CESifo WORKING PAPERS

10734
2023

October 2023

To Redistribute or to Predistribute? The Minimum Wage versus Income Taxation When Workers Differ in Both Wages and Working Hours

Aart Gerritsen

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 10734*

To Redistribute or to Predistribute?
The Minimum Wage versus Income Taxation When Workers Differ in Both Wages and Working Hours

Abstract

I consider the case for the minimum wage alongside (optimal) income taxes when workers differ in both wages and working hours, such that a given level of income corresponds to multiple wage rates. The minimum wage is directly targeted at the lowest-wage workers, while income taxes are at most targeted at all low-income workers, regardless of their hourly wage rates. This renders the minimum wage unambiguously desirable in a discrete-type model of the labor market. Desirability of the minimum wage is a priori ambiguous in a continuous-type model of the labor market. Compared to the minimum wage, income taxes are less effective in compressing the wage distribution but more effective in redistributing income. Desirability of the minimum wage depends on this trade-off between the "predistributional advantage" of the minimum wage and the "redistributional advantage" of the income tax. I derive a desirability condition for the minimum wage and write it in terms of empirical sufficient statistics. A numerical application to the US suggests a strong case for a higher federal minimum wage – especially if social preferences for the lowest-wage workers are relatively strong and the wage elasticity of labor demand relatively small.

JEL-Codes: H210, J380.

Keywords: minimum wage, income taxation, optimal redistribution, multidimensional heterogeneity.

Aart Gerritsen
Erasmus University Rotterdam
PO Box 1738
The Netherlands – 3000 DR Rotterdam
agerritsen@ese.eur.nl
Internet: https://sites.google.com/site/aartgerritsen/

October 2023
I thank Marcelo Arbex, Pierre Boyer, Dana Sisak, Robert Dur, Antoine Ferey, Albert Jan Hummel, Bas Jacobs, Hans van Kippersluis, Adam Lavecchia, Etienne Lehmann, Kevin Spiritus, Otto Swank, Damián Vergara, and Nicolas Werquin for inspiring conversations and helpful comments.

1 Introduction

The minimum wage is a politically popular policy instrument meant to improve living standards of the working poor. Virtually all rich countries have a legal minimum wage and political debates over raising the minimum wage are everywhere a recurring phenomenon.[1] Empirical studies on the impact of the minimum wage have greatly contributed to the popularity of the minimum wage. A large and growing number of studies suggests that small increases in the minimum wage generate at most modest adverse effects on employment.[2] This abundance of empirical findings stands in stark contrast to the scarcity of theoretical justifications for the minimum wage. Regardless of the magnitude of the employment response, most theoretical studies have a hard time finding a useful role for the minimum wage if the government can also set income taxation.

Existing theoretical studies generally assume a one-to-one correspondence between equilibrium wages and income. As a result, the exact same people that are targeted by the minimum wage could also be targeted by the income tax. It is no wonder, then, that theoretical studies often fail to find a role for the minimum wage alongside the income tax. In reality, there is no one-to-one correspondence between wages and income because workers differ widely in both hourly wages and working hours. People with low income may be full-time workers with a minimum wage or part-time workers with a higher wage rate. Figure 1 – which decomposes the lower tail of the US income distribution into different wage categories – clearly shows that any given level of income corresponds to a wide range of hourly wage rates.

In this paper, I take this empirical fact into account and reappraise the relative merits of the minimum wage versus income taxation. I consider an economy in which people differ in both wages and working hours, such that a given level of income corresponds to multiple hourly wage rates. The government can enforce a nonlinear income tax and a binding minimum wage, but it cannot condition taxes on hourly wages.[3] A minimum wage is then better targeted at low-wage workers than the income tax. This yields a potential justification for the minimum wage if the government values redistribution from high- to low-wage workers. I derive conditions under which the minimum wage is on the margin superior to income taxation, write these conditions in terms of empirically observable

[1] Countries without a legal minimum wage typically have union-negotiated wage floors on sectoral levels. Examples of recently enacted minimum wage policies are the EU directive on adequate minimum wages, the binding minimum wage in Germany, the living wage in the United Kingdom, and minimum-wage increases on state, county, and even city levels in the United States.

[2] For a non-exhaustive list of recent contributions, see Cengiz et al. (2019) for the United States, Harasztosi and Lindner (2019) for Hungary, and Dustmann et al. (2022) for Germany. Adverse employment effects may well be more significant for larger increases in the minimum wage, see Clemens and Strain (2021).

[3] I motivate this by actually observed government policy, and by the reasoning, from Lee and Saez (2012), that the enforcement of a binding minimum wage does *not* require costless information on individual wage rates as long as sufficient whistle-blower protection is in place.

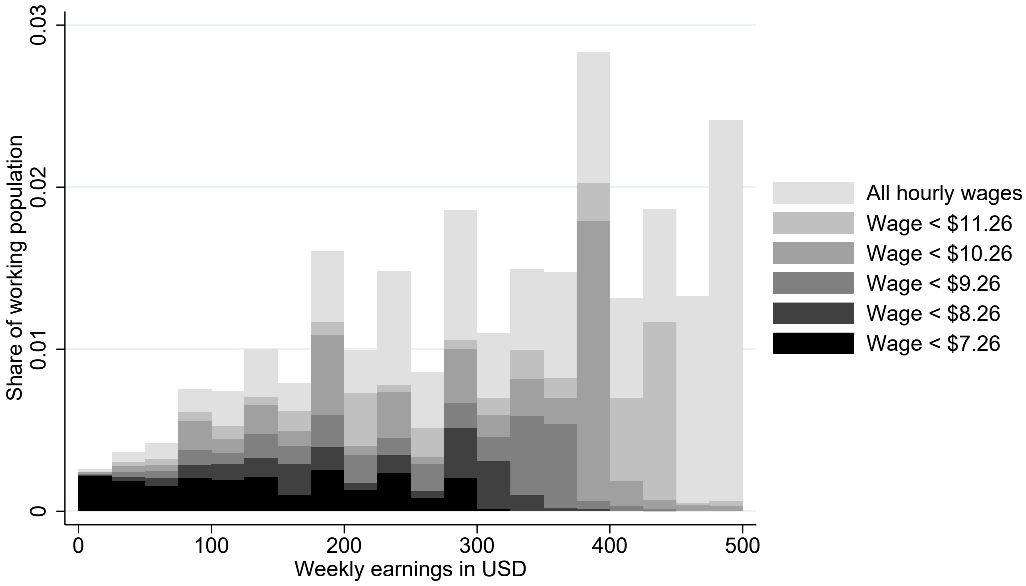

Figure 1: Share of working population by earnings bin and wage category

Notes: Each bar corresponds to a weekly earnings bin of \$25. Each shade corresponds to a different wage category. The lowest wage category corresponds to workers who earn at most the federal minimum wage (\$7.25). The graph is based on the micro-data from the NBER Outgoing Rotation Group of the Current Population Survey for 2019, available at https://www.nber.org/morg/. Sampling weights are used to determine shares of the total working population.

sufficient statistics, and calibrate these conditions using data on the United States.

The first part of the paper considers a highly stylized competitive labor market with a discrete number of worker types à la Stiglitz (1982) and Stern (1982). This is meant to illustrate the basic mechanism that makes a minimum wage potentially better targeted than the income tax. It also revisits the classic case *against* the minimum wage that was made within the context of the same type of models (Allen, 1987; Guesnerie and Roberts, 1987). I show that a minimum wage is unambiguously desirable if the government values redistribution from high- to low-wage workers, rationing takes place on the intensive margin, and the lowest income type consists of both minimum-wage and higher-wage workers.

This is almost trivially true if there are only two types of workers with identical income but different wages. In that case, an income tax is of little use as both workers have the same income. But a minimum wage *is* able to redistribute from high- to low-wage workers. More surprisingly, this unambiguous result in favor of the minimum wage survives in the presence of a discrete number of additional worker types with higher levels of income. Thus, in a model with a discrete number of worker types, wage inequality among low-income workers completely overturns the classic argument against the minimum wage.

This illustration is useful in illuminating the potential role for the minimum wage. However, the assumptions under which a discrete number of worker types pool at the same income level are rather stark. The second part of the paper therefore considers a more realistic model with a continuum of income levels à la Mirrlees (1971). I extend this workhorse model of optimal taxation in two main directions. First of all, I allow for a mass of workers that earn the lowest hourly wage rate and face a finite elasticity of demand – as is empirically realistic (e.g., Cengiz et al., 2019). Second, I assume that workers are heterogeneous in wage rates *and* preferences for work. This generates wage inequality even conditional on income.

I then study two small policy reforms. The first reform is an increase in a marginally binding minimum wage. The second reform is an increase in marginal taxes for the working poor. As the intercept of the tax function adjusts to make the reforms budget neutral, the tax increase should really be seen as an increase of in-work benefits that is phased out with income – not unlike the earned income tax credit (EITC) in the US. Thus, within a US context, the welfare comparison of the two reforms is suggestive about the relative merits of the minimum wage versus the EITC. In a more general context, the comparison shines a light on whether the working poor are better helped by an expansion of the minimum wage or income-conditional transfer programs. I find that the answer to this question is *a priori* ambiguous and subject to a trade-off.

Intuitively, an increase in the minimum wage yields distributional benefits by raising the lowest wage rate. It generates distortionary costs by reducing demand for lowest-wage workers. The same welfare effects could be obtained by raising marginal taxes for low incomes. Higher taxes generate distortionary costs by reducing *supply* of (among others) the lowest-wage workers, which in turn raises their wage rates through general-equilibrium effects. However, the tax reform yields two additional welfare effect. On the one hand, it yields additional distortions as it also reduces labor supply of low-income workers with higher wage rates. This implies a "*pre*distributional advantage" of the minimum wage over taxation: the minimum wage raises pre-tax wages against lower distortionary costs. On the other hand, unlike the minimum wage, the tax reform raises revenue from higher-income workers that could be redistributed to the poor. This implies a "*re*distributional advantage" of the income tax. The desirability of the minimum wage hinges on the trade-off between its predistributional advantage and its redistributional disadvantage.

A comparison of the two reforms yields a condition under which an increase in the minimum wage is more desirable than a tax reform with comparable effects on the wage distribution. I write this desirability condition in terms of empirically observable sufficient statistics. It shows that a minimum wage is more likely to be desirable if income is less informative about wage rates – making the income tax worse targeted at the lowest wage rates. Moreover, desirability of the minimum wage is declining in the elasticity of labor demand and increasing in the elasticity of labor supply. Intuitively, the more elastic demand (supply), the higher the distortionary costs of the minimum wage (income tax). This implies that empirical studies on the employment effects of the minimum wage are of direct relevance to the desirability of the minimum wage. Indeed, the minimum wage becomes unambiguously desirable if the elasticity of demand approaches zero.

I numerically calibrate the desirability condition of a binding minimum wage for the United States. For this, I consider two different social welfare functions. In the first, government *only* cares about workers with the lowest wage rates. In the second, government only cares about workers with low levels of income, *regardless* of their wage rates. Under either social welfare function, I can write the desirability condition solely in terms of characteristics of the income distribution and the ratio of demand and supply elasticities. I calibrate the characteristics of the income distribution on the basis of microdata from the Current Population Survey of 2019 – the same data used by Cengiz et al. (2019) in their study on the employment effects of the minimum wage.

If government cares only about the lowest-wage workers, I find that a binding minimum wage is desirable if the ratio of elasticities of labor demand and supply is less than five ($e_D/e_S < 5$, where both elasticities are positively defined). The empirical literature on supply and demand elasticities suggests that this desirability condition is easily satisfied.[4] Thus, I find that a binding US minimum wage is desirable if the government cares only about redistributing towards the lowest-wage workers. I then consider the case in which government cares about *all* low-income workers. In that case, I find that a binding minimum wage is desirable if the ratio of elasticities is less than about 1.4. Although this is clearly a much higher hurdle to pass, it is not entirely implausible either – especially given recent estimates of small employment responses to the minimum wage. I conclude that realistic inequality in *both* wages and income provides a clear justification for a binding minimum wage – especially if government cares more about low-*wage* workers than about low-*income* workers per se, and if an increase in the minimum wage has only modest effects on labor demand.

Related literature There is a relatively small literature that focuses on the desirability of minimum wages alongside (optimal) income taxes. The classic references are Allen (1987) and Guesnerie and Roberts (1987). They conclude that the minimum wage is a redundant policy instrument; it could achieve nothing that could not also be achieved by income taxes. Thus, they pose the essential challenge for anyone who wants to build a case for the minimum wage: *what does a minimum wage do that cannot be done by income taxation?* The answer that I provide in this paper is that the minimum wage targets redistribution at low-income workers *with low wage rates*. Income taxation, on the other hand, may only target support at *all* low-income workers, regardless of their wage rates.

[4] A meta-study by Lichter, Peichl, and Siegloch (2015) shows that the vast majority of estimates of demand elasticities are between 0 and 1. Typical estimates for supply elasticities, meanwhile, tend to center around 0.3 (e.g., Chetty, 2012; Saez, Slemrod, and Giertz, 2012).

Other papers have provided different answers. While Allen (1987) and Guesnerie and Roberts (1987) assume that labor rationing occurs on the intensive margin, Marceau and Boadway (1994) argue that the minimum wage – unlike income taxation – may lead to involuntary unemployment on the extensive margin. This may be desirable if it generates a positive fiscal externality, i.e., if the unemployed pay more taxes than the employed. Similarly, Lee and Saez (2012) argue that a minimum wage may only be optimal in combination with an Earned Income Tax Credit that is generous enough so that low-skilled employment is subsidized on a net basis.[5] Gerritsen and Jacobs (2020) argue that involuntary low-skilled unemployment may generate a positive fiscal externality by incentivizing human capital formation. Boadway and Cuff (2001) show that involuntary unemployment – and therefore the minimum wage – may be desirable if the government can differentiate transfers between the involuntarily and the voluntarily unemployed. Each of these papers require involuntary low-skilled unemployment to be a net benefit to society for the minimum wage to be desirable. Moreover, none of their results depend on the magnitude of the employment effects of the minimum wage – generating a disconnect between the normative theoretical literature and the focus of most empirical studies.

A second strand of the normative literature focuses on the role of the minimum wage in the presence of labor market imperfections. Cahuc and Laroque (2013) consider a monopsonistic labor market and find a similar result to Allen (1987) and Guesnerie and Roberts (1987), namely that the minimum wage is superfluous with sufficient tax instruments. Hungerbühler and Lehmann (2009) and Lavecchia (2020) study a framework with search frictions and show that the minimum wage may be desirable if low-skilled workers have inefficiently little bargaining power. Vergara (2023) also studies a framework with search frictions, and argues that a binding minimum wage may be desirable as a way to indirectly tax profits if profit taxation is imperfect. Ahlfeldt, Roth, and Seidel (2022), Berger, Herkenhoff, and Mongey (2022), and Hurst et al. (2023) are three recent papers that study the implications of minimum wages in quantitative general equilibrium models of the labor market. They find that the minimum wage potentially yields gains in equity and efficiency but do not consider the optimal taxation of income.

This paper is also related to the larger literature on optimal taxation when individuals are heterogeneous across multiple dimensions. Many of these papers focus on the optimal taxation of a single income category when people differ in wages and preferences (e.g., Choné and Laroque, 2010; Rothschild and Scheuer, 2013; Sachs, Tsyvinski, and Werquin, 2020; Jacquet and Lehmann, 2021a,b; Bergstrom and Dodds, 2021). Others extend the policy space by additionaly considering taxation of commodities, capital, or spousal income (e.g., Saez, 2002; Kleven, Kreiner, and Saez, 2009; Diamond and Spinnewijn, 2011; Spiritus et al., 2022; Ferey, Lockwood, and Taubinsky, 2022). My paper is the first to

[5]This argument could also be extended to the question of whether labor unions are desirable, see Hummel and Jacobs (2023).

add minimum wages to the policy space, considering their desirability alongside income taxation when workers differ across multiple dimensions, such that a given income level corresponds to multiple wage rates.

Methodologically, the paper relies on a comparison of two policy perturbations: an increase in the minimum wage and a tax reform with similar effects on the wage distribution. As such, the paper fits within a tradition that applies tax perturbations to derive conditions for optimal policy – see especially Saez (2001). It is a common strategy to derive a desirability condition for a given policy by comparing a perturbation of that policy with a similar reform of the income tax schedule. For examples of other applications, see Christiansen (1981) on public good provision, Christiansen (1984) and Saez (2002) on commodity taxation, Gerritsen et al. (2022) on capital taxation, and Kaplow (2008) on the general principle of combined tax reforms.

Road map The next Section introduces income-conditional wage inequality within a discrete-type labor market and shows that a minimum wage is, in that case, unambiguously desirable. Section 3 presents a continuous-type labor market, and derives and discusses the policy trade-off that determines desirability of the minimum wage. Section 4 presents the empirical application, and Section 5 concludes.

2 Economies with a discrete number of income levels

2.1 A two-type labor market

I first consider a highly stylized economy in which a binding minimum wage is unambiguously part of the policy optimum. This helps in developing the intuition behind later results. The economy has two types of individuals, denoted by $i = \{A, B\}$. There are n^i individuals of each type and individuals supply h^i hours of work – yielding aggregate labor supply of type i equal to $L^i \equiv n^i h^i$. The two types are imperfect substitutes in production. In particular, I assume that the output of a representative firm is given by the following Cobb-Douglas production function:

$$Y = F(L^A, L^B) = (L^A)^\alpha (L^B)^{1-\alpha}, \tag{1}$$

with $\alpha \in (0, 1)$ a share parameter. Wages for both types are given by w^i and the price of output is normalized to 1. Profit maximization ensures that marginal productivity and wage rates are equated for both types, $F_{L^i}(L^A, L^B) = w^i$. I chose a Cobb-Douglas production function because of its attractive feature of fixed income shares α and $1 - \alpha$ for labor types A and B. This allows me to calibrate the model in such a way that both types earn the same equilibrium income. In particular, I set the share parameter at $\alpha = n^A/(n^A + n^B)$, such that a share $n^A/(n^A + n^B)$ of income is paid to type-A workers. As a result, equilibrium income is the same for both types of workers and equal to $z^i \equiv w^i h^i = Y/(n^A + n^B)$ for both i.

An individual's tax burden is given by a potentially nonlinear function of income $T(z^i)$. Labor is the only source of income and all income is spent on consumption c^i. The budget constraint of worker i is thus given by $c^i = w^i h^i - T(w^i h^i)$. Individuals enjoy consumption but dislike labor supply. This is reflected by their utility function $U^i = u^i(c^i, h^i)$, which is increasing in c^i, decreasing in h^i, concave in both arguments, homogeneous within types, and heterogeneous across types. Equilibrium labor supply is implied by equating the marginal rate of substitution of leisure for consumption with the marginal net-of-tax wage rate:

$$(2) \qquad MRS^i(c^i, h^i) \equiv \frac{-u^i_h(c^i, h^i)}{u^i_c(c^i, h^i)} = (1 - T'(z^i))w^i.$$

I assume that $MRS^B(c, h) > MRS^A(c, h)$ for any given bundle of consumption and labor supply $\{c, h\}$. This implies that type-B individuals value leisure more than type-A individuals. Because profit maximization implies that both types earn the same income, it must therefore be the case that type-B workers earn higher wage rates while working less hours than type-A workers: $h^A > h^B$ and $w^A < w^B$. Intuitively, because type-B individuals have a relative distaste for work, their labor is scarcer and therefore more productive on the margin.

2.2 The government's instrument set

The government can condition its taxes on labor income ($z^i = w^i h^i$) but not on wage rates (w^i). The optimal tax literature tends to rationalize this with informational constraints: the government can observe labor income on the individual level, but it cannot observe wage rates or working hours separately. It can therefore tax labor income but not wage rates (Mirrlees, 1971). Although the government does not observe wage rates, I assume that it *can* enforce a minimum wage. Guesnerie and Roberts (1987) refer to this as a "somewhat mixed observability assumption." Nevertheless, this assumption is in line with the practical observation that actual governments typically do not condition taxes on wage rates, but they do implement and enforce both minimum wages and income taxes.[6] Moreover, Lee and Saez (2012) suggest that minimum wages are enforceable without actually observing individual wage rates if (a) wages can in principle be observed at some cost, (b) workers are rewarded for truthful whistle blowing and (c) workers cannot

[6] Of course, neither instrument is enforced *perfectly*. Much has been written on income-tax evasion (e.g., Slemrod, 2019). Less is known about minimum-wage enforcement, but Clemens and Strain (2022) suggest that noncompliance with minimum-wage legislation is empirically nontrivial, even though compliance is the norm. In this paper, I abstract from both forms of noncompliance.

credibly commit to not whistle blow when offered a job.

2.3 The government's objective

I assume that the social planner wants to maximize the utility of type-A workers so that we can write the social welfare function as $\mathcal{W} = n^A U^A$. This could be rationalized in different ways. First, there is a long tradition in political philosophy that espouses redistribution to compensate individuals for differences in their opportunities but not their preferences (Dworkin, 2000; Roemer, 1996; Fleurbaey, 2008). As long as type-A workers face a lower wage rate than type-B workers, their opportunities to advance in life are worse because they need to work more hours to obtain the same level of consumption. At the same time, preference heterogeneity should in itself not affect the redistributive preferences of the government. Thus, even though both types of workers earn the same income, workers of type A deserve compensation because their low wage rates keep them from earning a higher income, whereas workers of type B do not deserve compensation because they are materially held back only because of their strong preference for leisure. Second, depending on the cardinalization of the two utility functions, one could argue that type-A workers are worst off because they work more hours than type-B workers even though they enjoy the same amount of consumption. Thus, from a more traditionally welfarist point of view, a Rawlsian (maximin) government would also be intent on maximizing the utility of type-A workers.[7]

The government's budget constraint stipulates that tax revenue must be sufficient to cover some exogenously given revenue requirement R, such that $(n^A + n^B)T(z^A) = R$, where I used the fact that $z^B = z^A$. Denoting the Lagrange multiplier of the budget constraint by λ, the government's objective function is given by the following Lagrangian:

$$\mathcal{L} = n^A u^A(w^A h^A - T(w^A h^A), h^A) + \lambda((n^A + n^B)T(z^A) - R), \tag{3}$$

where I substituted the budget constraint into the utility function. The Lagrangian allows me to decompose the marginal social-welfare effects of any policy reform into utility effects and budgetary effects. The first term in eq. (3) shows that a policy reform yields utility gains only to the extent that it leads to higher type-A wages w^A. The envelope theorem tells us that policy-induced changes in working hours do not directly affect utility as long as workers are free to choose their labor hours such that they are on the margin indifferent between working more or less. The second term in eq. (3) shows that a policy reform yields budgetary gains to the extent that it expands the tax bases z^i (provided that marginal taxes are positive, $T'(z^i) > 0$; if $T'(z^i) < 0$, a policy yields budgetary gains

[7] While I assume for simplicity that the government *only* cares about workers of type A, the results in this Section only require that the government attaches a larger marginal social welfare weight to workers of type A than to workers of type B.

if it *reduces* tax bases z^i).[8]

2.4 Desirability of a binding minimum wage

In a competitive labor market, a binding minimum wage causes labor rationing by reducing labor demand to a level below nominal labor supply. I assume that workers are rationed on the intensive margin so that minimum-wage workers are forced to work less hours than they desire. In the absence of rationing, utility maximization implies that workers are on the margin indifferent between working more or less hours. As a result, rationing is efficient in the sense that marginal utility losses associated with rationing are of second order when evaluated at an allocation without rationing. The assumption of efficient rationing is typical in theoretical work on minimum wages but not necessarily realistic.[9] I briefly discuss the implications of inefficient rationing at the end of Section 3.

To determine the desirability of a binding minimum wage, I evaluate the social-welfare effects of a small increase in the low wage rate w^A at an allocation without a binding minimum wage. These welfare effects can be decomposed into utility effects and budgetary effects. I then compare this reform to a tax reform that yields identical budgetary effects. If the increase in the minimum wage yields a larger utility gain than the tax reform, then the government can achieve a social-welfare improvement by implementing a binding minimum wage while offsetting any budgetary effects by an appropriate adjustment in taxes. Moreover, if this holds at the allocation in which taxes are set optimally, then a binding minimum wage must be part of the overall policy optimum. Pursuing this line of analysis in the Appendix yields the following Proposition.

Proposition 1 *In an economy with two types of workers who earn the same income but differ in wages, a binding minimum wage is unambiguously part of the policy optimum.*

Proof. See Appendix A. ■

A marginally binding minimum wage raises the wage rate of type-A workers. As a result, type-A labor demand declines, reducing type-A working hours. This triggers general equilibrium effects as the decline in type-A employment reduces marginal productivity and therefore wages of type-B workers.[10] As long as consumption and leisure are normal goods, the decline in type-B wages reduces labor earnings z^B. And since equilibrium income is the same for both types, the reform also reduces type-A labor earnings z^A.

[8] Obviously, a policy reform also affects utility and revenue if it consists of a change in the level of taxes $T(z^i)$. For my results, however, I only need to consider a policy reform of *marginal* taxes $T'(z)$.

[9] Empirical evidence on the efficiency of rationing is scarce but Luttmer (2007) finds mixed results. Gerritsen (2018) and Gerritsen and Jacobs (2020) discuss policy implications of inefficient rationing.

[10] This in turn affects type-B labor supply, triggering further general equilibrium effects on marginal productivity and therefore labor demand for type-A workers, etcetera until a new equilibrium is reached.

Summing up, a marginal increase in a binding minimum wage yields utility gains through its increase in w^A and budgetary losses through its negative effect on tax bases $z^A = z^B$.[11]

The same reduction in tax bases could be achieved by raising the marginal tax rate $T'(z^i)$ – inducing reductions in labor supply for both types. However, the type-A utility gains associated with such an increase in marginal taxes are always smaller than the utility gains of the increase in the minimum wage. To see this, first consider the case in which compensated labor-supply elasticities are the same for both types of workers. In that case, an increase in the marginal tax rate reduces labor supply of both types in the same proportion, leaving marginal productivity and thus wage rates of both types unaltered. Thus, with identical elasticities, an increase in marginal taxes yields budgetary losses without any offsetting type-A utility gains. The only case in which the increase in the marginal tax rate could raise type-A wages and utility is if compensated elasticities are larger for type-A workers than for type-B workers. In that case, there is a relatively strong reduction in type-A labor supply, yielding an increase in type-A productivity and wages. Nevertheless, because the increase in the marginal tax rate also discourages labor supply of type-B workers – yielding a countervailing effect on type-A wages – the marginal tax rate can never be as effective as the minimum wage in raising type-A utility. Hence, for the same budgetary effect, a minimum wage always yields greater type-A utility gains than the marginal tax rate. Thus, a binding minimum wage must be part of the policy optimum.

2.5 Three types

It seems almost trivial that a binding minimum wage is a more desirable instrument for redistribution than a marginal tax rate on income when individuals differ in their wage rates but have the same level of income. After all, the *prima facie* case for an income tax is not particularly strong when there is no income inequality. It is therefore useful to show that the results from Proposition 1 are robust to adding another worker type with a higher level of income. In particular, assume that there are also n^C workers of type C. Type-C workers enter the production function linearly, earn a higher wage rate than the other two types ($w^C > w^B > w^A$), and have the same utility function as type-A workers ($u^C(\cdot) = u^A(\cdot)$, i.e., type C also has a relatively weak preference for leisure). As a result, type-C income is higher than that of the other two types ($z^C > z^B = z^A$).

The government still wants to maximize type-A utility subject to a budget constraint that now also includes revenue from type-C workers. In addition to the budget constraint, the government faces an incentive constraint that requires workers of type C to prefer

[11] Assuming that $T'(z^i) > 0$; note that if $T'(z^i) \leq 0$, then a marginal increase in the minimum wage yields utility gains without budgetary losses and is therefore unambiguously desirable.

"their" equilibrium level of income over that of types A and B:

$$u^C(z^C - T(z^C), z^C/w^C) \geq u^C(z^A - T(z^A), z^A/w^C). \tag{4}$$

This yields an economy that is comparable to the two-type economy of Stiglitz (1982) and Stern (1982), except that the low level of income is earned by both high- and low-wage workers. Denoting the Lagrange multiplier of the incentive constraint by γ, the government's objective function is given by the following Lagrangian:

$$\tilde{\mathcal{L}} = \mathcal{L} + \lambda n^C T(z^C) + \gamma(u^C(z^A - T(z^A), z^A/w^C) - u^C(z^C - T(z^C), z^C/w^C)), \tag{5}$$

where $\tilde{\mathcal{L}}$ refers to the new Lagrangian and $\mathcal{L}$ to the original Lagrangian in eq. (3). The following Proposition establishes the robustness of Proposition 1.

Proposition 2 *In an economy with three types of workers where two types earn the same income but differ in wages and a third type earns more, a binding minimum wage is unambiguously part of the policy optimum.*

Proof. See Appendix A. ∎

The intuition behind Proposition 2 is straightforward. A binding minimum wage and an increase in the marginal tax rate at $z^A = z^B$ both yield a reduction in taxable income z^A. The incentive constraint in eq. (4) is relaxed by a reduction in taxable income z^A, regardless of whether this reduction took place because of an increase in the minimum wage or an increase in the marginal tax rate. Thus, the minimum wage and marginal taxes are both equally effective in relaxing the incentive constraint. However, as was shown in the proof of Proposition 1, for a given reduction in taxable income z^A, a minimum wage raises $\mathcal{L}$ more than an increase in the marginal tax rate $T'(z^A)$. Thus, even with income inequality, an increase in the minimum wage – evaluated in the absence of a binding minimum wage and at any given tax schedule – is still unambiguously more desirable than an increase in marginal taxes. This implies that a binding minimum wage must be part of the policy optimum.

Proposition 2 stands in stark contrast to the results in Allen (1987) and Guesnerie and Roberts (1987), who conclude that a binding minimum wage is undesirable in the discrete-type optimal tax framework of Stiglitz (1982) and Stern (1982). The only material difference between their model and mine is that I allow for wage heterogeneity among low-income workers. This small and empirically realistic alteration of the model yields diametrically opposing results. Instead of being unambiguously undesirable, the minimum wage turns out to be unambiguously desirable.

Nevertheless, the stylized nature of the model – discrete rather than continuous types, and the knife-edge calibration of the production function – means that Propositions 1 and

2 should be interpreted with caution. In particular, the welfare calculus of marginal taxes is quite different in a setting with a continuous rather than a discrete income distribution. With a discrete number of income levels, the marginal tax rate at one income level does not immediately affect the tax burden at the higher income level. This is because any such effect could always be undone by an adjustments of the marginal taxes in between the two levels of income. In contrast, if the income distribution is continuous, an increase in the marginal tax rate *does* mechanically lead to an increase in tax burdens for workers with a higher level of income. The next Section shows that, in that case, desirability of a binding minimum wage is no longer unambiguous.

3 Economy with a continuous distribution of income

3.1 The model

I now consider a model with a continuum of income levels as in Mirrlees (1971). I extend the standard Mirrleesian model in two directions. First of all, the standard model features a linear production technology. A minimum wage would in that case simply destroy all jobs for workers whose marginal productivity falls short of the minimum wage (Boadway and Cuff, 2001). This is empirically implausible (e.g., see Cengiz et al., 2019, who find strong evidence of bunching around the minimum wage). Instead, I assume that production is strictly concave in the supply of a mass of the least productive workers. I do retain the assumption that production is linear in the supply of more productive workers. Second, I allow for heterogeneity in both earning capacity and preferences for work as in Jacquet and Lehmann (2021a). This generates heterogeneity in income even among workers with the same hourly wage rates.

I consider a mass-one continuum of individuals $\mathcal{I}$. It is again useful to think of two separate types of workers. There is a continuum $\mathcal{A}$ of type-A workers that end up earning the minimum wage, and a continuum $\mathcal{B}$ of type-B workers that earn higher wages. The total population consists of the union of both types, such that $\mathcal{A} \cup \mathcal{B} = \mathcal{I}$. Individuals $i \in \mathcal{I}$ supply h^i working hours. Type-A workers are homogeneous in their productivity; their aggregate labor supply is given by $L^A = \int_{\mathcal{A}} h^i \mathrm{d}i$.[12] Type-$B$ workers are heterogeneous in their hourly productivity θ^i; their aggregate *effective* labor supply is written as $L^B = \int_{\mathcal{B}} \theta^i h^i \mathrm{d}i$. Total production is given by:

$$Y = F(L^A) + L^B, \tag{6}$$

with $F'(\cdot) > 0$ and $F''(\cdot) < 0$. Thus, production is strictly concave in type-A labor supply

[12] I assume that all the relevant variables – in this case labor hours h^i – are integrable over the type space.

and linear in type-B labor supply. Profit maximization implies that the equilibrium hourly wage rate for type-A workers is given by $w^i = w^A \equiv F'(L^A)$ for all $i \in \mathcal{A}$. The equilibrium hourly wage rates for type-B workers are given by $w^i = \theta^i$ for all $i \in \mathcal{B}$. I only consider equilibriums in which $F'(L^A) < \theta^i$ for all $i \in \mathcal{B}$, so that type-A workers earn the lowest wage rate. Thus, a minimum wage will only be binding for type-A workers. The production function features decreasing returns to scale, which implies equilibrium profits. I assume that these profits are fully taxed away. As I show later, relaxing this assumption would strengthen the case for a binding minimum wage.

Utility is given by $U^i = u^i(c^i, h^i)$, with $c^i = w^i h^i - T(w^i h^i)$ consumption and $T(w^i h^i)$ a nonlinear income tax. As before, utility maximization implies that individuals equate marginal rates of substitution and net marginal wage rates:

$$MRS^i(c^i, h^i) \equiv \frac{-u^i_h(c^i, h^i)}{u^i_c(c^i, h^i)} = (1 - T'(z^i))w^i, \tag{7}$$

which repeats eq. (2). I abstract from income effects on labor supply by assuming that utility is linear in consumption ($u^i_c = 1$ for all $i \in \mathcal{I}$). Workers of type A earn identical wage rates but differ in their preferences to work and therefore in their equilibrium hours worked. Workers of type B differ in both wage rates and working preferences. Hence, in equilibrium, there is income inequality within and between both types of workers. Importantly, despite the multidimensional heterogeneity, I assume that marginal changes in the tax system only cause marginal changes in economic behavior. As a result, given the absence of income effects, a marginal increase of the marginal tax rate around some income level z^* only causes marginal labor supply responses of individuals with income around z^*.[13]

The cumulative distribution function of the resulting income distribution is denoted by $G(z)$ with probability density function $g(z) \equiv G'(z)$. The income-contingent share of type-A earners is denoted by $\sigma(z)$. Thus, a share $\sigma(z)$ of people with income level z earn the minimum wage. The highest level of type-A income is denoted by $y \equiv \max\{z^i : i \in \mathcal{A}\}$. Collectively, I refer to people with income below y as the working poor.

3.2 Elasticity concepts

The distortionary impact of minimum wages and taxes depend on the responsiveness of labor demand and supply. It is useful to express this in terms of elasticities. The

[13]In other words, eq. (7) pegs down the unique utility-maximizing number of working hours, and the utility function and tax schedule are well-behaved, such that equilibrium labor supply is differentiable in the tax schedule. Formally ruling out discrete jumps in labor supply requires additional structure on individual preferences. Here I simply assume that labor supply is well-behaved without further exploring the necessary structural assumption that would guarantee this. Also see Jacquet and Lehmann (2021a,b) for further discussion on these issues.

wage-elasticity of type-A labor demand is defined as:

$$e_D \equiv -\frac{\mathrm{d}L^A/L^A}{\mathrm{d}w^A/w^A} = \left(\frac{-L^A F''}{F'}\right)^{-1} > 0. \tag{8}$$

The elasticity is defined positively so it gives the percentage *reduction* in type-A labor demand in response to a percent *increase* in the type-A wage rate. I assume that every type-A worker experiences the same micro-level elasticity of demand. That is, in response to an increase in the minimum wage, demand for each type-A worker's labor hours declines in proportion to e^D.

The compensated net wage elasticity of labor supply for individual i is given by:

$$e_S^i \equiv \frac{\mathrm{d}h^i}{\mathrm{d}\left((1-T')w^i\right)} \frac{(1-T')w^i}{h^i} = \left(\frac{h^i u_{hh}^i}{u_h^i}\right)^{-1}, \tag{9}$$

where the final equation follows from eq. (7) and the assumption of quasi-linear utility. The elasticity gives the percentage increase in labor supply in response to a percent increase in the net-wage rate. In the remainder, I assume that the elasticity of labor supply is homogeneous across the population, such that $e_S^i = e_S$ for all $i \in \mathcal{I}$.[14,15]

3.3 The government's objective

As before, the government maximizes type-A utility, so that social welfare is given by:

$$\mathcal{W} = \int_{\mathcal{A}} u^i(c^i, h^i)\mathrm{d}i. \tag{10}$$

Since $u_{cc}^i = 0$, the government does not care about redistribution within the group of type-A workers. The social-welfare function can again be rationalized by theories of justice that emphasize compensation for differences in opportunities but not for differences in preferences.

The government budget constraint is given by:

$$\int_0^\infty T(z)\mathrm{d}G(z) + F(L^A) - w^A L^A - R = 0. \tag{11}$$

[14]If $T'' \neq 0$, a change in labor supply leads to a change in the marginal tax rate and thus a change in the marginal net wage rate, which in turn affects equilibrium labor supply, etc. These circular effects are not taken into account in the definition of the elasticity. In terms of Jacquet and Lehmann (2021a), e_S^i is a "direct elasticity" rather than a "total elasticity." Because the "total elasticity" depends on the second derivative of the tax schedule, it makes more sense to assume that "direct elasticities" are homogeneous than to assume that "total elasticities" are homogeneous.

[15]This does not violate the assumption of heterogeneous preferences for work. The marginal rate of substitution $(-u_h^i)$ may vary across individuals even if its elasticity $(h^i u_{hh}^i/u_h^i)$ does not. Consider, for example, the utility function $u^i(c^i, h^i) = c^i - \psi^i (h^i)^{1+1/\eta}/(1+1/\eta)$, with ψ^i an individual-specific taste parameter, and $e_S^i = \eta$ for all i.

It consists of revenue from the income tax (first term), revenue from the profit tax (second and third terms), and exogenous expenditures R. To keep derivations tractable, I assume that marginal tax rates are constant for the working poor, such that $T''(z) = 0$ for all $z \leq y$. I denote the constant marginal tax rate for the working poor by $T'(z) = \tau$ for $z \leq y$. I reflect on this and other simplifying assumptions at the end of this Section.

3.4 Determining desirability

In the remainder, I determine the desirability of a binding minimum wage. I do so by comparing two different policy reforms. I first consider an increase in the minimum wage w^A, evaluated at an allocation with market-clearing wages. Such reform yields distributional benefits by raising type-A wages and lowering profits. It also generates distortionary costs by reducing type-A labor demand. I assume that any revenue effect is absorbed by means of a lump-sum tax or transfer (the intercept of the tax function). Dividing the distributional benefits by the distortionary costs provides a measure of the *bang for the buck* of a minimum-wage increase, $\mathcal{BB}_{w^A}$.

Considered in isolation, whenever the *bang* exceeds the *buck*, an increase in the minimum wage will raise social welfare. Nevertheless, outside the tax optimum, it is possible that a reform of the income tax yields an even higher bang for the buck. To test whether minimum wages are superior or inferior to taxation, I compare an increase of the minimum wage with a comparable reform of the tax schedule, that also reduces type-A employment and raises type-A wages.

In particular, I consider an increase in marginal taxes for the working poor, i.e., for income levels below y. As revenue effects are absorbed by adjusting the intercept of the tax function, this reform could be interpreted as an increase in government transfers that is phased out with (low) income. The increase in marginal taxes reduces type-A labor supply. This in turn raises type-A wages through general-equilibrium effects. Thus, like the minimum wage, the tax reform raises type-A wages and reduces type-A employment. Unlike the minimum wage, the tax reform yields additional distortionary costs by also reducing labor supply of the working poor of type B. Moreover, the tax reform yields additional distributional benefits by raising and redistributing tax revenue. Dividing total distributional benefits of the tax reform by the total distortionary costs yields a measure of the *bang for the buck* of the tax reform, $\mathcal{BB}_\tau$.

An increase in the minimum wage is relatively desirable if its bang for the buck exceeds that of the tax reform, such that $\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$. In that case, the minimum wage is the more cost-effective tool to support low-wage workers. Moreover, if the desirability condition holds at the tax optimum, in which case $\mathcal{BB}_\tau = 1$ by definition, then a binding minimum wage is necessarily part of the full policy optimum. In what follows, I consider the two reforms in detail and write the desirability condition ($\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$) in terms of empirically measurable sufficient statistics. In the main text, I heuristically derive the bang-for-the-buck measures. I present formal derivations in Appendix B.

3.5 Bang for the buck of a minimum-wage increase

Consider a perturbation of the minimum wage by $\mathrm{d}w^A > 0$. The welfare-relevant effects of this perturbation can be decomposed into utility gains, mechanical revenue losses (i.e., revenue losses absent behavioral effects), and behavioral revenue losses.[16] The utility gains minus mechanical revenue losses jointly represent the distributional gains of the minimum wage. The behavioral revenue losses represent its distortionary costs.

First consider the utility gains of a higher minimum wage. The perturbation raises wages of all type-A workers, which raises their utility. With quasi-linear utility, the utility gain for any individual $i \in \mathcal{A}$ simply corresponds to the mechanical increase in labor income, given by $(1-\tau)h^i \mathrm{d}w^i$. Integrating over all type-A workers yields the total type-A utility gains associated with the perturbation:

$$\mathcal{U}_{w^A} \equiv \int_{\mathcal{A}} ((1-\tau)h^i \mathrm{d}w^A)\mathrm{d}i = \int_0^y \sigma(z)(1-\tau)z\mathrm{d}G(z)\frac{\mathrm{d}w^A}{w^A}. \tag{12}$$

The mechanical changes in tax revenue are twofold. On the one hand, an increase in the minimum wage mechanically reduces profits, and therefore revenue from the profit tax. The mechanical reduction in profits is equal to the mechanical increase in wage costs: $L_A \mathrm{d}w^A$. On the other hand, an increase in the minimum wage mechanically raises gross income $z^i \equiv w^A h^i$ for individuals $i \in \mathcal{A}$. As a result, the government mechanically raises $\tau h^i \mathrm{d}w^A$ from every type-A worker. The net mechanical revenue loss is given by:

$$\mathcal{M}_{w^A} \equiv \int_{\mathcal{A}} h^i \mathrm{d}i \mathrm{d}w^A - \int_{\mathcal{A}} (\tau h^i \mathrm{d}w^A)\mathrm{d}i = \int_0^y \sigma(z)(1-\tau)z\mathrm{d}G(z)\frac{\mathrm{d}w^A}{w^A}. \tag{13}$$

Note that the utility gains are equal to the mechanical revenue loss, $\mathcal{U}_{w^A} = \mathcal{M}_{w^A}$. This is because the utility gains stem from a redistribution of resources from the government to lowest-wage workers. It represents a social-welfare gain as long as type-A utility gains are, at the margin, valued more than government revenue.

Finally, the perturbation of the minimum wage leads to behavioral revenue losses due to rationing of labor hours. In particular, labor hours of individuals $i \in \mathcal{A}$ change by $(\mathrm{d}h^i/\mathrm{d}w^A)\mathrm{d}w^A < 0$. As a result, they pay less taxes. The change in their tax payments is given by $\tau w^A(\mathrm{d}h^i/\mathrm{d}w^A)\mathrm{d}w^A$. Integrating over all type-A individuals yields aggregate

[16] Also see Saez (2001), who derives optimal income taxes by means of a similar decomposition of the welfare effects of a tax perturbation.

behavioral revenue losses due to the perturbation of the minimum wage:

$$\mathcal{R}_{w^A} \equiv -\int_{\mathcal{A}} \left(\tau w^A \frac{\mathrm{d}h^i}{\mathrm{d}w^A}\mathrm{d}w^A\right) \mathrm{d}i = e_D \int_0^y \sigma(z)\tau z \mathrm{d}G(z)\frac{\mathrm{d}w^A}{w^A}, \tag{14}$$

where I used the definition of the elasticity of labor demand from eq. (8), and the assumption that every type-A worker faces the same micro-level elasticity of demand.

Notice that these are the only welfare-relevant effects associated with the perturbation of the minimum wage. In particular, the envelope theorem implies that a marginal change in employment has no first-order effect on profits. The envelope theorem, combined with the assumption of efficient rationing and the fact that we evaluate the perturbation at a market-clearing allocation, implies that a marginal change in employment has no first-order effect on utility either.

Revenue is recycled back into the economy through a lump-sum grant or reduction in the tax schedule's intercept $T(0)$. Every unit of government revenue therefore yields an additional utility gain of $\int_0^y \sigma(z)\mathrm{d}G(z)$, the share of type-$A$ individuals in lump-sum transfers. This allows us to meaningfully compare utility gains and revenue losses. We then obtain the following bang-for-the-buck measure for the minimum wage.

Lemma 1 *A marginal increase in the minimum wage by* $\mathrm{d}w^A$*, evaluated at an initial allocation in which the labor market clears, yields distributional benefits and distortionary costs. The bang for the buck is given by the distributional benefits per unit of distortionary costs. It is equal to:*

$$\mathcal{BB}_{w^A} \equiv \frac{\mathcal{U}_{w^A} - \int_0^y \sigma(z)\mathrm{d}G(z)\mathcal{M}_{w^A}}{\int_0^y \sigma(z)\mathrm{d}G(z)\mathcal{R}_{w^A}} = \frac{1-\int_0^y \sigma(z)\mathrm{d}G(z)}{\int_0^y \sigma(z)\mathrm{d}G(z)} \frac{1}{\frac{\tau}{1-\tau}e_D}. \tag{15}$$

Proof. The first equation gives the definition of the bang for the buck. The second equation follows from substituting eqs. (12)–(14). Appendix B formally proves that eq. (15) gives the distributional benefits per unit of distortionary costs of raising the minimum wage w^A. ■

3.6 Bang for the buck of a comparable tax reform

Next, consider a perturbation of the tax schedule that raises marginal tax rates over the income range $[0, y]$ by $\mathrm{d}\tau > 0$. The welfare implications of this reform can again be decomposed into utility, mechanical revenue, and behavioral revenue effects. The utility and mechanical revenue effects represent the distributional gains; the behavioral revenue effects represent the distortionary costs. The bang for the buck is obtained by dividing distributional benefits by distortionary costs.

The first thing to note is that the increase in marginal tax rates leads to an equiproportional reduction in net wages for all type-A workers. This causes an equiproportional reduction in their labor supply, in turn resulting in an increase in type-A wages due to general equilibrium effects. The welfare implication of these effects are equivalent to those of an increase in the minimum wage. That is, both the minimum wage and marginal taxes generate an increase in w^A and an equiproportional reduction in h^i for $i \in \mathcal{A}$. Hence, the behavioral revenue losses of the tax-induced reduction of type-A employment are equivalent to those of the minimum wage and given by $\mathcal{R}_{w^A}$. Similarly, the tax-induced increase in wages yields utility gains $\mathcal{U}_{w^A}$ and mechanical revenue losses $\mathcal{M}_{w^A}$.[17]

But the increase in marginal taxes does not *merely* replicate an increase in minimum wages. Conditional on income, marginal taxes cannot differentiate between high- and low-wage workers. Thus, the increase in marginal taxes reduces labor supply of type-B workers as well as type-A workers. Moreover, contrary to the minimum wage, an increase in marginal tax rates at a given level of income raises revenue from everyone who earns more. This generates both utility losses and mechanical revenue gains.

Starting with the utility losses, the tax perturbation mechanically reduces net income of type-A workers by $z^i \mathrm{d}\tau$ for all $i \in \mathcal{A}$. With quasi-linear utility, the utility loss perfectly corresponds to the loss in net income. Integrating over all type-A workers, and subtracting the utility gains associated with the tax-induced increase in wages, the total type-A utility loss of the tax perturbation is given by:

$$\mathcal{U}_\tau \equiv \int_0^y \sigma(z) z \mathrm{d}G(z)\mathrm{d}\tau - \mathcal{U}_{w^A}. \tag{16}$$

The perturbation mechanically raises revenue from all individuals. The working poor face an increase in their tax rate by $\mathrm{d}\tau$, which mechanically raises their tax payments by $z^i \mathrm{d}\tau$ for all $i : z^i \leq y$. As marginal taxes are only raised for income levels below y, anyone who earns more than y faces the same tax increase of $y\mathrm{d}\tau$. Subtracting the mechanical revenue losses associated with the tax-induced increase in wages, the total mechanical revenue gain of the tax perturbation equals:

$$\mathcal{M}_\tau \equiv \int_0^y z \mathrm{d}G(z)\mathrm{d}\tau + (1 - G(y))y\mathrm{d}\tau - \mathcal{M}_{w^A}. \tag{17}$$

Finally, the tax perturbation leads to behavioral revenue losses as it reduces labor supply of both types of working poor. The revenue losses associated with reduced type-A

[17] To see this clearly, note that we can rewrite eqs. (12)–(14) by dividing and multiplying by $\mathrm{d}\tau$:

$$\begin{aligned}
\mathcal{U}_{w^A} &\equiv \int_{\mathcal{A}} ((1-\tau)h^i \mathrm{d}w^A)\mathrm{d}i = \int_{\mathcal{A}} (1-\tau)h^i \mathrm{d}i \left(\frac{\mathrm{d}w^A}{\mathrm{d}\tau}\right)\mathrm{d}\tau,\\
\mathcal{M}_{w^A} &\equiv \int_{\mathcal{A}} h^i \mathrm{d}i \mathrm{d}w^A - \int_{\mathcal{A}} (\tau h^i \mathrm{d}w^A)\mathrm{d}i = \int_{\mathcal{A}} (1-\tau)h^i \mathrm{d}i \left(\frac{\mathrm{d}w^A}{\mathrm{d}\tau}\right)\mathrm{d}\tau,\\
\mathcal{R}_{w^A} &\equiv \int_{\mathcal{A}} \left(\tau w^A \frac{\mathrm{d}h^i}{\mathrm{d}w^A}\mathrm{d}w^A\right)\mathrm{d}i = \int_{\mathcal{A}} \left(\tau w^A \frac{\mathrm{d}h^i}{\mathrm{d}\tau}\mathrm{d}\tau\right)\mathrm{d}i.
\end{aligned}$$

labor supply is equivalent to the behavioral revenue losses of the minimum wage, $\mathcal{R}_{w^A}$. On top of that, the tax perturbation reduces labor supply by $\mathrm{d}h^i/\mathrm{d}\tau$ for all $i \in \mathcal{B} : z^i \leq y$. This generates behavioral revenue losses equal to $\tau w^i \mathrm{d}h^i/\mathrm{d}\tau$. Integrating over type-$B$ workers with low income, and adding the behavioral revenue losses associated with type-A workers, yields the behavioral revenue losses of the tax perturbation:

$$\mathcal{R}_\tau \equiv \frac{\tau}{1-\tau} e_S \int_0^y (1-\sigma(z)) z \mathrm{d}G(z) \mathrm{d}\tau + \mathcal{R}_{w^A}, \tag{18}$$

where I used the definition of the elasticity of labor supply from eq. (9).

I again assume that revenue is recycled back into the economy through an adjustment in the tax schedule's intercept $T(0)$. Thus, the marginal social value of government revenue equals $\int_0^y \sigma(z)\mathrm{d}G(z)$. This yields the following bang-for-the-buck measure of a policy reform that raises marginal taxes for the working poor.

Lemma 2 *A marginal increase in tax rates over the income range* $[0, y]$ *by* $\mathrm{d}\tau$*, evaluated at an initial allocation without a binding minimum wage, yields distributional benefits and distortionary costs. The bang for the buck is given by the distributional benefits per unit of distortionary costs. It is equal to:*

$$\begin{aligned} \mathcal{BB}_\tau &\equiv \frac{\int_0^y \sigma(z)\mathrm{d}G(z)\mathcal{M}_\tau - \mathcal{U}_\tau}{\int_0^y \sigma(z)\mathrm{d}G(z)\mathcal{R}_\tau} \\ &= \frac{\int_0^y \sigma(z)\mathrm{d}G(z)\left(\int_0^y z\mathrm{d}G(z) + (1-G(y))y - \frac{\mathcal{M}_{wA}}{\mathrm{d}\tau}\right) - \int_0^y \sigma(z) z \mathrm{d}G(z) + \frac{\mathcal{U}_{wA}}{\mathrm{d}\tau}}{\int_0^y \sigma(z)\mathrm{d}G(z)\left(\frac{\tau}{1-\tau} e_S \int_0^y (1-\sigma(z)) z \mathrm{d}G(z) + \frac{\mathcal{R}_{wA}}{\mathrm{d}\tau}\right)}. \end{aligned} \tag{19}$$

Proof. The first equation gives the definition of the bang for the buck. The second equation follows from substituting eqs. (16)–(18). Appendix B formally proves that eq. (19) gives the distributional benefits per unit of distortionary costs of raising the tax rate τ. ■

3.7 Desirability of a binding minimum wage

We have seen that both minimum wages and income taxation may generate increases in the lowest wage rate w^A. However, relative to the minimum wage, income taxation yields additional distortionary costs *and* additional distributional benefits. Taxation yields more distortionary costs because it is not perfectly targeted at type-A workers with the lowest wage rates but also reduces labor supply of type-B workers with higher wage rates. This constitutes the "*predistributional advantage*" of the minimum wage: it achieves the same increase in type-A wages against lower distortionary costs.

At the same time, taxation yields more distributional benefits because it raises revenue that can be redistributed through lump-sum grants. This constitutes the "*redistributional* *advantage*" of taxation. Thus, the minimum wage harbors both advantages and disadvantages relative to taxation. Its relative desirability is therefore *a priori* ambiguous.

A minimum wage is superior to taxation if it generates more bang for the buck, thus if $\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$. To gain more insight into the relative desirability of the minimum wage, we can write this desirability condition in terms of empirically measurable sufficient statistics. I first do this for the special case in which the share of minimum-wage workers with income below y is independent of the level of income, such that $\sigma(z) = \sigma$ for all $z \leq y$. This yields a simple desirability condition with an intuitive appeal. Later, I derive the desirability condition for any general function $\sigma(z)$.

3.7.1 ...for constant $\sigma(z) = \sigma$ for all $z \leq y$

The following Proposition establishes the condition under which a minimum wage is superior to taxation if the share of minimum wage workers is constant for all incomes below y.

Proposition 3 *In an economy with a continuous distribution of income and a clearing labor market, in which $T'(z) = \tau$ and $\sigma(z) = \sigma$ for all $z \leq y$, an increase in the minimum wage is superior to a comparable tax reform if and only if:*

$$\frac{1 - \sigma G(y)}{1 - G(y)} > \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{y - \bar{y}}{\bar{y}}\right)\frac{e_D}{e_S}, \tag{20}$$

where I define $\bar{y}$ as the average income for the working poor, i.e., $\bar{y} \equiv \int_0^y z \mathrm{d}G(z)/G(y)$.

Proof. By definition, the minimum wage is superior to taxation if and only if $\mathcal{BB}_{w_L} > \mathcal{BB}_\tau$. Substitute for eqs. (15) and (19), and for $\sigma(z) = \sigma$. Rearrange to obtain eq. (20). ■

Eq. (20) writes the relative desirability of a minimum wage increase in terms of a number of empirical parameters. These consist of the share of minimum-wage workers ($\sigma G(y)$), the share of the working poor ($G(y)$), the share of the working poor that earn a minimum wage (σ), the highest income for lowest-wage workers (y) relative to the average income of the working poor ($\bar{y}$), and the elasticities of labor demand (e_D) and supply (e_S).[18] I briefly consider each term.

Evaluating the different terms in the desirability condition First of all, desirability of the minimum wage is declining in the share of minimum wage workers ($\sigma G(y)$). The reason for this is that the minimum wage redistributes from the government budget (through reduced profit tax revenue) directly to type-A workers (through higher wages).

[18] Obviously, the first three terms consist of only two mutually independent elements (σ and $G(y)$) but it is helpful to discuss all three terms in isolation.

Thus, for every unit increase in type-A income, the state loses one unit of revenue. The costs of the revenue loss is equivalent to type-A workers' share of the revenue, $\sigma G(y)$. The larger this share, the lower the distributional gains of the minimum wage, and thus, the less desirable the minimum wage.

Second, desirability of the minimum wage is increasing in the share of the working poor ($G(y)$). Intuitively, the tax reform raises revenue from everyone with income above y in order to redistribute to type-A workers with income below y. The redistributional benefits of the tax reform are therefore increasing in the share of people that earn a relatively high income ($1 - G(y)$). Thus, a larger share of the working poor ($G(y)$) reduces the distributional benefits of taxation, and thereby raises the relative desirability of the minimum wage.

Third, desirability of the minimum wage is declining in the share of poor workers that earn a minimum wage (σ). The reason why the minimum wage may be desirable at all is because it is better targeted at lowest-wage workers than the income tax. That is, while marginal taxes may also raise wages w^A by reducing labor supply of type-A workers, such taxes additionally distort labor supply of low-income workers with higher wages. As a result, marginal income taxes generate more distortions for the same increase in type-A wages. If the share of minimum wage workers among the working poor is smaller, then the income tax is even worse targeted relative to a minimum wage. Thus, the relative desirability of the minimum wage is declining in this share.

Fourth, desirability of the minimum wage is declining in the highest income of the working poor (y) relative to the average income of the working poor ($\bar{y}$). Low wages w^A can be raised through either minimum wages or through higher marginal taxes among the working poor. Higher taxes generate a redistributional advantage over minimum wages by raising revenue that could be redistributed through a lump-sum transfer. Recall that such tax reform raises revenue from the non-poor by $y\mathrm{d}\tau$. At the same time, it also raises revenue from minimum-wage workers by, on average, $\bar{y}\mathrm{d}\tau$. While the former constitutes a welfare gain, the latter constitutes a welfare loss. Hence, the distributional benefits of taxation are increasing in y and declining in $\bar{y}$. The relative desirability of the minimum wage is therefore declining in y and increasing in $\bar{y}$.

Fifth and final, the desirability of the minimum wage is declining in the elasticity of labor demand (e_D) relative to the elasticity of labor supply (e_S). The reason for this is intuitive. The distortionary costs of the minimum wage are increasing in the elasticity of labor *demand*. The distortionary costs of taxation are increasing in the elasticity of labor *supply*. Hence, the relative desirability of the minimum wage is declining in the elasticity of labor demand and increasing in the elasticity of labor supply. This finding is striking because earlier studies on the desirability of the minimum wage typically do not find an important role for the elasticity of labor demand.[19] Contrary to these earlier studies,

[19]See, for example, Allen (1987); Guesnerie and Roberts (1987); Marceau and Boadway (1994); Boad-

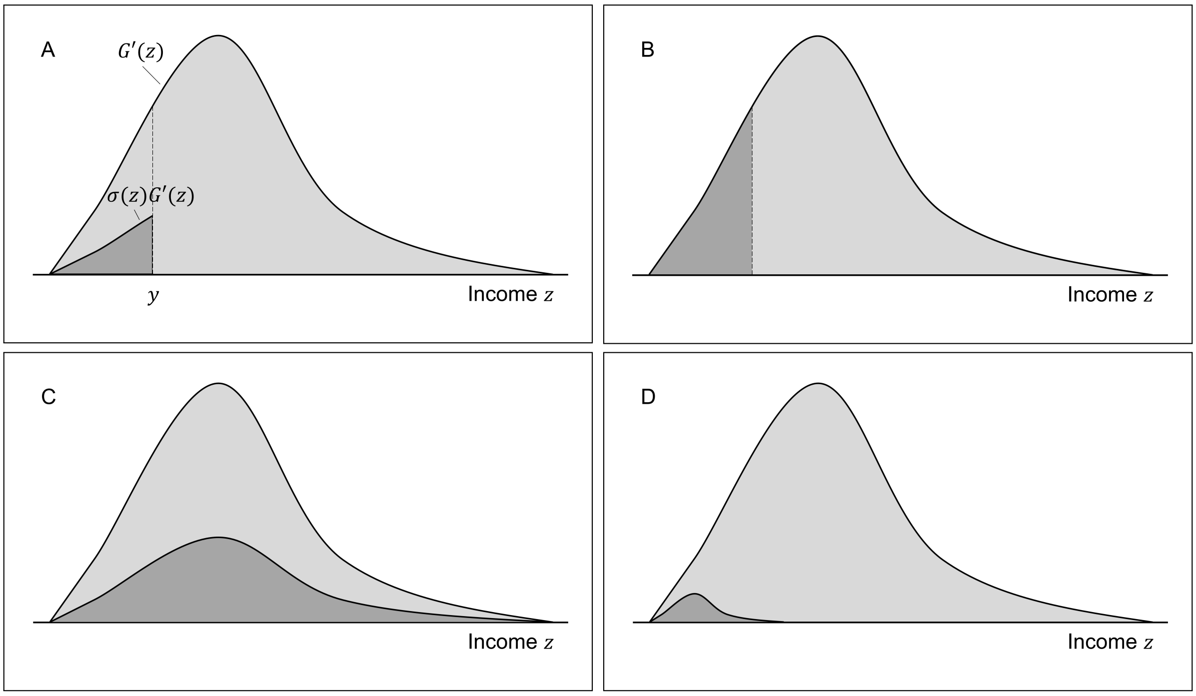

Figure 2: Illustrations of income distributions by wage type

Notes: The Figure gives income densities for various assumptions on the income-specific share of minimum-wage workers $\sigma(z)$. Dark shaded areas represent minimum-wage workers; light-shaded areas represent higher-wage workers. Panels A-C show the case of a fixed $\sigma(z) = \sigma$ for $z \leq y$. Panel A has $\sigma < 1$ and $G(y) < 1$. Panel B has $\sigma = 1$ and $G(y) < 1$. Panel C has $\sigma < 1$ and $G(y) = 1$. Panel D shows the case for some general function $\sigma(z)$.

Proposition 3 implies that empirical studies on the employment effects of the minimum wage are of direct relevance for the desirability of the minimum wage.

Two extreme cases Two highly unrealistic special cases are helpful in explaining the intuition behind Proposition 3. These special cases represent extreme cases of the distribution of wages and income, and are illustrated in Figure 2. Panel A shows a "typical" distribution, with income level y in the interior and minimum-wage workers overlapping in income with higher-wage workers. Panels B and C represent the two extreme cases.

In panel B, $\sigma = 1$, so that minimum-wage workers do *not* overlap with higher-wage workers. As a result, income taxes are equally well targeted as the minimum wage. Compared to the minimum wage, higher marginal taxes among the poor then do *not* yield any additional distortionary costs but *do* yield additional redistributional benefits. As a result, the minimum wage has no predistributional advantage over the income taxation, but the income tax does have a redistributional advantage over the minimum wage. Hence, a minimum wage is unambiguously inferior to taxation. This is confirmed by substituting $\sigma = 1$ into eq. (20). Thus, the case for the minimum wage versus income

way and Cuff (2001); Lee and Saez (2012); Gerritsen and Jacobs (2020).

taxation relies on the existence of income-conditional wage inequality. Only with income-conditional wage inequality is the minimum wage better targeted than the income tax.

In panel C, $G(y) = 1$, so that there is a constant share of minimum-wage workers *at each level of income*. In this case, income taxes no longer yield any redistributional advantages over the minimum wage. After all, minimum-wage workers now share equally in both tax payments and lump-sum transfers. However, compared to minimum wages, income taxes *do* generate additional labor supply distortions. Hence, the minimum wage does have a predistributional advantage over the income tax and is therefore unambiguously superior to taxation. This is confirmed by substituting $G(y) = 1$ into eq. (20). Thus, any case *against* the minimum wage relies on the fact that income taxes generate revenue to redistribute from rich to poor – thereby yielding additional redistributional benefits compared to the minimum wage.

Summing up The two extreme cases of panels B and C – though wildly unrealistic – clearly illustrate the trade-off between minimum wages and income taxes. The minimum wage generates less distortions for a given increase in type-A wages. Thus, the predistributional advantage of the minimum wage over income taxation. At the same time, income taxes yield revenue that could be redistributed to the poor, including the low-wage poor. Thus, the redistributional advantage of taxation over the minimum wage. This trade-off between the predistributional benefits of the minimum wage and the redistributional benefits of taxation ultimately determines the desirability of the minimum wage.

3.7.2 ...for general $\sigma(z)$

The desirability condition in Proposition 3 is attractive in its simplicity. It only depends on a small number of statistics that are more or less easy to verify empirically. But it also relies on the strong assumption that the share of minimum wage workers is constant for low levels of income. In reality, the share of minimum wage workers tends to decline with income – as illustrated by panel D of Figure 2. The following Proposition establishes the condition under which a minimum wage is superior to taxation for any general function $\sigma(z)$. This is also the condition that I calibrate in the next Section's numerical application to the US

Proposition 4 *In an economy with a continuous distribution of income and a clearing labor market, in which $T'(z) = \tau$ for all $z \leq y$, an increase in the minimum wage is* *superior to a comparable tax reform if and only if:*

$$
\text{(21)} \qquad \frac{1-\bar{\sigma}G(y)}{1-G(y)} > \left(\frac{\bar{\sigma}}{1-\bar{\sigma}}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\frac{e_D}{e_S} - \left(1-\bar{\sigma}G(y)-\frac{e_D}{e_S}\right)\left(\frac{1}{1-G(y)}\right)\mathrm{cov}_{z\leq y}\left[\frac{1-\sigma(z)}{1-\bar{\sigma}},\frac{z}{\bar{y}}\right],
$$

where $\bar{\sigma} \equiv \int_0^y \sigma(z)\mathrm{d}G(z)/G(y)$ *is the aggregate share of minimum-wage workers among the working poor, and* $\mathrm{cov}_{z\leq y}\left[\frac{1-\sigma(z)}{1-\bar{\sigma}},\frac{z}{\bar{y}}\right]$ *is a normalized covariance between* $1-\sigma(z)$ *and* z *over the income range* $[0,y]$.[20]

Proof. By definition, the minimum wage is superior to taxation if and only if $\mathcal{BB}_{w_L} > \mathcal{BB}_{\tau}$. Substitute for eqs. (15) and (19). Rearrange to obtain eq. (21). ■

Compared to Proposition 3, Proposition (4) adds one term to the desirability condition of the minimum wage. The first line of eq. (21) is virtually identical to the desirability condition in eq. (20). The only difference is that it includes the average share of minimum-wage workers ($\bar{\sigma}$) instead of a constant share (σ). The interpretation of this first line is entirely equivalent to the earlier interpretation of eq. (20).

The second line in eq. (21) is new and contains the normalized covariance between the share of non-minimum wage poor workers ($1-\sigma(z)$) and income (z). The income-conditional share of minimum-wage workers ($\sigma(z)$) can be expected to decline with income so that this covariance is positive. This is confirmed by Figure 1. Eq. (21) shows that this has an ambiguous effect on the desirability of the minimum wage, depending on the sign of the term $1-\bar{\sigma}G(y)-e_D/e_S$. The reason for this is that a positive covariance between $1-\sigma(z)$ and z raises both the distributional benefits *and* the distortionary costs of marginal taxes on the poor. Distributional benefits are increased because marginal taxes raise less revenue from type-A workers if they are mostly concentrated along very low levels of income. However, this also implies that marginal taxes lead to larger distortions among poor type-B workers. The net effect on the desirability of the minimum wage is positive if the ratio of elasticities (e_D/e_S) is sufficiently small. This once more confirms the importance of empirical findings on the employment effects of the minimum wage. In fact, Proposition 4 reconfirms that a minimum wage is unambiguously desirable if the elasticity of labor demand goes to zero.

[20] To be precise, by the definition of the covariance, it equals the difference between the average of the product and the product of the averages of the terms $(1-\sigma)/(1-\bar{\sigma})$ and $z/\bar{y}$:

$$
\mathrm{cov}_{z\leq y}\left[\frac{1-\sigma(z)}{1-\bar{\sigma}},\frac{z}{\bar{y}}\right] \equiv \frac{1}{G(y)}\int_0^y \left(\frac{1-\sigma}{1-\bar{\sigma}}\right)\left(\frac{z}{\bar{z}}\right)\mathrm{d}G(z) - 1.
$$

3.8 Discussion

I made a number of assumptions to derive the desirability conditions in Propositions 3 and 4. Relaxing these assumptions is likely to make the minimum wage either more or less desirable. To get a better understanding of how various assumptions strengthen or weaken the case for a binding minimum wage, I briefly discuss the most important assumptions below.

Government only cares about type-A workers Above, I assumed that the social welfare function only attaches a positive weight to workers with the lowest wage rate, i.e., to type-A workers. As I discussed in Section 2.3, such a social welfare function could be rationalized on the basis of existing theories of distributive justice (Rawls, 1971; Dworkin, 2000; Roemer, 1996; Fleurbaey, 2008). Nevertheless, other notions of distributive justice may object to an exclusive focus on the lowest-wage workers.

In Appendix C, I show that the case for a minimum wage is weakened if the government also attaches a positive welfare weight to the working poor of type B. In particular, I assume that the government attaches the same positive social welfare weight to *everyone* with income below y and a zero social welfare weight to everyone else. Considering the case that $\sigma(z) = \sigma$ for all $z \leq y$, I obtain the following desirability condition for the minimum wage:

$$\sigma > \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\frac{e_D}{e_S}. \tag{22}$$

Notice that the right-hand side perfectly coincides with the desirability condition in eq. (20). Moreover, the left-hand side is strictly smaller than in eq. (20).[21] Hence, attaching a positive social welfare weight to both types of working poor weakens the case for the minimum wage. Intuitively, it reduces the social value of distinguishing between type-A and type-B workers. At the same time, it does not completely destroy the case for the minimum wage. The minimum wage is still unambiguously desirable if the demand elasticity (e_D) approaches zero. Intuitively, with a zero demand elasticity, the minimum wage constitutes a policy instrument that generates redistribution without distortions – which cannot be replicated by the income tax as long as $\sigma < 1$.[22]

[21] The left-hand side in eq. (20) could be rewritten as $(1-\sigma G(y))/(1-G(y)) = \sigma + (1-\sigma)/(1-G(y))$.

[22] This contrasts with results in Section 2, which considered models with discrete types. In particular, Propositions 1 and 2 require the government to value low-wage workers strictly more than higher-wage workers with the same income. The reason for this is that there are no profits in those models. Instead, the minimum wage redistributes from high-wage workers to low-wage workers with the same income. Such redistribution is socially worthless if the government does not attach a higher welfare weight to low-wage workers. In the continuous-type model, however, the minimum wage redistributes from (taxed) profits to low-wage workers. This redistribution is socially valued even if the government values all low-income workers equally.

Full taxation of profits The economy generates profits because of decreasing returns to type-A labor. I have assumed that these profits are fully taxed away. As a result, higher type-A wages imply redistribution from the government (taxed profits) to type-A workers. The marginal value of such redistribution equals $1 - \int_0^y \sigma(z)\mathrm{d}G(z)$; the first term being the marginal value of resources in the hands of type-A workers, the second term being the marginal value of resources in the hands of the government (the share of type-A workers). These marginal distributional gains resurface in the numerators on the left-hand sides of the desirability conditions in eqs. (20) and (21).

Alternatively, we could assume that profits are not taxed but paid out to shareholders. If shares are uniformly distributed across the population, then type-A workers would earn a proportion $\int_0^y \sigma(z)\mathrm{d}G(z)$ of all profits. Obviously, this would yield the same results as in eqs. (20) and (21). Perhaps more realistically, if shares are owned by people who earn more than the minimum wage, then the marginal social value of untaxed profits are zero. This increases the distributional benefits of the minimum wage. In Appendix C, I derive the following desirability condition for the case in which $\sigma(z) = \sigma$ for all $z \leq y$:

$$\frac{1 + \sigma G(y)\frac{\tau}{1-\tau}}{1 - G(y)} > \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{y - \bar{y}}{\bar{y}}\right)\frac{e_D}{e_S}. \tag{23}$$

This condition is similar to eq. (20), except for a larger numerator on the left-hand side. Thus, the case for the minimum wage is strengthened if we assume that profits are untaxed and earned by the non-poor. Intuitively, redistribution from profits to minimum-wage workers is more valuable if profits are paid out to people that earn more than the minimum wage.[23]

Linear taxes for the working poor I have so far assumed that marginal taxes among the working poor are constant, such that $T''(z) = 0$ for all $z \leq y$. This assumption greatly simplifies the analysis and keeps the welfare effects of the perturbations tractable. The reason for this is that a perturbation of the tax schedule leads to changes in behavior and wages, which both lead to further changes in marginal taxes if $T'' \neq 0$, and therefore to second-round effects on behavior and wages. These second-round effects are absent when considering a perturbation of the minimum wage, because in that case behavioral effects are demand- rather than supply-driven, and thus unaffected by endogenous changes in marginal tax rates. This complicates the comparison of the two perturbations. In the words of a giant, it "produces a completely unwieldy expression" for the desirability condition of the minimum wage (Vickrey, 1945).

In Appendix C, I provide an *approximation* of the desirability condition by ignoring second-round effects. Naturally, this should not be taken as anything more than an

[23] This finding is in line with Vergara (2023), who finds – in a very different setting but for the same reason – that the optimal minimum wage is declining in the corporate income tax rate.

approximation, but it does provide some guidance as to how results are affected by non-constant marginal tax rates for the poor. I show that the distributional benefits of both perturbations are larger (smaller) if marginal tax rates are declining (increasing) over the income range $[0, y]$.[24] As a result, the existence of nonconstant marginal tax rates has an ambiguous effect on the relative desirability of a binding minimum wage. Nevertheless, the minimum wage is still unambiguously desirable if the elasticity of labor demand approaches zero.

No complementarity or substitutability between type-A and type-B workers The production function in eq. (6) implies that the two types of workers are neither complements nor substitutes in production. That is, increased supply of one type of workers does not affect the marginal productivity of the other type of workers. This contrasts with the discrete-type models in which I assumed that type-A and type-B working poor are complements in production. It is useful to briefly reflect on how the analysis of the policy perturbations may change if type-A and type-B working poor *are* complements or substitutes.[25] Recall that an increase in marginal taxes reduces both type-A and type-B labor supply. In the above analysis, the reduction in type-B labor supply had no impact on the wage distribution. However, if both types of workers are complements, a reduction in type-B labor supply would tend to reduce type-A wages. As a result, complementarity between both types increases the predistributional disadvantage of income taxation – thereby strengthening the case for the minimum wage. The same logic would imply that substitutability of both types weakens the case for the minimum wage.

Income taxes cannot be conditioned on wage rates The assumption that income taxes cannot be conditioned on wage rates is crucial for the results. To see this, imagine that the government were able to levy type-A specific marginal income taxes. It could then perfectly replicate the minimum wage by raising marginal taxes for type-A workers only. This would reduce type-A employment and raise their wages through general-equilibrium effects – exactly like the minimum wage would. In other words, if taxes could be targeted at type-A workers, then the minimum wage ceases to be better targeted than taxation. The minimum wage is then redundant, as in Allen (1987) and Guesnerie and

[24] Intuitively, the minimum wage raises income more for type-A workers that work many hours and thus earn a relatively high income. Thus, distributional gains are larger if type-A workers with a relatively high income face a lower marginal tax rate. Similarly, to ensure equiproportional changes in labor supply among type-A workers, the tax perturbation raises marginal tax rates proportional to the net-of-tax rate $1 - T'(z)$. Thus, if marginal taxes are declining with income, the marginal tax hikes are largest for the highest incomes in the range $[0, y]$. This implies that a larger part of the tax increase is paid by workers with income $z > y$ rather than workers with income $z \leq y$. Thus, the distributional gains of the tax perturbation are also larger if marginal taxes decline with income.

[25] Also see Sachs, Tsyvinski, and Werquin (2020) on optimal nonlinear taxation with endogenous wages and cross-wage effects of labor supply.

Roberts (1987). Nevertheless, the idea that wage-conditional taxes cannot be enforced is a cornerstone of the literature on optimal taxation since Mirrlees (1971). Moreover, the observation that actual income taxes are generally not conditioned on hourly wages lends credence to the assumptions made in this paper.

Efficient rationing I assumed that rationing is efficient and takes place on the intensive margin. As a result, a small amount of rationing does not cause any first-order utility losses. If rationing *were* to generate first-order utility losses, this would reduce the utility gains associated with an increase in the minimum wage. This would naturally weaken the case for the minimum wage. Nevertheless, any utility losses associated with inefficient rationing are going to be proportional to the magnitude of demand responses to the minimum wage. Thus, the utility losses associated with inefficient rationing go to zero if the elasticity of demand approaches zero. This once more emphasizes the normative importance of empirical studies on the employment effects of the minimum wage.

Desirability versus the optimal level of the minimum wage Finally, all four Propositions in this paper evaluate the comparison between the minimum wage and income taxation *at an initial equilibrium without labor rationing*. This implies that the desirability conditions are only applicable to the question of whether there should be a binding minimum wage in the first place. They are not immediately informative about the desirability of raising the minimum wage beyond a marginally binding level. By focusing on an initial equilibrium without rationing, I could ignore the utility losses associated with an increase in rationing. Further increases in the minimum wage *will* yield first-order utility losses. The optimal level of the minimum wage then trades off its predistributional advantages against the redistributional advantages of the income tax *and* the utility losses associated with rationing.

4 A numerical application to the United States

4.1 Data and assumptions

In this Section, I numerically calibrate the desirability condition for the United States. The US federal minimum wage is by far the lowest minimum wage in the developed world if measured relative to mean or median wages.[26] Thus, of all national minimum wages, the US federal minimum wage is most likely to be non-binding. This allows me

[26] The federal minimum wage equals 20% of the average hourly wage rate and 29% of the median hourly wage rate. Latvia is runner-up with 34% and 42%. OECD averages correspond to 43% and 55%. See Organisation for Economic Co-operation and Development (2022) or OECD statistics at https://stats.oecd.org/Index.aspx?DataSetCode=MIN2AVE.

to calibrate the desirability condition – which is derived at an equilibrium without labor rationing – on the basis of US data.

I calibrate two desirability conditions. First, I calibrate eq. (21) from Proposition 4. This desirability condition is based on a social welfare function that only counts the utility of lowest-wage workers. The condition only depends on characteristics of the income and wage distribution, and on the ratio between demand and supply elasticities (e_D/e_S). Thus, with data on the US income and wage distribution, I can derive the values for the elasticity ratio under which an increase in the federal minimum wage is desirable. Second, I calibrate the desirability condition that is based on a social welfare function that counts the utility of *all* low-income workers equally, regardless of their wage rate. This condition is the analogue of (22) but with a potentially non-constant share $\sigma(z)$. The full condition is given in Appendix D, along with more details on the empirical calibration. Like eq. (21), it only depends on characteristics of the income and wage distribution, and the elasticity ratio (e_D/e_S). This second calibration is meant to illustrate the importance of the government's distributive preferences over low-wage versus other low-income workers.

I calibrate the characteristics of the income and wage distribution by using microdata from the Current Population Survey of 2019.[27] This is the same data that produced Figure 1. For hourly workers, it provides data on hourly wage rates and usual weekly hours of work, which is used to obtain usual weekly earnings. For other workers, it provides data on usual weekly hours of work and weekly earnings, which is used to obtain an hourly wage rate. I thus obtain individual data on both hourly wages and weekly earnings.

To determine the relevant characteristics of the wage and income distribution, I need to make two more decision. First of all, I need to decide which individuals count as type-A workers, i.e., who are the "lowest-wage workers." In the strictest definition of lowest-wage workers, I only consider individuals who earn at most the federal minimum wage of \$7.25 per hour. But I consider workers with wages up to \$8.25 per hour as an alternative definition of lowest-wage workers.

Second, I need to decide whether and how to deal with outliers in usual weekly working hours, which has a maximum value of 99. Keeping these outliers in the analysis strengthens the case for the minimum wage because it makes income less informative about hourly wages, thus making the income tax relatively less well targeted. Nevertheless, while 99 seems an implausibly high number of usual hours worked, it is unclear where to draw the line. I therefore consider four different cases in which I truncate working hours above the 90th percentile (50 hours), the 95th percentile (55 hours), the 99th percentile (70 hours), and not at all.

[27] In particular, I use the NBER Outgoing Rotation Group of the Current Population Survey for 2019, available at https://www.nber.org/morg/.

Table 1: Calibration results

Panel A: SWF 1		Hours truncated above...			
		50 (p.90)	55 (p.95)	70 (p.99)	no trunc.
Max wage rate type A	\$7.25	6.1	7.1	11.8	18.1
	\$8.25	5.2	5.4	8.3	13.5
Panel B: SWF 2		**Hours truncated above...**			
		50 (p.90)	55 (p.95)	70 (p.99)	no trunc.
Max wage rate type A	\$7.25	1.4	1.4	1.7	1.6
	\$8.25	1.5	1.4	1.6	1.5

Notes: An increase in the minimum wage is more desirable than a comparable tax reform if the ratio of demand and supply elasticities (e_D/e_S) is smaller than the value reported in the table. Panel A (SWF 1) provides results for a social welfare function that only attaches positive weight to the utility of lowest-wage workers. Panel B (SWF 2) provides results for a social welfare function that only attaches positive weight to the utility of low-income workers, regardless of their hourly wage rates. Columns differ in the degree to which the working hours variable has been truncated. Rows differ in the definition of lowest-wage workers. Also see Appendix D for more details on the numerical calibration.

4.2 Results

Table 1 provides the results of the calibration exercise. An increase in the minimum wage is more desirable than a comparable tax reform if the ratio of demand and supply elasticities (e_D/e_S) is smaller than the number that is reported in the table. Numbers are reported for different definitions of lowest-wage workers (in rows), and different truncations of working hours (in columns). The upper panel A ("SWF 1") considers a social welfare function that only values lowest-wage workers; the lower panel B ("SWF 2") considers a social welfare function that values all low-income workers.

The most conservative results in panel A indicate that the desirability condition for the minimum wage is satisfied if the ratio of demand and supply elasticities is at most 5. The desirability condition becomes significantly less strict if I truncate working hours *less* or if I adopt a stricter definition of "lowest-wage workers." Intuitively, in both cases, income is less informative about whether someone earns the lowest wage rate. Hence, a minimum wage is better targeted as compared to the income tax. Typical estimates for the elasticity of labor supply are centered around 0.3 (e.g., Chetty, 2012; Saez, Slemrod, and Giertz, 2012), while the vast majority of estimates for the elasticity of labor demand lie between 0 and 1 (Lichter, Peichl, and Siegloch, 2015). This implies that the desirability condition for the minimum wage is easily satisfied if the government exclusively cares about the lowest-wage workers. Thus, in the absence of pre-existing rationing, an increase in the minimum wage is socially preferable over a comparable reform of the income tax.

Panel B indicates that it is harder to satisfy the desirability condition for the minimum wage if the government equally cares about all working poor. In the most conservative calibrations, the ratio of demand and supply elasticities may be at most 1.4 for the minimum wage to be more desirable than a comparable tax reform.[28] Demand elasticities may well exceed the supply elasticity by a factor larger than 1.4, which would make the minimum wage undesirable. Nevertheless, recent estimates on the employment effects of the minimum wage tend to be concentrated around the lower end of the $[0, 1]$ range. This implies that there may be a plausible case for the minimum wage even if the government cares equally much about all working poor.

Summing up, we may draw three conclusions from the numerical calibrations. First, there is a strong case to be made for an increase in the US federal minimum wage. Second, this case is strengthened by distributive preferences that favor poor workers with low wage rates over poor workers with higher wage rates. Third, the case for the minimum wage is further strengthened by recent empirical estimates of small demand elasticities.

5 Conclusion

Much of the previous literature on the desirability of the minimum wage assumes a one-to-one correspondence between wages and income. This implies that the income tax can target the exact same people that are targeted by the minimum wage. As a result, the literature has found it exceedingly difficult to come up with a robust justification for a binding minimum wage. In this paper, I show that the case for the minimum wage is considerably strengthened if (i) we consider an economy in which any given level of income corresponds to multiple hourly wage rates, and (ii) the government values redistribution from workers with a high wage rate to workers with a low wage rate – even if they earn the same income.

Under these conditions, a binding minimum wage is unambiguously desirable in discrete-type models of the labor market. In continuous-type models of the labor market, the minimum wage yields “predistributional advantages” over income taxation because its impact on the wage structure comes with lower distortionary costs. At the same time, income taxation yields “redistributional advantages” over the minimum wage by raising revenue that can be redistributed. The case for the minimum wage hinges on the trade-off between the predistributional advantages of the minimum wage and the redistributional advantages of income taxation. I capture this trade-off with a desirability condition in

[28]This critical value is not very sensitive to the definition of lowest-wage workers or hours truncation. Intuitively, targeting becomes less of an issue if the government cares equally much about the lowest-wage poor and the higher-wage poor.

terms of empirical sufficient statistics. Calibration on the basis of US data suggests that a strong case can be made for an increase in the federal minimum wage.

As a closing thought, I like to emphasize that the minimum wage is only one of many policies of predistribution. Government policy on education, labor market institutions and wage bargaining, international trade, industrial policies, and technological change all affect the pre-tax distribution of wages and income (Piketty et al., 2020; Blanchet, Chancel, and Gethin, 2022). A more comprehensive comparison of redistribution versus predistribution would account for a broader set of predistribution policies. Even if a binding minimum wage were to be undesirable, other forms of predistribution policies may be usefully employed alongside income taxation. It is also important to understand whether different predistribution policies act as complements or substitutes. For example, extensive provision of public education may either strengthen or weaken the case for a binding minimum wage. I see further analysis of the relative merits and demerits of different pre- and redistriutional policies as an important avenue for future research.

References

Ahlfeldt, Gabriel, Duncan Roth, and Tobias Seidel. 2022. "Optimal minimum wages in spatial economies." CEPR Discussion Paper No. 16913.

Allen, Stephen P. 1987. "Taxes, redistribution, and the minimum wage: A theoretical analysis." *Quarterly Journal of Economics* 102 (3):477–489.

Berger, David W., Kyle F. Herkenhoff, and Simon Mongey. 2022. "Minimum wages, efficiency and welfare." NBER Working Paper 29662.

Bergstrom, Katy and William Dodds. 2021. "Optimal taxation with multiple dimensions of heterogeneity." *Journal of Public Economics* 200:104442.

Blanchet, Thomas, Lucas Chancel, and Amory Gethin. 2022. "Why is Europe more equal than the United States?" *American Economic Journal: Applied Economics* 14 (4):480–518.

Boadway, Robin and Katherine Cuff. 2001. "A minimum wage can be welfare-improving and employment-enhancing." *European Economic Review* 45 (3):553–576.

Cahuc, Pierre and Guy Laroque. 2013. "Optimal taxation and monopsonistic labor market: Does monopsony justify the minimum wage?" *Journal of Public Economic Theory* 16 (2):259–273.

Cengiz, Doruk, Arindrajit Dube, Attila Lindner, and Ben Zipperer. 2019. "The effect of minimum wages on low-wage jobs." *Quarterly Journal of Economics* 134 (3):1405–1454.

Chetty, Raj. 2012. "Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply." *Econometrica* 80 (3):969–1018.

Choné, Philippe and Guy Laroque. 2010. "Negative marginal tax rates and heterogeneity." *American Economic Review* 100 (5):2532–47.

Christiansen, Vidar. 1981. "Evaluation of public projects under optimal taxation." *Review of Economic Studies* 48 (3):447–457.

———. 1984. "Which commodity taxes should supplement the income tax?" *Journal of Public Economics* 24 (2):195–220.

Clemens, Jeffrey and Michael R. Strain. 2021. "The heterogeneous effects of large and small minimum wage changes: Evidence over the short and medium run using a pre-analysis plan." NBER Working Paper No. 29264.

———. 2022. "Understanding "wage theft": Evasion and avoidance responses to minimum wage increases." *Labour Economics* 79 (102285):1–27.

Diamond, Peter and Johannes Spinnewijn. 2011. "Capital income taxes with heterogeneous discount rates." *American Economic Journal: Economic Policy* 3 (4):52–76.

Dustmann, Christian, Attila Lindner, Uta Schönberg, Matthias Umkehrer, and Philipp Vom Berge. 2022. "Reallocation effects of the minimum wage." *Quarterly Journal of Economics* 137 (1):267–328.

Dworkin, Ronald. 2000. *Sovereign Virtue*. Cambridge, MA: Harvard University Press.

Ferey, Antoine, Benjamin B. Lockwood, and Dmitry Taubinsky. 2022. "Sufficient statistics for nonlinear tax systems with general across-income heterogeneity." Mimeo.

Fleurbaey, Marc. 2008. *Fairness, Responsibility, and Welfare*. Oxford: Oxford University Press.

Gerritsen, Aart. 2018. "Equity and efficiency in rationed labor markets." *Journal of Public Economics* 153:56–68.

Gerritsen, Aart and Bas Jacobs. 2020. "Is a minimum wage an appropriate instrument for redistribution?" *Economica* 87:611–637.

Gerritsen, Aart, Bas Jacobs, Kevin Spiritus, and Alexandra Rusu. 2022. "Optimal taxation of capital income with heterogeneous rates of return." Mimeo.

Guesnerie, Roger and Kevin Roberts. 1987. "Minimum wage legislation as a second best policy." *European Economic Review* 31 (1-2):490–498.

Harasztosi, Péter and Attila Lindner. 2019. "Who pays for the minimum Wage?" *American Economic Review* 109 (8):2693–2727.

Hummel, Albert Jan and Bas Jacobs. 2023. "Optimal income taxation in unionized labor markets." *Journal of Public Economics* forthcoming.

Hungerbühler, Mathias and Etienne Lehmann. 2009. "On the optimality of a minimum wage: New insights from optimal tax theory." *Journal of Public Economics* 93 (3):464–481.

Hurst, Erik, Patrick Kehoe, Elena Pastorino, and Thomas Winberry. 2023. "Optimal income taxation in unionized labor markets." NBER Working Paper 30294.

Jacquet, Laurence and Etienne Lehmann. 2021a. "Optimal taxation with composition effects." *Journal of the European Economic Association* 19 (1):1299–1341.

———. 2021b. "Optimal taxation with multidimensional heterogeneity: A mechanism design method." *Social Choice and Welfare* forthcoming.

Kaplow, Louis. 2008. *The Theory of Taxation and Public Economics*. Princeton University Press.

Kleven, Henrik Jacobsen, Claus Thustrup Kreiner, and Emmanuel Saez. 2009. "The optimal income taxation of couples." *Econometrica* 77 (2):537–560.

Lavecchia, Adam M. 2020. "Minimum wage policy with optimal taxes and unemployment." *Journal of Public Economics* 190:1–20.

Lee, David and Emmanuel Saez. 2012. "Optimal minimum wage policy in competitive labor markets." *Journal of Public Economics* 96 (9–10):739–749.

Lichter, Andreas, Andreas Peichl, and Sebastian Siegloch. 2015. "The own-wage elasticity of labor demand: A meta-regression analysis." *European Economic Review* 80:94–119.

Luttmer, Erzo F. P. 2007. "Does the Minimum Wage Cause Inefficient Rationing?" *BE Journal of Economic Analysis & Policy* 7 (1).

Marceau, Nicolas and Robin Boadway. 1994. "Minimum wage legislation and unemployment insurance as instruments for redistribution." *Scandinavian Journal of Economics* 96 (1):67–81.

Mirrlees, James A. 1971. "An exploration in the theory of optimum income taxation." *Review of Economic Studies* 38 (2):175–208.

Organisation for Economic Co-operation and Development. 2022. "Minimum wages in times of rising inflation." OECD Policy Brief, December 2022.

Piketty, Thomas, Antoine Bozio, Bertrand Garbinti, Jonathan Goupille-Lebret, and Malka Guillot. 2020. "Predistribution vs. redistribution: Evidence from France and the U.S." CEPR Discussion Paper No. 15415.

Rawls, John. 1971. *A Theory of Justice*. Cambridge, MA: Harvard University Press.

Roemer, John E. 1996. *Theories of Distributive Justice*. Cambridge, MA: Harvard University Press.

Rothschild, Casey and Florian Scheuer. 2013. "Redistributive taxation in the Roy model." *Quarterly Journal of Economics* 128 (2):623–668.

Sachs, Dominik, Aleh Tsyvinski, and Nicolas Werquin. 2020. "Nonlinear tax incidence and optimal taxation in general equilibrium." *Econometrica* 88 (2):469–493.

Saez, Emmanuel. 2001. "Using elasticities to derive optimal income tax rates." *Review of Economic Studies* 68 (1):205–229.

———. 2002. "The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes." *Journal of Public Economics* 83 (2):217–230.

Saez, Emmanuel, Joel Slemrod, and Seth H. Giertz. 2012. "The elasticity of taxable income with respect to marginal tax rates: A critical review." *Journal of Economic Literature* 50 (1):3–50.

Slemrod, Joel. 2019. "Tax compliance and enforcement." *Journal of Economic Literature* 57 (4):904–954.

Spiritus, Kevin, Etienne Lehmann, Sander Renes, and Floris Zoutman. 2022. "Optimal taxation with multiple incomes and types." Tinbergen Institute Discussion Paper TI 2022-006/VI.

Stern, Nicholas. 1982. "Optimum taxation with errors in administration." *Journal of Public Economics* 17 (2):181–211.

Stiglitz, Joseph E. 1982. "Self-selection and pareto efficient taxation." *Journal of Public Economics* 17 (2):213–240.

Vergara, Damián. 2023. "Minimum wages and optimal redistribution." Mimeo.

Vickrey, William. 1945. "Measuring marginal utility by reactions to risk." *Econometrica* 13 (4):319–333.

A Proofs of Propositions 1 and 2

A.1 Proof of Proposition 1

I prove Proposition 1 in four steps. First, I present the key equations of the model and their total derivatives. Second, I use these total derivatives to show that an increase in the minimum wage and an increase in the marginal tax rate both lead to a reduction in individual labor earnings. Third, I take the total derivative of the government's goal function to show that, in a clearing labor market, a given reduction in individual labor earnings yield greater social-welfare gains if achieved through a higher minimum wage rather than a higher marginal tax rate. This implies that the implementation of a binding minimum wage is part of a social-welfare enhancing policy reform for any given tax schedule – including the optimum tax schedule. As a result, a binding minimum wage must be part of the policy optimum.

A.1.1 Comparative statics of key model equations

Write the tax function as $T(w^i h^i) = \tau w^i h^i + \mathcal{T}$, with τ the marginal tax rate and $\mathcal{T}$ a lump-sum component of the tax schedule. This parameterization of the tax schedule is without loss of generalization because there is only one income level. As a result, the relevant features of the tax schedule can be summarized by just two parameters: the marginal tax rate τ, and the intercept of the tax schedule $\mathcal{T}$. The key equations of the model can then be summarized as follows.

$$F(n^A h^A, n^B h^B) = w^A n^A h^A + w^B n^B h^B, \tag{24}$$

$$w^A h^A = w^B h^B, \tag{25}$$

$$MRS^i(c^i, h^i) \equiv \frac{-u_h^i((1-\tau)w^i h^i - \mathcal{T}, h^i)}{u_c^i((1-\tau)w^i h^i - \mathcal{T}, h^i)} = (1-\tau)w^i, \quad i = \{A, B\}. \tag{26}$$

The first equation implies that firms run zero profits, which follows from the competitive nature of the labor market and first-degree homogeneity of the production function (constant returns to scale). The second equation implies that labor income of both types are equal, which follows from the Cobb-Douglas nature of production and the particular calibration of the income shares α. The third equation is the first-order condition for labor supply, in which I have substituted the budget constraint. The latter equation allows me to write equilibrium supply of working hours as $h^i = h^i(w^i, \tau, \mathcal{T})$.

The total derivative of the first equation yields:

$$\frac{\mathrm{d}w^B}{w^B} = -\frac{n^A}{n^B}\frac{\mathrm{d}w^A}{w^A}, \tag{27}$$

where I substituted for the firm's first-order conditions ($F_{L^i} = w^i$) and eq. (25). The percentage change in type-B wages is negatively proportional to the percentage change in type-A wages. Intuitively, any increase in type-A wages are paid for by reductions in type-B wages and vice versa.

The total derivative of the second equation yields:

$$\left(1+\frac{n^A}{n^B}\right)\frac{\mathrm{d}w^A}{w^A}=\frac{\mathrm{d}h^B}{h^B}-\frac{\mathrm{d}h^A}{h^A}, \tag{28}$$

where I substituted for the comparative statics in eq. (27). This equation has two different interpretations. In the case of clearing labor markets, it shows how type-A wages (LHS) adjust to accomodate relative changes in both types' labor supply (RHS). In the case of a binding minimum wage, it shows how type-A labor demand adjusts to accomodate relative changes in the minimum wage and type-B labor supply.

The total derivative of the third equation yields:

$$\frac{\mathrm{d}h^i}{h^i}=-e_c^i\frac{\mathrm{d}\tau}{1-\tau}+e_u^i\frac{\mathrm{d}w^i}{w^i}+\eta^i\left(\mathrm{d}\mathcal{T}+w^ih^i\mathrm{d}\tau\right), \tag{29}$$

with e_c^i and e_u^i the compensated and uncompensated wage elasticities of labor supply, and η^i the income semi-elasticity of labor supply. The (semi-)elasticities are defined as:

$$\begin{aligned}
(30)\quad e_c^i &\equiv \left(\frac{\partial h^i(w^i,\tau,\mathcal{T})}{\partial w^i}+(1-\tau)h^i\frac{\partial h^i(w^i,\tau,\mathcal{T})}{\partial \mathcal{T}}\right)\frac{w^i}{h^i}\\
&= \left((1-\tau)w^ih^i\frac{MRS_c^i}{MRS^i}+\frac{MRS_h^ih^i}{MRS^i}\right)^{-1}>0,\\
(31)\quad \eta^i &\equiv \frac{\partial h^i(w^i,\tau,\mathcal{T})}{\partial \mathcal{T}}\frac{1}{h^i}=\frac{MRS_c^i}{MRS^i}e_c^i\geq 0.\\
(32)\quad e_u^i &\equiv \frac{\partial h^i(w^i,\tau,T)}{\partial w^i}\frac{w^i}{h^i}=e_c^i-(1-\tau)w^ih^i\eta^i.
\end{aligned}$$

A.1.2 Comparative statics of individual labor earnings

Recall that individual labor earnings are defined as $z^i\equiv w^ih^i=z$ for both i. Taking the total derivative and substituting for eqs. (27) and (29) for $i=B$ yields:

$$\frac{\mathrm{d}z}{z}=\frac{\mathrm{d}h^B}{h^B}+\frac{\mathrm{d}w^B}{w^B}=-\left(1+e_u^B\right)\frac{n^A}{n^B}\frac{\mathrm{d}w^A}{w^A}-e_c^B\frac{\mathrm{d}\tau}{1-\tau}+\eta^B\left(\mathrm{d}\mathcal{T}+z\mathrm{d}\tau\right). \tag{33}$$

When considering an increase in the minimum wage, all right-hand side differentials can be considered exogenous. If leisure and consumption are normal goods, the uncompensated wage elasticities of labor supply must exceed minus one, $e_u^B>-1$. The above equation therefore immediately proves that an increase in the minimum wage $\mathrm{d}w^A>0$ causes a reduction in individual labor earnings $\mathrm{d}z<0$.

When considering a change in taxes, the wage differential $\mathrm{d}w^A$ is endogenous to the tax change. Further substituting for eqs. (28) and (29) yields:

$$\text{(34)}\quad \frac{\mathrm{d}z}{z} = -\left(\frac{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}\frac{e_c^A}{e_c^B}}{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}}\right)e_c^B\frac{\mathrm{d}\tau}{1-\tau} + \left(\frac{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}\frac{\eta^A}{\eta^B}}{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}}\right)\eta^B\left(\mathrm{d}\mathcal{T}+w^Ah^A\mathrm{d}\tau\right).$$

This proves that any *compensated* increase in the marginal tax rate, such that $\mathrm{d}\tau > 0$ and $\mathrm{d}\mathcal{T} + w^Ah^A\mathrm{d}\tau = 0$, causes a reduction in individual labor earnings $\mathrm{d}z < 0$. Thus both minimum wages and marginal taxes can be used to reduce taxable income. The welfare-relevant question is: which instrument yields a larger welfare gain for a given reduction in income?

A.1.3 Marginal social-welfare effects of a policy reform

Recall that the government objective is given by the following Lagrangian:

$$\text{(35)}\qquad \mathcal{L} = n^Au^A((1-\tau)w^Ah^A - \mathcal{T}, h^A) + \lambda((n^A+n^B)(\tau z + \mathcal{T}) - R).$$

The total derivative of the Lagrangian yields:

$$\text{(36)}\quad \mathrm{d}\mathcal{L} = (1-\tau)zn^Au_c^A\left(\frac{\mathrm{d}z}{z} - \frac{\mathrm{d}h^A}{h^A}\right) + \tau z(n^A+n^B)\lambda\frac{\mathrm{d}z}{z} + \left((n^A+n^B)\lambda - n^Au_c^A\right)(z\mathrm{d}\tau + \mathrm{d}\mathcal{T}),$$

where I used the envelope theorem for the derivative of type-A utility and substituted for $\mathrm{d}w^A/w^A = \mathrm{d}z/z - \mathrm{d}h^A/h^A$. There are three terms in this derivative. The first term indicates that an increase in type-A wages raises type-A utility and thus social welfare. The second term indicates that an increase in taxable income yields budgetary gains if the marginal tax rate is positive. And the third term indicates that an increase in individual tax burdens yields both budgetary gains and utility losses.

Further substitute for $\mathrm{d}h^A/h^A$ by using eqs. (27), (28), and (29) for $i = B$ yields:

$$\text{(37)}\quad \mathrm{d}\mathcal{L} = \frac{(1-\tau)zn^Bu_c^A}{1+e_u^B}\left(\frac{-\mathrm{d}z}{z} - e_c^B\frac{\mathrm{d}\tau}{1-\tau}\right) - \tau z(n^A+n^B)\lambda\frac{-\mathrm{d}z}{z} + \left((n^A+n^B)\lambda - n^Au_c^A + \frac{(1-\tau)zn^Bu_c^A}{1+e_u^B}\eta^B\right)(z\mathrm{d}\tau + \mathrm{d}\mathcal{T}).$$

Now consider two different reforms. One reform raises the minimum wage ($\mathrm{d}w^A > 0$) and keeps taxes fixed. The other reform raises the marginal tax rate and keeps the tax burden fixed ($\mathrm{d}\tau > 0$ and $\mathrm{d}\mathcal{T} = -z\mathrm{d}\tau$). Substituting this into the derivative of the Lagrangian yields the following two equations for the two different reforms:

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} = \frac{(1-\tau)n^B u_c^A}{1+e_u^B} - \tau(n^A+n^B)\lambda, \tag{38}$$

$$\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathcal{T}}\right) = \frac{(1-\tau)n^B u_c^A}{1+e_u^B} - \tau(n^A+n^B)\lambda - \frac{zn^B u_c^A e_c^B}{1+e_u^B}\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}. \tag{39}$$

The first equation gives us the social-welfare effects of raising the minimum wage such that taxable income declines by one unit (hence the division by $-\mathrm{d}z/\mathrm{d}w^A > 0$ on the left-hand side). The second equation gives us the social welfare effects of raising the marginal tax rate such that taxable income declines by the same one unit (hence the division by $-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T}) > 0$ on the left-hand side).

The first equation tells us that the minimum wage raises type-A utility (first term) at the cost of a decline in the tax base and thus tax revenue (second term). The first two terms in the second equation are identical. They tell us that an increase in the marginal tax rate raises type-A wages and utility by reducing type-A labor supply (first term) and reduces the tax base and thus tax revenue (second term). However, there is a strictly negative third term. This term represents the fact that a higher tax rate also reduces type-B labor supply, which has the opposite effect on type-A wages and thus *reduces* type-A utility.

Taken together, the two equations prove Proposition 1. They show us that an increase in the minimum wage and an increase in marginal taxes can yield identical reductions in taxable income. But the minimum wage is more effective in raising type-A wages and thus in raising type-A utility. The reason is that the marginal tax rate not only discourages type-A labor supply (raising type-A wages) but also type-B labor supply (reducing type-A wages). Finally, notice that the second equation must equal zero in the tax optimum. As a result, in the tax optimum we have:

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} = \frac{zn^B u_c^A e_c^B}{1+e_u^B}\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})} > 0. \tag{40}$$

Hence, a binding minimum wage can improve upon the tax optimum and is therefore necessarily part of the overall policy optimum.

A.2 Proof of Proposition 2

The tax schedule can now be parameterized such that $T(z^i) = \tau z^i + \mathcal{T}$ for $i = \{A, B\}$ and $T(z^C) = \tau^C z^C + \mathcal{T}^C$, where h^C solves for the first-order condition of type-C labor supply:

$$MRS^C(c^C, h^C) \equiv \frac{-u_h^C((1-\tau^C)z^C - \mathcal{T}^C, z^C/w^C)}{u_c^C((1-\tau^C)z^C - \mathcal{T}^C, z^C/w^C)} = (1-\tau^C)w^C. \tag{41}$$

Again, this parameterization is without loss of generality because, with two levels of income, only four parameters are needed to fully describe the relevant features of the tax schedule: two marginal tax rates and two "virtual" intercepts of the tax schedule.

Recall that the government objective is given by the following Lagrangian:

$$\begin{aligned} \tilde{\mathcal{L}} = {} & \mathcal{L} + \lambda n^C(\tau^C z^C + \mathcal{T}^C) \\ & +\gamma\left(u^C\left((1-\tau)z - \mathcal{T}, z/w^C\right) - u^C\left((1-\tau^C)z^C - \mathcal{T}^C, z^C/w^C\right)\right). \end{aligned} \tag{42}$$

The total derivative for given levels of τ^C and $\mathcal{T}^C$ is given by:

$$\mathrm{d}\tilde{\mathcal{L}} = \mathrm{d}\mathcal{L} + \gamma u_c^C\left((1-\tau)z - \mathcal{T}, z/w^C\right)\left((1-\tau)z\frac{\mathrm{d}z}{z} - (z\mathrm{d}\tau + \mathrm{d}\mathcal{T})\right). \tag{43}$$

Consider again an increase in the minimum wage ($\mathrm{d}w^A > 0$) versus a compensated increase in the marginal tax rate at the lower level of income ($\mathrm{d}\tau > 0$ and $\mathrm{d}\mathcal{T} = -z\mathrm{d}\tau$). Substitute this into the derivative of the Lagrangian and rearrange to get:

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}w^A} = \frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} - (1-\tau)\gamma u_c^C\left((1-\tau)z - \mathcal{T}, z/w^C\right), \tag{44}$$

$$\begin{aligned} \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\mathcal{T}}\right) = {} & \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathcal{T}}\right) \\ & -(1-\tau)\gamma u_c^C\left((1-\tau)z - \mathcal{T}, z/w^C\right). \end{aligned} \tag{45}$$

From the Proof of Proposition 1, we know that

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} > \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathcal{T}}\right), \tag{46}$$

such that

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}w^A} > \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\mathcal{T}}\right). \tag{47}$$

The left-hand side of the inequality gives the marginal effect on the Lagrangian of a unit reduction in taxable income caused by an increase in the minimum wage. The right-hand side gives the marginal effect on the Lagrangian of a unit reduction in taxable income caused by an increase in marginal taxes. The inequality implies that the increase in the minimum wage is more desirable than the increase in the tax rate. Hence, by the same reasoning as in Proposition 1, a minimum wage must be part of the policy optimum. This proves Proposition 2.

B Deriving bang-for-the-buck measures from the social-welfare function

Individual consumption is given by:

$$c^i = w^i h^i - T(w^i h^i) = w^i h^i - T(0) - \int_0^{z^i} T'(z)dz, \tag{48}$$

where the latter equation decomposes one's tax burden into the intercept of the tax schedule and its marginal tax rates. This allows us to write social welfare as:

$$\mathcal{W} = \int_{\mathcal{A}} u^i \left((1-\tau)w^A h^i - T(0), h^i\right) \mathrm{d}i, \tag{49}$$

where I imposed a constant linear tax rate τ for income levels between 0 and y. The government's budget constraint is given by:

$$\begin{aligned} &\int_0^\infty T(z)\mathrm{d}G(z) + F(L^A) - w^A L^A - R \\ &= T(0) + \int_0^\infty \int_0^{z^j} T'(z)\mathrm{d}z\mathrm{d}G(z^j) + F(L^A) - w^A L^A - R \\ &= T(0) + \tau \int_0^y z^j \mathrm{d}G(z^j) + (1-G(y))\tau y + \int_y^\infty \int_y^{z^j} T'(z)\mathrm{d}z\mathrm{d}G(z^j) + F(L^A) - w^A L^A - R = 0. \end{aligned} \tag{50}$$

The second equation decomposes the income tax into an intercept and marginal tax rates; the third equation imposes a constant linear tax rate τ for income levels between 0 and y. This allows us to write social welfare as:

$$\begin{aligned} \mathcal{W} = \int_{\mathcal{A}} u^i \Big(&(1-\tau)w^A h^i + \tau \int_0^y z^j \mathrm{d}G(z^j) \\ &+ (1-G(y))\tau y + \int_y^\infty \int_y^{z^j} T'(z)\mathrm{d}z\mathrm{d}G(z^j) + F(L^A) - w^A L^A - R, h^i \Big) \mathrm{d}i. \end{aligned} \tag{51}$$

Equilibrium labor hours h^i are determined by either the supply or the demand curve in case of market clearing wages. They are determined by the demand curve in case of a binding minimum wage. Individuals with income below y face a linear tax rate τ. For them, I define the net wage rate as $\omega^i \equiv (1-\tau)w^i$, and write the labor supply function as $h^i_S(\omega^i)$, as implied by the first-order condition of the individual in eq. (7). I write the labor demand function as $h^i_D(w^i)$, as implied by the firm's first-order condition, $w^A = F'(L^A) = F'(\int_{\mathcal{A}} h^i \mathrm{d}i)$, along with the assumption of uniform rationing along the intensive margin. The wage rate w^A follows either exogenously in case of a binding minimum wage, or endogenously if the minimum wage is not binding. In the latter case, I write the equilibrium wage function as $w^A(\tau)$, as implied by the market clearing condition: $h^i_S((1-\tau)w^A) = h^i_D(w^A)$ for all $i \in \mathcal{A}$. For future reference, notice that the total derivative of the market clearing condition yields:

$$\frac{\partial h_S^i}{\partial \omega^i}\omega^i\left(\frac{\mathrm{d}w^A}{w^A}-\frac{\mathrm{d}\tau}{1-\tau}\right)=\frac{\partial h_D^i}{\partial w^A}w^A\frac{\mathrm{d}w^A}{w^A} \iff \frac{\mathrm{d}w^A/w^A}{\mathrm{d}\tau/(1-\tau)}=\frac{e_S}{e_S+e_D}, \tag{52}$$

where we substituted for the definitions of the supply and demand elasticities.

Substituting the demand function for individuals $i\in\mathcal{A}$ and the supply function for individuals $i\in\mathcal{B}$, we can write the social welfare function as:

(53)

$$\begin{aligned}\mathcal{W}(\tau,w^A)=\int_{\mathcal{A}}u^i\Bigg((1-\tau)w^Ah^i+\tau\int_{\mathcal{A}}w^Ah_D^j(w^A)\mathrm{d}j+\tau\int_{\mathcal{B}:z^j<y}\theta^jh_S^j(\tau)\mathrm{d}j\\ +(1-G(y))\tau y+\int_y^\infty\int_y^{z^j}T'(z)\mathrm{d}z\mathrm{d}G(z^j)+F(L^A)-w^AL^A-R,h^i\Bigg)\mathrm{d}i.\end{aligned}$$

Notice that the social-welfare function incorporates the government's budget constraints. Thus, a simple derivative with respect to a policy parameter yields the marginal net social-welfare effect of the policy. Further note that the derivatives with respect to h^i and L^A are zero by virtue of the envelope theorem.

Taking the derivative of the social-welfare function, I obtain the following expression for the minimum wage:

$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial w^A/w^A}&=\int_{\mathcal{A}}\left((1-\tau)w^Ah^i+\tau\int_{\mathcal{A}}w^Ah^j\mathrm{d}j-w^AL^A-\tau\int_{\mathcal{A}}w^A\frac{\partial h_D^j(w^A)}{\partial w^A}\mathrm{d}j\right)\mathrm{d}i\\ &=\int_{\mathcal{A}}\left((1-\tau)z^i-(1-\tau)\int_0^y\sigma(z)z\mathrm{d}G(z)-\tau e^D\int_0^y\sigma(z)z\mathrm{d}G(z)\right)\mathrm{d}i\\ &=(1-\tau)\int_0^y\sigma(z)z\mathrm{d}G(z)-(1-\tau)\int_0^y\sigma(z)\mathrm{d}G(z)\int_0^y\sigma(z)z\mathrm{d}G(z)\\ &\qquad-\tau e^D\int_0^y\sigma(z)\mathrm{d}G(z)\int_0^y\sigma(z)z\mathrm{d}G(z),\end{aligned}\tag{54}$$

where I used the assumption $u_c^i=1$ in the first equation, and rewrote the domains of the inner integrals, wrote out $L^A=\int_{\mathcal{A}}h^j\mathrm{d}j$, and substituted for the definition of the demand elasticity in the second equation, and rewrote the domain of the outer integral in the third equation. The three terms on the right-hand side correspond to the utility effects (first term), mechanical revenue effects (second term), and the behavioral revenue effects (third term) of raising the minimum wage. Dividing eq. (54) by the revenue effect of raising the minimum wage yields:

$$\frac{w^A\partial\mathcal{W}/\partial w^A}{\tau e^D\int_0^y\sigma(z)\mathrm{d}G(z)\int_0^y\sigma(z)z\mathrm{d}G(z)}=\frac{1-\int_0^y\sigma(z)\mathrm{d}G(z)}{\frac{\tau}{1-\tau}e^D\int_0^y\sigma(z)\mathrm{d}G(z)}-1=\mathcal{BB}_{w^A}-1, \tag{55}$$

where the last equation follows from the definition of the bang for the buck of the minimum wage, see eq. (15). Thus, $\mathcal{BB}_{w^A}$ can be interpreted as the welfare gain per unit of distortionary costs of raising the minimum wage. An increase in the minimum wage enhances social welfare as long as $\mathcal{BB}_{w^A} > 1$.

Now consider an increase in the tax rate τ, evaluated at an equilibrium with market-clearing wages. This yields:

$$
\begin{aligned}
(56)\quad \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\tau} &= \frac{\partial \mathcal{W}}{\partial w^A}\frac{\mathrm{d}w^A}{\mathrm{d}\tau} + \frac{\partial \mathcal{W}}{\partial \tau} \\
&= \frac{\partial \mathcal{W}}{\partial w^A}\frac{\mathrm{d}w^A}{\mathrm{d}\tau} + \int_{\mathcal{A}} \Big(- w^A h^i + \int_{\mathcal{A}\cup\mathcal{B}: z^j < y} w^j h^j \mathrm{d}j + (1 - G(y))y \\
&\qquad\qquad + \tau \int_{\mathcal{B}: z^j < y} \theta^j \frac{\partial h_B^j(\tau)}{\partial \tau} \mathrm{d}j \Big) \mathrm{d}i \\
&= \frac{\partial \mathcal{W}}{\partial w^A}\frac{\mathrm{d}w^A}{\mathrm{d}\tau} + \int_{\mathcal{A}} \Big(- z^i + \int_0^y z \mathrm{d}G(z) + (1 - G(y))y \\
&\qquad\qquad - \frac{\tau}{1-\tau} e^S \int_0^y (1 - \sigma(z)) z \mathrm{d}G(z) \Big) \mathrm{d}i \\
&= \frac{\partial \mathcal{W}}{\partial w^A}\frac{\mathrm{d}w^A}{\mathrm{d}\tau} - \int_0^y \sigma(z) z \mathrm{d}G(z) + \int_0^y \sigma(z)\mathrm{d}G(z) \Big(\int_0^y z \mathrm{d}G(z) + (1 - G(y))y \Big) \\
&\qquad\qquad - \frac{\tau}{1-\tau} e^S \int_0^y \sigma(z)\mathrm{d}G(z) \int_0^y (1 - \sigma(z)) z \mathrm{d}G(z),
\end{aligned}
$$

where I used the assumption $u_c^i = 1$, applied eq. (52), and rewrote the domains of the inner integrals. The first term on the right-hand side represents the welfare effects of a higher tax rate that run via its effect on equilibrium wages. The remaining terms represent, for given wages, the utility losses (second term), mechanical revenue gains (third term), and the behavioral revenue losses (fourth term) associated for a tax increase. Using eq. (54) and the definitions in eqs. (12)–(14), I can rewrite the first term as:

$$
(57)\qquad \frac{\partial \mathcal{W}}{\partial w^A}\frac{\mathrm{d}w^A}{\mathrm{d}\tau} = \frac{\mathcal{U}_{w^A}}{\mathrm{d}\tau} - \int_0^y \sigma(z)\mathrm{d}G(z) \left(\frac{\mathcal{M}_{w^A}}{\mathrm{d}\tau} + \frac{\mathcal{R}_{w^A}}{\mathrm{d}\tau} \right).
$$

Substituting this back into eq. (56) yields:

$$
\begin{aligned}
(58)\quad \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\tau} &= - \left(\int_0^y \sigma(z) z \mathrm{d}G(z) - \frac{\mathcal{U}_{w^A}}{\mathrm{d}\tau} \right) + \int_0^y \sigma(z)\mathrm{d}G(z) \Big(\int_0^y z \mathrm{d}G(z) + (1 - G(y))y - \frac{\mathcal{M}_{w^A}}{\mathrm{d}\tau} \Big) \\
&\qquad - \int_0^y \sigma(z)\mathrm{d}G(z) \left(\frac{\tau}{1-\tau} e^S \int_0^y (1 - \sigma(z)) z \mathrm{d}G(z) + \frac{\mathcal{R}_{w^A}}{\mathrm{d}\tau} \right).
\end{aligned}
$$

The total utility effects of a higher tax rate – including its effects through a change in wages – are now captured by the first right-hand side term, the total mechanical revenue effect by the second term, and the total behavioral revenue effect by the third term. Dividing by the behavioral revenue effect, I obtain:

$$
(59)\qquad \frac{\mathrm{d}\mathcal{W}/\mathrm{d}\tau}{\int_0^y \sigma(z)\mathrm{d}G(z) \left(\frac{\tau}{1-\tau} e^S \int_0^y (1 - \sigma(z)) z \mathrm{d}G(z) + \frac{\mathcal{R}_{w^A}}{\mathrm{d}\tau} \right)} = \mathcal{BB}_\tau - 1,
$$

where I used the definition of $\mathcal{BB}_\tau$ from eq. (19). Thus, $\mathcal{BB}_\tau$ can be interpreted as the welfare gain per unit of distortionary costs of raising the tax rate τ. An increase in the tax rate enhances social welfare as long as $\mathcal{BB}_\tau > 1$. The minimum wage yields more welfare gains for the same distortionary losses if $\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$.

C Extensions of the baseline model

C.1 Desirability if government also cares about type-B poor

Assume that the government cares equally much about the working poor with the lowest wages *and* the working poor with higher wage rates. In other words, the marginal social welfare of everyone with status-quo income below y equals 1, whereas the marginal social welfare of everyone else equals 0. We can reappraise the same perturbations as in the main text and evaluate utility, mechanical revenue, and behavioral revenue effects. Since the minimum wage does not affect type-B individuals directly, $\mathcal{U}_{w^A}$, $\mathcal{M}_{w^A}$, and $\mathcal{R}_{w^A}$ remain unaffected and given by eqs. (12)–(14). However, the marginal value of public funds is now given by $G(y)$, the working poor share of the lump-sum grant. Thus, the bang for the buck is now given by:

$$\mathcal{BB}_{w^A} \equiv \frac{\mathcal{U}_{w^A} - G(y)\mathcal{M}_{w^A}}{G(y)\mathcal{R}_{w^A}} = \frac{1-G(y)}{G(y)} \frac{1}{\frac{\tau}{1-\tau}e_D}. \tag{60}$$

Turning to the tax perturbation, the mechanical and behavioral revenue gains ($\mathcal{M}_\tau$ and $\mathcal{R}_\tau$) are independent of social welfare weights and are thus still given by eqs. (17) and (18). However, utility losses now also reflect the fact that the tax reform raises utility from type-B working poor. Hence, it is given by:

$$\mathcal{U}_\tau = \int_0^y z\mathrm{d}G(z)\mathrm{d}\tau - \mathcal{U}_{w^A}. \tag{61}$$

The new bang for the buck represents this adjustment and the fact that the marginal value of revenue is now equal to $G(y)$:

$$\begin{aligned} \mathcal{BB}_\tau &\equiv \frac{G(y)\mathcal{M}_\tau - \mathcal{U}_\tau}{G(y)\mathcal{R}_\tau} \\ &= \frac{G(y)\left(\int_0^y z\mathrm{d}G(z) + (1-G(y))y - \frac{\mathcal{M}_{wA}}{\mathrm{d}\tau}\right) - \int_0^y z\mathrm{d}G(z) + \frac{\mathcal{U}_{wA}}{\mathrm{d}\tau}}{G(y)\left(\frac{\tau}{1-\tau}e_S\int_0^y(1-\sigma(z))z\mathrm{d}G(z) + \frac{\mathcal{R}_{wA}}{\mathrm{d}\tau}\right)}. \end{aligned} \tag{62}$$

Substituting into the desirability condition $\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$ and rearranging yields:

$$1 > \left(\frac{1}{1-\bar{\sigma}}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\frac{e_D}{e_S} - \mathrm{cov}_{z<\bar{y}}\left[\frac{1-\sigma(z)}{1-\bar{\sigma}}, \frac{z}{\bar{y}}\right]. \tag{63}$$

Or, imposing $\sigma(z) = \sigma$ for all $z \leq y$:

$$\sigma > \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\frac{e_D}{e_S}. \tag{64}$$

C.2 Desirability if profits are untaxed and earned by non-poor

In this case, an increase in the minimum wage no longer reduces government revenue from the profit tax. Thus, instead of eq. (13), the mechanical revenue loss is now given by:

$$\mathcal{M}_{w^A} = -\int_0^y \sigma(z)\tau z \mathrm{d}G(z)\frac{\mathrm{d}w^A}{w^A}. \tag{65}$$

An increase in the minimum wage mechanically *raises* revenue because it mechanically raises labor income $z^i = w^A h^i$ for $i \in \mathcal{A}$. The updated bang for the buck is given by:

$$\mathcal{BB}_{w^A} = \frac{1 + \int_0^y \sigma(z)\mathrm{d}G(z)\frac{\tau}{1-\tau}}{\int_0^y \sigma(z)\mathrm{d}G(z)\frac{\tau}{1-\tau}e_D}. \tag{66}$$

The bang for the buck of the tax reform remains unaffected. Substituting for eqs. (19) and (66) into the desirability condition $\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$, and rearranging yields:

$$\begin{aligned}\frac{1+\bar{\sigma}G(y)\frac{\tau}{1-\tau}}{1-G(y)} > &\left(\frac{\bar{\sigma}}{1-\bar{\sigma}}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\frac{e_D}{e_S} \\ &-\left(1+\frac{\bar{\sigma}G(y)\tau}{1+\tau}-\frac{e_D}{e_S}\right)\left(\frac{1}{1-G(y)}\right)\mathrm{cov}_{z\le y}\left[\frac{1-\sigma(z)}{1-\bar{\sigma}},\frac{z}{\bar{y}}\right].\end{aligned} \tag{67}$$

Or, imposing $\sigma(z) = \sigma$ for all $z \le y$:

$$\frac{1+\sigma G(y)\frac{\tau}{1-\tau}}{1-G(y)} > \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\frac{e_D}{e_S}. \tag{68}$$

C.3 Desirability with nonlinear taxes on the poor

C.3.1 Bang for the buck of a minimum-wage increase

Now consider the case in which $T'(z)$ is not necessarily constant for $z \le y$. The welfare analysis of the minimum-wage perturbation remains largely the same as in the main text. An increase in the minimum wage yields utility gains, mechanical revenue losses and behavioral revenue losses. These effects are still given by eqs. (12)–(14), except that marginal taxes equal $T'(z)$ instead of τ. The updated bang-for-the-buck measure is thus given by:

$$\mathcal{BB}_{w^A} = \left(\frac{1-\int_0^y \sigma(z)\mathrm{d}G(z)}{\int_0^y \sigma(z)\mathrm{d}G(z)}\right)\left(\frac{\int_0^y (1-T'(z))\sigma(z)z\mathrm{d}G(z)}{\int_0^y T'(z)\sigma(z)z\mathrm{d}G(z)}\right)\frac{1}{e_D}. \tag{69}$$

Note that the bang for the buck collapses into eq. (15) if we were to write $T'(z) = \tau$.

C.3.2 Bang for the buck of a comparable tax reform

Next consider a comparable perturbation of the tax schedule, in the sense that it leads to an equiproportional reduction in employment for all type-A workers – just like the minimum wage. Unfortunately, the design of such a tax reform is complicated by the fact that any behavioral change in income – due to changes in either working hours or wages – is going to endogenously affect marginal tax rates, leading to second-round behavioral effects. Ensuring that every type-A worker reduces their labor supply in the same proportion, then, typically necessitates a highly complicated reform of the tax schedule. This in turn yields unwieldy expressions for the welfare effects of such tax reform.

To sidestep this issue, I approximate the welfare effects of the tax perturbation by ignoring second-round behavioral effects induced by endogenous changes in marginal tax rates. This simplifies the design of the tax reform significantly. In particular, it allows me to focus on a relatively simple tax reform that raises marginal taxes in proportion to $1 - T'(z)$ for all incomes $z \leq y$. That is, I consider a change in marginal tax rates $\mathrm{d}T'(z) > 0$ such that I can write $\mathrm{d}T'(z)/(1 - T'(z)) \equiv \mathrm{d}\tau > 0$, which is identical for all levels of income $z \leq y$. This results in the same (first-round) proportional change in net-of-tax rates for all poor workers. Hence, it also leads to the same (first-round) proportional change in labor supply. To the extent that the reform reduces labor supply of type-A workers, the welfare effects are equivalent to those of an increase in the minimum wage: utility increases by $\mathcal{U}_{w^A}$, mechanical revenue declines by $\mathcal{M}_{w^A}$, and behavioral revenue losses equal $\mathcal{M}_{w^A}$. This follows from the same logic as in the main text.

As in the main text, the tax perturbation produces two more effects. First of all, it reduces labor supply of type-B working poor, which yields additional behavioral revenue losses. Second, the perturbation raises tax revenue from all workers. This generates both mechanical revenue gains and type-A utility losses.

I first consider the utility losses. A household i with income $z^i \leq y$ faces an increase in tax payments equal to $\int_0^{z^i} \mathrm{d}T'(x)\mathrm{d}x$, where $\mathrm{d}T'(x)$ denotes the exogenous (i.e., first-round) change in marginal taxes at income level x. Because the increase in marginal taxes is the same proportion of $1 - T'$ for all income levels, we can write the tax increase for worker i as $\int_0^{z^i}(1 - T'(x))\mathrm{d}x\frac{\mathrm{d}T'(x)}{1-T'(x)} = \int_0^{z^i}(1 - T'(x))\mathrm{d}x\mathrm{d}\tau$. Given quasi-linear utility, the utility loss for type-A workers is equal to the increase in taxes. Integrating the tax increase over all type-A workers, and subtracting $\mathcal{U}_{w_L}$, the total type-A utility loss associated with the tax perturbation is given by:

$$
\begin{aligned}
(70) \qquad \mathcal{U}_\tau &= \int_0^y \left(\int_0^z (1 - T'(x))\mathrm{d}x \right) \sigma(z)\mathrm{d}G(z)\mathrm{d}\tau - \mathcal{U}_{w^A} \\
&= \int_0^y (1 - \bar{T}'(z))\sigma(z) z \mathrm{d}G(z)\mathrm{d}\tau - \mathcal{U}_{w^A},
\end{aligned}
$$

where I defined $\bar{T}'(z) \equiv \int_0^z T'(x)\mathrm{d}x/z$ as the average marginal tax rate for income up to z. Although I do not consider extensive-margin labor supply decisions, this "average marginal tax rate" corresponds to what is typically known as the participation tax rate: $\bar{T}'(z) \equiv \int_0^z T'(x)\mathrm{d}x/z = (T(z) - T(0))/z$.

The tax perturbation raises mechanical revenue from both poor and non-poor. Starting with the poor, the perturbation raises additional taxes $\int_0^{z^i} \mathrm{d}T'(x)\mathrm{d}x$ from every $i : z^i \leq y$. Substituting for $\mathrm{d}\tau = \mathrm{d}T'(x)/(1 - T'(x))$, and for the definition of the average marginal tax rate, yields $\int_0^{z^i} \mathrm{d}T'(x)\mathrm{d}x = (1 - \bar{T}'(z^i))z^i\mathrm{d}\tau$. Integration over all individuals with income below y yields the mechanical revenue gains from the working poor. At the same time, the non-poor each face an increase in their tax burden of $\int_0^y \mathrm{d}T'(x)\mathrm{d}x = (1 - \bar{T}'(y))y\mathrm{d}\tau$. Multiplying by the number of rich, $1 - G(y)$, yields the mechanical revenue gains from the non-poor. Further subtracting the mechanical revenue loss associated with increased wages ($\mathcal{M}_{w^A}$) yields:

$$\mathcal{M}_\tau = \int_0^y (1 - \bar{T}'(z))z\mathrm{d}G(z)\mathrm{d}\tau + (1 - G(y))(1 - \bar{T}'(y))y\mathrm{d}\tau - \mathcal{M}_{w^A}. \tag{71}$$

Finally, the tax perturbation reduces labor supply of type-B working poor – leading to behavioral revenue losses. A worker with income $z < y$ faces an increase in the marginal tax rate by $(1 - T'(z^i))\mathrm{d}\tau$. This results in a behavioral reduction in labor income of $z^i e_S \mathrm{d}\tau$, and thus to behavioral revenue losses equal to $T'(z^i)z^i e_S \mathrm{d}\tau$. Integrating over all individuals $i \in \mathcal{B} : z^i \leq y$, and adding the behavioral revenue losses associated with type-A workers ($\mathcal{R}_{w^A}$), yields:

$$\mathcal{R}_\tau = e_S \int_0^y T'(z)(1 - \sigma(z))z\mathrm{d}G(z)\mathrm{d}\tau + \mathcal{R}_{w^A}. \tag{72}$$

Taken together, this yields to the following updated measure for the tax reform's bang for the buck:

$$\begin{aligned} \mathcal{BB}_\tau &\equiv \frac{\int_0^y \sigma(z)\mathrm{d}G(z)\mathcal{M}_\tau - \mathcal{U}_\tau}{\int_0^y \sigma(z)\mathrm{d}G(z)\mathcal{R}_\tau} \\ &= \frac{\int_0^y \sigma(z)\mathrm{d}G(z)\left(\int_0^y (1 - \bar{T}'(z))z\mathrm{d}G(z) + (1 - G(y))(1 - \bar{T}'(y))y - \frac{\mathcal{M}_{wA}}{\mathrm{d}\tau}\right)}{\int_0^y \sigma(z)\mathrm{d}G(z)\left(e_S \int_0^y T'(z)(1 - \sigma(z))z\mathrm{d}G(z) + \frac{\mathcal{R}_{wA}}{\mathrm{d}\tau}\right)} \\ &\quad - \frac{\int_0^y \sigma(z)(1 - \bar{T}'(z)z\mathrm{d}G(z) + \frac{\mathcal{U}_{wA}}{\mathrm{d}\tau}}{\int_0^y \sigma(z)\mathrm{d}G(z)\left(e_S \int_0^y T'(z)(1 - \sigma(z))z\mathrm{d}G(z) + \frac{\mathcal{R}_{wA}}{\mathrm{d}\tau}\right)}. \end{aligned} \tag{73}$$

C.3.3 Desirability of a binding minimum wage

An increase in the minimum wage – evaluated at clearing labor markets – is more desirable than a comparabl tax reform if and only if $\mathcal{BB}_{w^A} > \mathcal{BB}_\tau$. I focus on the case in which

$\sigma(z) = \sigma$ for all $z \leq y$, as in Proposition 3. Substituting for eqs. (69) and (73) into the desirability condition, and rearranging, yields:

$$
\begin{aligned}
(74)\quad & \frac{1-\sigma G(y)}{1-G(y)}\left(1+\mathrm{cov}_{z\leq y}\left[\frac{1-T'(z)}{1-\bar{T}'(y)},\frac{z}{\bar{y}}\right]\right) > \\
& \frac{e_D}{e_S}\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{y-\bar{y}}{\bar{y}}+\frac{1}{G(y)}\int_0^y\left(\frac{(1-\bar{T}'(y))-(1-\bar{T}'(z))}{1-\bar{T}'(y)}\right)\left(\frac{z}{\bar{y}}\right)\mathrm{d}G(z)\right).
\end{aligned}
$$

Comparison with eq. (20) in Proposition 3 reveals two novel terms, one on each side of the desirability condition. I discuss both in turn.

On the left-hand side, we see the normalized covariance between the net-of-tax rate $1-T'(z)$ and income z. A positive covariance implies larger distributional gains from the minimum wage. After all, a percent increase in wages yields utility gains proportional to $(1-T'(z^i))z^i$ for $i \in \mathcal{A}$. The income gain is strongest for type-A workers with a relatively high income; thus the distributional gains are stronger if type-A workers with high income z^i face a relatively high marginal retention rate $1-T(z^i)$. Thus, this novel term suggest that the minimum wage is *more* desirable if marginal taxes are decreasing with income. In the same vain, it suggests that the minimum wage is *less* desirable if marginal taxes are increasing with income.

On the right-hand side, we see a new term that depends on the difference between the "average marginal net-of-tax rate" for income level y and the "average marginal net-of-tax rate" for income levels below y. Like the novel term on the left-hand side, this term is positive if marginal taxes are declining with income, such that $T'(y) < T'(z)$ for $z < y$. This is because declining marginal tax rates imply larger distributional benefits of the tax reform. After all, marginal taxes are increased in proportion to $1-T'(z)$. Thus, if $T'(z)$ is declining in z, then the marginal tax increases are increasing with income. In that case, revenue is raised relatively less from the poor than from the rich. Thus, the distributional benefits of the tax perturbation are higher if marginal taxes for the poor are declining with income. This novel term thus suggests that the minimum wage is *less* desirable if marginal taxes are decreasing with income. In the same vain, it suggests that the minimum wage *more* desirable if marginal taxes are increasing with income.

Notice that the two novel terms have opposing implications for the desirability of the minimum wage. If marginal taxes are declining (increasing) with income, the distributional benefits of both the minimum-wage increase *and* the tax reform are larger (smaller).[29] Thus, the implications of non-constant marginal tax rates for the working poor are *a priori* ambiguous. As before, however, regardless of the shape of the tax schedule, the minimum wage is unambiguously desirable if the elasticity of labor demand

[29]It can furthermore be shown that the distortionary costs of the minimum wage and the tax reform are also both smaller if marginal taxes are declining with income. However, these effects on the distortionary costs of the two policy perturbations exactly cancel out.

approaches zero.[30]

[30] Note that the left-hand side of eq. (74) is unambiguously positive, regardless of the covariance between marginal taxes and income. To see this, note that we can rewrite the bracketed term on the left-hand side as:

$$1 + \text{cov}\left[\frac{1-T'(z)}{1-\bar{T}'(y)}, \frac{z}{\bar{y}}\right] = \frac{1}{G(y)} \int_0^y \left(\frac{1-T'(z)}{1-\bar{T}'(y)}\right)\left(\frac{z}{\bar{y}}\right) \mathrm{d}G(z) > 0.$$

D Numerical calibration

To obtain the results in Table 1, I take a number of steps. First, I rewrite the desirability condition from eq. (21) as follows:

$$\frac{e_D}{e_S} < \frac{\left(1-\int_0^y \sigma(z)\mathrm{d}G(z)\right)\left(1-\frac{\int_0^y \sigma(z)z\mathrm{d}G(z)}{\int_0^y z\mathrm{d}G(z)}\right)}{\int_0^y \sigma(z)\mathrm{d}G(z)\left(1+\frac{1-G(y)}{G(y)}\frac{y}{\int_0^y z\mathrm{d}G(z)/G(y)}\right)-\frac{\int_0^y \sigma(z)z\mathrm{d}G(z)}{\int_0^y z\mathrm{d}G(z)}}. \tag{75}$$

Similarly, I rewrite the desirability condition for the case in which the government cares equally about all working poor. This condition is derived in Appendix C and rewritten as:

$$\frac{e_D}{e_S} < \left(1-\frac{\int_0^y \sigma(z)z\mathrm{d}G(z)}{\int_0^y z\mathrm{d}G(z)}\right)\left(\frac{\int_0^y z\mathrm{d}G(z)/G(y)}{y-\int_0^y z\mathrm{d}G(z)/G(y)}\right). \tag{76}$$

The right-hand sides of these equations only depend on characteristics of the income and wage distribution. In particular, it only depends on the following five characteristics.

- $\int_0^y \sigma(z)\mathrm{d}G(z)$ is the share of minimum wage workers in the working population.
- y is the highest weekly earnings level of lowest-wage workers.
- $G(y)$ is the proportion of workers with weekly earnings below y.
- $\frac{\int_0^y \sigma(z)z\mathrm{d}G(z)}{\int_0^y z\mathrm{d}G(z)}$ is total weekly earnings of lowest-wage workers divided by total weekly earnings of all households with income below y.
- $\int_0^y z\mathrm{d}G(z)/G(y)$ is the average weekly earnings of the working poor.

In the most conservative calibration, I truncate working hours above 50 and define lowest-wage workers as those with an hourly wage of at most \$8.25, see Table 1. Under this calibration, I find the following values for these characteristics:

$$\begin{aligned}
\int_0^y \sigma(z)\mathrm{d}G(z) &= 0.038,\\
y &= 403.84,\\
G(y) &= 0.18,\\
\frac{\int_0^y \sigma(z)z\mathrm{d}G(z)}{\int_0^y z\mathrm{d}G(z)} &= 0.15,\\
\int_0^y z\mathrm{d}G(z)/G(y) &= 257.64.
\end{aligned}$$