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Sinking Land: Optimal Control of Subsidence

Abstract

Land subsidence threatens the living conditions of roughly 1.2 billion people worldwide in deltaic regions characterized by soft top soil. Economic activity in deltaic regions requires lowering the groundwater levels to keep the land sufficiently dry to maintain productivity, which, however, leaves future generations worse off by accelerating subsidence and increasing future costs. The current policymaking is often myopic by ignoring this intertemporal trade-off. This paper provides a model recognizing this trade-off: we integrate the dynamics of land subsidence and groundwater management to derive optimal paths for controlling the groundwater level. Applying our model to the paradigm case of Dutch agricultural peatlands, we find that the welfare costs of ignoring dynamic efficiency can be in the order of 10 percent of the land value. Our results support current proposals to slow down subsidence by increasing the groundwater levels even in the absence of its social benefits such as avoided carbon dioxide emissions.

JEL-Codes: C610, Q150, Q240, Q250, Q500.

Keywords: land subsidence, agricultural production, intertemporal trade-offs, water management, optimal control.

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1 Introduction

Land subsidence, the downward movement of land surface, is a global threat of paramount importance. Recent estimates indicate that 19 percent of the global population faces a high likelihood of being affected by land subsidence (Herrera-García et al., 2021). Subsidence of soft soils is a major problem in delta regions with high population densities, such as the West and the North of the Netherlands, the Mekong Delta hosting Ho Chi Minh City, the Chao Phraya Delta hosting Bangkok, or the Ciliwung Delta hosting Jakarta. The soft upper soil in these regions compacts and decomposes over long time horizons leading to gradual subsidence. This process is often exacerbated by anthropogenic factors such as intensive agriculture and urbanization (Koster and Ommeren, 2015). The consequences can be severe and include reduction in agricultural productivity, damages to the built environment and infrastructure, and increase in flood risk (Pelsma et al., 2020). For example, deltaic regions typically sink much faster than the sea level rises due to climate change (Shirzaei and Bürgmann, 2018; Nicholls et al., 2021). Hence, subsidence poses severe risks to life and property in delta regions.

How should land subsidence be managed? Keeping soils in subsiding delta areas sufficiently dry for economic activity requires lowering the groundwater levels artificially (*drainage*), which in turn speeds up subsidence, as dry soils compact and decompose easier than wet soils. The continuous cycle of groundwater drainage and subsidence depletes the fertile upper soil layer and increases costs of maintaining groundwater levels. Hence, current productivity gains from deeper ground water levels come at the expense of future costs due to subsidence: less top soil for future generations and higher future costs of pumping out the groundwater and maintaining the water infrastructure. Therefore, policymakers responsible for groundwater and land-use management face an intertemporal trade-off between the costs of land subsidence and the costs of mitigating subsidence through maintaining higher groundwater levels, which is often ignored in current policymaking.

This paper recognizes this intertemporal trade-off in a modelling framework. We offer an analytically tractable dynamic model with an optimal control approach. Our model integrates the dynamics of land subsidence and groundwater management to derive socially optimal paths for groundwater levels. We show that the current policy of aligning the speed of drainage with that of subsidence ignores these intertemporal trade-offs, hence it is *myopic*. We compare this myopic benchmark with the *forward-looking* groundwater policy that takes into account the intertemporal trade-off. Applying our model to the paradigm case of

Dutch agricultural peatlands, we find that ignoring dynamic efficiency can lead to substantial welfare costs of around 10 percent of the potential net gains from agriculture on those peatlands.

Since the Middle Ages, low-lying peatlands of the Netherlands have been drained to allow for agriculture, and even more intensely in the last century (Erkens et al., 2016). As a result, these peatlands are sinking by 8 mm annually on average (Van den Born et al., 2016). The Dutch policy on subsidence has focused on adaptation: lowering groundwater levels periodically at the same rate as the subsidence to maintain productivity levels (Council for the Environment and Infrastructure, 2020; Gils et al., 2021). We show that such a policy cannot be optimal from a forward-looking perspective. Our results suggest to increase groundwater levels compared to this myopic benchmark. Indeed, alternative management strategies have been under consideration in the Netherlands since many years (Querner et al., 2012; Hardeveld et al., 2018). Only recently, a new policy initiative called for increasing groundwater levels to slow down subsidence (Harbers, 2023). The main motivation of the proposal is to mitigate the CO_2 emissions associated with land subsidence. Our results suggest that increasing the groundwater levels can yield significant benefits even in the absence of any benefits from avoided carbon emissions.

In this paper, we focus on subsidence management in agricultural areas. The specific hydrologic, geologic and economic dynamics of subsidence depend on land-use and geographical context. Our model can easily be adapted to analyze other settings, for example urban areas. In this initial step, we ignore various externalities associated with land subsidence, such as the CO_2 emissions and flood risks. Therefore, our estimated welfare costs represent a lower bound. Evaluating the effects of these externalities is an important avenue for future research.

There is a large literature on physical and technical aspects of land subsidence including drivers, measurement and monitoring, future projection, and technical mitigation (Querner et al., 2012; Fokker et al., 2018; Asselen et al., 2018; Shirzaei et al., 2020; Lizárraga and Buscarnera, 2020; Wu et al., 2022). Project appraisal studies have quantified costs and benefits of suggested measures (Warren et al., 1975; Hardeveld et al., 2018; Wade et al., 2018; Kok and Costa, 2021). Our paper is related to the literature on the intertemporal allocation of scarce resources, such as fossil fuel resources (Dasgupta and Heal, 1974; Stiglitz, 1976; Salant, 1976; Hoel, 1978; Sweeney, 1993; Heal, 1993) or groundwater (Gisser and Sanchez, 1980; Casola et al., 1986; Ben-Gal et al., 2013; Quintana Ashwell et al., 2018; Reinelt, 2020). In this paper, we treat the subsiding peat soil as a scarce resource whose intertemporal allocation can be managed by groundwater regulation.

To our best knowledge, Goetz and Zilberman (1995) is the only study so far to point out the Hotelling dynamics arising from the scarcity of peat. Our model, on the other hand, is the first to reflect the unique dynamics and trade-offs stemming from endogenous changes in the costs of groundwater management due to subsidence. This novel mechanism is the main factor driving our robust empirical result that the cost of ignoring dynamic efficiency is substantial, while Goetz and Zilberman (1995) find that these costs are negligible. Unlike Goetz and Zilberman (1995) we do not fix the planning horizon. Our model sheds light on the long-term subsidence effects of groundwater management and we are able to determine the time horizon of the use of peatlands for agriculture endogenously.

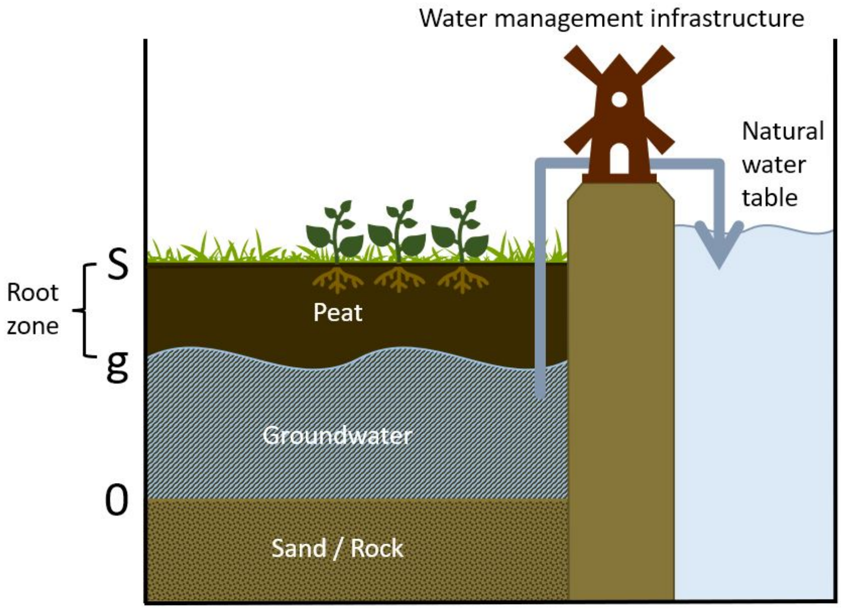
The rest of the paper is organized as follows: We present our model in Section 2. In Section 3, we provide the myopic solution as the benchmark, and derive optimal forward-looking paths in Section 4. Next, we present the optimal paths and their welfare implications by calibrating our model to the Dutch agricultural peatlands in Section 5. In these sections, we focus on a case characterized by a thick peat layer where the terminal time is determined by exhaustion of net economic benefits, which is not the case when the peat layer is thin. We present our results for the case of a thin peat layer in Section 6. The welfare gains from following a forward-looking policy instead of a myopic one crucially depend on the thickness of the peat layer, which we analyze in Section 7. Section 8 concludes.

2 Model

The single most important determinant of the speed of land subsidence is the groundwater level. We devise a deterministic model for a single agricultural plot of land in which the groundwater level can be fully controlled by water pumping and other water management practices. While we focus on subsidence management in agricultural areas, our model can be easily adapted to land subsidence associated with other economic activities, such as urban development.

Physical environment. Figure 1 describes the key elements of our model. The upper soil layer consists of peat, potentially mixed with clay, which we briefly refer to as the peat layer. It is fertile, but subsides gradually over time when drained. We measure the levels of peat surface $S(t)$ and groundwater $g(t)$ at time t in reference to the border where the upper and lower soil layers meet. For notational convenience, we will often ignore the time argument. The lower

Figure 1: Key elements of the model



Notes: The figure illustrates a schematic vertical cross section of a plot of peat land together depicting the key elements of the model: the soil height S, groundwater level g and root zone $R = S - g$. The blue arrows indicate water flows.

layer does not subside, and is relatively less fertile, such as a rocky layer where agricultural production is not possible, or a sandy layer with a low productivity. Without drainage, the groundwater level rises above the surface of the peat layer preventing agricultural activity. This case is the natural equilibrium state of the system where there is no subsidence. Agricultural production, however, requires pumping out the groundwater to leave a positive root zone $R = S - g$ between soil surface and groundwater level.

Yield and costs. The agricultural yield from a given plot of land is a function of the root zone, such that $y = y(R)$. Based on insights from soil and agronomic sciences (Wessiling, 1974; Mulder et al., 2021), we specify a quadratic form for the yield function as follows:

$$y(R) = \psi R(2\kappa - R),$$

where parameter κ is the yield maximizing root zone. The productivity parameter ψ captures agricultural prices. Thus we refer to $y(R)$ as yield or revenues interchangeably. Marginal yield with respect to groundwater level is given by $y_g = 2\psi (R - \kappa)$ and is negative and increasing in the relevant range $R \in [0, \kappa]$. Throughout the paper, we may indicate partial derivatives with a subscript. Then the maximum attainable yield is given by $\bar{y} = \psi\kappa^2$ and the corresponding root zone is $\bar{R} = \kappa$.

The costs of water management are the pumping effort and the investment in and management of the water infrastructure to reach a certain groundwater depth. Costs are determined by the depth of the groundwater level relative to the natural (initial) water table, such that $c = c(g)$. Let S_0 be the initial level of the soil surface of a given plot of land. The water management costs are then given by:

$$c(g) = c_0 + \gamma(S_0 - g),$$

where c_0 stands for the fixed costs and $\gamma > 0$ is the marginal cost of lowering the groundwater level. Here, we assume that costs are linear in groundwater depth, in accordance with most short- and medium-term subsidence cost estimates (Van den Born et al., 2016; Pelsma et al., 2020). It is also plausible that marginal costs are increasing in depth in the long term, as we need broader dikes, more infrastructure and have to pump the groundwater higher up to prevent flooding (Van den Born et al., 2016). Therefore, our predicted welfare gains from adopting a forward-looking policy represent a lower bound.

Lowering the groundwater level to create a root zone for agricultural production leads to subsidence as follows:

$$\dot{S} = -\alpha R,$$

where $\dot{S}$ is the total time derivative of S, which is the vertical shrinkage of the upper soil layer at a point in time. Hence subsidence is proportional to the root zone. We refer to α as the subsidence rate. Here, we assume regeneration of the peat soil is negligible and ignore potential subsidence in deeper geological formations. These simplifications provide analytical tractability to gain insights into the dynamics of our model. Incorporating such considerations is straightforward and can provide further empirical precision.

Myopic versus forward-looking policymaking. The myopic policymaker chooses the groundwater level to maximize instantaneous net benefits $\Pi(S, g)$ at each point in time without accounting for the effect of subsidence on future costs and benefits. The myopic policy maker's problem is:

$$
\begin{aligned}
&\max_{g(t)} \Pi\big(S(t), g(t)\big),\\
&\qquad\qquad \text{where } \Pi\big(S(t), g(t)\big) = y(S, g) - c(g),\\
&\qquad \text{subject to } \dot{S}(t) = -\alpha\big(S(t) - g(t)\big),\\
&\qquad\qquad S(0) = S_0, \quad S(t) \geq 0, \quad 0 \leq g(t) \leq S_0,\\
&\qquad\qquad \text{for all } t \in [0, T].
\end{aligned}
\tag{1}
$$

The forward-looking policymaker, on the other hand, chooses the path of groundwater level g and the terminal time T to maximize the sum of net benefits V over the entire planning horizon $t \in [0, T]$, by discounting future net benefits at rate δ and taking the subsidence dynamics into account. Formally, the forward-looking policymaker solves

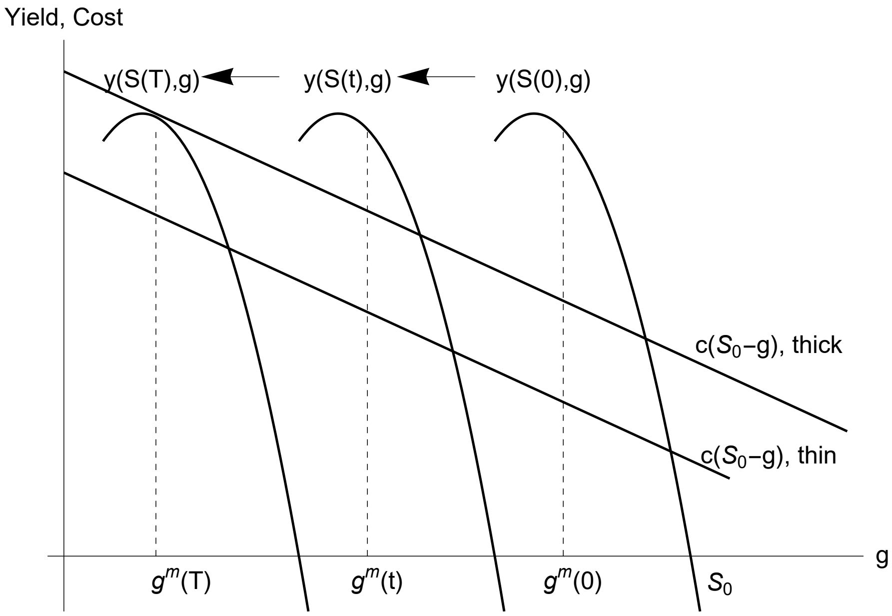
$$
\begin{aligned}
&\max_{g(t),\, T} V\big[S(t), g(t), t\big],\\
&\qquad\qquad \text{where } V\big[S(t), g(t), t\big] = \int_0^T \Pi\big(S(t), g(t)\big) e^{-\delta t} dt,\\
&\qquad \text{subject to } \dot{S}(t) = -\alpha\big(S(t) - g(t)\big),\\
&\qquad\qquad S(0) = S_0, \quad S(t) \geq 0, \quad 0 \leq g(t) \leq S_0,\\
&\qquad\qquad \text{for all } t \in [0, T].
\end{aligned}
\tag{2}
$$

At any point in time, a decision maker can opt out from using the land by stopping groundwater maintenance, such that $g(t) = S_0$. Then the plot fully submerges yielding zero net instantaneous benefits. Therefore, both the myopic and forward-looking policymakers maintain a positive root zone until a terminal (stopping) time T with $\Pi(T) = 0$, after which the land is abandoned. In the forward-looking case, the terminal (stopping) time T is an endogenous variable to be chosen optimally together with $g(t)$, while $\Pi(T) = 0$ can be considered a surprise for the myopic policymaker.

3 Benchmark: myopic policy

We first describe our benchmark policy scenario, namely the myopic policymaking which ignores the effect of current decisions on future conditions. To provide an intuitive understanding of the dynamics in our model, we start with illustrating the myopic policymaking in Figure 2. The figure illustrates the yield function

Figure 2: Myopic policymaking



Notes: This figure illustrates myopic policymaking (m) for two case: high and low fixed costs labelled as thick and thin peat cases. Agricultural yield y and drainage costs c are depicted as a function of groundwater level g.

as a function of the groundwater level g for a given soil surface level S. As long as $g(t) < S(t)$ at time t, there is subsidence, and the yield function moves leftwards. The policymaker initially chooses a $g(0)$ below S_0 by leaving a root zone $R(0) = S_0 - g(0)$ to produce $y(R(0))$. While a positive root zone is necessary input for agricultural production, excessive drainage of the groundwater, for example too low $g(0)$ with a too high $R(0)$, also leads to lower agricultural yield, as plants do not receive sufficient water. Cost of lowering the groundwater $c(g)$ is linear and increasing in depth $S_0 - g$. Therefore, the costs are decreasing in g for a given S_0, as depicted for cases with high and low (fixed) costs.

First, consider the high cost case in Figure 2. The myopic policy ignores what will happen tomorrow and chooses $g(0)$ by maximizing the agricultural yield net of the costs, such that $g^m(0)$, the myopic (m) groundwater level at $t = 0$, maximizes the distance between the yield and the cost curve which occurs where their slopes are equal. Creating a root zone by lowering the groundwater level leads to subsidence, which is given by the change in the peat level as $\dot{S}(t) = -\alpha R(t)$, where α is the subsidence rate. Hence, subsidence is faster when the root zone is larger. In Figure 2, subsidence is represented by the leftward movement of the yield function. Hence, the right branch of the yield function traces $S(t)$ over time on the horizontal axis. The yield function in the middle represents the situation at an arbitrary point in time t. As the shape of the yield function does not change over time and the slope of the cost function is given, the optimal myopic root zone remains constant.

In Figure 2, the yield function on the left represents the point where the myopic policy stops drainage. Agricultural activity beyond this point does not yield further net benefits. Therefore, the myopic policymaker abandons the land avoiding any further costs. However, if the costs were described by the lower cost function, the net benefits would still be positive when the groundwater level reaches to the zero reference point. In this case, the agricultural activity would still go on, but switches into a new phase where the dynamics and optimal behaviour depend on the remaining peat and the characteristics of the lower soil layer. As in the figure, we label the former situation as the *thick peat* case, where the stopping time is determined by the exhaustion of net economic benefits while we are still exploiting the upper soil layer. We refer to the latter case as the *thin peat* case, where, once the zero groundwater level is reached, a new phase starts. In the following, we first focus on the thick peat case.

The solution to myopic policymaking is straightforward. Still, for the clarity of the exposition, we summarize the solution in a proposition as follows and provide the derivations in the Appendix:

Proposition 1. *There exist a unique critical S_0 level $\hat{S}_0$, such that if $S_0 \geq \hat{S}_0$, then there exists a unique myopic policy $g^m(t) > 0$ for all $t \in [0, T]$ solving Equation (1). Respectively, the myopic groundwater level, peat height, root zone, terminal time, and the critical initial soil height are given by*

$$
\begin{aligned}
g^m(t) &= S_0 - R^m(\alpha t + 1) \\
S^m(t) &= S_0 - \alpha R^m t \\
R^m &= \kappa - \frac{\gamma}{2\psi}, \\
T^m &= \frac{1}{\alpha}\left(\frac{y^m - c_0}{\gamma R^m} - 1\right), \\
\hat{S}_0 &= \frac{y^m - c_0}{\gamma},
\end{aligned}
$$

where we denote $y^m = y(R^m)$.

Proof. See Appendix A. □

The results in Proposition 1 reflects what we observe in Figure 2. The groundwater level decreases at the same speed as the soil level leading to a root zone that remains constant over time. The marginal agricultural returns to root zone are given by $2\psi(\kappa - R)$, which is increasing in κ and ψ. Hence, the myopic root zone is increasing in κ and ψ. On the other hand, a higher marginal cost of lowering the groundwater level γ leads to a smaller root zone, as it reduces the net benefits from agricultural production.

The myopic policy abandons the land when the agricultural returns does not recover the costs of maintaining the groundwater level, as reflected in the expression for the terminal time T^m. A higher root zone speeds up subsidence leading to a faster exploitation of the peat layer.

In this section, our assumption is that the peat layer is sufficiently thick for given yield and cost structures, such that $S_0 > \hat{S}_0$. The critical soil surface level $\hat{S}_0$ is endogenous and determined by the yield and cost structure. Hence, even if the peat layer is physically thin, a sufficiently low agricultural productivity and/or high groundwater management costs can yield $S_0 \geq \hat{S}_0$ leading to the situation which we label as the thick peat case. In this case, the optimal groundwater levels are always positive. The thin peat case arises when $S_0 < \hat{S}_0$ which we analyze in Section 6.

4 Forward-looking policy

In this section, we present the solutions to the forward-looking paths in thick peat. In this optimal control problem given by equation (2), the control variable is the groundwater level g and the state variable is the height of the peat layer S. Hence, the planner steers subsidence by managing groundwater levels. The current value Hamiltonian is given by

$$H_c = [\psi(S - g)(2\kappa - (S - g))] - [c_0 + \gamma(S_0 - g)] - \alpha\lambda(S - g),$$

where λ is the costate variable representing the shadow price of depleting the peat layer. The optimum path must satisfy the FOCs of the maximum principle given by

$$
\begin{aligned}
\frac{\partial H_c}{\partial \lambda} = 0 \Rightarrow & \dot{S} = -\alpha(S-g) && (3)\\
\frac{\partial H_c}{\partial g} = 0 \Rightarrow & 2\psi(S-g-\kappa) = -\gamma - \alpha\lambda && (4)\\
\dot{\lambda} - \delta\lambda = -\frac{\partial H_c}{\partial S} \Rightarrow & \dot{\lambda} = (\delta+\alpha)\lambda + 2\psi(S-g-\kappa) && (5)\\
\text{Transversality C. 1} \Rightarrow & \lambda(T)e^{-\delta T} = 0 && (6)\\
\text{Transversality C. 2} \Rightarrow & H_c(T)e^{-\delta T} = 0, && (7)
\end{aligned}
$$

where the transversality conditions arise from having free terminal time and state, respectively. The first and third FOCs represent the canonical system: the law of motion for the state (S) and costate (λ) variables, respectively. The former is simply a restatement of the subsidence equation. The second FOC is the static optimality condition that the current value Hamiltonion must be maximized with respect to the groundwater level at each point in time, which implies that, at each point in time, the current profit effects of changing the groundwater level must be equal to its effect on the value of remaining peat layer height.

We summarize the solution to these FOCs for the thick peat case in the following proposition:

Proposition 2. *There exists a critical S_0 level $\hat{S}_0$, such that, if $S_0 \geq \hat{S}_0$, there exists a unique forward-looking policy $g^f(t) > 0$ for all $t \in [0, T]$. Respectively, the forward-looking groundwater level, peat height, root zone, and shadow price are given by*

$$
\begin{aligned}
g^f(t) &= \left(S_0 - R^m + \Lambda\left(1 + \frac{\alpha}{\delta}e^{-\delta T^f}\right)\right) - \alpha\left(R^m - \Lambda\right)t - \Lambda\left(1 + \frac{\alpha}{\delta}\right)e^{\delta\left(t - T^f\right)}\\
S^f(t) &= \left(S_0 + \Lambda\frac{\alpha}{\delta}e^{-\delta T^f}\right) - \alpha\left(R^m - \Lambda\right)t - \Lambda\frac{\alpha}{\delta}e^{\delta\left(t - T^f\right)}\\
R^f(t) &= R^m - \Lambda\left(1 - e^{\delta\left(t - T^f\right)}\right)\\
\lambda^f(t) &= \frac{\gamma}{\delta}\left(1 - e^{\delta(t - T^f)}\right),
\end{aligned}
$$

where $\Lambda = \gamma\alpha/(2\psi\delta)$ and T^f is the forward looking terminal time which obeys a transversality condition:

$$
\begin{aligned}
&TVC(T^f) = 0,\\
&\qquad \text{where } TVC(T^f) = \left(R^m - \frac{y^m - c_0}{\gamma}\right) + \Lambda\frac{\alpha}{\delta}\left(1 - e^{-\delta T^f}\right) + \alpha\left(R^m - \Lambda\right)T^f.
\end{aligned}
$$

Proof. See Appendix B for the derivation of the solution. See Appendix D.5 for its uniqueness property. □

In this solution, there are several important points worth attention. First, it is clear that the forward looking root zone is always smaller than the myopic root zone until the terminal time at where they are equal. Therefore, the forward-looking planner keeps the subsidence slower by choosing a path with higher groundwater levels at all times. Second, it is also easy to verify that the groundwater level and the peat layer height of the two policies are equal at the terminal times. The reason is that the zero profit condition determining the terminal time is the same for both policy types. The difference is that the forward-looking planner extends the exploitation time with a smaller root zone by taking into account the scarcity of the peat layer. Finally, while the uniqueness of myopic solution is self-evident, the forward-looking terminal time is given by an implicit function, and it appears in the expressions for all endogenous variables. We refer to Appendix D.5 for the uniqueness of forward-looking policy.

We summarize the above discussion in the following proposition:

Proposition 3. *In the thick peat case, (i) the forward-looking groundwater level is higher than the myopic one, leading to slower subsidence, and smaller root zone. Formally,*

$$g^f(t) > g^m(t), \quad S^f(t) > S^m(t), \quad \textit{and} \quad R^f(t) > R^m.$$

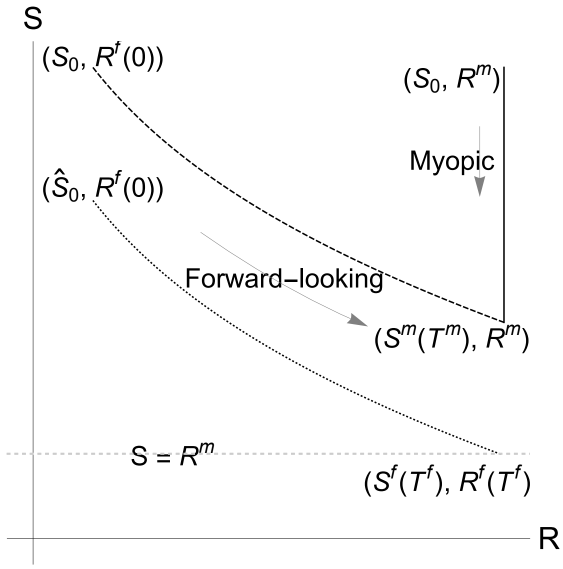
(ii) The exploitation time in the forward looking policy is longer compared to the myopic policy. Formally, we have $T^f > T^m$. (iii) The forward-looking and myopic paths end with same groundwater level, peat layer height, and root zone.

$$g^f(T^f) = g^m(T^m), \quad S^f(T^f) = S^m(T^m), \quad \textit{and} \quad R^f(T^f) = R^m.$$

Proof. The proof follows the above discussion. □

To provide the intuition underlying these results, Figure 3 describes the forward-looking policy in reference to the myopic policy. It compares the myopic and forward-looking policies on the $\{R(t), S(t)\}$ plane for the thick peat case. The myopic paths start at (S_0, R^m) and end at $(S(T^m), R^m)$. Hence the root zone is constant as described earlier. The forward-looking policy starts with a smaller root zone at $(S_0, R^f(0))$ for two reasons: First, it accounts for the fact that subsidence will lead to higher costs in the future. Second, it gives higher weight to present compared to future net benefits by applying a constant discount rate δ. As the

Figure 3: Myopic versus forward-looking policies



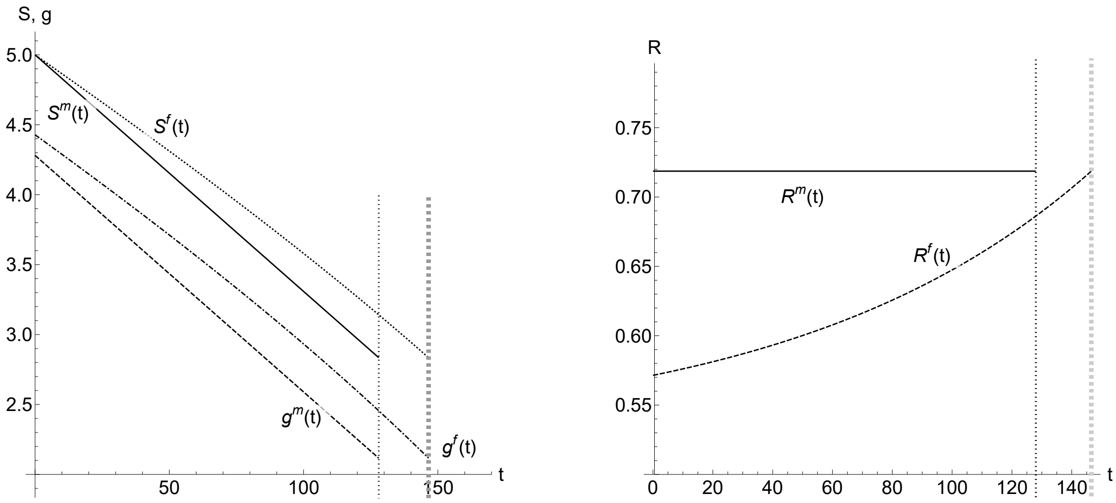
Notes: This figure compares myopic (m) vs forward-looking (f) policies, depicting myopic (solid vertical) and forward looking (dashed) paths of root zone R and soil surface S; from $t = 0$ to endogenous terminal time T.

forward-looking root zone is smaller $R^f(0) < R^m$, subsidence is slower. Hence, the forward looking policy exploits the peat over a longer time horizon. The length of the paths, as depicted in Figure 3, are proportional to the time spans that the peat is exploited. The forward-looking root zone increases gradually reaching the myopic root zone at the stopping time, as it is determined by the exhaustion of net economic benefits, which is the same for both policies.

Figure 3 also illustrates a forward-looking path starting at $(\hat{S}_0, R_f(0))$ which represents the borderline case between the thick and thin peat cases. On this path, economic exhaustion happens at the time when we reach the zero groundwater level. Ceteris paribus, starting with $S_0 > \hat{S}_0$ leads to the thick peat case. Otherwise, we are in the thin peat case.

In the previous section, we have shown that the myopic root zone is constant. A constant root zone is an observational fact in the Netherlands where groundwater levels are lowered at the same rate as the land subsides. In the model, the constancy of the myopic root zone follows from the linearity of the cost function, which seems to be a good approximation. Our argument that current policymaking is likely to be myopic follows from another theoretical finding: intertemporal optimality rules out a constant root zone over time. Hence, current policymaking cannot be forward-looking and can be at best myopic. This result holds under fairly general conditions without parametrizing the yield and cost function. We

Figure 4: Myopic versus forward-looking paths in thick peat



(a) Top soil and groundwater height (b) Root zones

Notes: This figure illustrates the myopic (m) and forward-looking (f) paths in thick peat applied to Dutch peatlands over time t in years. See Appendix C for the baseline values of the parameters. Vertical lines represent stopping times. Panel (a) presents the optimal top soil and groundwater (S and g) heights in meters. Panel (b) presents optimal root zones R in meters.

state this result in the following proposition:

Proposition 4. *Suppose that yield function is concave and the cost function $c(g)$ is weakly convex in g. Then, the root zone solving the forward-looking problem in equation (2) cannot be constant.*

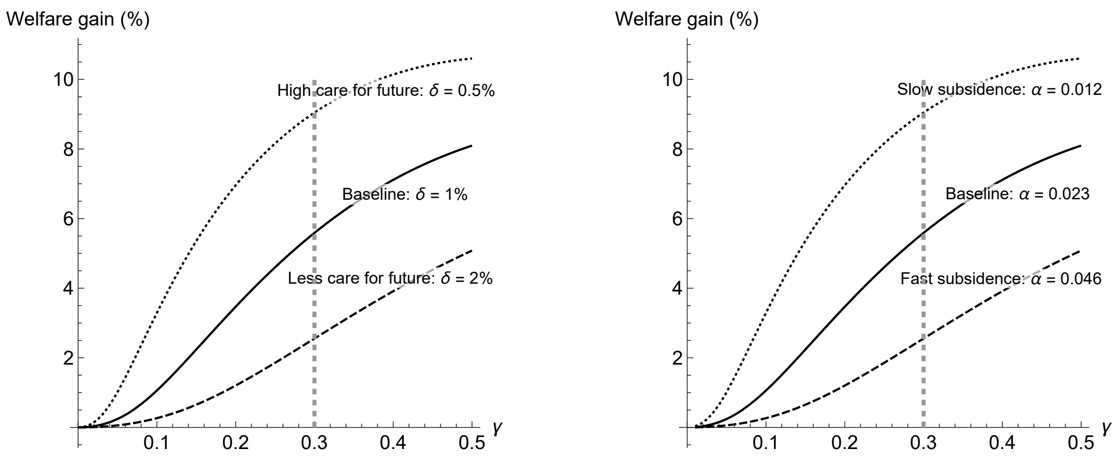
Proof. See Appendix E. □

5 Calibration: optimal paths and welfare

In this section, we apply our model to the Dutch drained peatlands based on parameter values obtained from the literature. We provide a detailed Appendix C for the parameter values). First, we present the optimal paths for the groundwater height and corresponding soil height and root zone under myopic and forward-looking behaviour. Next, we analyze the welfare implications of these paths. As stated earlier, we focus on the thick peat case. We present the results for the thin peat case in the next section.

Optimal paths. Figure 4a shows the myopic (m) and forward-looking (f) paths for the groundwater $g(t)$ and soil surface $S(t)$ levels when the peat layer is very

Figure 5: Welfare gains



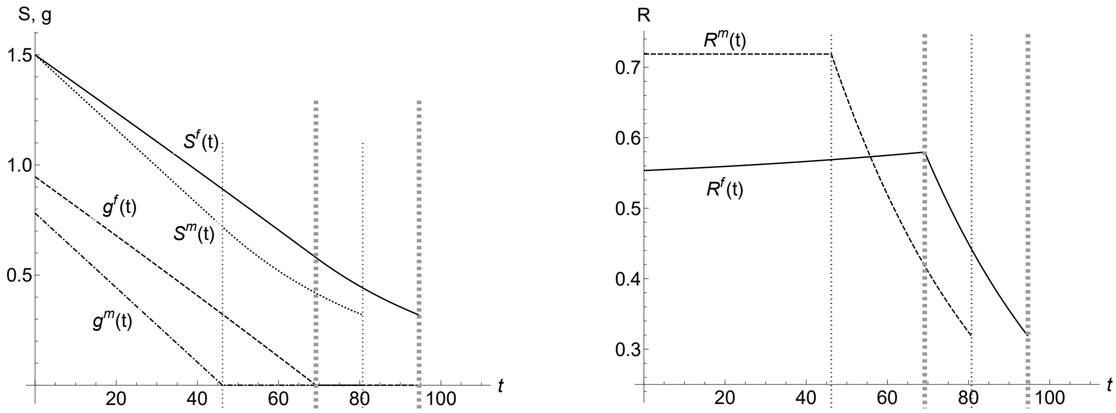
(a) Sensitivity to the discount rate δ (b) Sensitivity to the subsidence rate α

Notes: This figure shows the welfare gains from forward-looking policy compared to the myopic benchmark in percentage points over a range of levels of marginal cost γ, calibrated for Dutch peatlands. See Appendix C for the baseline values of the parameters. Vertical dashed lines represent the baseline value of γ. Panel (a) and panel (b) present sensitivity analyses with respect to the discount rate δ and subsidence rate α, respectively.

thick and given by 5 meters. Forward-looking policy starts with a higher groundwater and levels off slower, exploiting the fertile upper soil for a longer time horizon, almost 20 additional years. Hence, subsidence is slower, and the soil surface level is always higher compared to that of the myopic policy. The right panel shows the corresponding root zones $R(t)$. The initial root zone in the forward-looking case is around 20 cm smaller, and remains smaller throughout the planning horizon. As explained earlier, exhaustion of net benefits determining the stopping time happens at the same root zone depth, while the forward-looking policy reaches that level much later.

Welfare gains. For both the myopic and forward-looking policies, the welfare represents the sum of net benefits (profits) from the initial to the terminal time. Figure 5 shows that the welfare from following the forward-looking policy is around 6 percent higher compared to that from the myopic policy with our baseline parameter choices. When the marginal cost γ is higher, the welfare gains can be over 8 percent. The figure presents two sensitivity analyses: On the left panel, the welfare gains are around 10 percent, when we apply a lower discount rate to the future benefits and costs, which reflects weighing future generations' well-being more. On the right panel, the welfare gains again are around 10 percent when the subsidence rate is slower compared to our benchmark case. In both cases, the

Figure 6: Myopic versus forward-looking paths in thin peat



(a) Top soil and groundwater height

(b) Root zones

Notes: This figure illustrates the myopic (m) and forward-looking (f) paths in thin peat applied to Dutch peatlands over time t in years. See Appendix C for the baseline values of the parameters. Vertical lines represent stopping times. Panel (a) presents the optimal top soil and groundwater (S and g) heights in meters. Panel (b) presents optimal root zones R in meters.

welfare gains can be higher than 10 percent when the marginal costs are higher than our benchmark value. Our finding is remarkably different from Goetz and Zilberman (1995) who do "not [find] a big difference between short- and farsighted optimal behavior". Note that their result is obtained for a fixed time horizon of 30 years.

6 Thin peat

Until this section, we have assumed that the initial peat layer is sufficiently thick, such that the groundwater levels are still within the peat layer at the terminal time. When this condition does not hold, the characteristics of the lower soil level is binding for the optimal paths. For expositional brevity, we assume that the lower base layer is unproductive. From a welfare comparison perspective, the difference with the case of a productive lower base layer is small which we discuss later in this section. When the base layer is unproductive and the peat layer is not sufficiently thick, we may observe three further cases: First, *Thin peat case*, when $S_0 < \hat{S}_0$. In this case, we start with a positive groundwater level, but we reach a zero groundwater level while land use is still profitable. Second, *Very thin peat case* when S_0 is very small, where we start and end with zero groundwater level. Third, *Non-utilization case* when S_0 is too small, such that there are no net benefits to be exploited. The solutions for the latter two cases are straightforward. In this section, we present the calibration results for the optimal paths in the the thin peat case. We provide the solutions and their derivations for all cases in Appendix D.

Figure 6 illustrates the myopic and forward-looking paths for a thin peat layer of 1.5 meters. The forward-looking policy lowers the groundwater level much slower, slowing down the subsidence and reaching the unproductive base layer around 20 years later than the myopic policy. At this phase switch, the remaining peat layer is smaller with the forward-looking policy. There is an important behavioural difference leading to the drastic differences between the phase-switching times and remaining peat layers: the forward-looking policy chooses the phase-switching time and how much peat to leave for the second phase by trading off its effects on the net benefits in the first and second phases. On the other hand, phase-switching time can be considered a surprise for the myopic policymaker.

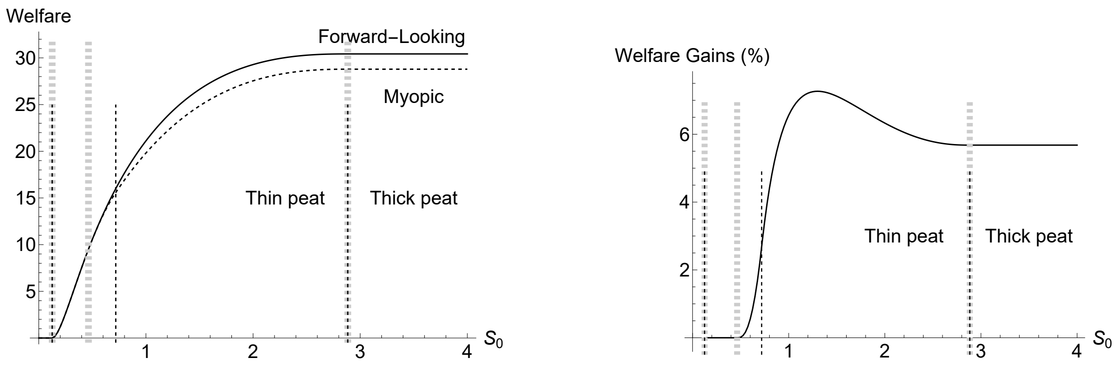
Despite the difference in the timing of second phase and remaining top soil level at the switching time, the dynamics in the second phase is the same for both policies: the groundwater level is kept at $g = 0$, and soil surface is allowed to subside until net benefits are exhausted. The duration of the second phase is longer with the myopic policy, as it reaches to the zero groundwater level very early by ignoring the intertemporal trade-offs that the forward-looking policy takes into account. The right panel of Figure 6 shows the corresponding root zones. The forward-looking policy starts with a root zone around 15 cm smaller which increases gradually over time, while the myopic root zone is constant. At the phase switching time, the forward-looking root zone is smaller, as it leaves less peat for the second phase as explained earlier.

Note that, when the lower soil level is not fertile, economic exhaustion must happen at some point in time, as the upper soil level gradually becomes thinner leading to lower returns in terms of agricultural yield, while we still experience the high costs of maintaining the groundwater level at zero. On the other hand, if the lower base layer were productive, agricultural activity might be maintained forever. However, such a policy is optimal only if the peat layer is sufficiently thin, so that the cost in this final phase is not too high. Hence, in this case there are no drastic differences in terms of welfare evaluation.

7 Heterogeneity in welfare gains

The most important observational variability determining the optimal decision is the initial soil surface level. Figure 7 illustrates the welfare levels for given initial soil heights in the left panel. The right panel illustrates the welfare gains from

Figure 7: Welfare comparison by peat thickness



(a) Welfare levels (b) Welfare gains

Notes: This figure shows the welfare comparison by thickness of peat layer, calibrated for Dutch peatlands. See Appendix C for the baseline values of the parameters. Vertical lines indicate threshold levels of $\hat{S}_0$ leading to different cases (short-thin lines for the myopic and tall-thick lines for the forward-looking policies). Panel (a) provides the net present values of welfare levels in 1000 Euros per hectare as a function of initial top soil height S_0 in meters. Panel (b) shows the welfare gains from the forward-looking policy in reference to myopic policy in percent as a function of initial top soil height S_0 in meters.

following the forward-looking policy. The vertical lines are the critical S_0 levels for different cases. As mentioned earlier, an unproductive base layer entails four cases: 1) Thick peat. 2) Thin peat. 3) Very thin peat. 4) No utilization. The critical S_0 values may be different for the myopic (thin vertical lines) and forward-looking policy (thick vertical lines).

The welfare functions differ between policies. For both the myopic and forward-looking policies, we calculate the intertemporal net benefits (profits) as

$$V^i = \int_0^{T^i} \Pi^i(t) e^{-\delta t} dt,$$

for $i = \{m, f\}$. Here, T^i represents the overall terminal time which is given by the end of profitable use. In the thin peat case profitable use stretches over two phases, before and after a zero ground water level is reached. The instantaneous profit functions differ by case and are given in the proofs of corresponding propositions. Therefore, $\Pi^i(t)$ is not necessarily a continuous function. For example, in the thin peat case, where there are three phases, V^i is given by the sum of welfare under the first and second phases.

As can be seen in the left panel of Figure 7, welfare does not depend on S_0 in the thick peat case, as economic net benefits are exhausted within the upper soil layer. On the other hand, welfare is increasing in S_0 in thin peat. The right panel shows that the welfare gains from following the forward-looking policy is around

6 percent in the thick peat case. In the thin peat, these gains can be around 8 percent. In the thin peat case, the welfare gains are inverted-U shaped, which is driven by two opposing adjustments with respect to a change in S_0: A higher S_0 leads to higher profits given the terminal time and leads to a longer extraction given the profits. Welfare is increasing in both adjustments. However, while the former can reduce the relative advantage of a forward-looking policy, the latter always favours it.

Figure 7 shows that welfare gains are smaller when the peat is very thin where the characteristics of the lower soil layer matters. For this reason, assuming a productive lower soil layer does not make any significant difference in this welfare comparison within reasonable parameter ranges.

8 Discussion and conclusion

This paper offers a workhorse model for groundwater management in subsiding peatlands which accounts for the intertemporal trade-offs in the management of land subsidence. Applying our model to the paradigm case of peatlands in the Netherlands, we show that the welfare gains from adopting a forward-looking policy instead of a myopic one can be in the order of 10 percent of the value of the land, depending on the thickness of the initial peat layer. This prediction is conservative for several reasons: First, the increase in future costs could be more drastic than suggested by our linear cost function due to water volumes, energy requirements, and infrastructure costs that are increasing in the depth of groundwater levels (Van den Born et al., 2016), which might suggest a convex cost function instead. Second, uncertainty in rainfall patterns and variation in exterior water pressure can drive up groundwater management costs. Therefore extending our model with stochastic elements representing groundwater variability can lead to additional gains from forward-looking management. Third, our welfare evaluations are based on the assumption that marginal well-being derived from agricultural yield is constant, such as a linear utility function which is a one-to-one mapping from agricultural yield to consumer preferences. A concave utility function would put more weight on the marginal yield when the consumption level is small due to subsidence. Fourth, peatlands are large carbon stores, and current subsidence is responsible for 3% of Dutch CO_2 emissions (Van den Akker et al., 2008). An important step forward would be to account for the associated social costs of subsidence. Overall, the gains from applying a forward-looking policy are likely to be even higher than our conservative prediction.

Our results show that both the intertemporal trade-off between short-term and long-term production losses as well as water management costs can be rationales for slowing down land subsidence over time, even when we disregard the social costs of subsidence, for example, from CO_2 emissions, which are generally considered the main reason for subsidence mitigation. Our model can provide valuable input for decision-makers in the design of more efficient long-term policies for groundwater and subsidence management. We have shown that the current common water policy for Dutch peatlands is likely to be short-sighted. Our results support the calls to raise relative groundwater levels to stretch out the exploitation of the peat over a longer time period and delay higher future costs.

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Appendix

A Proof of Proposition 1

The first order condition (FOC) for myopic optimality is given by $\Pi_g = 0$. This FOC is sufficient to show that myopic root zone R^m is constant over time and given by the expression in the proposition. For $R^m > 0$, we must have $\kappa > \gamma/2\psi$. Then the myopic yield is positive and given by $y^m = \bar{y} - \gamma^2/4\psi > 0$. There is subsidence following $\dot{S} = -\alpha R^m = -\alpha(\kappa - \gamma/(2\psi))$. Solving this simple differential equation together with the initial condition $S(0) = S_0$ gives the myopic path for the peat height $S^m(t)$ in the proposition. Using $g = S - R$ leads to the myopic groundwater path $g^m(t)$. As $g^m(0) = S_0 - R^m$, we must have $S_0 > R^m$, which defines the critical level $\hat{S}_0 = R^m$.

The net benefits Π^m follows from substituting $g^m(t)$ and $S^m(t)$:

$$\Pi^m = y^m - (c_0 + \gamma R^m(\alpha t + 1)) \text{ for } h = \{1, 2\}.$$

The myopic terminal time T^m is the point in time where instantaneous net benefit is zero, such that $\Pi^m(T^m) = 0$, which leads to the expression for T^m in the proposition.

The derivation of the critical surface level requires further results from the thin peat case, which we analyze in Appendix D; see the proof of Proposition 5.

B Proof of Proposition 2

Substituting Equation (4) in (5) gives $\dot{\lambda} = \delta\lambda - \gamma$. The solution to this linear first order differential equation is given by

$$\lambda = \frac{\gamma}{\delta} + K_1 e^{\delta t}, \tag{8}$$

where K_1 is an undetermined coefficient. Substituting this expression and $R = S - g$ in Equation (4) yields

$$R = R^m - \frac{\alpha}{2\psi}\left(\frac{\gamma}{\delta} + K_1 e^{\delta t}\right), \tag{9}$$

where we simplify the expression by using the myopic root zone in thick peat denoted here with R^m. Substituting this expression in Equation (3) gives $\dot{S} =$

$-\alpha R^m + (\alpha^2/2\psi)\,\lambda$. Substituting Equation (8), and solving the resulting differential equation gives

$$S = K_2 + \left(-\alpha R^m + \frac{\alpha^2}{2\psi}\frac{\gamma}{\delta}\right) t + \frac{\alpha^2}{2\psi}\frac{K_1}{\delta} e^{\delta t}, \tag{10}$$

where K_2 is an undetermined coefficient. By using the initial condition $S(0) = S_0$, we eliminate K_2, which leads to

$$S = \left(S_0 - \frac{\alpha^2}{2\psi}\frac{K_1}{\delta}\right) + \left(-\alpha R^m + \frac{\alpha^2}{2\psi}\frac{\gamma}{\delta}\right) t + \frac{\alpha^2}{2\psi}\frac{K_1}{\delta} e^{\delta t}. \tag{11}$$

Substituting the final expressions for S and R in $g = S - R$ gives

$$g = \left(S_0 - \frac{\alpha^2}{2\psi}\frac{K_1}{\delta} + \frac{\alpha}{2\psi}\frac{\gamma}{\delta} - R^m\right) + \left(-\alpha R^m + \frac{\alpha^2}{2\psi}\frac{\gamma}{\delta}\right) t + \left(\frac{\alpha^2}{2\psi}\frac{1}{\delta} + \frac{\alpha}{2\psi}\right) K_1 e^{\delta t}. \tag{12}$$

Substituting Equation (8) in the first transversality condition and solving for the undetermined coefficient K_1 leads to $K_1 = -\frac{\gamma}{\delta} e^{-\delta T}$. Substituting this expression back in those derived so far for S, g, R and λ leads to the forward-looking paths provided in the proposition.

The forward-looking terminal time can be derived by using the second transversality condition. The Hamiltonian at the terminal time is given by

$$\begin{aligned} H_c(T)e^{-\delta T} &= \Bigg[\underbrace{\psi R(T)(2\kappa - R(T))}_{y^m}\Bigg] e^{-\delta T} - \left[c_0 + \gamma(S_0 - g(T))\right] e^{-\delta T} - \underbrace{\alpha\lambda(T)e^{-\delta T} R(T)}_{=0 \text{ as } \lambda(T)e^{-\delta T}=0} \\ &= y^m e^{-\delta T} - \left[c_0 + \gamma(S_0 - g(T))\right] e^{-\delta T}, \end{aligned}$$

The second line follows from the first transverality condition and its implication $R(T) = R^m$. In addition, we simply substitute myopic yield for brevity. By applying the second transversality condition with this expression above and substituting $g(T)$, we reach the expression in the proposition which implicitly defines the terminal time.

We provide its derivation in the proof of Proposition 6 in Appendix D, as its derivation requires further results from the thin peat case.

C Parameter values in the application

We obtain the parameter values from multiple data sets and existing applications of subsidence models to the Dutch drained peat grasslands. The parameter values of the yield function $y(S, g, t)$ are from a number of studies on the effect of changes in groundwater levels on yield and farm profit, which are based on the 'Waterwijzer Landbouw' tool and WOFOST models (Mulder et al., 2021). We use cost estimates from existing cost-benefit analyses and water authority projections of subsidence impacts on water management to assign values to the cost function $c(g, t)$. Finally, we use the empirical estimation on the relation between subsidence and groundwater levels by Van den Akker et al. (2008) and adapted by Geisler (2015) to assign values to the parameters of the subsidence function $\dot{S}(S, g, t)$. Table C.1 shows the baseline parameter values and value ranges used in our numerical analysis and provides more details including the source studies.

Marginal subsidence rate. Our baseline value is taken from the empirical relationship found by Van den Akker et al. (2008) based on field experiments in Dutch peatlands. This rate applies to an area of peat without clay cover and with negligible peat regrowth. When clay is mixed through the peat, subsidence is significantly slower, so we apply half the baseline value as our lower bound. Geisler (2015) estimates a higher subsidence rate under the climate scenario with the heighest temperature increases (W+) of the Dutch Meteorological Institute, which we take as our upper bound. These empirical values are obtained in relation to the mean lowest groundwater depth (GLG), defined as the average over 3 lowest measurements every year for 8 years. Since our model is continuous, we change this variable to the continuous depth of the groundwater table, which in our model is equivalent to the root zone, in a 1:1 ratio, in line with the Phoenix subsidence model of Putte (2020).

Fixed costs. Fixed costs depend on hydrologic circumstances and soil composition in the initial situation. Ruijgrok and Van Tuinen (2019) estimate the reference management costs for a pure peat plot at €308/ha/yr, which we use as a baseline. We use their highest cost estimate, for a peat plot with thick clay layer (€494/ha/yr) as our upper value. We use the cost assessment of Wetterskip Fryslan (2014) of €10 million/yr for 52,000 ha peat grassland(= €192/ha/yr) as a lower bound.

Table C.1: Parameters of the model

	Description	Baseline value	Upper bound	Lower bound	Source
α	marginal subsidence rate [m/yr]	0.02354	0.046	0.012	(Van den Akker et al., 2008; Geisler, 2015; Putte, 2020)
c_0	fixed water management cost [€1000/ha]	0.308	0.494	0.192	(Ruijgrok and Van Tuinen, 2019; Wetterskip Fryslan, 2014)
γ	marginal cost of lowering groundwater level [€1000/m/ha]	0.305	0.95	0.2	(Stowa, 2021; Ruijgrok and Van Tuinen, 2019; Hartman et al., 2012)
κ	yield maximizing root zone [m]	0.8	0.6	1.2	(Daatselaar and Prins, 2020; Mulder et al., 2021)
ψ	productivity rate [€1000/m/ha]	1.875	4.6875	0.7	(Ruijgrok and Van Tuinen, 2019; Stowa, 2021; Kroes et al., 2015)[e]
δ	social discount rate	0.01	0.02	0.005	

Notes: This table presents the benchmark parameter values used in the applications to Dutch peatlands.

Fixed costs of water management. γ is the marginal yearly water management costs per meter of drop in the water level relative to the initial soil level. It consists of infrastructure investments, yearly management expenses and energy costs of pumping. In reality, some costs are related to the depth of the groundwater (energy and pumping infrastructure), while other costs are related to the depth of the soil level (landscaping infrastructure). Since the soil level always follows the water level – the maximum difference cannot be more than κ – expressing costs as a function of soil or water depth does not make a large difference, so we have lumped those costs together here. For one-off investments, we use adequate amortization periods to convert these into annual maintenance costs. We used infrastructure prices and investment requirements provided in table 4.1 of Stowa (2021) and water infrastructure cost estimates of Ruijgrok and Van Tuinen (2019). Energy cost estimates are provided by Hartman et al. (2012). γ has a high uncertainty range, resulting from different amortization periods, high local heterogeneity of infrastructure needs and lack of reliable cost data.

Marginal cost of lowering the groundwater level. The yield maximizing root zone depends on local hydrologic and physical circumstances. Based on the WWL-table produced by Mulder et al. (2021) we can find that for peatlands the lowest yield losses from either dry or wet circumstances occur when the root zone is between 0.6 and 1.2 meters. Daatselaar and Prins (2020) show that a root zone of 0.8 meters results in highest revenues in the grass peatlands of the Groene Hart area (South-Holland).

Yield maximizing root zone. Parameter ψ scales the parabolic net revenue function. Its peak value is dependent on both the yield maximizing root zone (κ) and the maximum net revenues obtained with that root zone. The baseline value for ψ corresponds to $\kappa = 0.8$ and maximum net revenues of €1050/ha/yr, which is an estimate of Ruijgrok and Van Tuinen (2019) for Frisian peatland farmers excluding all government subsidies, based on the WaterWijzer Landbouw model (Kroes et al., 2015). The lower bound for ψ corresponds to a higher κ of 1.2. The upper bound corresponds to a higher maximum net revenue of €3100/ha/yr. This is an estimate by Stowa (2021) of the average profits over all Dutch dairy farms - not just those on peatlands - and is likely to include government subsidies.

D Full solution

In the main text, we primarily focused on the thick peat case and provide only the calibration results for the thin peat case. In this section, we provide the full solution of our model covering all potential cases. First, we start with outlining all potential cases and phases entailed by each case. Second, we introduce further notation to distinguish optimal paths for each case and phase. Third, we provide the full solution for the myopic case. Fourth, we solve optimal control models that cover the cases of thick peat, thin peat, and very thin peat.

D.1 Cases, phases and notation

In the following, we derive optimal paths for the two policy types indexed with $i = \{m, f\}$, where m and f indicate myopic and forward-looking optimality, respectively. Initially, we assume that the lower soil layer is unproductive, such as a rocky layer, so that the root zone is constrained by the upper layer. In this case, an optimal path can have at most two phases (or regimes) $r = \{r1, r2\}$. In Phase 1, the groundwater levels are positive. If the peat layer is sufficiently high, then net economic benefits are exhausted during Phase 1, before reaching the zero groundwater level, which we call the *thick peat case*. If the initial peat layer is not that high, then Phase 1 ends with zero groundwater level, where the optimal policy switches to Phase 2 where the groundwater level is kept at zero until the net economic benefits are exhausted. We refer to this case as the *thin peat case*. There are two further cases characterized by very thin peat: one with a zero groundwater policy throughout, and another one without any agricultural activity. We will refer to the former case as *very thin peat case*, and the latter as the *non-utilization case*. To summarize, given the cost and yield structure, with an unproductive lower soil layer, we distinguish four cases depending on the initial peat height indexed $h = \{h1, h2, h3, h4\}$ and labeled thick, thin, very thin, and non-utilization case, respectively.

In the following, we indicate optimal paths with a superscript $\{i, r, h\}$ indicating the policy type, phase, and case. For example, $g^{m,r1,h1}$ is the optimal myopic groundwater management path in Phase 1 of Case 1 (thick peat). We may ignore some or all of the superscripts when there is no risk of confusion. We will parameterize the borders between different cases in a critical initial surface level $\hat{S}_0$. We will have three such critical levels denoted with $\{\hat{S}_0^{c1}, \hat{S}_0^{c2}, \hat{S}_0^{c3}\}$ constituting the border surface levels between the four cases we may observe (see Figure 7).

D.2 Myopic policy

We state our results for the myopic policy for all possible cases in a single proposition. To clarify the notation and to be precise in distinguishing the cases, we include the results in Proposition 1 in the following proposition.

Proposition 5. *There exists a myopic policy $g^m(t)$ for all $t \in [0, T]$ solving Equation (1). There exist unique critical S_0 levels $\{\hat{S}_0^{c1}, \hat{S}_0^{c2}, \hat{S}_0^{c3}\}$, such that*

$$\begin{aligned}
\textit{if } S_0 \geq \hat{S}_0^{c1} &\Rightarrow \textit{Case 1 (Thick peat): } g^m(0) > 0 \textit{ and } g^m(T^m) \geq 0,\\
\textit{if } \hat{S}_0^{c1} > S_0 \geq \hat{S}_0^{c2} &\Rightarrow \textit{Case 2 (Thin peat): } g^m(0) > 0 \textit{ and } g(T^m) = 0,\\
\textit{if } \hat{S}_0^{c2} > S_0 \geq \hat{S}_0^{c3} &\Rightarrow \textit{Case 3 (Very thin peat): } g^m(0) = 0 \textit{ and } g(T^m) = 0,\\
\textit{if } \hat{S}_0^{c3} > S_0 \geq 0 &\Rightarrow \textit{Case 4 (non-utilization): } g^m(t) \geq S_0.
\end{aligned}$$

Part I *In Cases 1 and 2 ($S_0 \geq \hat{S}_0^{c2}$), there exists an initial phase (Phase 1), where, respectively, the myopic groundwater level, peat height, and root zone are given by*

$$\begin{aligned}
g^{m,r1,h}(t) &= S_0 - R^m\,(\alpha t + 1)\\
S^{m,r1,h}(t) &= S_0 - \alpha R^m t\\
R^{m,r1,h} &= R^m = \kappa - \frac{\gamma}{2\psi},\\
&\textit{where } h = \{h1, h2\},\\
&\textit{and } \hat{S}_0^{c2} = R^m.
\end{aligned}$$

Here, and also in the remainder of the paper, we denote $R^{m,r1,h}$ for $h = \{h1, h2\}$ with R^m as we will use it frequently.

Part II *In Case 1 ($S_0 \geq \hat{S}_0^{c1}$), the myopic terminal time T^m is given by the end of Phase 1 as follows:*

$$T^{m,r1,h1} = \frac{1}{\alpha}\left(\frac{y^m - c_0}{\gamma R^m} - 1\right).$$

Here, and also in the remainder of the paper, we denote $y^{m,r1,h} = y(R^m)$ with y^m. The critical initial peat height is given by

$$\hat{S}_0^{c1} = \frac{\psi R^m(2\kappa - R^m) - c_0}{\gamma} = \frac{y^m - c_0}{\gamma}.$$

Part III *In Case 2 ($\hat{S}_0^{c1} > S_0 \geq \hat{S}_0^{c2}$), the end of Phase 1 is given by*

$$T^{m,r1,h2} = \frac{1}{\alpha}\left(\frac{S_0}{R^m} - 1\right),$$

which is the time that the myopic policy enters Phase 2. In Phase 2, the myopic groundwater level, peat height, and root zone are given by

$$g^{m,r2,h2} = 0$$
$$S^{m,r2,h2}(t) = R^{m,r2,h2}(t) = R^m e^{-\alpha\tau}$$

where we define $\tau = t - T^{m,r1,h2}$. In Case 2, the myopic terminal time T^m is given by the end of Phase 2 as follows

$$T^{m,r2,h2} = T^{m,r1,h2} - \frac{1}{\alpha}\ln\left(\frac{\kappa}{R^m}\left(1 - \sqrt{1 - \frac{c_0 + \gamma S_0}{\bar{y}}}\right)\right).$$

Part IV *In Case 3 ($\hat{S}_0^{c2} > S_0 \geq \hat{S}_0^{c3}$), Phase 1 paths are the same with Phase 2 paths of Case 2, except that the initial peat height is given by S_0, while the initial peat height in Phase 2 of Case 2 is R^m. Hence,*

$$g^{m,r1,h3} = 0$$
$$S^{m,1,3}(t) = R^{m,r1,h3}(t) = S_0 e^{-\alpha t}$$

and the myopic terminal time T^m is given by the end of Phase 1 as follows

$$T^{m,r1,h3} = -\frac{1}{\alpha} \ln\left(\frac{\kappa}{S_0}\left(1 - \sqrt{1 - \frac{c_0 + \gamma S_0}{\bar{y}}}\right)\right).$$

The critical initial peat height is given by

$$\hat{S}_0^{c3} = \kappa\left(1 - \sqrt{\left(1 - \frac{c_0 + \gamma S_0}{\bar{y}}\right)}\right).$$

Part V *In Case 4 ($\hat{S}_0^{c3} > S_0 \geq 0$), the plot of land is not utilized.*

Proof. We start with the results in Part I for Phase 1 of Cases 1 and 2. The expressions for $g^{m,r1,h}(t)$, $S^{m,r1,h}(t)$, and R^m are the same with the expressions provided in Proposition 1 and proven in Appendix A. Here, we just clarify that these expressions also apply to Phase 1 of Case 2 (thin peat).

As $g^{m,r1,h}(0) = S_0 - R^m$, we must have $S_0 > R^m$, which defines the critical level $\hat{S}_0^{c2} = R^m$ stated in Part I, above, in which we have either the thick peat or thin peat cases.

The net benefits $\Pi^{m,r1,h}$ follow from substituting $g^{m,r1,h}(t)$ and $S^{m,r1,h}(t)$:

$$\Pi^{m,r1,h} = y^m - (c_0 + \gamma R^m (\alpha t + 1)) \text{ for } h = \{1, 2\}.$$

Phase 1 ends when economic benefits are exhausted or the groundwater level hits zero, whatever happens first. In Case 1 it ends at $T^{m,r1,h1}$ when the optimal path in Phase 1 yields zero instantaneous net benefit, such that $\Pi^{m,r1,h1}(T^{m,r1,h1}) = 0$, which leads to the expression for $T^{m,r1,h1}$ in Part II. We have stated this expression in Proposition 1. Second, it may end at $T^{m,r1,h2}$ (Case 2) when the optimal path reaches to zero groundwater level, such that $g^{m,r1,h2}(T^{m,r1,h2}) = 0$, which yields the expression for $T^{m,r1,h2}$ in Part III.

In the thick peat case (Case 1), the initial peat surface is very high given a yield and cost structure. Therefore, the net economic benefits are exhausted before reaching the zero groundwater level, and the plot of land is abandoned at $T^{m,r1,h1}$. In the thin peat case (Case 2), the net economic benefits are still positive at the zero groundwater level. Hence, the economic activity switches to a new phase. We have thin peat if $T^{m,r1,h2} < T^{m,r1,h1}$. Solving $T^{m,r1,h1} = T^{m,r1,h2}$ for S_0 gives the critical initial peat height $\hat{S}_0^{c1}$ in Part II, above which we have thick peat. This critical level is the same as the one stated in Proposition 1.

In the thin peat case, we switch to Phase 2 at $T^{m,r1,h2}$ after which the groundwater level is kept at zero as long as the net instantaneous benefits are positive. Therefore, subsidence is given by $\dot{S} = -\alpha S$. For notational convenience, define $\tau = t - T^{m,r1,h2}$. It is straightforward to show that $g^{m,r2,h2} = 0$ and $R^{m,r2,h2}(t) = S^{m,r2,h2}(t) = R^m e^{-\alpha\tau}$ as given in Part III.

The resulting instantaneous net benefits are given by

$$\Pi^{m,r2,h2}(t) = \psi(S^{m,r2,h2}(t))(2\kappa - S^{m,r2,h2}(t)) - (c_0 + \gamma S_0)\,.$$

The abandonment time marking the end of Phase 2 is given by $\Pi^{m,r2,h2}(T^{m,r2,h2}) = 0$, which leads to the expression for $T^{m,r2,h2}$ in Part III. Here, $\Pi^{m,r2,h2}(T^{m,r2,h2}) = 0$ leads to two real roots, but only one of them is in the admissible space.

We go on with Part IV. The third critical peat height in Part III can be derived by using $\hat{S}_0^{c3} = S^{m,r2,h2}(T^{m,r2,h2})$, under which we are in the non-utilization case. In Case 3 (very thin peat), we have $\hat{S}_0^{c2} > S_0 \geq \hat{S}_0^{c3}$. It is easy to see that there is a corner solution, such that $g^{m,r1,h3}(t) = 0$ for all $t \in [0, T^{m,r1,h3}]$, and the peat level follows $S^{m,r1,h3}(t) = R^{m,r1,h3}(t) = S_0 e^{-\alpha t}$. The only difference to Phase 2 of Case 2 is that the starting time is $t = 0$ instead of $T^{m,r1,h2}$ and the initial peat height is S_0 instead of R^m. Therefore, simply replacing these values in the expression for $T^{m,r2,h2}$ gives the expression for $T^{m,r1,h3}$ in the proposition, which is the end of Phase 1 and also the myopic terminal time for Case 3. $\square$

D.3 Forward-looking policy in thin peat.

Our results for the forward looking policy in thin peat are summarized in the following proposition followed by its proof.

Proposition 6. *There exists a critical initial surface level $\hat{S}_0^{c2}$, such that if $\hat{S}_0^{c1} > S_0 \geq \hat{S}_0^{c2}$, there exists an initial phase (Phase 1) with a unique forward-looking policy $g^{f,r1,h2}(t)$ for all $t \in [0, T]$. Respectively, the forward-looking groundwater* *level, peat height, root zone, and shadow price are given by*

$$g^{f,r1,h2}(t) = \left(S_0 - R^m + \Lambda\left(1 - \frac{\alpha}{\gamma}K(\overline{T})\right)\right) - \alpha\left(R^{m,1,2} - \Lambda_1\right)t + \Lambda_1\left(\frac{\alpha+\delta}{\gamma}\right)K(\overline{T})e^{\delta t}$$

$$S^{f,r1,h2}(t) = \left(S_0 - \Lambda\frac{\alpha}{\gamma}K(\overline{T})\right) - \alpha\left(R^{m,1,2} - \Lambda\right)t + \Lambda\frac{\alpha}{\gamma}K(\overline{T})e^{\delta t}$$

$$R^{f,r1,h2}(t) = R^m - \Lambda\left(1 + \frac{\delta}{\gamma}K(\overline{T})e^{\delta t}\right)$$

$$\lambda^{f,r1,h2}(t) = \frac{\gamma}{\delta} + K(\overline{T})e^{\delta t},$$

$$\text{where } \Lambda = \frac{\gamma}{2\psi}\frac{\alpha}{\delta},$$

$$K(\overline{T}) = \frac{(S_0 - R^m + \Lambda_1) - \alpha\left(R^m - \Lambda_1\right)\overline{T}}{\Lambda\frac{\alpha}{\gamma} - \Lambda\left(\frac{\alpha+\delta}{\gamma}\right)e^{\delta\overline{T}}}$$

Here we denote $T^{f,r1,h2}$ with $\overline{T}$. The switching time from Phase 1 to Phase 2 is given by a salvage value condition (SVC) as follows

$$SVC(\overline{T}) = 0, \text{ where we define}$$

$$SVC(\overline{T}) := \frac{\gamma}{\delta} + K(\overline{T})e^{\delta\overline{T}} - \left(\frac{2\kappa\psi}{(\alpha+\delta)} - \frac{2\psi S_{\overline{T}}}{(2\alpha+\delta)}\right) + 2S_{\overline{T}}^{\left(-\frac{\alpha+\delta}{\alpha}\right)}\Theta,$$

$$\text{with the parameter } \Theta = \left(\hat{S}_0^{c3}\right)^{\left(\frac{\delta}{\alpha}\right)}\left(\frac{(\gamma S_0 + c_0)(\alpha+\delta) - \delta\kappa\psi\hat{S}_0^{c3}}{(\alpha+\delta)(2\alpha+\delta)}\right).$$

Here we denote $S^{f,r1,h2}(\overline{T})$ with $S_{\overline{T}}$, and $\hat{S}_0^{c3}$ is the critical level for the non-utilization case derived earlier for the myopic case, which is the same here in the forward-looking case. Note that we redefine $\overline{T}$ and $S_{\overline{T}}$ in each proposition to denote the end of Phase 1 values.

In Phase 2, the forward-looking groundwater level, peat height, and root zone are given by

$$g^{f,r2,h2} = 0$$

$$S^{f,r2,h2}(t) = R^{f,r2,h2}(t) = S_{\overline{T}}e^{-\alpha\tau},$$

where $\tau = t - \overline{T}$, and recall that we here denote $T^{f,r1,h2}$ with $\overline{T}$, and $S_{\overline{T}}$ stands for $S^{f,r1,h2}(\overline{T})$.

The forward-looking terminal time T^f is given by the end of Phase 2 as follows:

$$T^{f,r2,h2} = \overline{T} - \frac{1}{\alpha} \ln \left(\frac{\kappa}{S_{\overline{T}}} \left(1 - \sqrt{1 - \frac{c_0 + \gamma S_0}{\bar{y}}} \right) \right).$$

Finally, the critical level $\hat{S}_0^{c2}$ is given by $SVC(0) = 0$.

Proof. The expressions for the forward-looking paths of S, g, R, and λ in Phase 1 in thick peat given respectively by Equations (12), (10), (9), and (8) in the proof of Proposition 2 also apply here in the thin peat case. The only difference is that the undetermined coefficient K_1 is replaced by $K(T)$. The reason is that the relation between K_1 and the terminal time of Phase 1 are different as the transversality and/or stopping conditions are different. The end of Phase 1 in thin peat is given by $g(T) = 0$, which, together with Equation (12), leads to the expression for $K(T)$ in the proposition.

The second phase dynamics is the same as the corresponding myopic one, given by

$$S(t) = S_T e^{-\alpha(t-T)},$$

where, for convenience, we denote the initial peat height in Phase 2 with S_T. The expression for the abandonment time of the myopic case (Case 2) applies also here, as the only difference is the peat height S_T. Therefore, the length of Phase 2 and the peat height abandoned is given by

$$\begin{aligned} \overline{\tau} &= -\frac{1}{\alpha} \ln \left(\frac{\kappa}{S_T} \left(1 - \sqrt{1 - \frac{c_0 + \gamma S_0}{\bar{y}}} \right) \right) \\ S_{\overline{\tau}} &= \kappa \left(1 - \sqrt{1 - \frac{c_0 + \gamma S_0}{\bar{y}}} \right). \end{aligned}$$

Note that $S_{\overline{\tau}}$ is the same with $\hat{S}_0^{c3}$ determining the non-utilization case. For the next step of the derivations, note the following expressions:

$$\overline{\tau} = -\frac{1}{\alpha} \ln \frac{S_{\overline{\tau}}}{S_T}, \ \frac{\partial \overline{\tau}}{\partial S_{\overline{T}}} = \frac{1}{\alpha S_{\overline{T}}} \text{ and } e^{-\alpha \overline{\tau}} = \frac{S_{\overline{\tau}}}{S_T} \tag{13}$$

The stopping time in Phase 1 is given by a salvage value condition. The salvage value is the total continuation value after Phase 1 given by

$$\begin{aligned}\phi(S_T,T) &= \int_T^{T+\overline{\tau}} \left(\psi S(2\kappa - S) - c_0 - \gamma S_0\right) e^{-\delta(t-T)} dt \\ &= \int_0^{\overline{\tau}} \left(\psi S(2\kappa - S) - c_0 - \gamma S_0\right) e^{-\delta\tau} d\tau \\ &= \frac{2\kappa\psi S_T}{\alpha+\delta}\left[1 - e^{-(\alpha+\delta)\overline{\tau}}\right] - \frac{\psi S_T^2}{2\alpha+\delta}\left[1 - e^{-(2\alpha+\delta)\overline{\tau}}\right] - \frac{(c_0+\gamma S_0)}{\delta}\left[1 - e^{-\delta\overline{\tau}}\right] + K_3,\end{aligned}$$

where K_3 is an undetermined coefficient. Note that, for brevity, we are denoting the stopping time of Phase 1 with T. Hence, S_T represents the initial peat height at the beginning of Phase 2. Furthermore, we define $\tau = t - T$ and $\overline{\tau}$ as the duration of Phase 2, which provides a convenient way to rearrange the salvage value in line 1 as in line 2. The third line simply follows from evaluating the integral.

Substituting Equation (13) in the salvage value, we have

$$\begin{aligned}\phi(S_T) = &-\frac{\psi}{2\alpha+\delta} S_T^2 + \left(\frac{2\kappa\psi}{\alpha+\delta} + \frac{\psi S_{\overline{\tau}} e^{\frac{2\alpha+\delta}{\alpha}}}{2\alpha+\delta}\right) S_T \\ &+ \frac{S_{\overline{\tau}}}{S_T}\frac{(c_0+\gamma S_0)}{\delta} e^{\frac{\delta}{\alpha}} - \left(\frac{2\kappa\psi}{\alpha+\delta} S_{\overline{\tau}} e^{\frac{\alpha+\delta}{\alpha}} + \frac{(c_0+\gamma S_0)}{\delta} - K_4\right)\end{aligned}$$

The salvage value condition (SVC) is

$$\frac{\partial\phi(S_T)}{\partial S_T} = \lambda(T).$$

Using Equation (13), the left-hand side of the SVC can be expressed as follows

$$\frac{\partial\phi(S_T)}{\partial S_T} = \left(\frac{2\kappa\psi}{(\alpha+\delta)} - \frac{2\psi S_{\overline{\overline{T}}}}{(2\alpha+\delta)}\right) - 2S_T^{-\left(\frac{\alpha+\delta}{\alpha}\right)} S_{\overline{\tau}}^{\left(\frac{\delta}{\alpha}\right)} \left(\frac{(\gamma S_0 + c_0)(\alpha+\delta) - \delta\kappa\psi S_{\overline{\tau}}}{(\alpha+\delta)(2\alpha+\delta)}\right).$$

We have already provided the solution for $\lambda(T)$ on the right-hand side of the SVC. Using these two expressions and substituting $\hat{S}_0^{c3}$ for $S_{\overline{\tau}}$, we reach the expression implicitly defining the end point of Phase 1 in the proposition.

$\hat{S}_0^{c2}$ is defined as the critical level between thin and very thin peat cases, such that $T^{f,r1,h2} = 0$ at $S_0 = \hat{S}_0^{c2}$. The salvage value condition $SVC(T^{f,r1,h2}|S_0) = 0$ provides the implicit association between $T^{f,r1,h2}$ and S_0. If $SVC(0|S_0) = 0$, then $g(0) = 0$, meaning that we are on the threshold between thin and very thin peat. Therefore, $\hat{S}_0^{c2}$ is given by $SVC(0|\hat{S}_0^{c2}) = 0$. □

D.4 Forward-looking policy in very thin peat and the case of non-utilization.

All the critical values leading to these cases have been provided. For these cases, the optimal paths and terminal time are the same as in the myopic case, as there are no further intertemporal considerations in these cases.

D.5 Uniqueness of the solutions

The solutions provided in Propositions 1, 2, 5 and 6 are all unique. The uniqueness of the myopic solution is self evident. For the forward-looking solutions, the paths of endogenous variables (S, g, R, λ) are unique for given $\overline{T}$ (stopping time of Phase 1), as our setup satisfies the Mangasarian sufficiency conditions with a concave objective function and linear law of motion (subsidence equation). It is also easy to verify that the solution for Phase 2 in the thin peat case is also unique for given $\overline{T}$, as we have explicit solutions for the terminal values at the end of Phase 2. Therefore, the only complication in the uniqueness proof is to show that $\overline{T}$, given implicitly by the transversality or salvage value condition, is unique. These conditions do not lead to explicit solutions for $\overline{T}$. Our results presented in the previous sections rest on numerical solutions to those equations. In the following, we depict the most important elements of the proof that these solutions are indeed unique which provides analytical generality to our results. The full proof is available upon request.

The thin or thick peat cases require positive initial net benefits by definition, such that $\Pi(0) = \psi R(0)(2\kappa - R(0)) - (c_0 + \gamma R(0)) \geq 0$. This inequality, quadratic in $R(0)$, leads to the following condition:

$$R_m - \sqrt{R_m^2 - \frac{c_0}{\psi}} < R(0) < R_m + \sqrt{R_m^2 - \frac{c_0}{\psi}}, \tag{14}$$

which has to be satisfied by both myopic and forward-looking policies. By substituting $R(0) = R^m$, it is easy to verify that the myopic root zone satisfies this inequality.

Secondly, we also impose $S_0 \geq R(0) > 0$, which is also a necessary condition to have thick or thin peat cases. By substituting the initial forward-looking root zone in this inequality, we obtain the following condition:

$$S_0 + \Lambda\frac{\delta}{\gamma}K(\overline{T}) > R^m - \Lambda > \Lambda\frac{\delta}{\gamma}K(\overline{T}), \tag{15}$$

which applies for both thick and thin peat cases. In the thin peat case $K(\overline{T})$ is provided in Proposition 6. In the thick peat case, $K(\overline{T}) = -(\gamma/\delta)\, e^{-\delta\overline{T}}$.

Under these conditions, it can be shown that $TVC(\overline{T})$ has the following properties: (i) $TVC(0) < 0$, (ii) $TVC(\overline{T})$ is a $C1$ (i.e., at least once continuously differentiable) function of and monotonically increasing in $\overline{T}$, and (iii) $\lim_{\overline{T}\to\infty} TVC(\overline{T}) = \infty$. As a result, there exists at most one solution to $TVC(\overline{T}) = 0$. Finally, it can be verified that this solution features $g(0) > g(\overline{T}) > 0$. As a result, the forward-looking solution in the thick peat case is unique. The uniqueness of $\overline{T}$ in the thin peat case can be proved by showing that the salvage value condition $SVC(\overline{T})$ shows similar properties to $TVC(\overline{T})$ under the conditions given by Inequalities (14) and (15). Note that, in the thin peat case, we must have $g(0) > g(\overline{T}) = 0$ and $\lambda(\overline{T}) > 0$ by definition.

E Proof of Proposition 4

We have shown that the myopic root zone is constant over time in the thick and thin peat cases, while the forward-looking root zone is not. Here, we show that the forward-looking root zone cannot be constant under fairly general conditions: more precisely, without parametric assumptions for the yield and cost functions. We only assume that the cost function $c(g)$is weakly convex in g. The Hamiltonian is given by $H_c = y(R) - c(g) - \alpha\lambda(S - g)$. The maximum principle yields the following three FOCs:

$$\dot{S} = -\alpha R,$$
$$-y'(R) = c'(g) - \alpha\lambda,$$
$$\dot{\lambda} - (\delta + \alpha)\lambda = -y'(R).$$

Taking the total derivative of the second FOC gives

$$\dot{R} = \frac{-c''(g)\dot{S} + \alpha\dot{\lambda}}{y''(R) - c''(g)}.$$

Substituting $\dot{\lambda}$ from the third FOC and $\dot{S}$ from first FOC leads to

$$\dot{R} = \frac{c''(g)\alpha R - \alpha y'(R) + \alpha(\delta + \alpha)\lambda}{y''(R) - c''(g)}.$$

Now assume $\dot{R} = 0$, which requires

$$c''(g)R + (\delta + \alpha)\lambda = y'(R).$$

Substitute $y'(R)$ from the second FOC, which gives

$$c''(g)R + \delta\lambda = -c'(g).$$

The left-hand-side of this expression is positive, while the right-hand-side is negative, which is a contradiction. Therefore, the forward-looking root zone cannot be constant. This result shows that the current policymaking is very likely to be myopic at best. Our results on welfare gains from adopting a forward-looking policy assume myopic optimality as a benchmark. If the current policymaking is not myopic, adopting the forward-looking policy against such a non-optimal benchmark yields even higher welfare gains.