CESifo Working Papers

10460
2023

May 2023

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Paul Gaggl, Aspen Gorry, Christian vom Lehn

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
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CESifo *Working Paper No. 10460*

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Abstract

This paper studies structural change in production networks for intermediate inputs (input-output network) and new capital (investment network). For each network, we document a declining fraction of production by goods sectors and a rising fraction of production by services sectors. We develop a multisector growth model that admits structural change in production networks along the balanced growth path to study these trends. Disaggregated final expenditure data reveal that inputs to investment production are substitutes, rather than strong complements as suggested by existing work. Hence, resources endogenously reallocate toward the fastest growing producers of investment. Growth accounting exercises demonstrate that investment-specific technical change has risen in importance for aggregate U.S. growth over time, with 20-25% of aggregate growth after 2000 stemming from reallocation induced by structural change. At the same time, productivity growth within the input-output network has stagnated, contributing to the recent slowdown in aggregate growth.

JEL-Codes: E230, O140, O400, O410.

Keywords: structural change, input-output network, investment network, economic growth, technical change, balanced growth.

Paul Gaggl
Department of Economics
University of North Carolina at Charlotte
USA – Charlotte, NC 28223-0001
pgaggl@charlotte.edu

Aspen Gorry
Department of Economics
Clemson University
USA – Clemson, SC, 29634
cgorry@clemson.edu

Christian vom Lehn
Department of Economics
Brigham Young University
USA – Provo, UT 84602
cvomlehn@byu.edu

We are grateful for useful comments from Ben Bridgman, Maya Eden, John Fernald, Jeremy Greenwood, Basile Grassi, Berthold Herrendorf, Chris Herrington, Danial Lashkari, Rachel Ngai, Todd Schoellman, Mike Sposi, and Thomas Winberry and valuable feedback from participants at the 2022 Clemson Growth Conference, Spring 2022 I-85 Macro Workshop, Spring 2022 Midwest Macro Conference, Summer 2022 North American Meetings of the Econometric Society, 2022 SED Meetings, 2022 IZA Workshop on the Macroeconomics of Productivity, and numerous seminar presentations. We also thank Peter Call for providing excellent research assistance with the project.

1. Introduction

Production networks—the sectoral distributions of the production and purchases of commodities used in production (e.g., intermediates, investment)—play a central role in shaping economic fluctuations and growth.[1] However, with changes in technology and the organization of production, these production networks change over time. In this paper, we study structural change in production networks and its implications for economic growth.

Our analysis proceeds in four steps. First, we show that since 1947, production networks in both intermediate inputs and investment experienced a substantial decline in output produced by goods sectors (i.e., manufacturing, construction), offset by an increase in production by services sectors. Second, we develop a multi-sector model that allows for structural change in these production networks and characterize an aggregate balanced growth path within this environment. Third, we calibrate our model to match observed patterns of structural change using final expenditure price data, finding that inputs produced by different sectors are substitutes in the production of investment and complements in the production of intermediates and consumption. Finally, we use the model to perform growth accounting exercises for the U.S. over the period 1947-2020, both along and off the balanced growth path, analyzing the contributions of exogenous technical change and endogenous reallocation to aggregate growth over time, including its recent slowdown.

Our first step uses national accounting data to document how production patterns in input-output and investment networks have changed over time.[2] In each network, goods sectors produce a smaller share of output over time, offset by increased production in service sectors. The magnitude of these changes is large, similar to structural change observed in consumption expenditures, and these changes are widespread, occurring in nearly all sectors and in many countries.

We study these changes using an extension to the multi-sector neoclassical growth model, which has been commonly applied to study structural transformation in consumption expenditures (see Herrendorf, Rogerson and Valentinyi, 2014, for a survey). In our model, each sector produces gross output using a

[1] Among many possible examples, see Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Baqaee and Farhi (2019), Foerster, Hornstein, Sarte and Watson (2019), Kopytov, Mishra, Nimark and Taschereau-Dumouchel (2021), vom Lehn and Winberry (2022).

[2] Berlingieri (2013) and Galesi and Rachedi (2018) provide some evidence that services sectors have been rising in importance for intermediates production, while Herrendorf, Rogerson and Valentinyi (2021) and García-Santana, Pijoan-Mas and Villacorta (2021) present similar evidence for investment production. We show stylized facts for both networks in a unified framework with greater detail and at a more disaggregated level.

combination of capital, labor, and intermediate inputs. Production networks are modeled as each sector's intermediates and investment being a bundle of inputs purchased from many different sectors. Intermediate and investment inputs are bundled for each sector using constant elasticity of substitution functions, meaning that changes in relative prices across sectors, induced by changes in technology, can generate changes in the composition of intermediates and investment production—i.e., changes in production networks.

We characterize an aggregate balanced growth path (ABGP) that admits structural change in production networks, along which all aggregates denoted in units of the numeraire grow at a constant rate. Similar to Herrendorf et al. (2021), the existence of an ABGP only requires constant aggregate TFP growth, while the underlying sectoral TFP growth rates need not be constant.[3] Along the ABGP, aggregate growth is attributable to either productivity growth in the production of investment or productivity growth in the production of intermediates used to produce investment. The composition of these two forces in aggregate growth depends on the elasticities of substitution between inputs used in the production of investment and intermediates.

The structure of our model and its ABGP has two crucial implications for calibration. First, because we explicitly model the input-output network, it is appropriate to use final expenditure prices (which are affected by the prices of intermediate inputs) to study structural change (see the discussion in Herrendorf, Rogerson and Valentinyi, 2013 and Herrendorf et al., 2014). Second, in our model, a sector's contribution to aggregate growth depends on both exogenous productivity growth and the extent to which it produces consumption, investment, and intermediates. This highlights that previous work, focusing on a two-sector calibration (goods and services), may miss the role of heterogeneous productivity growth within goods and services across subsectors specializing in producing different products. To allow for such heterogeneity, our calibration considers six sectors, where each sector is defined as either a goods or services sector that produces exclusively consumption, intermediates, or investment. This approach preserves the parsimony of the two-sector representation while allowing for heterogeneity in prices and productivity growth specific to each product.

Our calibration requires accurate measures for the price of goods and services inputs to consumption,

[3] This contrasts with Ngai and Pissarides (2007), who assume that all sectoral TFP terms grow at constant rates, precluding the coexistence of balanced growth and structural change in intermediates production. In fact, our framework requires that TFP growth in at least one sector is non-constant for an aggregate balanced growth path to exist.

investment, and intermediates. To this end, we construct series for the prices of consumption and investment produced by goods and services sectors using final expenditure prices on 68 detailed consumption commodities and 30 detailed investment commodities. The production of each commodity is mapped to either goods or services sectors using bridge files published by the Bureau of Economic Analysis (BEA). The resulting price series for consumption and investment produced by goods and services allow us to infer intermediates prices from income-side accounting data, as sectoral gross output prices are a weighted average of consumption, investment, and intermediates prices. These new price series for intermediates produced by goods and services cover the period 1947-2020 and are consistent with published producer price indices (PPI) in the years those are available.

Using these new price series, our calibration implies that goods and services inputs are complements (Leontief) in both consumption and intermediates production, but are substitutes in the production of investment. This means that productivity growth in the production of investment will endogenously comprise a larger share of aggregate growth over time, as substitutability induces reallocation to the fastest-growing inputs, "frontiers," and complementarity in intermediates leads to reallocation toward the slowest-growing inputs, "bottlenecks." Our findings provide optimism about future growth relative to Baumol (1972), as endogenous reallocation of investment production toward services can accelerate aggregate growth.

The result on substitutability of investment inputs is novel, as previous studies find that the best fit is achieved when investment inputs are complements. This difference is primarily due to aggregation bias in price measurement used in previous work. For example, Herrendorf et al. (2021), García-Santana et al. (2021), and Sposi, Yi and Zhang (2021) all consider a single price for all products produced by a sector. However, the price trends of consumption, investment, and intermediates produced by goods and services are highly heterogeneous. For example, the services sector produces both final consumption products of health care and education (consumption)—whose prices have grown more than 40-fold since 1947—and software (investment)—whose price has fallen over time. Aggregation can thus mask these differential relative price trends across different final products. We show that such bias is still present when constructing relative prices using detailed income-side data on gross output prices. For example, the finest level of sectoral disaggregation that covers the postwar period still aggregates the differing price trends of consumption of legal services (increasing price) and investment in software (falling price) into a single sector price (professional and technical services).

Finally, we use the calibrated model to quantify the contribution of productivity growth in the production of investment and intermediate goods to aggregate growth. Along the balanced growth path, investment-specific technical change has become increasingly important over time, accounting for roughly 60% of aggregate TFP growth before and more than 80% after 2000. Furthermore, in counterfactuals with a unitary elasticity of substitution between investment inputs, growth in aggregate TFP since 2000 would be 25% lower, implying that structural change in the investment network has played a substantial role in recent productivity growth.

To study the implications of our model for aggregate growth off the balanced growth path, we derive an expression for aggregate GDP in the model measured as an index number, consistent with national accounting practices. This expression implies that aggregate growth is primarily determined by a combination of Domar-weighted productivity growth (i.e., Hulten's Theorem, Hulten, 1978; Baqaee and Farhi, 2019) and growth in aggregate TFP consistent with balanced growth. We use this expression to decompose the contributions of productivity growth in the production of consumption, investment, and intermediates to aggregate growth. The growth slowdown in the 1970s reflects slowing growth in both investment and intermediates-specific technical change, while the slowdown since 2000 is primarily attributable to stagnant exogenous productivity growth in the production of intermediates. Furthermore, absent reallocation of investment production to services sectors with high productivity growth, the slowdown in aggregate GDP growth would have been even worse, 20% slower than observed. Thus, in contrast to Gordon (2016), who argues that recent slowdowns in aggregate growth represent a trend dating back to the 1970s, our analysis suggests that the growth slowdown of the 1970s is distinct from that of the 2000s. Moreover, although significant attention has been given to productivity disruptions along supply chains during the COVID-19 pandemic, weak productivity growth in the intermediates network predates the pandemic by 20 years.

Related Literature. Our paper is at the intersection of two large literatures—the study of production networks as propagation mechanisms for economic fluctuations and growth and the study of long-run structural transformation from goods to services. Papers in the production networks literature have emphasized the role of static production networks in shaping fluctuations. The literature on structural change has typically focused on multi-sector models that either abstract from production networks (or treat them implicitly in a "value added" specification) or do not allow these networks to change over time. Our contribution lies in studying the intersection of these two phenomena.

While a significant literature has focused on how production networks propagate short-run fluctuations over the business cycle, their role in shaping long-run growth has also gained attention (e.g., Ngai and Samaniego, 2009; Moro, 2015; Foerster et al., 2019; Valentinyi, 2021). Our paper is closely related to Ngai and Samaniego (2009), who focus on how technical change in sectors producing intermediate goods can impact the composition of long-run growth via investment-specific technical change (as in Greenwood, Hercowitz and Krusell, 1997). Also closely related is recent work by Foerster et al. (2019), who focus on how production networks, especially the investment network, influence which sectors' TFP growth accounts for recent slowdowns in aggregate growth. Similar to Foerster et al. (2019), we find evidence that the proximate forces driving growth slowdowns in the 1970s and since 2000 are distinct. While some of our insights are similar, we highlight two important distinctions: first, our study allows for production networks to change endogenously over time, and second, our framework with structural change leads us to analyze patterns of growth across sectors defined not just by their product market (i.e., manufacturing, construction, services), but by the types of products they supply (i.e., consumption, investment, intermediates).

A second strand of literature focuses on structural change, but abstracts from explicitly modeling production networks. For example, Herrendorf et al. (2013) introduce value-added measures of structural change that implicitly embed indirect contributions to final production occurring via the input-output network. Herrendorf et al. (2021) use this approach to study structural change in consumption and investment across two sectors along a balanced growth path. We document that structural change in the network of intermediates production accounts for nearly half of the measured structural change in consumption and investment, motivating the importance of explicitly modeling the input-output network. Our model extends Herrendorf et al. (2021) to an arbitrary number of sectors and explicitly incorporates the input-output network into the model. Furthermore, explicitly modeling the input-output network allows for an internally consistent calibration using final expenditure prices, which reveals that goods and services are substitutes in investment production.

Our paper also relates to recent literature studying further disaggregation of the services sector. For example, recent work has split services into high-skill and low-skill services (Buera and Kaboski, 2012; Buera, Kaboski, Rogerson and Vizcaino, 2022), tradable and non-tradable services (Eckert et al., 2019), traditional and non-traditional services (Duarte and Restuccia, 2020), and stagnant and progressive services

(Duernecker, Herrendorf and Valentinyi, 2021). Our six sector disaggregation considers a different division of services sectors, distinguishing subsectors by the type of product they produce and emphasizing the elasticity of substitution between goods and services in the production of each product. Closest to our work, Duernecker et al. (2021) also provides some optimism about future growth, arguing that detailed services sectors are substitutes with each other within services consumption, implying reallocation towards high productivity growth services sectors.

Finally, our work relates to recent papers by Sposi (2019) and Sposi et al. (2021), who also explicitly incorporate structural change in intermediates production. These papers seek to explain the hump-shaped rise and fall of manufacturing and the global distribution of manufacturing. In contrast, we focus on the role of structural change in production networks for the composition and level of aggregate growth on and off a balanced growth path.

2. Data & Empirical Evidence

Our study of structural change focuses on two production networks—the production and purchases of intermediate inputs (the input-output network) and the production and purchases of new capital (the investment network). These networks are measured as matrices, where element (i, j) of the matrix reports expenditures by sector j on intermediates or investment produced by sector i.

2.1. Data

Our primary data source for measuring these production networks in the U.S. are the Make and Use Tables from the BEA Input Output Database, which contains data for each sector on the value of gross output, value added, intermediate input purchases (from each other sector), and final uses (consumption, investment, etc.) of each sector's production.[4] The database begins in 1947, and we study patterns running through 2020. For measuring the investment network, we also use data from vom Lehn and Winberry (2022), who use BEA Input Output and BEA Fixed Assets data to construct a time series of the investment network that covers a similar time frame and level of sector detail; we extend their data to run through

[4]This data includes imported intermediates, consumption, and investment. We do not attempt to remove these because our primary interest is in the sectors producing inputs to intermediates and investment and not in the country of origin for production.

Table 1: Average Share of Intermediates and Investment Production: Avg. 1947-2019

Goods Producing Sectors (NAICS Codes)	% of Prod. Int.	% of Prod. Inv.	Service Producing Sectors (NAICS Codes)	% of Prod. Int.	% of Prod. Inv.
Agriculture, forestry, fishing and hunting (11)	5.8	0.0	Wholesale trade (42)	5.8	3.8
Mining, except oil and gas (212)	1.1	0.2	Retail trade (44-45)	1.9	1.4
Support activities for mining (213)	0.0	0.3	Transport and warehousing (48-49, minus 491)	5.6	0.9
Construction (23)	1.9	37.0	Information (51)	4.6	6.2
Wood products (321)	1.5	0.5	Finance and insurance (52)	7.8	0.1
Non-metallic mineral products (327)	1.6	0.1	Real estate (531)	4.9	1.8
Primary metals (331)	4.8	0.1	Rental and leasing services (532-533)	1.1	0.0
Fabricated metal products (332)	4.2	1.1	Professional and technical services (54)	5.4	10.3
Machinery (333)	1.7	8.6	Management of companies and enterprises (55)	3.1	0.1
Computer and electronic products (334)	2.5	6.7	Administrative support and waste services (56)	3.3	0.1
Electrical equipment manufacturing (335)	1.2	1.0	Educational services (61)	0.3	0.2
Motor vehicles manufacturing (3361-3363)	3.3	9.1	Health services (62)	0.3	0.1
Other transportation equipment (3364-3369)	1.5	4.7	Arts, entertainment and recreation services (71)	0.4	0.1
Furniture and related manufacturing (337)	0.3	1.2	Accommodation services (721)	0.5	0.0
Misc. manufacturing (339)	0.9	1.3	Food services (722)	1.0	0.0
Food and beverage manufacturing (311-312)	4.9	0.1	Other private services (81)	1.7	0.1
Textile manufacturing (313-314)	2.0	0.3	Federal government (n/a, but incl. 491)	1.1	0.5
Apparel manufacturing (315-316)	0.7	0.0	State and local government (n/a)	1.4	0.9
Paper manufacturing (322)	2.6	0.0			
Printing products manufacturing (323)	1.2	0.6			
Chemical manufacturing (325)	4.6	0.4			
Plastics and rubber products (326)	1.8	0.1			

Notes: The table reports the average share of total intermediate and investment production by 40 consistent sectors. Individual components may not exactly sum to totals due to rounding. Sectors are classified according to the NAICS-based BEA codes, with 2007 NAICS codes listed in parentheses. Government sectors as defined by the BEA do not have naturally corresponding NAICS codes. See Appendix A for details of the data construction.

2020. Additional details regarding data sources and measurement of production networks are available in Appendix A.

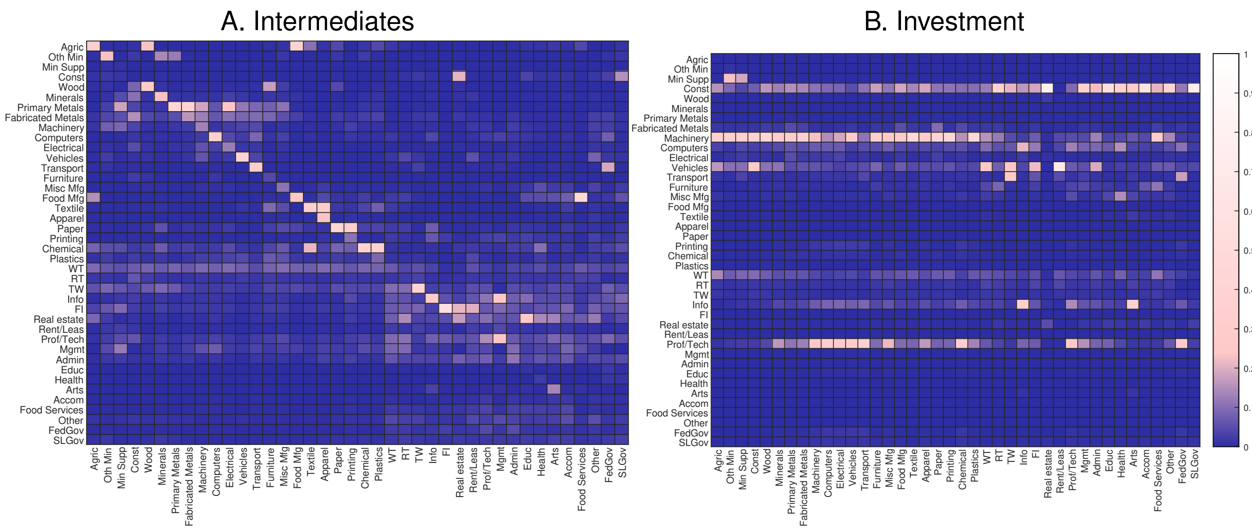
Our data provide consistent coverage of 40 NAICS-defined sectors of the economy, including agriculture and government; Table 1 lists each of the 40 sectors and their corresponding NAICS codes.[5] Our analysis excludes three sectors whose outcomes are highly volatile due to fluctuations in the price of oil—oil and gas extraction, utilities, and petroleum and coal manufacturing.[6] However, we show that these sectors exhibit no long-run trends in the production of total intermediates or investment, and that aggregated patterns of structural change are not significantly impacted by the inclusion of these sectors in Appendix A.

We consider evidence both at the 40-sector level of disaggregation and at a more aggregated two-sector

[5]More recent vintages of the BEA Input Output database allow for greater sectoral detail, but given our interest in structural change over the long run, we focus on these 40 sectors, which can be observed from 1947-2020.

[6]These sectors' prices are very volatile over short-run and medium-run horizons which can obfuscate long-run trends in relative prices of intermediate inputs. We consider how their inclusion affects price measurement in Appendix C.

Figure 1: Heatmaps of Average Scaled Production Networks, 1947-2020



Notes: Panel A shows the scaled input-output network for intermediate goods while panel B shows the scaled investment network for new capital goods. Each (i, j) element in the matrix shows the fraction of total expenditure by sector j (columns) coming from producing sector i (rows).

level. When we aggregate to two sectors, we define "goods" sectors as all agriculture, mining, construction, and manufacturing sectors (22 in total) and define "services" as all remaining sectors (18 in total).

To visualize the average distribution of producers of intermediate and investment goods, Figure 1 plots a heatmap of the average input-output network (panel A) and investment network (panel B) from 1947-2020. To focus on the sectors which are particularly important producers, the heatmap follows a standard convention in the networks literature and scales each column so that the rows sum to 1; thus, each entry (i, j) in Figure 1 represents the share of total expenditures in sector j produced by sector i. This scaling abstracts from information regarding the distribution of total expenditures across sectors. However, this information is implicit in the average share of total intermediates and investment produced by each sector from 1947-2020, reported in Table 1, and we use this information when assessing structural change in these networks.

The heatmaps show that both the input-output and investment networks are sparse; for any given sector, the majority of investment and intermediates are purchased from a small set of sectors. For the investment network, the distribution of investment producers is fairly similar across sectors—most sectors purchase investment goods from a collection of prominent investment hubs (see vom Lehn and Winberry (2022) for further discussion). However, for the input-output network, there is much more sector-specificity as to which sectors are important suppliers of intermediates. In particular, we observe significant homophily in the input-output network—goods sectors play a large role as intermediates suppliers for goods sectors and services sectors play a large role as intermediates suppliers for services sectors.

2.2. Structural Change in Production Networks

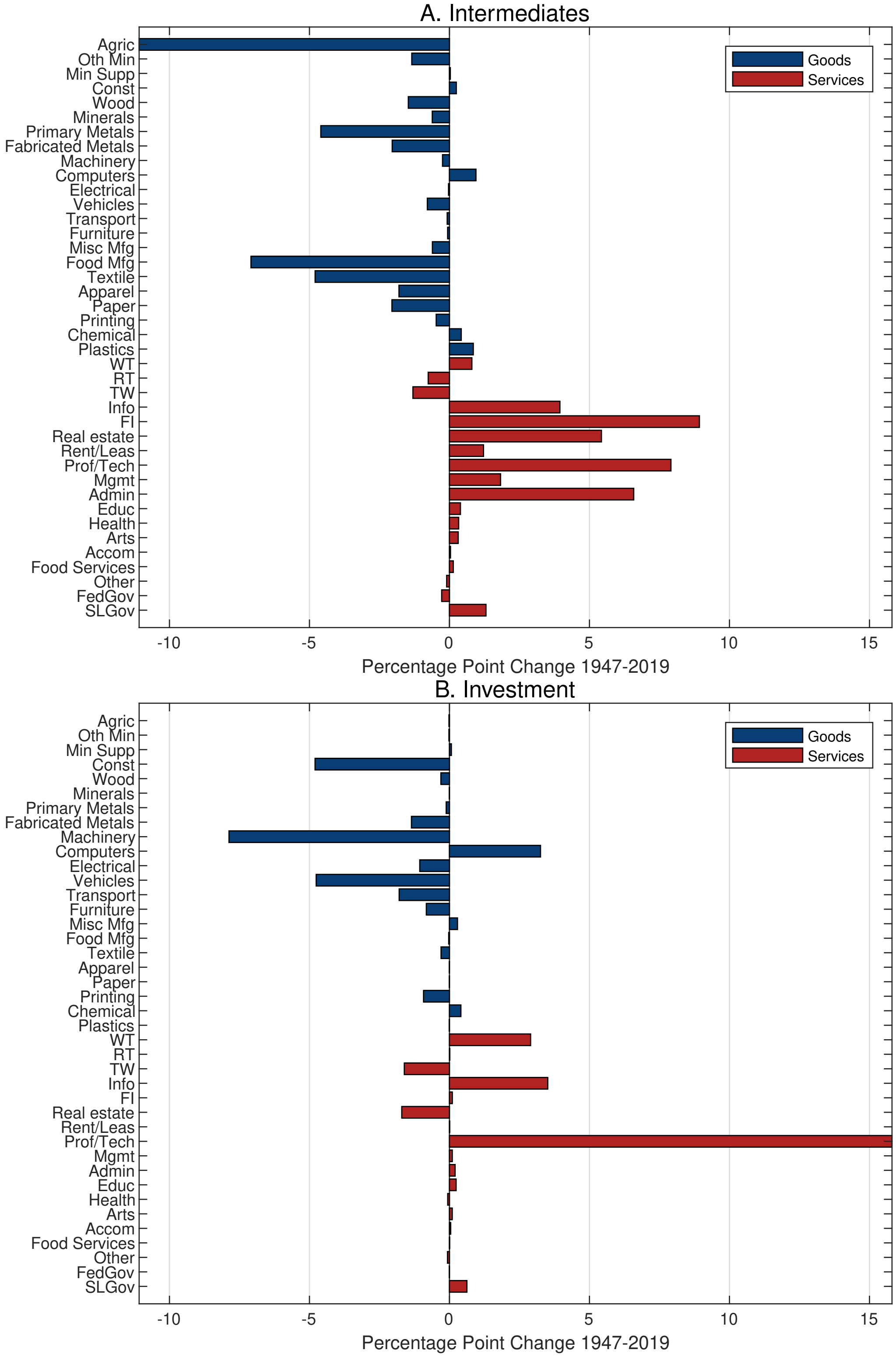
To measure structural change in these production networks, we compute the share of total production value (measured in current dollars) of intermediates or investment that was produced by each sector.[7] We compute the change in this share of production from 1947 to 2019 (not 2020, to avoid any unusual endpoint effects with the onset of the COVID-19 pandemic).

Figure 2 plots the difference between 1947 and 2019 in the share of total production value coming from each sector in the input-output network (panel A) and the investment network (panel B). Each network has seen significant structural change, with a significant increase in the share of production coming from services sectors (red bars) and an offsetting decrease in the share of production from goods sectors (blue bars). For intermediate goods, the largest increases in production share are in information services, finance/insurance, real estate, professional/technical services, and administrative and waste services; the largest decreases occurred in agriculture, primary metals, food and beverage manufacturing, and textile manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest decreases were in machinery, construction, and motor vehicle manufacturing.

Given that changes in these production networks are primarily characterized by a shift in production from goods to services sectors, we aggregate the data into two sectors: goods and services. This facilitates a parsimonious illustration of how the production of intermediates and investment has changed over time. Figure 3 plots the time series of the share of total production value of consumption, intermediates, and investment produced by goods sectors and services sectors. However, given that there is significant variation in which goods and services subsectors produce these final products, we revisit the underlying heterogeneity

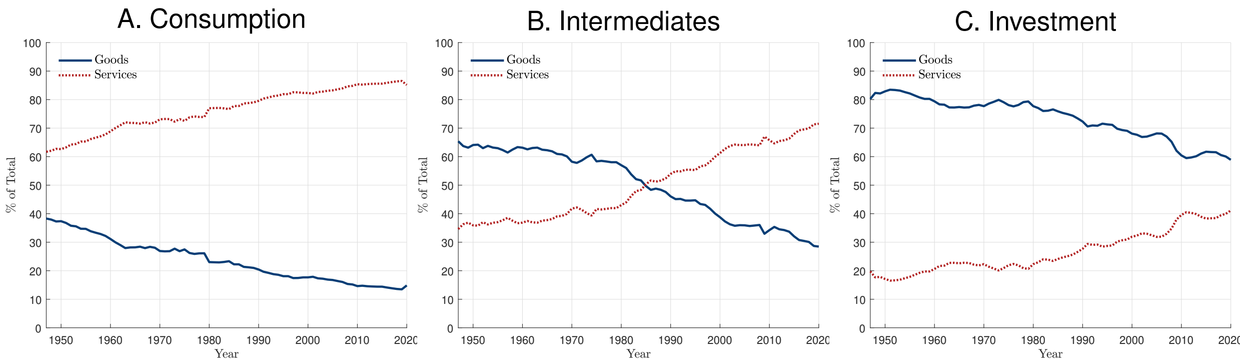
[7] In each year, we take the unscaled version of the matrices plotted in Figure 1, sum along all columns to compute total production of intermediates or investment by each sector, and then compute each producing sector's share of total production value.

Figure 2: Changes in Production Share of Intermediates and Investment: 1947-2019



Notes: Each bar represents the change in the share of intermediates (panel A) or investment panel A) produced by each sector between 1947 and 2019. Blue bars: goods sectors; red bars: services sectors.

Figure 3: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, 1947-2020



Notes: Each plots the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue, solid line) and the services sector (red, dotted line). A full listing of which sectors are included in the goods and services sectors can be seen in Table 1.

in goods and services sectors when we discuss model calibration and measurement in Section 5.

While there is heterogeneity in the level of how much of consumption, intermediates, and investment are produced by services, the trends in these shares over time are similar. Services sectors have long produced the majority of consumption expenditures. In contrast, services has only produced the majority of intermediates since the mid-1980s and still produces the minority of investment. For each product, the fraction produced by the services sector is rising over time. These changes are substantial in magnitude, with the services share of investment and consumption rising by 20-25 percentage points and the services share in intermediates increasing by more than 35 percentage points.

The rising share of services in the production of intermediates and investment reflects two changes. First, changes within sectors generate a rising services share, as sectors reallocate spending on intermediates and investment from goods to services producers. Second, changes in the distribution of total spending on intermediates and investment across sectors increase the services share, as the total spending by services sectors has risen over time and services sectors are key suppliers for services production (primarily for intermediates).[8]

We quantify the importance of these two channels for structural changes in production networks by doing a shift-share decomposition of the services production share, isolating changes occurring within and

[8]For this channel, we focus solely on intermediates and investment because there does not exist a "between sector" component to structural change for consumption, since final users of consumption are households, who are not naturally identified as connected to a particular production sector.

Table 2: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment

	1947	2019	Δ	Decomposition: within	Decomposition: between
Intermediates	0.35	0.71	0.37	0.19 (53%)	0.17 (47%)
Investment	0.20	0.40	0.20	0.21 (104%)	-0.01 (-4%)

Notes: The table reports the results of a shift-share decomposition of the share of services production over time. We decompose changes in the fraction of total intermediates (or investment) spending being produced by services, $Serv_t$, into changes in the share of spending on services within each purchasing industry j, $Serv_{jt}$, and the importance of industry j's spending on intermediates as a fraction of all intermediates expenditures, ω_{jt}. This decomposition can be expressed as:

$$\Delta Serv_t = \underbrace{\sum_j (\overline{\omega}_j \Delta Serv_{jt})}_{\text{within}} + \underbrace{\sum_j (\overline{Serv}_j \Delta \omega_{jt})}_{\text{between}},$$

where $\Delta x = x_{2019} - x_{1947}$ is the change in x and $\overline{x} = \frac{x_{2019}+x_{1947}}{2}$ is the average of x in the two periods 1947 and 2019. Results using all 40 sectors are similar and are available in Appendix A. Individual components may not exactly sum to totals due to rounding.

between sectors. The results of this decomposition are presented in Table 2. For investment, all changes over time in the services production share are due to changes within sectors; for intermediates, the within-sector component contributes roughly half of the change over time.[9] This evidence accords with the production network patterns shown in Figure 1, which showed more sector-specificity in intermediate suppliers than investment suppliers across sectors, consistent with a more significant role for between-sector changes in the services share of production.

A potential concern about the observed pattern of structural change in intermediates is that it may reflect outsourcing of services tasks, resulting in a change of *where* services tasks are performed and how they are recorded rather than a change in the actual *amount* of services tasks performed. That is, it may be that firms originally produced services tasks internally to produce final output, and then outsourced those tasks to other firms, with the outsourced tasks now being measured as services intermediates.

We provide three pieces of evidence to suggest that such outsourcing of services does not drive the

[9] We show in Appendix A that when the services production share is decomposed using heterogeneity across all 40 sectors, roughly 75% of the changes in investment are attributed to within-sector changes. The within-sector change component for intermediates is only slightly lower, still accounting for about 50% of the change.

observed structural change in intermediates. First, the national accounting data we use measures economic activity at the establishment level. Thus, services provided to establishments within a firm by separate administrative offices and headquarters are already classified as intermediates produced by services. If an establishment now receives these services from a different firm, it would not impact the measured share of intermediates produced by services. Second, the average ratio of spending on intermediates relative to gross output (across sectors) has remained between 42-46% for nearly the entire sample window of 1947 to 2020, with little trend and there is no correlation between increased intermediate spending (relative to gross output) and increases in sectors' share of intermediates purchased from services. Thus, structural change in the production of intermediates does not appear to have coincided with increased aggregate spending on intermediates.[10] Third, in Appendix A, we extend some of the arguments by Duernecker and Herrendorf (2022) and analyze structural transformation of the occupational distribution. Within all but one sector (agriculture), there is a systematic rise in workers employed in services occupations over the period 1950-2019, rather than a decline, which would result from systematic outsourcing of services tasks from goods to services sectors. Further, there is no relationship between a sector's within-sector structural change in services occupations and changes in the share of intermediates purchased from services sectors.

Finally, we analyze the importance of changes in the input-output network for "value-added" measures of structural change in consumption and investment. As explained in Herrendorf et al. (2013), a value-added approach to measuring sectoral production of consumption and investment focuses not only on the set of sectors that produce the final product but the network of sectors contributing intermediate inputs needed to produce the product. Thus, value-added measures of structural change implicitly include structural change in intermediates in addition to structural change in final producers of consumption and investment. In Appendix A, we provide details of a decomposition that isolates the contribution of structural change in intermediates to the observed value-added measures of structural change. Structural change in intermediates accounts for 40-50% of structural change in consumption and investment value-added, suggesting an important role for intermediates structural change in generating value-added structural change.

[10] A limitation of this argument is that the ratio of spending on intermediates to total gross output is not exactly the cost share of intermediates in production due to the presence of markups. If markups rise over time, a constant ratio of intermediates spending to gross output could imply a rising cost share of intermediates, possibly reflecting increased outsourcing. However, the evidence on the markup trends is mixed (see Basu (2019)), making it difficult to know if a constant ratio of intermediates spending to gross output is masking a rise in cost shares.

Appendix A contains additional empirical results regarding structural change in production networks, including a time series of expenditure shares for sectors whose production share of intermediates or investment has risen or fallen the most and detailed results on where within-sector changes in purchases from services sectors have been the largest. We also use data from the World Input Output Database (WIOD: Timmer, Dietzenbacher, Los, Stehrer and de Vries, 2015; Woltjer, Gouma and Timmer, 2021) to show that these changes in production networks are not unique to the United States but are observed more broadly throughout other high-income nations in Europe and Asia.

3. Model

To account for the patterns of structural change observed in the previous section, we now describe an N sector extension of the neoclassical growth model, which explicitly incorporates the production networks for intermediate inputs and sectoral investment. We first consider a general version of the model, but discuss additional assumptions necessary for a balanced growth path in Section 4.

3.1. Technology

For each sector j, gross output, Q_{jt}, is produced using capital, K_{jt}, labor L_{jt}, and a bundle of intermediate goods M_{jt} according to the following Cobb-Douglas production function:

$$Q_{jt} = A_{jt} \left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j} \right)^{\alpha_j} M_{jt}^{1-\alpha_j}, \tag{1}$$

where A_{jt} is exogenous TFP in sector j.

The intermediates bundle for each sector, M_{jt}, is produced by an "intermediates bundling" sector for sector j, which aggregates intermediate goods from all sectors using a CES technology:

$$M_{jt} = A_{jt}^{M} \left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}} \right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}, \tag{2}$$

where ϵ_{Mj} is the elasticity of substitution between sectoral inputs in the production of intermediate goods for sector j, $\omega_{Mij} \in (0,1)$ (with $\sum_i \omega_{Mij} = 1$) determines the relative importance of inputs from each sector in producing intermediates, M_{ijt} represents intermediate inputs used in sector j purchased from sector i at time t, and A^M_{jt} represents exogenous intermediates-bundling technical change for sector j. We specify the bundling of intermediate inputs as a separate sector with its own technical change to allow for relative price movements in the overall bundle of intermediate inputs beyond the implied price index based on the price of each individual intermediate input.

Capital is sector-specific and follows a standard law of motion, with depreciation rate δ_j in sector j:

$$K_{jt+1} = (1 - \delta_j)K_{jt} + X_{jt}. \tag{3}$$

Investment, X_{jt}, is produced in an "investment bundling" sector for sector j's capital with the following aggregation technology:

$$X_{jt} = A^X_{jt} \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}} \right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}, \tag{4}$$

where ϵ_{Xj} is the elasticity of substitution between sectors in the production of investment in sector j and $\omega_{Xij} \in (0, 1)$ (with $\sum_i \omega_{Xij} = 1$) determines the relative importance of inputs from each sector in producing investment. A^X_{jt} represents exogenous investment-bundling technical change for sector j.

3.2. Preferences

There is an infinitely lived representative household with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t U(\{C_{it}\}_{i=1}^N), \tag{5}$$

where $0 < \beta < 1$ is the discount rate and C_{it} is consumption produced by sector i. We assume that the period utility function, $U(\{C_{it}\}_{i=1}^N)$ follows a log CES structure, with

$$U(\{C_{it}\}_{i=1}^N) = \ln \left(\left[\sum_i \omega_{Ci}^{1/\epsilon_C} C_{it}^{\frac{\epsilon_C - 1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C - 1}} \right), \tag{6}$$

where ϵ_C represents the elasticity of substitution between consumption goods and $\omega_{Ci} \in (0, 1)$ (with $\sum_i \omega_{Ci} = 1$) determines the relative importance of consumption goods from each sector in aggregate consumption. Our framework abstract from preferences over leisure; we assume the household inelastically supplies one unit of labor each period.

Alternatively, household preferences could be specified with a PIGL indirect utility function, as introduced to the structural change literature in Boppart (2014). However, along the balanced growth path, the form of household preferences has no impact on either the growth rate of GDP or structural change in production networks.[11] Given this result and the focus of our paper, we keep the structure of household preferences as simple as possible and adopt the more tractable log CES case as our baseline.[12]

3.3. Equilibrium

We study the competitive equilibrium of this economy with representative profit-maximizing firms in all markets. The price of final output in each production sector j is denoted by P_{jt}; the price of the intermediates bundle is given by P_{jt}^M. The household owns the capital stock and accumulates capital in sector j by purchasing new investment goods from the investment bundling firm for sector j at price P_{jt}^X. The household rents sector-specific capital to each sector j at a rental price R_{jt}. Since labor is common to each sector and freely mobile, there is a single wage paid to the household, denoted by W_t. We provide a full listing of equilibrium conditions in Appendix B.

In equilibrium, all markets (final output, labor, capital, intermediate bundling, and investment bundling) clear. Final output produced by each sector can be used as consumption for the household, or as an input to intermediate and investment bundles across sectors, implying a market clearing relationship of:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{7}$$

Given constant returns and competitive markets, it is straightforward to show that the price indices for

[11] The aggregate balanced growth path derived in the subsequent section is identical under PIGL preferences where we restrict the number of sectors in preferences to two because the PIGL specification only allows for two distinct income elasticities.

[12] More recently, Comin, Lashkari and Mestieri (2021) consider structural change under a non-homothetic CES preference structure. However, as shown in their paper, this specification is only consistent with a balanced growth path (or "constant growth path") as $t \to \infty$.

the bundle of intermediate goods, P^M_{jt}, and the bundle of investment goods, P^X_{jt}, will be given by:

$$P^M_{jt} = \frac{1}{A^M_{jt}} \left(\sum_i \omega_{Mij} P_{it}^{1-\epsilon_{Mj}} \right)^{\frac{1}{1-\epsilon_{Mj}}} \tag{8}$$

$$P^X_{jt} = \frac{1}{A^X_{jt}} \left(\sum_i \omega_{Xij} P_{it}^{1-\epsilon_{Xj}} \right)^{\frac{1}{1-\epsilon_{Xj}}} . \tag{9}$$

Furthermore, straightforward manipulation of the first order conditions for each sector's production generates the following expression for the price of final output produced by each sector j:

$$P_{jt} = \frac{1}{A_{jt}} \left(\frac{R_{jt}}{\theta_j \alpha_j} \right)^{\theta_j \alpha_j} \left(\frac{W_t}{(1-\theta_j)\alpha_j} \right)^{(1-\theta_j)\alpha_j} \left(\frac{P^M_{jt}}{1-\alpha_j} \right)^{1-\alpha_j} . \tag{10}$$

Finally, we describe the equilibrium conditions that dictate structural change in production networks. Manipulating first order conditions for the intermediates and investment bundling sectors, the share of expenditures by the bundling sectors for sector j on inputs purchased from sectors i can be written as:

$$s^M_{ijt} \equiv \frac{P_{it} M_{ijt}}{P^M_{jt} M_{jt}} = \omega_{Mij} \left(\frac{P_{it}}{A^M_{jt} P^M_{jt}} \right)^{1-\epsilon_{Mj}} , \tag{11}$$

$$s^X_{ijt} \equiv \frac{P_{it} X_{ijt}}{P^X_{jt} X_{jt}} = \omega_{Xij} \left(\frac{P_{it}}{A^X_{jt} P^X_{jt}} \right)^{1-\epsilon_{Xj}} . \tag{12}$$

The share of intermediates (investment) expenditures by sector j on sector i's output is defined as s^M_{ijt} (s^X_{ijt}), and depends on the relative prices of each sector's output and the CES scale and elasticity parameters in the bundling sectors. Empirically, s^M_{ijt} (s^X_{ijt}) corresponds to a column j in the scaled input-output (investment) network plotted in Figure 1. Thus, movements in relative prices across sectors can induce structural change in production networks.

The total share of intermediates and investment produced by each sector will also depend on the distribution of total intermediates and investment spending across sectors. That is, the share of total intermediate and investment purchases produced by sector i (defined as s^M_{it} and s^X_{it}, respectively) are given by:

$$s^M_{it} = \sum_j \frac{P^M_{jt} M_{jt}}{\sum_k P^M_{kt} M_{kt}} s^M_{ijt}, \tag{13}$$

$$s_{it}^{X} = \sum_{j} \frac{P_{jt}^{X} X_{jt}}{\sum_{k} P_{kt}^{X} X_{kt}} s_{ijt}^{X}. \tag{14}$$

Changes in s_{it}^{M} and s_{it}^{X} over time (like those plotted in Figure 2) thus reflect both changes in production processes within sectors (changes in s_{ijt}^{M} and s_{ijt}^{X}) and changes in the composition of spending across all sectors. As our shift-share decomposition in Section 2.2 reveals, the latter margin is particularly important for structural change in intermediates.

4. Balanced Growth Path

We now consider the joint evolution of economic growth and structural change along an aggregate balanced growth path (ABGP). We focus on an aggregate balanced growth path for two reasons: (1) an aggregate balanced growth path is generally consistent with empirical U.S. economic growth since the Industrial Revolution, and (2) the aggregate growth path representation facilitates a clearer understanding of the relationships between structural change and aggregate economic growth. In Section 6.3, we consider the model's implications for economic growth without some of the restrictions necessary to obtain an ABGP.

We study an aggregate balanced growth path in this economy in three steps. First, we describe the necessary assumptions for such a path to exist and the implications of these assumptions for prices and aggregate quantities. Second, we state and discuss a proposition establishing the existence and nature of an ABGP consistent with structural change in production networks. Finally, we analyze the connection between the aggregate growth rate of the economy and structural change in production networks in a particular case that allows for greater analytical tractability.

4.1. Assumptions and Model Implications

While the rich heterogeneity of our model in the previous section is appealing, it is also generally inconsistent with the existence of an aggregate balanced growth path—for example, Acemoglu and Guerrieri (2008) argue that differential capital and labor intensities only generate balanced growth in the limit.[13] Thus, an aggregate balanced growth path requires assumptions that impose homogeneity in production functions

[13] That said, Herrendorf, Herrington and Valentinyi (2015) argue that capital share heterogeneity (as well as capital-labor substitution) is of second-order importance for explaining quantitative patterns of structural change.

across sectors and the bundling of new investment (but not in the bundling of intermediates).

Assumption 1. *The parameters of the sectoral production functions are the same across all sectors, i.e.,* $\alpha_j = \alpha$ *and* $\theta_j = \theta$ *for all* j.

Assumption 2. *The parameters governing the evolution of capital—both parameters of the investment bundling sectors and the depreciation rate—are the same for all sectors* j*, i.e.,* $\delta_j = \delta$, $\omega_{Xij} = \omega_{Xi}$ *and* $\epsilon_{Xj} = \epsilon_X$ *for all* j. *Furthermore, technical change in each sector's investment bundling is the same, i.e.,* $A^X_{jt} = A^X_t$.

These two assumptions allow us to simplify equilibrium price expressions. First, with common parameters in the investment bundling sectors, equation (9) implies that the price of new investment will be equated across sectors, i.e., $P^X_{jt} = P^X_t$. Furthermore, since the parameters governing the evolution of capital are also equated across sectors, there is now a single type of capital in the economy with a single rental rate, i.e., $R_{jt} = R_t$. Finally, given common production function parameters (and a common wage by assumption), equation (10) implies that the relative price of final output in sectors i and j depends only on sectoral TFP differences and differences in the price of the intermediates bundles in those two sectors:

$$\frac{P_{jt}}{P_{it}} = \frac{A_{it}/\left(P^M_{it}\right)^{1-\alpha}}{A_{jt}/\left(P^M_{jt}\right)^{1-\alpha}}. \tag{15}$$

Lemma 1 relates relative prices of final output in each sector to the price of intermediates and investment bundles:

Lemma 1. *Given assumptions 1 and 2, the price of sector* j*'s product relative to the price of the bundle of intermediates in each sector,* P^M_{it}*, and to the price of the bundle of investment,* P^X_t*, can be written as:*

$$\frac{P_{jt}}{P^X_t} = \frac{\tilde{B}^X_t}{\tilde{A}_{jt}} \tag{16}$$

$$\frac{P_{jt}}{P^M_{it}} = \frac{\tilde{B}^M_{it}}{\tilde{A}_{jt}} \tag{17}$$

where $\tilde{A}_{jt} \equiv \frac{A_{jt}}{\left(P^M_{jt}\right)^{1-\alpha}}$, $\tilde{B}^M_{it} \equiv A^M_{it}\left(\sum_k \omega_{Mki}\tilde{A}_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ *and* $\tilde{B}^X_t \equiv A^X_t\left(\sum_k \omega_{Xk}\tilde{A}_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$.

Proof. See Appendix B. □

Given the relationship between relative prices and technology, $\tilde{B}_t^X$ and $\tilde{B}_{it}^M$ represent investment-production and intermediate-production TFP. In each case, the expression contains an exogenous bundling component, A_t^X or A_{it}^M, and an endogenous component constructed from the sum of adjusted sectoral productivities, $\tilde{A}_{it}$, weighted in proportion to that sector's role as a producer of investment or intermediates. These adjusted sectoral productivities capture both direct advances in technical change in the form of sector-specific TFP in final production sectors, A_{it}, and indirect advances in technical change, captured in the prices of intermediate goods used to produce final investment or intermediates. Without further assumptions, we cannot solve for the price of intermediates in closed form, but we consider a special case where a closed-form representation exists at the end of this section.[14]

To derive an expression for aggregate GDP in this economy, nominal sectoral value added in sector j is defined as nominal sectoral gross output minus expenditures on intermediates:

$$P_{jt}^V V_{jt} = P_{jt} Q_{jt} - P_t^M M_{jt} \tag{18}$$

where V_{jt} represents real value added in sector j and P_{jt}^V is the price of value added. Thus, aggregate GDP, Y_t, denoted in units of the numeraire, is given by $Y_t = \sum_i P_{it}^V V_{it}$. We take aggregate investment as the numeraire in the economy.

Given the above assumptions and price relationships, the following lemma presents a closed-form expression for aggregate GDP in the economy:

Lemma 2. *Given Assumptions 1 and 2, aggregate GDP, denoted in units of the numeraire (aggregate investment), is given by:*

$$Y_t = \sum_i P_{it}^V V_{it} = \mathcal{A}_t K_t^\theta \tag{19}$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}}$ *and* $K_t = \sum_j K_{jt}$*. Furthermore, the following aggregate equilibrium*

[14] If we take aggregate investment as the numeraire, the price of the intermediate bundle in sector i is given by:

$$P_{it}^M = \frac{A_t^X}{A_{it}^M} \left(\sum_k \omega_{Xk} \left(\frac{A_{kt}}{(P_{kt}^M)^{1-\alpha}} \right)^{\epsilon_X - 1} \right)^{\frac{1}{\epsilon_X - 1}} \left(\sum_k \omega_{Mki} \left(\frac{A_{kt}}{(P_{kt}^M)^{1-\alpha}} \right)^{\epsilon_{Mi} - 1} \right)^{\frac{1}{1-\epsilon_{Mi}}}.$$

This produces a system of N non-linear equations that can be solved to obtain the price of each intermediates bundle. Without further assumptions, this system of nonlinear equations does not have a closed-form solution.

conditions hold:

$$R_t = \theta \mathcal{A}_t K_t^{\theta-1} \tag{20}$$

$$W_t = (1-\theta)\mathcal{A}_t K_t^{\theta} \tag{21}$$

Proof. See Appendix B. □

The representation of aggregate GDP in Lemma 2 follows the same production structure as sectoral value added. With aggregate investment as the numeraire, aggregate TFP, $\mathcal{A}_t$, only depends on investment-production TFP, $\tilde{B}_t^X$. Thus, growth in aggregate GDP only depends on technical change in the production of investment, either directly at final producers of investment or at their intermediate suppliers.[15]

4.2. An Aggregate Balanced Growth Path

We adopt the same aggregate balanced growth path (ABGP) definition as in Ngai and Pissarides (2007) and Herrendorf et al. (2021), where all aggregates denoted in units of the numeraire (aggregate investment) must grow at a constant rate. This means that K_t, Y_t, W_t, R_t, and X_t will grow at a constant (though not necessarily equal) rate. Total consumption expenditures and total intermediates expenditures are defined in units of the numeraire, with $E_t^C = \sum_i P_{it} C_{it}$ and $E_t^M = \sum_i P_{it}^M M_{it}$. Thus, E_t^C and E_t^M also grow at a constant rate along the ABGP.

For any variable X_t, the gross growth rate between time periods t and $t+1$ is defined as $\gamma_{t+1}^X \equiv \frac{X_{t+1}}{X_t}$. Along the ABGP, we drop the time subscripts for variables growing at a constant rate. With these definitions, we state the following proposition:

Proposition 1. *Assume that Assumptions 1 and 2 hold and that $\gamma_t^{\mathcal{A}} > \frac{1-\delta}{\beta}\ \forall t$. An aggregate balanced growth path exists where*

$$\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^{E^M} = \gamma^W = \left(\gamma_t^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{22}$$

[15] We consider an alternate representation of GDP in Section 6.2 that also allows for productivity growth in consumption production to affect aggregate GDP growth.

and $\gamma^R = 0$ *if and only if* γ_t^A *is constant.*

Proof. See Appendix B. □

Given the aggregate production function derived in Lemma 2, the result for the aggregate growth rate of the economy is unsurprising. As in the one-sector growth model, the economy's aggregate growth rate only depends on the growth rate of aggregate TFP. The requirement that $\gamma_t^A > \frac{1-\delta}{\beta}$ is standard and holds for most reasonable parameter values.[16]

Similar to the aggregate balanced growth path in Ngai and Pissarides (2007) when including intermediate inputs, the aggregate growth rate of the economy with investment as the numeraire only depends on technical change that increases the production frontier for investment, either through directly expanding production at final investment producers or through reducing costs of intermediate inputs for final investment producers. However, our aggregate balanced growth path differs from that of Ngai and Pissarides (2007), who argue that endogenous structural change in production networks is not possible along the aggregate balanced growth path. The key distinction between our two frameworks that allows for structural change in production networks along the balanced growth path is that we do not require that all exogenous technical change terms grow at a constant rate. As discussed in more detail in Herrendorf et al. (2021), at least one of the exogenous technical change processes *must* grow at a particular non-constant rate to satisfy the necessary and sufficient condition that γ_t^A remains constant. Imposing constant growth in all exogenous technical change terms would restore the result of Ngai and Pissarides (2007).

We emphasize that Proposition 1 requires no restrictions on the intermediates bundling processes across sectors; the balanced growth path admits arbitrary heterogeneity in the input-output network. However, with heterogeneity in the bundling process for intermediates across sectors, it is not possible to separate in closed form the contribution of the direct and indirect portions of investment-specific TFP growth. The following assumption and subsequent lemma consider a case in which such separation is possible, allowing for a separate assessment of the contributions of technical change at final investment producers and technical change at intermediate suppliers of investment producers to aggregate growth.

[16] For example, with annual depreciation of about 10% and annual discounting at a 3% real rate, this condition would require that $\gamma_t^A - 1 > 0.9/(1/(1+0.03)) - 1 = -0.073$ or that aggregate TFP growth exceeds -7%.

Assumption 3. *The parameters of the intermediates bundling sectors are the same for all sectors j, i.e., $\omega_{Mij} = \omega_{Mi}$ and $\epsilon_{Mj} = \epsilon_M$ for all j and that technical change in each sector's intermediates bundling is the same, i.e., $A^M_{jt} = A^M_t$*

Lemma 3. *Assume that assumptions 1-3 hold. Then the implied aggregate technical change term, $\mathcal{A}_t$ can be written as $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B^X_t \left(B^M_t\right)^{\frac{1-\alpha}{\alpha}}$, where $B^M_t \equiv A^M_t \left(\sum_k \omega_{Mk} A^{\epsilon_M - 1}_{kt}\right)^{\frac{1}{\epsilon_M - 1}}$ and $B^X_t \equiv A^X_t \left(\sum_k \omega_{Xk} A^{\epsilon_X - 1}_{kt}\right)^{\frac{1}{\epsilon_X - 1}}$. Further, with a single investment product and a single intermediate product, the relative productivities in investment production and intermediates production are equal to the inverse ratio of relative prices, $\frac{B^X_t}{B^M_t} = \frac{P^M_t}{P^X_t} = P^M_t$ (given the choice of aggregate investment as the numeraire).*

Proof. See Appendix B. □

Abstracting from heterogeneity in intermediates bundling allows us to write aggregate TFP as a function of multiplicatively separable contributions of investment-production technical change, B^X_t and intermediates-production technical change, B^M_t. However, even in this case, because of the CES nature of intermediates and investment bundling and the heterogeneous elasticities of substitution in these two processes, the two forces driving aggregate growth remain distinct. This is in contrast to a setting where intermediates and investment are bundled using Cobb-Douglas production technologies, in which case the growth contribution of each production sector via both networks can be combined into a single multiplier effect. Thus, throughout the remainder of our analysis of aggregate growth, we separately emphasize growth originating from the production of intermediates and the production of investment.

4.3. Implications of Balanced Growth for Structural Change and Growth Rates

A key feature of the ABGP is that it allows for structural change in production networks while the aggregate growth rate of the economy remains constant. Because the ABGP admits arbitrary heterogeneity in the input-output network, aggregate structural change in intermediates retains both a within-sector and a between-sector channel, following equation (13). However, in the case of investment, because assumptions 1 and 2 imply that each sector's production technology is identical and that there is a single investment good used for all sectors, structural change is identical across all sectors (i.e., $s^X_{it} = s^X_{ijt}\ \forall j$). Therefore, the ABGP rules out the composition channel for structural change in the investment network. As observed in Table 2, this channel played a minimal role in the observed patterns of structural change in the investment network.

Importantly, structural change in production networks will play a role in shaping the composition of the aggregate growth rate of the economy. For illustrative purposes, we discuss the case where assumption 3 holds, implying no heterogeneity in intermediates bundling technologies. This case helps build intuition for the role that structural change plays in shaping the aggregate growth rate, but we do not impose these simplifying assumptions in our quantitative exercises.

In this case, the growth rate of aggregates is given by $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} = \left(\gamma_t^{B^X}\left(\gamma_t^{B^M}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\theta}}$ along the ABGP. The dependence of the aggregate growth rate on both $\gamma_t^{B^X}$ and $\gamma_t^{B^M}$ establishes a direct connection between the aggregate growth rate and structural change in production networks. With some straightforward algebra, these two aggregate growth rates can be expanded as follows:

$$\gamma_t^{B^M} = \gamma_t^{A^M}\left(\sum_i s_{it-1}^M (\gamma_{it}^A)^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}} \tag{23}$$

$$\gamma_t^{B^X} = \gamma_t^{A^X}\left(\sum_i s_{it-1}^X (\gamma_{it}^A)^{\epsilon_X - 1}\right)^{\frac{1}{\epsilon_X - 1}} \tag{24}$$

These expressions establish that, in addition to the production share parameters on capital (θ) and value-added (α), the growth rate of aggregates depends on four components: growth in intermediates-bundling technical change ($\gamma_t^{A^M}$), growth in investment-bundling technical change ($\gamma_t^{A^X}$), and two weighted sums of technical change in each individual production sector i, where the weights are determined by the composition of intermediates production (s_{it}^M) and the composition of investment production (s_{it}^X). Of course, the very definition of the ABGP implies that γ^Y is a constant. Thus, along the ABGP, structural change in production networks will not impact the aggregate growth rate. But the composition of the growth rate depends on potentially differential rates of technical change in intermediates and investment production.

We make two additional points about how the CES structure for intermediates and investment matters for the composition of economic growth. First, at any point in time, the distribution of intermediates and investment expenditures across different producing sectors i, s_{it}^M and s_{it}^X, influences the aggregate growth rate. For example, if the share of intermediates expenditure is high in sectors experiencing rapid technological change, this will lead to faster economic growth. The underlying share parameters ω_{Mi} and ω_{Xi} in the CES aggregators for intermediates and investment will thus be essential for shaping the composition of economic growth.

Second, as relative prices change over time due to different rates of productivity growth across sectors, the distribution of intermediates and investment expenditures will change. The rate at which this distribution changes will depend on the elasticities of substitution in the CES aggregators for intermediates and investment, ϵ_M and ϵ_X. We establish how these impact aggregate growth with the following lemma:

Lemma 4. *Assume that assumptions 1-3 hold. Further, assume that there is weak positive dependence between the log of sectoral TFP and TFP growth across sectors (formally,* $\mathbb{E}\left[ln(A_{it}) \mid \gamma_{it}^A = a\right]$ *is weakly increasing in* a *with a probability measure across sectors of* s_{it-1}^M*).*

Holding all other parameters fixed, the growth rate of intermediates-production technical change, $\gamma_t^{B^M}$ *is weakly increasing in* ϵ_M*. The same result holds for the growth rate of investment-production technical change,* $\gamma_t^{B^X}$*; all other parameters fixed, it is increasing in* ϵ_X*.*

Proof. See Appendix B. □

Lemma 4 establishes that the higher the elasticity of substitution in either intermediates or investment, the faster intermediates- or investment-production technical change will grow. The intuition for this result can be seen from considering the limiting cases for these CES functions of growth rates when sector TFP growth rates are constant over time.[17] For example, if $\epsilon_M < 1$, implying gross complements, then as $t \to \infty$, the growth rate of intermediates-production technical change will converge to the *slowest* TFP growth rate among producers of intermediates. This is because movements in relative prices will ultimately cause s_{it}^M to converge to one for the slowest growing sector. In contrast, in the gross substitutes case (i.e., $\epsilon_M > 1$), this growth rate will converge to the *fastest* TFP growth rate, as all expenditures ultimately become concentrated on this sector.

Another implication of Lemma 4 and the above discussion occurs when one production network features complementarity in sectoral inputs and the other features substitutability. In this case, whichever network features gross substitutability will become endogenously more important for aggregate growth over time, as

[17] Lemma 4 does not require that sectoral TFP growth rates be constant over time; this is a convenient framing for intuition. The requirement is that there is weak positive dependence, which imposes that, on average, sectors with high TFP are also growing fast. Provided initial levels of TFP are normalized to 1, this implies that relative growth rates across sectors must be generally stable; if there are dramatic reversals in sectoral TFP growth rates, it is not possible to generally characterize the impact of ϵ_M and ϵ_X on the growth rates of $\gamma_t^{B^M}$ and $\gamma_t^{B^X}$.

resources are reallocated toward the fastest growing producers in that network ("frontiers"), while resources are allocated to the slowest growing producer in the other network ("bottlenecks"). We discuss this implication further once we obtain numerical values for these elasticities in Section 5 and analyze long-run growth patterns in Section 6.

5. Measurement, Calibration, and Structural Change

To analyze the patterns of structural change and forces driving aggregate growth described in the previous section, we now turn to the calibration of model parameters and the measurement of prices which feature prominently in that calibration. This section focuses on the general procedures we use to calibrate the model for analyses consistent with the aggregate balanced growth path (i.e., when assumptions 1 and 2 hold); additional measurement and calibration details are described in Appendix C.

The first question we must confront with regard to measurement and calibration is the level of aggregation. While our model puts no restriction on the number of sectors we can analyze, we choose a level of aggregation that balances two considerations. On the one hand, as Section 2 highlights, the main patterns of structural change can conveniently be summarized within a parsimonious two-sector framework focusing on goods and services. This approach allows for straightforward interpretation of production elasticities and limits the number of parameters to be calibrated. On the other hand, our model emphasizes the potential importance of heterogeneity in productivity growth and aggregation across consumption, investment, and intermediates. Further, evidence from the 40-sector data discussed in Section 2 suggests that, although services sectors increasingly produce all of these products, there is significant heterogeneity in which services subsectors are producing consumption, investment, and intermediates.

To balance these considerations, we focus on a six-sector aggregation, where each sector is defined by the interaction of the product market each sector operates in—goods or services—with the type of product it produces—consumption, investment, or intermediates. The resulting six sectors are goods consumption (e.g., books, toys, food), goods investment (e.g., buildings, machines, vehicles), goods intermediates (e.g., primary metals, chemicals), services consumption (e.g., education, health care), services investment (e.g., software, R&D), and services intermediates (e.g., financial services, wholesale trade). As a result, each sector in the model only produces consumption, investment, or intermediates, allowing for heterogeneity in productivity growth within goods and services sectors based on the final product they produce.

5.1. Calibration Strategy

The non-CES aggregator parameters in the model are calibrated following a standard approach. We use expenditure data from 1947-2020 at the 40 sector level from the BEA Input-Output Database to calibrate production function parameters: α_j is calibrated using sectoral data on the ratio of nominal value added to nominal gross output; θ_j is calibrated using sectoral data on labor compensation (adjusted for taxes and self-employment) relative to nominal value added. Depreciation rates are calibrated using data on implied depreciation rates by sector as reported in the BEA Fixed Asset Accounts. With common production function parameters and depreciation rates across sectors (i.e. assumptions 1 and 2), we set α, θ, and δ to the average of these expenditure ratios across sectors. This yields values of $\alpha = 0.54$, $\theta = 0.32$, and $\delta = 0.08$. Finally, we set $\beta = 0.96$.

Lastly, we calibrate the key CES aggregator parameters—the share parameters (ω_{Ci}, ω_{Xi}, ω_{Mij}) and the elasticity parameters (ϵ^C, ϵ^X_{ij}, ϵ^M_{ij}). The conceptual procedure is fairly standard: first, for each sector, j, the share parameters are set to match the initial fraction of expenditures on consumption, intermediates, or investment purchased from sector i in the year 1947.[18] Given values for these share parameters, we identify the elasticity parameters by estimating equations (11) and (12) (and an analogous equation for consumption expenditure shares) via non-linear least squares using annual data moments on expenditure shares and prices, with only one set of CES parameters for investment, given assumption 2.

5.2. Measuring Prices

To estimate the elasticity parameters for our six-sector aggregation, we require separate price series for consumption, investment, and intermediates produced by goods and services sectors. We first describe how we use final expenditure data to measure these prices and then discuss the novel insights from this approach relative to alternative measurement methods. Additional details are available in Appendix C.

[18] In the case of intermediates, this requires observing the intermediate expenditure patterns for our six sector partition of the economy. We describe in Appendix C how we generate these by aggregating the intermediate expenditure patterns across our observed 40 sectors in proportion to each sector's role in producing consumption, investment, or intermediates within goods or services sectors.

We measure consumption and investment prices for goods and services using expenditure-side data from the U.S. NIPA, which consistently covers expenditures and prices for 68 consumption commodities and 30 investment commodities from 1947-2020. Crucially, we also utilize "bridge files" published by the BEA (and extended by vom Lehn and Winberry, 2022), which report the extent to which each of these commodities is produced by goods or services sectors.

Formally, we measure price growth for consumption, C (or alternatively, investment, X), produced by goods or services ($j \in \{Goods, Services\}$) using price and spending data on commodities (P_{kt}^C and $P_{kt}^C Q_{kt}^C$ for $k \in \{1, ..., K\}$) according to the formula:

$$\Delta \ln(P_{jt}^C) = \sum_{k=1}^{K} \frac{\xi_{jkt}^C P_{kt}^C Q_{kt}^C}{\sum_{\ell=1}^{L} \xi_{j\ell t}^C P_{\ell t}^C Q_{\ell t}^C} \Delta \ln(P_{kt}^C) \tag{25}$$

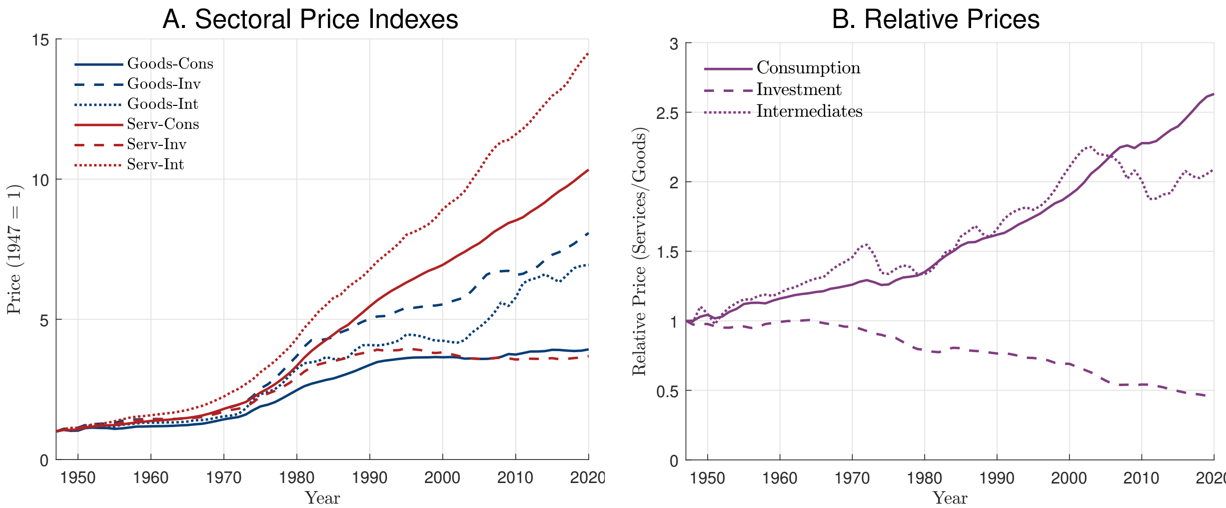
where ξ_{jkt}^C represents the entries of the bridge files, which identify the fraction of spending on commodity k that can be traced back to production by sector j (averaged across years $t-1$ and t).[19]

Measuring intermediates prices from expenditure side data is significantly more involved than measuring consumption or investment prices. Since intermediate inputs are not counted in GDP, expenditure-side national accounting data does not provide a detailed accounting of the price of intermediate commodities. In principle, the Producer Price Index (PPI) series published by the U.S. Bureau of Labor Statistics (BLS) provide purchasers' prices for intermediate inputs which is precisely what we would want, but this data has incomplete coverage of services sectors (roughly 85% of the services sectors producing intermediates) and prices for intermediates produced by the services sector are only available starting in 2009. However, we show in Appendix C that the procedure we use to identify intermediates prices generates time series of intermediates prices that line up almost perfectly with the published PPI data for prices of both goods- and services-produced intermediates.

Although we do not have direct data on the prices paid for intermediate products produced by goods and services, we can infer these prices using gross output prices for goods and services sectors from the BEA's GDP by Industry database in combination with our novel expenditure-side price series for investment and

[19] For example, in 1997, the bridge file for consumption commodities shows that of the 162 billion dollars spent on the commodity of new motor vehicles, 73% of that value came from the motor vehicle manufacturing sector, 23% of that value came from the retail trade sector, and 3% came from wholesale trade, and 1% came from transportation and warehousing services.

Figure 4: Prices of Goods and Services Consumption, Investment and Intermediates, 1947-2020



Notes: Panel A shows the time series of prices for each product (consumption, investment, or intermediates) produced by each sector (goods or services). Panel B shows the price of services divided by the price of goods for each commodity (consumption, investment, or intermediates).

intermediates produced by goods and services. Gross output prices for each sector are implicitly an average of the price of consumption, investment, and intermediates produced by that sector. Thus, using the price of consumption and investment produced by goods or services, we can identify the price of intermediates produced by goods or services as the residual in gross output prices. With this approach, we construct the price of intermediates produced by sector $j \in \{Goods, Services\}$ as:

$$\Delta \ln(P_{Mjt}) = \frac{1}{\zeta_{jt}^M} \left(\Delta \ln P_{jt}^{GO} - \zeta_{jt}^C \Delta \ln P_{Cjt} - \zeta_{jt}^X \Delta \ln P_{Xjt} \right), \tag{26}$$

where ζ_{jt}^i represents the average share (between $t-1$ and t) of total gross output of sector j used for commodity i, with $\zeta_{jt}^M + \zeta_{jt}^C + \zeta_{jt}^X = 1$, and $\Delta \ln P_{jt}^{GO}$ represents the log change of the gross output price for sector j.[20]

Figure 4 plots our price series for consumption, investment, and intermediates produced by goods and service sectors; panel A plots the raw prices, normalized to one in 1947 and panel B plots the relative price (services divided by goods) for each product. Unsurprisingly, the price of services consumption relative to

[20] As described in Appendix C, we adjust gross output prices (and to a lesser extent consumption and investment prices) for oil/energy price spillovers before performing this procedure. The qualitative patterns of relative price movements across goods and services sectors are robust to not making these corrections.

goods rises significantly over time. This is consistent with existing literature using relative prices to explain structural transformation in consumption. We also observe that the relative price of services intermediates is rising significantly. Given the rising share of expenditures on services products, rising relative prices are consistent with complementarity between goods and services inputs to consumption and intermediates.

In stark contrast, Figure 4 shows that the relative price of services investment is falling. Given rising expenditures on services investment, these falling prices suggest that goods and services inputs to investment are *substitutes*. The intuition for this finding is that investment inputs produced by services sectors are primarily information technology and intellectual property products (e.g., software and R&D), whose price has fallen significantly relative to the price of goods investment (e.g., equipment or structures).

This finding is robust to a wide variety of alternative specifications: we aggregate investment prices with user cost weights instead of investment expenditures as recommended by Holden, Gourio and Rognlie (2020); we focus exclusively on equipment and software, which feature the most overlap between goods and services production; we quality adjust investment prices as in Cummins and Violante (2002); and we use alternative implementations of bridge files for consumption and investment prices. In all cases, the price of investment by services sectors is declining relative to the price of investment produced by goods sectors (see Appendix C).

One concern about our measured prices is that the bridge file procedure could generate bias from inaccurately attributing the price growth of a single final commodity to both goods and services inputs, understating price growth in one sector, and overstating it in another. The high level of disaggregation for consumption and intermediates commodities partially ameliorates this concern—at this level of disaggregation, nearly two-thirds of all commodities are produced in large majority by either goods or services (more than 80%). The primary instance where both goods and services sectors contribute significantly to a consumption or investment commodity is when delivery of the commodity to the final user involves significant "margins" due to transportation, wholesale trade, or retail trade. This typically happens when the final product is itself a physical good. However, if we reclassify these margin sectors as goods sectors, then over 90% of all commodities would be produced in large majority by either goods or services. Appendix C shows that under this reclassification, the relative price of investment produced by services still declines.

Our finding of a declining relative price coincident with a rising expenditure share of services investment contrasts with the elasticities calibrated by Herrendorf et al. (2021), García-Santana et al. (2021) and Sposi et al. (2021), who use a single price for goods and services, respectively, to estimate that these inputs are complements in investment production. Their estimates differ from our findings because of aggregation bias. Figure 4 highlights significant heterogeneity in price growth among goods and services sectors. When aggregating these prices to a single goods and a single services price (e.g., the respective gross output prices published by the BEA), on average, goods are becoming less expensive relative to services. However, this masks a decline in the relative price of services investment, especially since investment is a small share of total services production.

This aggregation bias even persists when using heterogeneous gross output prices at the 40 sector level, drawing on Input Output data to determine each sector's contribution to producing consumption, investment, and intermediates (see Appendix C). The challenge with income-side accounting data at this level of disaggregation is that there is still significant aggregation bias within sectors, particularly services sectors. For 29 of the 40 sectors we observe, at least 15% of sectoral output is dedicated to at least two of the final uses—consumption, investment, and intermediates. For example, the professional and technical services sector produces significant amounts of consumption (e.g., legal and veterinary services), investment (e.g., custom computer programming and R&D), and intermediates (e.g., advertising and public relations services) commodities, each of which has very different price trends. A single price for this sector thus does not accurately represent price growth over time for the consumption, investment, and intermediate commodities it produces.

5.3. Elasticities of Substitution

Based on our data for sector-specific expenditures (Figure 3) and prices (Figure 4), we use the procedure described in Section 5.1 to calibrate parameter values for the CES bundling aggregators as displayed in Table 3. The table reports the parameters for the single CES aggregator in consumption and investment (given Assumption 2) and the parameters for each of the six sectors' intermediates bundling technology. As anticipated in our above discussion of the general time series patterns for relative prices and expenditures, the calibrated values for the elasticity parameters confirm that goods and services are complements in the production of consumption and intermediates (elasticities less than one), but substitutes in the production of investment (elasticities greater than one).

Table 3: Calibrated Parameters of CES Bundling Aggregators

Bundling Technology	Elasticity of Subsitution (ϵ)	Goods Share (ω)
Consumption	0	0.38
Investment	2.36	0.80
Intermediates		
Goods-Consumption	0.25	0.81
Goods-Investment	0.49	0.71
Goods-Intermediates	0.41	0.77
Services-Consumption	0	0.43
Services-Investment	0	0.36
Services-Intermediates	0	0.34

Notes: The table reports the calibrated parameter values for the CES aggregator parameters corresponding to equations (2), (4), and (6). For each type of product (consumption, investment, intermediates), the parameter ϵ represents the elasticity of substitution between goods and services inputs, and the parameter ω represents the share parameter attached to goods inputs (with one minus that parameter being the share parameter attached to services). Given Assumption 2, there is only a single set of parameters for investment.

The elasticities for consumption and intermediates imply strong complementarity between goods and services, with the best fit often given by Leontief aggregation, consistent with existing literature on structural change (e.g., Herrendorf et al., 2021; García-Santana et al., 2021; Sposi et al., 2021). However, the best fit for patterns of structural change in intermediates within goods sectors implies less complementarity than the Leontief specification.[21]

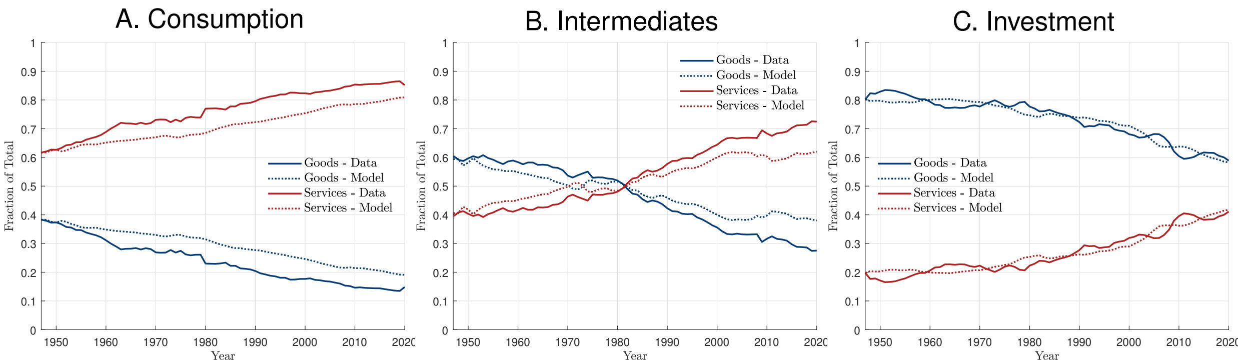
5.4. Structural Change

Given our calibration, we present the model's performance in accounting for patterns of structural change in Figure 5, taking as given measured price trends. Figure 5 presents the economy-wide goods and services expenditure shares in intermediates, constructed using the expenditure shares for each of our six sectors, aggregated according to equation (13).[22] We present the model fit for each of the six sectors' intermediates expenditure patterns and additional computational details in Appendix C.

[21] We have also explored estimating investment aggregation elasticity parameters separately for each of our six sectors, and find that goods and services inputs to investment are substitutes in each sector, albeit with slightly lower elasticity values for services sectors. These results are available upon request.

[22] Because we are considering the balanced growth equilibrium with common production parameters across sectors, each sector's share of total intermediate expenditures, $\frac{P_{jt}^M M_{jt}}{\sum_k P_{kt}^M M_{kt}}$, is equal to that sector's share of aggregate gross output. For consistency with the model, we aggregate sector-specific services expenditures shares using gross output weights; this generates slightly different empirical patterns of structural change than seen in Figure 3.

Figure 5: Model Calibration Fit to Structural Change Patterns in Consumption, Intermediates and Investment, 1947-2020



Notes: Each plots the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue lines) and the services sector (red lines). Data series are solid lines; model series are dashed lines.

Overall, the calibrated model provides a good fit to patterns of structural change, though a few comments are warranted. First, the model series for structural change in consumption does not match the entire rise in the share of services. This is perhaps unsurprising, given that household preferences in our model do not feature any income effects, which are commonly argued to be important for explaining structural change in consumption (e.g., Boppart, 2014; Comin et al., 2021). Given that income effects may be most important for explaining the declining role of agriculture among goods sectors, it is notable that the greatest departure between the model and the data series occurs in the earliest part of the sample, when the decline in agricultural consumption was the largest.

Second, the model closely matches the long-run patterns in structural change in investment. Given that our model abstracts from adjustment costs and uncertainty about the future price of investment, it is not surprising that the model does not generate the short and medium-run dynamics observed in the data.

Finally, the model reproduces the majority (approx. 2/3) of the rising share of intermediates produced by services. The model fit is even stronger before 2009, explaining roughly 80% of the overall increase, but fails to capture a substantial portion of the increased share of services intermediates after 2009.[23] We show in Appendix C that the model results reproduce the contribution of within-sector and between-sector forces to structural change in intermediates. While the discrepancy in the aggregate may reflect the presence of income effects (i.e., scale effects) in intermediates structural change, the lack of perfect fit to the data may

[23] As shown in Appendix C, the change in model fit from 2009-2020 mostly occurs within goods sectors; the model consistently accounts for roughly 2/3 of the rising services share throughout the sample.

instead reflect the data limitations regarding intermediates prices. Thus, given the challenges in measuring intermediates prices, the model does a good job of capturing the overall pattern of structural change in intermediates.

6. Growth Accounting with Changing Production Networks

Using our model calibration, we conduct two sets of growth accounting exercises to study the role of changing production networks in accounting for the U.S. growth experience since 1947. First, we decompose the evolution of aggregate TFP along the ABGP, denoted $\mathcal{A}_t$ in Section 4, analyzing how the aggregate growth contribution of technical change in the production of investment and intermediates evolves over the postwar sample. Second, we characterize and decompose the growth rate of aggregate GDP measured in a way that is more consistent with national accounting conventions (as opposed to measuring GDP in units of investment). We find that growth in this GDP measure is primarily driven by a combination of a Domar-weighted average of sectoral TFP growth (as per Hulten's Theorem, 1978) and growth in $\mathcal{A}_t$. While this characterization does require assumptions 1 and 2, it does not impose that $\mathcal{A}_t$ grow at a constant rate, allowing us to study not only the composition of aggregate growth over the postwar period but also what forces account for its slowdown in the 1970s and since the year 2000.

6.1. Measuring Technology

Before considering any growth accounting exercises, we must measure the exogenous processes of technical change in the model. Specifically, there are two types of exogenous technical change to be measured: exogenous technical change in each of the six sectors' production technology (sectoral TFP) and the exogenous technical change in the bundling technology (bundling TFP) for intermediate and investment inputs. We describe how we measure each of these in turn. Additional details are available in Appendix D.

First, we quantify growth in intermediates bundling TFP using a log first-order approximation of the equilibrium intermediate input price (equation (8)), for years t and $t-1$.[24] When log-linearized around the average expenditure share in these two years, the resulting Tornqvist index can be rearranged to yield the

[24] Herrendorf et al. (2021) use a similar approach to measuring exogenous investment TFP in their paper.

following expression for growth in intermediates bundling TFP for sector i:

$$\Delta \ln(A_{it}^{M}) = -\left(\Delta \ln(P_{it}^{M}) - \sum_{j=g-m,s-m} \left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^{M}M_{it}}}\right) \Delta \ln(P_{jt})\right), \tag{27}$$

where $g-m$ and $s-m$ are the goods-intermediates and services-intermediates sectors, $\left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^{M}M_{it}}}\right)$ is the average between t and $t-1$ of the share of intermediate spending by sector i purchased from sector j, $\Delta \ln(P_{it}^{M})$ is the price growth in the bundle of intermediates purchased by sector i, and $\Delta \ln(P_{jt})$ is the price growth in intermediates produced by sector j. We measure $\Delta \ln(A_{it}^{M})$ using BEA GDP by Industry data on the price of intermediate bundles by sector, aggregated to the 6 sector level.

The appeal of this approach to measuring intermediates bundling TFP is that it is measured independently of the model's calibrated parameters and fit. However, constructing intermediates bundling TFP using equation (8) generates nearly identical results. Furthermore, we expect growth in intermediates bundling TFP to be small, because measuring bundling TFP as a residual of observed intermediates bundle prices implies that the resulting series reflects additional heterogeneity (or mismeasurement) in intermediate input bundle prices beyond the weighted average of measured prices of intermediates produced by goods or services.[25]

Second, we can construct a series for growth in investment bundling TFP using an analogous procedure, building on equation (9). However, because we use the same source data to construct the price of the investment bundle as we do to measure investment prices produced by each sector, there is approximately zero growth in investment-bundling TFP. Thus, we set investment bundling TFP to be constant.[26]

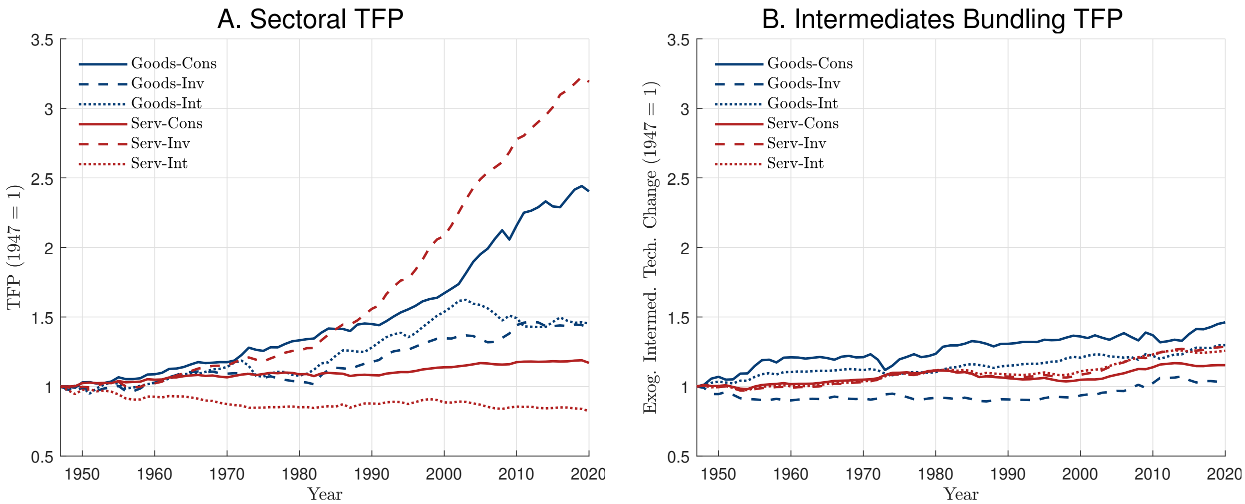
Finally, we measure sectoral TFP to be consistent with relative prices in the model, similar to García-Santana et al. (2021). That is, for five of our six sectors, we invert equation (15) and back out the sectoral TFP series that generate the relative price series observed in the data.[27] Since relative prices can only define technical change for all but one of our sectors, we construct the TFP series for the services-consumption

[25] The BEA Intermediate Inputs Bundle prices are generally constructed using a weighted average of sectoral gross output prices. Thus, there may be aggregation bias and measurement error in these prices.

[26] We have also explored measuring heterogeneity in investment-bundling TFP across sectors should assumption 2 be relaxed. There is little growth in investment-bundling TFP in each sector. Results are available upon request.

[27] To do this, we use the observed sectoral prices, P_{it}, and construct the price of the bundle of intermediate inputs implied by the model in equation (8), P_{it}^{M}, given these prices and the calibrated parameters reported in Table 3.

Figure 6: Technological Change by Sector, 1947-2020



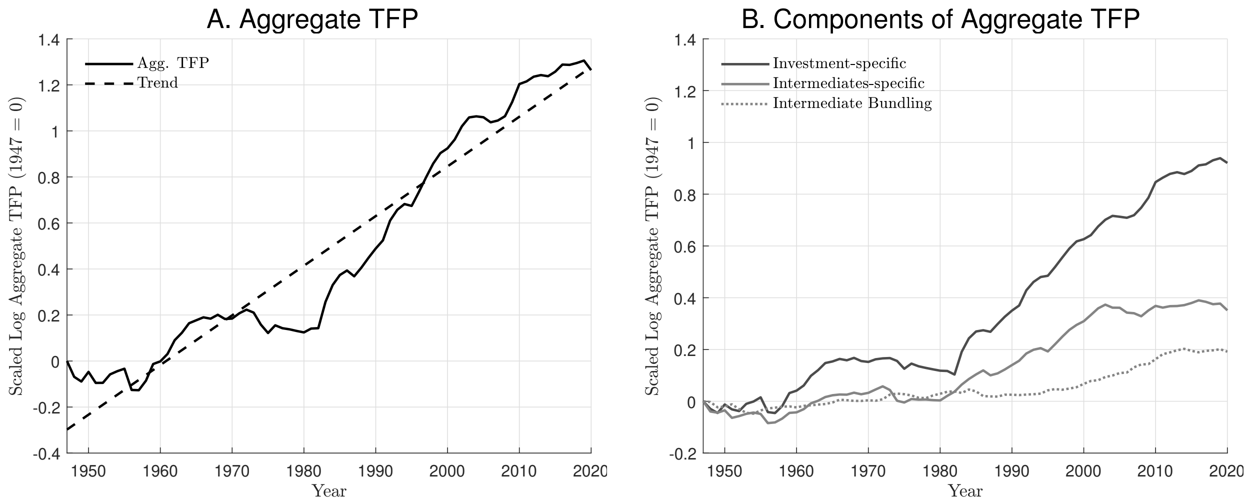
Notes: Panel A shows the time series of sectoral TFP for each sector, A_{it}; panel B shows the time series of intermediates-bundling technical change, A^M_{it}.

sector as a weighted average of Solow residuals from all 40 sectors in our data, where the weights correspond to each sector's share in producing services consumption as a final commodity. To construct the Solow residual for each of our 40 sectors, we follow the approach of vom Lehn and Winberry (2022) and compute real gross output net of the primary inputs in log differences.

Normalizing the level of all TFP terms to be 1 in 1947, Figure 6 displays sectoral TFP (panel A) and intermediates bundling TFP (panel B) for each sector over time. Given that sectoral TFP is calibrated using relative price data, it is unsurprising that growth in sectoral TFP illustrated in panel A of Figure 6 follows nearly the opposite ranking of growth in each sector's observed prices (panel A of Figure 4). That said, because the price of intermediates produced by services is growing faster than that produced by goods and because there is significant homophily in the input-output network (as seen in Section 2), the underlying technology growth in services sectors is faster than what is observed with relative prices alone. This explains, for example, why technical change in services-investment is significantly higher than in goods-consumption, despite the two sectors showing very similar price patterns in panel A of Figure 4. This highlights the importance of accounting for underlying production networks in accurately identifying sectoral TFP growth.

On average, intermediates bundling TFP growth is low (0.3% a year, averaged across sectors), and there

Figure 7: Aggregate Technological Change and Its Composition, 1947-2020



Notes: Panel A shows the time series of aggregate TFP, $\mathcal{A}_t$, in logs (normalized to zero in 1947), with a linear trend line drawn through it; panel B shows the counterfactual evolution of aggregate TFP for three cases: (1) only technological change among investment producers ("investment-specific technical change"), (2) only technological change in intermediates ("intermediates-specific technical change"), both at intermediates producers and from intermediates bundling, and (3) only intermediates-bundling technical change. For comparison to later results, all series are scaled by the coefficient $\frac{1}{1-\theta}$ (which is how aggregate TFP growth matters for aggregate GDP growth along the balanced growth path, as described in Proposition 1).

is little heterogeneity in this growth across sectors (see panel B of Figure 4).

6.2. Growth Accounting along the ABGP

Using our series for sectoral TFP and technical change in intermediates bundling, we compute the time series for aggregate TFP, $\mathcal{A}_t$, as defined in Lemma 2. The growth patterns of the resulting series are illustrated in panel A of Figure 7. Given that GDP grows in proportion to $\mathcal{A}_t^{\frac{1}{1-\theta}}$, we find it convenient to plot $\frac{1}{1-\theta} \ln \mathcal{A}_t$, facilitating easy comparison with our simulations for GDP growth in Section 6.3. Similar to the aggregate technical change series constructed by Herrendorf et al. (2021), the long-run growth rate of $\mathcal{A}_t$ is approximately constant, although with significant medium-run fluctuations, most notably during the 1970s—potentially induced by the real consequences of oil volatility of that decade.

In panel B of Figure 7, we illustrate the growth patterns of three counterfactual series for $\mathcal{A}_t$, which are constructed analogously to those in panel A, but under alternative paths for the underlying TFP processes. First, we construct $\mathcal{A}_t$ only using sectoral TFP growth in the goods and services sectors producing investment ("investment-specific technical change"), holding all other technical change series fixed at their initial

Table 4: TFP Growth Decomposition, 1947-2019

	Scaled Aggregate TFP Growth: $\Delta \ln(x) = \frac{1}{1-\theta} \Delta \ln \mathcal{A}_t$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%
All	1.31	100	0.13	100	0.80	100	0.38	100
Investment-Specific	0.94	72	0.08	61	0.51	63	0.31	82
Intermediates-Specific	0.38	29	0.05	37	0.31	38	0.07	18
Intermed. Bundling	0.20	15	0.05	42	0.04	5	0.13	35

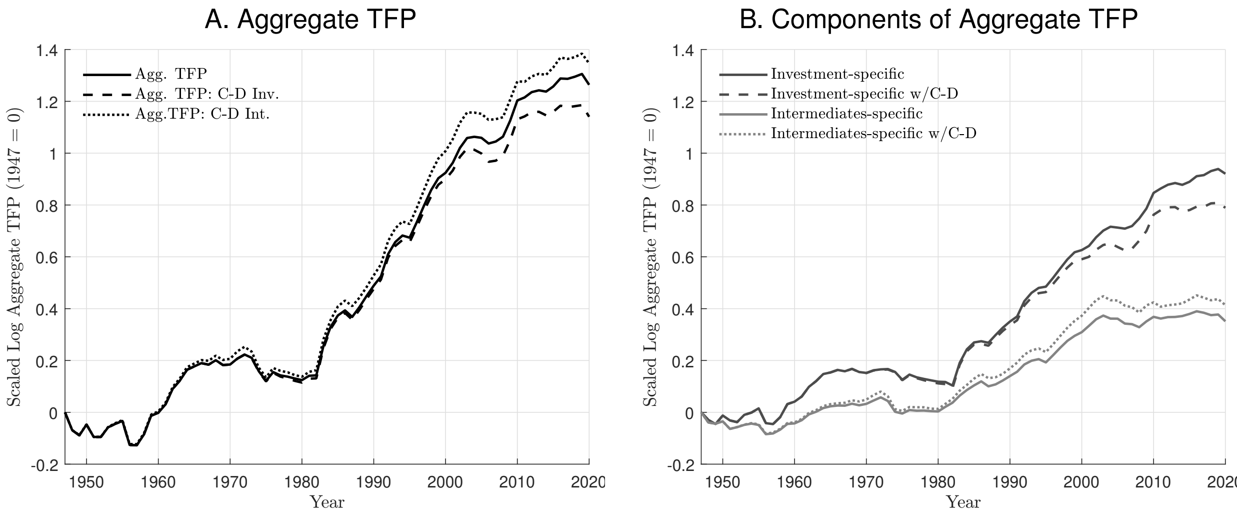
Notes: The table shows long-run log changes in scaled aggregate TFP, $\frac{1}{1-\theta} \Delta \ln(\mathcal{A}_t)$, across different periods for four alternative simulations: (1) the full model simulation with all measured TFP series; three counterfactual simulations with technological change respectively stemming only from (2) sectoral TFP growth among investment producers ("investment-specific technical change"), (3) sectoral TFP growth among intermediates producers and intermediates-bundling technical change in each sector ("intermediates-specific technical change"), and (4) intermediates-bundling technical change. Counterfactual changes are also expressed as a percent of the change from the full model simulation; these may not exactly sum to 1, given the nonlinear relationships between individual technology series and the aggregates. For each period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

values. Second, we construct $\mathcal{A}_t$ using sectoral TFP growth in the goods and services sectors producing intermediates and technical change in intermediates bundling ("intermediates-specific technical change"), while holding fixed sectoral TFP for sectors producing investment. Finally, we construct $\mathcal{A}_t$ only using technical change in intermediates bundling. Table 4 presents the corresponding numerical growth decomposition both for the entire sample and for three (approximately) twenty-year intervals beginning in 1960.

We make three observations about the contributions of investment- and intermediates-specific technical change to aggregate TFP growth. First, both sources of technical change significantly contribute to aggregate productivity growth. From Table 4, we see that over the entire sample of 1947-2019, investment-specific technical change contributes approximately 70% of aggregate TFP growth while intermediates-specific technical change accounts for approximately 30%.

Second, for most of the sample, intermediates bundling technical change only contributes a modest amount to aggregate TFP growth, consistent with the small amount of growth observed in these series (panel B of Figure 6). Table 4 shows that this source of technical change contributes about 15% of total TFP growth in the postwar period. However, it has been rising in importance over time, accounting for 35% of productivity growth since the year 2000. Furthermore, since intermediates bundling technical change is larger than total intermediates-specific technical change since 2000, the contribution of the endogenous

Figure 8: Aggregate TFP Growth, Cobb-Douglas Counterfactuals, 1947-2020



Notes: Panel A shows the time series of aggregate TFP, $\mathcal{A}_t$, in logs (normalized to zero in 1947), with two counterfactuals: one where the aggregation of investment inputs is Cobb-Douglas and one where the aggregate of intermediates inputs is Cobb-Douglas. Panel B shows the evolution of the components of aggregate TFP (either investment-specific technical change or intermediates-specific technical change) and how these change when aggregation is Cobb-Douglas in nature. When analyzing aggregate TFP growth from only investment, we only impose that investment aggregation is Cobb-Douglas and when analyzing aggregate TFP growth from only intermediates, we only assume that intermediates technical change is Cobb-Douglas. For comparison to later results, all series are scaled by the coefficient $\frac{1}{1-\theta}$, highlighting how aggregate TFP growth matters for aggregate GDP growth along the balanced growth path, as described in Proposition 1.

components of technical change in the production of intermediates is negative over this period.

Third, the importance of investment-specific technical change has been rising over time. Table 4 shows that this source of technical change accounts for up to 63% of aggregate productivity growth before 2000, but accounts for about 82% of aggregate productivity growth since 2000; in Appendix D, we show that this result still holds when analyzing productivity growth decade by decade. Furthermore, Figure 7 and Table 4 highlight that intermediates-specific technical change effectively stagnates after 2000. We revisit this observation when discussing the recent growth slowdown in Section 6.3.

As discussed in Section 4.3, the rising importance of investment-specific technical change could either reflect changing growth rates of underlying productivity series or the endogenous reallocation of resources across sectors. To explore the importance of endogenous reallocation, we consider a set of counterfactuals in which the investment or intermediates bundling technologies are Cobb-Douglas. This specification implies unitary elasticities of substitution and fixed expenditure shares, ruling out structural change.[28] Figure 8

[28] We take the limit as the elasticity of substitution goes to 1, using the calibrated values for the share parameters in the CES

Table 5: TFP Growth Decomposition, Cobb-Douglas Counterfactuals, 1947-2019

	Scaled Aggregate TFP Growth: $\Delta \ln(x) = \frac{1}{1-\theta} \Delta \ln \mathcal{A}_t$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%
A. All Sources of TFP Growth								
Baseline	1.31	100	0.13	100	0.80	100	0.38	100
Investment Cobb-Douglas	1.19	91	0.12	92	0.78	98	0.29	75
Intermediates Cobb-Douglas	1.38	106	0.13	106	0.87	109	0.38	99
B. Investment-Specific Technical Change								
Baseline	0.94	100	0.08	100	0.51	100	0.31	100
Investment Cobb-Douglas	0.81	86	0.07	91	0.48	94	0.22	70
C. Intermediates-Specific Technical Change								
Baseline	0.38	100	0.05	100	0.31	100	0.07	100
Intermediates Cobb-Douglas	0.44	116	0.05	113	0.36	118	0.06	94

Notes: The table reports log changes in scaled aggregate TFP, $\frac{1}{1-\theta} \Delta \ln(\mathcal{A}_t)$, across different periods, and log changes in investment-specific and intermediates-specific technical change. The table also reports counterfactuals for the cases where either investment or intermediates aggregation is Cobb-Douglas, ruling out structural change in that network. For each period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

presents the resulting counterfactual aggregate TFP series, illustrating how much of aggregate growth occurs when the corresponding production network is held fixed. The contribution of reallocation to aggregate productivity growth due to non-unitary elasticities of substitution is 100% minus the percent contributions reported in the second column of each panel in Table 5.

Figure 8 and Table 5 show that aggregate productivity growth is different under the Cobb-Douglas counterfactuals. When the aggregation of investment inputs is Cobb-Douglas, meaning that resources no longer reallocate to the fastest growing sector, over the entire postwar period, investment-specific technical change is 14% lower (panel B), and aggregate TFP growth is 9% lower (panel A). In contrast, when the aggregation of intermediates is Cobb-Douglas for each sector, intermediates-specific technical change is 16% higher (panel C) and aggregate TFP growth is 6% higher (panel A).

aggregator as the Cobb-Douglas exponents. When intermediates aggregation is Cobb-Douglas, we also recalibrate technology to match the relative prices observed in the data. Results without recalibrating are similar to those in Figure 8.

The importance of reallocation for productivity growth is more pronounced in recent years. Investment-specific technical change is 30% lower from 2000-2019 under the Cobb-Douglas counterfactual (panel B) and aggregate TFP growth is 25% lower (panel A), suggesting that the substitutability of investment inputs is important in accounting for recent aggregate growth. In contrast, the importance of reallocation forces in intermediates for economic growth is almost zero in the last 20 years.

6.3. GDP Growth Accounting off the ABGP

One potential shortcoming of the above analysis is that aggregate TFP, $\mathcal{A}_t$, corresponds to the aggregate growth rate of GDP only when GDP is measured in units of the numeraire—aggregate investment. As summarized in Lemma 2, measuring GDP this way eliminates any role for technical change in the production of consumption in aggregate growth. Aggregate growth only depends on direct technical change in the production of investment or on indirect technical change among the intermediate input suppliers to investment producers. Alternatively, we could choose aggregate consumption as the numeraire (as in Greenwood et al., 1997, or Foerster et al., 2019), but then any aggregate balanced growth path would be inconsistent with the Kaldor facts (see Duernecker et al., 2021). However, defining aggregate GDP in units of consumption or investment as the numeraire is inconsistent with how GDP is measured in national accounts, which utilize an index number based on growth in all subcomponents of GDP.

We develop an index-number representation of GDP that is more comparable to empirical measures of GDP in national accounting data. To do so, we define aggregate GDP in our model as a Tornqvist index, Y_t^{Index}, with aggregate GDP growth given by

$$\Delta \ln(Y_t^{Index}) = \sum_i \overline{\frac{P_{it}^V V_{it}}{P_t^Y Y_t}} \Delta \ln(V_{it}), \tag{28}$$

where $\overline{\frac{P_{it}^Y V_{it}}{P_t^Y V_t}}$ is the average share of aggregate GDP from sector i in years t and $t-1$ and $\Delta \ln(V_{it})$ is real value added growth in sector i between t and $t-1$.[29]

Given that sectoral value added in our model is given by $V_{it} = A_{it}^{\frac{1}{\alpha_i}} K_{it}^{\theta_i} L_{it}^{1-\theta_i}$, assumptions 1 and 2 and

[29] The U.S. national accounting system uses a Fisher ideal index number instead of a Tornqvist index. The two indices produce nearly identical results for U.S. data, and the Tornqvist index provides an expression for aggregate growth that is easier to analyze theoretically.

the results of Lemma 2 (but no further balanced growth restrictions) imply that we can rewrite the above expression for GDP as:

$$
\begin{aligned}
\Delta \ln(Y_t^{Index}) &= \sum_i \overline{\frac{P_{it}^V V_{it}}{P_t^Y Y_t}} \Delta \ln(V_{it}) \\
&= \sum_i \alpha \overline{\frac{P_{it} Q_{it}}{P_t^Y Y_t}} \left(\frac{1}{\alpha} \Delta \ln(A_{it}) + \theta \Delta \ln(K_{it}/L_{it}) + \Delta \ln(L_{it}) \right) \\
&\approx \sum_i \left(\overline{\frac{P_{it} Q_{it}}{P_t^Y Y_t}} \Delta \ln(A_{it}) \right) + \theta \Delta \ln(K_t) \\
&\approx \underbrace{\sum_i \left(\overline{\frac{P_{it} Q_{it}}{P_t^Y Y_t}} \Delta \ln(A_{it}) \right)}_{\text{(A) Hulten's Theorem term}} + \underbrace{\frac{\theta}{1-\theta} \left(\Delta \ln(\mathcal{A}_t) - \Delta \ln(R_t) \right)}_{\text{(B) Aggregate capital growth}} \qquad (29)
\end{aligned}
$$

The approximation in the above expression follows from the approximation $\sum_i \overline{\frac{P_{it}^V V_{it}}{P_t^Y Y_t}} \Delta \ln(L_{it}) \approx 0$, consistent with our assumption of a fixed aggregate labor supply.

Equation (29) decomposes aggregate GDP growth into two terms: (A) a "Hulten's Theorem" term (Hulten, 1978; Baqaee and Farhi, 2019), where productivity growth in each sector is aggregated proportional to its Domar weight (sectoral gross output divided by GDP), and (B) the growth rate of aggregate capital, which is given by the difference between the growth rate of aggregate TFP, $\mathcal{A}_t$, and the growth rate of the rental rate of capital, R_t.

Equation (29) holds for all equilibrium paths, not just balanced growth paths.[30] For example, an ABGP requires the additional assumption of constant aggregate TFP growth, which implies that the growth in the rental rate of capital is zero. Proposition 1 does not imply that growth in the aggregate GDP index, $\Delta \ln(Y_t^{Index})$, need be constant; constant aggregate growth in the GDP index would require the additional assumption that the Hulten's Theorem term also grows at a constant rate. As highlighted by expression (29),

[30] An interesting question is how subsequent results change if we analyze GDP as an index number without imposing assumptions 1 or 2, requiring a common production structure and a single type of capital. Absent these assumptions, such an analysis would require numerical simulation along a transition path. We expect that results of such an exercise would not yield materially different results for two reasons. First, the evidence presented in Section 2 suggests that 2 is approximately satisfied in the U.S., given the high degree of similarity in the composition of investment purchases across all sectors. Second, as we show in Appendix C, if we construct production parameters for each of the six sectors in our study, capital share parameters are fairly similar across sectors, suggesting a limited role for heterogeneity. The one notable source of heterogeneity worth studying further would be that intermediate inputs receive a much higher Cobb-Douglas weight in goods sectors than in services sectors. For example, as argued in Moro (2015), this observation coupled with structural change would imply a lower aggregate growth rate with a reduction in the input-output multiplier as the economy transitions from goods to services.

Hulten's Theorem does not hold in our model because aggregate capital and aggregate GDP can potentially grow at different rates. This is likely because the production of aggregate capital uses inputs from a different mix of sectors than aggregate GDP and aggregate capital growth is impacted by intermediates-bundling technical change, which is not present in the Hulten's Theorem term.

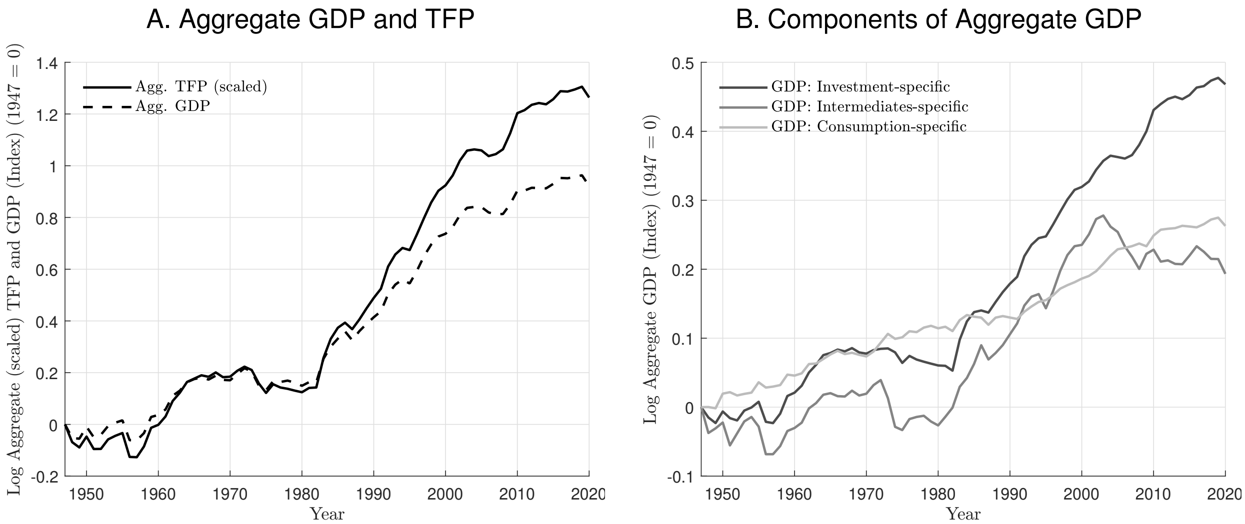
Given this more empirically consistent expression for aggregate GDP growth, we now revisit our decomposition of aggregate growth in Figure 7 and Table 4. We do so under the additional assumption that the change in the rental rate of capital is zero, which is empirically consistent with the U.S. experience over a long horizon (see for example Jones, 2016). We also update our definitions of investment-specific technical change and intermediates-specific technical change by scaling aggregate TFP growth (with only technical change in investment or intermediates) as in equation (29) and by adding in the relevant component of the Hulten's Theorem term (weighted TFP growth in sectors producing either investment or intermediates).[31] In addition to the two sources of technical change considered in Section 6.2 we also define "consumption-specific technical change" as the sum of sectoral TFP growth in goods and services sectors producing consumption, weighted by their Domar weights.

Figure 9 and Table 6 summarize our results. Panel A of Figure 9 shows that the two alternative GDP measures—log of the GDP index and scaled aggregate TFP—track each other closely through about the year 1990, after which the index-number measure of GDP observes a substantial growth slowdown relative to our aggregate TFP series.

To understand the evolution of growth in the GDP index and its slowdown over time, panel B of Figure 9 and Table 6 document how technical change in consumption, investment, and intermediates contributes to growth in the GDP index over time. Given that consumption-specific technical change now contributes to aggregate growth and that intermediates-specific technical change receives extra weight in the GDP index (compared to aggregate TFP), the overall fraction of GDP growth accounted for by investment-specific technical change is smaller; 50% over the post-war period. However, the importance of investment-specific technical change is still rising over time, accounting for 70% of growth post-2000 compared to 35-44% from 1960-2000. In Appendix D, we show that a significant fraction (approximately 20%) of aggregate

[31] Given Assumptions 1 and 2, the share of consumption, investment, and intermediates in aggregate gross output will be constant over time; thus, the Domar weights on each sector's productivity growth will only change over time due to structural change within consumption, investment, and intermediates.

Figure 9: Aggregate GDP Growth (Index Number) and Its Composition, 1947-2020



Notes: Panel A shows the time series of aggregate GDP measured as an Index number, compared to aggregate GDP measured in units of aggregate investment, both are measured in logs (normalized to zero in 1947); panel B shows counterfactual evolutions of aggregate GDP for three cases: (1) only technological change among investment producers ("Investment-specific"), (2) only technological change in intermediates, both at intermediates producers and from intermediates bundling technical change ("Intermediates-specific"), and (3) only technological change among consumption producers ("Consumption-specific").

Table 6: GDP Growth Decomposition, 1947-2019

	Aggregate GDP (Index) Growth: $\Delta\ln(x) = \Delta\ln(Y_t^{Index})$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%
All	0.96	100	0.11	100	0.59	100	0.23	100
Investment-Specific	0.48	50	0.04	35	0.26	44	0.16	70
Intermediates-Specific	0.21	22	0.00	3	0.26	45	-0.02	-9
Consumption-Specific	0.27	29	0.07	61	0.07	12	0.09	39

Notes: The table shows long-run log changes in aggregate GDP measured as an index number, $\Delta\ln(Y_t^{Index})$ (following equation (29)), across different periods for four alternative simulations: (1) the full model simulation with all measured TFP series; three counterfactual simulations with technological change respectively stemming only from (2) investment producers ("investment-specific technical change"), (3) intermediates producers and exogenous intermediates-bundling technical change ("intermediates-specific technical change"), and (4) consumption producers ("consumption-specific technical change"). Counterfactual changes are also expressed as a percent of the change from the full model simulation; these may not exactly sum to 1, given the nonlinear relationships between individual technology series and the aggregates. For each time period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

GDP growth post-2000 comes from reallocation forces in investment due to a non-unitary elasticity of substitution. Thus, absent endogenous reallocation of investment production to services sectors, the aggregate slowdown since 2000 would have been even worse. Interestingly, the more pronounced slowdown in aggregate GDP growth, especially post-2000, appears to be driven by stagnating intermediates-specific technical change.

This leads us to three observations that are potentially useful for the ongoing debate over the sources of slowing growth during the 1970s and since 2000, documented in a number of countries including the United States (see Syverson, 2017, for a recent review). First, growth in consumption-specific technical change is stable throughout the entire period since 1947; this does not appear to be a source of growth slowdown in any time period. Second, slowing productivity growth in both production networks accounts for the slowdown in aggregate growth during the 1960s and 1970s. Third, the slowdown after 2000 is primarily attributable to stagnating intermediates-specific technical change. In contrast to the arguments by Gordon (2016), this suggests that the recent slowdown in aggregate growth post-2000 may have different underlying causes than the slowdowns observed in earlier decades. Thus, future work aimed at understanding the origins of the slowdown in intermediates-technical change post-2000 is important for understanding the recent slowdown in aggregate growth.

7. Conclusion

This paper studies the intersection of structural change in production networks and economic growth. We document that the sectoral distribution of production in both intermediate and investment production networks has evolved, with services sectors producing a larger share of both intermediate and investment. To understand these patterns, we develop a framework that allows us to study structural change in both consumption and these production networks.

Explicitly modeling intermediates allows us to use final-expenditure prices, rather than gross output prices, to discipline the model in a way that is internally consistent (as discussed by Herrendorf et al., 2014). Specifically, we construct novel price series for goods and services, split by their use as final consumption, intermediates, or investment. Together with our stylized fact on the rising services share within investment production, these disaggregated price series imply that goods and services are substitutes in the production of investment, rather than complements, as found in previous studies.

This finding has important implications for economic growth and provides useful insights regarding a concern initially brought up by Baumol (1972)—that structural change leads to a systematic reallocation from productive/innovative goods sectors to less innovative services sectors, eventually leading to an economy where the least productive sector dictates all economic progress. While the intermediates network in our framework does indeed appear to suffer from Baumol's "cost disease", we find the investment network to be the primary engine of growth precisely *because* it is systematically shifting toward the most productive *services* (e.g., software development and R&D). Intuitively, complementarity in intermediates production leads to "bottleneck" growth, governed by the least productive sector, while substitutability in investment production generates "frontier growth", driven by the most productive sector. Our findings, similar to Duernecker, Herrendorf and Valentinyi (2017), thus provide some optimism for the impact of sectoral reallocation on aggregate economic growth.

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Appendix A. Measurement Details and Additional Empirical Results on Structural Change

Appendix A.1. Measurement Details

Our primary data source for measuring production networks and related data is the BEA Input-Output database, specifically, the time series of Make and Use tables from 1947-2020 and the time series of the investment network generated in vom Lehn and Winberry (2022). The Make and Use tables from the BEA Input Output database can be downloaded at the BEA's website here: `https://www.bea.gov/industry/input-output-accounts-data`; data from vom Lehn and Winberry (2022) containing the time series of the investment network can be found here: `https://doi.org/10.7910/DVN/CALDHX`. These data provide details for 40 NAICS-defined sectors of the economy, including agriculture and government (43 if energy/oil-intensive sectors are included); Table 1 lists each of the 40 sectors and their corresponding NAICS codes. More recent vintages of the BEA Input Output database allow for greater sectoral detail (and it is possible to construct more detailed investment networks for recent years), but given our interest in structural change over the long run, data on these 40 sectors are available since 1947.

We use the Make and Use tables from the BEA to measure input-output relationships in the following way. The core of the Use table is a square matrix that reports intermediate input expenditures by different sectors (organized along columns) on specific commodities (organized along rows). These commodities are named and assigned NAICS codes based on which sectors are major producers of the given commodity, but more than one sector may be involved in the production of a given commodity. The mix of sectors that produce a given amount of each commodity is observed in the Make table, which is a square matrix reporting commodities along columns and the amounts of each commodity produced by each sector along rows. The final input-output matrix in each year is the matrix product of a scaled Make table, where each column is scaled by its sum (thus summing to 1) and the unscaled Use table.

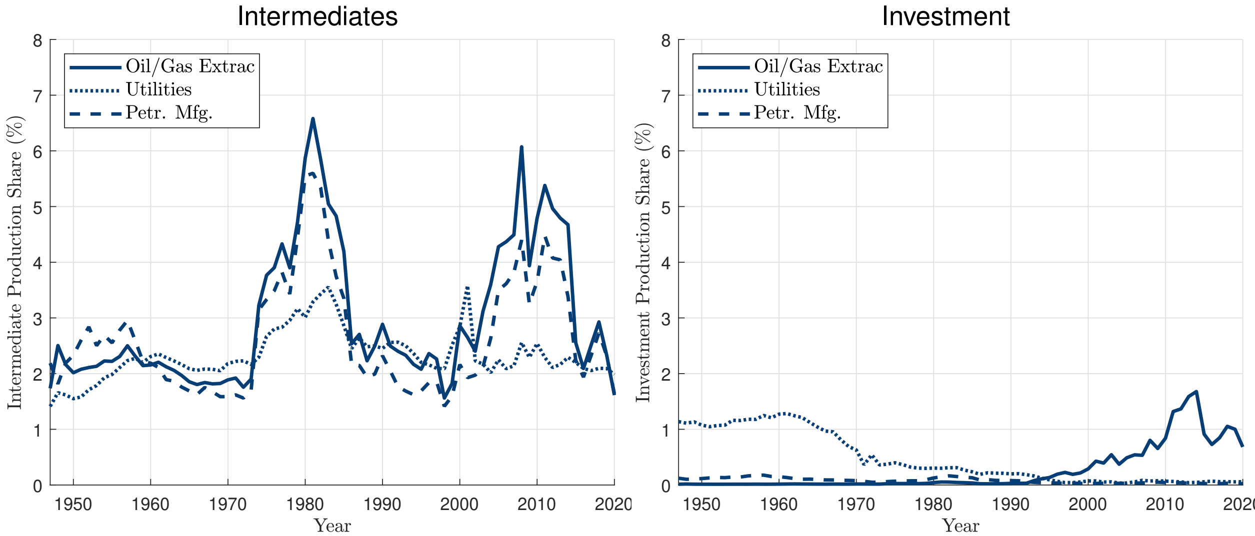
The investment network data from vom Lehn and Winberry (2022) reports the matrix of sectoral spending and production of new investment; see that paper for construction details. We follow the same approach to extend the investment network through the year 2020. However, the raw investment matrix from vom Lehn and Winberry (2022) is still organized with commodities along each row, not sectors, and needs to also be adjusted using the Make matrix. Thus, the final investment network data we use is the product of the scaled Make matrix and the unscaled investment network data from vom Lehn and Winberry (2022).

The Use tables from the BEA also contain information on the final uses of each commodity produced, including consumption. To measure structural change in consumption, we construct final consumption produced by each sector as the product of the scaled Make matrix and the private final consumption vector in the Use table. We do not include government consumption or make adjustments for imports/exports in computing final consumption, though this would only have a minor impact on our results. The Use table also has information on the final use of each commodity as new investment, however, we use sums of the data from the investment network (which is closely tied to this figure) to compute the total production of investment by each sector.

Appendix A.2. Additional Detail on Section 2 Results

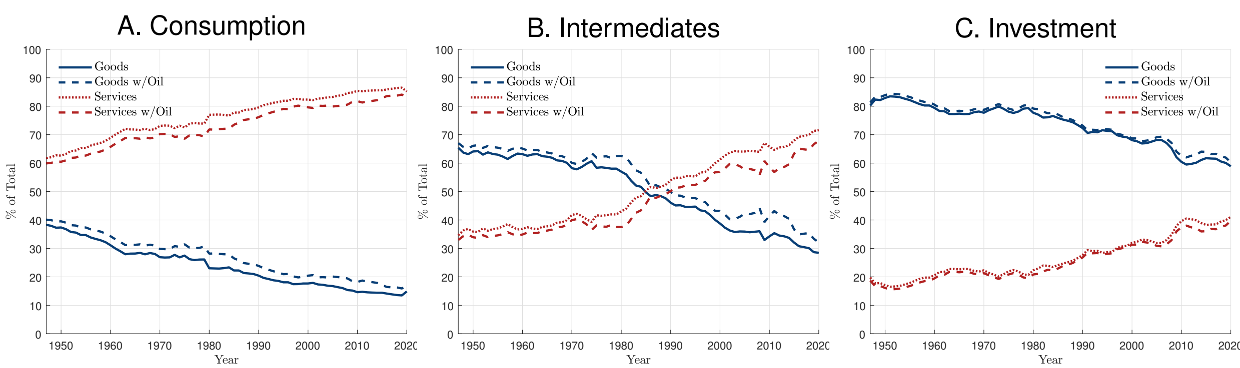
In this subsection, we report four additional empirical results: data on energy/oil-intensive sectors' contributions to intermediates and investment production over time, time series detail for sectors whose production share of investment or intermediates has increased or decreased the most, detail on how the services share of intermediates and investment is changing within all 40 sectors in our data, and the shift-share decomposition for the share of services in the production of investment or intermediates using data from all 40 sectors.

Figure A.1: Shares of Intermediates and Investment Produced by Oil-Intensive Sectors, 1947-2020



Notes: Panel A shows the share of total intermediates produced by oil-intensive sectors while panel B shows the share of investment produced by oil-intensive sectors.

Figure A.2: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, With and Without Oil-Intensive Sectors, 1947-2020

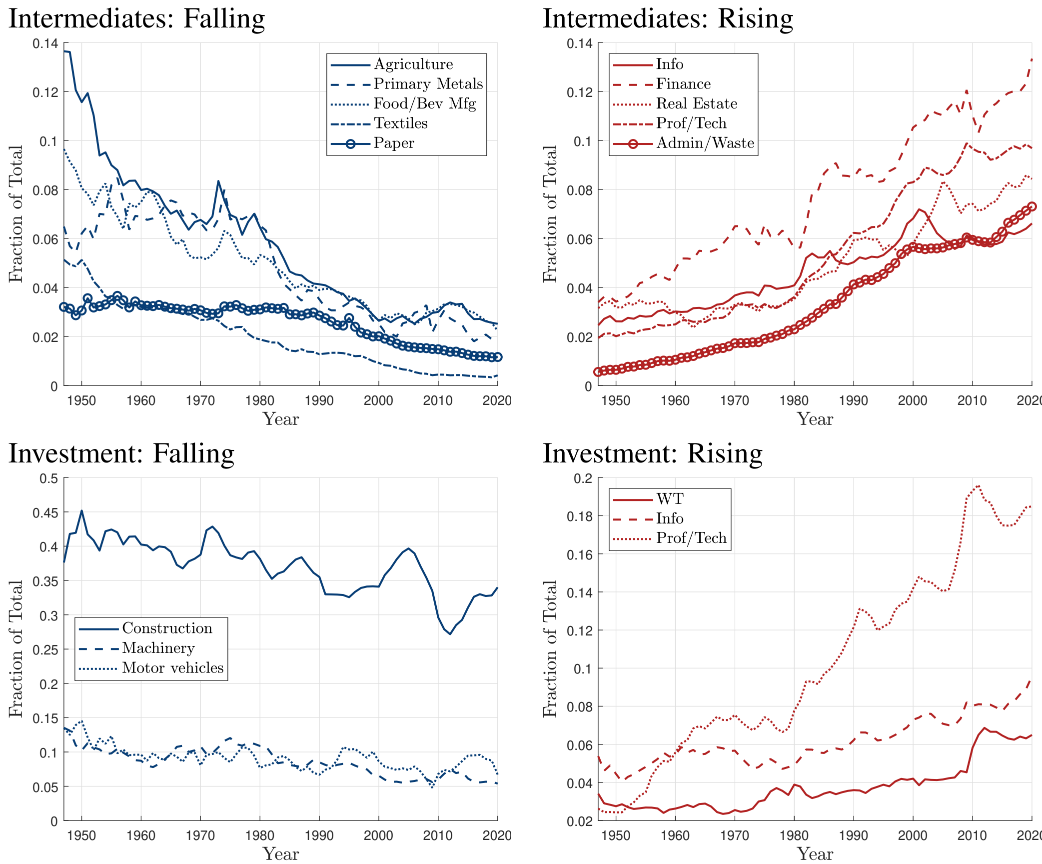


Notes: Each plots the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue lines) and the services sector (red lines). The dashed lines indicate these same fractions when oil-intensive sectors are included in the analysis (all part of the goods sector).

First, Figure A.1 reports the shares of intermediates and investment produced by each of the three sectors omitted from our analysis – oil/gas extraction, utilities, and petroleum manufacturing. Although there are large medium-run swings in these shares (particularly for intermediates), there is no long-run trend in how much these sectors produce of intermediates and only very slight (and off-setting) long-run trends in investment. Thus, as can be seen in Figure A.2, whether these sectors are included as part of the total goods sector or not, there are not large changes in the long-run structural change trends observed in consumption, intermediates, and investment.

Second, the four panels of Figure A.3 report the time series patterns of sectors whose share of production

Figure A.3: Time Series Changes in Production Share of Intermediates and Investment, Additional Sector Detail



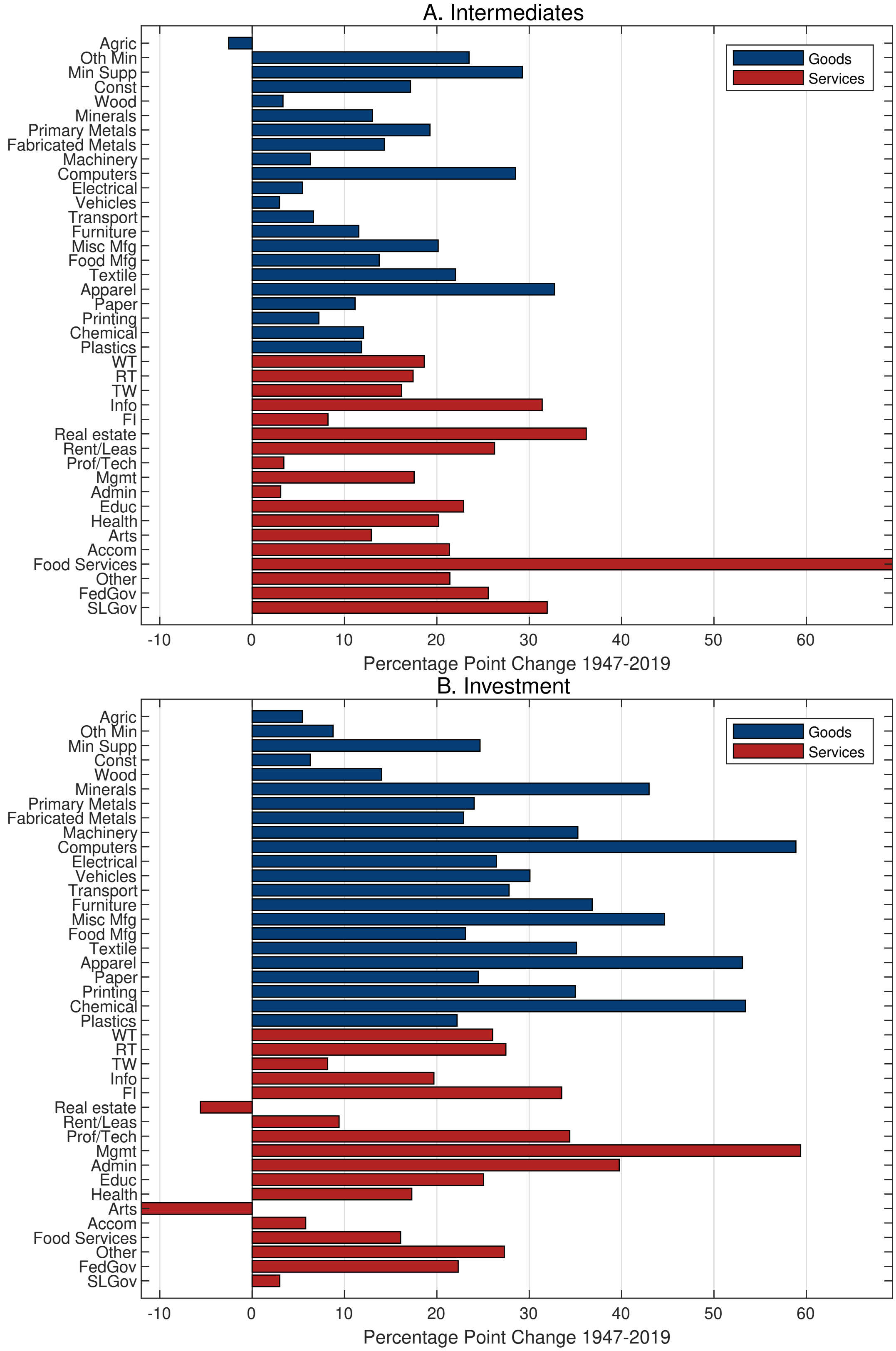
Notes: Each line represents a given sector's share of total production of intermediates (top rows) or investment (bottom rows). Right panels with red lines show sectors whose production share has increased the most; left panels with blue lines show sectors whose production share has decreased the most.

of intermediates or investment has increased the most (right panels) or decreased the most (left panels). As reported in Section 2, for intermediate goods, the largest increases in production share are in information services, finance/insurance, real estate, professional/technical services, and administrative support services; the largest decreases occurred in agriculture, primary metals, food and beverage manufacturing, textile manufacturing, and paper manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest decreases are in machinery, construction, and motor vehicle manufacturing. While there is certainly heterogeneity in the changes over time in each of these production shares, each sector's changes in production shares appear to be part of a gradual long-run trend and not some permanent spike occurring in a particular year.

Third, in Figure A.4, we present bar charts showing the change in the services share of production of intermediates (top panel) and investment (bottom panel) within all 40 sectors in our data between 1947 and 2019. Although there is significant heterogeneity in how much the services share of production of intermediates or investment has changed in each sector, it is increasing in the vast majority of sectors.

Finally, in Table A.1, we report the shift-share decomposition (Table 2) where instead of looking at

Figure A.4: Changes in Services Production Share of Intermediates and Investment Purchased by Each Sector: 1947-2019



Notes: Each bar represents the change in the services production share of intermediates (upper panel) or investment (lower panel) purchased by each sector between 1947 and 2019. Blue bars: goods sectors; red bars: services sectors.

Table A.1: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment, 40 Sector Detail

				Decomposition	
	1947	2019	Δ	within	between
Intermediates	0.35	0.71	0.37	0.18	0.19
				(49%)	(51%)
Investment	0.20	0.40	0.20	0.15	0.05
				(73%)	(27%)

Notes: The table reports the shift-share decomposition described in Table 2 for the production share of services, but with within-sector and between-sector changes across all 40 sectors in our data. Individual components may not exactly sum to totals due to rounding.

within and between sector changes for just 2 sectors, goods and services, we decompose the within and between sector changes in the services production share across all 40 sectors in our data. With this additional detail, the between component of changes in sectoral composition explains a larger fraction of the rise in the services share of production in both intermediates and investment. However, the vast majority of the rising services share in investment production is occurring within sectors (nearly 75%, as opposed to 100% in Table 2) and the within-sector component of the increase in the services share of intermediates production is about half.

Appendix A.3. Within-Sector Structural Transformation of Occupations

This appendix provides labor market evidence that the patterns of structural change we observe in the intermediates network are not driven by the outsourcing of labor services from goods sector establishments. The concern is that the growth in services in intermediates is driven by goods producing establishments outsourcing services tasks such as janitorial, legal, or accounting services to services establishments. If this were the primary driver of structural change in intermediates, the following two patterns would be prevalent in data. First, we would expect a decline in service task intensive occupations within goods producing industries. Second, we would expect that changes in service task intensity within sectors should be negatively correlated with changes in purchases of services intermediate inputs.

To investigate this concern, we utilize data from decennial U.S. Censuses for the years 1950-2010 and the 2019 American Community Survey (both provided by IPUMS 13.0) to document trends in the industry-specific concentration of service task intensive occupations. We consider service task intensive occupations as (1) managerial/professional/specialty, (2) technical/sales/admin, and (3) service occupations based on the IPUMS 13.0 `OCC1990` classification.

Using the IPUMS 13.0 `IND1990` industry aggregation, we construct 32 consistent sectors over the period 1950-2019 (following vom Lehn, 2018), as listed in Table A.2.[32] These sectors are a direct aggregation of the 40 NAICS 2007 sectors used in our main analysis and can therefore be directly compared over the entire sample. For each of these 32 sectors we construct two measures for the service-task intensity: the services employment share (total employment in service-task intensive occupations divided by total employ-

[32] IPUMS data does not allow us to distinguish the information services sector from the printing and publishing manufacturing sector throughout the postwar sample. We thus combine these two sectors into one and list it as a goods sector (though listing it as a services sector does not alter our results).

Table A.2: Change in Occupational Service Task Intensity Within Sectors, 1950-2019

	Perc. Pt. Ch.			Perc. Pt. Ch.	
Goods Producing Sectors (NAICS Codes)	Emp.	Earn.	Service Producing Sectors (NAICS Codes)	Emp.	Earn.
Ag./forestry/fishing/hunting (11)	-7.0	-8.6	Wholesale trade (42)	4.7	16.0
Mining, except oil and gas (212)	16.6	25.0	Retail trade (44-45)	5.7	9.2
Construction (23)	11.4	24.0	Transport and warehousing (48-49, minus 491)	9.7	20.2
Wood products (321)	9.3	21.3	Finance and insurance (52)	1.0	1.3
Non-metallic mineral products (327)	15.8	31.5	Real estate (531)	3.4	6.3
Primary and Fabricated Metals (331,332)	13.4	25.4	Prof./Tech./Rent./Mgmt/Admin. (54-56,532-533)	8.2	12.0
Machinery (333)	16.4	31.1	Educational services (61)	1.6	2.9
Computer and Electronic Mfg (335,334)	35.8	53.8	Health services (62)	2.3	4.2
Motor Vehicles Mfg (3361-3363)	12.9	30.3	Arts, ent. and rec. services (71)	5.3	4.4
Other transp. equipment (3364-3369)	28.9	43.8	Accommodation services (721)	1.5	3.3
Furniture and related MfG (337)	13.9	30.2	Food services (722)	-2.5	-2.4
Misc. manufacturing (339)	28.9	49.2	Other private services (81)	11.9	14.1
Food and Beverage Mfg (311-312)	8.0	23.3	Fed/State/Local Government (n/a, but incl. 491)	12.0	13.8
Textile manufacturing (313-314)	20.1	37.0			
Apparel manufacturing (315-316)	24.7	47.1			
Paper manufacturing (322)	11.5	25.4			
Printing and Information (51,323)	16.2	23.1			
Chemical manufacturing (325)	21.2	37.7			
Plastics and rubber products (326)	8.8	25.8			

Notes: The table reports the percentage point change in occupational employment and earnings share bteween 1950 and 2019 within 32 consistent sectors. The sectors are a direct aggregation of the 40 NAICS 2007 sectors listed in Table 1 to map into the IPUMS IND1990 classification as suggested by vom Lehn (2018). Service intensive occupations are (1) managerial/professional/specialty, (2) technical/sales/admin, and (3) service occupations based on the IPUMS 13.0 OCC1990 classification. Data on employment are taken from the U.S. Census obtained from IPUMS USA.

ment within the sector); and the services earnings share (total earnings of service-task intensive occupations divided by total earnings within the sector).

Table A.2 illustrates that employment and earnings shares of service intensive occupations within all but two (agriculture, and food services) of the 32 broad NAICS sectors have substantively increased over the period 1950-2019. This implies that the structural change patterns we document for the intermediates network do not coincide with a reduction in services workers in goods sectors.

An additional piece of suggestive evidence that outsourcing is not driving the patterns of structural change patterns in intermediates is that changes in occupational service-task intensity are not negatively correlated with changes in the purchases of intermediates produced by services sectors. To show this, we construct changes in our two measures of within-sector service intensity by decade and correlate these with changes in the each sector's share of intermediates expenditures produced by services. Specifically, we regress the change by decade in each measure of sectoral services task intensity on a constant, a full set of time effects (decade dummies), and the sectoral change by decade in the share of intermediates expenditures produced by services sectors.

Table A.3 summarizes the regression estimates, suggesting that within industry changes in the share of intermediates purchased from services appear to have no systematic correlation with concurrent changes in the service task intensity. If anything, while none of these correlations are statistically significant, the correlation for intermediates appears to be positive, rather than negative. This suggests that sectors experiencing more structural transformation in intermediates are not more likely to see a systematic decline of employment in service-task intensive occupations.

Table A.3: Changes in Service Task Intensity and Structural Change

	ΔService Task Intensity	
	Employment Share (1)	Earnings Share (2)
Δ Share of Intermediates Expenditures from Services	2.036 (3.207)	2.806 (4.680)
Obs.	224	224

Notes: The table reports results from linear regressions, where the left-hand side is the decadal change in the employment (column 1) or earnings share (column 2) of service task intensive occupations within 32 consistent NAICS sectors (listed in Table A.2). The regressors are a constant, a full set of time effects (coefficients not reported), and the decadal change in the share of intermediates purchased from services sectors. Standard errors are reported in parentheses underneath the coefficients. The decadal changes are based on the years 1949, 1959, 1969, 1979, 1989, 1999, 2009, 2018. Data on employment and earnings are taken from the decadal U.S. Censuses for the years 1950-2010. For the final decade we prefer the 2019 ACS rather than the 2020 U.S. Census, to avoid the impact of the COVID-19 pandemic. All U.S. Census data are taken from IPUMS 13.0. Earnings and employment are reported for the previous year in each survey. Service intensive occupations are (1) managerial/professional/specialty, (2) technical/sales/admin, (3) service occupations based on the IPUMS 13.0 OCC1990 classification. Standard errors are reported in parentheses and confidence levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Appendix A.4. Decomposing Value-Added Measures of Structural Change

We now analyze the importance of changes in the input-output network for "value-added" measures of structural change in consumption and investment. As explained in Herrendorf et al. (2013), a value-added approach to measuring sectoral production of consumption and investment focuses not only on the set of sectors producing the final product but also on the network of sectors contributing intermediate inputs needed to produce the product. Thus, value-added measures of structural change implicitly include structural change in intermediates and structural change in final producers of consumption and investment.

Value-added vectors of sectoral production of consumption and investment (denoted in current dollars), $\mathbf{c^{VA}}$ and $\mathbf{x^{VA}}$, are constructed using input-output data using the following equations:

$$\mathbf{c^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{-\mathbf{1}}\mathbf{c} \tag{A.1}$$

$$\mathbf{x^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{-\mathbf{1}}\mathbf{x} \tag{A.2}$$

where $\mathbf{c}$ and $\mathbf{x}$ are vectors of final production of consumption and investment, respectively, by each sector, $\mathbf{I}$ is the identity matrix, $\mathbf{v}$ is a diagonal matrix of the share of value added in gross output in each sector, and $\mathbf{\Gamma}$ is a matrix of input-output relationships, where the $(i, j)th$ element of $\mathbf{\Gamma}$ is the ratio of intermediates purchased by sector j from sector i to the total gross output in sector j.[33]

We use data from the BEA Make and Use tables to construct value-added measures of consumption and investment (as described in equations (A.1) and (A.2). As noted above, the final vectors of consumption and investment are available from the Use tables and the investment network data. To compute the fraction

[33] We first compute all of these objects at the 40 sector level and then analyze structural change at the two sector level, aggregated up from consumption value added and investment value-added constructed at the 40 sector level.

Table A.4: Decomposing Structural Change in Services Share of Value Added Measures of Consumption and Investment

Services Share of:	1947	2019	Δ	% of Total
Consumption Value Added	0.62	0.86	0.24	
Final Prod. only	0.62	0.79	0.17	70%
Input-Output only	0.62	0.73	0.11	45%
VA share only	0.62	0.60	-0.02	-7%
Investment Value Added	0.36	0.54	0.18	
Final Prod. only	0.36	0.50	0.14	75%
Input-Output only	0.36	0.46	0.09	52%
VA share only	0.36	0.33	-0.03	-19%

Notes: This table reports the share of value-added based consumption and investment produced by the services sector and how this changes over time due to changes in each component of the value-added measure (as seen in Equations (A.1) and (A.2)). Changes generated by each of the three components—final producers ("Final Prod. only"), the Total Requirements Matrix ("Input-Output only"), and value-added shares of gross output ("VA shares only")—are computed by holding fixed all other components at their values in 1947. The "% of total" column refers to the change in each component divided by the change in total consumption or investment value added. Because of the non-linear nature of the decomposition, the total of each component will not sum to the actual total.

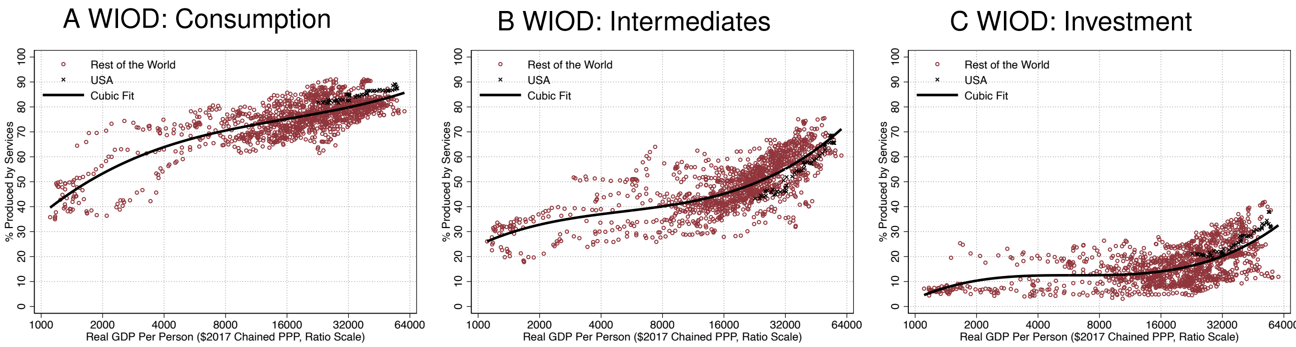
of value added in gross output, we use data in the Use table on nominal value added and nominal gross output for each sector. To compute the total requirements matrix, or Leontief inverse, we scale our final input-output data (adjusted by the Make matrix) by the total gross output of each sector (as opposed to the total spending on intermediates), which gives us the matrix $\mathbf{\Gamma}$. Because we initially adjust both final consumption and the input-output data by the Make matrix, the formulas in equations (A.1) and (A.2) look slightly different from those reported in Herrendorf et al. (2013), but the methods are identical.[34]

Structural change in consumption value added or investment value added can occur because of changes in $\mathbf{v}$, the ratio of value added to gross output, changes in the total requirements matrix $(\mathbf{I}-\mathbf{\Gamma})^{-1}$, the input-output network, or changes in $\mathbf{c}$ or $\mathbf{x}$, the final producers of consumption or investment goods. We consider a counterfactual decomposition where we allow only one of these three components to vary over time and hold the two other components fixed at their values in the initial year of our data, 1947. Since the construction of consumption and investment value-added is non-additive, the contributions of each of these three terms will not necessarily sum to one.

Table A.4 presents the decomposition, highlighting the contribution of each of these three forces to the change in the share of consumption value-added and investment value-added produced by the services sector between 1947 and 2019. Changes in the input-output network account for 45-55% of the rising share of services production of consumption and investment value added. This total contribution is potentially slightly inflated because the contribution of each component sums to more than the total change in the services share of consumption and investment value added. However, if we compute the contribution of changes in the input-output network as a fraction of the sum of changes in each component, input-output changes still account for 40-50% of structural change in consumption and investment value added.

[34] In principle, the total requirements matrix might change over time because the Make matrix has changed. We have explored counterfactuals holding the Make matrix fixed and found that changes in this matrix have virtually no impact on value-added measures of structural change.

Figure A.5: International Trends in Goods/Service Production Share of Consumption, Intermediates and Investment (1965-2011)



Notes: Panels A-C display the share of services in the production of consumption, investment, and intermediates using the use tables from the World Input Output Database (WIOD, see Timmer et al., 2015; Woltjer et al., 2021) plotted against real GDP per capita (taken from the Penn World Table 10.0: Feenstra, Inklaar and Timmer, 2015). Panels A-C additionally show a fitted cubic polynomial and also highlight the data points for the USA (black Xs).

Appendix A.5. International Evidence

This appendix illustrates that the broad patterns of structural transformation in the production of consumption, intermediates, and investment that we find in BEA data for the United States are in fact a widespread phenomenon, occurring in many countries around the world. To do so, we obtain use tables analogous to those of the BEA for 41 countries (including the United States) from two waves of the World Input Output Database (WIOD: Timmer et al., 2015; Woltjer et al., 2021). The first wave reports data from 1965-2000 for 25 countries and 23 sectors, while the second covers data for 40 countries and 35 sectors over the period 1995-2011. The international use tables adhere to the 1993 version of the System of National Accounts (SNA), a comprehensive conceptual and accounting framework for compiling and reporting macroeconomic statistics developed by the United Nation's (UN) Intersecretariat Working Group on National Acccounts (ISWGNA). Industries are classified according to the International Standard Industrial Classification revision 3 (ISIC Rev. 3) and we aggregate these industries to 21 consistently defined sectors that can be grouped into goods and services. As in our main analysis, we exclude oil and utilities producing industries and split total expenditure on consumption, intermediates, and investment into the portion supplied by goods industries and the portion supplied by services industries.

To harmonize the two WIOD waves we measure the service/goods production shares starting with the level observed in the first year of the data and then construct shares in subsequent years by cumulating observed annual growth rates in these shares. For countries that span both WIOD waves, we use the growth rates from the 2013 WIOD starting with the year 2000. We drop two countries: Hong Kong, because its data ends in 1999 and Luxembourg because its services share of investment is an outlier (about twice as large as that of countries at similar levels of development, such as the United states).

Figure A.5 plots structural change in consumption, investment, and intermediates using the WIOD data. To facilitate comparison across countries, we plot the share of consumption, intermediates, and investment produced by the service sector against real GDP per capita (in $2017 chained PPP on a ratio scale), similar to illustrations provided by Galesi and Rachedi (2018). This figure reveals several notable insights. First, we highlight data for the United States (marked with black Xs) to illustrate that the range of values for the U.S. service shares constructed from WIOD data are very similar to those we observe from BEA data in Figure 3. Second, the fitted cubic trend lines suggest that the time series patterns observed in the United

States are representative of the typical experience in other countries at similar levels of development (as measured by real GDP per capita). Third, the stylized fact of increasing services shares in the production of consumption, intermediates, and investment is present at all levels of development within the WIOD database, ranging from countries with initial GDP per person as low as 1,271 ($2017 chained PPP) to as high as 22,746 in 1965. It appears that structural transformation in the production of consumption is faster at lower levels of development, while that for intermediates and investment appears to accelerate at higher levels of development.

Appendix B. Equilibrium Conditions, Derivations and Proofs

Appendix B.1. Full Listing of Equilibrium Conditions

Appendix B.1.1. Household Problem

The household's problem is

$$\max_{C_{jt},K_{jt+1}} \sum_{t=0}^{\infty} \beta^t log \left(\left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C-1}} \right),$$

$$\text{s.t. } \sum_j P_{jt}C_{jt} + \sum_{j=1}^{N} P_{jt}^X \left(K_{jt+1} - (1-\delta_j)K_{jt}\right) \leq W_t + \sum_j R_{jt}K_{jt}.$$

The first order conditions for this problem, $\forall j$, are

$$\frac{P_{jt}^X}{E_t^C} = \frac{\beta}{E_{t+1}^C} \left(R_{jt+1} + P_{jt+1}^X(1-\delta_j) \right) \tag{B.1}$$

$$\frac{P_{jt}}{E_t^C} = \left(\omega_{Cj} \frac{C_t}{C_{jt}} \right)^{\frac{1}{\epsilon_C}} \frac{1}{C_t} \tag{B.2}$$

where total consumption, C_t, and total expenditures (denoted in units of the numeraire), E_t^C, are given by:

$$C_t \equiv \left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C-1}} \tag{B.3}$$

$$E_t^C = \sum_j P_{jt}C_{jt}. \tag{B.4}$$

Appendix B.1.2. Production Firm Problem

The profit maximization problem for the representative production firm in sector j is given by

$$\max_{L_{jt},K_{jt},M_{jt}} P_{jt}Q_{jt} - W_tL_{jt} - R_{jt}K_{jt} - P_{jt}^M M_{jt}.$$

where $Q_{jt} = A_{jt} \left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j} \right)^{\alpha_j} M_{jt}^{1-\alpha_j}$.

The first order conditions for this problem are

$$W_t = \alpha_j(1-\theta_j)\frac{P_{jt}Q_{jt}}{L_{jt}} \tag{B.5}$$

$$R_{jt} = \alpha_j\theta_j\frac{P_{jt}Q_{jt}}{K_{jt}} \tag{B.6}$$

$$P^M_{jt} = (1-\alpha_j)\frac{P_{jt}Q_{jt}}{M_{jt}}. \tag{B.7}$$

Appendix B.1.3. Bundling Firm Problems

For each sector j, there are two bundling firms: one that produces the intermediate good used by sector j, M_{jt}, and one that produces the investment (purchased by the household) for capital specific to sector j, X_{jt}.

The profit maximization problem of the intermediates bundling firm for sector j is given by:

$$\max_{M_{ijt}} P^M_{jt}M_{jt} - \sum_i P_{it}M_{ijt},$$

where the bundle of intermediates used by sector j, M_{jt} is given by:

$$M_{jt} = A^M_{jt}\left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}}\right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}. \tag{B.8}$$

The first order conditions for this problem are, for each sector i:

$$P_{it} = P^M_{jt}\left(A^M_{jt}\right)^{1-\frac{1}{\epsilon_{Mj}}}\left(\omega_{Mij}\frac{M_{jt}}{M_{ijt}}\right)^{\frac{1}{\epsilon_{Mj}}}. \tag{B.9}$$

Obtaining the expression for the price of the intermediates bundle sold to sector j, P^M_{jt}, as reported in equation (8), follows from solving the first order conditions for M_{ijt}, plugging into the expression for M_{jt}, and solving for P^M_{jt}.

The profit maximization problem and first order conditions for the investment bundling are symmetric and are given by:

$$\max_{X_{ijt}} P^X_{jt}X_{jt} - \sum_i P_{it}X_{ijt}$$

$$P_{it} = P^X_{jt}\left(A^X_{jt}\right)^{1-\frac{1}{\epsilon_{Xj}}}\left(\omega_{Xij}\frac{X_{jt}}{X_{ijt}}\right)^{\frac{1}{\epsilon_{Xj}}} \tag{B.10}$$

where the bundle of investment for capital specific to sector j, X_{jt} is given by:

$$X_{jt} = A^X_{jt}\left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}}\right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}. \tag{B.11}$$

Similarly, the expression for the price of the investment bundle for sector j's capital shown in equation (9) can be obtained by solving the first order conditions for X_{ijt}, plugging into the expression for X_{jt}, and solving for P^X_{jt}.

Appendix B.1.4. Market Clearing Conditions

In equilibrium, each labor, capital, intermediate bundling and investment bundling market clears. To conserve on notation, market clearing is built into how the capital, intermediates and investment problems have been written down. With the household inelastically providing unitary labor supply each period, labor market clearing is simply given by $\sum_j L_{jt} = 1$. That leaves market clearing for final production in each sector j, which is given by:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{B.12}$$

We also note that evolution of capital in each sector is given by the standard accumulation equation:

$$K_{jt+1} = (1 - \delta_j)K_{jt} + X_{jt}. \tag{B.13}$$

Appendix B.1.5. Sectoral Value Added and Prices

For each production sector j, constant returns to scale implies

$$W_t L_{jt} + R_{jt} K_{jt} + P^M_{jt} M_{jt} = P_{jt} Q_{jt}. \tag{B.14}$$

Therefore, the accounting definition of nominal value added is simply

$$P^V_{jt} V_{jt} = P_{jt} Q_{jt} - P^M_{jt} M_{jt} = W_t L_t + R_{jt} K_{jt}. \tag{B.15}$$

To obtain real value added, we use a discrete time application of the Divisia index definition, which differentiates the accounting definition of nominal value added holding prices fixed:

$$\begin{aligned} P^V_{jt} V_{jt} \Delta \ln V_{jt} &= P_{jt} Q_{jt} \Delta \ln Q_{jt} - P^M_{jt} M_{jt} \Delta \ln M_{jt} \\ \alpha_j \Delta \ln V_{jt} &= \Delta \ln Q_{jt} - (1 - \alpha_j) \Delta \ln M_{jt} \\ \Delta \ln V_{jt} &= \frac{1}{\alpha_j} \Delta \ln A_{jt} + \theta_j \Delta \ln K_{jt} + (1 - \theta_j) \Delta \ln L_{jt}. \end{aligned}$$

Cumulating this expression yields that real value added is given by $V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}$.[35]

Finally, we can write the price index for value added in sector j, P^V_{jt}, as follows:

$$\begin{aligned} P^V_{jt} &= \frac{P_{jt} Q_{jt} - P^M_{jt} M_{jt}}{V_{jt}} \\ &= \frac{P_{jt} V_{jt}^{\alpha_j} \left(\left(\frac{(1-\alpha_j) P_{jt}}{P^M_{jt}} \right)^{\frac{1}{\alpha_j}} V_{jt} \right)^{1-\alpha_j} - P^M_{jt} \left(\frac{(1-\alpha_j) P_{jt}}{P^M_{jt}} \right)^{\frac{1}{\alpha_j}} V_{jt}}{V_{jt}} \\ &= P_{jt}^{\frac{1}{\alpha_j}} \left(P^M_{jt} \right)^{1-\frac{1}{\alpha_j}} (1 - \alpha_j)^{\frac{1}{\alpha_j}} \left(\frac{1}{1 - \alpha_j} - 1 \right) \end{aligned}$$

[35]We have normalized the implicit time-invariant constant in cumulating this expression to 1.

$$= \quad \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}} \left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P_{jt}^M} \right)^{\frac{1-\alpha_j}{\alpha_j}}$$

where we use the fact that $Q_{jt} = V_{jt}^{\alpha_j} M_{jt}^{1-\alpha_j}$ and the fact that $M_{jt} = \left(\frac{(1-\alpha_j)P_{jt}}{P_{jt}^M} \right)^{\frac{1}{\alpha_j}} V_{jt}$ (from the first order conditions for intermediates, shown in equation (B.7)).

Appendix B.2. Proof of Lemma 1

With Assumptions 1 and 2, we have:

$$\frac{P_{jt}}{P_{it}} = \frac{A_{it}/\left(P_{it}^M\right)^{1-\alpha}}{A_{jt}/\left(P_{jt}^M\right)^{1-\alpha}} = \frac{\tilde{A}_{it}}{\tilde{A}_{jt}}$$

where $\tilde{A}_{jt} \equiv \frac{A_{jt}}{\left(P_{jt}^M\right)^{1-\alpha}}$.

With this relationship, the lemma is straightforward to prove by manipulation of the expression for the price of investment (equation (9), though now common to all sectors due to Assumptions 1 and 2):

$$\begin{aligned} P_t^X &= \frac{1}{A_t^X} \left(\sum_k \omega_{Xi} P_{kt}^{1-\epsilon_X} \right)^{\frac{1}{1-\epsilon_X}} \\ &= P_{jt} \frac{1}{A_t^X} \left(\sum_k \omega_{Xk} \left(\frac{P_{kt}}{P_{jt}} \right)^{1-\epsilon_X} \right)^{\frac{1}{1-\epsilon_X}} \\ &= P_{jt} \frac{1}{A_t^X} \left(\sum_k \omega_{Xk} \left(\frac{\tilde{A}_{jt}}{\tilde{A}_{kt}} \right)^{1-\epsilon_X} \right)^{\frac{1}{1-\epsilon_X}} \\ &= P_{jt} \tilde{A}_{jt} \frac{1}{A_t^X} \left(\sum_k \omega_{Xk} \left(\tilde{A}_{kt} \right)^{\epsilon_X - 1} \right)^{\frac{1}{1-\epsilon_X}} . \end{aligned}$$

Hence,

$$P_{jt} A_{jt} = P_t^X A_t^X \left(\sum_k \omega_{Xk} \left(\tilde{A}_{kt} \right)^{\epsilon_X - 1} \right)^{\frac{1}{\epsilon_X - 1}} .$$

Defining $\tilde{B}_X(t) \equiv A_t^X \left(\sum_k \omega_{Xk} \tilde{A}_{kt}^{\epsilon_X - 1} \right)^{\frac{1}{\epsilon_X - 1}}$, the result of the lemma obtains.

The proof for the price and technical change in intermediate goods follows identical steps as the above, with $\tilde{B}_{it}^M \equiv A_{it}^M \left(\sum_k \omega_{Mki} \tilde{A}_{kt}^{\epsilon_{Mi} - 1} \right)^{\frac{1}{\epsilon_{Mi} - 1}}$.

Appendix B.3. Proof of Lemma 2

Aggregate GDP, Y_t, denoted in units of the numeraire, is given by:

$$Y_t = \sum_i P_{it}^V V_{it}$$

where V_{it} is real value added in sector i and P_{it}^V is the price of value added in sector i.
As shown in Appendix B.1, sectoral real value added and its price, can be written as:

$$V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}$$
$$P_{jt}^V = \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}} \left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P_{jt}^M} \right)^{\frac{1-\alpha_j}{\alpha_j}}$$

Given Assumptions 1 and 2 (implying that $\alpha_j = \alpha$ for all sectors), and these expressions for real value added and its price, we can write aggregate GDP as:

$$\begin{aligned} Y_t &= \sum_i P_{it}^V V_{it} \\ &= \sum_i \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{A_{it} P_{it}}{(P_{it}^M)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \left(\frac{K_{it}}{L_{it}} \right)^{\theta} L_{it} \end{aligned}$$

Because of the common rental rate and wage, the capital to labor ratios will be equated across sectors, and with an aggregate labor supply of 1, will simply be equal to the aggregate stock of capital, $K_t = \sum_i K_{it}$. Further, from Lemma 1 and our choice of the price of investment as our numeraire, we have that $\frac{A_{it}P_{it}}{(P_{it}^M)^{1-\alpha}} = P_{it}\tilde{A}_{it} = \tilde{B}_t^X$. With this, we can rewrite the above expression for GDP as:

$$\begin{aligned} Y_t &= \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X \right)^{\frac{1}{\alpha}} (K_t)^{\theta} \sum_i L_{it} \\ &= \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X \right)^{\frac{1}{\alpha}} K_t^{\theta} \\ &= \mathcal{A}_t K_t^{\theta} \end{aligned}$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X \right)^{\frac{1}{\alpha}}$.

First order conditions for capital in each production sector give us $R_t = \theta\alpha \frac{P_{jt}Y_{jt}}{K_{jt}}$. Using our expressions for real sectoral value added and its price, as well as the result of Lemma 2, we can rewrite the this first order condition as:

$$\begin{aligned} R_t &= \theta\alpha \frac{P_{jt}Y_{jt}}{K_{jt}} \\ &= \theta\alpha \frac{\frac{1}{\alpha} P_{jt}^V V_{jt}}{K_{jt}} \end{aligned}$$

$$
\begin{aligned}
&= \theta \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{K_{jt}}{L_{jt}}\right)^{\theta-1} \left(\frac{P_{jt}A_{jt}}{(P_t^M)^{1-\alpha}}\right)^{\frac{1}{\alpha}} \\
&= \theta \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} (\tilde{B}_t^X)^{\frac{1}{\alpha}} \left(\frac{K_t}{L_t}\right)^{\theta-1} \\
&= \theta \mathcal{A}_t K_t^{\theta-1}
\end{aligned}
$$

Applying the same algebraic steps to the first order condition for labor demand (equation (B.5)) generates the other equation in the lemma, $W_t = (1-\theta)\mathcal{A}_t K_t^{\theta}$.

Appendix B.4. Proof of Proposition 1

Given the if and only if statement in the proposition, we must prove both the necessary and sufficient directions. We start with the necessary direction, showing if an ABGP exists, this requires that $\gamma^{\mathcal{A}}$ is constant and that $\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^{E^M} = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$

The requirement that $\gamma^{\mathcal{A}}$ be constant follows immediately from the aggregate production function expression from Lemma 2, $Y_t = \mathcal{A}_t K_t^{\theta}$. If Y_t and K_t grow at constant rates, that means that $\mathcal{A}_t$ must as well. Thus, the remainder of this direction of the proof entails showing that the growth rates of K_t, Y_t, W_t, X_t, E_t^C and E_t^M are all equal to $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and that the growth rate of R_t is zero.

Taking the Euler equation from the household's problem (see Appendix B.1), we have that:

$$
\frac{E_{t+1}^C}{E_t^C} = \gamma_{t+1}^{E^C} = \beta\left(R_{t+1} + 1 - \delta\right) \tag{B.16}
$$

This implies that a constant growth rate in household expenditures implies a constant rental rate of capital, R_t, along the ABGP.

Taking the ratio of first order conditions for capital in each sector, we have that:

$$
\frac{K_{jt}}{L_{jt}} = \frac{\theta}{1-\theta}\frac{W_t}{R_t} \tag{B.17}
$$

With our assumptions of common parameters across sectors, capital to labor ratios are equated, and since $\sum_j L_{jt} = 1$, we can write the aggregate capital stock, K_t, as $K_t = \frac{\theta}{1-\theta}\frac{W_t}{R_t}$. Since R_t is constant along the ABGP, this implies that $\gamma^K = \gamma^W$.

Likewise, if we take the ratio of first order conditions for capital and intermediates at production firms, and rearrange, we have:

$$
K_{it} = \frac{\alpha\theta}{1-\alpha}\frac{1}{R_t}P_{it}^M M_{it} \tag{B.18}
$$

Summing this equation across sectors i, we have that:

$$
K_t = \frac{\alpha\theta}{1-\alpha}\frac{1}{R_t}\sum_i P_{it}^M M_{it} \tag{B.19}
$$

Thus, total expenditures on intermediates, $E_t^M \equiv \sum_i P_{it}^M M_{it}$, will grow at the same rate as the aggregate capital stock.

From Lemma 2, we have that $R_t = \theta \mathcal{A}_t K_t^{\theta-1}$. Taking the ratio of this simplified first order condition for capital across time periods yields:

$$\frac{K_{t+1}}{K_t} = \left(\frac{R_{t+1}}{R_t}\frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\right)^{\frac{1}{1-\theta}} \tag{B.20}$$

$$\gamma^K = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{B.21}$$

where the last step holds given the constant rental rate of capital along the ABGP.

Taking the ratio of the aggregate production function from Lemma 2 across time periods yields:

$$\frac{Y_{t+1}}{Y_t} = \frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\left(\frac{K_{t+1}}{K_t}\right)^{\theta} \tag{B.22}$$

$$\gamma^Y = \gamma^{\mathcal{A}}\left(\gamma^{\mathcal{A}}\right)^{\frac{\theta}{1-\theta}} = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{B.23}$$

which thus implies $\gamma^Y = \gamma^K$.

Now, turning to the capital accumulation equation, if we divide by K_t, we have:

$$\gamma^K = (1-\delta) + \frac{X_t}{K_t}$$

Since γ^K is a constant, this requires that the RHS be constant, or in other words, $\gamma^X = \gamma^K$.

The only remaining condition to verify here is that aggregate consumption expenditures, E_t^C, grow at the same rate as aggregate capital. We can write GDP using expenditure side accounting as $Y_t = E_t^C + X_t$, since all these aggregates are denoted in units of the numeraire. Since we know that $\gamma^Y = \gamma^K = \gamma^X$ and γ^{E^C} is constant, then $\gamma^{E^C} = \gamma^Y = \gamma^K$. This finishes the necessity direction of the proof.

We now consider the sufficiency direction required for the proof. We now show that if $\gamma^{\mathcal{A}}$ is constant, then an ABGP exists. We do this by construction. We set $\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^{E^M} = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and we set R_t to be a constant such that the Euler Equation holds:

$$R_{t+1} = \frac{1}{\beta}\gamma^{E^C} - (1-\delta) \tag{B.24}$$

Given our assumption that $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} > \frac{1-\delta}{\beta}$, this will produce a non-negative rental rate for capital.

Then, given an initial value of $\mathcal{A}_t$, this value of R implies a unique value for K_0 from (the rewritten first order conditions). It is then straightforward to construct X_0 to satisfy capital accumulation, given K_0 and γ^K, and to construct E_0^M using equation (B.19) above. Finally, we can determine the initial condition for expenditures, using the expenditure side accounting relationship, with

$$\begin{aligned} E_0^C &= Y_0 - X_0 \\ &= \mathcal{A}_0 K_0^{\theta} - K_0\left(\gamma^K - (1-\delta)\right). \end{aligned}$$

Lastly, to show that transversality holds, we need that

$$lim_{t\to\infty}\beta^t\frac{K_t}{E_t^C} = 0 \tag{B.25}$$

Given that we have constructed the path such that $\gamma^K = \gamma^{E^C}$,$\frac{K_t}{E_t^C}$ will be a constant along this path and thus the limit will be satisfied. This completes the proof in the sufficiency direction.

Appendix B.5. Proof of Lemma 3

Given the assumption 3, that the parameters of the intermediates bundling sectors are the same for all sectors j, i.e. $\omega_{Mij} = \omega_{Xi}$ and $\epsilon_{Mj} = \epsilon_M$ for all j and that technical change in each sector's intermediates bundling is the same, i.e. $A^M_{jt} = A^M_t$, we start by revisiting the result of Lemma 1. Assumption 3 implies that there is now a single intermediate good in the economy, with a single price, P^M_t. As a result, given our definition of $\tilde{A}_{it} \equiv \frac{A_{jt}}{\left(P^M_{jt}\right)^{1-\alpha}}$, we have that

$$\frac{P_{jt}}{P_{it}} = \frac{\tilde{A}_{it}}{\tilde{A}_{jt}} = \frac{A_{it}}{A_{jt}}$$

Thus, we now have that $\tilde{B}^M_{it} = \tilde{B}^M_t$ and by the same logic as the above and the proof of Lemma 1, we have that:

$$\frac{B^X_t}{B^M_t} = \frac{P^M_t}{P^X_t}$$

where $B^M_{it} \equiv A^M_{it}\left(\sum_k \omega_{Mki} A^{\epsilon_{Mi}-1}_{kt}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ and $B^X_t \equiv A^X_t\left(\sum_k \omega_{Xk} A^{\epsilon_X-1}_{kt}\right)^{\frac{1}{\epsilon_X-1}}$.

The final part left to show is that $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B^X_t \left(B^M_t\right)^{\frac{1-\alpha}{\alpha}}$. Given that, in the more general case, $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}}\left(\tilde{B}^X_t\right)^{\frac{1}{\alpha}}$, this amounts to showing that $\tilde{B}^X_t = \left(B^X_t\right)^{\alpha}\left(B^M_t\right)^{1-\alpha}$. Given the definition of $\tilde{B}^X_t$, we have that:

$$\begin{aligned}
\tilde{B}_X(t) &= A^X_t\left(\sum_k \omega_{Xk}\tilde{A}^{\epsilon_X-1}_{kt}\right)^{\frac{1}{\epsilon_X-1}} \\
&= A^X_t\left(\sum_k \omega_{Xk}\left(\frac{A_{kt}}{\left(P^M_t\right)^{1-\alpha}}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}} \\
&= \frac{B^X_t}{\left(P^M_t\right)^{1-\alpha}} \\
&= \frac{B^X_t}{\left(\frac{B^X_t}{B^M_t}\right)^{1-\alpha}} \\
&= \left(B^X_t\right)^{\alpha}\left(B^M_t\right)^{1-\alpha}
\end{aligned}$$

This completes the proof.

Appendix B.6. Proof of Lemma 4

We prove the result for intermediates and ϵ_M first; the result for investment with ϵ_X follows by the symmetry of the CES functions.

For ease of exposition of the proof, we define $g^M(\epsilon_M)$ as follows:

$$g^M(\epsilon_M) = \left(\sum_i s^M_{it-1}(\gamma^A_{it})^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}}$$

Thus, the objective is to show that $g^M(\epsilon_M)$ is weakly increasing in ϵ_M. For ease of exposition, we also suppress the A superscript on γ^A_{it} and define $\gamma_i \equiv \gamma_{it}$.

We observe that $g^M(\epsilon_M)$ depends on ϵ_M in two ways—both directly, as an exponent on γ^A_{it} and in the exponent for the overall sum, but also indirectly, through its impact on s^M_{it-1}, which is itself a function of ϵ_M. With assumptions 1-3, s^M_{it-1} can be written as a function of exogenous values, including ϵ_M:

$$s^M_{it-1}(\epsilon_M) = \omega_{Mi} \frac{A^{\epsilon_M - 1}_{it-1}}{\sum_j^N \omega_{Mj} A^{\epsilon_M - 1}_{jt-1}}$$

Our goal is to show that for any $\epsilon_1 > \epsilon_2$, $g^M(\epsilon_1) \geq g^M(\epsilon_2)$. We show this in two steps. First, we define the function $\tilde{g}(\sigma, \epsilon_M) = \left(\sum_i s^M_{it-1}(\sigma)\gamma^{\epsilon_M - 1}_{it}\right)^{\frac{1}{\epsilon_M - 1}}$. We first show that for fixed σ and $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma, \epsilon_1) \geq \tilde{g}(\sigma, \epsilon_2)$. The second step defines the function $\hat{g}(\epsilon_M, \sigma) = \left(\sum_i s^M_{it-1}(\epsilon_M)\gamma^{\sigma - 1}_{it}\right)^{\frac{1}{\sigma - 1}}$ and shows that $\hat{g}(\epsilon_1, \sigma) \geq \hat{g}(\epsilon_2, \sigma)$. Then, given these two substeps, the final result follows from the following sequence of inequalities:

$$g(\epsilon_1) = \tilde{g}(\epsilon_1, \epsilon_1) \geq \tilde{g}(\epsilon_1, \epsilon_2) = \hat{g}(\epsilon_1, \epsilon_2) \geq \hat{g}(\epsilon_2, \epsilon_2) = g(\epsilon_2) \tag{B.26}$$

We note that the lemma makes the assumption of positive dependence in the form of $\mathbb{E}\left[ln(A_{it}) \mid \gamma^A_{it} = a\right]$ being weakly increasing in a. This assumptions is not needed until Step 2, and so we demonstrate Step 1 for the more general case without this assumption.

Step 1: For $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma, \epsilon_1) \geq \tilde{g}(\sigma, \epsilon_2)$. This first step of the proof follows from an application of Jensen's inequality. Jensen's inequality tells us that for any convex function, $\phi(x)$, any real valued function $h(x)$, and any set of non-negative weights a_i with $\sum_i a_i = 1$, $\sum_i a_i \phi(h(x_i)) \geq \phi(\sum_i a_i h(x_i))$. The inequality is reversed in the case where $\phi(x)$ is concave.

First, begin with the case where $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $\frac{\epsilon_1 - 1}{\epsilon_2 - 1} > 1$. Define $\phi(x) = x^{\frac{\epsilon_1 - 1}{\epsilon_2 - 1}}$. This function is convex because $\frac{\epsilon_1 - 1}{\epsilon_2 - 1} > 1$. Define $h(x) = x^{\epsilon_2 - 1}$ and $a_i = s^M_{it-1}(\sigma)$.

Jensen's inequality thus implies the following result:

$$\left(\sum_i s^M_{it-1}(\sigma)\gamma^{\epsilon_2 - 1}_{it}\right)^{\frac{\epsilon_1 - 1}{\epsilon_2 - 1}} = \tilde{g}(\sigma, \epsilon_2)^{\epsilon_1 - 1} \leq \left(\sum_i s^M_{it-1}(\sigma)\gamma^{\epsilon_1 - 1}_{it}\right) = \tilde{g}(\sigma, \epsilon_1)^{\epsilon_1 - 1}$$

Exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1 - 1}$, which is a positive exponent, completes the result for this case.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 < \frac{\epsilon_1 - 1}{\epsilon_2 - 1} < 1$, then it must be that $\epsilon_1 < 1$. In this case, $\phi(x)$ is now concave, which reverses the above inequality. However, because $\epsilon_1 < 1$, the step of exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1 - 1}$ again reverses the inequality and ensures the result holds.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 > \frac{\epsilon_1 - 1}{\epsilon_2 - 1}$, then $\epsilon_2 < 1$ and $\epsilon_1 > 1$, and $\phi(x)$ is again convex and $\frac{1}{\epsilon_1 - 1}$ is a positive exponent, so the result still holds.

Finally, consider the case where either $\epsilon_1 = 1$ or $\epsilon_2 = 1$. Although $\tilde{g}(\epsilon_M)$ is undefined in this case, we consider instead the limiting result, defining $\tilde{g}(\sigma, 0) = \prod_i \gamma_{it}^{s^M_{it-1}(\sigma)}$. Here we apply Jensen's inequality using $\phi(x) = \ln(x)$ and $h(x) = x^{\epsilon_1 - 1}$. If $\epsilon_2 = 1$ and $\epsilon_1 > 1$, then we have that:

$$(\epsilon_1 - 1)\ln(\tilde{g}(\sigma, \epsilon_1)) = \ln(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_1 - 1}) \geq (\epsilon_1 - 1)\sum_i s^M_{it-1}(\sigma)\ln(\gamma_{it}) = (\epsilon_1 - 1)\ln(g(\sigma, 0))$$

Dividing both sides by $\epsilon_1 - 1$ and exponentiating both sides of the inequality yields the result.

In the case where $\epsilon_1 = 2$ and $\epsilon_2 < 1$, the same steps can be followed, replacing ϵ_1 with ϵ_2, but now since $\epsilon_2 < 1$, the final step of dividing both sides by $\epsilon_2 - 1$ will reverse the inequality, proving the result.

Thus completes step 1.

Step 2: For $\epsilon_1 > \epsilon_2$, $\hat{g}(\epsilon_1, \sigma) \geq \hat{g}(\epsilon_2, \sigma)$. To prove this inequality, we show that $\frac{\partial \hat{g}(\epsilon_M, \sigma)}{\partial \epsilon_M} \geq 0$.

Taking the partial derivative, we obtain the following result:

$$\begin{aligned}
\frac{\partial \hat{g}(\epsilon_M, \sigma)}{\partial \epsilon_M} &= \frac{1}{\sigma - 1}\hat{g}(\epsilon_M, \sigma)^{2-\sigma}\sum_i \frac{\partial s^M_{it-1}(\epsilon_M)}{\partial \epsilon_M}\gamma_{it}^{\sigma-1} \\
&= \frac{1}{\sigma - 1}\hat{g}(\epsilon_M, \sigma)^{2-\sigma}\left(\sum_i s^M_{it-1}\gamma_{it}^{\sigma-1}\ln(A_{it}) - \left(\sum_i s^M_{it-1}\gamma_{it}^{\sigma-1}\right)\left(\sum_i s^M_{it-1}\ln(A_{it})\right)\right) \\
&= \frac{1}{\sigma - 1}\hat{g}(\epsilon_M, \sigma)^{2-\sigma} COV\left(\ln(A_{it}), \gamma_{it}^{\sigma-1}\right)
\end{aligned}$$

where $COV(\ln(A_{it}), \gamma_{it}^{\sigma-1})$ is the covariance between $\ln(A_{it})$ and $\gamma_{it}^{\sigma-1}$ where probability weights across sectors are defined by the shares s^M_{it-1}. Given the weak positive dependence assumption, that $\mathbb{E}\left[ln(A_{it}) \mid \gamma^A_{it} = a\right]$ is weakly increasing in a, the sign of this covariance term will be the the sign of $\sigma - 1$.[36] Thus, since this covariance has the same sign as $\sigma - 1$, the result will go through. Since we know that $\hat{g}(\epsilon_M, \sigma)^{2-\sigma} > 0$ and the entire expression is multiplied by $\frac{1}{\sigma-1}$, this ensures that $\frac{\partial \hat{g}(\epsilon_M, \sigma)}{\partial \epsilon_M} \geq 0$. This completes step 2 of the proof.

Given the successful completion of steps 1 and 2, the proof for intermediates follows from the inequalities in equation (B.26) and the proof for investment follows by symmetry.

Appendix C. Price Measurement Details and Additional Calibration Results

In this Appendix, we provide further detail regarding price measurement, robustness of our measurement procedures, and additional calibration details and results for Section 5.

[36] This can be seen by an iterated expectations argument. Consider two random variables X and Y which have, without loss of generality, zero mean. The covariance of X and Y is $COV(X, Y) = \mathbb{E}[XY] = \mathbb{E}[X\mathbb{E}[Y \mid X]] = COV(X, \mathbb{E}[Y \mid X])$. If we assume that $\mathbb{E}[Y \mid X = x]$ is increasing in x, then, since the covariance of two increasing functions of X is positive, we have the result. Note that if the conditional expectation of Y is weakly increasing in X, then it will be weakly increasing in $X^{\sigma-1}$ if $\sigma - 1 > 0$ and weakly decreasing in X if $\sigma - 1 < 0$. When $\sigma - 1 < 0$, then we have the covariance of an increasing and a weakly decreasing function of X, which is weakly negative.

Appendix C.1. Measuring Consumption and Investment Prices by Sector

We adopt a final expenditure approach to measuring the price of consumption and the price of investment produced by goods and services sectors. We start with final expenditure data on the prices of and total expenditures on 68 consumption commodities (NIPA Tables 2.4.4 and 2.4.5) and 30 investment commodities (NIPA Tables 5.3.4, 5.3.5, 5.5.4, 5.5.5, 5.6.4). The 68 consumption commodities are the finest level of consumption commodity disaggregation possible over the postwar period (after omitting commodities produced primarily by the energy-intensive sectors omitted from our analysis: motor vehicle fuels, fuel oils and other fuels, water supply and sanitation services, electricity, and natural gas). The 30 investment commodities (two structures commodities, 24 equipment commodities, three intellectual property commodities, and residential commodities) are a subset of the 33 investment commodities used in vom Lehn and Winberry (2022). Due to data limitations, primarily on detailed investment prices (which were not studied in vom Lehn and Winberry (2022)), we combine light trucks and other trucks into a single commodity, all software (prepackaged and custom) into a single commodity, and residential structures and residential equipment into a single commodity.[37]

Given data on the expenditures on and prices of these detailed commodities, we measure price growth for consumption, C (or alternatively, investment, X) produced by goods or services ($j \in Goods, Services$) using price and spending data on commodities (P^C_{kt} and $P^C_{kt}Q^C_{kt}$ for $k \in 1, ..., K$) according to the formula:

$$\Delta \ln(P^C_{jt}) = \sum_{k=1}^{K} \frac{\xi^C_{jkt} P^C_{kt} Q^C_{kt}}{\sum_{\ell=1}^{L} \xi^C_{j\ell t} P^C_{\ell t} Q^C_{\ell t}} \Delta \ln(P^C_{kt}) \tag{C.1}$$

where ξ^C_{jkt} represents the entries of "bridge files," which identify the fraction of spending on commodity k that can be traced back to production by sector j (averaged across years $t-1$ and t).

For consumption commodities, we use the BEA's published bridge file (available at `https://www.bea.gov/products/industry-economic-accounts/underlying-estimates`); for investment commodities, we use the bridge files constructed in vom Lehn and Winberry (2022), which we extend forward through the year 2020 using the most recent bridge file data from the BEA. For both consumption and investment commodities, we multiply the raw data in the bridge files by the Make table to accurately recover the sectors producing each commodity (though this step has minimal final impact). Both bridge files are published at a high level of sectoral detail. Since we are interested primarily in the differences between goods and services production, we aggregate the final bridge files to two sectors, goods and services (as defined in Table 1).

One limitation of the consumption bridge file published by the BEA is that it begins in 1997. As a result, we apply the 1997 bridge file data to all years 1947-1996. However, there are minimal differences in our measured consumption prices if we use a fixed bridge file, averaged over the entire 1947-2020 sample, suggesting that movements in the bridge file are negligible in generating long-run trends. Appendix A of vom Lehn and Winberry (2022) makes a similar point for the investment bridge file (which does vary over the entire sample), showing that only a small fraction of variance in the time series changes of the investment network can be attributed to changes in the bridge file. Instead, changes in the network over time

[37]The 33 investment commodities in vom Lehn and Winberry (2022) are approximately the finest level of disaggregation of investment commodities possible while accurately tracking each commodity's production by different sectors over time. It is possible to study patterns in more finely disaggregated non-mining structures, but the mix of sectors producing these structures is the same for each commodity. Thus, further disaggregation does not yield additional insight. For this case, we aggregate prices for detailed structures to a single investment price for non-mining structures using a Tornqvist index.

are generated by changes in the distribution of spending across investment commodities.

We also modify the bridge files for investment from vom Lehn and Winberry (2022) by assuming that goods produce the entirety of structures (including residential investment). In the original bridge files from vom Lehn and Winberry (2022), a small fraction of structures (roughly 7% of non-mining structures and 11% of residential investment) is produced by services, primarily margin contributions from sectors like real estate and finance. However, because overall investment production by services is low compared to goods, especially early in the sample (as seen in Figure 3) and because investment spending on structures is large, absent any adjustment, a sizable portion of the services investment price is determined by structures prices, although the amount of total services investment production coming from structures is falling over time, from 1/3 in 1947 to less than 10% by 2020. This is unappealing because 1) it is unlikely that the price of structures investment reflects the price of these margins and 2) because the exact contribution of these services margins is determined in part by imputation (see Appendix A of vom Lehn and Winberry (2022) for further details). Thus, we set the services contribution to the production of structures equal to zero in our final bridge files. However, even without this adjustment, Appendix C.3 documents that services investment prices are still significantly decreasing relative to goods.

As discussed in Section 5, one concern with the final expenditure approach to measuring the price of consumption and investment produced by goods and services is that there may be bias introduced when both goods and services contribute to the production of a commodity, but the final contributions of goods and services sectors have different prices. Fortunately, the high level of disaggregation we have for consumption and intermediates commodities partially ameliorates this concern – at this level of disaggregation, nearly 2/3 of all commodities are produced in large majority by either goods or services (more than 80%). The primary instance where both goods and services sectors contribute significantly to a consumption or investment commodity is when delivery to the final user involves significant "margins" due to transportation, wholesale trade, or retail trade. This is more common with consumption than with investment commodities. If these margin sectors were reclassified as goods sectors, however, then over 90% of all commodities would be produced in large majority by either goods or services. Appendix C.3 documents that our observed patterns of relative prices are robust to such a reclassification.

For consistency with how we measure intermediates prices, we adjust final consumption and investment prices to remove the input-output network transmission of changes in oil/energy prices. We already omit consumption commodities where a large portion of the commodity is produced by one of the energy-intensive sectors. However, oil/energy price fluctuations may significantly impact the final prices of sectors that heavily rely on energy as an intermediate input. Thus, they may influence the final prices of goods or services consumption or investment.

We use the following approach to make this adjustment. In the following subsection, we describe a procedure for "purging" gross output prices (at the 40 sector level) of the impact of oil/energy prices transmitted through the input-output network. This procedure yields an adjusted gross output price for each sector j, $\tilde{P}_{jt}$, and a "adjustment term," given by the difference between the adjusted price and the original gross output price, $\tilde{P}_{jt} - P_{jt}$. With these sector-specific adjustment terms, we adjust the final price of each commodity using a weighted sum of sectoral adjustment terms, weighted by each sector's position in the bridge file for each commodity. Formally, the final commodity prices for each consumption (or investment) commodity k are given by $\tilde{P}_{kt} = P_{kt} + \sum_{j=1}^{N} \xi^{C}_{jkt} \left(\tilde{P}_{jt} - P_{jt}\right)$, where ξ^{C}_{jkt} is the final bridge file for commodity k (in the case of consumption). The impacts of this adjustment are generally small, as can be seen in the subsequent subsection when we show price trends leaving in all oil/energy sectors.[38]

[38]Leaving in oil/energy sectors generates slightly faster price growth in goods consumption, but this is because of the inclusion

The final time series of consumption and investment prices are shown in Figure 4.

Appendix C.2. Measuring Intermediates Prices

Appendix C.2.1. General Methodology

As described in Section 5, there is limited data availability for measuring intermediates prices by producing sector using the final expenditure approach. Data from the Producer Price Index (PPI) is, in principle, the ideal source of measuring intermediates prices at the final user level, but it has incomplete coverage of services sectors (roughly 85% of the services sectors producing intermediates) and data on services sectors' intermediate prices is only available starting in 2009.

As a result, we measure intermediates prices using the procedure described in Section 5. Gross output prices by sector are implicitly an average of the price of consumption, investment, and intermediates produced by that sector. Thus, using the price of consumption and investment produced by goods or services, we can identify the price of intermediates produced by goods or services as the residual in gross output prices. Formally, the price of intermediates produced by sector $j \in \{Goods, Services\}$ is:

$$\Delta \ln(P_{Mjt}) = \frac{1}{\zeta_{jt}^{M}} \left(\Delta \ln P_{jt}^{GO} - \zeta_{jt}^{C} \Delta \ln P_{Cjt} - \zeta_{jt}^{X} \Delta \ln P_{Xjt}\right) \tag{C.2}$$

where ζ_{jt}^{i} represents the average share (between time periods $t-1$ and t) of total gross output of sector j used for commodity i, with $\zeta_{jt}^{M} + \zeta_{jt}^{C} + \zeta_{jt}^{X} = 1$, and $\Delta \ln P_{jt}^{GO}$ represents the log change of the gross output price for sector j.

Appendix C.2.2. Additional Adjustments for Oil/Energy Price Fluctuations

Although we omit sectors closely tied to market fluctuations in oil/energy prices from our analysis, these fluctuations may have a nontrivial impact on gross output prices via intermediate inputs. That is, because oil/energy is an intermediate input for many sectors, the final price of those sectors' output may reflect fluctuations in these prices. To abstract from fluctuations in the price of oil/energy in analyzing long-run price trends, we adjust gross output prices for the impact of oil/energy prices operating through intermediate input prices.

Consider the following representation of the evolution of gross output and intermediates bundle prices for sector j:

$$\begin{aligned} \Delta \ln(P_{jt}) &= \alpha_{jt} \Delta \ln(P_{jt}^{Y}) + (1-\alpha_{jt}) \Delta \ln(P_{jt}^{M}) \\ \Delta \ln(P_{jt}^{M}) &= \sum_{i \notin E} s_{ijt}^{M} \Delta \ln(P_{it}) + \sum_{i \in E} s_{ijt}^{M} \Delta \ln(P_{it}) \end{aligned}$$

where P_{jt} is the gross output price of sector j, P_{jt}^{Y} is the (implicit) price of value added in sector j, P_{jt}^{M} is the price of the intermediates bundle purchased by sector j, and s_{ijt}^{M} represents elements of the input-output matrix for sector j at time t. The set E describes a set of sectors we wish to "exclude," yet still impact intermediate bundle prices and thus gross output prices in each sector.[39]

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of consumption commodities primarily produced by oil/energy sectors, not these adjustments.

[39]Technically, the above representation only approximates how price measurement is done at the BEA for two reasons. First, the BEA uses Fisher indices instead of Tornqvist indices to compute price growth over time. However, the practical differences between these methodologies are negligible; the Tornqvist index is easier to analyze. Second, the above representation abstracts from imported intermediates, considering the gross output price of a sector j and an intermediate input made by that sector to be

The following dynamics define the "adjusted" price series we seek to obtain:

$$\Delta \ln(\tilde{P}_{jt}) = \left(\alpha_{jt} + (1-\alpha_{jt})\sum_{i \notin E} s^M_{ijt}\right)\Delta \ln(P^Y_{jt}) + (1-\alpha_{jt})(1-\sum_{i \notin E} s^M_{ijt})\Delta \ln(\tilde{P}^M_{jt})$$

$$\Delta \ln(\tilde{P}^M_{jt}) = \sum_{i \notin E} \frac{s^M_{ijt}}{1-\sum_{k\in E} s^M_{kjt}}\Delta \ln(\tilde{P}_{it})$$

These two linear systems can be solved using matrix algebra, implying that the adjusted gross output price series can be written as a function of the original gross output prices, expenditure shares and the prices of the excluded sectors:

$$\Delta\overrightarrow{ln(\tilde{P}_t)} = \Phi\left(\Delta\overrightarrow{ln(P_t)} - (I - diag(1-\alpha_t)\Gamma'_t)^{-1} diag(1-\alpha_t)\left(\overrightarrow{s^{M,E}_t}\right)'\Delta\overrightarrow{P^E_t}\right)$$

where

$$\Phi = (I - diag(1-\alpha_t)\Gamma'_t)^{-1} diag\left(1 + \frac{(1-\alpha_t)\sum_{i\notin E} s^M_{it}}{\alpha_t}\right)(I - diag(1-\alpha_t)\Gamma'_t),$$

$\overrightarrow{ln(\tilde{P}_t)}$ is a vector of adjusted gross output prices, $\overrightarrow{ln(P_t)}$ is a vector of observed gross output prices, $diag(1-\alpha_t)$ is a diagonal matrix with $1-\alpha_{jt}$ along the diagonal, Γ_t is the input-output network with ij-th element s^M_{ijt}, $\overrightarrow{s^{M,E}_t}$ is a vector of the input-output network elements corresponding to excluded sectors, and $\overrightarrow{P^E_t}$ is a vector of prices of sectors to be excluded.

When constructing gross output prices at the sector level, from which intermediates prices will be inferred, we aggregate up these adjusted gross output prices across sectors. We then apply corrections to the consumption and investment prices (as described in the previous subsection) using the difference between adjusted and unadjusted gross output prices. The final intermediates prices for goods and services are then constructed as residuals of the difference in adjusted gross output prices and adjusted consumption and investment prices.

Figure C.1 plots the price of consumption, investment, and intermediates produced by goods and services sectors when oil/energy sectors are excluded and included. The impact of including oil sectors is greatest on goods sectors – the price of goods consumption rises more with the inclusion of energy-intensive commodities, and the price of goods intermediates is much more volatile. However, including oil sectors does not change the relative price patterns observed in the data and has a negligible impact on investment prices.

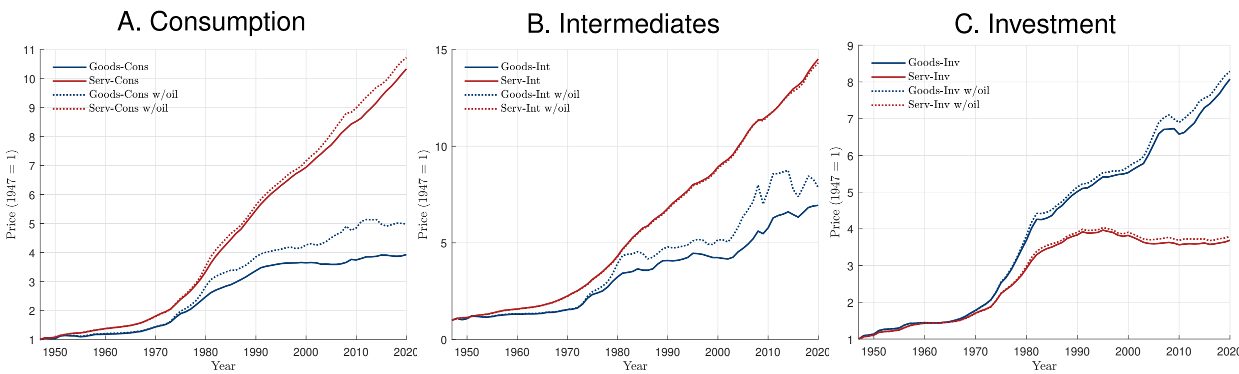
Appendix C.2.3. Comparison to PPI Data

Although data from the PPI is ill-suited to our purposes, we can still compare our measured intermediates prices for the periods intermediate input prices are available from the PPI to validate our procedure.

The PPI publishes the prices of four broad types of intermediate inputs: processed goods, unprocessed goods, services, and construction. To construct a goods intermediate price from these data, we aggregate

the same. This approximation is necessary to make the adjustments we propose. However, at least in the case of gasoline and oil, global movements and domestic movements in prices are very similar, suggesting minimal bias from ignoring the potential for differential oil prices from overseas.

Figure C.1: Sector Prices With and Without Oil Sectors Included



Notes: Solid lines denote baseline prices (as originally observed in Figure 4); dotted lines denote prices measured with oil/energy sectors included.

processed goods and unprocessed goods. We abstract from construction intermediates prices because they comprise only about 1% of all intermediates, and data on construction intermediates are only available beginning in 2009.[40] As PPI data is available monthly, we average monthly prices to the annual frequency to compare with our annual data.

Figure C.2 plots the goods and services intermediates price we obtain from our measurement procedure with the intermediates prices constructed from PPI data. We use our price series without adjustments for oil/energy prices (including all oil/energy sectors) because applying our adjustment procedures to the PPI data is impossible. For services intermediates prices from the PPI, we normalize prices in the first year they are available (2009) to match our services intermediates price series (for comparison). For both series, our inferred intermediates prices align closely with those published in the PPI.

Appendix C.3. Robustness of Relative Price Measurement

This section considers the robustness of our relative price measures to various modifications to our measurement procedures. We first consider modifications that affect all three relative price series (consumption, investment, and intermediates). We then consider modifications specific to the measurement of investment prices.

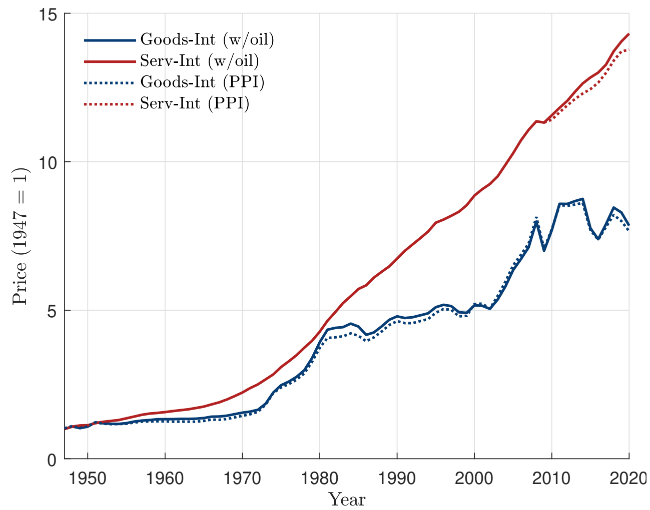
First, we consider the following three robustness exercises: 1) not omitting any sectors producing oil/energy products or making any adjustment for oil/energy price fluctuations, 2) reclassifying margin contributions (i.e., wholesale trade, retail trade, and transportation/warehousing margins) to production as goods sectors, and 3) setting bridge files to their average value throughout the entire sample (i.e., no time variation in bridge files). The relative prices of services to goods in consumption, investment, and intermediates in these three cases are compared to our baseline relative prices in Figure C.3.

In each case, the alternative relative price series are qualitatively similar to our baseline measures.[41]

[40] The BLS chooses not to publish aggregate weights that would facilitate aggregation of these price series to a single price series for goods (or all intermediates in general). However, in correspondence with economists at the BLS, we found out that the approximate weight on processed goods is 80%, and the approximate weight on unprocessed goods is 20%. We use these approximate weights to aggregate the two price series.

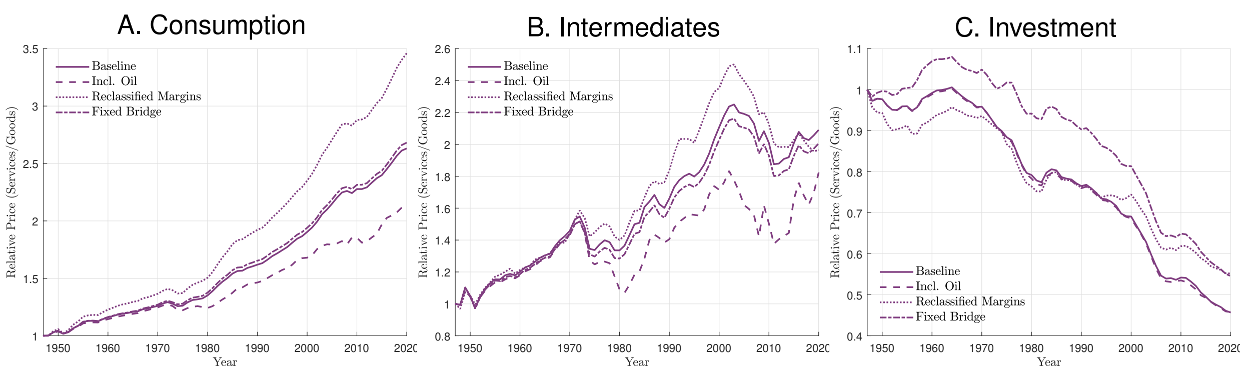
[41] The slightly different dynamics in the relative price of investment when holding bridge files fixed are largely attributable to changes in the importance of various margin sectors over time. If we both hold bridge files fixed and reclassify margins sectors,

Figure C.2: Comparison of Measured Intermediates Prices to PPI Data



Notes: Solid lines denote measured prices including oil/energy sectors; dotted lines denote prices obtained from the Producer Price Index (PPI). PPI data on services intermediates prices are only available beginning in 2009.

Figure C.3: Relative Price (Services/Goods) Robustness for Consumption, Investment and Intermediates

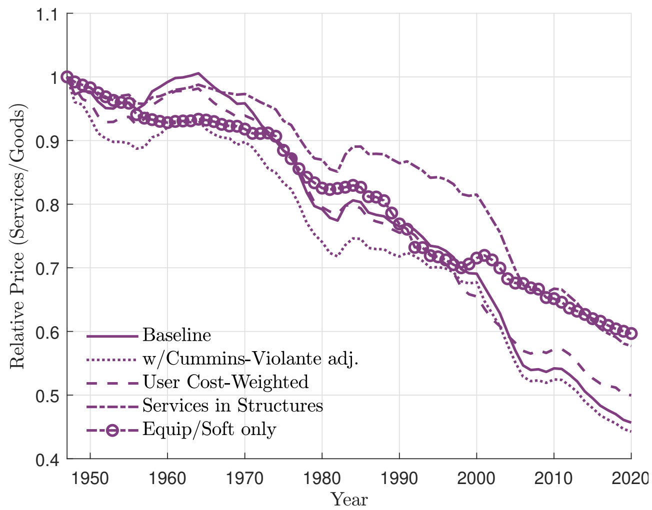


Notes: Solid lines denote baseline relative prices (as originally observed in the right panel Figure 4); dashed lines are prices inclusive of all oil/energy sectors (excluded in the baseline), dotted lines denote prices where margin sectors have been reclassified as goods sectors, and dash-dotted lines indicate prices measured when the bridge file mapping production of each commodity into production sectors is held fixed at its average value across the years 1947-2020.

Second, we consider the following five robustness checks more focused on the relative price of investment: 1) using user cost weights to aggregate investment prices (i.e., rental services measures, constructed originally in vom Lehn and Winberry (2022) and extended through the year 2000), 2) applying price quality adjustments to investment prices based on an extension to Cummins and Violante (2002), 3) allowing for structures to be partially produced by services sectors (as discussed in Appendix C.1), and 4) focusing exclusively on investment prices for equipment and software (where there is the most overlap of investment

there is little discernible impact on the relative price of investment from holding the bridge file fixed over time.

Figure C.4: Robustness of Investment Price Measurement



Notes: The solid line denotes the baseline relative price of investment (services/goods); the dotted line is the relative price with quality adjustments to equipment prices as presented in Cummins and Violante (2002), the dashed line is the relative price where prices are aggregated with user cost of capital weights, the dash-dotted line is the relative price when services are allowed to contribute to the production of structures, and the line with circle markers is the relative price when only consider equipment and software investment prices.

being produced by both goods and services sectors).[42] We plot the relative price of services investment to goods investment under each of these four cases and our baseline results in Figure C.4. Under each modification, the relative price of services investment falls relative to the price of goods investment.[43]

Appendix C.4. Comparison to Gross Output Price Measurement

As discussed in Section 5, an alternative way to measure consumption, investment, and intermediates prices is to use gross output prices exclusively. The simplest approach to measuring the price of products produced by goods and services is to construct a single price for goods and a single price for services and assume this price applies equally to consumption, investment, and intermediates products. Such price series are constructed by separately aggregating up sectoral prices for goods and services sectors. This parsimonious approach is used by Herrendorf et al. (2021), García-Santana et al. (2021), and Sposi et al. (2021).[44]

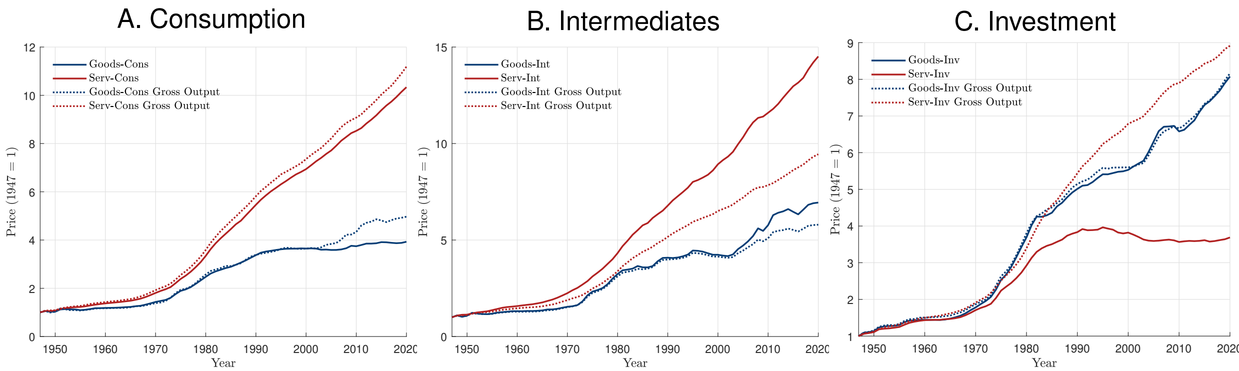
With a single price for goods sectors and a single price for services sectors, a long literature has documented that the relative price of services is rising over time. This is unsurprising given our findings in

[42] To extend the equipment investment quality adjustments presented in Cummins and Violante (2002), we take the percent difference between the quality-adjusted prices of each commodity and the actual prices from 1995-2000 and apply these to all years since 2000.

[43] Even if we focus only on equipment investment produced by goods and services, the relative price of investment produced by services still falls, including when reclassifying margin sectors as goods sectors.

[44] Herrendorf, Rogerson and Valentinyi (Forthcoming) measure the prices of goods and services in a "value-added" sense, embedding the input-output structure of the economy in how services and goods prices are aggregated. However, the overall trends in relative prices are qualitatively similar when not making this adjustment.

Figure C.5: Constructing Sector Prices Using Gross Output Data



Notes: Solid lines denote baseline prices; dotted lines denote prices constructed using gross output data according to equation (C.3).

Figure 4. Services prices are rising significantly faster than goods prices in intermediates and consumption, and these two uses make up the majority of gross output. Thus, aggregating the prices of consumption, investment, and intermediates into a single price masks heterogeneity in the price trends across different final uses.

However, it is possible to obtain consumption-, investment- and intermediates-specific prices for goods and services just using gross output data. Using input-output data, we can observe how each subsector within goods and services contributes to producing consumption, investment, and intermediates. We can then aggregate up detailed sectoral prices in proportion to how much each sector produces of consumption, investment, and intermediates. Formally, the price growth in use $U \in C, X, M$ produced by sector $j \in g, s$ can be measured from gross output data as:

$$\Delta \ln(P^u_{jt}) = \sum_{i \in j} s^{u,j}_{it} \Delta \ln(P_{it}) \tag{C.3}$$

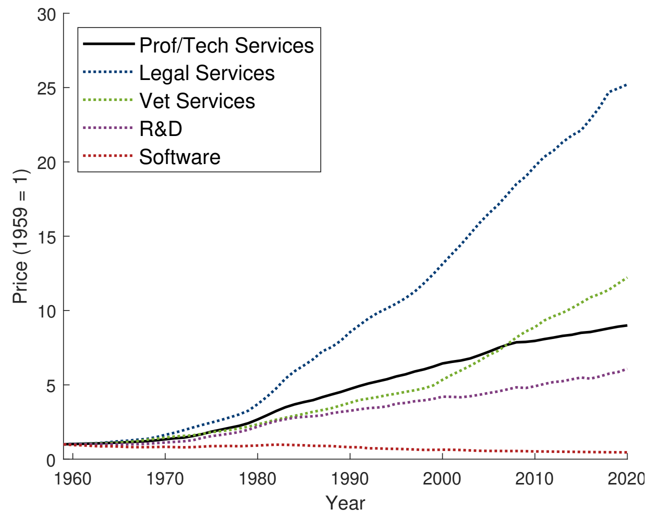
where P^u_{jt} is price of use u produced by sector j at time t, i denotes the individual subsectors within j, $s^{u,j}_{it}$ is the share of all of sector j's production of use u that is produced by subsector i, and P_{it} is the gross output price of sector i.

With this approach, variation in goods or services prices across consumption, investment, and intermediates is driven by heterogeneity in which subsectors produce each final product and heterogeneity in price growth in those subsectors. In principle, this gross output procedure could exactly replicate the prices we construct using final expenditure data if each detailed subsector only produces one of consumption, investment, and intermediates or if each of those products has the same final price. The equivalence of this gross output measurement approach and our final expenditure measurement approach is ultimately a question of how disaggregated the source data is.

Figure C.5 plots the consumption, investment, and intermediates prices for goods and services sectors constructed using gross output data according to equation (C.3).[45] These gross output prices are noticeably different than those we construct using final expenditure data, particularly for investment. Using gross

[45] We use the oil/energy price adjusted gross output prices for consistency with how we measure our baseline prices.

Figure C.6: Heterogeneous Prices Within the Professional and Technical Services Sector



Notes: The solid line denotes the gross output price for the professional and technical services sector, obtained from the BEA GDP by Industry database. The dotted lines are the prices of varied commodities produced in large part or exclusively by the professional and technical services sector. These data are obtained from Tables 2.4.4U and 5.6.4 of the NIPA.

output prices at this 40 sector level, the relative price of investment produced by services is rising over time.

One reason why the gross output prices do not replicate the prices we obtain using final expenditure data is that, at the 40 sector level of disaggregation, for 29 of the 40 sectors we observe, at least 15% of sectoral output is dedicated to at least of two of consumption, investment, and intermediates. As a result, the potential for aggregation bias is significant, given the maximum available level of disaggregation in gross output data.

For example, consider the professional and technical services sector. This sector is a key producer of services investment (i.e., custom computer programming and R&D), producing, on average, 33% of all services investment each year and 45% since the year 2000. And yet, the professional and technical services sector also produces significant amounts of consumption (i.e., legal services, veterinary services) and intermediates (i.e., advertising and public relations services) commodities, which are likely to have very different price trends than the investment produced by this sector. A single price for this sector would not accurately represent price growth over time for the consumption, investment, and intermediate commodities it produces.

To illustrate this, Figure C.6 plots final expenditure prices for several commodities predominantly (if not exclusively) produced by this sector – veterinarian services, legal services, R&D investment, and software investment – and compares these price trends to the single gross output price.[46] Figure C.6 shows that there is a lot of heterogeneity in the price trends of commodities this sector produces. This example illustrates the potential for aggregation bias in gross output prices, particularly for a sector that plays a crucial role in producing investment. Hence, the degree of disaggregation in gross output data is too coarse to accurately recover the price of services investment, explaining why we obtain different results using final expenditure data.

[46] As prices for some of these detailed commodities are only available starting in the year 1959, the figure begins in that year.

Table C.1: Six-Sector Parameter Calibration

	θ	α	δ
Aggregate	0.35	0.54	0.08
A. Goods Sectors			
Consumption	0.36	0.36	0.09
Intermediates	0.21	0.46	0.12
Investment	0.35	0.42	0.09
B. Services Sectors			
Consumption	0.40	0.67	0.05
Intermediates	0.40	0.68	0.09
Investment	0.37	0.66	0.07

Notes: This table reports the calibrated parameter values for the Cobb-Douglas exponents in production (θ for capital within value-added, α for the value-added portion of gross output) and the depreciation rate of capital in each of the six sectors we study and the average of those used in the baseline calibration.

Appendix C.5. Calibration Details and Additional Results

We first describe in more detail how all model parameters are calibrated, then provide additional calibration results and extensions.

Appendix C.5.1. Calibration Procedure Details

For our baseline calibration, we have the following parameters to calibrate: α, the Cobb-Douglas exponent on the intermediates bundle in each sector; θ, the Cobb-Douglas exponent on capital in each sector; δ, the depreciation rate; and the CES share and elasticity parameters in aggregating consumption, investment, and intermediates ($\omega_{Ci}, \omega_{Xi}, \omega_{Mij}, \epsilon^C, \epsilon^X$, and ϵ^M_j for $i \in g, s$ and $j \in g-c, g-x, g-m, s-c, s-x, s-m$. Even though our exercises primarily focus on balanced growth settings with reduced heterogeneity, it is useful to understand the degree of underlying heterogeneity that we are abstracting from. As a result, we describe how we calibrate each of these parameters for each sector j in our data; for our baseline calibration, the parameter values we use are the unweighted averages of these calibrated parameters across sectors.

To calibrate model parameters for α_j, θ_j, and δ_j at the six-sector level, we first obtain implicit values for each model parameter at the 40-sector level (as described in Table 1) and then aggregate those values up to the six sectors we focus on in our calibration. To obtain implicit values for α_j, θ_j, and δ_j at the 40 sector level, we follow the procedures of vom Lehn and Winberry (2022). We calibrate α_j and θ_j using expenditure data from 1947-2020 at the 40 sector level from the BEA Input-Output Database and historical extensions to that database. We calibrate α_j using the ratio of nominal value added to nominal gross output in each sector; we calibrate θ_j using one minus the ratio of nominal labor compensation in each sector to nominal value added (minus taxes and adjusted for self-employment, as in vom Lehn and Winberry (2022)). We also follow the procedures of vom Lehn and Winberry (2022) in calibrating δ_j for each sector, using the implied depreciation rates for each NAICS sector published in the BEA Fixed Assets database.

Given implicit parameter values at the 40 sector level, we then aggregate up to the six sectors we consider in our calibration: goods-consumption ($g-c$), goods-investment ($g-x$), goods-intermediates ($g-m$), services-consumption ($s-c$), services-investment ($s-x$), and services-intermediates ($s-m$). We construct aggregate parameter values for each of our six sectors as a weighted average of the parameter values across the 40 observed NAICS sectors, where each sector is weighted according to the amount of each final product – consumption, investment, or intermediates – it produces among goods or services sectors. For example, to aggregate up parameter values for the goods-consumption sector, we weight parameter values for each goods subsector k with the weight $\frac{P_{kt}C_{kt}}{\sum_{l\in\text{Goods}} P_{lt}C_{lt}}$, where $P_{kt}C_{kt}$ is the total nominal production of consumption by goods subsector k.[47] After we aggregate parameter values for each sector in each year, we take the average of these sectoral parameter values over each year from 1947-2020 and use that as the calibrated value of each parameter.

Table C.1 reports the calibrated parameter values for α_j, θ_j, and δ_j for each of the six sectors in our model; the calibrated values reported in Section 5 are simple averages of the value of each parameter across all six sectors.

The remaining parameters to be calibrated involve the CES aggregators for consumption, investment, and intermediates, with share parameters (ω) and elasticity parameters (ϵ) for each aggregator. Since we assume these share parameters sum to one (i.e. for consumption, $\omega_{Cg} + \omega_{Cs} = 1$), we then only need to calibrate the share parameter corresponding to the goods sector. We calibrate them to match the initial fraction of expenditures on consumption, intermediates, or investment purchased from the goods sector in the year 1947.[48] In the case of intermediates, there are six share parameters to calibrate, ω_{Mgjt} for $j \in \{g-c, g-x, g-m, s-c, s-x, s-m\}$; the share of intermediates produced by the goods sector in each of our six sectors is computing by aggregating up expenditure shares in each of our 40 sectors proportionally to each sector's role in producing intermediates among either goods or services sectors (the same as how we aggregate up parameter values across 40 sectors for α_j, θ_j, and δ_j).

Finally, given values for the share parameters, we identify elasticity parameters by estimating equations the following three equations via non-linear least squares using annual data moments on expenditure shares and annual data on the price of consumption, investment, and intermediates produced by goods and services sectors:

$$\frac{P_{g-c,t}C_{g-c,t}}{P_{g-c,t}C_{g-c,t} + P_{s-c,t}C_{s-m,t}} = \frac{\omega_{Cg}P_{g-c,t}^{1-\epsilon^C}}{\omega_{Cg}P_{g-c,t}^{1-\epsilon^C} + (1-\omega_{Cg})P_{s-c,t}^{1-\epsilon^C}}$$

$$\frac{P_{g-x,t}X_{g-x,t}}{P_{g-x,t}X_{g-x,t} + P_{s-x,t}X_{s-x,t}} = \frac{\omega_{Xgj}P_{g-m,t}^{1-\epsilon^X}}{\omega_{Xg}P_{g-x,t}^{1-\epsilon^X} + (1-\omega_{Xg})P_{s-x,t}^{1-\epsilon^X}}$$

$$\frac{P_{g-m,t}M_{g-m,jt}}{P_{g-m,t}M_{g-m,jt} + P_{s-m,t}M_{s-m,jt}} = \frac{\omega_{Mgj}P_{g-m,t}^{1-\epsilon_j^M}}{\omega_{Mgj}P_{g-m,t}^{1-\epsilon_j^M} + (1-\omega_{Mgj})P_{s-m,t}^{1-\epsilon_j^M}}$$

The final calibrated parameter values for these CES aggregator parameters can be seen in Table 3.

To present the aggregate structural change in intermediates predicted by the model in Figure 5, we aggregate up structural change in intermediates occurring in each of the three sectors (as in equations (11) and (12)). In our model, because the production technologies are identical across sectors, each sector's total share of intermediates expenditures, $\frac{P_{jt}^M M_{jt}}{\sum_i P_{it}^M M_{it}}$, is simply equal to that sector's share of total gross output,

[47]This approach to aggregation is subject to the same aggregation bias concerns raised about price measurement in the previous subsection of this Appendix. However, there is no final expenditure analog to production data that allows us to have more precise measurement of these parameters. We leave explorations for improving such measurement for future work.

[48]With prices normalized to one in the year 1947, the value of these share parameters is exactly the share of consumption, investment, or intermediates produced by the goods sector in 1947.

$\frac{P_{jt}Q_{jt}}{\sum_i P_{it}Q_{it}}$. Using the resource constraint, we can write the nominal gross output of any sector j as:

$$\begin{aligned} P_{jt}Q_{jt} &= P_{jt}C_{jt} + \sum_{i=1}^{N} P_{jt}M_{jit} + \sum_{i=1}^{N} P_{jt}X_{jit} \\ &= \left(\xi_{jt} + \Lambda_{jt}\frac{X_t}{P_t^C C_t}\right) P_t^C C_t + \sum_{i=1}^{N} \Gamma_{jit}P_{it}Q_{it} \end{aligned}$$

where $\xi_{jt} = \frac{P_{jt}C_{jt}}{P_t^C C_t}$ equals the fraction of all consumption expenditures produced by sector j, $\Lambda_{jt} = \frac{\sum_{i=1}^{N} P_{jt}X_{jit}}{X_t}$ equals the fraction of all investment expenditures produced by sector j, and $\Gamma_{jit} = (1-\alpha)\frac{P_{jt}M_{jit}}{P_{it}^M M_{it}}$ is the fraction of all investment expenditures by sector i produced by sector j (scaled by $1-\alpha$).

In matrix notation, we can write this as:

$$\overrightarrow{PQ} = (I-\Gamma)^{-1}\left(\overrightarrow{\xi} + \overrightarrow{\Lambda}\frac{X_t}{P_t^C C_t}\right) P_t^C C_t.$$

Given that $P_t^C C_t$ is common to all sectors, all we need to obtain the gross output shares for each sector is Γ, $\overrightarrow{\xi}$, $\overrightarrow{\Lambda}$, and $\frac{X_t}{P_t^C C_t}$. With the exception of $\frac{X_t}{P_t^C C_t}$, the other objects are all outputs of the calibration procedure, determined solely by prices and CES parameters. The ratio of investment spending to consumption spending, $\frac{X_t}{P_t^C C_t}$, will be constant along the balanced growth path (given that investment and consumption expenditures grow at the same rate, as established in Proposition 1). Drawing from the details of the proof to Proposition 1 in Appendix B, we can write the equilibrium investment to consumption expenditures ratio as:

$$\frac{X_t}{P_t^C C_t} = \frac{\gamma^K - (1-\delta)}{\left(\frac{1}{\theta\beta}-1\right)\gamma^K - \left(\frac{1}{\theta}-1\right)(1-\delta)}.$$

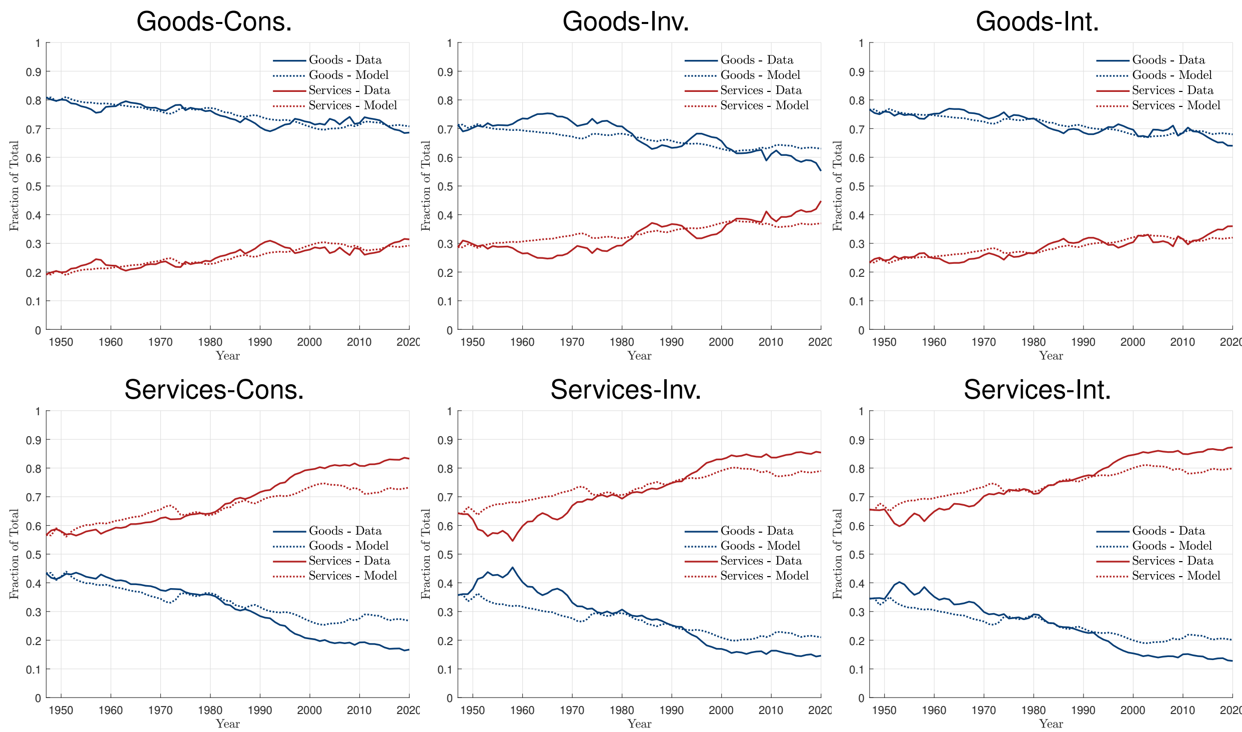
Given values of δ, θ, β, and γ^K we can compute this ratio and thus compute the gross output shares for each sector along the balanced growth path. Note that γ^K is not a parameter we can set in advance, but a realized equilibrium objects based on the technology processes fed into the model. Since these gross output shares are not needed to compute the balanced growth path growth rate, we compute these ex-post using the observed average growth rate of the capital stock along the balanced growth path implied by the technology series.[49]

Because the model has no production function heterogeneity along the aggregate balanced growth path, we can use gross output shares to aggregate intermediate expenditures in each sector. We then aggregate the comparison data presented in Figure 5 using gross output shares instead of intermediates expenditure shares. This generates a slight difference compared to the observed patterns of structural change in intermediates shown in Figure 3.[50]

[49]The variation in gross output shares induced by even sizable differences in the aggregate growth rate of capital is not particularly large, and thus these shares are robust to a wide range of potential technology series.

[50]If we were to consider a transition path exercise allowing for additional heterogeneity, the model would also have heterogeneity in intermediates expenditure shares within sectors and could be more directly compared to (and better replicate) the structural

Figure C.7: Model Calibration Fit to Structural Change Patterns in Intermediates, 6 Sector Level, 1947-2020



Notes: Each panel plots the fraction of intermediates purchased from goods (blue lines) and services (red lines) Data series are solid lines; model series are dashed lines.

Appendix C.5.2. Additional Calibration Results

Our baseline calibration involves six sectors, each with a unique distribution of intermediates spending across goods and services intermediates. In Section 5, we reported the calibration fit to the model for aggregate intermediates expenditures. Here, we first report the model's fit to intermediate expenditure patterns in all six sectors and how the calibration compares to the shift-share decomposition exercises presented in Section 2.

In Figure C.7, we present the fit of the calibration to the share of intermediates purchased from goods and services sectors in each of our six sectors. In general, the model fit to structural change in intermediates is better in our three goods sectors before 2009; the model series account for 90% of the structural change in intermediates for goods sectors over through 2009, compared to 69% for services sectors. However, the decline in fit quality post-2009 appears to be primarily concentrated among goods sectors. The overall fit to structural change patterns through 2020 is similar across sectors, with the model explaining 2/3 of intermediates structural change in both goods and services over the entire sample.

In Table C.2, we compare a shift-share decomposition of our model's results for structural change in investment to a comparable shift-share decomposition on the data. Although the model does not replicate the entire rise in the share of intermediates produced by services, it does closely match the composition of changes over time due to within and between sector forces. As noted above in Appendix C.5.1, when

change patterns documented for intermediates in Section 2.

Table C.2: Shift-Share Decomposition of Services Share of Production of Intermediates, Model vs. Data

				Decomposition	
	1947	2019	Δ	within	between
Data	0.39	0.73	0.33	0.20 (59%)	0.14 (41%)
Model	0.41	0.62	0.21	0.12 (59%)	0.09 (41%)

Notes: The table reports the results of a shift-share decomposition of the share of services production of intermediates over time. See notes to Table 2 for details. However, data figures are different from those in Table 2 to be consistent with aggregation in the model; see the discussion in Appendix C.5.1.

comparing the model results to the data, we aggregate intermediate expenditures in each sector slightly differently than in the empirical results presented in Appendix C. Because there is no heterogeneity in the share of total intermediate spending in gross output across sectors, we use gross output shares to aggregate intermediate expenditure shares in the data; this accounts for the fact that the between sector contribution falls from 47% in Table 2 to 41% in Table C.2.

Appendix D. Details/Extensions to Growth Accounting Exercises

Appendix D.1. Measuring Technical Change

Here we provide additional details for how we measure the exogenous technical change processes that we feed into our model in Section 6.

First, we measure growth in intermediates bundling TFP using a log first-order approximation of the equilibrium intermediate input price (equation (8)), for years t and $t-1$.[51] Formally, we use the following expression for growth in intermediates bundling TFP for sector i:

$$\Delta \ln(A_{it}^M) = -\left(\Delta \ln(P_{it}^M) - \sum_{j=g-m,s-m} \left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^M M_{it}}}\right) \Delta \ln(P_{jt})\right) \tag{D.1}$$

where $g{-}m$ and $s{-}m$ are the goods-intermediates and services-intermediates sectors respectively, $\left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^M M_{it}}}\right)$ is the average between t and $t-1$ of the share of intermediate spending by sector i purchased from sector j (measured as described in Appendix C), $\Delta \ln(P_{it}^M)$ is the price growth in the bundle of intermediates purchased by sector i, and $\Delta \ln(P_{jt})$ is the price growth in intermediates produced by sector j.

We measure $\Delta \ln(A_{it}^M)$ using BEA GDP by Industry data on the price of intermediate bundles by sector, aggregated up to the 6 sector level. The BEA only publishes price indices for the entire bundle of intermediates, not for individual intermediate inputs; thus, this data is not directly useful for measuring intermediates prices. Furthermore, these price indices are computed using gross output prices, meaning they may be subject to the aggregation bias concerns described in Section 5. However, the BEA uses internal data on gross output prices which are much more disaggregated than what is publicly available and consistently observ-

[51] Herrendorf et al. (2021) use a similar approach to measuring exogenous investment TFP in their paper.

able across time, which ameliorates some concern about bias in measurement of these intermediate price bundles.

To aggregate up intermediate prices bundles observed at the 40 sector level to the 6 sector level, we construct a weighted average of intermediate bundle price growth across goods or services sectors weighted by both that sector's total intermediate expenditures and role as a producer of consumption, investment or intermediates. For example, growth in the intermediate price bundle for services-consumption sector, $\Delta ln(P^M_{s-c,t})$ is aggregated up according to the equation:

$$\Delta ln(P^M_{s-c,t}) = \sum_{i \in \{Services\}} \frac{\frac{P_{it}C_{it}}{\sum_{k \in \{Services\}} P_{kt}C_{kt}} P^M_{it} M_{it}}{\sum_{l \in \{Services\}} \frac{P_{lt}C_{lt}}{\sum_{k \in \{Services\}} P_{kt}C_{kt}} P^M_{lt} M_{lt}} \Delta ln(P^M_{it})$$

where $P_{it}C_{it}$ is the value of consumption produced by sector i, $P^M_{lt} M_{lt}$ is the value of intermediates purchased by sector i, and P^M_{it} is the price of the intermediates bundle purchased by sector i. Similar to how we measure prices for consumption, intermediates, and investment, we adjust the intermediates price bundle for each to remove the impact of oil/energy price fluctuations. We do this by subtracting off the price of omitted sectors' output in proportion to each sector's use of omitted sectors as intermediates and by subtracting off a correction term for all other sectors' gross output prices (in proportion to each sector's use as an intermediate).

We measure sectoral TFP so that it is consistent with relative prices in the model, similar to García-Santana et al. (2021). That is, for five of our six sectors, we invert equation (15) and back out the sectoral TFP series that generate the relative price series observed in the data. To do this, we use the observed sectoral prices to also construct the intermediate inputs bundle prices implied by the model, as in equation (8), divided by the price of investment (the numeraire), defined by (9). However, since relative prices can only define technical change for all but one sector, we construct the TFP series for the services-consumption sector as a weighted average of Solow residuals from all 40 sectors in our data, where the weights correspond to each sector's share in producing services consumption as a final commodity. To construct the Solow residual for each of our 40 sectors we follow the approach of vom Lehn and Winberry (2022) and compute real gross output net of the primary inputs in log differences.

Formally, we construct sectoral TFP growth at the 40 sector level as:

$$\Delta \ln(A_{jt}) = \Delta \ln(Q_{jt}) - \alpha_j \theta_j \Delta \ln(K_{jt}) - \alpha_j(1 - \theta_{jt}) \ln L_{jt} - (1 - \alpha_j) \ln M_{jt}. \quad \text{(D.2)}$$

Measures of real gross output, real intermediates, and employment are taken from the BEA GDP by Industry database and the real capital stock is constructed using the perpetual inventory method using sectoral real investment, implied depreciation rates, and initial values of the capital stock from the BEA Fixed Asset Accounts. Values of the production parameters are assigned using evidence on cost shares observed in BEA Input Output data—α_j using the ratio of nominal value added to nominal gross output in each sector and $1 - \theta_j$ using the sectoral share of compensation in value added. To more precisely isolate changes in technical change from changes in the production technology or depreciation rates, we allow α_j, θ_j, and δ_j to vary over time in measurement. For the growth rate of TFP in any two years, we use the average of cost shares or implied depreciation rates for those two years. We normalize the level of all technological change terms to be 1 in 1947.

We then construct growth in sectoral TFP for the services-consumption sector by averaging the growth rates of sectoral TFP for all services, weighted in proportion to each sector's share of consumption production (among services sectors).

Table D.1: Aggregate GDP Growth, Cobb-Douglas Counterfactuals, 1947-2020

	Scaled Aggregate GDP Growth: $\Delta\ln(x) = \Delta\ln Y_t^{Index}$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%
Baseline	0.96	100	0.11	100	0.59	100	0.23	100
Investment Cobb-Douglas	0.90	94	0.11	96	0.58	98	0.18	79
Intermediates Cobb-Douglas	1.03	107	0.12	108	0.64	109	0.23	100
Consumption Cobb-Douglas	1.03	107	0.12	106	0.60	102	0.27	118

Notes: The table shows long-run log changes in aggregate GDP measured as an index number, $\Delta\ln(Y_t^{Index})$, following equation (29), for different time periods. The table also reports counterfactuals for the cases where either investment, intermediates, or consumption aggregation is Cobb-Douglas, ruling out structural change in each network or the composition of consumption. For each time period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

Appendix D.2. Additional Growth Accounting Results

In this section, we present two additional results. First, we report an extension of Table 4 and Table 6 where we report the composition of aggregate TFP (GDP) growth decade by decade, further highlighting the rising importance of investment-specific technical change over time. Second, we report an extension of Table 5 where we report counterfactuals with a unitary elasticities of substitution (i.e. Cobb-Douglas) in our GDP index number growth accounting framework (introduced in equation (29)).

Panel A of Table D.2 shows decade by decade log changes in aggregate TFP, $\mathcal{A}_t$, and in the same sets of counterfactuals presented in Table 4—investment-specific technical change, intermediates-specific technical change, and only intermediates-bundling technical change. Panel B reports decade by decade log changes in the aggregate GDP Index, Y_t^{Index}, and in the same sets of counterfactuals as presented in Table 6—investment-specific, intermediates-specific, and consumption-specific technical change. In some decades, log changes in GDP or its components are very low and/or negative, complicating interpretation; this is especially true in the years 1950-1960 and 1970-1980. However, we observe a clear upward trend in the importance of investment-specific technical change over time, explaining as much as 90% of all growth between 2010-2019. In particular, notice that intermediates-specific technical change contributes negatively during the 2010s while investment-specific technical change overcompensates for the missing growth from the intermediates network.

Table D.1 shows the counterfactual log changes in GDP growth (measured as an index number) and its components where there is a unitary elasticity of substitution in different aggregation technologies – consumption, investment, and intermediates. In the counterfactuals featuring unitary elasticities of substitution, GDP growth over the entire sample 1947-2019 would be either 7% higher (when consumption or intermediates are Cobb-Douglas) or 6% lower (when investment is Cobb-Douglas). We also see that the importance of reallocation forces due to non-unitary elasticities is rising in recent decades—roughly 20% of aggregate GDP growth since 2000 is attributable to the reallocation of resources within investment, and growth would have been 20% higher over this period had there been no reallocation of resources within consumption.

Table D.2: Growth Decomposition by Decade, 1950-2020

	1950s		1960s		1970s		1980s		1990s		2000s		2010s	
Sources	Δ	%	Δ	%	Δ	%	Δ	%	Δ	%	Δ	%	Δ	%
A. Growth Decomposition of Aggregate TFP along the ABGP: $\Delta = \frac{1}{1-\theta}\Delta \ln \mathcal{A}_t$														
All	0.05	100	0.19	100	-0.06	100	0.36	100	0.44	100	0.28	100	0.10	100
Investment-Specific	0.05	117	0.11	60	-0.03	56	0.23	64	0.28	63	0.22	79	0.09	90
Intermediates-Specific	-0.01	-17	0.08	40	-0.03	49	0.14	37	0.17	39	0.06	21	0.01	9
Intermed. Bundling	-0.00	-8	0.03	15	0.03	-42	-0.00	-1	0.04	10	0.10	34	0.04	37
B. Growth Decomposition of the GDP Index: $\Delta = \Delta \ln Y_t^{Index}$														
All	0.05	100	0.13	100	-0.02	100	0.26	100	0.32	100	0.17	100	0.06	100
Investment-Specific	0.03	60	0.06	42	-0.02	80	0.12	45	0.14	43	0.11	67	0.05	79
Intermediates-Specific	-0.01	-17	0.05	37	-0.05	216	0.13	50	0.13	40	-0.01	-4	-0.01	-23
Consumption-Specific	0.03	57	0.03	21	0.04	-192	0.02	6	0.06	17	0.06	37	0.03	44

Notes: The Table reports decade-by-decade versions of Tables 4 (panel A) and 6 (panel B).