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Abstract

We present a general and tractable oligopoly model of multi-sided platforms with endogenous side and platform choices of heterogeneous end-users, considering any mix of single-homing and multi-homing platforms and in which participating on one side could preclude doing so on others. We show the existence of a unique equilibrium number of end-users and characterize optimal platform pricing. Using the equilibrium conditions, we formally derive (across sides and platforms) switching effects that distort optimal pricing, which can lead to markups exceeding the Lerner index and rule out the classical "cross-subsidization" result. We then provide a unifying framework to analyze multi-sided platform mergers, which rationalizes mixed results from the previous literature by providing, based on the switching effects, a set of conditions that predict the upward pricing pressure post-merger. We show that while optimal pricing is determined by the nature of end-users' side choices, their platform choices are crucial for merger analysis.

JEL-Codes: D430, G340, L110, L130, L220, L860.

Keywords: multi-sided markets, heterogeneous end-users, endogenous side choice, mergers of platforms, digital platforms.

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1 Introduction

Multi-sided markets have become ubiquitous. The remarkable rise in information technologies has facilitated users to participate seamlessly in multiple segments of these markets. Prime examples of multi-sided markets include Amazon and eBay, which enable users to buy and sell goods, and Airbnb, where guests, private hosts, and lodgings interact (Farronato and Fradkin, 2022). As digitization progresses, users enjoy even more immediate and flexible platform choices and side-switching options, frequently guided by market forces. For example, apps like Uber and Bolt now allow car owners to drive passengers, deliver food, or even rent out their cars with just a tap of their fingertips; Rover and Wag allow dog owners to switch from having their dogs taken for a walk to walking someone else's dog; and platforms such as Piclo (UK) and Drift (USA) are leading a pioneering trend that allows households either to buy or sell energy based on their needs and on demand and supply conditions (IRENA, 2020). In this paper, we build a tractable general model of multi-sided markets that captures these increasingly observed features of digital platforms and use the model to study optimal pricing and mergers.

In several dimensions, our model generalizes recent seminal contributions to this literature by introducing *both* endogenous side and platform choices of end-users in a richer set of scenarios. By contrast, Tan and Zhou (2021) study price competition and market entry with the assumption that end-users cannot choose sides (*fixed sides*) on single-homing platforms. Under full coverage and symmetry, prices increase to the detriment of end-users as additional platforms enter. Jullien and Pavan (2019) show that, under imperfect information, end-users' beliefs about participation affect demand elasticities and prices when the preferences of end-users are correlated with their beliefs.[1] The authors consider two-sided platforms with fixed sides, also limiting the role of end-users' choices.

Instead, we study optimal pricing in a model in which end-users choose the side(s) that they join on both multi- and single-homing platforms, obtaining results that, in some cases, markedly depart from those of previous contributions. These results reveal the importance of considering the sides of platforms (rather than entire platforms) as a fundamental unit of analysis in multi-sided

[1] Despite playing a significant role in certain classic examples of two sided-markets—such as video-games or operating systems—that require undertaking substantial sunk cost investments, beliefs are not especially relevant for an ample set of digital platforms in which end-users can easily appear on one side/platform or another.

markets. Then, we apply our theoretical framework to platform mergers. As discussed by Rysman (2009), the implications of mergers for end-users are very different in multi-sided markets than in other markets.[2] The generality of our model allows us to rationalize seemingly contradictory results from previous empirical studies on platform mergers that focus on particular industries (Fan, 2013; Song, 2021). We thus provide a unifying framework that can accommodate mergers in a broad range of multi-sided markets with increasingly flexible choices and that predicts post-merger prices based on observable parameters before the merger occurs. By placing an emphasis on sides, we shed light on the harmful, though usually perceived as innocuous, effects of mergers of seemingly unrelated platforms that are connected through one side—exemplified by Ola Cabs' takeover of Foodpanda, a food delivery app whose couriers often deliver by car—and other *conglomerate mergers*. We demonstrate that, while end-users' side choices crucially affect optimal pricing, the nature of platform choices (single- vs. multi-homing) is key to determine the effects of mergers.

In the spirit of the seminal contributions of this literature, our model builds on and extends the micro-founded decision-making process of end-users. We derive a unique subgame perfect equilibrium number of end-users and characterize the optimal pricing of platforms for different cases. Specifically, we consider *(i)* that end-users choose whether they join one platform or no platform (single-homing) or even several platforms (multi-homing), in which case, some of the end-users may still endogenously choose to single-home;[3] *(ii)* that end-users can participate on either one side or multiple sides at once, wherein some of the end-users may opt for single side participation; and *(iii)* that cross-group externalities can be either positive or, conversely, negative (*congestion effect*). In all these cases, we assume that for the end-users, platform-specific idiosyncratic valuations for joining each side[4] are drawn from a general, well-behaved density function. These valuations are key in determining participation decisions given the distinct constraints specific to each of the cases.

First, we consider that end-users can join any side on any multi-homing platform. In this setting, each end-user independently decides based on *participation constraints* (PC) her participation on

[2] Nocke and Whinston (2013) provide a framework for the antitrust authority's optimal merger approval policy, while Evans and Schmalensee (2013) show why traditional antitrust analysis fails in the case of multi-sided platforms.

[3] Previous authors frequently restrict their analysis to multi-homing platforms (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006). Moreover, most of the literature, except for Gao (2018) and Choi and Zennyo (2019), assumes fixed sides, ignoring end-users' side choices.

[4] This approach is commonly used to capture end-user heterogeneity (Armstrong, 2006; Correia-da Silva et al., 2019; Jullien and Pavan, 2019; Tan and Zhou, 2021). Other authors alternatively (or additionally) introduce heterogeneity in cross-group network externalities (Rochet and Tirole, 2003, 2006; Weyl, 2010).

each side and platform: an end-user joins a side on a platform if the sum of the valuation of joining it and the cross-group externality, net of the price (also called fee), is larger than the outside option. Using PCs, we obtain closed-form expressions for the demand for sides, from which we then derive price elasticities. In line with the intuition of previous authors, we formally show that the own-price elasticity is negative; that is, an increase in the fee on one side reduces participation on it. The effect on all other sides, measured by the cross-price elasticity, is ambiguous though, as it depends on the network effect: it is negative under positive cross-group externalities, and it turns positive under congestion effects. This is also reflected in the characterization of the Lerner index, which, in the former case, is dragged down by the markup charged to end-users on other sides—due to the *opportunity cost*[5] in terms of revenue raised from end-users on the other sides—while increasing in the latter case.

We then consider that participation is limited to a single side on certain platforms, such as Uber and Bolt, where a car owner faces the decision whether to rent her vehicle, use it to provide rides, or engage in food delivery. In this case, endogenous side choices are further determined by *incentive compatibility constraints* (ICC): an end-user joins a side if the utility gained is greater than the outside option (PC) and greater than the utility obtained by joining any other mutually exclusive side (ICC). The latter yields a novel *switching-side effect* that we formally characterize, by which an increase in the fee on one side induces some end-users to switch to another side. Thus, increasing the fee has two (potentially countervailing) effects: it negatively (under positive cross-group externalities) or positively (under congestion) affects participation on the other sides through the network effect and also increases participation on the other sides due to the switching-side effect, making the sign of the cross-price elasticity (ex-ante) ambiguous. This critically affects platform pricing as, contrary to previous results, the markup is greater than the Lerner index if the switching-side effect dominates the network effect, even under positive cross-group externalities. This stands in stark contrast to previous authors who show that additional participation and higher platform profits following an increase in the fee on one side can occur only under congestion.

Finally, with some subtle yet important particularities, our main results prove robust if we assume that a subset of platforms are single-homing; i.e., end-users are allowed to join only one

[5] Rochet and Tirole (2006) coined this term. Unlike them, we capture not only the markup lost from participation on other sides but also the resulting network effect on the side where the price increased.

platform on each side. This occurs, for example, if car owners on Uber or Bolt are barred from renting their cars on Turo or Getaround and/or from providing rides through Lyft. This case is micro-founded by introducing additional ICCs, which give rise to *switching-platform effects*.[6] We formally derive these effects, show how they interact with both the network effect and the switching-side effect discussed above, and examine their impact on the unique participation equilibrium and optimal platform pricing. Finally, we remark that our results hold for any number of (asymmetric) platforms and also extend to no double-counting of network externalities and to end-users' random demand, as we formally show in Appendix B.

Two of the key features of our model (end-user endogenous platform and side choices) were previously only partially considered in more specific settings. For example, according to Bakos and Halaburda (2020), Jeitschko and Tremblay (2020), and Teh et al. (2023), end-users who belong to one of two (fixed) sides simply decide which platform(s) they join. Jeitschko and Tremblay (2020) and Teh et al. (2023) derive an equilibrium in which some end-users single-home, while others multi-home. Bakos and Halaburda (2020) study two platforms competing à la Hotelling to show that, under full coverage and multi-homing, the classical result of cross-subsidization breaks down. Gao (2018) studies bundling in a monopoly platform that allows end-users to choose sides and offers a discount to those who join both sides. Choi and Zennyo (2019) instead model a two-sided platform duopoly on a Hotelling line in which end-users choose a platform and a side. However, Choi and Zennyo (2019) restrict to single-homing with mutually exclusive sides and full coverage. With the exception of Gao (2018), none of these authors consider congestion effects.

Our approach also contrasts with pioneer contributions to this literature (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006; Weyl, 2010), which, to capture end-user demand, rely on the trade-off between the fees incurred and the benefit obtained by joining the platform. As in any standard market model, participation decreases as own-side fees increase and increases with the number of end-users on the other side. These models provide an appropriate toolkit to characterize the classic examples of two-sided markets (e.g., video games, shopping malls, credit cards), considering that *(i)* end-users—at least on one side—are unlikely to be widely heterogeneous (game developers, retailers, advertising agencies), and *(ii)* end-users on each side appear as clearly differentiated groups (players/developers, card holders/merchants,

[6] Chandra and Collard-Wexler (2009) and Affeldt et al. (2013) intuitively identify these effects.

readers/advertisers) with little or no relevant possibility of inter-group participation choices and side overlap. There is now a wide range of easy-to-access digital platforms that allow heterogeneous end-users to actively choose and switch between the side(s) of the market they join.

Given platforms' optimal pricing and the corresponding unique participation equilibrium, we then study mergers of (asymmetric) platforms by building on the work of Affeldt et al. (2013). These authors adapt Farrell and Shapiro (2010)'s Upward Pricing Pressure (UPP) measure of mergers to multi-sided markets and show that the incentives to increase prices depend on the diverted sales that the merged entity recaptures. The value of these diverted sales are formally characterized by *diversion ratios* (ratios of own- and cross-price elasticities), which can be measured and estimated (Conlon and Mortimer, 2021). We extend Affeldt et al. (2013)'s analysis,[7] as our micro-founded model provides additional insights into the direction and strength of diversion ratios in a broader set of cases that capture the reality of many different platforms—single- vs. multi-homing, mutually exclusive sides, congestion effects, etc. Since these ratios and prices are measurable and publicly observable, the likely UPP of platform mergers can be predicted before the merger takes place.

Therefore, another contribution of this paper is to provide a unified framework for evaluating the welfare consequences of multi-sided platform mergers that reveals the importance of effects *across platform sides* for this analysis. This framework thus allows us to understand the disparate results of previous authors who studied mergers in two-sided markets. Correia-da Silva et al. (2019) provide an overview of this literature. They show theoretically—as do Chandra and Collard-Wexler (2009)—that a merger has ambiguous effects on prices, depending on the existence of cross-subsidization. We show that cross-subsidization is a key element that determines in which direction post-merger prices change, although the different effects we formally identify also matter.

From an empirical point of view, Fan (2013) finds by using data on newspapers that ad rates and subscription prices post-merger tend to move in opposite directions. Jeziorski (2014) concludes that for mergers in the radio industry, while ad quantity goes down (thus increasing the listeners' welfare), ad prices increase after the merger. Along the same lines, Song (2021) uses data from TV magazines to estimate a model of a two-sided market and finds that post-merger prices might go either up or down, with prices on different sides usually moving in opposite directions. Using diversion ratios, we set out the conditions under which these results hold, and we present intuition

[7] Cosnita-Langlais et al. (2021) incorporate cross-side effects into the UPP for a merger of two two-sided platforms.

on the forces and effects that yield them. Bearing in mind the well-acknowledged features of media markets, our merger analysis also explicitly considers congestion effects (Anderson and Peitz, 2020).

Most papers on platform mergers focus on and impose the assumptions that best suit specific markets (e.g., magazines, radio, newspapers). Instead, we not only provide a general toolkit for mergers of platforms offering similar services, but we also apply it to mergers of platforms that, despite offering different services, are "somehow related" through one or more sides; for example, a food-delivery platform and a ride-hailing platform (which clearly offer different services) both allow drivers to choose at their convenience to deliver food or to give rides to passengers. Since end-users' side choices are at the heart of our analysis, this framework permits us to unmask the impact that digital conglomerate mergers and other agreements have on prices that seem to be perceived as innocuous by antitrust authorities—such as the mergers of Ola Cabs and Foodpanda, Uber and JUMP, AirBnB and HotelTonight, and the partnership agreement between Sixt and Tier.

The rest of the paper is as follows. We set up the model in Section 2. In Section 3, we provide the system of equations that characterize market demand for each of the cases we study. Section 4 proves the existence and uniqueness of participation equilibria for each of the cases, characterizes the equilibria and optimal pricing, and discusses certain features of the equilibria and optimal pricing. In Section 5, we study platform mergers, while Section 6 concludes. All proofs are in Appendix A, and some extensions are in Appendix B.

2 Model setup

We consider an oligopoly with a set of m platforms, denoted by $\mathcal{M}$ and indexed by $q \in \mathcal{M} := \{1, ..., m\}$. Each platform q enables interaction between end-users on $i \in \mathcal{D}^q := \{1, ..., d^q\}$ different sides. We assume that $d^q \geq 2$ for all $q \in \mathcal{M}$. The super-index q reflects that the number of sides need not be identical across platforms. However, for simplicity, we drop this super-index from now on.[8] A unit-measure continuum of end-users choose to join side i on any of the m platforms or to join no platform at all—sometimes we allow end-users to join multiple sides on multiple platforms. Slightly abusing notation, we denote by N_i^q the set of end-users that join side i on platform q.

Each end-user obtains an idiosyncratic surplus upon joining side i on platform q, denoted by v_i^q.

[8]If one side is present on one platform but not on others, this poses no problem for our analysis.

This surplus parameter, also called *gross membership benefit*, is independent of the number of end-users on q's other sides and, therefore, of the market size or the number of transactions performed. Let $\mathbf{v} := (v_1^1, ..., v_d^m) \in \mathbb{R}_+^{m\times d}$ be the set of possible combinations of surplus parameters. Without loss of generality, we let $\mathbf{v} \in [0,1]^{m\times d}$, since only the relative relationship between the parameters is relevant. We assume that end-users are heterogeneous in v_i^q for all i and q, whose realization is independently drawn from a continuous, strictly increasing joint cumulative distribution function $F(\mathbf{v})$, with an associated joint density function $f(\mathbf{v})$, which has full and bounded support.

We denote the *cross-group (network) externalities* by $\phi_i^q(\mathbf{N}_{\text{-}i}^q)$, which captures how end-users on platform q's side i are affected by participation $\mathbf{N}_{\text{-}i}^q$ on all sides other than i on q.[9] That is, $\phi_i^q(\cdot)$ is a continuously differentiable, strictly monotone mapping, $\phi_i^q : [0,1]^{d-1} \to \mathbb{R}$, with $\phi_i^q(\mathbf{0}) = 0$ (normalization). If $\phi_i^q(\mathbf{N}_{\text{-}i}^q) > 0$, then there are *positive cross-group externalities* and, thus, on platform q, end-users on side i enjoy meeting those on other sides—for example, the presence of buyers is beneficial for sellers on e-commerce websites. Conversely, if $\phi_i^q(\mathbf{N}_{\text{-}i}^q) < 0$, then there are *congestion effects*: those on side i do not enjoy meeting end-users on other sides—for example, readers dislike advertising on media outlets. While we do not impose a specific functional form on $\phi_i^q(\cdot)$, we assume, in line with previous authors, that the aggregate impact of an (arbitrarily) small change in $\mathbf{N}_{\text{-}i}^q$ on $\phi_i^q(\cdot)$ is bounded.

Assumption 1. *(Bounded feedback loop)* $\sum_{j\in\mathcal{D}\setminus\{i\}} \left| \frac{\partial \phi_i^q(\mathbf{N}_{\text{-}i}^q)}{\partial N_j^q} \right| \in (0,1)$ *for all* $i \in \mathcal{D}$ *and* $q \in \mathcal{M}$.

Assumption 1 ensures that changes in participation on one side of the platform do not trigger disproportionately larger changes in participation on the other sides. If this assumption is not satisfied, then the "feedback loop" stemming from a change in market participation generates increasingly amplified effects on participation on other sides, ultimately resulting in corner solutions where end-users either join only one side or none (outcomes that lack meaningful analytical insights).

Finally, we assume that end-users who join side i on platform q pay a fixed membership or *participation fee*, usually called price and denoted by p_i^q.[10] We let $\mathbf{p}^q := (p_1^q, ..., p_d^q) \in \mathbb{R}^d$ be the

[9] With few exceptions, such as Tan and Zhou (2021), previous authors assume that these effects are linear in $\mathbf{N}_{\text{-}i}^q$; i.e., $\phi_i^q(\cdot) = \sum_j \alpha_{ij}^q N_j^q$ for some $\alpha_{ij}^q \in \mathbb{R}$, for all $i, j \in \mathcal{D}$, $i \neq j$, and for all $q \in \mathcal{M}$ (see, e.g., Armstrong, 2006).

[10] End-users pay no per-interaction fees. For monopoly platforms, Armstrong (2006) shows that lump-sum fees are equivalent to per-transaction fees, as discussed by Rochet and Tirole (2003). In our model, Bajo-Buenestado and Kinateder (2019) arrive at a similar conclusion, which extends to a combination of both fee types given the same transaction count.

vector of prices charged by platform $q \in \mathcal{M}$, and we denote by $\mathbf{p} := (\mathbf{p}^1, ..., \mathbf{p}^m) \in \mathbb{R}^{m \times d}$ the vector of prices charged by all platforms.

The timing in our model is as follows. First, nature chooses end-users' idiosyncratic surplus parameters $\mathbf{v}$, and the platforms choose prices $\mathbf{p}$. Second, end-users learn the surplus parameters and prices and decide which side(s) on which platform(s) to join. Finally, end-users access the corresponding side(s), and payoffs are realized.

Given $\mathbf{v}$, $\mathbf{p}$, $\mathbf{N}^q_{\text{-}i}$, and $\phi^q_i(\mathbf{N}^q_{\text{-}i})$, an end-user who joins side i on platform q obtains a utility of

$$u^q_i := v^q_i + \phi^q_i(\mathbf{N}^q_{\text{-}i}) - p^q_i. \tag{1}$$

If platforms are multi-homing, (1) implies *double-counting of cross-group effects*, as end-users on different sides realize the network externality every time they meet on different platforms. This is the case, for example, when buyers benefit from meeting the same seller on multiple e-commerce websites, which occurs if the buyers are interested in a seller's reviews on different websites (Bakos and Halaburda, 2020). However, in other cases, only the first interaction matters. For example, advertisers may value the first impression of an ad on a viewer, while subsequent impressions on other platforms are not valued (Ambrus et al., 2016; Anderson et al., 2018). In Appendix B, we extend our model to no double-counting of cross-group externalities and show that we capture the key results in this literature (incremental pricing; lower or no cross-subsidization).

Unlike end-users from the existing literature, end-users in our model endogenously decide not only whether to join a platform[11] but also which side(s) to join. In particular, to join side i on platform q, the participation constraint PC^q_i requires that the utility of an end-user is larger than her outside option (normalized to 0), i.e., $u^q_i \geq 0$. Moreover, we consider platforms where, for various reasons (examples of which are provided below), simultaneous participation on multiple sides is unfeasible (mutually exclusive sides). In such cases, we additionally require that the utility of joining one side is larger than that of joining any other; i.e., to participate on side i on platform q, the incentive compatibility constraint ICC^q_{ij} requires that $u^q_i \geq u^q_j$ for all $j \neq i$. We consider these features in two standard contexts: multi-homing and single-homing platforms.

[11] Previous authors, such as Bakos and Halaburda (2020), Choi and Zennyo (2019), and Tan and Zhou (2021), usually assume full coverage. In our equilibrium analysis, whether there is full or less than full coverage arises endogenously.

3 Characterization of the demand for sides

In this section, we derive expressions characterizing end-user demand for each side and platform, taking into account the particularities of the different cases we consider. We begin by introducing our baseline case, which constitutes a micro-founded version of the framework frequently employed by previous authors. Subsequently, we introduce modifications for specific sides and platforms due to various constraints, inspired by particularly notable examples. Importantly, we present all our findings with the assumption that sides and platforms may exhibit any combination of these distinct cases we consider. As shown below, this highlights the importance of examining sides (or subsets of sides) rather than entire platforms as the fundamental unit of analysis.

Baseline case. First, we assume that end-users can join any side on any platform. This baseline case, which aligns with the classic framework examined by previous researchers, not only facilitates the formal proof of intuitive conclusions drawn by several of the researchers but also serves as the building block for our subsequent analysis. In this case, end-users' participation decisions on different sides and platforms are independent, and consequently, each platform solves its own profit maximization problem (Belleflamme and Peitz, 2019b). These features are observed, for example, on e-commerce platforms such as eBay, Facebook Marketplace, and Taobao, where end-users engage in simultaneous selling, buying, and advertising of different items across multiple platforms without any exclusivity constraints. Similarly, classified ads websites like Craigslist, Recycler, and Oodle follow the same setup, as each end-user can make independent choices regarding her participation across different sides on the platforms.

Formally, given the realization of gross membership benefits $\mathbf{v}$ and the vector of prices $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}$ on platform $q \in \mathcal{M}$ if, and only if,

$$(PC_i^q) \quad v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q \quad \geq \quad 0.$$

From PC_i^q, the demand for side i on platform q can be obtained as follows:

$$N_i^q(\mathbf{N}_{\text{-}i}^q, \mathbf{p}^q) = pr\big(v_i^q \geq p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)\big). \tag{2}$$

Let us denote by $f_i^q(v_i^q)$ the marginal density function of the valuation of side i on platform q, which need not follow the same (marginal) density as that of any other side on any platform. Then, the number of end-users joining side i on platform q can be written as

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)}^{1} f_i^q(v_i^q) dv_i^q,$$

or, equivalently,

$$N_i^q(\cdot) = 1 - F_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)\big), \tag{3}$$

where $F_i^q(\cdot)$ is the corresponding (marginal) distribution function of v_i^q. □

Mutually exclusive sides. We now modify the previous baseline case and consider a scenario in which a subset of sides (possibly all), denoted as $\mathcal{D}' \subseteq \mathcal{D}$ with $|\mathcal{D}'| \geq 2$, are mutually exclusive.[12] In this context, end-users cannot simultaneously be on sides i and j within any platform, for all $i, j \in \mathcal{D}'$, $i \neq j$, requiring the end-users to make a choice between one side or the other. This exclusivity among sides, which can arise from various factors—including physical constraints or economic (opportunity cost-related) reasons—is a common feature observed in prominent digital peer-to-peer platforms.

For example, in app-based transportation companies like Uber and Bolt, an end-user can either request a ride or rent someone's vehicle (possibly with valet service) to complete her trip. This decision may be influenced by market conditions and the end-user's opportunity cost. However, performing both activities concurrently is physically unfeasible. Likewise, in addition to renting out their vehicles, vehicle owners can offer rides to passengers or engage in food delivery services. These choices are guided by similar trade-offs based on market conditions and opportunity cost. Nevertheless, undertaking these activities simultaneously is physically impractical. This also occurs in dog boarding and walking service platforms like Rover and Wag: an end-user might use both platforms to walk dogs but cannot plausibly have her dog taken for a walk while walking someone else's dog. The same concept applies to many other platforms that require the physical presence of an end-user who either provides or seeks a service.

Formally, given certain mutually exclusive sides, say, i and j, end-users do not just decide

[12] Our model can also accommodate a platform-specific set of mutually exclusive sides. In this case, we would introduce a super-index q in the set $\mathcal{D}'$. For expositional clarity, we omit this super-index.

whether to join platform q but also which of the sides (if any) to join. We capture this feature by introducing ICCs; that is, given $\mathbf{v}$ and $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}' \subseteq \mathcal{D}$ on platform $q \in \mathcal{M}$ if, and only if, PC_i^q defined above holds and

$$(ICC_{ij}^q) \quad v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q \quad \geq \quad v_j^q + \phi_j^q(\mathbf{N}_{\text{-}j}^q) - p_j^q,$$

also holds for all $j \in \mathcal{D}' \setminus \{i\}$, specifically if an end-user is better off by joining side i than side j on platform q.

Together these conditions yield the number of end-users on platform q's side i:

$$N_i^q(\mathbf{N}_{\text{-}i}^q, \mathbf{p}^q) = pr\left(v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q \geq \max\left\{\max_{j \in \mathcal{D}' \setminus \{i\}} \left\{v_j^q + \phi_j^q(\mathbf{N}_{\text{-}j}^q) - p_j^q\right\}, 0\right\}\right). \tag{4}$$

Given $f_i^q(v_i^q)$ and considering that the valuations of different mutually exclusive sides are independent, the demand for side i is characterized by

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)}^{1} f_i^q(v_i^q) \prod_{j \in \mathcal{D}' \setminus \{i\}} \int_0^{v_i^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q)} f_j^q(v_j^q) dv_j^q dv_i^q,$$

or, equivalently,

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\cdot)}^{1} f_i^q(v_i^q) \prod_{j \in \mathcal{D}' \setminus \{i\}} F_j^q\big(v_i^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q)\big) dv_i^q, \tag{5}$$

where $F_j^q\big(v_i^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q)\big) = \int_0^{v_i^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q)} f_j^q(v_j^q) dv_j^q$ is the (marginal) cumulative distribution function of v_j^q. $\square$

Single-homing platforms. Departing again from the baseline case, we now assume instead that a subset of platforms (possibly all), denoted by $\mathcal{M}' \subseteq \mathcal{M}$ with $|\mathcal{M}'| \geq 2$, are single-homing.[13] In this scenario, an end-user cannot simultaneously join any side i on platforms q and r, for all $q, r \in \mathcal{M}'$, $q \neq r$. As a result, the end-user must choose whether to join each side on one platform or another. This exclusivity among platforms can arise due to various reasons, including legal (contractual), technical, or cost-related constraints.

[13] Our model can accommodate the case in which the single-homing constraint applies to a subset of sides. This would require the introduction of a sub-index d in the set $\mathcal{M}'$, omitted for the sake of expositional clarity.

For example, peer-to-peer car rental platforms like Turo impose explicit restrictions on car owners who list their vehicles in the US and Australia, prohibiting the vehicles from being listed on multiple platforms simultaneously. Similarly, certain food delivery apps incorporate exclusivity clauses into their contracts, preventing a restaurant from delivering through multiple platforms—in several countries, this exclusivity extends to couriers, who are restricted from working for various apps.[14] For content development, platform choices also hinge on technical constraints related to differing programming languages; for example, Android developers must employ a C/C++-based environment, whereas iOS developers use a Java-based ecosystem (Shekhar, 2021). Another case in point is wireless technology standards, such as IEEE 802.11 or 3GPP (Farrell and Simcoe, 2012). Both chip manufacturers and portable device companies must adhere to one standard since developing products compatible with multiple standards would be extremely costly. Consequently, in all these cases, end-users' decisions to participate on side i on different platforms are no longer independent.

Formally, given $\mathbf{v}$ and $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}$ on platform $q \in \mathcal{M}' \subseteq \mathcal{M}$ if, and only if, PC_i^q defined above holds and

$$(ICC_i^{qr})\ v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q \quad \geq \quad v_i^r + \phi_i^r(\mathbf{N}_{\text{-}i}^r) - p_i^r,$$

also holds for all $r \in \mathcal{M}' \setminus \{q\}$, where $\mathcal{M}' \subseteq \mathcal{M}$ is the subset of single-homing platforms. The condition ICC_i^{qr} arises due to single-homing: participating on side i on platform q precludes an end-user from joining side i on any other platform $r \neq q$, in the subset of single-homing platforms.

Putting these conditions together yields the number of end-users on platform q's side i:

$$N_i^q(\cdot) = pr\left(v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q \geq \max\left\{\max_{r\in\mathcal{M}'\setminus\{q\}} \{v_i^r + \phi_i^r(\mathbf{N}_{\text{-}i}^r) - p_i^r\}, 0\right\}\right).$$

Thus, given $f_i^q(v_i^q)$, the number of end-users on platform q's side i is characterized by

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)}^{1} f_i^q(v_i^q) \prod_{r\in\mathcal{M}'\setminus\{q\}} \int_0^{v_i^q - \phi_i^r(\mathbf{N}_{\text{-}i}^r) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q)} f_i^r(v_i^r) dv_i^r dv_i^q,$$

[14] In practice, this occurred in Spain after the 2021 Rider Act. Previously, riders were considered independent contractors and could deliver for multiple platforms. However, after 2021, they became workers of a specific platform.

or, equivalently, noting that $F_i^r\big(v_i^q - \phi_i^r(\mathbf{N}^r_{\text{-}i}) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}^q_{\text{-}i})\big) = \int_0^{v_i^q - \phi_i^r(\mathbf{N}^r_{\text{-}i}) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}^q_{\text{-}i})} f_i^r(v_i^r) dv_i^r$,

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\cdot)}^1 f_i^q(v_i^q) \prod_{r \in \mathcal{M}' \setminus \{q\}} F_i^r\big(v_i^q - \phi_i^r(\mathbf{N}^r_{\text{-}i}) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}^q_{\text{-}i})\big) dv_i^q. \qquad \square$$

Single-homing platforms with mutually exclusive sides. Finally, for the sake of completeness, we now consider the most restrictive scenario in which some or all platforms are single-homing and, simultaneously, some or all sides are mutually exclusive.

This scenario is exemplified by several of the previously mentioned transportation companies in jurisdictions that require contractual exclusivity for drivers: e.g., in certain EU countries, the drivers have the status of employees.[15] In this case, drivers providing rides on one platform can neither do so on other platforms (single-homing) nor can they engage in renting their cars (mutually exclusive sides). This situation also finds parallels in the increasingly popular platform-based energy communities and utilities, such as Piclo (UK), Powerledger (Australia), and Drift (US) (IRENA, 2020; Baake et al., 2023). Upon joining one of the utilities—securing electricity from two utilities is unworkable—households equipped with solar panels and domestic batteries can decide whether to buy or sell electricity based on their needs (e.g., appliances, electric vehicles charge, etc.) and demand-supply conditions. However, engaging in both selling and buying energy simultaneously is unfeasible.

Formally, this double exclusivity (within and across certain platforms) requires the introduction of additional ICCs. More precisely, given $\mathbf{v}$ and $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}'$ on platform $q \in \mathcal{M}'$ if, and only if, PC_i^q, ICC_{ij}^q, and ICC_i^{qr} defined above hold and

$$(ICC_{ij}^{qr})\ v_i^q + \phi_i^q(\mathbf{N}^q_{\text{-}i}) - p_i^q \quad \geq \quad v_j^r + \phi_j^r(\mathbf{N}^r_{\text{-}j}) - p_j^r,$$

also holds for all $j \in \mathcal{D}' \subseteq \mathcal{D}$, $i \neq j$ and for all $r \in \mathcal{M}' \subseteq \mathcal{M}$, $q \neq r$. This latter constraint implies that an end-user is better off by joining platform q's side i than platform r's side j. Together these

[15] Belleflamme and Peitz (2019a) note that third-party applications conveniently display offers from different companies to drivers on a single screen. However, in certain countries, exclusivity policies restrict such practices.

conditions yield the number of end-users on platform q's side i:

$$N_i^q(\cdot) = pr\left(v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q \geq \max \left\{ \max_{\substack{j\in\mathcal{D}' \\ r\in\mathcal{M}'}} \left\{ v_j^r + \phi_j^r(\mathbf{N}_{\text{-}j}^r) - p_j^r \right\}, 0 \right\} \right).$$

We can then use the marginal density functions of the surplus parameters and their corresponding marginal distribution functions to rewrite participation on platform q's side i as follows:

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)}^1 f_i^q(v_i^q) \prod_{r\in\mathcal{M}'\setminus\{q\}} F_i^r\big(v_i^q - \phi_i^r(\cdot) + p_i^r - p_i^q + \phi_i^q(\cdot)\big)$$
$$\prod_{j\in\mathcal{D}'\setminus\{i\}} F_j^q\big(v_i^q - \phi_j^q(\cdot) + p_j^q - p_i^q + \phi_i^q(\cdot)\big) \prod_{j\in\mathcal{D}'\setminus\{i\}} \prod_{r\in\mathcal{M}'\setminus\{q\}} F_j^r\big(v_i^q - \phi_j^r(\cdot) + p_j^r - p_i^q + \phi_i^q(\cdot)\big) dv_i^q.$$

□

By capturing the number of end-users on each side and platform while considering the exclusivity that might exist across and within platforms, we can then build a system of $m \times d$ equations that captures the number of end-users on all sides and platforms, denoted as $\mathbf{N}(\cdot)$. Importantly, as explained above, this system can accommodate any combination of all the distinct cases examined in this section, where all, some, or none of the sides are mutually exclusive and in which some, all, or none of the platforms are single-homing.

4 Participation equilibrium and platform-optimal pricing

We solve our model backward to characterize a subgame perfect equilibrium. We begin with the second stage, where end-users decide which side(s) and platform(s) to join, given any $\mathbf{p}$. We formally prove the existence and uniqueness of the participation equilibrium, and then we characterize the solutions that are non-empty, i.e., those in which there are end-users on all sides and platforms. Finally, we study the platform's optimal pricing and derive explicit price elasticity formulas, discussing their implications tailored to the four scenarios introduced in Section 3.

4.1 Participation equilibrium: existence, uniqueness, and characterization

In Proposition 1, the proof of which can be found in Appendix A, we show the existence and uniqueness of a participation equilibrium, $\mathbf{N}^*(\mathbf{p})$, given $\mathbf{p}$. This result holds if Assumption 1 and an additional standard condition related to the maximum change in N_i^q resulting from a small change in N_j^q are satisfied.

Proposition 1. *Given* $\mathbf{p}$*, there is a unique participation equilibrium* $\mathbf{N}^*(\mathbf{p})$*, if Assumption 1 and the following condition holds:*

$$\beta := \underbrace{\max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \left|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\right|}_{\beta_1=\text{condition on dispersion of } v_i\text{'s}} \times \underbrace{\max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \left(\sum_{j\in\mathcal{D}\setminus\{i\}} \sup_{\mathbf{N}^q_{\text{-}i}\in[0,1]^{d\text{-}1}} \left|\frac{\partial \phi_i^q(\mathbf{N}^q_{\text{-}i})}{\partial N_j^q}\right| \right)}_{\beta_2=\text{condition on externalities}} < 1.$$

If all sides are mutually exclusive, Proposition 1 holds under Assumption 1 alone. However, if some sides are non-mutually exclusive, the additional condition $\beta_1 < \beta_2^{-1}$ must be satisfied. Intuitively, β_1 bounds the maximum change in N_i^q resulting from a small change in N_j^q, among all possible $\mathbf{N}^q$ vectors. The absolute value of the derivative of N_i^q with respect to N_j^q is a marginal density $f_{ij}^q(\cdot) \in [0,1]$. In case there are some non-mutually exclusive sides, then the sum of these marginal densities for all $j \in \mathcal{D}$ may exceed 1 since each end-user can join multiple sides within the same platform. By contrast, when all sides are mutually exclusive, end-users can join only one side at most, ensuring that this sum remains below 1; consequently, $\beta_1 < 1$ is trivially satisfied. Condition $\beta_2 < 1$, which bounds the feedback loop of cross-group externalities, holds by Assumption 1. Therefore, in cases where all sides are mutually exclusive, $\beta < 1$ follows directly from Assumption 1, but if some sides are non-mutually exclusive, we additionally require $\beta_1 < \beta_2^{-1}$.[16]

Next, we provide necessary and sufficient conditions for the existence of a non-empty participation equilibrium, which is an equilibrium with at least one end-user on each side and platform. A non-empty equilibrium requires that the price, net of the cross-group externalities, must be below 1 for all sides and platforms. If this condition is not satisfied for a specific side and platform, no end-user would opt to join it—as end-user valuations are at most 1. Under multi-homing, if all sides are non-mutually exclusive, this condition stands as both necessary and sufficient for the existence of a non-empty equilibrium. However, in all other cases, this condition together with binding ICC conditions, which prevent a particular side and/or platform from consistently yielding the highest utility in comparison to others, are both necessary and sufficient.

[16]Note that $\beta < 1$ is a sufficient condition, also fulfilled at non-empty corner solutions, in which all end-users join at least one side. Tan and Zhou (2021) also impose a very similar condition to ensure uniqueness under single-homing and fixed sides.

First, when mutually exclusive sides are present, the ICCs across all mutually exclusive sides for all platforms must be binding. Second, in case all sides are non-mutually exclusive but some platforms are single-homing, then the ICCs must be binding across all sides for each single-homing platform. Lastly, if there are single-homing platforms with some mutually exclusive sides, then both sets of conditions described above must be met simultaneously, and the ICCs of all mutually exclusive sides across each single-homing platform must also be binding. All these conditions are formally provided in Proposition 2, the proof of which is in Appendix A.

Proposition 2. *Given* $\mathbf{p}$*, there exists a non-empty participation equilibrium* $\mathbf{N}^*(\mathbf{p})$ *if, and only if,*

a) (PC) $\left[\phi_i^q(\cdot) - p_i^q\right] \geq -1$ *holds for all* $i \in \mathcal{D}$ *and all* $q \in \mathcal{M}$ *in the baseline case;*

b) (PC) and (ICC 1) $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^q(\cdot) - p_j^q\right] \in [-1, 1]$ *hold for all* $i, j \in \mathcal{D}'$*,* $i \neq j$*, and all* $q \in \mathcal{M}$*, in case sides* $i, j \in \mathcal{D}' \subseteq \mathcal{D}$ *are mutually exclusive;*

c) (PC) and (ICC 2) $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_i^r(\cdot) - p_i^r\right] \in [-1, 1]$ *hold for all* $i \in \mathcal{D}$ *and all* $q, r \in \mathcal{M}'$*,* $q \neq r$*, in case platforms* $q, r \in \mathcal{M}' \subseteq \mathcal{M}$ *are single-homing;*

d) (PC), (ICC 1), (ICC 2), and (ICC 3) $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^r(\cdot) - p_j^r\right] \in [-1, 1]$ *hold for all* $i, j \in \mathcal{D}'$*,* $i \neq j$*, and all* $q, r \in \mathcal{M}'$*,* $q \neq r$*, in case platforms* $q, r \in \mathcal{M}' \subseteq \mathcal{M}$ *are single-homing and sides* $i, j \in \mathcal{D}' \subseteq \mathcal{D}$ *are mutually exclusive.*

Throughout the rest of the paper, we focus our analysis on the set of non-empty equilibria as characterized in Proposition 2.[17] These equilibria are unique, provided the conditions outlined in Proposition 1 are satisfied. We also emphasize that non-empty equilibria should not be confused with solutions involving full market coverage, which is an assumption frequently made by previous authors. In our framework, it can be easily shown that having at least one PC slack for all end-users serves as a sufficient condition for $\mathbf{N}(\cdot)$ to yield full market coverage.

[17] Note that if participation is empty on one side or platform, our analysis extends to the remaining $d-1$ sides or $m-1$ platforms, respectively.

4.2 Platform optimal pricing

In the first stage, platform q maximizes its profit π^q by choosing prices for all sides. Assuming a cost per end-user of $c_i^q > 0$ on side i, platform q's profit maximization problem is given by

$$\max_{\{p_1^q,\cdots,p_d^q\}} \quad \pi^q := \sum_{i\in\mathcal{D}} (p_i^q - c_i^q) N_i^q. \tag{6}$$

At an interior solution, the first-order-condition (FOC) of (6) that characterizes platform q's optimal price for side i, denoted by $\hat{p}_i^q$, is given by

$$N_i^q + (\hat{p}_i^q - c_i^q)\frac{\partial N_i^q}{\partial p_i^q} + \sum_{j\in\mathcal{D}\setminus\{i\}} (\hat{p}_j^q - c_j^q)\frac{\partial N_j^q}{\partial p_i^q} = 0, \tag{7}$$

where N_i^q and its derivatives are evaluated at the optimal price $\hat{p}_i^q$. By the extreme value theorem, given the continuity of the profit function, generically there exists a vector of optimal prices that solves (7) for all $i \in \mathcal{D}$ and all $q \in \mathcal{M}$.[18] The left-hand-side (LHS) of (7) is the usual FOC with respect to p_i^q for any profit-maximizing firm, augmented by $\sum_{j\in\mathcal{D}\setminus\{i\}}(\hat{p}_j^q - c_j^q)\frac{\partial N_j^q}{\partial p_i^q}$. This sum captures the fact that increasing $\hat{p}_i^q$ also affects participation on all other sides through different cross-group effects, formally identified below.

When (7) is rearranged, optimal pricing can be rewritten in the familiar Lerner index notation:

$$\frac{\hat{p}_i^q - c_i^q}{\hat{p}_i^q} = \frac{1}{|\varepsilon_i^q|} + \sum_{j\in\mathcal{D}\setminus\{i\}} \lambda_{ij}^q \frac{(\hat{p}_j^q - c_j^q)}{\hat{p}_i^q}, \tag{8}$$

where $\varepsilon_i^q := \frac{\partial N_i^q}{\partial p_i^q}\frac{p_i^q}{N_i^q}$ is the own-price elasticity of demand for platform q's side i, which is evaluated at $\hat{p}_i^q$, and $\lambda_{ij}^q := \left[\frac{\partial N_j^q}{\partial p_i^q} / \left|\frac{\partial N_i^q}{\partial p_i^q}\right|\right]$ is the *diversion ratio*:[19] it reflects the change in the number of end-users on platform q's side j relative to the change on side $i \neq j$ when p_i^q increases. Hence, the diversion ratio captures the extent to which platform q internalizes the different effects that a price change has on the number of end-users on other sides. To quantify these effects, in the following subsection, we characterize own- and cross-price elasticities for the different combinations of the

[18] The optimal price vector is unique under additional standard assumptions, such as log-concavity of $f_i^q(\cdot)$. This assumption is sufficient for uniqueness and is commonly made. In a related context, it is made by Zhou (2017), Choi et al. (2018), and (for some results) Tan and Zhou (2021).

[19] Due to Shapiro (1996), this term is widely used in the anti-trust literature (see Conlon and Mortimer, 2021).

scenarios considered. In both the following subsection and Section 5, without loss of generality, we restrict the analysis to the case in which platforms charge positive prices[20]—which, however, does not preclude platforms from cross-subsidizing (i.e., pricing below marginal cost), if they find that doing so is optimal.

4.3 Price elasticities and implications for optimal platform pricing

4.3.1 Baseline case

First, we consider the baseline case in which end-users can join any side on any platform. Given the marginal density functions of valuations of any pair of sides, say, i and j, in platform q, $f_i^q(\cdot)$ and $f_j^q(\cdot)$ respectively, the derivatives of N_i^q and N_j^q with respect to p_i^q can be formally obtained from the expressions that characterize the number of end-users in (3):

$$\frac{\partial N_i^q}{\partial p_i^q} = -\underbrace{f_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)\big)}_{\text{price effect}} + \underbrace{f_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{\text{-}i}^q)\big)\frac{\partial \phi_i^q(\mathbf{N}_{\text{-}i}^q)}{\partial p_i^q}}_{\text{network effect}}, \tag{9}$$

$$\frac{\partial N_j^q}{\partial p_i^q} = \underbrace{f_j^q\big(p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q)\big)\frac{\partial \phi_j^q(\mathbf{N}_{\text{-}j}^q)}{\partial p_i^q}}_{\text{network effect}}. \tag{10}$$

The own-price derivative (9) can be decomposed into a *price effect*, which arises from the negative impact of p_i^q on PC_i^q, and a *network effect*. Under positive cross-group externalities, the decrease in N_i^q due to the negative price effect reduces participation on all sides except side i, which further decreases N_i^q. Under congestion, the effect is opposite in sign, and N_i^q increases. The cross-price derivative of N_j^q with respect to p_i^q is given by (10). In this case, only a network effect arises: the decrease in N_i^q due to the price effect changes end-user utility on side j through the cross-group externality, thus altering N_j^q. In Lemma 1, we formally show that, given a non-empty interior equilibrium with $N_i^q \in (0,1)$ for all i and q,[21] the sign of (10) depends on the nature of the network effect: it is negative under positive cross-group externalities and positive under congestion. We also

[20] This restriction simplifies the interpretation of the own- and cross-price elasticities and the matrix of relative prices defined in Section 5. Our analysis extends to the case in which any platform charges a strictly negative price on some side(s).

[21] If all end-users join a side in a platform (i.e., $N_i^q = 1$ for some i and q), an arbitrarily small change in p_i^q (or p_j^q) does not affect participation on that side and platform (i.e., $N_i^q = 1$ still holds). Consequently, own- and cross-price derivatives are zero. These non-empty corner equilibria, in which all end-users join side i in platform q, cannot occur in all other cases considered since side i is mutually exclusive and/or platform q is single-homing.

show that, despite the ambiguity in the sign of the network effect, an increase in p_i^q always decreases N_i^q—the price effect dominates the network effect under Assumption 1. From these results, the signs of the own- and the cross-price elasticities then follow.

Lemma 1. *Consider that end-users can join any side on any platform (baseline case). Given a non-empty interior equilibrium* $\mathbf{N}^*(\mathbf{p})$*, the own-price elasticity* $\varepsilon_i^q := \frac{\partial N_i^q}{\partial p_i^q}\frac{p_i^q}{N_i^q} < 0$*, the cross-price elasticity* $\varepsilon_{ij}^q := \frac{\partial N_i^q}{\partial p_j^q}\frac{p_j^q}{N_i^q} < 0$ *under positive cross-group externalities, and* $\varepsilon_{ij}^q > 0$ *under congestion.*

Lemma 1's proof can be found in Appendix A. It implies that λ_{ij}^q is negative under positive cross-group externalities and positive under congestion. Therefore, the optimal pricing rule captures several results that are in line with previous studies. First, if cross-group externalities are positive, the optimal markup on one side is adjusted downward by the opportunity cost in terms of the revenue from all other sides, resulting in a markup lower than the usual Lerner index. Second, this optimal pricing rule may involve *cross-subsidization*; that is, for some parameters, platform q finds it optimal to set a price below marginal cost on one side, resulting in a strictly positive profit on another side. For example, if $|\varepsilon_i^q|$ is sufficiently large, we have $\hat{p}_i^q - c_i^q < 0$, requiring $\hat{p}_j^q - c_j^q > 0$ for some side j.[22]

4.3.2 Mutually exclusive sides

We now consider that a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ are mutually exclusive. That is, end-users face a choice between joining either side i or j, for all $i, j \in \mathcal{D}'$, $i \neq j$. In this case, the same price and network effects (stemming from the PCs) as those described above occur following a change in p_i^q. However, there is an additional effect through the ICC(s): an increase in p_i^q induces some end-users, who are on side i but are close-to-indifferent between joining side i or j, to switch from the former to the latter. We call this the *switching-side effect* and show its existence in Proposition 3, the proof of which can be found in Appendix A.

Proposition 3. *Consider that a subset of sides* $\mathcal{D}' \subseteq \mathcal{D}$*,* $|\mathcal{D}'| \geq 2$*, in platform* q *are mutually exclusive. Given* $\mathbf{N}^*(\mathbf{p})$*, suppose that platform* q *changes prices from* $\mathbf{p}^q$ *to* $\tilde{\mathbf{p}}^q$*, where* $\tilde{p}_k^q = p_k^q$ *for all* $k \in \mathcal{D} \setminus \{i\}$*, and* $\tilde{p}_i^q = p_i^q + \varepsilon$ *for some* $i \in \mathcal{D}'$ *and* $\varepsilon > 0$*. Then some end-users switch from side*

[22] Only when we additionally impose that end-users' valuations are uniformly distributed does the characterization of optimal prices in our model yield those obtained, for example, by Armstrong (2006) and Rochet and Tirole (2006).

i to all other mutually exclusive sides $j \in \mathcal{D}' \setminus \{i\}$ on platform q, while no end-user changes from any side j to side i.

Therefore, an increase in p_i^q has a twofold impact on participation on sides i and j. First, N_i^q decreases due to the price and the switching-side effects. Second, N_j^q increases due to the switching-side effect and further increases (decreases) under congestion (positive cross-group externalities). To illustrate the interplay of these effects, we provide a numerical example next.

Example 1. Consider a platform with two mutually exclusive sides, and let end-users' valuations be jointly uniformly distributed; i.e., for each end-user, any $(v_i, v_j) \in [0,1]^2$ arises with equal probability. Let the cross-group externality be $\phi_i(N_j) = N_j$ and $\phi_j(N_i) = N_i$, and suppose that $p_i = p_j = .4$. Then, PC_i is given by $v_i - .4 + N_j \geq 0$, while ICC_{ij} is given by $v_i - .4 + N_j \geq v_j - .4 + N_i$. It can be easily shown that there is a unique participation equilibrium with full market coverage in which half of the end-users join side i, and the other half join side j (since the cross-group externality $N_i = N_j = .5$ is larger than the price .4).

Given this equilibrium, suppose now that the platform increases p_i to .42. Then, a new equilibrium arises with full market coverage. However, because joining side i is now relatively more expensive than joining side j, some end-users switch from side i to side j. This is captured by ICC_{ij}, which becomes $v_i - .42 + N_j \geq v_j - .4 + N_i$. To find the end-user who is indifferent between joining side i or j, we let ICC_{ij} hold with equality and find that $N_j = .51$ and $N_i = .49$; that is, the switching-side effect makes .01 of end-users on side i drop it and switch to side j.

However, $N_j = .51$ and $N_i = .49$ cannot be the final solution: if it were, this would imply that in equilibrium, $v_i = v_j$ holds, in which case, half of the end-users join each side (which is a contradiction). In fact, we also need to take into account the subsequent network effect, which prevents some end-users from leaving side i due to the additional end-users that switched to side j and, likewise, reduces participation on side j due to the decrease in participation on side i.

To find the new equilibrium, we calculate the valuation for side i, denoted by $\bar{v}_i$, for an end-user with $v_j = 0$, such that she is indifferent to joining side i or j at prices $p_i = .42$ and $p_j = .4$; that is, $\bar{v}_i$ captures the higher valuation for side i that the indifferent end-user needs to join side i rather than side j, given the higher p_i. For such an indifferent end-user, the following equality must hold:

$$\bar{v}_i - .42 + N_j = -.4 + N_i. \tag{11}$$

Considering that the valuations are jointly uniformly distributed, N_i and N_j can be calculated from $N_i = .5(1-\bar{v}_i)^2$, and due to full market coverage, $N_j = 1 - .5(1-\bar{v}_i)^2$ must also hold. Replacing these conditions in (11) yields the following:

$$\begin{aligned} \bar{v}_i - .42 + 1 - .5(1-\bar{v}_i)^2 &= -.4 + .5(1-\bar{v}_i)^2, \\ \bar{v}_i - .02 + 1 - (1-\bar{v}_i)^2 &= 0. \end{aligned}$$

Solving for $\bar{v}_i$ yields two solutions, namely, one that is larger than 1 and thus discarded and another that is $\bar{v}_i = .0067$. Hence, at the new participation equilibrium, $N_i = .4933$ and $N_j = .5067$.

In summary, following the increase of p_i from .4 to .42 (with p_j unchanged at .4), first there is a switching-side effect that decreases N_i from .5 to .49 and increases N_j from .5 to .51. Then, the network effect reduces N_j again from .51 to .5067 and increases N_i from .49 to .4933. □

Next, we analytically derive both the own- and the cross-price elasticity for any pair of mutually exclusive sides, say, i and j. Regarding the own-price elasticity, one might conclude that its sign is ambiguous: since N_j^q can either increase (as shown in Example 1) or decrease (as we explain below) following an increase in p_i^q, the network effect on N_i^q could be either positive or negative, potentially offsetting the price effect. However, once again, we show that the own-price elasticity is always negative. By contrast, determining the implications of the different effects on the cross-price elasticity is not as straightforward. Hence, the sign of the cross-price elasticity depends on the model's parameters and on the features of the participation equilibrium.

More precisely, if there is full participation on platform q, such as in Example 1, an increase in p_i^q does not induce any end-user to drop the platform (i.e., PC_i^q remains slack), but some of the end-users will switch from side i to a different mutually exclusive side, say, j. In this context, the cross-price elasticity is unambiguously positive—regardless of whether the cross-group externalities are positive (which attenuates the increase in N_j^q) or negative (which reinforces the increase in N_j^q). If participation is less than full (i.e., PCs are binding), the effect of this decrease in N_i^q on N_j^q depends on the nature of the cross-group externalities. Under congestion, the cross-price derivative is unambiguously positive because both the network and the switching-side effect increase participation on side j. However, if the cross-group externalities are positive, both effects go in opposite directions, and the impact on N_j^q can be either positive or negative (depending on which effect dominates). These results are formally presented in Theorem 1, the proof of which can be found in Appendix A.

Theorem 1. *Given a subset $\mathcal{D}'$ of mutually exclusive sides on platform q, at a non-empty equilibrium $\mathbf{N}^*(\mathbf{p})$, $\varepsilon_i^q < 0$ for all $i \in \mathcal{D}'$. For $j \in \mathcal{D}' \setminus \{i\}$, $\varepsilon_{ij}^q > 0$ if $\sum_{i \in \mathcal{D}'} N_i^q = 1$ or if there is congestion, and ε_{ij}^q can be either positive or negative if $\sum_{i \in \mathcal{D}'} N_i^q < 1$ and cross-group externalities are positive.*

Theorem 1 captures a novel result that is worth highlighting, namely, that an increase in p_i^q can increase participation on all sides other than i, even in the presence of positive cross-group externalities (and regardless of whether market coverage is full or not). Intuitively, consistent with the previous literature, a decrease in the number of end-users on one side would typically reduce the number of end-users on all other sides through the network effect. However, in our model, an increase in p_i^q has a negative impact on side j only if the network effect dominates the switching-side effect; otherwise, N_j^q increases. Previous authors do not formally find this switching-side effect.

Finally, we examine the implications of these results for optimal platform pricing. As shown, the Lerner index (8) depends on the sign of the diversion ratio λ_{ij}^q for all $i, j \in \mathcal{D}$, $i \neq j$. If all sides are non-mutually exclusive (Section 4.3.1), the sign of λ_{ij}^q is solely determined by whether there are positive cross-group externalities or congestion. However, when sides i and j are mutually exclusive, determining the sign of λ_{ij}^q is more intricate, as follows from Theorem 1.

If the participation equilibrium yields full coverage, then those end-users that drop side i following an increase in p_i^q are *diverted* to another (mutually exclusive) side j, and $\lambda_{ij}^q > 0$. Consequently, the optimal markup is greater than the usual Lerner index. The same result is obtained in the presence of congestion effects as, in this case, an increase in p_i^q also increases N_j^q. However, with positive cross-group externalities and less than full coverage, an increase in p_i^q has two opposite effects on N_j^q, and thus, the result is ambiguous. Namely, if the switching-side effect dominates the network effect, increasing p_i^q raises participation on side j (since $\lambda_{ij}^q > 0$), and hence, the optimal markup is greater than the Lerner index. Conversely, if the network effect dominates the switching-side effect, $\lambda_{ij}^q < 0$, and consequently, the optimal markup is smaller than the usual Lerner index. This ambiguity under positive cross-group externalities is formally stated in Corollary 1.

Corollary 1. *Given a subset $\mathcal{D}'$ of mutually exclusive sides on platform q, at a non-empty equilibrium $\mathbf{N}^*(\mathbf{p})$, the vector of optimal platform prices, denoted by $\hat{\mathbf{p}}$, is characterized by the Lerner index (8), adjusted downward or upward by* $\sum_{j \in \mathcal{D} \setminus \{i\}} (\hat{p}_j^q - c_j^q)\lambda_{ij}^q$.

Compared to the previous literature, the characterization of optimal prices in Corollary 1 differs in two important aspects. First, all previous authors show that the optimal prices are adjusted downward (absent congestion) after a price increase, resulting in a markup lower than the usual Lerner index (e.g., Armstrong, 2006 and Rochet and Tirole, 2006). This is due to the fact that an increase in p_i^q has an opportunity cost for the platform in terms of side j revenue. To the contrary, we show that for some parameters, the impact of an increase in p_i^q on side j's participation is positive, which results in optimal platform markups higher than the usual Lerner index (even with positive cross-group externalities). Second, in the latter case, we find that it is never optimal for the platform to cross-subsidize—a result that also stands in stark contrast with the conclusions of previous authors.[23]

4.3.3 Single-homing platforms

Parting again from the baseline case, in which end-users can join multiple sides within a platform, we now assume that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$ are single-homing. This implies that each end-user faces a choice between joining side i either on platform q or r, for all $q, r \in \mathcal{M}'$, $q \neq r$, and for all $i \in \mathcal{D}$. Thus, each end-user joins side i on the platform that yields the highest utility (if any) through ICCs. This introduces an additional effect that we call *switching-platform effect*: an increase in p_i^q induces some end-users on side i on platform q, who are close-to-indifferent between joining side i on platform q or r, to switch from q to r. We now formally show the existence of this effect.

Proposition 4. *Consider that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$, $|\mathcal{M}'| \geq 2$, are single-homing. Given $\mathbf{N}^*(\mathbf{p})$, suppose that platform $q \in \mathcal{M}'$ changes prices from $\mathbf{p}^q$ to $\tilde{\mathbf{p}}^q$, where $\tilde{p}_j^q = p_j^q$ for all $j \in \mathcal{D} \setminus \{i\}$, and $\tilde{p}_i^q = p_i^q + \varepsilon$ for some $i \in \mathcal{D}$ and $\varepsilon > 0$. Then, on side i, some end-users switch from platform*

[23] A notable exception is Bakos and Halaburda (2020), who show under several additional assumptions that if there are multi-homing end-users on all sides, it is never optimal for the platform to cross-subsidize, as the interdependence between the different sides—traditionally captured in the literature by the network effect—falls apart. In our model, the transmission channel is slightly different: when the interdependence of sides (through the network effect) becomes less important *vis-à-vis* our novel switching-side effect, then cross-subsidization does not arise.

q to all other single-homing platforms $r \in \mathcal{M}' \setminus \{q\}$, while no end-user changes from any r to q.

The proof of this proposition follows that of Proposition 3 but employs condition ICC_i^{qr} instead of ICC_{ij}^q. Importantly, the switching-platform effect identified in Proposition 4 occurs regardless of whether there are positive cross-group externalities or congestion. Hence, this effect further decreases the number of end-users on platform q's side i following an increase in p_i^q relative to the baseline case. Table 1 summarizes and compares all the effects that occur when platforms q and r are multi-homing, relative to the case in which they are single-homing, assuming that sides i and j are non-mutually exclusive.

Table 1: Effects that occur following an increase in p_i^q on end-user participation on sides i and j (non-mutually exclusive)

	Platforms q and r are multi-homing		**Platforms q and r are single-homing**	
	Effect	**Sign**	**Effect**	**Sign**
Side N_i^q	Price effect Network effect	– – (+)	Price effect Network effect Switching-platform effect	– – (+) –
Side N_j^q	Network effect	– (+)	Network effect	– (+)
Side N_i^r	$\emptyset$		Switching-platform effect	+

Note: The signs of the different effects indicated are those under positive cross-group externalities, while the signs in parenthesis are those under congestion (indicated only for the cases in which they differ).

In both cases, Lemma 1 applies regarding the sign of the own- and cross-price derivatives. However, these derivatives are "larger" in the single-homing case because the switching-platform effect amplifies the negative impact of an increase in p_i^q on platform q's participation. In any case, platform q's profit maximization problem is similar to the one in Section 4.3.1. Thus, the usual Lerner index charged to end-users on one side is dragged down by the markup charged to end-users on the other sides under positive cross-group externalities,[24] and the usual Lerner index increases in the case of congestion.

[24] Due to the negative impact on the markup charged, platforms might have the incentive to segment the market in order to limit the switching-platform effect (Karle et al., 2020).

4.3.4 Mutually exclusive sides and single-homing platforms

In the final case, we consider that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$ are single-homing, and a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ are mutually exclusive. Therefore, each end-user is restricted to joining one of the mutually exclusive sides (say, i or j) on one of the single-homing platforms (say, q or r). In this setup, the switching-side and the switching-platform effects derived above occur. In addition, there exists an end-user who is close-to-indifferent between joining side i on platform q or side j on platform r—e.g., a car owner might be nearly indifferent between renting her car on Uber or providing rides on Bolt. Thus, through ICC_{ij}^{qr}, an increase in p_i^q induces some end-users to switch from platform q's side i to platform r's side j. We call this last and novel effect *switching-side-and-platform effect*, formally stated in Proposition 5, the proof of which is analogous to that of Proposition 3 but uses ICC_{ij}^{qr} instead of ICC_{ij}^{q}.

Proposition 5. *Consider that a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$, $|\mathcal{D}'| \geq 2$, are mutually exclusive and that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$, $|\mathcal{M}'| \geq 2$, are single-homing. Given $\mathbf{N}^*(\mathbf{p})$, suppose that platform $q \in \mathcal{M}'$ changes prices from $\mathbf{p}^q$ to $\tilde{\mathbf{p}}^q$, where $\tilde{p}_k^q = p_k^q$ for all $k \in \mathcal{D} \setminus \{i\}$, and $\tilde{p}_i^q = p_i^q + \varepsilon$ for some $i \in \mathcal{D}'$ and $\varepsilon > 0$. Then some end-users switch from side i on q to all other mutually exclusive sides $j \in \mathcal{D}' \setminus \{i\}$ on all other single-homing platforms $r \in \mathcal{M}' \setminus \{q\}$, while no end-user changes to platform q's side i.*

If platforms q and r are multi-homing, participation on side i decreases following an increase in p_i^q, as we show in Section 4.3.2. When these platforms are single-homing, this negative impact on participation on side i is "larger" due to two additional effects: the switching-platform and the switching-side-and-platform effects. Consequently, the own-price elasticity is negative. By contrast, the effect of an increase in p_i^q on participation on another mutually exclusive side j on platform q remains ambiguous. When q and r are multi-homing, this ambiguity is driven by two opposite effects, namely, the switching-side effect—which increases participation on side j—and the network effect, which decreases it absent congestion. If these platforms are single-homing, both the switching-platform and switching-side-and-platform effects unambiguously increase participation on all mutually exclusive sides of all single-homing platforms except q. These effects are summarized in Table 2, which provides a comparison between the case in which platforms q and r are multi-homing and single-homing, respectively, provided that sides i and j are mutually exclusive.

Table 2: Effects that occur following an increase in p_i^q on end-user participation on sides i and j (mutually exclusive)

	Platforms q and r are multi-homing		**Platforms q and r are single-homing**	
	Effect	**Sign**	**Effect**	**Sign**
Side N_i^q	Price effect	$-$	Price effect	$-$
	Network effect	$-$ $(+)$	Network effect	$-$ $(+)$
	Switching-side effect	$-$	Switching-side effect	$-$
			Switching-platform effect	$-$
			Switching-side-and-platform effect	$-$
Side N_j^q	Network effect	$-$ $(+)$	Network effect	$-$ $(+)$
	Switching-side effect	$+$	Switching-side effect	$+$
Side N_i^r	$\emptyset$		Switching-platform effect	$+$
Side N_j^r	$\emptyset$		Switching-side-and-platform effect	$+$

Note: The signs of the different effects indicated are those under positive cross-group externalities, while the signs in parenthesis are those under congestion (indicated only for the cases in which they differ).

Finally, as in Section 4.3.2, the platform's optimal markup on each side is larger or smaller than the Lerner index depending on whether there is congestion or not and, if not, on whether the switching-side effect dominates the network effect. The role of the switching-platform and switching-side-and-platform effects on prices will become relevant, though, in the following section.

5 Platform merger analysis

We rely on the results derived above to study the consequences of a merger of platforms for markups. For that purpose, we extend the analysis by Affeldt et al. (2013), who adapted Farrell and Shapiro (2010)'s UPP impact of mergers of firms to the mergers of two two-sided platforms. Farrell and Shapiro (2010) show that the changes in prices following a merger depend on the value of the diverted sales that the merged entity recaptures, formally characterized by diversion ratios.

The importance of diversion ratios to understand the consequences of mergers has also gained momentum among practitioners in competition and antitrust; e.g., the US Federal Trade Commission's 2010 merger guidelines (currently in force) and the 2023 draft of the merger guidelines (currently under consideration) use diversion ratios as key statistics to measure unilateral price effects of mergers. The reason for this is that diversion ratios synthesize the degree to which the merged entity internalizes the substitution effect between its jointly owned products. Diversion ratios are not only acknowledged as a powerful tool to analyze mergers, particularly in the context of non-homogeneous goods, but also as a practical one, as they can be estimated using market, scanner, or survey data—see Conlon and Mortimer (2021) and Section 2.2 in Affeldt et al. (2013).

Our theoretical framework provides deeper insights on the direction of diversion ratios, which are based on the different effects we identify. Therefore, our framework serves as an appropriate toolkit to analyze mergers by considering which of the different effects occur across sides and/or platforms within the merged entity in different scenarios. We thus provide a general and unifying framework to anticipate the UPP of platform mergers in a multi-sided market context. Equipped with this framework, we rationalize the mixed results obtained by authors that were, until now, given only in more specific contexts.[25]

For example, Chandra and Collard-Wexler (2009) study theoretically the merger of two two-sided platforms that compete à la Hotelling. In their model, the authors show that a switching of end-users occurs across platforms (on the same side, as in Section 4.3.3), which is internalized if both platforms merge. The authors then find that the merged entity increases or decreases prices depending solely on whether the platforms cross-subsidize before the merger or not. Correia-da Silva et al. (2019) also find an ambiguous effect on prices depending on the existence of cross-subsidization, assuming instead that platforms compete à la Cournot. By contrast, we show that the final impact on prices depends not only on the existence of cross-subsidization before the merger but also on the relative strength of the network effect *vis-à-vis* the switching-side effect.

Using data on TV magazines, Song (2021) finds empirically that post-merger prices might either go up or down, with prices on different sides usually moving in opposite directions. Similarly, using data on US daily newspapers, Fan (2013) shows that ad rates and subscription prices post-merger tend to move in opposite directions.[26] Considering that an increase in subscription fees (in ad rates) by magazines and newspapers is implausible to induce a reader to *switch side* and become

[25]We abstract from certain features that are relevant for mergers of platforms in particular industries, such as political motives in the media industry (Anderson and McLaren, 2012), "time paying attention" by users in digital outlets and social networks (Prat and Valletti, 2022), or content variety in the radio industry (Sweeting, 2010).

[26]Jeziorski (2014), who studies mergers in the radio industry, documents a similar trade-off that the merged firms face between exercising market power on one side of the market (advertisers) or the other (listeners).

an advertiser (or vice versa), and bearing in mind that cross-group externalities are positive on the advertising side, our model rationalizes these empirical results. Indeed, our model predicts that when the network effect across different sides on the same platform dominates the switching-side effect, as is the case in the examples under consideration, post-merger prices on both sides do move in opposite directions.[27]

5.1 General case

We consider the same theoretical framework as in Section 2 but additionally assume that a subset of platforms, denoted by $\mathcal{N} \subseteq \mathcal{M}$, merge, where $\mathcal{N} = \{1, ..., n\}$ with $2 \leq n \leq m$ (as long as $m \geq 2$). Then, the profit maximization problem for the merged firms is

$$\max_{\{p_1^1,\cdots,p_d^n\}} \quad \sum_{s\in\mathcal{N}} \pi^s := \sum_{s\in\mathcal{N}} \sum_{i\in\mathcal{D}} (p_i^s - c_i^s) N_i^s.$$

In the most general case, i.e., considering all potential across and within platform effects that may exist, the optimal price that the merged entity charges to end-users on side i of platform s at an interior solution is characterized by the following FOC:

$$\frac{(p_i^s - c_i^s)}{p_i^s} = \frac{1}{|\varepsilon_i^s|} + \sum_{j\in\mathcal{D}\setminus\{i\}} \lambda_{ij}^s \frac{(p_j^s - c_j^s)}{p_i^s} + \underbrace{\sum_{t\in\mathcal{N}\setminus\{s\}} \sum_{j\in\mathcal{D}} \lambda_{ij}^{st} \frac{(p_j^t - c_j^t)}{p_i^s}}_{\text{across-platform diversion effects}}, \tag{12}$$

where λ_{ij}^s is the diversion ratio defined above and $\lambda_{ij}^{st} := \left[\frac{\partial N_j^t}{\partial p_i^s} / \left|\frac{\partial N_i^s}{\partial p_i^s}\right|\right]$ for all $s, t \in \mathcal{N}$, $s \neq t$, and where the derivatives are evaluated at the optimal solution. Note that the right-hand side (RHS) of (12) adds an additional term relative to the RHS of (8), which characterizes optimal pricing before the merger. This last term in (12) appears since the merged entity internalizes the different across-platform effects (if any), which now determine the UPP of the merger.

As is customary in the literature (Affeldt et al., 2013; Miller et al., 2017; Conlon and Mortimer, 2021), we compute the UPP on the optimal markup for each side and platform, holding the prices of all sides and platforms in the RHS of (12) fixed at the pre-merger optimal level. That is, our goal is to predict the post-merger impact on the markup of side i and platform s based on pre-merger

[27] As we show later, if the switching-side effect dominates the network effect or if there are congestion effects on both sides of a platform, then prices on both sides will unambiguously increase after the merger.

optimal prices $\hat{p}_i^t$ for all $i \in \mathcal{D}$ and all $t \in \mathcal{N}$ from (8) and the pre-merger corresponding parameters ε_i^s, λ_{ij}^s, and λ_{ij}^{st}, in each of the cases studied in previous sections. To do so, it is convenient to present a "stacked" version of (12) for all platforms that merge. For that purpose, let us denote the matrices of the (pre-merger) *within* and *across* platform diversion ratios, respectively, by

$$\Lambda^s = \begin{pmatrix} 1 & -\lambda_{12}^s & \cdots & -\lambda_{1d}^s \\ -\lambda_{21}^s & 1 & \cdots & -\lambda_{2d}^s \\ \vdots & & \ddots & \vdots \\ -\lambda_{d1}^s & -\lambda_{d2}^s & \cdots & 1 \end{pmatrix} \qquad \text{and} \qquad \Lambda^{st} = \begin{pmatrix} -\lambda_{11}^{st} & -\lambda_{12}^{st} & \cdots & -\lambda_{1d}^{st} \\ -\lambda_{21}^{st} & -\lambda_{22}^{st} & \cdots & -\lambda_{2d}^{st} \\ \vdots & & \ddots & \vdots \\ -\lambda_{d1}^{st} & -\lambda_{d2}^{st} & \cdots & -\lambda_{dd}^{st} \end{pmatrix},$$

and let the corresponding matrices of (pre-merger) relative optimal prices be

$$\Theta^s = \begin{pmatrix} 1 & \frac{\hat{p}_2^s}{\hat{p}_1^s} & \cdots & \frac{\hat{p}_d^s}{\hat{p}_1^s} \\ \frac{\hat{p}_1^s}{\hat{p}_2^s} & 1 & \cdots & \frac{\hat{p}_d^s}{\hat{p}_2^s} \\ \vdots & & \ddots & \vdots \\ \frac{\hat{p}_1^s}{\hat{p}_d^s} & \frac{\hat{p}_2^s}{\hat{p}_d^s} & \cdots & 1 \end{pmatrix} \qquad \text{and} \qquad \Theta^{st} = \begin{pmatrix} \frac{\hat{p}_1^t}{\hat{p}_1^s} & \frac{\hat{p}_2^t}{\hat{p}_1^s} & \cdots & \frac{\hat{p}_d^t}{\hat{p}_1^s} \\ \frac{\hat{p}_1^t}{\hat{p}_2^s} & \frac{\hat{p}_2^t}{\hat{p}_2^s} & \cdots & \frac{\hat{p}_d^t}{p_2^s} \\ \vdots & & \ddots & \vdots \\ \frac{\hat{p}_1^t}{\hat{p}_d^s} & \frac{\hat{p}_2^t}{\hat{p}_d^s} & \cdots & \frac{\hat{p}_d^t}{\hat{p}_d^s} \end{pmatrix},$$

for all $s, t \in \mathcal{N}$, $s \neq t$.

Then, optimal pricing for the merged platforms is given by the following (block) matrix equation:

$$\underbrace{\left(\begin{array}{c|c|c|c} \Lambda^1 \odot \Theta^1 & \Lambda^{12} \odot \Theta^{12} & \cdots & \Lambda^{1n} \odot \Theta^{1n} \\ \hline \Lambda^{21} \odot \Theta^{21} & \Lambda^2 \odot \Theta^2 & \cdots & \Lambda^{2n} \odot \Theta^{2n} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \Lambda^{n1} \odot \Theta^{n1} & \Lambda^{n2} \odot \Theta^{n2} & \cdots & \Lambda^n \odot \Theta^n \end{array} \right)}_{\mathcal{H}} \begin{pmatrix} \boldsymbol{\mu}^1 \\ \boldsymbol{\mu}^2 \\ \vdots \\ \boldsymbol{\mu}^n \end{pmatrix} = \begin{pmatrix} \mathcal{E}^1 \\ \mathcal{E}^2 \\ \vdots \\ \mathcal{E}^n \end{pmatrix},$$

where $\odot$ denotes the Hadamard product of two matrices; ${}^T\boldsymbol{\mu}^s := \left(\frac{p_1^s - c_1^s}{p_1^s}, \cdots, \frac{p_d^s - c_d^s}{p_d^s} \right) \in \mathbb{R}^d$ is the vector of optimal markups for each merging platform s that result from incorporating the predicted UPP post-merger; and $\mathcal{E}^s$ is the vector of (pre-merger) inverse own-price elasticities on platform s, where ${}^T\mathcal{E}^s := \left(\frac{1}{|\varepsilon_1^s|}, \cdots, \frac{1}{|\varepsilon_d^s|} \right) \in \mathbb{R}_+^d$. Therefore, the UPP impact of the merger of n platforms on their markups depends on the diversion ratios across platforms, i.e., on Λ^{st} for all $s, t \in \mathcal{N}$, $s \neq t$.[28] We now analyze the merger's impact for each of the possible cases considered in Section 3.

[28] Following Song (2021), we ignore any efficiency gains that arise from the merger, for example, from an increase in productivity (Braguinsky et al., 2015) or innovation (Cabral, 2021). However, we can accommodate efficiency gains along the lines of Affeldt et al. (2013).

Multi-homing platforms. In this case, there are no switching-platform effects, and consequently, across-platform diversion ratios are zero (regardless of whether sides are mutually exclusive or not). Therefore, the merger does not affect markups.

Proposition 6. *If all merged platforms are multi-homing, then* $\mathcal{H} = \bigoplus_{s \in \mathcal{N}} (\Lambda^s \odot \Theta^s)$*, all off-diagonal block matrices in* $\mathcal{H}$ *are null, and the merger does not change platforms' markups relative to those in the pre-merger case.*

Under multi-homing, there is no UPP post-merger. This result follows from the *independent* nature of platforms (Belleflamme and Peitz, 2019b). To illustrate this, consider e-commerce platforms (e.g., Taobao, Facebook Marketplace, or eBay), on which end-users may sell and buy goods. Suppose that an end-user sells an item through several e-commerce platforms. Then, an increase in the selling fee charged by one of the platforms might induce the end-user to drop it. However, this does not induce the end-user to drop the other platforms. In other words, changes in prices do not trigger a switching of end-users across platforms, and hence, there are no across-platform diversion effects to be internalized by the merged entity.[29] Therefore, if some of the platforms merge, the optimal prices are identical to those charged by the platforms separately. Consistently, Farronato et al. (2023) empirically find that fees did not increase in the context of a multi-homing platform merger between the two dog boarding/walking companies Rover and DogVacay.

Single-homing platforms. We now consider that a subset of the merging platforms, denoted by $\mathcal{N}' \subseteq \mathcal{N}$ with $|\mathcal{N}'| \geq 2$, are single-homing. In this case, end-users must choose whether they join one platform or another for each of their sides. Unlike in the previous case, where joining multiple platforms was possible, the exclusivity that single-homing platforms impose on end-users triggers different across-platform effects, which are internalized when they merge. As a consequence, optimal markups before and after the merger generically differ.

Suppose first that all platforms' sides are non-mutually exclusive. In this scenario, there are only switching-platform effects across the single-homing platforms, and the following properties regarding matrix $\mathcal{H}$ immediately follow.

[29] We implicitly assume that end-users have no time or budget constraints. Otherwise, price changes may induce substitution effects, and end-users may switch platforms. In this case, the single-homing framework would apply.

Lemma 2. *Consider that a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing, in which all sides are non-mutually exclusive. Then, all off-diagonal block matrices of $\mathcal{H}$ are non-positive. In particular, they are equal to $\Lambda^{st} \odot \Theta^{st} = diag\left(-\lambda_{ii}^{st}\frac{\hat{p}_i^t}{\hat{p}_i^s}\right)$ for all $i \in \mathcal{D}$ and all $s, t \in \mathcal{N}'$, $s \neq t$, and equal to the null matrix for all the other cases.*

Since switching-side-and-platform effects are null under non-mutually exclusive sides, all the off-diagonal elements in Λ^{st} are zero. Thus, the post-merger UPP on side i of single-homing platform s is solely determined by the diversion ratios that capture the switching-platform effects (λ_{ii}^{st}) and their interaction with the markups charged by the other single-homing platforms $t \neq s$ on side i. That is, the last element on the RHS of (12) boils down to $\sum_{t \in \mathcal{N}' \setminus \{s\}} \lambda_{ii}^{st} \frac{(p_i^t - c_i^t)}{p_i^s}$, which (evaluated at optimal pre-merger prices) determines the UPP on side i on platform s. Bearing in mind that λ_{ii}^{st} is unambiguously positive for all $i \in \mathcal{D}$ and for all $s, t \in \mathcal{N}'$, $s \neq t$,[30] it then follows that the markup on side i on platform s can go up or down post-merger. The direction of this change depends on the sign of $(\hat{p}_i^t - c_i^t)$ for all $t \in \mathcal{N}'$, $t \neq s$, i.e., on whether the other single-homing platforms charge a pre-merger price above or below marginal cost to end-users on side i.

If cross-group externalities are negative (congestion), then the network effect is positive, and as a consequence, optimal (pre-merger) prices are always above marginal cost, i.e., $(\hat{p}_i^t - c_i^t) > 0$ for all $t \in \mathcal{N}'$. Hence, the merger unambiguously increases prices on all sides for all single-homing platforms. However, if cross-group externalities are positive, optimal pre-merger markups are adjusted downward, and cross-subsidization may take place; i.e., $(\hat{p}_i^t - c_i^t)$ may be negative for some $t \in \mathcal{N}'$. Thus, the UPP of the merger is (ex-ante) indeterminate. For example, if end-users on a side of a single-homing platform are cross-subsidized before the merger, then the price on this side further decreases post-merger. This also implies that this platform charges a positive pre-merger markup to end-users on another side, who experience an increase in the markup after the merger. By contrast, if the platform's markups are positive before the merger on all sides, then post-merger prices unambiguously increase.

Proposition 7. *Consider that all sides are non-mutually exclusive. Under congestion effects, the merger increases markups on all sides of the merged single-homing platforms. If cross-group*

[30] This holds regardless of the sign of the cross-group externalities. As shown in Section 4.3.3, the switching-platform effect is unambiguously positive both under positive cross-group externalities and under congestion effects.

externalities are positive and $\sum_{t\in\mathcal{N}'\setminus\{s\}} \lambda_{ii}^{st}\frac{(\hat{p}_i^t-c_i^t)}{\hat{p}_i^s} > 0$ $\Big($*conversely* $\sum_{t\in\mathcal{N}'\setminus\{s\}} \lambda_{ii}^{st}\frac{(\hat{p}_i^t-c_i^t)}{\hat{p}_i^s} \leq 0\Big)$*, the merger increases (conversely weakly decreases) the markup on side* $i \in \mathcal{D}$ *of platform* $s \in \mathcal{N}'$*.*

To illustrate this result, consider the merger of two single-homing platforms A and B, each with two sides. If pre-merger prices are above marginal cost on both sides of both platforms, then the merger unambiguously raises prices. However, this is not true if the platforms cross-subsidize end-users on one side (which requires the cross-group externalities to be positive), say, on side 1. In this case, the merger further decreases markups on side 1 for both platforms, while the merger raises markups on side 2 for both platforms. This result is in line with Song (2021), who empirically shows that for the newspaper industry, ad prices post-merger tend to move in the opposite direction of copy prices (consistent with the fact that readers are usually cross-subsidized and single-home). However, prices sometimes increase on both sides after the merger, which is the case if newspapers are able to charge positive markups to advertisers and readers.

Finally, for the sake of completeness, we consider that a subset of sides, denoted by $\mathcal{D}' \subseteq \mathcal{D}$, with $|\mathcal{D}'| \geq 2$, of the single-homing platforms that merge are mutually exclusive. Contrary to the previous case, some of the off-diagonal elements in matrix Λ^{st} are now strictly negative due to the presence of the additional switching-side-and-platform effect identified in Section 4.3.4.

Lemma 3. *Consider that a subset of sides* $\mathcal{D}' \subseteq \mathcal{D}$ *of the merged platforms are mutually exclusive and that a subset of the merged platforms,* $\mathcal{N}' \subseteq \mathcal{N}$*, are single-homing. Then, all off-diagonal block matrices of* $\mathcal{H}$ *are non-positive. In particular,* $\Lambda^{st} \odot \Theta^{st}$ *is a non-diagonal, non-positive matrix for all* $s, t \in \mathcal{N}'$*,* $s \neq t$*, and it is equal to the null matrix for all the other cases.*

Based on this result, we show that the UPP on a merging platform's markups depends not only on the nature of the cross-network externalities (as in the previous case) but also, more strikingly, on the strength of the switching-side effects of the other merging platforms. To observe this, we consider any of the merging single-homing platforms, say, $s \in \mathcal{N}'$. The UPP of the merger on this platform is determined by the interaction of the diversion ratios that capture both the switching-platform and the switching-side-and-platform effects with the corresponding (pre-merger) optimal markups charged by all the other single-homing merging platforms $t \in \mathcal{N}' \setminus \{s\}$. As follows from Propositions 4 and 5, both the switching-platform effects (captured by λ_{ii}^{st} for all $i \in \mathcal{D}$) and the switching-side-and-platform effects (captured by λ_{ij}^{st} for all $i, j \in \mathcal{D}'$, $i \neq j$) are positive for all

$s, t \in \mathcal{N}'$, $s \neq t$.[31] However, the markups charged by any other merging platform $t \in \mathcal{N}' \setminus \{s\}$ can be either positive or negative, depending on *(i)* the sign of the cross-group externalities and *(ii)* the strength of the different effects across sides within the platform $t \in \mathcal{N}' \setminus \{s\}$.

More precisely, if cross-group externalities are negative (congestion), both the switching-side effect between any pair of mutually exclusive sides, say, i and j, and the network effect go in the same direction, and hence, the diversion ratio across these sides within platform $t \in \mathcal{N}' \setminus \{s\}$ (captured by λ_{ij}^{t}) are unambiguously positive. In this case, optimal pre-merger prices on platform $t \in \mathcal{N}' \setminus \{s\}$ are always above marginal cost. This also occurs if cross-group externalities are positive and if the switching-side effect between all pairs of mutually exclusive sides dominates the network effect on platform $t \in \mathcal{N}' \setminus \{s\}$. Consequently, in both cases, the merger unambiguously increases markups on all sides of platform s. Conversely, with positive cross-group externalities, if the network effect dominates the switching-side effect for a pair of mutually exclusive sides, say, i and j, on platform $t \in \mathcal{N}' \setminus \{s\}$, then λ_{ij}^{t} is negative, and hence, this platform might find it optimal to cross-subsidize (albeit not necessarily) some end-users. Thus, the UPP of the merger on platform s is ambiguous and depends on the model's parameters.

Proposition 8. *Consider that a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ are mutually exclusive. If $\lambda_{ij}^{s} \geq 0$ for all $i, j \in \mathcal{D}$, $i \neq j$, and for all $s \in \mathcal{N}'$, the merger increases markups on all the mutually exclusive sides of the merged single-homing platforms. Otherwise, if $\sum_{t \in \mathcal{N}' \setminus \{s\}} \left(\lambda_{ii}^{st} \frac{(\hat{p}_i^t - c_i^t)}{\hat{p}_i^s} + \sum_{j \in \mathcal{D}' \setminus \{i\}} \lambda_{ij}^{st} \frac{(\hat{p}_j^t - c_j^t)}{\hat{p}_i^s} \right) > 0$ $\big($conversely $\leq 0\big)$, the merger increases (conversely weakly decreases) the markup on side $i \in \mathcal{D}'$ of platform $s \in \mathcal{N}'$.*

By combining these findings with the optimal platform pricing results presented in Section 4, we can derive readily applicable, policy-relevant implications of mergers for welfare—defined as the sum of end-users' surplus (given by the valuation of those who join a side on a platform, net of the network effect and price paid) and platforms' profits. These implications are relevant for mergers involving single-homing platforms because, in these cases, there are post-merger price changes triggered by the different across-platform switching effects that are internalized by the platforms

[31] Note that switching-platform effects between side i in platform s and side i in platform t occur for all sides $i \in \mathcal{D}$ due to the single-homing nature of the platforms (e.g., a rider choosing between Uber and Bolt). However, switching-side-and-platform effects between side i in platform s and side j in platform t exclusively occur across the subset of mutually exclusive sides $i, j \in \mathcal{D}', i \neq j$ (e.g., a passenger deciding to get a ride with Bolt or opting to rent a car using Uber).

that merge.[32] Therefore, the overall impact on welfare hinges *(i)* on the magnitude of the across-platform switching effects (how many end-users switch platforms), which increase merged platforms' profits at the expense of end-users' surplus, and *(ii)* on the magnitude of the own-price effect (how many end-users drop/join the platforms), which negatively or positively impacts end-users' surplus, depending on the sign of the markups charged by the platforms.

First, if there are congestion effects or if there are positive cross-group externalities and the switching-side effect dominates the network effect, cross-subsidization does not occur. Consequently, there is a strictly positive UPP post-merger, leading to an unambiguous decrease in end-user surplus and an unambiguous increase in platforms' profits. However, the net impact on overall welfare is negative. This is because, on the one hand, platforms obtain an additional profit (via the internalized across-platform switching effects) at the expense of end-users' surplus, but on the other hand, some end-users (on some sides/platforms) drop out due to higher prices, resulting in an overall negative impact on welfare.[33] However, when there are positive cross-group externalities and the network effect dominates the switching-side effect, cross-subsidization on a side may occur. In such instances, the merger leads to lower prices on the cross-subsidized sides—thereby increasing participation—while the prices on other sides rise, causing decreased participation, resulting in an ambiguous effect on end-user surplus (Song, 2021). Consequently and since platforms' profits also increase post-merger (otherwise, platforms would not set the new prices), this case yields an overall (ex-ante) ambiguous impact on welfare.[34]

In summary, our general framework accommodates the analysis of mergers involving various platform types, regardless of whether some of their sides are mutually exclusive or not. In each case, the presence of non-zero diversion ratios in Λ^{st} plays a pivotal role in shaping the resulting impact on post-merger prices and welfare—in particular, in the presence of single-homing platforms (diagonal elements) with mutually exclusive sides (off-diagonal elements). In the following subsection, we show how this framework also provides a useful workhorse for analyzing the merger of platforms

[32] We remark again that these implications are also relevant for multi-homing platforms in cases where end-users effectively single-home due to, e.g., budget or time constraints. In such instances, price changes may also induce across-platform effects, and consequently, it would be more appropriate to apply the single-homing framework.

[33] This result is consistent with the literature on platform mergers involving substitutes, such as Nevo (2000), which is a framework that closely resembles the two cases discussed here.

[34] Our analysis abstracts away from other issues, such as regressive effects arising when one side of the market is "excessively subsidized" at the expense of others (Sarin, 2020) and the potential for platforms to implement "own-rules" that enhance welfare, as explored in Johnson et al. (2023), that lie beyond the scope of this paper.

that offer "seemingly unrelated" services but that are connected through some sides.

5.2 Special case: merger of "seemingly unrelated" platforms

For now, we have referred mainly to previous studies of mergers among multi-sided platforms within the same industry (e.g., newspapers, radio stations, etc.) or to platforms that offer similar services (e.g., dog walking/boarding). However, our main results above highlight that the relevant unit of analysis is not a platform but a side (and its potential overlap along with the corresponding effects). Consequently, we finally show that our framework also provides a useful toolkit to analyze mergers of platforms that offer different services (potentially in different industries)—usually called *conglomerate mergers*—but in which at least one of the sides is somehow connected through one of the previously identified effects.[35]

For example, this is the case if a peer-to-peer ride-sharing platform merges with another platform that offers food delivery services. These platforms not only offer unrelated services but also belong to very different sectors. However, they are connected through one of the sides if food couriers deliver orders from restaurants to customers by car: in this case, the courier might choose either to deliver food or to drive passengers (i.e., there is a switching-platform effect between both platforms on the drivers' side). A case in point is provided by Ola Cabs' acquisition of the food delivery platform Foodpanda—whose drivers often deliver food by car—in December 2017.

Another example is the acquisition of HotelTonight—a platform that connects travelers and hotels for very last-minute bookings—by AirBnb in March 2019. Again, although these platforms offer different services (vacation rentals vs. hotel rooms), travelers who search for apartments using AirBnB might switch and check for hotels at HotelTonight's website (in particular, if AirBnB users are last-minute travelers). There is again a switching-platform effect that the acquirer firm internalizes. A similar internalization of the switching-platform effect can be expected in other mergers of platforms, such as that of Lyft and Motivate (a US bicycle sharing platform) and that of Uber and JUMP (a scooter sharing system), as users might choose either to receive a ride by car or to complete their trips by finding a nearby bike or scooter to use.

Formally, in all these examples, end-users on a side, say, side i, on each platform are unrelated,

[35] Even though conglomerate mergers are becoming increasingly popular in (digital) platform markets, previous studies discuss only the potential harmful effects of conglomerate mergers in other (standard) markets (Garcia and de Azevedo, 2019).

which implies that $\frac{\partial N_i^s}{\partial p_i^t} = 0$ for all $s, t \in \mathcal{N}$, $s \neq t$ (e.g., riders are not directly affected by a change in the fee charged to those who order food, and those who order food are not affected by a change in the fee charged for a ride), which in turn implies that $\lambda_{ii}^{st} = 0$.[36] However, the cross-price derivative will be different from 0 for end-users on another side, say, side j, through which platforms s and t are related (e.g., some last minute travelers might drop HotelTonight and instead search for an apartment on AirBnb if the fees charged by HotelTonight increase, and Uber users might choose to use a JUMP scooter instead of an Uber vehicle if fees charged by the latter service are relatively high). In these cases, $\lambda_{jj}^{st} > 0$, and consequently, $\mathcal{H}$ is a diagonal matrix in which some of the diagonal elements are equal to 0 (for the unrelated sides), while others are strictly lower than 0 (for the related sides).

Suppose that platforms s and t are unrelated on side i but related through side j. Then, optimal pricing for the merged platform is characterized by[37]

$$\frac{(p_j^s - c_j^s)}{p_j^s} = \frac{1}{|\varepsilon_j^s|} + \lambda_{ij}^s \frac{(p_i^s - c_i^s)}{p_i^s} + \underbrace{\lambda_{jj}^{st} \frac{(p_j^t - c_j^t)}{p_j^s}}_{\text{across-platform diversion effects}} . \tag{13}$$

Therefore, if the price-cost markup charged by platform t on side j is strictly positive (i.e., in the absence of cross-subsidization), the merger is beneficial for the merging platforms (to the detriment of end-users), as it increases side j's markups by the across platforms diversion effects indicated in (13). This occurs because these platforms internalize the switching-platform effect on the side through which they are related (i.e., side j in the example).[38] This result holds even though platforms s and t offer different services (they do not share the same end-users), and therefore, its consequences might potentially receive less attention from anti-trust authorities—e.g., in countries that investigate mergers only if the market share of the combined entity is above a certain threshold (Motta and Peitz, 2021; Nocke and Whinston, 2022). Indeed, antitrust and competition authorities did not actively investigate as potentially harmful some of the mergers and acquisitions that we

[36] For simplicity, we assume here that there are positive cross-group externalities and no switching-side-and-platform effects (if there are no same side effects across platforms, *a fortiori*, it is unlikely that effects across sides and platforms will be observed). However, our general framework can easily accommodate the case in which $\lambda_{ij}^{st} \neq 0$.

[37] This result easily generalizes to more than two multi-sided platforms. In this case, the post-merger effect in (13) should include the sum for all other platforms t whose sides j relate to side j of platform s.

[38] Argentesi et al. (2021) note that most acquisitions by Amazon, Facebook, or Google target companies offering a wide range of products and services complementary to those supplied (i.e., those facilitating the switching effects).

mention above, such as that of Foodpanda and OlaCabs or that of HotelTonight and AirBnb.

6 Conclusions

This paper presents a general micro-founded model of multi-sided markets in which heterogeneous end-users endogenously choose both which side and which platform they join. These decisions are considered with the assumption that participation on one side of a platform sometimes precludes participation on another side and that platforms can be either single- or multi-homing.

From the model's primitives, we derive the demand of the different sides for the various scenarios considered. For each of the scenarios, we find a unique participation equilibrium and then derive platforms' optimal prices, from which closed-form expressions for own- and cross-price elasticities arise. Using the closed-form expressions, we formally characterize a set of novel switching effects, by which an increase in the fee on one side of a platform induces some end-users to switch to different sides and/or platforms. Optimal prices differ mainly along the dimension of whether the sides of platforms are mutually exclusive or not.

Then, we analyze the consequences of multi-sided platform mergers. We show that for end-users, mergers of multi-homing platforms are innocuous, as they do not affect optimal pricing, while mergers of single-homing platforms do affect optimal pricing (and there is a nuanced difference depending on whether the platform sides are mutually exclusive or not). Thus, we provide a unifying framework to analyze platform mergers, which allows us to explain the mixed results obtained by previous authors.

We illustrate that both dimensions—single- vs. multi-homing and mutually exclusive sides or otherwise—are important in our framework. The first determines whether mergers between multi-sided platforms have harmful effects for end-users through post-merger price increases, and the second determines whether platforms' optimal markups can be larger than the usual Lerner index.

The tractability of the general theoretical model we provide will be useful for applied and future empirical research, in particular, to study antitrust issues. We believe, moreover, that the model will be an appealing and convenient starting point for authors interested in more specific applications that can be obtained by modifying our relatively general assumptions.

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Appendix A. Proofs

Proof of Proposition 1. Let $\mathbf{p}$ be given. Define set $N^{m+1} := \{v_i^q \mid pr\big(v_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q < 0\big)$ for all $i \in \mathcal{D}$ and all $q \in \mathcal{M}\}$, i.e., any end-user in set N^{m+1} joins no platform at all since her valuation net of price and cross-group externality is less than zero. Then, let $\tilde{N} := (\mathbf{N}^1, ..., \mathbf{N}^m, N^{m+1})$, where $\mathbf{N}^q := (N_1^q, ..., N_d^q)$ for all $q \in \mathcal{M}$, and define the mapping $\psi : \tilde{N} \to \tilde{N}$, such that $\psi : [0,1]^{m\times d+1} \to [0,1]^{m\times d+1}$. By Brouwer's fixed point theorem, ψ has a fixed point given that it is a continuous function from a compact and convex set to itself. Denote such a fixed point by $\mathbf{N}^*(\mathbf{p})$. This shows existence of a participation equilibrium $\mathbf{N}^*(\mathbf{p})$.

To show uniqueness of the participation equilibrium,[39] suppose that Assumption 1 holds and note that the L^∞ norm on some vector $\mathbf{X}$ is defined as $\|X\|_\infty = \max\limits_{i\in\mathcal{D},q\in\mathcal{M}} |x_i^q|$. Then, $\|\mathbf{N}-\mathbf{N'}\|_\infty$ denotes the infinity norm of the distance between vectors $\mathbf{N}$ and $\mathbf{N'}$. We show uniqueness by using the contraction mapping theorem, and thus, need to show that for all $\mathbf{N}, \mathbf{N'}$,

$$\|\tilde{N}(\mathbf{N}) - \tilde{N}(\mathbf{N'})\|_\infty \leq \beta \|\mathbf{N} - \mathbf{N'}\|_\infty. \tag{A.1}$$

Let

$$\beta := \max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \left| \frac{\partial N_i^q(\cdot)}{\partial N_j^q} \right| \times \max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \left(\sum_{j\in\mathcal{D}\setminus\{i\}} \sup_{\mathbf{N}_{\text{-}i}^q\in[0,1]^{d-1}} \left| \frac{\partial \phi_i^q(\mathbf{N}_{\text{-}i}^q)}{\partial N_j^q} \right| \right) < 1.$$

To show that (A.1) holds, we use the following result which is an immediate consequence of the mean value theorem: for any continuously differentiable function $g(a_1, ..., a_n)$ on a convex domain $E \subset \mathbf{R}^n$, and for any vectors $\mathbf{a} = (a_1, ..., a_n)$ and $\mathbf{b} = (b_1, ..., b_n)$,

$$|g(\mathbf{a}) - g(\mathbf{b})| \leq \left(\max_i |a_i - b_i|\right) \left(\sup_{\mathbf{e}\in E} \sum_{j=1}^n \left| \frac{\partial g(\mathbf{e})}{\partial a_j} \right| \right),$$

where $\max_i|a_i - b_i|$ is L^∞ norm of the vector $\mathbf{a}-\mathbf{b}$, and $\left(\sup_{\mathbf{e}\in E} \sum_{j=1}^n \left|\frac{\partial g(\mathbf{e})}{\partial a_j}\right|\right) \leq \sum_{j=1}^n \sup_{\mathbf{e}\in E} \left|\frac{\partial g(\mathbf{e})}{\partial a_j}\right|$, which is the supremum of the L^1 norm of the gradient ∇g over E.

Applying these results to any i and q and all $\mathbf{N}, \mathbf{N'}$, yields $\left|N_i^q\big(\phi_i^q(\mathbf{N}_{\text{-}i}^q)\big) - N_i^q\big(\phi_i^q(\mathbf{N}_{\text{-}i}^{q'})\big)\right| \leq$

[39] Our proof of uniqueness follows a similar line of argument as Tan and Zhou (2021). Yet, they consider only single-homing and mutually exclusive (fixed) sides and also assume full coverage. For more details about the mathematical arguments we use, any advanced book on mathematical analysis is helpful, such as Dieudonné (1964).

$$\leq \max_{i\in\mathcal{D}} \left|\phi_i^q(\mathbf{N}_{\text{-}i}^q) - \phi_i^q(\mathbf{N}_{\text{-}i}^{q'})\right| \times \max_{i\in\mathcal{D}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \left|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\right| \leq$$

$$\max_{i\in\mathcal{D}} \left(\sum_{j\in\mathcal{D}\setminus\{i\}} \sup_{\mathbf{N}_{\text{-}i}^q\in[0,1]^{d-1}} \left|\frac{\partial \phi_i^q(\mathbf{N}_{\text{-}i}^q)}{\partial N_j^q}\right| \right) \|\mathbf{N}^q - \mathbf{N}^{q'}\|_\infty \left(\max_{i\in\mathcal{D}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \left|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\right| \right)$$

$$\leq \beta\|\mathbf{N} - \mathbf{N}'\|_\infty,$$

which holds for any $\mathbf{N}$, $\mathbf{N}'$, any $i \in \mathcal{D}$, and $q \in \mathcal{M}$. $\square$

Proof of Proposition 2. Let $\mathbf{p}$ be given. Then, to show that there is a non-empty participation equilibrium $\mathbf{N}^*(\mathbf{p})$, consider first case a), the baseline case. Suppose *ad absurdum* that $\left[\phi_i^q(\cdot) - p_i^q\right] < -1$ for some $i \in \mathcal{D}$ and $q \in \mathcal{M}$, i.e., PC is not fulfilled. Then for an end-user with the highest valuation $v_i^q = 1$ it holds that $v_i^q + \phi_i^q(\cdot) - p_i^q < 0$, and she does not join side i on platform q, and thus, the equilibrium is empty, a contradiction. Therefore, PC is necessary for the equilibrium to be non-empty. Suppose next that $\mathbf{N}^*(\mathbf{p}) \neq \emptyset$, i.e., there is some end-user on each side i on each platform q. Since $v_i^q = 1$ is the highest valuation for any side i on any platform q, this implies that $\left[\phi_i^q(\cdot) - p_i^q\right] \geq -1$ for all $i \in \mathcal{D}$ and all $q \in \mathcal{M}$, and so PC is sufficient for the equilibrium to be non-empty.

Consider next case b) of multi-homing with sides $i, j \in \mathcal{D}' \subseteq \mathcal{D}$, $i \neq j$, being mutually exclusive. From case a), it follows that $\mathbf{N}^*(\mathbf{p}) = \emptyset$, when PC is not fulfilled. Suppose next *ad absurdum* that $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^q(\cdot) - p_j^q\right] \notin [-1, 1]$ for $i, j \in \mathcal{D}'$, $i \neq j$ and $q \in \mathcal{M}$. If $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^q(\cdot) - p_j^q\right] < -1$, then even an end-user with $v_i^q = 1$ and $v_j^q = 0$ does not join side i on q, and if $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^q(\cdot) - p_j^q\right] > 1$, then even an end-user with $v_i^q = 0$ and $v_j^q = 1$ does not join side j on q. Both end-users exist by full support. Hence, the equilibrium is empty, a contradiction. Therefore PC and ICC 1 are necessary conditions. Suppose next that $\mathbf{N}^*(\mathbf{p}) \neq \emptyset$. Then it follows from case a) that $\left[\phi_i^q(\cdot) - p_i^q\right] \geq -1$ for all i and q, and moreover, by full support the existence of the two end-users just found implies that $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^q(\cdot) - p_j^q\right] \in [-1, 1]$ holds for $i, j \in \mathcal{D}'$, $i \neq j$, and $q \in \mathcal{M}$. Therefore, PC and ICC 1 are sufficient for the equilibrium to be non-empty.

Consider next case c) of non-mutually exclusive sides and that platforms $q, r \in \mathcal{M}' \subseteq \mathcal{M}$, $q \neq r$, are single-homing. From case a), it follows that $\mathbf{N}^*(\mathbf{p}) = \emptyset$, when PC is not fulfilled. Suppose next *ad absurdum* that $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_i^r(\cdot) - p_i^r\right] \notin [-1, 1]$ for $i \in \mathcal{D}$ and for $q, r \in \mathcal{M}'$, $q \neq r$. If

$\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_i^r(\cdot)-p_i^r\right]<-1$, then even an end-user with $v_i^q=1$ and $v_i^r=0$ does not join side i on q, and if $\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_i^r(\cdot)-p_i^r\right]>1$, then even an end-user with $v_i^q=0$ and $v_i^r=1$ does not join side i on r. Both end-users exist by full support. Hence, the equilibrium is empty, a contradiction. Therefore PC and ICC 2 are necessary conditions. Suppose next that $\mathbf{N}^*(\mathbf{p})\neq\emptyset$, then it follows from case a) that $\left[\phi_i^q(\cdot)-p_i^q\right]\geq -1$ for all i and q, and moreover, by full support the existence of the two end-users just found implies that $\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_i^r(\cdot)-p_i^r\right]\in[-1,1]$ for $i\in\mathcal{D}$ and for $q,r\in\mathcal{M}'$, $q\neq r$. Therefore, PC and ICC 2 are sufficient for the equilibrium to be non-empty.

Finally, consider case d) of mutually exclusive sides $i,j\in\mathcal{D}'\subseteq\mathcal{D}$, $i\neq j$, and single-homing platforms $q,r\in\mathcal{M}'\subseteq\mathcal{M}$, $q\neq r$. From above, it follows that $\mathbf{N}^*(\mathbf{p})=\emptyset$, when PC, ICC 1 or ICC 2 is not fulfilled. Suppose next *ad absurdum* that $\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_j^r(\cdot)-p_j^r\right]\notin[-1,1]$ for $i,j\in\mathcal{D}'$, $i\neq j$, and $q,r\in\mathcal{M}'$, $q\neq r$. If $\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_j^r(\cdot)-p_j^r\right]<-1$, then even an end-user with $v_i^q=1$ and $v_j^r=0$ does not join side i on q, and if $\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_j^r(\cdot)-p_j^r\right]>1$, then even an end-user with $v_i^q=0$ and $v_j^r=1$ does not join side j on r. Both end-users exist by full support. Hence, the equilibrium is empty, a contradiction. Therefore PC, ICC 1, ICC 2 and ICC 3 are necessary conditions. Suppose next that $\mathbf{N}^*(\mathbf{p})\neq\emptyset$, then it follows from above that PC, ICC 1 and ICC 2 hold, and moreover, by full support the existence of the two end-users just found implies that $\left[\phi_i^q(\cdot)-p_i^q\right]-\left[\phi_j^r(\cdot)-p_j^r\right]\in[-1,1]$ for $i,j\in\mathcal{D}'$, $i\neq j$, and $q,r\in\mathcal{M}'$, $q\neq r$. Therefore, PC, ICC 1, ICC 2 and ICC 3 are sufficient for the equilibrium to be non-empty. □

Proof of Lemma 1. For ease of notation, we drop superscript q for the platform. Given the price effect in (9), $\Delta N_i^{PE}:=-f_i(\cdot)<0$, we first solve the sign of the cross-price derivative (10):

$$\frac{\partial N_j}{\partial p_i}=\frac{\partial N_j}{\partial N_i}\frac{\partial N_i}{\partial p_i}=\frac{\partial(1-F_j(p_j-\phi_j(\mathbf{N}_{\text{-}j})))}{\partial N_i}\Delta N_i^{PE}=$$

$$=\frac{\partial(1-F_j(p_j-\phi_j(\mathbf{N}_{\text{-}j})))}{\partial\phi_j(\mathbf{N}_{\text{-}j})}\sum_{i\neq j}\frac{\partial\phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}\Delta N_i^{PE}, \tag{A.2}$$

where we just apply the chain rule. Note that $\frac{\partial(1-F_j(p_j-\phi_j(\mathbf{N}_{\text{-}j})))}{\partial\phi_j(\mathbf{N}_{\text{-}j})}=f_j(p_j-\phi_j(\mathbf{N}_{\text{-}j}))\in(0,1)$, and $\sum_{i\neq j}\left|\frac{\partial\phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}\right|\in(0,1)$ by Assumption 1. It thus follows that $\frac{\partial N_j}{\partial p_i}\in(-\Delta N_i^{PE},\Delta N_i^{PE})$, that is, the absolute value of the cross-price derivative is smaller than that of the price effect in the own-price derivative. Finally, the sign of $\frac{\partial N_j}{\partial p_i}$ is determined by the sign of $\sum_{i\neq j}\frac{\partial\phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}$, i.e., $\frac{\partial N_j}{\partial p_i}<0$ under positive cross-group externalities and $\frac{\partial N_j}{\partial p_i}>0$ under congestion effects.

Given that $\sum_{i\neq j} \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i} \Delta N_i^{PE} = \sum_{i\neq j} \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i} \frac{\partial N_i}{\partial p_i} = \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial p_i}$, rewriting (A.2) yields (10) as required:

$$\frac{\partial N_j}{\partial p_i} = f_j\big(p_j - \phi_j(\mathbf{N}_{\text{-}j})\big)\frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial p_i}.$$

Now we show formally that $\frac{\partial N_i}{\partial p_i} < 0$. From (9) we obtain

$$\frac{\partial N_i}{\partial p_i} = -f_i(\cdot) + f_i(\cdot)\frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial p_i} = f_i(\cdot)\left(-1 + \sum_{j\neq i} \frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial N_j}\frac{\partial N_j}{\partial p_i}\right).$$

By Assumption 1, $\sum_{j\neq i} \left|\frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial N_j}\right| \in (0,1)$, and as just shown, $\left|\frac{\partial N_j}{\partial p_i}\right| < \left|\Delta N_i^{PE}\right| = f_i(\cdot) \in (0,1)$. Therefore, $\sum_{j\neq i} \frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial N_j}\frac{\partial N_j}{\partial p_i} \in (-1,1)$, and thus the magnitude of the price effect in (9) is strictly larger than that of the network effect which implies that $\frac{\partial N_i}{\partial p_i} < 0$, and the result follows. □

Proof of Proposition 3. Given $\mathbf{p}$ and $\mathbf{N}^*(\mathbf{p})$, consider end-user #1 with valuations $\bar{v}_i^q$ and $\bar{v}_j^q$, who joins side $i \in \mathcal{D}'$ on platform q, but who would also benefit, though by less, from joining side $j \in \mathcal{D}' \setminus \{i\}$, on platform q, i.e., for her $\bar{v}_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q > \bar{v}_j^q + \phi_j^q(\mathbf{N}_{\text{-}j}^q) - p_j^q > 0$. Rewriting this yields

$$\bar{v}_j^q - \bar{v}_i^q < \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q + p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q). \tag{A.3}$$

Next consider end-user #2 with valuations $\tilde{v}_i^q$ and $\tilde{v}_j^q$, who joins side j on platform q, but who would also benefit, though by less, from joining side i on platform q, i.e., for her $\tilde{v}_j^q + \phi_j^q(\mathbf{N}_{\text{-}j}^q) - p_j^q > \tilde{v}_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q > 0$. Rewriting this yields

$$\tilde{v}_j^q - \tilde{v}_i^q > \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q + p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q). \tag{A.4}$$

Both end-users exist by full support and as shown in Proposition 2, case b). Combining (A.3) and (A.4) yields:

$$\tilde{v}_j^q - \tilde{v}_i^q > \phi_i^q(\mathbf{N}_{\text{-}i}^q) - p_i^q + p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q) > \bar{v}_j^q - \bar{v}_i^q. \tag{A.5}$$

Given $\mathbf{p}^q$ and $\tilde{\mathbf{p}}^q$ such that $\tilde{p}_k^q = p_k^q$ for all $k \in \mathcal{D}\setminus\{i\}$, and $\tilde{p}_i^q = p_i^q + \varepsilon$ for some $i \in \mathcal{D}'$, where $\varepsilon > 0$, if end-user #1 switches to side j, then end-user #2 does not switch to side i. To show this, we state the new ICCs for both end-users. For end-user #1, this is $\bar{v}_j^q + \phi_j^q(\mathbf{N}_{\text{-}j}^q) - p_j^q > \bar{v}_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - \tilde{p}_i^q$.

Rewriting this yields

$$\bar{v}_j^q - \bar{v}_i^q > \phi_i^q(\mathbf{N}_{\text{-}i}^q) - \tilde{p}_i^q + p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q). \tag{A.6}$$

Next consider end-user #2 and suppose *ad absurdum* that she switches to side i on platform q. If this were true, then for her it holds that $\tilde{v}_i^q + \phi_i^q(\mathbf{N}_{\text{-}i}^q) - \tilde{p}_i^q > \tilde{v}_j^q + \phi_j^q(\mathbf{N}_{\text{-}j}^q) - p_j^q$. Rewriting yields

$$\tilde{v}_j^q - \tilde{v}_i^q < \phi_i^q(\mathbf{N}_{\text{-}i}^q) - \tilde{p}_i^q + p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q). \tag{A.7}$$

Analogously as before, combining (A.6) and (A.7) yields:

$$\bar{v}_j^q - \bar{v}_i^q > \phi_i^q(\mathbf{N}_{\text{-}i}^q) - \tilde{p}_i^q + p_j^q - \phi_j^q(\mathbf{N}_{\text{-}j}^q) > \tilde{v}_j^q - \tilde{v}_i^q. \tag{A.8}$$

Clearly this contradicts (A.5) which yields $\tilde{v}_j^q - \tilde{v}_i^q > \bar{v}_j^q - \bar{v}_i^q$. Therefore, we have shown that in case of switching (following a price change in a given equilibrium), this always takes place from the side on which the price increases to another mutually exclusive side. End-users from the side on which the price did not change never switch to the side on which it increases. $\square$

Proof of Theorem 1. Consider first full coverage, i.e., that $\sum_{i\in\mathcal{D}'} N_i^q = 1$. Without loss of generality pick side $i \in \mathcal{D}'$, and for ease of notation, drop superscript q. Given our assumption of full support, from ICC_{ij} it follows that for some end-user on side $i \in \mathcal{D}'$, $v_i = v_j + \phi_j(\cdot) - p_j + p_i - \phi_i(\cdot)$, for all $j \in \mathcal{D}' \setminus \{i\}$. Then, as p_i increases, any such end-user is strictly better off by joining side j instead. Since $\sum_{i\in\mathcal{D}'} N_i = 1$, and by Proposition 3 it follows that N_i decreases while N_j increases for all $j \in \mathcal{D}' \setminus \{i\}$ after an increase in p_i, then $\varepsilon_i < 0$ and $\varepsilon_{i,j} > 0$ for all $j \in \mathcal{D}' \setminus \{i\}$.

Suppose next that $\sum_{i\in\mathcal{D}'} N_i < 1$. Given own-price elasticity $\varepsilon_i := \frac{\partial N_i}{\partial p_i}\frac{p_i}{N_i}$, we show next that $\frac{\partial N_i}{\partial p_i} < 0$. The number of side-$i$ end-users in equilibrium is given by (5). Taking the derivative w.r.t. p_i yields

$$\begin{aligned}\frac{\partial N_i}{\partial p_i} = \; & \overbrace{\left(-1 + \frac{\partial \phi_i(\cdot)}{\partial p_i}\right) f_i(p_i - \phi_i(\mathbf{N}_{\text{-}i})) \prod_{j\in\mathcal{D}'\setminus\{i\}} F_j\left(p_j - \phi_j(\mathbf{N}_{\text{-}j})\right)}^{\text{price and network effect from } PC_i \Rightarrow \text{ end-users that drop the platform if } p_i \text{ increases}} \quad + \\ & \underbrace{\int_{p_i - \phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i) \frac{\partial}{\partial p_i} \prod_{j\in\mathcal{D}'\setminus\{i\}} F_j\left(v_i - \phi_j(\mathbf{N}_{\text{-}j}) + p_j - p_i + \phi_i(\mathbf{N}_{\text{-}i})\right) dv_i}_{\text{price and network effect from } ICC_{ij} \text{ for all } j\in\mathcal{D}'\setminus\{i\} \Rightarrow \text{ end-users that switch away from side } i \text{ if } p_i \text{ increases}} \quad .\end{aligned} \tag{A.9}$$

Now, we solve the derivative in the second line of (A.9):

$$\frac{\partial}{\partial p_i}\prod_{j\in\mathcal{D}'\setminus\{i\}} F_j\left(v_i-\phi_j(\mathbf{N}_{\text{-}j})+p_j-p_i+\phi_i(\mathbf{N}_{\text{-}i})\right)=\sum_{j\in\mathcal{D}'\setminus\{i\}}\frac{\partial F_j(\cdot)}{\partial p_i}\prod_{k\in\mathcal{D}'\setminus\{i,j\}}F_k(\cdot)=$$

$$=\sum_{j\in\mathcal{D}'\setminus\{i\}}\left(-\frac{\partial\phi_j(\cdot)}{\partial p_i}-1+\frac{\partial\phi_i(\cdot)}{\partial p_i}\right)f_j(\cdot)\prod_{k\in\mathcal{D}'\setminus\{i,j\}}F_k(\cdot).$$

Let $C:=\int_{p_i-\phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i)\sum_{j\in\mathcal{D}'\setminus\{i\}}\left(-\frac{\partial\phi_j(\cdot)}{\partial p_i}\right)f_j(\cdot)\prod_{k\in\mathcal{D}'\setminus\{i,j\}}F_k(\cdot)dv_i$, which captures the network effect that follows from the switching-side effect (i.e., the network impact generated by those that switch) on all other sides $k\in\mathcal{D}'\setminus\{i,j\}$. By Assumption 1, since the cross-group externalities are bounded, it cannot be larger than the switching-side effect itself. Formally, $\frac{\partial\phi_j(\cdot)}{\partial p_i}=\sum_{k\in\mathcal{D}'\setminus\{j\}}\frac{\partial\phi_j(\cdot)}{\partial N_k}\frac{\partial N_k}{\partial p_i}$ and by Assumption 1, $\sum_{k\in\mathcal{D}'\setminus\{j\}}\frac{\partial\phi_j(\cdot)}{\partial N_k}\in(-1,1)$. If this expression as well as $\sum_{k\in\mathcal{D}'\setminus\{i,j\}}\frac{\partial N_k}{\partial p_i}$ have the same sign, then $\frac{\partial\phi_j(\cdot)}{\partial p_i}>0$ and $C<0$. Otherwise, $\frac{\partial\phi_j(\cdot)}{\partial p_i}<0$ and $C>0$. We now show that this implies that $\frac{\partial N_i}{\partial p_i}<0$. Since $\sum_{k\in\mathcal{D}'\setminus\{i,j\}}\frac{\partial N_k}{\partial p_i}=\sum_{k\in\mathcal{D}'\setminus\{i,j\}}\frac{\partial N_k}{\partial N_i}\frac{\partial N_i}{\partial p_i}$, given positive cross-group externalities $\sum_{k\in\mathcal{D}'\setminus\{j\}}\frac{\partial\phi_j(\cdot)}{\partial N_k}\in(0,1)$ and $\sum_{k\in\mathcal{D}'\setminus\{i,j\}}\frac{\partial N_k}{\partial N_i}>0$, so for $\frac{\partial\phi_j(\cdot)}{\partial p_i}<0$ to hold requires that $\frac{\partial N_i}{\partial p_i}<0$; while under congestion $\sum_{k\in\mathcal{D}'\setminus\{j\}}\frac{\partial\phi_j(\cdot)}{\partial N_k}\in(-1,0)$, and $\sum_{k\in\mathcal{D}'\setminus\{i,j\}}\frac{\partial N_k}{\partial N_i}<0$ and for $\frac{\partial\phi_j(\cdot)}{\partial p_i}<0$ to hold requires that $\frac{\partial N_i}{\partial p_i}<0$. In both cases, the own-price elasticity needs to be negative since otherwise a contradiction arises.

Let $A:=f_i\big(p_i-\phi_i(\mathbf{N}_{\text{-}i})\big)\prod_{j\in\mathcal{D}'\setminus\{i\}}F_j\big(p_j-\phi_j(\mathbf{N}_{\text{-}j})\big)$, and note that $A\in(0,1)$ is a measure of end-users that drop side i due to the shift in PC_i. Then, replacing A and the derivative into (A.9) yields

$$\frac{\partial N_i}{\partial p_i}=\left(-1+\frac{\partial\phi_i(\cdot)}{\partial p_i}\right)A+\int_{p_i-\phi_i(\mathbf{N}_{\text{-}i})}^{1}f_i(v_i)\sum_{j\in\mathcal{D}'\setminus\{i\}}\left(-\frac{\partial\phi_j(\cdot)}{\partial p_i}-1+\frac{\partial\phi_i(\cdot)}{\partial p_i}\right)f_j(\cdot)\prod_{k\in\mathcal{D}'\setminus\{i,j\}}F_k(\cdot)dv_i.$$

By the Fubini/Tonelli theorem, the previous expression can be rewritten as follows

$$\frac{\partial N_i}{\partial p_i}=\left(-1+\frac{\partial\phi_i(\cdot)}{\partial p_i}\right)A+\sum_{j\in\mathcal{D}'\setminus\{i\}}\left(-\frac{\partial\phi_j(\cdot)}{\partial p_i}-1+\frac{\partial\phi_i(\cdot)}{\partial p_i}\right)\int_{p_i-\phi_i(\mathbf{N}_{\text{-}i})}^{1}f_i(v_i)f_j(\cdot)\prod_{k\in\mathcal{D}'\setminus\{i,j\}}F_k(\cdot)dv_i.$$

Let $B:=\sum_{j\in\mathcal{D}'\setminus\{i\}}\int_{p_i-\phi_i(\mathbf{N}_{\text{-}i})}^{1}f_i(v_i)f_j(\cdot)\prod_{k\in\mathcal{D}'\setminus\{i,j\}}F_k(\cdot)dv_i$, and note that $B\in(0,1)$ is a measure of end-users that drop side i due to the shift in ICC_{ij} for all $j \in \mathcal{D}' \setminus \{i\}$. Then,

$$\frac{\partial N_i}{\partial p_i} = \left(\underbrace{-1}_{\text{price effect } \Delta N_i^{PE}} + \underbrace{\frac{\partial \phi_i(\cdot)}{\partial p_i}}_{\text{network effect}} \right) (A + B) + C. \tag{A.10}$$

Since sides are mutually exclusive and the total measure of end-users is 1, it follows that $(A+B) \in (0,1)$, since A arises from PC_i, while B arises from ICC_{ij} for all $j \in \mathcal{D}' \setminus \{i\}$, i.e., A and B measure different end-users that drop side $i \in \mathcal{D}'$. Hence, $\frac{\partial N_i}{\partial p_i} < 0$ as long as $\frac{\partial \phi_i(\cdot)}{\partial p_i} \in (-1,1)$, which we show next. Note that $\frac{\partial \phi_i(\cdot)}{\partial p_i} = \sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \frac{\partial N_j}{\partial N_i} \frac{\partial N_i}{\partial p_i}$, with $\frac{\partial N_i}{\partial p_i} = \Delta N_i^{PE} := -(A+B) \in (-1,0)$, i.e., this is the initial drop due to the price effect which triggers all other changes. Hence, $\frac{\partial \phi_i(\cdot)}{\partial p_i} = \sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \frac{\partial N_j}{\partial N_i} \Delta N_i^{PE}$ and $\sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \frac{\partial N_j}{\partial N_i} \leq \sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \sum_{i \in \mathcal{D}' \setminus \{j\}} \frac{\partial N_j}{\partial N_i}$, which is in $(-1,1)$ by Assumption 1 and since sides are mutually exclusive. Finally, since $C < 0$ or if $C > 0$, then $\frac{\partial N_i}{\partial p_i} < 0$, and $\varepsilon_i < 0$ holds.

Before showing the sign of $\frac{\partial N_j}{\partial p_i}$, we remark that the change in p_i affects N_j for all $j \in \mathcal{D}' \setminus \{i\}$ and the changes in N_k for $k \in \mathcal{D}' \setminus \{i,j\}$ have feedback effects on N_j as well which, however, are bounded; i.e., their sum is smaller in magnitude than the effect of p_i on N_j, given Assumption 1 and since sides are mutually exclusive (so each end-user appears at most on one side on the platform). Hence, to show the sign of $\frac{\partial N_j}{\partial p_i}$, we abstract from these higher order effects whereby changes in N_k for $k \in \mathcal{D}' \setminus \{i,j\}$ (due to the change in p_i) lead again to changes in N_j.

The number of side-j end-users in equilibrium is given by (5). Taking the derivative w.r.t. p_i yields

$$\begin{aligned} \frac{\partial N_j}{\partial p_i} = & \overbrace{\frac{\partial \phi_j(\cdot)}{\partial p_i} f_j(v_j) \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big)}^{\text{network effect from } PC_j} + \\ & \underbrace{\int_{p_j - \phi_j(\mathbf{N}_{\text{-}j})}^{1} f_j(v_j) \frac{\partial}{\partial p_i} \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big) dv_j}_{\text{network effect from } ICC_{jk} \text{ for all } k \in \mathcal{D}' \setminus \{j\}}. \end{aligned} \tag{A.11}$$

Solving for the partial derivative in the second line of (A.11) yields

$$\frac{\partial}{\partial p_i} \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big) = \sum_{k \in \mathcal{D}' \setminus \{j\}} \frac{\partial F_k(\cdot)}{\partial p_i} \prod_{l \in \mathcal{D}' \setminus \{k,j\}} F_l(\cdot)$$

Focusing on the effect with respect to i and, as discussed above, abstracting from higher order effects through ICC_{jk} for $k \in \mathcal{D}' \setminus \{i,j\}$, we obtain the following expression,

$$\frac{\partial F_i(\cdot)}{\partial p_i} \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) = \left(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i} + \frac{\partial \phi_j(\cdot)}{\partial p_i}\right) f_i(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot).$$

Let $X := f_j(v_j) \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big)$ and note that $X \in (0,1)$. Then, replacing X and the partial derivative in (A.11), and applying the Fubini/Tonelli theorem yields

$$\frac{\partial N_j}{\partial p_i} = \left(\frac{\partial \phi_j(\cdot)}{\partial p_i}\right) X + \left(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i} + \frac{\partial \phi_j(\cdot)}{\partial p_i}\right) \int_{p_j - \phi_j(\mathbf{N}_{\text{-}j})}^{1} f_j(v_j) f_i(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_j.$$

Now replace $Y := \int_{p_j - \phi_j(\mathbf{N}_{\text{-}j})}^{1} f_j(v_j) f_i(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_j$, note that $Y \in (0,1)$, and simplify:

$$\frac{\partial N_j}{\partial p_i} = \frac{\partial \phi_j(\cdot)}{\partial p_i}(X+Y) + \left(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i}\right) Y. \tag{A.12}$$

The second term on the RHS of (A.12) captures end-users switching from side i to side j after p_i increases; this term is positive since $\frac{\partial \phi_i(\cdot)}{\partial p_i} \in (-1,1)$, as shown above in the first part of the proof. Regarding the first term on the RHS of (A.12) (the network effect), as also follows from above, $\frac{\partial \phi_j(\cdot)}{\partial p_i} \in (-1,1)$ and this derivative is negative if side i exerts positive cross-group externalities on side j, while it is positive if there are congestion effects. Moreover, $(X+Y) \in (0,1)$ since sides are mutually exclusive. Therefore, summing up the two terms on the RHS of (A.12), under congestion, the cross-price derivative is strictly positive, while under positive cross-group externalities, the negative network effect is compensated by the positive switching-side effect, and the cross-price derivative is positive if $\left(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i}\right) Y > \left|\frac{\partial \phi_j(\cdot)}{\partial p_i}\right| (X+Y)$, or $\left(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i} - \left|\frac{\partial \phi_j(\cdot)}{\partial p_i}\right|\right) Y > \left|\frac{\partial \phi_j(\cdot)}{\partial p_i}\right| X$, and negative otherwise. □

Proof of Proposition 6. If all platforms are multi-homing, there are neither switching-platform effects nor switching-side-and-platform effects; that is, $\lambda_{ij}^{st} = 0$ for all $i,j \in \mathcal{D}$ and for all $s,t \in \mathcal{N}$, $s \neq t$. Consequently, Λ^{st} is a null matrix, for all $s,t \in \mathcal{N}$, $s \neq t$. Therefore, markups are given by $\boldsymbol{\mu}^s = (\Lambda^s \odot \Theta^s)^{-1} \mathcal{E}^s$ for all $s \in \mathcal{N}$, which are identical to the markups in the pre-merger case. □

Proof of Lemma 2. If a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing and in which all sides are non-mutually exclusive, then there are switching-platform effects (by Proposition 4), but there are no switching-side-and-platform effects. That is, for all $s, t \in \mathcal{N}'$, $s \neq t$, it holds, *(i)* $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$; while *(ii)* $\lambda_{ij}^{st} = 0$ for all $j \in \mathcal{D}$, $i \neq j$. Therefore, $\Lambda^{st} \odot \Theta^{st}$ is a diagonal matrix. This matrix is non-positive since $\lambda_{ii}^{st} > 0$, as follows from Proposition 4. Finally, for all other platforms $s, t \in \mathcal{N} \setminus \mathcal{N}'$ (i.e., those that are not single-homing), there are neither switching-platform effects nor switching-side-and-platform effects. That is, $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$ and for all $s, t \in \mathcal{N} \setminus \mathcal{N}'$, $s \neq t$ and, consequently, the corresponding Λ^{st} matrices are null. $\square$

Proof of Proposition 7. From the proof of Lemma 2, it follows that $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$, $i \neq j$, and for all $s, t \in \mathcal{N}'$, $s \neq t$. Moreover, by Proposition 4, $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$ and for all $s, t \in \mathcal{N}$, $s \neq t$. Thus, after incorporating all pre-merger optimal prices in the RHS of (12), this expression becomes

$$\frac{(p_i^s - c_i^s)}{p_i^s} = \frac{1}{|\varepsilon_i^s|} + \sum_{j \in \mathcal{D} \setminus \{i\}} \lambda_{ij}^s \frac{(\hat{p}_j^s - c_j^s)}{\hat{p}_i^s} + \sum_{t \in \mathcal{N}' \setminus \{s\}} \lambda_{ii}^{st} \frac{(\hat{p}_i^t - c_i^t)}{\hat{p}_i^s}. \tag{A.13}$$

Therefore, relative to the pre-merger case given by (8), the last term on the RHS of (A.13) additionally appears post-merger. Hence, the sign of this last term determines whether the markup of platform $s \in \mathcal{N}'$ on side $i \in \mathcal{D}$ increases or decreases after the merger. If $(\hat{p}_i^t - c_i^t) > 0$ for all $i \in \mathcal{D}$ and for all $t \in \mathcal{N}'$, $s \neq t$ (which unambiguously occurs if there are congestion effects), then this additional term is unambiguously positive. However, if $(\hat{p}_i^t - c_i^t) < 0$ for some $i \in \mathcal{D}$ and $t \in \mathcal{N}'$, $s \neq t$ (which only may occur if there are positive cross-group externalities), then this additional term can be either positive or negative. $\square$

Proof of Lemma 3. If a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ of the merged platforms are mutually exclusive, and a subset of the merged platforms $\mathcal{N}' \subseteq \mathcal{N}$ are single-homing, then there are both switching-platform effects (by Proposition 4) and switching-side-and-platform effects (by Proposition 5). That is, for all $s, t \in \mathcal{N}'$, $s \neq t$, it holds, *(i)* $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$, and *(ii)* $\lambda_{ij}^{st} > 0$ for all $i, j \in \mathcal{D}'$, $i \neq j$. Therefore, $\Lambda^{st} \odot \Theta^{st}$ is non-positive for all $s, t \in \mathcal{N}'$, $s \neq t$. Moreover, since $\lambda_{ij}^{st} > 0$ for all $i, j \in \mathcal{D}'$, $i \neq j$, and for all $s, t \in \mathcal{N}'$, $s \neq t$, some of the off-diagonal elements in Λ^{st} are strictly negative and, consequently, $\Lambda^{st} \odot \Theta^{st}$ is non-diagonal. Finally, for all other platforms $s, t \in \mathcal{N} \setminus \mathcal{N}'$ (i.e., those that are not single-homing), there are neither switching-platform effects nor switching-side-and-platform effects. That is, $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$ and for all $s, t \in \mathcal{N} \setminus \mathcal{N}'$, $s \neq t$ and, consequently, the corresponding Λ^{st} matrices are null. $\square$

Proof of Proposition 8. From the proof of Lemma 3, it follows that for all $s, t \in \mathcal{N}'$, $s \neq t$, *(i)* $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$, and *(ii)* $\lambda_{ij}^{st} > 0$ for all $i, j \in \mathcal{D}'$, $i \neq j$. Then, after incorporating all pre-merger optimal prices in the RHS of (12), this expression becomes

$$\frac{(p_i^s - c_i^s)}{p_i^s} = \frac{1}{|\varepsilon_i^s|} + \sum_{j \in \mathcal{D} \backslash \{i\}} \lambda_{ij}^s \frac{(\hat{p}_j^s - c_j^s)}{\hat{p}_i^s} + \sum_{t \in \mathcal{N}' \backslash \{s\}} \left(\lambda_{ii}^{st} \frac{(\hat{p}_i^t - c_i^t)}{\hat{p}_i^s} + \sum_{j \in \mathcal{D}' \backslash \{i\}} \lambda_{ij}^{st} \frac{(\hat{p}_j^t - c_j^t)}{\hat{p}_i^s} \right). \quad \text{(A.14)}$$

Therefore, relative to the pre-merger case given by (8), the last term on the RHS of (A.14) (i.e., the whole summation in parenthesis) additionally appears post-merger. Hence, the sign of this last term determines whether the markup of platform $s \in \mathcal{N}'$ on side $i \in \mathcal{D}'$ increases or decreases after the merger. If $(\hat{p}_i^t - c_i^t) > 0$ for all $i \in \mathcal{D}$ and for all $t \in \mathcal{N}'$, $t \neq s$ (which unambiguously occur if $\lambda_{ij}^t \geq 0$ for all $i, j \in \mathcal{D}$, $i \neq j$), then this additional term is unambiguously positive. However, if $(\hat{p}_i^t - c_i^t) < 0$ for some $i \in \mathcal{D}$ and $t \in \mathcal{N}'$, $t \neq s$ (which only may occur in all other cases not included in the previous condition), then this additional term can be either positive or negative. $\square$

Appendix B: Model extensions (for online publication)

B.1 Partial or no double-counting with multi-homing platforms

In Section 3, we characterize the demand for sides assuming that end-users on one side, say, side i, equally value all those on another side, say, side j, on the same platform. However, as discussed by some authors (Bakos and Halaburda, 2020), this assumption might not always be appropriate in the context of multi-homing platforms. For example, buyers on e-commerce platforms benefit from viewing a certain product on one platform, but viewing the same product on another does not add extra value for them. Other authors note that this also applies to media markets: for advertisers, media end-users that can be reached through a single outlet are much more valuable than those that can be reached through multiple ones (Ambrus et al., 2016; Anderson et al., 2018). Thus, for the sake of completeness, in this appendix, we extend our model to accommodate this alternative assumption.

Recall that, for multi-homing platforms and non-mutually exclusive sides, the demand function for side i on platform q is given by (2), while (4) characterizes the demand for that side and platform under mutually exclusive sides. Note that (4) is similar to (2) but augmented by a term that captures the ICC on the RHS. In our analysis, we primarily focus on the former, although similar conclusions apply to the latter. By the law of total probability, the previous expression can be decomposed as

$$N_i^q(\cdot) = \underbrace{pr\big(v_i^q \geq p_i^q - \phi_i^q(\cdot) \cap v_i^r < p_i^r - \phi_i^r(\cdot)\big)}_{\text{single-homing end-users } (\hat{N}_i^q)} + \underbrace{\big(v_i^q \geq p_i^q - \phi_i^q(\cdot) \cap v_i^r \geq p_i^r - \phi_i^r(\cdot)\big)}_{\text{multi-homing end-users } (\bar{N}_i^q)}$$

for all $i \in \mathcal{D}$ and all $q, r \in \mathcal{M}, q \neq r$. That is, the total number of end-users on side i on platform q is equal to the sum of two groups: end-users who join this side and platform but do not join side i on any other platform r (single-homers), and those who also join side i on some other platform r (multi-homers). Let us denote the former set of end-users by $\hat{N}_i^q$ and the latter one by $\bar{N}_i^q$.

Under the partial or no double-counting assumption, end-users fully value the presence of those on other sides on the same platform who single-home, giving little (or no) value to those who multi-home. The network effect in this case is expressed as $\phi_i^q(\hat{\mathbf{N}}_{\text{-}i}^q + \boldsymbol{\sigma}_{\text{-}i}^q \cdot \bar{\mathbf{N}}_{\text{-}i}^q)$, where $\hat{\mathbf{N}}_{\text{-}i}^q$ (resp., $\bar{\mathbf{N}}_{\text{-}i}^q$) is the vector of single-homing (resp., multi-homing) end-users on all sides other than i on platform q. The vector $\boldsymbol{\sigma}^q_{\text{-}i}$ contains $(d-1)$ parameters in the interval $[0,1)$, where each parameter, like σ^q_j, can be interpreted either as the probability with which end-users on side i interact with multi-homing end-users on side j on platform q for the first time or as a parameter that captures the lower cross-group network externality associated with multi-homing end-users on side j. This setup also accommodates the extreme case in which end-users on other sides shared with more than one platform have no market value, i.e., $\sigma^q_j = 0$, as proposed by Anderson et al. (2018).

In this alternative case, the derivatives of N^q_i and N^q_j with respect to p^q_i differ slightly from those obtained in Sections 4.3.1 and 4.3.2. These derivatives, which can be formally obtained using the modified version of (3) that incorporate the new network effect indicated above, are as follows:

$$\frac{\partial N^q_i}{\partial p^q_i} = -\underbrace{f^q_i\big(p^q_i - \phi^q_i(\mathbf{N}^q_{\text{-}i})\big)}_{\text{price effect}} + \underbrace{f^q_i\big(p^q_i - \phi^q_i(\mathbf{N}^q_{\text{-}i})\big)\left[\frac{\partial \phi^q_i(\hat{\mathbf{N}}^q_{\text{-}i})}{\partial p^q_i} + \boldsymbol{\sigma}^q_{\text{-}i} \cdot \frac{\phi^q_i(\bar{\mathbf{N}}^q_{\text{-}i})}{\partial p^q_i}\right]}_{\text{network effect}}, \tag{B.1}$$

$$\frac{\partial N^q_j}{\partial p^q_i} = \underbrace{f^q_j\big(p^q_j - \phi^q_j(\mathbf{N}^q_{\text{-}j})\big)\left[\frac{\partial \phi^q_j(\hat{\mathbf{N}}^q_{\text{-}j})}{\partial p^q_i} + \boldsymbol{\sigma}^q_{\text{-}j} \cdot \frac{\phi^q_j(\bar{\mathbf{N}}^q_{\text{-}j})}{\partial p^q_i}\right]}_{\text{network effect}}. \tag{B.2}$$

These derivatives differ from (9) and (10) in Section 4.3.1 only in the network effect, which now depends not only on the total number of end-users on other sides on platform q but also on the composition of these end-users (i.e., on how many of them are single-homers and multi-homers, respectively). The impact that this new expression characterizing the network effect has on our analysis depends on the type of market solution we encounter.

In case all end-users multi-home, (B.2) boils down to $\frac{\partial N^q_j}{\partial p^q_i} = f^q_j\big(p^q_j - \phi^q_j(\mathbf{N}^q_{\text{-}j})\big)\left[\boldsymbol{\sigma}^q_{\text{-}j} \cdot \frac{\phi^q_j(\bar{\mathbf{N}}^q_{\text{-}j})}{\partial p^q_i}\right]$. In this case, the network effect plays a less prominent role relative to that obtained in Sections 4.3.1 and 4.3.2, as its magnitude is significantly reduced by $\boldsymbol{\sigma}^q_{\text{-}i}$. In fact, in the limiting case in which $\boldsymbol{\sigma}^q_{\text{-}i} \to \mathbf{0}$, the network effect vanishes. These results have different implications (but similar conclusions) in the two cases that we consider in Sections 4.3.1 and 4.3.2.

First, in the case of platforms with non-mutually exclusive sides—as in Section 4.3.1—, if the network effect vanishes, then the cross-price derivative becomes small. In this scenario, optimal pricing, as characterized by (7), closely resembles the usual Lerner pricing formula for a standard monopolist, as the third element on the LHS of (7) converges to zero; that is, optimal pricing depends exclusively on the value associated with single-homing end-users. Therefore, as noted in Bakos and Halaburda (2020), cross-subsidization—where different sides of a platform are interdependent through the network effect—does not occur. Importantly, since the cross-price derivative is relatively close to zero, so is λ_{ij}^q.

Second, if platforms' sides are mutually exclusive and cross-group externalities are positive, as discussed in Section 4.3.2, increasing p_i^q has two countervailing effects on side j. Namely, it decreases participation on that side (due to the network effect), but it also increases it (due to the switching-side effect). In the limiting case where $\boldsymbol{\sigma}_{\text{-}i}^q \to \mathbf{0}$, the network effect vanishes, and the switching-side effect is more likely to dominate the network effect. In this scenario, the cross-price derivative will likely be unambiguously positive (and so is λ_{ij}^q), resulting in an optimal markup greater than the usual Lerner index, even in the absence of congestion. This optimal pricing rule depends mostly on the value associated to single-homing end-users and, to a lesser degree, on the marginal value associated to multi-homers—given by the extent to which some of them switch sides in response to a marginal change in price. This observation is consistent with the so-called "principle of incremental pricing" (Anderson et al., 2018). Obviously, cross-subsidization cannot occur in this case.

Finally, in case there is full market participation with both single-homing and multi-homing end-users, and also in case there is less than full participation—implying the coexistence of both types of end-users due to the full support and continuity of the distribution of valuations—the outcomes are similar. These cases represent intermediate cases of the previous ones, wherein the network effect is attenuated as the presence of multi-homers increases. It becomes akin to what is found in Sections 4.3.1 and 4.3.2 as the number of multi-homing end-users decreases.

B.2 Random demand

In our main analysis, every end-user on the platform is "always" interacting with the end-users on the other sides. However, there are cases where an end-user who joins the platform is not always *active*, which is they are not always interacting with end-users on the other sides. Consider, for example, an Uber driver. She does not offer her services 24/7, but rather decides when to be active; sometimes she may decide to earn money as an Uber driver and sometimes she may enjoy her free time.

In order to capture this, suppose that an end-user who joins platform q's side i is expected to be active on side i with probability $\theta_i^q \in (0,1)$. With the complementary probability $1-\theta_i^q$ she is not (even though she joined that side and paid the corresponding fee p_i^q). This probability can be interpreted as the ratio at which end-users are usually active or as the quantity below 1 that they exchange on the platform. While intuitively both cases have slightly different interpretations, formally they are equivalent. Correspondingly, an end-user is active on platform q's side i with probability $\theta_i^q \in (0,1)$. These ex-ante probabilities are resolved before an end-user interacts on the platform. In this case, an end-user that joins side i on platform q obtains a utility of

$$u_i^q := \theta_i^q \left[v_i^q + \phi_i^q(\boldsymbol{\theta}_{\text{-}i}^q \cdot \mathbf{N}_{\text{-}i}^q)\right] - p_i^q, \tag{B.3}$$

where $\boldsymbol{\theta}_{\text{-}i}^q$ is the vector of probabilities of being active on all sides other than i on platform q. The utility a representative end-user obtains upon joining side i is first multiplied by the probability with which she is expected to be active on side i on platform q. If she is active, then, as before, she obtains v_i^q in addition to the network effect multiplied by the expected number of end-users she encounters on the other sides. Finally, the corresponding fee to join a side is paid for sure by the end-user (even if she interacts less than all the time or trades less than one unit of the good or service on the platform).

Now we introduce three changes of variables that will simplify our analysis:

1. Define a new random variable for any end-user i: $\check{v}_i^q = \theta_i^q v_i^q$.
2. Define a new random variable $\check{\mathbf{N}}_{\text{-}i}^q = \boldsymbol{\theta}_{\text{-}i}^q \cdot \mathbf{N}_{\text{-}i}^q$.
3. Finally, define a new random variable $\check{\phi}_i^q(\check{\mathbf{N}}_{\text{-}i}^q) = \theta_i^q\left[\phi_i^q(\boldsymbol{\theta}_{\text{-}i}^q \cdot \mathbf{N}_{\text{-}i}^q)\right]$ (since end-users' demand is random, their expected cross-group externality is also a random variable with expected value).

Therefore, the expression that captures the utility for an end-user becomes

$$u_i^q := \check{v}_i^q + \check{\phi}_i^q(\check{\mathbf{N}}_{\text{-}i}^q) - p_i^q. \tag{B.4}$$

Under the multi-homing assumption, if platforms' sides are non-mutually exclusive, then the participation constraint that yields sides' demand is as follows

$$(PC_i^q) \quad \check{v}_i^q + \check{\phi}_i^q(\check{\mathbf{N}}_{\text{-}i}^q) - p_i^q \quad \geq \quad 0,$$

and, if platforms' sides i and j are mutually exclusive for $i, j \in \mathcal{D}' \subseteq \mathcal{D}$, then the demands are characterized additionally by the following incentive compatibility constraints

$$(ICC_{i,j}^q) \quad \check{v}_i^q + \check{\phi}_i^q(\check{\mathbf{N}}_{\text{-}i}^q) - p_i^q \quad \geq \quad \check{v}_j^q + \check{\phi}_j^q(\check{\mathbf{N}}_{\text{-}j}^q) - p_j^q.$$

These conditions resemble the corresponding ones provided in Section 3, and similar ICCs can be obtained for the case in which platforms are single-homing. Therefore, all results obtained above hold here as well after applying the change of variables, although all variables are "smaller" since they are random.