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Precautionary Savings, Loss Aversion, and Risk: Theory and Evidence

Abstract

We consider a simple, two period, consumption-savings model with future income uncertainty. We are interested in the relation between the micro and macro motives for dealing with uncertainty. These include risk aversion, loss aversion, and precautionary savings. We provide the relevant theory, followed by empirical tests based on subject-specific savings choices, and the measurement of subject-specific behavioral parameters such as loss aversion and present bias. We predict, and show empirically, that loss aversion reduces savings, and that those who are more loss averse are less likely to engage in precautionary savings. Present-bias reduces savings. We also show that decision makers save more in response to a mean preserving spread of future random incomes, and this response is strengthened by loss aversion. We term this as the loss aversion-hedging motive.

JEL-Codes: D010, D910.

Keywords: income uncertainty, precautionary savings, loss aversion, loss aversion-hedging.

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1 Introduction

A central theme of economics is the study of how decision makers respond to risk. We consider a two period consumption-savings problem, a single good, but no financial assets. First period income is non-stochastic, but second period income is stochastic; it can either be low (bad state) or high (good state). A key insight from macroeconomic models is that under such conditions, individuals might save (i) in order to smooth consumption over time and/or (ii) use precautionary savings to hedge against the uncertainty caused by stochastic future income.

It is pedagogically useful to distinguish between the 'micro' approach and the 'macro' approach.

1. In microeconomics and in behavioral economics, the two key motivations to deal with uncertainty are *risk aversion* and *loss aversion*. Loss aversion, which applies under certainty and under uncertainty, is a central concept in behavioral economics, and underpins some of the most successful applications of behavioral economics in humans and animals.[1] The literature has also attempted to derive a link between loss aversion and savings (Aizenman, 1998; Bowman et al., 1999; Siegmann, 2002; Koszegi and Rabin, 2009; Park, 2016; Pagel, 2017; Ibanez and Schneider, 2023); we note our differences from this literature below.

2. In macroeconomics, the focus has been on identifying the *precautionary savings motive* for hedging against future income uncertainty.[2] It is well known that an increase in risk aversion in such a model does not lead to unambiguous predictions about savings.[3] Leland (1968) showed in a two period model that if the third derivative of the utility function is strictly positive ($u''' > 0$), then the decision maker engages in *precautionary savings*. This takes the form of increasing current savings at the expense of current consumption, relative to the certainty equivalent result (see Section 4 below), hence, leading to higher consumption growth.[4]

This micro/macro dichotomy gives rise to a range of questions that speak to the very foundations of macroeconomic models. What effect does an increase in loss aversion have on precautionary

[1] Loss aversion implies that losses are relatively more aversive than the satisfaction derived from gains of the same magnitude. The mean and the median values of loss aversion typically cluster around a value of 2, i.e., losses bite on average about two times, equivalent gains. For surveys of the rich applications of loss aversion, see Kahneman and Tversky (2000) and Dhami (2019, Vol. 1). Indeed, associating risk aversion with the shape of the utility function alone, as is the case under expected utility theory, is empirically refuted on account of non-linear probability weighting (Kahneman and Tversky, 2000; Dhami, 2019, Vol. 1). There is also a claim that risk aversion has little empirical power to explain cautionary behavior once loss aversion is fully taken into account (Novemsky and Kahneman, 2005).

[2] In models with a richer menu of assets than ours, hedging against future income uncertainty can take many other forms. This includes, but is not restricted to, investing in assets with correlated returns, and taking either a short or a long position in substitute securities. We abstract from these considerations by constructing a theoretical model that can be faithfully taken, as it is, to lab experiments. This allows for a clear and stringent test of the theory.

[3] The reason is that changes in risk aversion are not sufficient to determine changes in the relevant Euler equation, but the third derivative of the utility function is sufficient (Proposition 2(iii) below). However, under certain conditions, risk aversion may lead to an increase in savings in the presence of loss aversion (Bowman et al., 1999).

[4] This result was extended to multiple periods (Sibley, 1975; Miller, 1976). Other extension included the implications for savings from different sources of income (Sandmo, 1970; Skinner, 1988); relation between risk preferences and precautionary savings (Drèze and Modigliani, 1972; Kimball, 1990); and the computation of numerical solutions (Zeldes, 1989). There are several applications of these ideas; see Carroll and Kimball (2008) and Lugilde et al. (2019) for surveys. The classical precautionary savings model has also been adapted in several other directions such as financial inclusion, subjective expectations, migrant networks, on the job search, illiquid assets and portfolio choice (Giles and Yoo, 2007; Lise, 2013; Deidda, 2013; Bayer et al., 2019; Christelis et al., 2020), but this is not the focus of our work.

savings? Are more loss averse decision makers more/less likely to engage in precautionary savings? How are savings influenced by mean preserving spreads in stochastic income? Does loss aversion mediate the effects of such mean preserving spreads in income? Do individuals who exhibit preference reversals save more or less, under these circumstances? The aim of our paper is to answer these questions.

We aim to answer these questions using Köszegi-Rabin preferences (Köszegi and Rabin, 2006, 2009).[5] In our two period model, decision makers receive an initial endowment of a single good at time $t = 1$ and must decide between current consumption and savings. At time $t = 2$, the income of the decision maker comprises of savings carried over from time $t = 1$, and a random income component. Random income can take two possible values– a negative value (negative shock) with probability $0 < p < 1$, and a positive value (positive shock) with probability $1 - p$. At time $t = 1$, the decision maker only knows the distribution of the shocks. Thus, at time $t = 2$, in the event of a negative shock to income, the decision maker may be in the *domain of losses,* relative to the reference point, where loss aversion bites.

Loss aversion has two opposing temporal effects in our model. At time $t = 1$, reducing current consumption to increase savings by a unit is aversive to loss averse subjects. Indeed, this channel forms the basis of the Thaler and Benartzi (2004) SMarT pension scheme. They write (p. S169-70): "Loss aversion affects savings because once households get used to a particular level of disposable income, they tend to view reductions in that level as a loss. Thus, households may be reluctant to increase their contributions to the savings plan because they do not want to experience this cut in take-home pay." At time $t = 2$, losses only occur with probability $0 < p < 1$ in the event of a negative shock to income; and only in this state does loss aversion bite. Hence, the time $t = 2$ loss aversion offsets time $t = 1$ loss aversion, in marginal utility terms, by a diminished amount due to $0 < p < 1$. Thus, on net, the time $t = 1$ effect dominates, and an increase in loss aversion is predicted to reduce current savings (Proposition 2(i)). Furthermore, we show that loss averse subjects are less likely to engage in precautionary savings relative to loss tolerant subjects (Corollary 1) and this provides us with an important method of directly testing our predictions.

By observing the optimal saving choices of subjects, we are able to compute the gap between optimal consumption at time $t = 1$, c_1^*, and expected second period consumption at time $t = 2$, Ec_2^*. Under the classical certainty equivalence result, that arises under quadratic utility, which has a zero third derivative of the utility function (Section 4), we have $c_1^* = Ec_2^*$. The precautionary savings motive gives an additional inducement to save, so that $c_1^* < Ec_2^*$; and we refer to the flip side, $c_1^* > Ec_2^*$, as 'reckless undersaving.' This allows us to classify each subject in our experiments into one of these three categories.

Risk aversion is predicted to have an ambiguous effect on savings. The reason is that the effect of risk on the Euler equation depends on whether the marginal utility of income is convex or concave, which depends on the third derivative of the utility function; risk alone, through the second

[5]We use endogenous reference points in a limited sense, but we do not use the Köszegi and Rabin (2006, 2009) stochastic state-dependent reference points. The rationality assumptions behind the three proposed rational expectations-based equilibrium concepts in Köszegi and Rabin are fairly strong and are unlikely to be supported by the evidence on human behavior in one-shot lab experiments in the absence of learning and feedback opportunities, such as ours (Dhami and Sunstein, 2022). Since we are interested in closely testing the predictions of our theoretical model against the data, we do not wish to burden our one-shot game subjects with such rationality requirements.

derivative is unable to determine the effect on savings. It is important to note that the predictions of our model hold for 'each' decision maker. This requires us to estimate subject-specific loss aversion in order to conduct a stringent test of our theory.[6] We also check for preference reversals among subjects in order to test for the effects on savings of departures from the standard model of time preferences.

1.1 Experiments

We conduct a suitably incentivized lab experiment with 79 students at an Experimental Economics Lab in the UK in March 2023. Our empirical tests are theory-driven, direct, stringent, and based on subject-specific behavioral data. Subjects in our experiments engage in two different tasks. In the *consumption-savings task*, we confront subjects in our two period model with second period income uncertainty. In the *lottery choice task*, subjects make choices between risky lotteries that allows us to estimate the parameter of loss aversion for each subject in our experiment. Our method has similarities with the methods in Abdellaoui (2000) and Gächter et al. (2022). We also classify subjects into those who are present-biased or not, based on whether they exhibit preference reversals or not.

1.2 Findings

The mean subject-specific loss aversion parameter in our experiment is 1.6571 and the median value is 1.6609.[7] On average, 46% of the savings choices in our experiment are consistent with the precautionary saving motive; 18% with the classic certainty equivalence result; and 36% with reckless undersaving.

Our empirical findings largely confirm our theoretical model. An increase in loss aversion decreases individual savings. Loss aversion, relative to loss tolerance, decreases the odds of precautionary saving by 56%, on average. Being loss averse, relative to loss tolerant, decreases the probability of precautionary saving behavior by 20%. This demonstrates a link between the micro and macro motives that was mentioned above. To the best of our knowledge this is a novel theoretical channel and empirical finding in the literature based on directly measured subject-specific loss aversion. This is the first main objective of our paper.

Present-biased individuals are less likely to engage in precautionary saving behavior, as one would expect. This result speaks to the growing literature on the joint interplay of risk and time preferences; for a survey, see Dhami (2019, Vol. 3). Being male, relative to female, decreases the odds of precautionary saving behavior by 39%. This result speaks to the large literature on the

[6] Testing the classical prediction that relates precautionary savings to the sign of the third derivative of the utility function would require determining the sign of this derivative for each subject; an exercise we do not engage in. No such direct tests, based on the third derivative of the utility function, have been conducted in the literature. Several indirect tests based on approximations of the Euler equation, regression analysis, and structural models based on micro data have been conducted (Guiso et al., 1992; Carol and Samwick, 1998; Lusardi, 1998; Engen and Gruber, 2001; Gourinchas and Parker, 2002; Cagetti, 2003). For a survey, see Carroll and Kimball (2008) who point out the relative advantages and disadvantages about each of the methods in terms of the empirical proxies used, truly exogenous variation in uncertainty, and the ability to control for potential confounds.

[7] In their meta study, Brown et al. (2023) find that the mean loss aversion coefficient is 1.955. Gächter et al. (2022) find that the mean subject-specific loss aversion for riskless choice is 2.12 and the median is 1.73; for risky choice, median loss aversion is 1.33 and the loss aversion parameters for risky and riskless choice are correlated.

greater risk-seeking and overconfidence among men, relative to women; for a literature survey, see Dhami (2016).

Older individuals and those who spend more time deliberating on saving decisions are more likely to engage in precautionary saving. Keeping fixed the expected value of the shock at time $t = 2$, we show that precautionary savings are more likely to arise when the magnitude of the negative shock in the bad state is higher.

Finally, and in our second main result, we show that a mean preserving spread of the random time $t = 2$ income induces decision makers to save more. However, unlike the classical explanation based on risk-aversion, we show that the effect of mean preserving spreads of income on savings is mediated by loss aversion. Even when the shocks to random income are symmetric, loss averse decision makers perceive the downside risk to be higher than it actually is, and save more. We term this the *loss averse-hedging motive*, unlike risk-hedging. For instance, the most extreme shock (gain or lose 2000 units with equal probability) relative to our reference shock (gain or lose 500 units with equal probability) induces a loss averse decision maker to save an extra 302 units relative to a loss tolerant decision maker; and this difference is statistically significant. We show that it is difficult to reconcile this empirical finding with the main theoretical models in the literature.

1.3 Related Literature

In our model, the decision maker experiences loss aversion from sacrificing current consumption at time $t = 1$. As noted, this channel was also used by Thaler and Benartzi (2004) as the 'basis' of their influential SMart savings plan. As described above, this led to our prediction that loss aversion reduces savings, which is confirmed by our empirical results. However, several papers with a dynamic structure ignore the effect of loss aversion at time $t = 1$ and take account of 'only' the effect of loss aversion in mitigating the effect of the bad income state in the future at time $t = 2$ (Aizenman, 1998; Siegmann, 2002; Koszegi and Rabin, 2009; Park, 2016; Pagel, 2017; Ibanez and Schneider, 2023). By ignoring the effect of loss aversion on current marginal utility of consumption at time $t = 1$, but taking account of only the future benefits, these models predict that loss aversion will increase savings; we are not able to confirm this finding for our data. Of the papers cited in this subsection, only Ibanez and Schneider (2023) provide an empirical test supporting their prediction using observational savings data from low income individuals in Bogotá, Colombia. However, their use of observational data poses challenges in measuring the relevant variables, and there are important differences in their paper from ours.[8]

[8]Ibanez and Schneider (2023) take the existing value of the stock of an individual's monetary assets as a measure of savings, made over several periods. It is not clear if the assumptions in their theoretical model are met with their observational data (e.g., stationarity of the income distribution). They measure 'income risk' from the self-reported risk from becoming unemployed. An advantage of our lab experiments is that we can tightly control for savings and the 'income risk' to correspond exactly to the sense used in our theoretical model to derive the predictions. They identify precautionary savings with a positive third derivative of the utility function, which is unobserved. By contrast, and as discussed above, we use the fundamental result on the sign of $c_1^* - Ec_2^*$ to classify subjects as precautionary savers; this is clearly difficult with observational data. At the very minimum, we complement other field studies with our tightly controlled lab study that has unprecedented internal validity while it might not have the same external validity as observational studies.

2 The model

Consider a decision maker who lives for two time periods, where time $t = 1, 2$.[9] The decision maker has initial non-stochastic income, $y > 0$, at time $t = 1$, and stochastic income z at time $t = 2$ (specified below). At time $t = 1$, income y can be either consumed, c_1, or saved, $s \geq 0$. All income at time $t = 2$ is fully consumed, c_2. We assume that the interest rate on savings equals zero.[10] Thus, the budget constraints at times $t = 1$ and $t = 2$ are given, respectively, by

$$c_1 + s = y, \tag{2.1}$$

$$c_2 = s + z. \tag{2.2}$$

The stochastic income, z, at time $t = 2$, takes two possible values.

1. In the *bad state,* which occurs with probability $p \in (0, 1)$, $z = \varepsilon_l < 0$.

2. In the *good state*, which occurs with probability $1 - p$, $z = \varepsilon_h > 0$.

Thus, in the bad state, the lowest possible value of c_2 is ε_l (if $s = 0$) and in the good state, the highest possible value of c_2 is $y + \varepsilon_h$ (if $s = y$). Hence, the set of all possible outcomes for c_2 is given by the compact set $X = [\varepsilon_l, y + \varepsilon_h]$. The expected value of the random second period income is

$$Ez \equiv \overline{z} = p\varepsilon_l + (1 - p)\, \varepsilon_h, \tag{2.3}$$

where E is the time expectation operator, conditional on the information set at $t = 1$, that captures uncertainty with respect to the realization of the random income z.

2.1 Preferences

In each of the two time periods, the decision maker has an instantaneous utility function that is of the Köszegi-Rabin form. However, we do not use endogenous stochastic state-dependent reference points in the sense used in Köszegi and Rabin (2006, 2009) and we do not use their equilibrium concepts for reasons that we explain below. Let ω_1 and ω_2 be, respectively, the reference points for consumption at times $t = 1$ and $t = 2$; we specify reasonable bounds on the reference points in Section 2.2 below. Hence, the decision maker maximizes the following, undiscounted, two period utility function[11]

$$U = v(c_1; \omega_1) + E\left[v(c_2; \omega_2)\right];\ c_1 \in [0, y]\,, c_2 \in X, \tag{2.4}$$

[9] Our motivation is to construct a theoretical model that we can take 'as it is' to the experiments, rather than a 'just so' theoretical model. Hence, we sometimes sacrifice generality in favor of a stripped-down version that can be understood clearly by our experimental subjects.

[10] Alternatively, in the analysis of precautionary savings, we could have assumed a positive interest rate that equaled the discount rate so that they both cancel out in the relevant Euler equation (Blanchard and Fischer, 1989). But this requires assuming a steady state, which might be an unreasonable assumption in a one shot experiment. It is more persuasive to make the assumption of zero interest rate that can be easily implemented in experiments.

[11] The absence of discounting is required to ensure that the discount rate equals the zero interest rate, which, as noted above, is critical to define precautionary savings. Our results go through with a positive discount rate and a positive interest rate if we make the steady state assumption that they are identical. The time gap between the two time periods in our experiment is 1 month, hence, the discount rate over this interval is likely to be reasonably small. Furthermore, we demonstrate the robustness of our methods of measuring the behavioral parameters to considerations of a positive discount rate below.

where

$$v(c_t;\omega_t) = u(c_t) + \mu g\left(c_t;\omega_t\right);\ \mu \geq 0,\ t = 1, 2. \tag{2.5}$$

We now explain (2.4), (2.5). In Köszegi-Rabin preferences in (2.5), the utility function from the 'absolute level' of consumption at time $t = 1, 2$, $u : \Re \rightarrow \Re$, is twice continuously differentiable, strictly increasing, and strictly concave ($u' > 0$, $u'' < 0$). The second term on the RHS in (2.5) is known as *gain-loss utility* and $\mu \geq 0$ is the relative weight on gain-loss utility. Gain-loss utility, g, at time $t = 1, 2$ depends on the value of consumption, c_t, relative to the reference point, ω_t. The neoclassical model, without reference dependent preferences, is recovered as a special case when $\mu = 0$. The 'non-linear form' of gain-loss utility is given by

$$g\left(c_t, \omega_t\right) = \begin{cases} \left(c_t - \omega_t\right)^{\beta} & \text{if} \quad c_t \geq \omega_t \\ -\lambda\left(\omega_t - c_t\right)^{\beta} & \text{if} \quad c_t < \omega_t \end{cases},\ 0 < \beta \leq 1\,, t = 1, 2. \tag{2.6}$$

In (2.6), λ is the parameter of loss aversion and this requires $\lambda > 1$. In other words, losses are more aversive than equivalent gains, and this has massively expanded the explanatory power of economic theory.[12] However, recent experiments have also found the presence of a significant number of loss-tolerant subjects (Chapman et al., 2022; Dhami et al., 2022); this corresponds to the case $\lambda < 1$. We allow for both cases in our empirical analysis but we continue to refer to λ as the parameter of loss aversion.[13] The function in (2.6) is strictly concave in the domain of gains and strictly convex in the domain of losses. This allows for diminishing sensitivity to gains and losses; the analogue of the concept of diminishing marginal utility.

For small stake gambles, as in the typical lab experiments such as ours, gain-loss utility is linear, i.e., $\beta = 1$ in (2.6) (Köszegi and Rabin, 2006, 2009). The linear form of gain-loss utility, which we use in our model, and denote by $\overline{g}\left(c_t, \omega_t\right)$, in order to differentiate from the non-linear case, is given by

$$\overline{g}\left(c_t, \omega_t\right) = \begin{cases} \left(c_t - \omega_t\right) & \text{if} \quad c_t \geq \omega_t \\ -\lambda\left(\omega_t - c_t\right) & \text{if} \quad c_t < \omega_t \end{cases},\ t = 1, 2. \tag{2.7}$$

We shall employ the non-linear version of gain-loss utility in (2.6) 'only' in Section 5.4 in order to extend the predictions of Köszegi and Rabin preferences to mean preserving spreads of stochastic time $t = 2$ income. Otherwise, throughout our paper, we follow the linear version.

2.2 Reference points

At time $t = 1$, the individual receives the riskless or certain income, y. There is extensive evidence that under certainty, the status-quo provides a good reference point (Kahneman and Tversky, 2000; Dhami, 2019, Vol. 1). For this reason, we take the time $t = 1$ reference point, ω_1, as the status-quo income, thus

$$\omega_1 = y. \tag{2.8}$$

Thus, any savings that are made from current income are classed as a loss relative to the current reference point of y. Indeed, this was one of the important building blocks in the SMarT savings

[12] For extensive applications of loss aversion, see Kahneman and Tversky (2000), and Dhami (2019, Vol. 1).

[13] In classical prospect theory preferences, we have $v(c_t;\omega_t) = g\left(c_t;\omega_t\right)$. In neoclassical economics, we have $v(c_t;\omega_t) = u(c_t)$. Hence, Köszegi-Rabin preferences take a weighted average of the two.

proposal in Thaler and Benartzi (2004). In neoclassical preferences (i.e., $\mu = 0$), any savings made in the first period from income y, even for the very first unit of savings, constitutes a loss, but in different units, that of the marginal utility of savings.[14]

However, income at time $t = 2$, given by $s+z$, is uncertain because z is stochastic. It is less clear how reference points are formed under uncertainty. Proposals include the rational expectations of income, the expected value of the future income, or a fraction (greater than or less than 1) of the expected value, and finally stochastic endogenous state-dependent reference points (as in Köszegi and Rabin, 2006); for a survey, see Dhami (2019, Vol. 1). In order to enhance the generality of our results, yet respect the bounded rationality of economic agents (see discussion below) we do not specify an exact reference point. Rather, we specify plausible bounds on ω_2 and thereby demonstrate the robustness of our results to a range of reference points. We discuss this below.

The decision maker knows, at time $t = 1$, that *for a given choice of* $s \in [0, Y]$, the time $t = 2$ income is $s + z$. At time $t = 1$, the decision maker formulates the time $t = 2$ reference point, conditional on each time $t = 1$ savings decision $s \in [0, Y]$. Let ω_2 be the reference point corresponding to a particular choice of $s \in [0, Y]$; in full notation, we would have written this as $\omega_2(s)$. It is not plausible to assume that ω_2 exceeds the highest possible time $t = 2$ income, which arises in the good state when $z = \varepsilon_h$. Hence, we assume that

$$\omega_2 < s + \varepsilon_h = c_2, \text{ for a given } s \in [0, Y]. \tag{2.9}$$

Thus, when time $t = 2$ income is the highest possible, the individual is always in the domain of gains for any $s \in [0, Y]$.

Analogously, we assume that when time $t = 2$ income is the lowest possible, i.e., in the bad state when $z = \varepsilon_l < 0$, for any level of $s \in [0, Y]$, the decision maker at time $t = 1$ imagines that he/she is in the domain of losses, so

$$\omega_2 > s + \varepsilon_l = c_2, \text{ for a given } s \in [0, Y]. \tag{2.10}$$

We allow for any time $t = 2$ reference point, ω_2, that satisfies (2.9), (2.10). Both conditions are eminently sensible.

Example 1 *Suppose, for instance,* ω_2 *equals an arbitrarily weighted average of second period income across the two states, i.e.,* $\omega_2 = \alpha (s + \varepsilon_l) + (1 - \alpha) (s + \varepsilon_h)$, *where* $0 < \alpha < 1$. *In the special case* $\alpha = p$, *the reference point is* $\omega_2 = s + \overline{z}$, *where* $\overline{z} = p\varepsilon_l + (1 - p) \varepsilon_h$ *is the average time* $t = 2$ *income. In this case,* ω_2 *is the expected value of time* $t = 2$ *income; which incidentally is also the rational expectations of second period income, conditional on the information set at time* $t = 1$. *Thus, our bounds in (2.9), (2.10) also hold for the rational expectations of income from the perspective of time* $t = 1$, *and conditional on a given level of* $s \in [0, Y]$. *But we allow for any* $0 < \alpha < 1$, *which accommodates a large range of possible human behaviors. Hence, our results are*

[14] Under neoclassical preferences, the first term in the Euler equation that captures the current sacrifice in consumption on account of savings is $-u'(y - c_1)$; and his holds true even for the first unit of savings. Thus, there is nothing perverse in the suggestion of Thaler and Benartzi (2004). They simply add an extra cost, in terms of loss aversion, to any extra unit of savings. Our results generalize to the case where $\omega_1 = \kappa y$, where $0 < \kappa < 1$; however, in this case, the results need to be stated differently and in a more cumbersome manner, taking into account the different domains of gains and losses.

not dependent on a particular value of the reference point. We are agnostic about what is the true value of $0 < \alpha < 1$.

Further discussion on endogenous reference points: The bounds in (2.9), (2.10) depend on savings, s, made at time $t = 1$, which is endogenous in the model. In this sense, our approach lies within the framework of endogenous reference points, and this is a sensible assumption. At time $t = 1$,the decision maker must consider which domain they are likely to fall into at time $t = 2$, conditional on their savings choice $s \in [0, Y]$. However, we do not impose the rational expectations conditions embodied in the 3 equilibrium concepts in Köszegi and Rabin (2006, 2009). Any one of these conditions would have required us to assume state dependent reference points (a state in our model is given by the set of all possible time $t = 2$ incomes, i.e., the set $\Gamma = [0, Y] \times \{\varepsilon_l, \varepsilon_h\}$). We would then need to determine an endogenous probability distribution over Γ that actually holds true in equilibrium. In one shot experiments, and lack of learning/feedback, this requires incredible and implausible levels of cognitive sophistication on the part of decision makers, which sits uneasily with the evidence on bounded rationality (Dhami and Sunstein, 2022).[15] In other settings where subjects have greater learning opportunities under repeated play, and possibly feedback, Köszegi and Rabin reference points are likely to be a better approximation.

One can also reconcile our approach with the traditional approach to reference points over lotteries in a static model (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). Consider time $t = 2$ in isolation where, given the predetermined savings, s, the decision maker faces the following lottery $(s + \varepsilon_l, p; s + \varepsilon_h, 1 - p)$. The bounds in (2.9), (2.10) then specify the reference point ω_2 to lie anywhere between the two extreme levels of incomes, $s + \varepsilon_l$ and $s + \varepsilon_h$, such that the decision maker is in gains in the good state and losses in the bad state.

3 General solution and some benchmark results

Substituting the budget constraints from (2.1) and (2.2) into the objective function in (2.4), and using (2.9), (2.10), the unconstrained optimization problem of the decision maker is

$$s^* \in argmax\ U = [u(y - s) + \mu \overline{g}\,(y - s; \omega_1)] + [Eu\,(s + z) + \mu E\overline{g}\,(s + z; \omega_2)]\,,\ s \in [0, y]\,, \quad (3.1)$$

where

$$\overline{g}\,(y - s; \omega_1) = -\lambda s, \quad (3.2)$$

$$Eu(c_2) \equiv Eu\,(s + z) = pu(s + \varepsilon_l) + (1 - p)\,u(s + \varepsilon_h), \quad (3.3)$$

$$E\overline{g}\,(s + z; \omega_2) = -p\lambda\,(\omega_2 - (s + \varepsilon_l)) + (1 - p)\,(s + \varepsilon_h - \omega_2)\,. \quad (3.4)$$

We now explain each of the expressions in (3.1)–(3.4). In (3.1), intertemporal utility is the sum of the Köszegi-Rabin utilities in each time period. Using (2.1) and (2.8), we have $c_1 - \omega_1 = y - s - y = -s < 0$. Thus, using the second row in (2.7), the time $t = 1$ gain-loss utility is $\overline{g}\,(y - s, \omega_1) = -\lambda s$.

The expected utility of time $t = 2$ consumption, $Eu(c_2)$, in (3.3), takes expectations over the realizations of the random income, z, over the two states of the world, conditional on the time

[15]The evidence does, however, support that decision makers take outcomes and probabilities into account in forming reference points, as in expected incomes (Dhami, 2019, Vol. 1), but this is allowed by our specification.

$t = 1$ information set. Finally, in the determination of expected second period gain-loss utility, $E\overline{g}(s+z,\omega_2)$, we take account of the relation between c_2 and ω_2 specified in (2.2), (2.9), and (2.10). With probability p, the bad state occurs and $s+\varepsilon_l < \omega_2$ so the second row of (2.7) applies; and with probability $1-p$, the good state occurs and $s+\varepsilon_h > \omega_2$, so the first row of (2.7) applies.

Differentiating (3.1) with respect to s, we get

$$\frac{\partial U}{\partial s} = -u'(y-s) - \mu\lambda + E\left[u'(s+z)\right] + \mu\left(p\lambda + (1-p)\right). \tag{3.5}$$

The first and third terms on the RHS of (3.5) comprise the Euler equation under standard preferences ($\mu = 0$). They capture, respectively, the current marginal cost and future marginal benefits from an extra unit of savings, measured in terms of the marginal utility of consumption. The second and fourth terms on the RHS of (3.5) arise from marginal changes in gain-loss utility, arising from an extra unit of savings. The second term shows the current marginal sacrifice in gain-loss utility arising from loss aversion, as in Thaler and Benartzi (2004).[16] The fourth term shows the marginal benefit in gain-loss utility at time $t = 2$ from an extra unit of savings that arises from the following two sources. (i) A reduction of loss aversion when there is a bad state of the world at time $t = 2$ that arises with probability p, and (ii) the extra consumption made possible in the good state of the world at time $t = 2$ that arises with probability $1-p$.

The second order condition is

$$\frac{\partial^2 U}{\partial s^2} = u''(y-s) + E\left[u''(s+z)\right] < 0. \tag{3.6}$$

Since the objective function is strictly concave and defined over a compact set, $s \in [0, y]$, there is a unique solution found by solving the first order condition. Thus, at an interior solution, $s = s^*$, we have

$$\frac{\partial U}{\partial s} = -u'(y-s^*) - \mu\lambda + E\left[u'(s^*+z)\right] + \mu\left(p\lambda + (1-p)\right) = 0. \tag{3.7}$$

4 Certainty equivalence and precautionary savings ($\mu = 0$)

The neoclassical certainty equivalent result relies on two assumptions that are a special case of the model that we have described above. First, the utility function is quadratic. Second, there is no gain-loss utility, so $\mu = 0$. We now consider this case.

The utility function is given by

$$u(c_t) = c_t - \frac{a}{2}c_t^2,\ a > 0,\ c \geq 0,\ t = 1, 2. \tag{4.1}$$

Using (2.2) and (4.1), $Eu'(c_2) = p(1 - a(s+\varepsilon_l)) + (1-p)(1 - a(s+\varepsilon_h))$, or

$$Eu'(c_2) = 1 - as - a\overline{z} = u'(Ec_2), \tag{4.2}$$

[16] Suppose that, unlike Thaler and Benartzi (2004), in (2.8) we had made the assumption that the time $t = 1$ reference point is $\omega_1 = \kappa y$ where $0 < \kappa < 1$. Then the analysis is modified as follows. The objective function in (3.1) becomes piecewise linear. We then need to write the first order condition for three cases: $s < \kappa y$, $s = \kappa y$, and $s > \kappa y$; compute the maximum value of the objective function separately in each domain; and then pick the domain in which the value of the objective function highest. In terms of the comparative static effects in Proposition 2, the results would need to be stated separately for each of the three domains. For instance, the negative effect of loss aversion on savings (Proposition (i)) would need to be stated for the domain $s < \kappa y$. However, we believe that the assumption in Thaler and Benartzi (2004) is persuasive and produces cleaner predictions.

where $\overline{z}$ is defined in (2.3).[17]

Using the Euler equation (3.7), the intermediate result in (4.2), the budget constraints, (2.1), (2.2), restricting $\mu = 0$, at an interior solution we have

$$u'(c_1) = Eu'(c_2) = u'(Ec_2). \tag{4.3}$$

$$\Rightarrow c_1 = Ec_2. \tag{4.4}$$

We can also solve out for the optimal level of savings. We have $Ec_2 = s + \overline{z}$, so using (2.1), we can rewrite (4.4) as $y - s = s + \overline{z}$, which can be solved out for the optimal savings level, superscripted with two stars to distinguish this special case,

$$s^{**} = \frac{y - \overline{z}}{2}. \tag{4.5}$$

Optimal savings are positive if $y \geq \overline{z}$, and negative if $y < \overline{z}$. The level of savings s^{**} reflects a pure consumption smoothing model. When $\overline{z} = 0$, we have $s^{**} = \frac{y}{2}$, so that the decision maker smoothes income equally between the two time periods.

This analysis gives rise to the *certainty equivalent result* because the optimal solution to savings in the following two cases is identical.

1. *Certainty*: The decision maker receives at time $t = 2$, the non-stochastic income $s + \overline{z}$ with certainty.

2. *Uncertainty*: The decision maker receives, as in our model, at time $t = 2$, the stochastic income z, in addition to first period savings, s, so that the second period income, $s + z$, is random.

Remark 1 *From the Euler equation under quadratic utility in (4.3), $u'(c_1) = Eu'(c_2)$, consider an increase in the variance of consumption, to reflect an increase in uncertainty. We have that $Eu'(c_2) = 1 - as - a\overline{z}$ is independent of the variance of consumption. In this case, from (4.4), $c_1 = Ec_2$. The key is that for quadratic utility, derivatives higher than order 2 equal zero, and in particular $u''' = 0$.*
Consider the the Euler equation $u'(c_1) = Eu'(c_2)$ and now suppose instead that $u''' \neq 0$ (which rules out quadratic utility). The effect of a change in the variance in consumption on the Euler equation, $u'(c_1) = Eu'(c_2)$, now depends on how $Eu'(c_2)$ is influenced by this change. It turns out that the key determinant is the sign of u''' (Proposition 1, below).

The main implication of the absence of the precautionary savings motive is given in (4.4), i.e., $c_1 = Ec_2$. Whether the decision maker consumes a smaller amount (or saves more) is not determined by risk aversion alone (sign of u'') but by the sign of u''' (Remark 1). Hence, the literature uses departures from the condition $c_1 = Ec_2$, to identify *precautionary savings*, and its flip side, *reckless undersaving*. We summarize the standard result in the next definition.

Definition 1 *Let s^{**} be given in (4.5).*
(i) If $c_1 < Ec_2$ ($\Leftrightarrow s > s^{**}$) then the individual engages in precautionary savings.
(ii) If $c_1 > Ec_2$ ($\Leftrightarrow s < s^{**}$) then the individual engages in reckless undersaving.

[17] In (4.2), we have used the fact that $u'(c_2) = 1 - ac_2$ and $Ec_2 = s + \overline{z}$.

Thus, the precautionary savings motive induces the decision maker to save an amount even greater than s^{**}, over and above what is required to perform the income smoothing role; while reckless undersavings induces them to save less.

Proposition 1 *Suppose that* $\mu = 0$.
(a) If for all $x \in X$ *we have* $u'''(x) > 0$*, then the decision maker engages in precautionary savings* $(c_1 < Ec_2)$.
(b) If, for all $x \in X$ *we have* $u'''(x) < 0$*, then the decision maker engages in reckless undersaving* $(c_1 > Ec_2)$.

From Proposition 1, when $u''(x) < 0$ and $u'''(x) > 0$, the decision maker reduces first period consumption relative to expected second period consumption. This gives rise to precautionary savings.

5 Optimal savings in the Köszegi-Rabin model, $\mu > 0$

In this section, throughout, we allow for gain-loss utility, so that $\mu > 0$.

5.1 The effect of loss aversion on savings

The comparative static effects of loss aversion on savings in the Köszegi-Rabin model are stated in the next proposition, followed by a discussion.

Proposition 2 *Consider the optimization problem with Köszegi-Rabin preferences in (3.1).*
(i) (Loss aversion, λ*) Optimal savings are decreasing in the magnitude of loss aversion,* λ.
(ii) (Relative weight on gain-loss utility, μ*) If the decision maker is loss averse* $(\lambda > 1)$*, then optimal savings are 'decreasing' in* μ*. The reverse is true for loss tolerant decision makers* $(\lambda < 1)$.
(iii) (Risk) For the CRRA utility function, $u(c) = \frac{1}{1-\gamma}c^{1-\gamma}; \gamma > 0, \gamma \neq 1$*, risk aversion has ambiguous effects on optimal savings.*

Discussion of Proposition 2: From the first order condition (3.7), loss aversion has two opposing intertemporal effects. Suppose that time $t = 1$ savings increase by 1 unit. This results in a time $t = 1$ loss in utility of $\mu\lambda$ (second term on the RHS of (3.7)). However, in the future, at time $t = 2$, losses only occur with probability $0 < p < 1$ and it is only in this state of the world that loss aversion bites. Hence, a unit increase in current savings offsets future loss aversion, in marginal utility terms, by $p\mu\lambda < \mu\lambda$. Thus, on net, an increase in loss aversion reduces current savings (Proposition 2(i)).[18]

From the first order condition (3.7), a unit increase in the parameter of gain-loss utility, μ, leads to two effects. It increases current loss in utility from an extra unit of savings by λ. But it also leads to an increase in future marginal utility, from the extra unit of savings, equal to $p\lambda + (1 - p)$. The net effect is a change in marginal utility by the term $(1 - p)(1 - \lambda)$ which is positive if $\lambda < 1$

[18]This effect would be even stronger if future utilities were discounted. If the discount factor were $0 < \delta < 1$, then the analogous condition is $\delta p\mu\lambda < p\mu\lambda < \mu\lambda$.

(loss tolerant) and negative if $\lambda > 1$ (loss averse); this is the content of Proposition 2(ii). However, we do not measure subject-specific parameter values for μ, hence, we do not test Proposition 2(ii).

Using the intuition in Remark 1, the result in Proposition 2(iii) is not surprising because risk aversion alone is not sufficient to tell us how the Euler equation will be influenced by changes in the variance of consumption. We need, in addition, the sign of the third derivative (Proposition 1).

From Proposition 2(i), the optimal savings of loss averse subjects ($\lambda > 1$) are predicted to be lower than the optimal savings of loss tolerant subjects ($\lambda < 1$). Hence, a potential implication of this result is that loss averse decision makers might be less likely to engage in precautionary savings, relative to loss tolerant decision makers (see Corollary 1 below). However, we also directly test the prediction in Proposition 2(i).

We can, for each subject, observe their optimal savings, s^*. This allows us to compute their optimal first period consumption c_1^*, and their expected second period consumption Ec_2^*. Using Definition 1, we can then test if $c_1^* = Ec_2^*$ (certainty equivalence), $c_1^* < Ec_2^*$ (precautionary savings), or $c_1^* > Ec_2^*$ (reckless undersaving).

5.2 The effect of loss aversion on precautionary savings

Recall that the precautionary savings result requires us to explicitly assume that the interest rate and the discount rate are identical. In our experiments, the interest rate is zero. However, some subjects in our experiments might have a strictly positive discount rate. Indeed, for about 28% of our subjects, we find the phenomena of *preference reversals*. Preference reversals occur, for instance, under quasi-hyperbolic discounting, but not under exponential discounting. One can show that, even in the presence of quasi-hyperbolic discounting, precautionary savings are more likely to arise if the parameter of loss aversion is lower; which is consistent with Proposition 2(i). Or, that more loss averse subjects are less likely to engage in precautionary savings.

Corollary 1 *Precautionary savings are more likely to arise if the parameter of loss aversion is lower. In particular, loss averse subjects ($\lambda > 1$) are less likely to engage in precautionary savings as compared to loss tolerant ($\lambda < 1$) subjects. This result holds true in the presence of time discounting (as in models of quasi-hyperbolic discounting), and also in the absence of time discounting.*

We use Corollary 1 to test the predictions of our model on precautionary savings.

5.3 Optimal savings and means preserving spreads of income

In our experiments, we also consider the effects of mean preserving spreads of the random time $t = 2$ income, z. This gives rise to two questions. First, what is the predicted effect on optimal savings when a mean preserving spread of incomes takes place. Second, how does loss aversion mediate the effects of such a mean preserving spread of incomes. For instance, does higher loss aversion enhance, or mitigate, the effects of a mean preserving spread of incomes on savings?

Consider a mean preserving spread in the distribution of the time $t = 2$ random income, z. In our experiments, in 3 out of 4 cases, we consider a family of symmetric shocks of the form $\varepsilon_l = -\varepsilon_h$, and each of these shocks occurs with probability 0.5 (i.e., $p = 1 - p = 0.5$), so that

$Ez = p\varepsilon_l + (1-p)\varepsilon_h = 0$. We index the size of the shocks by a real-valued parameter $\theta > 0$ that gives successive members of this family of shocks, starting from a baseline shock. Thus, in the good state, the shock is $\theta\varepsilon_h$, and in the bad state the shock is $\theta\varepsilon_l$ such that the following two conditions hold.

$$\theta\varepsilon_l = -\theta\varepsilon_h, \tag{5.1}$$

and

$$p = 1 - p = 0.5, \tag{5.2}$$

so that $Ez = 0$. Increases in the size of θ preserve $Ez = 0$ but entail a larger, symmetric, spread of the stochastic time $t = 2$ income.[19] Hence, we define a mean preserving spread of incomes, in our model, as an increase in the parameter θ.

Proposition 3 *Suppose that the decision maker has Köszegi-Rabin preferences given in (3.1), with linear gain-loss utility as in (2.7). Consider a mean preserving spread of the time $t = 2$ stochastic income, z, captured by an increase in the size of the parameter θ.*
(a) If $u''' > 0$, then optimal savings are increasing in θ. The reverse is true when $u''' < 0$.
(b) The response of optimal savings to a mean preserving spread of the time $t = 2$ income is independent of the parameter of loss aversion.

Discussion of Proposition 3: From Proposition 3(a), if $u''' > 0$, which is the condition for precautionary savings in the neoclassical model (Proposition 1), then optimal savings increase in response to a mean preserving spread of incomes. The intuition is similar to the one outlined in Remark 1. A mean preserving spread of income increases the risk facing the decision maker and precautionary savers in the classical model respond to this risk by saving more. Note that the condition $u'' < 0$ is not sufficient to determine the effect on the relevant Euler equation. However, if marginal utility is convex in consumption, i.e., $u''' > 0$, then the decision maker responds by saving more on account of the precautionary motive when a mean preserving spread takes place.

In contrast, to the theoretical prediction in Proposition 3(b), of zero effect, we find in our data that loss aversion 'increases' the response of savings to mean preserving spreads of incomes. The prediction in Proposition 3(b) arises from the additive separability of absolute and relative utility in Köszegi-Rabin preferences given in (3.1) and linear gain-loss utility. In Section 5.4, below, we check if an extension of Köszegi-Rabin preferences to non-linear gain-loss utility (which subsumes classical prospect theory) is able to explain the data.

5.4 Extension of the model to non-linear gain-loss utility

In this section, we extend our model to nonlinear gain-loss utility, using (2.6). We show that, like the predictions in the linear gain-loss case in Proposition 3(b), the predicted mediating effects of loss aversion on mean preserving spreads of income are also not supported by our data.[20] Hence,

[19] Here is one hypothetical example that uses (5.1). For $\theta = 0.2$, $(\theta\varepsilon_l, \theta\varepsilon_h) = (-0.2\varepsilon_h, 0.2\varepsilon_h)$; for $\theta = 0.5$, $(\theta\varepsilon_l, \theta\varepsilon_h) = (-0.5\varepsilon_h, 0.5\varepsilon_h)$; and so on.

[20] We are giving this result as an extension because at the time of our original IRB application, we used the linear form of gain-loss utility. This extension was written after analyzing our data and this is a case of empirics feeding back into the theory, and we are not claiming a reverse-engineered victory for our theoretical model.

this suggests that there is an essential element of the problem that is not captured by Köszegi-Rabin preferences, and it appears to come from the assumption of additivity of preferences over absolute and relative levels of consumption.[21]

Under nonlinear gain-loss utility, given in (2.6), we can rewrite the objective function of the decision maker in (3.1)–(3.4), as follows.[22] The decision maker chooses $\widetilde{s} \in [0, y]$ to maximize

$$\widetilde{U} = \left[u(y-s) - \mu\lambda\left(s\right)^{\beta}\right] + Eu\left(s+z\right) + \mu\left[\left(1-p\right)\left(s+\theta\varepsilon_h - \omega_2\right)^{\beta} - p\lambda\left(\omega_2 - \left(s-\theta\varepsilon_h\right)\right)^{\beta}\right]. \quad (5.3)$$

Proposition 4 *Consider the optimization problem given in (5.3). Suppose that the second order condition holds.*
(a) Optimal savings, $\widetilde{s}$, can be either increasing or decreasing in the parameter θ, which captures mean preserving spreads in income. In the special case of $\mu \to 0$, we have that $\widetilde{s}$ is increasing in θ if $u''' > 0$.
(b) An increase in loss aversion, λ, 'reduces' the response of optimal savings $\widetilde{s}$ to θ.

Discussion of Proposition 4: From Proposition 4(a), a mean preserving spread has indeterminate effects on savings. A sufficient condition for optimal savings to increase in the mean preserving spread of incomes can be given for the special case of $\mu \to 0$ and $u''' > 0$. From Proposition 4(b), loss aversion 'reduces' the savings response to a mean preserving spread of incomes. By contrast, under linear gain-loss utility there was a zero predicted mediating effect of loss aversion (Proposition 3(b)). Both predictions are rejected by our data, which shows that loss aversion 'increases' the savings response to a mean preserving spread of incomes

5.5 Sufficiency of $u'''(x) > 0$ for precautionary savings?

In this section, we briefly outline the relation between loss aversion and precautionary savings in the presence of gain-loss utility. We show that for loss tolerant subjects ($\lambda < 1$), a strictly positive third derivative of the utility function, $u'''(x) > 0$, is a sufficient condition for precautionary savings. However, the condition $u'''(x) > 0$ is neither necessary nor sufficient for precautionary savings for loss averse subjects ($\lambda > 1$).

Proposition 5 *(a) If the decision maker is loss tolerant ($\lambda < 1$), then, the condition $u'''(x) > 0$ is sufficient for the existence of precautionary savings.*
(b) If the decision maker is loss averse ($\lambda > 1$), then all three outcomes are possible: certainty equivalence, precautionary savings, and reckless undersaving.

Since we do not directly measure the third derivative in our data, the result in Proposition 5 is purely of theoretical interest.

[21] Indeed, by implication, our data is also not consistent with the identical predictions of classical prospect theory (Kahneman and Tversky, 1979) on the mediating effects of loss aversion on mean preserving spreads of income. These extra calculations are available on request from the authors.

[22] In order to distinguish the intertemporal utility function in the case of nonlinear gain-loss utility, we use the notation $\widetilde{U}$ instead of U for the utility function. We also use tildes on the relevant variables, such as the optimal savings level, $\widetilde{s}$.

6 Experiments and data

6.1 Experimental Design

To test the predictions of our theoretical model, we designed and conducted an incentivized experiment. We developed a two-part, within-subjects, experimental design. Part 1 of the experiment was designed to study savings behavior in a two period model, identical in all details, to the one used to derive our theoretical predictions. Part 2 of the experiment was designed to measure subject-specific loss aversion. This allows us to formally test the relationship between loss aversion and the decision to engage in precautionary savings, which is one of the key channels of exploration in our paper. All payoffs were expressed in units of an experimental currency, EC, and converted into real money according to the exchange rate: $1000EC = £1$. All units below are expressed in terms of EC. We now explain both parts in detail.

1. Part 1 (Optimal consumption/savings choice decision): Subjects face a two-period consumption-savings problem where they are asked to allocate a given amount of money now (the time $t = 1$ endowment, y, in the theoretical model) to current consumption, c_1, and future consumption, c_2, in one month's time (time $t = 2$). Thus, the time gap between two successive periods in our experiment is one month. We varied the endowment y in different sub-cases. Whatever amount is allocated to consumption at time $t = 1$, c_1, is paid to the subjects on the same day. The amount saved towards consumption in one month, s, is added to a random income, z, at time $t = 2$. So at time $t = 2$, the subjects receive the random income $s + z$ and it is paid out at time $t = 2$. At time $t = 1$, subjects received a £6 participation, or show-up, fee in the experiment, on the day of the experiment.

   At time $t = 1$, the subjects were informed that they will receive a random income at time $t = 2$ that could be either positive or negative (the analogues of $\varepsilon_h > 0$ and $\varepsilon_l < 0$ in our model), with equal chances; the exact amounts of ε_h and ε_l vary in different sub-cases. If a subject had not saved enough at time $t = 1$ to cover for losses at time $t = 2$ (e.g., $s > 0$ is too low relative to $\varepsilon_l < 0$), she was informed that losses would be subtracted from a guaranteed amount to be paid as a lumpsum at date $t = 2$. The lumpsum of £6 is paid at $t = 2$ in order to ensure that none of our subjects is out of pocket at the end of the experiment; this may be viewed as the second part of our participation fee (£6 each at dates $t = 1$ and $t = 2$).

   We had three levels of endowments ($y = 2000, 3000, 4000$) in different sub-cases. We had 4 kinds of shocks, which constituted our random income, z. Three of the shocks were symmetric ($\varepsilon_l = -\varepsilon_h$), where the subject could win or lose an amount with a 50–50 chance (i.e., p in our theoretical model equals 0.5). For these symmetric shocks $\varepsilon_h \in [500, 1000, 2000]$, so essentially we have the case of a mean preserving spread in risk as ε_h increases (Sections 5.3 and 5.4 summarize the predictions in this case). The fourth shock was the only asymmetric shock, where $p = 0.5$ but $\varepsilon_h = 500$ and $\varepsilon_l = -1000$.

   Thus, subjects had to make a consumption/savings choice at time $t = 1$ for $3 \times 4 = 12$ sub-cases in Part 1 (3 levels of y and 4 levels of z). In effect, the sub-cases constitute the use of the strategy method for different levels of incomes, applied to Part 1. The full set of sub-cases is provided in Table 1. One of these sub-cases was played out for real to determine the payoffs from Part 1. Subjects were encouraged to assume that they have no other outside-the-lab source of income, consumption, or saving and they completed all the tasks without receiving any feedback between rounds.

Table 1: Optimal savings elicitation tasks

Task	Endowment	Positive shock	Negative shock
1	2000	500	-500
2	3000	1000	-1000
3	4000	2000	-2000
4	2000	1000	-1000
5	3000	2000	-2000
6	4000	500	-500
7	2000	2000	-2000
8	3000	500	-500
9	4000	1000	-1000
10	2000	500	-1000
11	3000	500	-1000
12	4000	500	-1000

The Table lists the 12 tasks (or sub-cases) used in Part 1 of the experiment. All payoffs are expressed in terms of experimental currency (EC) used in the experiment, with 1000 EC = £1. In each sub-case there is a 50% probability of the positive and negative shocks.

2. Part 2 (Elicitation of subject-specific loss aversion): The second part of the experiment was designed to elicit a measure of loss aversion at the individual level. We adopted a method similar to Gächter et al. (2022), but used the bisection procedure (Abdellaoui, 2000). We gave subjects a balanced risk lottery of the form $(x, 0.5; -y, 0.5)$, where $x > 0$ is a gain relative to the reference point, and $-y < 0$ is a loss relative to the reference point.[23] Given a value of x, we then elicited the value of the loss, $-y$, that makes the certainty equivalent of the given lottery equal to zero; i.e., a value of $-y$ such that $(x, 0.5; -y, 0.5) \sim (0, 1)$, where '$\sim$' is the indifference relation. Loss aversion is determined by the ratio $\frac{x}{y}$; this presupposes a linear prospect theory utility function over small stakes that has a kink at the reference point.

   In these tasks, the gain, x, is fixed and y is elicited through $k = 1, 2, ..., 6$ lottery choices, where k is the iteration number. In iteration k, subjects had to choose between a lottery of the form $(x, 0.5; -y_k, 0.5)$ and a sure amount of zero, where the amount y_k is determined from the previous $k - 1$ choices. However, in the very first lottery choice (i.e., $k = 1$), $-y_1$ is the midpoint of $[-1.4x, -0.1x]$, the feasible interval containing $-y_1$.[24]

   If in the first iteration $k = 1$, the lottery is chosen over a sure amount of zero, we make the lottery less attractive in the second iteration, $k = 2$. We do this by having $-y_2$ as the midpoint

[23] We use the standard terminology, so that the lottery $(x, 0.5; -y, 0.5)$ means a 50-50 chance of gaining an amount x and losing an amount y. Our method resembles the switching choice in Gächter et al. (2022), however, our bisection method allows for more variation across subjects.

[24] These amounts are proportional to the gain amount in the lottery, x, such that the upper bound of loss aversion parameter is set at $\frac{x}{0.1x} = 10$. Empirically, the median value of loss aversion is around 2 (Kahneman and Tversky, 2000; Dhami, 2019, Vol. 1).

of the reduced feasible interval $[-1.4x, -y_1]$, otherwise, $-y_2$ is the midpoint of $[-y_1, -0.1x]$; this bisecting process lies at the heart of the bisection method. The third iteration, $k = 3$, is contingent on the choices in the first two iterations, creating four possibilities. If a subject prefers zero to both lotteries in the first two iterations, $-y_3$ is the midpoint of $[-y_2, -0.1x]$. If a subject prefers lotteries to zero in the first two iterations, then $-y_3 \in [-1.4x, -y_2]$. If a subject first prefers zero at $k = 1$ and then the lottery in the second iteration at $k = 2$, then $-y_3 \in [-y_1, -y_2]$, otherwise, $-y_3$ is the midpoint of $[-y_2, -y_1]$. Hence, the interval containing y shrinks in the remaining choices by replacing the lower or the upper bound of the feasible interval in each iteration based on the subjects' previous choices.

We use three levels of gains $x \in [2000, 3000, 4000]$, the same as the endowments in Part 1 (see Table 1). Therefore, subjects faced $3 \times 6 = 18$ iterations in total; one of these iterations was chosen at random to be paid off for real. As noted earlier, any losses were covered by a show-up fee paid on the day.

Finally, we included two non-incentivized questions to gather information on subjects' time preferences. A key element of the modern time discounting literature is the recognition of *preference reversals*, as in models of hyperbolic discounting. By contrast, under exponential discounting, preference reversals cannot arise.[25] We identify present-biased preferences with choices that exhibit preference reversals.[26] We ask subjects to choose between (i) receiving 2500 in one month vs. receiving 2000 today, and (ii) receiving 2500 in 11 months vs. receiving 2000 in 10 months. Subjects exhibit preference reversals if they pick '2000 today' in (i) and they pick '2500 in 11 months' in (ii). We classify such subjects as present-biased.

6.2 Procedures

The subjects were 79 students from a UK Experimental Economics Lab standard subject pool (mostly undergraduate students, 57% female, average age 20.7). The experiment took place in March 2023 and there were in total 6 sessions with 12 to 16 subjects participating in each session. The average payment was £15.65 (£7.71 on the day of the experiment and £7.96 one month later). The sessions lasted 35 minutes, on average, including the time for the instructions and the comprehension test.[27] To cover potential losses, there was a guaranteed show-up fee of £6 paid on the day, and a guaranteed amount of £6 paid in one month.

The experiment was computerized using the LIONESS Lab platform (Giamattei et al. , 2020) and the recruitment took place via ORSEE (Greiner , 2015). Subjects were randomly seated to individual PCs where they could complete the task at their own pace. They were given written instructions, and before they were able to begin the experiment, they had to go through an extensive comprehension questionnaire. To ensure that payments could be delivered precisely on time, we completed the payments using Amazon pre-paid cards which were sent to the participants via email immediately after the session and one month later. After completing Parts 1 and 2

[25] This is on account of the *stationarity axiom*. However, alternatives to exponential discounting, such as hyperbolic discounting relax this axiom. For a formal treatment, see Dhami (2019, Vol. 3).

[26] An alternative method would be to use the Convex Time Budgets method in Andreoni and Springer (2012), but we did not use this method because the estimation of the discount rate is not critical to our problem.

[27] The experimental instructions can be found in the supplementary section.

of the experiment, subjects were asked to complete a questionnaire to elicit their demographic characteristics. Subjects could not communicate with each other, and their PCs were separated by privacy screens. All subjects participated only once to the experiment.

6.3 Data

We have a sample of 79 students. We elicit loss aversion for three levels of gains $x \in [2000, 3000, 4000]$, the same as the endowments in Part 1 of our experiment.[28] Averaged across all three levels of gains, the mean loss aversion parameter is 1.6571 with a median value of 1.6609; the minimum and maximum values (across all subjects and levels of gains) are, respectively, 0.7196 and 4.1080.

There is no significant difference in the estimated parameter of loss aversion among the three levels of gains. Hence, for each subject, we use the mean value of loss aversion across all three levels of gains, to categorize subjects as either loss averse ($\lambda > 1$) or loss tolerant ($\lambda \leq 1$). This approach is likely to reduce measurement errors and alleviate the assumption of a linear utility function, and is also the approach used in Chapman et al. (2022).

We classify the savings choices of subjects as precautionary savings if the consumption at time $t = 1$ is less than the expected time $t = 2$ consumption, i.e., $c_1 < Ec_2$ (see Definition 1). On average, 46% of the choices exhibit precautionary saving behavior; 36% of the choices indicate reckless undersaving behavior, $c_1 > Ec_2$ (see Definition 1); while the remaining 18% choices exhibit the classic certainty equivalence result $c_1 = Ec_2$. Table 2 shows the number and percentage of observed choices reflecting precautionary saving behavior for each of the 12 sub-cases, which have varying amounts of endowment y and stochastic income, z. The results of the One-Way ANOVA test indicate that there is no significant difference in the mean of savings across the four levels of stochastic income (p-value: 0.971).

Table 2: Precautionary Saving behavior

	Stochastic Income			
Endowment	(500,0.5; -500,0.5)	(1000,0.5; -1000,0.5)	(2000,0.5; -2000,0.5)	(500,0.5; -1000,0.5)
2000	26 (33%)	32(41%)	45 (57%)	35 (44%)
3000	32(41%)	34(43%)	54(68%)	37(47%)
4000	37(47%)	32(41%)	40(51%)	33(42%)

Table shows the number and percentage of subjects engaging in precautionary saving.

The variable 'Shock' refers to the value taken by the random income at time $t = 2$, and it has 4 categories of which three are symmetric and one is asymmetric (see Tables 1 and 2). Ignore for the moment, the last category in the last column in Table 2, which is an asymmetric shock. For the remaining three symmetric shocks, where the expected value of stochastic income is zero in all cases, and the probability of the bad states is fixed at 0.5, we have the following three cases. The variable 'Shock' takes the value 0 for the reference category with a symmetric shock of 500 (i.e., an income of 500 in the good state and -500 in the bad state); the value 1 for a symmetric shock of 1000; and the value 2 for a symmetric shock of 2000.

As the absolute value of the shock increases, we have a mean preserving spread of the time $t = 2$ random income distribution, as outlined in Section 5.3. Most reasonable decision theories

[28] Recall from the description of Part 2 of our task, above, that x refers to the gain in the lottery $(x, 0.5; -y, 0.5)$.

will require savings to increase with an increase in the absolute value of the shock, and this also serves as a consistency check on the data. In these three cases we have, respectively, the following mean values of savings measured in EC: 1552.312 (shock value 0), 1639.19 (shock value 1), and 1854.599 (shock value 2).

7 Regression Results

From Proposition 2(i), loss aversion 'directly' reduces savings. As far as we know, we are the first to test this prediction in an explicitly dynamic model, where 'individual-specific loss aversion' is directly measured.[29] As noted in Corollary 1, a direct implication is that the precautionary savings of loss averse subjects ($\lambda > 1$) are likely to be lower than the precautionary savings of loss tolerant subjects ($\lambda < 1$). We test these predictions in this section.

Another important set of predictions that we test in this section relate to the effects of mean preserving spreads of time $t = 2$ income on optimal savings. We show that Köszegi-Rabin preferences (i) explain well the effects of mean preserving spreads on savings (Proposition 3(a)), but (ii) are unable to explain why loss aversion strengthens the response of savings to mean preserving spreads of income either under linear gain-loss utility (Proposition 3(b)) or nonlinear gain-loss utility (Proposition 4(b)).

7.1 Determinants of Precautionary Savings

Recall that an individual engages in precautionary saving if $c_1^* < Ec_2^*$; reckless undersavings if $c_1^* > Ec_2^*$; and certainty equivalence if $c_1^* = Ec_2^*$. Therefore, our dependent variable is binary and we employ the following logit model to analyze the determinants of precautionary saving behavior:

$$P(Y = 1|X) = P(\beta X + u > 0) = F(\beta X) = \frac{1}{1 + \frac{1}{e^{\beta X}}}, \tag{7.1}$$

where $Y = 1$ indicates precautionary saving behavior ($c_1^* < Ec_2^*$) and $Y = 0$, its absence. X is a vector of explanatory variables and β is a vector of coefficients. The explanatory variables used in (7.1), with the corresponding names given in Table 3, and the basic data on the individual categories, are as follows.

- 'Loss aversion': Dummy variable that takes the value 1 if the subject is loss averse ($\lambda > 1$) and 0 if the subject is loss tolerant ($\lambda \leq 1$). 63/79 (80%) of subjects are loss averse.[30]
- 'Present bias': Dummy variable that takes the value 1 if the subject is present biased and 0 otherwise (see Section 6 for an explanation). 22/79 (28%) of subjects are present biased.
- 'Age' gives the self-reported age of subjects. The minimum and maximum age is, respectively, 18 and 35; the mean and median age is, respectively, 20.72 and 19; and the standard deviation is 3.36.

[29] The only exception that we are aware of is Ibanez and Schneider (2023) who use observational data. However, there are extensive differences between their paper and ours which offers a complementary analysis in the lab with tighter controls and more precise implementation of the theoretical model that is possible only in a lab; these differences were noted in the introduction.

[30] We follow here the distinction between loss averse and loss tolerant subjects in Chapman et al. (2022), which also reduces measurement error. With a continuous measure of loss aversion, the results are qualitatively similar, although it reduces the statistical significance of loss aversion.

• 'Gender' is a dummy variable for gender and takes the value 1 for male and 0 for female. 34/79 subjects (43%) are males and 45/79 subjects (57%) are females.

• 'Education' is a dummy variable. It equals 1 for Masters/PhD students, and 0 otherwise. 62/79 subjects (78%) are undergraduates, 16/79 (20%) are masters students and there is 1 PhD student.

• 'Income' is a self declared outside-the-lab monthly expense that is used as a proxy for the real world income of the subjects. The mean income is £425, and the median is £300. The standard deviation is £334.

• 'Time' indicates the length of time taken for the completion of the experiment.

• 'Endowment' is a categorical variable that captures the subject's endowment in each question. It takes the value 0 for the reference category when the endowment is 2000, 1 for an endowment of 3000, and 2 for an endowment of 4000 (this corresponds to distinct values of y, the first period income endowment in our model).

• 'Shock' is a categorical variable that indicates the stochastic income at time $t = 2$ (see details in Table 2 and it corresponds to stochastic income z in our model). The probability of the *good* and *bad* states is fixed at 0.5 in all cases. 'Shock' takes the value 0 for the reference category with a symmetric shock of 500 when the expected value of stochastic income is zero (i.e., $z = \theta\varepsilon_h = 500$ in the *good state* and $z = \theta\varepsilon_l = -500$ in the *bad state* so that $\theta = 1$ and both (5.1), (5.2) are satisfied). It takes the value 1 for a symmetric shock of 1000 ($\theta = 2$), 2 for a symmetric shock of 2000 ($\theta = 4$), and 3 for an asymmetric shock with a negative expected value, where the income in the *good state* is 500 and in the *bad state* is -1000.

In Table 3, we present the logit regression results; the dependent variable is $P(Y = 1|X)$ given in (7.1). In the second column, we present the coefficient estimates; the third column gives the Odds-Ratio; and the last column gives the marginal effects from the logit model.

From Table 3, loss aversion decreases the odds of precautionary saving by 56% ($(1-0.440)\times 100$). In other words, the odds of precautionary saving behavior are 0.440 times lower for loss averse individuals compared to loss tolerant ones (keeping all other predictors constant). This finding aligns with our theoretical prediction that loss tolerant subjects are more likely to engage in precautionary savings (Corollary 1). The last column of Table 3 reports the marginal effects. Being loss averse, relative to loss tolerant, decreases the probability of precautionary saving behavior by almost 20%. As far as we are aware, this is the first empirical demonstration of this result with subject-specific directly measured loss aversion.

Present-biased individuals exhibit a reduced likelihood of engaging in precautionary saving behavior.[31] The effect of present-bias is comparable to that of loss aversion, and both effects are significant at the 10% level, but the magnitude of the effects is relatively large. This is an important result and speaks to the growing literature on the joint effects of risk and time preferences; for a survey, see Dhami (2019, Vol. 3).

[31] Recall our definition of present bias, which is consistent with a violation of the stationarity axiom for the axioms of rationality under time discounting (Dhami, 2019, Vol. 3). Hence, one cannot simply derive the theoretical implications of present bias by introducing a simple discount factor, δ, to discount future utilities. At a minimum, this would require setting up a model of quasi-hyperbolic discounting, but that requires a minimum of three time-dated values of consumption, while we have two time periods.

Table 3: Logit Regression results

	Dependent variable is a dummy for pre cautionary saving		
	Estimates	Odds-Ratio	Marginal Effects
Loss aversion	-0.821^{*}	0.440^{***}	-0.202***
	(0.456)	(0.201)	(0.047)
Present Bias	-0.792^{**}	0.453^{***}	-0.190***
	(0.360)	(0.163)	(0.038)
Endowment=3000	0.273^{**}	1.314^{*}	0.068
	(0.125)	(0.164)	(0.042)
Endowment=4000	0.058	1.060	0.014
	(0.124)	(0.131)	(0.042)
Shock=1	0.058	1.060	0.014
	(0.150)	(0.159)	(0.049)
Shock=2	0.832^{***}	2.299^{**}	0.205***
	(0.233)	(0.536)	(0.047)
Shock=3	0.192	1.211	0.048
	(0.164)	(0.199)	(0.049)
Age	0.204	1.227	0.051
	(0.544)	(0.667)	(0.059)
Age^2	-0.003	0.997	-0.001
	(0.011)	(0.011)	(0.001)
Gender	-0.491	0.612^{*}	-0.121***
	(0.363)	(0.222)	(0.038)
Education	0.304	1.355	0.075
	(0.546)	(0.740)	(0.059)
Income	-0.001^{***}	0.999^{*}	-0.0002***
	(0.0005)	(0.0005)	(0.0001)
Time	0.002	1.002	0.0004 ***
	(0.001)	(0.001)	(0.0001)
Constant	-2.871	0.057	
	(2.701)	(0.153)	
Observations	948	948	948
Log Likelihood	-599.286		
Akaike Inf. Crit.	1,226.573		

Note: *p<0.1; **p<0.05; ***p<0.01. Standard errors are clustered at the individual level.

Compared to females, males are less likely to engage in precautionary savings; being male decreases the odds of such behavior by almost 39%. This result speaks to the large literature on the relatively greater risk-seeking and overconfidence among men, relative to women; for a literature survey, see Dhami (2016).

Recall from Table 1 that in 3 out of 4 cases, we had symmetric shocks to random income at time $t = 2$, so that the expected value of the shock equals 0. However, the magnitude of the shocks varies; and the negative shock takes respective values, -500, -1000, -2000 in the first three sub-cases. The magnitude of the negative shock in the bad state has more influence than the expected value of the stochastic income (which is zero in all cases). When the size of the loss increases to 2000 (Shock equals 2) compared to the reference category of Shock (a loss of 500 in the bad state), the odds of precautionary saving increase by almost 130%. This also accords well with one's intuition and with our model as we explain below.

Similar results are observed within subsets of our data. Table 4 presents the logit regression results within each level of the time $t = 1$ endowment, as well as the pooled results across all endowment levels. In all of these regressions, the coefficient of loss aversion is consistently negative and significant at the 5% level. As far as we are aware, this is the first demonstration of such a result with directly measured subject-specific loss aversion.

Table 4: Determinants of precautionary saving behavior within each endowment

	Dependent variable is a dummy for pre cautionary saving			
	(Endowment of 2000)	(Endowment of 3000)	(Endowment of 4000)	(Pooled)
Loss aversion	-0.796^{**}	-0.733^{**}	-0.971^{**}	-0.819^{***}
	(0.381)	(0.358)	(0.390)	(0.213)
Present Bias	-0.899^{***}	-0.810^{***}	-0.701^{**}	-0.789^{***}
	(0.290)	(0.294)	(0.297)	(0.166)
Shock=1	0.361	0.114	−0.293	0.058
	(0.346)	(0.333)	(0.339)	(0.192)
Shock=2	1.096^{***}	1.262^{***}	0.173	0.830^{***}
	(0.359)	(0.368)	(0.352)	(0.202)
Shock=3	0.533	0.282	−0.233	0.191
	(0.341)	(0.330)	(0.342)	(0.191)
Age	0.110	0.157	0.350	0.204
	(0.422)	(0.485)	(0.484)	(0.263)
Age^2	−0.001	−0.002	−0.006	−0.003
	(0.009)	(0.010)	(0.010)	(0.005)
Gender	−0.171	-0.478^{*}	-0.841^{***}	-0.489^{***}
	(0.274)	(0.275)	(0.282)	(0.156)
Education	0.305	0.297	0.335	0.303
	(0.397)	(0.389)	(0.378)	(0.218)
Income	-0.001^{***}	−0.001	-0.001^{**}	-0.001^{***}
	(0.0004)	(0.0004)	(0.0004)	(0.0002)
Time	0.001	0.002	0.002	0.002^{***}
	(0.001)	(0.001)	(0.001)	(0.001)
Constant	−2.135	−2.452	−3.751	−2.751
	(4.772)	(5.519)	(5.453)	(2.983)
N	316	316	316	948

Note: *p<0.1; **p<0.05; ***p<0.01. Standard errors are clustered at the individual level.

We find support for another prediction of our model, which suggests that among subjects in the precautionary saving category, those who are less loss averse demonstrate a higher difference between expected consumption in period 2, Ec_2^*, and current consumption, c_1^*; this follows from the results derived in the lead up to Corollary 1. This indicates a negative correlation between loss aversion and the difference between expected consumption in period 2 and current consumption ($Ec_2^* - c_1^*$). For choices belonging to the precautionary saving category, Pearson's correlation coefficient between loss aversion and $Ec_2^* - c_1^*$ is -0.087 ($p - value = 0.0675$). On the other hand, for the choices of subjects who exhibit reckless undersaving or certainty equivalence, this correlation is 0.075 ($p - value = 0.0869$).

7.2 Determinants of savings

In Table 5, the dependent variable is the actual savings by individuals. We present two different models. Model 1 contains the same explanatory variables as those used in Table 3. Model 2 adds interaction terms between loss aversion and the variable 'Shock' in order to explain why a mean preserving spread of incomes elicits greater savings.[32]

The general results parallel those on precautionary savings. Consider the estimates in Model 2. Loss averse subjects save, on average, 440 units less than loss tolerant subjects. This confirms the prediction in Proposition 2(i) that an increase in loss aversion 'directly' reduces savings. The effect of loss aversion on savings is larger than the impact of present bias on savings, which decreases savings by 253 units (while holding all other predictors constant). Moreover, while the loss aversion coefficient is significance at the 5% level, present bias is statistically significant at the 10% level. In comparison to females, males tend to save less; and being male is associated with a decrease in savings by 172 units. Older individuals and those who spend more time deliberating on the saving decisions tend to save slightly more. Higher levels of initial time $t = 1$ endowments have a substantial positive effect on saving that is significant at the 1% level.

When comparing different categories of shocks, we again get results similar to those for precautionary savings. Compared to the reference category of shock (Shock = 0), an increase in the size of the loss to 2000 units (Shock = 2) leads to a savings increase of 302 units in Model 1, and this effect is significant at the 1% level. This is consistent with the predictions of the optimal response of savings to mean preserving spreads of income (Proposition 3(a)). Furthermore, in Model 1, for an asymmetric shock with a negative expected value (Shock = 3), savings increase by 117 units relative to the reference category, and this result is significant at the 5% level.[33]

Once we add the interaction terms in Model 2, these results change. None of the shock variables, by themselves, have statistical significance anymore. However, the following two findings are of interest.

1. The interaction term "Loss aversion: Shock=2" is now positive and significant at the 10% level. In other words, being loss averse relative to being loss tolerant, increases savings by $481 - 440 = 41$ units in the case of Shock=2, relative to the reference category where Shock=0.

[32] When we interact loss aversion with the variable 'Shock' in Table 3, none of the coefficients is significant, hence, we do not report those results. But these results are available on request.

[33] Note that our predictions on the optimal response of savings to mean preserving spreads do not cover the case of Shock = 3 because in this case the mean and the variance of the shocks changes simultaneously.

Table 5: OLS regressions on savings

	Dependent variable is saving:	
	(1)	(2)
Loss aversion	-288.026^{*}	-440.349^{**}
	(173.204)	(218.622)
Present Bias	-253.078^{*}	-253.078^{*}
	(140.863)	(141.090)
Age	190.585	190.585
	(229.735)	(230.105)
Age^2	-3.177	-3.177
	(4.800)	(4.807)
Gender	-172.601	-172.601
	(138.117)	(138.339)
Education	56.357	56.357
	(170.729)	(171.004)
Time	0.238	0.238
	(0.500)	(0.501)
Income	-0.212	-0.212
	(0.196)	(0.197)
Endowment=3000	526.171^{***}	526.171^{***}
	(41.528)	(41.595)
Endowment=4000	$1{,}021.136^{***}$	$1{,}021.136^{***}$
	(66.517)	(66.624)
Shock=1	86.878^{*}	44.583
	(47.119)	(93.287)
Shock=2	302.287^{***}	-81.208
	(83.441)	(280.080)
Shock=3	117.435^{**}	57.333
	(55.198)	(127.156)
Loss aversion: Shock=1		53.036
		(107.863)
Loss aversion: Shock=2		480.891^{*}
		(289.141)
Loss aversion: Shock=3		75.365
		(141.115)
Constant	$-1{,}090.200$	-968.727
	(2,563.760)	(2,550.976)

Note: *p<0.1; **p<0.05; ***p<0.01. Standard errors are clustered at the individual level.

Thus, being loss averse, relative to loss tolerant, sharpens the response of savings to mean preserving spreads of time $t = 2$ random income.

2. Conditional on a subject being loss averse, Shock=2 induces a decision maker to save an extra 481 units relative to the reference category of Shock=0 and this is statistically significant.

Both effects have reasonably high magnitudes. Thus, a mean preserving spread in income may be said to induce a *loss averse-hedging motive*. The traditional explanation for this phenomenon is in terms of risk aversion. Our results indicate that ignoring loss aversion may lead to misleading results and an over emphasis on the importance of risk aversion.

The loss averse-hedging motive is difficult for economic theory to explain. Köszegi-Rabin preferences cannot explain this finding with linear gain-loss utility (see Proposition 3b); and by default, standard expected utility theory cannot either. When combined with nonlinear gain-loss utility, Köszegi-Rabin preferences in fact make the opposite prediction to the one that we find in our data, namely that loss aversion should reduce the savings response to mean preserving spreads (see Proposition 4(b)). Loss aversion is not a part of the repertoire of several other behavioral models, such as models of salience, limited attention, and hyperbolic discounting. Hence, these models cannot provide a resolution to this empirical finding either. Köszegi-Rabin preferences that are not additively separable might be one possible resolution. It is also possible that models of bounded rationality and mental accounting might provide a potential resolution. For instance, decision makers might have a mental account for consumption from stochastic incomes and use simple rules of thumb mediated by loss aversion to minimize the variance of consumption. Using such models, and exploring their underlying transmission mechanisms to explain our data, would be potentially a fruitful task for future research.

8 Conclusions

Microeconomic theory focuses on risk aversion and loss aversion as the key determinants of hedging against future income risk in dynamic models. By contrast, macroeconomic theory focusses on precautionary savings. We construct a simple macroeconomic model with future income uncertainty, that we replicated as it is in our experiments, and demonstrate that loss aversion and precautionary motives are related.

Our theoretical model predicts that loss averse decision makers save less, and are less likely to engage in precautionary savings. Our empirical results are consistent with this prediction. We also show that optimal savings increase in response to mean preserving spreads of income, and this is also consistent with our data. However, loss averse decision makers are found to respond even more strongly to a mean preserving spread of the random future income. We term this as the *loss averse-hedging motive* relative to the standard risk-hedging motive in classical theory. However, it is difficult for economic theory to explain this finding, and we show that it cannot be 'directly' explained by Köszegi-Rabin preferences, prospect theory preferences, salience theory, models of limited attention, or by hyperbolic discounting, among others. Replicating this finding, and if established, explaining this finding by using the relevant theory can be a fruitful avenue for future research.

We take account of the effects of loss aversion on current and future consumption in making our predictions on the effects of loss aversion on savings. This approach is consistent with the work of Thaler and Benartzi (2004). However, a body of theoretical work takes account of the effects of loss aversion 'only' on future consumption and predicts that loss aversion will increase current savings. This prediction is not consistent with our data. We also show that the effects of risk aversion on savings are ambiguous in our model. But the effects of present-bias, albeit a bit smaller in magnitude than the effects of loss aversion, are also statistically significant in reducing savings.

Appendix

Proofs of Results

Proof of Proposition 1: Substitute $\mu = 0$ in (3.7). At an interior solution, and using the two budget constraints, (2.1) and (2.2), we get

$$u'(c_1) = E\left[u'(c_2)\right]. \tag{8.1}$$

Suppose that the utility function satisfies the restriction $u'''(x) > 0$, for all $x \in X$. Then, u' is a strictly convex function for all $x \in X$. From Jensen's inequality, it follows that $u'(Ec_2) < E\left[u'(c_2)\right]$. Thus, using (8.1), we get

$$u'(Ec_2) < u'(c_1). \tag{8.2}$$

By assumption, $u''(x) < 0$ for all $x \in X$, thus, it follows from (8.2) that $c_1 < Ec_2$ (precautionary savings). If, on the other hand, $u'''(x) < 0$ for all $x \in X$, then Jensen's inequality implies that $u'(Ec_2) > E\left[u'(c_2)\right]$, so the condition $u''(x) < 0$ for all $x \in X$ implies that $c_1 > Ec_2$ (reckless undersaving). ■

Proof of Proposition 2: Applying the implicit function theorem successively to the first order condition (3.7), and using (3.6), we get

(i)

$$\frac{\partial s}{\partial \lambda} = -\mu\left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1}(1-p) < 0.$$

(ii)

$$\frac{\partial s}{\partial \mu} = \left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1}\left[(1-p)(1-\lambda)\right].$$

It follows that if the decision maker is loss averse, $\lambda > 1$, then $\frac{\partial s}{\partial \mu} < 0$; and if the decision maker is loss tolerant, $\lambda < 1$, then $\frac{\partial s}{\partial \mu} > 0$.

(iii) For the CRRA utility function, $u(c) = \frac{1}{1-\gamma}c^{1-\gamma}; \gamma > 0$, $\gamma \neq 1$, the foc in (3.7) is

$$-(y-s)^{-\gamma} - \mu\lambda + \left[p(s+\varepsilon_l)^{-\gamma} + (1-p)(s+\varepsilon_h)^{-\gamma}\right] + \mu\left[p\lambda + (1-p)\right] = 0.$$

Implicitly differentiating with respect to γ, we get

$$\frac{\partial s}{\partial \gamma} = \left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1}\left[(y-s)^{-\gamma}ln(y-s) - p(s+\varepsilon_l)^{-\gamma}ln(s+\varepsilon_l) - (1-p)(s+\varepsilon_h)^{-\gamma}ln(s+\varepsilon_h)\right] \gtreqless 0.$$

■

Proof of Corollary 1: Suppose that subjects in our experiments discount the time $t = 2$ utility by a discount factor $0 < \delta \leq 1$. The case $\delta = 1$ subsumes our model and all results below also hold for the case $\delta = 1$. Consider the variable $\chi(\lambda) = c_1^* - \delta E c_2^*$. Substituting $c_1^* = y - s^*$ and $E c_2^* = s^* + \overline{z}$, we get $\chi = (y - s^*) - \delta(s^* + \overline{z})$, or

$$\chi(\lambda) = y - (1+\delta) s^*(\lambda) - \delta \overline{z},$$

where we have suppressed dependence of savings on factors other than loss aversion, λ. Since $s^*(\lambda)$ is continuously differentiable in λ (theorem of the maximum), χ is a continuously differentiable function of λ. Let $\lambda = \widehat{\lambda}$ be defined such that $\chi\left(\widehat{\lambda}\right) = 0$, i.e, when $\lambda = \widehat{\lambda}$, we have $c_1^* = \delta E c_2^*$ (certainty equivalence). We have

$$\frac{\partial \chi}{\partial \lambda} = -(1+\delta)\frac{\partial s^*}{\partial \lambda} > 0, \tag{8.3}$$

where the sign of (8.3) follow directly from Proposition 2(i). This also holds if, as in our theoretical model, $\delta = 1$. Hence, using the continuity and monotonicity of χ, we have that for all $\lambda < \widehat{\lambda}$, it must be the case that $\chi(\lambda) < 0$, or $c_1^* < \delta E c_2^*$ (precautionary savings), and for all $\lambda > \widehat{\lambda}$, it must be the case that $\chi(\lambda) > 0$, or $c_1^* > \delta E c_2^*$ (reckless undersaving). ■

Proof of Proposition 3: (a) Using (5.1) we can write the first order condition in (3.7) for an interior solution as (recall from (5.2) that $p = 1 - p = 0.5$).

$$\frac{\partial U}{\partial s} = \left[-u'(y - s^*) - \mu\lambda + \mu\left(p\lambda + (1-p)\right)\right] + p\left[u'(s^* - \theta\varepsilon_h) + u'(s^* + \theta\varepsilon_h)\right] = 0.$$

Given our assumptions, the optimal savings function is continuously differentiable, hence, using the implicit function theorem, we get

$$\frac{\partial s^*}{\partial \theta} = \left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1} p\varepsilon_h\left[-u''(s^* - \theta\varepsilon_h) + u''(s^* + \theta\varepsilon_h)\right]. \tag{8.4}$$

The sign of (8.4) is determined by the sign of the term in the square brackets on the RHS that is, in general, indeterminate. Using a first order Taylor series approximation

$$u''(s^* + \theta\varepsilon_h) \approx u''(s^* - \theta\varepsilon_h) + u'''(s^* - \theta\varepsilon_h)\left[(s^* + \theta\varepsilon_h) - (s^* - \theta\varepsilon_h)\right],$$

or

$$u''(s^* + \theta\varepsilon_h) - u''(s^* - \theta\varepsilon_h) \approx 2\theta\varepsilon_h u'''(s^* - \theta\varepsilon_h).$$

Thus, if $u''' > 0$, then $u''(s^* - \theta\varepsilon_h) < u''(s^* + \theta\varepsilon_h)$, which implies that the term in the square brackets in (8.4) is positive, so $\frac{\partial s^*}{\partial \theta} > 0$. Otherwise, if $u''' < 0$, then we have $\frac{\partial s^*}{\partial \theta} < 0$.

(b) The RHS of (8.4) is independent of the parameter of loss aversion, λ. Hence, $\frac{\partial}{\partial \lambda}\left(\frac{\partial s^*}{\partial \theta}\right) = 0$.

■

Proof of Proposition 4: Using (5.3) and (5.1), (5.2) so that $p = 1 - p$, the relevant first order condition (the analogue of the first order condition for Köszegi-Rabin preferences in (3.7)) is

$$\begin{aligned}\frac{\partial \widetilde{U}}{\partial s} &= -u'(y - s) - \mu\lambda\beta\,(s)^{\beta-1} + p\left[u'(s - \theta\varepsilon_h) + u'(s + \theta\varepsilon_h)\right] \\ &\quad + \mu p\left[\beta\left(s + \theta\varepsilon_h - \omega_2\right)^{\beta-1} + \lambda\beta\left(\omega_2 - (s - \theta\varepsilon_h)\right)^{\beta-1}\right] = 0.\end{aligned} \tag{8.5}$$

Suppose that the second order condition holds, so that at the optimal solution, $\frac{\partial^2 \widetilde{U}}{\partial s^2} < 0$.[34] Under the given assumptions, $\widetilde{s}$ is continuously differentiable. Hence, using the implicit function theorem, we get

$$\frac{\partial \widetilde{s}}{\partial \theta} = \left[p\varepsilon_h \left[-u''(s - \theta\varepsilon_h) + u''(s + \theta\varepsilon_h) \right] + \mu p \beta (\beta - 1) \varepsilon_h \left[(s + \theta\varepsilon_h - \omega_2)^{\beta-2} + \lambda (\omega_2 - (s - \theta\varepsilon_h))^{\beta-2} \right] \right] \times \left(-\frac{\partial^2 \widetilde{U}}{\partial s^2} \right)^{-1} \gtreqless 0. \tag{8.6}$$

In general, we cannot sign this expression. But if $\mu \to 0$ then the second term in the square brackets, corresponding to gain-loss utility, disappears and a simple adaption of the proof of Proposition 3(a) shows that $\widetilde{s}$ is increasing in θ if $u''' > 0$.

(b) Differentiating (**??**) with respect to the parameter of loss aversion, λ, we get

$$\frac{\partial}{\partial \lambda} \left(\frac{\partial \widetilde{s}}{\partial \theta} \right) = \left(-\frac{\partial^2 \widetilde{U}}{\partial s^2} \right)^{-1} \mu p \varepsilon_h \beta (\beta - 1) (\omega_2 - (s - \theta\varepsilon_h))^{\beta-2} < 0. \because \beta < 1. \blacksquare \tag{8.7}$$

Proof of Proposition 5: (a) Using the budget constraints, (2.1), (2.2), we can rewrite (3.7),

$$E\left[u'(c_2)\right] = u'(c_1) + \mu\lambda - \mu (p\lambda + (1 - p)). \tag{8.8}$$

If $u'''(x) > 0$, then u' is a strictly convex function, thus, it follows from Jensen's inequality that

$$u'(Ec_2) < E\left[u'(c_2)\right]. \tag{8.9}$$

We have $\mu\lambda - \mu (p\lambda + (1 - p)) = \mu(1 - p)(\lambda - 1)$. For loss tolerant subjects $\lambda < 1$. In this case we have from (8.8), (8.9) that

$$u'(Ec_2) < u'(c_1) + \mu(1 - p)(\lambda - 1) < u'(c_1). \tag{8.10}$$

We have $u'' < 0$. Hence, from (8.10), we get $c_1 < Ec_2$, which implies that a loss tolerant decision maker engages in precautionary savings (see Definition 1).

(b) If the decision maker is loss averse, $\lambda > 1$, then from (8.10) $c_1 \gtreqless Ec_2$, all three outcomes are possible: certainty equivalence, precautionary savings, and reckless undersaving. $\blacksquare$

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[34] We have by direct differentiation of (8.5)

$$\frac{\partial^2 \widetilde{U}}{\partial s^2} = u''(y - s) + Eu''(s + z) + \mu\beta(\beta - 1) \left[-\lambda (s)^{\beta-2} + (1 - p)(s + \theta\varepsilon_h - \omega_2)^{\beta-2} - p\lambda (\omega_2 - (s - \theta\varepsilon_h))^{\beta-2} \right].$$

The first two terms are negative but in the third term, that captures gain-loss utility, there are both positive and negative terms. Thus, a-priori the second order condition is not guaranteed to hold.

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