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Abstract

This paper explores the relationship between market accessibility and various participants' welfare in an intermediated directed-search market. For a general class of meeting technologies, we provide a necessary and sufficient condition under which efficiency requires imperfect accessibility, such that each seller's listing is only observed by some but not all buyers. We show that the platform optimally implements the efficient outcome, but fully extracts surplus from the transactions it intermediates. We also find that in general, buyers prefer to minimize market accessibility, while sellers prefer a weakly greater accessibility level than that which is socially efficient. The efficiency of imperfect accessibility is robust to the introduction of a second chance for unmatched buyers to search.

JEL-Codes: D830, J640, M370.

Keywords: meeting technology, directed search, platform, intermediation, accessibility.

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1 Introduction

In many search markets mediated by digital platforms such as labour, housing, or everyday task outsourcing, participants often face a strict cap on the number of options they may consider. For example, Oneflare, an online marketplace for service providers (e.g., plumbers, electricians, pet groomers and interior designers), allows a customer who posts a job request to be approached by a maximum of only three service providers. This is not very surprising if displaying a large number of options substantially increases the platform's cost, as with traditional means of advertising. However, digital platforms' costs of managing meetings between participants are extremely low, if not zero. Instead, given that platforms as profit-maximizing intermediaries, it is more likely that participants' market accessibility is limited due to strategic reasons.

The goal of this paper is to explore the relationship between market accessibility and various participants' welfare in the presence of search frictions. Is it good or bad for overall efficiency if buyers' search opportunities are deliberately limited? If a profit-maximizing platform intermediates transactions, what level of market accessibility should it choose, and will the resulting allocation be efficient? Do buyers and sellers always prefer better accessibility to the market, given that market volume is often touted as a key means of overcoming search friction? We answer these questions by studying a parsimonious duopoly model with homogeneous goods and directed search frictions, but expect the main mechanism we uncover to be also in effect in more general environments.

The key ingredient of our model is the platform's endogenous choice of meeting technology. The platform chooses from a general set of meeting technologies that determine not only the allocation of meeting opportunities with sellers among buyers, e.g., which buyers observe a seller's ads, but also the total number of meeting opportunities with each seller.[1] As a consequence, the platform's choice of meeting technology directly determines the *market accessibility*, i.e., the number of buyers who observe an individual seller. This differs from standard directed-search models (e.g., Peters, 1984a, Julien et al., 2000, Burdett et al., 2001), where market accessibility is restricted to being equal to the total number of buyers. Through this, we are able to delineate the role of market accessibility on market participants' welfare.

Several key assumptions on the platform's choice of meeting technology connect

[1] The terminology "meeting" has been used in the literature of directed search, e.g., Eeckhout and Kircher (2010). See Section 6.2 for the detailed discussion.

our analysis to the circumstances faced by platforms in reality. First, there is no waste in the allocation of meeting opportunities: each buyer is not given more than one meeting opportunity with the same seller. This contrasts sharply with classic advertisement models (e.g. Butters, 1977) where buyers can receive multiple "'wasted" ads from the same seller, and better reflects how online platforms' algorithms minimize duplication in ad views by buyers.[2] Second, meeting opportunities with sellers are assigned among buyers either jointly, i.e., as a pair, or separately. We take the share of each type of meetings exogenously. One interpretation of this is that some customers have pre-existing relationships with both sellers, which results in a joint meeting opportunity. Otherwise, i.e., among the buyers without such pre-existing relations, meetings with each seller are allocated separately and independently. Another interpretation would be that some buyers are attentive, who pay attention to two prices when both are present, but others are inattentive, who pay attention to only one price even if both are present. Meetings with attentive buyers correspond to joint meetings, whereas meetings with inattentive buyers correspond to separate meetings, where the latter can still observe both prices if the information is presented to them repeatedly.

Our analysis uncovers a novel channel through which changes in the market accessibility of the platform's meeting technology help to mitigate endogenous search frictions. An unmatched seller exists if and only if (i) every buyer allocated at least one meeting opportunity observes both sellers, i.e., is fully informed, and (ii) all buyers select the same seller. A higher level of market accessibility reduces the likelihood of (ii), increasing efficiency. Meanwhile, its effect on (i) is non-monotone. Due to the no waste and no coordination properties, allocating more meeting opportunities is unlikely to generate more fully informed buyers if there are currently many uninformed buyers, which occurs when the market accessibility level is low. Conversely, allocating more meeting opportunities is likely to generate more fully informed buyers if the current market accessibility level is high.

We find that an intermediate level of market accessibility is often socially efficient, providing a good balance in minimizing the probabilities of events (i) and (ii). In particular, we show that perfect market accessibility is efficient if and only if there are relatively many joint meetings. This insight extends to the case when unmatched buyers are allowed to search one more time. There, we show that imperfect accessibility is efficient if the number of buyers is sufficiently large. As a

[2] This assumption is also relevant to many of the previously stated examples, e.g. buyers often never receive duplicate job interviews or house selling information.

direct implication of these findings, policies that promote greater accessibility to the marketplace may not always work in the direction of improving efficiency.

Next, we show that a profit-maximizing platform will choose the efficient level of market accessibility, which is often imperfect. This is consistent with anecdotal evidence such as the Oneflare example mentioned earlier and the recent empirical findings in the online labour market.[3] Through a field experiment, Horton and Vasserman (2021) find that some jobs receive too many applications and a cap on the application number would significantly reduce such congestion—a worker's hiring rate conditional on applying to a given job, would increase by 17%.

We also characterise the market accessibility levels that are optimal for buyers and sellers, and show that these often diverge from the efficient accessibility level. On one hand, buyers prefer a minimal level of market accessibility. This is because it allows buyers not only to avoid competition with each other but also to induce sellers to more aggressively compete for the smaller pool of buyers. On the other hand, sellers prefer a weakly greater level of market accessibility than the efficient level. Here, the reverse logic applies: higher levels of market accessibility increase the probability of greater competition between buyers for the seller's product, leading to increased profits. To our knowledge, we are the first to offer such a characterisation of buyer- and seller-optimal market accessibility levels in markets with directed-search frictions.

Two recent branches of the directed search literature are relevant. The first branch addresses the issue of market accessibility. These include Peters (1984b) who allows sellers to send costless advertisements, Lester (2011) who introduces heterogeneous search costs so that some buyers can observe all posted prices and other buyers can only observe one price, and Gomis-Porqueras et al. (2017) who study costly stochastic advertising that generates dispersed information among buyers. None of these papers establish the relationships between market accessibility and various welfare measures, which is the main focus of our paper. For example, Lester (2011) does not allow for uninformed buyers who do not observe any sellers at all, and so an increase in market accessibility cannot reduce the number of uninformed buyers, which is an important margin for efficiency in our model. In contrast, our approach directly allows the platform to determine the level of market accessibility via the choice of meeting technology which can be measured by a single parameter.

The second branch, of directed-search literature, including Kennes and Schiff

[3]Li and Netessine (2019) show that on an online peer-to-peer holiday property rental platform, doubling market size leads to a 5.6% *reduction* of matches.

(2008) and Gautier et al. (2019), considers fee-setting intermediaries but does not study the implication of market accessibility. Fee-setting intermediaries have been systematically studied in the literature of two-sided markets, e.g., Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006). A recent trend is to incorporate buyer search in the study of intermediaries as we do in this paper. That includes sequential search models pioneered by Wolinsky (1986) and Anderson and Renault (1999), and those in Eliaz and Spiegler (2011), de Cornière (2016), Wang and Wright (2016, 2020) and Teh and Wright (2020). Our paper complements these works by focusing on trade-offs associated with the other important choice made by platforms: the level of market accessibility.

In different contexts, several papers show that in multi-sided markets, limiting one side's access to information about the other side can improve matching efficiency. These include Calvó-Armengol and Zenou (2005) in the context of job network formation, Casadesus-Masanell and Halaburda (2014) for network goods, Halaburda et al. (2018) in the presence of competing dating platforms, and Glebkin et al. (2021) for financial intermediaries in over-the-top (OTC) markets. Our approach points to a novel source of limiting participants' choices to improve efficiency: full market accessibility can give rise to an excessive amount of search externalities and a less-than-efficient number of matches.[4]

Finally, our paper is related to the burgeoning literature that studies the information design problem of platforms, often in an environment with differentiated products but without search friction. The recent contributions include Armstrong and Zhou (forthcoming), Johnson et al. (2020), and Teh (2020).[5] In our model, the platform's design choice is with respect to the market accessibility level. We contribute to the literature by showing that, in the presence of directed search friction, the efficient accessibility level in a homogenous good market can still be decentralized by the platform' profit-maximizing choice.

[4]A large body of literature, pioneered by Diamond (1982), considers exogenous matching functions with increasing returns, resulting in thin market externalities (for a recent survey, see Stevens, 2007). The search externality, however, is endogenous in a directed-search framework, and we show that it can also be affected by the market accessibility level.

[5]See also Armstrong and Vickers (2019), Bergemann et al. (2021), and Shi and Zhang (2020) who study market segmentation through information provision in models where sellers sell homogeneous goods and can price discriminate between captive and contested buyers.

2 The model

A platform offers a unit mass of symmetric and independent product categories. Within each product category, two sellers sell homogenous products. This market structure is consistent with the observation that, although platforms often list many items, competition only exists among a small number of sellers.[6] We can thereby focus on a representative product category with two sellers, indexed by $s \in \{1, 2\}$, and $B \geq 3$ homogeneous buyers.[7] Each seller is endowed with one unit of the good, with the consumption value normalized to one for buyers and zero for sellers. Buyers and sellers can trade only through the platform that charges a per-transaction seller fee f, and offers a meeting technology that determines which buyer observes which seller. We shall refer to a buyer who observes both sellers as a *fully informed* buyer, a buyer who observes one seller as a *partially informed buyer*, and a buyer who does not observe any seller as an *uninformed buyer*.

□ **Trading protocols**. Sellers sell goods using first-price auctions.[8] Each seller $s \in \{1, 2\}$ posts a reserve price denoted by r_s. The reserve price r_s is honoured only if exactly one buyer participates in seller s's auction. If more than one buyer participates, the participating buyers bid for trade. If multiple buyers submit the same highest bid then each of them obtains the product with equal probabilities. Auctions with reserve prices capture the idea that on many digital platforms sellers only have limited commitment power with respect to the posted prices.

When attending the auction of seller s, a buyer's bidding strategy depends on the posted reserve price, r_s, and the observed number of buyers at seller s, denoted by m_s. Bertrand type of reasoning yields the optimal bidding strategy for buyers

$$\tilde{b}(r_s, m_s) = \begin{cases} r_s & \text{if} \quad m_s = 1; \\ 1 & \text{if} \quad m_s > 1. \end{cases} \tag{1}$$

[6]This modelling approach was also adopted for example by Karle et al. (2020). In the example of Oneflare mentioned in the introduction, while there can be many registered customers, only a small number of them will request assistance in a specific neighbourhood on a specific day.

[7]The model's tractability worsens substantially if we allow for more than two sellers. This is as with more than two sellers, a partially informed buyer may know more than one seller, and so has a non-trivial decision of which seller to visit. Nevertheless, in the Online Appendix (here), we consider a three seller, three buyer example, and show the key insights we derive continue to hold.

[8]A second-price auction will yield the same outcome in this environment.

Given (1), Seller s's realized profit is then

$$\tilde{\pi}(r_s, m_s) = \begin{cases} 0 & \text{if } \; m_s = 0; \\ r_s & \text{if } \; m_s = 1; \\ 1 & \text{if } \; m_s > 1. \end{cases}$$

□ **Buyers' search**. As common in the directed-search framework, a buyer can attend at most one seller's auction. The buyer can do so only if she observes this seller, and in the case where she observes two sellers, she needs to decide which seller to select. Buyers cannot coordinate with each other over which seller to select. In particular, we assume fully informed buyers use symmetric strategies following an observed pair (r_1, r_2). The possible mis-coordination among buyers represents the endogenous search (or coordination) frictions.[9]

□ **Meeting technology**. Each seller can be observed by N buyers. That is, there are N meeting opportunities among buyers with each seller. Meeting opportunities with each seller can be allocated either jointly, i.e., a meeting opportunity with seller 1 is bundled with a meeting opportunity with seller 2 and allocated to one buyer, or separately, i.e., a meeting opportunity with only one seller $s \in \{1, 2\}$ is allocated to a buyer. The platform's meeting technology determines both the total number of meeting opportunities N, and how joint and separate meeting opportunities are allocated among buyers. The platform chooses the meeting technology subject to always allocating an exogenously-determined $N_J \in \{0, ..., B\}$ number of joint meeting opportunities[10] (so $N - N_J$ is the number of separate meeting opportunities with each seller), and other constraints imposed by coordination frictions.

More precisely, a meeting opportunity allocation for a buyer $b \in \{1, ..., B\}$ is captured by the triplet $\boldsymbol{n}^b \equiv (n_1^b, n_2^b, n_J^b)$, where $n_s^b \in \{0, .., N\}$ is the number of separate meeting opportunities with seller $s \in \{1, 2\}$ allocated to b, and $n_J^b \in \{0, .., N\}$ is the number of joint meeting opportunities allocated to b. A vector of meeting-opportunity allocations across buyers is denoted by $\boldsymbol{n} = (\boldsymbol{n}^1, ..., \boldsymbol{n}^B)$. With a slight abuse of notation, we also write $\boldsymbol{n} = (\boldsymbol{n}_1, \boldsymbol{n}_2, \boldsymbol{n}_J)$, where $\boldsymbol{n}_s \equiv (n_s^1, ..., n_s^B)$ is the allocations of separate meeting opportunities with seller s, and $\boldsymbol{n}_J \equiv (n_J^1, ..., n_J^B)$ is the allocation of joint meeting opportunities across buyers. Then, the set of

[9]See Wright et al. (2021) for a comprehensive overview of this literature and the rationale for assuming this type of friction.

[10]Refer to Section 1 for an interpretation of this exogenous constraint on the allocation of joint meeting opportunities.

possible allocations of N meeting opportunities, subject to always allocating N_J number of joint meeting opportunities, is

$$\mathcal{N}_{N,N_J} \equiv \{\boldsymbol{n} : \sum_{b=1}^{B}(n_1^b + n_J^b) = \sum_{b=1}^{B}(n_2^b + n_J^b) = N \text{ and } \sum_{b=1}^{B} n_J^b = N_J\}$$

A *meeting technology* is a pair (N, P), where $N \in \{N_J, .., B\}$, and $P \in \Delta(\mathcal{N}_{N,N_J})$ is a distribution over allocations with N total meeting opportunities. Given $\boldsymbol{n}_J$, let $P(\boldsymbol{n}_1, \boldsymbol{n}_2 | \boldsymbol{n}_J)$ denote the marginal probability of allocating separate meeting opportunities $(\boldsymbol{n}_1, \boldsymbol{n}_2)$ among sellers, and $P_s(\boldsymbol{n}_s|\boldsymbol{n}_J)$ the marginal probability of allocating separate meeting opportunities $\boldsymbol{n}_s$ with seller s. We restrict the platform to choosing among meeting technologies that satisfy the properties listed in Assumption 1 below. These impose restrictions on the platform's algorithm for allocating meeting opportunities. Roughly speaking, they require that the probability for a buyer to meet a seller cannot depend on their identities, any buyer cannot have more than one meeting opportunity with a given seller, and the algorithm does not completely eliminate search frictions.

Assumption 1. (N, P) *satisfies the following three properties.*

1. **Symmetry**. *For all* $(\boldsymbol{n}^1, ..., \boldsymbol{n}^B) \in \mathcal{N}_{N,N_J}$ *and permutations* g *of* $\{1, .., B\}$, $P(\boldsymbol{n}^1, ..., \boldsymbol{n}^B) = P(\boldsymbol{n}^{g(1)}, .., \boldsymbol{n}^{g(B)})$.

2. **No Waste**. *For all* $(\boldsymbol{n}^1, ..., \boldsymbol{n}^B) \in \text{supp}(P)$, $b \in \{1, .., B\}$ *and* $s \in \{1, 2\}$, $n_s^b + n_J^b \in \{0, 1\}$.

3. **No Coordination**. *For all* $(\boldsymbol{n}_1, \boldsymbol{n}_2, \boldsymbol{n}_J) \in \mathcal{N}_{N,N_J}$, $P(\boldsymbol{n}_1, \boldsymbol{n}_2|\boldsymbol{n}_J) = P_1(\boldsymbol{n}_1|\boldsymbol{n}_J) \times P_2(\boldsymbol{n}_2|\boldsymbol{n}_J)$.

The *symmetry* assumption implies that the meeting technology treats the (homogeneous) buyers identically, so the associated distribution P over allocations is invariant to permutations of buyers. Symmetry also rules out the trivial case that the buyers' probability of receiving meeting opportunities with each seller can depend on their identity, which helps remove search frictions. The *no waste* assumption ensures that each buyer is allocated at most one meeting with each seller. In particular, this rules out the possibility that a buyer may be allocated both a joint meeting and a separate meeting with some seller. Finally, the *no coordination* assumption preserves the endogenous search frictions: separate meeting opportunities are allocated independently across sellers, ruling out the possibility for buyers to coordinate their strategies in selecting sellers. The no-waste assumption is natural, given our focus on platforms whose algorithms can prevent buyers from viewing duplicate ads, while the other two assumptions are commonly assumed in the directed-search literature. Further implications of Assumption 1 are discussed in Section 6.2.

Given non-negative integers $x > y$, let $C_x^y \equiv x!/(y!(x-y)!)$. Further let $\widehat{\mathcal{N}}_{N,N_J} \subseteq \mathcal{N}_{N,N_J}$ denote the subset of allocations of meeting opportunities which incorporates the no-waste assumption, i.e.,

$$\widehat{\mathcal{N}}_{N,N_J} \equiv \left\{ \boldsymbol{n} \in \mathcal{N}_N : \exists \mathcal{B} \subseteq \{1, ..., B\} \text{ s.t. } \begin{array}{c} |\mathcal{B}| = N_J, \\ \forall b \in \mathcal{B},\ n_s^b = 0 \text{ and } n_J^b = 1, \\ \forall b \in \{1, .., B\} \backslash \mathcal{B},\ n_s^b \in \{0,1\} \text{ and } n_J^b = 0 \end{array} \right\} \tag{2}$$

In Proposition 6 in the Appendix, we show that there exists a unique meeting technology (N, P_{N,N_J}) that satisfies Assumption 1, where

$$P_{N,N_J}(\boldsymbol{n}) = \begin{cases} \frac{1}{(C_B^{N_J})(C_{B-N_J}^{N-N_J})^2}, & \boldsymbol{n} \in \widehat{\mathcal{N}}_{N,N_J} \\ 0, & \boldsymbol{n} \notin \widehat{\mathcal{N}}_{N,N_J} \end{cases} \tag{3}$$

This result has two implications. First, the platform's choice of meeting technologies reduces to choosing among the class $\{(N, P_{N,N_J})\}_{N=N_J}^B$. That is, for a given N_J, the platform's choice of meeting technology can be fully captured by the parameter N. Second, N exactly coincides with the number of buyers who observe a given seller. Put differently, under Assumption 1, there is a one-to-one relationship between the platform's choice of meeting technology, and that of the market accessibility level. Hence, we refer to N as the *accessibility level* of the platform's meeting technology, $N = B$ as *perfect accessibility* and $N < B$ as *imperfect accessibility.*

In practice, meeting technologies of the form described in Proposition 6 can be implemented via a two-stage process. First, the platform identifies a random subset of N_J number of buyers among all B buyers, and allocates to each such buyer a joint meeting opportunity. Second, for seller 1, the platform identifies a random subset of $N - N_J$ number of buyers among the remaining $B - N_J$ buyers, and allocates to each such buyer a meeting opportunity with seller 1. The process is then repeated to allocate the separate meeting opportunities with seller 2.

Two prominent examples from the directed search literature fit the requirements we impose above on meeting technologies. The first, which we call *fully-separate* *meeting technology*, concerns the case when $N_J = 0$. There, all N meeting opportunities with a seller are allocated independently (and uniformly randomly) among buyers. This meeting technology captures real-world situations such as job interview scheduling,[11] and is the reminiscence of the matching technology where the short side of the market is always cleared (Stevens, 2007). The second, which we call *fully-joint meeting technology*, concerns the meeting process which arises when $N_J \geq 1$ and the platform chooses $N = N_J$. There, every meeting opportunity with seller 1 is always jointly allocated with a meeting opportunity with seller 2. One example can be price comparison sites, which often return a search result containing multiple listings that buyers can see at once. By allowing for the fixed number of joint meetings N_J to be strictly between 0 and N, the meeting technologies we consider generalize these examples.

□ **Timing of the game**. The timing of the game is as follows. In the first stage, the platform publicly sets a seller fee f and chooses the meeting technology, i.e., the accessibility level. In the second stage, buyers and sellers decide whether to join the platform. Each participating seller s sets a reserve price r_s. Then, given the accessibility level chosen by the platform, participating buyers' information regarding sellers, i.e., the meeting opportunities allocated to each buyer, are realized. Fully or partially informed buyers choose a seller to visit. Finally, sellers and informed buyers trade using auctions. The equilibrium concept we use is subgame-perfect Nash equilibrium.

3 Efficient market accessibility

Consider the problem of a social planner who aims to maximize expected total surplus, subject to directed search frictions, and to choosing a meeting technology that satisfies Assumption 1. Holding fixed a number of joint meeting opportunities $N_J \in \{0, ..., B\}$, the discussion in Section 2 implies that the planner's problem is equivalent to selecting an accessibility level $N \in \{N_J, ..., B\}$ to maximize the expected total number of matches between sellers and buyers.

Let m_s denote the number of buyers who select seller s. Then, the expected total number of matches is given by

$$\text{Pr.}[m_1 \geq 1] \cdot \text{Pr.}[m_2 = 0] + \text{Pr.}[m_1 = 0] \cdot \text{Pr.}[m_2 \geq 1] + 2 \cdot \text{Pr.}[m_1 \geq 1] \cdot \text{Pr.}[m_2 \geq 1].$$

[11] Each job seeker is scheduled, at most, to attend one interview for each vacancy, and the interviews for vacancy 1 are separately scheduled from the interviews for vacancy 2.

To compute these probabilities, we define $\Gamma_{N_J}(k|N,B)$ as the probability of having $k = 0,...,N$ fully informed buyers when there are in total B buyers and N $(= 0,1,...,B)$ meeting opportunities with each seller, among which N_J $(= 0,1,...,N)$ are joint meeting opportunities. By definition, any meeting technology generates at least N_J fully informed buyers. Meanwhile, the probability of having N fully informed buyers is given by the aggregate probability of drawing an allocation of meeting opportunities where every buyer who is allocated a separate meeting opportunity with seller 1 is allocated a separate meeting opportunity with seller 2. Applying (3), this is[12]

$$\Gamma_{N_J}(N|N,B) = \sum_{\boldsymbol{n}\in\widehat{\mathcal{N}}_{N,N_J}:\boldsymbol{n}_1^b=\boldsymbol{n}_2^b\ \forall b\in\mathcal{B}} P_{N,N_J}(\boldsymbol{n}) = \frac{1}{C_{B-N_J}^{N-N_J}}. \tag{4}$$

The number of fully informed buyers is critical in determining market efficiency. If there are N fully informed buyers, which occurs with probability $\Gamma_{N_J}(N|N,B)$, all buyers randomize over which seller to visit, such that each buyer visits each seller with probability $1/2$. There will be only one match with probability $2(1/2)^N = (1/2)^{N-1}$, and two matches with probability $1-(1/2)^{N-1}$. If there are less than N fully informed buyers, i.e., there exist partially informed buyers, then there is at least one buyer who observes only seller 1 and another buyer who only observes seller 2. Hence, if partially informed buyers exist, which occurs with probability $1-\Gamma_{N_J}(N|N,B)$, each seller meets at least one buyer and there will be two matches with probability one. As a result, the accessibility level N generates the expected total number of matches as

$$\begin{aligned} T_{N_J}(N) &\equiv \Gamma_{N_J}(N|N,B)\left[\left(\frac{1}{2}\right)^{N-1} + 2\left(1-\left(\frac{1}{2}\right)^{N-1}\right)\right] + 2(1-\Gamma_{N_J}(N|N,B)) \\ &= 2\left[1-\left(\frac{1}{2}\right)^{N}\Gamma_{N_J}(N|N,B)\right]. \end{aligned} \tag{5}$$

[12] For obtaining an allocation $\boldsymbol{n}_J$ with N_J joint meetings, there are $C_B^{N_J}$ possibilities. Then, there are a total of $C_{B-N_J}^{N-N_J}$ possible allocations of $N-N_J$ separate meetings with each seller among the remaining $B-N_J$ buyers such that each buyer who obtains a separate meeting with seller 1 also obtains a separate meeting with seller 2. Hence,

$$\sum_{\boldsymbol{n}\in\widehat{\mathcal{N}}_{N,N_J}:\boldsymbol{n}_1^b=\boldsymbol{n}_2^b\ \forall b\in\mathcal{B}} P_{N,N_J}(\boldsymbol{n}) = C_B^{N_J}\times C_{B-N_J}^{N-N_J}\times\left(\frac{1}{(C_B^{N_J})(C_{B-N_J}^{N-N_J})^2}\right) = \frac{1}{C_{B-N_J}^{N-N_J}}$$

Let

$$N^e_{N_J} = \underset{N \in \{N_J, \ldots, B\}}{\arg\max}\; T_{N_J}(N) \tag{6}$$

denote the efficient accessibility level given N_J, and let $\lfloor x \rfloor$ denote the greatest non-negative integer smaller than $x \in \mathbb{R}_+$.

Proposition 1. *The efficient accessibility level is unique and is given by*

$$N^e_{N_J} = \left\lfloor \frac{2(B+1)+N_J}{3} \right\rfloor \tag{7}$$

The key implication of Proposition 1 is that perfect market accessibility is often inefficient unless sufficiently many meeting opportunities are joint.

Corollary 1. $N^e_{N_J} = B$ *if and only if* $N_J \geq B-2$.

Observe that for large B, $N^e_{N_J} < B$ holds for a wide range of N_J. Why then does market efficiency often call for imperfect accessibility? Search frictions can lead to less than two total matches only when exactly N buyers are fully informed. The probability of having N fully informed buyers is given by the term $\Gamma_{N_J}(N|N,B)$ in (5), which we refer to as the *extensive margin of search friction*. On the other hand, the term $(1/2)^N$ measures the mis-coordination (selecting the same seller) among the N fully informed buyers, and we refer to it as the *intensive margin of search friction*. A higher degree of market accessibility, measured by larger N, strictly decreases the intensive margin of search friction, which helps increase efficiency. However, its effect on the extensive margin of search friction is less clear-cut.

As an example, consider the case of fully separate meeting technologies, i.e., $N_J = 0$. Here, there are in total C^N_B possible cases in terms of which buyers observe an individual seller. In addition, there are in total C^N_B cases of exactly N buyers being fully informed. Therefore, $\Gamma_0(N|N,B) = C^N_B/(C^N_B)^2 = 1/C^N_B$, and

$$T_0(N) = 2\left[1 - \left(\frac{1}{2}\right)^N \frac{1}{C^N_B}\right]. \tag{8}$$

While an increase in N still decreases the intensive margin of search friction, the effect on the extensive margin of search friction is non-monotonic as $\Gamma_0(N|N,B) = 1/C^N_B$ initially decreases and then increases in N. The intuition of such non-monotonicity is clear. When N is small compared to B, there are plenty of uninformed buyers. Due to the no-waste assumption, an additional meeting opportunity with each seller is thus unlikely to reach the same buyer. Conversely, when N is close to B, most buyers are either fully or partially informed, so an additional meeting opportunity with each seller is very likely to create an additional fully informed buyer. Hence, maximizing efficiency requires an intermediate degree of market accessibility.

Why is perfect market accessibility efficient only when N_J is sufficiently large? By definition, a meeting technology with N_J joint meeting opportunities automatically generates N_J fully informed buyers. A larger N_J thus leads to larger search frictions, leading to greater mis-coordination between buyers, and thus a high probability of having just one match. In turn, it becomes increasingly important to generate a larger number of separate meeting opportunities $N - N_J$ to reduce the probability of a single match. This is accomplished by further increasing the market accessibility level, and so $N^e_{N_J}$ is increasing in N_J. This also explains why, when N_J is too large such that search frictions are sufficiently large, it is efficient to have perfect market accessibility, i.e., $N^e_{N_J} = B$.

We conclude by considering the impact of a change in the market size, i.e., the number of buyers B, on the efficient accessibility level. By (7), notice that the efficient *proportion of market accessibility*, i.e.,

$$x^e_{N_J} \equiv \frac{N^e_{N_J}}{B} = \frac{1}{B}\left\lfloor \frac{2(B+1)+N_J}{3} \right\rfloor$$

is *decreasing* in the market size. This is as an increase in B only leads to a (further) increase in the reduction of the extensive margin of search frictions $\Gamma_{N_J}(N|N,B)$ from small increases in the market accessibility level N.

4 Equilibrium market accessibility

In this section, we characterize the equilibrium of the game. Sections 4.1 and 4.2 begin by studying the buyer-seller continuation games induced by the platform choosing $N \geq 2$ and $N = 1$ respectively when the platform charges $f = 0$.[13] The equilibrium platform fee and market accessibility level are derived in Section 4.3.

[13]We separate the $f = 0$ analysis from the $f > 0$ analysis, as the former will be used in the upcoming analysis of seller- and buyer-optimal accessibility levels in Section 5.

4.1 Buyer-seller games when $N \geq 2$

Suppose that $N \geq 2$. We work backward and start with the buyers' search problem. Except for the extreme case of perfect accessibility, i.e., $N = B$, buyers' information will be dispersed. That is, there potentially exists buyers who observe no seller, one seller, and two sellers. If a buyer observes no seller, then she has no one to visit and her payoff is zero. If a buyer observes one seller, she is partially informed and can only visit the seller she observes. Her payoff then depends on whether the seller in question is visited by other buyers.

We next describe the symmetric equilibrium strategy of a fully informed buyer who observes both sellers. Let $\sigma_1(r_1, r_2) \in (0, 1)$ denote the symmetric equilibrium probability that a fully informed buyer attends seller 1's auction. The buyer obtains a positive payoff from seller 1 if and only if she is the only one to visit seller 1, because otherwise, the *ex post* competition would leave the winning buyer with zero surpluses. If partially informed buyers exist, then each seller receives at least one buyer. Hence, a necessary condition for a fully informed buyer to be the only visitor of seller 1 is that all other informed buyers are fully informed. Conditional on that there exist other $N - 1$ fully informed buyers, a fully informed buyer obtains a positive payoff from selecting seller 1 if and only if none of the other fully informed buyers selects seller 1, which happens with probability $(1 - \sigma_1(r_1, r_2))^{N-1}$.

Let $\widetilde{\Gamma}_{N_J}(N - 1|N, B)$ denote the probability that there exist other $N - 1$ fully informed buyers from a fully informed buyer's perspective.[14] Her expected payoff for selecting seller 1, who posts a reserve price r_1, is therefore given by

$$u_{N_J|1}(r_1, r_2|N) = (1 - r_1)(1 - \sigma_1(r_1, r_2))^{N-1}\widetilde{\Gamma}_{N_J}(N - 1|N, B). \tag{9}$$

Analogously, her expected payoff for selecting seller 2 is

$$u_{N_J|2}(r_1, r_2|N) = (1 - r_2)(\sigma_1(r_1, r_2))^{N-1}\widetilde{\Gamma}_{N_J}(N - 1|N, B). \tag{10}$$

Closely examining (9) and (10) reveals that for any $(r_1, r_2) \neq (1, 1)$, the symmetric

[14] When $0 < N_J < N$, there is uncertainty in how a buyer becomes fully informed. It can be because she receives a joint meeting opportunity, or because she receives separately one meeting opportunity with each seller. A fully informed buyer needs to take these possibilities into account when she calculates $\widetilde{\Gamma}_{N_J}(N - 1|N, B)$. Luckily, $\widetilde{\Gamma}_{N_J}(N - 1|N, B)$ cancels out when computing the equilibrium in the stage where buyers select sellers to visit.

directed search equilibrium is unique, and is given by[15]

$$\sigma_1(r_1, r_2) = \begin{cases} \frac{A(r_1,r_2)}{1+A(r_1,r_2)}, & r_2 < 1 \\ 1, & r_1 < 1 = r_2 \end{cases} \tag{11}$$

where $A(r_1, r_2) \equiv \left(\frac{1-r_1}{1-r_2}\right)^{\frac{1}{N-1}}$. We further note that unless either $r_1 = 1$ or $r_2 = 1$, the unique equilibrium $\sigma_1(r_1, r_2)$ must be in mixed strategies.

We now establish the symmetric equilibrium strategies of sellers, fixing buyers' symmetric equilibrium strategy at $\sigma_1(r_1, r_2)$. Seller 1's expected profit is

$$\begin{aligned}\pi_{N_J}(r_1, r_2|N) &= r_1 \cdot \text{Pr.}[m_1 = 1] + \text{Pr.}[m_1 > 1] \\ &= 1 - \text{Pr.}[m_1 = 0] - \text{Pr.}[m_1 = 1] \cdot (1 - r_1).\end{aligned}$$

$m_1 = 0$ arises when there are N fully informed buyers and none of them select seller 1. The probability of this event is $\Gamma_{N_J}(N|N, B)(1 - \sigma_1)^N$. Meanwhile, $m_1 = 1$ holds when (i) there are N fully informed buyers but only one of them selects seller 1, which happens with probability $\Gamma_{N_J}(N|N, B)N\sigma_1(1 - \sigma_1)^{N-1}$; or (ii) there are $N - 1$ fully informed buyers (and therefore two partially informed buyers) but none of them selects seller 1, which happens with probability $\Gamma_{N_J}(N-1|N, B)(1-\sigma_1)^{N-1}$, where[16]

$$\Gamma_{N_J}(N - 1|N, B) = \frac{C_{B-N_J}^{N-N_J-1} C_{B-N+1}^1 C_{B-N}^1}{(C_{B-N_J}^{N-N_J})^2} = \frac{(N - N_J)(B - N)}{C_{B-N_J}^{N-N_J}}$$

In this case, it is a partially informed buyer who only observes seller 1 that partici-

[15] In the proof of Theorem 1, we show that there is no symmetric equilibrium in which both sellers set a reserve price of 1. Thus, we omit $(1, 1)$ from our exposition here.

[16] For an allocation of N_J joint meetings, there are $C_B^{N_J}$ possibilities. Then, to have $N - 1$ fully informed buyers, $N - N_J - 1$ buyers not allocated a joint meeting must be allocated a separate meeting both with seller 1 and seller 2, of which there are $C_{B-N_J}^{N-N_J-1}$ possibilities. Among the remaining $B - N + 1$ buyers, one buyer must be allocated a separate meeting only with seller 1, and another a separate meeting only with seller 2, of which there are $C_{B-N+1}^1 C_{B-N}^1$ possibilities. Since the meeting technology satisfies (3), the probability of having $N - 1$ fully informed buyers is

$$C_B^{N_J} \times C_{B-N_J}^{N-N_J-1} \times C_{B-N+1}^1 C_{B-N}^1 \times \left(\frac{1}{(C_B^{N_J})(C_{B-N_J}^{N-N_J})^2}\right) = \frac{C_{B-N_J}^{N-N_J-1} C_{B-N+1}^1 C_{B-N}^1}{(C_{B-N_J}^{N-N_J})^2}$$

pates in seller 1's auction. Hence,

$$\pi_{N_J}(r_1, r_2|N) = 1 - \Gamma_{N_J}(N|N,B)(1-\sigma_1(r_1,r_2))^N \\ - [\Gamma_{N_J}(N|N,B)N\sigma_1(r_1,r_2) + \Gamma_{N_J}(N-1|N,B)](1-\sigma_1(r_1,r_2))^{N-1}(1-r_1). \tag{12}$$

where $\sigma_1(r_1, r_2)$ is as defined in (11).

Differentiating $\pi_{N_J}(r_1, r_2|N)$ with respect to r_1 and setting $r_1 = r_2$, it is straightforward to verify that for $N \geq 2$, $r_{N_J}(N)$ defined below is the unique interior $r \in (0,1)$ that satisfies the first-order conditions for seller 1's profit, subject to seller 2 setting the same reserve price:

$$\begin{aligned} r_{N_J}(N) &= 1 - \frac{\Gamma_{N_J}(N|N,B)N}{\Gamma_{N_J}(N|N,B)N^2 + 2\Gamma_{N_J}(N-1|N,B)(N-1)} \\ &= 1 - \frac{1}{N + (2(N-N_J)(N-1)(B-N)/N)}, \end{aligned} \tag{13}$$

where the second equality follows from $\Gamma_{N_J}(N|N,B) = 1/C_{B-N_J}^{N-N_J}$ and $\Gamma_{N_J}(N-1|N,B) = (N-N_J)(B-N)/C_{B-N_J}^{N-N_J}$. We further show in the Appendix that (i) there is no equilibrium where sellers set $r_1 = r_2 = 0$ or $r_1 = r_2 = 1$, and (ii) $\pi_{N_J}(r_1, r_{N_J}(N)|N)$ is strictly single-peaked at $r_1 = r_{N_J}(N)$. Combined, these imply that $r_{N_J}(N)$ is the unique symmetric equilibrium reserve price.

Theorem 1. *Suppose that $N \geq 2$. Then, a (pure-strategy) symmetric directed search equilibrium exists and is unique. Furthermore, the equilibrium reserve price $r_{N_J}(N)$ is given by (13).*

From (13), we further observe that $r_{N_J}(N)$ strictly decreases in N_J and increases in B. An increase in N_J creates more fully informed buyers, intensifying competition between sellers and thus leading to a decrease in $r_{N_J}(N)$. Meanwhile, on the aggregate level, the sellers' potential demand increases as B becomes larger, allowing them to increase their reserve prices $r_{N_J}(N)$ in response.

4.2 Buyer-seller games when $N = 1$

The analysis in Section 4.1 does not readily extend to the case of $N = 1$. Here, we have either $N_J = 0$ or $N_J = 1$. When $N_J = 1$, a single buyer becomes fully informed while all the other buyers are uninformed. The fully informed buyer thus visits the seller who sets a lower reserve price, leading to Bertrand competition between sellers and zero profit to each seller.

The case of $N_J = 0$ is more involved. There are two possible scenarios regarding buyers' information: (i) a single buyer observes both sellers, and (ii) one buyer observes only seller 1 and another buyer observes only seller 2. If a buyer is fully informed, she will select the seller with the lower reserve price. If a buyer is partially informed, she will select the observed seller provided the reserve price is no greater than 1. Note that *ex post* bidding never takes place when $N = 1$, as each seller can meet at most one buyer.

Now, if both sellers set some $r_1 = r_2 > 0$, one seller can slightly undercut the other seller's reserve price, obtaining a fully informed buyer with probability one (if there is one). Meanwhile, neither seller will set a zero reserve price since they can set a positive reserve price and sell only to a partially informed buyer with a positive probability. Consequently, the symmetric equilibrium behaviour of sellers must involve the use of a mixed strategy.

Denote the symmetric equilibrium mixed strategy of sellers by a distribution function $F(r)$. By the standard argument given in Varian (1980), there is no gap and no mass point in the support of $F(r)$, such that its support is given by $[\underline{r}, \overline{r}]$ for some $0 \leq \underline{r} \leq \overline{r} \leq 1$. Furthermore, note that if $\overline{r} < 1$, then only a partially informed buyer will buy when faced with a reserve price of $\overline{r}$. If so, then a seller can instead redistribute the mass on $\overline{r}$ to $r = 1$ without losing demand while strictly increasing her profit. Hence, $\overline{r} = 1$ must hold.

To derive the symmetric equilibrium price distribution F and the lower bound $\underline{r}$, we first consider the expected profit of seller 1 from charging some $r_1 \in [0, 1]$. Notice that there is a fully informed buyer with probability $\Gamma_0(1|1, B)$. Furthermore, given that seller 2 randomizes over reserve prices according to F, seller 2's reserve price is higher than r_1 with probability $1 - F(r_1)$, in which case the fully informed buyer will buy from seller 1. Meanwhile, with probability $\Gamma_0(0|1, B)$, there is a partially informed buyer who can only buy from seller 1. Hence, seller 1's expected profit from setting a reserve price r_1 is

$$\pi_1(r_1, F(r)) = r_1[\Gamma_0(1|1, B)(1 - F(r_1)) + \Gamma_0(0|1, B)].$$

Recall that if F is an equilibrium price distribution, then seller 1 must be indifferent between any $r \in [\underline{r}, 1)$ and $r = 1$, the latter of which yields an expected profit

$\Gamma_0(0|1, B)$. This yields the indifference condition

$$r[\Gamma_0(1|1, B)(1 - F(r)) + \Gamma_0(0|1, B)] = \Gamma_0(0|1, B)$$

which can be rearranged to obtain the equilibrium price distribution

$$F(r) = 1 - \frac{\Gamma_0(0|1, B)}{\Gamma_0(1|1, B)} \left(\frac{1}{r} - 1\right) = 1 - B\left(\frac{1}{r} - 1\right). \tag{14}$$

The lower bound can then be identified by solving $F(\underline{r}) = 0$. This yields $\underline{r} = \Gamma_0(0|1, B)/(\Gamma_0(0|1, B) + \Gamma_0(1|1, B)) = B/(B+1)$, which is also equal to each firm's expected equilibrium profit.

Theorem 2. *Suppose that $N = 1$*

1. *If $N_J = 1$, then in any symmetric directed search equilibrium, both sellers obtain zero profits.*
2. *If $N_J = 0$, then a symmetric directed search equilibrium exists and is unique. The equilibrium is characterized by a non-degenerate distribution F over reserve prices on $[B/(B+1), 1]$, where F is given by (14).*

4.3 Profit-maximizing platform

We now turn to the platform's equilibrium behaviour. The platform sets a transaction fee f and a market accessibility level $N \in \{N_J, ..., B\}$ to maximize its own profit, given by $\Pi_{N_J}(N) = f \cdot T_{N_J}(N)$, where $T_{N_J}(N)$ is the expected total number of trades in equilibrium given in (5), subject to the buyers' and the sellers' (i) participation constraint, i.e., non-negative payoffs, and (ii) equilibrium behaviour described in Sections 4.1 and 4.2. Following the platform's choice of f and N, sellers play the same game as discussed in Sections 4.1 and 4.2, except that their profit margins will reduce by f.

Observe that for a given (r_1, r_2), buyers' equilibrium behaviours are unaffected by the transaction fee f. Meanwhile, with $N \geq 2$, seller 1's expected profit is now

$$\begin{aligned}\pi_{N_J}(r_1, r_2, f|N) &= (r_1 - f) \cdot \text{Pr.}[m_1 = 1] + (1 - f) \cdot (1 - \text{Pr.}[m_1 = 0] - \text{Pr.}[m_1 = 1]) \\ &= (1 - f)(1 - \text{Pr.}[m_1 = 0]) - (1 - r_1) \cdot \text{Pr.}[m_1 = 1].\end{aligned}$$

and seller 2's profit can be derived in a similar fashion. Following an identical logic to that in Section 4.1, we can derive the symmetric equilibrium reserve price for a given f and N by

$$r_{N_J}(f|N) = 1 - \frac{1-f}{N + (2(N-N_J)(N-1)(B-N)/N)}.$$

Notice that $r_{N_J}(f|N) = 1$ whenever $f = 1$, which is the highest possible fee the platform can charge without causing sellers and buyers to withdraw. Furthermore, the choice of fee does not influence the total number of matches in any symmetric equilibrium where fully informed buyers select each seller with probability 1/2. Thus, the platform optimally selects $f^* = 1$ for any given $N \geq 2$, and obtains a payoff of $T_{N_J}(N)$.

From here, we recall that $T_{N_J}(N)$ is uniquely maximized at $N^e_{N_J} > 1$ defined in (7) for $N \in \{N_J, ..., B\}$. Furthermore, when $N = 1$, the platform's payoff from choosing any fee f must be bounded above by the total surplus $T_{N_J}(1) \leq T_{N_J}(N^e_{N_J})$. Thus, the platform's optimal accessibility level is simply $N^*_{N_J} = N^e_{N_J}$. Furthermore, the platform sets $f^* = 1$ and $N^*_{N_J} = N^e_{N_J}$ in equilibrium, obtaining an equilibrium profit of $T_{N_J}(N^e_{N_J})$, and fully extracting all expected surplus, leaving both buyers and sellers with zero gains from trade.

Proposition 2. *The platform sets the efficient level of market accessibility, i.e., $N^*_{N_J} = N^e_{N_J}$ and charges the maximum transaction fee $f^* = 1$ in equilibrium.*

The key insight of Proposition 2 is that the efficient accessibility level, i.e., that which maximizes the expected total surplus, can be implemented by a profit-maximizing platform that sets a simple transaction fee for its intermediation service. In other words, the constrained-efficient outcome, i.e., subject to search frictions, can also be achieved in a decentralized setting. From Corollary 1, our model predicts imperfect accessibility to arise in a wide range of environments. However, the benefits of attaining an efficient outcome do *not* necessarily translate into benefits for the users of the platform as the platform's transaction fee induces full surplus extraction.[17]

[17] Through allowing the platform to fully extract surplus from buyers and sellers, the simple transaction fee structure weakly outperforms all other types of fees (e.g., fixed fee, percentage fee).

5 Buyer- and Seller-optimal market accessibility

A key insight of Section 4 is that the platform chooses the socially efficient market accessibility level in equilibrium. A natural question then is whether the platform's choice (also) coincides with that which is optimal for either trading party. We address this question by characterizing the seller-optimal and buyer-optimal market accessibility levels. Throughout, we assume away the platform fee, i.e., set $f = 0$.

5.1 Seller-optimal market accessibility

Let $\pi_{N_J}(N)$ denote the sellers' equilibrium expected profit given $N_J \in \{0, ..., B\}$ and $N \in \{N_J, ..., B\}$. We call any

$$N^{\mathcal{S}} \in \underset{N\in\{N_J,...,B\}}{\arg\max}\ \pi_{N_J}(N)$$

a *seller-optimal accessibility level*.

To identify $N^{\mathcal{S}}$, for $N \geq 2$, by applying $r_1 = r_2 = r_{N_J}(N)$ identified in (13) to $\pi_{N_J}(r_1, r_2|N)$, we obtain the seller's equilibrium expected profit

$$\pi_{N_J}(N) = 1 - \left(\frac{1}{2}\right)^{N-1} \frac{1}{C_{B-N_J}^{N-N_J}} \left[1 + \frac{(N-N_J)(B-N)}{N^2 + 2(N-1)(N-N_J)(B-N)}\right] \tag{15}$$

Meanwhile, using the results from Section 4.2, it is immediate that for $N = 1$,

$$\pi_0(1) = \frac{B}{B+1}, \qquad \pi_1(1) = 0$$

By comparing these profits, we provide a characterization of $N^{\mathcal{S}}$ below.

Proposition 3. *Take any seller-optimal accessibility level $N^{\mathcal{S}}$. Then,*

1. *For all $N_J \in \{0, ..., B\}$, $N^{\mathcal{S}} \geq N^e_{N_J}$, i.e., any seller-optimal market accessibility level is always weakly greater than the efficient level.*

2. *If N_J is sufficiently small, then $N^{\mathcal{S}} < B$, i.e., perfect accessibility is strictly not seller-optimal.*

Proposition 3 delivers two key insights. First, sellers weakly prefer a greater level of market accessibility than the efficient level. To understand this result, recall that a seller's payoff from being visited by two or more buyers is 1, while being visited by a single buyer yields a profit of $r_{N_J}(N) < 1$. Hence, unlike matching efficiency, a seller strictly benefits from having multiple visiting buyers over having a single visiting buyer. Conditional on having at least one visiting buyer, the probability in which two or more buyers visit a seller is increasing in N. It is this additional benefit from raising N that leads to $N^{\mathcal{S}} \geq N^{e}_{N_J}$.

Despite sellers' preferences for a larger market accessibility level, the second part of Proposition 3 states that imperfect accessibility can still be optimal for sellers when N_J is low. Here, the logic mirrors that regarding the efficient meeting accessibility level discussed in Proposition 1. Recall that a seller cannot sell if there are N fully informed buyers and all of them select the rival seller. Furthermore, when N_J is sufficiently small and N is relatively large, the probability of having N fully informed buyer increases with N. Thus, like the efficient accessibility level, $N^{\mathcal{S}} < B$ still holds when N_J is close to zero.

5.2 Buyer-optimal market accessibility

Let $u_{N_J}(N)$ denote the buyer's expected payoff given $N_J \in \{0, ..., B\}$ and $N \in \{N_J, ..., B\}$. We call any

$$N^{\mathcal{B}} \in \underset{N \in \{N_J, ..., B\}}{\arg\max}\ u_{N_J}(N)$$

a *buyer-optimal accessibility level*.

To identify a buyer-optimal accessibility level, we note that each buyer's expected payoff can be written as the difference between total surplus $T_{N_J}(N)$ as defined in (5), and sellers' total profits π_N defined in Section 5.1 (depending on whether $N \geq 2$ or $N = 1$), divided by the number of buyers B. Therefore,

$$u_{N_J}(N) = \frac{1}{B}\Big(T_{N_J}(N) - 2\pi_{N_J}(N)\Big) \tag{16}$$

When $N \geq 2$, then using (15), a buyer's payoff is

$$u_{N_J}(N) = \frac{1}{BC^{N-N_J}_{B-N_J}}\left(\frac{1}{2}\right)^{N-1}\left(1 + \frac{2(N-N_J)(B-N)}{N^2 + 2(N-1)(N-N_J)(B-N)}\right) \tag{17}$$

On the other hand, when $N = 1$ such that there is no competition between buyers, then if $N_J = 0$, a buyer's expected payoff is

$$u_0(1) = \frac{1}{B}\left(T_0(1) - \frac{2B}{B+1}\right) = \frac{B-1}{B^2(B+1)}. \tag{18}$$

If $N_J = 1$ such that sellers' profits are zero, then a buyer's expected payoff is

$$u_1(1) = \frac{1}{B}\left(T_1(1) - 0\right) = \frac{1}{B} \tag{19}$$

By comparing these payoffs, we fully characterize the buyer-optimal accessibility level below.

Proposition 4. *The buyer-optimal market accessibility $N^{\mathcal{B}}$ is as follows.*

1. *If either $N_J > 0$ or $B > 3$, then $N^{\mathcal{B}} = \max\{1, N_J\}$, i.e., the minimal market accessibility level is uniquely buyer-optimal.*
2. *If $N_J = 0$ and $B = 3$, then both $N^{\mathcal{B}} = 2$ and $N^{\mathcal{B}} = 3$ are buyer optimal.*

The main insight of Proposition 4 is that excluding the extreme case in which no joint meetings are allocated and there are a minimal number of buyers, i.e., $N_J = 0$ and $B = 3$, a buyer always strictly prefers being given the *minimum* access to sellers in a matching market. Consequently, when $B > 3$, the buyer-optimal market accessibility level is increasing in the number of joint meeting opportunities N_J, and does not vary in the size of the market B. Meanwhile, when $B = 3$, the buyer-optimal market accessibility level first decreases in N_J (from $N^{\mathcal{B}} = 3$ when $N_J = 0$, to $N^{\mathcal{B}} = 1$ when $N_J = 1$), and then increases in N_J (for $N_J > 1$). Furthermore, perfect accessibility can only be optimal for buyers in either the extreme case above, or the trivial event when all meetings must be allocated jointly, i.e., $N_J = B$.

The key to understanding this counter-intuitive result lies in the competition among buyers. As an example, when $N = 1$, a buyer is either partially informed of a seller and no other buyers know this seller or fully informed and there is no other informed buyer. Therefore, buyers do not face any ex post competition after being informed and thus always obtain a positive utility. Proposition 4 shows that, except in the extreme case of $B = 3$ and $N_J = 0$, the benefits from avoiding competition with other buyers outweigh the cost of being less frequently informed. Hence, buyers' payoffs are maximized at the minimum accessibility level.

6 Discussion

In this section, we elaborate upon several of the assumptions made in the model. Section 6.1 investigates the robustness of our key insight, i.e., that imperfect market accessibility is efficient, by allowing unmatched buyers to search again. Section 6.2 connects the market accessibility level to key properties of meeting technologies previously emphasized in the literature, and discusses the implications of relaxing several of the assumptions made on the platform's choice of meeting technology (Assumption 1).

6.1 A second chance to search

We now study a variant of our model that incorporates an additional period for buyers and sellers to interact. Our main goal is to show that imperfect market accessibility can still be efficient. For simplicity, we assume throughout that $N_J = 0$, and let N^e denote the efficient accessibility level.

There are two periods, 1 and 2. To avoid unnecessary complications, we assume that the supply of goods is constant across periods, i.e., each seller has a unit to sell in both periods. Unlike sellers, buyers exit the market upon successful trade. The meeting parameter N is chosen at the beginning of period 1 and buyers observe the same set of sellers in both periods. We assume that buyers are myopic and will not strategically delay their search.[18] Under this assumption, the equilibrium in each period is the same as in our original setting except that there are fewer buyers in period 2. We assume a common discount factor $\delta \in [0, 1]$ for the value derived in period 2.

The intuition can be best understood by considering a simple case with only three buyers. We will generalize the results to arbitrary B at the end of the section. Suppose $N = 1$. As in Sections 3-5, we will use $T(N)$ to denote the discounted total expected values generated from both periods. There exists a fully informed buyer, which generates one match, with a probability $1/3$, and two partially informed buyers, which generate two matches, with a probability $2/3$. All matches are formed in period 1. So the discounted total value is

$$T(1) = \frac{1}{3} + 2 \times \frac{2}{3} + \delta \times 0 = \frac{5}{3}. \tag{20}$$

Suppose $N = 3$ and therefore all buyers are fully informed. There is one match with probability $1/4$ and two matches with probability $3/4$ in period 1. So the

[18] If buyers are sophisticated, they want to delay their participation if everyone else does not delay. This is because the number of participating buyers is always smaller in period 2 than in period 1, which causes sellers to set a lower equilibrium reserve price in period 2. However, if everyone else delays their participation, sellers will set a high reserve price in period 2 and an informed buyer's optimal response might be not to delay.

expected value in period 1 is $7/4$. If there is only one match in period 1, there is one match with probability $1/2$ and two matches with probability $1/2$ in period 2. If there are two matches in period 1, there will be for sure one match in period 2. So the expected value generated in period 2 is $(1/4)\times(1/2+2\times(1/2))+(3/4)\times 1 = 9/8$. So the discounted total value is

$$T(3) = \frac{7}{4} + \delta\left(\frac{9}{8}\right). \tag{21}$$

Suppose $N = 2$. With probability $1/3$, there are two fully informed buyers. With probability $2/3$, there are one fully informed buyer and two partially informed buyers. In the former case, there will be two matches with probability $1/2$ and only one match with probability $1/2$ in period 1. In the latter case, there are two matches in period 1. The expected value generated in period 1 is $(1/3)\times((1/2)+2\times(1/2)) + (2/3)\times 2 = 11/6$. If there is only one match in period 1, there will be one match in period 2. If there are two matches in period 1, there will be an additional match in period 2 if there exist partially informed buyers, and no match in period 2 otherwise. The expected value generated in period 2 is $(1/3)\times(1/2)+(2/3) = 5/6$. So the discounted total value is

$$T(2) = \frac{11}{6} + \delta\left(\frac{5}{6}\right). \tag{22}$$

Let us compare (20), (21), and (22). It is clear that $N = 1$ generates the lowest level of total efficiency. When $N = 1$, having a second chance to search does not add any value to matching as all the informed buyers are matched in period 1. According to our results in the static setting, $N = 2$ always generates the highest value in period 1. The question is then whether $N = 3$ can generate a higher value in period 2. By comparing the second terms in (21) and (22), perfect accessibility, i.e., $N = 3$, indeed dominates in period 2. We can conclude that perfect accessibility is efficient if participants are patient enough, i.e. $\delta > 2/7$, and imperfect accessibility is efficient otherwise.

The reason why, unlike in the static case, $N = 2$ can be less efficient relative to perfect accessibility $N = 3$ in this example is because there is a strictly positive probability that there exist uninformed buyers who cannot participate. This is a serious problem for efficiency, particularly given that there are only three buyers. This issue, however, is less severe when the number of buyers is large enough, as our next result demonstrates.

Proposition 5. *Suppose the market operates in both periods 1 and 2 and participants have a common discount factor $\delta \in [0, 1]$. Then, there exists a $\widehat{B} > 3$ and $\widehat{\delta} \in (0, 1)$ such that*

1. *If $B \leq \widehat{B}$ and $\delta > \widehat{\delta}$, then $N^e = B$.*

2. *If $B > \widehat{B}$, or $B \leq \widehat{B}$ and $\delta < \widehat{\delta}$, then $1 < N^e < B$.*

The disadvantage of enforcing imperfect accessibility is that some consumers might be uninformed and therefore cannot participate in both periods. This disadvantage is severe when the number of buyers B is small. When there are a large number of buyers, this disadvantage will be mitigated as long as the accessibility level N is not too small relative to B. This is because a sufficient number of buyers will be either partially or fully informed, and whether there exist uninformed buyers becomes irrelevant for the total number of matches. Together with the fact that imperfect accessibility always dominates in period 1, we can conclude that imperfect accessibility is optimal in generating matches even in the dynamic setting provided that B is sufficiently large.

To illustrate the above, let $T^2(N)$ denote the expected number of matches generated in period 2 given the accessibility level N. Figure 1 plots $T^2(B)$ (the red dots) and $T^2(B-1)$ (the blue dots) for $B \in \{3, ..., 10\}$. Observe that the number of matches in period 2 is maximized by full accessibility only when $B = 3$ or 4. Meanwhile, imperfect accessibility, i.e., under $N = B - 1$, achieves more matches when B becomes larger. Thus, our main insight derived in the static setting continues to hold in the presence of additional search opportunities.

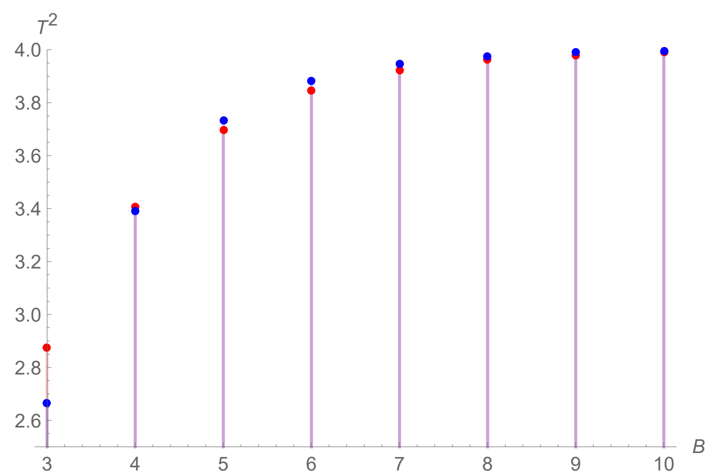

Figure 1: Comparison of $T^2(B)$ and $T^2(B-1)$

6.2 Discussion of meeting technologies

The distinction between meetings and matching in the directed search framework was first established by Eeckhout and Kircher (2010) (see also Lester et al., 2015). The meetings considered in their model is ex post in the sense that buyers first search submarkets where individual sellers post prices. Meetings then take place within each submarket according to the meeting technology. In contrast, the meeting technology proposed in our model generates submarkets or a network of contacts between buyers and sellers within which buyers can search for individual sellers.

According to Eeckhout and Kircher (2010), a meeting techonology is *non-rival* if each additional meeting with a seller allocated to a buyer does not affect other buyers' chances of meeting with the same seller, *rival* if there is at most one buyer at any time at any given seller, so allocating a meeting with a seller to a buyer completely removes other buyers' chances of meeting with the seller, and *partially rival* otherwise. As in Eeckhout and Kircher (2010), our meeting technologies capture each of these various degrees of rivalry. The meeting technology is rival if $N = 1$, as a meeting opportunity between a seller and a buyer implies that any other buyers do not have the opportunity to meet the same seller. Meanwhile, the meeting technology is non-rival when $N = B$, as every buyer has the opportunity to meet with every seller at the same time. Finally, the meeting technology is partially rival if $1 < N < B$, as a meeting opportunity between a seller and a buyer reduces (but does not completely eliminate) the opportunity for other buyers to meet the seller.

Unlike in Eeckhout and Kircher (2010) where meeting opportunities are exogenously allocated, the platform in our model affects the meeting outcome by controlling market accessibility, which endogenously affects whether the meeting technology is rival, non-rival or partially rival. Indeed, Corollary 1 states that the platform's optimal accessibility level $N^*_{N_J}$ (which by Proposition 2 coincides with the efficient accessibility level) is equal to B if and only if N_J is no less than $B - 2$. In other words, the optimal meeting technology for the platform is non-rival if and only if the ratio N_J/B is sufficiently large, and partially rival otherwise. Notably, since $N^*_{N_J} > 1$ always holds, a fully rival meeting technology is never optimal for the platform (nor socially efficient). This remains true with respect to seller-optimality (Proposition 3), while in contrast, buyers can find a fully rival meeting technology optimal (Proposition 4).

Another important property of meeting technologies discussed in the literature is that of *invariance*. According to Lester et al. (2015), a meeting technology is invariant if a buyer's decision to visit the mechanism posted by a seller does not interfere with the process that the seller meets with other buyers. In our setup with a finite number of buyers, the meeting technology satisfies invariance exactly at the minimal market accessibility level $N = 1$. Notably, since the platform's optimal accessibility level always satisfies $N^*_{N_J} = N^e_{N_J} > 1$, an invariant meeting technology is never optimal for the platform (and never socially efficient nor seller-optimal), but can be buyer-optimal.

We now discuss other aspects of our meeting technologies. First, we require that the platform must allocate a total of N meeting opportunities with each seller. Without this assumption, it is possible for sellers to extract all surplus from buyers, even in the presence of search frictions. As an example, suppose that there are two buyers, and consider any meeting technology which always allocates one meeting opportunity with seller 1 and two with seller 2. Then, seller 2 knows that there always exists at least one buyer who only receives one meeting opportunity with him, and so he can set $r_2 = 1$ to fully extract the surplus of that buyer. Knowing this, seller 1 will then find it optimal to set $r_1 = 1$. Under this pair of reserve prices, there exists an equilibrium in the buyer game under which the fully informed buyer visits seller 1 and the partially informed buyer visits seller 2. There are two matches even in the presence of search frictions and sellers fully extract buyer surplus.

It is important to recognize that if there exists a meeting opportunity with seller 1, then there must also exist a corresponding meeting opportunity with seller 2. Together with this, the no-waste property rules out the possibility for a seller to be visited by no buyers when partially informed buyers exist. If there exist m partially informed buyers who only know seller 1, there must also exist m partially informed buyers who only know seller 2. No waste prevents a buyer from receiving more than one meeting opportunity with a seller.[19]

7 Conclusion

This paper shows that an increase in market accessibility has a profound impact on matching efficiency, seller profits and buyer surplus in a directed search equilibrium. Using a model with a continuum of duopoly product categories, we manage to identify the levels of market accessibility which are optimal for various policy goals. In particular, we show that full accessibility often leads to a less desirable outcome,

[19]A related analysis of advertising in the directed search environment that violates the "no waste" assumption can be found in Gomis-Porqueras et al. (2017).

not only for efficiency but also for all other participant groups. We further consider a profit-maximizing platform that can centrally control market accessibility and charge fees for its intermediation service. We show that the platform implements efficient meeting allocation and fully extracts surplus by choosing an intermediate level of market accessibility.

In order to show that there is a straightforward rationale for why platforms may want to restrict participants' meeting choices, the current analysis excludes various market characteristics such as entry and exit on both sides, ex-ante heterogeneity among participants, and idiosyncratic match values. These additional market characteristics should be incorporated into the exercise if and when more tractable analytical tools become available in the future. In particular, it would be interesting to study the optimal fee structure, along with the optimal market accessibility, when the buyer side is featured with heterogeneous outside options. In such an extension, a change in market accessibility affects not only buyers' information about the market and their search strategies, but also the total number of buyers who participate in the market. While we expect imperfect market accessibility to continue yielding the greatest efficiency in many environments, we conjecture that a profit-maximizing platform is less likely to implement the efficient meeting allocation and extract all the surplus.

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Appendix

Proposition 6. *For each $N \in \{N_J, ..., B\}$, there exists a unique meeting technology (N, P_{N,N_J}), with $P_{N,N_J}(\boldsymbol{n})$ given in (3), that satisfies Assumption 1.*

Proof of Proposition 6. Given any $N \in \{N_J, ..., B\}$, it is straightforward to verify that the meeting technology (N, P_{N,N_J}), with P_{N,N_J} defined by (3), satisfies Assumption 1. Hence, we are left to show that any meeting technology (N, P) which satisfies Assumption 1 satisfies (3) for all $\boldsymbol{n} \in \mathcal{N}_{N,N_J}$. Taking any such meeting technology (N, P) and any $\boldsymbol{n} \in \mathcal{N}_{N,N_J}$, there are two cases to consider.

Case 1: Suppose $\boldsymbol{n} \notin \widehat{\mathcal{N}}_{N,N_J}$. Then, either one of three possibilities occur. First, the number of buyers offered (more than) one joint meeting is not equal to N_J. That is, there exists a subset $\mathcal{B}' \subset \{1, .., B\}$ with $|\mathcal{B}'| \neq N_J$ such that $n^b_J > 0$ if and only if $b \in \mathcal{B}'$. Second, exactly N_J number of buyers are offered one joint meeting but one such buyer is allocated (more than) one separate meetings with a seller. That is, there exists a subset $\mathcal{B}' \subset \{1, .., B\}$ with $|\mathcal{B}'| = N_J$ and $n^b_J > 0$ if and only if $b \in \mathcal{B}'$ (which implies $n^b_J = 1$ if and only if $b \in \mathcal{B}'$), but there exists $b \in \mathcal{B}'$ with $n^b_s > 0$ for some $s \in \{1, 2\}$. Third, exactly N_J number of buyers are offered one joint meeting and no separate meetings, but there exists a buyer not offered a joint meeting which is allocated more than one meeting with a seller. That is, there exists a subset $\mathcal{B}' \subset \{1, .., B\}$ with $|\mathcal{B}'| = N_J$ and $n^b_J = 1$ if and only if $b \in \mathcal{B}'$, and $n^b_s = 0$ for all $b \in \mathcal{B}'$ and $s \in \{1, 2\}$, but there exists $b \in \{1, ..., B\} \backslash \mathcal{B}'$ such that $n^b_s > 1$ for some $s \in \{1, 2\}$. As P satisfies no waste, any of these imply $P(\boldsymbol{n}) = 0$.

Case 2: Suppose $\boldsymbol{n} = (\boldsymbol{n}_1, \boldsymbol{n}_2, \boldsymbol{n}_J) \in \widehat{\mathcal{N}}_{N,N_J}$. Notice that

$$P(\boldsymbol{n}) = P_J(\boldsymbol{n}_J) \times P(\boldsymbol{n}_1, \boldsymbol{n}_2 | \boldsymbol{n}_J) = P_J(\boldsymbol{n}_J) \times P_1(\boldsymbol{n}_1 | \boldsymbol{n}_J) \times P_2(\boldsymbol{n}_2 | \boldsymbol{n}_J)$$

where $P_J(\boldsymbol{n}_J)$ denotes the marginal probability of allocating joint meetings according to $\boldsymbol{n}_J$, and the second equality holds as P satisfies no-coordination. We proceed by identifying $P_J(\boldsymbol{n}_J)$ and $P_s(\boldsymbol{n}_s|\boldsymbol{n}_J)$ for each seller $s \in \{1,2\}$.

Part 1: $P_J(\boldsymbol{n}_J)$ By the definition of $\widehat{\mathcal{N}}_{N,N_J}$, notice that $\boldsymbol{n}_J$ is drawn from the set

$$\widehat{\mathcal{N}}_{(N,N_J)|J} \equiv \left\{ \boldsymbol{n}_J : \exists \mathcal{B}' \subseteq \{1,..,B\} \text{ s.t. } |\mathcal{B}'| = N_J \text{ and } n_J^b = \begin{cases} 1, & b \in \mathcal{B}' \\ 0, & b \in \{1,...,B\} \backslash \mathcal{B}' \end{cases} \right\}$$

We now show that every element of $\widehat{\mathcal{N}}_{(N,N_J)|J}$ must be drawn with equal probability under P_J. Let $\mathcal{B}(\boldsymbol{n}_J)$ denote the set of buyers under which $n_J^b = 1$ if and only if $b \in \mathcal{B}(\boldsymbol{n}_J)$. Take any $\boldsymbol{n}_J, \tilde{\boldsymbol{n}}_J \in \widehat{\mathcal{N}}_{(N,N_J)|J}$, and let g be any permutation of $\{1,...,B\}$ under which permuting the indices of $\boldsymbol{n}_J$ yields $\tilde{\boldsymbol{n}}_J$, i.e., $(n_J^{g(1)}, \cdots, n_J^{g(B)}) = \tilde{\boldsymbol{n}}_J$. Observe that $(\boldsymbol{n}_J, \boldsymbol{n}_1, \boldsymbol{n}_2) \in \widehat{\mathcal{N}}_{(N,N_J)}$ if and only if $(\boldsymbol{n}_1, \boldsymbol{n}_2) \in \widehat{\mathcal{N}}_{(N,N_J)|1}(\boldsymbol{n}_J) \times \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J) \equiv \widehat{\mathcal{N}}_{(N,N_J)|S}(\boldsymbol{n}_J)$, where for each seller $s \in \{1,2\}$,

$$\widehat{\mathcal{N}}_{(N,N_J)|s}(\boldsymbol{n}_J) \equiv \left\{ \boldsymbol{n}_s : \sum_{b=1}^{B} n_s^b = N - N_J \text{ and } n_s^b \begin{cases} = 0, & b \in \mathcal{B}(\boldsymbol{n}_J) \\ \in \{0,1\}, & b \in \{1,..,B\} \backslash \mathcal{B}(\boldsymbol{n}_J) \end{cases} \right\}$$

Furthermore, for any $(\boldsymbol{n}_1, \boldsymbol{n}_2) \in \widehat{\mathcal{N}}_{(N,N_J)|S}(\boldsymbol{n}_J)$, symmetry implies

$$P(\boldsymbol{n}_J, \boldsymbol{n}_1, \boldsymbol{n}_2) = P(\tilde{\boldsymbol{n}}_J, g_1(\boldsymbol{n}_1), g_2(\boldsymbol{n}_2))$$

where $g_s(\boldsymbol{n}_s) \equiv (n_s^{g(b)})_{b=1}^B$, i.e., $g_S(\boldsymbol{n}_s)$ is obtained from $\boldsymbol{n}_s$ by permuting buyers' indices according to g, defines a bijection from $\widehat{\mathcal{N}}_{(N,N_J)|s}(\boldsymbol{n}_J)$ to $\widehat{\mathcal{N}}_{(N,N_J)|s}(\tilde{\boldsymbol{n}}_J)$. Combined with the observation that the support of P is concentrated on $\widehat{\mathcal{N}}_{(N,N_J)|S}$, this implies that by aggregating over all pairs $(\boldsymbol{n}_1, \boldsymbol{n}_2) \in \widehat{\mathcal{N}}_{(N,N_J)|S}(\boldsymbol{n}_J)$,

$$\begin{aligned} P(\boldsymbol{n}_J) = \sum_{(\boldsymbol{n}_1,\boldsymbol{n}_2) \in \widehat{\mathcal{N}}_{(N,N_J)|S}(\boldsymbol{n}_J)} P(\boldsymbol{n}_J, \boldsymbol{n}_1, \boldsymbol{n}_2) &= \sum_{(\boldsymbol{n}_1,\boldsymbol{n}_2) \in \widehat{\mathcal{N}}_{(N,N_J)|S}(\boldsymbol{n}_J)} P(\tilde{\boldsymbol{n}}_J, g_1(\boldsymbol{n}_1), g_2(\boldsymbol{n}_1)) \\ &= \sum_{(\tilde{\boldsymbol{n}}_1,\tilde{\boldsymbol{n}}_2) \in \widehat{\mathcal{N}}_{(N,N_J)|S}(\boldsymbol{n}_J)} P(\tilde{\boldsymbol{n}}_J, \tilde{\boldsymbol{n}}_1, \tilde{\boldsymbol{n}}_2) = P(\tilde{\boldsymbol{n}}_J) \end{aligned}$$

Thus, since $\widehat{\mathcal{N}}_{(N,N_J)|J}$ has exactly $C_B^{N_J}$ number of elements, $P_J(\boldsymbol{n}_J) = \frac{1}{C_B^{N_J}}$ holds.

Part 2: $P_s(\boldsymbol{n}_s|\boldsymbol{n}_J)$ By the definition of $\widehat{\mathcal{N}}_{N,N_J}$, $\boldsymbol{n}_s$ is drawn from the set $\widehat{\mathcal{N}}_{(N,N_J)|s}(\boldsymbol{n}_J)$. We will show that every element of $\widehat{\mathcal{N}}_{(N,N_J)|s}(\boldsymbol{n}_J)$ must be drawn with equal probability under $P_s(\cdot|\boldsymbol{n}_J)$, focusing on seller $s = 1$ for now. Take any $\boldsymbol{n}_1, \tilde{\boldsymbol{n}}_1 \in \widehat{\mathcal{N}}_{(N,N_J)|1}(\boldsymbol{n}_J)$. Let g be any permutation of $\{1,...,B\}$ under which (i) permuting the indices of $\boldsymbol{n}_1$

yields $\tilde{\boldsymbol{n}}_1$, i.e., $(n_1^{g(1)}, ..., n_1^{g(B)}) = \tilde{\boldsymbol{n}}_1$, and (ii) the allocation of joint meetings does not change under g, i.e., $g(b) = b$ for all $b \in \mathcal{B}(\boldsymbol{n}_J)$ so $\boldsymbol{n}_J = (n_J^{g(b)})_{b=1}^B$ (one such permutation exists as $\tilde{\boldsymbol{n}}_1 \in \widehat{\mathcal{N}}_{(N,N_J)|1}(\boldsymbol{n}_J)$). For a given $\boldsymbol{n}_1 \in \widehat{\mathcal{N}}_{(N,N_J)|1}(\boldsymbol{n}_J)$, a necessary condition for $(\boldsymbol{n}_1, \boldsymbol{n}_2)$ to be drawn with positive probability under $P(\cdot|\boldsymbol{n}_J)$ is for $\boldsymbol{n}_2 \in \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J)$. Furthermore, for any $\boldsymbol{n}_2 \in \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J)$, symmetry implies

$$P(\boldsymbol{n}_J, \boldsymbol{n}_1, \boldsymbol{n}_2) = P((n_J^{g(b)})_{b=1}^B, \tilde{\boldsymbol{n}}_1, g_2(\boldsymbol{n}_2)) = P(\boldsymbol{n}_J, \tilde{\boldsymbol{n}}_1, g_2(\boldsymbol{n}_2))$$

where g_2 is constructed from permutation g in the manner described in Part 1. Since $P(\boldsymbol{n}_J) > 0$, this implies

$$P(\boldsymbol{n}_J)P(\boldsymbol{n}_1, \boldsymbol{n}_2|\boldsymbol{n}_J) = P(\boldsymbol{n}_J)P(\tilde{\boldsymbol{n}}_1, g_2(\boldsymbol{n}_2)|\boldsymbol{n}_J) \Rightarrow P(\boldsymbol{n}_1, \boldsymbol{n}_2|\boldsymbol{n}_J) = P(\tilde{\boldsymbol{n}}_1, g_2(\boldsymbol{n}_2)|\boldsymbol{n}_J)$$

From here, aggregating over all $\boldsymbol{n}_2 \in \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J)$ yields

$$\begin{aligned} P_1(\boldsymbol{n}_1|\boldsymbol{n}_J) = \sum_{\boldsymbol{n}_2 \in \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J)} P(\boldsymbol{n}_1, \boldsymbol{n}_2|\boldsymbol{n}_J) &= \sum_{\boldsymbol{n}_2 \in \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J)} P(\tilde{\boldsymbol{n}}_1, g_2(\boldsymbol{n}_2)|\boldsymbol{n}_J) \\ &= \sum_{\tilde{\boldsymbol{n}}_2 \in \widehat{\mathcal{N}}_{(N,N_J)|2}(\boldsymbol{n}_J)} P(\tilde{\boldsymbol{n}}_1, \tilde{\boldsymbol{n}}_2|\boldsymbol{n}_J) = P_1(\tilde{\boldsymbol{n}}_1|\boldsymbol{n}_J) \end{aligned}$$

Thus, since $\widehat{\mathcal{N}}_{(N,N_J)|1}(\boldsymbol{n}_J)$ has exactly $C_{B-N_J}^{N-N_J}$ number of elements, $P_1(\boldsymbol{n}_1|\boldsymbol{n}_J) = \frac{1}{C_{B-N_J}^{N-N_J}}$ holds. By a symmetric argument for seller 2, $P_2(\boldsymbol{n}_2|\boldsymbol{n}_J) = \frac{1}{C_{B-N_J}^{N-N_J}}$ holds.

Combining Parts 1 and 2, we obtain

$$P(\boldsymbol{n}) = \frac{1}{C_B^{N_J}} \times \frac{1}{C_{B-N_J}^{N-N_J}} \times \frac{1}{C_{B-N_J}^{N-N_J}} = \frac{1}{(C_B^{N_J})(C_{B-N_J}^{N-N_J})^2}$$

as required. ■

Proof of Proposition 1. Fix a $N_J \in \{0, .., B\}$. Observe that for any $N \in \{N_J + 1, ..., B\}$,

$$\begin{aligned} T_{N_J}(N) - T_{N_J}(N-1) &= -\left(\frac{1}{2}\right)^{N-1} \frac{1}{C_{B-N_J}^{N-N_J}} + \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_J}^{N-1-N_J}} \\ &= \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_J}^{N-1-N_J}} \left[1 - \frac{N - N_J}{2(B - N + 1)}\right], \end{aligned}$$

where the second equality follows from $\frac{C^{N-1-N_J}_{B-N_J}}{C^{N-N_J}_{B-N_J}} = \frac{\frac{B-N_J!}{N-1-N_J!B-N+1!}}{\frac{B-N_J!}{N-N_J!B-N!}} = \frac{N-N_J}{B-N+1}$. Thus,

$$T_{N_J}(N) \geq T_{N_J}(N-1) \text{ if and only if } N \leq \frac{2(B+1)+N_J}{3}$$

where the LHS inequality holds with equality if and only if the RHS inequality holds with equality.

Now, suppose that $N_J < B-2$. Since $B > \frac{2(B+1)+N_J}{3} > N_J + \frac{5}{3}$, $N^e_{N_J}$ must be the unique value of $N \in \{N_J, .., B\}$ that satisfies $N \leq \frac{2(B+1)+N_J}{3} < N+1$, i.e.,

$$N^e_{N_J} = \left\lfloor \frac{2(B+1)+N_J}{3} \right\rfloor$$

Next, suppose $N_J \geq B-2$. Then, $\frac{2(B+1)+N_J}{3} \geq B$ holds, so the efficient accessibility level is B. Thus,

$$N^e_{N_J} = B = \left\lfloor \frac{2(B+1)+B}{3} \right\rfloor = \left\lfloor \frac{2(B+1)+N_J}{3} \right\rfloor$$

This completes the proof of Proposition 1. ■

Proof of Corollary 1. Follows from Proposition 1. ■

Proof of Theorem 1. We first show that there is no equilibrium where sellers both set $r_1 = r_2 = 1$. First, note that any $\sigma_1(1,1) \in [0,1]$ constitutes a symmetric directed search equilibrium among the buyers upon observing sellers set $r_1 = r_2 = 1$. Thus, suppose $\sigma_1(1,1) \in [0,1/2]$ is played following $r_1 = r_2 = 1$ (the case for $\sigma_1(1,1) > 1/2$ can be argued similarly but from seller 2's perspective). Fixing $r_2 = 1$ and applying (12), we see that

$$\pi_{N_J}(1,1|N) = 1 - \Gamma_{N_J}(N|N,B)(1-\sigma_1(1,1))^N < 1 = \pi_{N_J}(1/2,1|N)$$

and so, seller 1 strictly prefers deviating to $r_1 = 1/2$ over setting $r_1 = 1$.

We now characterise the unique symmetric equilibrium reserve price. By the above, we need only consider $r_1 = r_2 < 1$. Fixing $r_2 < 1$, the partial derivative of

$\pi_{N_J}(r_1, r_2|N)$ with respect to r_1 is given by

$$-\underbrace{\left(\frac{A(r_1,r_2)}{(1-r_1)(N-1)(1+A(r_1,r_2))^{n-1}}\right)}_{\equiv B_1(r_1,r_2)} \times \underbrace{\left(\begin{array}{c}\Gamma_{N_J}(N|N,B)N(1-N(1-r_1)) \\ -\Gamma_{N_J}(N-1|N,B)(N-1)(1-r_1)(1+A(r_1,r_2))\end{array}\right)}_{\equiv B_2(r_1,r_2)} \tag{23}$$

(23) is strictly positive for $r_1 = r_2 = 0$, so sellers cannot (both) set a reserve price of zero in any symmetric equilibrium. Meanwhile, if $r = r_1 = r_2 \in (0,1)$ is a symmetric equilibrium, then it must set (23) to zero. It is simple to verify that for any $r_1 = r_2$, $B_1(r_1, r_2) > 0$, while $r_{N_J}(N)$ defined in (13) is the unique $r \in (0,1)$ which solves $B_2(r,r) = 0$. Meanwhile, fixing $r_2 = r_{N_J}(N)$, we see that $B_1(r_1, r_{N_J}(N)) \geq 0$ for all $r_1 \in [0,1]$ (and strictly so when $r_1 < 1$), while as $N \geq 2$,

$$\frac{\partial B_2}{\partial r_1} = \Gamma_{N_J}(N|N,B)N^2 + \Gamma_{N_J}(N-1|N,B)\Big(N-1+(N-2)A(r_1,r_2)\Big) > 0$$

Hence, $B_2(r_1, r_{N_J}(N))$ is strictly negative on $[0, r_{N_J}(N))$ and strictly positive on $(r_{N_J}(N), 1)$. Combined, these imply that $\pi_{N_J}(r_1, r_{N_J}(N)|N)$ is strictly single-peaked in r_1 on $[0,1]$ at $r_{N_J}(N)$, so $r_{N_J}(N)$ is seller 1's best-response to seller 2 setting $r_{N_J}(N)$. Thus, $r_{N_J}(N)$ is the unique symmetric equilibrium reserve price. ■

Proof of Theorem 2.
In text. ■

Proof of Proposition 2.
In text. ■

Proof of Proposition 3. We begin with two preliminary observations. First, it is never seller-optimal to choose $N = 1$. This is clearly the case when $N_J = 1$, as sellers' profits are equal to zero then. Meanwhile, when $N_J = 0$, we notice that

$$\pi_0(B) - \pi_0(1) = \frac{1}{B} - \left(\frac{1}{2}\right)^{B-1} > 0$$

as $B \geq 3$. Hence, it without loss to focus on $N \geq 2$ throughout, such that the seller's profit is always captured by (15).

Next, note on that for all $N \geq 3$,

$$\begin{aligned}&\pi_{N_J}(N) - \pi_{N_J}(N-1)\\&= \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_J}^{N-1-N_J}} \left[1 - \frac{N-N_J}{2(B-N+1)} + K(N-1) - K(N)\frac{N-N_J}{2(B-N+1)}\right]\end{aligned} \tag{24}$$

where

$$K(N) \equiv \left[\frac{(N-N_J)(B-N)}{N^2 + 2(N-1)(N-N_J)(B-N)}\right]$$

is strictly decreasing in N, bounded above by $1/2$, and $K(B) = 0$.

We now prove the claims in Proposition 3, beginning with showing that B cannot be seller optimal for N_J sufficiently small. Substituting $N = B$ into (24) yields

$$\pi_{N_J}(B) - \pi_{N_J}(B-1) = \left(\frac{1}{2}\right)^{B-2} \frac{1}{C_{B-N_J}^{B-1-N_J}} \left[1 - \frac{B-N_J}{2} + K(B-1)\right]$$

A simple computation yields that this is strictly less than zero if and only if

$$0 \leq N_J < \frac{(11 - 16B + 5B^2)}{4(B-2)} - \frac{1}{4}\sqrt{\frac{B^4 - 10B^2 + 16B - 7}{(B-2)^2}}$$

Since the RHS is strictly greater than zero, B is strictly not buyer-optimal whenever N_J is sufficiently small.

Next, we prove that $N^{\mathcal{S}} \geq N^e_{N_J}$. Given the proof of Proposition 1, this is equivalent to showing that

$$N \leq \frac{2(B+1)+N_J}{3} \quad \Rightarrow \quad \pi_{N_J}(N) \geq \pi_{N_J}(N-1)$$

By (24), if $N \leq \frac{2(B+1)+N_J}{3}$, such that $\frac{N-N_J}{2(B-N+1)} \leq 1$, then

$$\pi_{N_J}(N) - \pi_{N_J}(N-1) \geq \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_J}^{N-1-N_J}} \left[1 - \frac{N-N_J}{2(B-N+1)} + K(N-1) - K(N)\right] \geq 0$$

as required. ■

Proof of Proposition 4 We split the proof into three parts.

Part 1: First, suppose that $N_J \geq 2$. If $N_J = B$, then $N = B$ is trivially uniquely buyer-optimal. If $N_J = B - 1$, then

$$u_{B-1}(B-1) = \frac{1}{B}\left(\frac{1}{2}\right)^{B-2} > \frac{1}{B}\left(\frac{1}{2}\right)^{B-1} = u_{B-1}(B)$$

so $N_J = B - 1$ is uniquely buyer-optimal. Finally, if $N_J < B - 1$, then observe that

$$1 + \frac{2(N - N_J)(B - N)}{N^2 + 2(N-1)(N - N_J)(B - N)} \leq 2$$

while $\frac{1}{C_{B-N_J}^{N-N_J}} \leq 1$. Therefore, for any $N > N_J + 1$,

$$\begin{aligned} u_{N_J}(N) &= \frac{1}{BC_{B-N_J}^{N-N_J}}\left(\frac{1}{2}\right)^{N-1}\left(1 + \frac{2(N - N_J)(B - N)}{N^2 + 2(N-1)(N - N_J)(B - N)}\right) \\ &\leq \frac{1}{B}\left(\frac{1}{2}\right)^{N-2} < \frac{1}{B}\left(\frac{1}{2}\right)^{N_J-1} = u_{N_J}(N_J) \end{aligned}$$

Meanwhile, for $N = N_J + 1$, noting that

$$\left(1 + \frac{2(B - N_J - 1)}{(N_J+1)^2 + 2N_J(B - N_J - 1)}\right) < 2$$

we have

$$u_{N_J}(N_J + 1) < \frac{1}{B}\left(\frac{1}{2}\right)^{N_J-1} = u_{N_J}(N_J)$$

Hence, N_J is uniquely buyer-optimal.

Part 2: Next, suppose $N_J = 1$. Using the same logic as the proof of Part 1, for all $N \geq 3$,

$$u_1(N) \leq \frac{1}{B}\left(\frac{1}{2}\right)^{N-2} < \frac{1}{B} = u_1(1) \tag{25}$$

Meanwhile, for $N = 2$, since $B \geq 3$ such that $\frac{1}{C_B^1} \leq \frac{1}{3}$

$$u_1(2) \leq \frac{2}{BC_B^1} = \frac{2}{3B} < \frac{1}{B} = u_1(1)$$

Hence, $N^{\mathcal{B}} = 1 = N_J$ is uniquely buyer-optimal.

Part 3: Finally, suppose that $N_J = 0$. We have several subcases to consider

1. Suppose $B = 3$. Then, $u_0(1) = \frac{1}{18}$, $u_0(2) = u_0(3) = \frac{1}{12}$. Thus, the buyer-optimal accessibility level is either $N^{\mathcal{B}} = 2$ or $N^{\mathcal{B}} = 3$.

2. Suppose $B = 4$. Then, $u_0(1) = \frac{3}{80}$, $u_0(2) = \frac{5}{144}$, $u_0(3) = \frac{9}{448}$ and $u_0(4) = \frac{1}{32}$. Thus, $N^{\mathcal{B}} = 1$ is uniquely buyer-optimal.

3. Suppose $B = 5$. Then, $u_0(1) = \frac{2}{75}$, $u_0(2) = \frac{7}{400}$, $u_0(3) = \frac{3}{440}$, $u_0(4) = \frac{3}{500}$ and $u_0(5) = \frac{1}{80}$. Thus, $N^{\mathcal{B}} = 1$ is uniquely buyer-optimal.

4. Suppose $B = 6$. Then, $u_0(1) = \frac{5}{252}$, $u_0(2) = \frac{1}{100}$, $u_0(3) = \frac{7}{2400}$, $u_0(4) = \frac{1}{576}$, $u_0(5) = \frac{5}{2496}$ and $u_0(6) = \frac{1}{196}$. Thus, $N^{\mathcal{B}} = 1$ is uniquely buyer-optimal.

5. Suppose $B \geq 7$. Then, for all $N \geq 3 + \frac{\log(B)}{\log(2)}$, where $\frac{\log(B)}{\log(2)} < B - 1$,

$$\left(\frac{1}{2}\right)^{N-2} - \frac{B-1}{B(B+1)} \leq \left(\frac{1}{2}\right)\left[\left(\frac{1}{2}\right)^{N-3} - \frac{1}{B}\right] < 0$$

Therefore, recalling the discussion in Part 1,

$$u_{N_J}(N) \leq \frac{1}{B}\left(\frac{1}{2}\right)^{N-2} < \frac{B-1}{B(B+1)} = u_0(1)$$

where we note that this implies $N = B$ is strictly not buyer-optimal. Meanwhile, for any $N \in \{3, ..., \lfloor 3 + \frac{\log(B)}{\log(2)} \rfloor\}$, we notice that $C_{B-N_J}^{N-N_J} = C_B^N > C_B^1 = B$. As a result, if $N \geq 3$,

$$\begin{aligned} u_{N_J}(N) &= \frac{1}{BC_{B-N_J}^{N-N_J}}\left(\frac{1}{2}\right)^{N-1}\left(1 + \frac{2(N-N_J)(B-N)}{N^2 + 2(N-1)(N-N_J)(B-N)}\right) \\ &< \frac{1}{B^2}\left(\frac{1}{2}\right)^{N-2} \leq \frac{1}{B^2}\frac{1}{2} \leq \frac{B-1}{B^2(B+1)} = u_0(1) \end{aligned}$$

Finally, for $N = 2$, since $B \geq 7$ such that $\frac{1}{C_B^2} = \frac{2}{B-1} \leq \frac{1}{3}$,

$$u_0(2) \leq \frac{1}{BC_B^2} = \frac{2}{B^2(B-1)} < \frac{1}{B^2}\frac{1}{2} \leq \frac{B-1}{B^2(B+1)} = u_0(1)$$

Hence, $N^{\mathcal{B}} = 1$ is uniquely buyer-optimal.

Combined, Parts 1, 2 and 3 prove Proposition 4. ■

Proof of Proposition 5 We begin by supposing that $N = 1$. Then, all possible matches are formed in period 1, so the discounted total value is

$$T(1) = \frac{5}{3} + \delta \times 0 = \frac{5}{3}.$$

Next, suppose $N = B$. First, consider what happens in period 1. This is a market with two sellers and B fully informed buyers. With probability $2 \times (1/2)^B = (1/2)^{B-1}$, there is only one match. With probability $1 - (1/2)^{B-1}$, there are two matches. So, we expect $(1/2)^{B-1} + 2[1 - (1/2)^{B-1}] = 2[1 - (1/2)^B]$ matches. The period-2 market is exactly the same as in period 1 except the number of buyers reduces to $B-1$ or $B-2$. With $B-1$ buyers left, the expected number of matches in period 2 is $2[1 - (1/2)^{B-1}]$. With $B-2$ buyers left, the expected number of matches in period 2 is $2[1-(1/2)^{B-2}]$. So the expected value generated in period 2 is $2(1/2)^{B-1}[1 - (1/2)^{B-1}] + 2(1 - (1/2)^{B-1})[1 - (1/2)^{B-2}] = 2[1 - (1/2)^{B-1}]^2$. So the discounted total value is

$$T(B) = 2[1 - (1/2)^B] + 2\delta[1 - (1/2)^{B-1}]^2.$$

Finally, suppose $N \in (1, B)$. If there are less than $N - 1$ fully informed buyers, there must exist two partially informed buyers who only know firm 1 and two other partially informed buyers who only know firm 2. Then, there will always be two matches in each period. So if there are less than N fully informed buyers, the discounted total value is

$$2 + 2\delta.$$

With probability $1/C_B^N$ there are N fully informed buyers. In period 1, there are $2[1 - (1/2)^N]$ matches. In period 2, depending on whether there were one (with probability $(1/2)^{N-1}$) or two matches (with probability $1 - (1/2)^{N-1}$) formed in period 1, there are $2[1 - (1/2)^{N-1}]$ or $2[1 - (1/2)^{N-2}]$ matches respectively. So in case there are $N - 1$ fully informed buyers, the discounted total value is

$$\begin{aligned}
&2\left[1-\left(\frac{1}{2}\right)^{N}\right]+\delta\left[\left(\frac{1}{2}\right)^{N-1}\times 2\left(1-\left(\frac{1}{2}\right)^{N-1}\right)+\left(1-\left(\frac{1}{2}\right)^{N-1}\right)\times 2\left[1-\left(\frac{1}{2}\right)^{N-2}\right]\right]\\
=&2\left[1-\left(\frac{1}{2}\right)^{N}\right]+2\delta\left[1-\left(\frac{1}{2}\right)^{N-1}\right]^{2}
\end{aligned}$$

With probability $N(B-N)/C_B^N$ there are $N-1$ fully informed buyers, which means there will be two matches in period 1. In period 2, if both partially informed buyers were matched in period 1, there will be $2[1-(1/2)^{N-1}]$ matches in period 2. The probability that both partially informed buyers are matched in period 1 is

$$
\begin{aligned}
&\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}\times 1\times\frac{1}{N}+C_{N-1}^{1}\times\frac{1}{2}\times\frac{1}{N-1}+...+C_{N-1}^{N-1}\times\frac{1}{N}\times 1\right]\\
=&\left(\frac{1}{2}\right)^{N-1}(C_{N+1}^{1}+C_{N+1}^{2}+...+C_{N+1}^{N})\frac{1}{N(N+1)}\\
=&\left(\frac{1}{2}\right)^{N-1}\frac{2(2^{N}-1)}{N(N+1)}.
\end{aligned}
$$

To derive the second equality above, we use $C_{N+1}^{1}+C_{N+1}^{2}+...+C_{N+1}^{N}=2^{N+1}-C_{N+1}^{0}-C_{N+1}^{N+1}$. If only one partially informed buyer was matched in period 1, there is only one match with probability $(1/2)^{N-2}$ and two matches with probability $1-(1/2)^{N-2}$ in period 2. The probability that only one partially informed buyer is matched in period 1 is

$$
\begin{aligned}
&2\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}1\frac{N-1}{N}+C_{N-1}^{1}\frac{1}{2}\frac{N-2}{N-1}+C_{N-1}^{2}\frac{1}{3}\frac{N-3}{N-2}+...+C_{N-1}^{N-1}\frac{1}{N}0\right]\\
=&2\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}1+C_{N-1}^{1}\frac{1}{2}+C_{N-1}^{2}\frac{1}{3}+...+C_{N-1}^{N-2}\frac{1}{N-1}\right]\\
&-2\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}\frac{1}{N}+C_{N-1}^{1}\frac{1}{2}\frac{1}{N-1}+C_{N-1}^{2}\frac{1}{3}\frac{1}{N-2}+...+C_{N-1}^{N-2}\frac{1}{N-1}\frac{1}{2}\right]\\
=&2\left(\frac{1}{2}\right)^{N-1}\frac{1}{N}\left[C_{N}^{1}+C_{N}^{2}+C_{N}^{3}+...+C_{N}^{N-1}\right]\\
&-2\left(\frac{1}{2}\right)^{N-1}\frac{1}{N(N+1)}\left[C_{N+1}^{0}+C_{N+1}^{1}+C_{N+1}^{2}+...+C_{N+1}^{N-2}\right]\\
=&\left(\frac{1}{2}\right)^{N-1}\frac{2(2^{N}-1)(N-1)}{N(N+1)}.
\end{aligned}
$$

To derive the third equality above, we use $C_{N}^{1}+C_{N}^{2}+C_{N}^{3}+...+C_{N}^{N-1}=2^{N}-C_{N}^{0}-C_{N}^{N}$ and $C_{N+1}^{0}+C_{N+1}^{1}+C_{N+1}^{2}+...+C_{N+1}^{N-2}=2^{N+1}-C_{N+1}^{0}-C_{N+1}^{N}-C_{N+1}^{N+1}$. If no partially informed buyers were matched in period 1, there will be two matches in period 2. The probability of having no partially informed buyers being matched in period 1 is

$$
1-\left(\frac{1}{2}\right)^{N-1}\frac{2(2^{N}-1)}{N(N+1)}-\left(\frac{1}{2}\right)^{N-1}\frac{2(2^{N}-1)(N-1)}{N(N+1)}=1-\left(\frac{1}{2}\right)^{N-1}\frac{2N(2^{N}-1)}{N(N+1)}.
$$

Therefore, following having $N-1$ fully informed buyers, the discounted value is

$$
\begin{aligned}
&2+\delta\Bigg[\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)}{N(N+1)}2\left[1-\left(\frac{1}{2}\right)^{N-1}\right]\\
&+\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)(N-1)}{N(N+1)}\left[\left(\frac{1}{2}\right)^{N-2}+2\left(1-\left(\frac{1}{2}\right)^{N-2}\right)\right]\\
&+\left(1-\left(\frac{1}{2}\right)^{N-1}\frac{2N(2^N-1)}{N(N+1)}\right)2\Bigg]\\
=&2+\delta\left[2-\frac{16\left[(1/2)^N-(1/2)^{2N}\right]}{N+1}\right]
\end{aligned}
$$

Thus, for any $N\in(1,B)$, the generated value is

$$
\begin{aligned}
T(N)=&2\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\right]+\delta\Bigg[\left(1-\frac{1}{C_B^N}-\frac{N(B-N)}{C_B^N}\right)2\\
&+\frac{2}{C_B^N}\left(1-\left(\frac{1}{2}\right)^{N-1}\right)^2+\frac{N(B-N)}{C_B^N}\left(2-\frac{16\left[\left(\frac{1}{2}\right)^N-\left(\frac{1}{2}\right)^{2N}\right]}{N+1}\right)\Bigg]\\
=&2\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\right]+\delta\left[2-\frac{1}{C_B^N}\left(\left(\frac{1}{2}\right)^{N-2}-\frac{16N(B-N)}{N+1}\left(\left(\frac{1}{2}\right)^N-\left(\frac{1}{2}\right)^{2N}\right)\right)\right]\\
=&2\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\right]+2\delta\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\left(1-\left(\frac{1}{2}\right)^N\right)\frac{4(N+1)+8N(B-N)}{N+1}\right]
\end{aligned}
$$

Let $T^2(N)$ be the total matches in period 2. We next compare $T^2(B)$ and $T^2(B-1)$. We have

$$
T^2(B-1)=2\left[1-\left(\frac{1}{2}\right)^{B-2}\frac{4B+8(B-1)}{2B^2}+\left(\frac{1}{2}\right)^{2B-2}\frac{4B+8(B-1)}{B^2}\right].
$$

Then,

$$
T^2(B)-T^2(B-1)=2\left(\frac{1}{2}\right)^{B-2}\left[\frac{4B+8(B-1)}{B^2}\left(\frac{1}{2}-\left(\frac{1}{2}\right)^B\right)-1+\left(\frac{1}{2}\right)^B\right]. \tag{26}
$$

Note that in (26), when B becomes large, the term $\frac{4B+8(B-1)}{B^2}$ strictly decreases in B and approaches 0, the term $\frac{1}{2}-\left(\frac{1}{2}\right)^B$ is bounded from above by $\frac{1}{2}$, and the term $\left(\frac{1}{2}\right)^B$ approaches zero. So when B is sufficiently large the sign of (26) is entirely governed by the term -1, which is negative. So we can conclude that $T^2(B) - T^2(B-1) < 0$ when B is sufficiently large. This implies imperfect accessibility dominates full accessibility even in period 2 if B is large enough. ■