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Vasiliki Dimakopoulou, George Economides, Apostolis Philippopoulos, Vanghelis Vassilatos

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Abstract

We investigate what happens when the fiscal authorities do not react to rising public debt so that the unpleasant task of fiscal sustainability falls upon the Central Bank (CB). In particular, we explore whether the CB's bond purchases in the secondary market can restore stability and determinacy in an otherwise unstable model. This is investigated in a DSGE model calibrated to the Euro Area (EA) and where monetary policy is conducted subject to the numerical rules of the Eurosystem (ES). We show that given the recent situation in the ES, and to the extent that a relatively big shock hits the economy and fiscal policy remains active, there is no room left for further quasi-fiscal actions by the ECB; there will be room only if the ES' rules are violated. We then search for policy mixes that can respect the ES's rules and show that debt-contingent fiscal and quantitative monetary policies can reinforce each other; this confirms the importance of policy complementarities. On the negative side, bond purchases by the CB worsen income inequality and the unavoidable reversal, in the form of QT, will come at a real cost.

JEL-Codes: E500, E600, O500.

Keywords: central banking, fiscal policy, debt stabilization, Euro Area.

Vasiliki Dimakopoulou
Athens University of Economics and Business
Athens / Greece
dimakokv@aueb.gr

George Economides
Athens University of Economics and Business
Athens / Greece
gecon@aueb.gr

*Apostolis Philippopoulos**
Athens University of Economics and Business
Athens / Greece
aphil@aueb.gr

Vanghelis Vassilatos
Athens University of Economics and Business
Athens / Greece
vvassila@aueb.gr

*corresponding author

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1 Introduction

In the rich literature on the interaction between fiscal and monetary policies,[1] the conventional policy assignment is the one in which fiscal policy ensures public debt sustainability (typically meaning that tax-spending policy instruments respond to public debt imbalances), while monetary policy controls inflation (typically meaning that the Central Bank sets its policy nominal interest rates as a function of inflation according to a Taylor rule). This policy assignment, also known as passive fiscal policy and active monetary policy (Leeper, 1991), usually delivers macroeconomic stability and determinacy (see e.g. Leeper, 2022, for a recent review).

However, in practice, for a variety of reasons (political factors, being in a recession, etc.), fiscal authorities may not be willing, or able, to reduce public spending or raise tax rates as a reaction to rising public debt, so that monetary policy can be called upon to play a more direct fiscal role. Actually, this has been the case most of the time since the eruption of the global financial crisis in 2008. Since 2008, most central banks have been employing quantitative policies like large-scale purchases of government bonds in the secondary market (known as quantitative easing, QE) financed mainly by the issuance of interest-bearing reserves held by private banks at the Central Bank. As a result, at the end of 2022, the Eurosystem's cumulated net holdings of sovereign bonds were 31.8% of total public debt, while this share was negligible before 2008. At the same time, the Euro Area's total public debt, as share of its total GDP, has increased from 69% in 2008 to 92% in 2022, while, in most member-countries, there is no evidence of systematic stabilizing response of national fiscal policies to rising public debt.[2] All this seems to imply that, practically, it is quantitative monetary policies that have been contingent on the situation of public finances. This resembles what Leeper (1991) has described as active fiscal policy and passive monetary policy or what is known as fiscal dominance.[3]

[1] See e.g. Leeper (1991, 2021, 2023), Leith and Wren-Lewis (2008), Kirsanova et al (2009), Leeper et al (2010), Davig et al (2010), Davig and Leeper (2011), Canzoneri et al (2011), Canova and Pappa (2011), Reis (2013, 2017), Bassetto and Messer (2013), Del Negro and Sims (2015), Benigno and Nisticò (2020), Sims and Wu (2020, 2021), Bernanke (2020), Bassetto and Sargent (2020), Bianchi et al (2021), Buiter (2021), Chadha et al (2021), Hall and Sargent (2022), Kurovskiy et al (2022), Hooley et al (2023), etc. See below for details and how our work differs.

[2] According to our calculations, during 2001-2022, the correlation coefficient between current public debt as share of GDP and next period's primary fiscal surplus as share of GDP has been -0.20 in the EA as a whole; that is, fiscal policy has been destabilizing in the sense that primary surpluses fall, or primary deficits rise, in response to higher inherited debt. As expected, this average masks big differences across member countries. This coefficient is -0.55 in Italy, -0.44 in France and -0.21 in Spain. By contrast, it is 0.39, namely stabilizing, in Germany. The data are from Eurostat. Details are available upon request.

[3] For a review of the early literature, see e.g. Walsh (2017, chapter 4). See also e.g.

In this paper, we investigate what happens when the fiscal authorities do not react to rising public debt so that the politically unpleasant task of debt stabilization and fiscal sustainability falls upon the Central Bank (CB). We explore the possibility that the CB's bond purchases in the secondary market can guarantee stability and determinacy in an otherwise dynamically unstable environment where the path of public debt would have been explosive. That is, we study whether quantitative monetary policies can be a substitute for tax-spending public debt stabilization policies and, if they can, under what circumstances. We do so in a DSGE model calibrated to the Euro Area (EA).[4]

We deliberately employ a standard general equilibrium model. The private sector consists of households, firms and banks. Households are all Ricardian (at least in the base model). Firms are modeled as in the New Keynesian literature and in addition face a financial constraint when they borrow from private banks. Private banks are modeled as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), which means that there is an extra financial friction in the form of moral hazard in banking.[5] Regarding the policy sector, following a big part of the literature since 2008, we treat the Treasury and the CB as different policy entities with separate budget constraints. The Treasury finances its spending by various taxes, a transfer from the CB, and by issuing bonds purchased by private banks in the primary market. The CB's balance sheet includes the main items in the consolidated financial statements of the ES and its monetary policy instruments include the policy nominal interest rates on reserves held by private banks and on loans to private banks, the transfer to the Treasury and the fraction of outstanding government bonds purchased from private banks in the secondary market.[6]

Reis (2017), Bassetto and Sargent (2020), Buiter (2021), Hall and Sargent (2022), Leeper (2023) and Hooley et al (2023) for the role of central banks as quasi-fiscal actors.

[4]For the ECB's policies, see e.g. Hartmann and Smets (2018), Rostagno et al (2019, 2021), Coenen et al (2020), Fabiani et al (2021), Havlik and Heinemann (2020), Benigno et al (2022), Belhocine et al (2023), etc. Papers with DSGE models for the EA include Smets and Wouters (2003), Angelini et al (2019), Hohberger et al (2019a, 2019b), Darracq Paries et al (2019), Coenen et al (2020), Rostagno et al (2021), Bankowski et al (2021), Hauptmeier et al (2022), Kabaca et al (2023), Mackowiak and Schmidt (2023), Bankowski (2023), Gomes and Seoane (2023), etc. See below for details and how our work differs.

[5]As is known, quantitative monetary policies, and QE in particular, can have real effects if there are financial frictions that overturn Wallace's (1981) irrelevance proposition. The model by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) is one of the main devices that open the door for such effects. Note that Villa (2013) augmented the Smets and Wouters (2003) model with the Gertler-Kiyotaki-Karadi model and found it was empirically relevant for the EA. Hence, here we use this model. Another popular device is the model of Cúrdia and Woodford (2011) that employs real transaction costs associated with financial items. See e.g. Beck et al (2014) and Walsh (2017, chapter 11.5) for reviews of this literature.

[6]Subsection 2.8 below summarizes how monetary (interest rate and quantitative) policies are transmitted to the real economy in our model.

Policy is conducted by "simple and implementable" feedback rules (see Schmitt-Grohè and Uribe (2007)) according to which the independently set policy instruments can react to a small number of indicators. In particular, regarding fiscal policy, tax-spending instruments can react to public debt, while, regarding monetary policy, policy interest rates can react to inflation á la Taylor and the CB's sovereign bond purchases are allowed to react to public debt similarly to fiscal instruments. Thus, we study whether, and under what circumstances, debt-contingent fiscal and QE monetary policies can be substitutable in terms of public debt stability and determinacy. We start by studying one instrument at a time so as to search which one can restore stability and determinacy in an otherwise unstable system and, if it can, at what cost. In addition, to mimic the conduct of monetary policy in the Eurosystem (ES), we restrict the fraction of sovereign bonds in the hands of the CB not to exceed an upper limit and we also exclude the possibility of transfers from the fiscal authorities to the CB. Regarding the upper limit, our base simulations assume 50% (a loose upper limit like this does not do our results any favors).[7] Regarding the transfer, we assume that it cannot be negative because fiscal support of the ECB is, in practice, very difficult in the ES.[8]

We solve the model by using parameter values that match specific characteristics of the EA over 2002-2022 and by setting the policy (fiscal and monetary) instruments at their recent values in the year 2022. Then, departing from 2022, we shock the model by assuming, for example, an adverse supply shock that is big enough to generate a recession and, at the same time, a rise in the public debt to GDP ratio in the short term (although we have experimented with various shocks and the main results do not change). We start by switching on Taylor type rules only, according to which the policy interest rates react to inflation. This macroeconomy, if left on its own, is dynamically unstable because of explosive public debt. Thus, a policy instrument needs to react to outstanding public debt to restore stability. We then experiment with different policy instruments, both fiscal and monetary, by setting the associated feedback policy coefficients on debt at the minimum value required for stability in each experiment studied.

[7] We choose 50% because the ES's upper issuer limit is not well defined at least in practice. Although the official PSPP limit of 33% is still in place, bond purchases under PEPP have not counted as part of 33%. At the same time, there is a 50% upper limit that applies to entities listed as "supranationals located in the EA" like the ESM, the EIB, etc. We therefore choose 50% which seems to be the highest possible value at the moment.

[8] A negative transfer is interpreted as "fiscal support", or "loss of independence", or even "insolvency" of the CB by various authors (see e.g. Del Negro and Sims (2015) and Reis (2017)). Focusing on the ES, as Reis (2017) argues, the charters of the ECB have no explicit allowance for fiscal support of the ECB so that, for the ECB to receive a transfer from the national fiscal authorities, an agreement by all member-countries is needed which is politically difficult. For details, see the "Protocol of the statute of the ES of CBs and of the ECB" available at the site of the ECB.

To put our work in the context of the literature, we start with something standard. We show that public debt, and hence macroeconomic, stability can be restored when at least one fiscal (tax-spending) instrument reacts to the public debt gap. This is in accordance with the conventional policy regime mentioned in the opening paragraph above, in the sense that fiscal policy instruments are assigned to debt stabilization and at least one of the policy interest rates is assigned to inflation stabilization. Noticeably, under this regime, bond purchases by the CB and the transfer from the CB to the government respect the numerical rules of the ES mentioned above.

We then move on the main part of the paper. We now switch off debt stabilization through fiscal policy (i.e. fiscal policy is now "active" in the Leeper sense) and investigate whether quantitative monetary policy in the form of debt-contingent sovereign bond purchases can do the unpleasant job and thereby free the hands of the Treasury. Our simulations show that, given the current situation in the ES, this can be done only if both rules of the ES are violated; in particular, only if the fraction of bonds in the hands of the CB is unrestricted and if fiscal support of the CB is possible at least in some periods. Robustness checks, on the other hand, show that this is not a generic result. In particular, and as is perhaps expected, if the fraction of sovereign bonds held initially by the CB were less than in the current situation (as said, it was 31.8% in 2022),[9] or if we were willing to make the counter-factual - but popular in the academic literature - assumption that the CB participated in the primary sovereign bond market, then it would be possible for the CB to stabilize the economy on its own via debt-contingent QE and still respect the rules of the ES. The same applies if the shock that triggers dynamics is relatively small, although even a small shock can result in violation of the ES's rules under QE-type stabilization when the upper limit on bond holdings by the ES is set at lower, more realistic, values than 50%.

It is therefore fair to claim that, given the current situation in the ES, and to the extent that relatively big shocks keep hitting the European economy (like the ongoing cost of living crisis, the war in Ukraine, natural disasters, etc) and that fiscal policy remains unresponsive to public debt imbalances, there is no room left for further quasi-fiscal actions by the ECB under the self-imposed rules of the ES. And, as our results show, all this holds under a loose upper limit of sovereign bond holdings (i.e. 50%). But, if this is the case, a natural question arises. If, in practice, we do not observe any systematic fiscal reaction to debt imbalances and, at the same time QE cannot restore debt stability without violating the rules of the ES, quoting Leeper et al (2010) in their study for the US economy, it is natural to ask ourselves

[9] This can contribute to explaining why the ECB's QE policy was successful in the aftermath of the global financial crisis in the previous decade; at that time, the ECB had much more space to manoeuvre the economy.

"Why do forward-looking agents continue to purchase bonds with relatively low interest rates and bond prices don't plummet?". A possible answer to this - to the extent that we want to maintain the assumption of rationality - is that agents believe that current fiscal inaction is temporary only and it will be replaced by necessary corrections in the future. We therefore simulate an extra scenario with policy mixes where fiscal corrections start after, say, 10 years from now and this is complemented by mild debt-contingent QE policy. Now, our simulations show that such a joint use of policies can restore stability and this can happen respecting the rules of the ES. In other words, if there is the announcement to credibly implement fiscal corrections in the near future, there is room for complementarities. Debt-contingent fiscal and QE monetary policies can create space for each other.

We will close the paper by addressing some common concerns about the use of QE as a means of public debt stabilization. For instance, by adding a second type of households, we show that QE is accompanied by worse income inequality relative to other instruments used for the same policy task. We also show that the unavoidable reversal of QE, in the form of QT, will come at a real cost over time.

Literature and how we differ As already said, there is a rich literature on the nexus between fiscal and monetary policies in general and the same applies to papers on the EA in particular. Within this literature, since 2008, many authors have built DSGE models with financial frictions to study the effects of quantitative monetary policies on the macroeconomy. Such applications to the EA include e.g. Hohberger et al (2019a, 2019b), Darracq Paries et al (2019), Kabaca et al (2023) and Mackowiak and Schmidt (2023).[10] Our paper differs mainly because here we address a different issue: we investigate the role of such policies as a means of public debt stabilization and we do so within the institutional framework of the ES.[11] In particular, building a model that includes the main items in the consolidated financial statements of the ES, we show that the implications of sovereign bond purchases by the CB in the secondary market, and their ability to restore stability and determinacy, depend crucially on the institutional restrictions under which monetary policy is conducted and, specifically, on whether there are upper limits to the fraction of bonds that the CB can hold and whether a fiscal support from the Treasury to the CB is possible in case of need. Another difference between our paper and most of the above

[10] By contrast, focusing on DSGE papers on the EA, Villa (2013), Angelini et al (2019), Bankowski et al (2021), Hauptmeier et al (2022), Gomes and Seoane (2023), etc, include interest rate policy only.

[11] Kurovskiy et al (2022) address a similar issue in a model for the US. However, they assume that the CB purchases sovereign bonds in the primary market, and, as we show, this matters to the results. In addition, since they calibrate their model to the US economy, naturally, they do not investigate whether and when extra numerical rules like those of the ES are violated when the CB exercises its QE type policies.

literature is that the latter assumes that the CB purchases sovereign bonds in the primary market; as we show, this counter-factual assumption makes a difference.

The rest of the paper is as follows. Section 2 presents the model. Parameterization and solution for the year 2022 are in section 3. Sections 4 and 5 present simulation results when we depart from the year 2022. Section 6 closes the paper. An Appendix includes algebraic details.

2 Model

This section constructs a medium-scale micro-founded macroeconomic model that embeds most of the macroeconomic policies observed in practice in the EA. We will start with an informal description of the model.

2.1 Informal description of the model

Households We start with a single type. These households consume, work, hold currency and keep deposits at private banks. They also own the private firms and banks and so receive their profits. Households are modeled in subsection 2.2.

Private firms A single final good is produced by final good firms which act competitively using differentiated intermediate goods as inputs. The latter are produced by intermediate goods firms which act monopolistically à la Dixit-Stiglitz and face nominal rigidities à la Rotemberg as in the New Keynesian literature. Intermediate goods firms choose labor and capital and also make use of productivity-enhancing public goods. On the financial side, these firms can borrow from private banks subject to a working capital constraint. Firms are modeled in subsection 2.3.

Private banks On their asset side, private banks make loans to private firms, hold interest-bearing reserves at the CB and purchase government bonds in the primary market. On the side of liabilities, they receive deposits from households and loans from the CB. Also, when we allow for a secondary market for government bonds, private banks can sell to the CB a fraction of the bonds they have previously purchased in the primary market. This asset-liability mix is embedded into the banking model of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013). Private banks are modeled in subsection 2.4 in the case in which both private banks and the CB participate in the primary sovereign bond market and in subsection 2.7 in the main case in which it is only private banks that participate in this market.

Treasury On the revenue side, the Treasury, or the government, taxes households' income and consumption as well as firms' and banks's profits, receives a transfer from the CB and issues bonds. On the expenditure side, the Treasury spends on public investment, public consumption and income transfers to households. The Treasury is modeled in subsection 2.5.

Central Bank On the side of assets, the CB makes loans to private banks and holds government bonds. On the side of liabilities, its monetary base consists of banknotes and interest-bearing reserves. Given this balance sheet, the policy instruments of the CB are the nominal interest rate on reserves held by private banks at the CB, the nominal interest rate on loans to private banks, the transfer to the government, as well as the fraction of new sovereign bonds purchased in the primary market, or alternatively the fraction of outstanding sovereign bonds purchased from private banks in the secondary market. The CB is modeled in subsection 2.6 in the counterfactual case in which it participates in the primary sovereign bond market and in subsection 2.7 in the case in which this happens in the secondary market.

2.2 Households

There are N identical households indexed by subscript $h = 1, 2, ..., N$. Each h maximizes discounted lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{h,t}, u_{h,t}; g_t^c\right) \tag{1}$$

where $c_{h,t}$ and $u_{h,t}$ are respectively h's consumption and leisure, g_t^c is per capita public spending on utility-enhancing goods provided by the government and $0 < \beta < 1$ is households' time discount factor.

For our numerical solutions, we will use a simple log-linear function (taking calibration into account, our results do not depend on the functional form used) :

$$u\left(c_{h,t}, u_{h,t}; g_t^c\right) = \mu_1 \log c_{h,t} + \mu_2 \log u_{h,t} + \mu_3 \log g_t^c$$

where $0 < \mu_1$, μ_2, $\mu_3 < 1$ are preference parameters with $\mu_1 + \mu_2 + \mu_3 = 1$.

At each t, the time constraint of each h is:

$$l_{h,t} + u_{h,t} \equiv 1 \tag{2a}$$

where $l_{h,t}$ is work hours.

The period budget constraint of each h written in real terms is:

$$(1 + \tau_t^c)c_{h,t} + j_{h,t} + m_{h,t} \equiv$$

$$\equiv (1 - \tau_t^y)w_t l_{h,t} + (1 + i_t^d)\frac{p_{t-1}}{p_t} j_{h,t-1} + \frac{p_{t-1}}{p_t} m_{h,t-1} + \pi_{h,t} + g_t^t \tag{2b}$$

where $j_{h,t}$ is the real value of end-of-period bank deposits earning a nominal interest rate i_{t+1}^d in the next period, $m_{h,t}$ is the real value of end-of-period currency carried over from t to $t+1$, w_t is the real wage rate, p_t is the price level of the final good, $\pi_{h,t}$ are net funds transferred from firms and banks to the household, g_t^t is a transfer from the government, and $0 \leq \tau_t^c$, $\tau_t^y < 1$ are tax rates on consumption and labor income income respectively.

To give a role to currency, we assume a cash-in-advance constraint like:

$$m_{h,t} \geq (1+\tau_t^c) c_{h,t} \tag{2c}$$

Each household h chooses $\{c_{h,t}, l_{h,t}, u_{h,t}, j_{h,t}, m_{h,t}\}_{t=0}^{\infty}$ to maximize (1) subject to (2a-c). The first-order conditions are in Appendix A.1.

2.3 Private firms and production

Firms are modeled as in the New Keynesian literature. That is, there is a single final good produced by competitive final good firms which use differentiated intermediate goods as inputs à la Dixit-Stiglitz. Then, each differentiated intermediate good is produced by an intermediate good firm that acts as a monopolist in its own product market facing Rotemberg-type nominal price fixities[12] and a financial constraint.

2.3.1 Final good firms

There are N identical final good firms indexed by subscript $f = 1, 2, ..., N$. Each f produces $y_{f,t}$ by using intermediate goods according to the Dixit-Stiglitz aggregator:

$$y_{f,t} = \left[\sum_{i=1}^{N} \frac{1}{N^{1-\theta}} (y_{f,i,t})^{\theta} \right]^{\frac{1}{\theta}} \tag{3}$$

where $y_{f,i,t}$ is the quantity of intermediate good of variety $i = 1, 2, ..., N$ used by each firm f and $0 \leq \theta \leq 1$ is a parameter, where $1/(1-\theta)$ measures the degree of substitutability between intermediate goods. Note that we use $\frac{1}{N^{1-\theta}}$ to avoid scale effects in equilibrium (for similar modelling, see e.g. Blanchard and Giavazzi (2003)).[13]

The firm's real profit is:

$$\pi_{f,t} \equiv y_{f,t} - \sum_{i=1}^{N} \frac{p_{i,t}}{p_t} y_{f,i,t} \tag{4}$$

where $p_{i,t}$ is the price of each intermediate good i.

Each f chooses $y_{f,i,t}$ to maximize (4) subject to (3). The familiar first-order condition is in Appendix A.2.

[12] Using Calvo-type nominal rigidities would not affect our main results.

[13] That is, since $y_{f,i,t} = \frac{y_{i,t}}{N}$, where $y_{i,t}$ is the output of each intermediate good firm i, in a symmetric equilibrium we will simply have $y_{f,t} = y_{i,t}$.

2.3.2 Intermediate goods firms

There are N differentiated intermediate goods firms indexed by the subscript $i = 1, 2, ..., N$. These firms own the stock of capital, make investment and other factor decisions, and face Rotemberg-type price adjustment costs. New investment is financed by retained earnings and loans from private banks where these loans are subject to a working capital constraint.

Firm i's real net dividend, $\pi_{i,t}$, is defined as:

$$\pi_{i,t} \equiv (1-\tau_t^{\pi})\left(\frac{p_{i,t}}{p_t}y_{i,t} - w_t l_{i,t}\right) - x_{i,t} -$$

$$-\frac{\xi^p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 \overline{y}_{i,t} + \left(L_{i,t} - \left(1+i_t^l\right)\frac{p_{t-1}}{p_t}L_{i,t-1}\right) \tag{5}$$

where $l_{i,t}$ is units of labor input used by firm i, $x_{i,t}$ is i's investment in capital goods, $L_{i,t}$ is the real value of end-of-period loans received from private banks on which the firm pays a nominal interest rate, i_{t+1}^l, in the next period, $0 \leq \tau_t^{\pi} < 1$ is the tax rate on gross profits, $\xi^p \geq 0$ is a parameter measuring Rotemberg-type price adjustment costs, and $\overline{y}_{i,t}$ is average output.[14]

The law of motion of each i's physical capital stock, $k_{i,t}$, is (without capital adjustment costs, the relative price of capital will be 1):

$$k_{i,t} = (1-\delta)\, k_{i,t-1} + x_{i,t} \tag{6}$$

where $0 \leq \delta \leq 1$ is the capital depreciation rate.

For the firm's production function, we adopt the form: :

$$y_{i,t} = A_t (k_{i,t-1}^{\alpha} l_{i,t}^{1-\alpha})^{1-\varepsilon} \left(k_{t-1}^g\right)^{\varepsilon} \tag{7}$$

where k_{t-1}^g is per firm public infrastructure capital, $0 < a < 1$ and $0 \leq \varepsilon < 1$ are technology parameters, and A_t obeys an $AR(1)$ process defined below.

These firms are subject to a working capital constraint.[15] That is, they have to finance a fraction of their payments to labor with loans from private banks:

$$L_{i,t} \geq \eta_i w_t l_{i,t} \tag{8}$$

where the parameter $\eta_i \geq 0$ measures the tightness of borrowing conditions faced by firms.

[14] Thus, Rotemberg-type costs associated with price changes are assumed to be proportional to average output, $\overline{y}_{i,t}$, which is taken as given by each i. This is not important but helps in producing smooth dynamics.

[15] See also e.g. Walsh (2017, section 5.3) and Uribe and Schmitt-Grohé (2017, section 6.4). That is, the idea is that firms pay wages before selling their product. Note that we could assume different types of financial constraints, like collateral borrowing constraints as in e.g. Gertler and Karadi (2011) and Sims and Wu (2021); we report that our main results do not depend on this.

Each firm i maximizes the discounted sum of net-of-tax dividends distributed to households:

$$\sum_{j=0}^{\infty} \beta_{t,t+j} \pi_{i,t+j} \tag{9}$$

where, since firms are owned by households, we will ex post postulate that $\beta_{t,t+j}$ equals households' marginal rate of substitution between consumption at t and $t+j$. That is, $\beta_{t,t} \equiv 1$, $\beta_{t,t+j} \equiv \beta^{j} \frac{\lambda_{h,t+j}}{\lambda_{h,t}}$, etc., where $\lambda_{h,t}$ is the Lagrange multiplier associated with households' budget constraint above.

Each firm i chooses $\{l_{i,t},\ k_{i,t},\ L_{i,t}\}_{t=0}^{\infty}$ to maximize (9) subject to (5), (6)-(8) and the demand function for its product coming from the final good firm's optimization problem (see Appendix A.2 for the latter). The first-order conditions are in Appendix A.3.

2.4 Private banks

There are N identical private banks indexed by the subscript $p = 1, 2, ..., N$. In addition to their standard role, which is the provision of intermediation by converting deposits by households into loans to firms, we allow private banks to hold interest-bearing reserves at the CB, to purchase government bonds and to borrow from the CB. In other words, on the asset side of banks, we have loans to firms, reserves and government bonds, while, on the liability side, we have deposits obtained from households and loans taken from the CB. Hence, each private bank p enters period t with predetermined assets in the form of reserves, $m_{p,t-1}$, government bonds, $b_{p,t-1}$, and loans to firms, $L_{p,t-1}$, as well as with preexisting obligations in the form of deposits from households, $j_{p,t-1}$, and loans from the CB, $z_{p,t-1}$.

For expositional convenience, we start with the counter-factual case in which both private banks and the CB participate in the primary market for sovereign bonds; the case in which only private banks participate in the primary market, while the CB purchases sovereign bonds from private banks in the secondary market, is modeled in subsection 2.7 below.

This financial mix is embedded into the popular banking model of Kiyotaki-Gertler-Karadi. Omitting details, this means the following.

The balance sheet of each private bank p at the end of t is:

$$L_{p,t} + b_{p,t} + m_{p,t} \equiv j_{p,t} + z_{p,t} + n_{p,t} \tag{10}$$

where $n_{p,t}$ is the bank's after-tax net worth defined as:

$$n_{p,t} \equiv \frac{p_{t-1}}{p_t} \{[1 + (1 - \tau_t^{\pi})\, i_t^{l}] L_{p,t-1} + [1 + (1 - \tau_t^{\pi})\, i_t^{b}] b_{p,t-1} + \tag{11}$$

$$+ [1 + (1 - \tau_t^{\pi})\, i_t^{r}] m_{p,t-1} - [1 + (1 - \tau_t^{\pi})\, i_t^{d}] j_{p,t-1} - [1 + (1 - \tau_t^{\pi})\, i_t^{z}] z_{p,t-1}\}$$

As in the above papers, it is assumed that after period t there is a probability $(1-\sigma)$ that a banker will exit the sector at $t+1$ transferring his/her wealth to households and, at the same time, the same fraction of households will enter the banking sector transferring their money to this sector. Given that the bank pays dividends only when it exits, its objective at the end of t is to maximize its value, $V_{p,t}$, which is equal to the present discounted value of future dividends:

$$V_{p,t} \equiv \max \sum_{j=1}^{\infty} (1-\sigma)\, \sigma^{j-1} \beta_{t,t+j} n_{p,t+j} \tag{12}$$

where the discount factor $\beta_{t,t+j}$ has been defined above.

Also, again as in the above papers, it is assumed that banks can divert a fraction, $0 \leq \vartheta \leq 1$ of their "divertable" net assets to their owners, the households, and, hence, may go bankrupt. Given this possibility, for the bank to keep operating, its value, $V_{p,t}$, has to be equal to, or greater than, the amount it can divert. Hence, the bank faces the incentive constraint at each t:

$$V_{p,t} \geq \vartheta(L_{p,t} + N^b b_{p,t} + N^m m_{p,t} - N^z z_{p,t}) \tag{13}$$

where N^b, N^z, N^m are parameters associated respectively with the bank's loans to firms, bond holdings, reserves at the CB and loans obtained from the CB, so as to capture the idea that the ease of diverting different types of assets and liabilities differs across them; typically in this literature, $0 \leq N^b \leq 1$ meaning that it is easier to divert private loans than sovereign bonds. As the first-order conditions can show, these parameters drive interest rate spreads or asset pricing wedges and will therefore be calibrated to give interest rate differentials as in the data.

Before we move on, it is worth reminding the implications of the incentive constraint (13).[16] When binding, this constraint opens the door through which QE type policies by the CB can affect the credit policies of private banks. In particular, an increase in the CB's holdings of sovereign bonds, when translated to lower bond holdings by private banks, $b_{p,t}$, can - other things equal - raise the supply of loans to firms, $L_{p,t}$, and this can in turn ease the loan constraint faced by production firms. Note however that this is "other things equal"; here, the incentive constraint includes other items like reserves held at the CB, $m_{p,t}$, and loans obtained from the CB, $z_{p,t}$, so that, even when $b_{p,t}$ falls, this may not necessarily raise $L_{p,t}$. We believe this can contribute to explaining why, while the CB's balance sheet policies can affect interest rates, their general equilibrium effects on the real economy may be weak (more on this later).

Each p chooses $\{L_{p,t}, b_{p,t}, m_{p,t}, z_{p,t}\}_{t=0}^{\infty}$ to solve the above problem. The solution of the banks' problem in the case in which the CB also participates

[16] See e.g. Walsh (2017, chapter 11.5) and Sims and Wu (2021) for detailed discussions.

in the primary bond market is in Appendix A.4, while, the factual case in which only private banks do so is presented in subsection 2.7 below and solved in Appendix A.5.

2.5 The Treasury and fiscal policy instruments

The Treasury, or the fiscal branch of government, uses revenues from various taxes, the issuance of new bonds and a direct transfer from the CB to finance its spending activities. Its flow budget constraint written in per capita and real terms is:

$$g_t^c + g_t^g + g_t^t + (1 + i_t^b)\frac{p_{t-1}}{p_t}b_{t-1} \equiv b_t + \eta_t + \frac{T_t}{N} \tag{14}$$

where g_t^c, g_t^g and g_t^t are spending on public consumption, public investment and transfer payments respectively, b_t is the end-of-period total public debt, η_t is a transfer from the CB to the Treasury,[17] and $\frac{T_t}{N}$ is per capita and real tax revenues defined as:

$$\frac{T_t}{N} \equiv \tau_t^c c_{h,t} + \tau_t^y w_t l_{h,t} + \tau_t^\pi (y_{i,t} - w_t l_{i,t}) + \tag{15}$$

$$+\tau_t^\pi \frac{p_{t-1}}{p_t}(i_t^l L_{p,t-1} + i_t^r m_{p,t-1} + i_t^b b_{p,t-1} - i_t^z z_{p,t-1} - i_t^d j_{p,t-1})$$

Public investment, g_t^g, augments public capital whose motion is:

$$k_t^g = (1 - \delta^g)k_{t-1}^g + g_t^g \tag{16}$$

where $0 \leq \delta^g \leq 1$ is the public capital depreciation rate.

In our solutions below, to be closer to the data, instead of working with the levels of public spending, we will work with their GDP shares, s_t^c, s_t^g, s_t^t, where $g_t^c = s_t^c y_{f,t}$, $g_t^g = s_t^g y_{f,t}$ and $g_t^t = s_t^t y_{f,t}$. One of the fiscal variables must follow residually to close the Treasury's budget constraint; along the transition path, we will assume that this role is played by the end-of-period public debt, b_t, so that the rest of the fiscal policy variables, $x_t \equiv (s_t^c$, s_t^g, s_t^t, τ_t^c, τ_t^y, $\tau_t^\pi)$, can be set independently.

Following most of the related literature,[18] we will allow the independently set fiscal policy instruments, x_t, to follow feedback, or state-contingent, simple rules according to which, in addition to an exogenous $AR(1)$ component, they can also react to the beginning-of-period public debt to GDP ratio as a deviation from its steady state value. As is well recognized and as we will shall see below, this can restore dynamic stability. Thus,

[17] As pointed out by Reis (2017), the charters of the ECB state that it must rebate its net profit to the national CBs of the ES every year and most of them, in turn, are required by national law to send them as dividends to their respective fiscal authorities.

[18] For a recent paper that also reviews the literature on state-contingent fiscal rules, see e.g. Malley and Philippopoulos (2022).

$$x_t = \rho^x x_{t-1} + (1-\rho^x)\, x + \gamma^{x,b}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) \tag{17}$$

where $\gamma^{x,b}$ s are feedback policy coefficients, $0 \leq \rho^x \leq 1$ are persistence parameters, and variables without time subscripts denote steady state values.

2.6 The Central Bank and monetary policy instruments

The assets of the CB include loans to private banks and government bonds, while, on the side of liabilities, we have banknotes held by households and interest-bearing reserves held by private banks. Note that these are also the largest (asset and liability) items in the financial statements of the ES.

As said above, for expositional convenience, we first model the case in which the CB participates in the primary market for government bonds (the secondary market is modeled in subsection 2.7 below). Then, the flow budget constraint of the CB linking changes in assets and liabilities written in real and per capita terms is:

$$b_{cb,t} + z_{p,t} + i_t^r \frac{p_{t-1}}{p_t} m_{p,t-1} + \eta_t \equiv$$

$$\equiv (1+i_t^b)\frac{p_{t-1}}{p_t} b_{cb,t-1} + (1+i_t^z)\frac{p_{t-1}}{p_t} z_{p,t-1} + m_t - \frac{p_{t-1}}{p_t} m_{t-1} \tag{18}$$

where $m_t \equiv m_{h,t} + m_{p,t}$ is the monetary base as the sum of banknotes and reserves, while $b_{cb,t}$ is the amount of government bonds purchased by the CB at the end of the period. Since, at this stage, we assume that the CB participates in the primary market of those bonds, and since b_t is the total amount of these bonds (see equation (14) above), we denote $b_{cb,t} \equiv (1-\Lambda_t)\, b_t$ for the bonds held by the CB and $b_{p,t}^T \equiv \Lambda_t b_t$ for total bonds held by private banks, where $(1-\Lambda_t)$ is a quantitative monetary policy instrument.[19]

Similarly to fiscal policy, we need to model the independently set monetary policy instruments, $(i_t^r,\ i_t^z,\ \eta_t,\ 1-\Lambda_t)$. Starting with the nominal interest rates on reserves held by private banks at the CB and on CB loans obtained by private banks, i_t^r and i_t^z, we assume Taylor-type rules like:

$$\log(1+i_t^z) = (1-\rho^z)\log(1+i^z) + \rho^z \log\left(1+i_{t-1}^z\right) + \gamma^{z,\pi}\log(\pi_t/\pi) \tag{19a}$$

$$\log(1+i_t^r) = (1-\rho^r)\log(1+i^r) + \rho^r \log\left(1+i_{t-1}^r\right) + \gamma^{r,\pi}\log(\pi_t/\pi) \tag{19b}$$

[19] That is, the market-clearing condition will be $b_t = b_{p,t}^T + b_{cb,t} \equiv \Lambda_t b_t + (1-\Lambda_t)\, b_t$ (see Appendix A.6 for details).

where $\pi_t \equiv \frac{p_t}{p_{t-1}}$, $\gamma^{z,\pi}$, $\gamma^{r,\pi} \geq 0$ are feedback policy coefficients, $0 \leq \rho^z$, $\rho^r \leq 1$ are persistence parameters, and i^z, i^z denote exogenous steady state values. Note that since the policy rates are not negative in our solutions, we do not include an explicit zero lower bound (ZLB) constraint.

Regarding the transfer from the CB to the government, η_t, following e.g. Reis (2017) and Sims and Wu (2021), we assume a policy rule like:

$$\eta_t = \left(m_{h,t} - \frac{p_{t-1}}{p_t} m_{h,t-1} \right) + (1 + i_t^z) \frac{p_{t-1}}{p_t} z_{p,t-1} + \\ + \left(1 + i_t^b\right) \frac{p_{t-1}}{p_t} b_{cb,t-1} - (1 + i_t^r) \frac{p_{t-1}}{p_t} m_{p,t-1} \tag{20a}$$

But, as said above, since in practice the ES does not allow for the possibility of support from the national fiscal authorities, we rule out negative transfers so that:[20]

$$\eta_t \geq 0 \tag{20b}$$

Regarding $(1 - \Lambda_t)$, since we want to explore the possibility that QE monetary policy can be a substitute for fiscal policy regarding public debt stabilization, we allow $(1 - \Lambda_t)$ to follow the feedback policy rule:

$$1 - \Lambda_t = \left(1 - \rho^\Lambda\right)(1 - \Lambda) + \rho^\Lambda (1 - \Lambda_{t-1}) + \gamma^\Lambda \left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y} \right) \tag{21a}$$

where $\gamma^\Lambda \geq 0$ is a feedback policy coefficient, $0 \leq \rho^\Lambda \leq 1$ is a persistence parameter and Λ is the exogenous steady state value.

But, as said above, according to the rules of the ES, this is subject to an upper limit so that:

$$(1 - \Lambda_t) \leq (1 - \Lambda)^{\max} \tag{21b}$$

where the value of the policy parameter, $(1 - \Lambda)^{\max}$, is specified in subsection 3.2 below.[21]

2.7 Adding a secondary market in sovereign bonds

In practice, CBs do not participate in the primary sovereign bond market. Instead, private banks have the right to sell to the CB in the secondary

[20] Thus, in the code, we define an auxiliary variable $1 - \eta\_temp_t$ given by equation (20a) and then set $\eta_t = \max(0, 1 - \eta\_temp_t)$.

[21] Thus, in the code, we define an auxiliary variable $1 - \Lambda\_temp_t$ given by equation (21a) and then set $1 - \Lambda_t = \min((1 - \Lambda)^{\max}, 1 - \Lambda\_temp_t)$.

market a fraction of the bonds they have previously purchased in the primary market. We now augment the above model to allow for this possibility; we do so in a simple way.

We imagine that in the beginning of each period, each private bank, p, keeps a fraction, $0 \leq \Lambda_t \leq 1$, of the bonds, $b_{p,t-1}$, it purchased at $t-1$, and sells the rest, $0 \leq 1-\Lambda_t \leq 1$, at a price Φ_t to the CB in the secondary market. In other words, for each bond it sells, the private bank receives Φ_t in exchange for $1+(1-\tau_t^{\pi})\, i_t^b$, which is the net-of-tax return on these bonds if held to maturity. Clearly, the private bank will exercise this exchange, or option, only if $\Phi_t \geq 1+(1-\tau_t^{\pi})\, i_t^b$. In other words, to acquire bonds, the CB has to pay a premium to private banks. Actually, this is similar to Gertler Kiyotaki (2010, section 3.3) who assume that the government or the CB have to pay a price above the market price to acquire bank equity; they call this premium a "gift" to private banks. Without loss of generality, we rewrite this inequality as an equality, $\Phi_t \equiv \kappa[1+(1-\tau_t^{\pi})\, i_t^b]$, where the value of the parameter $\kappa \geq 1$ will be specified in subsection 3.2 below.[22]

The rest of this subsection presents what changes relative to above.

2.7.1 Private banks

The above transaction shows up in the banks' net worth, $n_{p,t}$, which changes from (11) to:

$$n_{p,t} = \frac{p_{t-1}}{p_t}\{[1+(1-\tau_t^{\pi})\, i_t^l]L_{p,t-1} + \Big[\Lambda_t[1+(1-\tau_t^{\pi})\, i_t^b] + (1-\Lambda_t)\,\Phi_t\Big]\, b_{p,t-1} + \tag{22}$$

$$+[1+(1-\tau_t^{\pi})\, i_t^r]m_{p,t-1} - [1+(1-\tau_t^{\pi})\, i_t^d]j_{p,t-1} - [1+(1-\tau_t^{\pi})\, i_t^z]z_{p,t-1}\}$$

A detailed solution of banks' new problem is in Appendix A.5.

2.7.2 The Central Bank

The CB is on the other side of the market. Thus, its budget constraint changes from (18) to:

$$\Phi_t(1-\Lambda_t)\frac{p_{t-1}}{p_t}b_{p,t-1} + z_{p,t} + i_t^r\frac{p_{t-1}}{p_t}m_{p,t-1} + \eta_t \equiv$$

[22] Our pricing formula is not inconsistent with the formula used by the option theory. For the pricing of put options, i.e. options that give an asset holder the right to sell the asset, see e.g. Hull (2008, chapter 12). Using the language of this literature, $1+(1-\tau_t^{\pi})\, i_t^b$ is the market price and Φ_t is the strike price, so that the put option will be exrcised only if $\Phi_t \geq 1+(1-\tau_t^{\pi})\, i_t^b$. We would like to add here that working as in this literature and if we assume that the risk free return is, say, the interest rate on reserves held at the CB, i_t^r, it follows $\Phi_t = [1+(1-\tau_t^{\pi})\, i_t^b] + i_t^r > 1+(1-\tau_t^{\pi})\, i_t^b$. We report that our results do not change if we use the latter in the final system.

$$\equiv (1-\Lambda_t)(1+i_t^b)\frac{p_{t-1}}{p_t}b_{p,t-1}+(1+i_t^z)\frac{p_{t-1}}{p_t}z_{p,t-1}+m_t-\frac{p_{t-1}}{p_t}m_{t-1} \quad (23)$$

and the associated transfer from the CB to the government, η_t, changes from (20a) to:

$$\eta_t = \left(m_{h,t}-\frac{p_{t-1}}{p_t}m_{h,t-1}\right)+(1+i_t^z)\,\frac{p_{t-1}}{p_t}z_{p,t-1}+ \quad (24)$$
$$+\left[\left(1+i_t^b\right)-\Phi_t\right](1-\Lambda_t)\frac{p_{t-1}}{p_t}b_{t-1}-(1+i_t^r)\,\frac{p_{t-1}}{p_t}m_{p,t-1}$$

2.7.3 The Treasury

The treasury's budget constraint remains as in (14) except that now bonds are sold to private banks only.[23] Also, the definition for the Treasury's tax revenues changes from (15) to:

$$\frac{T_t}{N}\equiv \tau_t^c c_{h,t}+\tau_t^y w_t l_{h,t}+\tau_t^\pi(y_{i,t}-w_t l_{i,t})+ \quad (25)$$

$$+\tau_t^\pi\frac{p_{t-1}}{p_t}(i_t^l L_{p,t-1}+i_t^r m_{p,t-1}+i_t^b\Lambda_t b_{p,t-1}-i_t^z z_{p,t-1}-i_t^d j_{p,t-1})$$

since now only a fraction, Λ_t, of income from bonds is taxable.

2.8 Macroeconomic system, monetary policy transmission and what comes next

Collecting equations, Appendix A.6 presents the macroeconomic system in the counter-factual case in which the CB purchases sovereign bonds in the primary market, while Appendix A.7 does the same when the CB purchases these bonds in the secondary market. While we focus on the latter, the former will be used for comparison. All this is given the paths of exogenous variables and policy instruments whose "long-run" values will be set as in the data.

Before we move on, it is useful to clarify the channels through which monetary policy can have real effects in general equilibrium. Regarding interest rate policies, these policies can have real effects because of nominal

[23] That is, the market-clearing condition in the primary bond market will be $b_t=b_{p,t}^T$, while $b_{p,t-1}^T\equiv\Lambda_t b_{p,t-1}^T+(1-\Lambda_t)\,b_{p,t-1}^T=\Lambda_t b_{p,t-1}^T+b_{cb,t-1}$ in the secondary market (see Appendix A.7 for details).

rigidities as is common in the New Keynesian literature. Regarding quantitative monetary policies, they can have real effects because of the moral hazard problem in the Gertler-Kiyotaki-Karadi setup employed here and the working capital constraint faced by firms. In other words, the moral hazard problem opens the door through which quantitative monetary policies affect the credit policy of private banks and, in turn, the working capital constraint faced by firms opens the door through which private banks' credit policy can affect the production sector. More specifically, these policies lower interest rates in certain securities and hence trigger portfolio-rebalancing effects that can affect the yields of other securities too; the same policies can also act more directly on the supply side of credit by easing the constraints faced by private banks, so that the latter can increase their supply of credit to the private economy (see e.g. Walsh (2017, chapter 11.5), Sims and Wu (2021) and Benigno et al (2022) for details). Nevertheless, the general equilibrium effects of such policies on the real economy, as well as on public finances, are naturally a quantitative matter. As we shall see, they also depend on whether the CB participates in the secondary market for sovereign bonds.

In the next sections, we will parameterize the model, present data, and solve the model numerically under various policy scenaria. In particular, we will work as follows. After calibrating the model to EA data, we will get an initial "steady state" solution using the calibrated parameter values and data of the year 2022 for the exogenous variables; all this is in the next section 3. Then, in the remaining sections, 4 and 5, departing from this initial solution for 2022, we will shock the model and investigate which fiscal and/or monetary policies can ensure dynamic stability and determinacy and, if yes, under what conditions. In our solutions, we assume that all is common knowledge so that we solve the model under perfect foresight by using a non-linear Newton-type method implemented in Dynare.

3 Parameter values, policy variables and solution for 2022

This section first parameterizes the model using annual data of the EA over the euro period 2002-2022 (unless otherwise stated), then presents the values of the model's exogenous variables and, finally, solves for the model's "initial steady state" defined as a situation in which variables do not change and exogenous policy variables are set as in the most recent data. As we shall see, this solution can match reasonably well the current key features of the EA and can thus serve as a reasonable departure point for the policy experiments in the next sections, 4 and 5.

3.1 Parameter values

Parameter values, either calibrated or set, are listed in Table 1. Starting with preference parameters, private agents' time discount factor, β, is calibrated from the steady state version of the Euler equation for domestic deposits (equation (A.7.3) in Appendix A.7). We assume that the deposit rate equals the reserves rate set by the CB in September 2022, 2%, which in turn implies $\beta = 0.9804$. The weights given to private consumption and leisure, μ_1 and μ_2, in the households' utility function are calibrated, for given μ_3, from the steady state versions of equations (A.7.1) and (A.7.2) in Appendix A.7) using data for the share of private consumption to GDP (0.543), the labour income share (0.471), the percentage of time devoted to leisure (0.682) and the effective labour income and consumption tax rates (0.38 and 0.165 respectively).[24] The obtained values of μ_1 and μ_2, after setting $\mu_3 = 0.05$,[25] are 0.477 and 0.473 respectively.

Continuing with technology parameters in the production function of goods, the exponent on labor, $1 - \alpha$, is calibrated from the expression $(1 - \alpha)(1-\epsilon) = 0.471$, where 0.471 is the above mentioned average labour income share in the data and ϵ measures the contribution of productivity-enhancing public goods/services in private production. Following e.g. the early paper by Baxter and King (1993) but also more recent work of Ramey (2020) and many others, we set ϵ equal to 0.05.[26] This value for ϵ implies that α, which is the exponent on capital in the Cobb-Douglas production function, equals 0.454. The private and government capital depreciation rates, δ and δ^g respectively, are both set equal to 0.046 (see *Monthly Bulletin*, ECB, 2006). The steady state TFP parameter, A, is set at 1. Regarding the Dixit-Stiglitz parameter measuring imperfect competition in the product market, θ, we use information from Eggertson et al (2014), who report that the gross markup in traded goods is around 1.15 in EA countries; the latter implies $\theta = 0.85$. We also set the parameter associated with Rotemberg-type price adjustment costs, ξ^p, at 3, which is a value within commonly used ranges. Finally, we set the coefficient, η_i, in the firms' financial constraint (8) at 0.3 which is as in e.g. Korinek and Mendoza (2014).

Continuing with the banking sector, we set the parameters in the banks'

[24]Data on EA's private consumption to GDP ratio are taken from the database of Federal Reserve Bank of St. Louis and cover the period 2002-2022. Data on EA's labour income share, again for the period 2002-2022, are taken from Eurostat, whereas data on average total hours worked within a year, for the period 2002-2021, are taken from OECD. Notice here that, following usual practice, we have defined total hours available on a yearly basis as $52 \times 14 \times 7 = 5096$. Finally, the series of the effective tax rates are taken from *Taxation Trends in the European Union* (European Commission, 2022).

[25]We report that our main results are robust to changes in μ_3, namely, the weight given to utility-enhancing public services, whose value is relatively agnostic and is usualy set between 0 and 0.1 (see e.g. Baxter and King (1993) and Baier and Glomm (2001)).

[26]We report that our main results are robust to changes in the value of ϵ.

incentive constraint (13) so as to match the main interest rates at the end of 2022 (see Appendix A.4 on how the parameters in (13) are translated into parameters ξ^l, ξ^b, ξ^z and ξ^m, reported here). In particular, we calibrate the parameters associated with banks' loans to firms, ξ^l, and with government bonds, ξ^b, using the steady state version of private banks' first-order conditions for loans and government bonds (equations (A.7.15) and (A.7.16) in Appendix A.7), so as to match the EA's lending and government bond rates at the end of 2022 ($i^l = 3.41\%$ and $i^b = 3.40\%$ where the data are from the site of the ECB); the resulting values are $\xi^l = 0.60$ and $\xi^b = 0.65$. To hit the above, we also need to set the parameter associated with loans provided by the CB, ξ^z, at 0.2, while, for simplicity, we set the parameter associated with reserves, ξ^m, at 0.[27] Regarding the banks' survival rate, σ, and the proportional transfer of entering banks, γ, they are calibrated to match banks' reserves at the CB as a percentage of GDP at the end of 2022 ($m_p/y = 30\%$) and to get a reasonable value of banks' total net worth as share of GDP (around 45%); the resulting values are $\sigma = 0.92$ and $\gamma = 0.017$.

Table 1

Baseline parameterization

Parameter	Description	Value	
μ_1	weight of consumption in utility	0.477	calibr
μ_2	weight of leisure in utility	0.473	calibr
μ_3	weight of public goods in utility	0.05	calibr
β	time discount factor	0.9804	calibr
δ and δ^g	depreciation rate of priv and pub capital	0.046	calibr
A	TFP	1	set
α	share of capital in production	0.454	calibr
ε	contribution of public capital in production	0.05	set
θ	substitutability parameter of intermediate goods	0.85	calibr
ξ^p	price adjustment cost parameter	3	set
ξ^l	parameter associated with banks' loans to firms	0.6	calibr
ξ^b	parameter associated with banks' gov bonds	0.65	calibr
ξ^z	parameter associated with banks' loans from the CB	0.2	set
ξ^m	parameter associated with banks' reserves at the CB	0	set
σ	bankers' survival rate	0.92	set
γ	proportional transfer to entering bankers	0.017	calibr
η_i	coeff. in working capital constraint	0.3	set

[27] See also Sims and Wu (2021). We report that our results are not sensitive to the parameter value of ξ^m.

3.2 Policy variables

Policy variables, as well as parameters and feedback coefficients included in policy rules, are listed in Table 2. The values of policy variables correspond to their most recent values in the data. Regarding fiscal policy, the recent data values of s_t^c, s_t^g, s_t^t, τ_t^c, τ_t^y and τ_t^π, namely, public spending on consumption, investment and transfer payments all three as shares of GDP, as well as the effective tax rates on consumption, personal income and corporate profits, are 0.22, 0.03, 0.22, 0.165, 0.385 and 0.206 respectively.[28] Regarding monetary policy, we set the nominal interest rates on reserves held at the CB, i_t^r, at 2%, which was its value at the end 2022, while, the nominal interest rate at which banks borrow from the CB, i_t^z, is set at 2.5% which is higher than the reserves rate and close to the rate on MROs in the ES.[29] The fraction of sovereign bonds held by the ES, $1-\Lambda$, is set at 31.8%, which was the sum of of PSPP and PEPP stocks relative to member-countries national debts in 2022.[30] In our baseline solutions, we will set the upper issue limit in (21b), $(1-\Lambda)^{\max}$, at 50% (see the discussion in the Introduction above). Finally, we set the parameter that quantifies the premium paid by the CB to private banks when purchasing bonds in the secondary market, κ, at 1.01, which is the highest possible value under which the CB's transfer to the Treasury does not turn to negative in the base long run solution.

Regarding the $AR(1)$ persistence parameters in policy rules, we set them at the common value of 0.8 across all instruments. The feedback coefficients on inflation in the Taylor rules for the policy rates are both set at 1.5 (i.e. $\gamma^{r,\pi} = \gamma^{z,\pi} \equiv \gamma^\pi = 1.5$), except otherwise said. The values of the feedback policy coefficients on public debt imbalances will be specified below since they change across policy experiments; in each case, they will be set at the minimum value required to guarantee that public debt remains on a stable path so there is a unique determinate equilibrium.

[28] The source of the spending instruments is Eurostat while the tax rates are from *Taxation Trends in the EU* (European Commission (2022)). Note that the effective tax rates on consumption and labor income are at their 2020 values, while, the effective tax rate on corporate profits as well as government consumption and investment as shares to GDP are set at their 2021 values.

[29] The two primary lending policies of the ECB are its refinancing operations (MROs, LTROs, TLTROs) and the marginal lending facility used for overnight liquildity. See the site of the ECB for details.

[30] There are two active asset purchase programs today. The APP that started in late 2014 and whose biggest item has been the PSPP, and the PEPP that was a response to the covid-19 pandemic and hence it is temporary. Since March 2023, the ES only partially reinvests the principal payments from maturing PSPP securities. Regarding the PEPP, the ECB discontinued its net asset purchases in March 2022 but the maturing principal payments will be reinvested until at least the end of 2024. See the site of the ECB for details.

Table 2. Policy variables			
Parameter	Description	Value	
s^g	gov investment to GDP	3%	data
s^c	gov consumption to GDP	22%	data
s^t	gov transfers to GDP	22%	data
τ^c	consumption tax rate	16.5%	data
τ^y	personal income tax rate	38.5%	data
τ^π	corporate tax rate	20.6%	data
i^r	interest rate on reserves	2.00%	data
i^z	interest rate on CB's loans to banks	2.50%	data
$1-\Lambda$	CB's gov bonds' holdings	31.8%	data
$(1-\Lambda)^{\max}$	CB's gov bonds' holdings threshold	50%	set
κ	parameter in pricing function of bonds in secondary market	1.01	calibrated
$\rho^{g,g}$	persistence of gov investment	0.8	set
$\rho^{g,c}$	persistence of gov consumption	0.8	set
$\rho^{g,t}$	persistence of gov transfers	0.8	set
$\rho^{\tau,c}$	persistence of consumption tax rate	0.8	set
$\rho^{\tau,y}$	persistence of personal income tax rate	0.8	set
$\rho^{\tau,\pi}$	persistence of corporate tax rate	0.8	set
ρ^r	persistence of reserves rate	0.8	set
ρ^z	persistence of CB lending rate	0.8	set
$\rho^{1-\Lambda}$	persistence of CB's bond holdings	0.8	set
$\gamma^{r,\pi}$	coefficient on inflation in Taylor rule for reserves rate	1.5	set
$\gamma^{z,\pi}$	coefficient on inflation in Taylor rule for lending rate	1.5	set

3.3 Solution for the year 2022

Table 3 reports the values of the main endogenous variables produced by the model's solution when we use the parameter values in Table 1 and the policy instruments and coefficients in Table 2. In this solution, variables do not change so this is what we call the initial steady state. As can be seen, the model's solution can mimic reasonably well the situation in the EA in 2022 and can therefore serve as a departure point from what will follow next. Note that for this initial steady state solution only, the GDP share of government transfers, s_t^t, plays the role of the residually determined public financing instrument that closes the government budget constraint with the public debt to GDP ratio being set at its data value (91.6%); this gives $s_t^t = 9.81\%$ which is much lower than in the data (22%); this provides a first indication that some kind of fiscal correction will be unavoidable sooner or later[31] and this will be confirmed below when we shock the model and study transition paths.

Table 3 Model's solution for key endogenous variables in 2022			
Variable	Description	Model	Data
b/y	public debt to GDP	91.6%	91.6%
c/y	private consumption to GDP	55%	54%
inv/y	private investment to GDP	20%	
k/y	private capital to output	4.41	
L/y	private banks' loans to GDP	13%	
j/y	private deposits to GDP	61%	
m_p/y	private banks' reserves to GDP	30%	30%
i^l	interest rate on bank loans	3.5%	3.41%
i^d	interest rate on bank deposits	2%	1.45%
i^b	interest rate on government bonds	3.21%	3.31%
l	work hours	0.29	0.32

4 Main results

To trigger transition dynamics, we assume that a negative 10% TFP shock hits the economy at the initial steady state. Using the assumed $AR(1)$ process for TFP, we set the persistence parameter, ρ^A, at 0.5 so that the shock vanishes within approximately 10 periods. To the extent that we get a transition solution, these numbers generate an economic downturn, combined with a rising public debt to GDP ratio, in around the first 5 periods. This can somehow mimic the various crises since 2008. Note that, at this early stage, only the Taylor rules, according to which the two policy interest rates react to inflation, are switched on (see equations (19a)-(19b) above). Also recall that the CB participates in the secondary market for government bonds (however, for comparison, we will also report below what happens in the counter-factual case of primary market participation).

Our experiments imply that when none of the other policy instruments (namely, tax-spending instruments and quantitative monetary policy instruments) react to public debt, the model is dynamically unstable and cannot produce a transition solution. We will therefore investigate which policies can restore dynamic stability and determinacy. Before we proceed, it is worth reporting that we have experimented with various types of shocks (in addition to TFP), and of various signs and sizes, and the above qualitative result remains the same; namely, if a shock hits the EA economy, and if there is no some kind of systematic policy reaction to public debt, the path of the

[31] D'Erasmo et al (2016) call this "the classic debt sustainability analysis". Since it focuses only on the long run implications of fiscal policies, its main flaw is that it cannot guarantee that the inherited public debt is sustainable.

latter is explosive and hence a solution does not exist over the transition.[32]

4.1 The conventional policy assignment

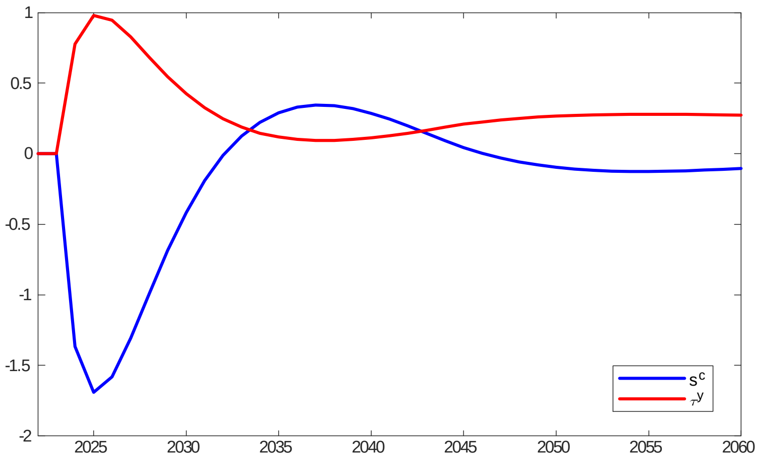
To put our results in the context of the academic literature, we start with the standard case in which the fiscal authorities do their job. Dynamic stability and determinacy are restored when at least one fiscal (tax-spending) instrument, $x_t \equiv (s_t^c, s_t^g, s_t^t, \tau_t^c, \tau_t^y, \tau_t^\pi)$, reacts systematically to the public debt gap by following (17), while QE monetary policy remains as it was at the end of 2022. Specifically, in this set of simulations, we set the feedback fiscal policy coefficient on the public debt gap, $\gamma^{x,b}$, at 0.02 in (17), which is the minimum value that ensures stability of debt across this set of policy experiments, while, at the same time, we set $\gamma^\Lambda = 0$ in the rule for QE monetary policy in (21a). Our simulations show that in this case η_t and, by construction, $1-\Lambda_t$ remain within their ES ranges as in equations (20b) and (21b). In other words, a stable ES can be guaranteed when policy interest rates react to inflation and at least one of the tax-spending instruments reacts to public debt imbalances. This is in accordance with Leeper's (1991) policy mix of passive fiscal policy and active monetary policy, as well as with the result in Kirsanova et al (2009) who refer to this policy mix as the "consensus assignment", although here we also have quantitative monetary policies. Regarding interest rate policy under this regime, we have set $\gamma^\pi = 1.5$ (although, we report that our results do not change even when the Taylor principle is not satisfied; in particular, they hold for $\gamma^\pi \geq 0.3$).

Graph 1 plots the time-paths of some commonly used debt-contingent fiscal policies. Results are expressed as percentage deviations from their departure 2022 values. In this graph, we include the path of one public spending instrument, say, public consumption as share of GDP (s_t^c), and the path of one tax instrument, say, the personal income tax rate (τ_t^y).[33] As expected, the spending share has to be reduced, while the tax rate has to rise to restore stability.

[32] It is worth pointing out that such instability arises even in the case of positive public investment shocks; in other words, the usual claim by politicians that if pubic spending is on productive activities, it can be self-financing - in the sense that no spending cuts and/or tax rises will be necessary in the future - is not supported by our model (see also Malley and Philippopoulos (2022) for the recent US infrastructure stimulus).

[33] The related paths of other fiscal (tax-spending) instruments used for debt stabilization are available upon request. Their message is the same as that from s_t^c and τ_t^y.

Graph 1
Public spending to GDP and tax rates used for debt stabilization (percentage deviation from 2022)

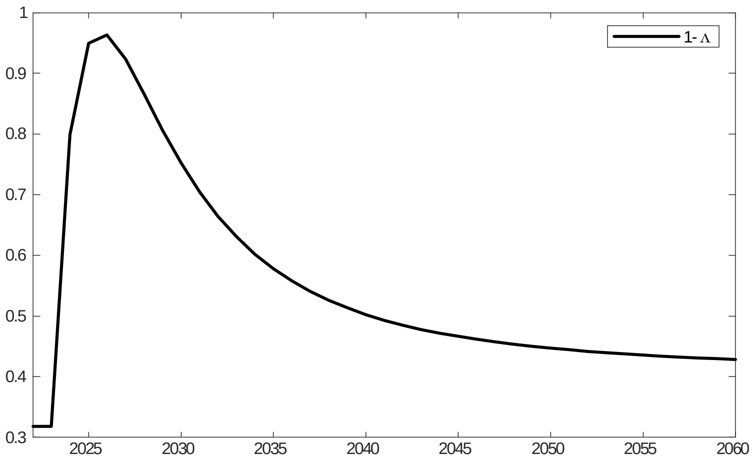


4.2 Can quantitative monetary policy do the unpleasant job?

We now switch off any kind of fiscal policy response to public debt and instead investigate what happens when this task is assigned to the CB. Bond purchases by the latter in the secondary market, $1 - \Lambda_t$, are now assumed to be contingent on the public debt gap as in the feedback policy rule (21a), that is, now $\gamma^{\Lambda} > 0$, while, at the same time, we set $\gamma^{x,b} = 0$ in (17). This is like a regime of active fiscal policy and passive monetary policy (see Leeper (1991)).

We start by assuming away the ES-type restrictions on the conduct of quantitative monetary policy. In other words, we start by assuming that, over time, there is no upper limit to the fraction of bonds, $1-\Lambda_t$, that the CB can hold, to the extent of course that this fraction does not exceed 1, and that the CB's transfer to the Treasury, η_t, is free to also take negative values if this is needed. In this unrestricted case, that resembles the conduct of monetary policy in the US, our simulations imply that stability and determinacy can be restored when the feedback policy coefficient on the public debt gap, γ^{Λ}, is set at a relatively high value, around 3, which implies that $1 - \Lambda_t$ rises a lot in some time periods, sometimes as high as around 95% in this set of experiments. Regarding interest rate policy, here we again set $\gamma^{\pi} = 1.5$ (although, we again report that these results hold for $\gamma^{\pi} \geq 0.3$). Graph 2 shows the time-path of $1 - \Lambda_t$ under this policy scenario. The CB needs to increase its QE a lot and for many periods to ensure public debt and macroeconomic stability.

Graph 2
QE used for debt stabilization
(in levels)



It then naturally follows that if, other things equal, we impose the ES-type restrictions, namely, $1-\Lambda_t \leq 0.5$ and $\eta_t \geq 0$, the model fails to give a transition solution.[34] Actually, both restrictions, $1-\Lambda_t \leq 0.5$ and $\eta_t > 0$, are violated. Focusing on $1-\Lambda_t \leq 0.5$, this should not come as a surprise: after the additional government bonds purchased by the ES under the PEPP in the years of the pandemic, the recent average holdings, 31.8%, are closer to the upper limit of 50%. There is no much space left for a further significant increase in $1-\Lambda_t$.

Summing up, we have so far studied two polar cases in which macroeconomic stability and determinacy are restored either by fiscal reaction to public debt or by debt-contingent QE monetary policy, although, in the latter case, QE can do the job only if it is allowed to violate the rules of the ES. Before examining the robustness of this result and fiscal-monetary mixes that can perhaps allow debt-contingent QE policy to respect the rules of the ES, it is critical to understand the macroeconomic implications of the above two polar cases. This is what we do next.

[34] We report that this result is robust to changes in the banking sector parameters, e.g. ξ^l, γ, σ. We also report that there is a small region of γ^π (i.e. $0.3 \leq \gamma^\pi \leq 0.5$) that seems to deliver stability under QE without violating the ES restrictions; on the other hand, the associated impulse response functions show that this comes at the cost of relatively big changes in inflation and especially in $1-\Lambda_t$ over time.

4.3 Macroeconomic implications of the above policies

In this subsection, we show how the above two polar cases (debt stabilization via fiscal adjustmen, or via QE without the ES's restrictions) affect public finances and the real economy.

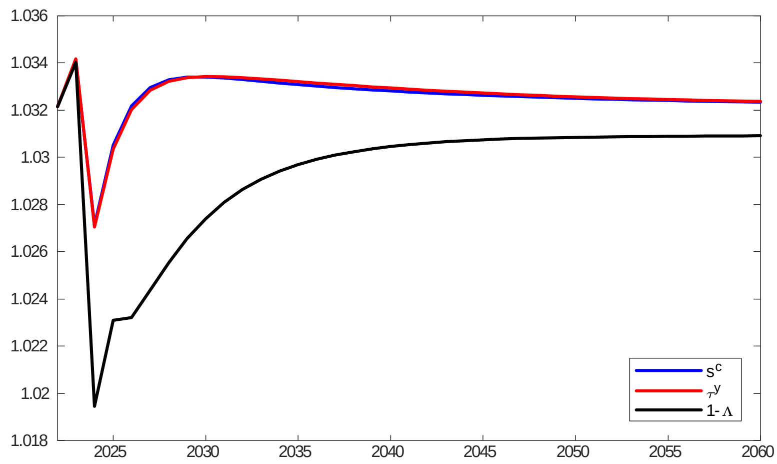
4.3.1 Implications for public finances

To understand how different policies manage to restore public debt stability, we compute the public finance implications of three alternative policies: the case in which public debt stabilization is achieved by adjustments in a public spending instrument, say, public consumption as share of GDP, s_t^c, the case in which this is achieved by adjustments in a tax instrument, say, the personal income tax rate, τ_t^y, and the case in which this is achieved by free adjustments in the share of sovereign bonds purchased by the CB in the secondary market, $1-\Lambda_t$. For each policy, we will compute the resulting paths of the real gross interest rate on sovereign bonds, $(1+i_t^b)\frac{p_{t-1}}{p_t}$, the transfer from the CB to the Treasury, η_t, and the primary fiscal surplus, $\frac{T_t}{N}-\left(g_t^c+g_t^g+g_t^t\right)$; these three endogenous variables shape the dynamics of public debt in the Treasury's budget constraint (14).[35] The corresponding graphs are Graphs 3, 4 and 5. Again, the departure values are those of the year 2022.

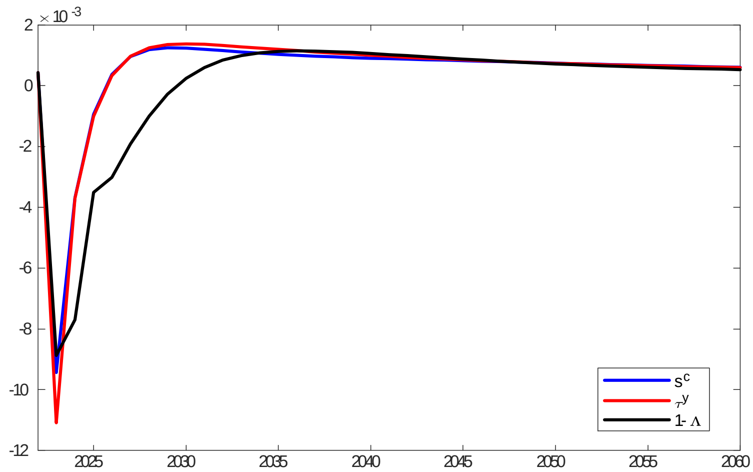
Inspection of Graphs 3, 4 and 5 implies that the use of QE policies through the endogenous adjustment of $1-\Lambda_t$ to the public debt gap is superior in terms of reducing the real interest rate (see Graph 3) which is the coefficient on inherited public debt in the difference equation (14), but is inferior to spending cuts and tax rises in terms of the transfer from the CB to the Treasury (see Graph 4) as well as in terms of the primary fiscal surplus (see Graph 5). We believe these are intuitive results and consistent with the general belief that the main benefit of QE has been to reduce sovereign yields and calm financial markets rather than to generate extra resources for the fiscal authorities (or, quoting Reis (2017), rather than to "alleviate fiscal burdens"). Finally, we report that, regarding the transition path of public debt, we observe a slower convergence when it is QE that reacts to it, while, when we use cuts in government consumption, debt falls relatively fast.

[35] That is, in general equilibrium models, where the sovereign real interest rate, public spending, tax revenues, etc, are all endogenous variables depending, among other things, on inherited public debt, dynamic stability is a more complex issue than in simple debt arithmetic calculations where debt stability depends only on the differerence between the exogenous real interest rate and the exogenous growth rate, i.e. the so-called $r-g$ differential.

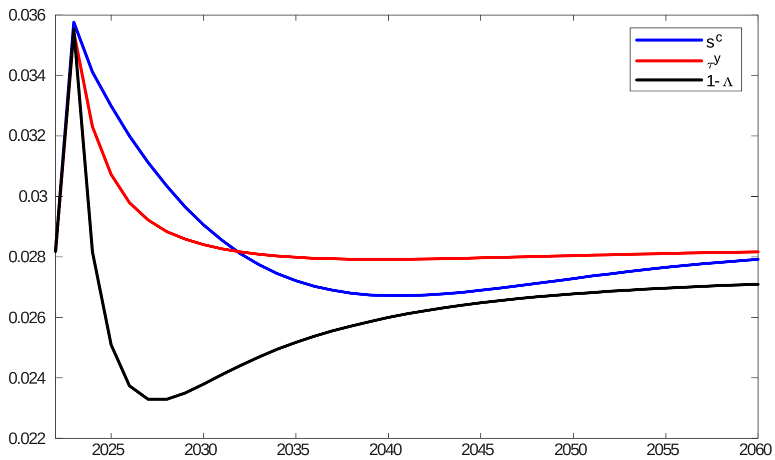
Graph 3
Gross real interest rate on government bonds
(in levels)



Graph 4
The CB's transfer to the Treasury
(in levels)



Graph 5
Primary fiscal surplus
(in levels)



4.3.2 Implications for the real economy

Here we compute the path of cumulative discounted output, as difference from its departure value in 2022, under the same three alternative public debt stabilization policies studied in the previous subsection, namely, the case in which stabilization is restored by adjustments in s_t^c, the case in which this is achieved by adjustments in τ_t^y and, finally, the case in which this is achieved by free adjustments in $1 - \Lambda_t$. For each case, we compute the value of φ_t, which is defined as:

$$\varphi_t \equiv \sum_{s=0}^{t} \frac{y_s - y}{(1 + i^b)^s}$$

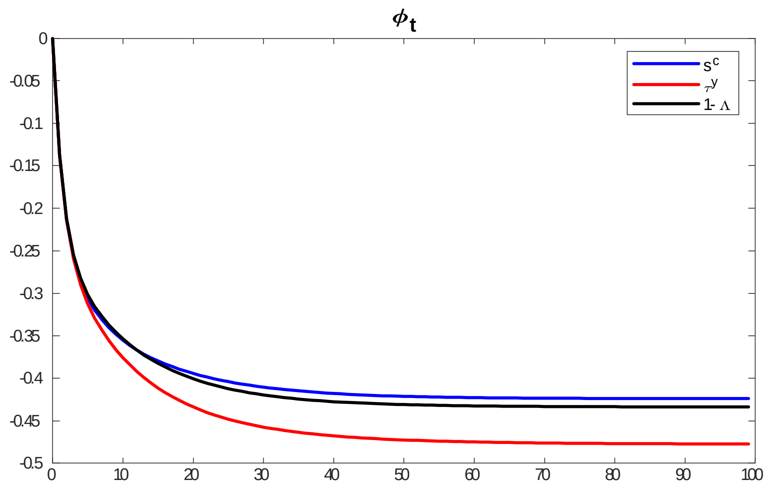
where y is the value of output in the initial steady state and i^b is the steady state value of the interest rate on sovereign bonds (we report that using the time-path of the interest rate, instead of its steady state value, for discounting is not important for our results).

The three paths of φ_t are illustrated in Graph 6. As can be seen, the fall in output - triggered by the adverse TFP shock - is bigger when it is the income tax rate that reacts to stabilize the public debt trajectory than when this is achieved by cuts in public consumption spending or by further purchases of sovereign bonds by the CB in the secondary market.[36] Perhaps it looks a bit surprising that bond purchases by the CB are not

[36] We report that the lower the value of the parameter κ, which quantifies the premium paid by the CB to private banks when purchasing their bonds in the secondary market,

superior to cuts in public consumption spending. Nevertheless, this happens because, although bond purchases by the CB create an excess demand for these bonds that clearly brings down their yield (see Graph 3 above), this is not translated to lower yields in other markets too due to the presence of asset pricing wedges. Hence, the portfolio reallocation effects are relatively small and, in any case, the extra income obtained by private banks from the CB is not automatically used to finance more bank loans to production firms; it can also be used, jointly with loans obtained from the CB at a low policy interest rate, to increase interest-bearing reserves held by banks at the CB. Also recall that, in this class of models, public consumption provides only welfare services so its cut is not damaging the supply side of the economy.

Graph 6
Output gap under alternative debt stabilization policies



4.4 How general are the above results?

To check the robustness of our results, we investigate whether debt-contingent QE policy could restore stability on its own and still respect the rules of the ES, if there were different conditions. In particular, if the CB faced more favorable initial conditions than 31.8% and/or different upper limits from the 50% assumed so far, or if the economy were hit by a relatively small shock.

For example, imagine that, other things equal, the CB starts with a smaller amount of public debt than in the data, say, 21%, which is the fraction of the PSPP in the data in 2022 ignoring the rest 10.8% (where

the smaller are the effects of QE policy on the economy, and in particular on the sovereign bond's interest rate. This is intuitive.

31.8-21=10.8) that has to do with the PEPP; then, resolving the model, QE can restore stability on its own and this can happen without violating the ES rules. On the other hand, if we start with 21%, but the upper limit is 33% which is the official upper limit of PSPP, instead of the loose 50% assumed so far, we go back to the main result above, namely, the ES restrictions need to be violated for QE to be able to do the unpleasant job. In other words, the initial stock of bonds in the hands of CB does matter for the success of QE policies and this can perhaps contribute to explaining why the ECB's intervention during the global financial crisis of the previous decade was successful; at that time, the ECB had much more space to manoeuvre the economy.

We have also experimented with a smaller TFP shock than the 10% shock assumed so far (see at the very start of this section). If the adverse shock is, say, 5% only, and the upper limit remains at 50%, QE can do the job without violating the ES restrictions. On the other hand, if the shock is 5% but the upper limit is 33%, the ES restrictions need to be violated.

Summing up, as is perhaps expected, if the starting situation were more favorable than that at the end of 2022, or if the shocks triggering dynamics were relatively mild, it would be possible for the CB to stabilize the economy on its own via debt-contingent QE and still respect the numerical rules of the ES. Nevertheless, the main result does not change. Namely, given the current situation, if a relatively big shock hits the European economy and fiscal policy remains active, there is no room left for further quasi-fiscal actions by the ECB to the extent that the ES's rules are respected.

4.5 The importance of a popular, although counter-factual, assumption

Before we proceed to study policy mixes, it is important to compare our results to most of the related literature. That is, we now solve the model in the counter-factual, although popular in the literature, case in which the CB participates in the primary sovereign bond market like private banks do (as said, modelling details are in Appendices A.4 and A.6 which can be compared to Appendices A.5 and A.7 for the secondary market).

We report that the main qualitative result is that now, other things equal, QE monetary policy can be a substitute for fiscal policy regarding debt stabilization and, at the same time, respect the rules of the ES in (20b) and (21b), i.e. now $\eta_t \geq 0$ and $1 - \Lambda_t \leq 0.5$. The latter happens because now the adjustment of $1 - \Lambda_t$ that can do the job can be achieved by much lower values of γ^{Λ} than in the case in which this is done in the secondary market (for example, now $\gamma^{\Lambda} = 0.9$ while we had $\gamma^{\Lambda} = 3$ above).[37] This

[37] Fiscal policy reaction to debt and interest rate reaction to inflation are as in subsection 4.2 above. Namely, $\gamma^{\tau^y,b} = 0.02$ and $\gamma^{\pi} = 1.5$. We also report that, to match the data with the new model specification, we have: (a) changed the parameter γ included in

makes the necessary increase in $1-\Lambda_t$ smaller so that the latter can remain within its ES range. The general idea is that the CB has a more direct control over public finances if it can purchase bonds in the primary market. Specifically, now, QE type policies, which increase $1-\Lambda_t$ and thus decrease Λ_t, can directly ease the incentive constraint faced by private banks, so that the latter are expected to increase more strongly their supply of credit to the private economy and this can benefit the real economy when it is financially constrained (see e.g. Walsh (2017, chapter 11.5.4) for details). Also, $1-\Lambda_t$ now enters directly the Treasury's budget constraint.

Summing up, the usual assumption that the CB purchases government bonds in the primary market is not innocent when the issue is the stability of public debt.

4.6 Policy mixes and complementarities

So far we have studied polar cases. We have seen that stability and determinacy can be restored either by conventional fiscal corrections, or by debt-contingent QE monetary policy, although in the latter case the ES's rules are violated. If this is the case, and since big shocks keep hitting the European economy since the start of 2020, it is natural to ask ourselves a question, which is similar to that asked by Leeper et al (2010) in their study for the sustainability of public debt in the US economy. In particular, if so far we cannot observe any systematic fiscal reaction to public debt imbalances and, at the same time, the space for further QE monetary policy has been exhausted given the self-imposed ES's restrictions, then, quoting Leeper and his co-authors, a natural question to ask ourselves is "Why do forward-looking agents continue to purchase bonds with relatively low interest rates and bond prices don't plummet?". As Leeper and his co-authors argue, a natural answer to this - to the extent that we want to maintain the assumption of rationality - could be that private agents believe that the current fiscal inaction is temporary only and it will be replaced by necessary fiscal corrections of some kind in the future. In other words, the belief is that the necessary fiscal reaction has been just backloaded.

To address this possible scenario, we now allow for fiscal reaction to public debt after, say, 10 periods, complemented by QE in the secondary market in the sense that $1-\Lambda_t$ also helps by reacting to public debt, say, from the very beginning. In particular, we set $\gamma^{\tau^y,b} = 0.02$ after 10 periods and zero before, while γ^{Λ} and γ^{π} are set as in subsections 4.1 and 4.2 above.[38] Now we do get stability and determinacy and, in addition, quantitative monetary policy respects the rules of the ES. Therefore, although QE policy

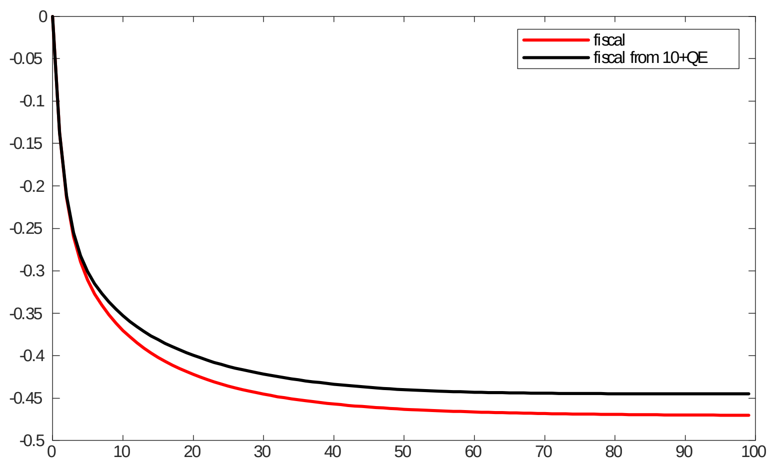
households' transfer to entering bankers so as to better match reserves and CB loans as shares of GDP and (b) slightly adjusted government transfers as share of GDP so as to match the debt-to-GDP ratio.

[38] Namely, $\gamma^{\Lambda} = 3$ and $\gamma^{\pi} = 1.5$.

cannot on its own restore stability and determinacy and at the same time respect the rules of the ES, it can do so if there is the anticipation of fiscal reaction to public debt in the near future and this anticipation proves to be credible.

Regarding real implications, Graph 7 shows the path of the output gap, φ_t, in the case of such a policy mix, in particular, when, for instance, we use the income tax rate as the fiscal instrument that reacts to public debt after 10 periods. This graph also includes, for comparison, the path of φ_t in the case in which public debt would be stabilized by fiscal policy only and from the very beginning. As is shown, the recession is smaller in the former case in which QE policy complements the backloaded fiscal policy. In other words, the adverse real effects of the negative TFP shock are mitigated when the CB gives the Treasury a hand through debt-contingent QE policy even if the latter is a mild one. Therefore, fiscal and monetary policy reinforce each other by creating space for each other (for similar synergies between fiscal and monetary policies in the EA, see also e.g. Bankowski et al (2021) although in a model without quantitative monetary policy).

Graph 7
Output gap with a delayed fiscal reaction and a policy mix



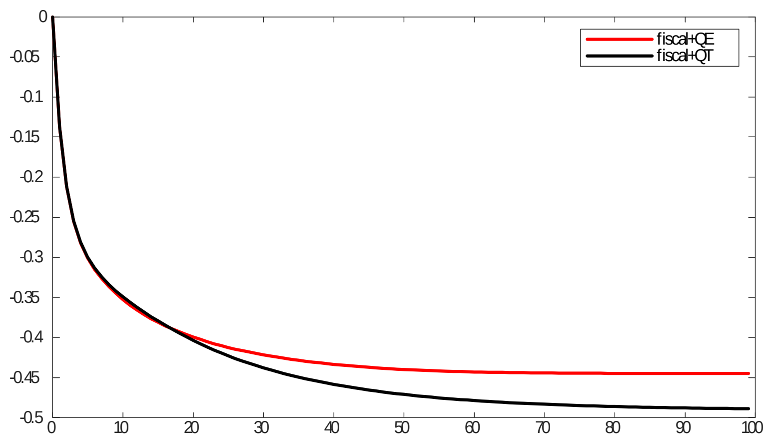
5 Downsides to using QE policies

The literature has stressed several downsides to using large-scale asset purchase programmes (see e.g. the discussion in Benigno et al (2022)). Here, we will address two of them: first, the implications of an unavoidable (sooner or later) policy reversal or what is known as quantitative tightening and, second, the distributional implications of such policies. In doing so, we keep assuming that the CB purchases sovereign bonds in the secondary market.

5.1 Unwinding QE

In this subsection, we compare two cases: (a) Fiscal policy reaction to public debt after, say, period 10 being complemented by QE, in the sense that $1-\Lambda_t$ also reacts to public debt from the very beginning (which was the regime in subsection 4.6 above). (b) Fiscal policy as in (a) except that now we have quantitative gradual tightening, in the sense that now, instead of being accommodative, $1-\Lambda_t$ exogenously and gradually decreases over time from 31.8% (initial steady state) to say 5% (new, terminal steady state). Regarding fiscal policy, we focus on the case in which it is the income tax rate that reacts to public debt imbalances. Regarding interest rate policy, we again set $\gamma^{\pi}=1.5$. The paths of the output gap in these two cases are shown in Graph 8. As can be seen, case (b) is more recessionary than (a) confirming the common fear that QT will not be without real costs.

Graph 8
Output gap with QE and with QT

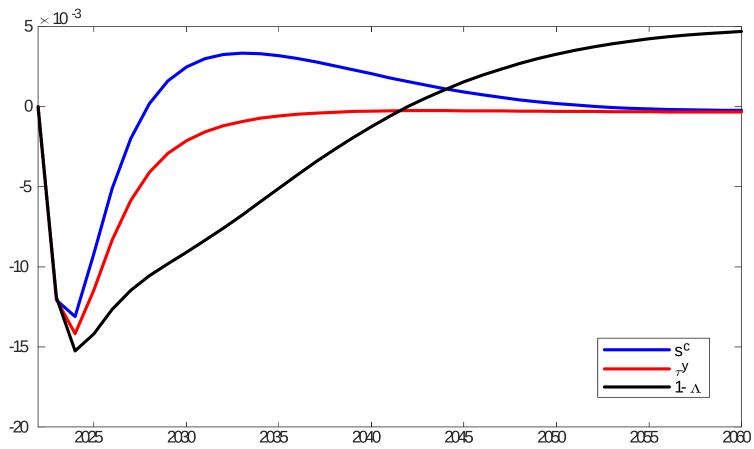


5.2 Inequality and QE

One of the possible reservations in the literature is that QE policies can worsen income inequality. In this subsection, we try to give a quantitative answer to this argument. To do so, we add household heterogeneity in the simplest possible way: we distinguish between Ricardian households and hand-to-mouth households. The former are as in subsection 2.2 above which means that these households save and own the private firms and banks. The latter just live on their labor income and government transfers.

The augmented model is presented in Appendix A.8. Here, we just present the final numerical results for inequality as measured by the net income ratio of the two income groups, namely, the net income of hand-to-mouth households to the net income of Ricardian households, so an increase in this ratio translates to lower income inequality. Graph 9 plots this ratio as deviation from its initial steady solution and it does so under the same debt stabilization policies studied so far: the case in which debt stabilization is restored by adjustments in public consumption as share of GDP (s_t^c), the case in which this is achieved by adjustments in the income tax rate (τ_t^y) and, finally, the case in which this is achieved by adjustments in the share of sovereign bonds purchased by the CB in the secondary market ($1 - \Lambda_t$); note that, in this set of policy experiments as it was also the case in subsection 4.2 above, quantitative monetary policy is not restricted by ES type constraints.[39] Regarding interest rate policy, as above, $\gamma^\pi = 1.5$. As can be seen, the use of $(1 - \Lambda_t)$ is accompanied by higher income inequality in the short- and medium-term.[40] This mainly happens because bond purchases by the CB boosts their prices, and this benefits the Ricardian households as bank owners and thus bond holders. Notice also that the use of $(1 - \Lambda_t)$ is characterized by slower convergence to the steady state.

Graph 9
Net income ratio (deviation from 2022 value)



[39]For these experiments, we set the associated feedback policy coefficients on debt at the minimum value required for stability in each case studied. This means $\gamma^\Lambda = 3.5$, $\gamma^{s^c,b} = 0.02$ and $\gamma^{\tau^y,b} = 0.02$.

[40]Inequality decreases after some time with $(1 - \Lambda_t)$ but this happens only because QE in the short- and medium-term is being followed by QT.

6 Caveats and extensions

In this paper, we investigated whether QE type policies can substitute spending cuts and/or tax rises for public debt stabilization in an otherwise unstable model. Our answer is a qualified "yes" in general, although the ES seems to have exhausted much of its room for further fiscal-type manoeuvre given its self-imposed upper limit on sovereign bond holdings and the non-allowance of fiscal support.

Since the main results have already been listed in some detail in the Introduction, we close with caveats and possible extensions. Here, we have used an aggregate model for the EA. Such a model can easily mask differences across member countries and their relevance to the general equilibrium effects of quantitative monetary policies, where the latter can be either country-specific or one-size-fits-all. We leave the study of these issues within a two-region model for the EA for future research.

References

[1] Alesina A., I. Angeloni and F. Etro (2005): International unions, American Economic Review, 95, 602-615.

[2] Angelini E., K. Bokan, K. Christoffel, M. Ciccarelli and S. Zimic (2019): Introducing ECB-BASE: The blueprint of the new ECB semi-structural model of the euro area, European Central Bank, Working Paper, no. 2315.

[3] Baier S. and G. Glomm (2001): Long-run growth and welfare effects of public policies with distortionary taxation, Journal of Economic Dynamics and Control, 25, 2007-2042.

[4] Bankowski K., K. Christoffel and T. Faria (2021): Assessing the fiscal-monetary policy mix in the euro area, European Central Bank, Working Paper, no. 2623.

[5] Bankowski K. (2023): Fiscal policy in the semi-structural model ECB-BASE, European Central Bank, Working Paper, no. 2802.

[6] Bassetto M. and T. Messer (2013): Fiscal consequences of paying interest on reserves, Fiscal Studies, 34, 413-436.

[7] Bassetto M. and T. Sargent (2020): Shotgun wedding: Fiscal and monetary policy, NBER Working Paper, no. 27004.

[8] Baxter M. and R. King (1993): Fiscal policy in general equilibrium, American Economic Review 83, 315–334.

[9] Beck T., A. Colciago and D. Pfajfar (2014): The role of financial intermediaries in monetary policy transmission, Journal of Economic Dynamics and Control, 43, 1-11.

[10] Belhocine N., A. vir Bhatia and J. Frie (2023): Raising rates with a large balance sheet: The ES's net income and its fiscal implications, International Monetary Fund, Working Paper, no. WP/23/145.

[11] Benigno P. and Nistico (2020): Non-neutrality of open market operations, American Economic Journal: Macroeconomics, 12, 175-226.

[12] Benigno P., P. Canofari, G. Di Bartolomeo and M. Messori (2022): The ECB's asset purchase programme: Theory, effects and risks, Journal of Economic Surveys, 1-25.

[13] Bernanke B. (2020): The new tools of monetary policy, American Economic Review, 110, 943-983.

[14] Bianchi F., R. Faccini and L. Melosi (2021): Monetary and fiscal policies in times of large debt: Unity is strength, mimeo.

[15] Blanchard O. and F. Giavazzi (2003): Macroeconomic effects of regulation and deregulation in goods and labor markets, Quarterly Journal of Economics, 118, 879-907.

[16] Blanchard O. (2019): Public debt and low interest rates, American Economic Review, 109, 1197-1229.

[17] Borio C. and P. Disyatat (2009): Unconventional monetary policies: An appraisal, Bank for International Settlements Working Paper, no. 292.

[18] Buiter W. (2021): Central Banks as Fiscal Players, Cambridge University Press.

[19] Canova F. and E. Pappa (2011): Fiscal policy, pricing frictions and monetary accommodation, Economic Policy, 26, 555-598.

[20] Canzonei M., R. Cumby and B. Diba (2011): The interaction between monetary and fiscal policy, in Handbook of Monetary Economics, volume 3B, edited by B. Friedman and M. Woodford, North Holland.

[21] Chadha J., L. Corrado, J. Meaning and T. Schuler (2021): Monetary and fiscal complementarity in the covid-19 pandemic, European Central Bank, Working Paper no. 2588.

[22] Chen X., E. Leeper and C. Leigh (2019): US monetary and fiscal policies - conflict or cooperation? Adam Smith Business School University of Glasgow, Working Paper, no. 2020-04.

[23] Coenen G., P. Karadi, S. Schmidt and A. Warne (2018): The New Area-Wide Model II: An extended version of the ECB's micro-founded model for forecasting and policy analysis with a financial sector, European Central Bank, Working Paper, no. 2200, revised in December 2019 .

[24] Coenen G., C. Montes-Galdon and F. Smets (2020): Effects of state-dependent forward guidance, large-scale asset purchases and fiscal stimulus in a low-interest-rate environment, European Central Bank, Working Paper no. 2352.

[25] Corsetti G., K. Kuester, A. Meir and G. Muller (2013): Sovereign risk, fiscal policy and macroeconomic stability, Economic Journal, 123, F99-F132.

[26] Cúrdia V. and M. Woodford (2011): The central-bank balance sheet as an instrument of monetray policy, Journal of Monetary Economics, 58, 54-79.

[27] Darracq Paries M., J. Korner and N. Papadopoulou (2019): Empowering central bank asset purchases: The role of financial policies, European Central Bank, Working Paper, no. 2237.

[28] Davig T., E. Leeper and T. Walker (2010): Unfunded liabilities and uncertain fiscal financing, Journal of Monetary Economics, 57, 600-619.

[29] Davig T. and E. Leeper (2011): Monetary-fiscal policy interactions and fiscal stimulus, European Economic Review, 55, 211-227.

[30] Del Negro M. and C. Sims (2015): When does a central bank's balance sheet require fiscal support? Journal of Monetary Economics, 73, 1-19.

[31] D' Erasmo P., E. Mendoza and J. Zhang (2015): What is sustainable public debt?, in Handbook of Macroeconomics, volume 2, edited by J. Taylor and H. Uhlig, North Holland.

[32] Eggertsson G., A. Ferrero and A. Raffo (2014): Can structural reforms help Europe?, Journal of Monetary Economics, 61, 2-22.

[33] Fabiani J., M. Fidora, R. Setzer, A. Westphal and N. Zorell (2021): Sudden stops and asset purchase programmes in the Euro Area, European Central Bank, Working Paper, no. 2597.

[34] Galí J. (2019): The effects of a money-financed fiscal stimulus, NBER Working Paper, no. 26249.

[35] Gertler M. and P. Karadi (2011): A model of unconventional monetary policy, Journal of Monetary Economics, 58, 17-34.

[36] Gertler M. and P. Karadi (2013): QE 1 vs. 2 vs. 3 ...: A Framework for analyzing large-scale asset purchases as a monetary policy tool, International Journal of Central Banking, 9, 5-53.

[37] Gertler M. and N. Kiyotaki (2010): Financial intermediation and credit policy in business cycle analysis, in Handbook of Monetary Economics, volume 3A, edited by B. Friedman and M. Woodford, North-Holland.

[38] Gomes P. and H. Seoane (2023): Made in Europe: Monetary-fiscal policy mix with financial frictions, mimeo.

[39] Hall G. and T. Sargent (2022): Three world wars: Fiscal and monetary consequences, mimeo.

[40] Hartmann P. and F. Smets (2018): The first 20 years of the ECB: Monetary policy, European Central Bank, Working Paper, no. 2219.

[41] Hauptmeier S., C. Kamps and L. Radke (2022): Counter-cyclical fiscal rules and the ZLB, European Central Bank, Working Paper, no. 2715.

[42] Havlik A. and F. Heinemann (2020): Sliding down the slippery slope? Trends in the rules and country allocations of the ES's PSPP and PEPP, EconPol Policy Report, 21/2020.

[43] Hohberger S., R. Priftis and L. Vogel (2019a): The macroeconomic effects of QE in the euro area: Evidence from an estimated DSGE model, Journal of Economic Dynamics and Control, 108, 1-31.

[44] Hohberger S., R. Priftis and L. Vogel (2019b): The distributional implications of conventional monetary policy and QE: Evidence from an estimated DSGE model, Bank of Canada Staff, Working Paper, no. 2019-6.

[45] Hooley J., A. Khan, C. Lattie, I. Mak, N. Salazar, A. Sayegh and P. Stella (2023): Quasi-fiscal implications of central bank crisis interventions, IMF, Working Paper, no. 23/114.

[46] Hull J. (2008): Fundamentals of Futures and Options Markets, Pearson Prentice Hall, sixth edition.

[47] Jiang W., T. Sargent, N. Wang and J. Yang (2022): A p theory of government debt and taxes, mimeo.

[48] Kabaca S., R. Maas, K. Mavromatis and R. Priftis (2023): Opitmal quantitative easing in a monetary union, European Economic Review, forthcoming.

[49] Kirsanova T., C. Leigh and S. Wren-Lewis (2009): Monetary and fiscal policy interaction: The current consensus assignment in the light of recent developments, Economic Journal, 199, F482-F496.

[50] Korinek A. and E. Mendoza (2014): From sudden stops to Fisherian deflation: Quantitative theory and policy, Annual Review of Economics, 6, 299-332.

[51] Kurovskiy G., L. Lambertini and S. Tsiaras (2022): Active and passive (un)conventional monetary and fiscal policies for debt stability, Ecole Polytechnique Federale de Lausanne, mimeo.

[52] Leeper E. (1991): Equilibria under "active" and "passive" monetary and fiscal policies, Journal of Monetary Economics, 27, 129-147.

[53] Leeper E. (2016): Why central banks should care about fiscal rules, Sveriges Riksbank Economic Review, 3, 109-125.

[54] Leeper E., M. Plante and N. Traum (2010): Dynamics of fiscal financing in the U.S., Journal of Econometrics, 156, 304-321.

[55] Leeper E., T. Walker and S. Yang (2010): Government investment and fiscal stimulus, Journal of Monetary Economics, 57, 1000-1012.

[56] Leeper E. (2021): Shifting policy norms and policy interactions, mimeo.

[57] Leeper E. (2023): Fiscal dominance: How worried should we be?, Policy Brief, Mercatus Centre, George Mason University.

[58] Leith C. and S. Wren-Lewis (2008): Interactions between monetary and fiscal policy under flexible exchange rates, Journal of Economic Dynamics and Control, 32, 2854-2882.

[59] Malley J. and A. Philippopoulos (2022): The macroeconomic effects of funding US infrastructure, forthcoming in European Economic Review.

[60] Mackowiak B. and S. Schmidt (2023): Passive monetary policy and active fiscal policy in a monetary union, European Central Bank, Working Paper no. 2781.

[61] Miao J. (2014): Economic Dynamics in Discrete Time, MIT Press.

[62] Philippopoulos A., V. Vassilatos and P. Varthalitis (2017): Fiscal consolidation and its cross-country effects, Journal of Economic Dynamics and Control, 83, 55-106.

[63] Ramey V. (2020): The macroeconomic consequences of infrastructure investment, NBER, Working Paper no. 27625.

[64] Reis R. (2013): The mystique surrounding the central bank's balance sheet, applied to the European crisis, American Economic Review, 103, 135-140.

[65] Reis R. (2017): Can the central bank alleviate fiscal burdens? CESifo, Working Paper, no. 6604-2017. Published in The Economics of Central Banking, edited by Mayes, Siklos and Sturm, Oxford University Press, 2019.

[66] Rostagno M., C. Altavilla, G. Carboni, W. Lemke, R. Motto, A. Saint Guilhem and J. Yiangou (2019): A tale of two decades: The ECB's monetary policy at 20, European Central Bank, Working Paper no. 2346.

[67] Rostagno M., C. Altavilla, G. Carboni, W. Lemke, R. Motto and A. Saint Guilhem (2021): Combining negative rates, forward guidance and asset purchases: Identification and impacts of the ECB's unconventional policies, European Central Bank, Working Paper no. 2564.

[68] Schmitt-Grohé S. and M. Uribe (2007): Optimal simple and implementable monetary and fiscal rules, Journal of Monetary Economics, 54, 1702-1725.

[69] Sims E. and J. Wolff (2017): State-dependent fiscal multipliers: Calvo vs. Rotemberg, Economics Letters,159, 190-194.

[70] Sims E. and J. Wolff (2018): The state-dependent effects of tax shocks, European Economic Review, 107, 57-85.

[71] Sims E. and J.C. Wu (2020): Are QE and conventional monetary policy substitutable?, International Journal of Central Banking, 16, 195-230.

[72] Sims E. and J.C. Wu (2021): Evaluating central banks' tool kit: past present and future, Journal of Monetary Economics, 118, 135-160.

[73] Smets F. and R. Wouters (2003): An estimated dynamic stochastic general equilibrium model of the Euro Area, Journal of the European Economic Association, 1, 1123-1175.

[74] Uribe M. and S. Schmitt-Grohé (2017): Open Economy Macroeconomics, Princeton University Press.

[75] Villa S. (2013): Financial frictions in the euro area: A Bayesian assessment, European Central Bank, Working Paper no. 1521.

[76] Wallace N. (1981): A Modigliani-Miller theorem for open market operations, American Economic Review, 71, 267-274.

[77] Walsh C. (2017): Monetary Theory and Policy, MIT Press, 4th edition.

[78] Wu J.C. and Y. Xie (2022): (Un)Conventional monetary and fiscal policy, mimeo.

Appendices

A.1 Solution of households' problem

The first-order conditions of each h for $c_{h,t}$, $l_{h,t}$, $j_{h,t}$, $m_{h,t}$ are respectively:

$$\frac{\mu_1}{c_{h,t}} = (\lambda_{h,t} + \psi_{h,t})(1 + \tau_t^c) \tag{A.1a}$$

$$\frac{\mu_2}{(1 - l_{h,t})} = \lambda_{h,t}(1 - \tau_t^y) w_t \tag{A.1b}$$

$$\lambda_{h,t} = \beta \lambda_{h,t+1}(1 + i_{t+1}^d) \frac{p_t}{p_{t+1}} \tag{A.1c}$$

$$\lambda_{h,t} - \psi_{h,t} = \beta \lambda_{h,t+1} \frac{p_t}{p_{t+1}} \tag{A.1d}$$

where $\lambda_{h,t}$ and $\psi_{h,t}$ are Lagrangean multipliers associated with the budget constraint and the cash-in-advance constraint respectively and we also have:

$$\psi_{h,t}[(1 + \tau_t^c) c_{h,t} - m_{h,t}] = 0 \tag{A.1e}$$

Leisure hours, $u_{h,t}$, will follow residually from the time constraint, eq. (2a) in the main text.

A.2 Solution of final good firms' problem

Final good firms act competitively. The first-order condition of each f for $y_{f,i,t}$, and since $y_{f,i,t} = \frac{y_{i,t}}{N}$, gives the standard demand function:

$$p_{i,t} = p_t \left(\frac{y_{i,t}}{y_{f,t}} \right)^{\theta - 1} \tag{A.2}$$

That is, in a symmetric equilibrium, we simply have $y_{f,t} = y_{i,t}$, $p_t = p_{i,t}$ and $\pi_{f,t} = 0$.

A.3 Solution of intermediate goods firms' problem

Intermediate goods firms act monopolistically in their own product market. The first-order conditions of each i for $l_{i,t}$, $k_{i,t}$ and $L_{i,t}$ are respectively:

$$(1-\tau_t^\pi) w_t + N_{i,t} \eta_i w_t = [(1-\tau_t^\pi)\theta \left(\frac{y_{i,t}}{y_{f,t}} \right)^{\theta-1} - \xi^p \left(\frac{p_{i,t}}{p_{i,t-1}} - 1 \right) \frac{p_t}{p_{i,t-1}} (\theta-1) \left(\frac{y_{i,t}}{y_{f,t}} \right)^{\theta-1} \frac{\overline{y}_{i,t}}{y_{i,t}} +$$

$$+ \frac{\beta \lambda_{h,t+1}}{\lambda_{h,t}} \xi^p \left(\frac{p_{i,t+1}}{p_{i,t}} - 1 \right) \frac{p_{i,t+1}}{(p_{i,t})^2} p_t (\theta - 1) \left(\frac{y_{i,t}}{y_{f,t}} \right)^{\theta-1} \frac{\overline{y}_{i,t+1}}{y_{i,t}}] \frac{\partial y_{i,t}}{\partial l_{i,t}} \tag{A.3a}$$

$$1 = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}[1-\delta+(1-\tau^{\pi}_{t+1})\theta\left(\frac{y_{i,t}}{y_{f,t}}\right)^{\theta-1}\frac{\partial y_{i,t+1}}{\partial k_{i,t}}]-$$

$$-\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^{p}\left(\frac{p_{i,t+1}}{p_{i,t}}-1\right)\frac{p_{t+1}}{p_{i,t}}(\theta-1)\frac{\overline{y}_{i,t+1}}{y_{i,t+1}}\left(\frac{y_{i,t+1}}{y_{f,t+1}}\right)^{\theta-1}\frac{\partial y_{i,t+1}}{\partial k_{i,t}}+$$

$$+\frac{\beta^{2}\lambda_{h,t+2}}{\lambda_{h,t}}\xi^{p}\left(\frac{p_{i,t+2}}{p_{i,t+1}}-1\right)\frac{p_{i,t+2}}{(p_{i,t+1})^{2}}p_{t+1}(\theta-1)\frac{\overline{y}_{i,t+2}}{y_{i,t+1}}\left(\frac{y_{i,t+1}}{y_{f,t+1}}\right)^{\theta-1}\frac{\partial y_{i,t+1}}{\partial k_{i,t}} \tag{A.3b}$$

$$1+N_{i,t} = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\left(1+i^{l}_{t+1}\right)\frac{p_t}{p_{t+1}} \tag{A.3c}$$

where $N_{i,t}$ is the multiplier associated with the firm's working capital constraint.and we also have:

$$N_{i,t}(L_{i,t}-\eta_i w_t l_{i,t}) = 0 \tag{A.3d}$$

Finally, the TFP, A_t, is assumed to follow an $AR(1)$ process of the form:

$$A_t = A_t^{\rho^A} A^{1-\rho^A} + \varepsilon^A_t \tag{A.3e}$$

where A denotes the steady state value, $0 < \rho^A < 1$ is the persistence parameter, and ε^A_t is a shock term.

A.4 Solution of private banks' problem

We solve private banks' maximization problem working as in Sims and Wu (2021). Each p's value function satisfies the Bellman:

$$V_{p,t} = \max\left(1-\sigma\right)\beta_{t,t+1}n_{p,t+1} + \sigma\beta_{t,t+1}V_{p,t+1} \tag{A.4a}$$

Using the bank's balance sheet in (10) to substitute out $j_{p,t}$, we can rewrite the bank's net worth in (11) as:

$$n_{p,t} = \frac{p_{t-1}}{p_t}\{(1-\tau^{\pi}_t)(i^l_t - i^d_t)L_{p,t-1} + (1-\tau^{\pi}_t)\left(i^b_t - i^d_t\right)b_{p,t-1} + \tag{A.4b}$$

$$+(1-\tau^{\pi}_t)(i^r_t - i^d_t)m_{p,t-1} - (1-\tau^{\pi}_t)(i^z_t - i^d_t)z_{p,t-1} +$$

$$+\left[1+(1-\tau^{\pi}_t)i^d_t\right]n_{p,t-1}\}$$

so that (A.4a) becomes:

$$V_{p,t} = \max\left(1-\sigma\right)\beta_{t,t+1}\frac{p_t}{p_{t+1}}\{\left(1-\tau^{\pi}_{t+1}\right)(i^l_{t+1} - i^d_{t+1})L_{p,t} + \tag{A.4c}$$

$$+\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^{b}-i_{t+1}^{d}\right)b_{p,t}+\left(1-\tau_{t+1}^{\pi}\right)(i_{t+1}^{r}-i_{t+1}^{d})m_{p,t}-$$

$$-\left(1-\tau_{t+1}^{\pi}\right)(i_{t+1}^{z}-i_{t+1}^{d})z_{p,t}+\left[1+\left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^{d}\right]n_{p,t}\}+\sigma\beta_{t,t+1}V_{p,t+1}$$

which is like equation (A.10) in Sims and Wu (2021).

In what follows, since θ is a constant, for notational simplicity we rewrite the bank's incentive constraint in (13) as:

$$V_{p,t}\geq\xi^{l}L_{p,t}+\xi^{b}b_{p,t}+\xi^{m}m_{p,t}-\xi^{z}z_{p,t}\tag{A.4d}$$

where $\xi^{l}\equiv\vartheta$, $\xi^{b}\equiv\vartheta N^{b}$, $\xi^{m}\equiv\vartheta N^{m}$ and $\xi^{z}\equiv\vartheta N^{z}$, where the ξ's will be calibrated to match intrerest rate differentials as in the data. Note that we assume that $\xi^{l}L_{p,t}+\xi^{b}b_{p,t}+\xi^{m}m_{p,t}-\xi^{z}z_{p,t}<L_{p,t}+b_{p,t}+m_{p,t}-z_{p,t}$ and this will be confirmed in equilibrium.

The Lagrangean of this problem, including the constraint (A.4d), is:

$$\mathcal{L}_{p,t}\equiv(1+\zeta_{t})\left\{(1-\sigma)\beta_{t,t+1}\frac{p_{t}}{p_{t+1}}\left\{\left(1-\tau_{t+1}^{\pi}\right)(i_{t+1}^{l}-i_{t+1}^{d})L_{p,t}+\right.\right.$$

$$+\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^{b}-i_{t+1}^{d}\right)b_{p,t}+\left(1-\tau_{t+1}^{\pi}\right)(i_{t+1}^{r}-i_{t+1}^{d})m_{p,t}-$$

$$-\left(1-\tau_{t+1}^{\pi}\right)(i_{t+1}^{z}-i_{t+1}^{d})z_{p,t}+\left[1+\left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^{d}\right]n_{p,t}\}$$

$$\left.+\sigma\beta_{t,t+1}V_{p,t+1}\right\}-\zeta_{t}\left(\xi^{l}L_{p,t}+\xi^{b}b_{p,t}+\xi^{m}m_{p,t}-\xi^{z}z_{p,t}\right)$$

where ζ_{t} is the multiplier associated with (A.4d).

Each p's first-order conditions for $L_{p,t}$, $b_{p,t}$, $z_{p,t}$, $m_{p,t}$ are respectively:

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_{t}}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)(i_{t+i}^{l}-i_{t+i}^{d})=\frac{\zeta_{t}}{(1+\zeta_{t})}\xi^{l}\tag{A.4e}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_{t}}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)(i_{t+i}^{b}-i_{t+i}^{d})=\frac{\zeta_{t}}{(1+\zeta_{t})}\xi^{b}\tag{A.4f}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_{t}}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^{z}-i_{t+1}^{d}\right)=\frac{\zeta_{t}}{(1+\zeta_{t})}\xi^{z}\tag{A.4g}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_{t}}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^{r}-i_{t+1}^{d}\right)=\frac{\zeta_{t}}{(1+\zeta_{t})}\xi^{m}\tag{A.4h}$$

where Ω_{t+1} is defined below and $\beta_{t,t+1}$ equals the household's marginal rate of substitution between consumption at t and $t+1$, i.e. $\beta_{t,t+1} \equiv \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}$.

To derive an expression for Ω_t, since the underlying problem is linear, we guess that the value function is linear in net worth:

$$V_{p,t} = \phi_t n_{p,t} \tag{A.4i}$$

so that Ω_{t+1} is:

$$\Omega_{t+1} \equiv 1 - \sigma + \sigma\phi_{t+1} \tag{A.4j}$$

Using (A.4i) and (A.4j), we rewrite (A.4a) as:

$$\phi_t n_{p,t} = (1-\sigma)\,\beta_{t,t+1} n_{p,t+1} + \sigma\beta_{t,t+1}\phi_{t+1} n_{p,t+1} = \tag{A.4k}$$

$$= \beta_{t,t+1} n_{p,t+1}\,(1-\sigma+\sigma\phi_{t+1}) =$$

$$= \beta_{t,t+1} n_{p,t+1}\Omega_{t+1}$$

To generate the RHS of (A.4k), we move (A.4b) one period forward and multiply by $\beta_{t,t+1}\Omega_{t+1}$. Then, using the first-order conditions above, we get:

$$\beta_{t,t+1}\Omega_{t+1} n_{p,t+1} = \frac{\zeta_t}{(1+\zeta_t)}(\xi^l L_{p,t} + \xi^b b_{p,t} + \xi^m m_{p,t} - \xi^z z_{p,t}) + \tag{A.4l}$$

$$+\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1 + \left(1-\tau^\pi_{t+1}\right) i^d_{t+1}\right] n_{p,t}$$

which holds when the incentive constraint binds, $V_{p,t} = \xi^l L_{p,t} + \xi^b b_{p,t} + \xi^m m_{p,t} - \xi^z z_{p,t}$.

Then, if we combine $(A.4i)$, $(A.4k)$ and $(A.4l)$, we get:

$$V_{p,t} = \phi_t n_{p,t} =$$

$$= \beta_{t,t+1} n_{p,t+1}\Omega_{t+1} =$$

$$= \frac{\zeta_t}{(1+\zeta_t)}(\xi^l L_{p,t} + \xi^b b_{p,t} + \xi^m m_{p,t} - \xi^z z_{p,t}) + \beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1 + \left(1-\tau^\pi_{t+1}\right) i^d_{t+1}\right] n_{p,t} =$$

$$= \frac{\zeta_t}{(1+\zeta_t)} V_{p,t} + \beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1 + \left(1-\tau^\pi_{t+1}\right) i^d_{t+1}\right] n_{p,t} =$$

$$= \frac{\zeta_t}{(1+\zeta_t)}\phi_t n_{p,t} + \beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1 + \left(1-\tau^\pi_{t+1}\right) i^d_{t+1}\right] n_{p,t}$$

And, after some calculations, we get for ϕ_t:

$$\phi_t = (1+\zeta_t)\,\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1+\left(1-\tau^{\pi}_{t+1}\right)i^d_{t+1}\right] \tag{A.4m}$$

which is similar to equation (2.15) in Sims and Wu (2021).

Aggregation: Aggregate the balance sheet condition of private banks in (10):

$$L^T_{p,t} + b^T_{p,t} + m^T_{p,t} = j^T_{p,t} + z^T_{p,t} + N^T_{p,t} \tag{A.4n}$$

where $N^T_{p,t}$ is the total net worth of private banks in the beginning of t. We can derive an equation of motion for $N^T_{p,t}$, by first recognizing that it is the sum of the net worth of "surviving" bankers and the net worth of "entering" bankers. The latter is equal to the "start up" funds provided by households, $\gamma(L_{p,t-1}+b_{p,t-1}+m_{p,t-1})$, where γ is a parameter (see also Gertler and Karadi (2011)). Thus, we have:

$$N^T_{p,t} = \sigma n^T_{p,t} + \gamma\frac{p_{t-1}}{p_t}\left\{L^T_{p,t-1}+b^T_{p,t-1}+m^T_{p,t-1}\right\} \tag{A.4o}$$

where the first term on the RHS is the net worth of banks that stay in the market and the second term is households' transfers to new bankers.

Aggregating ($A.4b$), the net worth of banks that remain in the market, $\eta^T_{p,t}$, is given by:

$$n^T_{p,t} = \frac{p_{t-1}}{p_t}\{(1-\tau^{\pi}_t)\,(i^l_t - i^d_t)L^T_{p,t-1} + (1-\tau^{\pi}_t)\left(i^b_t - i^d_t\right)b^T_{p,t-1} + \tag{A.4p}$$

$$+(1-\tau^{\pi}_t)\,(i^r_t - i^d_t)m^T_{p,t-1} - (1-\tau^{\pi}_t)\,(i^z_t - i^d_t)z^T_{p,t-1} +$$

$$+(1+(1-\tau^{\pi}_t)i^d_t)n^T_{p,t-1}\}$$

Banks' profits transferred to households are:

$$\pi^T_{p,t} = (1-\sigma)\,n^T_{p,t} \tag{A.4q}$$

which is the wealth of exiting banks.

Aggregating ($A.4h$), we have:

$$V^T_{p,t} = \phi_t N^T_{p,t} \tag{A.4r}$$

Aggregating ($A.4d$), we have:

$$V^T_{p,t} \geq \xi^l L^T_{p,t} + \xi^b b^T_{p,t} + \xi^m m^T_{p,t} - \xi^z z^T_{p,t} \tag{A.4s}$$

which, as said above, is assumed to hold with equality.

Therefore, in this block of the model, we have 12 variables, $V^T_{p,t}$, $L^T_{p,t}$, $b^T_{p,t}$, $m^T_{p,t}$, $z^T_{p,t}$, $j^T_{p,t}$, ζ_t, $n^T_{p,t}$, $N^T_{p,t}$, $\pi^T_{p,t}$, ϕ_t, Ω_t, in 12 equations, ($A.4e$)-($A.4h$), ($A.4j$) and ($A.4m$)-($A.4s$).

A.5 Solution of private banks' problem when they sell bonds to the CB

In this appendix, we present the banks' problem when they can sell bonds to the CB in the secondary market. We will present what changes relative Appendix A.4.

The equation for net worth is now:

$$n_{p,t} = \frac{p_{t-1}}{p_t}\{(1-\tau_t^\pi)\,(i_t^l - i_t^d)L_{p,t-1} + (1-\tau_t^\pi)\,(i_t^r - i_t^d)m_{p,t-1} + \tag{A.5a}$$

$$+\left[\Lambda_t\left(1+(1-\tau_t^\pi)\,i_t^b\right) + (1-\Lambda_t)\,\Phi_t - \left(1+(1-\tau_t^\pi)\,i_t^d\right)\right] b_{p,t-1} -$$

$$-(1-\tau_t^\pi)\,(i_t^z - i_t^d)z_{p,t-1} + \left(1+(1-\tau_t^\pi)\,i_t^d\right) n_{p,t-1}\}$$

Working as in Appendix A.4, the four optimality conditions for $L_{p,t}$, $b_{p,t}$, $z_{p,t}$, $m_{p,t}$, are given by:

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^\pi\right)(i_{t+i}^l - i_{t+i}^d) = \frac{\zeta_t}{(1+\zeta_t)}\xi^l \tag{A.5b}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[\Lambda_{t+1}(1+\left(1-\tau_{t+1}^\pi\right)i_{t+1}^b) + \Phi_{t+1}\,(1-\Lambda_{t+1}) - (1+\left(1-\tau_{t+1}^\pi\right)i_{t+1}^d)\right] = \tag{A.5c}$$

$$= \frac{\zeta_t}{(1+\zeta_t)}\xi^b$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^\pi\right)\left(i_{t+1}^z - i_{t+1}^d\right) = \frac{\zeta_t}{(1+\zeta_t)}\xi^z \tag{A.5d}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^\pi\right)\left(i_{t+1}^r - i_{t+1}^d\right) = \frac{\zeta_t}{(1+\zeta_t)}\xi^m \tag{A.5e}$$

where $\Omega_{t+1} = 1-\sigma+\sigma\phi_{t+1}$ and $\Phi_t \equiv \kappa[1+(1-\tau_t^\pi)\,i_t^b]$.

Aggregation: The total net worth of private banks in the beginning of period t is now given by:

$$N_{p,t}^T = \sigma n_{p,t}^T + \gamma\frac{p_{t-1}}{p_t}\left\{L_{p,t-1}^T + \Lambda_t b_{p,t-1}^T + m_{p,t-1}^T\right\} \tag{A.5f}$$

where the net worth of banks that stay in the market, $\eta_{p,t}^T$, is:

$$n_{p,t}^T = \frac{p_{t-1}}{p_t}\{(1-\tau_t^\pi)\,(i_t^l - i_t^d)L_{p,t-1}^T + (1-\tau_t^\pi)\,(i_t^r - i_t^d)m_{p,t-1}^T + \tag{A.5g}$$

$$+\left[\Lambda_t\left(1+(1-\tau_t^{\pi})\,i_t^b\right)+(1-\Lambda_t)\,\Phi_t-\left(1+(1-\tau_t^{\pi})\,i_t^d\right)\right]b_{p,t-1}^T-$$

$$-(1-\tau_t^{\pi})(i_t^z-i_t^d)z_{p,t-1}^T+(1+(1-\tau_t^{\pi})i_t^d)n_{p,t-1}^T\}$$

The rest of equations are as in Appendix A.4.

A.6 Macroeconomic system (when the CB participates in the primary bond market)

A.6.1 Market-clearing conditions

In the market for dividends:

$$\pi_{h,t}=\pi_{i,t}+\pi_{p,t}-\gamma(L_{p,t-1}+b_{p,t-1}+m_{p,t-1}) \tag{1}$$

In the labor market:

$$l_{h,t}=l_{i,t}=l_t \tag{2}$$

In the market for bank deposits:

$$j_{h,t}=j_{p,t}^T=j_t \tag{3}$$

In the market for bank loans:

$$L_{i,t}=L_{p,t}^T=L_t \tag{4}$$

In the primary bond market:

$$b_{p,t}^T+b_{cb,t}=b_t \tag{5}$$

where $b_{p,t}^T=\Lambda_t b_t$ and $b_{cb,t}=(1-\Lambda_t)\,b_t$.

In the money market:

$$m_t=m_{h,t}+m_{p,t}^T \tag{6}$$

A.6.2 Equations and unknowns

Collecting equations, the macroeconomic system that we solve numerically consists of the following equations:

Households

$$\frac{\mu_1}{c_{h,t}} = (\lambda_{h,t} + \psi_{h,t})(1+\tau_t^c) \tag{A.6.1}$$

$$\frac{\mu_2}{(1-l_t)} = \lambda_{h,t}(1-\tau_t^y)w_t \tag{A.6.2}$$

$$\lambda_{h,t} = \beta\lambda_{h,t+1}(1+i_{t+1}^d)\frac{p_t}{p_{t+1}} \tag{A.6.3}$$

$$\lambda_{h,t} - \psi_{h,t} = \beta\lambda_{h,t+1}\frac{p_t}{p_{t+1}} \tag{A.6.4}$$

$$\psi_{h,t}\left((1+\tau_t^c)c_{h,t} - m_{h,t}\right) = 0 \tag{A.6.5}$$

$$(1+\tau_t^c)c_{h,t} + j_t + m_{h,t} \equiv$$

$$\equiv (1-\tau_t^y)w_t l_t + (1+i_t^d)\frac{p_{t-1}}{p_t}j_{t-1} + \frac{p_{t-1}}{p_t}m_{h,t-1} + \pi_{h,t} + g_t^t \tag{A.6.6}$$

Firms In a symmetric equilibrium, $y_{f,t} = y_{i,t} \equiv y_t$, $k_{i,t} \equiv k_t$ and $p_{i,t} = p_t$. Thus,

$$\pi_{i,t} = (1-\tau_t^\pi)(y_t - w_t l_t) - x_t - \frac{\xi^p}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 y_t + \left(L_t - \left(1+i_t^l\right)\frac{p_{t-1}}{p_t}L_{t-1}\right) \tag{A.6.7}$$

$$k_t = x_t + (1-\delta)k_{t-1} \tag{A.6.8}$$

$$y_t = A\left(k_{t-1}^g\right)^\varepsilon (k_{t-1}^\alpha l_t^{1-\alpha})^{1-\varepsilon} \tag{A.6.9}$$

$$(1-\tau_t^\pi)w_t + N_{i,t}\eta_i w_t = [(1-\tau_t^\pi)\theta - \xi^p\left(\frac{p_t}{p_{t-1}} - 1\right)\frac{p_t}{p_{t-1}}(\theta-1) +$$
$$+\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t}\frac{(\theta-1)y_{t+1}}{y_t}]\frac{\partial y_t}{\partial l_t} \tag{A.6.10}$$

$$1 = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}[1-\delta+(1-\tau_{t+1}^\pi)\theta\frac{\partial y_{t+1}}{\partial k_t}] -$$
$$-\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t}(\theta-1)\frac{\partial y_{t+1}}{\partial k_t} +$$

$$+\frac{\beta^2\lambda_{h,t+2}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+2}}{p_{t+1}}-1\right)\frac{p_{t+2}}{p_{t+1}}(\theta-1)\frac{y_{t+2}}{y_{t+1}}\frac{\partial y_{t+1}}{\partial k_t} \tag{A.6.11}$$

$$1+N_{i,t}=\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\left(1+i^l_{t+1}\right)\frac{p_t}{p_{t+1}} \tag{A.6.12}$$

$$N_{i,t}\left(L_t-\eta_i w_t l_t\right))=0 \tag{A.6.13}$$

Private banks

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\left(1-\tau^\pi_{t+1}\right)\frac{p_t}{p_{t+1}}(i^l_{t+i}-i^d_{t+i})=\frac{\zeta_t}{(1+\zeta_t)}\xi^l \tag{A.6.14}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\left(1-\tau^\pi_{t+1}\right)\frac{p_t}{p_{t+1}}(i^b_{t+i}-i^d_{t+i})=\frac{\zeta_t}{(1+\zeta_t)}\xi^b \tag{A.6.15}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^\pi_{t+1}\right)\left(i^z_{t+1}-i^d_{t+1}\right)=\frac{\zeta_t}{(1+\zeta_t)}\xi^z \tag{A.6.16}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^\pi_{t+1}\right)\left(i^r_{t+1}-i^d_{t+1}\right)=\frac{\zeta_t}{(1+\zeta_t)}\xi^m \tag{A.6.17}$$

$$V^T_{p,t}=\phi_t N^T_{p,t} \tag{A.6.18}$$

$$V^T_{p,t}=\xi^l L_t+\xi^b\Lambda_t b_t+\xi^m m^T_{p,t}-\xi^z z^T_{p,t} \tag{A.6.19}$$

$$n^T_{p,t}=\frac{p_{t-1}}{p_t}\{(1-\tau^\pi_t)(i^l_t-i^d_t)L_{t-1}+(1-\tau^\pi_t)\left(i^b_t-i^d_t\right)\Lambda_{t-1}b_{t-1}+ \tag{A.6.20}$$

$$+(1-\tau^\pi_t)(i^r_t-i^d_t)m^T_{p,t-1}-(1-\tau^\pi_t)(i^z_t-i^d_t)z^T_{p,t-1}+$$

$$+\left[1+(1-\tau^\pi_t)i^d_t\right]n^T_{p,t-1}\}$$

$$N^T_{p,t}=\sigma n^T_{p,t}+\gamma\frac{p_{t-1}}{p_t}\left\{L_{t-1}+\Lambda_{t-1}b_{t-1}+m^T_{p,t-1}\right\} \tag{A.6.21}$$

$$L_t+\Lambda_t b_t+m^T_{p,t}=j_t+z^T_{p,t}+N^T_{p,t} \tag{A.6.22}$$

$$\pi^T_{p,t}=(1-\sigma)\,n^T_{p,t} \tag{A.6.23}$$

$$\phi_t=(1+\zeta_t)\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1+\left(1-\tau^\pi_{t+1}\right)i^d_{t+1}\right] \tag{A.6.24}$$

$$\Omega_t=1-\sigma+\sigma\phi_t \tag{A.6.25}$$

Treasury

$$g_t^c + g_t^g + g_t^t + (1 + i_t^b)\frac{p_{t-1}}{p_t}b_{t-1} = b_t + \frac{T_t}{N} + \eta_t \tag{A.6.26}$$

$$\frac{T_t}{N} \equiv \tau_t^c c_{h,t} + \tau_t^y w_t l_t + \tau_t^\pi (y_t - w_t l_t) + \tag{A.6.27}$$

$$+\tau_t^\pi \frac{p_{t-1}}{p_t}(i_t^l L_{t-1} + i_t^r m_{p,t-1}^T + i_t^b \Lambda_{t-1} b_{t-1} -$$

$$-i_t^z z_{p,t-1}^T - i_t^d j_{t-1})$$

$$k_t^g = (1 - \delta^g)k_{t-1}^g + g_t^g \tag{A.6.28}$$

Central Bank

$$(1 - \Lambda_t)b_t + z_{p,t}^T + i_t^r \frac{p_{t-1}}{p_t} m_{p,t-1}^T + \eta_t \equiv$$

$$\equiv (1 + i_t^b)(1 - \Lambda_{t-1})\frac{p_{t-1}}{p_t}b_{t-1} + (1 + i_t^z)\frac{p_{t-1}}{p_t}z_{p,t-1}^T +$$

$$+ \left(m_{h,t} + \ m_{p,t}^T\right) - \frac{p_{t-1}}{p_t}\left(m_{h,t-1} + \ m_{p,t-1}^T\right) \tag{A.6.29}$$

Dividends

$$\pi_{h,t} = \pi_{i,t} + \pi_{p,t} - \gamma \frac{p_{t-1}}{p_t}\left\{L_{t-1} + \Lambda_{t-1} b_{t-1} + m_{p,t-1}^T\right\} \tag{A.6.30}$$

Money market

$$m_t = m_{h,t} + \ m_{p,t}^T \tag{A.6.31}$$

Endogenous and exogenous variables This is a dynamic system of 31 equations in 31 variables which are $\{c_{h,t},\, j_t,\, m_{h,t},\, l_t,\, \pi_{h,t}\}_{t=0}^{\infty}$, $\{\lambda_{h,t},\, \psi_{h,t},\, N_{i,t}\}_{t=0}^{\infty}$, $\{\pi_{i,t},\, y_t,\, x_t,\, k_t,\, L_t\}_{t=0}^{\infty}$, $\{\pi_{p,t}^T,\, z_{p,t}^T,\, m_{p,t}^T,\, V_{p,t}^T,\, \zeta_t,\, n_{p,t}^T,\, N_{p,t}^T,\, \phi_t,\, \Omega_t\}_{t=0}^{\infty}$, $\{\frac{T_t}{N}\}_{t=0}^{\infty}$, $\{b_t\}_{t=0}^{\infty}$, $\{m_t\}_{t=0}^{\infty}$, $\left\{k_{g,t}^g\right\}_{t=0}^{\infty}$, $\{p_t/p_{t-1},\ i_t^b,\ i_t^d,\ i_t^l,\ w_t\}_{t=0}^{\infty}$. This is given the paths/rules of fiscal policy instruments, $\{\tau_t^c,\, \tau_t^y,\, \tau_t^\pi,\, s_t^c,\, s_t^g,\, s_t^t,\}_{t=0}^{\infty}$ and monetary policy instruments, $\{i_t^z,\, i_t^r,\, \eta_t,\, (1 - \Lambda_t)\}_{t=0}^{\infty}$. In the steady state only, b_t and s_t^t change places.

A.7 Macroeconomic system (when the CB participates in the secondary bond market)

A.7.1 Market-clearing conditions

The only market clearing that changes relative to above is the one referring to government bonds in the primary market, which now is:

$$b_{p,t}^{T} \equiv b_t \tag{7}$$

A.7.2 Equations and unknowns

Collecting equations, the macroeconomic system that we solve numerically consists of the following equations:

Households

$$\frac{\mu_1}{c_{h,t}} = (\lambda_{h,t} + \psi_{h,t})(1 + \tau_t^c) \tag{A.7.1}$$

$$\frac{\mu_2}{(1 - l_t)} = \lambda_{h,t}(1 - \tau_t^y)w_t \tag{A.7.2}$$

$$\lambda_{h,t} = \beta\lambda_{h,t+1}(1 + i_{t+1}^d)\frac{p_t}{p_{t+1}} \tag{A.7.3}$$

$$\lambda_{h,t} - \psi_{h,t} = \beta\lambda_{h,t+1}\frac{p_t}{p_{t+1}} \tag{A.7.4}$$

$$\psi_{h,t}\left((1 + \tau_t^c)c_{h,t} - m_{h,t}\right) = 0 \tag{A.7.5}$$

$$(1 + \tau_t^c)c_{h,t} + j_t + m_{h,t} \equiv$$

$$\equiv (1 - \tau_t^y)w_t l_t + (1 + i_t^d)\frac{p_{t-1}}{p_t}j_{t-1} + \frac{p_{t-1}}{p_t}m_{h,t-1} + \pi_{h,t} + g_t^t \tag{A.7.6}$$

Firms

$$\pi_{i,t} = (1-\tau_t^\pi)(y_t - w_t l_t) - x_t - \frac{\xi^p}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 y_t + \left(L_t - \left(1 + i_t^l\right)\frac{p_{t-1}}{p_t}L_{t-1}\right) \tag{A.7.7}$$

$$k_t = x_t + (1 - \delta)k_{t-1} \tag{A.7.8}$$

$$y_t = A\left(k_{t-1}^g\right)^\varepsilon \left(k_{t-1}^\alpha l_t^{1-\alpha}\right)^{1-\varepsilon} \tag{A.7.9}$$

$$(1-\tau_t^{\pi})w_t + N_{i,t}\eta_i w_t = [(1-\tau_t^{\pi})\theta - \xi^p\left(\frac{p_t}{p_{t-1}}-1\right)\frac{p_t}{p_{t-1}}(\theta-1)+$$

$$+\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t}-1\right)\frac{p_{t+1}}{p_t}\frac{(\theta-1)y_{t+1}}{y_t}]\frac{\partial y_t}{\partial l_t} \tag{A.7.10}$$

$$1 = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}[1-\delta+(1-\tau_{t+1}^{\pi})\theta\frac{\partial y_{t+1}}{\partial k_t}]-$$

$$-\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t}-1\right)\frac{p_{t+1}}{p_t}(\theta-1)\frac{\partial y_{t+1}}{\partial k_t}+$$

$$+\frac{\beta^2\lambda_{h,t+2}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+2}}{p_{t+1}}-1\right)\frac{p_{t+2}}{p_{t+1}}(\theta-1)\frac{y_{t+2}}{y_{t+1}}\frac{\partial y_{t+1}}{\partial k_t} \tag{A.7.11}$$

$$1+N_{i,t} = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\left(1+i_{t+1}^l\right)\frac{p_t}{p_{t+1}} \tag{A.7.12}$$

$$N_{i,t}\left(L_t - \eta_i w_t l_t)\right) = 0 \tag{A.7.13}$$

Private banks

$$\Phi_t = \kappa\left[1+(1-\tau_t^{\pi})i_t^b\right] \tag{A.7.14}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\left(1-\tau_{t+1}^{\pi}\right)\frac{p_t}{p_{t+1}}(i_{t+i}^l - i_{t+i}^d) = \frac{\zeta_t}{(1+\zeta_t)}\xi^l \tag{A.7.15}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[\Lambda_{t+1}(1+\left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^b)+\Phi_{t+1}\left(1-\Lambda_{t+1}\right)-(1+\left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^d)\right] = \frac{\zeta_t}{(1+\zeta_t)}\xi^b \tag{A.7.16}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^z - i_{t+1}^d\right) = \frac{\zeta_t}{(1+\zeta_t)}\xi^z \tag{A.7.17}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^r - i_{t+1}^d\right) = \frac{\zeta_t}{(1+\zeta_t)}\xi^m \tag{A.7.18}$$

$$V_{p,t}^T = \phi_t N_{p,t}^T \tag{A.7.19}$$

$$V_{p,t}^T = \xi^l L_t + \xi^b b_t + \xi^m m_{p,t}^T - \xi^z z_{p,t}^T \tag{A.7.20}$$

$$n_{p,t}^T = \frac{p_{t-1}}{p_t}\{(1-\tau_t^\pi)\,(i_t^l - i_t^d)L_{t-1} + (1-\tau_t^\pi)\,(i_t^r - i_t^d)m_{p,t-1}^T + \tag{A.7.21}$$

$$+\left[\Lambda_t\left(1+(1-\tau_t^\pi)\,i_t^b\right) + (1-\Lambda_t)\,\Phi_t - \left(1+(1-\tau_t^\pi)\,i_t^d\right)\right]b_{t-1} -$$

$$-(1-\tau_t^\pi)\,(i_t^z - i_t^d)z_{p,t-1}^T + \left(1+(1-\tau_t^\pi)\,i_t^d\right)n_{p,t-1}^T\}$$

$$N_{p,t}^T = \sigma n_{p,t}^T + \gamma\frac{p_{t-1}}{p_t}\left\{L_{t-1} + \Lambda_t b_{t-1} + m_{p,t-1}^T\right\} \tag{A.7.22}$$

$$L_t + b_t + m_{p,t}^T = j_t + z_{p,t}^T + N_{p,t}^T \tag{A.7.23}$$

$$\pi_{p,t}^T = (1-\sigma)\,n_{p,t}^T \tag{A.7.24}$$

$$\phi_t = (1+\zeta_t)\,\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}(1+\left(1-\tau_{t+1}^\pi\right)i_{t+1}^d) \tag{A.7.25}$$

$$\Omega_t = 1-\sigma+\sigma\phi_t \tag{A.7.26}$$

Treasury

$$g_t^c + g_t^g + g_t^t + (1+i_t^b)\frac{p_{t-1}}{p_t}b_{t-1} = b_t + \frac{T_t}{N} + n_t \tag{A.7.27}$$

$$\frac{T_t}{N} \equiv \tau_t^c c_{h,t} + \tau_t^y w_t l_t + \tau_t^\pi(y_t - w_t l_t) + \tag{A.7.28}$$

$$+\tau_t^\pi\frac{p_{t-1}}{p_t}(i_t^l L_{t-1} + i_t^r m_{p,t-1}^T + \Lambda_t i_t^b b_{t-1} -$$

$$-i_t^z z_{p,t-1}^T - i_t^d j_{t-1})$$

$$k_t^g = (1-\delta^g)k_{t-1}^g + g_t^g \tag{A.7.29}$$

Central Bank

$$\Phi_t(1-\Lambda_t)\frac{p_{t-1}}{p_t}b_{t-1} + z_{p,t}^T + i_t^r\frac{p_{t-1}}{p_t}m_{p,t-1}^T + \eta_t \equiv$$

$$\equiv (1-\Lambda_t)(1+i_t^b)\frac{p_{t-1}}{p_t}b_{t-1} + (1+i_t^z)\frac{p_{t-1}}{p_t}z_{p,t-1}^T +$$

$$+\left(m_{h,t} + m_{p,t}^T\right) - \frac{p_{t-1}}{p_t}\left(m_{h,t-1} + m_{p,t-1}^T\right) \tag{A.7.30}$$

Dividends

$$\pi_{h,t} = \pi_{i,t} + \pi_{p,t} - \gamma \frac{p_{t-1}}{p_t} \left\{ L_{t-1} + \Lambda_t b_{t-1} + m^T_{p,t-1} \right\} \tag{A.7.31}$$

Money market

$$m_t = m_{h,t} + m^T_{p,t} \tag{A.7.32}$$

Endogenous and exogenous variables We therefore have a dynamic system of 32 equations in 32 variables which are $\{c_{h,t},\ j_t,\ m_{h,t},\ l_t,\ \pi_{h,t}\}_{t=0}^{\infty}$, $\{\lambda_{k,t},\ \psi_{h,t},\ N_{i,t}\}_{t=0}^{\infty}$, $\{\pi_{i,t},\ y_t,\ x_t,\ k_t,\ L_t\}_{t=0}^{\infty}$, $\{\pi^T_{p,t},\ z^T_{p,t},\ m^T_{p,t},\ V^T_{p,t},\ \zeta_t,\ \eta^T_{p,t},\ N^T_{p,t},\ \phi_t,\ \Omega_t\}_{t=0}^{\infty}$, $\{\frac{T_t}{N}\}_{t=0}^{\infty}$, $\{b_t\}_{t=0}^{\infty}$, $\{m_t\}_{t=0}^{\infty}$, $\left\{k^g_{g,t}\right\}_{t=0}^{\infty}$, $\{p_t/p_{t-1},\ i^b_t,\ i^d_t,\ i^l_t,\ w_t,\ \Phi_t\}_{t=0}^{\infty}$. This is given the paths/rules of fiscal policy instruments, $\{\tau^c_t,\ \tau^y_t,\ \tau^\pi_t,\ s^c_t,\ s^g_t,\ s^t_t,\}_{t=0}^{\infty}$ and monetary policy instruments, $\{i^z_t,\ i^r_t,\ n_t,\ (1-\Lambda_t)\}_{t=0}^{\infty}$. In the steady state only, b_t and s^t_t change places.

A.8 Adding hand-to-mouth households

We now assume that the economy is populated by two types of households: savers and hand-to-mouth consumers. For simplicity, we assume that both types of households are of the same size, N. We will model what changes only relative to above.

A.8.1 Households as savers

Households as savers are modelled as in Section 2.2 in the main text. Thus the budget constraint of each h written in real terms is:

$$(1 + \tau^c_t) c_{h,t} + j_{h,t} + m_{h,t} \equiv$$

$$\equiv (1 - \tau^y_t) w_t l_{h,t} + (1 + i^d_t) \frac{p_{t-1}}{p_t} j_{h,t-1} + \frac{p_{t-1}}{p_t} m_{h,t-1} + \pi_{h,t} + \widetilde{g}^t_t \tag{A.8a}$$

where $\widetilde{g}^t_t$ is the real average per capita transfer payment.

Then, savers' net-of-taxes income is:

$$y_{h,t} \equiv (1 - \tau^y_t) w_t l_{h,t} + (1 + i^d_t) \frac{p_{t-1}}{p_t} j_{h,t-1} + \frac{p_{t-1}}{p_t} m_{h,t-1} + \pi_{h,t} + \widetilde{g}^t_t - \tau^c_t c_{h,t} \tag{A.8b}$$

A.8.2 Hand-to-mouth households

There are N identical hand-to-mouth households indexed by subscript $m = 1, 2, ..., N$. These households are like savers but they choose consumption and money holdings only, so that their income consists of labor income and government transfers.

The period budget constraint of each m written in real terms is:

$$(1+\tau_t^c)c_{m,t} + m_{m,t} \equiv (1-\tau_t^y)w_t l_{m,t} + \frac{p_{t-1}}{p_t} m_{m,t-1} + \widetilde{g}_t^t \tag{A.8c}$$

where $c_{m,t}$ and $l_{m,t}$ are respectively m's consumption and work hours, and $m_{m,t}$ is the real value of end-of-period currency carried over from t to $t+1$.

Thus, we have 5 additional equations:

$$\frac{\mu_1}{c_{m,t}} = (\lambda_{m,t} + \psi_{m,t})(1+\tau_t^c) \tag{A.8d}$$

$$\frac{\mu_2}{(1-l_{m,t})} = \lambda_{m,t}(1-\tau_t^y)w_t \tag{A.8e}$$

$$\lambda_{m,t} - \psi_{m,t} = \beta\lambda_{m,t+1}\frac{p_t}{p_{t+1}} \tag{A.8f}$$

$$\psi_{m,t}[(1+\tau_t^c)c_{m,t} - m_{m,t}] = 0 \tag{A.8g}$$

$$(1+\tau_t^c)c_{m,t} + m_{m,t} \equiv (1-\tau_t^y)w_t l_{m,t} + \frac{p_{t-1}}{p_t} m_{m,t-1} + \widetilde{g}_t^t \tag{A.8h}$$

in 5 additional endogenous variables, $\{c_{m,t},\ l_{m,t},\ m_{m,t},\ \lambda_{m,t},\ \psi_{m,t}\}$, where $\lambda_{m,t}$ and $\psi_{m,t}$ are Lagrangean multipliers associated with the budget constraint and the cash-in-advance constraint respectively.

Then, hand-to-mouth consumers' net-of-taxes income is:

$$y_{m,t} \equiv (1-\tau_t^y)w_t l_{m,t} + \frac{p_{t-1}}{p_t} m_{m,t-1} + \widetilde{g}_t^t - \tau_t^c c_{m,t} \tag{A.8i}$$

A.8.3 The Treasury

Total tax revenues are now:

$$\frac{T_t}{N} \equiv \tau_t^c(c_{h,t} + c_{m,t}) + \tau_t^y w_t(l_{h,t} + l_{m,t}) + \tau_t^\pi(y_{i,t} - w_t(l_{h,t} + l_{m,t})) + \tag{A.8j}$$

$$+\tau_t^\pi\frac{p_{t-1}}{p_t}(i_t^l L_{p,t-1} + i_t^r m_{p,t-1} + i_t^b b_{p,t-1} - i_t^z z_{p,t-1} - i_t^d j_{p,t-1})$$

A.8.4 Market-clearing conditions

The market clearing conditions for the labor and money markets change to:

In the labor market:

$$l_{h,t} + l_{m,t} = l_{i,t} \equiv l_t \tag{A.8k}$$

In the money market:

$$m_t = m_{h,t} + m_{m,t} + m_{p,t}^{T} \tag{A.8l}$$