# cesifo Working PAPERS 

# Strategic Complementarities in a Model of Commercial Media Bias <br> Anna Kerkhof, Johannes Münster 

## Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded

- from the SSRN website: www.SSRN.com
- from the RePEc website: www.RePEc.org
- from the CESifo website: https://www.cesifo.org/en/wp


# Strategic Complementarities in a Model of Commercial Media Bias 


#### Abstract

Media content is an important privately supplied public good. While it has been shown that contributions to a public good crowd out other contributions in many cases, the issue has not been thoroughly studied for media markets yet. We show that in a standard model of commercial media bias, qualities of media content are strategic complements, whereby investments into quality crowd in further investments and engage competitors in a race to the top. Therefore, financially strong public service media can mitigate commercial media bias: the content of commercial media can be more in line with the preferences of the audience and less advertiser-friendly in a dual (mixed public and commercial) media system than in a purely commercial media market.


JEL-Codes: C700, H410, L130, L510, L820.
Keywords: commercial media bias, public service media, advertising two-sided markets, supermodular games, strategic complements, public goods.

Anna Kerkhof<br>ifo Institute - Leibniz Institute for Economic<br>Research at the University of Munich<br>Munich / Germany<br>kerkhof@ifo.de

Johannes Münster<br>University of Cologne<br>Cologne / Germany<br>johannes.muenster@uni-koeln.de

October 25, 2023
We thank Lara Mai for excellent research assistance. This project is funded by the Bavarian State Ministry of Science and the Arts in the framework of the bidt Graduate Center for Postdocs. Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany‘s Excellence Strategy - EXC 2126/1-390838866.

## 1 Introduction

Media content belongs to the most important cases of privately supplied public goods. Its consumption is non-rival, and in many cases like free TV or freely available Internet content no exclusion is taking place. Media content differs markedly from other public goods, though, because of the importance of advertising revenue for media markets, and the economic analysis of the private supply of these public goods must take the multi-sided nature of media markets into account (Anderson and Coate 2005). Recent literature has made major progress in this research area (see Anderson and Jullien 2015 and Jullien, Pavan, and Rysman 2021).

One important result from the theory of private public good supply is that, under fairly general conditions, private contributions to a public good are strategic substitutes, whereby high contributions are crowding out others (see Batina and Ihori, 2005, Chapter 6, and Bucholz and Sandler, 2021, Finding F9, for overviews). Surprisingly, this issue has not been thoroughly examined for media markets, even though it is highly relevant for the welfare analysis of media policy. E.g., in discussions about the proper role and scope of public service media (PSM), one crucial question is whether raising the program quality of a regulated (public) broadcaster will increase or decrease the program quality of its commercial competitors. ${ }^{1}$

There are two conflicting views. On the one hand, PSM could crowd out private investment and innovation in media markets. E.g., the existence of PSM may lead to less entry of commercial media; see Berry and Waldfogel (1999) for empirical evidence. Similarly, Armstrong and Weeds (2007a) show that in a duopoly where a public and a commercial broadcaster compete, raising the program quality of PSM partially crowds out the commercial broadcaster and lowers its program quality. This reasoning is echoed by regulation authorities like Ofcom in the UK and the Scientific advisory board at the Federal Ministry of Finance in Germany (Ofcom 2004, Wissenschaftlicher Beirat beim Bundesministerium der Finanzen 2014.)

However, PSM might also foster a "competition for quality", whereby public and commercial media compete for audiences. This reasoning goes back to Coase (1947), pondering that PSM might induce a "natural rivalry to furnish the most attractive programs" (p.197). Indeed, recent empirical evidence suggests that in countries where PSM invest in high-quality media content, program quality of commercial media tends to be high, too (Simon 2013). Similarly, Sehl et al. (2020) find that, controlling for

[^0]GDP, per capita revenues of PSM and commercial broadcasters are positively correlated across EU countries. ${ }^{2}$

In this paper, we demonstrate that in a model of commercial media bias, program qualities in terms of the media's reporting accuracy are strategic complements rather than strategic substitutes. Reporting accuracy here refers to media content that fully and truthfully reports facts as opposed to hiding information or dumbing down content. ${ }^{3}$ Viewers prefer high, while advertisers prefer lower reporting accuracy. The strategic complementarity stems from the media's fundamental trade-off in these models: Increasing reporting accuracy increases the value of the media content for the audience but decreases the willingness to pay of the advertisers to reach consumers. The latter effect becomes less important when a media company has a smaller audience; hence, its incentives to increase reporting accuracy are higher. Thus, in a media market with both public and commercial media, raising the PSM's reporting accuracy reduces the commercial media's audiences and thereby also their implicit cost of increasing their own reporting accuracy. As a result, the PSM crowd in reporting accuracy and engage the commercial media in a race to the top.

Our main model focuses on media content that is freely available. We show that our results generalize to a model featuring both pay media and free media, however, if reporting accuracy is about revealing information that the media already posses. We also discuss conditions under which our findings generalize to multidimensional strategy spaces, spillover effects of reporting accuracy on advertising revenue of other media outlets, different demand functions, income effects of taxes or license fees used to finance the PSM, endogenous entry and exit, and biases of Public Service Media.

Our paper contributes to four strands of literature. First, we contribute to the literature on commercial media bias. Several empirical papers document the effect of

[^1]advertising on media coverage in terms of mutual fund recommendations (Reuter and Zitzewitz 2006), product mentions (Gambaro and Puglisi 2015), coverage of government scandals (Di Tella and Franceschelli 2011) and climate change (Beattie, 2020). We present a fairly standard model of commercial media bias. Our model is in many ways similar the models studied by Ellman and Germano (2009), Germano and Meier (2013), and Kerkhof and Münster (2015); it captures bias through distorted reporting accuracy that caters to the preferences of advertisers rather than consumers. Our paper is especially close to Germano and Meier (2013) who show that competition on media market mitigates commercial bias, and to Kerkhof and Münster (2015) who find that competition between media outlets makes it more likely that a cap on advertising quantities is welfare enhancing. Relatedly, Blasco et al. (2016) find that if the media can raise their audience share through increasing their reporting accuracy, then competition in the market may also increase the expected accuracy of reports. ${ }^{4}$ These predictions are in line with the empirical results of Beattie et al. (2021) who find that newspapers provide less coverage of car recalls by their advertisers, but competition for readers raises reporting accuracies and thus mitigates bias. Similarly, Focke et al. (2016) show that commercial media bias is likely mitigated by reputational concerns on behalf of the media, e.g., if they face a demanding audience.

Second, we advance the broad research on the private supply of public goods (Bergstrom et al., 1986, Batina and Ihori, 2005). The provision of public goods via advertising is studied by Luski and Wettstein (1994) and Anderson and Coate (2005). These papers do not study media bias, however.

Third, we add to the literature on supermodular games, i.e., games in which the best response of any player is increasing in the actions of its competitors (Topkis 1979, Milgrom and Roberts, 1990, Vives 1985, 1990, 2005a, 2005b, Van Zandt and Vives, 2007, Frankel et al., 2003). Leveraging the theory of supermodular games allows us to obtain fairly general results in a model with many asymmetric media outlets. Specifically, we show that reporting accuracies are strategic complements rather than strategic substitutes.

Fourth, as a consequence of strategic complementarities, public investments into program quality induce commercial media to provide high quality, too. Hence, our results support media policies that advocate financially strong PSM. In this way, we also contribute to the economics literature on public service media (see Armstrong and

[^2]Weeds 2007b, Strömberg 2015, and Weeds 2020 for surveys). To the best of our knowledge, the issue how public media affect the content of commercial media has not been studied yet in the literature on commercial media bias. Other aspects of this debate have of course been analyzed; in addition to the empirical literature referenced above, several theoretical studies on the market impact of public media exist. Armstrong and Weeds (2007a) study investments in a vertical quality dimension. Richardson (2006) investigates how a publicly-provided radio station offering local content affects the provision of local content by commercial stations. Garcia Pires (2016) compares media diversity in commercial versus mixed public and private duopolies. Our paper complements this line of research by studying commercial media bias.

The remainder of this paper is structured as follows. Section 2 introduces our theoretical framework. In Section 3, we demonstrate that program qualities in terms of the media's reporting accuracy are strategic complements, which is our main finding, and describe the implications for crowding in effects of Public Service Media. Section 4 considers the case where some commercial media are pay media. Section 5 discusses several extensions of our model. Section 6 concludes.

## 2 Model

In this section we introduce a fairly standard model of commercial media bias (Ellman and Germano 2009, Germano and Meier 2013, Kerkhof and Münster 2015, see Blasco and Sobbrio 2012 for a survey). Consider a model with $n$ commercial media denoted by $1, \ldots, n$ and $m$ public service media (PSM) denoted $n+1, \ldots, n+m$. The set of commercial media is denoted by $C=\{1, \ldots, n\}$, the set of PSM is $P=\{n+1, \ldots, n+m\}$. Each media outlet $i \in C \cup P$ chooses a reporting accuracy $v_{i} \in V_{i} \subseteq \mathbb{R}_{+}$. (An extension to multidimensional strategy spaces is considered in Section 5.) Reporting accuracy $v_{i}$ is about fully and truthfully reporting facts, as opposed to hiding information or dumbing down content. The audience prefers higher reporting accuracy, whereas advertisers prefer lower reporting accuracy. We assume that the strategy sets $V_{i}$ are compact and contain $v_{i}=0$.

A consumer's utility from consuming outlet $i$ is $u_{i}=f_{i}\left(v_{i}\right)$, where $f_{i}$ is continuous, strictly increasing, and satisfies $f_{i}(0)=0$. Unless otherwise noted, we simply assume $f_{i}\left(v_{i}\right)=v_{i}$. In Sections 2 and 3, nothing is lost in setting $u_{i}=v_{i}$; the distinction between utility $u_{i}$ and reporting accuracy $v_{i}$ becomes important when con-
sidering pay media or multidimensional strategies. For a commercial outlet $i \in C$, let $u_{-i}^{C}=\left(u_{1, \ldots}, u_{i-1}, u_{i+1}, \ldots, u_{n}\right)$ denote the vector of the utilities of $i$ 's commercial competitors, $u^{P}=\left(u_{n+1}, \ldots ., u_{n+m}\right)$ the vector of utilities of the public service media, and $u_{-i}=\left(u_{-i}^{C}, u^{P}\right)$.

The size of the audience of a media outlet is denoted by $s_{i}$. We impose the following assumptions. ${ }^{5}$

Assumption (1) For all $i \in C, s_{i}$ is positive, continuous, increasing in $u_{i}$, and decreasing in $u_{j}$ for all $j \in P \cup C \backslash\{i\}$.

Assumption (2) For all $i \in C, s_{i}$ has increasing differences in $\left(u_{i}, u_{-i}\right)$.
Assumptions (1) to (2) are fulfilled by several standard demand formulations, including linear demand functions, quadratic demand functions where the coefficients of the interaction terms are positive, and the Hotelling, Salop, and Spokes (Chen and Riordan 2007) model in the relevant range where all market shares are interior. The logit model violates Assumption (2) whenever there are $n \geq 2$ commercial outlets; in Section 5 we give a sufficient condition for our results to hold when Assumption (2) is violated.

Denote the advertising revenue of outlet $i$, per member of the audience, by $R_{i}$. A crucial assumption in models of advertiser bias is that, for a given audience, ad revenue depends negatively on reporting accuracy:

Assumption (3) For all $i \in C, R_{i}$ is positive, continuous, decreasing in $v_{i}$, and independent of $v_{j}$ for all $j \neq i$.

By Assumption (3), $R_{i}$ is independent from the reporting in other outlets as in Ellman and Germano (2009) and Kerkhof and Münster (2015). Germano and Meier (2013) model spillover effects of reporting accuracy on the advertising revenue of other outlets; we will discuss spillover effects in Section 5.

Each media outlet $i$ has a cost $c_{i}\left(v_{i}\right)$ that may depend on its reporting accuracy. ${ }^{6}$

[^3]Assumption (4) For all media outlets $i \in C \cup P, c_{i}$ is continuous, increasing in $v_{i}$, and zero at $v_{i}=0$.

We distinguish between two cases. First, reporting accuracy could be about faithfully reporting information that the media already have. In this case, the only cost of reporting accurately is lower advertising revenue, but there is no additional direct cost of obtaining the information in the first case. Formally, in the current model it means that $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$. We refer to this case of withholding information as "dumbing down content". Second, reporting accuracy could also be about investigative journalism, about establishing new facts and information. Then it seems plausible that $c_{i}\left(v_{i}\right)$ is strictly increasing in $v_{i}$. For example, the media might have to hire more journalists to increase reporting accuracy (see Hamilton 2016 for a detailed description of the economics of investigative journalism). We refer to this case as "investigative journalism".

Arguably, dumbing down content is highly relevant for the study of commercial media bias; indeed several papers in the literature focus on this case (Ellman and Germano 2009, Germano and Meier 2013, Kerkhof and Münster 2015, Blasco, Pin and Sobbrio 2016). For example, two important topics where advertising has influenced editors are the health risks of smoking (e.g. Bagdikian 2004 Chapter 12) and climate change (Beattie 2020, Boykoff and Boykoff 2004). The scientific facts about these topics were long well established and easily accessible, but media coverage and public perception significantly lagged in time behind the scientific consensus. Moreover, as shown in Beattie (2022), commercial media bias in the tone of coverage about climate change, measured based on comparisons of environmental and skeptical texts, can have important behavioral consequences, and merely changing the tone of coverage does not impact its cost.

On the other hand, pressure from advertisers may also deter media from investigative journalism. Our main results on free media do not depend on whether we study dumbing down content or investigative journalism. For pay media, we show that the distinction matters.

Commercial media in our main model are funded by advertising and their content is freely available for consumers; pay media will be considered in Section 4. The profit of a commercial media outlet $i=1, \ldots, n$ is (substituting $v_{i}=u_{i}$ and $v_{-i}=u_{-i}$ into $s_{i}$ )

$$
\pi_{i}\left(v_{i}, v_{-i}\right)=s_{i}\left(v_{i}, v_{-i}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}\right) .
$$

Commercial outlet $i$ maximizes $\pi_{i}\left(v_{i}, v_{-i}\right)$ by choosing $v_{i} \in V_{i}$. Note that we disregard fixed costs which could be saved by going out of business; we defer a discussion of exit and entry to Section 5.

The public service media (PSM) in our model are not-for-profit and financed independent of advertising. Their content is freely available for all consumers. ${ }^{7}$ The budget of PSM $i \in P$ is $b_{i}$. We assume the PSM spend their budget to maximize consumer utility by choosing $v_{i} \in V_{i}$ subject to $c_{i}\left(v_{i}\right) \leq b_{i}$. The feasible sets $V_{i} \subseteq \mathbb{R}_{+}$are compact, contain $v_{i}=0$, and may depend on the budget $b_{i}$. We assume that a larger budget enlarges the feasible set: if $b_{i}<b_{i}^{\prime}$, then $V_{i}\left(b_{i}\right) \subseteq V_{i}\left(b_{i}^{\prime}\right)$. The model allows for inefficiencies of PSM, since different media outlets can have different cost functions and different feasible sets of reporting accuracies. We discuss potential biases of PSM in Section 5.

Some (but not all) of our considerations below impose the additional assumption that a sufficiently high reporting accuracy is necessary for a PSM to attract an audience. To express this formally, for $i \in P$ let $v_{-i}^{P}=\left(v_{n+1}, \ldots, v_{i-1}, v_{i+1}, \ldots, v_{n+m}\right)$ denote the vector of reporting accuracies of the other PSM.

Assumption (5) A PSM outlet with zero reporting accuracy ( $v_{i}=0$ ) attracts no audience: $s_{i}\left(0, v^{C}, v_{-i}^{P}\right)=0$ for all $i \in P$ and $\left(v^{C}, v_{-i}^{P}\right)$, and demand for the other media outlets is as if outlet $i$ did not exist.

Assumption (5) seems reasonable when the audience has a sufficiently attractive outside option not to consume any media. Note that under Assumption (5), a PSM with an insufficient budget cannot produce a content that attracts any audience; then the game reduces to a game between the remaining media outlets only. We will explicitly indicate where we use Assumption (5).

## 3 Main results

Consider the PSM first.

## Proposition 1 A PSM i chooses

$$
v_{i}=\bar{v}_{i}\left(b_{i}\right):=\max _{v_{i} \in V_{i}\left(b_{i}\right)}\left\{v_{i} \mid c_{i}\left(v_{i}\right) \leq b_{i}\right\}
$$

[^4]Moreover, $\bar{v}_{i}\left(b_{i}\right)$ is increasing in $b_{i}$, and independent of the strategies of the other media outlets.

Proof. Outlet $i \in P$ solves

$$
\max _{v_{i} \in V_{i}} v_{i} \text { s.t. } c_{i}\left(v_{i}\right) \leq b_{i} .
$$

An increase of $b_{i}$ relaxes the PSMs budget constraint and enlarges the feasible set $V_{i}$, hence $\bar{v}_{i}$ is increasing in $b_{i}$. Moreover $\bar{v}_{i}$ is unique and independent of the strategies chosen by the other media outlets.

We now turn to the commercial media. Proposition 1 allows us to view the game between the commercial media as parameterized by the budgets of the PSM $b:=$ $\left(b_{n+1}, \ldots, b_{n+m}\right)$. Let $\bar{v}^{P}(b)=\left(\bar{v}_{i}\left(b_{i}\right)\right)_{i=n+1}^{m}$ denote the vector of reporting accuracies chosen by the PSMs. For $i \in C$, let

$$
\tilde{\pi}_{i}\left(v_{i}, v_{-i}^{C}, b\right):=\pi_{i}\left(v_{i}, v_{-i}^{C}, \bar{v}^{P}(b)\right)
$$

and let $\Gamma_{b}=\left(C,\left(\tilde{\pi}_{i}\right)_{i=1}^{n}, \Pi_{i=1}^{n} V_{i}\right)$ denote the resulting game between the commercial media outlets: the set of players is $C$, payoff functions are $\tilde{\pi}_{i}$, and strategy spaces are $V_{i}$.

Proposition $2 \Gamma_{b}$ is a parameterized supermodular game. ${ }^{8}$
Proof. The strategy spaces $V_{i} \subseteq \mathbb{R}_{+}$are compact by assumption, hence compact lattices, and the objective functions $\tilde{\pi}_{i}$ are continuous in $v_{i}$ for fixed $v_{-i}$ and $b$.

Next, we show that $\pi_{i}$ has increasing differences in $\left(v_{i}, v_{-i}\right)$. For simplicity of the exposition, we will assume here that the functions $R_{i}, s_{i}$ and $c_{i}$ are differentiable; Appendix $A$ gives the proof without assuming differentiability. From

$$
\pi_{i}\left(v_{i}, v_{-i}\right)=s_{i}\left(v_{i}, v_{-i}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}\right)
$$

we obtain

$$
\frac{\partial \pi_{i}}{\partial v_{i}}=\frac{\partial s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{i}} R_{i}\left(v_{i}\right)+s_{i}\left(v_{i}, v_{-i}\right) R_{i}^{\prime}\left(v_{i}\right)-c_{i}^{\prime}\left(v_{i}\right)
$$

and

$$
\begin{equation*}
\frac{\partial^{2} \pi_{i}}{\partial v_{j} \partial v_{i}}=\frac{\partial^{2} s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j} \partial v_{i}} R_{i}\left(v_{i}\right)+\frac{\partial s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j}} R_{i}^{\prime}\left(v_{i}\right) \geq 0, j \neq i \tag{1}
\end{equation*}
$$

[^5]where the inequality follows because of $\frac{\partial^{2} s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j} \partial v_{i}} \geq 0$ by Assumption (2), $\frac{\partial s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j}} \leq 0$ by Assumption (1), and $R_{i}^{\prime}\left(v_{i}\right) \leq 0$ by Assumption (3).

We have shown that $\pi_{i}$ has increasing differences in $\left(v_{i}, v_{-i}\right)$. Therefore, $\tilde{\pi}_{i}$ has increasing differences in $\left(v_{i}, v_{-i}^{C}\right)$. Moreover, $\pi_{i}$ has increasing differences in $\left(v_{i}, v^{P}\right)$. It remains to show that $\tilde{\pi}_{i}$ has increasing differences in $\left(v_{i}, b\right)$. By Proposition 1, $\bar{v}_{k}\left(b_{k}\right)$ is increasing in $b_{k}$, while $\bar{v}_{k^{\prime}}$ does not depend on $b_{k}$ for $k^{\prime} \neq k$. Since $\pi_{i}$ has increasing differences in $\left(v_{i}, v^{P}\right)$, it follows that $\tilde{\pi}_{i}$ has increasing differences in $\left(v_{i}, b\right)$.

Proposition 2 shows that the reporting accuracies are strategic complements. The economics of the result is straightforward. The fundamental trade-off for a commercial outlet in a model of commercial media bias is as follows: providing content in line with the preferences of the audience attracts a bigger audience, but leads to lower advertising revenue per consumer. If the reporting accuracies of competing media increase, the audience of a given outlet is smaller, hence also the implicit cost of increasing its own reporting accuracy. The logic is closely related to the finding in Germano and Meier (2013) that underreporting typically increases with the concentration of ownership on the media market, which has found empirical support in Beattie et al. (2021).

Leveraging the theory of supermodular games (see Vives 2005 or Sarver 2023 for expositions) allows us to generate fairly general results in our model featuring many asymmetric media outlets. Denote a strategy profile in game $\Gamma_{b}$ by $v^{C}=\left(v_{1}, \ldots, v_{n}\right)$. Proposition 2 implies that $\Gamma_{b}$ has, for any $b$, a lowest equilibrium $v^{C, l o w}$ and a highest equilibrium $v^{C, h i g h}$, such that any equilibrium $v^{C}$ satisfies $v^{C, l o w} \leq v^{C} \leq v^{C, h i g h}$. Since $\tilde{\pi}_{i}$ is continuous in $\left(v_{i}, v_{-i}\right)$ for all $i \in C$, Milgrom and Roberts (1990) applies and the set strategy combinations that survive iterated elimination of strictly dominated strategies has smallest and largest elements $v^{C, l o w}$ and $v^{C, h i g h}$.

Turning to comparative statics, the equilibria $v^{C, l o w}$ and $v^{C, h i g h}$ are monotone increasing in $b$. When the equilibrium is unique, a stronger monotone comparative static result is available, which we highlight the following Proposition 3.

Proposition 3 Suppose $\Gamma_{b}$ has a unique equilibrium. Then the equilibrium reporting accuracy of each commercial media outlet $i \in C$ is increasing in the budget $b_{j}$ of any $P S M j \in P$.

Proposition 3 states that PSM crowd in reporting accuracy, in line with the idea that PSM engage commercial media in a race to the top. To illustrate the result, we compare a "dual" (or mixed public and commercial) media market, featuring both

PSM and $n$ commercial media outlets, with a purely commercial media market consisting only of the same $n$ commercial outlets, postponing considerations about entry to Section 5. Under Assumption (5), Proposition 3 implies that in a "dual" media market, the content of the commercial media outlets is more audience-friendly and less advertiser-friendly than when there are only the $n$ commercial media. To see this, recall that when $b_{i}$ is insufficient, the PSM $i$ cannot attract any audience in our model by Assumption (5), and then the resulting competition between the remaining media is as if PSM $i$ was not on the market. Applying Proposition 3 shows that the reporting accuracies of the commercial media will be lower in this situation than when there are viable PSM.

## 4 Pay media

This section studies an extension to the model where some commercial media outlets are pay media, i.e. earn revenue from direct payments from consumers. Suppose that media outlets $i \in C^{f}=\left\{1, \ldots, n_{f}\right\}$ are free media: they are funded solely by advertising and 'free' in the sense that consumers do not pay a monetary price for consumption. Outlets $i \in C^{\text {pay }} \in\left\{n_{f}+1, \ldots, n\right\}$ are pay media. The set of all commercial media is $C=C^{f} \cup C^{\text {pay }} .{ }^{9}$ As above, outlets $i \in P=\{n+1, \ldots m\}$ are PSMs.

A pay media outlet $i \in C^{\text {pay }}$ chooses reporting accuracy $v_{i} \in V_{i}$ and price $p_{i} \in P_{i} \subseteq$ $\mathbb{R}_{+}$, where $P_{i}$ contains $p_{i}=0$ and is compact. ${ }^{10}$ Utility from outlet $i$ is $u_{i}=v_{i}-p_{i}$, utility from an outlet $j \in C^{f} \cup P$ is simply $u_{j}=v_{j}$.

The profit of a pay media outlet $i \in C^{p a y}$ is

$$
\pi_{i}\left(v_{i}, p_{i}, u_{-i}\right)=s_{i}\left(u_{i}, u_{-i}\right)\left(R_{i}\left(v_{i}\right)+p_{i}\right)-c_{i}\left(v_{i}\right),
$$

where $u_{-i}$ is the vector of utilities offered by the other outlets. The profit of a free media outlet $i \in C^{f}$ is

$$
\pi_{i}\left(u_{i}, u_{-i}\right)=s_{i}\left(u_{i}, u_{-i}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}\right) .
$$

[^6]In all other respects, the model is as in Section 2 above.
For pay media, results depend on whether we consider dumbing down content or investigative journalism (see the discussion after Assumption (4) above). Results are clear cut in the case of dumbing down content, where $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$. We show that in this case, when some competitor $j \neq i$ increases the utility $u_{j}$, ceteris paribus outlet $i$ will also offer a higher utility: pay media keep their reporting accuracy constant but lower their price, while free media increase their reporting accuracy. We briefly discuss the case of investigative journalism towards the end of this section.

Proposition 4 Assume that $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$ for all $i \in C^{\text {pay }}$. Then outlet $i \in C^{\text {pay }}$ will choose reporting accuracy

$$
v_{i}=\bar{v}_{i}:=\arg \max _{v_{i} \in V_{i}}\left(R_{i}\left(v_{i}\right)+v_{i}\right)
$$

Moreover, $\bar{v}_{i}$ is independent of price $p_{i}$ chosen by outlet $i$, and independent of the strategies of the other media outlets.

Proof. We adapt a technique from Armstrong (2006) and proceed in two steps to solve the maximization problem of $i \in C^{p a y}$. The first step maximizes profits by choosing $v_{i}$, holding consumer utility $u_{i}=v_{i}-p_{i}$ constant at a given level $\bar{u}_{i}$ by implicitly adjusting the price. The second step maximizes by choosing the price, or equivalently consumer utility. Formally, the first step is

$$
\max _{v_{i} \in V_{i}} s_{i}\left(\bar{u}_{i}, u_{-i}\right)\left(R_{i}\left(v_{i}\right)+v_{i}-\bar{u}_{i}\right)-c_{i}\left(v_{i}\right)
$$

In this maximization problem, consumer utility is constant, hence demand $s_{i}$ is constant as well. Because $c_{i}\left(v_{i}\right)$ is constant, the profit maximizing $v_{i}$ does not depend on $c_{i}$, and $i$ maximizes $s_{i}\left(\bar{u}_{i}, u_{-i}\right)\left(R_{i}\left(v_{i}\right)+v_{i}-\bar{u}_{i}\right)$. Moreover, $s_{i}$ is just a multiplicative constant in this objective function that does not change the profit maximizing $v_{i}$. Hence the solution is $v_{i}=\bar{v}_{i}$ as defined in the proposition. Note that $\bar{v}_{i}$ is independent of the price $p_{i}$ and the other firms' strategies.

The reporting accuracy $\bar{v}_{i}$ might be so high that outlet $i \in C^{\text {pay }}$ has no advertising revenue and is funded only by payments from its audience. On the other hand, if $R_{i}\left(\bar{v}_{i}\right)>0$, then outlet $i$ has two sources of revenue, advertisers and consumers.

Proposition 4 allows us to consider the game as a game where the pay media have only one choice variable, their price. To find the profit maximizing prices, substitute
$v_{i}=\bar{v}_{i}$ into the profit function of outlet $i \in C^{p a y}$. Instead of maximizing profits by choosing $p_{i}$, we can equivalently think of firm $i$ as choosing utility $u_{i}$, taking into account that $u_{i}=\bar{v}_{i}-p_{i}$ so $p_{i}=\bar{v}_{i}-u_{i}$. The set of utilities that outlet $i$ can choose from is $U_{i}:=\left\{u_{i} \mid u_{i}=\bar{v}_{i}-p_{i}, p_{i} \in P_{i}\right\}$.

The objective function is

$$
\hat{\pi}_{i}\left(u_{i}, u_{-i}\right):=s_{i}\left(u_{i}, u_{-i}\right)\left(R_{i}\left(\bar{v}_{i}\right)+\bar{v}_{i}-u_{i}\right)-c_{i}\left(v_{i}\right)
$$

Mirroring our definitions leading to Proposition 2 above, let

$$
\tilde{\pi}_{i}\left(u_{i}, u_{-i}^{C}, b\right)=\left\{\begin{array}{lc}
\pi_{i}\left(u_{i}, u_{-i}^{C}, \bar{v}^{P}(b)\right), & \text { if } i \in C^{f} \\
\hat{\pi}_{i}\left(u_{i}, u_{-i}^{C}, \bar{v}^{P}(b)\right), & \text { if } i \in C^{\text {pay }}
\end{array}\right.
$$

Let $\Gamma_{b}^{\text {pay }}=\left(C,\left(\tilde{\pi}_{i}\right)_{i=1}^{n}, \Pi_{i=1}^{n_{f}} V_{i} \times \Pi_{i=n_{f}+1}^{n} U_{i}\right)$ denote denote the resulting game between the commercial media outlets: the set of players is $C=C^{f} \cup C^{\text {pay }}$, payoff functions are $\tilde{\pi}_{i}$, outlets $i \in C^{f}$ chooses $u_{i} \in V_{i}$, outlets $i \in C^{\text {pay }}$ choose $u_{i} \in U_{i}$ while their reporting accuracy is fixed at $\bar{v}_{i}$ by Proposition 4, and as above the utilities offered by the PSM are given by $\bar{v}^{P}(b)$.
 parameterized supermodular game.

Proof. To begin with, since $P_{i} \subseteq \mathbb{R}_{+}$is compact, the set of feasible utilities $U_{i}:=$ $\left\{u_{i} \mid u_{i}=\bar{v}_{i}-p, p_{i} \in P_{i}\right\} \subseteq \mathbb{R}$ is compact as well.

For simplicity, we will only give the proof assuming differentiability. For $i \in C^{\text {pay }}$ and $j \neq i$,

$$
\begin{aligned}
\frac{\partial^{2} \hat{\pi}_{i}}{\partial u_{j} \partial u_{i}} & =\frac{\partial}{\partial u_{j}}\left(\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}}\left(R_{i}\left(\bar{v}_{i}\right)+\bar{v}_{i}-u_{i}\right)-s_{i}\left(u_{i}, u_{-i}\right)\right) \\
& =\frac{\partial^{2} s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{j} \partial u_{i}}\left(R_{i}\left(\bar{v}_{i}\right)+\bar{v}_{i}-u_{i}\right)-\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{j}} \geq 0
\end{aligned}
$$

where the inequality follows because of $\frac{\partial^{2} s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{j} \partial u_{i}} \geq 0$ by Assumption (2), $R_{i}\left(\bar{v}_{i}\right) \geq 0$ and $\bar{v}_{i}-u_{i}=p_{i} \geq 0$, and $\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{j}} \leq 0$ by Assumption (1).

The remainder of the proof is similar to the proof of Proposition 2.
Proposition 5 shows that $\Gamma_{b}^{p a y}$ has increasing reaction functions. That is, when some competitor of a commercial media outlet $i$ increases the utility it offers, ceteris paribus
outlet $i$ will also offer a higher utility. For a pay media outlet $i \in C^{\text {pay }}$, reporting accuracy is constant by Proposition 4, but $i$ lowers its price. The economics behind the result is straightforward: tougher competitors reduce residual demand, and as a reaction firm $i$ charges a lower price. Proposition 5 also shows that the free media will, as in Section 3 above, ceteris paribus react to increases in an competitor's utility by increasing their reporting accuracy.

As above, we can leverage the theory of supermodular games to obtain results on $\Gamma_{b}^{p a y}$. In particular, Proposition 3 generalizes in the following way: ${ }^{11}$

Proposition 6 Assume that $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$ for all $i \in C^{\text {pay }}$, and suppose that $\Gamma_{b}^{p a y}$ has a unique equilibrium. Then an increase of the utility $u_{j}$ of a PSM $j \in P$ will increase the utilities $u_{i}$ offered by all commercial outlets: the reporting accuracies of free media increase, the reporting accuracies of pay media stay constant but their prices decline.

To conclude this section, we briefly consider to the case of investigative journalism. We point out that the assumption that $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$ for all $i \in C^{\text {pay }}$ is crucial for our proofs of Propositions 4 to 6. If $c_{i}$ is strictly increasing in $v_{i}$, then the equilibrium reporting accuracy of firm $i$ can be strictly decreasing in $u_{-i}$, and the total effect of $u_{-i}$ on $u_{i}$ is ambiguous, as we show in example in Appendix B below. We leave a full exploration of pay media in the case of investigative journalism for future research.

## 5 Discussion

Of course, our model abstracts away from several issues that could potentially be relevant. In this section, we discuss multidimensional strategy spaces, spillover effects of reporting accuracy on the advertising revenue of other media outlets, demand functions with decreasing differences, income effects, entry, and potential biases in PSM. We focus on free media, and assume differentiability wherever convenient for expositional simplicity.

[^7]
### 5.1 Multidimensional strategy spaces

Media outlets may choose different reporting accuracies for different topics, and may choose other dimensions of program quality that are less of a concern for advertisers.

Suppose that outlet $i$ reports about $k_{i}$ topics, and let $v_{i, k}$ denote reporting accuracy about topic $k$. Outlet $i$ chooses a vector $v_{i} \in V_{i} \subseteq \mathbb{R}_{+}^{k_{i}}$ of reporting accuracies. We assume $V_{i}$ is compact and contains 0 . Consumer utility from outlet $i$ is $u_{i}=f_{i}\left(v_{i}\right)$, where $f_{i}$ is continuous, strictly increasing, and satisfies $f_{i}(0)=0$. Advertising revenue per consumer is $R_{i}\left(v_{i}\right)$, where $R_{i}: V_{i} \rightarrow \mathbb{R}_{+}$is positive, continuous, decreasing in $v_{i}$, and independent of $v_{-i}$. The profit of $i \in C$ is $\pi_{i}=s_{i}\left(u_{i}, u_{-i}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}\right)$. Turning to the PSM, suppose that $i \in P$ chooses $v_{i} \in V_{i}\left(b_{i}\right)$ subject to $c_{i}\left(v_{i}\right) \leq b_{i}$. As above, a higher budget may enlarge the feasible set $V_{i}$.

Dumbing down In the case of dumbing down content, where there are no direct costs of raising accuracy so $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$ for all $i \in C$, our results generalize in a similar way as in the case of pay media considered above.

To see why, decompose the profit maximization problem of a commercial outlet into two steps. In the first step, the vector of reporting accuracies is chosen to maximize advertising revenue per consumer, subject to the constraint that the utility of the consumer is at least equal to some given $u_{i}$. The maximal value of advertising revenue under this constraint is

$$
R_{i}^{*}\left(u_{i}\right)=\max _{v_{i} \in V_{i}}\left\{R_{i}\left(v_{i}\right) \mid f_{i}\left(v_{i}\right) \geq u_{i}\right\} .
$$

We show in Appendix C. 1 that $R_{i}^{*}$ has all the features assumed about $R_{i}$ in our main model (see Assumption (3)): $R_{i}^{*}$ is positive, continuous, decreasing in $u_{i}$ and independent of $u_{-i}$.

The second step then optimizes over $u_{i}$. The choice set is

$$
U_{i}:=\left\{u_{i} \in \mathbb{R}_{+} \mid \exists v_{i} \in V_{i}: \quad f_{i}\left(v_{i}\right) \geq u_{i}\right\}
$$

We show in Appendix C. 1 that $U_{i}$ has all the features assumed about the strategy set $V_{i}$ in our main model.

This two-step procedure allows us to consider the interaction between the commercial outlets as a game where each outlet $i \in C$ has one decision variable $u_{i}$, choice set
$U_{i}$, and payoff function

$$
\pi_{i}\left(u_{i}, u_{-i}\right)=s_{i}\left(u_{i}, u_{-i}\right) R_{i}^{*}\left(u_{i}\right) .
$$

From here, the analysis is as in our main model above. In particular, it follows that if the equilibrium is unique, an increase of the budget of a PSM increases the utilities offered by all commercial outlets.

Investigative reporting Consider next the case of investigative reporting. Trivially, as long as the game remains supermodular, our results generalize. A relevant concern is, however, whether the profit of an outlet will be supermodular in its own choice variables.

We illustrate this with a two-dimensional case inspired by Germano and Meier (2013). Instead of denoting the choice variable of outlet $i$ by $\left(v_{i 1}, v_{i 2}\right)$, we denote it by $\left(v_{i}, y_{i}\right)$ to avoid notational clutter, slightly abusing notation. Suppose that outlet $i$ chooses reporting accuracy $v_{i} \in V_{i} \subseteq \mathbb{R}_{+}$and quality $y_{i} \in Y_{i} \subseteq \mathbb{R}_{+}$, where $V_{i}$ and $Y_{i}$ are compact and contain 0 . The quality $y_{i}$ does not affect $R_{i}$. Consumer utility from outlet $i$ is $u_{i}=f_{i}\left(v_{i}, y_{i}\right)$, where $f_{i}$ is a continuous and strictly increasing function with $f_{i}(0,0)=0$. The profit of $i \in C$ is

$$
\pi_{i}=s_{i}\left(u_{i}, u_{-i}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}, y_{i}\right)
$$

PSM $i \in P$ maximizes $u_{i}=f_{i}\left(v_{i}, y_{i}\right)$ subject to $c_{i}\left(v_{i}, y_{i}\right) \leq b_{i}$ by choosing $v_{i} \in V_{i}$ and $y_{i} \in Y_{i}$. As above, a higher budget may enlarge the feasible sets $V_{i}$ and $Y_{i}$.

We show in Appendix C. 2 that $\pi_{i}$ has increasing differences in $\left(\left(v_{i}, y_{i}\right),\left(v_{-i}, y_{-i}\right)\right)$. For a supermodular game, however, $\pi_{i}$ also needs to be supermodular in $\left(v_{i}, y_{i}\right)$. Whether or not this is the case depends on the $\operatorname{cost}$ function $c_{i}$ and the utility function $f_{i}$.

If there are sufficiently strong economies of scope in producing $\left(v_{i}, y_{i}\right)$, or if there are sufficiently strong complementarities between $v_{i}$ and $y_{i}$ for the consumers, then $\pi_{i}$ is supermodular in $\left(v_{i}, y_{i}\right)$. If this is the case for all commercial outlets $i \in C$, our results generalize. In particular, Proposition 3 generalizes in the sense that an increase in the budget of a PSM increases both $y_{i}$ and $v_{i}$ of all commercial outlets $i \in C$, and the utility $u_{i}$ of all consumers increases.

On the other hand, $\pi_{i}$ will be submodular in $\left(y_{i}, v_{i}\right)$ if $s_{i}$ is concave in $u_{i}$ and there
are no complementarities stemming from the cost function $c_{i}$ or the utility function $f_{i}$. In this case, the effect of a higher budget for the PSM on the utility of the audience of commercial media is ambiguous, as we show in an example in Appendix C.3.

### 5.2 Spillover effects of reporting accuracies on advertising revenue of other media outlets

Our main model assumes that the advertising revenue of a commercial media outlet depends on its own reporting accuracy, but not on the reporting accuracies of other media outlets. Arguably, advertising revenue of all outlets might be negatively affected when some outlets report about deficiencies of a product, hence there may be spillover effects as in Germano and Meier (2013). Our results are robust when these spillover effects are small compared to the direct effect of an outlet's own reporting on its advertising revenue. To make this precise, replace Assumption (3) by

Assumption (3') For all $i \in C, R_{i}$ is positive, continuous, decreasing in $\left(v_{i}, v_{-i}\right)$, and has increasing differences in $\left(v_{i}, v_{-i}\right)$.

Note that, since $R_{i}$ is decreasing in $v_{i}$, increasing differences here mean that advertising revenue is not as severely affected by an increase in $v_{i}$ when other outlets have a high reporting accuracy, which seems a reasonable assumption. We show in Appendix D. 1 that, under Assumptions (1), (2), (3'), and (4), a sufficient condition for $\Gamma_{b}$ to be a parameterized supermodular game is that spillover effects are small in the sense that

$$
\frac{\left|\frac{\partial R_{i}}{\partial v_{j}}\right|}{\left|\frac{\partial s_{i}}{\partial v_{j}}\right|} \leq \frac{\left|\frac{\partial R_{i}}{\partial v_{i}}\right|}{\frac{\partial s_{i}}{\partial v_{i}}}
$$

for all $i \in C$ and all $j \neq i$.
If spillover effects were as strong as the direct effects, however, the strategic complementarities between the outlets' reporting accuracies might cease to exist, and reporting accuracies might become independent of each other. We illustrate this in an example in Appendix D.2.

### 5.3 Demand functions with decreasing differences: logit model

Violations of Assumption (2) do not overturn our results when the elasticity of advertising revenue with respect to reporting accuracy is sufficiently high. To see this, note that the crucial inequality (1) in the proof of Proposition 2 holds if

$$
\frac{\left|R_{i}^{\prime}\left(v_{i}\right)\right|}{R_{i}\left(v_{i}\right)} \geq \frac{\frac{\partial^{2} s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j} \partial v_{i}}}{\frac{\partial s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j}}} .
$$

Under Assumption (2), the right hand side is negative hence the above inequality is always satisfied; when Assumption (2) is violated advertising revenue must react sufficiently strong to reporting accuracy for the inequality to hold.

To illustrate, suppose there is mass of consumers normalized to one, and the market share of outlet $i$ is

$$
s_{i}\left(v_{i}, v_{-i}\right)=\frac{f_{i}\left(v_{i}\right)}{\sum_{j=0}^{n+m} f_{j}\left(v_{j}\right)},
$$

where the functions $f_{i}\left(v_{i}\right)$ are strictly positive and strictly increasing, and $v_{0}$ is the utility of the outside option. We allow (but do not require) the functions $f_{i}$ to differ across media outlets. The logit model is a special case where $f_{i}\left(v_{i}\right)=\exp \left(\mu v_{i}\right)$ for some exogenous parameter $\mu>0$.

This demand function satisfies Assumption (1), but in general violates Assumption (2). In particular, if there are two or more commercial media outlets, $s_{i}$ cannot have increasing differences for all $i \in C$, as we show in Appendix E. There we also prove, however, that a sufficient condition for $\Gamma_{b}$ to be a supermodular game is that

$$
\frac{\left|R_{i}^{\prime}\left(v_{i}\right)\right|}{R_{i}\left(v_{i}\right)} \geq \frac{f_{i}^{\prime}\left(v_{i}\right)}{f_{i}\left(v_{i}\right)}
$$

for all $i \in C$. In the logit model, this sufficient condition reduces to $\left|R_{i}^{\prime}\left(v_{i}\right)\right| / R_{i}\left(v_{i}\right) \geq \mu$ for all $i \in C$.

This illustration shows that, while Assumption (2) is restrictive, decreasing differences in the demand functions do not necessarily overturn our results when advertising revenue reacts strongly on reporting accuracy.

### 5.4 Income effects

Consumers in our model have to pay taxes or licence fees to cover the budgets of the PSM. These payments are independent of individual media consumption. They could, however, affect demand via income effects. Such income effects can strengthen our main results, however, when the media are normal goods, i.e. demand increases in income.

Suppose that for $i \in C, s_{i}\left(v_{i}, v_{-i}, b\right)$ is decreasing in $b$ (the higher $b$, the lower consumers' remaining income; if media are normal goods, demand is lower). Moreover, suppose that $s_{i}$ has increasing differences in $\left(v_{i}, b\right)$, i.e. demand reacts more on quality differences when income is lower. The strategic complementarities between the commercial media are not affected by the income effects. For $i \in C$ and $j \in P$, consider the cross-partial

$$
\begin{aligned}
\frac{\partial^{2} \tilde{\pi}_{i}}{\partial b_{j} \partial v_{i}} & =\left(\frac{\partial^{2} s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j} \partial v_{i}} R_{i}\left(v_{i}\right)+\frac{\partial s_{i}\left(v_{i}, v_{-i}\right)}{\partial v_{j}} R_{i}^{\prime}\left(v_{i}\right)\right) \bar{v}_{j}^{\prime}\left(b_{j}\right) \\
& +\frac{\partial^{2} s_{i}\left(v_{i}, v_{-i}, b\right)}{\partial b_{j} \partial v_{i}} R_{i}\left(v_{i}\right)+\frac{\partial s_{i}\left(v_{i}, v_{-i}, b\right)}{\partial b_{j}} R_{i}^{\prime}\left(v_{i}\right) .
\end{aligned}
$$

The first line describes the effects studied in our main model above: an increase of $b_{j}$ increases $\bar{v}_{j}$ and this has the effects studied above (the terms in the bracket are the same as in inequality (1) in the proof of Proposition 2). The second line stems from the income effect. Note that $\frac{\partial^{2} s_{i}\left(v_{i}, v_{-i}, b\right)}{\partial b_{j} \partial v_{i}} \geq 0$ because $s_{i}$ has increasing differences in $\left(v_{i}, b\right)$, and $\frac{\partial s_{i}\left(v_{i}, v_{-i}, b\right)}{\partial b_{j}} \leq 0$ because good $i$ is normal; hence the second line is positive. This shows that income effects strengthen the strategic complementarities that drive our results.

On the other hand, PSM might lead commercial media to exit the market. Income effects can strengthen this type of crowding out: the PSM do not only offer competing products, but also lower demand for commercial media via income effects. We discuss entry and exit next.

### 5.5 Entry and exit

Our results on strategic complementarities apply to situations where the PSM do not induce any of the commercial outlets to exit the market. In reality, when the PSM budgets are sufficiently increased, commercial outlets may be driven out of business.

By the same token, if PSM were scaled back, this could trigger entry of additional commercial outlets. The new entrants would have to provide sufficiently high quality in order to overturn the results in Proposition 3, however.

To illustrate, suppose the PSM were abolished in favor of a purely commercial media market. Without additional entry, our results above predict that reporting accuracy of the commercial outlets would decline. Entry of $m$ additional commercial outlets would keep the number of media outlets constant. If these entrants provide lower reporting accuracy than the PSM used to, however, incumbent commercial media will still provide lower reporting accuracy than before the commercialization of the media market, and so all consumers are negatively affected by lower reporting accuracies. Whether there will be sufficiently many entrants with sufficiently high reporting accuracy to overcome this negative effect on consumers will depend, among other things, on barriers to entry, the revenue potential of the market, the PSMs' budgets, and possibly cost advantages or disadvantages of new entrants.

### 5.6 Biases in PSM

Our model allows PSM to be cost-inefficient but assumes them to be unbiased. Of course, PSM may themselves be biased as well (see e.g. Crawford and Levonyan 2018). Commercial biases can exist in PSM, as they also engage in advertising or product placement. In some countries, governments are major advertisers themselves (Di Tella and Franceschelli 2011, Szeidl and Szucs 2021), and PSM may be especially susceptible to government influence. Moreover, advertising revenue helps against other sources of biases in media content (see for example Besley and Prat 2006 and Petrova 2011).

While such considerations are clearly important, we point out that our results can allow for some biases in PSM. The strategic complementarities between the reporting accuracies of commercial media do not depend on assumptions about the PSM at all. Concerning the impact of PSM budgets on commercial media, the key issue is whether a higher budget of a PSM will translate into more or less severe biases of this PSM outlet. As long as the reporting accuracies of PSM are increasing in their budgets, Propositions 2 and 3 are robust to biases in the PSM.

As in our discussion of entry above, any evaluation of the impact of potentially biased PSM on the content of commercial media outlets crucially depends on what the alternatives to PSM are. PSM are typically not for profit, and they often face tighter
limitations on advertising than commercial outlets (see e.g. Crawford et al. 2017), which may counteract commercial media bias (Kerkhof and Münster 2015). Indeed, PSM typically have a higher share of hard news and socially relevant topics in their program, so their commercial biases may be lower (see Cushion 2017 for a wide ranging review).

## 6 Conclusion

This paper shows that in a standard model of commercial media bias, program qualities in terms of the media's reporting accuracy are strategic complements rather than strategic substitutes. The strategic complementarity stems from the media's fundamental trade-off in these models: Increasing reporting accuracy increases the value of the media content for the audience but decreases the willingness to pay of the advertisers to reach consumers. The latter effect becomes less important when a media company has a smaller audience; hence, its incentives to increase reporting accuracy are higher. Thus, in a media market with both public service media (PSM) and commercial media, raising the PSMs' reporting accuracy reduces the commercial media's audiences and thereby also the implicit costs of increasing their own reporting accuracy. As a result, the PSM crowd in reporting accuracy and engage the commercial media in a race to the top. This is in line with recent empirical evidence on public and private investments into program quality and on the impact of competition in media markets.

Our finding contributes to recurrent media policy debates about the proper role and scope of PSM. While several regulation authorities fear that raising the program quality of PSM could crowd out private investments into program quality, our results support policies that advocate strong and financially well-equipped PSM.

## A Increasing differences without differentiability

In the proof of Proposition 2, we assumed that the functions $s_{i}, R_{i}$, and $c_{i}$ are differentiable in order to prove that $\pi_{i}$ has increasing differences in $\left(v_{i}, v_{-i}\right)$. In this appendix we give the proof without assuming differentiability. Consider one outlet $j \neq i$, hold all other $v_{k}(k \neq i, j)$ constant and suppress them in the formulas to avoid notational
clutter. Then

$$
\pi_{i}\left(v_{i}, v_{j}\right)=s_{i}\left(v_{i}, v_{j}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}\right)
$$

Suppose that $v_{i}^{h}>v_{i}^{l}$ and $v_{j}^{h}>v_{j}^{l}$. Then

$$
\pi_{i}\left(v_{i}, v_{j}^{h}\right)-\pi_{i}\left(v_{i}, v_{j}^{l}\right)=\left(s_{i}\left(v_{i}, v_{j}^{h}\right)-s_{i}\left(v_{i}, v_{j}^{l}\right)\right) R_{i}\left(v_{i}\right)
$$

and

$$
\begin{aligned}
& \pi_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-\pi_{i}\left(v_{i}^{h}, v_{j}^{l}\right)-\left(\pi_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-\pi_{i}\left(v_{i}^{l}, v_{j}^{l}\right)\right) \\
& =\left(s_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{h}, v_{j}^{l}\right)\right) R_{i}\left(v_{i}^{h}\right)-\left(s_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)\right) R_{i}\left(v_{i}^{l}\right) \\
& =\left(s_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{h}, v_{j}^{l}\right)-\left(s_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)\right)\right) R_{i}\left(v_{i}^{h}\right) \\
& +\left(s_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)\right)\left(R_{i}\left(v_{i}^{h}\right)-R_{i}\left(v_{i}^{l}\right)\right)
\end{aligned}
$$

By Assumption (2), $s_{i}$ has increasing differences in $\left(v_{i}, v_{j}\right)$, i.e.

$$
s_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{h}\right) \geq s_{i}\left(v_{i}^{h}, v_{j}^{l}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)
$$

or equivalently

$$
s_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{h}, v_{j}^{l}\right) \geq s_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right) .
$$

Since $R_{i}\left(v_{i}^{h}\right) \geq 0$, it follows that

$$
\left(s_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{h}, v_{j}^{l}\right)-\left(s_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)\right)\right) R_{i}\left(v_{i}^{h}\right) \geq 0
$$

Moreover, by (2) $s_{i}\left(v_{i}^{l}, v_{j}^{h}\right) \leq s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)$ and by (3), $R_{i}\left(v_{i}^{h}\right) \leq R_{i}\left(v_{i}^{l}\right)$, thus

$$
\left(s_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-s_{i}\left(v_{i}^{l}, v_{j}^{l}\right)\right)\left(R_{i}\left(v_{i}^{h}\right)-R_{i}\left(v_{i}^{l}\right)\right) \geq 0
$$

It follows that

$$
\pi_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-\pi_{i}\left(v_{i}^{h}, v_{j}^{l}\right) \geq \pi_{i}\left(v_{i}^{l}, v_{j}^{h}\right)-\pi_{i}\left(v_{i}^{l}, v_{j}^{l}\right)
$$

or equivalently

$$
\pi_{i}\left(v_{i}^{h}, v_{j}^{h}\right)-\pi_{i}\left(v_{i}^{l}, v_{j}^{h}\right) \geq \pi_{i}\left(v_{i}^{h}, v_{j}^{l}\right)-\pi_{i}\left(v_{i}^{l}, v_{j}^{l}\right)
$$

i.e. $\pi_{i}$ has increasing differences in $\left(v_{i}^{h}, v_{j}^{h}\right)$.

## B Pay media and investigative reporting: an example

In this appendix we consider an example of a pay media outlet in the case of investigative journalism. Consider a Hotelling duopoly. Outlet 1 is a pay media outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to $u_{2}$; for this exercise it does not matter whether outlet 2 is another commercial (pay or free) media outlet or a PSM.

Example 1 Suppose that $V_{1}=\mathbb{R}_{+}, c_{1}\left(v_{1}\right)=k v_{1}^{2} / 2$ where $k$ is a parameter, and $R_{1}\left(v_{1}\right)=\max \left\{1-\beta v_{1}, 0\right\}$ with $0<\beta<1$. The total audience has a fixed size normalized to 1 , and the market share of outlet 1 is given by the Hotelling specification

$$
s_{1}\left(v_{1}, p_{1}, u_{2}\right)=\left\{\begin{array}{cc}
0, & \text { if } \frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau} \leq 0 \\
\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau}, & \text { if } 0<\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau}<1, \\
1, & \text { otherwise }
\end{array}\right.
$$

The profit of outlet 1 is

$$
\pi_{1}\left(v_{1}, p_{1}, u_{2}\right)=s_{1}\left(v_{1}, p_{1}, u_{2}\right)\left(R_{1}\left(v_{1}\right)+p_{1}\right)-\frac{k v_{1}^{2}}{2}
$$

Assume that

$$
\begin{equation*}
4 k \tau>(1-\beta)^{2} \tag{2}
\end{equation*}
$$

in order that $\pi_{1}$ is strictly concave in $\left(v_{1}, p_{1}\right)$ in the relevant range. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_{1}>0, p_{1}>0$, $R_{1}>0$ and $0<s_{1}<1 .{ }^{12}$

Note that for this example $k$ has to be sufficiently high for the second order condition to hold, hence the case of dumbing down is not a limit case of this example.

Remark 1 In Example 1, $v_{1}$ and $p_{1}$ are strictly decreasing in $u_{2}$. Moreover, $u_{1}=v_{1}-p_{1}$ is strictly increasing in $u_{2}$ if $2 k \tau>(1-\beta)^{2}$, and strictly decreasing if $2 k \tau<(1-\beta)^{2}$.

[^8]Proof. In the relevant range,

$$
\pi_{1}\left(v_{1}, p_{1}, u_{2}\right)=\left(\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau}\right)\left(1-\beta v_{1}+p_{1}\right)-\frac{k v_{1}^{2}}{2} .
$$

The partial derivatives are

$$
\begin{aligned}
& \frac{\partial \pi_{1}}{\partial p_{1}}=-\frac{1}{2 \tau}\left(1-\beta v_{1}+p_{1}\right)+\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau} \\
& \frac{\partial \pi_{1}}{\partial v_{1}}=\frac{1}{2 \tau}\left(1-\beta v_{1}+p_{1}\right)-\beta\left(\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau}\right)-k v_{1} .
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
\frac{\partial^{2} \pi_{1}}{\partial p_{1}^{2}} & =-\frac{1}{\tau}<0 \\
\frac{\partial^{2} \pi_{1}}{\partial v_{1}^{2}} & =-\frac{\beta}{\tau}-k<0 \\
\frac{\partial^{2} \pi_{1}}{\partial p_{1} \partial v_{1}} & =\frac{1+\beta}{2 \tau}
\end{aligned}
$$

Hence the determinant of the Hessian is

$$
\frac{1}{\tau}\left(\frac{\beta}{\tau}+k\right)-\left(\frac{1+\beta}{2 \tau}\right)^{2}>0
$$

iff $4 k \tau>(1-\beta)^{2}$. This shows $\pi_{1}$ is strictly concave in the relevant range if inequality (2) holds.

The first order conditions for an interior solution are

$$
\begin{aligned}
\frac{1}{2 \tau}\left(1-\beta v_{1}+p_{1}\right) & =\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau} \\
\frac{1}{2 \tau}\left(1-\beta v_{1}+p_{1}\right) & =\beta\left(\frac{1}{2}+\frac{v_{1}-p_{1}-u_{2}}{2 \tau}\right)+k v_{1}
\end{aligned}
$$

Solving the first order conditions gives

$$
\begin{aligned}
& v_{1}^{*}\left(u_{2}\right)=\frac{\left(\tau+1-u_{2}\right)(1-\beta)}{4 k \tau-(1-\beta)^{2}}, \\
& p_{1}^{*}\left(u_{2}\right)=\frac{(\beta(1-\beta)+2 k \tau)\left(\tau-u_{2}\right)+1-\beta-2 k \tau}{4 k \tau-(1-\beta)^{2}} .
\end{aligned}
$$

Differentiate

$$
\begin{aligned}
& \frac{\partial v_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=-\frac{1-\beta}{4 k \tau-(1-\beta)^{2}}<0 \\
& \frac{\partial p_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=-\frac{\beta(1-\beta)+2 k \tau}{4 k \tau-(1-\beta)^{2}}<0
\end{aligned}
$$

Moreover, from $u_{1}^{*}\left(u_{2}\right)=v_{1}^{*}\left(u_{2}\right)-p_{1}^{*}\left(u_{2}\right)$,

$$
\frac{\partial u_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=\frac{2 k \tau-(1-\beta)^{2}}{4 k \tau-(1-\beta)^{2}}
$$

Therefore, $u_{1}^{*}\left(u_{2}\right)$ is strictly increasing in $u_{2}$ if $2 k \tau>(1-\beta)^{2}$, and $u_{1}^{*}\left(u_{2}\right)$ is strictly decreasing in $u_{2}$ if $2 k \tau<(1-\beta)^{2}$.

It remains to check under which parameter constellations an interior solution exists. Note that $v_{1}^{*}\left(u_{2}\right)>0$ iff

$$
\begin{equation*}
\tau+1>u_{2} \tag{3}
\end{equation*}
$$

and $p_{1}^{*}\left(u_{2}\right)>0$ iff

$$
\begin{equation*}
\tau+\frac{1-\beta-2 k \tau}{(\beta(1-\beta)+2 k \tau)}>u_{2} \tag{4}
\end{equation*}
$$

Note that

$$
\frac{1-\beta-2 k \tau}{(\beta(1-\beta)+2 k \tau)}<1
$$

by inequality (2). Thus inequality (3) is implied by inequality (4).
We turn to advertising revenue next. Note that

$$
R_{1}\left(v_{1}^{*}\left(u_{2}\right)\right)=1-\beta \frac{\left(\tau+1-u_{2}\right)(1-\beta)}{4 k \tau-(1-\beta)^{2}}
$$

is strictly positive iff

$$
\begin{equation*}
u_{2}>\tau+1-\frac{4 k \tau-(1-\beta)^{2}}{\beta(1-\beta)} \tag{5}
\end{equation*}
$$

Inequalities (4) and (5) hold simultaneously iff

$$
\begin{equation*}
\tau+\frac{1-\beta-2 k \tau}{(\beta(1-\beta)+2 k \tau)}>u_{2}>\tau+1-\frac{4 k \tau-(1-\beta)^{2}}{\beta(1-\beta)} \tag{6}
\end{equation*}
$$

By inequality (2), the right hand side is strictly smaller than the left hand side; therefore (6) is satisfied in an nonempty open set of values for $u_{2}$.

Finally, we need to make sure that $0<s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right)<1$. This is the case iff

$$
0<\frac{1}{2}+\frac{v_{1}^{*}\left(u_{2}\right)-p_{1}^{*}\left(u_{2}\right)-u_{2}}{2 \tau}<1
$$

or equivalently

$$
-\tau<v_{1}^{*}\left(u_{2}\right)-p_{1}^{*}\left(u_{2}\right)-u_{2}<\tau
$$

We have

$$
\begin{aligned}
& v_{1}^{*}\left(u_{2}\right)-p_{1}^{*}\left(u_{2}\right)-u_{2} \\
& =\frac{\left(\tau+1-u_{2}\right)(1-\beta)}{4 k \tau-(1-\beta)^{2}}-\frac{(\beta(1-\beta)+2 k \tau)\left(\tau-u_{2}\right)+1-\beta-2 k \tau}{4 k \tau-(1-\beta)^{2}}-u_{2} \\
& =\tau \frac{2 k-2 k \tau+(1-\beta)^{2}-2 k u_{2}}{4 k \tau-(1-\beta)^{2}}
\end{aligned}
$$

Thus $0<s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right)<1$ iff

$$
-1<\frac{2 k-2 k \tau+(1-\beta)^{2}-2 k u_{2}}{4 k \tau-(1-\beta)^{2}}<1
$$

or equivalently

$$
\begin{equation*}
-\left(4 k \tau-(1-\beta)^{2}\right)<2 k-2 k \tau+(1-\beta)^{2}-2 k u_{2}<4 k \tau-(1-\beta)^{2} \tag{7}
\end{equation*}
$$

The expression in the middle is a strictly decreasing function of $u_{2}$.
Since $u_{2}<\tau+1$ by (3),

$$
\begin{aligned}
2 k-2 k \tau+(1-\beta)^{2}-2 k u_{2} & >2 k-2 k \tau+(1-\beta)^{2}-2 k(\tau+1) \\
& =-\left(4 k \tau-(1-\beta)^{2}\right)
\end{aligned}
$$

thus the first inequality in (7) holds.

Similarly, by (5),

$$
\begin{aligned}
& 2 k-2 k \tau+(1-\beta)^{2}-2 k u_{2} \\
& <2 k-2 k \tau+(1-\beta)^{2}-2 k\left(\tau+1-\frac{4 k \tau-(1-\beta)^{2}}{\beta(1-\beta)}\right) \\
& =\frac{1}{\beta(1-\beta)}\left(\beta^{2}-\beta+2 k\right)\left(-\beta^{2}+2 \beta+4 k \tau-1\right)
\end{aligned}
$$

Therefore, a sufficient condition for the second inequality in (7) is that

$$
\begin{aligned}
& \left(4 k \tau-(1-\beta)^{2}\right)-\left(\frac{1}{\beta(1-\beta)}\left(\beta^{2}-\beta+2 k\right)\left(-\beta^{2}+2 \beta+4 k \tau-1\right)\right) \\
& =2(\beta(1-\beta)-k) \frac{4 k \tau-(1-\beta)^{2}}{\beta(1-\beta)}>0
\end{aligned}
$$

which is true iff

$$
\begin{equation*}
\beta(1-\beta)>k \tag{8}
\end{equation*}
$$

We have established that the problem has an interior solution under the conditions (8), (2), and (6), which we repeat here for convenience:

$$
\begin{aligned}
\beta(1-\beta) & >k \\
4 k \tau & >(1-\beta)^{2} \\
\tau+\frac{1-\beta-2 k \tau}{(\beta(1-\beta)+2 k \tau)} & >u_{2}>\tau+1-\frac{4 k \tau-(1-\beta)^{2}}{\beta(1-\beta)}
\end{aligned}
$$

To see they can be satisfied simultaneously, first choose $\beta$ and $k$ such that the first line holds. Then choose $\tau$ such that the second line holds; note that depending on how you choose $\tau$, either $2 k \tau>(1-\beta)^{2}$ or $2 k \tau<(1-\beta)^{2}$. Finally, choose $u_{2}$ for the last line.

A numerical example that satisfies all the constraints may be reassuring. Let $\beta=$ $0.5, \tau=1.25$, and $u_{2}=1.5$. For $k=0.11,2 k \tau=2 * 0.11 * 1.25=0.275>(1-\beta)^{2}=$ 0.25 and $u_{1}^{*}\left(u_{2}\right)$ is strictly increasing in $u_{2}$. For $k=0.09,2 k \tau=2 * 0.09 * 1.25=0.225<$ $0.25<4 k \tau=0.45$, and $u_{1}^{*}\left(u_{2}\right)$ is strictly decreasing.

Within our parameter restrictions, $u_{1}^{*}\left(u_{2}\right)$ is strictly increasing in $u_{2}$ if $k$ is large. An economic intuition is that the marginal costs of $v_{1}$ are rapidly increasing if $k$ is large, and hence then the falling price dominates the decrease in reporting accuracy.

To give more details, recall that $v_{1}^{*}\left(u_{2}\right)$ and $p_{1}^{*}\left(u_{2}\right)$ are strictly decreasing in $u_{2}$. If $k$ is large, the effect of $u_{2}$ on $v_{1}^{*}\left(u_{2}\right)$ becomes less important (smaller in absolute value):

$$
\frac{\partial}{\partial k} \frac{\partial v_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=\frac{4 \tau(1-\beta)}{\left(4 k \tau-(1-\beta)^{2}\right)^{2}}>0
$$

On the other hand, the effect of $u_{2}$ on $p_{1}^{*}\left(u_{2}\right)$ also becomes less important:

$$
\begin{aligned}
\frac{\partial}{\partial k} \frac{\partial p_{1}^{*}\left(u_{2}\right)}{\partial u_{2}} & =\frac{\partial}{\partial k}\left(-\frac{\beta(1-\beta)+2 k \tau}{4 k \tau-(1-\beta)^{2}}\right) \\
& =\frac{2 \tau\left(1-\beta^{2}\right)}{\left(4 k \tau-(1-\beta)^{2}\right)^{2}}>0
\end{aligned}
$$

But note that $4 \tau(1-\beta)-2 \tau\left(1-\beta^{2}\right)=2 \tau(1-\beta)^{2}>0$, thus

$$
\frac{\partial}{\partial k} \frac{\partial v_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}>\frac{\partial}{\partial k} \frac{\partial p_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}
$$

That is, if $k$ increases, the change of $v_{1}^{*}\left(u_{2}\right)$ in $u_{2}$ is vanishing quicker than the change of $p_{1}^{*}\left(u_{2}\right)$ in $u_{2}$. For large enough $k, u_{1}^{*}\left(u_{2}\right)$ increases in $u_{2}$ because the falling price overcompensates for the falling accuracy.

## C Multidimensional strategy spaces

## C. 1 Dumbing down

This appendix considers multidimensional strategy spaces in the case of dumbing down, where $c_{i}\left(v_{i}\right)$ is constant in $v_{i}$ for all $i \in C$. As in the main text, let

$$
U_{i}:=\left\{u_{i} \in \mathbb{R}_{+} \mid \exists v_{i} \in V_{i}: \quad f_{i}\left(v_{i}\right) \geq u_{i}\right\}
$$

We show that $U_{i}$ has the properties assumed about the choice set $V_{i}$ in our main model. That is, we show that $U_{i} \subseteq \mathbb{R}_{+}$is compact, and $U_{i}$ contains zero. Since $0 \in V_{i}$ and $f_{i}(0)=0,0 \in U_{i}$. Moreover, since $V_{i}$ is compact and $f_{i}$ is continuous, by the Weierstrass Theorem a maximum achievable utility exists, thus $U_{i}=\left[0, \max _{v_{i} \in V_{i}} f_{i}\left(v_{i}\right)\right]$ is compact.

As in the main text, let $R_{i}^{*}: U_{i} \rightarrow \mathbb{R}_{+}$be defined by

$$
R_{i}^{*}\left(u_{i}\right)=\max _{v_{i} \in V_{i}}\left\{R_{i}\left(v_{i}\right) \mid f_{i}\left(v_{i}\right) \geq u_{i}\right\} .
$$

We show that the function $R_{i}^{*}$ has all the properties required in Assumption (3).
First, $R_{i}^{*}$ is positive since $R_{i}$ is positive by assumption.
Second, we use the Maximum Theorem to show that $R_{i}^{*}$ is continuous. $R_{i}$ is continuous by assumption. It remains to show that the constraint correspondence, which gives for any $u_{i} \in U_{i}$ the set of reporting accuracies that achieve utility at least equal $u_{i}$, is continuous. Let $g_{i}: U_{i} \rightarrow V_{i}, g_{i}\left(u_{i}\right)=\left\{v_{i} \in V_{i} \mid f_{i}\left(v_{i}\right) \geq u_{i}\right\}$, denote the constraint correspondence. The range of $g_{i}$ is $V_{i}$, which is compact by assumption. Moreover, $g_{i}$ is upper hemicontinuous by continuity of $f_{i},{ }^{13}$ and $g$ is lower hemicontinuous by standard arguments establishing continuity of the expenditure function via the Maximum Theorem (see e.g. Kreps 2013, page 237).

Third, $R_{i}^{*}$ is decreasing in $u_{i}$. To see this, suppose to the contrary that $u_{i}^{1} \geq u_{i}^{0}$ but $R_{i}^{*}\left(u_{i}^{1}\right)>R_{i}^{*}\left(u_{i}^{0}\right)$. Then there exists $v_{i}^{1} \in V_{i}$ such that $f_{i}\left(v_{i}^{1}\right) \geq u_{i}^{1}$ and $R_{i}^{*}\left(u_{i}^{1}\right)=$ $R_{i}\left(v_{i}^{1}\right)$. But since $u_{i}^{1} \geq u_{i}^{0}$, it is also true that $f_{i}\left(v_{i}^{1}\right) \geq u_{i}^{0}$, and therefore

$$
R_{i}^{*}\left(u_{i}^{0}\right)=\max _{v_{i} \in V_{i}}\left\{R_{i}\left(v_{i}\right) \mid f_{i}\left(v_{i}\right) \geq u_{i}^{0}\right\} \geq R_{i}\left(v_{i}^{1}\right)=R_{i}^{*}\left(u_{i}^{1}\right),
$$

contradicting the assumption that $R_{i}^{*}\left(u_{i}^{1}\right)>R_{i}^{*}\left(u_{i}^{0}\right)$.
Fourth, $R_{i}^{*}$ is obviously independent of $v_{-i}$.

## C. 2 Investigative reporting: conditions for a supermodular game

This appendix studies conditions for a supermodular game when outlet $i$ chooses reporting accuracy $v_{i}$ and a quality $y_{i}$ which does not affect $R_{i}$. Recall that consumer utility from outlet $i$ is $u_{i}=f_{i}\left(v_{i}, y_{i}\right)$. For $i \in C$,

$$
\pi_{i}=s_{i}\left(u_{i}, u_{-i}\right) R_{i}\left(v_{i}\right)-c_{i}\left(v_{i}, y_{i}\right) .
$$

[^9]We first show that $\pi_{i}$ has increasing differences in $\left(\left(v_{i}, y_{i}\right),\left(v_{-i}, y_{-i}\right)\right)$.

$$
\begin{aligned}
& \frac{\partial \pi_{i}}{\partial v_{i}}=\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i}} R_{i}\left(v_{i}\right)+s_{i}\left(u_{i}, u_{-i}\right) R_{i}^{\prime}\left(v_{i}\right)-\frac{\partial c_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i}} \\
& \frac{\partial \pi_{i}}{\partial y_{i}}=\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} R_{i}\left(v_{i}\right)-\frac{\partial c_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} .
\end{aligned}
$$

Note that for all $j \neq i$ and $x_{j} \in\left\{v_{j}, y_{j}\right\}$,

$$
\frac{\partial}{\partial x_{j}} \frac{\partial \pi_{i}}{\partial v_{i}}=\left(\frac{\partial^{2} s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{j} \partial u_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i}} R_{i}\left(v_{i}\right)+\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{j}} R_{i}^{\prime}\left(v_{i}\right)\right) \frac{\partial f_{j}\left(v_{j}, y_{j}\right)}{\partial x_{j}} \geq 0
$$

and

$$
\frac{\partial}{\partial x_{j}} \frac{\partial \pi_{i}}{\partial y_{i}}=\frac{\partial^{2} s_{i}\left(f_{i}\left(v_{i}, y_{i}\right), u_{-i}\right)}{\partial u_{j} \partial u_{i}} \frac{\partial f_{j}\left(v_{j}, y_{j}\right)}{\partial x_{j}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} R_{i}\left(v_{i}\right) \geq 0,
$$

so $\pi_{i}$ has increasing differences in $\left(\left(v_{i}, y_{i}\right),\left(v_{-i}, y_{-i}\right)\right)$.
For a supermodular game, however, $\pi_{i}$ also needs to be supermodular in $\left(v_{i}, y_{i}\right)$. To study when this is the case, calculate the cross-partial

$$
\begin{aligned}
\frac{\partial^{2} \pi_{i}}{\partial y_{i} \partial v_{i}} & =\frac{\partial^{2} s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}^{2}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i}} R_{i}\left(v_{i}\right)+\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}} \frac{\partial^{2} f_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i} \partial y_{i}} R_{i}\left(v_{i}\right) \\
& +\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} R_{i}^{\prime}\left(v_{i}\right)-\frac{\partial^{2} c_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i} \partial v_{i}} .
\end{aligned}
$$

Thus $\pi_{i}$ will be supermodular in $\left(v_{i}, y_{i}\right)$ if there are pronounced economies of scope in producing $\left(v_{i}, y_{i}\right)$ so that $\frac{\partial^{2} c_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i} \partial v_{i}}$ is sufficiently negative, or if $v_{i}$ and $y_{i}$ are strong complements for the consumers so that $\frac{\partial^{2} f_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i} \partial y_{i}}$ is sufficiently positive.

But note that the third term in the above formula for the cross-partial of $\pi_{i}$,

$$
\frac{\partial s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} R_{i}^{\prime}\left(v_{i}\right)
$$

is negative. Moreover the first term

$$
\frac{\partial^{2} s_{i}\left(u_{i}, u_{-i}\right)}{\partial u_{i}^{2}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial y_{i}} \frac{\partial f_{i}\left(v_{i}, y_{i}\right)}{\partial v_{i}} R_{i}\left(v_{i}\right)
$$

is negative if $s_{i}$ is concave in $u_{i}$. Therefore, $\pi_{i}$ will be submodular in $\left(y_{i}, v_{i}\right)$ if there are no complementarities stemming from $c_{i}$ and $f_{i}$, and $s_{i}$ is concave in $u_{i}$. In this case, the effect of a higher budget for the PSM on the utility of the audience of commercial
media is ambiguous, as we show in an example in Appendix C.3.

## C. 3 Investigative reporting: an example with a submodular profit function

In this appendix, we show by example that $\pi_{i}$ may be strictly submodular in the choice variables of outlet $i$, and that in this case the effect of PSM on the utility of the product offered by outlet $i$ is ambiguous. We consider a Hotelling duopoly. Outlet 1 is a commercial outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to $u_{2}$; for this exercise it does not matter whether outlet 2 is another commercial (pay or free) media outlet or a PSM.

Example 2 Consider a duopoly where outlet 1 is a commercial outlet. Suppose $V_{1}=$ $Y_{1}=\mathbb{R}_{+}$. The audience has a fixed total size of 1 , and market share of outlet 1 is given by the Hotelling demand specification

$$
s_{1}\left(u_{1}, u_{2}\right)=\left\{\begin{array}{cc}
0, & \text { if } \frac{1}{2}+\frac{u_{1}-u_{2}}{2 \tau} \leq 0 \\
\frac{1}{2}+\frac{u_{1}-u_{2}}{2 \tau}, & \text { if } 0<\frac{1}{2}+\frac{u_{1}-u_{2}}{2 \tau}<1 \\
1, & \text { otherwise }
\end{array}\right.
$$

for $i=1,2$. Consumer utility from the commercial media outlet is $u_{1}=f_{1}\left(v_{1}, y_{1}\right)=$ $v_{1}+y_{1}$ and the cost function is $c_{1}\left(v_{1}, y_{1}\right)=k y_{1}^{2} / 2$. Advertising revenue per consumer is $R_{1}\left(v_{1}\right)=\max \left\{1-\beta v_{1}, 0\right\}$ where $\beta>0$ is an exogenous parameter. We assume that

$$
\begin{equation*}
4 k \tau>\beta \tag{9}
\end{equation*}
$$

in order that the $\pi_{1}$ is strictly concave in $\left(v_{1}, y_{1}\right)$ in the relevant range. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_{1}>0$, $y_{1}>0, R_{1}>0$ and $0<s_{1}<1 .{ }^{14}$

Note that in Example 2, there are no complementarities between $v_{1}$ and $y_{1}$ stemming from the cost function $c_{1}$ or the utility function $f_{1}$. Moreover, the demand function is linear in $u_{1}$ in the relevant range. As a consequence, $\pi_{1}$ is submodular in $\left(v_{1}, y_{1}\right)$ in the relevant range.

[^10]Remark 2 In Example 2, $v_{1}$ is strictly increasing in $u_{2}$, and $y_{1}$ is strictly decreasing in $u_{2}$. The utility offered by the commercial outlet, $u_{1}$, is strictly increasing in $u_{2}$ if $2 k \tau>\beta$, and strictly decreasing in $u_{2}$ if $2 k \tau<\beta$.

Proof. In the relevant range,

$$
\pi_{1}=\left(\frac{1}{2}+\frac{v_{1}+y_{1}-u_{2}}{2 \tau}\right)\left(1-\beta v_{1}\right)-\frac{k}{2} y_{1}^{2} .
$$

The first order conditions are

$$
\begin{aligned}
& \frac{\partial \pi_{1}}{\partial v_{1}}=\frac{1}{2 \tau}\left(1-\beta v_{1}\right)-\beta\left(\frac{1}{2}+\frac{v_{1}+y_{1}-u_{2}}{2 \tau}\right)=0 \\
& \frac{\partial \pi_{1}}{\partial y_{1}}=\frac{1}{2 \tau}\left(1-\beta v_{1}\right)-k y_{1}=0
\end{aligned}
$$

The second derivatives are

$$
\begin{aligned}
\frac{\partial^{2} \pi_{1}}{\partial v_{1}^{2}} & =-\frac{\beta}{\tau}<0 \\
\frac{\partial^{2} \pi_{1}}{\partial y_{1}^{2}} & =-k<0 \\
\frac{\partial^{2} \pi_{1}}{\partial v_{1} \partial y_{1}} & =-\frac{\beta}{2 \tau}<0 .
\end{aligned}
$$

The last inequality shows that $\pi_{1}$ is strictly submodular in $\left(v_{1}, y_{1}\right)$ in the relevant range. The determinant of the Hessian matrix is

$$
\frac{k \beta}{\tau}-\frac{\beta^{2}}{4 \tau^{2}}=\frac{\beta}{\tau}\left(k-\frac{\beta}{4 \tau}\right)>0
$$

iff $4 k \tau>\beta$; i.e. $\pi_{1}$ is strictly concave in the relevant range if inequality (9) holds.
Assuming an interior solution, the best reply function is

$$
\begin{aligned}
& v_{1}^{*}\left(u_{2}\right)=\frac{1}{\beta(4 k \tau-\beta)}\left(-\beta+2 k \tau-2 k \beta \tau^{2}+2 k \beta \tau u_{2}\right), \\
& y_{1}^{*}\left(u_{2}\right)=\frac{1}{4 k \tau-\beta}\left(\beta \tau-\beta u_{2}+1\right) .
\end{aligned}
$$

Note that, since $4 k \tau>\beta$ by assumption (9),

$$
\begin{aligned}
& \frac{\partial v_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=\frac{2 k \tau}{4 k \tau-\beta}>0, \\
& \frac{\partial y_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=\frac{-\beta}{4 k \tau-\beta}<0 .
\end{aligned}
$$

The utility offered by 1 is $u_{1}^{*}\left(u_{2}\right)=v_{1}^{*}\left(u_{2}\right)+y_{1}^{*}\left(u_{2}\right)$. Thus

$$
\frac{\partial u_{1}^{*}\left(u_{2}\right)}{\partial u_{2}}=\frac{2 k \tau-\beta}{4 k \tau-\beta}
$$

which is strictly positive if $2 k \tau>\beta$, but strictly negative if $2 k \tau<\beta$.
It remains to establish conditions on the fundamentals such that the solution is interior. Note that $v_{1}^{*}\left(u_{2}\right)>0$ if $u_{2}$ is sufficiently large, and $y_{1}^{*}\left(u_{2}\right)>0$ when $u_{2}$ is sufficiently small. We show that there exists a non-empty open interval of values for $u_{2}$ such that both $v_{1}^{*}\left(u_{2}\right)>0$ and $y_{1}^{*}\left(u_{2}\right)>0$. We have $v_{1}^{*}\left(u_{2}\right)>0$ iff

$$
u_{2}>\frac{1}{2 k \beta \tau}\left(2 k \beta \tau^{2}-2 k \tau+\beta\right)
$$

and $y_{1}^{*}\left(u_{2}\right)>0$ iff

$$
u_{2}<\frac{\beta \tau+1}{\beta} .
$$

Moreover,

$$
\frac{\beta \tau+1}{\beta}>\frac{1}{2 k \beta \tau}\left(2 k \beta \tau^{2}-2 k \tau+\beta\right),
$$

since by inequality (9)

$$
\frac{\beta \tau+1}{\beta}-\frac{1}{2 k \beta \tau}\left(2 k \beta \tau^{2}-2 k \tau+\beta\right)=\frac{1}{2 k \beta \tau}(4 k \tau-\beta)>0 .
$$

Therefore, whenever

$$
\begin{equation*}
\frac{1}{2 k \beta \tau}\left(2 k \beta \tau^{2}-2 k \tau+\beta\right)<u_{2}<\frac{\beta \tau+1}{\beta}, \tag{10}
\end{equation*}
$$

we have both $v_{1}^{*}\left(u_{2}\right)>0$ and $y_{1}^{*}\left(u_{2}\right)>0$.

We also need to make sure that $R_{1}\left(v_{1}^{*}\left(u_{2}\right)\right)>0$ and $s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right) \in(0,1)$.

$$
\begin{aligned}
R_{1}\left(v_{1}^{*}\left(u_{2}\right)\right) & =1-\frac{1}{(4 k \tau-\beta)}\left(-\beta+2 k \tau-2 k \beta \tau^{2}+2 k \beta \tau u_{2}\right) \\
& =\frac{2 k \tau}{4 k \tau-\beta}\left(\beta \tau+1-\beta u_{2}\right)>0,
\end{aligned}
$$

which is strictly positive because $\beta \tau+1>\beta u_{2}$ by (10).
Moreover,

$$
s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right)=\frac{k\left(\beta \tau+1-\beta u_{2}\right)}{\beta(4 k \tau-\beta)}>0
$$

by inequality (10). It remains to check whether $s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right)<1$. Note that $s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right)$ is strictly decreasing in $u_{2}$. By inequality (10),

$$
\frac{k\left(\beta \tau+1-\beta u_{2}\right)}{\beta(4 k \tau-\beta)}<\frac{k\left(\beta \tau+1-\beta\left(\frac{1}{2 k \beta \tau}\left(2 k \beta \tau^{2}-2 k \tau+\beta\right)\right)\right)}{\beta(4 k \tau-\beta)}=\frac{1}{2 \beta \tau}
$$

so a sufficient condition for $s_{1}\left(u_{1}^{*}\left(u_{2}\right), u_{2}\right)<1$ is that

$$
\begin{equation*}
2 \beta \tau>1 \tag{11}
\end{equation*}
$$

We have shown that the maximization problem has an interior solution if inequalities (11) (9), and (10) hold, which we repeat here for convenience:

$$
\begin{aligned}
2 \beta \tau & >1, \\
4 k \tau & >\beta, \\
\frac{1}{2 k \beta \tau}\left(2 k \beta \tau^{2}-2 k \tau+\beta\right) & <u_{2}<\frac{\beta \tau+1}{\beta} .
\end{aligned}
$$

To see they can be simultaneously satisfied, first choose $\beta$ and $\tau$ to satisfy the first inequality. Then choose $k$ to satisfy the second inequality; note that depending on how you choose $k$, you can have either $2 k \tau>\beta$ or $2 k \tau<\beta$. Finally, choose $u_{2}$ to satisfy the third inequality.

A numerical example that satisfies all the constraints may be reassuring. Let $\beta=$ $2 / 3, \tau=1$, and $u_{2}=2.2$. For $k=1,2 k \tau=2>\beta$ so $u_{1}$ is strictly increasing in $u_{2}$. For $k=\frac{2}{10}, 4 k \tau=\frac{8}{10}>\frac{2}{3}=\beta>\frac{4}{10}=2 k \tau$ so $u_{1}$ is strictly decreasing.

Within our parameter restrictions, $u_{1}$ is strictly increasing in in $u_{2}$ if $k$ is sufficiently large. An economic intuition for this is that, if $k$ is large, the marginal costs of $y_{1}$ are rapidly increasing, hence the positive impact of $u_{2}$ on $v_{1}$ overcompensates the negative impact of $u_{2}$ on $y_{1}$. In more detail, the reaction of $v_{1}^{*}$ and $y_{1}^{*}$ to changes in $u_{2}$ both become smaller in absolute value when $k$ increases:

$$
\begin{aligned}
\frac{\partial}{\partial k} \frac{\partial v_{1}^{*}\left(u_{2}\right)}{\partial u_{2}} & =\frac{-2 \beta \tau}{(4 k \tau-\beta)^{2}}<0 \\
\frac{\partial}{\partial k} \frac{\partial y_{1}^{*}\left(u_{2}\right)}{\partial u_{2}} & =\frac{4 \beta \tau}{(4 k \tau-\beta)^{2}}>0
\end{aligned}
$$

But $k$ affects the reaction of $y_{1}^{*}$ to changes in $u_{2}$ more than it affects the reaction of $v_{1}^{*}$, so for large enough values of $k$, the reaction of $v_{1}^{*}$ dominates the effect of $u_{2}$ on $u_{1}^{*}\left(u_{2}\right)$.

## D Spillover effects of reporting accuracies on advertising revenue of other media outlets

## D. 1 Small spillover effects

This appendix proves the claim that under Assumptions (1), (2), (3'), and (4), $\Gamma_{b}$ is a parameterized supermodular game if spillover effects are small in the sense that

$$
\frac{\left|\frac{\partial R_{i}}{\partial v_{j}}\right|}{\left|\frac{\partial s_{i}}{\partial v_{j}}\right|} \leq \frac{\left|\frac{\partial R_{i}}{\partial v_{i}}\right|}{\frac{\partial s_{i}}{\partial v_{i}}} .
$$

Differentiate

$$
\pi_{i}=s_{i}\left(v_{i}, v_{-i}\right) R_{i}\left(v_{i}, v_{-i}\right)-c_{i}\left(v_{i}\right)
$$

to obtain

$$
\begin{aligned}
\frac{\partial \pi_{i}}{\partial v_{i}} & =\frac{\partial s_{i}}{\partial v_{i}} R_{i}+s_{i} \frac{\partial R_{i}}{\partial v_{i}}-\frac{\partial c_{i}}{\partial v_{i}}, \\
\frac{\partial^{2} \pi_{i}}{\partial v_{j} \partial v_{i}} & =\frac{\partial^{2} s_{i}}{\partial v_{j} \partial v_{i}} R_{i}+\frac{\partial s_{i}}{\partial v_{i}} \frac{\partial R_{i}}{\partial v_{j}}+\frac{\partial s_{i}}{\partial v_{j}} \frac{\partial R_{i}}{\partial v_{i}}+s_{i} \frac{\partial^{2} R_{i}}{\partial v_{j} \partial v_{i}} .
\end{aligned}
$$

We have $\frac{\partial^{2} s_{i}}{\partial v_{j} \partial v_{i}} \geq 0$ by Assumption (2) and $\frac{\partial^{2} R_{i}}{\partial v_{j} \partial v_{i}} \geq 0$ by Assumption (3'). Therefore, $\frac{\partial^{2} \pi_{i}}{\partial v_{j} \partial v_{i}} \geq 0$ holds if

$$
\frac{\partial s_{i}}{\partial v_{i}} \frac{\partial R_{i}}{\partial v_{j}}+\frac{\partial s_{i}}{\partial v_{j}} \frac{\partial R_{i}}{\partial v_{i}} \geq 0
$$

Rearranging completes the proof.

## D. 2 Large spillover effects: an example

In this appendix, we show by example that $\pi_{i}$ may have constant differences in $\left(v_{i}, v_{-i}\right)$ if spillover effects are large. We consider a Hotelling duopoly. Outlet 1 is a commercial outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to $v_{2}$; for this exercise it does not matter whether outlet 2 is another free commercial media outlet or a PSM.

Example 3 Suppose that $V_{1}=\mathbb{R}_{+}$,

$$
R_{1}\left(v_{1}, v_{2}\right)=\max \left\{1-\left(\alpha v_{1}+\beta v_{2}\right), 0\right\}
$$

where $\alpha>0$ and $\beta>0$ are exogenous parameters, $s_{1}$ is given by a Hotelling specification. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_{1}>0, R_{1}>0$, and $0<s_{1}<1$. ${ }^{15}$

Note that $R_{1}$ in Example 3 satisfies Assumption (3').
Remark 3 Consider Example 3. If $\alpha>\beta$, then $\pi_{1}$ has strictly increasing differences in $\left(v_{1}, v_{2}\right)$ in the relevant range. If $\alpha=\beta$, then $\pi_{1}$ has constant differences in $\left(v_{1}, v_{2}\right)$ in the relevant range.

Proof. In the relevant range,

$$
s_{1}\left(v_{1}, v_{2}\right)=\frac{1}{2}+\frac{v_{1}-v_{2}}{2 \tau}
$$

hence

$$
\frac{\partial^{2} s_{1}\left(v_{1}, v_{2}\right)}{\partial v_{2} \partial v_{1}}=0
$$

[^11]and
$$
\frac{\partial s_{1}\left(v_{1}, v_{2}\right)}{\partial v_{1}}=-\frac{\partial s_{1}\left(v_{1}, v_{2}\right)}{\partial v_{2}}=\frac{1}{2 \tau} .
$$

The profit of commercial outlet 1 is

$$
\pi_{1}\left(v_{1}, v_{2}\right)=s_{1}\left(v_{1}, v_{2}\right)\left(1-\left(\alpha v_{1}+\beta v_{2}\right)\right)-c_{1}\left(v_{1}\right)
$$

Hence

$$
\frac{\partial \pi_{1}}{\partial v_{1}}=\frac{1}{2 \tau}\left(1-\left(\alpha v_{1}+\beta v_{2}\right)\right)-\alpha s_{1}\left(v_{1}, v_{2}\right)-\frac{\partial c_{1}\left(v_{1}\right)}{\partial v_{1}}
$$

and

$$
\frac{\partial^{2} \pi_{1}}{\partial v_{1} \partial v_{2}}=\frac{\alpha-\beta}{2 \tau} .
$$

Therefore, if $\alpha>\beta, \pi_{1}$ has strictly increasing differences in $\left(v_{1}, v_{2}\right)$. On the other hand, if $\alpha=\beta$, then $\pi_{1}$ has constant differences in $\left(v_{1}, v_{2}\right)$.

An implication of Remark 3 is that, if the cost function $c_{1}$ is strictly convex and twice differentiable, the profit maximizing reporting accuracy $v_{1}^{*}\left(v_{2}\right)$ is strictly increasing in $v_{2}$ if $\alpha>\beta$, and $v_{1}^{*}\left(v_{2}\right)$ is constant in $v_{2}$ if $\alpha=\beta .{ }^{16}$

To conclude this appendix, we assume a quadratic cost function to illustrate that all the assumptions in Example 3 are consistent with each other. Suppose that $c_{1}\left(v_{1}\right)=$ $k v_{1}^{2} / 2, k>0$. Then the best reply function is

$$
v_{1}^{*}\left(v_{2}\right)=\frac{\frac{1}{2 \tau}-\frac{\alpha}{2}+\frac{\alpha-\beta}{2 \tau} v_{2}}{\left(\frac{\alpha}{\tau}+k\right)}
$$

Example 3 assumed an interior solution with $v_{1}>0, R_{1}>0$ and $s_{1} \in(0,1)$. To see these assumptions are consistent with each other, consider the symmetric case
${ }^{16}$ To see this, note the first order condition for an interior solution is

$$
\frac{\partial \pi_{1}\left(v_{1}, v_{2}\right)}{\partial v_{1}}=0
$$

The second order condition holds since

$$
\frac{\partial^{2} \pi_{1}\left(v_{1}, v_{2}\right)}{\partial v_{1}^{2}}=-c_{1}^{\prime \prime}\left(v_{1}\right)<0
$$

By the implicit function rule,

$$
\operatorname{sign}\left(\frac{d v_{1}^{*}\left(v_{2}\right)}{d v_{2}}\right)=\operatorname{sign}\left(\frac{\partial^{2} \pi_{1}}{\partial v_{1} \partial v_{2}}\right)=\operatorname{sign}(\alpha-\beta) .
$$

where both firms are commercial media and have the same cost and advertising revenue functions. In a symmetric equilibrium,

$$
v_{1}=v_{2}=\frac{1-\alpha \tau}{\alpha+\beta+2 k \tau}>0
$$

iff $\alpha \tau<1$. Moreover, for $i=1,2$,

$$
R_{i}\left(v_{i}\right)=1-\frac{(\alpha+\beta)(1-\alpha \tau)}{\alpha+\beta+2 k \tau}=\frac{\tau\left(\alpha^{2}+\alpha \beta+2 k\right)}{\alpha+\beta+2 k \tau}>0
$$

Finally, by symmetry $s_{1}\left(v_{1}, v_{2}\right)=s_{2}\left(v_{1}, v_{2}\right)=1 / 2$.

## E Demand functions with decreasing differences: the logit model

Suppose that

$$
s_{i}\left(v_{i}, v_{-i}\right)=\frac{f_{i}\left(v_{i}\right)}{\sum_{j=0}^{n+m} f_{j}\left(v_{j}\right)},
$$

where the functions $f_{i}\left(v_{i}\right)$ are strictly positive and strictly increasing, and $v_{0}$ is the utility of the outside option. For $k \neq i$,

$$
\begin{aligned}
\frac{\partial s_{i}}{\partial v_{k}} & =\frac{-f_{i}\left(v_{i}\right) f_{k}^{\prime}\left(v_{k}\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{2}} \\
\frac{\partial^{2} s_{i}}{\partial v_{i} \partial v_{k}} & =\frac{f_{k}^{\prime}\left(v_{k}\right) f_{i}^{\prime}\left(v_{i}\right)\left(f_{i}\left(v_{i}\right)-\sum_{j \neq i} f_{j}\left(v_{j}\right)\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{3}}
\end{aligned}
$$

This implies that, if there are two or more commercial outlets, $s_{i}$ cannot have increasing differences for all $i \in C$, so Assumption (2) is violated.

Moreover,

$$
\begin{aligned}
\frac{\partial^{2} \pi_{i}}{\partial v_{i} \partial v_{k}} & =\frac{f_{k}^{\prime}\left(v_{k}\right) f_{i}^{\prime}\left(v_{i}\right)\left(f_{i}\left(v_{i}\right)-\sum_{j \neq i} f_{j}\left(v_{j}\right)\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{3}} R_{i}\left(v_{i}\right)-\frac{f_{i}\left(v_{i}\right) f_{k}^{\prime}\left(v_{k}\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{2}} R_{i}^{\prime}\left(v_{i}\right) \\
& =\frac{f_{k}^{\prime}\left(v_{k}\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{2}}\left(\frac{f_{i}^{\prime}\left(v_{i}\right)\left(f_{i}\left(v_{i}\right)-\sum_{j \neq i} f_{j}\left(v_{j}\right)\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)} R_{i}\left(v_{i}\right)-f_{i}\left(v_{i}\right) R_{i}^{\prime}\left(v_{i}\right)\right) \\
& >\frac{f_{k}^{\prime}\left(v_{k}\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{2}}\left(f_{i}^{\prime}\left(v_{i}\right) \frac{\left(-\sum_{j \neq i} f_{j}\left(v_{j}\right)\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)} R_{i}\left(v_{i}\right)-f_{i}\left(v_{i}\right) R_{i}^{\prime}\left(v_{i}\right)\right) \\
& >\frac{f_{k}^{\prime}\left(v_{k}\right)}{\left(\sum_{j=1}^{n+m} f_{j}\left(v_{j}\right)\right)^{2}}\left(-f_{i}^{\prime}\left(v_{i}\right) R_{i}\left(v_{i}\right)-f_{i}\left(v_{i}\right) R_{i}^{\prime}\left(v_{i}\right)\right)
\end{aligned}
$$

so a sufficient condition for $\pi_{i}$ to have increasing differences in $\left(v_{i}, v_{-i}\right)$ is that

$$
-f_{i}\left(v_{i}\right) R_{i}^{\prime}\left(v_{i}\right) \geq f_{i}^{\prime}\left(v_{i}\right) R_{i}\left(v_{i}\right)
$$

or equivalently

$$
\frac{\left|R_{i}^{\prime}\left(v_{i}\right)\right|}{R_{i}\left(v_{i}\right)} \geq \frac{f_{i}^{\prime}\left(v_{i}\right)}{f_{i}\left(v_{i}\right)}
$$

In the logit model, $f_{i}\left(v_{i}\right)=\exp \left(\mu v_{i}\right)$ and hence $f_{i}^{\prime}\left(v_{i}\right) / f\left(v_{i}\right)=\mu$.

## References

[1] Armstrong, M. (2006). Competition in two-sided markets. The RAND Journal of Economics, 37(3), 668-691.
[2] Armstrong, M., \& Weeds, H. (2007a). Programme quality in subscription and advertising-funded television. Working paper.
[3] Armstrong, M., \& Weeds, H. (2007b). Public service broadcasting in the digital world. In: Seabright, P., \& Von Hagen, J. (Eds.). (2007). The Economic Regulation of Broadcasting Markets: Evolving Technology and Challenges for Policy. Cambridge University Press.
[4] Anderson, S. P., \& Jullien, B. (2015). The advertising-financed business model in two-sided media markets. In Handbook of media economics (Vol. 1, pp. 41-90). North-Holland.
[5] Anderson, S. P., \& Coate, S. (2005). Market provision of broadcasting: A welfare analysis. The Review of Economic Studies, 72(4), 947-972.
[6] Bagdikian, B. H. (2004). The new media monopoly. Beacon Press Boston.
[7] Batina, R. G., \& Ihori, T. (2005). Public goods: theories and evidence. Springer Science \& Business Media.
[8] Beattie, G. (2022). Measuring social benefits of media coverage: How coverage of climate change affects behavior. Available at SSRN 4211879.
[9] Beattie, G. (2020). Advertising and media capture: The case of climate change. Journal of Public Economics, 188, 104219.
[10] Beattie, G., Durante, R., Knight, B., \& Sen, A. (2021). Advertising spending and media bias: Evidence from news coverage of car safety recalls. Management Science, 67(2), 698-719.
[11] Bergstrom, T., Blume, L., \& Varian, H. (1986). On the private provision of public goods. Journal of Public Economics, 29(1), 25-49.
[12] Berry, S. T., \& Waldfogel, J. (1999). Public radio in the United States: does it correct market failure or cannibalize commercial stations?. Journal of Public Economics, 71(2), 189-211.
[13] Besley, T., \& Prat, A. (2006). Handcuffs for the grabbing hand? Media capture and government accountability. American Economic Review, 96(3), 720-736.
[14] Blasco, A., Pin, P., \& Sobbrio, F. (2016). Paying positive to go negative: Advertisers' competition and media reports. European Economic Review, 83, 243-261.
[15] Blasco, A., \& Sobbrio, F. (2012). Competition and commercial media bias. Telecommunications Policy, 36(5), 434-447.
[16] Boykoff, M. T., \& Boykoff, J. M. (2004). Balance as bias: Global warming and the US prestige press. Global environmental change, 14(2), 125-136.
[17] Buchholz, W., \& Sandler, T. (2021). Global public goods: a survey. Journal of Economic Literature, 59(2), 488-545.
[18] Chen, Y., \& Riordan, M. H. (2007). Price and variety in the spokes model. The Economic Journal, 117(522), 897-921.
[19] Coase, R. H. (1947). The origin of the monopoly of broadcasting in Great Britain. Economica, 14(55), 189-210.
[20] Crawford, G. S., Deer, L., Smith, J., \& Sturgeon, P. R. (2017). The Regulation of Public Service Broadcasters: Should there be more advertising on television?.
[21] Crawford, G. S., \& Levonyan, V. (2018). Media Bias in Public Service Broadcasting: Evidence from the BBC.
[22] Cushion, S. (2017). The democratic value of news: Why public service media matter. Bloomsbury Publishing.
[23] Ellman, M., \& Germano, F. (2009). What do the papers sell? A model of advertising and media bias. The Economic Journal, 119(537), 680-704.
[24] Focke, F., Niessen-Ruenzi, A., \& Ruenzi, S. (2016). A friendly turn: Advertising bias in the news media. Available at SSRN 2741613.
[25] Frankel, D. M., Morris, S., \& Pauzner, A. (2003). Equilibrium selection in global games with strategic complementarities. Journal of Economic Theory, 108(1), 144.
[26] Gambaro, M., \& Puglisi, R. (2015). What do ads buy? Daily coverage of listed companies on the Italian press. European Journal of Political Economy, 39, 41-57.
[27] Garcia Pires, A. J. (2016). Media plurality: private versus mixed duopolies. Journal of Public Economic Theory, 18(6), 942-960.
[28] Germano, F., \& Meier, M. (2013). Concentration and self-censorship in commercial media. Journal of Public Economics, 97, 117-130.
[29] Hamilton, J. T. (2016). Democracy's detectives: The economics of investigative journalism. Harvard University Press.
[30] Jullien, B., Pavan, A., \& Rysman, M. (2021). Two-sided markets, pricing, and network effects. In Handbook of Industrial Organization (Vol. 4, No. 1, pp. 485592). Elsevier.
[31] Kerkhof, A., \& Münster, J. (2015). Quantity restrictions on advertising, commercial media bias, and welfare. Journal of Public Economics, 131, 124-141.
[32] Kreps, D. M. (2013). Microeconomic foundations I: choice and competitive markets (Vol. 1). Princeton university press.
[33] Luski, I., \& Wettstein, D. (1994). The provision of public goods via advertising: existence of equilibria and welfare analysis. Journal of Public Economics, 54(2), 309-321.
[34] Mas-Colell, A., Whinston, M. D., \& Green, J. R. (1995). Microeconomic theory (Vol. 1). New York: Oxford university press.
[35] Milgrom, P., \& Roberts, J. (1990). Rationalizability, learning, and equilibrium in games with strategic complementarities. Econometrica, Vol. 58, No. 6, 1255-1277.
[36] Nielsen, R. K., Fletcher, R., Sehl, A., \& Levy, D. (2016). Analysis of the relation between and impact of public service media and private media. Available at SSRN 2868065.
[37] Ofcom (2004). Ofcom review of public service television broadcasting (PSB). URL: https://www.ofcom.org.uk/consultations-and-statements/category-2/psb
[38] Petrova, M. (2011). Newspapers and parties: How advertising revenues created an independent press. American Political Science Review, 105(4), 790-808.
[39] Reuter, J., \& Zitzewitz, E. (2006). Do ads influence editors? Advertising and bias in the financial media. The Quarterly Journal of Economics, 121(1), 197-227.
[40] Richardson, M. (2006). Commercial broadcasting and local content: cultural quotas, advertising and public stations. The Economic Journal, 116(511), 605-625.
[41] Sehl, A., Fletcher, R., \& Picard, R. G. (2020). Crowding out: Is there evidence that pub-lic service media harm markets? A cross-national comparative analysis of commercial television and online news providers. European Journal of Communication, 35(4), 389-409.
[42] Simon, J. (2013). Public and private broadcasters across the world: The race to the top. Inside the BBC. London: BBC.
[43] Sarver, Todd (2023). Microeconomic theory lecture notes.
[44] Strömberg, D. (2015). Media coverage and political accountability: Theory and evidence. In Handbook of Media Economics (Vol. 1, pp. 595-622). North-Holland.
[45] Szeidl, A., \& Szucs, F. (2021). Media capture through favor exchange. Econometrica, 89(1), 281-310.
[46] Tella, R. D., \& Franceschelli, I. (2011). Government advertising and media coverage of corruption scandals. American Economic Journal: Applied Economics, 3(4), 119-151.
[47] Topkis, D. M. (1979). Equilibrium points in nonzero-sum n-person submodular games. Siam Journal on control and optimization, 17(6), 773-787.
[48] Van Zandt, T., \& Vives, X. (2007). Monotone equilibria in Bayesian games of strategic complementarities. Journal of Economic Theory, 134(1), 339-360.
[49] Vives, X. (2005a). Complementarities and games: New developments. Journal of Economic Literature, 43(2), 437-479.
[50] Vives, X. (2005b). Games with strategic complementarities: New applications to industrial organization. International Journal of Industrial Organization, 23(7-8), 625-637.
[51] Vives, X. (1990). Nash equilibrium with strategic complementarities. Journal of Mathematical Economics, 19(3), 305-321.
[52] Vives, X. (1985). On the efficiency of Bertrand and Cournot equilibria with product differentation. Journal of Economic Theory, 36(1), 166-175.
[53] Weeds, H. (2020). Rethinking Public Service Broadcasting for the Digital Age.
[54] Wissenschaftlicher Beirat beim Bundesministerium der Finanzen (2014). Öffentlich-rechtliche Medien - Aufgabe und Finanzierung. URL: https://www.bundesfinanzministerium.de/Content/DE/Downloads/Ministerium /Wissenschaftlicher-Beirat/Gutachten/2014-12-15-gutachten-medien.pdf? _blob=publicationFile\&v=11


[^0]:    ${ }^{1}$ In this paper, "commercial media" refers are all profit-maximizing media outlets.

[^1]:    ${ }^{2}$ These correlations are in line with a crowding in effect of PSM. They might, however, also be driven by unobserved confounding factors such as high preferences for television, and do not allow to infer causality. Weeds (2020) reviews the literature on the question whether Public Service Broadcasters crowd out or crowd in private programming and concludes that "further research is needed in this area before firm conclusions can be drawn" (Weeds 2020, p. 10). Similarly, Nielsen et al. (2016) review academic publications and studies funded by stakeholders such as government agencies and public or private media organizations. They point out that there is little research on the market impact of public service media, and conclude that "existing studies provide little evidence for a negative market impact of public service media upon domestic private sector media" (p. 17).
    ${ }^{3}$ E.g., advertisers might prefer the media not to report critically about their products. Moreover, studies from marketing have shown that advertisers prefer light genres like comedy that put consumers in a more ad-receptive mood, whereas consumers prefer action and news. See Ellman and Germano (2009) and Kerkhof and Münster (2015) for extensive discussion.

[^2]:    ${ }^{4}$ Blasco and Sobbrio (2012) provide a survey on competition and commercial media bias.

[^3]:    ${ }^{5}$ In this paper, unless otherwise stated, "increasing" means "weakly increasing" and "positive" means "non-negative". A similar remark aplies for "decreasing" and "negative".
    ${ }^{6}$ The cost $c_{i}$ does not dependend on audience size; it can be thought of as "first copy costs". In online and broadcast media (radio and TV), once distribution channels are in place, marginal costs of an additional audience is basically zero. In printed newspaper markets the costs for paper, printing and delivery are substantial. Any constant marginal cost of an additional consumer can be thought of as incorporated in the function $R_{i}$, which then gives advertising revenue per consumers net of marginal costs per consumer. Note that with this interpretation of $R_{i}$, it could become strictly negative for high values of $v_{i}$ (since advertising revenue might be strictly smaller than marginal costs), in conflict with Assumption (3). Such values of $v_{i}$, however, lead to losses and hence are dominated; they can be eliminated from the strategy set $V_{i}$, restoring Assumption (3).

[^4]:    ${ }^{7}$ Consumers have to pay taxes or licence fees that are used to finance the PSM, but these payments are independent of personal media consumption. These payments could affect media demand via income effects. We discuss income effects in Section 5.

[^5]:    ${ }^{8}$ See e.g. Sarver (2023) Chapter 3 for the definition of a parameterized supermodular games.

[^6]:    ${ }^{9}$ We assume $n_{f}<n$. The case $n_{f}=n$ is the case without pay media studied above. We can allow for the case where all commercial outlets are pay media $\left(n_{f}=0\right)$. The most relevant case is when pay media and free media co-exist $\left(0<n_{f}<n\right)$. Note we assume that outlets $i \in C^{f}$ provide their content for free to consumers. Similarly, we assume that outlets in $C^{\text {pay }}$ choose strictly positive prices.
    ${ }^{10}$ This implies that there is a highest feasible price that firms cannot exceed, which is without loss of generality if consumers stop buying if the price is too high.

[^7]:    ${ }^{11}$ Of course, if $c_{j}\left(v_{j}\right)$ is identically zero for the public media as well, and $V_{j}$ does not depend on $b_{j}$, then changes in the budgets of PSM have no effects in our model. Even if $c_{j}\left(v_{j}\right)$ is identically zero for $j \in P$, however, $\bar{v}_{j}\left(b_{j}\right)$ will be strictly increasing if a larger budget strictly enlarges the feasible set $V_{j}$ by allowing strictly higher acccuracies. Moreover, a larger budget may allow the PSM to increase their attractiveness in other ways that are not related to reporting accuracy.

[^8]:    ${ }^{12}$ Conditions on the fundamentals such that the solution is interior will be given in the proof below.

[^9]:    ${ }^{13}$ Since the range of $g_{i}$ is compact, $g_{i}$ is upper hemicontinuous if for any two sequences $u_{i}^{m} \rightarrow u_{i} \in U_{i}$ and $v_{i}^{m} \rightarrow v_{i}$, with $u_{i}^{m} \in U_{i}$ and $v_{i}^{m} \in g_{i}\left(u_{i}^{m}\right)$ for all $m$, we have $v_{i} \in g_{i}\left(u_{i}\right)$ (see e.g. Mas-Colell, Whinston, and Green 1995, Section M.H). Since $v_{i}^{m} \in g_{i}\left(u_{i}^{m}\right)$ for all $m, f_{i}\left(v_{i}^{m}\right) \geq u_{i}^{m}$ for all $m$, hence $f_{i}\left(v_{i}\right) \geq u_{i}$ by continuity of $f_{i}$. Moreover, $v_{i}^{m} \in g_{i}\left(u_{i}^{m}\right)$ for all $m$ implies $v_{i}^{m} \in V_{i}$ for all $m$, and since $V_{i}$ is compact, $v_{i} \in V_{i}$. This completes the proof that $v_{i} \in g_{i}\left(u_{i}\right)$.

[^10]:    ${ }^{14}$ We provide conditions on the fundamentals where this is the case in the proof below.

[^11]:    ${ }^{15}$ We show by example after the proof that there are parameter constellations where the solution is interior.

