ is sufficiently large - i.e., $\gamma \geq \frac{1}{a}$ – an increase in the extensive margin allows firms to identify a larger set of consumers. This reduces the information advantage, represented by the DU area for firm A and the UD area for firm B and the overall effect on the aggregate profits becomes negative.

For the same parameter values, i.e., $a \leq 1$ and $\gamma \geq \frac{1}{a}$, the profit function is inverse U-shape in the intensive margin, when a is sufficiently small, and decreasing otherwise. It is worth noting that, in contrast with the extensive margin that may have a positive effect on the uniform price, the impact of the intensive margin on profits comes only from a change in the information set, because the uniform price is always decreasing in a. Indeed, a positive effect on profits occurs only if the increase in the DU area dominates the reduction in the UU area and the increase in the DD and UD areas. If a is sufficiently large, the opposite occurs.

When instead the parameter values are larger, i.e., $a > 1$ and $\gamma \in [\frac{1}{2a}, \frac{1}{2a-2}]$, the profit function π_A^{22} is always decreasing both in the intensive and the extensive margin. The intuition is the following. If $a > 1$, an increase in both margins contributes mainly to enlarge the region of identified consumers, where competition between firms is tough. Moreover, in this region of parameters, an increase in γ always has a decreasing effect on the uniform price. Therefore, the combination of the impacts on prices and on information sets makes the overall effect on profits negative.

4.3 Case 3: totally identified indifferent consumers

Finally, we consider the case where all the indifferent consumers are identified by both firms so that the UU area disappears. This case occurs when $a \geq 1$ and $\gamma \geq \frac{1}{2a-2}$ (Figures 2.8 and 2.9). It is also worth noting that there is a threshold of γ such that the market square shows only the DD area if $\gamma \geq \bar{\bar{\gamma}} \equiv \frac{1}{a-1}$. In this limit case, any further increase of γ has no impact on the market configuration and, consequently, on the equilibrium prices and profits. Clearly, by symmetry, there is also a threshold of a such that, if $a \geq \bar{\bar{a}} \equiv \frac{1+\gamma}{\gamma}$, any further increase of a doesn't increase the firms' ability to recognize consumers. Instead, if these are not the cases, there still exist the DU and UD areas. Each firm charges a price equal to zero to the consumers identified only by the rival, which replicates the tough competition in the DD area. Hence, in this region of parameters, firms compete through personalized prices for any consumer in the market square. The resulting profits are equal to $\pi_A^3 = \pi_B^3 = \frac{t}{4}$ as in the standard Lederer and Hurter (1986) model with perfect information.

Therefore, we can state the following lemma.

Lemma 4: If the indifferent consumers are totally identified, firms' profits are independent of the intensive and extensive margin of information.

The above Lemma provides an interesting result. If all the consumers in the market are identified at least by one firm, the competition becomes so tough that any variation in the intensive or extensive margin of information doesn't affect the firms' pricing behaviour. They simply post the most competitive price for the consumers they cannot identify, so exerting a downward pressure on the personalized prices. The lack of an area of unidentified consumers shrinks the firms' prices. This means that, in order to foster an aggressive pricing strategy, to the benefit of consumers, it is not necessary to make the market completely transparent, but it is sufficient to set a level of intensive and extensive margin of information such that just the indifferent consumers are identified by the firms.

4.4 Comparative statics

In this Section, we consider the overall relationship between the extensive and intensive margin of information and the firms' profits, by looking at the whole range of parameters. This allows us to identify the effect of any non-marginal variation in γ and a on firms' profits.

Let us consider first the extensive margin, γ. As discussed in the previous sections, an increase in γ enlarges the share of consumers that firms are able to identify, so shrinking the UU area and expanding the DU and UD areas if γ is sufficiently small. However, when γ is large, a further increase in γ expands the DD area at the expenses of the more profitable DU and UD areas. This clearly affects firms' profits because, by applying discriminatory prices, any area can be deemed as a distinct market and a variation in its size can be considered as a demand variation. The impact on the size of the different areas is coupled with the effect on personalized and uniform prices. In particular, we have already proved that an increase in γ has a negative effect on the uniform prices, apart from the case in which the indifferent consumers are partially identified and the intensive margin is sufficiently small.

In the following proposition, we characterize the impact of any increase in the extensive margin on firms' profits, by studying their shape in the whole range of parameters.

Proposition 1. *For* $a \in [0, 1]$, *firms' profits are strictly decreasing if* γ *is sufficiently small and inverse U-shape otherwise. For* $a \in (1,2)$, *firms' profits are:*

- *strictly decreasing if γ is sufficiently small;*
- *inverse U-shape if γ is intermediate;*
- *independent of γ if γ is sufficiently large.*

For $a \geq 2$, firms' profits are strictly decreasing if γ is sufficiently small and independent of γ otherwise.

For any $a > 0$, firms' profits are globally maximized at $\gamma = 0$.

This proposition shows that though $\gamma = 0$ corresponds to the global maximum, firms' profits might be non-monotonic in the extensive margin and, precisely, there is an intermediate range of γ where firms' profits are increasing in this information feature if the intensive margin is not too large. This is illustrated in Figure 3.

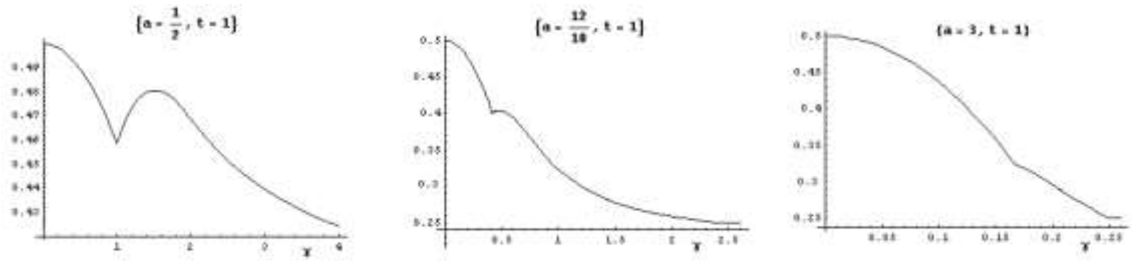

Figure 3: Impact of γ on firm A's profit in an unconstrained information regime.

This result highlights a novel characteristic of an increase in the firms' available information. According to a well-known result in the literature (Thisse and Vives 1988) only a monopolistic firm charging discriminatory prices can benefit from a more accurate level of information. In oligopoly, on the contrary, a larger share of identified consumers enlarges the area in which firms compete for each consumer individually, to the detriment of their profits. This is not the case in our model when firms have limited information about the closest consumers and the increase in the available information is more focused on the further rather than the closer consumers. In this event, if, for instance, a minimum level of the extensive margin is impossible to prevent (e.g., for geographical reasons or frequency of purchase), firms might have an incentive to invest in such an information technology in order to identify an intermediate share of consumers, corresponding to the local maximum when firms' profits are inverse U-shape. Moreover, if firms achieve that information technology simultaneously, this can also be seen as a form of collusion, which could be of interest and subject to investigation by the Antitrust Authorities.

Now, let us focus on the intensive margin a. An increase in the intensive margin enlarges the size of the group of identified consumers both with high and low willingness to pay as long as $a \leq 1$. Instead, when $a > 1$, an increase in the intensive margin enlarges the size of identified consumers that are located far from the firms, because the closest consumers are already recognized. It is worth noting that, in the latter case, the effect on the firms' information set is similar to an increase in the extensive margin as it allows to identify a larger share of further rather than closer consumers. Hence, the difference in the impacts of the extensive and intensive margins tapers off as a increases.

Moreover, likewise the extensive margin, an increase in a reduces the size of the UU area, it expands the DD area if a is sufficiently large and it enlarges the DU and UD areas as long as a is sufficiently small (otherwise they are replaced by the DD area). However, in contrast with the extensive margin, the impact of an increase in a on the uniform price is always decreasing, which translates into a downward pressure on the personalized prices. Therefore, any positive effect on firms' profits derives from an increase in the areas in which firms charge personalized prices.

The following proposition characterizes the impact of any increase in the intensive margin on firms' profits, by showing that a positive effect may exist only if the indifferent consumers are partially identified.

Proposition 2. *For $\gamma \in [0, \frac{1}{2}]$, firms' profits are strictly decreasing if a is sufficiently small and independent of a otherwise. For $\gamma \geq \frac{1}{2}$, firms' profits are:*

- *strictly decreasing if a is sufficiently small;*
- *inverse U-shape if a is intermediate;*
- *independent of a if a is sufficiently large.*

For any $\gamma > 0$, firms' profits are globally maximized at $a = 0$.

The above proposition shows that firms' profits are non-monotonic also in the intensive margin if γ is sufficiently large. Figure 4 illustrates how firms' profits varies according to the size of the intensive margin.

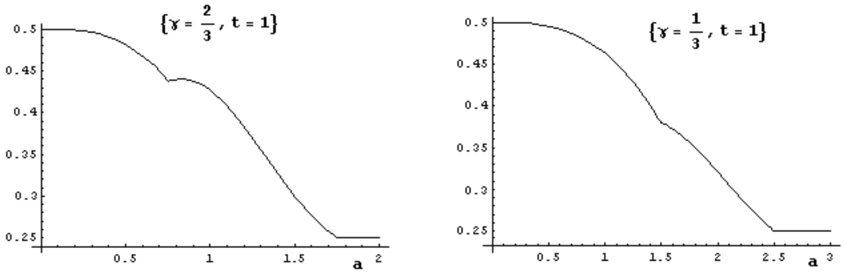

Figure 4: Impact of a on firm A's profit in an unconstrained information regime.

Interestingly, there is an intermediate range of a, in which firms' profits increase in the intensive margin if this information feature is not too large, that is, as shown in the Appendix, lower than 1. This implies that enlarging the size of identified consumers can increase firms' profits only if firms cannot recognize the entire set of consumers, including those that are located close to one of the two firms. The reason is the following. In this case, an increase in the intensive margin allows firms to enlarge the share of consumers with the highest willingness to pay, because of their locations in the Hotelling line. Instead, if $a > 1$, an increase in the intensive margin doesn't rise the share of consumers from which firms can extract more surplus, but it allows firms to identify a larger quota of the further consumers with a milder effect on their profits.

The inverse U-shape relationship between profits and information for intermediate values of the two margins echoes the shape of the relation between profits and consumer addressability in Chen and Iyer (2002). Our model shares with theirs the assumption that an imperfect information technology allows firms to identify consumers with a probability smaller than 1, which is constant in their model. An important difference between our model and theirs is the role played by those consumers which are unidentified by both firms; they are not served in Chen and Iyer (2002), while in our model firms compete in uniform prices for these consumers. As a result, the opportunity-cost of increasing the set of identified consumers is zero in Chen and Iyer (2002), while it corresponds to the difference between the earnings from uniform and personalized pricing in our model. Thus, the presence in our model of the highly profitable UU area makes our setting much less favorable to personalized pricing than theirs: nevertheless, for some parameter values more information can have a positive effect on profits.

Finally, we conclude this section by underlining an important difference between the intensive and extensive margin. These two margins of information allow us to isolate the impact of changes in the *size* and in the *composition* of the information set. If personalized prices are viable, firms are more interested in recognizing the closer rather than the further consumers, as they have a higher willingness to pay. Therefore, it is not surprising that firms are eager to trade a lower extensive margin for a higher intensive margin, as reported in the following remark.

Remark 1. Keeping constant the size of the firms' information sets, the optimal composition of the intensive and extensive margins of information involves the maximization of the intensive margin.

To verify this trade-off, we need to analyze the firm's profits by keeping constant their information set.[17] For the sake of simplicity, let us consider the case in which all the indifferent consumers are unidentified. An increase in the extensive margin and a concurrent decrease in the intensive margin (in order to keep the information set constant) imply substituting a share of closer with distant consumers in the group of identified consumers. Of course, this change in the composition of the identified consumers reduces the firms' profits, though the size of the information set remains constant.

With the same logic, we can argue that this result holds in the remaining two cases as well with an important difference in the areas involved in the trade-off. In the case of unidentified indifferent consumers, the trade-off between the two margins corresponds to the trade-off between closer and more distant consumers in the same DU or UD area. By contrast, in the other cases the trade-off is between closer consumers in the DU or UD

[17] A formal proof for the first case, which is the most controversial as argued later, is provided in the Appendix.

areas and more distant consumers, some of them in the same DU or UD area and a growing fraction of them in the DD area, which is characterized by tough competition between firms. This further reduces profits and shows that firms prefer increasing the intensive margin and reducing the extensive margin rather than the opposite.

Consider for example the impact of the GDPR on the airline industry empirically analyzed by Aridor *et al*. (2020). They found that such a regulation induced more consumers to deny consent to data collection but, at the same time, it made the remaining consumers more easily identifiable. These contrasting effects cannot be tackled in a standard model that uses only one dimension of the available information. Conversely, our model can accommodate a simultaneous decrease in the intensive margin and an increase in the extensive margin. Furthermore, we know that these changes, if the size of information set remains about constant, have a negative impact on firm profits.

5. Constrained regime

If firms collect information on consumer' preferences, for example through internet search on firms' website or previous purchases, it is reasonable to assume that each firm can gather information only on its closer consumers.[18] In this Section, we suppose that firms shared the market equally, so that each firm has information only on the consumers in the half market closer to its location. Doing so, we restrict x_A, the last consumer on the right whose location is known by firm A, to belong to the interval $\left[0, \frac{1}{2}\right]$. This prevents firm A from using personalized prices to compete for consumers closer to firm B (and *viceversa*). The peculiar characteristic of this regime is the absence of an area in which both firms can adopt personalized prices. Hence, as intuition suggests, the impact of a and γ on firms' profits differs from the previous analysis. In the following, we characterize the equilibrium outcomes, and we study how profits vary according to the size of the intensive and extensive margins of information (Section 5.1). Then, we analyse how the firms' competitive behaviour change in the neighbourhood of $a = 1$

[18] Indeed, it is reasonable to assume that consumers are more likely to browse firms' website for which they have a preference. Moreover, it is possible that online browsing alone can foster brand loyalty towards the product that users searched for (Colombo and Pignataro, 2022).

when the extensive margin tends to be perfect, such that all the consumers are recognized by one of the two firms (Section 5.2).

5.1 Equilibrium analysis

In this subsection, we characterize the equilibrium outcomes in this constrained regime (CR). Of course, if the indifferent consumers are totally unidentified, the equilibrium analysis has no differences with respect to the previous Section. Instead, when the indifferent consumers are totally identified, the market square features only the DU and the UD areas. Hence, any further increase of a or γ has no impact on the market configuration and the equilibrium prices and profits. Therefore, we focus on an intermediate region of the parameters in which the indifferent consumers are partially identified.[19] In this regime, there are only two relevant market structures which are illustrated in Figure 2.10 and 2.11.

The pricing strategy used in the UU area is the same as in the unconstrained regime, but now, any increase in the firms' information set leads to a reduction in the UU area and an expansion in the DU and UD areas. Therefore, the main difference with respect to the previous regime consists basically in the shape of the information set, which leads to the following profit function:

$$\pi_A^{CR} = \begin{cases} \dfrac{t[4\gamma(1+2a)+12\gamma^2(2-a-a^2)-1]}{24\gamma[1+2\gamma(1-a)]} & if\ a \in [0,1]\ and\ \gamma \geq \dfrac{1}{2a} \\ \dfrac{t[6\gamma(1+a)+12a\gamma^2(1-a)-8(1-a)^3-1]}{24\gamma} & if\ a > 1\ and\ \gamma \in \left[\dfrac{1}{2a}, \dfrac{1}{2a-2}\right] \end{cases}$$ [20]

It is worth noting that, for $a \in [0,1]$ and $\gamma \geq \frac{1}{2a}$ (Figure 2.10), the profit function is increasing, while for $a > 1$ and $\gamma \in \left[\frac{1}{2a}, \frac{1}{2a-2}\right]$ (Figure 2.11), the profit function is U-shape in the extensive margin γ. Gathering Lemma 2 and 4 together with these comparative statics, we can state the following.

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[19] The thresholds coincide with those identified in the previous Section.

[20] The constraints on a and γ can be also written as $\gamma \geq 1/2$ and $a \in [\frac{1}{2\gamma}, 1]$ (Figure 2.10) and $\gamma \geq 1/2$ and $a \in [1, \frac{1+2\gamma}{2\gamma}]$, or $\gamma \leq 1/2$ and $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$ (Figure 2.11).

Proposition 3. *For $a \in [0,1]$, firms' profits are U-shape in γ if γ is sufficiently small and increasing otherwise. For $a > 1$, firms' profits are:*

- *U-shape if γ is sufficiently small;*
- *strictly decreasing if γ is intermediate;*
- *independent of γ if γ is sufficiently large.*

Firms' profits are globally maximized for $\gamma \to \infty$ if $a \in [0, 1)$ and at $\gamma = 0$ otherwise.

Figure 5 illustrates Proposition 3.

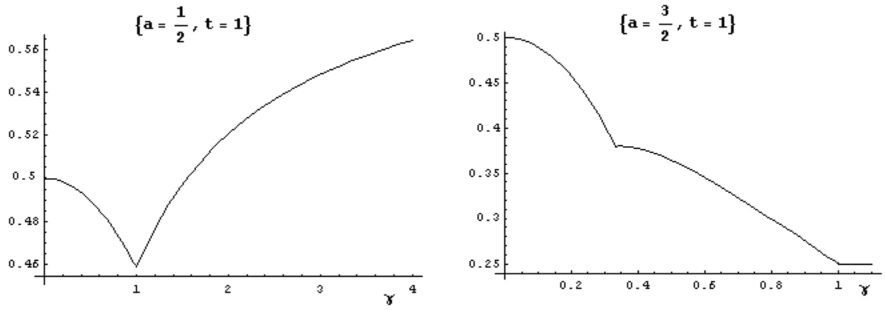

Figure 5: Impact of γ on firm A's profit in a constrained information regime.

Proposition 3 highlights the impact of an increase in the extensive margin when firms can use personalized pricing protected by the exclusive access to their information set. In this case, an increase in the extensive margin leads to a larger share of consumers for whom each firm has an information advantage over its rival, without the risk of creating a group of consumers identified by both firms and for whom the competition becomes very tough. Therefore, firms' profits are decreasing in γ as long as the negative effect in the UU area prevails on the positive impact on the DU/UD area. If γ is sufficiently large, the share of consumers identified only by one firm becomes so large to offset the reduction of profits from the group of unidentified consumers. Hence, firms' profits are maximized if the DU and UD areas are maximized as well – i.e., for $\gamma \to \infty$. However, if $a > 1$, an increase in γ leads the group of unidentified consumers to be localized in the middle of the market square, where firms' profits are low because of the consumers' transportation costs. This occurs as long as $\gamma < \frac{1}{2a-2}$. Indeed, for any $\gamma \geq \frac{1}{2a-2}$ firms recognize all the consumers belonging to their half of the market square and an increase in γ has no effect on firms' profits.

Let consider now the impact of the intensive margin a. With the same logic adopted to study how the extensive margin affects the firms' profits, let us disentangle the two cases illustrated in Figure 2.10 and 2.11. If $\gamma \geq 1/2$ and $a \in \left[\frac{1}{2\gamma}, 1\right]$ (Figure 2.10), the profit function is increasing (decreasing) in intensive margin if $a \leq (\geq) 1 + \frac{1-\sqrt{2\gamma}}{2\gamma}$. Instead, if $\gamma \geq 1/2$ and $a \in \left[1, \frac{1+2\gamma}{2\gamma}\right]$, or $\gamma \leq 1/2$ and $a \in \left[\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}\right]$ (Figure 2.11), the profit function is always decreasing in the intensive margin.

Gathering Lemma 2 and 4 together with these comparative statics, we can state the following.

Proposition 4. *For* $\gamma \in [0, \frac{1}{2}]$, *firms' profits are strictly decreasing if* a *is sufficiently small and independent of* a *otherwise. For* $\gamma \geq \frac{1}{2}$, *firms' profits are:*

- *strictly decreasing if* a *is sufficiently small;*
- *inverse U-shape if* a *is intermediate;*
- *independent of* a *if* a *is sufficiently large.*

Firms' profits are globally maximized at $a = 1 + \frac{1-\sqrt{2\gamma}}{2\gamma}$ *for any* $\gamma \geq \frac{2+\sqrt{3}}{3}$ *and at* $a = 0$ *otherwise.*

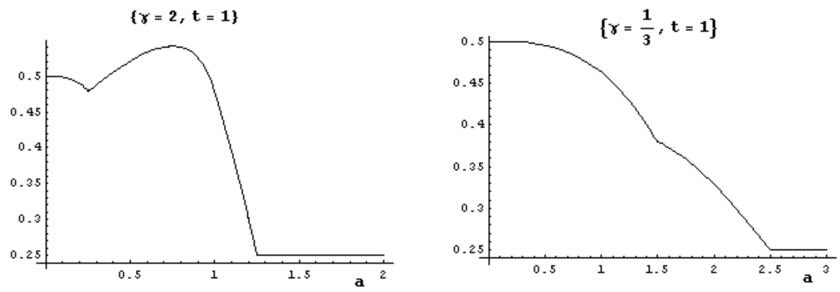

Figure 6: Impact of a on firm A's profit in a constrained information regime.

Figure 6 shows the results described in Proposition 4. It is worth noting that firms' profits follow a shape, according to the size of the intensive margin, reminiscent of that illustrated in the unconstrained regime. This differs from the impact of γ on firms' profits, which is qualitatively different between the unconstrained and constrained regime. However, the impact of a on firms' profits coincides only if the indifferent consumers are totally unidentified. Indeed, the constrained regime allows firms to prevent them from competing on personalized prices. Therefore, when the indifferent consumers are partially identified, any negative effect on firms' profits following an increase in the intensive margin is softened. In particular, when the extensive margin is sufficiently large, firms' profits are maximized if the intensive margin is intermediate. In fact, if the intensive margin is too large the UU area tends to disappear, so that firms' profits are minimized.

Finally, note that the optimal size of the intensive margin never coincides with perfect information. Specifically, firms' profits are maximized when there is no information at all or when firms recognize only a share of their consumers, such that there still exists a quota of consumers who are unidentified by both firms. Thus, in the constrained regime, the impact of the intensive and extensive margin on firms' profits are qualitative different. If the extensive margin is so small that firms cannot recognize a large quota of consumers, their profits are maximized if a uniform price is charged to all the consumers, as in the unconstrained regime. By contrast, if the extensive margin is sufficiently large, there is an intermediate value of a (lower than 1) that allows firms to maximize profits offering a mix of personalized and uniform prices. Instead, the optimal size of the extensive margin is always at the extremes. Specifically, if the intensive margin is imperfect, such that the extensive margin cannot get rid of the group of consumers unidentified by both firms, firms' profits are maximized when γ goes to infinity. Otherwise, non-personalized prices – i.e., uniform prices, when firms don't recognize consumers – maximize firms' profits. This provides two further results. First, it highlights that the extensive margin of information becomes much more relevant in the constrained rather than in the unconstrained regime, because now firms are protected against any rival's aggressive pricing strategy and a very large extensive margin increase their profits. Second, it shows that firms may optimally choose to maintain some degrees of uncertainty and a perfect information on all their consumers, even in a constrained regime, is never optimal.

Our constrained regime resembles the weak privacy regime where each firm can collect information on the preference of its own consumers analysed by Shy and

Stenbacka (2016). They compare the weak privacy regime with one where each firm collects information on its own customers, but it is mandated to share its information with the rival so that each has information on the consumers located on the whole Hotelling line. While Shy and Stenbacka (2016) find that firms' profits are the highest in the weak privacy regime, our model goes one step further and it shows that, while firms prefer the setting that protects them from aggressive competition, the optimal level of information is not the perfect information assumed in their model.[21]

5.2 Neighborhood of perfect information

Now, it remains to verify how profits change in the neighbourhood of perfect information. Let us consider the following three cases: (i) both the intensive and the extensive margins are imperfect and tend to be perfect; (ii) firms can perfectly recognize a strictly positive share of consumers through the intensive margin, while the extensive margin tends to be sufficiently large to allow firms to recognize all the consumers; (iii) the intensive margin is perfect, while the extensive margin tends to be perfect. Therefore, in the following, we study the profit functions when $a \to 1^-$ and $\gamma \to \infty$ (case (i)), $a \to 1^+$ and $\gamma \to \frac{1}{2a-2}$ (case (ii)) and $a = 1$ and $\gamma \to \infty$ (case (iii)).

In case (i), as $\gamma \to \infty$, the limit function is $\lim_{\gamma \to \infty} \pi_A^{CR} = \frac{t(2+a)}{4}$. Hence, the intensive margin has a positive effect on profits as it approaches 1 – i.e., the maximum level of customer recognition. This is not surprising given the result in Proposition 4, which states that firms' profits are globally maximized at $a = 1 + \frac{1-\sqrt{2\gamma}}{2\gamma}$ if the extensive margin is sufficiently large.[22] Therefore, we can conclude that firms' profits go to $\frac{3t}{4}$ as $a \to 1^-$. Instead, in case (ii), as $\gamma \to \frac{1}{2a-2}$, the limit function is $\lim_{\gamma \to \frac{1}{2a-2}} \pi_A^{CR} = \frac{t}{4}$. This is in accordance with the Figures 5 and 6: if the intensive margin is larger than 1 and the extensive margin is sufficiently large, firms' profits become equal to $\frac{t}{4}$ and independent of the margins of

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[21] Shy and Stenbacka (2016) assume that firms have perfect knowledge of consumer preferences in the information set, which is consistent with a given privacy regime, without looking at different degrees of information within a privacy regime.

[22] If $\gamma \to \infty$, the second term goes to zero.

information. The reason of the difference between case (i) and (ii) can be found in the variation of the UU area. If $a \to 1^-$, the existence of a UU area leads firms to post a uniform price which is strictly larger than zero, which allows firms to keep high personalized prices. Hence, in this case, an increase in the intensive margin simply enlarges the group of identified consumers and, in the constrained information regime, the information advantage of each firm. By contrast, if $a \to 1^+$, the UU area tends to disappear as the extensive margin increases. As we have discussed in the previous sections, this has a stronger negative impact on the uniform price, which becomes equal to zero as $\gamma \to \infty$, with a consequent downward pressure on personalized prices. This result echoes the findings of Thisse and Vives (1988) – i.e., identifying a larger share of consumers to charge personalized prices has a negative impact on profits, as it makes competition tough. However, in this constrained information regime, each firm can recognize only the half of consumers, who are close to its location, while in Thisse and Vives (1988), firms can recognize all the consumers in the market. Nevertheless, if information tends to be perfect, firms charge a uniform price equal to zero to the further consumers, so leading to an intense competition.

Finally, let us consider the case (iii). From a mathematical point of view, if $a = 1$ and $\gamma \to \infty$, the limit function is $\lim_{\gamma \to \infty} \pi_A^{CR} = \frac{t}{2}$, which coincides with the profit function when firms charge a uniform price. However, this is not the unique equilibrium outcome, and it isn't indicative of the most competitive pricing behaviour. Indeed, in this case, each firm serves only half of the market (the closer to its location) and can post any price to the rest of consumers with no impact on its profit. Hence, if we consider the most competitive (uniform) price that firms can charge to the further consumers – i.e., $p = 0$ – , the profit function becomes equal to $\frac{t}{4}$. By adopting the hypothesis of a competitive firms' behaviour, we can state the following proposition.

Proposition 5. *If firms adopt the most competitive pricing and the group of recognized consumers tends to be maximized, firms' profits are equal to*

- $\frac{3t}{4}$ *if both margins of information tend to be perfect;*

- $\frac{t}{4}$ *if firms can recognize all the consumers close to their location and the extensive margin tends to be perfect.*

The above Proposition provides a crucial result when information tends to be perfect. It highlights the discontinuity in the firms' pricing behaviour when a approaches 1, but it also points out the relevance in the sequential increase in the intensive or extensive margin of information. Indeed, if the extensive margin is maximized and the increase in the intensive margin follows, the impact on firms' profits is positive. By contrast, if the intensive margin is maximized and the extensive margin leads to recognize all the consumers in the market, the impact on firms' profits is negative. This provides a novel rationale for the optimal sequence of policy interventions that facilitate the information gathering process by affecting the intensive or the extensive margin.

Finally, we conclude this section by explaining why the analysis in the neighbourhood of perfect information is relevant in the constrained rather than in the unconstrained regime. In the unconstrained regime, firms can also recognize the further consumers and they can compete harshly in personalized pricing. Hence, the uniform price is not relevant, as in the constrained regime, for determining the personalized prices. Indeed, if $a < 1$, as $\gamma \to \infty$, the limit function is $\lim_{\gamma \to \infty} \pi_A^{UR} = \frac{t(2-a)}{4}$. Hence, the intensive margin has a negative effect on profits as it approaches 1. This is in contrast with the constrained regime, where the impact of an increase in the intensive margin rises firms' profits if the extensive margin tends to be perfect. Instead, if $a \geq 1$, the limit function becomes equal to $\frac{t}{4}$, so replicating the result of Thisse and Vives (1988) and showing the negative impact of personalized pricing on firms' profits when all the consumers are recognized by both firms. This avoids the discontinuity in $a = 1$, which is instead verified in the constrained regime.

6. Policy implications

In this paper, we characterize the impact of two distinct margins of information on firms' profits in two alternative information regimes. Specifically, we prove that the effects change dramatically if firms can have access to a vast set of information, also including the preferences of consumers located closer to the rival, or alternatively, they can only recognize those consumers that, other things being equal, prefer their product. This implies that, according to the source of information and its gathering process, some specific policy implications can be discussed.

For instance, if firms collect information through the location setting of consumers' smartphones, as well as their social media activities, it is reasonable to assume that they can reach even the more distant consumers. Similarly, any individual using online services to search for a product leaves some digital footprints that can be tracked quite easily by each firm. However, consumers are more willing to protect their privacy when they browse firms' websites for which they have not a strong brand loyalty. This leads us progressively from an unconstrained to a constrained regime. Indeed, when consumers have a noticeable understanding of how they can be tracked via their online browsing, it can be hard to assume that firms can identify the further consumers. Moreover, if firms collect information from a given consumers' past purchase behaviour, it might be difficult, or even impossible, to recognize consumers that have a horizontal preference for the product sold by the rival.

These results have important implications in the perspective of a regulatory policy with the aim of protecting sensitive consumers' data. A first straightforward implication is that a Regulatory Authority should never ignore the ways in which firms, or a third party, collect consumers' information. Indeed, the information regime in place determines the impact of the margins of information on firms' profits and consumers surplus consequently.[23] Then, we highlight that some policy interventions might differ in their effect on the intensive or the extensive margin of information. On the one hand, a legislation that imposes a high standard of all consumers' privacy protection, such as the European e-Privacy Directive that requires the use of online cookies to collect and

[23]Since demand is inelastic and the firms' locations are exogenously given, larger (lower) profits imply lower (greater) consumer surplus.

storage consumers' information, contributes to lower the intensive margin of information.

On the other hand, the geolocation data privacy regulations, such as the California Privacy Rights Act (CPRA) allows Californians to opt out of any service tracking their location, in this way denying their consent for the geolocation data being used for location-based or behaviour-based offers. Thus, this regulation makes it more difficult for firms to reach distant consumers, which implies a lower size of the extensive margin in our model. Therefore, according to the initial set of available information and the information regime, the above regulatory interventions might have contrasting effects on firms' profits both in terms of sign and size. Let us consider, for example, the constrained regime and suppose that the initial set of available information implies an imperfect degree of the intensive margin and a sufficiently large degree of the extensive margin. In this case, a marginal decrease in the intensive margin, following the enforcement of a regulation like the European e-Privacy Directive, rises firms' profits, as it brings α closer to its optimal value. Conversely, a decrease in the extensive margin, following the enforcement of a regulation like the CPRA, reduces firms' profits, as it brings γ closer to the value that minimizes firms' profits. This illustrative example shows the potential opposite effects of an increase (or decrease) of the intensive and extensive margins on firms' profits. This proves that: (i) these margins are not perfect substitutes in their effects on firms' and consumers' surplus; (ii) regulatory interventions aiming at protecting consumers' privacy might have different side effects according to the most pronounced impact on one of the two margins.

Furthermore, in both regimes, a large intensive margin minimizes firms' profits and, by symmetry, maximizes the aggregate consumer surplus. Hence, a Public Authority managing the intensive margin of information and aiming at protecting consumers should foster a widespread use of personalized prices. By contrast, the extensive margin that minimizes firms' profits is the largest as possible both in the unconstrained regime and in the constrained regime only if it allows firms to recognize all the consumers jointly. Otherwise, in the constrained regime, an intermediate value of the extensive margin minimizes firms' profits. Therefore, the optimal pricing policy is to induce firms to charge personalized prices in the former case and a mix of uniform and personalized prices in the latter case. This discussion is summarized in the following proposition.

Proposition 6. *In the constrained regime, if $\alpha < 1$, the aggregate consumer surplus is maximized if the extensive margin is intermediate, such that both personalized and uniform prices coexist. Otherwise, the optimal policy is to maximize α or γ, that is a policy that fosters the use of personalized prices to a large share of consumers.*

In sum, personalized prices are not always welfare detrimental for consumers, as it is commonly assumed, but in fact they can maximize their aggregate surplus by making the competition tough between firms. However, there is a case in which this is not true, that is in the constrained regime – i.e., when firms never compete on personalized prices – when the size of the intensive margin doesn't allow firms to recognize the entire set of consumers for any level of the extensive margin. In this event, lowering the share of consumers to be charged with personalized prices is optimal from the consumers' point of view. This extends the findings of Thisse and Vives (1988) and specifies that when a regulatory intervention cannot make the market completely transparent – i.e., when the degree of the intensive margin is imperfect and a Regulatory Authority can manage only the extensive margin – it is not optimal to maximize the share of consumers charged with personalized prices. The regulator should rather induce the firms to post both personalized and uniform prices.

Finally, we find that, especially in the constrained regime, firms may have incentive to enlarge their information set through both or one of the two margins. This can be done by improving their information gathering process or, alternatively, by buying information from a third party, like a data provider. This stresses the importance of regulating the trade of consumers' information, the consent to information sharing, and the consumers' literacy in digital privacy.

7. Conclusion

In this paper, we contribute to the growing debate on the degree of consumers' privacy protection, as shown for example by the recent Italian case about the closing of the website Chat GPT[24], and the use of personalized pricing. We do this by analysing a simple duopoly model where firms compete on prices along the Hotelling line. The novel aspect of the paper is the assumption that firms have less than perfect information on the consumers' location and that the set of identified consumers is decreasing with the distance from the firm's location. Therefore, firms are asymmetric in the groups of consumers they can identify. We argue that different market structures emerge according to the firms' available information and competition is significantly affected by the size and composition of identified consumers, to which firms charge personalized prices. Specifically, we model the firms' available information in two dimensions: the intensive margin, that represents how many consumers are identified in each single location, and the extensive margin, that represents how many locations the firm knows. This allows us to suggest a novel theoretical tool to analyse the effects of privacy rules and regulations on firms' profits and consumers surplus, which is particularly relevant when policy interventions might impact only one of the two dimensions.

We compare the equilibrium outcomes in two alternative information regimes: the unconstrained regime, in which firms can gather information on consumers irrespective of their brand preferences, and the constrained regime, in which each firm can collect information only about consumers in its own turf. In the unconstrained regime, if the intensive (extensive) margin is not too large (small), an increase in the extensive (intensive) margin may be profit enhancing, but uniform pricing is the profit maximizing strategy. In the constrained regime, instead, firms' profits are maximized if firms adopt both personalized and uniform pricing, which implies that they can recognize only a

[24] In March 2023, the Italian Data Protection Authority blocked the website Chat GPT and it has imposed several requirements on Open AI – the company that created Chat GPT –, including a detailed disclosure on the processing of users' data. The website was reinstated in Italy at the end of April 2023. On 15th May 2023, the Financial Times highlights how the regulatory approach to Artificial Intelligence in general, and Chat GPT in particular, differs across the world and how, so far, the discussion about its regulation has focused on individual rights to privacy, and protection from discrimination. See the article: "Is artificial intelligence the right technology for risk management?".

share of consumers. Indeed, firms' profits are increasing in the intensive (extensive) margin only if the extensive (intensive) margin is not too large. These results shed some doubts on the actual (policy) attitude against the use of personalized pricing and toward a strict protection of consumers' privacy regardless of the information regime in place.

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APPENDIX

Proof of Lemma 1.

Consider area DU. Since we are considering a symmetric case, area DU must lie at the left of ½. The utility from buying product A for a consumer located in $x \leq \frac{1}{2}$ is: $v - p_{A,x}^{DU} - tx$ where $p_{A,x}^{DU}$ is the personalized price charged by firm A in area DU. The utility of buying product B instead is: $v - p_B - t(1-x)$ with p_B is independent of x given that firm B has no knowledge of the consumers location in this area. Hence, the optimal price of firm A is: $p_{A,x}^{DU} = p_B + t(1-x) - tx$. Given that the lowest price that B can charge is 0, firm A sells to all consumers in this region and has positive profit. Indeed, if $p_B = 0$ the best-reply by firm A is $p_{A,x}^{DU} = t(1-x) - tx$ which is positive given that by symmetry the area DU is always characterized by $x \leq 1/2$. Hence, in equilibrium firm B does not sell to any consumer in the DU area. Of course, *mutatis mutandis*, the same holds for firm A in the area UD. ■

***Proof of lemma 2*:**

Note first that $\frac{\partial \pi_A^1}{\partial \gamma} = \begin{cases} -\frac{t\gamma a^3}{3} & if \quad a \leq 1 \\ -\frac{2t\gamma(1-3a+3a^2)}{3} & if \quad a \geq 1 \end{cases}$, therefore, $\frac{\partial \pi_A^1}{\partial \gamma} \leq 0$. Second, $\frac{\partial \pi_A^1}{\partial \gamma} = \begin{cases} -t\gamma^2 a^2 & if \quad a \leq 1 \\ -t\gamma^2(2a-1) & if \quad a \geq 1 \end{cases}$, so that $\frac{\partial \pi_A^1}{\partial a} \leq 0$. Thus, the firm's profit decreases as the region where the firm can price discriminate gets larger. ■

***Proof of Lemma 3*:**

We distinguish between two sub-cases: (1) $a \in [0,1]$ and $\gamma \geq \frac{1}{2a}$ (Figure 2.4 and 2.5) and (2) $a \geq 1$ and $\gamma \in [\frac{1}{2a}, \frac{1}{2a-2}]$ (Figure 2.6 and 2.7).

In the sub-case (1), the aggregate equilibrium profit of firm A is given by the following function:

$$\pi_A^{21} = \begin{cases} \frac{t[(20a-4)\gamma - 5 + 12\gamma^2(4+2a-7a^2) - 48(1-a)^2\gamma^3]}{48\gamma[1-2\gamma(1-a)]} & if \quad \gamma \in [\frac{1}{2a}, \frac{1}{a}] \\ \frac{t[(44a-32)\gamma + 7 + 24\gamma^2(2-3a+a^2)]}{48\gamma[1-2\gamma(1-a)]} & if \quad \gamma \geq \frac{1}{a} \end{cases}$$

where $\gamma \leq \frac{1}{a}$ guarantees that both $g(x)$ and $f(x)$ intersect the horizontal axis as illustrated in Figure 2.4 (otherwise they intersect the vertical axis as in Figure 2.5).

In the sub-case (2) instead, the aggregate profit of firm A is given by the following function:

$$\pi_A^{22} = \begin{cases} \frac{t[6(1+5a)\gamma-5+12a\gamma^2(4-5a)-8(2-6a+9a^2-5a^3)\gamma^3]}{48\gamma} & if \quad \gamma \in [\frac{1}{2a},\frac{1}{a}] \\ \frac{t[6(5-3a)\gamma+7-16\gamma^3(1-a^3)]}{48\gamma} & if \quad \gamma \in [\frac{1}{a},\frac{1}{2a-2}] \end{cases}.$$

Let us consider first the sub-case 1 when $\gamma \in [\frac{1}{2a},\frac{1}{a}]$. The partial derivative of the profit function with respect to γ is $\frac{\partial \pi_A^{21}}{\partial \gamma} = \frac{t[5+20\gamma(1-a)+4\gamma^2(14-16a-a^2)-96\gamma^3a^2(1-a)[1+\gamma(1-a)]}{48\gamma^2[1+2\gamma(1-a)]^2}$ which is positive (negative) if $\gamma \leq (\geq)\ \bar{\gamma}(a)$, where $\bar{\gamma}(a)$ is a root of a fourth-degree equation in a.[25] When instead $\gamma \geq \frac{1}{a}$, the partial derivative of the profit function with respect to γ is: $\frac{\partial \pi_A^{21}}{\partial \gamma} = -\frac{t[7+28\gamma(1-a)+40\gamma^2(1-a)^2]}{48\gamma^2[1+2\gamma(1-a)]^2} < 0$. Looking at the intensive margin a, when $\gamma \geq 1/2$ and $a \in [\frac{1}{2\gamma},1]$ the profit is π_A^{21}. If $\gamma \in [\frac{1}{2},1]$ and $a \in [\frac{1}{2\gamma},1]$, or $\gamma \geq 1$ and $a \in [\frac{1}{2\gamma},\frac{1}{\gamma}]$ (Figure 2.4), the partial derivative of the profit function with respect to a is: $\frac{\partial \pi_A^{21}}{\partial a} = \frac{t[5+4\gamma(4-7a)+4\gamma^2(6-18a+13a^2)-32\gamma^3a(1-a)[1+\gamma(1-a)]}{8[1+2\gamma(1-a)]^2}$, which is positive (negative) if $a \leq (\geq)\ \bar{a}(\gamma)$, where $\bar{a}(\gamma)$ is a root of a third-degree equation in γ.[26] If $\gamma \geq 1$ and $a \in [\frac{1}{\gamma},1]$ (Figure 2.5), the partial derivative of the profit function with respect to a is $\frac{\partial \pi_A^{21}}{\partial a} = -\frac{t[3+8\gamma(1-a)+8\gamma^2(1-a)^2]}{8[1+2\gamma(1-a)]^2} < 0$.

Now, let consider the sub-case 2 when $\gamma \in [\frac{1}{2a},\frac{1}{a}]$. The partial derivative of the profit function with respect to γ is: $\frac{\partial \pi_A^{22}}{\partial \gamma} = \frac{t}{48}\left[\frac{5}{\gamma^2} - 32\gamma + 80a^3\gamma + 48a(1+2\gamma) - 12a^2(5+12\gamma)\right]$, which is positive for small γ and negative otherwise. Hence, in the interval $\in \left[\frac{1}{2a},\frac{1}{a}\right]$, the function π_A^{22} is inverse U-shape, with a local maximum $\bar{\bar{\gamma}}(a)$.[27]. If $\gamma \in$

[25] The functional form of $\bar{\gamma}(a)$ is available from the authors upon request.
[26] The functional form of $\bar{a}(\gamma)$ is available from the authors upon request.
[27] The functional form of $\bar{\bar{\gamma}}(a)$ is available from the authors upon request.

$[\frac{1}{a}, \frac{1}{2a-2}]$, the partial derivative of the profit function with respect to γ is: $\frac{\partial \pi_A^{22}}{\partial \gamma} = \frac{t[-7-32\gamma^3(1-a)^3]}{48\gamma^2} < 0$.

Let us analyse the impact of the intensive margin on the profit function. When $\gamma \geq 1/2$ and $a \in [1, \frac{1+2\gamma}{2\gamma}]$, or $\gamma \leq 1/2$ and $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$ the profit is π_A^{22}. If $\gamma \in [\frac{1}{2}, 1]$ and $a \in [1, \frac{1}{2\gamma}]$ or $\gamma \leq 1/2$ and $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$ (Figure 2.6), the partial derivative of the profit function with respect to a is: $\frac{\partial \pi_A^{22}}{\partial a} = \frac{t}{8}[5 + \gamma(8 - 20a) + 4\gamma^2(2 - 6a + 5a^2)] < 0$. If $\gamma \in [\frac{1}{2}, 1]$ and $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$, or $\gamma \geq 1$ and $a \in [1, \frac{1+2\gamma}{2\gamma}]$ (Figure 2.7), the partial derivative of the profit function with respect to a is $\frac{\partial \pi_A^{22}}{\partial a} = \frac{t[-3-8\gamma^2(1-a)^2]}{8} < 0$. ■

Proof of Proposition 1:

Depending on the values of a and γ, there are three cases:

- All indifferent consumers are unidentified: only areas DU/UD and UU
- Partially identified indifferent consumers: all areas DU/UD, UU, and DD are present.
- All indifferent consumers are identified: only areas DU/UD and DD.

The parameter set sustaining each case have already been characterized in the proof of Lemma 2, Lemma 3 and Lemma 4, where we have also shown the relationship between firm's profits and extensive margin. Here we characterize the shape of the equilibrium profits for any possible parameter set.

Preliminary, it can be observed that the profits function is continuous in γ. This part of the proof is omitted, as it comes simply from checking the value of the profit function at the endpoints of the relevant parameter ranges.

- Suppose $a \in [0,1]$. From Lemma 2, we know that when all indifferent consumers are unidentified the profits are strictly decreasing in γ. Therefore, in the parameter range $\gamma \in [0, \frac{1}{2a}]$, there is a local maximum at $\gamma = 0$. When the indifferent consumers are partially identified and $\gamma \in [\frac{1}{2a}, \frac{1}{a}]$, we know from Lemma 3 (sub-case 1), that the profits are inverse U-shaped in γ. Hence, there is another local maximum at $\bar{\gamma}(a) \in$

$[\frac{1}{2a}, \frac{1}{a}]$. Finally, when the indifferent consumers are partially identified and $\gamma > \frac{1}{a}$, we know from Lemma 3 (sub-case 1), that the profits are strictly decreasing in γ. Since $\pi_A^1(\gamma = 0) > \pi_A^{21}(\gamma = \bar{\gamma}(a))$, we conclude that the profits are globally maximized at $\gamma = 0$.

- Suppose $a \in [1,2]$. From Lemma 2 we know that as long as all indifferent consumers are unidentified, the profits are strictly decreasing in γ. When indifferent consumers are partially identified we know from Lemma 3 (sub-case 2), that the profits are inverse U-shaped in γ. Hence, there is another local maximum at $\bar{\bar{\gamma}}(a) \in \left[\frac{1}{2a}, \frac{1}{a}\right]$. When all indifferent consumers are identified, we know from Lemma 4 that the profits are invariant in γ. Since $\pi_A^1(\gamma = 0) > \pi_A^{22}(\gamma = \bar{\bar{\gamma}}(a))$, we conclude that the profits are globally maximized at $\gamma = 0$.
- Suppose $a > 2$. From Lemma 2, we know that as long as all indifferent consumers are unidentified, the profits are strictly decreasing in γ. When indifferent consumers are partially identified we know from Lemma 3 (sub-case 2), that the profits are strictly decreasing in γ. When all indifferent consumers are identified, we know from Lemma 4 that the profits are invariant in γ. ■

Proof of Proposition 2:

As for Proposition 1, depending on the values of a and γ, there are three cases:

- All indifferent consumers are unidentified: only areas DU/UD and UU
- Partially identified indifferent consumers: all areas DU/UD, UU, and DD are present.
- All indifferent consumers are identified: only areas DU/UD and DD.

The parameter set sustaining each case have already been characterized in the proof of Lemma 2, Lemma 3 and Lemma 4, where we have also shown the relationship between firm's profits and intensive margin. Here we characterize the shape of the equilibrium profits for any possible parameter set.

Preliminary, it can be observed that the profits function is continuous in a. This part of the proof is omitted, as it comes simply from checking the value of the profit function at the endpoints of the relevant parameter ranges.

- Suppose $\gamma \geq 1$. From Lemma 2 we know that as long as all indifferent consumers are unidentified, the profits are strictly decreasing in a. Therefore, in the parameter range $a \in [0, \frac{1}{2\gamma}]$, there is a local maximum at $a = 0$. When $a \in [\frac{1}{2\gamma}, \frac{1}{\gamma}]$, we know from point Lemma 3 (partially identified indifferent consumers sub-case 1), that the profits are inverse U-shaped in a. Hence, there is another local maximum at $\bar{a}(\gamma) \in [\frac{1}{2\gamma}, \frac{1}{\gamma}]$. When $a \in [\frac{1}{\gamma}, 1]$, we know from point b) (partial identified indifferent consumers sub-case 1), that the profits are strictly decreasing in a. Finally, when all indifferent consumers are identified, we know from Lemma 4 that the profits are invariant in a. Since $\pi_A^1(a = 0) > \pi_A^{21}(a = \bar{a}(\gamma))$, we conclude that the profits are globally maximized at $a = 0$.
- Suppose $\gamma \in [\frac{1}{2}, 1]$. From Lemma 2 we know that as long as all indifferent consumers are identified, the profits are strictly decreasing in a. Therefore, in the parameter range $a \in [0, \frac{1}{2\gamma}]$, there is a local maximum at $a = 0$. When $a \in [\frac{1}{2\gamma}, 1]$, we know from Lemma 3 (partially identified indifferent consumers, sub-case 1), that the profits are inverse U-shaped in a. Hence, there is another local maximum at $\bar{a}(\gamma) \in [\frac{1}{2\gamma}, 1]$. When $a \in (1, \frac{1}{\gamma}]$, we know from Lemma 3 (partially identified indifferent consumers, sub-case 2), that the profits are strictly decreasing in a. Finally, when all indifferent consumers are identified, we know from Lemma 4 that the profits are invariant in a. Since $\pi_A^1(a = 0) > \pi_A^{21}(a = \bar{a}(\gamma))$, we conclude that the profits are globally maximized at $a = 0$.
- Suppose $\gamma \in [0, \frac{1}{2}]$. From Lemma 2 we know that as long as all indifferent consumers are unidentified, the profits are strictly decreasing in a. Therefore, in the parameter range $a \in [0, \frac{1}{2\gamma}]$, there is a local maximum at $a = 0$. When $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$, we know from Lemma 3 (sub-case 2), that the profits are strictly decreasing in a. Finally, when

all indifferent consumers are identified, we know from Lemma 4, that the profits are invariant in a. We conclude that the profits are globally maximized at $a = 0$. ■

Trade-off between $\boldsymbol{a}$ and $\boldsymbol{\gamma}$

The DU area is given by $A = \frac{(a \times a\gamma)}{2}$. Let us differentiate the profit function π_A^1, by keeping constant the area A, with respect to the intensive margin a. Doing this, we obtain the following: $\frac{\partial \pi_A^1}{\partial a} | \bar{A} = \frac{4t}{3} \frac{\bar{A}}{a^2} > 0$. Then, we replicate this profit differentiation with respect to the extensive margin γ. This leads to the following: $\frac{\partial \pi_A^1}{\partial \gamma} | \bar{A} = -\frac{t}{6}\sqrt{\frac{8\bar{A}^3}{\gamma}} < 0$. By studying the sign of the partial derivatives, we can state that firms' profits are positively (negatively) affected by an increase in the intensive (extensive) margin under the constraint of keeping constant the area of identified consumers.

Proof of Proposition 3:

In the constrained model (and differently form the unconstrained model), area DD never emerges. It is possible to distinguish between two cases, depending on the values of a and γ:

- if lines g and f do not intersect, we have the case discussed in Lemma 2;
- if lines g and f do intersect (from no onwards simply the *intersection* case) we have a case which is specific of the constrained information regime.

We characterize the shape of the equilibrium profits for any parameter range, and we characterize the global and local maxima. Preliminary, it can be observed that the profits function is continuous in γ. This part of the proof is omitted, as it comes simply from checking the value of the profit function at the endpoints of the relevant parameter ranges.

- Suppose $a \in [0,1]$.[28] When γ increases, the relevant figures are given by the following sequence: figure 2.1 ($\gamma = 0$), figure 2.2 ($\gamma \in (0, \frac{1}{2a}]$) and figure 2.10 ($\gamma > \frac{1}{2a}$).

[28] When a=0 only figure 2.1. emerges whatever the level of γ is.

From Lemma 2, we know that when all indifferent consumers are unidentified, the profits are strictly decreasing in γ. Therefore, in the parameter range $\gamma \in [0, \frac{1}{2a}]$, there is a local maximum at $\gamma = 0$. In the *intersection* case, we observe that $\frac{\partial \pi_A^{CR}}{\partial \gamma} = \frac{t[1+4\gamma(1-a)+4\gamma^2(4-5a+a^2)]}{24\gamma^2[1+2\gamma(1-a)]^2} > 0$. Therefore, the profits are U-shaped in γ. Since $\lim_{\gamma\to\infty} \pi_A^{CR} = \frac{t(2+a)}{4} > \frac{t}{2} = \pi_A^1(\gamma = 0)$, the profits are globally maximized when γ goes to infinite.

- Suppose $a > 1$. When γ increases, the relevant figures are given by the following sequence: figure 2.1 ($\gamma = 0$), figure 2.3 ($\gamma \in (0, \frac{1}{2a}]$), and figure 2.11 ($\gamma \in (\frac{1}{2a}, \frac{1}{2a-2}]$). From Lemma 2, we know that when all indifferent consumers are unidentified, the profits are strictly decreasing in γ. Therefore, in the parameter range $\gamma \in [0, \frac{1}{2a}]$, there is a local maximum at $\gamma = 0$. In the *intersection* case, we observe that $\frac{\partial \pi_A^{CR}}{\partial \gamma} = \frac{t[1-16\gamma^3+16a^3\gamma^3+12a\gamma^2(1+4\gamma)-12a^2\gamma^2(1+4\gamma)]}{24\gamma^2}$ which is positive up to $\tilde{\gamma}(a) \in (\frac{1}{2a}, \frac{1}{2a-2}]$ and then negative. Therefore, there is a local maximum at $\tilde{\gamma}(a)$. Finally, when all indifferent consumers are identified, we know from Lemma 4, that the profits are invariant in γ. Since $\pi_A^{CR}(\gamma = \tilde{\gamma}(a)) < t/2 = \pi_A^1(\gamma = 0)$, we conclude that the profits are globally maximized at $\gamma = 0$. ■

Proof of Proposition 4

When $a \in [0, \frac{1}{2\gamma}]$, we have figure 2.1, 2.2, and 2.3, and the analysis is the same as for the unconstrained information regime case. Therefore, in this parameter range, the profits are strictly decreasing in *a*. When $a > \frac{1}{2\gamma}$, we observe the intersection case, and we have figure 2.10 (when $\gamma > \frac{1}{2}$ and $a \in [\frac{1}{2\gamma}, 1]$) and figure 2.11 (when $\gamma > \frac{1}{2}$ and $a \in [1, \frac{1+2\gamma}{2\gamma}]$, or $\gamma \in [0, \frac{1}{2}]$ and $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$). In this case, when $\gamma > \frac{1}{2}$ and $a \in [\frac{1}{2\gamma}, 1]$, we have: $\frac{\partial \pi_A^{CR}}{\partial a} = \frac{t[1+2\gamma(1-2a)+4\gamma^2(1-a)^2]}{4[1+2\gamma(1-a)]^2}$ which is positive (negative) if $a \leq (\geq) \bar{a}^C(\gamma) \equiv 1 + \frac{1-\sqrt{2\gamma}}{2\gamma}$. On the other hand, when $\gamma > \frac{1}{2}$ and $a \in [1, \frac{1+2\gamma}{2\gamma}]$, or $\gamma \in [0, \frac{1}{2}]$ and $a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$, we have $\frac{\partial \pi_A^{CR}}{\partial a} = \frac{t[1+2\gamma(1-2a)+4\gamma^2(1-a)^2]}{4} < 0$. Finally, when $a > \frac{1+2\gamma}{2\gamma}$ the profits are constant and equal to $\frac{t}{4}$.

Here we characterize the shape of the equilibrium profits for any possible parameter set. Preliminary, it can be observed that the profits function is continuous in *a*. This part of the proof is omitted, as it comes simply from checking the value of the profit function at the endpoints of the relevant parameter ranges.

- Suppose $\gamma \geq \frac{1}{2}$. When *a* increases, the relevant figures are given by the following sequence: figure 2.1 ($a = 0$), figure 2.2 ($a \in [0, \frac{1}{2\gamma}]$), figure 2.10 ($a \in [\frac{1}{2\gamma}, 1]$), and figure 2.11 ($a \in [1, \frac{1+2\gamma}{2\gamma}]$). From Lemma 2 we know that when all indifferent consumers are unidentified, the profits are strictly decreasing in *a*. Therefore, in the parameter range $a \in [0, \frac{1}{2\gamma}]$, there is a local maximum at $a = 0$. From the discussion above, in the *intersection* case, the profits are inverse U-shaped in *a* with a local maximum at $\bar{a}^C(\gamma)$. Finally, when all indifferent consumers are identified, we know from Lemma 4 that the profits are invariant in *a*. Since $\pi_A^1(a = 0) \leq (\geq) \pi_A^{CR}(a = \bar{a}^C(\gamma))$ when $\gamma \leq (\geq) \frac{2+\sqrt{3}}{3}$ we conclude that the profits are globally maximized at $a = \bar{a}^C(\gamma)$ when $\gamma \geq \frac{2+\sqrt{3}}{3}$ and at $a = 0$ when $\gamma \leq \frac{2+\sqrt{3}}{3}$.
- Suppose $\gamma \in [0, \frac{1}{2}]$. When *a* increases, the relevant figures are given by the following sequence: figure 2.1 ($a = 0$), figure 2.2 ($a \in [0, 1]$), figure 2.3 ($a \in [1, \frac{1}{2\gamma}]$), and figure 2.11 ($a \in [\frac{1}{2\gamma}, \frac{1+2\gamma}{2\gamma}]$). From Lemma 2 we know that when all indifferent consumers are unidentified, the profits are strictly decreasing in *a*. Therefore, in the parameter range $a \in [0, \frac{1}{2\gamma}]$, there is a local maximum at $a = 0$. From the discussion above, in the *intersection* case the profits are decreasing in *a*. Finally, when all indifferent consumers are identified, we know from Lemma 4 that the profits are invariant in *a*. Therefore, the profits are globally maximized at $a = 0$.