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Supplier Encroachment with Mutual Outsourcing

Abstract

We examine the incentives and implications of supplier encroachment, when final good production requires the use of multiple complementary inputs and the entry of a supplier into the final good market gives rise to mutual outsourcing of inputs between the encroaching supplier and the incumbent. We show that, post encroachment, mutual outsourcing between the competing final good producers is indeed the equilibrium. We also show, contrary to existing results, that encroachment can raise the input price paid by the incumbent and reduce consumer surplus. Nevertheless, the incumbent can benefit from encroachment due to the generation of a new profits source: input sales to the encroaching supplier. It can benefit even without enjoying a cost or a first mover advantage. This would have been impossible in an environment with a single input and without mutual outsourcing. Our analysis yields novel managerial, empirical and policy implications.

JEL-Codes: D430, L110, L210, L220, L230.

Keywords: supplier encroachment, complementary inputs, mutual outsourcing, outsourcing, input pricing, market entry.

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1 Introduction

Production and commercialization of final goods requires the use of multiple complementary inputs.[1] Original brand manufacturers typically produce some of the required inputs in-house, while they procure the rest from external input suppliers. Input suppliers, however, occasionally expand into final good production and turn into rivals of their customers, a practice referred to as supplier encroachment.[2] Since final good production by the encroaching suppliers also requires the use of multiple complementary inputs, supplier encroachment commonly gives rise to situations where the encroaching suppliers procure inputs from their own input customers. Stated differently, encroachment often gives rise to mutual outsourcing between the encroaching suppliers and their customers-rivals in the final good market.

In the mobile phone market, for example, when Google introduced its self-branded Nexus smart phone, it outsourced its production to Samsung, to which Google continued to supply its Android operating system. In the aviation market, when Mitsubishi encroached in the production of commercial jets, Boeing, which has been sourcing components of its jets from Mitsubishi, agreed to provide to Mitsubishi "customer support including spare parts provisioning, service operations and field services".[3] Similarly, in the tablets market, when Microsoft released its own Surface tablet, it continued to provide its operating system to Dell, which in turn started providing sales and professional support services to Microsoft.[4]

The rapid developments of information technology and of flexible supply chains in recent years have facilitated the emergence of more complex distribution and sourcing strategies; it has made easier for suppliers (producers) to become distributors and for distributors to become suppliers (producers). Many suppliers (producers) have started selling their products through intermediate online sales platforms as well as they have established their own direct online shops, generating the opportunity for platforms to provide support services to the online shops of the encroached suppliers.[5]

[1] Multiple complementary inputs are used in the production of various final goods, such as in the production of automobiles, mobile phones and personal computers. As stated by Choi and Gerlach (2019, p.1): "It is increasingly common in today's high-tech industries that commercialization of new products requires applications of multiple technologies".

[2] Examples of supplier encroachment abound. For instance, in the electronics market, Acer, Asustek and Samsung operated initially as component suppliers for branded manufacturers of electronics, but eventually moved on to the production of competing electronics under their own brand names.

[3] https://boeing.mediaroom.com/2011-06-22-Mitsubishi-Aircraft-Corporation-Selects-Boeing-to-Provide-World-Class-Customer-Support-and-Services. More information is available at: The Mitsubishi spacejet.

[4] https://www.forbes.com/sites/aarontilley/2015/09/08/microsoft-partners-with-dell-and-hp-to-boost-surface-tablet-sales-in-enterprise.

[5] Amazon provides its so called order "fulfillment" services that include packaging, storage, and delivery, among

The above examples clearly demonstrate that a common implication of supplier encroachment is the transformation of final good incumbents both into competitors and into input suppliers of their own suppliers. The literature on supplier encroachment, focusing on environments in which a single input – the input of the encroaching supplier – is used in final good production and in which the incumbent operates purely as a final good producer, has neglected this.

We should note that mutual outsourcing, which has also become quite common in recent years, does not appear only in cases with supplier encroachment. There are many instances in which already established final good manufacturers outsource various aspects of their manufacturing processes to one another. For example, in the car market, Nissan uses Daimler's Mercedes front-wheel-drive architecture platform, while Daimler sources diesel and gas engines from Nissan. Similarly, Toyota is using Suzuki's platform to develop compact models and in return supplies Suzuki with engines and batteries. In the media market, two competing cable TV providers in the UK, Sky and BT, agreed the cross-supply of TV channels.[6] The literature on outsourcing, focusing also on environments in which a single input is used in final good production, has not paid attention to mutual outsourcing so far.

In this paper, we revisit the incentives and implications of supplier encroachment incorporating the fact that final good production requires the use of multiple complementary inputs which can be mutually outsourced. In addition, we explore whether indeed competing final good manufacturers have incentives for mutual outsourcing.

Our model features an input supplier and an incumbent final good producer. Two complementary inputs are used in the production of the final good. Each firm is specialized in the production of one of these inputs. In particular, each firm can produce one input in-house at zero marginal cost, while it has the capability to produce the complementary input at a higher cost. The input supplier considers encroaching in the final good market. When it does, the encroaching supplier and the incumbent produce compete in quantities in the final good market. Importantly, the encroaching supplier, similarly to the incumbent, decides whether it will produce the complementary input in-house or it will source it from its final good rival at the wholesale price offered by the latter.

We demonstrate that mutual outsourcing of complementary inputs is always an equilibrium

other things. More information is available at: https://supplychain.amazon.com. Furthermore, Amazon has introduced its own Amazon branded products in many product markets, including recently the grocery retail market, competing with product manufacturers to which it provides its online marketplace platform.

[6]See Pun (2015) for more examples of cross-supply.

post encroachment. That is, firms always decide to source the complementary input from their rival rather than to produce it in-house. In fact, mutual outsourcing is the unique equilibrium. Under mutual outsourcing, both firms have two sources of profits: profits from input sales and from final good sales. The former profits are due to the fact that each firm is more efficient than its rival in the production of one of the inputs. Neither firm has incentives to unilaterally deviate from mutual outsourcing and act only as a final good manufacturer by making an input offer to its rival that it will not be subsequently accepted. If it did so, it would loose one of its profits sources −its profits from input sales. This loss would not be offset by the surge in its profits from final good sales that occurs when (not always) the rival's cost for sourcing the complementary input increases.

We derive three main sets of results which manifest that the presence of mutual outsourcing alters, even reverses, conventional views on encroachment. First, we show that encroachment can raise the input price faced by the incumbent and reduce consumer surplus. This occurs when the marginal cost of in-house production is relatively high and in turn the equilibrium wholesale prices under encroachment are equal to it.

Second, we show that, in contrast to when a single input is used in final good production, encroachment does not always arise in equilibrium. The input supplier does not encroach when the in-house production of the complementary input is too costly. This is so because it then faces a high input price and in turn the profits that it generates from the final good market are low and do not suffice to offset the reduction in its profits from input sales.

Third, we show that the incumbent can benefit from encroachment even without enjoying a drop in its input cost. This is because encroachment generates a new profits source for the incumbent; profits from its input sales to the encroaching supplier. Hence, encroachment is more likely to be beneficial both for the encroacher and the incumbent when multiple inputs are used and mutual outsourcing is present. Indeed, then, encroachment can be mutually profitable in a setting with symmetry, without cost and first-mover advantages or prior investments by the incumbent, which are needed for the mutual profitability of encroachment when a single input is required in production. Interestingly, when encroachment is mutually beneficial, it has no effect on the input price.

While in our main analysis input trade takes place through linear wholesale price contracts, extending our model we show that when non-linear two-part tariff contracts are used, mutual outsourcing is still an equilibrium post encroachment. Furthermore, we show that encroachment always raises the incumbent's input sourcing cost and that it is less likely to occur.

The remaining of the paper proceeds as follows. Section 2 reviews the related literature and highlights our contribution. Section 3 presents our model. Section 4 includes the equilibrium analysis of the benchmark case of no encroachment and the case in which a single input is used in final good production. Section 5 demonstrates that mutual outsourcing is the unique equilibrium post encroachment. Section 6 explores the incentives and implications of encroachment in the presence of mutual outsourcing. Section 7 extends the analysis by considering two-part tariff contracts. Lastly, Section 8 concludes and summarizes the managerial and the empirical implications of our analysis. All proofs are in the Appendix, unless a proof is very straightforward in which case it is omitted.

2 Literature Review

Several papers in the economics, marketing, and operations management literature explore the incentives of an input supplier (final good manufacturer) to encroach into the final goods market (to open a direct distribution channel). They put forward various reasons for which, contrary to conventional wisdom, a final good incumbent can benefit from the intensification of competition in the final good market resulting from supplier's expansion. In Chiang et al. (2003) and Arya et al. (2007), the incumbent benefits because it enjoys a drop in the wholesale price that results from the effort of the encroaching supplier, which faces a cost disadvantage and a second-mover disadvantage (Stackelberg) in the final good market, to protect its profits from input sales.[7] In Tsay and Agrawal (2004), the incumbent benefits because the drop in the wholesale price resulting from encroachment reinforces its investments in sales effort, and in Yoon (2016) and Matsushima and Mizuno (2018), because it can free-ride on the encroaching supplier's investments in cost-reduction.[8]

Our work contributes to the literature on encroachment in four important aspects. First, while previous research on supplier encroachment neglects the frequently observed instances in which multiple inputs are used in final good production and in which encroachment triggers mutual outsourcing between the encroaching supplier and the incumbent, our paper focuses on such instances; it explores the implications of the multiple complementary inputs and mutual outsourcing for en-

[7] Wang et al. (2013) and Niu et al. (2015) explore the role of the sequential choice of quantities and prices respectively.

[8] Recent papers explore the welfare implications of supplier encroachment under non-linear contracts (Matsushima et al., 2018), the role of asymmetric information (Li et al., 2014 and 2015), of limited capacity (Yang et al., 2018), and of the presence of multiple suppliers (Hotkar and Gilbert, 2021).

croachment. Second, in contrast to commonly met conclusions in the encroachment literature, our paper shows that encroachment can raise the wholesale price and harm consumers in the presence of mutual outsourcing.[9] It also shows that the supplier's incentives to encroach crucially depend on the efficiency of in-house input production, while the literature has neglected the incumbent's capability of in-house input production. Third, our paper puts forward a novel explanation for the positive impact of encroachment on the incumbent. The incumbent can benefit from the generation of a new profits source – its input sales to the encroaching supplier. Fourth, instead of considering asymmetric market environments, as is common in the encroachment literature, our paper shows that the incumbent can benefit from encroachment (even) in a symmetric setting in which it does not enjoy a cost and a first mover advantage in the final good market.

Several papers in the outsourcing literature explore firms input sourcing strategies. In particular, they explore firms incentives to outsource input production rather than to undertake input production in-house. Within this literature, a number of papers focus on the incentives of competing firms to (strategically) outsource to a common external supplier (e.g., Cachon and Harker, 2002, Buehler and Haucap, 2006, Arya et al. 2008). Others, closer to our work, consider a firm's incentives to outsource to a rival (e.g., Lim and Tan, 2010, Wang et al., 2016, Mandal and Jain, 2021). Most of these papers, with a few exceptions (e.g., Mandal and Jain, 2021), do not also consider supplier encroachment. Importantly, these papers, in contrast to ours, focus on settings in which a single input is used in final good production and thus in settings in which the study of only unilateral outsourcing and not also mutual outsourcing is possible.[10] Our paper attempts to fill this gap in the literature by exploring the incentives for mutual outsourcing.[11]

In the licensing literature, a small number of papers studies cross-licensing of complementary technologies or patents between competitors (e.g., Fershtman and Kamien, 1992, Shapiro, 1985, Jeon and Lefouili, 2018, Choi and Gerlach, 2019). These papers focus on cross-licensing incentives

[9] Hotkar and Gilbert (2021) consider a different market environment in which competing suppliers sell through a common reseller and show that encroachment can increase the wholesale price of the encroaching supplier and decrease the wholesale price of the rival supplier. While we show that encroachment can hurt consumers, Hotkar and Gilbert (2021) do not deal with the impact of encroachment on consumer welfare.

[10] Pun (2015) examines a setting with two competing final good manufacturers and two inputs. In Pun (2015), in contrast to our paper, each manufacturer can produce only one of the inputs in-house and thus it must outsource the other input. Pun (2015) neither examines whether firms will outsource or not nor whether supplier encroachment will take place. His focus is on the choice between outsourcing to a rival or to an independent input supplier. Kopel et al. (2016) consider the outsourcing incentives of a firm that uses multiple inputs in final good production when outsourcing is to an external supplier.

[11] In a recent paper, Arei and Matsushima (2023) consider an environment with mutual outsourcing in the absence though of sourcing decisions. Their focus is the endogenization, not of mutual outsourcing, but of the organizational form (managerial delegation).

and the impact of cross-licensing on competition in an *exogenously* given final good market in which they preassume that patent holders (input suppliers) are active.[12] Instead, our paper focuses on the incentives of an input supplier to become active in the final good market and on the impact of encroachment.[13] The contribution of our paper to the cross-licensing literature is two-fold. First, we show that the emergence of a market structure with mutual outsourcing (cross-licensing) crucially depends on market features and in particular on the inefficiency of in-house production technology: an input supplier (a patent holder) can be better off acting exclusively as a supplier (patent licensee) when its input production efficiency is high (the patented technology is strong). Second, we characterize the equilibrium under mutual outsourcing (cross-licensing) when firms have the capability of producing the complementary technology in-house (the licensed patents are not essential) and the contract does not involve fixed fees.[14]

Some papers in the literature on vertical relationships show that the existence of a complementary input supplier, by influencing input pricing, can affect the incentives and implications of vertical integration (Laussel, 2008, Matsushima and Mizuno, 2012) and of contract exclusivity (Kitamura et al., 2018). But these papers consider settings in which at least one of the complementary input suppliers is not active in the final goods market and mutual outsourcing is absent. Our work contributes to this literature too by exploring how vertical trading affects and is affected by the expansion of a supplier of a complementary into the final good market.

3 Model

There are two firms in the market, an input supplier–manufacturer, firm M, and an incumbent final good producer–retailer, firm R. Initially, only firm R produces a final good. Final good production requires the use, in a one-to-one proportion, of two inputs, input r and input m. Each firm is specialized in the production of one of the two inputs, but also has the capability of producing the complementary input at a (weakly) higher cost. More specifically, firm R produces input r in-house at zero marginal cost, while it either produces input m in-house at marginal cost $c \geq 0$ or outsources input m to firm M, which produces it at zero marginal cost. When firm R outsources

[12] Fershtamn and Kamien (1992) also focus on the impact of cross-licensing on the innovation race.

[13] The literature on horizontal subcontracting/outsourcing also allows for input sourcing from a competitor (e.g., Kamien et al., 1989, Spiegel, 1993, Baake et al., 1999). But it does so in settings either without cross-supply (i.e., only one of the competitors sources from the other) or with cross-supply in different product markets (Baake et al., 1999). This literature also does not study supplier encroachment.

[14] Jeon and Lefouili (2018) consider only contracts that include fixed fees, while Choi and Gerlach (2019) consider only essential patents.

input m to firm M, it pays a per unit wholesale price, w_M, offered by firm M. The demand for firm R's final good is given by the following linear (inverse) demand function: $p_R(q_R) = a - q_R$, with $a > c$.

Firm M considers encroaching in the final good market, i.e., it considers establishing the possibility of final good production.[15] In order to produce its final good when it encroaches, firm M uses the in-house produced input m at zero cost and it either sources input r from firm R at a per unit wholesale price, w_R, or it produces input r in-house at marginal cost $c \geq 0$.[16] Firms M and R are symmetric in terms of retailing and other costs that they may face, which for simplicity we set equal to zero.[17] When firm M encroaches, the demand function for firm i's final product, with $i, j = M, R$ and $i \neq j$, is given by: $p_i(q_i, q_j) = a - q_i - q_j$.[18] The market structure without and with encroachment is depicted in Figures 1 and 2 respectively.

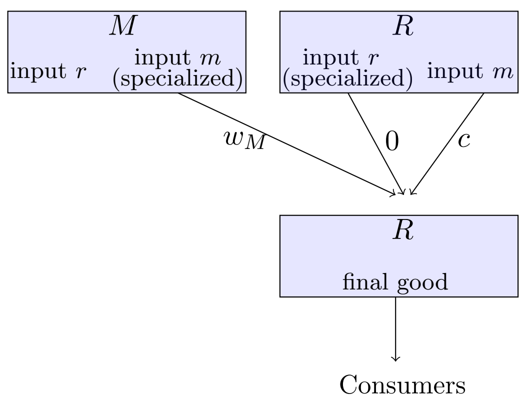

Figure 1: Market structure without encroachment

Firms play the following four-stage game with simultaneous and independent moves in each stage.

[15]To establish the possibility to produce the final good, firm M needs to open a new production line after incurring a fixed cost, which for simplicity we set equal to zero. Encroachment does not mean that the firm commits to producing a positive quantity of the final good.

[16]When $c > 0$, our analysis also applies to a situation where, if instead of firms being able to produce the complementary input in-house, there is a competitive fringe in the inputs market which can produce each of the inputs with narginal cost c.

[17]A common assumption in the encroachment literature (e.g., Chiang et al., 2003, Arya et al., 2007, Li et al., 2014, Hotkar and Gilbert, 2021) is that the encroaching firm has an exogenous cost disadvantage vis-a-vis the incumbent retailer. We assume, instead, that the encroaching firm can compete on par with the incumbent in the final good market. This allows us to focus on the analysis of the strategic incentives and implications of encroachment without the distortions generated by exogenous cost asymmetries. In addition, in our setting the cost that firm M faces when it encroaches corresponds to the cost of input r, which is endogenous when it outsources to firm R.

[18]For tractability reasons, we assume that the final goods are homogeneous. In Appendix B, we also consider what happens when the final goods of firm M and firm R are horizontally differentiated, focusing on mutual outsourcing post encroachment. We confirm our main results and conclude, not surprisingly, that encroachment is more likely to be mutually beneficial then.

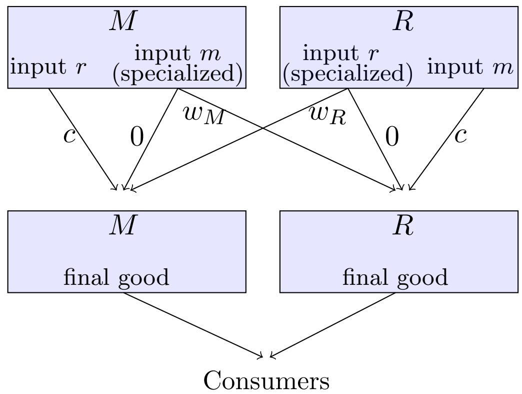

Figure 2: Market structure with encroachment

- **Stage 1:** Firm M decides whether or not to encroach.
- **Stage 2:** Wholesale price offer(s).
  - *No Encroachment*: Firm M chooses its wholesale price offer, $w_M \in \{\emptyset, \mathbb{R}_+\}$, to firm R for its specialized input m, where $\emptyset$ indicates no offer.[19]
  - *Encroachment*: Firm i chooses its wholesale price offer, $w_i \in \{\emptyset, \mathbb{R}_+\}$, to firm j, $i, j = M, R$, for its specialized input.
- **Stage 3:** Acceptance or not of wholesale price offer(s) if a firm has received an offer.
  - *No Encroachment*: Firm R, if it has received an offer from firm M, it accepts or not the offer.
  - *Encroachment*: Firm M and firm R, if they have received an offer from firm R and firm M respectively, accept or not the offer.

In both cases, if a firm accepts the offer, it uses the outsourced complementary input in the production of its final good in the following stage.[20] Instead, if a firm does not accept the offer, or has not received an offer, it uses the in-house produced complementary input in the following stage. Clearly, if both firms accept the offers, there is mutual outsourcing.

[19] We assume that if a firm makes a wholesale price offer it incurs a small cost $\varepsilon > 0$. This cost can be due, for example, to legal fees associated with drafting a contract and ensures that if an offer will not be accepted in equilibrium, then the firm is better off not making an offer at all.

[20] That is, the firm makes a binding commitment – it cannot reverse its sourcing decision in the following stage. This assumption is standard in the outsourcing literature where firms outsource although they have the in-house production capability (e.g., Arya et al., 2008, Feng and Lu, 2013, Colombo and Scrimitore, 2018). It can be justified, for instance, when firms include a penalty in their contract in case of contract violation, when firms undertake specific investments to make their final goods compatible with the external supplier's input, when they shut-down their own input production facilities, and when the application of procurement plans involves long lead times.

- **Stage 4:** Final good quantity decision(s).
  - *No Encroachment*: Firm R chooses the quantity of its final good.
  - *Encroachment*: Firm M and firm R choose the quantities of the final good.

We solve for the pure strategy subgame perfect equilibria and reason by backward induction. We assume throughout that when a firm is indifferent between outsourcing and insourcing the complementary input (i.e., accepting or not accepting the wholesale price offer), it opts for outsourcing. Under encroachment with mutual outsourcing, we focus on symmetric equilibria in wholesale prices and we use the Pareto dominance criterion to select a unique equilibrium when there are multiple symmetric equilibria in wholesale prices.[21] Throughout, we use superscripts B and E to denote the equilibrium values in the benchmark – the case of *no encroachment* – and in the case of *encroachment* respectively.

4 Benchmark Case of No Encroachment and Single Input Case

4.1 No Encroachment

We start by considering the benchmark case in which firm M operates only in the inputs market. In stage 4, firm R chooses q_R to maximize its monopoly profits, $\pi_R(q_R) = (p_R(q_R) - c_R)q_R$, where $c_R = w_M$ if it has accepted firm M's wholesale price offer and thus outsources input m, and $c_R = c$ if instead it has not accepted firm M's offer (or has not received an offer) and thus produces input m in-house. This results in $q_R(c_R) = \frac{a-c_R}{2}$.

In stage 3, firm R accepts firm M's offer if and only if $(p_R(q_R(w_M))-w_M)q_R(w_M) \geq (p_R(q_R(c))-c)q_R(c)$. It follows that firm R accepts firm M's offer if and only if $w_M \leq c$.

In stage 2, the unconstrained wholesale price that maximizes firm M's profits, $\pi_M(w_M) = w_M q_R(w_M)$, is $w_M = \frac{a}{2}$. This wholesale price satisfies $w_M \leq c$ if and only $\frac{c}{a} \geq \frac{1}{2}$. Firm M makes positive profits only if its offer is accepted. In light of this, the wholesale price that firm M offers to firm R in equilibrium and the subsequent equilibrium quantities and profits under no encroachment

[21] Hence, when we state that there exists a unique symmetric equilibrium regarding the wholesale prices under mutual outsourcing, it may be the case that it is unique after the application of our selection criterion.

are

$$w_M^B = \frac{a}{2}; \quad q_R^B = \frac{a}{4}; \quad \pi_M^B = \frac{a^2}{8}; \quad \pi_R^B = \frac{a^2}{16} \quad \text{if } \frac{c}{a} \geq \frac{1}{2} \tag{1}$$

$$w_M^B = c; \quad q_R^B = \frac{a-c}{2}; \quad \pi_M^B = \frac{(a-c)c}{2}; \quad \pi_R^B = \frac{(a-c)^2}{4} \quad \text{if } \frac{c}{a} < \frac{1}{2}. \tag{2}$$

4.2 Single Input

In order to assess the impact of multiple complementary inputs in the production process, we abstract now from our model and assume instead, as in much of the literature (e.g., Arya et. al., 2007), that final good production requires the use of a single input, input m. We continue to assume that firm M can produce input m in-house at zero cost, while firm R can either produce input m in-house at marginal cost c or source it from firm M at w_M. Unlike Arya et. al. (2007), we do not assume that firm M has a cost disadvantage in retailing, that firms choose their quantities sequentially post encroachment, and that firm R is unable to produce the input in-house.

In the case of no encroachment , equilibrium is given again by (1) and (2). In the case of encroachment, firms solve in stage 4 the following maximization problems

$$\max_{q_R} \ \pi_R(q_R, q_M) = (p_R(q_R, q_M) - c_R)q_R;$$
$$\max_{q_M} \ \pi_M(q_M, q_R) = p_M(q_M, q_R)q_M + \mathbb{1} w_M q_R,$$

where when firm R has accepted firm M's wholesale price offer, the indicator function $\mathbb{1}$ takes the value of 1 and $c_R = w_M$, while otherwise, the indicator function $\mathbb{1}$ takes the value of 0 and $c_R = c$. Clearly, when firm R does not accept firm M's wholesale price offer, firm M earns no revenue from input sales. The first order conditions result in $q_R(c_R) = \frac{a-2c_R}{3}$ and $q_M(c_R) = \frac{a+c_R}{3}$.

In stage 3, firm R accepts firm M's wholesale price offer if and only if $w_M \leq c$. In stage 2, the unconstrained wholesale price which maximizes firm M's profits, $\pi_M(w_M) = \frac{(a+w_M)^2}{9} + \frac{(a-2w_M)w_M}{3}$, is $w_M = \frac{a}{2}$. Firm M, taking into account that $w_M \leq c$ if and only if $\frac{c}{a} \geq \frac{1}{2}$, offers to firm R the following wholesale prices in equilibrium of the encroachment case with a single input

$$w_M^{ES} = \frac{a}{2} \quad \text{if } \frac{c}{a} \geq \frac{1}{2} \quad \text{and} \quad w_M^{ES} = c \quad \text{if } \frac{c}{a} < \frac{1}{2}.$$

Firm R accepts and the equilibrium profits in the encroachment case with a single input are

$$\pi_M^{ES} = \frac{a^2}{4}; \quad \pi_R^{ES} = 0 \quad \text{if } \frac{c}{a} \geq \frac{1}{2}$$
$$\pi_M^{ES} = \frac{a^2 + 5ac - 5c^2}{9}; \quad \pi_R^{ES} = \frac{(a-2c)^2}{9} \quad \text{if } \frac{c}{a} < \frac{1}{2}.$$

A straightforward observation is that encroachment has no impact on the input price, $w_M^{ES} = w_M^B$. Intuitively, firm M's encroachment gives rise to two opposite effects. On the one hand, it restricts firm R's quantity and drives firm M to lower its wholesale price. On the other hand, it transforms firm R to firm M's rival in the final good market and drives firm M to raise rival's cost, i.e., to increase its wholesale price. These two effects cancel each other out. As we will show, this is no longer true when multiple complementary inputs are required in final good production.

A further observation, which can be easily verified, is that for any $\frac{c}{a} \in [0, 1)$ encroachment is profitable for firm M and unprofitable for firm R. We summarize these conclusions in the next Proposition.

Proposition 1 *When final good production requires the use of a single input, encroachment has no impact on the wholesale price,* $w_M^{ES} = w_M^B$*, it always occurs in equilibrium,* $\pi_M^{ES} > \pi_M^B$*, and it hurts the incumbent final good producer,* $\pi_R^{ES} < \pi_R^B$*.*

Firm M's encroachment reduces its profits from input sales, but generates profits from final good sales. The latter implication always dominates as the final good profits of firm M are significant due to the fact that, in contrast to its rival in the final good market, it faces zero input cost. In fact, when firm R is too inefficient in the in-house input production (c is high), firm M fully forecloses firm R from the market ($q_R^E = 0$ if $\frac{c}{a} \geq \frac{1}{2}$) and enjoys the profits of a final good monopolist.

While the input supplier is better off when it encroaches, the opposite holds for the final good incumbent who suffers a reduction in its profits caused by the loss in its monopoly position due to encroachment. When a single input is used in final good production, there is no force in place to offset this reduction. This contrasts with Arya et al. (2007), who find that encroachment causes a drop in the input price that makes encroachment desirable for the incumbent when the encroaching firm faces an exogenous cost disadvantage relative to the incumbent and operates as a Stackelberg follower in the final good market.[22]

[22]Note that even if we assume that the encroaching manufacturer faces an exogenous cost disadvantage in the final good market, encroachment, in the single input case, still hurts the incumbent when the choice of quantities in the final good market takes place simultaneously.

5 Mutual Outsourcing under Encroachment

In this section, we examine whether or not when both firms are in the final good market, i.e., when encroachment has taken place, they opt in equilibrium for mutual outsourcing.

In stage 4, each firm i, with $i, j = M, R$ and $i \neq j$ solves

$$\max_{q_i} \pi_i(q_i, q_j) = \underbrace{(p_i(q_i, q_j) - c_i)q_i}_{\text{profit from final good sales}} + \underbrace{\mathbb{1} w_i q_j}_{\text{profit from input sales}}, \tag{3}$$

where the indicator function $\mathbb{1}$ takes the value of 1 when firm j has accepted firm i's wholesale price offer, while it takes the value of 0 when firm j has not accepted the offer or has not received an offer. Moreover, $c_i = w_j$ when firm i has accepted firm j's offer and thus outsources input j, and $c_i = c$ when firm i has not accepted firm j's offer or has not received an offer and thus produces input j in-house. Solving the system of first order conditions, we obtain

$$q_i(c_i, c_j) = \frac{a - 2c_i + c_j}{3}. \tag{4}$$

In stage 3, there are four different types of candidate equilibria: (i) *Mutual Outsourcing*, where both firms M and R have accepted the wholesale price offers that they have received, (ii) *Unilateral Outsourcing by firm M*, where only firm M has accepted the offer that it has received, (iii) *Unilateral Outsourcing by firm R*, where only firm R has accepted the offer that it has received, (iv) *No Outsourcing*, where neither of the two firms has accepted an offer or has received an offer. Cleary, mutual outsourcing can arise in equilibrium if both firms have received offers and neither firm has incentives to unilaterally deviate to a situation in which it does not accept the offer and produce the complementary input in-house. In the Appendix, we present in detail the possible deviations from each of these types of candidate equilibria.

In stage 2, each firm decides whether to make an offer and if so the level of the wholesale price. When firms mutually outsource, the best response functions that result from the maximization of firms' profits (3) are

$$w_i = \frac{a}{2} - \frac{w_j}{10}. \tag{5}$$

It follows that the (unconstrained) wholesale prices are strategic substitutes. This is due to the fact that when firm j raises w_j, firm i faces a higher marginal cost and in turn sells a lower quantity of its final good (4). Consequently, firm i's profits from the final good market shrink and its interest in its profits from the input market is augmented. To support its input profits, firm i drops its rival's input cost w_i. The resulting unconstrained wholesale prices and firms profits are

$$w_M = w_R = \frac{5a}{11} \quad \text{and} \quad \pi_M = \pi_R = \frac{14a^2}{121}. \tag{6}$$

Note that the unconstrained wholesale prices are lower than the unconstrained wholesale price in the case in which a single input is used in final good production, $w_M^{ES} = \frac{a}{2}$. Thus, input competition lowers the unconstrained wholesale prices.

Next, we examine whether the above unconstrained wholesale prices or other wholesale prices can support mutual outsourcing as an equilibrium as well as whether mutual outsourcing is the unique equilibrium. The derivation of these wholesale prices is not a straightforward task for two reasons. First, because the unconstrained wholesale prices do not necessarily satisfy the conditions for mutual outsourcing to be an equilibrium (i.e., there can be deviation incentives to not accept), and second because firms might have incentives to unilaterally deviate to wholesale prices that do not give rise to mutual outsourcing in stage 3. We perform these tasks in the proof of the following Proposition.

Proposition 2 *In the encroachment case, mutual outsourcing is the unique equilibrium for all values of $\frac{c}{a} \in [0,1)$. Under mutual outsourcing, the unique symmetric pure strategy equilibrium wholesale prices are given by*

$$w_M^E = w_R^E = \begin{cases} c, & \text{if } \frac{c}{a} \in \left[0, \frac{5}{11}\right] \\ \frac{5a}{11}, & \text{if } \frac{c}{a} \in \left(\frac{5}{11}, \frac{29}{44}\right) \\ \frac{4(a-c)}{3}, & \text{if } \frac{c}{a} \in \left[\frac{29}{44}, \frac{9+\sqrt{13}}{17}\right] \\ c, & \text{if } \frac{c}{a} \in \left(\frac{9+\sqrt{13}}{17}, 1\right), \end{cases} \tag{7}$$

and firm profits are given by

$$\pi_M^E = \pi_R^E = \begin{cases} \frac{(a+2c)(a-c)}{9}, & \text{if } \frac{c}{a} \in \left[0, \frac{5}{11}\right] \\ \frac{14a^2}{121}, & \text{if } \frac{c}{a} \in \left(\frac{5}{11}, \frac{29}{44}\right) \\ -\frac{11\left(a-\frac{8c}{11}\right)(a-4c)}{81}, & \text{if } \frac{c}{a} \in \left[\frac{29}{44}, \frac{9+\sqrt{13}}{17}\right] \\ \frac{(a+2c)(a-c)}{9}, & \text{if } \frac{c}{a} \in \left(\frac{9+\sqrt{13}}{17}, 1\right). \end{cases} \tag{8}$$

As Proposition 2 states, mutual outsourcing always arises in equilibrium post encroachment. In fact, mutual outsourcing is the unique equilibrium then. That is, both firms opt for sourcing the complementary input from their rival rather than producing it in-house. The main driving force of this finding is the generation of profits from input sales; both firms, facing a cost advantage in the production of one of the inputs, can increase their profits by turning their rival into also a customer. Neither firm has incentives to unilaterally deviate from mutual outsourcing and act only as a final good manufacturer by making an input offer to its rival that it will not be subsequently accepted. If it did so, it would loose one of its profits sources - its profits from input sales. This loss would not be offset by the surge in its profits from final good sales that occurs when as we will see below (not always) the rival's cost for sourcing the complementary input increases.

Under mutual outsourcing, the unconstrained wholesale prices (6) arise in equilibrium when c takes intermediate values. When, instead, firms are either very efficient or very inefficient in the in-house production of the complementary input, equilibrium wholesale prices equal c. In particular, when c is low, none of the firms has incentives to unilaterally raise its wholesale price offer above c. If it did, its offer would not be accepted and the firm would be worse off making no profits from input sales, while its profits from the final good sales would not be significant since its rival would not be relatively too inefficient. The same mechanism is in place in the single input case, where, as we saw, $w_M^{ES} = c$ when c is low.

When c is high, although the unconstrained wholesale prices are lower than c, they do not arise in equilibrium. A firm, say firm M, has incentives to deviate from the unconstrained wholesale prices and offer a higher wholesale price than (6) so that it induces non acceptance by firm R in stage 3. Doing so, firm M turns firm R into a very inefficient rival in the final good market (producing input m in-house with a high c), and thus, although firm M would make no profits from input sales, it would make high profits from the final good market. In contrast, none of the firms has incentives to deviate from mutual outsourcing at wholesale prices equal to a high c.

The mutual outsourcing equilibrium exhibits some interesting properties. First, firm R is never fully foreclosed, whereas in the single input case, the unconstrained wholesale price set by firm M fully forecloses firm R from the final good market when c is too high. Second, the profits that firms make in the equilibrium with mutual outsourcing (8) exceed the profits that they make in the no outsourcing candidate equilibrium, i.e., if both firms produce both inputs in-house, $\frac{(a-c)^2}{9}$, for $c > 0$. Therefore, firms are not trapped in a prisoner's dilemma opting for mutual outsourcing.

6 Incentives and Implications of Encroachment

We now compare encroachment with no encroachment and draw our conclusions regarding the incentives and the implications of encroachment.

6.1 Impact on Input Price

We start with the comparison of the wholesale prices.

Proposition 3 *Encroachment with mutual outsourcing of inputs:*

(i) increases the wholesale price that the incumbent retailer faces, $w_M^E = c > w_M^B = \frac{a}{2}$*, if* $\frac{c}{a} \geq \frac{2(9+\sqrt{13})}{34}$*,*

(ii) decreases the wholesale price that the incumbent retailer faces, $w_M^E < w_M^B$*, if* $\frac{5}{11} < \frac{c}{a} < \frac{2(9+\sqrt{13})}{34}$*,*

(iii) does not affect the wholesale price that the incumbent retailer faces, $w_M^E = w_M^B = c$*, if* $\frac{c}{a} \leq \frac{5}{11}$*.*

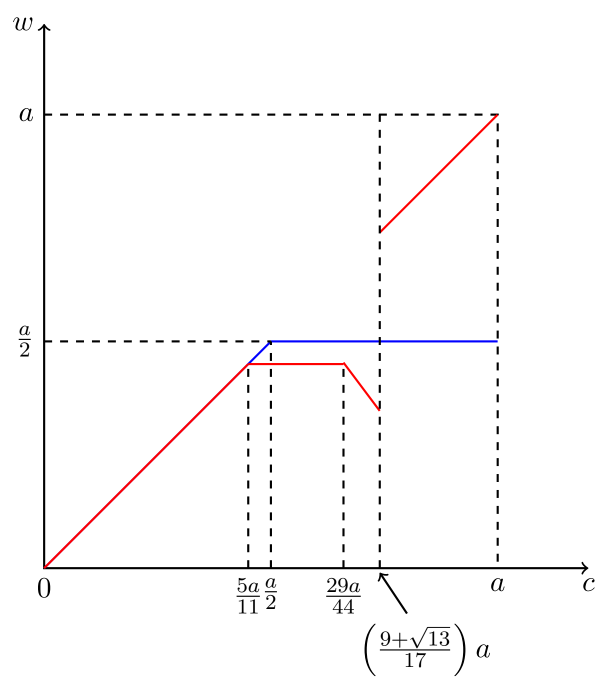

Figure 3: Equilibrium wholesale prices. In red are the wholesale prices under encroachment and in blue under no encroachment (the blue coincides with the red for $c \in \left[0, \frac{5a}{11}\right]$).

When firm M encroaches, the wholesale price that it charges to firm R can either increase, decrease or remain unchanged relative to the no encroachment case (see also Figure 3). Why is this so? Encroachment, as we also saw in the case in which a single input is used in final good production, brings about two key differences in the market. First, it restricts firm R's quantity, and second it generates profits of firm M from the final good market. While the first difference motivates firm M to lower the wholesale price its charges to firm R, the second motivates it to increase it.

When multiple inputs are used in final good production, encroachment with mutual outsourcing brings about an *additional* difference: it makes firm M a potential input customer of its customer-rival. An implication is the presence of strategic interaction in the setting of the wholesale prices by the two firms, in contrast to the no encroachment case and the single input case where firm M is the sole determinant of the wholesale price.

Recall that with a single input, when c is high, the unconstrained wholesale price of firm M forecloses firm R, so firm M has no incentive to raise it further. With multiple inputs, encroachment makes firms (potentially) symmetric and there is no foreclosure in equilibrium. When c is high, firms can sustain high equilibrium wholesale prices, higher than the unconstrained ones, and earn significant revenue from the input market.

6.2 Impact on Firm Profits

We now turn to stage 1 and examine whether firm M has incentives to encroach when encroachment is accompanied by mutual outsourcing. We reach the following conclusion.

Proposition 4 *Encroachment with mutual outsourcing of inputs arises in equilibrium,* $\pi_M^E > \pi_M^B$, *if and only if* $\frac{c}{a} \leq \frac{2}{5}$.

Encroachment incentives are not always present when multiple inputs are used in final good production and encroachment is accompanied by mutual outsourcing. Although encroachment generates profits for firm M from the final good market, these profits do not always offset the decrease in firm M's profits from the inputs market. This is so because when firm M encroaches, it outsources, similarly to its rival, the production of one of the inputs. Thus, in contrast to the single input case, it does not enjoy a cost-advantage in the final good market and in turn it does not extract a big share of the final good profits. Furthermore, the profits that firm M generates from the final good market are quite low when c is high since, as we saw in Proposition 2, the wholesale prices that firms face then equal c. Therefore, when c is high, and particularly when $\frac{c}{a} > \frac{2}{5}$, the final good profits of firm M do not suffice to make encroachment profitable.

We turn now to the desirability of encroachment from the incumbent retailer's viewpoint.

Proposition 5 *Encroachment with mutual outsourcing of inputs benefits the incumbent retailer, $\pi_R^E > \pi_R^B$, if and only if $\frac{c}{a} \geq \frac{5}{17}$.*

When encroachment occurs, the incumbent retailer can enjoy a second source of revenue from the sales of input r to the encroaching supplier. This additional revenue source offsets its lost profits from the final good market when firms are too inefficient in the in-house production of the complementary input. Thus, in contrast to the single input case where the incumbent is always worse off with encroachment, the incumbent can benefit from encroachment with multiple inputs.

Corollary 1 *Encroachment with mutual outsourcing of inputs is mutually beneficial for the manufacturer and the incumbent retailer when $\frac{c}{a} \in \left[\frac{5}{17}, \frac{2}{5}\right]$.*

As the above Corollary states, there exists a range of c values where both firms are better off with encroachment without side payments between them. When this holds, encroachment neither lowers nor raises the wholesale prices, $w_M^E = w_M^B = c$.

The above findings highlight the role of the multiple complementary inputs and mutual outsourcing. In sum, when multiple complementary inputs are used and mutual outsourcing is present, encroachment is less likely to occur than when a single input is used and it can benefit the incumbent even when the latter does not enjoy a cost and a first-mover advantage in the final good market.

6.3 Impact on Consumer Surplus

Having explored the desirability of encroachment for firms, we turn now to its desirability for consumers; we explore how encroachment affects consumer surplus.

Given that the final goods are homogenous, consumer surplus is given by the area of the triangle below the inverse demand, $p(Q) = a - Q$, and above any price; hence, $CS = \frac{(a-p)Q}{2} = \frac{Q^2}{2}$, where Q is the aggregate quantity in equilibrium. In particular, in the no encroachment case, the equilibrium quantity $Q^B = q_R^B$ is given by (1) and (2), and in turn, consumer surplus is

$$CS^B = \begin{cases} \frac{(a-c)^2}{8}, & \text{if } \frac{c}{a} < \frac{1}{2} \\ \frac{a^2}{32}, & \text{if } \frac{c}{a} \geq \frac{1}{2}. \end{cases}$$

Under encroachment with mutual outsourcing, using (4), we have $Q^E = q_M^E + q_R^E = \frac{2(a-w_R^E)}{3}$, where w_R^E is the equilibrium symmetric wholesale price given by (7). It follows that consumer surplus is then

$$CS^E = \begin{cases} \frac{2(a-c)^2}{9}, & \text{if } \frac{c}{a} \in \left[0, \frac{5}{11}\right] \\ \frac{8a}{121}, & \text{if } \frac{c}{a} \in \left(\frac{5}{11}, \frac{29}{44}\right) \\ \frac{2(a-4c)^2}{81}, & \text{if } \frac{c}{a} \in \left[\frac{29}{44}, \frac{9+\sqrt{13}}{17}\right] \\ \frac{2(a-c)^2}{9}, & \text{if } \frac{c}{a} \in \left(\frac{9+\sqrt{13}}{17}, 1\right). \end{cases}$$

The following conclusion is a straightforward implication of the comparison of consumer surplus in the two cases.

Proposition 6 *Encroachment with mutual outsourcing of inputs increases consumer surplus, $CS^E > CS^B$, if and only if $\frac{c}{a} < \frac{9+\sqrt{13}}{17}$.*

Encroachment generates two (possibly opposing) forces on consumer welfare. First, it intensifies competition, a fact that can result in a lower price for the final good. Second, it can lead to higher or lower equilibrium input prices that in turn can lead to a higher or a lower final good price. When the in-house production of the complementary input is sufficiently inefficient, and in particular when $\frac{c}{a} > \frac{2(9+\sqrt{13})}{34}$, as we saw in Proposition 3, encroachment leads to a higher input price. The increase in the input price then, and thus, the deterioration of firm's final good production efficiency offsets the intensification of competition and makes consumers worse off with encroachment. Clearly, this could not occur in the single input case since then encroachment introduces competition in the final good market without affecting the input price.

The above finding suggests that the entry of firms into a final good market should not always be welcomed or encouraged (e.g., through subsidies) by policy makers even when the entrants are not inefficient relative to the incumbents. In situations in which, pre-entry, the entrant operates as an input supplier and post-entry mutual outsourcing of inputs between the entrant and the incumbents can arise, authorities should take into account the impact of entry not only on the final good market but also on the pricing of inputs. Entry can cause an increase in the incumbent's input cost and thereby harm consumers.

Still, combining Corollary 1 and Proposition 6, we can conclude that when encroachment occurs in equilibrium without any policy intervention, consumers always become better off and the incumbent may also become better off.

7 Two-part Tariffs

In what follows, we examine the robustness of our main results when input trading takes place through two-part tariff contracts which, besides the wholesale price w_i, include a fixed fee, F_i, that firm j pays to firm i.

In the no encroachment (benchmark) case, firm M sets $w_M^{BT} = 0$, it maximizes total industry profits, and uses the fixed fee to extract all the profits of firm R except its outside option of producing all inputs in-house, $F_M^{BT} = \frac{a^2}{4} - \frac{(a-c)^2}{4} = \frac{c(2a-c)}{4}$. The resulting equilibrium profits are

$$\pi_M^{BT} = \frac{c(2a-c)}{4} \text{ and } \pi_R^{BT} = \frac{(a-c)^2}{4}.$$

In the encroachment case, we focus on a symmetric equilibrium where both firms accept the two-part tariff offers. For this to be the case, the following condition must be satisfied

$$\pi_i(w_j, w_i) + F_i - F_j \geq \pi_i(c, w_i) + F_i \Rightarrow F_j \leq \pi_i(w_j, c_i) - \pi_i(c, w_i). \tag{9}$$

In stage 2, the fixed fee is chosen so that (9) is satisfied with equality; hence, firm j sets $F_j = \frac{(4a-4c+w_i-4w_j)(c-w_j)}{9}$. Substituting the fixed fees in firm i's profits, we obtain

$$\pi_i(w_j, w_i) = \frac{4w_j^2 + (-4a + c - 2w_i)w_j + a^2 + (4c + w_i)a - 4c^2 - w_i^2}{9} - F_j.$$

Firm i chooses w_i, and hence F_i, to maximize its profits holding w_j and F_j fixed. The symmetric Nash equilibrium wholesale prices are $w_M^{ET} = w_R^{ET} = \frac{a}{4}$. They induce acceptance by both firms in stage 3 and result in the following equilibrium profits

$$\pi_M^{ET} = \pi_R^{ET} = \frac{a^2}{8}. \tag{10}$$

Two observations are in line. First, the equilibrium wholesale prices maximize total industry profits.[23] Second, the incumbent always faces a higher wholesale price with than without encroachment, $w_M^{ET} > w_M^{BT}$. Therefore, similarly to what happens under wholesale price contracts, under two-part tariffs too, encroachment can raise the incumbent's input cost. In fact, under two-part tariffs, this happens always and not only when c is high.

Firm M encroaches in equilibrium, $\pi_M^{ET} \geq \pi_M^{BT}$, if and only if $\frac{c}{a} \leq 1 - \frac{1}{\sqrt{2}}$. Intuitively, firm M extracts, through the fixed fee, a larger piece of the pie without than with encroachment. This is so because when it encroaches, firm M is in a symmetric market position with firm R, while when it operates exclusively as an input supplier, it has full control of the trading terms. Taking this into account, firm M does not encroach unless c is sufficiently low. In the latter case, firm M is unable to extract a relative big piece of the pie without encroachment since its customer's outside option of in-house production is notable. The fact that the pie is larger with encroachment dominates then and encroachment arises. This finding gives rise to the following implication regarding cross-licensing: when the licensed technology (patent) is strong, cross-licensing of complementary technologies between competitors is unlikely to arise since a patent holder can generate more profits by refraining to act as a competitor in the final goods market.

Since the condition for the encroachment to be profitable for firm M is stricter under two-part tariffs than the respective condition in Proposition 4, encroachment is less likely under two-part tariffs than under wholesale price contracts. This is driven by the fact that, as noted by Milliou et al. (2003 and 2009), an input supplier extracts a larger surplus share from its customers through a two-part tariff than through a wholesale price contract. This means that the fact that firm M extracts a larger piece of the pie without encroachment is more pronounced under two-part tariffs than under wholesale price contracts making encroachment less likely to occur with two-part tariffs.

Even though encroachment raises its input cost, firm R can be better off with encroachment. In particular, $\pi_R^{ET} \geq \pi_R^{BT}$ if and only if $\frac{c}{a} \geq 1 - \frac{1}{\sqrt{2}}$. Clearly, in contrast to wholesale price contracts, there is no range of c for which encroachment is beneficial for both firms under two-part tariffs.

[23] Jeon and Lefouili (2018, 2020) show that industry profit maximization can be achieved through cross-licensing with bilateral two-part tariffs, in a setting with more than two firms in the market. Our result is in line with their finding.

8 Conclusion

Supplier encroachment often triggers mutual outsourcing of complementary inputs between the encroaching supplier and its incumbent customer. In this paper, first, we demonstrated that this is indeed the case, and second, we examined how mutual outsourcing affects the supplier incentives to encroach into the final good market and the implications of encroachment on the incumbent and the consumers.

The dual roles that both the encroaching supplier and the incumbent have as input suppliers of one another and as final good rivals when encroachment occurs lead to findings that differ from the ones that commonly arise in the encroachment literature. We showed that the supplier foregoes encroachment when the inefficiency of in-house production of the complementary input is high. The supplier does not wish to sacrifice part of the high profits that it makes from its input sales in order to generate low profits from its final good sales then. We also showed that encroachment can raise the wholesale price faced by the incumbent when the inefficiency of in-house production of the complementary input is high. Clearly, when this happens, encroachment is unwelcome by the incumbent. Instead, the incumbent can benefit from encroachment when it causes a drop in the wholesale price but also when it leaves it unaffected. In the latter case, the elimination of the incumbent's monopoly position in the final good market can be offset by the profits that it attains from its input sales in its new role as an input supplier. Without the input profits, the incumbent would be worse off if the encroaching supplier had neither a cost nor a second-mover disadvantage in the final good market. We show in addition that consumers do not always benefit from encroachment. While they enjoy the intensification of competition in the final good market induced by encroachment, they can be hurt by the raise in the incumbent's input sourcing cost also induced by encroachment.

In our main analysis, inputs are traded through wholesale price contracts, a contractual form which, according to empirical studies, is used in various industries. We demonstrate that under two-part tariffs contracts, encroachment would occur under more stringent conditions due to the encroaching supplier's considerations regarding the surplus share that it extracts.

Besides theoretical relevance, our results can be of practical importance. They can offer guidance to managers of firms that operate as successful input producers and entertain the possibility of expanding in the production of final goods. As the results of our paper suggest, the managers who make such decisions have to consider whether they will be able to efficiently produce their final goods on their own or they will need to source complementary inputs from their customers-rivals in the final good market. In the latter case, their decision should depend on the competition intensity in the final market and the extend of the inefficiency of in-house production of the complementary inputs, i.e., on whether input production is too specialized. Our results also provide insights that can help guide the entrant's decisions regarding the wholesale pricing of the inputs.

A number of empirical predictions regarding encroachment can also be drawn on the basis of our results. These are summarized as follows. Encroachment is more likely to be observed in markets in which input trading takes place through linear wholesale price contracts than in markets in which more complex contracts, such as two-part tariffs, are used. Moreover, encroachment is more likely to lead to a higher wholesale price paid by the incumbent when two-part tariffs are used in input trade, or when firms are quite inefficient in the production of the complementary input in markets where wholesale price contracts are used.

Furthermore, considering what happens when the final goods of the encroaching supplier and the incumbent are differentiated, we confirm our main results and conclude that, not surprisingly, encroachment is more likely to be mutually beneficial then.

We have performed our analysis in a setting with one incumbent and have treated the in-house input production costs as exogenous. It may be of interest to examine the additional effects that arise in the presence of multiple competing incumbents, the implications of encroachment on firms' incentives to invest in reducing their costs of producing the complementary inputs in-house, as well as what would happen under sequential choice of quantities. It would also be of interest to examine whether our main findings could also arise not from supplier encroachment but from the entry of an additional final good producer in the market. We leave these for future work.

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A Appendix

A.1 Proof of Proposition 2

In stage 3 under encroachment, where each firm decides whether to accept or not the other firm's wholesale price offer (if an offer has been made), there are four possible candidate equilibria that are described below. In deriving the profits we use (3) and (4).

- *Mutual Outousourcing* or (*Accept, Accept*), where both firms accept each other's wholesale price offers. In this a case, there is mutual outsourcing. Firms' profits in terms of the wholesale prices are given by

$$\begin{aligned}\pi_M^{AA}(w_R, w_M) &= \frac{(5a - w_R)w_M}{9} - \frac{5w_M^2}{9} + \frac{(a - 2w_R)^2}{9};\\ \pi_R^{AA}(w_M, w_R) &= \frac{(5a - w_M)w_R}{9} - \frac{5w_R^2}{9} + \frac{(a - 2w_M)^2}{9}.\end{aligned} \tag{A.11}$$

- *Unilateral Outsourcing by firm M* or (*Accept, Not Accept*), where firm M accepts firm R's wholesale price offer and firm R does not accept firm M's offer, or firm M has not made an offer. Firms' profits are then

$$\pi_M^{AN}(w_R, c) = \frac{(a - 2w_R + c)^2}{9}; \quad \pi_R^{AN}(c, w_R) = \frac{5w_R^2}{9} + \frac{(5a - c)w_R}{9} + \frac{(a - 2c)^2}{9}. \tag{A.12}$$

- *Unilateral Outsourcing by firm R or* (*Not Accept, Accept*), where firm M does not accept firm R's wholesale price offer (or firm R has not made an offer) and firm R accepts firm M's offer. Firms' profits are then

$$\pi_M^{NA}(c, w_M) = \frac{5w_M^2}{9} + \frac{(5a-c)w_M}{9} + \frac{(a-2c)^2}{9}; \quad \pi_R^{NA}(w_M, c) = \frac{(a-2w_M+c)^2}{9}. \tag{A.13}$$

- *No Outsourcing* or (*Not Accept, Not Accept*), where neither firm accepts the other's offer (or no firm has made an offer) and firms' profits are

$$\pi_M^{NN}(c, c) = \pi_R^{NN}(c, c) = \frac{(a-c)^2}{9}. \tag{A.14}$$

In what follows, we derive the conditions under which each of the above four types of candidate equilibria can indeed be an equilibrium in stage 3.

- (*Accept, Accept*) is an equilibrium when firm M has no incentives to deviate to (*Not Accept, Accept*), i.e., when $\pi_M^{AA}(w_R, w_M) \geq \pi_M^{NA}(c, w_M)$, and firm R has no incentives to deviate to (*Accept, Not Accept*), i.e., when $\pi_R^{AA}(w_M, w_R) \geq \pi_R^{AN}(c, w_R)$. It follows that (*Accept, Accept*) is an equilibrium if (w_M, w_R) belong in one of the following sets

$$\begin{aligned}
\mathcal{E}_{AA}^{I} &\equiv \left\{(w_M, w_R) : w_M \leq c, w_R \leq c, w_M \leq (a-c) + \frac{w_R}{4}, w_M \geq -4(a-c) + 4w_R\right\} \\
\mathcal{E}_{AA}^{II} &\equiv \left\{(w_M, w_R) : w_M \geq c, w_R \geq c, w_M \geq (a-c) + \frac{w_R}{4}, w_M \leq -4(a-c) + 4w_R\right\} \\
\mathcal{E}_{AA}^{III} &\equiv \left\{(w_M, w_R) : w_M \geq c, w_R \leq c, w_M \geq (a-c) + \frac{w_R}{4}, w_M \geq -4(a-c) + 4w_R\right\} \\
\mathcal{E}_{AA}^{IV} &\equiv \left\{(w_M, w_R) : w_M \leq c, w_R \geq c, w_M \leq (a-c) + \frac{w_R}{4}, w_M \leq -4(a-c) + 4w_R\right\}.
\end{aligned} \tag{A.15}$$

Thus, when there is mutual outsourcing, there are four regions in (w_M, w_R) of possible equilibria in stage 3 captured by the above four sets. See also Figures 4, 5 and 6, where these four sets, along with the sets for the other equilibria, are depicted. There is a region of low wholesale prices for both firms, set $\mathcal{E}_{AA}^{I}$, and a region of high wholesale prices for both firms, set $\mathcal{E}_{AA}^{IV}$. In the former set the main source of revenue is the final product market, while in the latter it is the input market. Note that both firms can accept wholesale prices strictly higher than the cost of in-house production c, as in $\mathcal{E}_{AA}^{II}$, and this can be an equilibrium.[24] This can be understood as follows. If firm i does not accept a $w_j > c$, it will produce the complementary input in-house. Its production cost will decrease, it will produce more output and its profits from the product market will increase. But firm j will produce less output and hence i's profits from the input sales to j will decrease. The

[24] In Cases 2 and 3, in Section A.1.1, there exist such equilibria, although they are Pareto dominated by other equilibria with $w \leq c$.

decrease in profits from the input sales can offset the increase of the product market profits, when firm i expects firm j to accept its $w_i > c$ offer.

- (*Accept, Not Accept*) is an equilibrium when firm M has no incentives to deviate to (*Not Accept, Not Accept*), i.e., when $\pi_M^{AN}(w_R, c) \geq \pi_M^{NN}(c, c)$, and firm R has no incentives to deviate to (*Accept, Accept*), i.e., when $\pi_R^{AN}(c, w_R) > \pi_R^{AA}(w_M, w_R)$. Therefore, (*Accept, Not Accept*) is an equilibrium if (w_M, w_R) belong in one of the following sets

$$\begin{aligned}
\mathcal{E}_{AN}^{I} &\equiv \left\{(w_M, w_R) : w_M < c, w_R \leq c, w_M > (a-c) + \frac{w_R}{4}\right\} \\
\mathcal{E}_{AN}^{II} &\equiv \left\{(w_M, w_R) : w_M > c, w_R \leq c, w_M < (a-c) + \frac{w_R}{4}\right\}.
\end{aligned}$$

- (*Not Accept, Accept*) is an equilibrium if (w_M, w_R) belong in one of the following sets (the analysis coincides with that of (*Accept, Not Accept*) above with the roles of the two firms reversed)

$$\begin{aligned}
\mathcal{E}_{NA}^{I} &\equiv \{(w_M, w_R) : w_R < c, w_M \leq c, w_M < -4(a-c) + 4w_R\} \\
\mathcal{E}_{NA}^{II} &\equiv \{(w_M, w_R) : w_R > c, w_M \leq c, w_M > -4(a-c) + 4w_R\}.
\end{aligned}$$

- (*Not Accept, Not Accept*) is an equilibrium when firm M has no incentives to deviate to (*Accept, Not Accept*), i.e., when $\pi_M^{NN}(c, c) > \pi_M^{AN}(w_R, c)$, and firm R has no incentives to deviate to (*Not Accept, Accept*), i.e., when $\pi_R^{NN}(c, c) > \pi_R^{NA}(w_M, c)$. It follows that (*Not Accept, Not Accept*) is an equilibrium if (w_M, w_R) belong in the set $\mathcal{E}_{NN} \equiv \{(w_M, w_R) : w_M > c, w_R > c\}$.[25]

A.1.1 Mutual outsourcing equilibrium

We begin by searching for symmetric equilibria in w where both firms accept the wholesale price offers (mutual outsourcing). There are three cases consistent with this kind of equilibria, depending on the value of $\frac{c}{a} \in [0, 1)$. To better understand why the three cases arise, first note that $a - c < \frac{4(a-c)}{3} \Leftrightarrow \frac{c}{a} < 1$, which is always satisfied. As it will become clearer very soon, these thresholds determine the location and shape of the various $\mathcal{E}$ sets we derived above. Then, there are three

[25] A mixed strategy equilibrium (mse) could also exist. Let ν_j be the probability with which firm j accepts i's offer. Then, it can be easily confirmed that firm i is indifferent between accepting and not accepting if $\nu_j = \frac{4(a-w_j)}{4c-w_i}$. It then follows that $\nu_R < 1$ if and only if $w_M < 4w_R - 4(a-c)$ and $\nu_M < 1$ if and only if $w_M > \frac{w_R}{4} - (a-c)$. Therefore, the mixed strategy is an equilibrium if $(w_M, w_R) \in \mathcal{E}_{mse} \equiv \left\{(w_M, w_R) : w_M > (a-c) + \frac{w_R}{4}, w_M < -4(a-c) + 4w_R\right\}$. We assume that when a pure and a mixed strategy equilibrium co-exist, firms play the pure strategy. Firm M's expected equilibrium profits are (R's expected profits are symmetric to M's): $E\pi_M = [4c^3 + (4a - w_M - 12w_R)c^2 - 4\left(a + \frac{w_M}{2}\right)(a - 2w_R)c - (20a - 20w_R)w_M^2 + (19a^2 - 20aw_R)w_M]/(36c - 9w_M)$.

possibilities for the level of c relative to the $a-c$ and $\frac{4(a-c)}{3}$ thresholds.[26] In what follows we consider each case.

Case 1: $a-c < \frac{4(a-c)}{3} < c < a$.

This case is valid if $\frac{c}{a} \in \left(\frac{4}{7}, 1\right) \approx (0.571, 1)$. It corresponds to Figure 4, which depicts the regions in which combinations of (w_M, w_R) satisfy the conditions for the emergence of each type of stage 3 candidate equilibria, consistent with case 1.[27]

In this case, the unconstrained wholesale prices (6) cannot be in the $\mathcal{E}^{II}_{AA}$ region of Figure 4, since this would contradict the condition that $\frac{c}{a} > \frac{4}{7}$. The unconstrained wholesale prices (6) are strictly less than $\frac{4(a-c)}{3}$, the northeast vertex of the $\mathcal{E}^{I}_{AA}$ set, if and only if $\frac{c}{a} < \frac{29}{44} \approx 0.66$. When this is so, i.e., when $\frac{c}{a} < \frac{29}{44}$, a local deviation is by construction unprofitable. However, a firm, say firm M, can raise its wholesale price to the point that firm R does not accept the offer. This amounts to getting out of the $\mathcal{E}^{I}_{AA}$ set and into the adjacent set $\mathcal{E}^{I}_{AN}$. The difference between firm M's profits with the unconstrained wholesale prices and its deviation profits, (A.12), is $\frac{125}{1089}a^2 - \frac{2}{99}ac - \frac{1}{9}c^2$, which is positive if $\frac{c}{a} \leq \frac{(-1+3\sqrt{14})}{11} \approx 0.929$. Firm M can also lower its wholesale price if (6) is greater than $a-c$ (which is equivalent to $\frac{c}{a} > \frac{6}{11}$–a condition that is satisfied in this case) and move into the adjacent $\mathcal{E}^{I}_{NA}$ set. The difference now between its profits with the unconstrained wholesale prices (6) and its deviation profits, (A.13), is $\frac{49}{99}ac - \frac{145}{1089}a^2 - \frac{4}{9}c^2$, which is positive if and only if $\frac{c}{a} \in \left[\frac{5}{11}, \frac{29}{44}\right]$. Therefore, the unconstrained wholesale prices $w_R = w_M = \frac{5a}{11}$ are an equilibrium and both firms accept the offers if $\frac{c}{a} \in \left(\frac{4}{7}, \frac{29}{44}\right)$.

When, instead, $\frac{c}{a} \geq \frac{29}{44}$, and thus the unconstrained wholesale prices $w_R = w_M = \frac{5a}{11}$ are weakly higher than $\frac{4(a-c)}{3}$ the candidate equilibrium wholesale prices consistent with both firms accepting are now $w_R = w_M = \frac{4(a-c)}{3}$ and the profits with these wholesale prices are

$$\pi_M = \pi_R = -\frac{11\left(a - \frac{8c}{11}\right)(a-4c)}{81}. \tag{A.16}$$

Since the candidate wholesale prices are lower than the unconstrained wholesale prices, each firm may want to increase its wholesale price. A firm, say firm M, by unilaterally increasing its wholesale price, even infinitesimally, moves to the $\mathcal{E}^{I}_{AN}$ set and induces the other firm not to accept the offer. The difference between its profits with $w_R = w_M = \frac{4(a-c)}{3}$, (A.16), and its deviation profit, (A.12),

[26] Also, $\frac{4(a-c)}{3} < a \Leftrightarrow \frac{c}{a} > \frac{1}{4}$. This matters only in case 3, where $\frac{c}{a}$ is low and hence $\frac{4(a-c)}{3}$ can be higher than a. We return to this possibility when we analyze case 3.

[27] In this case, there exists a region where a pure strategy in stage 3 does not exist, see the white area in Figure 4. However, neither a symmetric equilibrium nor a deviation fall in this area when the mixed strategy is the unique equilibrium.

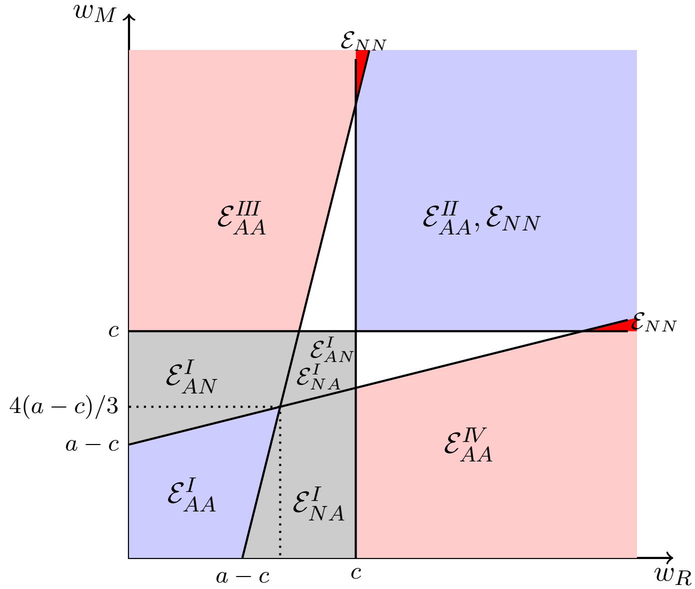

Figure 4: Case 1: $a - c < \frac{4(a-c)}{3} < c < a$. In the white area, there is no pure strategy equilibrium in stage 3, but a mixed strategy equilibrium exists (see footnote 25).

is $2ac - \frac{4}{9}a^2 - \frac{17}{9}c^2$, which is positive if and only if $\frac{c}{a} \in \left[2\left(\frac{9}{34} - \frac{\sqrt{13}}{34}\right), 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)\right] \approx [0.317, 0.741]$. Firm M can also decrease w_M and move to the $\mathcal{E}_{NA}^{I}$ set. The difference now between its pre- and post-deviation profits is zero. Therefore, $w_R = w_M = \frac{4(a-c)}{3}$ is an equilibrium and both firms accept the offers if $\frac{c}{a} \in \left[\frac{29}{44}, 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)\right]$.

We now check whether $w_M = w_R = c$ is an equilibrium. This is at the southwest vertex of the $\mathcal{E}_{AA}^{II}$ set and corresponds to the following profits

$$\pi_M = \pi_R = \frac{(a+2c)(a-c)}{9}. \tag{A.17}$$

We cannot have an equilibrium in the interior of the $\mathcal{E}_{AA}^{II}$ set because in the case 1 we are examining, the unconstrained wholesale prices $w_R = w_M = \frac{5a}{11}$ are strictly less than c. If candidate equilibrium wholesale prices were in the interior of the $\mathcal{E}_{AA}^{II}$ set, they would be strictly higher than c. A firm would deviate then locally while still inducing acceptance and would increase its profits. But we can have an equilibrium on the boundary of this set, i.e., $w_M = w_R = c$.[28] A firm, say M, could deviate from $w_M = w_R = c$ by lowering its wholesale price (or not making an offer) and moving to the $\mathcal{E}_{AN}^{I}$ set. The difference between its profits with $w_M = w_R = c$, (A.17), and its deviation profit, (A.12), is $\frac{(a-c)c}{3} > 0$.[29] Hence, $w_M = w_R = c$ is an equilibrium and both firms

[28] There are also asymmetric candidate equilibria in the neighborhood of $w_M = w_R = c$, on the edges of the $\mathcal{E}_{AA}^{II}$ set. We ignore these equilibria, since our focus is on symmetric ones.

[29] Note that the $\mathcal{E}_{NA}^{I}$ set does not contain its boundary $w_R = c$, so a deviation there is not possible. And we have

accept the offers for all values of c of this case.

To summarize, for $\frac{c}{a} \in \left(\frac{4}{7}, \frac{29}{44}\right)$ there are two equilibria, $w_R = w_M = \frac{5a}{11}$ and $w_M = w_R = c$. For $\frac{c}{a} \in \left[\frac{29}{44}, 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)\right]$ there are two equilibria, $w_R = w_M = \frac{4(a-c)}{3}$ and $w_M = w_R = c$. Finally, for $\frac{c}{a} \in \left(2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right), 1\right)$, the unique equilibrium is $w_M = w_R = c$. It can be easily shown that $w_M = w_R = c$ is Pareto dominated by either $w_R = w_M = \frac{5a}{11}$ or $w_R = w_M = \frac{4(a-c)}{3}$, when they co-exist. Therefore, after applying the Pareto refinement, the unique wholesale prices equilibrium is: (i) for $\frac{c}{a} \in \left(\frac{4}{7}, \frac{29}{44}\right)$, $w_R = w_M = \frac{5a}{11}$, (ii) for $\frac{c}{a} \in \left[\frac{29}{44}, 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)\right]$, $w_R = w_M = \frac{4(a-c)}{3}$, and (iii) for $\frac{c}{a} \in \left(2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right) 1, 1\right)$, $w_M = w_R = c$.

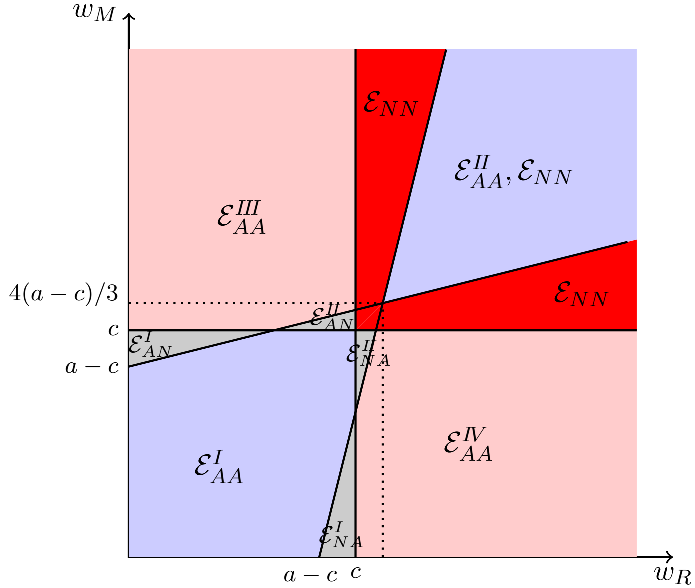

Figure 5: Case 2: $a - c < c \leq \frac{4(a-c)}{3} < a$.

Case 2: $a - c < c \leq \frac{4(a-c)}{3} < a$.

This case is valid if $\frac{c}{a} \in \left(\frac{1}{2}, \frac{4}{7}\right] \approx [0.5, 0.571]$ and corresponds to Figure 5. The unconstrained wholesale prices fall in the interior of the $\mathcal{E}_{AA}^{I}$ region. A deviation by firm M to a higher wholesale price so that we move to either the $\mathcal{E}_{AN}^{I}$ or the $\mathcal{E}_{AN}^{II}$ set is, as we know from the analysis in case 1, unprofitable if $\frac{c}{a} \leq \frac{-1+3\sqrt{14}}{11} \approx 0.929$. Hence, no incentives exist for such deviation. A deviation to a lower wholesale price by firm M so that we move to the $\mathcal{E}_{NA}^{I}$ set, if $\frac{5a}{11} > a - c$ and thus $\frac{c}{a} > \frac{6}{11}$ which is in the range of this case, as we also showed in Case 1, is unprofitable if $\frac{c}{a} \in \left[\frac{5}{11}, \frac{29}{44}\right]$, which is a superset of the case 2 range. Thus, no incentives for such a deviation as well. Moreover, as we showed in case 1, $w_M = w_R = c$, which is on the boundary of the $\mathcal{E}_{AA}^{I}$ set, is always an equilibrium.

also assumed that when a pure and a mixed strategy co-exist, firms play the pure strategy.

An equilibrium can also be at the southwest vertex of the $\mathcal{E}_{AA}^{II}$ set, given by $w_R = w_M = \frac{4(a-c)}{3}$.[30] A deviation in this case by a firm, say firm M, moves the wholesale prices to the $\mathcal{E}_{RR}$ set, so both firms would not accept the wholesale price offers. The difference between firm M's profits with $w_R = w_M = \frac{4(a-c)}{3}$, (A.16), and its deviation profits, (A.14), is $\frac{70}{81}ac - \frac{20}{81}a^2 - \frac{41}{81}c^2$, which is positive if and only if $\frac{c}{a} \in \left[2\left(\frac{35}{82} - \frac{9\sqrt{5}}{82}\right), 2\left(\frac{35}{82} + \frac{9\sqrt{5}}{82}\right)\right] \approx [0.362, 1.344]$. Hence, $w_R = w_M = \frac{4(a-c)}{3}$ is also an equilibrium.

To summarize, there are three equilibria, $w_R = w_M = \frac{5a}{11}$, $w_R = w_M = \frac{4(a-c)}{3}$ (which is higher than c), and $w_M = w_R = c$. The first in the interior of the $\mathcal{E}_{AA}^{I}$ set, the second on the southwest vertex of the $\mathcal{E}_{AA}^{II}$ set, and the third on the northeast vertex of the $\mathcal{E}_{AA}^{I}$ set. It can be shown that $w_R = w_M = \frac{5a}{11}$ Pareto dominates the other two if $\frac{c}{a} < \frac{29}{44}$, which holds in this case. Thus, after applying the Pareto refinement, the unique equilibrium is $w_R = w_M = \frac{5a}{11}$.

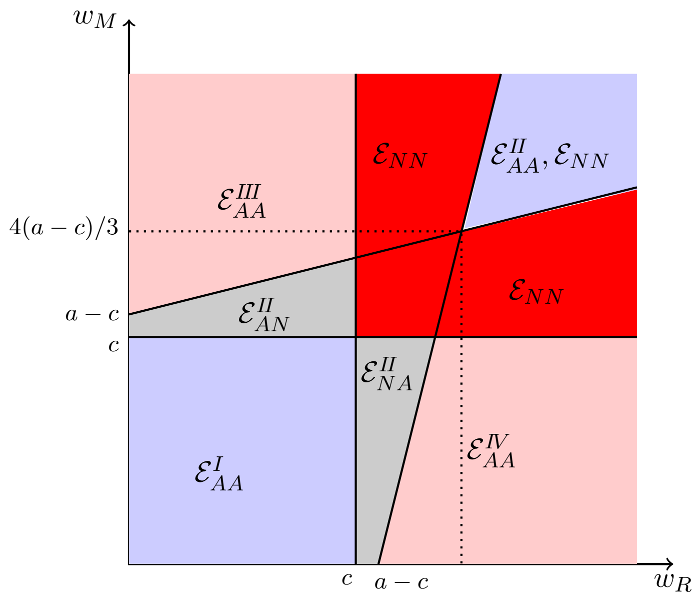

Figure 6: Case 3: $c \leq a - c < \frac{4(a-c)}{3} < a$.

Case 3: $c \leq a - c < \frac{4(a-c)}{3} < a$.[31]

This case is valid if $\frac{c}{a} \in \left[0, \frac{1}{2}\right]$ and corresponds to Figure 6. The unconstrained wholesale prices are in the $\mathcal{E}_{AA}^{I}$ set and are strictly less than c if $\frac{c}{a} > \frac{5}{11} \approx 0.454a$. As we showed in Case 1, a deviation by firm M to a higher w_M so that firm R does not accept, i.e., moving to the $\mathcal{E}_{AN}^{II}$ set, is unprofitable

[30] As in Case 1, it cannot be in the interior of that set.

[31] Note that in this case $\frac{4(a-c)}{3}$ can be greater than a when $\frac{c}{a} < \frac{1}{4}$. This, however, does not affect the analysis in any substantial way, because $w_M = w_R = \frac{4(a-c)}{3}$ is not an equilibrium in case 3, even when $\frac{4(a-c)}{3} < a$. When $\frac{4(a-c)}{3} > a$, there is no AA region when $w_M = w_R = a$ (a possible corner equilibrium) and therefore it ceases to be a candidate equilibrium.

if $\frac{c}{a} \leq \frac{-1+3\sqrt{14}}{11} \approx 0.929$. If now $\frac{c}{a} < \frac{5}{11}$, (6) falls outside of the $\mathcal{E}^{I}_{AA}$ set. For this range, we check whether $w_R = w_M = \frac{4(a-c)}{3}$, which is on the southwest vertex of the $\mathcal{E}^{II}_{AA}$ set, is an equilibrium. As we showed in Case 2, it is an equilibrium for $\frac{c}{a} \in \left[2\left(\frac{35}{82} - \frac{9\sqrt{5}}{82}\right), 2\left(\frac{35}{82} + \frac{9\sqrt{5}}{82}\right)\right] \approx [0.362, 1.344]$.[32] Also, as we showed in Case 1, $w_M = w_R = c$, which is on the northeast vertex of the $\mathcal{E}^{I}_{AA}$ set, is always an equilibrium.

To summarize, for $\frac{c}{a} \in \left[0, 2\left(\frac{35}{82} - \frac{9\sqrt{5}}{82}\right)\right)$, $w_M = w_R = c$ is the unique equilibrium. For $\frac{c}{a} \in \left(2\left(\frac{35}{82} - \frac{9\sqrt{5}}{82}\right), \frac{5}{11}\right)$, there are two equilibria, $w_M = w_R = c$ and $w_R = w_M = \frac{4(a-c)}{3}$ (which is higher than c). For $\frac{c}{a} \in \left[\frac{5}{11}, \frac{1}{2}\right]$, there are three equilibria, $w_R = w_M = \frac{5a}{11}$, $w_R = w_M = \frac{4(a-c)}{3}$ (which is higher than c) and $w_R = w_M = c$. It can be easily shown that $w_R = w_M = c$ is Pareto dominated by $w_R = w_M = \frac{5a}{11}$, but it Pareto dominates $w_R = w_M = \frac{4(a-c)}{3}$, when they co-exist. Finally, $w_M = w_R = c$ Pareto dominates $w_R = w_M = \frac{4(a-c)}{3}$ when they co-exist in case 3. Therefore, after applying the refinement, the unique equilibrium is: (i) for $c \in \left[0, \frac{5a}{11}\right]$, $w_R = w_M = c$, and (ii) for $c \in \left(\frac{5a}{11}, \frac{a}{2}\right]$, $w_R = w_M = \frac{5a}{11}$.

A.1.2 Unilateral outsourcing

We examine whether *(Accept, Not Accept)* can be supported as an equilibrium. The firms' profits are given by (A.12). Note that $\pi^{AN}_R(c, w_R)$ is increasing in w_R so in equilibrium firm R will set the maximum price consistent with this type of an equilibrium, $w_R = c$. Given that there is a cost $\varepsilon > 0$ of making an offer, firm M will not make an offer given that in this candidate equilibrium its offer is not accepted (note that profits do not depend on w_M). Firm M can deviate in stage 2 to making a wholesale price offer w_M that is accepted by R, i.e., that belongs to one of the $\mathcal{E}_{AA}$ sets given that $w_R = c$.

We check whether such deviation is profitable. The candidate equilibrium profit is $\pi^{AN}_M(c, c) = \frac{(a-c)^2}{9}$ and the deviation profit is given by (A.11), with $w_R = c$. The difference between firm M's post- and pre-deviation profits is $\frac{-5w_M^2 + (5a-c)w_M - 2ac + 3c^2}{9}$, which is strictly positive if and only if $w_M \in \left(\frac{5a-c-\sqrt{25a^2-50ac+61c^2}}{10}, \frac{5a-c+\sqrt{25a^2-50ac+61c^2}}{10}\right)$. It can be verified that $\frac{5a-c+\sqrt{25a^2-50ac+61c^2}}{10} \in (c, a)$, so there exist w_M's that fall in the $\mathcal{E}_{AA}$ sets, see Figures 4, 5 and 6, that make the deviation strictly profitable. Therefore, there does not exist an equilibrium where only one firm outsources.

[32] We cannot have an equilibrium in the interior of the $\mathcal{E}^{II}_{AA}$ set, because, as we have already argued in previous cases, a firm has a profitable deviation.

A.1.3 No outsourcing

Finally, we examine whether *(Not Accept, Not Accept)* can be supported as an equilibrium. The firms' profits are given by (A.14). Since each firm expects that its offer will not be accepted, no firm makes an offer. A firm, say firm R, can deviate in stage 2 by making an offer that is accepted, $w_R \leq c$. The deviation profit is given by (A.12). It can be easily verified that if, for example, $w_R = c$ this deviation is strictly profitable. Therefore, there does not exist an equilibrium where no firm outsources.

A.2 Proof of Proposition 3

The wholesale prices in the encroachment case are given by Proposition 2, while in the no encroachment case, they are given by (1) and (2). In particular:

(i) When $c \geq 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a$, in the no encroachment case the wholesale price faced by firm R is $w_M^B = \frac{a}{2}$, while in the encroachment case it is $w_M^E = c$. Since $2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a > \frac{a}{2}$, it follows that $w_M^E > w_M^B$ when $c \geq 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a$.

(ii) When $\frac{5a}{11} < c < \frac{a}{2}$, in the no encroachment case the wholesale price faced by firm R is $w_M^B = c$, while in the encrochmemt case it is $w_M^E = \frac{5a}{11}$. Thus, $w_M^E < w_M^B$ when $\frac{5a}{11} < c < \frac{a}{2}$.

Moreover, when $\frac{a}{2} < c < \frac{29a}{44}$, in the no encroachment case the wholesale price faced by firm R is $w_M^B = \frac{a}{2}$, while in the encrochmemt case it is $w_M^E = \frac{5a}{11}$. Since $\frac{5a}{11} < \frac{29a}{44}$, it follows that $w_M^E < w_M^B$ when $\frac{a}{2} < c < \frac{29a}{44}$. Finally, when $\frac{29a}{44} < c < 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)$, in the no encroachment case the wholesale price faced by firm R is $w_M^B = \frac{a}{2}$, while in the ncroachment case it is $w_M^E = \frac{4(a-c)}{3}$. We note that w_M^E decreases with c, hence, w_M^E takes its maximum value in this range. When $c \to \frac{29a}{44}$, we have $w_M^E < w_M^B$. Thus, $w_M^E < w_M^B$ when $\frac{29a}{44} < c < 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)$.

Summing up, $w_M^E < w_M^B$ when $\frac{5a}{11} < c < 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a$.

(iii) When $c \leq \frac{5a}{11}$, in the no encroachment case the wholesale price faced by firm R is $w_M^B = c$ and in the encroachment case it is $w_M^E = c$. Thus, $w_M^B = w_M^E$ when $c \leq \frac{5a}{11}$.

A.3 Proof of Proposition 4

The equilibrium profits under encroachment are given in Proposition 2, while under no encroachment they are given in (1) and (2).

- When $c \leq \frac{5a}{11}$, without encroachment profits are $\pi_M^B = \frac{(a-c)c}{2}$, while with encroachment profits are given by $\pi_M^E = \frac{(a+2c)(a-c)}{9}$. We find that $\pi_M^B - \pi_M^E \leq 0$ if and only if $c \leq \frac{2a}{5}$. Thus, $\pi_M^E \geq \pi_M^B$

if $c \leq \frac{2a}{5}$, while $\pi_M^E < \pi_M^B$ if $\frac{2a}{5} < c \leq \frac{5a}{11}$.

- When $\frac{5a}{11} < c < \frac{a}{2}$, without encroachment profits are $\pi_M^B = \frac{(a-c)c}{2}$, while with encroachment profits are given by $\pi_M^E = \frac{14a^2}{121}$. We find that $\pi_M^B - \pi_M^E > 0$.

- When $\frac{a}{2} < c < \frac{29a}{44}$, without encroachment profits are $\pi_M^B = \frac{a^2}{8}$, while with encroachment profits are given by $\pi_M^E = \frac{14a^2}{121}$. We find that $\pi_M^B - \pi_M^E > 0$.

- When $\frac{29a}{44} \leq c \leq 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a$, without encroachment profits are $\pi_M^B = \frac{a^2}{8}$, while with encroachment profits are given by $\pi_M^E = \frac{11\left(a-\frac{8c}{11}\right)(a-4c)}{81}$. We find that $\pi_M^B - \pi_M^E > 0$.

- When $2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a < c$, without encroachment profits are $\pi_M^B = \frac{a^2}{8}$, while with encroachment profits are given by $\pi_M^E = \frac{(a+2c)(a-c)}{9}$. We find that $\pi_M^B - \pi_M^E > 0$.

A.4 Proof of Proposition 5

The equilibrium profits of firm R under encroachment are given by (8). The equilibrium profits under no encroachment are given by (1) and (2).

- When $c \leq \frac{5a}{11}$, without encroachment profits are $\pi_R^B = \frac{(a-c)^2}{4}$, while with encroachment profits are given by $\pi_R^E = \frac{(a+2c)(a-c)}{9}$. We find that $\pi_R^B - \pi_R^E \leq 0$ if and only if $\frac{5a}{11} \geq c \geq \frac{5a}{17}$.

- When $\frac{5a}{11} < c < \frac{a}{2}$, without encroachment profits are $\pi_R^B = \frac{(a-c)^2}{4}$, while with encroachment profits are given by $\pi_R^E = \frac{14a^2}{121}$. We find that $\pi_R^B - \pi_R^E > 0$.

- When $\frac{a}{2} < c < \frac{29a}{44}$, without encroachment profits are $\pi_R^B = \frac{a^2}{16}$, while with encroachment profits are given by $\pi_R^E = \frac{14a^2}{121}$. We find that $\pi_R^B - \pi_R^E > 0$.

- When $\frac{29a}{44} \leq c \leq 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a$, without encroachment profits are $\pi_R^B = \frac{a^2}{16}$, while with encroachment profits are given by $\pi_R^E = \frac{11\left(a-\frac{8c}{11}\right)(a-4c)}{81}$. We find that $\pi_R^B - \pi_R^E > 0$.

- When $2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right)a < c$, without encroachment profits are $\pi_R^B = \frac{a^2}{16}$, while with encroachment profits are given by $\pi_R^E = \frac{(a+2c)(a-c)}{9}$. We find that $\pi_R^B - \pi_R^E > 0$.

B Appendix B: Product Differentiation

We examine next what happens when the final goods produced by firm M and firm R in the encroachment case are horizontally differentiated. To do so, we assume that the demand function for firm i's final product, with $i, j = M, R$ and $i \neq j$, is given by $p_i(q_i, q_j) = a - q_i - \gamma q_j$, where the parameter γ, with $\gamma \in (0, 1)$, denotes the degree of product differentiation, namely, the lower is γ, the more differentiated the final goods of firm i and firm j are. We repeat the whole equilibrium analysis and we derive the equilibrium symmetric wholesale prices that generate mutual outsourcing of inputs in equilibrium and the respective firm profits. We abstract though from performing a complete comparison between the no encroachment and the encroachment case as it would not produce any new insights. Still, we conclude that, not surprisingly, encroachment can be more likely to take place and to be mutually beneficial when final goods are differentiated. Intuitively, encroachment generates more profits for firm M from the final good market when final goods are differentiated as market competition in weaker then.

In the encroachment case, in stage 4, each firm solves

$$\max_{q_i} \ \pi_i(q_i, q_j) = (p_i(q_i, q_j) - c_i)q_i + \mathbb{1} w_i q_j, \tag{B.1}$$

where the indicator function $\mathbb{1}$ takes the value of 1 when firm j has accepted firm i's wholesale price offer, $c_j = w_i$, and 0 when it has not accepted the offer and produces the complementary input in-house, $c_j = c$. Solving the system of first order conditions, we obtain

$$q_i(c_i, c_j) = \frac{a(2-\gamma) - 2c_i + \gamma c_j}{4 - \gamma^2}. \tag{B.2}$$

In stage 3, there are four possible types of candidate equilibria: (i) (*Accept, Accept*), (ii) (*Accept, Not Accept*), (iii) (*Not Accept, Accept*), (iv) (*Not Accept, Not Accept)*, where the first entry in the parenthesis refers to firm M's decision and the second entry to firm R's decision. Next, we examine the conditions under which each of these candidate equilibria is indeed an equilibrium. First, we present the profits for each stage 3 action profile (where the first argument in $\pi_i(\cdot, \cdot)$ refers to the marginal cost of firm $i = M$ and the second to the marginal cost of firm $j = R$).

$$\begin{aligned}
\pi_i(w_j, w_i) &= \frac{w_j(a-w_j)\gamma^3 + (a-w_i)(a-3w_i)\gamma^2 - 4a(a-w_j)\gamma + 4a^2 + (8w_i - 8w_j)a - 8w_i^2 + 4w_j^2}{(4-\gamma^2)^2}, \ (A,\ A) \\
\pi_i(c, c) &= \frac{(a-c)^2}{(2+\gamma)^2}, \ (N,\ N) \\
\pi_i(w_j, c) &= \frac{((a-c)\gamma - 2a + 2w_j)^2}{(4-\gamma^2)^2}, \ (A,\ N) \\
\pi_i(c, w_i) &= \frac{w_i(a-c)\gamma^3 + (a-w_i)(a-3w_i)\gamma^2 - 4a(a-c)\gamma + 4a^2 + 8a(w_i - c) + 4c^2 - 8w_i^2}{(4-\gamma^2)^2}, \ (N,\ A).
\end{aligned}$$

(i) (*Accept, Accept*)

It is an equilibrium when firm M has no incentives to deviate to (*Not Accept, Accept*) and firm R has no incentives to deviate to (*Accept, Not Accept*). If firm i does not accept firm j's offer, then firm j earns no revenue from input sales and firm i produces the final good with both inputs produced in-house (one at 0 cost and the other at cost c). In particular, firm M does not have incentives to unilaterally deviate if

$$\pi_M(w_R, w_M) \geq \pi_M(c, w_M) \Rightarrow \frac{4\left(\frac{1}{4}w_M\gamma^3 - \gamma a + 2a - c - w_R\right)(c - w_R)}{(4-\gamma^2)^2} \geq 0$$

and firm R does not have incentives to unilaterally deviate if

$$\pi_R(w_M, w_R) \geq \pi_R(c, w_R) \Rightarrow \frac{4\left(\frac{1}{4}\gamma^3 w_R - \gamma a + 2a - c - w_M\right)(c - w_M)}{(4-\gamma^2)^2} \geq 0.$$

It follows from the above that (*Accept, Accept*) is an equilibrium if (w_M, w_R) belongs in one of the following sets

$$\begin{aligned}
\mathcal{E}^{I}_{AA} &\equiv \left\{(w_M, w_R) : w_M \leq c, w_R \leq c, w_M \leq (a(2-\gamma)-c) + \frac{\gamma^3 w_R}{4}, w_M \geq -4(a(2-\gamma)-c) + \frac{4w_R}{\gamma^3}\right\} \\
\mathcal{E}^{II}_{AA} &\equiv \left\{(w_M, w_R) : w_M \geq c, w_R \geq c, w_M \geq (a(2-\gamma)-c) + \frac{\gamma^3 w_R}{4}, w_M \leq -4(a(2-\gamma)-c) + \frac{4w_R}{\gamma^3}\right\} \\
\mathcal{E}^{III}_{AA} &\equiv \left\{(w_M, w_R) : w_M \geq c, w_R \leq c, w_M \geq (a(2-\gamma)-c) + \frac{\gamma^3 w_R}{4}, w_M \geq -4(a(2-\gamma)-c) + \frac{4w_R}{\gamma^3}\right\} \\
\mathcal{E}^{IV}_{AA} &\equiv \left\{(w_M, w_R) : w_M \leq c, w_R \geq c, w_M \leq (a(2-\gamma)-c) + \frac{\gamma^3 w_R}{4}, w_M \leq -4(a(2-\gamma)-c) + \frac{4w_R}{\gamma^3}\right\}.
\end{aligned}$$

(ii) (*Accept, Not Accept*)

It is an equilibrium when firm M has no incentives to deviate to (Not Accept, Not Accept) and firm R has no incentives to deviate to (Accept, Accept). It turns out that firm M has no incentive to unilaterally deviate if

$$\pi_M(w_R, c) \geq \pi_M(c, c) \Rightarrow \frac{4(c - w_R)(-(a-c)\gamma + 2a - c - w_R)}{(2+\gamma)^2(2-\gamma)^2} \geq 0.$$

Similarly, firm R has no incentive to unilaterally deviate if

$$\pi_R(c, w_R) > \pi_R(w_M, w_R) \Rightarrow \frac{(-\gamma^3 w_R + 4a\gamma - 8a + 4c + 4w_M)(c - w_M)}{(4-\gamma^2)^2} > 0.$$

Then (*Accept, Not Accept*) is an equilibrium if (w_M, w_R) belong in one of the following sets

$$\mathcal{E}^{I}_{AN} \equiv \left\{(w_M, w_R) : w_M < c, w_R \leq c, w_M > (a(2-\gamma)-c) + \frac{\gamma^3 w_R}{4}\right\}$$
$$\mathcal{E}^{II}_{AN} \equiv \left\{(w_M, w_R) : w_M > c, w_R \leq c, w_M < (a(2-\gamma)-c) + \frac{\gamma^3 w_R}{4}\right\}.$$

(iii) (*Not Accept, Accept*)

The analysis of this type of candidate equilibrium coincides with that of (*ii*) with the roles of the two firms reversed. Then (*Not Accept, Accept*) is an equilibrium if (w_M, w_R) belong in one of the following sets

$$\mathcal{E}^{I}_{NA} \equiv \left\{(w_M, w_R) : w_R < c, w_M \leq c, w_M < -4(a(2-\gamma)-c) + \frac{4w_R}{\gamma^3}\right\}$$
$$\mathcal{E}^{II}_{NA} \equiv \left\{(w_M, w_R) : w_R > c, w_M \leq c, w_M > -4(a(2-\gamma)-c) + \frac{4w_R}{\gamma^3}\right\}.$$

(iv) (*Not Accept, Not Accept*)

It is an equilibrium when firm M has no incentives to deviate to (*Accept, Not Accept*) and firm R has no incentives to deviate to (*Not Accept, Accept*). Firm M has no incentive to unilaterally deviate if

$$\pi_M(c,c) > \pi_M(w_R, c) \Rightarrow \frac{4(w_R-c)(a(2-\gamma)-c(1-\gamma)-w_R)}{(2+\gamma)^2(2-\gamma)^2} > 0$$

and firm R has no incentive to unilaterally deviate if

$$\pi_R(c,c) > \pi_R(w_M, c) \Rightarrow \frac{4(w_M-c)(a(2-\gamma)-c(1-\gamma)-w_M)}{(2+\gamma)^2(2-\gamma)^2} > 0.$$

Note that $a > w_i$ implies that $a(2-\gamma)-c(1-\gamma) > w_i$ for all $\gamma \in [0,1]$.

Thus, (N*ot Accept, Not Accept*) is an equilibrium if (w_M, w_R) belong in the following set

$$\mathcal{E}_{NN} \equiv \{(w_M, w_R) : w_M > c, w_R > c\}.$$

In stage 2 the firms choose their wholesale prices. It can be easily confirmed, using (B.1) and (B.2), that the solution to the system of the first order conditions of the maximization of profits with respect to wholesale prices, and assuming that both firms accept the offers, is

$$w_M = w_R = \frac{a(4+2\gamma-\gamma^2)}{8+4\gamma-\gamma^2}. \tag{B.3}$$

The profits are

$$\pi_M = \pi_R = \frac{2a^2(6 + 2\gamma - \gamma^2)}{(8 + 4\gamma - \gamma^2)^2}. \tag{B.4}$$

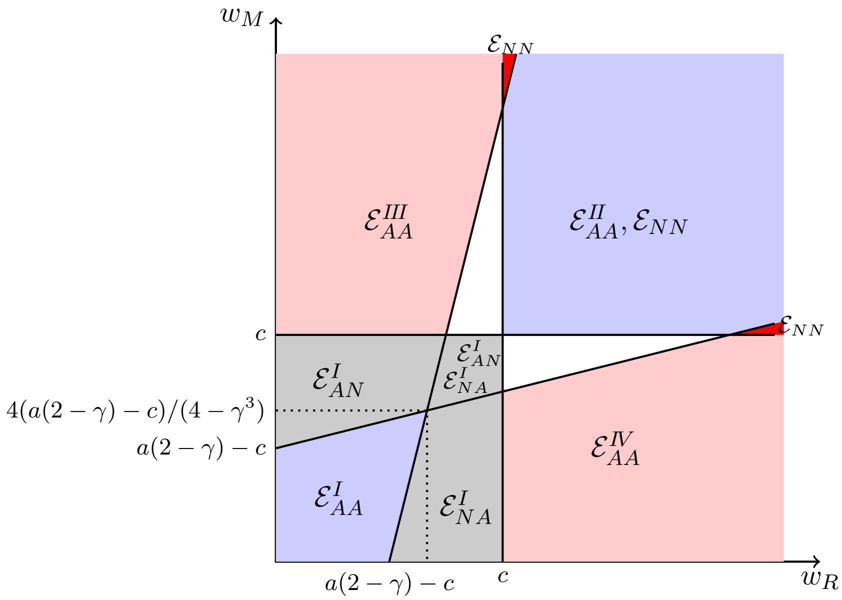

Figure 7: Case 1: $a(2-\gamma) - c < \frac{4(a(2-\gamma)-c)}{4-\gamma^3} < c < a$.

We search for symmetric equilibria in w where both firms accept the wholesale price offers. There are three cases consistent with this kind of equilibria, depending on the value of $\frac{c}{a}$ relative to γ. To better understand why the three cases arise, first note that $a(2-\gamma) - c < \frac{4(a(2-\gamma)-c)}{4-\gamma^3} \Leftrightarrow \frac{c}{a} < 2 - \gamma$, which is always satisfied. As it will become clearer very soon, these thresholds determine the location and shape of the various $\mathcal{E}$ sets we derived above. Then, there are three possibilities for the level of c relative to the $a(2-\gamma) - c$ and $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$ thresholds.[33] In what follows we consider each case. When there are multiple symmetric equilibria in (w_M, w_R) we use the Pareto dominance criterion to select a unique equilibrium.

Case 1: $a(2-\gamma)-c < \frac{4(a(2-\gamma)-c)}{4-\gamma^3} < c < a$. This case is valid if $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$, which corresponds to Figure 7.[34] Note first that in this case the unconstrained wholesale prices (B.3) cannot be in the $\mathcal{E}_{AA}^{II}$ set, since this would contradict the condition $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$. Then, (B.3) is strictly less than $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$, the northeast vertex of the $\mathcal{E}_{AA}^{I}$ set, if and only if $\frac{c}{a} < \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4\gamma^2-16\gamma-32}$, where

[33] Also, $\frac{4(a(2-\gamma)-c)}{4-\gamma^3} < a \Leftrightarrow \frac{c}{a} > 1 - \gamma + \frac{\gamma^3}{4}$. This matters only in case 3, where $\frac{c}{a}$ is low and hence $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$ can be higher than a.

[34] In this case, there exists a region where a pure strategy in stage 3 does not exist, see the white area in Figure 7. However, neither a symmetric equilibrium nor a deviation fall in this area. Thus, for brevity, we omit the derivation of the mixed strategy equilibrium when $\gamma < 1$.

$\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4\gamma^2-16\gamma-32} > \frac{4}{4+2\gamma+\gamma^2}$, for all $\gamma \in [0,1]$. This implies that under the latter condition, (B.3) lies strictly inside the $\mathcal{E}^I_{AA}$ set. In addition, $\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4\gamma^2-16\gamma-32} > 1$, if $\gamma < 0.5328$. This implies that for $\gamma < 0.5328$, (B.3) is a candidate equilibrium for all values of $\frac{c}{a}$ of case 1. A local deviation from (B.3) is by construction unprofitable. However, a firm, say firm M, can raise its wholesale price to the point that firm R does not accept the offer. This amounts to getting out of the $\mathcal{E}^I_{AA}$ set and into the adjacent set $\mathcal{E}^I_{AN}$. The profit difference between (B.4) and the deviation profit is given by

$$\frac{A}{(\gamma^2-4\gamma-8)^2(\gamma^2-4)^2}$$

where $A \equiv (-3a^2+2ac-c^2)\gamma^6+(12a^2-16ac+8c^2)\gamma^5+(20a^2+8ac)\gamma^4+(-80a^2+112ac-64c^2)\gamma^3+(-80a^2-64c^2)\gamma^2+128a(a-c)\gamma+128a^2$. The profit difference is positive if

$$\frac{c}{a} \leq \frac{\gamma^6-8\gamma^5+4\gamma^4+56\gamma^3-64\gamma+\sqrt{B}}{\gamma^2(\gamma^4-8\gamma^3+64\gamma+64)},$$

where $B \equiv -2\gamma^{12}+20\gamma^{11}-4\gamma^{10}-384\gamma^9+256\gamma^8+3136\gamma^7-576\gamma^6-11776\gamma^5-4096\gamma^4+16384\gamma^3+12288\gamma^2$.[35] If $\gamma < 0.9306$, then the above threshold is greater than 1, suggesting that a deviation is unprofitable for all $\frac{c}{a}$. Otherwise, it is less than one and when $\frac{c}{a}$ exceeds the above threshold a deviation is profitable.

Firm M can also lower its wholesale price, if (B.3) is greater than $a(2-\gamma)-c$, which is equivalent to $\frac{c}{a} > \frac{(\gamma^3-5\gamma^2-2\gamma+12)}{8-\gamma^2+4\gamma}$, and move into the adjacent $\mathcal{E}^I_{NA}$ set. The latter condition is satisfied if $\gamma > 0.956$, because $1 > \frac{4}{4+2\gamma+\gamma^2} > \frac{(\gamma^3-5\gamma^2-2\gamma+12)}{8-\gamma^2+4\gamma}$. If $\gamma < 0.513$, $1 < \frac{(\gamma^3-5\gamma^2-2\gamma+12)}{8-\gamma^2+4\gamma}$, and hence this condition is not satisfied. Therefore, this deviation will not happen. If $\gamma \in [0.513, 0.956]$, then $1 > \frac{(\gamma^3-5\gamma^2-2\gamma+12)}{8-\gamma^2+4\gamma} > \frac{4}{4+2\gamma+\gamma^2}$. The difference between (B.4) and the deviation profits is given by

$$-\frac{C}{(\gamma^2-4\gamma-8)^2(\gamma^2-4)^2}$$

where $C \equiv (a\gamma^5-2a\gamma^4-8a\gamma^3+(20a-4c)\gamma^2+(8a+16c)\gamma-48a+32c)((a-c)\gamma^2+(-2a+4c)\gamma-4a+8c)$. The profit difference is positive if and only if $\frac{c}{a} \in \left[\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}, \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}\right]$. It can be verified that $\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2} > \frac{4}{4+2\gamma+\gamma^2}$. It can also be verified that $\frac{\gamma^6-8\gamma^5+4\gamma^4+56\gamma^3-64\gamma+\sqrt{B}}{\gamma^2(\gamma^4-8\gamma^3+64\gamma+64)} > \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}$, for all $\gamma \in [0,1]$. Thus, $\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}$ is the relevant no deviation threshold, when a deviation to the $\mathcal{E}^I_{NA}$ set can take place.

Therefore, and summarizing the analysis so far, (B.3) is an equilibrium and both firms accept

[35] When $\gamma = 1$, the above inequality becomes $\frac{c}{a} \leq \frac{-1+3\sqrt{14}}{11}$.

the offers for the complementary input if $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$ and $\gamma < 0.5328$. If $\gamma \in [0.5328, 1]$ (B.3) is an equilibrium and both firms accept the offers for the complementary input if $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}\right)$, where the upper bound is less than 1, implying that for high c relative to a, (B.3) is not an equilibrium.[36]

Next, assume that $\frac{c}{a} \geq \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}$, so (B.3) is weakly higher than $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$. This case, as we have mentioned above, is valid for $\gamma > 0.5328$. The candidate equilibrium wholesale prices consistent with both firms accepting are

$$w_R = w_M = \frac{4(a(2-\gamma)-c)}{4-\gamma^3}. \tag{B.5}$$

The profits are

$$\pi_M = \pi_R = \frac{(a\gamma^3 - 4a\gamma + 4a - 4c)(a\gamma^3 + 4a\gamma^2 - 4(a-c)\gamma - 12a + 4c)}{(\gamma+2)^2(4-\gamma^3)^2}. \tag{B.6}$$

Since the candidate wholesale prices are lower than the unconstrained wholesale prices (B.3), each firm may want to increase its wholesale price. A firm, say M, by unilaterally increasing its price, even infinitesimally, moves to the $\mathcal{E}^I_{AN}$ set and induces the other firm not to accept the offer. The profit difference between (B.6) and the deviation profit is given by

$$\frac{D}{(2+\gamma)(4-\gamma^3)^2},$$

where $D \equiv (2ac - c^2)\gamma^5 + (4a^2 - 2c^2)\gamma^4 + (-16a^2 + 12ac - 8c^2)\gamma^3 - 8c(a+c)\gamma^2 + (32a^2 - 16c^2)\gamma - 32(a - c/2)(a - c)$. The above profit difference is positive if and only if

$$\frac{c}{a} \in \left[\frac{\gamma^5 + 6\gamma^3 - 4\gamma^2 - \sqrt{E} + 24}{\gamma^5 + 2\gamma^4 + 8\gamma^3 + 8\gamma^2 + 16\gamma + 16}, \frac{\gamma^5 + 6\gamma^3 - 4\gamma^2 + \sqrt{E} + 24}{\gamma^5 + 2\gamma^4 + 8\gamma^3 + 8\gamma^2 + 16\gamma + 16}\right],$$

where $E \equiv \gamma^{10} + 4\gamma^9 + 4\gamma^8 - 8\gamma^7 - 28\gamma^6 - 32\gamma^5 + 16\gamma^4 + 32\gamma^3 + 64\gamma^2 + 64$. It can be verified that $\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}$ is in the interior of the above set for all $\gamma \in [0, 1]$. Moreover $\frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16} > 1$ if and only if $\gamma < 0.6946$.

Firm M can also decrease w_M and move to the $\mathcal{E}^I_{NA}$ set. The difference between pre- and post-deviation profits is zero. Therefore, (B.5) is an equilibrium and both firms accept the offers for the complementary input if $\frac{c}{a} \in \left[\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}, \frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16}\right]$ and $\gamma > 0.6946$,

[36]When $\gamma = 1$, $\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)} = \frac{29}{44}$.

or $\frac{c}{a} \in \left[\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}, 1\right]$ and $\gamma < 0.6946$.

We now check whether

$$w_M = w_R = c \tag{B.7}$$

is an equilibrium. This is at the southwest vertex of the $\mathcal{E}^{II}_{AA}$ set. The profits are

$$\pi_M = \pi_R = \frac{(a-c)(a+c(1+\gamma))}{(2+\gamma)^2}. \tag{B.8}$$

Note that we cannot have an equilibrium in the interior of the $\mathcal{E}^{II}_{AA}$ set. In the case we are examining the unconstrained wholesale prices (B.3) are strictly less than c. If candidate equilibrium wholesale prices were in the interior of the $\mathcal{E}^{II}_{AA}$ set they would be strictly higher than c. A firm would deviate in its wholesale price locally while still inducing acceptance and would increase its profits. But we can have an equilibrium on the boundary of this set, i.e., (B.7). A firm, say M, can deviate from (B.7) by lowering its wholesale price and moving to the $\mathcal{E}^{I}_{AN}$ set. The difference in profits between (B.8) and the deviation profit is $\frac{(a-c)c}{2+\gamma} > 0$.[37] Hence, (B.7) is an equilibrium and both firms accept the offer for the complementary input for all values of c.

To summarize, first assume $\gamma < 0.5328$. There are two equilibria: (B.3) and (B.7), for all $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$.

Second, assume $\gamma \in [0.5328, 0.6946]$. For $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}\right)$ there are two equilibria: (B.3) and (B.7). For $\frac{c}{a} \in \left[\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}, 1\right)$ there are two equilibria: (B.5) and (B.7).

Third, assume $\gamma \in [0.6946, 1]$. For $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}\right)$ there are two equilibria: (B.3) and (B.7). For $\frac{c}{a} \in \left[\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}, \frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16}\right]$ there are two equilibria: (B.5) and (B.7). For $\frac{c}{a} \in \left[\frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16}, 1\right)$, (B.7) is the unique equilibrium.[38]

It can be shown that (B.3) Pareto dominates (B.7), if $\frac{c}{a} \geq \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, or $\frac{c}{a} \leq \frac{4-3\gamma^2-2\gamma}{\gamma^3-3\gamma^2-12\gamma-8}$. Moreover, the bound of case 1, $\frac{4}{4+2\gamma+\gamma^2}$, is greater than $\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, for all $\gamma \in [0, 1]$, so (B.3) Pareto dominates (B.7).

Second, it can be shown that (B.5) Pareto dominates (B.7) if $\frac{c}{a} \in \left[\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^4-4\gamma^2+8}{\gamma^3(\gamma+1)}\right]$. The upper bound of the latter interval is always greater than 1, so in case 1 (B.5) Pareto dominates (B.7).

The following Proposition summarizes the equilibria after we apply the Pareto refinement.

[37] Note that the $\mathcal{E}^{I}_{NA}$ set does not contain its boundary $w_R = c$, so a deviation there is not possible. We have also assumed that when a pure and a mixed strategy co-exist, firms play the pure strategy.

[38] When $\gamma = 1$, $\frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16} = 2\left(\frac{9}{34} + \frac{\sqrt{13}}{34}\right) \approx 0.741$.

Proposition 7 *Suppose the cost of producing the complementary input in-house is high,* $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$, *then, the unique symmetric equilibrium wholesale prices under mutial outsourcing, are described as follows:*

(a) *When* $\gamma < 0.5328$, *for* $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$, (B.3) *is the equilibrium.*

(b) *When* $\gamma \in [0.5328, 0.6946]$.

- *for* $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}\right)$, (B.3) *is the equilibrium.*
- *for* $\frac{c}{a} \in \left[\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}, 1\right)$, (B.5) *is the equilibrim.*

(c) *When* $\gamma \in [0.6946, 1]$.

- *for* $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}\right)$, (B.3) *is the equilibrium.*
- *for* $\frac{c}{a} \in \left[\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}, \frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16}\right]$, (B.5) *is the equilibrium.*
- *for* $\frac{c}{a} \in \left[\frac{\gamma^5+6\gamma^3-4\gamma^2+\sqrt{E}+24}{\gamma^5+2\gamma^4+8\gamma^3+8\gamma^2+16\gamma+16}, 1\right)$, (B.7) *is the equilibrium.*

Interestingly, as γ increases, high wholesale equilibrium prices become more likely. For example, (B.7), which is higher than (B.3), is an equilibrium only for relatively high γ.

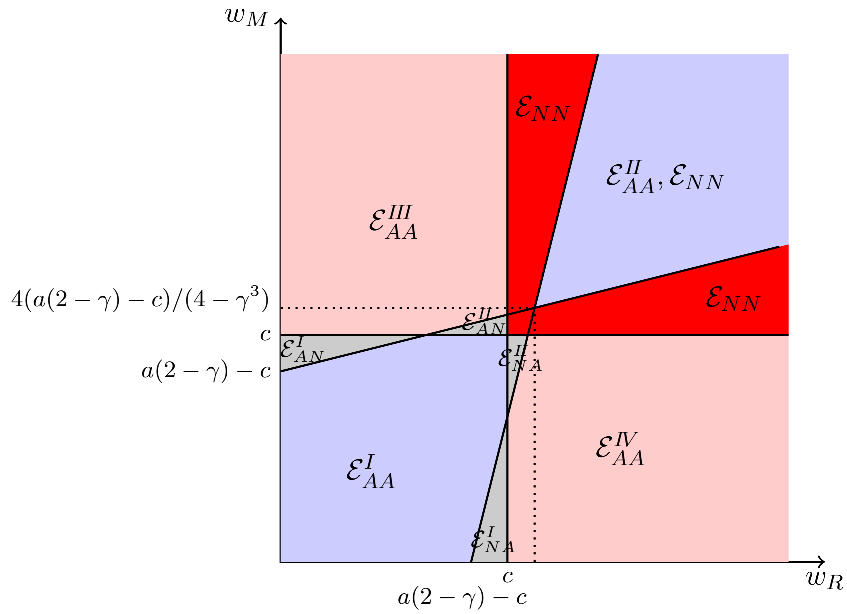

Figure 8: Case 2: $a(2-\gamma)-c < c \leq \frac{4(a(2-\gamma)-c)}{4-\gamma^3} < a$.

Case 2: $a(2-\gamma)-c < c \leq \frac{4(a(2-\gamma)-c)}{4-\gamma^3} < a$. This case is valid if $\frac{c}{a} \in \left(\frac{2-\gamma}{2}, \frac{4}{4+2\gamma+\gamma^2}\right]$, which corresponds to Figure 8. The unconstrained wholesale prices (B.3) fall in the interior of the $\mathcal{E}_{AA}^{I}$ region, because it can be shown that (B.3) is less than $\frac{a(2-\gamma)}{2}$ which is less than c. M can deviate to a higher wholesale price so that we move either to $\mathcal{E}_{AN}^{I}$ or the $\mathcal{E}_{AN}^{II}$ set. From the analysis in case 1, this deviation is unprofitable if

$$\frac{c}{a} \leq \frac{\gamma^6 - 8\gamma^5 + 4\gamma^4 + 56\gamma^3 - 64\gamma + \sqrt{B}}{\gamma^2(\gamma^4 - 8\gamma^3 + 64\gamma + 64)}.$$

It can be shown that the above threshold is higher than $\frac{4}{4+2\gamma+\gamma^2}$ for all $\gamma \in [0,1]$. Hence this deviation is unprofitable.

Also, a deviation to a lower wholesale price by M so that we move to the $\mathcal{E}^I_{NA}$ set, following the analysis in case 1, is unprofitable if

$$\frac{c}{a} \in \left[\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}, \frac{\gamma^5 - 2\gamma^4 - 8\gamma^3 + 20\gamma^2 + 8\gamma - 48}{4(\gamma^2 - 4\gamma - 8)}\right].$$

It can be shown that the range of case 2, $\left(\frac{2-\gamma}{2}, \frac{4}{4+2\gamma+\gamma^2}\right]$, is a strict subset of the above interval, so such a deviation is also unprofitable.

Also, as we showed in case 1, (B.7), which is on the boundary of the $\mathcal{E}^I_{AA}$ set, is always an equilibrium.

An equilibrium can also be at the southwest vertex of the $\mathcal{E}^{II}_{AA}$ set, given by (B.5).[39] A deviation in this case moves the wholesale prices to the $\mathcal{E}_{RR}$ set, so both firms would not accept the wholesale price offers. The difference in profits between (B.6) and the deviation profit is

$$\begin{aligned} &\frac{1}{(\gamma+2)^2(\gamma^3-4)^2}\Big((2ac-c^2)\gamma^6 + 4a^2\gamma^5 + (-8a^2+4ac)\gamma^4 + (-16a^2-16ac+8c^2)\gamma^3 \\ + \quad & 32a(a-c)\gamma^2 + 32(a-c/2)(a+c)\gamma - 64(a-c/2)(a-c)\Big), \end{aligned}$$

which is positive if and only if[40]

$$\frac{c}{a} \in \left[\frac{\gamma^6 + 2\gamma^4 - 8\gamma^3 - 16\gamma^2 + 8\gamma - \sqrt{F} + 48}{\gamma^6 - 8\gamma^3 + 16\gamma + 32}, \frac{\gamma^6 + 2\gamma^4 - 8\gamma^3 - 16\gamma^2 + 8\gamma + \sqrt{F} + 48}{\gamma^6 - 8\gamma^3 + 16\gamma + 32}\right],$$

where $F \equiv \gamma^{12} + 4\gamma^{11} - 4\gamma^{10} - 32\gamma^9 - 28\gamma^8 + 80\gamma^7 + 224\gamma^6 + 32\gamma^5 - 448\gamma^4 - 512\gamma^3 + 64\gamma^2 + 768\gamma + 256$.

It can be shown that the range of case 2, $\left(\frac{2-\gamma}{2}, \frac{4}{4+2\gamma+\gamma^2}\right]$, is a strict subset of the above interval, so such a deviation is unprofitable. Hence, (B.5) is also an equilibrium.

To summarize, there are three equilibria, (B.3), (B.5) and (B.7). The first in the interior of the

[39] As in case 1, it cannot be in the interior of that set.

[40] The interval below when $\gamma = 1$ becomes

$$\left[2\left(\frac{35-9\sqrt{5}}{82}\right), 2\left(\frac{35+9\sqrt{5}}{82}\right)\right] \approx [0.362, 1.344].$$

$\mathcal{E}^{I}_{AA}$ set, the second on the southwest vertex of the $\mathcal{E}^{II}_{AA}$ set and the third on the northeast vertex of the $\mathcal{E}^{I}_{AA}$ set.

It can be shown that (B.3) Pareto dominates (B.5), if $\frac{c}{a} \leq \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}$, or $\frac{c}{a} \geq -\frac{3\gamma^5+6\gamma^4-24\gamma^3-36\gamma^2+56\gamma+80}{4(\gamma^3-3\gamma^2-12\gamma-8)}$. Moreover, the upper bound of case 2, $\frac{4}{4+2\gamma+\gamma^2}$, is less than $\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4(\gamma^2-4\gamma-8)}$, for all $\gamma \in [0,1]$, so (B.3) Pareto dominates (B.5).

Furthermore, it can be shown that (B.3) Pareto dominates (B.7), if $\frac{c}{a} \geq \frac{\gamma^2-2\gamma-4}{\gamma^2-4\gamma-8}$, or $\frac{c}{a} \leq \frac{4-3\gamma^2-2\gamma}{\gamma^3-3\gamma^2-12\gamma-8}$. Moreover, the lower bound of case 2, $\frac{2}{2-\gamma}$, is greater than $\frac{\gamma^2-2\gamma-4}{\gamma^2-4\gamma-8}$, for all $\gamma \in [0,1]$, so (B.3) Pareto dominates (B.7).

The following Proposition summarizes the equilibria after we apply the Pareto refinement.

Proposition 8 *Suppose the cost of producing the complementary input in-house is intermediate,* $\frac{c}{a} \in \left(\frac{2-\gamma}{2}, \frac{4}{4+2\gamma+\gamma^2}\right]$*, then, the unique symmetric equilibrium wholesale prices under mutual outsourcing are given by* (B.3)*.*

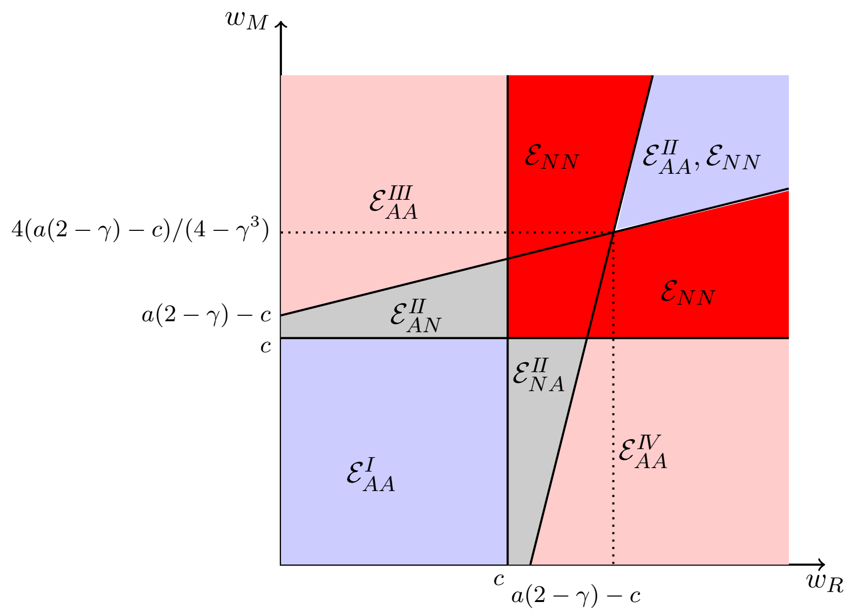

Figure 9: Case 3: $c \leq a(2-\gamma)-c < \frac{4(a(2-\gamma)-c)}{4-\gamma^3} < a$.

Case 3: $c \leq a(2-\gamma)-c < \frac{4(a(2-\gamma)-c)}{4-\gamma^3} < a$.[41] This case is valid if $\frac{c}{a} \in \left[0, \frac{2-\gamma}{2}\right]$, which corresponds to Figure 9. The unconstrained wholesale prices (B.3) are in the $\mathcal{E}^{I}_{AA}$ set and strictly less then c,

[41] Note that in this case $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$ can be greater than a when $\frac{c}{a} < 1-\gamma+\frac{\gamma^3}{4}$. This, however, does not affect the analysis in any substantial way, because $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$, that is (B.5), is not an equilibrium in case 3, even when $\frac{4(a(2-\gamma)-c)}{4-\gamma^3} < a$. When $\frac{4(a(2-\gamma)-c)}{4-\gamma^3} > a$, there is no AA region when $w_M = w_R = a$ and therefore it ceases to be a candidate equilibrium.

if $\frac{c}{a} > \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$. As we showed in case 1, a deviation by M to a higher w_M so that firm R does not accept, i.e., moving to the $\mathcal{E}^{II}_{AN}$ set, is unprofitable if

$$\frac{c}{a} \leq \frac{\gamma^6 - 8\gamma^5 + 4\gamma^4 + 56\gamma^3 - 64\gamma + \sqrt{B}}{\gamma^2(\gamma^4 - 8\gamma^3 + 64\gamma + 64)}.$$

The above threshold is always higher than $\frac{2-\gamma}{2}$, so such a deviation is unprofitable.

If now $\frac{c}{a} < \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, (B.3) falls outside the $\mathcal{E}^{I}_{AA}$ set and it may be in the $\mathcal{E}^{II}_{AA}$ set. This can happen if (B.3) is greater than $\frac{4(a(2-\gamma)-c)}{4-\gamma^3}$, which is the case if $\frac{c}{a} > \frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4\gamma^2-16\gamma-32}$. But as we know from case 1, $\frac{\gamma^5-2\gamma^4-8\gamma^3+20\gamma^2+8\gamma-48}{4\gamma^2-16\gamma-32} > \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, for all $\gamma \in [0,1]$, so both inequalities cannot hold simultaneously and hence (B.3) cannot be in the $\mathcal{E}^{II}_{AA}$ set. For the $\frac{c}{a} < \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$ range, we check whether (B.5), which is on the southwest vertex of the $\mathcal{E}^{II}_{AA}$ set, is an equilibrium. As we showed in case 2 it is for[42]

$$\frac{c}{a} \in \left[\frac{\gamma^6 + 2\gamma^4 - 8\gamma^3 - 16\gamma^2 + 8\gamma - \sqrt{F} + 48}{\gamma^6 - 8\gamma^3 + 16\gamma + 32}, \frac{\gamma^6 + 2\gamma^4 - 8\gamma^3 - 16\gamma^2 + 8\gamma + \sqrt{F} + 48}{\gamma^6 - 8\gamma^3 + 16\gamma + 32}\right],$$

where $\frac{2-\gamma}{2}$ falls within the above interval for all $\gamma \in [0,1]$.

It can be shown that $\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2} > \frac{\gamma^6+2\gamma^4-8\gamma^3-16\gamma^2+8\gamma-\sqrt{F}+48}{\gamma^6-8\gamma^3+16\gamma+32}$, if and only if $\gamma > 0.715$. Also, we can show that $\frac{2-\gamma}{2} > \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, for all $\gamma \in [0,1]$.

Next, as we showed in case 1, (B.7), which is on the northeast vertex of the $\mathcal{E}^{I}_{AA}$ set, is always an equilibrium.

To summarize, first assume that $\gamma > 0.715$. For $\frac{c}{a} \in \left[0, \frac{\gamma^6+2\gamma^4-8\gamma^3-16\gamma^2+8\gamma-\sqrt{F}+48}{\gamma^6-8\gamma^3+16\gamma+32}\right)$, (B.7) is the unique equilibrium. For $c \in \left(\frac{\gamma^6+2\gamma^4-8\gamma^3-16\gamma^2+8\gamma-\sqrt{F}+48}{\gamma^6-8\gamma^3+16\gamma+32}, \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}\right)$ there are two equilibria, (B.5) and (B.7). For $\frac{c}{a} \in \left[\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}, \frac{2-\gamma}{2}\right]$ there are three equilibria, (B.3), (B.5) and (B.7).

Second assume that $\gamma < 0.715$. For $\frac{c}{a} \in \left[0, \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}\right)$, (B.7) is the unique equilibrium. For $\frac{c}{a} \in \left[\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}, \frac{\gamma^6+2\gamma^4-8\gamma^3-16\gamma^2+8\gamma-\sqrt{F}+48}{\gamma^6-8\gamma^3+16\gamma+32}\right]$ there are two equilibria, (B.3) and (B.7). For

$$c \in \left(\frac{\gamma^6 + 2\gamma^4 - 8\gamma^3 - 16\gamma^2 + 8\gamma - \sqrt{F} + 48}{\gamma^6 - 8\gamma^3 + 16\gamma + 32}, \frac{2-\gamma}{2}\right)$$

there are three equilibria, (B.3), (B.5) and (B.7).

From the analysis in case 1, we know that (B.5) Pareto dominates (B.7) if $\frac{c}{a} \in \left[\frac{4}{4+2\gamma+\gamma^2}, \frac{\gamma^4-4\gamma^2+8}{\gamma^3(\gamma+1)}\right]$. In addition, $\frac{2-\gamma}{2} \leq \frac{4}{4+2\gamma+\gamma^2}$, so in case 3 (B.7) Pareto dominates (B.5), when they co-exist.

[42] We cannot have an equilibrium in the interior of the $\mathcal{E}^{II}_{AA}$ set, because, as we have already argued in previous cases, a firm has a profitable deviation.

From the analysis in case 2 we know that (B.3) Pareto dominates (B.7), if $\frac{c}{a} \geq \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, or $\frac{c}{a} \leq \frac{4-3\gamma^2-2\gamma}{\gamma^3-3\gamma^2-12\gamma-8}$. Hence, if $\frac{c}{a} < \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$ (B.7) Pareto dominates (B.3) and if $\frac{c}{a} > \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}$, it is the other way around.

The following Proposition summarizes the equilibria after we apply the Pareto refinement.

Proposition 9 *Suppose the cost of producing the complementary input in-house is low,* $\frac{c}{a} \in \left[0, \frac{2-\gamma}{2}\right]$*, then, the unique symmetric equilibrium wholesale prices under mutual outsourcing are described as follows:*

(a) When $\gamma > 0.715$*.*

- *for* $\frac{c}{a} \in \left[0, \frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}\right)$, (B.7) *is the equilibrium.*
- *for* $\frac{c}{a} \in \left[\frac{4+2\gamma-\gamma^2}{8+4\gamma-\gamma^2}, \frac{2-\gamma}{2}\right]$, (B.3) *is the equilibrium.*

(b) When $\gamma < 0.715$*.*

- *for* $\frac{c}{a} \in \left[0, \frac{\gamma^6+2\gamma^4-8\gamma^3-16\gamma^2+8\gamma-\sqrt{F}+48}{\gamma^6-8\gamma^3+16\gamma+32}\right)$, (B.7) *is the equilibrium.*
- *for* $\frac{c}{a} \in \left(\frac{\gamma^6+2\gamma^4-8\gamma^3-16\gamma^2+8\gamma-\sqrt{F}+48}{\gamma^6-8\gamma^3+16\gamma+32}, \frac{2-\gamma}{2}\right]$, (B.3) *is the equilibrium.*

We can now compare the equilibrium profits under no encroachment and encroachment. We will not perform a complete comparison, because it will not produce any new insights. Rather, we will demonstrate that, not surprisingly, encroachment is more likely to take place and be mutually beneficial when products are differentiated, $\gamma < 1$. Suppose $\frac{c}{a} \in \left(\frac{4}{4+2\gamma+\gamma^2}, 1\right)$, so we are in case 1. Recall that when $\gamma = 1$, and $\frac{c}{a}$ is high, M has no incentives to encroach. Let's assume that $\gamma < 0.53$, so (B.3) are the unique equilibrium wholesale prices under encroachment, see Proposition 7. It can be easily verified that the equilibrium profit under encroachment, (B.4), is higher than the profits under no encroachment, π_M^B and π_R^B given by (1), so M will encroach and also benefit R, for all $\gamma \in [0, 0.53]$.