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Trade-Offs in Choosing a College Major

Abstract

Recent empirical analyses reveal substantial differences in the choices of college majors between demographic and socio-economic groups that are further amplified upon students' adjustment of their educational choices in the course of studies. The best documented and salient are the differences between genders, whereby women tend to be significantly underrepresented in some quantitatively oriented academic fields such as STEM, Business, and Economics, which also happen to be associated with relatively more lucrative careers, and overrepresented in others, such as Humanities and Education. Among potential explanations for this gender imbalance, some scholars noted that those more lucrative fields tend to have a more competitive environment and assign, on average, lower grades and conjectured that female students exhibit stronger aversion to low grades, hence their relative aversion to low-grading disciplines. The empirical literature also brings up a competing reasoning that gender biases in the choices of disciplines are directly driven by differences in preferences toward fields and pecuniary as well as non-pecuniary aspects of careers associated with them. This paper develops a theoretical model, which proposes a foundation for the latter explanation as a predominant one and reconciling it with the empirical evidence of gender differences in responsiveness to grades mentioned above. The paper argues that a student's responsiveness to grades, in terms of the initial choice of and persistence in majors, is field-specific and is the stronger, the weaker is the student's preferential attachment to the field. A key implication is that categories of students who attach high importance to pecuniary benefits of post-college careers, will be more tolerant toward inferior grades they may receive in the disciplines which promise such lucrative careers. It further explains why such students also tend to exhibit higher dropout rates from college.

JEL-Codes: I230, I240, J240, D210.

Keywords: higher education, college major, switching majors, dropout, gender gap.

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1. Introduction

The century of American college education which featured uninterrupted dramatic expansion of enrollment and, for most of the post-war period, a strong overall trend in the growth of the average college wage premium (e.g., Goldin and Katz, 2008) is showing signs of tapering off in both dimensions (e.g., Kaganovich et al. 2021, Ashworth and Ransom, 2019). At the same time, the choice of college major has become a stronger determinant of the variation in career earnings than the choice between going to college or not. In other words, the variation in college *major* premia is overtaking the average college premium (see James, 2012, Hershbein and Kearney, 2014, Altonji et al., 2015). Accordingly, there is strong evidence that students' expectation of future earnings associated with college majors is a significant determinant of their decisions to choose among them: see Berger (1988), Montmarquette et al. (2002), Arcidiacono (2004), and Arcidiacono et al. (2012) among others. The literature finds, however, that these choices differ systematically across demographic and socio-economic groups, particularly across genders. For instance, Gemici and Wiswall (2014) document persistence of significant gender gaps in majoring in STEM and Business in favor of men, and in Social Sciences (excluding Economics) and Humanities in favor of women. This despite women now increasingly overtaking men in terms of the overall college enrollments, as well-documented by Goldin et al. (2006) and analyzed by Becker et al. (2010) among others. Women's higher rates of attrition from STEM and other quantitatively oriented majors (Chen and Soldner, 2013) exacerbate these gaps. Given the dramatic variation in college major premia, this obviously implies gender gap in future career earnings. Indeed, Sloane et al. (2021) document clustering of men and women in different categories of majors in a way that systematically lowers college educated women's future wages relative to their male counterparts.

The above facts motivate substantial literature seeking to understand and, in some instances, to offer policy prescriptions to address the phenomenon. A common thread in much of it (e.g., Achen and Courant, 2009) focuses on well-known differences in grading practices across disciplines: average grades are substantially lower in STEM and Business/Economics (which also happen to be more lucrative) than in Education, Humanities and Social Sciences, excluding Economics. This points to a potential trade-off for students deciding in favor of more lucrative majors (important positive for many) in terms of having to potentially contend with relatively lower grades or higher effort cost of getting good ones (an obvious negative). Some of the empirical literature indeed finds that poor grade performance, actual or expected, is a deterrent in the pursuit of a major and persistence in it and that, furthermore, there are differences across genders and other groups in such responsiveness to grades. For instance, Montmarquette et al. (2002) find that, controlling for perceived probability of success, women are less likely to choose lucrative majors than men. This can be taken to imply that women's attachment to lucrative majors is more contingent, than it is for men, on signals of success they receive while pursuing such majors, i.e., on grades earned there, as also suggested by some of the subsequent literature (e.g., Ahn et al., 2019, and references therein). This reasoning is supported by Chen and Soldner's (2013) observation of greater aversion on the part of women and minorities to an excessively “competitive climate” in STEM – an increasingly common suggestion in the literature (see, e.g., Niederle and Vesterlund, 2010) that finds much following among higher education administrators. Such reasoning offers an explanation for the stark gender gap in the choices of and persistence in majors based on postulating exogenous gender differences in responsiveness to grades as signals of performance, whereby women allegedly generally exhibit less tolerance for inferior grades. Thus, according to this conjecture, women exhibit relatively lower incidence of enrolling in and higher attrition from disciplines such as STEM and Business/Economics, because those tend to assign relatively low grades.

An alternative explanation for the gap in major choices and persistence emerges from recent empirical work (Zafar, 2013, Rendall and Rendall, 2014, Altonji et al., 2015) which finds evidence of systematic differences between male and female students in the importance they attach to different characteristics of their future occupations. According to this literature, women, on average, place lower weight on pecuniary benefits, and greater weight on the non-pecuniary ones, associated with a future career than do men in their choices of college majors. Arguably, this may be due to continued gender differences in family role expectations and norms (Blau and Kahn, 2017). This reasoning, furthermore, offers attractive alternative explanation for gender differences in students’ responsiveness to grades in their choices of or persistence in majors, which does not require postulating that they reflect differences in preferences for grades. Since more lucrative college majors do tend to assign relatively lower grades or, equivalently, require more effort to attain high grades (a trade-off that makes perfect economic sense), the individuals who are relatively less concerned about the lucrativity of future careers will be less inclined, *ceteris paribus*, to accept the disutility of relatively low grades that prevail in these majors or higher effort cost required to gain good grades there. It then follows that women (or any other demographic group, which is found to attach less importance to pecuniary benefits of post-college careers) would exhibit relatively stronger responsiveness to inferior grade performance in majors that offer more lucrative pay in exchange for poorer grades. On the flip side of this argument, men or other demographic groups who tend to attach relatively high value to future career earnings (see, e.g., Montmarquette et al., 2002, for evidence concerning minorities) would exhibit relatively stronger tolerance for low grades in their pursuit of lucrative majors. Thus, according to this argument, the strength of responsiveness to grades in students’ choices is *endogenous* and field-specific rather than universal, reflecting preferential attachments to academic disciplines, akin to the obvious fact that consumers’ responsiveness to prices depends on the strength of their preference for the corresponding products. In other words, it is the underlying weaker attachment to a field that makes a student more sensitive to grades received in it, not the other way around.

Kaganovich et al. (2023) find empirical evidence in support of this conjecture using a comprehensive data set of Indiana University’s student performance and choices. They study student responses to grades accumulated over the first two years in college in terms of their decisions to persist in the originally selected academic disciplines and provide estimates for the weights men and women place on their valuations of academic disciplines and grades. They find, somewhat unexpectedly, that (a) men exhibit stronger, relative to women, "taste" for good (aversion to bad) grades received in all the broad academic categories, (b) men and women differ in their "tastes" for academic disciplines, and (c) the overall responsiveness to grades by students is a combined result of these two factors. For instance, the study shows that women indeed tend to exhibit relatively stronger overall responsiveness to grades in their patterns of migration out of STEM, but that this is due to their significantly weaker "taste" for STEM, while they are less directly averse to low grades in STEM than are men.

In this paper, I develop a theoretical framework to analyze the above reasoning that students' responsiveness to grades, in terms of choosing and persisting in college majors, reflects their preferences for academic disciplines. I present a utilitarian model of students' decision-making about selecting a field of study and then either persisting there or switching to an alternative. Students differ in ability (pre-college preparation) and the importance they attach to future career earnings, study effort in college, and psychic benefits of (taste for) good grades. I then show that the population of students who give higher priority to future earnings will sort themselves more strongly toward more lucrative majors. In other words, this group will be characterized by lower average ability and lower grades in lucrative majors compared to the group of students who place relatively lower utility weight on pecuniary benefits of a career. In fact, the result of self-sorting across majors is that the average ability and grades of the former group will be lower within both majors (the more and the less lucrative). Accordingly, the group placing less weight on pecuniary benefits will exhibit "stronger" self-sorting across majors, such that they will feature relatively higher average ability in each major.[1]

I further apply the model to analyze student decisions whether to persist in an initially chosen major based, in the spirit of Manski (1989) and Altonji (1993), on the assumption that students' initial choice of majors is made with imprecise knowledge of their ability, which gets updated according to the grades received. Specifically, according to the model, if a student initially choses a lucrative major and receives a low grade in the first stage of studies there, this sends a signal of low ability and can compel the student to switch to an alternative major or to drop out of college, rather than persist, if the grade falls below certain cutoff. I show that the threshold grade level in question is in inverse relation to the weight a student places on pecuniary benefits of the career associated with a major. This result implies, in particular, that if women tend to place relatively lower weight on pecuniary benefits of the career associated with a chosen major, then threshold grade for deciding to switch out of initially chosen lucrative major will be higher for women than for men. This is well-aligned with the aforementioned empirical results of Kaganovich et al. (2023) and thus offers their plausible underlying mechanism.

An important additional insight of this analysis is that the decisions of choosing and persisting in

[1] Assuming that women do tend to place lower weight on pecuniary benefits of future careers, the above result predicts that women would exhibit "stronger" sorting, along the ability dimension, across majors than men would. This prediction is indeed consistent with the evidence presented by Kaganovich et al. (2023) showing women's remarkably superior grade performance to men's in all the academic disciplines at Indiana University.

a major contribute to the well-known significant gender gap, in favor of women, in college completion rates (among those matriculating). A common view in the literature, prominent in Stinebrickner and Stinebrickner (2012), is that men's significantly higher dropout rates are largely explained, besides their superior returns in non-college job market, by men's comparatively inferior academic performance, unanticipated due to comparatively high overconfidence. This paper underscores a contributing role of the mechanism that is driven by gender differences in preferences for academic majors along with the fact of different grading standards among them. Under the assumption that men give higher priority to more lucrative majors, the results detail the mechanism of men's and women's self-sorting bias into the majors with prevailing lower and higher grades, respectively. Among students who started out in more lucrative hence tougher grading such majors, women's decision against persisting will be triggered by higher grade cutoff than for male counterparts. Furthermore, non-persisting men will be more likely to drop out of college, whereas non-persisting women will tend to switch to a less lucrative major, given women's relatively stronger taste for such majors.

The paper is structured as follows. Section 2 presents a model of student decisions about selecting a field of study and persisting in it; it then derives baseline results in a framework where students make all-in-one college career decision, based on supposed complete information about their academic ability. Section 3 extend s the analysis to the main case of interest where the college study consists of two stages, the lower and the upper division, whereby students learn their true ability upon completing the former based on grade performance there and then decide whether to persist in their initial choice, switch to an academic alternative, or drop out. Section 4 concludes. Appendix contains the proofs of Section 2 results.

2. Single Stage Model: Decisions and Trade-offs under Complete Information

Manski (1989) followed by Altonji (1993) introduced a utilitarian framework with information updating to analyze individual sequential decisions whether to enroll and then to persist in college. I follow its spirit to incorporate students' *two-stage* decisions about initially choosing a college major and then potentially persisting in it based on characteristics of major disciplines and students' individual characteristics, such as abilities and preferences. Recent research shows that the prominent factors entering students' preferences include expected future career earnings (see Arcidiacono, 2004) as well as the effort required to pursue the corresponding studies (e.g., Montmarquette et al., 2002, Babcock and Marks, 2011).

First, throughout this section, I'll consider a model of a "one-shot deal" decision where student possesses complete information of his ability in advance and so is indeed able to make an optimal choice of a major (if any) and learning effort to exert there. In the next section, I'll extend the framework to that of two-stage education decision process. There, I'll presume, as did Manski (1989), that students' initial choices are based on imprecise beliefs about their academic ability, which get updated based on the outcomes of the first stage of studies. The new information then motivates students to either keep or change their initial choices accordingly, at the second stage of their studies.

I assume that a student's pre-college academic preparation is represented by a single variable *a,* academic ability, distributed between 0 and *A* where *A* may be finite or infinite. As noted, throughout this section only, I also assume that each student *i* has advance full knowledge of her academic ability $a(i)$ and is thus able to make the optimal choice of college career upfront as a "one-shot deal".

To keep matters simple, I assume that while in college, each student chooses to pursue his/her degree in one of two majors *m*=1,2, and that the human capital level student *i* will choose to attain will be major-specific: $h_{m,i}(a,e)$, which is an increasing function of student's ability *a* and the effort *e* he/she chooses to exert while pursuing the major. I assume that these two factors are complementary; for simplicity but without loss of generality, I use the following common functional form which meets this condition:

$$h_{m,i} = a_i e_{m,i} \tag{1}$$

I further posit that each major establishes a grading scale such that the major-specific GPA grade of student *i*, a continuous variable, is strictly increasing function of his/her human capital attainment in the major:

$$g_m(h_m)$$

Equivalently, each major has a scale of its academic standards corresponding to each GPA grade level *g*, such that $h_m(g)$ is the level of attainment of human capital specific to major *m* required for obtaining GPA grade *g* in the major. Thus, given continuous GPA grading scale, there is a one-to-one correspondence between human capital attainment in major *m* and GPA grade earned there, so student's choice of the former is equivalent to the choice of a GPA grade level to pursue in the major. For a student of given ability, these choices are in turn equivalent, according to (1), to his/her choice of the level of effort to exert in pursuing major *m*.

I posit that if an individual chooses to go to work without getting a college degree, which I denote as option *m*=0, his/her human capital level is fixed at $h_0 = 1$, independent of individual characteristics, so the career income will be also fixed at

$$I_0 = w_0$$

the "unskilled wage". In contrast, career income derived by student *i* from a degree in major *m* =1,2 depends in human capital attainment and is given by

$$I_{m,i} = w_m h_{m,i} \tag{2}$$

where w_m are exogenously given wage rates per unit of major-specific human capital. Thus, obtaining a college degree is potentially rewarded by job market premia, which depend on human capital attainment levels and differ by major. Specifically, I assume that the premium *rates* satisfy

$$w_1 > w_2 > w_0$$

so major 1 is more *lucrative* than major 2. It is then economically sensible and realistic to suppose that non-pecuniary benefits (for instance, flexible working schedule or stronger job security) are greater in careers associated with major 2. Such a situation would indeed help explain why some individuals of comparable levels of academic abilities may make different choices of majors: that would depend on how much weight they place on pecuniary and non-pecuniary benefits of the associated careers in their preferences.

Each student *i* makes educational choices about going to college (or not), choosing a major and human capital attainment in it to maximize the value of separable utility function across all the available options:

$$U_{m,i} = \alpha_i u(I) + t(h_m) - \beta_i v(e \mid a_i) + \gamma_i s(g_m(h_m)), \tag{3}$$

which, in the order of components in the expression, increases in career income *I* and in the non-pecuniary benefit of a career resulting from human capital attainment h_m in the chosen major, decreases in the educational effort *e* these decisions entail, and also incorporates psychic benefits of GPA grade *g* earned in college (when the grade is high) or, accordingly, a distress from a low grade. Note that all components of (3) depend on student's effort along with ability. Indeed, for a given effort higher ability yields superior outcomes, in terms of human capital attainment and grade earned. Although the disutility of study effort only depends on the level of effort, the study effort required for attaining a given outcome decreases in ability. Coefficients α_i, β_i, and γ_i stand for the weights student *i* attaches to the components of welfare associated with career income, the disutility of study effort, and the utility of GPA grade *g* received, respectively.[2] All the weights are measured as relative to the importance student attaches to the utility derived from non-pecuniary benefits of major-specific human capital. This means that all else equal, the fact that a student attaches greater value to non-pecuniary benefits of a career can be equivalently expressed as her putting lower weights on the other components of utility function (3), i.e., relatively smaller values of α_i, β_i, and γ_i.

To summarize, a student's decisions in the model can be reduced to two choices: of major *m* and of human capital level (equivalently, grade level) to attain in it. The human capital attainment in major *m* by student with given ability *a* and utility weight coefficients α, β, γ is in turn determined by his/her choice of the level of effort there. Thus, according to expressions (1) and (2), for a student who decides to go to college, the utility maximization problem can be

[2] If a student attaches no importance to the grade per se, i.e., $\gamma_i = 0$, which is an acceptable special case of the model for which the paper's results remain valid, she will retain incentives for human capital attainment embedded in the other components of utility, hence the attainment of appropriately high grades.

formulated in terms of student's choice of major m=1,2 and the level of educational effort e to exert in pursuing it:

$$\max_{m}\ \max_{e}\ U_m(a,e)=\alpha u(w_m ae)+t(ae)-\beta v\left(e\right)+\gamma s(g_m(ae)) \tag{4}$$

where student indexation has been dropped.

I further assume that functions u(.), t(.), s(.), and g(.) are strictly concave and increasing while the disutility of effort v(.) is strictly convex and increasing.

It is obvious that if individual chooses m=0, i.e., no college, then his optimal study effort is $e_0(a)\equiv 0.$ For simplicity and without loss of generality, I set the values of functions s(.), t(.), and v(.) corresponding to this choice to zero, so

$$U_{0,i}=\alpha_i u(w_0)$$

I shall now analyze the trade-offs a student of given *known* ability faces when choosing between majors m=1, 2, the first being more lucrative than the second.

It is a well-established fact (see, e.g., Aachen and Courant, 2009) that grade distributions vary considerably across academic disciplines. The most striking are the differences between relatively lower GPA levels common in STEM, which tend to be more lucrative, and much higher GPA levels in the predominantly less lucrative Humanities and Social Sciences (save for Economics). Such trade-off across the fields of study between expected future wage earnings associated with a discipline and its grading standards (and accordingly, the required study effort) can be well-understood in the context of labor market equilibrium. Indeed, higher expected returns of an occupation are potentially balanced out, for a marginal candidate, by the higher effort cost of qualifying for it in such equilibrium. Based on this reasoning, I posit that for each GPA grade level g, the corresponding academic standards for achieving this grade, $h_1(g)$ and $h_2(g)$, compare in the same order as the wage rates (the lucrativity) of the majors. In other words,

$$h_1(g)>h_2(g)$$

must hold, so attaining a particular grade level in a more lucrative major will take a student of given ability more effort than in a less lucrative one, a conclusion consistent with straightforward economic logic. I assume furthermore that grade standards are "*separated*" between the majors, such that human capital attainment standard for any passing grade in the more lucrative major exceeds the standards for any high grade in the less lucrative one.[3] It then follows that for a

[3] Kaganovich and Su (2020) argue that such "separation" emerges if the difference in wage rates between the lucrative and non-lucrative majors is substantial enough. They use a model of intra-university competition among academic departments to explain that the trade-off between major-specific expected career earnings and the grade standards they impose arises in equilibrium resulting from the units' competition for students; furthermore, when the wage rate differential in favor of the more lucrative major is sufficiently large, they show that respective

student of given ability and utility weight coefficients, the level of effort required in more lucrative major 1 is higher than that in major 2, regardless of the grades pursued there. In other words, I impose

Assumption 1: The more lucrative major has stricter grading standards. Moreover, the wage differential between w_1 and w_2 is large enough such that academic standards in the more lucrative major strongly dominate, i.e., for any grade levels g_1 and g_2 in the respective majors the corresponding academic standards compare as follows: $h_1(g_1) > h_2(g_2)$.

Assumption 1 directly implies that, controlling for individual ability *a*, a student would need to exert higher effort if pursuing major 1 vs. that in major 2. Since the objective function of problem (4) is concave in the effort variable *e*, the problem's first order conditions are sufficient for optimum. Based on this logic, also extended to the "no college" option, I obtain the following result proven in Appendix.

Lemma 1: Under the provisions of Assumption 1, there are cut-off ability levels

$$a_{1/2} = a_{1/2}(\alpha, \beta, \gamma), \quad a_{2/0} = a_{2/0}(\alpha, \beta, \gamma), \quad a_{1/0} = a_{1/0}(\alpha, \beta, \gamma) \tag{5}$$

such that, for a given set of their utility weight coefficients α, β, γ, students with ability below $a_{k/j}$ will prefer the less lucrative of the options *k* and *j* (specifically, major 2 over major 1, or no college vs. a particular major, respectively), while students with ability above the threshold will prefer the more lucrative option in a given binary choice.

Remark. The above result allows for trivial possibilities, such as $a_{2/0} = a_{1/0} = 0$ which would imply that all individuals with a given set of utility weight coefficients will prefer to go to college even if their academic ability is infinitesemal. Similarly, $a_{1/2} = 0$ and $a_{1/2} = A$ would imply, respectively, that individuals of any ability prefer major 1 over major 2 and vice versa. Although such scenarios may obrtain under sufficiently large (respectively, small) disparities between the wage rates associated with the corresponding options, it is realistic to assume such cases away and thus posit, without imposing further structure, that individuals of sufficiently low ability will prefer the "no college" option, and that under at least some combinations of utility weight coefficients individuals of sufficiently high ability will prefer major 1 over major 2.

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distributions of grade standards become "separated" (see also Kaganovich et al., 2021.) They further present empirical evidence of such trade-off based on Indiana University data; specifically, while more lucrative majors tend to maintain less generous grading standards, the less lucrative ones are prone to *grade inflation*. The academic standards for grades are chosen by individual departments and serve as their competitive instruments for attracting students to the corresponding disciplines or for deterring them, depending on academic eligibility. This leads to student self-sorting across the departments, and, in equilibrium, to a positive relationship between job market rewards of majors and their grading standards, as described above. These results are supported by the findings by Babcock and Marks (2011) that study time is persistently higher on average (i.e., without controlling for ability) in engineering and sciences compared to other disciplines. Assumption 1 of "separated" grading standards thus amounts to an underlying condition of sufficiently large disparity between the wage rates associated with the majors.

Let $V_m(a,\alpha,\beta,\gamma)$ stand for the value function of the problem of maximizing the utility function U_m within option m by an individual who has ability level a and weight coefficients α,β,γ, i.e., the highest level of welfare the individual could achieve were he/she to pursue option m=0,1,2. Then $V_0(a,\alpha,\beta,\gamma)\equiv\alpha u(I_0)$ whereas for m=1,2, $V_m(a,\alpha,\beta,\gamma)=\max_e\ U_m(a,e)$ as defined by expression (4).

Barring the trivial extreme cases outlined in the above Remark, the cut-off ability levels $a_{k/j}$ satisfy equalities

$$V_k(a_{k/j},\alpha,\beta,\gamma)=V_j(a_{k/j},\alpha,\beta,\gamma) \tag{6}$$

It is then straightforward to derive (see Appendix for details) from Lemma 1 the following comparative statics result:

<u>Lemma 2</u>:

(i) The ability thresholds $a_{k/j}$ defined above for $k=1,2,\ j=2,0,\ k\neq j$ for choosing a more lucrative option over the less lucrative one all decline in the weight α a student places on pecuniary benefits of a major, as well as in the weight γ of his/her utility received from GPA grades; they increase in the weight β a student places on the disutility of study effort in college.

(ii) Threshold $a_{1/2}$ declines in the wage rate of more lucrative major w_1, and increases in that of the less lucrative one w_2; $a_{2/0}$ declines in w_2 and $a_{1/0}$ declines in w_1.

Lemma 2 implies not only that individuals who value pecuniary benefits of a career more will prefer, obviously, a higher paying option *ceteris paribus*, but that *the ability sorting among such individuals will be more strongly biased in favor of a more lucrative option*. Likewise, ability sorting among individuals whose preferences exhibit relatively stronger aversion to study effort is more strongly biased, *ceteris paribus*, in favor of a less lucrative, hence "easier" option. According to part (ii) of the Lemma, the extent of the ability sorting bias with respect to either of these preference parameters will be the stronger the greater the lucrativity gap between the options.

A further implication of Lemma 2, in terms of the choice between college majors, is that there is a *trade-off between the lucrativity of a major and the grade level acceptable to a student there.* To show this, I first state the following auxiliary fact whose detailed proof is provided in Appendix:

<u>Lemma 3</u>. Individually optimal human capital attainment levels $h_m(a)$ by student of ability *a* in each major *m*=1,2 increase in ability.

Let now $g_{k/j}$ for $k = 1,2,\ j = 2,0,\ k \neq j$ stand for the cut-off GPA grade levels between choosing respective options by students who are characterized by a given bundle of utility weight coefficients α, β, γ. The grade cut-offs uniquely correspond to the respective ability thresholds defined in Lemma 1 for the subset of students with these utility weight coefficients. Specifically, I define $g_{1/2}$ and $g_{1/0}$ as the lowest GPA grades acceptable to a student, with the above utility coefficients, for choosing major 1 against the respective alternatives; $g_{2/0}$ is defined similarly, *mutatis mutandis*. Note that the grade performance $g_m(h_m(a))$ only depends on, and increases in, the human capital level the student attains in the major. Therefore, combining Lemmas 2 and 3 yields the following:

<u>Corollary 1</u>:

(i) The GPA grade thresholds $g_{k/j}$ defined above for $k = 1,2,\ j = 2,0,\ k \neq j$, for choosing a more lucrative option over a less lucrative one, decline in the weights α and γ and increase in β.

(ii) Grade threshold $g_{1/2}$ declines in the wage rate of more lucrative major w_1 and increases in that of the less lucrative one w_2; $g_{2/0}$ declines in w_2 and $g_{1/0}$ declines in w_1.

Thus, the higher the weight α a student of given ability attaches to pecuniary benefits of a major, the lower the acceptable grade level at which this student will still prefer the choice of the more lucrative major. In other words, students who place a higher weight on future income, which according to the studies referenced in the Introduction is more common for men, will endogenously exhibit higher tolerance for (or less aversion to) low grades in choosing a more lucrative major compared to their peers (disproportionately women, as claimed in the referenced studies) who are relatively less attracted to pecuniary benefits. Symmetrically, it is easy to demonstrate that those former type students will exhibit relatively less tolerance for low grades received in a less lucrative major when faced with the alternative to pursue a more lucrative one. Similar results obtain for students who place relatively lower weight on the disutility of study effort, other things equal.

<u>Remark</u>. I have assumed that w_0, the "no-college" wage rate, is the same for all types of students. It is well-known, however, that wages in this segment of labor market tend to be superior for men (which is often cited as a factor in higher college enrollment rates among women). It is then straightforward to extend the analysis to show that, when comparing a given college major to the no-college option, if w_0 is sufficiently higher for men than for women, then men will exhibit less tolerance for low grades in a less lucrative major than do women, particularly when it comes to decisions to drop out of college, which arise in the framework considered in the next section.

I'll now focus on the implications of the differing values of weight α , the strength of relative "taste" for pecuniary benefits of a future career, for decisions by individuals, across the ability spectrum, whether to attend college and what major to choose there.

To simplify the argument, I'll assume that the population consists of two distinct groups which differ only in terms of the weight α , while other preference parameters are identical for all and that the differences in weight α between the groups are uncorrelated with academic ability which is therefore distributed similarly in both groups. Specifically, I assume that all members of the first group, which I call group "H", place high weight $\alpha = \alpha^H$ on the utility of future career income, whereas such weight is relatively low, $\alpha = \alpha^L \ll a^H$ for members of the second group, "L".

I'll first analyze the optimal choices of members of group "H". Comparing maximum values for each option ("no college", major 1, and major 2) according to the respective expressions (4) and keeping in mind that the value associated with not going to college V_0 does not depend on ability, it is clear that for members of this group who possess sufficiently high ability the following ranking of the options, in terms of maximum values attainable there, obtains

$$V_1^H(a) > V_2^H(a) > V_0^H \qquad (7)$$

where I denote $V_m^H(a) = V_m(a, \alpha^H, \beta, \gamma)$ for *m*=0,1,2, having assumed, for simplicity and without loss of generality, that coefficients β and γ do to not vary across students, hence can be dropped from the expressions. Relationship (7) implies that for high ability individuals in group "H" attending college and pursuing the "lucrative" major 1 is the best option. The analysis of expressions (4) for each option makes it similarly clear that the best choice for individuals of sufficiently low ability in this group is "no college" because welfare level V_0 dominates the alternatives. Tracking the value rankings of student options along the entire student ability axis between the two extremes, I obtain the following result (see Appendix for details of the proof):

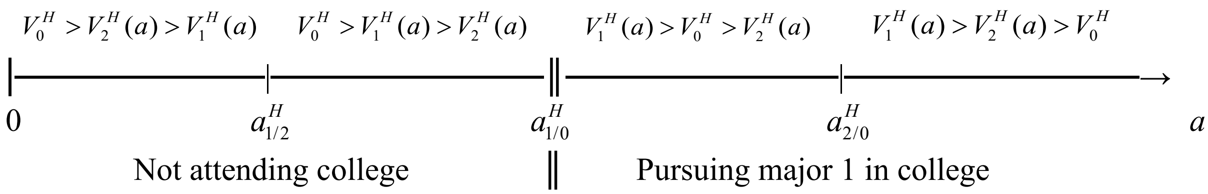
<u>Proposition 1</u>. Consider the sub-population "H" of individuals characterized by a high weight $\alpha = \alpha^H$ they attach to pecuniary benefits of future career and no variation in other preference parameters. If α^H is sufficiently high, then the group-specific ability thresholds, which determine individual's choices between the corresponding options, satisfy the following relationships:

$$a_{1/2}^H < a_{1/0}^H < a_{2/0}^H$$

where all the utility weight coefficients are omitted from the expressions. This implies that this sub-population partitions into two groups. Pursuing lucrative major 1 is the top choice for all those with ability above $a_{1/0}^H$. For those whose ability falls below this threshold, the "no college" option dominates.

The following figure specifies maximum value rankings (as detailed in the Proposition's proof in the Appendix) for students along the ability axis in the sub-population "H", which lead to the above result. It makes clear, in particular, that pursuing the less lucrative major 2 is not optimal for any member of this sub-population: it's either the lucrative major or no college at all.

Figure 1. Value comparisons across options and optimal choices as functions of ability for students in group "H" (placing high weight on career income)



Thus, an important takeaway from Proposition 1 is that for individuals who value pecuniary benefits of a career sufficiently strongly but whose ability is not high enough to pursue a lucrative major will prefer to not graduate from college at all over doing so with a non-lucrative major.

Applying similar reasoning to the sub-population "L", one easily obtains that the threshold ability level $a_{1/2}^L$ is high, if the weight α^L is sufficiently small. Further, along similar lines, denoting $V_m^L(a) = V_m(a, \alpha^L, \beta, \gamma)$ for *m*=0,1,2, I obtain

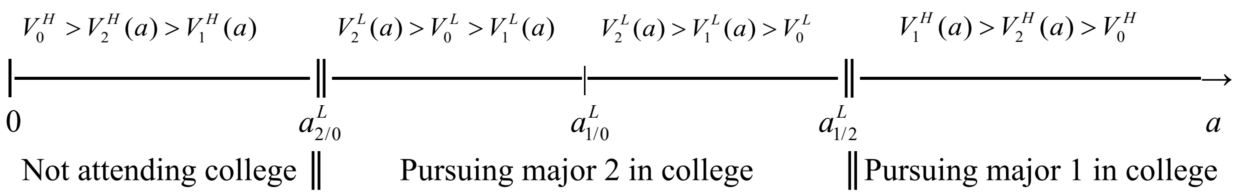
<u>Proposition 2</u>. Consider group "L" of individuals characterized by low weight $\alpha = \alpha^L \ll a^H$ they attach to pecuniary benefits of future career and no variation in other preference parameters. If α^L is sufficiently low, then the group-specific ability thresholds, which determine individual choices between the corresponding options, satisfy the following relationships:

$$a_{2/0}^L < a_{1/0}^L < a_{1/2}^L$$

where all the utility weight coefficients from the expressions. This implies that this sub-population partitions into three groups. Pursuing lucrative major 1 is the top choice for those with ability $a > a_{1/2}^L$. For those whose ability falls into the interval $a_{2/0}^L < a < a_{1/2}^L$, pursuing less lucrative major 2 is the best choice. Finally, the "no college" option dominates for those with ability below the lower threshold, $a_{2/0}^L$.

The following figure details maximum value rankings for students along the ability axis in group "L", which leads to the Proposition's result. Unlike for members of group "H", the less lucrative but "easier" major 2 emerges as an attractive option for students in the middle range of ability in this group. The threshold level of ability above which major 1 dominates is relatively high. This fact is indeed intuitive: for students of average ability in group "L" who do not place high weight on future earnings, which is the main draw of major 1, the need to invest relatively higher study effort to meet this major's higher standards is not sufficiently justfied by its future higher wages.

Figure 2. Value comparisons across options and optimal choices as functions of ability for students in group "L" (placing low weight on career income)



3. Two-Stage Decisions in College

The previous section's analysis proceeded under the benchmark scenario where students make final and irrevocable college enrollment and major choice decisions when matriculating, owing to the assumption that individuals possess accurate information about their academic ability from the outset. In this section, I'll demonstrate that these results can be extended to a more realistic framework where students first make their initial matriculation and major choice decisions under incomplete information about their ability, similar to the empirical framework of Arcidiacono (2004) and Kaganovich et al. (2023). According to this scenario, students can make adjustments to their course of study mid-way through college, based on the information about their true ability, which is revealed through their academic performance up to that point. Thus, at this point in their studies students are faced with an *ex post* choice among a new set of alternatives.

I thus consider the following two-stage scenario for a student who initially matriculated in college and chose a particular major *ex ante*, based on the belief held at the time about his/her ability. Specifically, suppose that the student believed at the time her ability level to be a^0 and assume, for the sake of argument, that based on this information the student chose the more lucrative major 1. Suppose further that having completed the first stage of studies, the student had been able to infer her true academic ability based on performance embodied in grades received, and is now contemplating her choices *ex post*, in transition to the second, "upper division" stage of college education.

The student's presumed *ex ante* choice of major 1 implies, according to Lemma 1, that inequality $a^0 > a_{1/2}(\alpha, \beta, \gamma)$ must have been true for her, given the weight coefficients α, β, γ in her preferences.[4] Suppose further, that based on GPA grades the student received in the first stage of

[4] This is obviously the case if students' belief about their initial ability signal a^0 is naïve, i.e., perceived as accurate. However, it is sufficient to assume that posterior expected ability level is monotone increasing in the ability signal, as is the case under reasonable assumptions about the testing system the ability signals are based on (see, e.g., Eckwert and Zilcha, 2020).

studies, her true ability level was revealed to be a'. The student now faces three *ex post* potential choices: (a) *persist* in the originally chosen major 1, (b) *switch* to a less lucrative (hence "easier", per Assumption 1) major 2, and (c) *drop out* of college. It is quite clear that if the revealed true ability was no lower than the *ex ante* belief, $a' \geq a^0$, this would have only reinforced the original choice, so the above dilemma arises in earnest only if the revealed ability is below the initial belief, i.e., $a' < a^0$.

The student will be compelled to choose option (a), *persisting* in major 1, if the updated ability is high enough to "beat" both alternative options: *switching* to major 2 or *dropping out* of college. According to Lemma 1, this will be the case iff [5]

$$a' \geq \max\left\{a_{1/2}(\alpha,\beta,\gamma),\ a_{1/0}(\alpha,\beta,\gamma)\right\}$$

If, however, the updated ability level falls below this threshold, Propositions 1 and 2 give guidance for further analysis in order to determine which of the alternatives, (b) or (c), prevails. According to the Propositions, this depends on the weight α the student attaches to the pecuniary benefits of a future career.

For simplicity, I continue to assume as in the framework of Propositions 1 and 2 that the population consists of two distinct groups, "H" and "L", which differ only in terms of the weights the individuals place on the pecuniary career benefits: high weight $\alpha = \alpha^H$ in the first group, and low weight $\alpha = \alpha^L \ll a^H$ in the second.

According to Lemma 2, threshold $a_{1/2}(\alpha,\beta,\gamma)$ decreases in the weight α, so

$$a_{1/2}^H < a_{1/2}^L \tag{8}$$

Moreover, according to Proposition 1, $a_{1/2}^H < a_{1/0}^H$, so

$$\max\left\{a_{1/2}^H, a_{1/0}^H\right\} = a_{1/0}^H$$

This means that if the updated ability a' happens to be below this threshold, dropping out of college becomes the best option, rather than switching to the "easier" and less lucrative major 2.

Along similar lines, Proposition 2 shows that when the weight α is low: $\alpha = \alpha^L$, then $a_{1/2}^L$ is relatively large, so

$$\max\left\{a_{1/2}^L, a_{1/0}^L\right\} = a_{1/2}^L$$

[5] It should be noted that in the single-stage college framework for which ability thresholds separating different best options were introduced and analyzed, the benefits and costs of pursuing either option entailed potential pursuit of each of the college option for the entire term (such as four years of college), whereas the mid-term choices considered in this two-stage framework apply to a shorter period. In applying the previous section's results here, I ignore this difference in the time cost accounting: this simplification helps streamline the exposition but does not diminish generality, since adjusting the analysis to the changes in the benefits and costs is straightforward.

and therefore, if the updated ability a' falls just below this threshold, then switching to major 2 is a better option than dropping out. The latter option becomes dominant only for students with substantially lower realizations of ability: $a' < a_{1/0}^{L}$.

I summarize the results of the above analysis as the following

Theorem. Consider a student who enrolled in lucrative major 1 based on the student's original belief that his/her academic ability level is a^0. If the student receives a downgraded realization of ability $a' < a^0$ through the GPA grade received in the first stage of studies, the following is the taxonomy of the student's actions depending on the weight α he/she places on pecuniary benefits of a major.

(i) If the weight is sufficiently high, $\alpha = \alpha^H$, then the ability threshold below which switching to major 2 becomes preferable to staying in major 1 is below that for dropping out of college:

$$a_{1/2}^{H} < a_{1/0}^{H} \tag{9}$$

so only the following two outcomes apply depending on the student's ability realization a':

- the student will choose to persist in lucrative major 1 if $a' > a_{1/0}^{H}$;
- the student will choose to drop out of college if $a' < a_{1/0}^{H}$,

whereby the latter possibility can only occur if the initial signal of academic ability was strongly biased upward, i.e., $a' < a^0$, granted that $a^0 > a_{1/0}^{H}$, i.e., the student's initial ability signal was high enough to compel him/her to matriculate.

(ii) If the weight α is sufficiently low, $\alpha = \alpha^L$, then $a_{1/2}^{H} < a_{1/2}^{L}$. Furthermore,

$$a_{1/2}^{L} > a_{1/0}^{L} \tag{10}$$

Thus, the following three outcomes apply depending on the student's ability realization a':

- the student will choose to persist in the lucrative major 1, if $a' > a_{1/2}^{L}$;
- the student will choose to switch to the less lucrative major 2, if $a_{2/0}^{L} < a' < a_{1/2}^{L}$;
- the student will choose to drop out of college if $a' < a_{2/0}^{L}$.

It is straightforward to translate the Theorem's results in terms of student responses to grade performance in the first stage of college relative to the respective grade cut-offs by applying the analysis of Corollary 1 of Lemmas 2 and 3:

Corollary 2. Consider a student in group "H", i.e., one who places high weight $\alpha = \alpha^H$ on pecuniary benefits of a career and matriculated in lucrative major 1 based on initial belief about his/her ability. The cutoff for GPA grade level the student must receive during the first stage of studies in major 1 in order to decide to persist in it will be lower compared to that of the otherwise similar peers in group "L". That is,

$$\max\left\{g_{1/2}^{H}, g_{1/0}^{H}\right\} = g_{1/0}^{H} < \max\left\{g_{1/2}^{L}, g_{1/0}^{L}\right\} = g_{1/2}^{L}$$

Therefore, controlling for the preferential aversion to study effort and bad grades (the utility weights β and γ), students in group "H" will

(a) exhibit stronger persistence in major 1 than group "L" students, i.e., will persist in the face of comparatively lower GPA grade in major 1, whereby this acceptable grade cutoff will be the lower (hence persistence the higher), the higher is the differential between wage rates and w_1 and w_2,

(b) be more likely to choose to drop out of college in response to poor GPA grade performance in major 1 compared to similarly underperforming group "L" students who will more likely switch to less lucrative major 2 and whose decision to drop out of college altogether will only occur in response to substantially lower grades compared to such cutoff for group "H" peers.

These results match the pattern of gender differences in the endogenous responsiveness to grades driven by the differences in tastes for fields of study demonstrated in the empirical analysis of Kaganovich et al. (2023), as discussed in the Introduction. Namely, assuming that men tend to place higher weight on pecuniary benefits of careers than do women, then the above Theorem along with Corollary 2 produce precisely such pattern. Specifically, they show, just as the aforementioned empirical analysis, that men will persist in lucrative majors more strongly than women when receiving somewhat discouraging ability signals through grades, whereas women are relatively more likely, compared to men, in such a situation to switch to a less lucrative major. Furthermore, when faced with strongly discouraging ability signals, i.e., substantially poor performance in a lucrative major such that persisting there becomes infeasible or excessively costly, men are significantly more likely than women to drop out of college altogether, whereas women are more likely than men to rather turn to a less lucrative major.

It is also noteworthy that this paper's theoretical results can offer a novel explanation to a known empirical phenomenon that women tend to outperform men in terms of GPA grades across the board, i.e., in all academic categories. While the dominant explanation in the literature is that women tend to exert more effort in academic tasks in college than men, the results obtained here show that such outcomes can be additionally explained by relatively stronger "sorting" women exhibit in leaving more lucrative, hence "tougher grading" majors in favor of the less lucrative and hence more lenient ones. Indeed, such stronger sorting on the part of women would follow from Lemma 3 and Corollaries 1 and 2 under the assumption that men tend to place higher weight on pecuniary benefits of careers than do women. These results also suggest that the differences in how men and women sort themselves across majors is primarily driven by gender differences in preferences for fields of study (and their associated career characteristics) rather than the attitude to grades *per se*.

5. Conclusion

The paper's results provide a theoretical foundation for the argument that stronger responsiveness to grades in students' choices of and persistence in academic fields, rather than being an exogenous characteristic specific to a demographic group (such as gender), is more likely to reflect the group differences in the underlying preferences for academic fields, whose existence in principle has been documented in the literature. Furthermore, the results suggest that it is the lower underlying preference of a student for a field of study that is likely to make him or her more responsive to grades received in it, rather than the response being driven by direct preference for good grades or study effort required to attain them. This argument runs counter to a commonly suggested understanding (including by college administrators trying to encourage student enrollment in certain disciplines) that it is an ingrained stronger direct sensitivity to grades, akin to a behavioral predisposition, characteristic of certain student types that makes them less attached to the academic disciplines which are known to assign lower grades. Our analysis therefore questions the effectiveness of college administrators' proposals to more demanding majors to catch up to the more lenient grading policies in other majors to make themselves more attractive to students.

Besides the paper's main focus on students' decisions whether to persist in their starting college discipline or to migrate to an alternative field, the analysis also offers interesting insights into decisions whether to persist in college until graduation or to drop out. A large magnitude of the latter phenomenon in the US higher education made it a subject of much attention of the empirical literatures, both in micro- and macro-economic contexts. The results of the Theorem and Corollary 2 predict that students who attach high value to post-college career earnings but perform too poorly to compel them to persist in a major that promises such high monetary returns, are more likely to drop out of college rather than switch to a less lucrative alternative major. This prediction indeed appears to be borne out in the data showing that such decision pattern is more frequent among men relative to women. For instance, Chen and Soldner's (2013) NCES Report states that of men initially pursuing STEM fields in bachelor's degree programs 24% left them by dropping out of college altogether compared to 14% such figure among women who started out in STEM. The present paper's results establish the mechanism of this phenomenon as driven by the differential value students attach to pecuniary benefits of careers, consistent with the empirical work referenced in the Introduction showing that men tend to value such benefits relatively more than women.

Appendix

Proof of Lemma 1.

Consider the optimization problem (4) solved by student of ability level a whose utility function has the weight coefficients α, β, γ. The problem entails choosing a major and the effort level to exert within it. Let $V_m(a, \alpha, \beta, \gamma)$ stand for the maximum value of the utility function U_m in problem (4) within major m for a student of ability level a and weight coefficients α, β, γ, i.e., the highest welfare the student can achieve were he/she to pursue major m. This means that problem (4) for the student can be stated as that of choosing across majors with the highest level of value $V_m(a, \alpha, \beta, \gamma)$:

$$\max_{m} V_m(a, \alpha, \beta, \gamma) = \max_{m, e} U = \alpha u(w_m ae) + t(ae) - \beta v(e) + \gamma s(g_m(ae)) \quad \text{(A.1)}$$

Let a student of ability level a' and utility function weight coefficients α, β, γ prefer major 1 over major 2 or be indifferent between the two, i.e.,

$$V_1(a', \alpha, \beta, \gamma) \geq V_2(a', \alpha, \beta, \gamma) \quad \text{(A.2)}$$

and let $a'' > a'$. I shall now prove that students of ability a'' with the same utility weights will strictly prefer major 1 over major 2.
Using the Envelope Theorem, I can write for major m=1, 2 :

$$\frac{\partial V_m}{\partial a} = \alpha u'(w_m a e_m(a)) w_m e_m(a) + t'(a e_m(a)) e_m(a) + \gamma s'(g_m(a e_m(a))) g'_m(a e_m(a)) e_m(a) \quad \text{(A.3)}$$

where $e_m(a)$ is the optimal level of effort in (A.1). Then, according to the first order conditions of optimum in (A.1), the expression (A.3) can be rewritten as:

$$\frac{\partial V_m}{\partial a} = \beta v'(e_m(a)) \frac{e_m(a)}{a} \quad \text{(A.4)}$$

This expression increases in the effort argument $e_m(a)$ since function v(.) is convex. It now remains to refer to Assumption 1, which implies that $e_1(a) > e_2(a)$, so according to (A.4), $\frac{\partial V_1}{\partial a} > \frac{\partial V_2}{\partial a}$. Combining this with (A.2), it is straightforward to conclude that $V_1(a'', \alpha, \beta, \gamma) \geq V_2(a'', \alpha, \beta, \gamma)$ for any $a'' > a'$, as required.

The proof of the complementing result: if a student of given ability prefers the less lucrative major 2 over major 1 or is indifferent between the two, then student of a lower ability will have the same preference, is completely analogous to the above.

I have thus established that controlling for the utility weight coefficients, the difference between a student's valuation of the two majors, $V_1(a, \alpha, \beta, \gamma) - V_2(a, \alpha, \beta, \gamma)$, strictly increases in student's ability. This completes the Lemma's proof regarding threshold $a_{1/2}$.

The proofs regarding $a_{1/0}$ and $a_{2/0}$ proceed along the same lines but simplified by the fact that $\frac{\partial V_0}{\partial a} = 0$ as well as the fact that study effort under the "no college" option $e_0(a) = 0$ for any level ability level a, hence $e_m(a) > e_0(a)$ is trivially the case for m=1, 2. $\blacksquare$

Proof of Lemma 2.

Let a student of ability level $a^0_{1/2}$ and utility function weight coefficients $\alpha^0, \beta^0, \gamma^0$ be indifferent between majors k and j, as per Lemma 1, i.e.,

$$V_1(a^0_{1/2}, \alpha^0, \beta^0, \gamma^0) = V_2(a^0_{1/2}, \alpha^0, \beta^0, \gamma^0) \tag{A.5}$$

According to the Envelope Theorem, $\frac{\partial V_m}{\partial \alpha} = u(w_m a e_m(a))$, $\frac{\partial V_m}{\partial \gamma} = s(g_m(a e_m(a)))$, $\frac{\partial V_m}{\partial \beta} = -v(e_m(a))$ for m=1,2. Therefore, since $e_1(a) > e_2(a)$ according to Assumption 1, we can write $\frac{\partial V_1}{\partial \alpha} > \frac{\partial V_2}{\partial \alpha}$, $\frac{\partial V_1}{\partial \beta} < \frac{\partial V_2}{\partial \beta}$, $\frac{\partial V_1}{\partial \gamma} > \frac{\partial V_2}{\partial \gamma}$ Therefore, along the lines of the proof of Lemma 1, for any $\alpha' > \alpha^0$, any $\beta' > \beta^0$, and any $\gamma' > \gamma^0$, I can state, respectively:

$$V_1(a^0_{1/2}, \alpha', \beta^0, \gamma^0) > V_2(a^0_{1/2}, \alpha', \beta^0, \gamma^0), \quad V_1(a^0_{1/2}, \alpha^0, \beta^0, \gamma') > V_2(a^0_{1/2}, \alpha^0, \beta^0, \gamma')$$

and

$$V_1(a^0_{1/2}, \alpha^0, \beta', \gamma^0) < V_2(a^0_{1/2}, \alpha^0, \beta', \gamma^0)$$

Applying the reasoning in the proof of Lemma 1 to these inequalities yields the results of part (i) of Lemma 2 for the threshold $a^0_{1/2}$. The proofs for the other thresholds $a_{k/j}$ is analogous.

To prove result (ii) observe that $\frac{\partial V_m}{\partial w_m} > 0$ for m=0, 1, 2 while $\frac{\partial V_k}{\partial w_j} = 0$ for $k = 1, 2,\ j = 2, 0,\ k \neq j$.

To obtain the result for threshold $a_{1/2}$, apply the above to see that the difference between a student's valuations of the two majors, $V_1(a, \alpha, \beta, \gamma) - V_2(a, \alpha, \beta, \gamma)$, strictly increases and decreases in the wage rates w_1 and w_2, respectively. It thus remains to apply the reasoning used in the proof of Lemma 1. The reasoning for the results concerning the thresholds $a_{2/0}$ and $a_{1/0}$ is analogous. $\blacksquare$

Proof of Lemma 3.

Consider the first order condition of optimum in problem (A.1) of a student of ability a determining his/her optimal level of effort $e_m(a))$ if they were to pursue major m.

$$[\alpha u'(w_m h_m(a))w_m + t'(h_m(a)) + \gamma s'(g_m(h_m(a)))g_m'(h_m(a))] \frac{\partial h_m(a,e)}{\partial e} = \beta v'(e_m(a)) \qquad \text{(A.6)}$$

Let the ability parameter *a* in the above relationship increase. Suppose contrary to the Lemma's assertion that for some level of ability *a* its increase results in a decrease of human capital attainment. This implies, according to expression (1), that the study effort must also decrease in response to the increase in ability. Therefore, the right-hand side of equation (A.6) will decrease with the rise in ability under consideration since *v*(.) is convex. However, the left-hand side of (A.6) increases in *a* since functions *u*(.), *t*(.), *s*(.), and *g*(.) are all increasing and concave and because $\partial h_m(a,e)/\partial e$ increases in a -- the latter is true whenever ability *a* and effort *e* are complementary in human capital production, which is certainly the case for the production function (1) hereby. This contradiction completes the proof. ∎

Proof of Proposition 1.

I will first focus on proving the first part $a_{1/2}^H < a_{1/0}^H$ of the Proposition's result. According to the expression (4) these two ability cutoffs are defined by the following equations, respectively:

$$V_1^H = \alpha^H u(w_1 a e_1) + t(a e_1) - \beta v(e_1) + \gamma s(g_1(a e_1)) = V_0^H \qquad \text{(A6)}$$

where $V_0^H = \alpha^H u(w_0)$ while e_1 is the optimal effort level of student of ability $a_{1/0}^H$ in major 1. Let e_2 be the optimal effort level for student of ability $a_{1/0}^H$ in major 2. Recall that $e_1 > e_2$ according to Assumption 1. Since effort level e_2 by student of ability $a_{1/0}^H$ in major 1 is suboptimal, we can write:

$$U_1^H(e_2) = \alpha^H u(w_1 a e_2) + t(a e_2) - \beta v(e_2) + \gamma s(g_1(a e_2)) < V_1^H$$

But at the same time,

$$U_1^H(e_2) > \alpha^H u(w_2 a e_2) + t(a e_2) - \beta v(e_2) + \gamma s(g_2(a e_2)) = V_2^H$$

because $w_1 > w_2$ and α^H is large. Combining these two relationships we obtain that for a student whose ability level is at the threshold $a_{1/0}^H$ the inequality $V_2^H < V_1^H$ holds true. This implies, according to Lemma 1, that $a_{1/2}^H < a_{1/0}^H$, as required.

Recall further that at the ability level $a_{1/0}^H$ equality (A6) holds by the definition of this threshold. Therefore, we have $V_2^H < V_0^H$ at this ability level, which according to Lemma 1 means that $a_{1/0}^H < a_{2/0}^H$, which completes the proof of the proposition. ∎

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