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# Partially Adaptive Econometric Methods and Vertically Integrated Majors in the Oil and Gas Industry

## Abstract

Regression model error assumptions are essential to estimator properties. Least squares model parameters are consistent and efficient when the underlying error terms are normally distributed but yield inefficient estimators when errors are not normally distributed. Partially adaptive and M-estimation are alternatives to least squares when regression model errors are not normally distributed. Vertically Integrated firms in the oil and gas industry is one industrial sector where error mis-specification is consequential. Equity returns are a common area where returns are not normally distributed, and inappropriate error distribution specification has substantive effect when estimating capital costs. Vertically Integrated Major equity returns and accompanying regression model error terms are not normally distributed, and this study considers error returns for Integrated oil and gas producers. Vertically Integrated firm returns and their regression model error are not normally distributed, and alternative estimators to least squares have desirable properties.

JEL-Codes: G120, L710, L720, Q400, Q410.

Keywords: partially adaptive regression models, oil and gas industry, Integrated Majors, vertical integration.

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## I. Introduction

Assumptions regarding equity return distributions and their errors are central to regression analysis, and equity return distributions vary across and within industries. There is a well-established literature on stock return distributions in general, and oil and gas industry equity returns have thick tails (leptokurtosis) that may be skewed. Various methods are used to model equity returns, and estimating non-normal OLS Capital Asset Pricing Model (CAPM) return coefficients and regression errors that are not appropriately weighted are inefficient estimators prone to larger errors and unreliable test statistics. Mandelbrot (1963) and Fama (1965) demonstrate that stock return distributions are not normally distributed and have leptokurtosis. Mohanty and Nandha (2011) and Carson (2022) evaluate oil and gas industry returns with equity and commodity variables, however, do not consider the assumptions regarding oil and gas error distributions. This study considers efficient, robust estimation with an application of CAPM-type models among Integrated oil and gas producers.

Publicly traded equities price risk across industries, and a firm's equity and commodity excess return coefficients assess equity funding costs relative to perceived market risks. Entrepreneurial financiers assess risk relative to returns, and information from equity markets provides valuable information to assess risk across and within industries. For equity holders, accurately estimating capital cost coefficients leads to more efficiently constructed portfolios that provide reliable information for risk in industrial economics. Equity and commodity market excess return coefficients are generally estimated with least squares methods that are inefficiently estimated for error distributions that have fat tails and leads to larger estimated coefficient standard deviations that are less reliable in inference. This study uses flexible probability density functions (pdfs) to efficiently estimate Integrated oil and gas producer equity

and commodity market coefficients across an array of estimation techniques that account for alternative regression error distributions. Flexible semi-parametric distributions included here include the Skewed Generalized T (SGT) distribution that nests restricted cases of the Skewed Error Distribution (SGED). While applications used in this study are limited to risk assessment for large Integrated oil and gas producers, the method applies to other equity market estimations.

## II. Model

Partially adaptive estimation is more flexible than parametric least squares estimation, where errors are assumed to be normal, independent, and identically distributed. A regression's standard linear form is:

$$Y_i = X_i\theta + \varepsilon_i \quad (1)$$

where  $Y_i$  is the  $i^{\text{th}}$  observed dependent variable vector associated with the  $X_i$ ,  $l \times k$  matrix of explanatory variables.  $\theta$  is a  $k \times 1$  vector of unknown equity and commodity return coefficients, and  $\varepsilon_i$  is an  $n \times 1$  error term vector. Various alternatives to least squares robustly estimate  $\theta$  that are less sensitive to the assumed normal error term distributions.

M-estimation is one alternative to least squares estimation, which minimizes the more flexible distribution of errors,  $\rho(\varepsilon)$ , in the parameter  $\theta$ ,

$$\hat{\theta}_m = \arg \min_{\theta} \sum_{i=1}^N \rho(Y_i - X\theta) \quad (2)$$

where  $\rho$  is assumed to be a differentiable function in  $\varepsilon$ .

M-estimation includes various parameters as special cases. For example, the  $L_p$  estimator is a special case in M-estimation, where  $\theta_m$  is defined by

$$\hat{\theta}_{L_p} = \arg \min_{\theta} \sum |Y_i - X\theta|^p \quad (3)$$

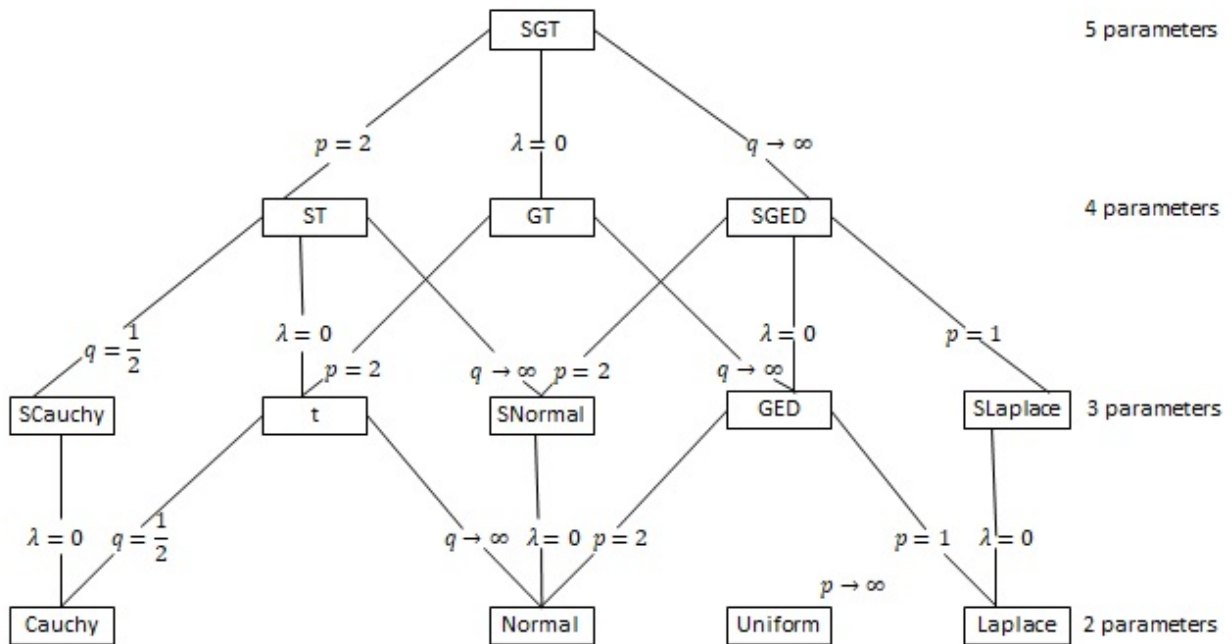
where least squares and least absolute deviations (LAD) estimators are limited cases of the  $L_p$ , when  $p=2$  and  $p=1$ , respectively.

Partially adaptive estimation uses a more flexible distribution than equation 1's error distribution, which is particularly suitable in equity market studies, where the distribution is kurtotic and skewed. Partially adaptive estimation is derived with  $\theta$  that minimizes the partially adaptive estimator,

$$\theta_{PAE} = \arg \min_{\theta, \Sigma} \sum_{i=1}^N -\ln f(Y_i - X\theta | \Sigma) \quad (4)$$

where  $f(\cdot)$  is a regression error term probability density function, and  $\Sigma$  is the vector of distributional parameters. If the pdf is correctly specified, the PAE is the maximum likelihood estimator (Davidson and McKennon, 2004, p. 399), and allows the error term's influence to adjust to data characteristics. If  $f(\cdot)$  is a flexible pdf corresponding to  $\theta$ , estimators can have more desirable estimator properties than least squares and alternative estimators.

This study considers the potential advantages of selecting  $f(\cdot)$  associated with a five-parameter distribution family, the skewed Generalized t (SGT) distribution, introduced by Theodossiou (1998). The SGT distribution family includes nested distributions for the Generalized T (McDonald and Newey (1988), the Skewed Generalized Error Distribution (SGED, Theodossiou, 2015), and the Generalized Error Distribution. Laplace, Normal, and Student T are further limiting cases. The SGT family relationships are described in Figure 1.



**Figure 1 Skewed Generalized T distribution and Restricted Relationships**

As a generalized distribution, the SGT and restricted regression error distributions in this study are the SGT, SGED, GT, and SGT. Partially adaptive estimation based on the SGT distribution estimate the vector of  $\theta$  and distribution parameters are  $\lambda$ ,  $\phi$ ,  $p$ , and  $q$ . Equation 5 is the SGT pdf.

$$SGT(y; m, \phi, p, q) = \frac{p}{\left[ 2\phi q^{1/p} B\left(\frac{1}{p}, q\right) \times \left(1 + [y - m]^p / (1 + \lambda \text{sign}(y - m)^p q \phi^p)\right)^{q+1/p} \right]} \quad -\infty < y < \infty \quad (5)$$

where  $B(\cdot, \cdot)$  is a beta function.  $m$  is the mode of the  $y$  oil and gas returns.  $\lambda$  is a shape parameter that measures error skewness, and  $|\lambda| < 1$ . The error distribution is symmetric when  $\lambda = 0$ , with the error's distribution skewness is determined by the sign of  $\lambda$ .  $\phi$  is a positive

scale parameter, where  $q$  and  $p$  control tail thickness and the distribution's height (Hansen, et al, 2010, p. 157).

From Figure 1, the SGT approaches the Skewed Generalized Error Distribution as  $p \rightarrow 0$  (SGED, Thodossiou, 2015),

$$SGED(y; m, \lambda, \phi, p) = \frac{pe^{-\left(|y-m|^p / \left(1 + \lambda \text{sign}(y-m)\right)^p \phi^p\right)}}{\left[2\phi\Gamma\left(\frac{1}{p}\right)\right]} \quad (6)$$

$$-\infty < y < \infty$$

where  $\Gamma\left(\frac{1}{p}\right)$  is the gamma function,  $p$  controls distribution kurtosis, and  $\lambda$  is the error distribution's skewness.

McDonald and Newey (1988) introduce the Generalized-T distribution, which is a limited case of the SGT as  $\lambda \rightarrow 0$ .

$$GT(y; \phi, p, q) = \frac{p}{\left[2\phi q^{1/p} B\left(\frac{1}{p}, q\right) \left(1 + |y|^p / q\phi^p\right)^{q+1/p}\right]} \quad (7)$$

$$-\infty < y < \infty$$

$\phi$  remains the positive scale parameter, and  $q$  and  $p$  control the error distribution's shape. As  $p$  and  $q$  increase, the error disruption's tail thickness decreases, and tail thickness increases as  $p$  and  $q$  decrease.

When  $p=2$ , the SGT converges on the Skewed T Distribution (Hansen, et al, 1994).

$$ST(y; \phi, p, q) = \frac{p}{\left[2\phi q^{1/2} B\left(\frac{1}{2}, q\right) \left(1 + [y]^2 / (1 + \lambda q\phi^2)\right)^{q+1/2}\right]} \quad (8)$$

$$-\infty < y < \infty$$



There are other limiting cases of the SGT distribution family. For example, when  $p \rightarrow 1$ , the SGED is the Skewed Laplace (SLaplace). When  $q \rightarrow \infty$ , the SGED is the Skewed Normal distribution (SNormal). As  $q \rightarrow \infty$ , the Generalized T converges to the GED, and the GED is also known as the Generalized Normal distribution. When  $\lambda \rightarrow 0$ , the SGED is the GED. The GT further reduces to the Student T distribution when  $p \rightarrow 2$ . As  $q \rightarrow \infty$ , the Skewed T is a Skewed Normal. When  $\lambda \rightarrow 0$ , the Skewed T becomes the Student T, and the ST is a Student T when  $\lambda \rightarrow 0$ , of the Skewed Cauchy distribution when  $q \rightarrow \frac{1}{2}$ .

These distributions have corresponding influence functions that measure their error's influence in estimation. An influence function is also more adaptive when there are more parameters, which allows adjusting a given error distribution to reflect tail behavior and error influence in estimation. Skewed Generalized T, Generalized T, Generalized Error, and Normal influence functions are

$$\psi_{SGT}(\varepsilon, \lambda, \phi, p, d) = (pq + 1) \text{sign}(\varepsilon) |\varepsilon|^{p-1} / \left[ q\phi^p \left( (1 + \lambda \text{sign}(\varepsilon))^p + |\varepsilon|^p \right) \right] \quad (9)$$

$$\psi_{GT}(\varepsilon, \phi, p, q) = (pq + 1) \text{sign}(\varepsilon) |\varepsilon|^{p-1} / (q\phi^p + |\varepsilon|^p) \quad (10)$$

$$\psi_{GED}(\varepsilon, p, \phi) = p |\varepsilon|^{p-1} \text{sign}(\varepsilon) / \phi^p \quad (13)$$

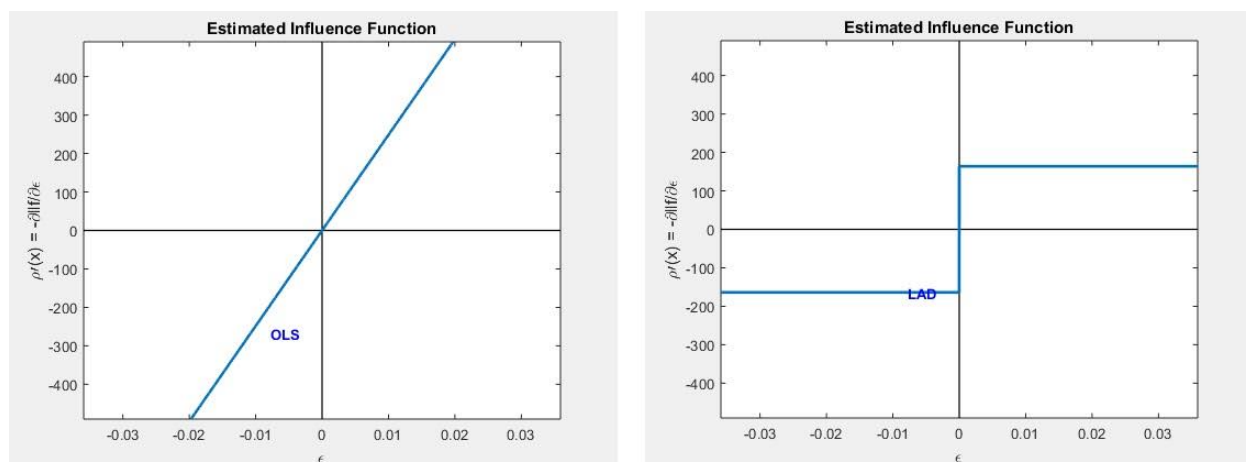
$$\psi_{Normal}(\varepsilon, \phi) = \frac{2\varepsilon}{\phi^2} \quad (14)$$

The corresponding influence function associated with flexible error distributions is

$$\psi(\varepsilon) = \frac{\partial \rho}{\partial \varepsilon} \quad (15)$$

An influence function in statistics is an estimator's dependence on the value of any one point in the sample, and ExxonMobil is generally the largest vertically Integrated oil and gas firm. Estimated ExxonMobil OLS and LAD return influence functions are presented in Figure 1.

Figure 2 illustrates that least square estimation is sensitive to outliers, whereas LAD is robust across the error distribution. Error term kurtosis is greater than the normal distribution when  $p$  is less than 2, and least squares gives greater weight to larger errors than the LAD. ExxonMobil returns illustrate the influence function's for the SGT and GT distributions in Figures 3.

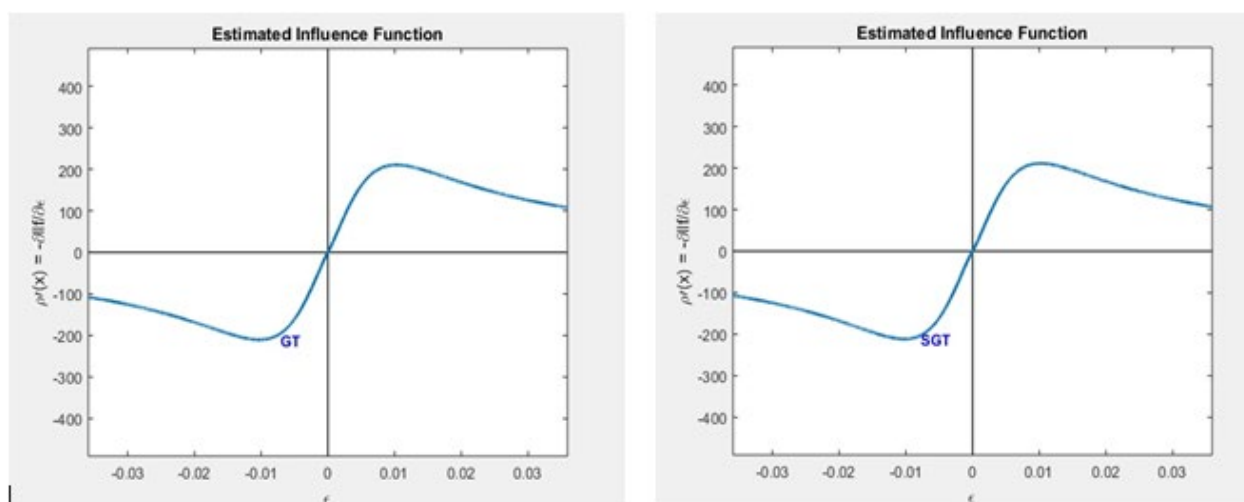


**Figure 2, Least Squares and Least Absolute Deviation Influence Functions**

Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

OLS and LAD influence functions provide useful comparisons for more flexible distribution's influence functions (Figure 2). Where OLS gives undue weight for larger errors, LAD estimators gives the same weight across error distributions. This is compared to the GT and SGT ExxonMobil influence functions that initially give errors greater weight and robustly

apply weights across errors. The GT and SGT influence functions increase over wider ranges and descend, which initially gives greater weight to smaller errors but discounts larger errors, with data determining when discounting begins (Figure 3).



**Figure 3, ExxonMobil Generalized and Skewed Generalized T Influence Function**

Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

### III. Literature Review

Partially adaptive estimation is broadly applied across various economic applications that include equity returns and housing markets data (Butler et al. 1990; McDonald et al. 2010), and violating least squares normality assumptions is important because estimated CAPMs are used in public utilities to estimate common equity costs (McDonald et al. 2010). Butler et al. (1990) use partially adaptive estimation to illustrate the effects of violating the normality assumption on

common equities. McDonald et al. (2010) indicate that CAPM normality assumption violations have efficiency consequences on utility return equities, which have thick tails and leads to null hypothesis that are less likely to be significant due to large standard deviations.

The equity market literature finds that asset return densities are skewed with leptokurtosis, and equity returns have fat tails with extreme outliers. However, least squares estimators are sensitive to extreme price and index surprises that inappropriately weigh extreme errors that decreases estimator efficiency. Efficient equity coefficient estimation is addressed by Fielits and Smith (1972), Francis (1975), Butler, McDonald and Nelson (1989), and McDonald and White (1993). Theodossiou (1998) rejects stock, exchange rates, and gold normality assumptions. Akgiray and Booth (1988, 1991) use normal distribution mixture models with non-normal probability density functions to estimate statistical properties, and Bali (2003) uses various non-normal alternative distributions to model extreme changes in the US Treasury market. Bali and Weinbaum (2007) reject equity return normality across various equity market indices. Oil and gas returns similarly have skewed densities that are leptokurtotic.

#### **IV. Oil and Gas Industry**

Firms are integrated when their internal structure is vertically organized to take advantage of internal scale to achieve cost savings by using infrastructure internally more efficiently compared to separated units. Internally organized vertically Integrated oil and gas companies are leading examples of a vertically Integrated industry. The international oil and gas industry is dominated by large state-owned vertically Integrated companies, which includes Saudi Aramco, Petróleos de Vanezuela, S. A. (PDVSA), Mexico's Pemex, and Russia's Gasprom. The oil and

gas industry is partitioned between Super-Majors and competitive Independents (Yergin, 1991; FTC, 1982). Independent producers are classified as upstream exploration & production and equipment & service, midstream transportation & pipeline, and downstream refining & marketing. Upstream exploration & production firms explore for and extract oil that increases their oil reserves by either exploring for and discovering new production or acquiring or merging with existing producers. While they provide equipment & services throughout each sector of the oil and gas industry, equipment & service firms have evolved to fracture and service upstream oil wells and are grouped here with upstream exploration & production firms. Once brought to the surface, midstream transportation & production firms transport and store oil and gas once brought into production. Because the price of crude can vary considerably between when transportation & production firms take delivery and deliver crude, transportation & pipeline and refining & marketing firms bear considerable greater equity and commodity risk because they take possession of oil and natural gas, when output can vary considerably while crude is held by downstream firms. The industry's market configuration is that large Integrated, multi-unit firms have infrastructure in each part of the oil and gas industry, and BP, Chevron, ConocoPhillips, Exxon, ENI, Royal Dutch Shell (Shell), and Total are the Integrated oil and gas producers in this study. Consequently, partially adaptive estimation is more flexible to return variation that accommodates large equity and commodity risk.

Firm equity returns vary by industry, across industries, time, and firm information, and the sample used here includes the post-financial market collapse between 2008 and 2020. Data to evaluate Integrated firms includes the S&P 500 as the market index, and firm size effects are estimated with Fama-French small-minus-big (SML). Firm value effects are estimated with high-minus-low (HML). Other equity market risk variables are conservative-minus-aggressive

(CMA) and robust-minus-weak (RMW). Carhart (1997) shows that momentum (MOM) affects stock returns, which is included here. Commodity excess returns are North Sea Brent and Henry Hub's natural gas returns.

**Table 1, Major Oil & Gas Rate of Returns**

	<i>N</i>	<i>Mean</i>	<i>Median</i>	<i>Standard Deviation</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Sharpe Ratio</i>	<i>JB</i>
BP	2,469	-.000331	-.000257	.019200	-.263582	13.5278	-.0172	
Chevron	2,470	-.000251	-.000122	.016547	-.123409	13.0117	-.0152	
ConocoPhillips	2,493	.000382	.000410	.020502	-.016884	10.8951	.0186	
ENI	2,316	.000246	.000727	.020909	.080253	8.6913	.0118	
Exxon	2,486	-.000371	-.000547	.015247	.574492	21.8873	-.0243	
Petrobras	2,490	-.000642	-.000395	.035031	.216490	9.1026	-.0183	
PetroChina	2,486	-.000451	-.001128	.022339	.260339	9.7724	-.0202	
Shell	2,493	.000375	.000675	.018046	.391126	13.8642	.0208	
Total SA	2,392	.000274	.000938	.019042	.022314	9.5453	.0144	
Total	2,455	-.000085	.000034	.020763	.126793	12.2553	-.0033	

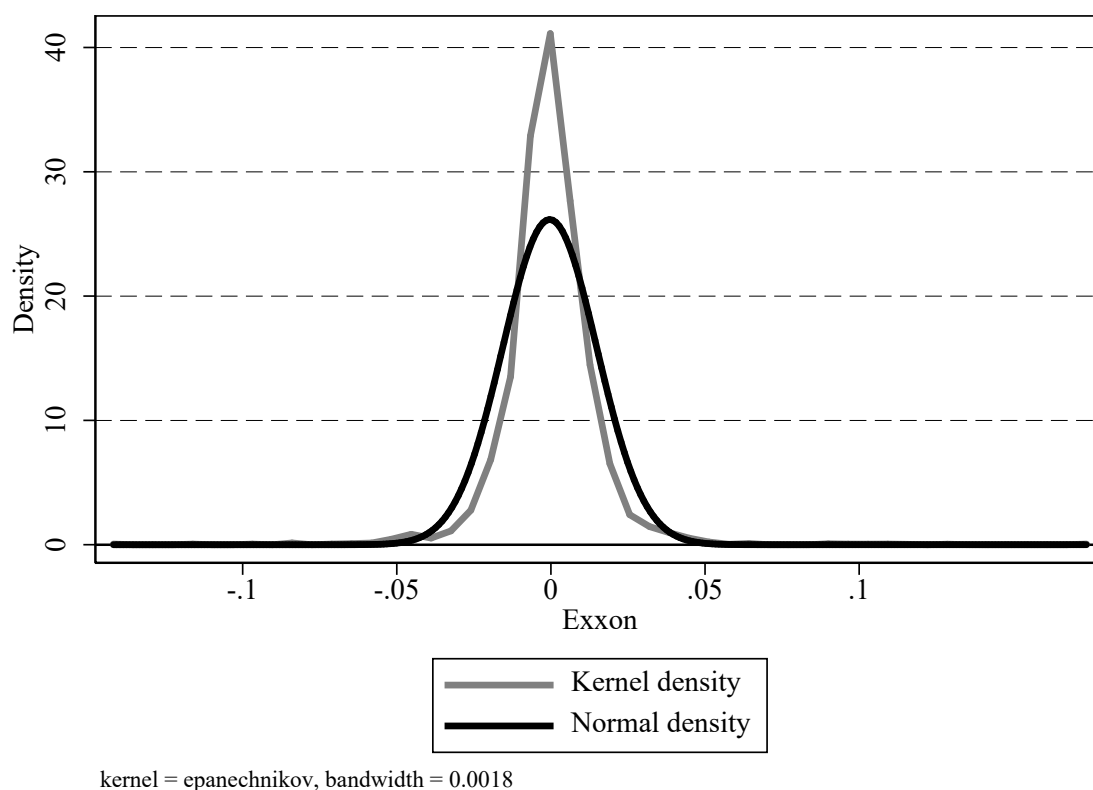
Source: Prices and adjusted rates of returns from Yahoo! Finance.

Evaluating Integrated producer returns with S&P 500, crude, and natural gas is insightful.

In this study, advantages are considered by using  $f(\cdot)$  as the SGT defined by Theodossiou (1998), a flexible 5-parameter distribution. The Generalized T (GT, McDonald and Newey, 1988), the Skewed Generalized Error Distribution (SGED, Theodossiou, 2015), the Generalized Error Distribution (GED), Normal, Laplace, and Student T distributions are limited cases.

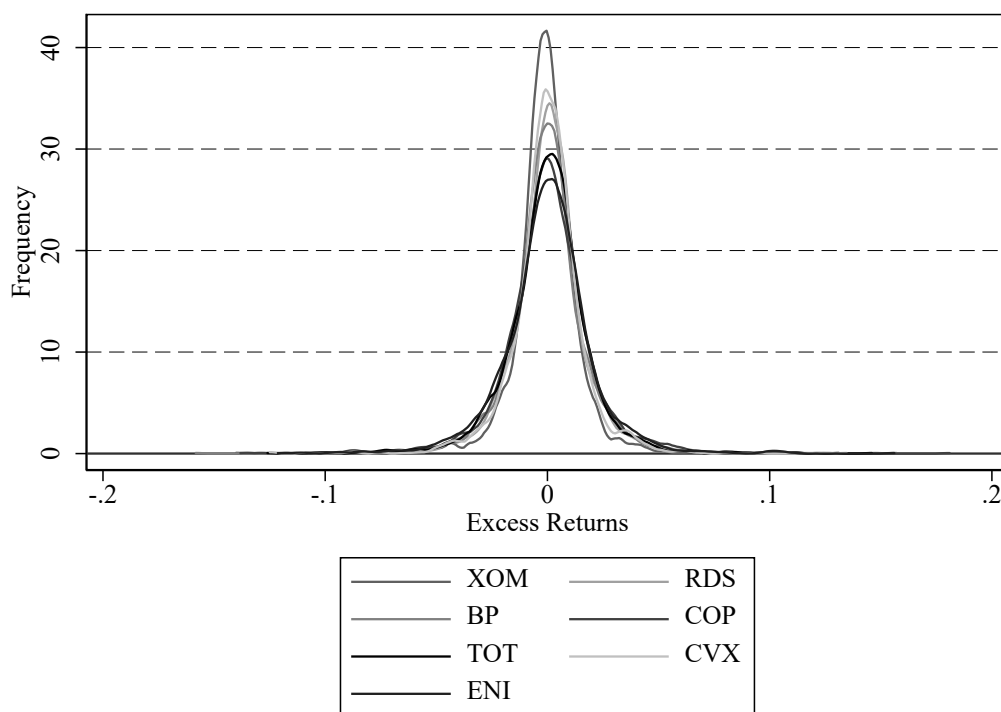
As a segment of the oil and gas industry, vertically Integrated expected returns varied little between ConocoPhillips' .000382 and ExxonMobil's excess returns of -.000371, and the most prominent ExxonMobil, BP, and Chevron had negative returns (Table 1). Risk, measured by return standard deviations, was lowest for ExxonMobil, the most stable Integrated over the period. The shape of equity excess return variation indicates much about firm returns and risk

(Figure 2 and Figure 3). A positively skewed distribution indicates a firm experiences positive price surprises, whereas negatively skewed distributions indicate firms experience negative price surprises. BP was the most likely to experience positive price surprises but also experienced negative price swings (Table 1). BP's interval includes 2010, which includes BP's Deep Water Horizon offshore well explosion, where its equity price decreased from a high of \$35 in December 2007 to a low of \$13.62 in June 2010, a decrease of 57 percent. Petrobras had the greatest risk. Chevron, Shell, and BP, three well-entrenched Majors, similarly had low risk.



**Figure 4, ExxonMobil Return Distribution**

Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>



**Figure 5, Integrated Excess Returns**

Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

The Sharpe ratio is returns per unit of risk, and Integrated oil and gas Sharpe ratios are higher than other oil and gas producers (Carson, 2022). Between August 2008 and July 2020, Shell Oil had the highest Sharp ratio, followed by ConocoPhillips, ENI, Chevron, BP, Petrobras, PetroChina, and ExxonMobil. Among Integrated firms, ConocoPhillips has traditionally taken on greater exploration & production risk than other Integrated firms. Alternatively, ExxonMobil has underperformed for returns relative to risk compared to other Integrated firms, where



ExxonMobil's management group has focused on growth rather than creating value and had a capital allocation strategy that was undisciplined, creating negative returns during periods of high oil prices. In sum, comparing Integrated excess return variation indicates large, well-capitalized ExxonMobil had lower risks but lower returns relative to risk.

## V. Estimating Model Parameters

Partially adaptive estimation is now used to evaluate vertically Integrated oil and gas producer returns to select the appropriate model to evaluate Major return variation.

$$R_{it} - R_{ft} = \alpha + \beta_1(R_{mt} - R_{ft}) + \beta_2(R_{ot} - R_{ft}) + \beta_3(R_{gt} - R_{ft}) + \beta_4SMB_t + \beta_5HML_t + \beta_6MOM_t + \varepsilon_t$$

(16)

$R_{it}$  is the daily rate of return on the  $i^{\text{th}}$  Integrated oil and gas producer.  $R_{ft}$  is the daily T-Bill rate of return. Over this interval, crude oil prices were high and stable, while the US T-bill rate remained low.  $R_{mt}$  is the Standard & Poor's daily adjusted equity return, and  $\beta_1$  is how the  $i^{\text{th}}$  firm's excess daily adjusted equity return varied with daily excess equity market excess returns.  $R_{ot}$  is the Brent North Sea daily rate of return, and  $\beta_2$  is the  $i^{\text{th}}$  firm's excess daily oil rate of return variation with Brent North Sea.  $R_{gt}$  is the natural gas daily rate of return, and  $\beta_3$  is the  $i^{\text{th}}$  firm's rate of return variation with natural gas's excess return variation. Together,  $\beta_1$ ,  $\beta_4$ , and  $\beta_6$  measure Integrated firm's equity market risk, while  $\beta_2$  and  $\beta_3$  measure Integrated firm's commodity market risk.  $SMB_t$ ,  $HML_t$ , and  $MOM_t$  are small-minus-big, high-minus-low, and momentum.

Table 2, Major Oil &amp; Gas Rate of Returns by Equity and Commodity Market Risk

<i>BP</i>	<i>OLS</i>	<i>LAD</i>	<i>GED</i>	<i>T</i>	<i>GT</i>	<i>SGED</i>	<i>ST</i>	<i>SGT</i>
Intercept	-.001 (.001)	-.001 (.001)	- (2.26 <sup>-4</sup> )	-.001 (.001)	-.001 (.001)	- (1.21 <sup>-4</sup> )	-.001 (.001)	-.001 (.001)
ExMarket	.825*** (.047)	.738*** (.024)	.731*** (.)	.740*** (.025)	.740*** (.025)	.731*** (.)	.745*** (.025)	.745*** (.025)
ExCrude	.169*** (.015)	.194*** (.010)	.194*** (.001)	.197*** (.010)	.197*** (.010)	.196*** (.)	.197*** (.010)	.196*** (.010)
ExGas	-4.19 <sup>-5</sup> (3.88 <sup>-5</sup> )	-4.07 <sup>-4</sup> (2.80 <sup>-5</sup> )	-4.05 <sup>-5</sup> (8.29 <sup>-7</sup> )	-1.43 <sup>-5</sup> (2.68 <sup>-5</sup> )	-1.37 <sup>-5</sup> (2.72 <sup>-5</sup> )	-4.32 <sup>-5</sup> (-.003)	-2.66 <sup>-5</sup> (2.75 <sup>-5</sup> )	-2.60 <sup>-5</sup> (2.78 <sup>-4</sup> )
SMB	.003*** (.001)	.003*** (.001)	.002*** (3.27 <sup>-5</sup> )	.002*** (.001)	.002*** (.001)	.002*** (.001)	.002*** (.001)	.002*** (.001)
HML	-3.39 <sup>-4</sup> (.001)	.002** (.001)	.002*** (3.57 <sup>-5</sup> )	.002*** (.001)	.002*** (.001)	.002*** (.001)	.002*** (.001)	.002*** (.001)
MOM	-.001* (.001)	- (3.97 <sup>-4</sup> )	- (1.72 <sup>-5</sup> )	- (4.03 <sup>-4</sup> )	- (4.03 <sup>-4</sup> )	- (4.03 <sup>-4</sup> )	-.001 (4.03 <sup>-4</sup> )	- (4.03 <sup>-4</sup> )
LR			7,552.99	7,594.47	7,594.48	7,553.46	7,596.51	7,596.52
<b>ExxonMobil</b>	<b>OLS</b>	<b>LAD</b>	<b>GED</b>	<b>T</b>	<b>GT</b>	<b>SGED</b>	<b>ST</b>	<b>SGT</b>
Intercept	-1.63 <sup>-4</sup> (1.79 <sup>-4</sup> )	- (1.37 <sup>-4</sup> )	- (2.52 <sup>-5</sup> )	-3.30** (1.39 <sup>-4</sup> )	-3.27 <sup>-4</sup> (1.40 <sup>-4</sup> )	-1.68 <sup>-4</sup> (2.63 <sup>-4</sup> )	-2.74 <sup>-4</sup> * (1.48 <sup>-4</sup> )	-2.12 <sup>-4</sup> (1.71 <sup>-4</sup> )
ExMarket	.888*** (.044)	.808*** (.014)	.808*** (.)	.783*** (.018)	.781*** (.018)	.811*** (.)	.782*** (.018)	.781*** (.018)
ExCrude	.113*** (.012)	.106*** (.006)	.106*** (.001)	.111*** (.007)	.111*** (.007)	.106*** (4.31 <sup>-4</sup> )	.111*** (.007)	.003 (.003)
ExGas	.004 (.003)	.003 (.003)	.003*** (.001)	.003 (.003)	.003 (.003)	.003*** (1.77 <sup>-4</sup> )	.003 (.003)	.003 (.003)
SMB	-1.03 <sup>-4</sup> (.001)	.001 (3.47 <sup>-4</sup> )	.002*** (.001)	3.87 <sup>-4</sup> (3.97 <sup>-4</sup> )	3.82 <sup>-4</sup> (3.97 <sup>-4</sup> )	.001*** (7.06 <sup>-5</sup> )	4.11 <sup>-4</sup> (3.99 <sup>-4</sup> )	4.05 <sup>-4</sup> (4.00 <sup>-4</sup> )
HML	3.72 <sup>-4</sup> (.001)	.002*** (4.16 <sup>-4</sup> )	.002*** (8.46 <sup>-5</sup> )	.001*** (.001)	.001*** (.001)	.002*** (1.67 <sup>-4</sup> )	.001** (.001)	.001*** (.001)
MOM	.002*** (.001)	4.26 <sup>-4</sup> * (2.43 <sup>-4</sup> )	4.28 <sup>-</sup> 4*** (7.00 <sup>-5</sup> )	3.80 <sup>-4</sup> (2.98 <sup>-4</sup> )	3.78 <sup>-4</sup> (2.99 <sup>-4</sup> )	3.64 <sup>-</sup> 4*** (-3.35 <sup>-5</sup> )	3.79 <sup>-4</sup> (3.00 <sup>-4</sup> )	3.78 <sup>-4</sup> (3.00 <sup>-4</sup> )
			8,472.74	8,510.25	8,510.43	8,475.03	8,510.96	8,511.10
<b>Chevron</b>	<b>OLS</b>	<b>LAD</b>	<b>GED</b>	<b>T</b>	<b>GT</b>	<b>SGED</b>	<b>ST</b>	<b>SGT</b>
Intercept	.001 (.001)	.001 (4.31 <sup>-4</sup> )	.001 (.)	.001 (.001)	.001 (.001)	.001*** (2.38 <sup>-4</sup> )	.001 (.001)	.001 (.001)
ExMarket	.928*** (.039)	.852*** (.015)	.853*** (.)	.847*** (.020)	.847*** (.020)	.855*** (.)	.847*** (.020)	.847*** (.020)
ExCrude	.165***	.154***	.152***	.157***	.157***	.153***	.157***	.157***

	(.012)	(.006)	(.)	(.001)	(.008)	(.)	(.008)	(.008)
ExGas	8.47 <sup>-6</sup>	9.13 <sup>-6*</sup>	9.32 <sup>-6***</sup>	9.20 <sup>-6</sup>	9.40 <sup>-6</sup>	9.50 <sup>-6</sup>	8.66 <sup>-6</sup>	9.04 <sup>-6</sup>
	(7.99 <sup>-6</sup> )	(5.02 <sup>-6</sup> )	(5.83 <sup>-7</sup> )	(5.66 <sup>-6</sup> )	(5.92 <sup>-6</sup> )	(1.59 <sup>-6</sup> )	(6.03 <sup>-6</sup> )	(5.82 <sup>-6</sup> )
SMB	7.82 <sup>-5</sup>	.001***	.001***	.001***	.001***	.001***	.001***	.001***
	(4.37 <sup>-4</sup> )	(1.74 <sup>-4</sup> )	(9.01 <sup>-5</sup> )	(2.35 <sup>-4</sup> )	(2.33 <sup>-4</sup> )	(1.39 <sup>-4</sup> )	(2.34 <sup>-4</sup> )	(.222 <sup>-4</sup> )
HML	3.73 <sup>-4</sup>	.001***	.001***	.001**	.001**	.001***	.001**	-.001**
	(.001)	(3.44 <sup>-4</sup> )	(3.14 <sup>-4</sup> )	(4.29 <sup>-4</sup> )	(4.26 <sup>-4</sup> )	(2.15 <sup>-4</sup> )	(4.30 <sup>-4</sup> )	(4.14 <sup>-4</sup> )
MOM	-1.98 <sup>-4</sup>	-	-	-	-	-	-	-
	(3.19 <sup>-4</sup> )	.001***	.001***	.001***	.001***	.001***	.001***	.001***
LR		(1.43 <sup>-4</sup> )	(3.52 <sup>-5</sup> )	(1.86 <sup>-4</sup> )	(1.82 <sup>-4</sup> )	(6.07 <sup>-5</sup> )	(1.85 <sup>-4</sup> )	(1.79 <sup>-4</sup> )
<b>Conoco-Phillips</b>			8,280.67	8,292.93	8,294.89	8,281.10	8,293.88	8,295.63
Intercept	OLS	LAD	GED	T	GT	SGED	ST	SGT
	1.11 <sup>-5</sup>	-1.82 <sup>-4</sup>	-2.04 <sup>-4***</sup>	-6.21 <sup>-5</sup>	-7.35 <sup>-5</sup>	6.88 <sup>-6</sup>	-3.58 <sup>-5</sup>	-9.38 <sup>-6</sup>
	(2.63 <sup>-4</sup> )	(2.02 <sup>-4</sup> )	(4.39 <sup>-5</sup> )	(1.99 <sup>-4</sup> )	(1.98 <sup>-4</sup> )	(5.53 <sup>-5</sup> )	(2.25 <sup>-4</sup> )	(2.54 <sup>-4</sup> )
ExMarket	.956***	.885***	.886***	.870***	.870***	.886***	.870***	.870***
	(.044)	(.019)	(.)	(.022)	(.022)	(.)	(.022)	(.022)
ExCrude	.247***	.215***	.215***	.224***	.223***	.213***	.224***	.223***
	(.018)	(.009)	(.001)	(.011)	(.011)	(.001)	(.011)	(.011)
ExGas	-.002	-.003	-	-.003	-.003	-	-.003	-.003
	(.005)	(.004)	(1.52 <sup>-4</sup> )	(.004)	(.004)	(.)	(.004)	(.004)
SMB	-.002**	-	-	-.001**	-.001**	-	-.001**	-.001**
	(.001)	(3.43 <sup>-4</sup> )	(6.94 <sup>-5</sup> )	(3.79 <sup>-4</sup> )	(3.75 <sup>-4</sup> )	(8.34 <sup>-5</sup> )	(3.79 <sup>-4</sup> )	(3.74 <sup>-4</sup> )
HML	3.90 <sup>-4</sup>	.001***	.001***	.001***	.001***	.001***	.001***	.001***
	(.001)	(3.71 <sup>-4</sup> )	(3.51 <sup>-5</sup> )	(4.23 <sup>-4</sup> )	(4.32 <sup>-4</sup> )	(3.66 <sup>-5</sup> )	(4.24 <sup>-4</sup> )	(4.30 <sup>-4</sup> )
MOM	-.001*	-	-	-	-	-	-	-
	(.001)	.002***	.002***	.001***	.001***	.002***	.001***	.001***
LR		(2.53 <sup>-4</sup> )	(3.30 <sup>-5</sup> )	(2.96 <sup>-4</sup> )	(3.03 <sup>-4</sup> )	(.)	(2.96 <sup>-4</sup> )	(3.03 <sup>-4</sup> )
<b>Royal Dutch Shell</b>			7,565.85	7,583.33	7,587.82	7,566.74	7,587.36	7,587.90
Intercept	OLS	LAD	GED	T	GT	SGED	ST	SGT
	9.05 <sup>-5</sup>	-5.63 <sup>-5</sup>	-5.20 <sup>-5</sup>	-2.36 <sup>-5</sup>	-2.43 <sup>-5</sup>	8.26 <sup>-5</sup>	1.46 <sup>-6</sup>	4.75 <sup>-5</sup>
	(2.04 <sup>-4</sup> )	(2.08 <sup>-4</sup> )	(9.18 <sup>-5</sup> )	(1.80 <sup>-4</sup> )	(1.80 <sup>-4</sup> )	(1.51 <sup>-4</sup> )	(1.86 <sup>-4</sup> )	(2.18 <sup>-4</sup> )
ExMarket	.808***	.799***	.800***	.793***	.794***	.797***	.793***	.794***
	(.040)	(.022)	(.030)	(.023)	(.023)	(.025)	(.023)	(.023)
ExCrude	.178***	.189***	.188***	.186***	.186***	.189***	.186***	.186***
	(.015)	(.009)	(.007)	(.010)	(.010)	(.002)	(.010)	(.010)
ExGas	-3.62 <sup>-4</sup>	.001	.002	.002	.003	.002	.002	.003
	(.005)	(.005)	(.006)	(.004)	(.005)	(.002)	(.004)	(.004)
SMB	-	-	-	-	.001	-	-	-
	.005***	.003***	.003***	.003***	.003***	.003***	.003***	.003***
	(8.24 <sup>-4</sup> )	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
HML	-.002	3.64 <sup>-4</sup>	3.52 <sup>-4</sup>	.001	-3.93 <sup>-4</sup>	.001***	.001	.001

MOM	(.001) -4.41 <sup>-4</sup>	(.001) -3.65 <sup>-4</sup>	(.001) -3.92 <sup>-4</sup>	(.001) -3.92 <sup>-4</sup>	(3.73 <sup>-4</sup> ) -3.93 <sup>-4</sup>	(1.79 <sup>-4</sup> ) -4.06 <sup>-4</sup> ***	(.001) -3.92 <sup>-4</sup>	(.001) -3.92 <sup>-4</sup>
LR	(.001)	(3.68 <sup>-4</sup> )	(4.34 <sup>-4</sup> ) 7,891.54	(3.72 <sup>-4</sup> ) 7,906.23	(3.73 <sup>-4</sup> ) 7,906.56	(1.94 <sup>-4</sup> ) 7,891.95	(3.72 <sup>-4</sup> ) 7,906.39	(3.69 <sup>-4</sup> ) 7,906.73

Source: Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural

Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

Notes: \*\*\* Significant at .01; \*\*Significant at .05; \* significant at .10.

Table 2 Continued

<i>Total SA</i>	<i>OLS</i>	<i>LAD</i>	<i>GED</i>	<i>T</i>	<i>GT</i>	<i>SGED</i>	<i>ST</i>	<i>SGT</i>
Intercept	9.37 <sup>-5</sup> (2.43 <sup>-4</sup> )	5.60 <sup>-4</sup> (2.65 <sup>-4</sup> )	8.96 <sup>-5</sup> (2.28 <sup>-4</sup> )	9.69 <sup>-5</sup> (2.17 <sup>-4</sup> )	9.67 <sup>-5</sup> (2.17 <sup>-4</sup> )	8.50 <sup>-5</sup> (2.39 <sup>-4</sup> )	1.15 <sup>-4</sup> (2.26 <sup>-4</sup> )	6.77 <sup>-4</sup> (2.37 <sup>-5</sup> )
ExMarket	.581*** (.039)	.601*** (.025)	.588*** (.027)	.569*** (.025)	.568*** (.025)	.588*** (.026)	.569*** (.025)	.567*** (.025)
ExCrude	.196*** (.015)	.188*** (.011)	.189*** (.009)	.194*** (.011)	.194*** (.011)	.189*** (.008)	.193*** (.011)	.194*** (.011)
ExGas	-.006 (.006)	-.007 (.007)	-.007 (.006)	-.007 (.005)	-.007 (.005)	-.007 (.006)	-.007 (.005)	-.007 (.005)
SMB	.013*** (.001)	.012*** (.001)	.012*** (.006)	.012*** (.001)	.012*** (.001)	.013*** (.001)	.012*** (.001)	.012*** (.001)
HML	.001 (.001)	.003*** (.001)	.002*** (.001)	.003*** (.001)	.003*** (.001)	.002*** (.001)	.003*** (.001)	.003*** (.001)
MOM	.001 (.001)	.001 (.001)	.001 (.001)	.001 (.001)	.001 (.001)	.001 (4.33 <sup>-4</sup> )	.001 (.001)	.001 (.001)
LR			7,323.26	7,338.11	7,338.21	7,323.26	7,338.15	7,338.25
<b>Sinopec</b>	<b>OLS</b>	<b>LAD</b>	<b>GED</b>	<b>T</b>	<b>GT</b>	<b>SGED</b>	<b>ST</b>	<b>SGT</b>
Intercept	.001 (.001)	-2.03 <sup>-4</sup> (.001)	-2.57 <sup>-4</sup> (5.77 <sup>-5</sup> )	-3.35 <sup>-4</sup> (3.95 <sup>-4</sup> )	-3.14 <sup>-4</sup> (3.95 <sup>-4</sup> )	.001** (2.72 <sup>-4</sup> )	2.21 <sup>-4</sup> (4.64 <sup>-4</sup> )	.001 (.001)
ExMarket	1.29*** (.070)	1.18*** (.048)	1.17*** (.)	1.19*** (.048)	1.18*** (.050)	1.15*** (.)	1.19*** (.048)	1.18*** (.053)
ExCrude	.044 (.027)	.048** (.020)	.045*** (.006)	.038* (.019)	.041** (.020)	.048*** (.)	.035* (.019)	.039** (.020)
ExGas	-.007 (.010)	.001 (.010)	- (.001)	.004 (.008)	.004 (.008)	- (.)	.005 (.001)	.003 (.009)
SMB	.002 (.002)	.002 (.001)	.001*** (1.41 <sup>-4</sup> )	.002 (.001)	.002 (.001)	.001*** (.)	.002 (.001)	.002 (.001)
HML	.003 (.002)	.002 (.001)	.002*** (2.24 <sup>-4</sup> )	.002* (.001)	.002 (.001)	.001*** (.)	.002* (.001)	.002 (.001)
MOM	-4.30 <sup>-5</sup> (.001)	-.002** (.001)	- (1.00 <sup>-4</sup> )	-.001 (.001)	-.001 (.001)	- (.)	-.001 (.001)	-.001 (.001)
LR			.002*** (5,846.19)	.002*** (5,855.77)	.002*** (5,859.67)	.001*** (5,850.06)	.002*** (5,859.02)	.002*** (5,863.02)

Source: Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural

Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

Notes: \*\*\* Significant at .01; \*\*Significant at .05; \* significant at .10.

Primary findings from Integrated oil and gas partially adaptive models with alternative error distributions are reported in Table 2. Coefficient estimates with their standard errors are the normal (OLS), Laplace, GED, Student t, GT, SGED, ST, and SGT error distribution models. Table 2's first five columns present assumed symmetric error distributions, whereas the last three columns make no such skewness assumptions. Comparing estimated coefficients across distributions is insightful, and smaller less well capitalized Sinopec had the largest OLS equity market coefficient, while French headquartered Total had the lowest equity market risk. Equity market returns also vary with commodity market risk, and smaller regionally Integrated firms had the highest commodity market risk, while large well-capitalized ExxonMobil had the lowest commodity market risk among Integrated firms. However, partially adaptive estimation is more sensitive to error weights, and Sinopec equity excess returns are comparable across return distributions with partially adaptive estimation. Partially adaptive estimation weights are lower for smaller commodity return variation, where ExxonMobil had the smallest commodity market risk.

If the errors are independently and identically distributed, least squares is the most efficient estimator and has the smallest errors among the class of unbiased estimators compared to other models, such as the SGT. However, least square's standard deviations are larger than more flexible SGT estimators, and the normal error regression assumption does not appropriately weigh Integrated oil and gas producer returns (Tables 2). Across distributional assumptions, least squares' standard deviations are nearly twice as large of more flexibly estimated partially

adaptive estimator standard deviations. For example, the least squares, normal estimator's Chevron equity market excess return standard deviation is .039 compared to Chevron's SGT equity market excess return standard deviation of .020. Results are similar across Integrated oil and gas producer returns and across regression model distributional assumptions. A Monte Carlo simulation would support the magnitude of the difference comparing alternative estimator distribution assumptions (McDonald and White, 1993). In sum, Integrated oil and gas OLS regression model assumption's are inappropriate, and flexible partially adaptive regression model error specifications are associated with more efficient estimation.

Table 3, Major's Estimated Distributional Parameters

<i>BP</i>	<i>GED</i>	<i>T</i>	<i>GT</i>	<i>SGED</i>	<i>ST</i>	<i>SGT</i>
Sigma	.0126	.0145	.0146	.0126	.0145	.0146
	.0003	.0010	.0014	.0003	.0010	.0014
Lambda				.0161	.0564	.0566
				.0016	.0279	.0280
P	.8946		2.029	.8958		2.033
	.0285		.2126	.0317		.2084
Q		1.376	1.340		1.380	1.340
		.0874	.2690		.0879	.2638
<b>Exxon</b>						
Sigma	.0086	.0092	.0093	.0086	.0091	.0093
	.0002	.0004	.0005	.0002	.0004	.0005
Lambda				.0384	.0324	.0319
				.	.0272	.0276
P	.9861		2.125	.9849		2.110
	.0321		.2193	.0320		.2189
Q		1.639	1.459		1.642	1.481
		.1162	.2988		.1166	.3068
<b>Chevron</b>						
Sigma	.0091	.0098	.0094	.0091	.0098	.0094
	.0002	.0005	.0003	.0002	.0045	.0004
Lambda				.0183	.0363	.0302
				.0154	.0264	.0246
P	.9839		1.625	.9865		.1686
	.0343		.1658	.0337		
Q		1.624	2.586		1.626	2.524
		.1201	.7076		.1201	.6864
<b>ConocoPhillips</b>						
Sigma	.0127		.0135	.0127	.0140	.0135
	.0003		.0007	.0004	.0008	.0007
Lambda				.0252	.0067	.0105
				.0026	.0265	.0260
P	.9564		1.801	.9539		1.789
	.0322		.1875	.0378		.1879
Q			1.867		1.506	1.893
			.4416		.1040	.4544
<b>Royal Dutch</b>						
Sigma	.0108	.0115	.0113	.0108	.0115	.0113
	.0002	.0005	.0004	.0002	.0005	.0004
Lambda				.0207	.0151	.0153
				.0088	.0269	.0263
P	1.058		1.827	1.058		1.824
	.0369		.2022	.0370		.2020
Q		1.734	2.123		1.736	2.134



<b>Total, SA</b>		.1318	.5825		.1321	.5877
Sigma	.0117	.0118	.0119	.0117	.0118	.0119
	.0002	.0003	.0003	.0002	.0003	.0003
Lambda				-.0008	-.0081	-.0086
				.0193	.0285	.0288
P	1.230		2.098	1.230		2.100
	.0451		.2309	.0450		.2314
Q		2.449	2.185		2.449	2.180
		.2395	.5928		.2393	.5911
<b>Sinopec</b>						
Sigma	.0251	.0281	.0260	.0250	.0280	.0260
	.0006	.0016	.0010	.0009	.0016	.0009
Lambda				.0714	.0662	.0650
				.0078	.0259	.0256
P	.9613		1.517	.9672		1.510
	.0326		.1447	.0401		.1329
Q		1.484	2.805		1.498	2.876
		.1048	.7752		.1066	.7376

Source: Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

Notes: \*\*\* Significant at .01; \*\*Significant at .05; \* significant at .10.

Table 3 presents estimated error distributional parameters. The Normal, LAD, GED, Student t, and Generalized t distributions impose regression error symmetry, where  $\lambda=0$ . The SGED, ST, and SGT relax regression error symmetry assumptions and allow for more flexibly estimated Integrated regression error skewness to allow for more realistic model assumptions that allow for more flexibly estimated Integrated regression errors. Subsequently, asymmetric regression error skewness is a more realistic model assumption for Integrated oil and gas models.

P and q control partially adaptive return's tail thickness, and Table 3 illustrates that Integrated firm's equity return errors have fat tails; yet, return kurtosis is not the same as regression error returns kurtosis. Normal, LAD, and Student t regression model assumptions

impose pre-determined regression error kurtosis, whereas the GED, GT, SGED, and SGT allow the data to determine kurtosis. Across Integrated return models, kurtosis exists, and the SGT, ST, and SGED allow for error return skewness. Consequently, across each of the Integrated

	<i>BP</i>	<i>ExxonMobil</i>	<i>Chevron</i>	<i>ConocoPhillips</i>	<i>Shell</i>	<i>TotalSA</i>	<i>Sinopec</i>
SGT=ST	.0252	.2702	3.498	1.082	.6710	.7388	7.9852
SGT=SGED	86.118	72.124	29.051	42.327	29.546	30.531	24.913
SGT=GT	4.079	1.335	1.479	.1614	.3384	.6296	6.681
SGT=t	4.104	76.723	5.399	1.146	.9850	.8196	14.496
SGT=GED	87.058	1.335	29.923	44.094	30.372	30.532	33.650

Majors, regression model errors are skewed and have excess kurtosis compared to the least square's normal distribution.

**Table 4, Vertically Integrated Partially Adaptive Likelihood Ratio Tests**

Source: Source: Prices and adjusted rates of returns from Yahoo! Finance. Crude and Natural Gas Prices are from Federal Reserve Economic Data. <https://fred.stlouisfed.org/>

Notes: \*\*\* Significant at .01; \*\*Significant at .05; \* significant at .10.

Integrated oil and gas error returns are asymmetric and kurtotic. For nested SGT error models, the likelihood ratio test distinguishes the statistically significant improvements of using more flexible, higher parameter error models. Among classical multiple restriction tests, the likelihood ratio test distinguishes the statistically significant improvements of using more flexible, higher parameter error models within the same distribution family to more restrictive models with fewer parameters. Among the multiple restriction tests, the likelihood ratio test is  $LR = 2(l - l^*)$ , where  $l$  is the optimized likelihood ratio value for the unrestricted and  $l^*$  is the likelihood ratio for the restricted models. Under general conditions, the LR test statistic is

asymptotically drawn from the  $\chi^2$  distribution, where the degrees of freedom are equal to the difference in the number of degrees of freedom in compared distributions. For example, a test between ExxonMobil's SGT and GED returns is  $2(l_{SGT}-l_{GED})=2(8,511.10-8,472.74)=76.72$ , which is distributed as a  $\chi^2(2)$  distribution. Table 4 indicates that the SGT with vertically Integrated oil and gas produces yields significant improvements over restricted parameters partially adaptive models.

Because of the strong overall SGT performance, we compare the more flexible SGT partially adaptive estimation results and its corresponding parameter estimates (Table 4). A leading interest with multi-factor models is equity return variation, and large Integrated firm returns positively vary in equity risk (Carson, 2023). Across density assumptions, more flexibly estimated equity market excess return variation is lower than least squares estimates. Integrated excess commodity return variation is also smaller than least squares. The exception is Shell, which was around the same equity return coefficient and larger commodity return coefficients. Subsequently, when more flexible probability density functions are used, partially adaptive estimation creates smaller coefficients with smaller variances.

## VI. Conclusion

Since the oil and gas industry's early years, large vertically Integrated firms have defined the industry and a widely recognized result is that equity market excess returns are not normally distributed but are kurtotic and skewed. Such asymmetries and fat tails extend to regression model errors, which makes estimated least squares equity market excess returns poorly estimated and inefficient. Traditional least squares estimation is minimum variance among all linearly unbiased estimates when regression model errors are independent and identically distributed, and

are efficient when errors are normally distributed. However, vertically Integrated firm returns and their regression model errors are not normally distributed, indicating that alternative estimators to least squares have desirable properties. Various alternative estimation techniques exist, and partially adaptive estimation techniques used here are grounded in more flexible regression error model assumptions. The Skewed Generalized T model is one flexible error distribution that outperforms the normal, and least squares estimator precision is improved with more flexible error distribution assumptions that account for varying returns and model error assumptions in the US oil and gas industry.

## References

- Akgiray, V. and G. G. Booth. (1988). “Mixed Diffusion-Jump Process Modeling of Exchange Rate Movements.” *The Review of Economics and Statistics*. 70. pp. 631-937.
- Akgiray, V. and G. G. Booth. (1991). “The Modelling of Stochastic Behavior of Canadian Foreign Exchange Rates.” *Journal of Multinational Financial Management*. 1, pp. 43-72.
- Bali, , T.G. (2003). “An Extreme Value Approach to Estimating Volativity and Value at Risk.” *The Journal of Business*.76, pp. 83-88.
- Bali, T.G. and D. Weinbaum. (2007). “A Conditional Extreme Value Volatitility Estimator Based on High Frequency Returns.” *Journal of Economic Dynamics and Control*. 31. pp. 361-397.
- Butler, Richard, James B. McDonald, Ray Nelson, and Steven B. White. (1990). “Robust and Partially Adaptive Estimation of Regression Models.” *Review of Economics and Statistics*, 72(2), pp. 321-327.
- Carson, Scott Alan (2020). “United States Oil and Gas Stock Returns with Multi-Factor Pricing Models: 2008-2018.” *North American Journal of Economics and Finance*, 54.  
<https://doi.org/10.1016/j.najef.2020.101236>
- Carson, Scott Alan (2022a). “Long Term Daily Equity Returns across Sectors of the Oil and Gas Industry, 2000-2019,” *Journal of Industry, Competition and Trade*. 22, pp. 125-143.

- Carson, Scott Alan (2022b). “Independent and Major Equity Market and Commodity Return Sources around the Time of Hydraulic Fracking and Horizontal Drilling Revolution: A Differences-in-Decompositions Approach.” *Economics of Innovation and New Technology*.
- Carson, Scott Alan (2023). “Quad O and Firm Ownership: The Effects of Oil and Gas Regulation on Firm Value,” *Journal of Environmental Economics and Policy*.
- Davidson, Russell and James MacKinnon. (2004). *Econometric Theory and Methods*. Oxford: University Press: Oxford.
- Fama, E. (1965). “The behavior of stock market prices.” *Journal of Business*, 38(1): 34-105.
- Fama, Eugene and Kenneth French. (1993). “Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33(1), pp. 3-56.
- Fama, Eugene and Kenneth French. (2004). “The Capital Asset Pricing Model: Theory and Evidence.” *Journal of Economic Perspectives* 18(3). pp. 25-46.
- Fama, Eugene and Kenneth French. (2015). “A Five-Factor Asset Pricing Model.” *Journal of Financial Economics*. 116, pp. 1-22.
- Federal Trade Commission. (September, 1982). *Merger in the Petroleum Industry*.  
<https://www.ftc.gov/sites/default/files/documents/reports/petroleum-industry-mergers-structural-change-and-antitrust-enforcement-report-staff-federal-trade/040813mergersinpetrol82.pdf> Accessed January 14<sup>th</sup>, 2022.
- Fielitz, B. D. and E. W. Smith. (1972). “Asymmetric Distributions of Stock Price Changes.” *Journal of the American Statistical Association*. 67. pp. 813-814.

- Francis, J. C. (1975). (1975). "Skewness and Investor Decisions." *Journal of Financial and Quantitative Analysis*. 10. pp. 163-172.
- Hansen, B. E. (1994), Autoregressive conditional density estimation, *International Economic Review*, 35(3), pp. 705–730.
- Hansen, Christian, James B. McDonald, and Whitney Newey. (2010). "Instrumental Variables Estimation with Flexible Distributions." *Journal of Business & Economic Statistics*, 28(1), pp. 13-25.
- Mandelbrot B (1963). The variation in certain specific prices. *Journal of Business*, 36(4): 394-419.
- Markowitz, Harry. (1952). "Portfolio Selection." *Journal of Finance*. 7(1), pp. 77-91.
- McDonald, James B. and Whitney K. Newey. (1988). "Partially Adaptive Estimation of Regression Models via the Generalized T Distribution. *Econometric Theory*, 4(3), pp. 428-457.
- McDonald, James B. and Steven White (1993). "A Comparison of Some Robust and Partially Adaptive Estimates of Regression Models." *Econometric Reviews* 12, pp. 103-124.
- McDonald, James B., Richard A. Michelfelder, and Panayiotis Theodossiou. "Robust estimation with flexible parametric distributions: estimation of utility stock betas." *Quantitative Finance* 10.4 (2010), pp. 375-387.
- Mohanty, Sunil, Mohan Nandha, Abdullah Turkistani, and Muhammed Alaitani. (2011). "Oil

Price Movements and Stock Market Returns: Evidence from Gulf Cooperation Council (GCC) Countries.” *Global Finance Journal* 22, pp. 42-55.

Theodossiou, Panayiotis (1998). “Financial Data and the Skewed Generalized T Distribution.” *Management Science* 44, pp. 1650-1661.

Theodossiou, Panayiotus (2015). “Skewed Generalized Error Distribution of Financial Assets and Option Pricing.” *Multinational Financial Journal* 19(4), pp. 223-266.