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CESIFO WORKING PAPER NO. 1082
CATEGORY 8: RESOURCES AND ENVIRONMENT
NOVEMBER 2003

CESifo *Working Paper No. 1082*

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Abstract

In this paper, we rank the relative importance of the exogenous parameters upon the optimal harvesting size in a stochastic rotation problem. We show that when the tree growth follows geometric Brownian motion, the harvesting size is most elastic to the harvesting cost, followed by the interest rate, and is least elastic to the parameters of tree growth. Similar ranking holds for the linear growth case. In both cases the harvesting size is increasing and concave in the harvesting cost, bounded between two parallel lines. The harvesting decision is made according to a stochastic extension of the Faustmann formula.

JEL Classification: Q23, O13.

Keywords: stochastic rotation problem, the Faustmann formula, site value, homogeneity conditions, and seedling value.

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I thank Nick Spulber for his suggestions, Manoj Atolia for a careful reading of the manuscript and useful discussion, and two anonymous referees of this Journal for their comments and suggestions leading to the improvement of the paper. Naturally, all remaining errors are mine.

1 Introduction

The Wicksellian tree-cutting problem addresses the issue of the optimal time to harvest a growing capital. A repeated tree-cutting problem (with replanting) is known as a rotation problem. In a recent article, Willassen (1998) used the theory of impulse control to put the stochastic rotation problem[1] on solid theoretical grounding, derived explicit solutions for the problem, and obtained more accurate valuation of the rotation. He also used simulation to investigate the sensitivity of the optimal harvesting size with respect to exogenous parameters. This raises an interesting question: To which exogenous parameter is the optimal harvesting size most elastic? In this paper, we shall rank theoretically the elasticities of the harvesting size with respect to the harvesting cost, the interest rate, the drift, and the variance of the growth pattern of a tree.

The answer to the proposed question has implications to other areas of economics beyond forest rotations. As pointed out in Miller and Voltaire

[1] Important contributions in this literature are made, for example, by Miller and Voltaire (1980, 1983), Malliaris and Brock (1982), Brock, Rothschild and Stiglitz (1988), Clarke and Reed (1989), Reed and Clarke (1990), and Clark (1990). It should be mentioned that the "maximum sustained yield" solution cherished by forest experts is closely related to the solution of the rotation problem [see, Mitra and Wan (1986)], and that other parameters would enter the harvesting rule if the capital market is not perfect [see Tahvonen, Selo and Kuuluvainen (2001)].

(1983), the tree paradigm can be applied to human capital growth models, job search models, a firm's entry/exit decision problem, and general renewable resources problems. Since the elasticity measures the sensitivity of the behavioral function with respect to an exogenous parameter, the ranking of the elasticities tells us the relative importance of each parameter in the analysis. The findings of this paper could potentially lead to new results in those areas.

The ranking of elasticities depends in general on the model specification. As demonstrated in Clarke and Reed (1989) and Reed and Clarke (1990), the harvesting rules depend on whether the tree growth is age-dependent or size-dependent. For this reason, we focus on the benchmark case in which the growth pattern of a tree follows geometric Brownian motion. In the Appendix we analyze the case of linear growth to strengthen the results.

Our approach to the proposed problem is not a new mathematical theory, but a return to the basics: exploring the first and the second order conditions of the stochastic rotation problem. As is common in this literature, the optimal harvesting size is *implicitly* defined by the first order condition (a stochastic extension of the Faustmann formula). Even without an explicit functional form, the optimal harvesting rule has many interesting built-in properties that are previously unknown. For example, there are homogeneity conditions among these elasticities that are inherent in the Faustmann formula. By way of Euler's theorem, we can partially rank some of the elasticities.

Next, we focus on the functional relationship between harvesting size and harvesting cost. We show that the harvesting size is a strictly increasing and strictly concave function of the harvesting cost, and is bounded between two parallel lines. It enables us to obtain estimates of the harvesting size and the rotation value that are sharper than Willassen's. For example, the harvesting size is almost linear in the harvesting cost and the estimate of the rotation value is almost exact if the seedling value is small. More important, we can rank the elasticities as follows. The optimal harvesting size is most elastic to the harvesting cost, followed by the interest rate. It is least elastic to the uncertainty elasticity, under certain assumption. We also show that, in the case of geometric Brownian growth, the optimal harvesting size is highly elastic to the harvesting cost if the harvesting cost is not much larger than the seedling value.

The paper is organized as follows. In section 2, we set up the model of forest rotations for geometric Brownian growth of a tree, and establish the stochastic version of the Faustmann formula. In section 3, we derive the major results of the rotation problem, in particular, we rank these elasticities. Similar results can be obtained for the case of linear growth, which are reported in the Appendix. In section 4, we draw some concluding remarks.

2 The Model

Throughout the paper, we consider only the case of $c > x$, where c is the harvesting cost (which include the cutting cost and the replanting cost) and $x > 0$ is the seedling value. Let the market interest rate r be constant over time, and let X_t be the size of a tree at time t. Assume the growth of a tree follows geometric Brownian motion

$$dX = \mu X dt + \sigma X dz, \tag{1}$$

where μ and σ are constants and z_t is the standard Wiener process. Assume also that the tree is cut when it reaches the size of b, with $b > x$. The time at which the tree is cut is a first passage time of (1),

$$\tau = \inf \left\{ t \geq 0 : X_t = b \right\}.$$

Let $G(b)$ be the current value of forest rotations. Based on Willassen's (1998) formulation, the value of the rotation can be written as the sum of the net present value of the tree up to the first passage time and the discounted value of future rotations after the first passage time, i.e.

$$G(b) = E_x\left[(X_\tau - c)\,e^{-r\tau}\right] + E_x\left[e^{-r\tau}\right]G(b)\,.$$

Since $X_\tau = b$, the present value of the rotation is

$$G(b) = \frac{(b-c)\,E_x\left[e^{-r\tau}\right]}{1 - E_x\left[e^{-r\tau}\right]}.$$

To ascertain the expected discounted factor $E_x\left[e^{-r\tau}\right]$, it is best to take the transformation $Y = \log X$. Then, by Ito's lemma,

$$dY = \left(\mu - \sigma^2/2\right)dt + \sigma dz. \tag{2}$$

If the tree is cut at $X = b$, then it is cut at $Y = \log b$, and the first passage time can be written as

$$\tau = \inf\left\{t \geq 0 : Y_t = \log b\right\}, \tag{3}$$

where Y_t follows (2) with the initial state $Y_0 = \log x$. It is well known that the expected discounted factor of (3) is

$$E_{\log x}\left[e^{-r\tau}\right] = e^{-\gamma(\log b - \log x)} = \left(\frac{x}{b}\right)^\gamma,$$

where γ is the positive root of the quadratic equation

$$r = q(\gamma) = \left(\mu - \sigma^2/2\right)\gamma + \left(\sigma^2/2\right)\gamma^2.$$

See, for example, Harrison (1985, p. 42). Clearly,

$$\gamma = \frac{-\left(\mu - \sigma^2/2\right) + \sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}}{\sigma^2} > 0. \tag{4}$$

Then the rotation problem is simplified as

$$\max_b G(b) = \max_b \left\{ \frac{(b-c)\,(x/b)^\gamma}{1-(x/b)^\gamma} \right\}. \tag{5}$$

The first order condition of (5) is

$$x^\gamma b^{1-\gamma} = \gamma c - (\gamma - 1)\, b, \tag{6}$$

which is equation 6.8 of Willassen (1998). Assume for the moment that there is a unique solution to equation (6). Denote the solution by b^*.

Equation (6) turns out to be the stochastic extension of the Faustmann formula for geometric Brownian growth of a tree. To prove this, we first recall that if $dX = \mu X dt$ (no uncertainty), then the formula is

$$\mu X(t) = X'(t) = r\left[X(t) - c\right] + \left(\frac{e^{-rt}}{1-e^{-rt}}\right) r\left[X(t) - c\right],$$

where the second term on the right-hand-side is known as the *site value*. See, for example, Clark (1990, p.270). The formula says that the marginal benefit of keeping the tree, $X'(t)$, is equal to the marginal cost of keeping the tree, which is measured in terms of forgone interest, $r\left[X(t) - c\right]$, *plus* the site value. At the optimal b^* and t^*, where t^* is defined by $b^* = xe^{\mu t^*}$, the formula is

$$\mu b^* = r\left(b^* - c\right) + \left(\frac{e^{-rt^*}}{1 - e^{-rt^*}}\right) r\left(b^* - c\right).$$

Next, we rewrite the first order condition (6) as

$$\mu b^* = \mu\gamma\left(b^* - c\right) + \left[\frac{\left(x/b^*\right)^\gamma}{1 - \left(x/b^*\right)^\gamma}\right] \mu\gamma\left(b^* - c\right). \tag{7}$$

Since $\left(x/b^*\right)^\gamma$ is the stochastic extension of the discount factor e^{-rt^*}, we only need to show that $\mu\gamma$ is the stochastic extension of r. To this end, we recall Newton's binomial series: For $|a| < 1$, and any real number y,

$$(1+a)^y = 1 + ya + \frac{1}{2!}y\left(y-1\right)a^2 + \frac{1}{3!}y\left(y-1\right)\left(y-2\right)a^3 + \ldots.$$

For given r and μ, $2r\sigma^2 < \left(\mu - \sigma^2/2\right)^2$ is valid for *small* σ^2. Therefore,

$$\begin{aligned} &\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2} \\ = \quad &\left(\mu - \sigma^2/2\right) + \frac{1}{2}\frac{2r\sigma^2}{\left(\mu - \sigma^2/2\right)} - \frac{1}{8}\frac{\left(2r\sigma^2\right)^2}{\left(\mu - \sigma^2/2\right)^3} + \frac{1}{16}\frac{\left(2r\sigma^2\right)^3}{\left(\mu - \sigma^2/2\right)^5} + \ldots, \end{aligned}$$

and

$$\gamma = \frac{-\left(\mu - \sigma^2/2\right) + \sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}}{\sigma^2} \to \frac{r}{\mu}, \text{ as } \sigma^2 \to 0.$$

In other words, $\mu\gamma$ plays the role of the market discount rate in this stochastic model. It should be noted that the second term on the right-hand-side of (7) is the stochastic extension of the site value for geometric Brownian growth of a tree.

The second order sufficient condition, at optimal b^*, is

$$G^{''}(b^*) = \frac{(x/b^*)^\gamma (1-\gamma)}{b^* [1-(x/b^*)^\gamma]} < 0,$$

if $\gamma > 1$ or, equivalently, if $\mu < r$. Thus, the solution will be *the* maximum once we show that the optimal solution uniquely exists. This condition ($\mu < r$) is most intuitive, because the benefit from cutting a tree is to sell it on the market and deposit the profit in a bank. The interest rate must be greater than the growth rate of a tree to warrant the harvesting.

The second order necessary condition is $G^{''}(b^*) \leq 0$, which implies $\gamma \geq 1$. If $\gamma < 1$, then the solution represents the *minimal* value, not the maximal value of the rotation. This second-order-condition argument provides an alternative explanation to why $\gamma < 1$ is "nonsensical." See Remark 6.2 of Willassen (1998).

To prove the existence of b^* and to facilitate the analysis, we shall make some change of variables. Let $B = b/x$, and $C = c/x$. Then equation (6) is simplified to

$$B^{1-\gamma} = \gamma C - (\gamma - 1) B. \tag{8}$$

We shall assume $C > 1$, since we are interested only in the case of $c > x$. In

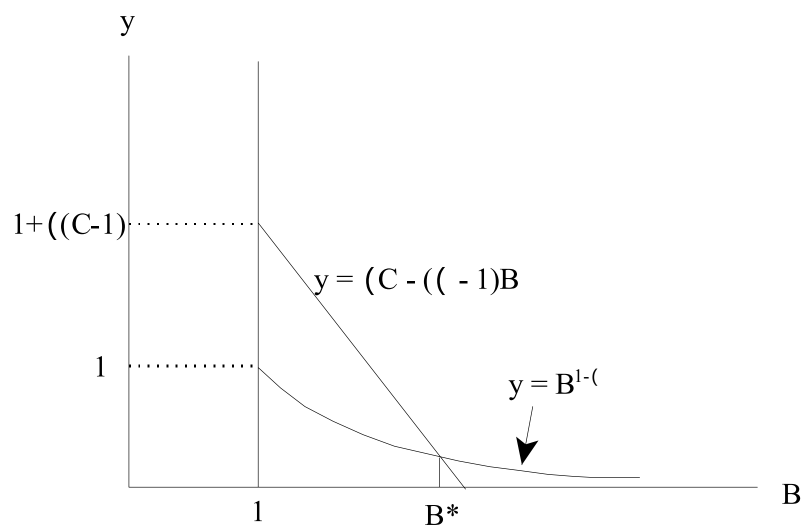

Figure 1: The determination of the harvesting size

the (B, y)-plane, the left-hand-side of (8) is a curve decreasing and convex in B, intercepting the vertical line $B = 1$ at $+1$, while the right-hand-side of (8) is a straight line of slope $-(\gamma - 1)$ intercepting the vertical line $B = 1$ at $1 + \gamma (C - 1)$. These two curves have a unique intersection as shown in Figure 1.

Thus, the optimal solution B^*, and hence b^*, is *uniquely* determined.

Our result of a unique solution may appear at odds with Willassen's Lemma 6.2 in which *two* solutions are asserted. The difference comes from the fact that Willassen set $b \in (0, \infty)$, instead of $b \in (x, \infty)$. Indeed, the

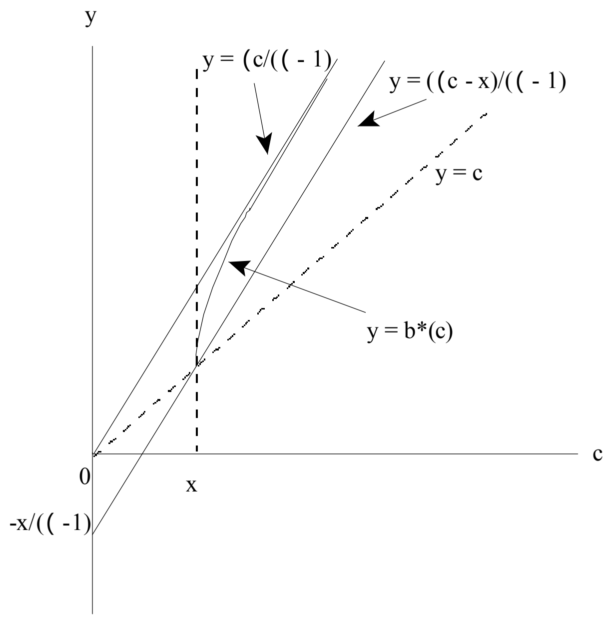

Figure 2: b* as a function of c

second solution derived therein lies in the interval $(0, x]$. While it might make mathematical sense to have this second solution, it makes no economic sense to cut a tree below its seedling size.

3 Properties of Harvesting Rules

3.1 A Sharper Estimate of the Harvesting Rule

To have a better understanding of the harvesting rule, we begin with treating b^* as a function of c, holding other parameters constant. The next proposition shows that b^*, as a function of harvesting cost, is well behaved and is depicted in Figure 2.

Proposition 1 *The optimal harvesting size b^* is strictly increasing and strictly concave in c satisfying*

$$\lim_{c\to x} b^*(c) = x \text{ and } \lim_{c\to\infty} b^*(c) = \infty, \tag{9}$$

$$\lim_{c\to x} \frac{\partial b^*}{\partial c} = \infty \text{ and } \lim_{c\to\infty} \frac{\partial b^*}{\partial c} = \left(\frac{\gamma}{\gamma-1}\right), \tag{10}$$

such that $b^*(c)$ is bounded between two parallel lines: $y = \left(\frac{\gamma}{\gamma-1}\right)c$ and $y = \left(\frac{\gamma}{\gamma-1}\right)c - \frac{x}{\gamma-1}$.

Proof. *A direct computation from* (8) *shows that*

$$\frac{\partial B^*}{\partial C} = \left(\frac{\gamma}{\gamma-1}\right)\frac{1}{1-(B^*)^{-\gamma}} > \frac{\gamma}{\gamma-1} > 1. \tag{11}$$

It follows that $\partial b^*/\partial c > 0$. Also, from (11)*, an increase in C raises B^* and hence lowers $\partial B^*/\partial C$. This shows the concavity of $b^*(c)$ in c. From Figure 1, a decrease in C produces a leftward shift of the line $y = \gamma C - (\gamma-1)B$. Since the curve $y = B^{1-\gamma}$ is convex and is of slope $-(\gamma-1)$ at $B = 1$, the optimal harvesting size B^* satisfies*

$$\lim_{C\to 1} B^*(C) = 1 \text{ and } \lim_{C\to\infty} B^*(C) = \infty, \tag{12}$$

which implies (9)*.*

Next, from (11)*, we have*

$$\lim_{C\to 1} \frac{\partial B^*}{\partial C} = \infty \text{ and } \lim_{C\to\infty} \frac{\partial B^*}{\partial C} = \left(\frac{\gamma}{\gamma-1}\right),$$

which implies (10)*. Thus,* $b^*(c)$ *is asymptotic to a straight line* $y = \left(\frac{\gamma}{\gamma-1}\right)c + a$*, for some constant* a*. Also, from* (10)*, for* $c > x$*,* $b^*(c)$ *is bounded from below by a straight line of slope* $-\gamma/(\gamma-1)$ *passing through the point* (x, x)*, i.e., by* $y = \left(\frac{\gamma}{\gamma-1}\right)c - \frac{x}{\gamma-1}$*. The distance between these two lines is* $a + \frac{x}{\gamma-1}$*, which is also the supremum of the distance between the curve* $y = b^*(c)$ *and and the line* $y = \left(\frac{\gamma}{\gamma-1}\right)c - \frac{x}{\gamma-1}$*, i.e.,*

$$\begin{aligned} a + \frac{x}{\gamma-1} &= \sup_{c>x}\left\{b^*(c) - \left(\frac{\gamma}{\gamma-1}\right)c + \frac{x}{\gamma-1}\right\} \\ &= \sup_{c>x}\left\{-\frac{x^{\gamma}\left[b^*(c)\right]^{1-\gamma}}{\gamma-1} + \frac{x}{\gamma-1}\right\} = \frac{x}{\gamma-1}, \end{aligned}$$

using (6)*. Thus,* $a = 0$*.* ■

Since $b^*(c)$ is bounded between $y = \left(\frac{\gamma}{\gamma-1}\right)c$ and $y = \left(\frac{\gamma}{\gamma-1}\right)c - \frac{x}{\gamma-1}$, we have

$$\left(\frac{\gamma}{\gamma-1}\right)c - \frac{x}{\gamma-1} \leq b^* \leq \left(\frac{\gamma}{\gamma-1}\right)c. \tag{13}$$

The estimation of the harvesting size is almost exact (and is linear in c) if the seedling value is very small. Inequality (13) also provides a sharper lower bound than Willassen's equation 6.13:

$$c \leq b^* \leq \left(\frac{\gamma}{\gamma-1}\right)c,$$

because

$$\frac{\gamma c - x}{\gamma - 1} > c \Leftrightarrow c > x.$$

Similarly, the net present value of the rotation is, using (6),

$$G(b^*) = \frac{(b^* - c)(x/b^*)^\gamma}{1 - (x/b^*)^\gamma} = \frac{b^*(b^* - c)(x/b^*)^\gamma}{\gamma(b^* - c)} = \frac{x^\gamma}{\gamma}\left(\frac{1}{b^*}\right)^{\gamma - 1}.$$

Using (13), we have

$$\left(\frac{c}{\gamma}\right)\left(\frac{x}{c}\right)^\gamma\left(\frac{\gamma - 1}{\gamma}\right)^{\gamma - 1} \le G(b^*) \le \left(\frac{c}{\gamma}\right)\left(\frac{x}{c}\right)^\gamma\left(\frac{\gamma c - c}{\gamma c - x}\right)^{\gamma - 1}.$$

It is obvious that this inequality is sharper than Willassen's equation 6.14: $0 < G(b^*) < c/\gamma$. The estimate of the rotation value is almost exact if the seedling value is small. It should be mentioned that, in the case of linear growth, harvesting size remains to be a strictly increasing and strictly concave function of the harvesting cost and is bounded by two parallel lines. See the Appendix for details.

3.2 Comparative Dynamics

Proposition 2 *(i) The comparative dynamics are:* $\partial b^*/\partial c > 0$, $\partial b^*/\partial x < 0$, $\partial b^*/\partial r < 0$, $\partial b^*/\partial \mu > 0$, *and* $\partial b^*/\partial \sigma^2 > 0$. *(ii) If* $J(c, r, \mu, \sigma^2)$ *is the value function of* (5), *then* $J_c < 0$, $J_x > 0$, $J_r < 0$, $J_\mu > 0$, *and* $J_{\sigma^2} > 0$.

Proof. *The inequality,* $\partial b^*/\partial c > 0$, *follows from* (11). *Similarly, from* (6),

$$\frac{\partial b^*}{\partial x} = -\left(\frac{\gamma}{\gamma - 1}\right)\frac{(x/b^*)^{\gamma - 1}}{1 - (x/b^*)^\gamma} < 0.$$

To derive other comparative dynamics, we note that, from (4),

$$\frac{\partial \gamma}{\partial \mu} = \frac{1}{\sigma^2}\left(-1 + \frac{\mu - \sigma^2/2}{\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}}\right) < 0, \tag{14}$$

$$\frac{\partial \gamma}{\partial \sigma^2} = \frac{\mu\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2} - \left[\mu\left(\mu - \sigma^2/2\right) + r\sigma^2\right]}{\sigma^4\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}} < 0, \tag{15}$$

if $r > \mu$, *and*

$$\frac{\partial \gamma}{\partial r} = \frac{1}{\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}} > 0, \tag{16}$$

we only need to determine the sign of $\partial b^*/\partial \gamma$. *Thus, the comparative dynamics will be complete if we can show that*

$$\frac{\partial B^*}{\partial \gamma} = \frac{C - B^* + \left(B^*\right)^{1-\gamma}\log B^*}{\left(\gamma - 1\right)\left[1 - \left(B^*\right)^{-\gamma}\right]} < 0. \tag{17}$$

To this end, denote the numerator of $\partial B^*/\partial \gamma$ *by*

$$g\left(C\right) = C - B^* + \left(B^*\right)^{1-\gamma}\log B^*,$$

which satisfies $g\left(1\right) = 0$, *by* (12). *Using* $\gamma > 1$, $B^* > 1$, *and* $\left(B^*\right)^{-\gamma} < 1$,

$$g'\left(C\right) = \frac{\left(B^*\right)^{-\gamma} - 1 - \gamma\left(\gamma - 1\right)\left(B^*\right)^{-\gamma}\log B^*}{\left(\gamma - 1\right)\left[1 - \left(B^*\right)^{-\gamma}\right]} < 0,$$

we can conclude that $g\left(C\right) < 0$ *for all* $C > 1$. *Thus,* $\partial B^*/\partial \gamma < 0$.

To prove (ii), we invoke the envelope theorem: $J_c < 0$, $J_x > 0$, $J_\gamma < 0$, *and then applying* (14), (15), *and* (16). ■

It is interesting to point out that some of the results in part (ii) were anticipated by Willassen using simulation method. In contrast, we have proved them theoretically.

3.3 Homogeneity Condition

The first order condition (6) implicitly defines some functional relationship between the harvesting size B^* and the exogenous parameters C, μ, r, and σ^2, which is summarized in the following proposition.

Proposition 3 *Let $\varepsilon(b^*, \theta)$ be the elasticity of the optimal harvesting size with respect to the exogenous variable θ. Then we have*

$$\varepsilon(b^*, \mu) + \varepsilon(b^*, r) + \varepsilon\left(b^*, \sigma^2\right) = 0. \tag{18}$$

Proof. First, we note that γ, as defined in (4), is homogeneous of degree 0 in (μ, r, σ^2). Then

$$\begin{aligned} B^*\left(C, \lambda\mu, \lambda r, \lambda\sigma^2\right) &= B^*\left(C, \gamma\left(\lambda\mu, \lambda r, \lambda\sigma^2\right)\right) = B^*\left(C, \gamma\left(\mu, r, \sigma^2\right)\right) \\ &= B^*\left(C, \mu, r, \sigma^2\right), \end{aligned}$$

i.e., B^* is homogeneous of degree 0 in $\left(\mu, r, \sigma^2\right)$. Then, by Euler's theorem, this homogeneity condition can be stated as

$$\varepsilon(B^*, \mu) + \varepsilon(B^*, r) + \varepsilon\left(B^*, \sigma^2\right) = 0, \tag{19}$$

Since $B = b/x$, $\varepsilon(B^*, \theta) = \varepsilon(b^*, \theta)$ for $\theta = \mu, r, \sigma^2$. Thus, we have (18). ∎

It should be mentioned that equation (18) remains valid when the tree growth is linear. In fact, there is *another* homogeneity condition in that case. This extra homogeneity condition makes it easier to rank the elasticities in the linear growth case than the geometric Brownian growth case. See the Appendix for details.

An immediate corollary of the proposition is the following ranking of elasticities:

Corollary 4 *The optimal harvesting size is more elastic to the interest rate than the growth pattern of a tree as represented by the drift and the variance of the growth process.*

Proof. It follows from

$$-\varepsilon(b^*, r) = \varepsilon(b^*, \mu) + \varepsilon\left(b^*, \sigma^2\right) > \max\left\{\varepsilon(b^*, \mu), \varepsilon\left(b^*, \sigma^2\right)\right\},$$

since $\varepsilon(b^*, \mu) > 0$ and $\varepsilon(b^*, \sigma^2) > 0$. ∎

Some comments are in order. Because of the built-in homogeneity in the Faustmann formula, we can partially rank some of the elasticities. As commented earlier, equation (18) remains valid in the case of linear growth. Therefore, the expected-growth-rate elasticity and the uncertainty elasticity remain smaller than the interest elasticity in that case. This corollary confirms the intuition that the interest rate is more important than the growth parameters of a tree in determining the harvesting rule. It remains to compare the relative importance of the interest rate with the harvesting cost.

3.4 Ranking the Elasticities

From (11), the harvesting-cost elasticity is

$$\varepsilon\left(b^{*},c\right)=\varepsilon\left(B^{*},C\right)=\frac{C}{B^{*}}\frac{\partial B^{*}}{\partial C}=\left(\frac{\gamma}{\gamma-1}\right)\frac{C}{B^{*}\left[1-\left(B^{*}\right)^{-\gamma}\right]}. \tag{20}$$

The next result shows that the harvesting-cost elasticity, as a function of the harvesting cost, is very well behaved.

Proposition 5 *The harvesting-cost elasticity $\varepsilon\left(b^{*},c\right)$ is a decreasing function of c (for $c>x$) with $\lim_{c\to x}\varepsilon\left(b^{*},c\right)=\infty$ and $\lim_{c\to\infty}\varepsilon\left(b^{*},c\right)=1$, i.e., it has an asymptotic limit 1.*

Proof. *A direct computation from (20) shows that $\partial\varepsilon\left(B^{*},C\right)/\partial C<0$ if*

$$B^{*}\left(1-\left(B^{*}\right)^{-\gamma}\right)<C\left(\frac{\gamma}{\gamma-1}\right)\left[1+\frac{\gamma\left(B^{*}\right)^{-\gamma}}{1-\left(B^{*}\right)^{-\gamma}}\right]. \tag{21}$$

From the first order condition (8), $C=B^{*}\left[\gamma-1+\left(B^{*}\right)^{-\gamma}\right]/\gamma$, inequality (21) becomes

$$\left(\gamma-1\right)\left[1-\left(B^{*}\right)^{-\gamma}\right]<\left[\gamma-1+\left(B^{*}\right)^{-\gamma}\right]\left[1+\frac{\gamma\left(B^{*}\right)^{-\gamma}}{1-\left(B^{*}\right)^{-\gamma}}\right],$$

which is easily verified to be true.

Using the fact that, as $C \to 1$*,* $B^* \to 1$ *and* $(B^*)^{-\gamma} \to 1$*, we have* $\lim_{C\to 1} \varepsilon(B^*, C) = \infty$*. To find* $\lim_{C\to\infty} \varepsilon(B^*, C)$*, we first note that*

$$\frac{\partial\left[B^*\left(1-(B^*)^{-\gamma}\right)\right]}{\partial C} = \left(\frac{\gamma}{\gamma-1}\right)\left[1+\frac{\gamma(B^*)^{-\gamma}}{1-(B^*)^{-\gamma}}\right].$$

Then, using the fact that, as $C \to \infty$*,* $B^* \to \infty$ *and* $(B^*)^{-\gamma} \to 0$*, we have*

$$\lim_{C\to\infty} \varepsilon(B^*, C) = \frac{1}{\lim_{(B^*)^{-\gamma}\to 0}\left[1+\frac{\gamma(B^*)^{-\gamma}}{1-(B^*)^{-\gamma}}\right]} = 1,$$

by l'Hopital's rule. The conversion to lowercase is straightforward. ■

To compare $-\varepsilon(B^*, r)$ with $\varepsilon(B^*, C)$, we note that, from (16) and (17), the interest elasticity is

$$-\varepsilon(b^*, r) = -\varepsilon(B^*, r) = \frac{r}{B^*}\frac{B^* - C - (B^*)^{1-\gamma}\log B^*}{(\gamma-1)\left[1-(B^*)^{-\gamma}\right]}\frac{1}{\sqrt{(\mu-\sigma^2/2)^2 + 2r\sigma^2}},$$

and the ratio of the two elasticities is

$$R = \frac{-\varepsilon(b^*, r)}{\varepsilon(b^*, c)} = \frac{-\varepsilon(B^*, r)}{\varepsilon(B^*, C)} = \frac{r\left[B^* - C - (B^*)^{1-\gamma}\log B^*\right]}{\gamma C\sqrt{(\mu-\sigma^2/2)^2 + 2r\sigma^2}}.$$

The numerator of R approaches the value of 0 when $c \to x$ (or $C \to 1$). Thus, we have

Proposition 6 *The ratio of the interest elasticity to the harvesting-cost elasticity satisfies*

$$\lim_{c\to x} \frac{-\varepsilon(b^*, r)}{\varepsilon(b^*, c)} = 0.$$

The proposition says that the interest elasticity is much smaller than the harvesting-cost elasticity if the harvesting cost is not very large. In practice, the harvesting cost would most likely fall in this range. It should be mentioned that the interest elasticity is *uniformly* dominated by the harvesting-cost elasticity in the case of linear growth without *any* restrictions. See the Appendix for details.

A natural question arises. When is the interest elasticity uniformly dominated by the harvesting-cost elasticity? The answer is the following

Proposition 7 *If the inequality*

$$r \leq \gamma (\gamma - 1) \sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2} \tag{22}$$

is satisfied, then $-\varepsilon(B^*, r) < \varepsilon(B^*, C)$ *and* $-\varepsilon(b^*, r) < \varepsilon(b^*, c)$*.*

Proof. *From the definition of* R*, we have*

$$\begin{aligned}
\frac{-\varepsilon(b^*, r)}{\varepsilon(b^*, c)} &= \frac{r\left[B^* - C - (B^*)^{1-\gamma}\log B^*\right]}{\gamma C\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}} < \frac{r(B^* - C)}{\gamma C\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}} \\
&\leq \frac{r}{\gamma(\gamma - 1)\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2}} \leq 1,
\end{aligned}$$

where the second inequality is obtained by invoking the upper bound of (13)*,* $B^* \leq [\gamma/(\gamma - 1)]\, C$*.* ■

Condition (22) is approximately equivalent to $\gamma \geq 2$ (or $r \geq 2\mu$) if σ^2 is small, because in that case $\gamma\mu \approx r$. It does not seem like a very stringent restriction.

To complete the ranking, we have the following

Proposition 8 *The uncertainty elasticity is smaller than the expected-growth-rate elasticity, i.e.,* $\varepsilon(b^*, \mu) \geq \varepsilon(b^*, \sigma^2)$ *if and only if* $(r - 2\mu)\sigma^2 \leq 4\mu^2$.

Proof. *Since* $\partial B^*/\partial\gamma < 0$, $\varepsilon(B^*, \mu) \geq \varepsilon(B^*, \sigma^2)$ *if and only if* $\mu(\partial\gamma/\partial\mu) \leq \sigma^2(\partial\gamma/\partial\sigma^2)$. *Then, from* (14) *and* (15), *it is equivalent to*

$$2\mu\left(\mu - \sigma^2/2\right) + r\sigma^2 \leq 2\mu\sqrt{\left(\mu - \sigma^2/2\right)^2 + 2r\sigma^2},$$

which can be simplified as $r\sigma^2 - 2\mu\sigma^2 \leq 4\mu^2$. ■

The proposition says that the ranking of the expected-growth-rate elasticity and the uncertainty elasticity depends on the parametric specification. The inequality in the proposition is easily satisfied if σ^2 is small.

4 Concluding Remarks

In this paper we have ranked the elasticities of the optimal harvesting size with respect to exogenous parameters, and obtained good estimates for the optimal harvesting size and the present value of the rotation when the tree growth is linear or follows geometric Brownian motion. We show that the harvesting size is strictly increasing and strictly concave in harvest cost, bounded between two parallel lines. We also show that the harvesting size is most elastic to the harvesting cost, followed by the interest rate, and is least elastic to the variance of the tree growth under certain conditions.

Our findings have interesting policy implications. Within the framework of profit maximization, the ranking of the elasticities suggests that the policies that raise the harvesting cost such as unit tax would be most effective in slowing down the logging because the most elastic parameter would have the largest impact. On the other hand, the rotation problem is a part of the economics of forestry. As pointed out in Samuelson (1976, p.469), the harvesting decision imposes externalities such as “flood control, pollution abatement, species preservation, vacationers’ enjoyment, etc.” Obviously, we should go beyond solving the rotation problem solely for its lumber value by explicitly including the externalities in the analysis so that the policy implications would be more accurate.

There are at least two directions to which this research can be potentially extended. We can specify the external effect of harvesting upon the rest of the economy. There is no doubt that the way to which the externality is specified changes the harvesting rule and the corresponding policy implications. The pioneering works in this area include Reed (1993) and Reed and Ye (1994) in which they explicitly considered environmental amenities of old-growth forests in their models and treated the harvesting of old-growth forests as a Wicksellian tree-cutting problem. Alternatively, we can examine the elasticities of the harvesting size to exogenous parameters when tree growth is age-dependent or when it is size-dependent. More structural restrictions will have to be imposed to come up with meaningful implications. These are, however, for future research.

5 Appendix

In this Appendix we analyze the case of linear growth. This case is well known (see, for example, Miller and Voltaire (1980, 1983)) and we shall report only new results. The proofs are similar to the geometric Brownian growth case and are left as an exercise.

Suppose the growth pattern of a tree is linear, i.e.

$$dX_t = \mu dt + \sigma dz.$$

Then

$$E_x\left[e^{-r\tau}\right] = e^{-\eta(b-x)},$$

where

$$\eta = \frac{-\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} > 0,$$

and the problem becomes

$$\max_b G(b) = \max_b \left\{ \frac{(b-c)\, e^{-\eta(b-x)}}{1 - e^{-\eta(b-x)}} \right\}. \tag{23}$$

The first order condition is

$$1 + \eta c - \eta b = e^{-\eta(b-x)}. \tag{24}$$

Equation (24) can be written as

$$\mu = \mu\eta\,(b-c) + \left[\frac{e^{-\eta(b-x)}}{1 - e^{-\eta(b-x)}}\right]\mu\eta\,(b-c)\,,$$

which is the stochastic version of the Faustmann formula when the tree growth is linear ($X'(t) = \mu$) because $\eta \to r/\mu$ as $\sigma^2 \to 0$. Using the change of variables, $B = b - x$ and $C = c - x$, (24) is changed to

$$1 + \eta C - \eta B = e^{-\eta B}. \tag{25}$$

The solution to (25) is well known.

1. Harvesting size as a function of harvesting cost

Proposition 9 *The optimal harvesting size b^* is increasing and concave in c satisfying (i) $\lim_{c \to x} b^*(c) = x$ and $\lim_{c \to \infty} b^*(c) = \infty$, (ii) $b^*(c)$ is bounded between two straight lines: $y = c$ and $y = c + 1/\eta$, (iii) $b^*(c)$ satisfies $\lim_{c \to x} \partial b^*(c)/\partial c = \infty$, and is asymptotic to $y = c + 1/\eta$.*

The proposition is best summarized in Figure 3.

2. Comparative Dynamics

Proposition 10 *(i) The comparative statics are: $\partial b^*/\partial c > 0$, $\partial b^*/\partial x < 0$, $\partial b^*/\partial r < 0$, $\partial b^*/\partial \mu > 0$, and $\partial b^*/\partial \sigma^2 > 0$. (ii) If $J(c, x, r, \mu, \sigma^2)$ is the value function of (23), then $J_c < 0$, $J_x > 0$, $J_r < 0$, $J_\mu > 0$, and $J_{\sigma^2} > 0$.*

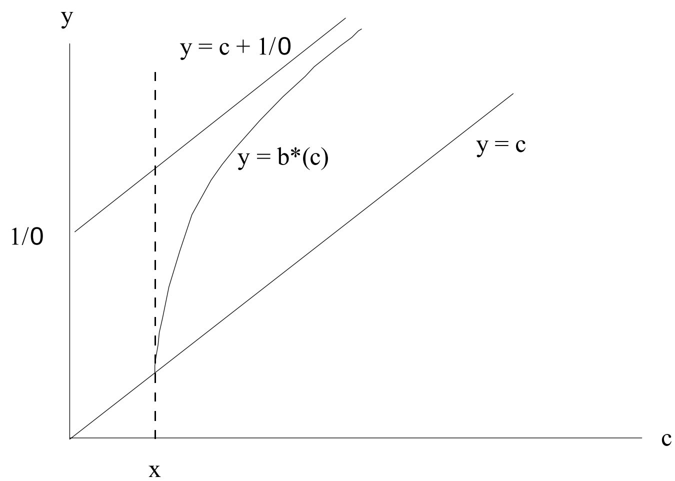

Figure 3: b* as a function of c

The proposition was also obtained and succinctly explained by Miller and Voltaire (1980, p.139). While their reasoning and their intuition were impeccable, one of their equations is incorrect. To prove $\partial b^*/\partial r < 0$, $\partial b^*/\partial \mu > 0$, and $\partial b^*/\partial \sigma^2 > 0$, it suffices to show that $\partial B^*/\partial \eta < 0$. A direct computation from (25) shows that

$$\frac{\partial B^*}{\partial \eta} = \frac{C - B^* + B^* e^{-\eta B^*}}{\eta \left(1 - e^{-\eta B^*}\right)}. \tag{26}$$

Instead, what Miller and Voltaire (1982) obtained is

$$\frac{\partial B^*}{\partial \eta} = \frac{C - B^* - B^* e^{-\eta B^*}}{\eta \left(1 + e^{-\eta B^*}\right)},$$

which makes the sign of $\partial B^*/\partial \eta$ obvious "by inspection." The correct proof goes as follows. Note that the function $f(C) = C - B^* + B^*\alpha$, where $\alpha = e^{-\eta B^*}$, satisfies $f(0) = 0$ (using $\lim_{C\to 0} B^* = 0$) and

$$f'(C) = 1 - \frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha} - \frac{\eta B^*\alpha}{1-\alpha} = -\frac{\eta B^*\alpha}{1-\alpha} < 0.$$

It follows that $f(C) < 0$ for all $C > 0$, and $\partial B^*/\partial\eta < 0$.

3. Homogeneity Conditions

Similar to the geometric Brownian growth case, we have (19). There is another homogeneity condition in this case, albeit it is less obvious. If we rewrite η as

$$\eta = \frac{-\mu + \sqrt{\mu^2 + 2\sigma^2/(1/r)}}{\sigma^2}$$

so that we can treat η as a function of $(\mu, 1/r, \sigma^2)$, then B^* is a function of $(C, \mu, 1/r, \sigma^2)$, i.e., $B^* = B^*(C, \mu, 1/r, \sigma^2)$. Note that if we double C, $1/r$, and σ^2, followed by a doubling of B^*, then equation (25) remains unchanged. In other words, B^* is homogeneous of degree 1 in $(C, 1/r, \sigma^2)$. By Euler's theorem and using the fact that $\varepsilon(B^*, 1/r) = -\varepsilon(B^*, r)$, we have

$$\varepsilon(B^*, C) - \varepsilon(B^*, r) + \varepsilon\left(B^*, \sigma^2\right) = 1. \quad (27)$$

4. Ranking the Elasticities

The ranking of the elasticities is similar to the case of geometric Brownian growth: the harvesting size is most elastic to the harvesting cost, followed by the interest rate; it is least elastic to the parameters of the growth process. What makes this case interesting is that the additional homogeneity condition (27) for the linear growth case enables us to show that the interest elasticity is dominated by the harvesting-cost elasticity without any restrictions and that we can find the range of each elasticity. Specifically, from (27) and (19), we have

$$1 - \varepsilon\left(B^*, C\right) = -\varepsilon\left(B^*, r\right) + \varepsilon\left(B^*, \sigma^2\right) = \varepsilon\left(B^*, \mu\right) + 2\varepsilon\left(B^*, \sigma^2\right).$$

It follows that

$$\max\left\{-\varepsilon\left(B^*, r\right), \varepsilon\left(B^*, \mu\right), 2\varepsilon\left(B^*, \sigma^2\right)\right\} < 1 - \varepsilon\left(B^*, C\right).$$

Our claims are verified once we have the following proposition describing $\varepsilon\left(B^*, C\right)$ as a function of C and its range.

Proposition 11 *The harvesting-cost elasticity* $\varepsilon\left(B^*, C\right)$ *is an increasing function of* C *with* $\lim_{C\to 0} \varepsilon\left(B^*, C\right) = 1/2$ *and* $\lim_{C\to\infty} \varepsilon\left(B^*, C\right) = 1$*, i.e., it satisfies* $1/2 < \varepsilon\left(B^*, C\right) < 1$ *and has an asymptotic limit* 1*. Consequently,* $-\varepsilon\left(B^*, r\right) < 1/2$*,* $\varepsilon\left(B^*, \mu\right) < 1/2$*, and* $\varepsilon\left(B^*, \sigma^2\right) < 1/4$*.*

Notice that the harvesting-cost elasticity is *increasing* in c in this case, which is opposite of the geometric Brownian growth case. The ranking between the expected-growth-rate elasticity and the uncertainty elasticity is dictated by the parameters of the tree growth.

Proposition 12 *The the uncertainty elasticity* $\varepsilon\left(B^{*},\sigma^{2}\right)$ *is smaller than expected-growth-rate elasticity* $\varepsilon\left(B^{*},\mu\right)$ *if and only if* $\mu/\sigma \geq \sqrt{r}/2$.

It is interesting to note that the inequality $\mu/\sigma \geq \sqrt{r}/2$ is trivially satisfied if σ^2 is small.

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