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The Politics of Bargaining as a Group

Abstract

We develop a dynamic model in which a group collectively bargains with an external party. At each date the group makes an offer to the external party (the 'agent') in exchange for a concession. Group members hold heterogeneous preferences over agreements and are uncertain about the agent's resolve. We show that all group members favor more aggressive proposals than they would if they were negotiating alone. By eliciting more information about the agent's resolve, these offers reduce the group members' uncertainty about the agent's preferences and therefore reduce the group's internal conflicts over its negotiating strategy. To mitigate the consequent risk that negotiations fail, decisive group members successively give up their influence over proposals: starting from any initially democratic decision process, the group eventually consolidates its entire negotiation authority into the hands of a single member.

JEL-Codes: D020, D710, D780, D820, L220.

Keywords: adverse selection, collective choice, political economy, dictatorship, bargaining.

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1. Introduction

How do groups negotiate? In almost every real-world bargaining context at least one of the negotiating parties is a group. For example, the Coordinated Bargaining Committee of General Electric Unions comprises eleven labor unions that jointly negotiate with the firm's management.[1] In 2019, ten US rail unions formed the Coordinated Bargaining Coalition to collectively negotiate a labor dispute between workers and freight railroads. In turn, more than thirty railroads bargained jointly from the other side of the negotiating table as the National Carriers Conference Committee. The dispute resolved in December 2022 only after presidential and congressional intervention. Further examples of groups that negotiate include households, suppliers' associations, consumer cooperatives, legislatures, trading blocks and international alliances.

New conflicts arise in bargaining when one or both sides is a group. This is because different members tend to have different preferences over the group's negotiating stance. When negotiating jointly with management some unions may have relatively larger strike funds and favor a tougher stance. When a trading block negotiates with a non-member, differences in the members' domestic politics or trading volumes with the non-member may similarly drive different preferences over a hard versus soft negotiating stance. How individual preferences are aggregated into a collective negotiating position determines which group members are decisive and which are instead sidelined, directly impacting the prospects for external agreement.

Motivated by these insights, our paper studies the dynamics of bargaining in a group. It asks: how do group members negotiate differently when they bargain collectively, as opposed to when they negotiate alone? How do within-group conflicts shape a group's external negotiating position? Conversely: how do external negotiations shape the internal dynamics of group decision-making?

We address these questions in an infinite-horizon model of negotiations between members of a group that jointly negotiate with an external party. We call this external party the *agent*. In each period, the members collectively make a proposal to the agent in exchange for a conces-

[1] For a history see Meyer (2001).

sion. The agent either accepts or refuses; if accepted the offer binds all members of the group. The group members derive heterogeneous payoffs from securing the agent's agreement, and they are uncertain about the agent's resolve, modeled as her privately known cost of concession.

Different group members hold different priorities in negotiations with the agent. Members with high benefits from an agreement prefer a generous offer the agent is sure to accept. Members with lower benefits instead prefer to gamble on the prospect of securing an agreement with a less generous offer. Thus, the group faces internal conflicts over their joint offer. This is moderated by the group's common uncertainty about the agent's resolve. More uncertainty enlarges the scope for disagreement about the group's bargaining stance. This uncertainty evolves with the path of offers and acceptance decisions, since today's negotiation outcomes inform the members' beliefs about the agent's resolve. So, initial negotiation outcomes shape future disagreements over the group's ongoing negotiating strategy.

The defining challenge facing the group is to collectively decide on its negotiation position i.e., its collective offer at each date. We model this process as a collective decision procedure that governs the order in which members can propose offers, and the voting rule used by the group to select the winning alternative. We allow for deterministic or random member recognition, and a wide array of voting rules including quotas, oligarchies, and rules with veto rights. Further, the collective decision process itself can be revised at the start of each period before that period's negotiations with the agent. Revisions to how the group decides are governed by the process inherited from the previous period.

We first study how the individual members' preferences over offers differ when they are in a group, instead of negotiating alone. We find that for *any* collective decision-making process under which a member anticipates that she may not be decisive over all future offers, she favors *more* aggressive proposals than she would in a stand-alone context.

To understand why, recognize that absent any uncertainty about the agent's resolve, all group members prefer the least generous offer that secures the agent's agreement. More uncertainty creates more scope for members to disagree on how to trade off the value of agreement with the probability of agreement. This disagreement harms a member whenever she cannot enforce her preferred offer on the remaining members. A group member therefore favors more aggressive offers that reveal more information about the agent's preferences, and which therefore reduce the scope for disagreement. This insures her against the risk that she cannot directly control future offers.

We then study the dynamics of authority within the group over the long run. We show that decisive coalitions of members successively trade away their decision-making authority until negotiation power is fully-consolidated into the hands of a single member. This phenomenon arises when (1) a group of collectively non-decisive members prioritize reaching an agreement and therefore favor an offer the agent is sure to accept, (2) a 'marginal' group member prefers a less generous offer under the inherited collective choice process, but would favor the most generous offer if she were decisive at all future periods, and (3) the union of non-decisive members and this marginal member is decisive. Complete concentration need not happen immediately: we illustrate the dynamics of how power increasingly and inexorably accumulates in the long run.

Our results and framework contribute to a number of literatures on group bargaining and collective decision-making.

Negotiating as a Group. We build on a small number of theoretical studies of group bargaining. Manzini and Mariotti (2005, 2009) show how a group can benefit from a unanimous voting rule that effectively delegates its bargaining posture to the group's most aggressive member. The benefits of implicit commitment power from negotiating as a group also appears in Bond and Eraslan (2010) and Konrad and Cusack (2014). In contrast with these papers, our framework and key trade-offs are dynamic and we focus on the evolution of the group's endogenous procedures as bargaining unfolds.

We find that group members' induced preferences over proposals are more aggressive than they would be if they were to negotiate alone. This corresponds to a phenomenon documented experimentally in settings that include ultimatum bargaining called the *group discontinuity effect* (Bornstein and Yaniv, 1998; Elbittar, Gomberg and Sour, 2011). The effect identifies the tendency of groups to behave more aggressively than individuals in otherwise similar circumstances. While typically conceived as a behavioral phenomenon, ours is the first account derived in a fully rational framework. In our group bargaining setting incentives to make more aggressive offers arise as a form of conflict management inside the group—they hasten the formation of a consensus amongst the group members over its future negotiating strategy. Crucially, this effect is distinct from the commitment power of strategic delegation (Persson and Tabellini, 1992; Harstad, 2008; Buisseret and Bernhardt, 2018). In these papers group members' induced preferences are fixed but they may delegate to players with different—i.e., more aggressive—induced preferences than their own. In our framework, every single group member's induced preferences over offers become more aggressive *solely* by virtue of negotiating in a group.

Games Played by Teams. Outside the bargaining context, two other papers also study preference inside teams of players that interact non-cooperatively with an external party. Duggan (2001) studies aggregation rules within groups whose members have heterogeneous preferences and provides conditions for equilibrium existence. Kim, Palfrey and Zeidel (2022) instead assume that team members have common preferences but each member observes the group's payoff from action profiles with noise. The set of individual payoff vectors is aggregated into the group's distribution over actions. Neither of these papers study which rules emerge endogenously. Widely-used solution concepts also implicitly presume an aggregation rule within a group, including Strong Nash Equilibrium and Coalition proof Nash equilibrium.

Power Concentration in Groups. Our paper provides a new rationale for the tendency of groups to concentrate decision-authority in the hands of ever-smaller subsets of members (Michels, 1959). Our account differs from existing theories in which centralization or delegation leverages an agent's expertise (Dessein, 2002), adapts decisions to local conditions (Liu and Migrow, 2022), improves coordination (Rantakari, 2008), or generates strategic pre-commitment against an external negotiating party (Besley and Coate, 1997). It also differs from studies (like all of these) that focus on static or stationary economies—implying either a static or stationary optimal allocation of authority—or in which authority is determined at a single point in time before conflicts of interest are known (Maggi and Morelli, 2006).

Our framework reconciles two seemingly divergent perspectives on the evolution of decision-making in groups and organizations. The first tradition—identified with March and Simon (1958) and Cohen, March and Olsen (1972)—views organizations as "constituted by shifting factions with differing interests that vie for control..." in which actions are viewed as "reflective of the preferences of a victorious coalition at a given point in time" (Tolbert and Hiatt, 2009, 175). Another perspective, summarized in Robert Michels' famous 'iron law', views organizations as inevitably drifting towards oligarchy and the concentration of power in the hands of the few. Both perspectives are correct in our model: at any date decisive coalitions are dominant, but over time the dynamics of shifting factional interests ensure that power is eventually consolidated into the hands of ever-smaller subsets of group members. Despite Williamson (1988)'s admonishment more than thirty-five years ago that "[t]he incentive literature makes no provision whatsoever for the possibility that oligarchy is a predictable process outcome" (p. 87), ours is the first theoretical framework to identify this possibility.

Non-Cooperative Refinement of Markov Voting Equilibria. We model the group's collective choice problems as an amendment agenda game (Duggan 2006, Austen-Smith and Banks 2005). This game is governed by a *procedure*, which specifies the order in which members can make proposals, and the voting rule used to select the winning alternative. We allow for deterministic or random recognition rules, and a wide array of voting rules, including quotas, oligarchies, and rules with veto rights. The sequences of offers made to the external party in our noncooperative equilibria constitute Markov voting equilibria à la Roberts (2015) or Acemoglu, Egorov and Sonin (2015). While the core is generally too permissive to make concrete predictions for some voting rules—such as large voting quotas—we show that Duggan (2006)'s amendment agenda game serves as a natural and effective approach to refine Markov voting equilibrium to a unique prediction under any procedure.

Our work relates more distantly to the literature on experimentation, e.g., Strulovici (2010), Anesi and Bowen (2021) and Bowen, Hwang and Krasa (2022); Freer, Martinelli and Wang (2020) survey recent contributions. Nevertheless, the strategic interaction with a privately informed external party in our model yields a learning technology that is fundamentally different from the experimentation literature. In those papers, a group collectively chooses between a risky reform and a safe status quo in a Poisson bandit framework with exogenous learning costs. Relative to a single-experimenter benchmark, individuals have insufficient incentives to learn in a group context.[2] In our setting, the learning cost is instead an opportunity cost of reaching an agreement with the agent. This opportunity cost is not fixed: it evolves with the preferences of decisive members and the collective choice process. We show that the fear of being marginalized in future rounds of bargaining gives individuals *excessive* incentives to learn in a group context. In turn, we show that members with very high (opportunity) costs of learning effectively delegate the decision to marginal members in order to deter them from excessive experimentation.

2. An Example

We begin with a two-period example that clarifies the main intuitions, before presenting our more general framework.

Basic Elements. We consider a two-period interaction between a group consisting of five members, $N \equiv \{1, \ldots, 5\}$, and an external party—the 'agent'. In each period $t = 1, 2$, the group can collectively make a demand from the agent in return for a policy concession, $x^t \in [0, 1]$. The agent accepts the demand ($a^t = 1$) or rejects it ($a^t = 0$). If the group does not make an offer ($x^t = \varnothing$), a status-quo policy of zero is implemented.

Group member $i \in N$'s period-t payoff is $a^t \left[b_i^t - x^t\right]$, where b_i^t is a stochastic benefit drawn at the start of every period from a c.d.f. F that is continuous and has full support on $[\underline{b}, \overline{b}]$. The realization $b^t = (b_1^t, \ldots, b_5^t)$ is publicly observed. The agent's period-t payoff is $a^t \left[x^t - c\right]$, where c is her privately observed cost from accepting the group's demand—her 'resolve'. The cost is drawn at the outset from $\{c_L, c_H\}$ and persists across both periods, with $\Pr(c = c_L) = p \in (0, 1)$, and $c_H < \underline{b}$. Players share a common discount factor $\delta \in (0, 1)$, and maximize average discounted payoffs.

Collective Choice. In each period $t \in \{1, 2\}$, after the group members' period-t benefits are re-

[2] Gieczewski and Kosterina (2020) obtain excessive experimentation in a setting where members can unilaterally take a safe outside option (i.e., exit).

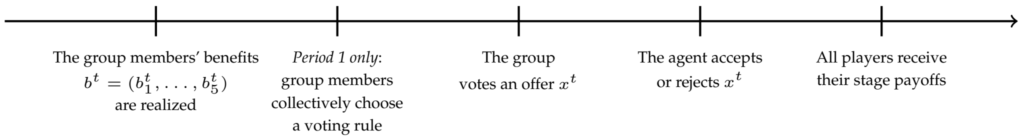

Figure 1 – Timing in each period $t = 1, 2$.

alized, they collectively vote an offer to the agent, x^t. The voting rule is modeled as a collection $\mathcal{D} \subseteq 2^N \setminus \{\emptyset\}$ of decisive coalitions that we only restrict to be monotonic (e.g., $C \in \mathcal{D}$ and $C \subseteq C'$ imply $C' \in \mathcal{D}$) and proper ($C, C' \in \mathcal{D}$ implies $C \cap C' \neq \emptyset$) — e.g., Austen-Smith and Banks (1999).

We also allow the group members to collectively select the voting rule $\mathcal{D}$ that they use to determine offers in each period. They choose this rule in period 1, after their benefits are realized and before they make their initial offer to the agent. They vote over the rule using the (exogenous) inherited rule $\mathcal{D}^0$, which we presume to be simple majority.[3] The timing is described in Figure 1.

Equilibrium. For this illustrative example, we assume that the group randomly selects an offers (uniformly) from the *core* of the voting rule $\mathcal{D}$ — that is, the set of offers that are undefeated in pairwise voting using $\mathcal{D}$. We impose this selection while still applying standard sequential-rationality and belief-consistency conditions. Our uniform selection is innocuous, and purely for exposition. In the sequel, we characterize the perfect Bayesian equilibria of a fully-fledged non-cooperative model of collective decision making among the group members. We further show how the group endogenously selects offers from the dynamic core in every equilibrium of the general model.

Analysis. We start with the second period. The agent with resolve c accepts the group's period-2 offer x^2 if and only if $x^2 \geq c$. On the path, the group members' common belief about the agent's type is either the prior, or degenerate. Define $b^* \equiv \frac{c_H - pc_L}{1-p}$. If the group learned that the agent's type is $c \in \{c_L, c_H\}$, they unanimously prefer to offer $x^2 = c$, which the agent accepts. Under the

[3] In the sequel the group can select a new rule in every period.

prior belief, instead, group member i with benefit b_i's preferred offer is

$$x^2(b_i) \equiv \begin{cases} c_H & \text{if } b_i \geq b^*, \\ c_L & \text{otherwise.} \end{cases}$$

Henceforth, we assume $\underline{b} < b^* < \overline{b}$. Thus, for any (equilibrium) belief of the group, it selects from at most two possible offers.

Given period-2 belief p^2 and voting rule $\mathcal{D}$, the *core* of the group's collective choice problem in period-2 is the set of offers that are undefeated in pairwise voting. We denote this set $\mathcal{K}(p^2, \mathcal{D})$: it is non-empty, and may contain either one or two offers. If the core contains two elements, our selection (solely for this example) presumes that each is equally likely to be chosen. This selection generates a unique equilibrium outcome in period 2 for any belief and voting rule.

We therefore turn to period $t = 1$. Let $\overline{x}^H$ denote a period-1 offer that both types accept, and $\overline{x}^L$ denote an offer that only the low-cost agent accepts. Then, $\overline{x}^L$ solves:

$$\overline{x}^L - c_L + \delta \times 0 = 0 + \delta(c_H - c_L).$$

If the agent accepts the offer, she reveals her cost of concession is low, and receives a payoff of zero at period 2. If she rejects the offer, the group members infer that the agent's cost is high, and she receives an offer of $x_2 = c_H$ at period 2. Similarly, $\overline{x}^H$ solves:

$$\overline{x}^H - c_H + \delta \times 0 = 0.$$

To see why, recognize that if the period-2 offer is c_L, the high-cost agent rejects and receives zero; if the period-2 offer is c_H, the agent's payoff is zero. Notice that the low-cost type agent also accepts this offer: if she rejects, her second-period payoff is bounded above for all possible beliefs of the group members by $\delta(c_H - c_L)$, which is strictly less than her payoff from accepting c_H, today.

We conclude that, in an equilibrium, the group faces a period-1 collective choice between two possible offers. As is standard, the group's optimal offer that the high-cost agent agent would accept coincides with that agent's static constraint, while the low type extracts a rent. This rent does not depend on the group's voting rule because the agent has only two possible types, so that the group's belief is degenerate after the agent either accepts or rejects the separating offer. As a consequence, the group members unanimously agree on their preferred period-2 offer after a period-1 offer that revealed the agent's cost, and the voting rule does not impact the agent's period-2 offer.

Individual versus group preferences over offers. We now study how different voting rules shape a group member's induced preferences over offers in period 1. Member i's continuation value from a period-1 separating offer is:

$$W^{\text{sep}} \equiv \mathbb{E}[b_i] - pc_L - (1-p)c_H.$$

When the group members' beliefs about the agent are degenerate, they unanimously agree on their preferred period-2 offer. As a consequence, the continuation value from an offer that reveals the agent's cost does not depend on the voting rule. Matters are different when the group members are uncertain about the agent's type. Let $\tau(b, \mathcal{D})$ denote the probability that the period-2 offer is c_H when the members hold prior belief p, the benefits realization is b, and the voting rule is $\mathcal{D}$. Member i's continuation value from a period-1 offer that reveals no information about the agent's cost (a *pooling* offer) is therefore:

$$W^{\text{pool}}(\mathcal{D}) \equiv \int_b \Big[\tau(b, \mathcal{D})(b_i - c_H) + \big[1 - \tau(b, \mathcal{D})\big]p(b_i - c_L)\Big]\, dF(b),$$

where $F(\cdot)$ is the joint distribution of the benefits profile $b = (b_1, \ldots, b_5)$. Member i with period-1 benefit b_i therefore prefers an offer that reveals the agent's cost—a *separating* offer—if and only if

$$b_i \leq \frac{1}{1-p}(c_H - p\overline{x}^L) + \frac{\delta}{1-p}\big[W^{\text{sep}} - W^{\text{pool}}(\mathcal{D})\big] \equiv \beta(\mathcal{D}).$$

We compare this threshold under two classes of voting rules. A voting rule $\mathcal{D}$ is a *dictatorship of* *member* i if:

$$\mathcal{D} = \{S \subseteq N : S \ni i\} \equiv \mathcal{D}^i.$$

Recognize that a dictatorship of member i is equivalent to a setting in which the group consists solely of member i.

Since the first period incentive constraints do not depend on the voting rule, and recalling $b^* \equiv \frac{c_H - pc_L}{1-p}$, we have that for any voting rule $\mathcal{D}$:

$$\beta(\mathcal{D}) - \beta(\mathcal{D}^i) = \delta(1-p)\int_b \left[\tau(b,\mathcal{D}^i) - \tau(b,\mathcal{D})\right](b_i - b^*)\ dF(b). \tag{1}$$

Setting aside the constant, and recognizing that $\tau(b, \mathcal{D}^i)$ takes the value 1 if $b_i \geq b^*$, and zero otherwise, for any $\mathcal{D} \neq \mathcal{D}^i$, the difference (1) must be weakly positive, and is in fact strictly positive:

$$\beta(\mathcal{D}) - \beta(\mathcal{D}^i) = \int_{\{b\,:\,b_i \geq b^*\}} \left[1 - \tau(b,\mathcal{D})\right](b_i - b^*)\ dF(b) - \int_{\{b\,:\,b_i < b^*\}} \tau(b,\mathcal{D})\,(b_i - b^*)\ dF(b) > 0. \tag{2}$$

To see why, recognize that since $\underline{b} < b^* < \bar{b}$ there is a positive probability realization of benefits in which either (1) all members other than i prefer separation at date 2, but i favors pooling: $b_j < b^* < b_i$ for all $j \neq i$, or (2) all members other than i prefer the pooling offer at date 2, but i favors separation: $b_i < b^* < b_j$ for all $j \neq i$. For these benefits realizations, i's losses from any $\mathcal{D} \neq \mathcal{D}^i$ are strictly positive. We therefore have the following observation.

Result 1. *For any $\mathcal{D} \neq \mathcal{D}^i$: $\beta(\mathcal{D}) > \beta(\mathcal{D}^i)$. That is: group members favor more aggressive proposals than they would if they were negotiating alone.*

A group member prefers to make more aggressive proposals than a single individual. The reason is that the separating offer reveals more information about the agent's resolve. This reduces the group members' scope for future conflicts over its negotiating strategy, which benefits a member that may not be decisive in the second round of bargaining.

Suppose member i's period-2 benefit is low (i.e,. $b_i^2 < b^*$), but that period's decisive member j has a high benefit, $b_j^2 > b^*$. Under the prior, high-benefit member j prioritizes agreement with the agent in period 2 by making the pooling offer c_H. This offer is excessively generous, from member i's perspective. A period-1 separating offer may reveal that the agent has a low cost, c_L. This reduces high-benefit j's most preferred offer, to low-benefit i's advantage.

Conversely, suppose member i's period-2 benefit is high (i.e., $b_i^2 > b^*$), but that period's decisive member j has a low benefit, $b_j^2 < b^*$. Under the prior, low-benefit member j prefers to gamble that the agent has a low cost of agreement by making the separating offer c_L. This offer risks that no agreement is reached the agent, which i prioritizes. Revealing that the agent has a high cost leads a future low-benefit decisive member *not* to gamble with a low offer in period 2, to the advantage of high-benefit member i.

Notice that if the group members' benefits from an agreement are constant across dates, then the value to any member of the first-period (majority) decisive coalition from pooling versus separating coincides with that member's net value when negotiating alone. This observation highlights that uncertainty about group members' preferences across periods generates uncertainty about which members will be decisive in the future. This uncertainty, in turn, generates a divergence between a member's preferred offer when negotiating alone versus negotiating in a group. Uncertainty about group members' future preferences is relevant in real-world contexts. For example, members may act as delegates on behalf of other group interests and face the prospect of replacement between periods—for example, due to an election.

Concentrating Power. Recall that the group selects the voting rule governing how it determines offers to the agent in each period. The members collectively select the rule in period 1, after their initial benefits are realized and before they vote their initial offer to the agent. They choose the rule under the status quo voting rule, $\mathcal{D}^0$, which we presume to be simple majority. We unearth a positive probability that a decisive coalition of members voluntarily cedes decision-making power, and opts to concentrate authority in a minority of members—possibly, a single member.

To see why, let $\overline{\beta}$ denote a group member's smallest possible pooling threshold under the prior:

$$\overline{\beta} = \min\left\{\beta(\mathcal{D}) : \mathcal{D} \neq \mathcal{D}^i\right\}. \tag{3}$$

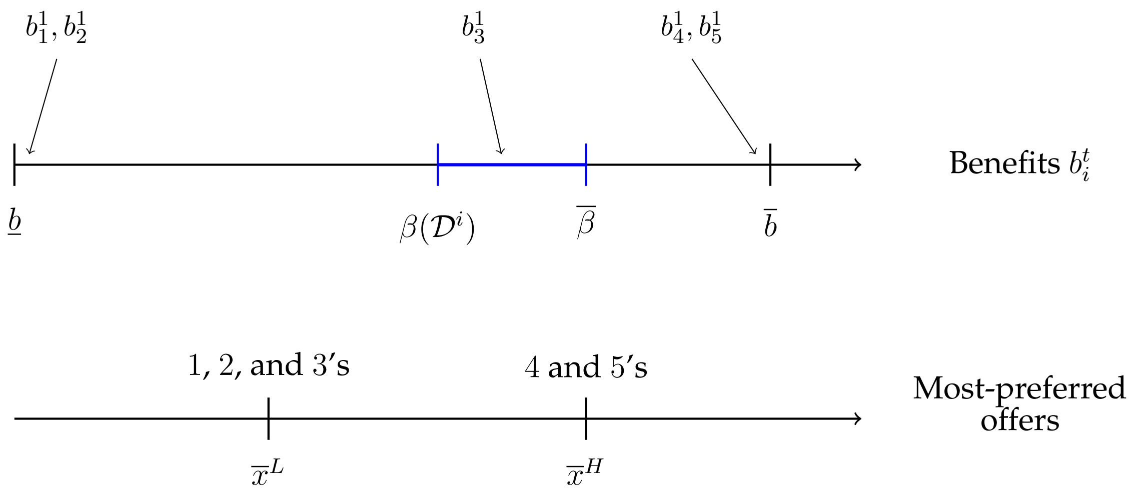

Figure 2 – The realization of group members' period-1 benefits described in text.

Assume $\underline{b} < \overline{\beta}$, so that the costs of separating the agent are so expensive that every group member prefers to pool in the first period. Consider the positive probability event—illustrated in Figure 2—in which the benefits realization b^1 is such that:

(i) b_1^1 and b_2^1 lie in a neighborhood of $\underline{b}$,

(ii) b_4^1 and b_5^1 lie in a neighborhood of $\overline{b}$, and

(iii) b_3^1 lies in $(\beta(\mathcal{D}^i), \overline{\beta})$.

Part (i) states that members 1 and 2 prefer the separating offer, but part (ii) states that members 4 and 5 prefer the pooling offer. Part (iii) states that member 3 prefers the pooling offer if she is a dictator; under any other voting rule, she prefers a separating offer (*Result 1*). It follows that under a simple majority voting rule, the group makes the separating offer in period 1.

If δ is small enough—or if $\overline{b}$ is large enough—high-benefit members 4 and 5 prioritize an agreement with the agent in period 1. Under the inherited simple majority rule, they cannot secure the pooling offer. Is there another voting rule that (1) guarantees the pooling offer will be made in period 1, and that (2) a majority of the group members would prefer to simple majority? The answer is *yes*: a majority of members strictly prefer a dictatorship of member 3, $\mathcal{D}^3$, to simple majority.

To see why, recognize that since members 4 and 5 prioritize agreement today, they strictly benefit from any change in the voting rule that guarantees a period-1 pooling offer. Since $\underline{b} < b_3^1 < \beta(\mathcal{D}^3)$, member 3 favors the pooling offer if and only if she has sole authority to decide the period2 offer. And, member 3 is trivially better off in both periods 1 and 2 when she is made a dictator. We obtain that a decisive coalition of today's group members—3, 4 and 5—strictly prefer 3's dictatorship to any voting rule that does not induce the pooling offer with probability one.

We conclude that for δ not too large, after this benefits realization, a voting rule lies in the core of the group's collective choice at the start of period 1 *only if* it induces the pooling offer in that period. A dictatorship is not the only rule that achieves this, however. Recognizing the inevitability of a period-1 pooling offer, members 1 and 2 could offer members 4 or 5 an alternative procedure that establishes this commitment: namely, an oligarchy of members 4 and 5: $\mathcal{D} = \big\{S \subseteq N \colon S \supseteq \{4,5\}\big\}$.

Besides a dictatorship or an oligarchy, no other voting rule guarantees the pooling offer, and thus no other voting rule commands the support of a majority.

Result 2. *If players care enough about period-1 outcomes, i.e., if δ is not too large, then there is a positive probability realization of benefits b^1 such that the only voting rules that belong to the core of the group's collective choice are:*

1. *an oligarchy: for some $i, j \in N$, $\mathcal{D} = \big\{S \subseteq N \colon S \supseteq \{i,j\}\big\}$, or*
2. *a dictatorship: for some $i \in N$, $\mathcal{D} = \mathcal{D}^i$.*

Our two-period model yields two insights. First, group members have incentives to make more aggressive proposals, relative to the proposals they would prefer it negotiating alone. Second, decisive group members may choose to consolidate negotiation authority in the hands of an oligarchy or even a single member.

The rest of the paper extends these insights to an infinite horizon model with any (finite) number of group members and agent types. We allow the group to reform its collective decision-making procedures at the start of every period, prior to negotiations with the agent. We further assume that the agent's type is re-drawn with positive probability in every period, ensuring that there is always scope for learning in future periods.

We first address the robustness of Result 1 that groups members favor more aggressive proposals than they would if they were negotiating alone. In our two-type example, a separating offer fully reveals the agent's cost and fully eliminates disagreement amongst group members. With more possible types, there are many partially-separating offers that leave residual uncertainty about the agent's resolve, and thus also leave scope for disagreement amongst the group over future offers. As a consequence, the members' continuation values from (partially) separating offers vary with the collective choice rule.

The agent's incentive constraints associated with separating offers may also vary with the collective choice rule. The reason is that the agent accounts for how information that she reveals today shapes future offers—possibly indirectly by triggering changes in the groups' choice rule. These changes in future offers affect her foregone rents from revealing information about her preferences. We verify that any wedge between a member's incremental benefit from learning the agent's type versus any associated incremental incentive costs remains positive across different choice procedures.

The sequel also extends our substantive finding about the group's concentration of decision-making authority in Result 2. Recall that in our two-period example the group starts with majority rule and makes at most one procedural reform decision; if, instead, members could reform their procedures more frequently, would the concentration of power stop, or would it continue, indefinitely? Does the answer depend on the initial inherited procedure We provide a strong answer to these questions by showing that *any* equilibrium sequence of procedures converges to the dictatorship of a single member almost surely. That is: starting from any initially democratic (i.e., non-dictatorial) decision process, the group eventually consolidates its entire negotiation authority into the hands of a single member.

The inevitability of dictatorship derives from three features of our model. First, the possibility that the agent's type is re-drawn in every period means that there is always a residual conflict of interest amongst the group members. Second, today's collective decision-making procedure is chosen under the inherited procedure from the previous period, which renders dictatorship absorbing. Third, we focus on settings where agents care enough about short-run outcomes—i.e., they have relatively low discount factors. This implies that members with a large instantaneous benefit from agreement prioritize the pooling over any offer that reveals information about the agent's preferences but risks rejection.

Finally, our uniform random selection from the core even in the two-period model highlights how the core may be too permissive to make concrete predictions for some voting rules, such as large quotas. Rather than imposing an arbitrary selection, we model the group's collective choice as an amendment agenda game (Duggan 2006, Austen-Smith and Banks 2005). The sequences of offers made to the agent in our noncooperative equilibria constitute Markov voting equilibria à la Roberts (2015) or Acemoglu, Egorov and Sonin (2015), and we show that the amendment agenda game refines Markov voting equilibrium to a unique prediction under any collective choice procedure.

3. Main Model

Main elements. A group of individuals, $N \equiv \{1, \ldots, n\}$, $n \geq 2$, bargain with an external party, indexed 0, over an infinite number of discrete periods. We call this external party the *agent*. In each period $t = 1, 2, \ldots$, the group can collectively make a demand to the agent, in exchange for a policy concession, x^t, chosen from a set $X \equiv [0, \hat{x}_0]$, where $\hat{x}_0 > 0$. The agent may concede to the demand, in which case we write $a^t = 1$, or not, in which case we write $a^t = 0$. If the members choose not to make any demand to the agent (i.e., $x^t = \varnothing$), then a status-quo policy 0 is implemented.

Group member i's stage payoff is $a^t \left[b_i^t - u(x^t)\right]$, where u is a convex, strictly increasing, continuously differentiable (dis)utility function on X, satisfying $u(0) = 0$; and b_i^t is a stochastic benefit chosen by Nature. We assume that each member i's benefit from agreement is drawn at the start of every period from a c.d.f. F_i that is continuous and has full support on some interval $B \equiv [\underline{b}, \overline{b}]$, with $\underline{b} < \overline{b}$. The benefit profile's realization $b^t = (b_1^t, \ldots, b_n^t)$ is publicly observed.[4]

The agent's stage payoff is $a^t \left[u_0(x^t) - c^t\right]$, where where u_0 is a concave, strictly increasing,

[4] As we later highlight, the assumption that members' benefits are independent across periods guarantees equilibrium existence. None of our characterization results rely on this assumption.

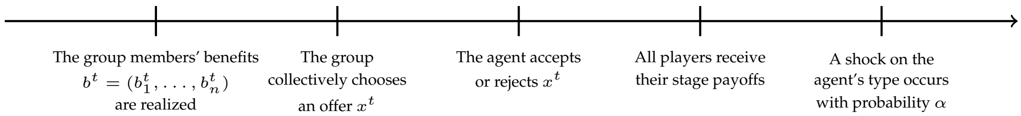

Figure 3 – Timing in each period $t = 1, 2, \ldots$.

continuously differentiable utility function on X, satisfying $u_0(0) = 0$; and c^t is her privately observed cost from conceding to the group's demand. We interpret this cost of concession as the agent's *resolve*. The cost is initially drawn by Nature from a finite set $C \equiv \{c_1, \ldots, c_K\}$, where $K \geq 2$ and $0 < c_1 < \cdots < c_K < u_0(\hat{x}_0)$, according to some nondegenerate distribution $p^0 \in \Delta(C)$. We assume that p^0 satisfies a local monotone hazard rate property: for every $\underline{k} = 1, \ldots, K-1$, the mapping $k \mapsto \sum_{\ell=\underline{k}}^{k} p^0(c_\ell)/p^0(c_{k+1})$ increases on $\{\underline{k}, \ldots, K-1\}$.[5]

Like the group members' benefits, we allow the agent's type to change across periods. Given our focus on learning, however, we assume some persistence. For simplicity, the agent's type evolves according to a marked point process: at the end of every period, the agent's type is re-drawn from C according to p^0 (and the group's common belief is correspondingly reset to p^0) with probability $\alpha \in (0, 1)$. Otherwise, the agent's type remains unchanged.[6]

All players share a common discount factor $\delta \in (0, 1)$, and seek to maximize their average discounted payoffs.

Payoff Restrictions. First, we assume that $u_0^{-1}(c_K) < u^{-1}(\underline{b})$, so that agreement is socially efficient, regardless of the agent's type.[7] Second, players are sufficiently concerned for short-run outcomes, in the sense that $\delta < \overline{\delta}$ for some appropriately chosen $\overline{\delta} > 0$. Third, in order to guarantee some conflict of interest amongst the group members we assume that $\underline{b}$ is not too large and that highest benefit $\overline{b}$ is not too close to $\underline{b}$. That is, we impose that $\underline{b} < \eta_1$ and $\overline{b} - \underline{b} > \eta_2$ for some appropriately chosen parameters η_1, $\eta_2 > 0$. The specific parameter thresholds $\overline{\delta}$, η_1, and η_2 are

[5] In fact, we only need this function not to decrease too fast. We could alternatively assume that $K = 2$ or that u is sufficiently convex, but we want to highlight that our results extend beyond the two-type case, and that they do *not* require the group members to be risk averse.

[6] We allow the agent's type to be re-drawn with positive probability at every period solely to ensure that the group's learning process never stops.

[7] Alternatively, we could assume that $F_i\left[u(u_0^{-1}(c_K))\right]$ is sufficiently small for all i. We discuss this further in the paper's concluding remarks.

defined precisely in the appendix.

Timing. The timing is described in Figure 3.

Collective decision making. After the group members' period-t benefits are realized, they collectively choose an offer x^t. The process of selecting an offer comprises two phases: an *organization* phase and a *negotiation phase*. Each phase is modeled as an amendment agenda game (Duggan, 2006, Austen-Smith and Banks, 2005). The agenda game is governed by a "procedure" that specifies the order in which the members can place alternatives on the agenda, and the voting rule they use to select a winning alternative from the agenda.

Formally, let I be the set of finite sequences of proposers $\iota_1, \ldots, \iota_m$, $m \geq n$, that include all the members (possibly with repetitions). A *procedure* consists of a probability distribution λ on I, and a collection $\mathcal{D} \subseteq 2^N \setminus \{\emptyset\}$ of decisive coalitions. We only restrict λ to belong to some (exogenously given) finite subset Λ of $\Delta(I)$; and $\mathcal{D}$ to be monotonic (e.g., $C \in \mathcal{D}$ and $C \subseteq C'$ imply $C' \in \mathcal{D}$) and proper ($C, C' \in \mathcal{D}$ implies $C \cap C' \neq \emptyset$) — e.g., Austen-Smith and Banks (1999). In what follows, we refer to any such a collection $\mathcal{D}$ as a *voting rule.* The family of procedures that satisfy these conditions is denoted by $\mathcal{P}$, with generic element $\wp = (\lambda, \mathcal{D})$.

Figure 4 illustrates the collective decision-making process. We describe each phase in detail.

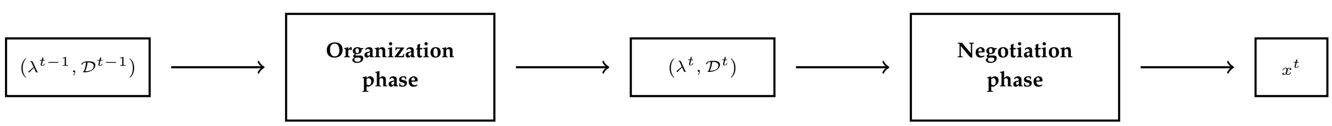

Figure 4 – The group's decision-making process.

Organization Phase. In period t, the group inherits a procedure $\wp^{t-1} = (\lambda^{t-1}, \mathcal{D}^{t-1})$ from the previous period—the procedure $\wp^0$ that prevails at the start of the first period is exogenously given. A finite sequence of proposers $\iota_1, \ldots, \iota_m$, $m \geq n$, is first drawn from I using λ^{t-1}. The proposers can then suggest, in that order, amendments to $\wp^{t-1}$; let $\wp_j$ be the procedure suggested by the j^{th} proposer. The group's final choice is determined by applying an amendment agenda to the resulting set of proposals: $\wp_m$ is pitted against $\wp_{m-1}$, the winner is then pitted against $\wp_{m-2}$, and so on, with the last remaining proposal $\wp_1$ pitted against the status quo, $\wp_0 = \wp^{t-1}$. In each round $j = 1, \ldots, m$ of the agenda, the members vote sequentially (in an arbitrary order) either for $\wp_{m-j+1}$

or for $\wp_{m-j}$. The outcome of each pairwise vote is decided by the ongoing voting rule $\mathcal{D}^{t-1}$.

Following Duggan (2006), we assume that procedural ties—situations in which none of the proposals in a pairwise vote is supported by a decisive coalition—are resolved in favor of the proposal made earlier. As a consequence, $\wp_{m-j}$ beats $\wp_{m-j+1}$ in the jth round if and only if a blocking coalition of members—i.e., a coalition S such that $N \setminus S \notin \mathcal{D}^{t-1}$—votes for $\wp_{m-j}$.

Let $\wp^t = (\lambda^t, \mathcal{D}^t)$ denote the outcome of the organization phase. The group subsequently moves to the negotiation phase.

Negotiation Phase. A new sequence of proposers $\jmath_1, \ldots, \jmath_{m'}$, $m' \geq n$, is drawn from I using λ^t. Then, the same process as in the previous phase repeats, except that proposals are now policies in X, and pairwise votes in the amendment agenda are decided by the newly adopted voting rule $\mathcal{D}^t$. The winner of the agenda, denoted x^t, is the offer submitted by the group to the agent.

Equilibrium. We study (pure-strategy) Markov perfect Bayesian equilibria of this game. Let Δ_{p^0} denote the set of probability distributions in $\Delta(C)$ that can be obtained from p^0 by Bayes updating, i.e.,

$$\Delta_{p^0} \equiv \left\{ p \in \Delta(C) \colon \exists C_0 \in 2^C \setminus \{\emptyset\} \text{ such that } p(c) = \frac{p^0(c)\mathbf{1}_{C_0}(c)}{\sum_{c' \in C_0} p^0(c')}, \forall c \in C \right\} ;$$

for every $p \in \Delta_{p^0}$, we define Δ_p in like manner. Equilibrium belief systems are required to satisfy the usual "no-signaling-what-you-don't-know condition," and to update any $p \in \Delta_{p^0}$ within Δ_p. Henceforth, we will refer to any Markov perfect Bayesian equilibrium that satisfies these restrictions more succinctly as an *equilibrium*.

Discussion. We impose that the group members' benefits are realized independently across periods solely to guarantee equilibrium existence. None of our equilibrium characterization results require this restriction. Alternatively, we could allow for serial correlation of the members' benefits and instead assume an arbitrarily large but finite horizon.

4. Preliminary Results

As a preliminary step, Lemma 1 establishes equilibrium existence, and Lemma 2 characterizes the outcome of any negotiation phase for a given period-t procedure. Lemma 3 then identifies the equilibrium offers generated by the group's collective choice procedure. This allows us to (i) generalize the index $\beta(\mathcal{D})$ that captures a group member's incentive to screen the agent, and (ii) identify a "decisive" member with whose induced negotiating preferences a generic member i might disagree.

Lemma 1. *An equilibrium exists.*

All equilibria of the negotiation phase have a simple structure.[8]

Lemma 2. *Let ϕ be any equilibrium. For any negotiation phase that begins with a procedure $\wp$ and a belief $p \in \Delta_{p^0}$, having support $\{c^1, \ldots, c^m\}$, $m \leq K$, there exist $\overline{x}^1 < \cdots < \overline{x}^m = u_0^{-1}(c^m)$ such that:*[9]

(i) regardless of the members' benefits and the sequence of proposers, the group's offer $x \in X$ must belong to $\{\overline{x}^1, \cdots, \overline{x}^m\}$; and

(ii) the type-c^ℓ agent accepts $\overline{x}^k$ if and only if $c^\ell \leq c^k$.

The group's offer is selected from a finite set of strictly increasing offers—one for each possible agent preference-type in their common belief's support. The largest offer $\overline{x}^m$ is accepted by all agent types, and we call this the *pooling* offer. For each remaining $k = 1, ..., m-1$, offer $\overline{x}^k$ separates agent-types $\{c_1, ..., c_k\}$ from remaining types $\{c_{k+1}, ..., c_{m-1}\}$. The agent's dynamic incentive constraints reflect that the group members' beliefs determine their future preferred offers, as well as the procedures the group uses to select from amongst those offers.

Which of the offers identified in Lemma 2 is chosen? Fix an equilibrium ϕ, and let $V_i^\phi(p; \lambda, \mathcal{D})$ denote group member i's continuation payoff at the start of every period that begins with belief p, and procedure (λ, $\mathcal{D}$). Lemma 2 yields that for any realization of the group members'

[8] Using a different refinement of PBE than Markov perfection, Acharya and Ortner (2017) obtain a similar equilibrium characterization for their single group-member framework. We stress that our proof, unlike theirs, relies on our restriction to small discount factors.

[9] To lighten notation, we omit the dependency of the $\overline{x}_k$'s on the equilibrium ϕ, procedure $\wp$, and belief p.

benefits from an agreement $b = (b_1, \ldots, b_n)$, the negotiation phase induces a collective choice problem amongst the members from the finite set of feasible alternatives $\{\overline{x}^1, \cdots, \overline{x}^m\}$. Member i's preferences over this set are given by the utility function

$$U_i^\phi(\overline{x}^k \mid p, b_i, \lambda, \mathcal{D}) \equiv (1-\delta)\big[b_i - u(\overline{x}^k)\big] \sum_{\ell=1}^{k} p(c^\ell) + \delta \mathbb{E}[V_i^\phi(\tilde{p}; \lambda, \mathcal{D})] \,, \tag{4}$$

for each $k = 1, \ldots, m$, where $\tilde{p}$ is a random variable corresponding to the members' belief at the start of the next period. The core $\mathcal{K}^\phi(p, b, \lambda, \mathcal{D})$ of this collective-choice problem can then be defined in the usual way: it is the subset of alternatives in $\{\overline{x}^1, \cdots, \overline{x}^m\}$ that cannot be defeated in a pairwise vote under the voting rule $\mathcal{D}$ (e.g., Austen-Smith and Banks 2005). In the Appendix, we verify that the members' induced preferences defined in (4) are single-peaked for almost all $b_i \in B$, yielding that the core is non-empty.

Building on this observation, our next lemma has two parts. First, it identifies the outcome of the negotiation phase, i.e., it identifies which offer the group actually makes. Second—for future reference—it identifies a necessary and sufficient condition for group member i to prefer the pooling offer.

Lemma 3. *Let ϕ be any equilibrium, let $p \in \Delta_{p^0}$ and $(\lambda, \mathcal{D}) \in \mathcal{P}$, and let $\overline{x}^1, \ldots, \overline{x}^m$ be defined as in Lemma 2. Then, in any negotiation phase that begins with belief p and procedure $(\lambda, \mathcal{D})$:*

(i) for almost all $b \in B^n$ and all $\iota \in I$, the group's offer when its members' realized benefits are b and the proposal sequence is ι solves

$$\max_x U_{\iota_1}^\phi(x \mid p, b, \lambda, \mathcal{D}), \text{ subject to } x \in \mathcal{K}^\phi(p, b, \lambda, \mathcal{D}) \,; \tag{5}$$

(ii) for every $i \in N$, there exists threshold $\beta_i^\phi(p; \lambda, \mathcal{D}) \in (\underline{b}, \overline{b})$ such that

$$\overline{x}^m = \arg\max_{x \in X} U_i^\phi(x \mid p, b, \lambda, \mathcal{D}) \tag{6}$$

if and only if $b_i > \beta_i^\phi(p; \lambda, \mathcal{D})$.

Recalling that ι_1 identifies the first proposer in the negotiation phase, Lemma 3 states that the group selects the first proposer's preferred offer from amongst the core alternatives of the collective choice problem.

The lemma also establishes an interior threshold on each member i's benefit such that her ideal offer—regardless of whether it lies in the core—is the pooling offer if and only if her benefit realization exceeds that threshold. In our earlier example with two types of agent, the group chooses whether to offer a separating contract, or a pooling contract. With $K \geq 3$ agent types, there are potentially many ways to partially separate the agent. The threshold $\beta_i^{\phi}(\cdot)$ can be interpreted as a heuristic that reflects a member's incentives to pursue any learning about the agent's type, instead of pursuing agreement by making an offer that all types accept: if $b_i > \beta_i^{\phi}(\cdot)$, the group member prefers to make a pooling offer that is always accepted; but single-peakedness implies that for any $b_i < \beta_i^{\phi}(\cdot)$, there exists $m \geq 1$ such that a member strictly prefers a contract that separates types $\{1, \ldots, m\}$ from the highest type $K - m$ types in the support of the members' beliefs.

We now define a dictatorship in our framework.

Definition 1.

(1) Procedure $(\lambda, \mathcal{D})$ is a *formal dictatorship* if the voting rule $\mathcal{D}$ is dictatorial, i.e., if there is some member i such that $\mathcal{D} = \{S \subseteq N : S \ni i\} \equiv \mathcal{D}^i$.

(2) Procedure $(\lambda, \mathcal{D})$ is an *informal dictatorship* if there is some $i \in \bigcap \mathcal{D}$ who proposes first with probability one under λ.

A procedure is a *dictatorship* if either (1) or (2) holds; otherwise, it is a *non-dictatorship*.

The first definition is standard: it identifies a unique individual that belongs to every decisive coalition and it corresponds to our two-period model. Nonetheless, a complete description of a "procedure" in our non-cooperative amendment agenda formulation includes not only a voting rule, but also the order in which proposers are recognized. Correspondingly, Lemma 3 suggests another way that procedures can concentrate authority. The lemma states that the first member recognized in the negotiation phase secures her preferred offer from amongst the alternatives in the core. Moreover, any veto player's preferred offer lies in the core. So, a procedure that gives a veto player first-proposer rights ensures her most-preferred offer, even if the voting rule does not explicitly make her a dictator.

While the specific definition of an informal dictatorship is closely tied to the details of our amendment agenda game, it more broadly captures real-world decision-making contexts in which veto power is jointly vested with agenda-setting power, or where formal rules grant outsized privileges to some individuals. For example, Ali, Bernheim and Fan (2019) show that predictability about the order of future proposers in the Baron-Ferejohn legislative bargaining framework ensures that the first proposer is tantamount to a dictator, while Bernheim, Rangel and Rayo (2006) obtain that the last proposer has pre-eminent decision-making power in the context of an evolving default option.

5. Individual versus Group Preferences Over Offers

Lemma 3 identifies a cut-off benefit $\beta_i^{\phi}(p;\wp)$ such that group member i prefers the pooling offer if and only if her realized benefit b_i exceeds $\beta_i^{\phi}(p;\wp)$. This cutoff can be loosely interpreted as reflecting a member i's incentive to learn the agent's type. Our earlier Result 1 from our two-period example highlighted that a member's benefit from learning the agent's type was higher under any rule that did not make her a dictator, relative to a rule that made her a dictator. We show that this result extends.

Proposition 1. *Let $\wp$ be any procedure in which member i is not a dictator, and let $\wp^i$ be any dictatorship in which i is a dictator. Then for any equilibria ϕ and φ, we have*

$$\beta_i^{\phi}(p,\wp^i) < \beta_i^{\varphi}(p,\wp)\,,$$

for all non-degenerate $p \in \Delta_{p^0}$.

Note that the comparison is strong, in the sense that it holds across *any* equilibria under either protocol. The intuition for i's benefit of learning about the agent under a non-dictatorship

$\varphi \neq \varphi^i$ is the same as in our two-period example: learning about the agent's preferences reduces the scope for conflict between the group members, and therefore insures i against the risks from not being decisive over future offers.

In our infinite horizon setting, however, there may also be *costs* of screening. To see why, suppose that the members' beliefs place positive probability on K types of agent. For any period-t procedure, suppose the group's period-t offer separates types $\{c_1, \ldots, c_{K-1}\}$ from $\{c_K\}$. Routine arguments establish that this offer, $\overline{x}^{K-1}$, is determined by type c_{K-1}'s binding incentive constraint:

$$(1-\delta)\left[u_0(\overline{x}^{K-1})-c_{K-1}\right]+\delta\begin{bmatrix}\text{type } c_{K-1}\text{'s expected}\\ \text{continuation payoff}\\ \text{from accepting } \overline{x}^2\end{bmatrix}=(1-\delta)\times 0+\delta\begin{bmatrix}\text{type } c_{K-1}\text{'s expected}\\ \text{continuation payoff}\\ \text{from rejecting } \overline{x}^{K-1}\end{bmatrix},$$

so that her period-t rent is

$$(1-\delta)\left[u_0(\overline{x}^{K-1})-c_{K-1}\right]=\delta\begin{bmatrix}\text{Expected difference in type } c_{K-1}\text{'s}\\ \text{continuation payoffs from}\\ \text{rejecting and accepting } \overline{x}^{K-1}\end{bmatrix}. \tag{7}$$

Recognize that any shock to the agent's type between periods t and $t+1$ has no bearing on the type c_{t+1} agent's period-t incentive constraint. The reason is that the shock resets the group members' common period-$t+1$ belief to p^0, and the period-t procedure persists at period $t+1$. Thus, the agent's period-$t+1$ continuation value after a shock at the end of the previous period is independent of her acceptance decision. It follows that the incentive constraint is:

$$(1-\delta)\left[u_0(\overline{x}^{K-1})-c_{K-1}\right]=\delta(1-\alpha)\begin{bmatrix}\text{Expected difference in type } c_{K-1}\text{'s continuation}\\ \text{payoffs from rejecting and accepting } \overline{x}^{K-1}\\ \text{conditional on no shock between } t \text{ and } t+1\end{bmatrix}.$$

The bracketed expression on the RHS accounts for both direct and indirect effects of the agent's period-t decision that can impact her offers at all future periods. To see why, notice that the agent's period-t acceptance decision changes the group's belief about her resolve and thus directly impacts group members' induced preferences over future offers for any fixed collective choice procedure. However, the period-t acceptance decision may also impact future offers indirectly by changing the collective choice procedures the group adopts at any future date.

Despite this complexity, we can verify that across *all* equilibria *any* variation in the bracketed expression on the RHS across procedures is $O(\delta)$. To see why, observe that

(1) if type c_{K-1} *accepts* $\overline{x}^{K-1}$, then conditional on no shock between t and $t+1$, hers is the highest possible type in the support of the group members' beliefs in $t+1$. Standard arguments yield that she obtains zero rent. This observation is invariant across procedures.

(2) If type c_{K-1} *rejects* $\overline{x}^{K-1}$, then conditional on no shock between t and $t+1$, the group members assign probability one to c_K, and offer $u_0^{-1}(c_K)$ in $t+1$. This observation, again, is invariant across procedures, since the group members unanimously prefer this offer.

Hence, any wedge in the type c_{K-1} agent's continuation value from accepting versus rejecting a period-t partially separating offer under different procedures happens *no sooner* than period $t+2$. Any such wedge—and therefore any incremental cost to the group across procedures—is scaled by δ^2 in the agent's period-t incentive constraints. The members' learning benefit is instead scaled by δ, since it accrues immediately from period $t+1$. We conclude that so long as δ is not too large, the incremental costs of learning are second-order to the benefits of learning.

6. Evolution of Collective Choice Procedures

In our two-period setting, Result 2 unearthed a positive probability that the groups moves from the status quo majority rule to a concentration of power either in the hands of an oligarchy or a dictator. In that example, however, the members only chose their procedure once—at the start of the first period. This raises an obvious question: what can be said about the long-run evolution of decision-making when the group can amend the status quo procedure in every period, and how does the answer depend on the initial rule at the start of their interaction with the agent?

Proposition 2. *Every equilibrium sequence of procedures* $\{(\lambda^t, \mathcal{D}^t)\}$ *converges to a dictatorship almost surely.*

To illustrate the theorem, we can extend our earlier example with five members, in which the ongoing procedure at the start of period t is simple majority rule, and the period-t belief is p^0 (e.g., a shock to the agent's type resets beliefs).

Fix an equilibrium, and let E denote the event "*the sequence of procedures starting in period* t *does not converge to a dictatorship.*" Suppose, contrary to Theorem 2, that $\Pr(E) > 0$, where probabilities are calculated according to the equilibrium strategies, and the distributions of member benefits and shocks on the agent's type. Let $\mathcal{P}_E$ denote the set of procedures that the group uses in event E, and $P(\lambda, \mathcal{D})$ denote a lower bound (to be determined) on the probability that the group adopts a dictatorship conditional on the arrival of a shock to the agent's type, given inherited procedure $(\lambda, \mathcal{D})$. Finally, let $\underline{P} \equiv \min\{P(\lambda, \mathcal{D}) : (\lambda, \mathcal{D}) \in \mathcal{P}_E\}$.[10]

To verify that $\underline{P} > 0$, let $\overline{\beta}_i$ denote group member i's smallest possible pooling threshold at belief p_0 in the event E, i.e.,

$$\overline{\beta}_i \equiv \min\left\{\beta_i^{\phi}(p^0; \lambda, \mathcal{D})\colon (\lambda, \mathcal{D}) \text{ is a non-dictatorship}\right\}, \tag{8}$$

where $\beta_i^{\phi}(p^0; \lambda, \mathcal{D})$ is defined in Lemma 3, and is the analogue of $\overline{\beta}$ defined in (3) of our earlier example. Proposition 1 yields that $\overline{\beta}_i > \underline{\beta}_i$, where $\underline{\beta}_i$ is i's pooling threshold when she is a dictator. Let F_1 denote the event described in our earlier example and highlighted in earlier Figure 2, in which

(i) b_1^t and b_2^t lie in a neighborhood of $\underline{b}$,

(ii) b_4^t and b_5^t lie in a neighborhood of $\overline{b}$, and

(iii) b_3^t lies in $(\underline{\beta}_3, \overline{\beta}_3)$.

Figure 5 replicates Figure 2, but extends the group members' induced preferences to account for $K \geq 3$ agent-types. Nonetheless, all the intuition from that example extends: members 4

[10] $\underline{P}$ is well-defined because $\mathcal{P}_E$ is finite.

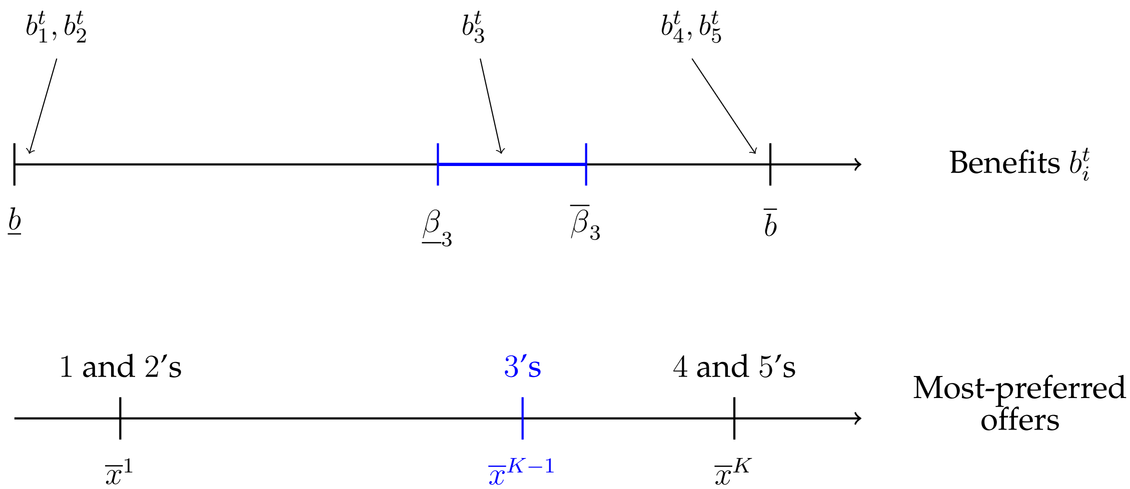

Figure 5 – The realization of the group members' period-t benefits in event F_1.

and 5 prioritize short-term agreement and are therefore willing to make member 3 a dictator—a procedure that 3 clearly welcomes. Nonetheless, we pointed out in our earlier example that 3's dictatorship is not the sole procedure that commits the group to a period-t pooling offer: Lemma 3 yields that the pooling offer is assured if and only if it is the first proposer's preference from amongst alternatives in the core. In fact, there are three classes of procedures $\wp^t$ that satisfy this requirement:

Class A: either member 3, 4 or 5 is a dictator, i.e., $\mathcal{D}^t = \mathcal{D}^i$ for $i \in \{3, 4, 5\}$,

Class B: members 4 and 5 are oligarchs, i.e., $\mathcal{D}^t = \big\{S \subseteq N\colon S \supseteq \{4, 5\}\big\}$,

Class C: members 4 and 5 are only blocking, i.e., $\{1, 2, 3\}, \{4, 5\} \notin \mathcal{D}^t$, and λ^t ensures that the first proposer is drawn from $\{4, 5\}$ with probability one, i.e., $\iota_1 \in \{4, 5\}$.

Note that Class C procedures were absent from our two-period example because we assumed that the members could only choose the voting rule. If the members adopt a procedure from class A, we set $P(\lambda^{t-1}, \mathcal{D}^{t-1}) = \Pr(F_1) > 0$.

Suppose, instead, the period-t organization phase yields a procedure from either classes B or C. Recognizing the inevitability of a period-t pooling offer, members 1 and 2 might prefer to offer members 4 or 5 a procedure that establishes this commitment without reverting immediately to a full-blown dictatorship. Suppose, for concreteness, that the members adopt a class-B

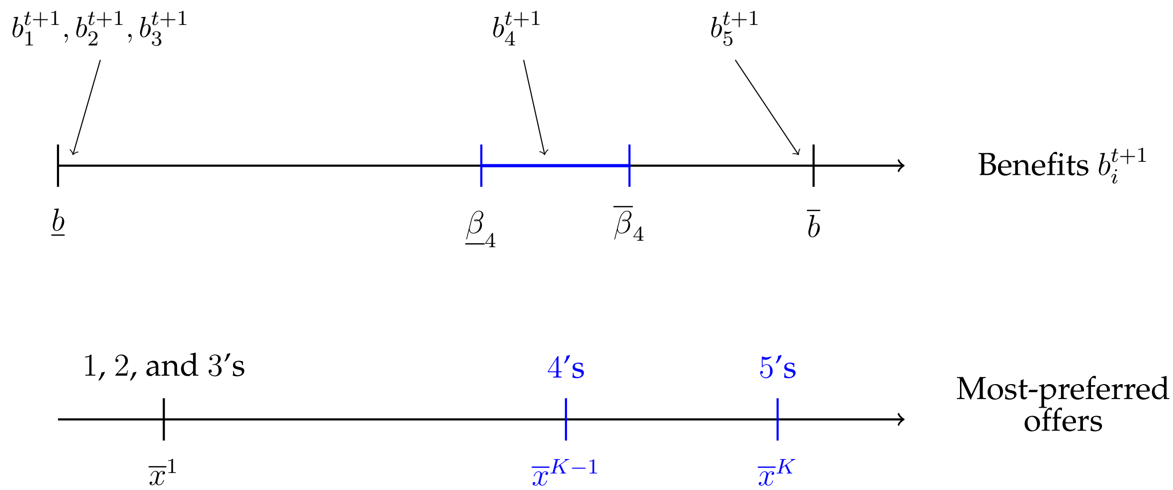

Figure 6 – The realization of member' benefits in period $t+1$ in event F_2.

procedure in period-t's organization phase, and which therefore persists to period $t+1$. Since the period-t negotiation phase yields the pooling offer, the members hold belief p^0 at period $t+1$ regardless of whether there is a shock to the agent's type.

Define the event F_2—illustrated in Figure 6—to be the conjunction of event F_1 in period t, followed by the following realization of benefits in period $t+1$:

(i) b_1^{t+1}, b_2^{t+1} and b_3^{t+1} lie in a neighborhood of $\underline{b}$,

(ii) b_5^{t+1} lies in a neighborhood of $\overline{b}$, and

(iii) b_4^{t+1} lies in $(\underline{\beta}_4, \overline{\beta}_4)$.

By a similar logic to the previous case, oligarch members 4 and 5 are assured of a procedure that guarantees a period-$t+1$ pooling offer. Now, however, any such procedure *must* make either 4 or 5 a dictator. We can therefore set $P(\lambda^t, \mathcal{D}^t) = \Pr(F_2) > 0$. Notice that the final possible class C procedure the members could adopt at period t follows a similar logic: while 4 and 5 are not oligarchs, whichever of these members is recognized in the period-$t+1$ organization phase to propose first can propose her ideal rule and then vote for it. We can again set $P(\lambda^t, \mathcal{D}^t) = \Pr(F_2) > 0$.

Since there are infinitely many shocks to the agent's type in event E, and each shock is followed by the adoption of dictatorship with probability at least $\underline{P} = \min\{P(\lambda, \mathcal{D}) : (\lambda, \mathcal{D}) \in \mathcal{P}_E\} > 0$, we obtain a contradiction that $\Pr(E) = 0$, and thus obtain our result.

While our example supposed that the group initially operates under a simple majority rule, Proposition 2 verifies that our argument also applies to any other inherited rule. To make this point concrete, notice that if the group inherits a unanimous rule we can amend the event F_1 in Figure 2 to the positive probability event in which *all* the members' benefits except for member 3's are in a neighborhood of $\bar{b}$. By the same logic as our earlier analysis under majority rule, the high-benefit members prioritize the agent's agreement. Since $b_3 \in (\underline{\beta}_3, \overline{\beta}_3)$, making member 3 a dictator switches her induced preference for a partially separating offer to a pooling offer. Since member 3 is strictly better off when made a dictator, and the remaining members are strictly better off from the pooling offer than any other, the only outcome of the organization phase is *some* procedure that ensures the pooling offer at the negotiation phase. But since the organization phase operates under unanimity rule, the *only* shift in procedures that commits the group to the pooling offer is 3's dictatorship. We therefore obtain the complete concentration of decision-authority in member 3, which persists through all future periods. This example highlights that reverting to a dictatorship can be Pareto-improving for the group.

7. Final Remarks

We introduce a framework that sheds light on the dynamics of negotiations when one of the negotiating parties is a group. We ask: how do group members negotiate differently when they bargain collectively, as opposed to when they negotiate alone? How do within-group conflicts shape a group's external negotiating position? Conversely: how do external negotiations shape the internal dynamics of group decision-making?

We find that individuals in a group setting tend to favor more aggressive negotiating strategies solely by virtue of the group setting. This reflects conflicts between group members that are shaped by their uncertainty about the preferences of the external negotiating party. This intra-group conflict is a novel channel that may heighten the risk that negotiations fail—even when every group member would prefer an agreement to no agreement. This risk may spur groups to consolidate authority into oligarchic or even dictatorial decision-making structures.

We see a number of interesting questions for future research. The most immediate one concerns how information increases or instead decreases conflict between group members. In our framework, better information about the agent's cost reduces disagreement between the group members. This is plausible in many negotiations contexts—for example, a union can calibrate its wage demands more effectively with better information about management's preferences. However, there may be other settings in which more information increases conflict between the members.

To see how this phenomenon could arise in an extension of our model, return to our leading example but instead of presuming that $c_H < \underline{b}$, suppose instead that $c_L < \underline{b} < c_H < \bar{b}$. The assumption that $\underline{b} < c_H$ implies that for some realizations of the group members' benefits, an agreement with the high-cost agent is not efficient. Let $\tau^p(b, \mathcal{D})$ denote the probability that the period-2 offer is c_H when the members assign probability p that the agent's type is c_L. In the Appendix, we generalize expression (1) by showing that a group member i's net incentive to learn the agent's type under non-dictatorship is:

$$\beta(\mathcal{D}) - \beta(\mathcal{D}^i) = \delta(1-p) \int_b \left[\tau^p(b, \mathcal{D}^i) - \tau^p(b, \mathcal{D}) + \tau^0(b, \mathcal{D}) - \tau^0(b, \mathcal{D}^i)\right](b_i - b^*)\, dF(b). \tag{9}$$

Our benchmark with $c_H < \underline{b}$ corresponds to $\tau^0(b, \mathcal{D}) = \tau^0(b, \mathcal{D}^i) = 1$. So long as the probability of a benefits realization for which the decisive member's benefit is below c_H isn't too large, (9) is strictly positive, yielding that group member i's induced preferences over offers are more aggressive in the group setting.

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Online Appendix

A. Proofs of Lemmas 1-3

We set $\overline{\delta} \equiv \min\{\overline{\delta}_0, \overline{\delta}_1, \overline{\delta}_2, \overline{\delta}_3, \overline{\delta}_4, \overline{\delta}_5, \overline{\delta}_6\}$, where the $\overline{\delta}_\ell$'s are upper bounds for the discount factor, defined below. We begin by establishing some notation and preliminary results. For each $k \in \{1, \ldots, K\}$, let

$$y_k^-(\delta) \equiv u_0^{-1}\left(c^k - \frac{\delta(1-\alpha)}{1-\delta} u_0(\hat{x}_0)\right)$$

and

$$y_k^+(\delta) \equiv u_0^{-1}\left(c^k + \frac{\delta(1-\alpha)}{1-\delta} u_0(\hat{x}_0)\right) .$$

Moreover, for every $p \in \Delta_{p^0}$, and each $c_k \in \operatorname{supp}(p)$, let $S_k^- \equiv \{c_1, \ldots, c_k\} \cap \operatorname{supp}(p)$ and $S_k^+ \equiv \{c_{k+1}, \ldots, c_K\} \cap \operatorname{supp}(p)$; let $p^{k-} \in \Delta_p$ be defined by

$$p^{k-}(c) \equiv \begin{cases} p(c)/p(S_k^-) & \text{if } c \in S_k^- , \\ 0 & \text{otherwise;} \end{cases}$$

let $p^{k+} \in \Delta_p$ be defined by

$$p^{k+}(c) \equiv \begin{cases} p(c)/p(S_k^+) & \text{if } c \in S_k^+ , \\ 0 & \text{otherwise,} \end{cases}$$

where $p(S_k^-) \equiv \sum_{c \in S_k^-} p(c)$ and $p(S_k^+) \equiv \sum_{c \in S_k^+} p(c)$. For every nondegenerate $p \in \Delta_{p^0}$, whose support is denoted $\{c^1, \ldots, c^m\}$, let $\beta_p \colon \{1, \ldots, m-1\} \to \mathbb{R}$ be defined by

$$\beta_p(k) \equiv u(x^{k+1}) + \left[u(x^{k+1}) - u(x^k)\right] \frac{\sum_{\ell=1}^k p(c^\ell)}{p(c^{k+1})} ,$$

for all $k \in \{1, \ldots, m-1\}$, where $x^\ell \equiv u_0^{-1}(c^\ell)$. This is the cutoff value of b_i that leaves each group member i indifferent between separating types at c_k and at c_{k+1}. All we need to ensure some conflict of interest among the group members (for low δ) is that $\underline{b} < \beta_p(k) < \overline{b}$, for some nondegenerate p and k. Without loss of generality, we will assume throughout that $\underline{b} < \min_p \beta_p(1) \equiv \underline{\beta}$

and $\overline{\beta} \equiv \max_p \beta_p(K-1) < \bar{b}$, where the minimum and the maximum are calculated over the nondegenerate type distributions in Δ_{p^0}. As β_p is strictly increasing function (see Lemma A1 below), we obtain these inequalities by setting $\eta_1 \equiv \underline{\beta}$ and $\eta_2 \equiv \overline{\beta} - \underline{\beta}$, and then imposing that $\underline{b} < \eta_1$ and $\bar{b} - \underline{b} > \eta_2$, as we do in the main text.

Finally, we say that a function $f\colon \{0,1,\ldots,K\} \to \mathbb{R}$ is *quasi-single-peaked* if: (i) $\left|\arg\max_k f(k)\right| \leq 2$; (ii) if $k, \ell \in \arg\max_k f(k)$, then $\ell \in \{k-1, k, k+1\}$; and (iii) $\ell_1 < \ell_2 \leq \min \arg\max_k f(k)$ implies $f(\ell_1) < f(\ell_2)$, and $\max \arg\max_k f(k) \leq \ell_2 < \ell_1$ also implies $f(\ell_1) < f(\ell_2)$. In words, f is quasi-single-peaked if it has a single maximizer and is single-peaked; or if it has two maximizers, which must be adjacent, and it is increasing "below" the maximizers and decreasing "above" them.

Lemma A1. For every nondegenerate $p \in \Delta_{p^0}$, with support $\{c^1, \ldots, c^m\}$, the function β_p is strictly increasing on $\{1, \ldots, m-1\}$.

Proof. Take any nondegenerate $p \in \Delta_{p^0}$, and let $\underline{k} \equiv \min \operatorname{supp}(p)$. For each $k = 1, \ldots, m-2$, we have

$$\begin{aligned}\beta_p(k+1) - \beta_p(k) &= \left[u(x^{k+2}) - u(x^{k+1})\right]\left(1 + \frac{\sum_{\ell=1}^{k+1} p(c^\ell)}{p(c^{k+2})}\right) - \left[u(x^{k+1}) - u(x^k)\right]\frac{\sum_{\ell=1}^{k} p(c^\ell)}{p(c^{k+1})} \\ &= \left[u(x^{k+2}) - u(x^{k+1})\right]\left(1 + \frac{\sum_{\ell=\underline{k}}^{k-\underline{k}+2} p^0(c_\ell)}{p^0(c_{k-\underline{k}+3})}\right) - \left[u(x^{k+1}) - u(x^k)\right]\frac{\sum_{\ell=1}^{k-\underline{k}+1} p^0(c_\ell)}{p^0(c_{k-\underline{k}+2})},\end{aligned}$$

so that β_p is strictly increasing if

$$\frac{\sum_{\ell=1}^{k-\underline{k}+1} p^0(c_\ell)/p^0(c_{k-\underline{k}+2})}{1 + [\sum_{\ell=\underline{k}}^{k-\underline{k}+2} p^0(c_\ell)/p^0(c_{k-\underline{k}+3})]} < \frac{u(x_{k+2}) - u(x_{k+1})}{u(x_{k+1}) - u(x_k)}\ .$$

By convexity of u, the ratio on the right-hand side is greater than or equal to one; and by the local monotone hazard rate property, the ratio on the left-hand side is strictly less than one. □

Lemma A2. There is $\bar{\delta}_0 > 0$ such that the following holds for all $\delta < \bar{\delta}_0$. Let $p \in \Delta_{p^0}$ be a belief whose support is denoted by $\{c^1, \ldots, c^m\}$, $1 \leq m \leq K$. Then, for every $i \in N$, $b_i \in B$, mapping

$W_i\colon \Delta_p \to \big[\underline{b} - u(\hat{x}_0), \overline{b}\big]$, $W_{i,0} \in \big[\underline{b} - u(\hat{x}_0), \overline{b}\big]$, and $(\overline{x}_1, \ldots, \overline{x}_m) \in X^m$ such that $\overline{x}_k \in \big[y_k^-(\delta), y_k^+(\delta)\big]$ for all $k = 1 \ldots, m$, the mapping $U_i(\cdot \mid b_i)\colon \{0, 1, \ldots, m\} \to \mathbb{R}$, defined by

$$
\begin{aligned}
U_i(0 \mid b_i) &\equiv \delta\big[(1-\alpha)W_i(p) + \alpha W_{i,0}\big] \, , \\
U_i(k \mid b_i) &\equiv (1-\delta)\big[b_i - u(\overline{x}_k)\big]p(S_k^-) + \delta\big[W_i(p^{k-})p(S_k^-) + W_i(p^{k+})p(S_k^+)\big] \\
&\qquad + \delta\alpha W_{i,0}, \, k \neq 0, m \, , \\
U_i(m \mid b_i) &\equiv (1-\delta)\big[b_i - u(x_m)\big] + \delta\big[(1-\alpha)W_i(p) + \alpha W_{i,0}\big] \, ,
\end{aligned}
$$

is quasi-single-peaked. Moreover, it is single-peaked for almost all $b_i \in B$.

Proof. Fix $p \in \Delta_{p^0}$. Consider first the mapping $U^p\colon \{0, 1, \ldots, m\} \times B \to \mathbb{R}$, defined by $U^p(0 \mid b) \equiv 0$, and $U^p(k \mid b) \equiv \big[b - u(x_k)\big]p(S_k^-)$, for all $k \in \{1, \ldots, m\}$ and $b \in B$. By definition, for any $k \in \{1, \ldots, m-1\}$, we have $U^p(k \mid b) \leq U^p(k+1 \mid b)$ if and only if $b \geq \beta_p(k)$ (and $U^p(k \mid b) > U^p(0 \mid b)$). As $\beta_p(k)$ is increasing in k (Lemma A1), the mapping $U^p(\cdot \mid b)$ is quasi-single-peaked, for all $b \in B$; and it is single-peaked for all $b \notin \big\{\beta_p(1), \ldots, \beta_p(m)\big\}$.

Now, let

$$
\beta_k^-(\delta) \equiv p(c_{k+1})^{-1}\left[u\big(y_{k+1}^-(\delta)\big)p(S_{k+1}^-) - u\big(y_k^+(\delta)\big)p(S_k^-) - \frac{\delta(1-\alpha)}{1-\delta}u(\hat{x}_0)\right]
$$

and

$$
\beta_k^+(\delta) \equiv p(c_{k+1})^{-1}\left[u\big(y_{k+1}^+(\delta)\big)p(S_{k+1}^-) - u\big(y_k^-(\delta)\big)p(S_k^-) + \frac{\delta(1-\alpha)}{1-\delta}u(\hat{x}_0)\right] ;
$$

and let $\overline{\beta}_k^i(\delta)$ be implicitly defined by $U_i\big(k \mid \overline{\beta}_k^i(\delta)\big) \equiv U_i\big(k+1 \mid \overline{\beta}_k^i(\delta)\big)$ for each $k \in \{1, \ldots, m-1\}$ — if $U_i(k \mid b_i) < U_i(k+1 \mid b_i)$ for all $b_i \in B$, then we set $\overline{\beta}_k^i(\delta) \equiv \underline{b}$; and if $U_i(k \mid b_i) > U_i(k+1 \mid b_i)$ for all $b_i \in B$, then $\overline{\beta}_k^i(\delta) \equiv \bar{b}$. By construction, for each k, $\overline{\beta}_k^i(\delta) \in \big[\beta_k^-(\delta), \beta_k^+(\delta)\big]$ and $\beta_k^-(\delta), \beta_k^+(\delta) \to \beta_p(k)$ as $\delta \to 0$. Hence, there exists $\overline{\delta}_p^i > 0$ such that $\overline{\beta}_k^i(\delta)$ is increasing in k and belongs to $(\underline{b}, \bar{b})$ whenever $\delta < \overline{\delta}_p^i$. This in turn implies that the mapping $U_i(\cdot \mid b_i)$ is quasi-single-peaked for all $b_i \in B$, whenever $\delta < \overline{\delta}_p^i$. Moreover, it is single-peaked for almost all $b_i \in B$, since indifference only occurs if b_i is equal to one of the $\overline{\beta}_k(\delta)$'s. As Δ_{p^0} and N are finite sets, we obtain the lemma by setting $\overline{\delta}_0 \equiv \min_{p\in\Delta_{p^0}, i\in N} \overline{\delta}_p^i$. □

For any set of alternatives $\{0, 1, \ldots, m\}$, $1 \leq m \leq K$, and any profile of utility functions $f = (f_1, \ldots, f_n)$ on $\{0, 1, \ldots, m\}$, we denote by $\mathrm{Core}(m, f)$ the core of the corresponding collective-choice problem. Given a sequence of proposers ι, let $\mathcal{A}(m, f, \iota)$ denote the (one-shot) amendment-agenda game in which the set of alternatives is $\{0, 1, \ldots, m\}$, alternative 0 is the status quo, and the group members' payoffs are given by f. The following lemma is a variant on Duggan's (2006) Theorem 6.

Lemma A3. Let $f = (f_1, \ldots, f_n)$ be a profile of single-peaked functions on $\{0, 1, \ldots, m\}$, $1 \leq m \leq K$. Then, any Markovian equilibrium outcome of the amendment-agenda game $\mathcal{A}(m, f, \iota)$ is a maximizer of f_{ι_1} on $\mathrm{Core}(m, f)$, for every realization of ι_1.

Proof. Consider any amendment-agenda game $\mathcal{A}(m, f, \iota)$. From the single-peakedness of the f_i's, $\mathrm{Core}(m, f)$ is nonempty, and all the alternatives in $\mathrm{Core}(m, f)$ must be adjacent. It follows that each group member i has a unique ideal alternative in $\mathrm{Core}(m, f)$, denoted $\hat{k}_i$. Suppose towards a contradiction that there is an equilibrium in which the chosen alternative, say k^*, is not $\hat{k}_{\iota_1}$. Then, the first proposer prefers k^* to $\hat{k}_{\iota_1}$; otherwise, she could profitably deviate from her equilibrium strategy by proposing $\hat{k}_{\iota_1}$, which would then be implemented — recall that procedural ties are resolved in favor of the alternatives proposed earlier. This in turn implies that k^* lies outside $\mathrm{Core}(m, f)$. There must therefore exist an alternative $k \in \{0, 1, \ldots, m\}$ and a decisive coalition S such that all members of S prefer k to k^*. Recall that all group members have an opportunity to propose. None of the members of S can propose before k^* is included in the agenda (on the equilibrium path); otherwise she could profitably deviate from the equilibrium by proposing k as soon as it is her turn to propose. Now consider the proposal by a member of S, say j, when k^* is the provisionally selected alternative. As the equilibrium is Markovian, she and all the other members of S know that k^* will be implemented if k^* remains the provisionally selected alternative after this round — at the start of any new round, the number of remaining rounds and the provisionally selected alternative are the only payoff-relevant variables. All the members of S would therefore be strictly better off accepting proposal k, and therefore, proposing k is a profitable deviation for proposer j; a contradiction. $\square$

A.1. Proof of Lemma 1

Let $\overline{\delta}_0$ be defined as in Lemma A2. Observe that there exists $\overline{\delta}_1 > 0$ such that

$$\frac{2\delta(1-\alpha)}{1-\delta}u_0(\hat{x}_0) \leq \min_{k\in\{1,\ldots,K-1\}}(c_{k+1}-c_k)\,,$$

for all $\delta < \overline{\delta}_1$. The upper bound $\overline{\delta}$ is chosen to be smaller than or equal to $\min\{\overline{\delta}_0, \overline{\delta}_1\}$, so that $\delta < \min\{\overline{\delta}_0, \overline{\delta}_1\}$.

Let $\mathfrak{D}$ be the set of monotonic, proper voting rules $\mathcal{D}$, and let $L \equiv |\Lambda \times \mathfrak{D}| < \infty$. We can thus label the set of feasible procedures $\{(\lambda_1, \mathcal{D}_1), \ldots, (\lambda_L, \mathcal{D}_L)\}$. Let $\mathcal{V} \equiv \left[0, u_0(\hat{x}_0) - c_1\right]^L \times \left[0, u_0(\hat{x}_0) - c_K\right]^L \times \left[\underline{b} - u(\hat{x}_0), \overline{b}\right]^{nL}$. In what follows, a typical element of $\mathcal{V}$ will be denoted $(\nu_0, \nu_1, \ldots, \nu_n)$, where $\nu_0 = (\nu_{0,1}, \ldots, \nu_{0,K})$ with $\nu_{0,k} \in \left[0, u_0(\hat{x}_0) - c_k\right]^L$, for each $k = 1, \ldots, K$; and $\nu_i \in \left[\underline{b} - u(\hat{x}_0), \overline{b}\right]^L$, for each $i \in N$. We will think of $\nu_{0,k}$ as the L-dimensional vector whose ℓth component, $\nu_{0,k,\ell}$, describes the continuation payoff of the type-c_k agent at the start of period that begins with procedure $(\lambda_\ell, \mathcal{D}_\ell)$ and belief p^0. The vector ν_i and its components, the $\nu_{i,\ell}$'s, will be interpreted in like manner.

Fix a degenerate belief p that assigns probability one to some type c_k, $k = 1, \ldots, K$. For each procedure $(\lambda_\ell, \mathcal{D}_\ell)$, we define the game $\mathcal{G}^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ among the group members as follows. Each period $t = 1, 2, \ldots$ begins with an ongoing procedure, say $(\lambda_l, \mathcal{D}_l)$. Then, events unfold as follows (if the game has not ended yet):

(1) The group members' benefit profile b^t is drawn according to the $F_i' s$, and the sequence of proposers ι^t according to λ_k.

(2) The organizational phase takes place as in the main game. Let $(\lambda_{l'}, \mathcal{D}_{l'})$ denote the resulting procedure.

(3) A shock on the agent's type occurs with probability α.

(4) If a shock occurred in the previous stage, then the game ends, and each group member i receives a payoff of $(1-\delta)\big[b_i^t - u(x_k)\big] + \delta\nu_{i,l'}$; otherwise, she receives a stage-payoff of $(1-\delta)\big[b_i^t - u(x_k)\big]$, and the game transitions to period $t+1$, which begins with procedure $(\lambda_{l'}, \mathcal{D}_{l'})$.

The (exogenously given) initial procedure at the start of period 1 is $(\lambda_\ell, \mathcal{D}_\ell)$. All group members seek to maximize their average discounted payoffs. This is a noisy stochastic game, in which action sets are finite, the noise component of the state (i.e., the group members' benefits) is generated by the continuous distributions $F_1, \ldots, F_n$ in every period, and the standard component (i.e., all the other payoff-relevant parameters) belongs to a finite set. It therefore admits a (possibly mixed) stationary Markov perfect equilibrium (Duggan, 2012). Let $V_i^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ denote group member i's equilibrium payoff. For future reference, we also define $V_{0,k}^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ as the corresponding expected payoff of the passive type-c_k agent.

Now fix $m = 2, \ldots, K$. Suppose that for every $p' \in \Delta_{p^0}$ with $\big|\mathrm{supp}(p')\big| \leq m-1$, we have defined a game $\mathcal{G}^{p'}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$, $\ell = 1, \ldots, L$, and corresponding continuation payoffs $V_i^{p'}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ and $V_{0,k}^{p'}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$, as above. Consider a belief $p \in \Delta_{p^0}$ such that $\big|\mathrm{supp}(p)\big| = m$. For (and only for) expositional ease, suppose that $\mathrm{supp}(p) = \{c_1, \ldots, c_m\}$. Observe that for every $k = 1, \ldots, m-1$, $\big|\mathrm{supp}(p^{k-})\big| \leq m-1$ and $\big|\mathrm{supp}(p^{k+})\big| \leq m-1$ and therefore, $V_i^{p^{k-}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$, $V_{0,k'}^{p^{k-}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$, $V_i^{p^{k+}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$, and $V_{0,k'}^{p^{k+}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$, are well-defined for all i, k', and ℓ. This allows us to (implicitly) define the policy $\chi_k(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ as the unique solution x to

$$(1-\delta)\big[u_0(x) - c_k\big]p(S_k^-) + \delta(1-\alpha)V_{0,k}^{p^{k-}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$$

$$= \delta(1-\alpha)V_{0,k}^{p^{k+}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)\,,$$

for each $k \leq m-1$, and $\chi_m(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n) \equiv x_m$. Observe that $\overline{x}_k$, $k < m$, is defined in such a way that the type-c_k is indifferent between revealing that her type belongs to S_k^- and pretending that her type belongs to S_k^+, given the continuation values obtained for the "continuation games" above.

Next, for each procedure $(\lambda_\ell, \mathcal{D}_\ell)$, we define the game $\mathcal{G}^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ among the group members as follows. Each period $t = 1, 2, \ldots$ begins with an ongoing procedure, say $(\lambda_l, \mathcal{D}_l)$. Then, events unfold as follows (if the game has not ended yet):

(1) The group members' benefit profile b^t is drawn according to the $F_i's$, and the sequence of proposers ι^t according to λ_k.

(2) The organizational phase takes place as in the main game. Let $(\lambda_{l'}, \mathcal{D}_{l'})$ denote the resulting procedure.

(3) The negotiation phase takes place as in the main game, but the group members are constrained to choose offers from the set $\{\chi_k(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n)\}_{k=1,\ldots,m}$. Let $\chi_{k'}(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n)$ denote the resulting offer to the agent.

(4) A shock on the agent's type occurs with probability α.

(5) If a shock occurred in the previous stage, then the game ends, and each group member i receives a payoff of $(1-\delta)\big[b_i^t - u\big(\chi_{k'}(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n)\big)\big] + \delta \nu_{i,l'}$; if a shock did not occur and $k' < m$, then the game ends, and she receives a payoff of $(1-\delta)\big[b_i^t - u\big(\chi_{k'}(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n)\big)\big] + \delta\big[p(S_{k'}^-)V_i^{p^{k'-}}(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n) + p(S_{k'}^+)V_i^{p^{k'+}}(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n)\big]$; otherwise, she receives a stage-payoff of $(1-\delta)\big[b_i^t - u\big(\chi_{k'}(\lambda_{l'}, \mathcal{D}_{l'} \mid \nu_0, \nu_1, \ldots, \nu_n)\big)\big]$, and the game transitions to period $t+1$, which begins with procedure $(\lambda_{l'}, \mathcal{D}_{l'})$.

The (exogenously given) initial procedure at the start of period 1 is $(\lambda_\ell, \mathcal{D}_\ell)$. All group members seek to maximize their average discounted payoffs. By the same logic as above, $\mathcal{G}^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ admits a stationary Markov perfect equilibrium, and we can define $V_i^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ as group member i's equilibrium payoff, and $V_{0,k}^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ as the (passive) type-c_k agent's corresponding payoff. Proceeding recursively, we thus obtain the functions $V_i^p(\cdot \mid \cdot)$ and $V_{0,k}^p(\cdot \mid \cdot)$ for $p = p^0$.

Consider the continuous function that maps every $(\nu_0, \nu_1, \ldots, \nu_n) \in \mathcal{V}$ into $\Big(\big(V_{0,k}^{p^0}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)\big)_{\substack{\ell=1,\ldots,L \\ k=1,\ldots,K}}, \big(V_i^{p^0}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)\big)_{\substack{i\in N \\ k=1,\ldots,K}}\Big) \in \mathcal{V}$. Applying Brouwer's fixed point theorem, we obtain a fixed point $(\nu_0^*, \nu_1^*, \ldots, \nu_n^*)$ for this function. Now, define the game Γ as follows. Each period $t = 1, 2, \ldots$ begins with a belief $p \in \Delta_{p_0}$ and a procedure $(\lambda, \mathcal{D}) \in \Lambda \times \mathfrak{D}$, inherited from the previous period. (The initial belief and procedure at the start of period 1 are as in our main game.) Then, events unfold as follows:

(1) The group members' benefit profile b^t is drawn according to the $F_i's$, and the sequence of proposers ι^t according to λ_k.

(2) The organizational phase takes place as in the main game. Let $(\lambda_{l'}, \mathcal{D}_{l'})$ denote the resulting procedure.

(3) The negotiation phase takes place as in the main game, but the group members are constrained to choose offers from the set $\{\chi_k(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)\}_{k=1,\ldots,m}$. Let $\chi_{k'}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0, \nu_1, \ldots, \nu_n)$ denote the resulting offer to the agent.

(4) A shock on the agent's type occurs with probability α.

(5) The game transitions to period $t+1$, which begins with ongoing procedure $(\lambda_{l'}, \mathcal{D}_{l'})$. If a shock occurred in the previous stage, then the belief at the start of $t+1$ is p^0; otherwise, it is p^{k-}.

It is easy to see that prescribing the group members to play as in the equilibrium of $\mathcal{G}^p(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)$ in every period that begins with belief p and procedure $(\lambda_\ell, \mathcal{D}_\ell)$, we obtain a stationary Markov perfect equilibrium ς for Γ. We now modify ς to a pure-strategy profile $\hat{\varsigma}$ as follows. Observe that the outcome of every period is a policy $\chi_{k'}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)$ in $\{\chi_k(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)\colon k = 1, \ldots, K \;\&\; \ell = 1, \ldots, L\}$ and a procedure $(\lambda_\ell, \mathcal{D}_\ell) \in \Lambda \times \mathfrak{D}$, yielding a payoff $(1-\delta)\big[b_i - u\big(\chi_{k'}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)\big)\big]p(S_{k'}^-) + \delta\big[\alpha V_i^{p^0}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*) + (1-\alpha)V_i^{p^{k-}}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)\big]$ to the benefit-b_i group member i. Thus, for any pair of outcomes o and o', there is a unique cutoff value of b_i, say $\beta_i(o, o')$, for which group member i is indifferent between o and o'. Given that the sets of group members and outcomes are finite (and the F_i's are continuous), the set of benefit profiles $(b_1, \ldots, b_n) \in B^n$ such that $b_i = \beta_i(o, o')$ for some group member i and outcome pair (o, o'), denoted B_0, is of measure zero. In any period that begins with a benefit profile in B_0, we modify the actions prescribed by ς to those prescribed by some pure-strategy Markov-perfect equilibrium of the corresponding one-period game, where payoffs are defined using the continuation values induced by ς. (Existence of such an equilibrium follows directly from backward induction. Note that to maintain Markov perfection in the entire Γ, one must change ς in the same way in all periods that start with the same belief, procedure, and proposer sequence.) As B_0 is a measure-zero event, those changes to ς do not affect the continuation values at the start of each period, which we obtained above. Therefore, the strategy profile thus obtained is still a Markov perfect equilibrium of Γ.

Now take any period in which the realization of the benefit profile lies outside B_0, so that no group member can be indifferent between any two possible outcomes in this period. In the final (voting) stage, if the active group member randomizes, then it must be that her choice has no impact on the final outcome — otherwise, she would not be indifferent and, consequently, would not randomize. It follows that we can replace her randomized choice by a pure one without affecting the period's outcome and, therefore, the equilibrium conditions in the other stages of the game. We can then apply the same logic recursively to the previous stage in both the organizational and negotiation phases; and repeat the same process in any such period to obtain a new pure-strategy Markovian strategy profile, $\hat{\varsigma}$. By construction, the latter is a Markov perfect equilibrium of Γ.

We are now in a position to construct a (putative) equilibrium strategy profile for our main game. We begin with group members' strategies $(\phi_1, \ldots, \phi_n)$. Fix any belief $p \in \Delta_{p^0}$, with support $\{c^1, \ldots, c^m\}$, and any ongoing procedure $(\lambda, \mathcal{D}) \in \mathcal{P}$. Given p and $(\lambda, \mathcal{D})$, $(\phi_1, \ldots, \phi_n)$ prescribes the group members to play exactly as in $\hat{\varsigma}$ in the organizational phase, for all realizations of the benefit profile and the sequence of proposers. Given the belief p, the benefit profile b, and the protocol $(\lambda', \mathcal{D}')$ inherited from the organizational phase, consider the (one-shot) amendment agenda game, in which: the set of alternatives is X; the sequence of proposers is drawn according to λ'; the voting rule is $\mathcal{D}'$; and each group member i's payoff from choosing x is given by $(1-\delta)\big[b_i - u(x)\big]p(S_k^-) + \delta(1-\alpha)V_i^{p^{k-}}(\lambda', \mathcal{D}' \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)$, where $k = 1, \ldots, m$ is the unique integer that satisfies $x \in \big[\chi_k(\lambda', \mathcal{D}' \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*), \chi_{k+1}(\lambda', \mathcal{D}' \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)\big)$. (If $x \geq \chi_m(\lambda', \mathcal{D}' \mid \nu_0^*, \nu_1^*, \ldots, \nu_n^*)$, then $k = m$.) It follows from Zermelo's theorem that this game has pure-strategy subgame-perfect equilibria; it is readily checked that in one of them, the group members make the same offers as those prescribed by $\hat{\varsigma}$ in the negotiation phase. Strategies $(\phi_1, \ldots, \phi_n)$ prescribe the same behavior as that equilibrium in the corresponding negotiation phase.

We now turn to the agent's strategy, σ. Given any belief $p \in \Delta_{p^0}$, with support $\{c^1, \ldots, c^m\}$, and any ongoing procedure $(\lambda, \mathcal{D}) \in \mathcal{P}$, the type-$c_l$ accepts an offer $x \in [\overline{x}_k, \overline{x}_{k+1})$ if and only if

$$\delta(1-\alpha)V_{0,l}^{p^{k+}}(\lambda, \mathcal{D} \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*) \leq (1-\delta)\big[u_0(x) - c_l\big] + \delta(1-\alpha)V_{0,l}^{p^{k-}}(\lambda, \mathcal{D} \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*)\,;$$

she accepts any offer $x \geq x_m$, and rejects any offer $x \in \big[0, \chi_1(\lambda, \mathcal{D} \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*)\big)$. Finally, beliefs are updated as follows: if the group members make no offer, or if they make an offer $x \in \big[0, \chi_1(\lambda, \mathcal{D} \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*)\big)$, then the belief remains equal to p, irrespective of the agent's response; and for each $k = 1, \dots, m-1$, if they make an offer $x \in \big[\chi_k(\lambda, \mathcal{D} \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*), \chi_{k+1}(\lambda, \mathcal{D} \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*)\big)$, then their belief becomes p^{k+} if the offer is accepted by the agent, and it becomes p^{k-} if it is rejected.

To complete the proof of the lemma, it remains to verify that the strategy profile and belief system constructed in the previous paragraph is an equilibrium of our main game. By construction (and the induction hypothesis), we can focus on periods that begin with belief p. First, optimality of the group members' choices follows by construction — if a group member i had a profitable deviation from ϕ_i in this game, then she would also have a profitable deviation in one of the equilibria constructed for the other games above. Moreover, it follows from the definition of the strategy profile that the type-c_k agent's equilibrium value function at belief p and procedure $(\lambda_\ell, \mathcal{D}_\ell)$ is given by $V_{0,\ell}(\cdot \mid c_k) \equiv V_{0,k}^{p}(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*)$. Therefore, it follows immediately from the definition of her strategy and the group members' belief-updating rule that deviations are unprofitable.

Finally, we must verify that the group members' belief-updating rule is consistent with Bayes' rule (whenever possible). Take any belief $p \in \Delta_{p^0}$, with support $\{c^1, \dots, c^m\}$, and any procedure $(\lambda_\ell, \mathcal{D}_\ell) \in \Lambda \times \mathfrak{D}$; and for notational ease, let $\overline{x}_k \equiv \chi_k(\lambda_\ell, \mathcal{D}_\ell \mid \nu_0^*, \nu_1^*, \dots, \nu_n^*)$, for each $k = 1, \dots, m$. Observe first that by definition of the $\overline{x}_k$'s, the type-c_k agent accepts the offer $\overline{x}_k$ from the group members in equilibrium. As her continuation values from accepting or rejecting any $x \in (\overline{x}_k, \overline{x}_{k+1})$ are equal to those from accepting or rejecting $\overline{x}_k$, and u_0 is an increasing function, she also accepts any $x \in (\overline{x}_k, \overline{x}_{k+1})$. This in turn implies that for all $c < c_k$, we have

$$
\begin{aligned}
(1-\delta)\big[u_0(x)-c\big] &+ \delta(1-\alpha)\big[V_{0,\ell}(p^{k-} \mid c) - V_{0,\ell}(p^{k+} \mid c)\big] \\
&\geq (1-\delta)\big[u_0(x)-c\big] + \delta(1-\alpha)\big[V_{0,\ell}(p^{k-} \mid c) - V_{0,\ell}(p^{k+} \mid c)\big] \\
&\quad - \Big[(1-\delta)\big[u_0(x)-c_k\big] + \delta(1-\alpha)\big[V_{0,\ell}(p^{k-} \mid c_k) - V_{0,\ell}(p^{k+} \mid c_k)\big]\Big] \\
&\geq (1-\delta)(c_k - c) - 2\delta(1-\alpha)u_0(\hat{x}_0) > 0\,,
\end{aligned}
$$

where the last inequality follows from $\delta < \overline{\delta} \leq \overline{\delta}_1$. Thus, all types $c \leq c_k$ accept any $x \in (\overline{x}_k, \overline{x}_{k+1})$. Moreover, for all $c > c_k$, the type-c agent's continuation value from accepting any $x \in (\overline{x}_k, \overline{x}_{k+1})$ is zero, conditional on no shock occurring on the path. As $(1-\delta)\big[u_0(x)-c\big] < 0 \leq \delta(1-\alpha)V_0(p^{k+} \mid c)$, her strategy then prescribes her to reject x. We conclude that the updating rule is consistent Bayes' rule following any offer $x \in (\overline{x}_k, \overline{x}_{k+1})$, $k = 1, \ldots, m-1$. By the same logic, it is also consistent Bayes' rule following offers in $[0, \overline{x}_1) \cap [x_m, \hat{x}_0]$. It is readily checked that group members' beliefs must belong to Δ_{p^0}, and that they satisfy the no-signaling-what-you-don't-know condition. This proves that the strategy profile and belief system constructed above constitute an equilibrium of the main game.

A.2. Proof of Lemma 2

Let $\overline{\delta}_1 > 0$ be defined as in the proof of Lemma 1. As $\delta \to 0$, $y_k^-(\delta), y_k^+(\delta) \to x_k \equiv u_0^{-1}(c_k)$. Therefore, there exists $\overline{\delta}_2 > 0$ such that $y_k^+(\delta) < y_{k+1}^-(\delta)$ for all $k = 1, \ldots, K-1$, whenever $\delta < \overline{\delta}_2$. For each $k \in \{1, \ldots, m-1\}$ and $i \in N$, let $\overline{\beta}_k^i(\delta)$ be defined as in the proof of Lemma A2. As we saw in that proof, $\overline{\beta}_k^i(\delta) \to \beta_p(k)$ as $\delta \to 0$. It follows that there exists a sufficiently small $\overline{\delta}_3 > 0$ such that $\overline{\beta}_{k+1}^i(\delta) - \overline{\beta}_k^i(\delta) \geq \big[\beta_p(k+1) - \beta_p(k)\big]/2$ for all $k \in \{1, \ldots, m-1\}$ and $i \in N$, whenever $\delta < \overline{\delta}_3$. We set $\overline{\delta} < \min\{\overline{\delta}_1, \overline{\delta}_2, \overline{\delta}_3\}$ and, henceforth, assume that $\delta < \overline{\delta}$.

Take any equilibrium, and let $p \in \Delta_{p^0}$. For notational ease, and without any loss of generality, assume that the support of p is $\{c_1, \ldots, c_m\}$, where $1 \leq m \leq K$. If the group members hold belief p and they make an offer that all agent types accept, then this offer must be x_m. To see this, observe first that as the type-c_{m-1} agent accepts any offer greater than or equal to

$y^+_{m-1}(\delta) < y^-_m(\delta) < x_m$ (where the first inequality follows from $\delta < \overline{\delta} \leq \overline{\delta}_2$), she must accept any offer $x \geq x_m$. As we showed in the proof of Lemma 1, $\delta < \overline{\delta} \leq \overline{\delta}_1$ then implies that all types $c < c_{m-1}$ also accept any such offer. This in turn implies that type c_m must accept any offer $x > x_m$ in equilibrium: if she rejected x, thus revealing her type to the group members, then she would receive a payoff of zero until the arrival of the next shock, as the group members would trivially offer her x_m in every period. Accepting x (thus receiving a positive payoff) would be a profitable deviation. Now suppose that the group members make an offer $x > x_m$ that is accepted by all agent types in equilibrium. The proposer who successfully proposed x in that period could then profitably deviate by proposing some $x' \in (x_m, x)$ instead. That policy would still be accepted by all agent types; all the group members' stage-payoffs would be increased; and their continuation values would remain unchanged, as the belief would remain the same. This is a contradiction, showing that an equilibrium offer that is accepted by all agent types must be x_m. Note in passing that this also shows that the group members never make an offer above x_m in equilibrium and, consequently, that the payoff to the highest type in the support of p must be zero until the arrival of the next shock.

Let $\sigma(p, \lambda, \mathcal{D}, x \mid c_k) \in \{0, 1\}$ be the type-c_k agent's response to an offer $x \in X$ when the group members hold belief p and the ongoing procedure is $(\lambda, \mathcal{D})$. As $\delta < \overline{\delta} \leq \overline{\delta}_2$, we have $y^+_\ell(\delta) < y^-_m(\delta)$, for all $\ell < m$. Hence, there exist offers that are accepted by all agent types but type c_m, i.e., the set $\big\{x \in X \colon \sigma(p, \lambda, \mathcal{D}, x \mid c_{m-1}) = 1 - \sigma(p, \lambda, \mathcal{D}, x \mid c_m) = 1\big\}$ is nonempty. Let $\overline{x}^{m-1}(p, \lambda, \mathcal{D}) \equiv \inf\big\{x \in X \colon \sigma(p, \lambda, \mathcal{D}, x \mid c_{m-1}) = 1 - \sigma(p, \lambda, \mathcal{D}, x \mid c_m) = 1\big\}$. Observe that $\overline{x}^{m-1}(p, \lambda, \mathcal{D})$ belongs to $\big[y^-_{m-1}(\delta), y^+_{m-1}(\delta)\big]$ and therefore, $\overline{x}^{m-1}(p, \lambda, \mathcal{D}) < \overline{x}^m(p, \lambda, \mathcal{D}) \equiv x_m$. By the same logic as in the previous paragraph, if the group members hold belief p and they make an offer that separates agent types in $\{c_1, \ldots, c_{m-1}\}$ from c_m, then this offer must be $\overline{x}_{m-1}(p, \lambda, \mathcal{D})$ — otherwise, it would have to be strictly higher than $\overline{x}_{m-1}(p, \lambda, \mathcal{D})$, and at least one group member could profitably deviate by inducing a slightly lower offer. Proceeding recursively, we define $\overline{x}_k(p, \lambda, \mathcal{D})$ for every $k = 1, \ldots, m-2$, in like manner.

To complete the proof of Lemma 2, it remains to establish that for each $k = 1, \ldots, m-1$, the group members separate agent types in $\{c_1, \ldots, c_k\}$ from those in $\{c_{k+1}, \ldots, c_m\}$, and that they pool agent types (with a successful offer), with positive probability in equilibrium. As $\delta < \overline{\delta} \le \overline{\delta}_3$, the open intervals $\big(\overline{\beta}^i_{k-1}(\delta), \overline{\beta}^i_k(\delta)\big)$ (or $(\overline{\beta}^i_{m-1}(\delta), \bar{b})$) are nonempty. For realizations $(b_1, \dots, b_n)$ of the group members' benefit profile such that $b_i \in \big(\overline{\beta}^i_{k-1}(\delta), \overline{\beta}^i_k(\delta)\big)$ for all i (an event that arises with positive probability), the group members unanimously agree that separating $\{c_1, \dots, c_k\}$ from $\{c_{k+1}, \dots, c_m\}$ is the best option, and must therefore do so in equilibrium by offering policy $\overline{x}_k(p, \lambda, \mathcal{D})$. Similarly, when all the members' benefits belong to $(\overline{\beta}^i_{m-1}(\delta), \bar{b})$, they all agree that pooling all the agent's types is the best option, so that the only possible outcome of the amendment-agenda game must be the offer x_m.

A.3. Proof of Lemma 3

The first part of the lemma is an immediate corollary of Lemmas 2, A2, and A3. The second part is directly obtained by defining $\beta^\phi_i(p, \lambda, \mathcal{D})$ as $\overline{\beta}^i_{m-1}(\delta)$ in the proof of Lemma A2 for the case where $W_i(p)$ is group member i's continuation value at belief p and ongoing procedure $(\lambda, \mathcal{D})$ under the equilibrium ϕ.

B. Proof of Proposition 1

For every equilibrium ϕ, let $V^\phi_i \colon \Delta_{p^0} \times \Lambda \times \mathfrak{D} \to \mathbb{R}$ be the value function of group member i induced by ϕ — i.e., for all $p \in \Delta_{p^0}$ and $(\lambda, \mathcal{D}) \in \Lambda \times \mathfrak{D}$, $V^\phi_i(p; \lambda, \mathcal{D})$ is i's expected continuation payoff at the start of any period that begins with belief p and procedure $(\lambda, \mathcal{D})$ (before the realization of the group members' benefit profile). Moreover, we denote by Γ the main game with endogenous procedures and for each $i \in N$, by Γ^i the benchmark game in which group member i is an (exogenously given) permanent dictator. For every equilibrium ϕ^i of the latter game, we denote by $W^{\phi^i}_i(p)$ dictator i's equilibrium continuation value at belief $p \in \Delta_{p^0}$. We begin by establishing a useful lemma.

Lemma B1. There exist $\kappa > 0$ and $\overline{\delta}_4 > 0$ such that the following holds for every $\delta < \overline{\delta}_4$, $i \in N$, and non-dictatorship $(\lambda, \mathcal{D})$. Let ϕ and ϕ^i be any equilibria of Γ and Γ^i, respectively; and let

$p \in \Delta_{p^0}$ be a belief whose support is denoted by $\{c^1, \ldots, c^m\}$. Then,

$$W_i^{\phi^i}(p) - V_i^{\phi}(p; \lambda, \mathcal{D}) - p(S_{m-1}^-)\big[W_i^{\phi^i}(p_k^-) - V_i^{\phi}(p_{m-1}^-; \lambda, \mathcal{D})\big]$$

$$-p(S_{m-1}^+)\big[W_i^{\phi^i}(p_{m-1}^+) - V_i^{\phi}(p_{m-1}^+; \lambda, \mathcal{D})\big] > \kappa \, .$$

Proof. Take any group member $i \in N$, non-dictatorship $(\lambda, \mathcal{D})$, and nondegenerate belief $p \in \Delta_{p^0}$, whose support is denoted by $\{c^1, \ldots, c^m\}$. Consider a period of game Γ that begins with belief p and procedure $(\lambda, \mathcal{D})$; and suppose for the time being that $\delta = 0$. For every $b_j \in B$, the payoff to the benefit-b_j group member j from offering policy $x_k \equiv u_0^{-1}(c_k)$, $k = 1, \ldots, m$, to the agent is given by $U^p(k \mid b_j)$, as defined in the proof of Lemma A2. It follows that if the group members do not amend the ongoing procedure $(\lambda, \mathcal{D})$ in the organizational phase, the offer made to the agent will be the ideal of the first proposer ι_1 in the core induced by $(\lambda, \mathcal{D})$. Moreover, since the shortsighted members' payoffs are independent of the ongoing procedure, it follows from the definition of the core that no procedure that would induce a different outcome may result from the organizational phase (in which $(\lambda, \mathcal{D})$ is the status quo).

For each $k = 1, \ldots, m$, let B_k be the set of realizations of the benefits and proposer sequences (at the start of the period) for which x_k is ι_1's ideal in the core, and let $\widehat{B}_k^i$ be those for which k is group member i's ideal in $\{1, \ldots, m\}$. We then have

$$\sum_{k=1}^{m} \Pr(\widehat{B}_k^i)\mathbb{E}\big[U^p(k \mid \tilde{b}_i) \mid \widehat{B}_k^i\big] = \sum_{k=1}^{m}\sum_{\ell=1}^{m} \Pr(\widehat{B}_k^i \cap B_\ell)\mathbb{E}\big[U^p(k \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] \, ,$$

and

$$\sum_{\ell=1}^{m} \Pr(B_\ell)\mathbb{E}\big[U^p(\ell \mid \tilde{b}_i) \mid B_\ell\big] = \sum_{k=1}^{m}\sum_{\ell=1}^{m} \Pr(\widehat{B}_k^i \cap B_\ell)\mathbb{E}\big[U^p(\ell \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] \, .$$

Let $\Delta_{k,\ell} \equiv \mathbb{E}\big[U^p(k \mid \tilde{b}_i) - U^p(\ell \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big]$. Since $\Delta_{k,\ell} = 0$ whenever $k = \ell$, we have

$$\sum_{k=1}^{m} \Pr(\widehat{B}_k^i)\mathbb{E}\big[U^p(k \mid \tilde{b}_i) \mid \widehat{B}_k^i\big] - \sum_{\ell=1}^{m} \Pr(B_\ell)\mathbb{E}\big[U^p(\ell \mid \tilde{b}_i) \mid B_\ell\big] = \sum_{k=1}^{m}\sum_{\ell \neq k} \Pr(\widehat{B}_k^i \cap B_\ell)\Delta_{k,\ell} \, .$$

Note that since the decision-making procedure $(\lambda, \mathcal{D})$ is not a dictatorship (and the F_i's have full support), there exist different k and ℓ such that $\Pr(\widehat{B}_k^i \cap B_\ell) > 0$.

Next, let $\Delta_{k,\ell}^- \equiv \mathbb{E}\big[U^{p^{(m-1)-}}(k' \mid \tilde{b}_i) - U^{p^{(m-1)-}}(\vec{\ell} \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big]$, where k' is a (random) maximizer of $U^{p^{(m-1)-}}(\cdot \mid \tilde{b}_i)$ — as above, we can ignore the measure-zero event in which i has two ideal alternatives — and $\vec{\ell}$ is the (random) alternative that satisfies $\phi(p^{(m-1)-}, \tilde{b}) = \overline{x}^{\vec{\ell}}$ (conditional on $\widehat{B}_k^i \cap B_\ell$). Observe that $U^{p^{(m-1)-}}(k, b) = U^p(k, b)/p(S_{m-1}^-)$, for all $k \in \{1, \ldots, m-1\}$ and $b \in B$. Thus, if $k, \ell \geq m-1$, then $k' = \vec{\ell} = m-1$ and therefore, $\Delta_{k,\ell}^- = 0$; if $k, \ell < m-1$, then $k' = k$ and $\vec{\ell} = \ell$, so that

$$\Delta_{k,\ell}^- = \mathbb{E}\big[U^p(k \mid \tilde{b}_i) - U^p(\ell \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] p(S_{m-1}^-)^{-1} \, ;$$

if $k < m-1 \leq \ell$, then $k' = k$ and $\vec{\ell} = m-1$, so that

$$\Delta_{k,\ell}^- \equiv \mathbb{E}\big[U^p(k \mid \tilde{b}_i) - U^p(m-1 \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] p(S_{m-1}^-)^{-1} \, ;$$

and, conversely, if $\ell < m-1 \leq k$, then

$$\Delta_{k,\ell}^- \equiv \mathbb{E}\big[U^p(m-1 \mid \tilde{b}_i) - U^p(\ell \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] p(S_{m-1}^-)^{-1} \, .$$

Hence, for all $k, \ell \in \{1, \ldots, m\}$ such that $k \neq \ell$, we have

$$\Delta_{k,\ell} - p(S_{m-1}^-)\Delta_{k,\ell}^- = \begin{cases} \mathbb{E}\big[U^p(k \mid \tilde{b}_i) - U^p(\ell \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] > 0 & \text{if } k, \ell \geq m-1 \, , \\ \mathbb{E}\big[U^p(m-1 \mid \tilde{b}_i) - U^p(\ell \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] > 0 & \text{if } k < m-1 \leq \ell \, , \\ \mathbb{E}\big[U^p(k \mid \tilde{b}_i) - U^p(m-1 \mid \tilde{b}_i) \mid \widehat{B}_k^i \cap B_\ell\big] > 0 & \text{if } \ell < m-1 \leq k \, , \\ 0 & \text{otherwise,} \end{cases}$$

where the inequalities follow from quasi-single-peakedness and the fact that by continuity of the F_i's, member i can only be indifferent between two offers with probability zero. Hence, there is a sufficiently small $\kappa_p^i(\lambda, \mathcal{D}) > 0$ such that

$$\sum_{k=1}^{m} \sum_{\ell \neq k} \Pr(\widehat{B}_k^i \cap B_\ell)\big[\Delta_{k,\ell} - p(S_{m-1}^-)\Delta_{k,\ell}^-\big] > \kappa_p^i(\lambda, \mathcal{D}) \, .$$

Now let $\Delta^+_{p_0}$ be the subset of nondegenerate probability distributions in Δ_{p^0}; and let $\kappa \equiv \min\{\kappa^i_p(\lambda, \mathcal{D})\colon p \in \Delta^+_{p^0}, i \in N, (\lambda, \mathcal{D}) \in \mathcal{P}\} > 0$. As the group members' continuation payoffs are (uniformly) bounded over all possible outcomes, and $\beta^-_k(\delta), \beta^+_k(\delta) \to \beta_p(k)$ as $\delta \to 0$ (so that the probability measure of benefit profiles for which dynamic preferences differ from static ones converges to zero), there exists a sufficiently small $\overline{\delta}_p > 0$ such that whenever $\delta < \overline{\delta}_p$, $\big|W^{\phi^i}_i(p) - V^\phi_i(p; \lambda, \mathcal{D}) - \sum_{k=1}^m \sum_{\ell \neq k} \Pr(\widehat{B}^i_k \cap B_\ell)\Delta_{k,\ell}\big| < \kappa/2$ and $\big|W^{\phi^i}_i(p^{(m-1)-}) - V^\phi_i(p^{(m-1)-}; \lambda, \mathcal{D}) - \sum_{k=1}^m \sum_{\ell \neq k} \Pr(\widehat{B}^i_k \cap B_\ell)\Delta^-_{k,\ell}\big| < \kappa/2$, for any $i \in N$ and any equilibria ϕ and ϕ^i of Γ and Γ^i. Let $\overline{\delta}_4 \equiv \min\{\overline{\delta}_p\colon p \in \Delta^+_{p^0}\}$.

Trivially, $W^{\phi^i}_i(p^{(m-1)+}) - V^\phi_i(p^{(m-1)+}; \lambda, \mathcal{D}) = 0$ — all group members agree on the best offer to the agent when their common belief is degenereate. Therefore, for any equilibria ϕ and ϕ^i of Γ and Γ^i, we have

$$\begin{aligned} W^{\phi^i}_i(p) - V^\phi_i(p; \lambda, \mathcal{D}) - p(S^-_{m-1})\big[W^{\phi^i}_i(p^-_k) - V^\phi_i(p^-_{m-1}; \lambda, \mathcal{D})\big] & \\ - p(S^+_{m-1})\big[W^{\phi^i}_i(p^+_{m-1}) - V^\phi_i(p^+_{m-1}; \lambda, \mathcal{D})\big] & \\ \geq \sum_{k=1}^m \sum_{\ell \neq k} \Pr(\widehat{B}^i_k \cap B_\ell)\big[\Delta_{k,\ell} - p(S^-_{m-1})\Delta^-_{k,\ell}\big] - \kappa > 0\,, & \end{aligned}$$

as desired. □

We now return to the proof of the main proposition. Let $\delta < \overline{\delta} < \min\{\overline{\delta}_1, \overline{\delta}_2, \overline{\delta}_3, \overline{\delta}_4\}$. Take any group member $i \in N$, non-dictatorship $(\lambda, \mathcal{D})$, and nondegenerate belief $p \in \Delta_{p^0}$, whose support is denoted by $\{c^1, \ldots, c^m\}$. Let ϕ and ϕ^i be equilibria of Γ and Γ^i, respectively.

Consider first a negotiation phase of Γ, in which the group members hold belief p and use procedure $(\lambda, \mathcal{D})$. Given the equilibrium ϕ, any group member i prefers separating the agent types in $\{c^1, \ldots, c^{m-1}\}$ from type m to pooling all types in this period if and only if

$$\begin{aligned} (1-\delta)\big[b_i - u(\overline{x}^m)\big] + \delta(1-\alpha)V^\phi_i(p; \lambda, \mathcal{D}) \leq (1-\delta)\big[b_i - u(\overline{x}^{m-1})\big]p(S^-_{m-1}) & \\ + \delta(1-\alpha)\big[p(S^-_{m-1})V^\phi_i(p^-_{m-1}; \lambda, \mathcal{D}) + p(S^+_{m-1})V^\phi_i(p^+_{m-1}; \lambda, \mathcal{D})\big]\,, & \end{aligned}$$

where $\overline{x}^{m-1}$ and $\overline{x}^m$ denote the equilibrium offers characterized in Lemma 2. In fact, by quasi-single-peakedness of continuation payoffs (Lemma A2), she prefers any separation of types to pooling all types if and only this inequality holds. It follows that

$$\beta_i^\phi(p;\lambda,\mathcal{D}) \equiv \left[(1-\delta)p(c^m)\right]^{-1}\Big[(1-\delta)\left[u(\overline{x}^m)-u(\overline{x}^{m-1})p(S_{m-1}^-)\right] \\ +\delta(1-\alpha)\left[p(S_{m-1}^-)V_i^\phi(p_{m-1}^-;\lambda,\mathcal{D})+p(S_{m-1}^+)V_i^\phi(p_{m-1}^+;\lambda,\mathcal{D})\right]-V_i^\phi(p;\lambda,\mathcal{D})\Big] .$$

By the same logic, given the equilibrium ϕ^i of Γ^i, we can define $\hat{\beta}_i^{\phi^i}(p)$ as

$$\hat{\beta}_i^{\phi^i}(p) \equiv \left[(1-\delta)p(c^m)\right]^{-1}\Big[(1-\delta)\left[u(\overline{x}^m)-u(\hat{x}^{m-1})p(S_{m-1}^-)\right] \\ +\delta(1-\alpha)\left[p(S_{m-1}^-)W_i^{\phi^i}(p_{m-1}^-)+p(S_{m-1}^+)W_i^{\phi^i}(p_{m-1}^+)\right]-W_i^{\phi^i}(p)\Big] ,$$

where $\hat{x}^{m-1}$ is the policy offered by dictator i when she seeks to separate the agent types in $\{c^1,\ldots,c^{m-1}\}$ from type m in ϕ^i. It then follows from Lemma B1 (and the fact that $\overline{x}^m=\hat{x}^m=u_0^{-1}(c^m)$) that $\hat{\beta}_i^{\phi^i}(p)<\beta_i^\phi(p)$ if

$$(1-\delta)\left[u(\overline{x}^{m-1})-u(\hat{x}^{m-1})\right]<\delta(1-\alpha)\kappa . \tag{B1}$$

Let $V_0^\phi(\cdot\mid c_{m-1})$ and $W_0^{\phi^i}(\cdot\mid c_{m-1})$ be the type-c_{m-1} agent's continuation values induced by ϕ and ϕ^i. Observe that $\overline{x}^{m-1}$ is the unique solution to

$$(1-\delta)\left[u_0(\overline{x}^{m-1})-c_{m-1}\right]+\delta(1-\alpha)V_0^\phi(p^{(m-1)-}\mid c_{m-1})=\delta(1-\alpha)V_0^\phi(p^{(m-1)+}\mid c_{m-1})$$

or, equivalently,

$$(1-\delta)\left[u_0(\overline{x}^{m-1})-c_{m-1}\right]=\delta(1-\alpha)\left[V_0^\phi(p^{(m-1)+}\mid c_{m-1})-V_0^\phi(p^{(m-1)-}\mid c_{m-1})\right] ,$$

where, for notational ease, we omit the dependency of $V_0^\phi(\cdot\mid c_{m-1})$ on $(\lambda,\mathcal{D})$. To see why this equation must hold in equilibrium, suppose towards a contradiction that the type-c_{m-1} agent is strictly better off accepting offer $\overline{x}_{m-1}$. By continuity of u_0, this implies that there exists a sufficiently small $\varepsilon > 0$ such that

$$(1-\delta)\big[u_0(\overline{x}^{m-1}-\varepsilon) - c_{m-1}\big] > \delta(1-\alpha)\big[V_0^{\phi}(p^{(m-1)+} \mid c_{m-1}) - V_0^{\phi}(p^{(m-1)-} \mid c_{m-1})\big] \, .$$

As $\overline{x}^{m-1}-\varepsilon < y_m^-(\delta)$, the type-$c_m$ agent would reject the offer $\overline{x}^{m-1}-\varepsilon$, so that the group members' updated beliefs would assign a probability of zero to types $c \geq c_m$ after observing a rejection of $\overline{x}^{m-1}-\varepsilon$. Hence, the type-$c_{m-1}$ agent would be strictly better off accepting $\overline{x}^{m-1}-\varepsilon$ than rejecting it, so that all the group members would be better off offering her $\overline{x}^{m-1}-\varepsilon$ rather than $\overline{x}^{m-1}$; a contradiction. By the same logic, $\hat{x}^{m-1}$ must satisfy

$$(1-\delta)\big[u_0(\hat{x}^{m-1}) - c_{m-1}\big] = \delta(1-\alpha)\big[W_0^{\phi^i}(p^{(m-1)+} \mid c_{m-1}) - W_0^{\phi^i}(p^{(m-1)-} \mid c_{m-1})\big] \, .$$

Let $v_0 \equiv u_0^{-1}$, $\overline{\Delta} \equiv V_0^{\phi}(p^{(m-1)+} \mid c_{m-1}) - V_0^{\phi}(p^{(m-1)-} \mid c_{m-1})$, and $\widehat{\Delta} \equiv W_0^{\phi^i}(p^{(m-1)+} \mid c_{m-1}) - W_0^{\phi^i}(p^{(m-1)-} \mid c_{m-1})$. Using the agent's incentive constraints above, we obtain:

$$\begin{aligned} u(\overline{x}^{m-1}) - u(\hat{x}^{m-1}) &\leq u'(\overline{x}^{m-1})(\overline{x}^{m-1} - \hat{x}^{m-1}) \\ &= u'(\overline{x}^{m-1}) \left[v_0\left(c_{m-1} + \frac{\delta(1-\alpha)}{1-\delta}\overline{\Delta}\right) - v_0\left(c_{m-1} + \frac{\delta(1-\alpha)}{1-\delta}\widehat{\Delta}\right)\right] \\ &\leq u'(\overline{x}^{m-1}) v_0'\left(c_{m-1} + \frac{\delta(1-\alpha)}{1-\delta}\overline{\Delta}\right)\frac{\delta(1-\alpha)}{1-\delta}(\overline{\Delta} - \widehat{\Delta}) \, , \end{aligned}$$

where the inequalities follow from the convexity of u and v_0. Thus, if $\overline{\Delta} \leq \widehat{\Delta}$, condition B1 holds and we obtain the proposition.

Now suppose that $\overline{\Delta} > \widehat{\Delta}$, so that $(1-\delta)\big[u(\overline{x}^{m-1}) - u(\hat{x}^{m-1})\big] \leq \delta(1-\alpha)u'(\hat{x}_0)v_0'(\hat{x}_0)(\overline{\Delta} - \widehat{\Delta})$; and condition B1 holds whenever $u'(\hat{x}_0)v_0'(\hat{x}_0)(\overline{\Delta} - \widehat{\Delta}) < \kappa$. By definition, $p^{(m-1)+}$ is the degenerate probability distribution that assigns probability one to type c^m. When the group members hold such a belief, they unanimously agree that the best offer to the agent $\overline{x}^m = u_0^{-1}(c^m)$. It follows that starting from belief $p^{(m-1)+}$, this is the offer that must be made in the current period — and, as long as no shock occurs, in every future period — regardless of the procedures in place. As this is the best offer that the agent can receive in the continuation game, it is always optimal for her to accept it. It follows that $\left|V_0^{\phi}(p^{(m-1)+} \mid c_{m-1}) - W_0^{\phi^i}(p^{(m-1)+} \mid c_{m-1})\right| \leq \delta u_0(\hat{x}_0)$. Moreover, in any equilibrium (of either game), the offer made to the agent must be lower than or equal to $x_{m-1} \equiv u_0^{-1}(c_{m-1})$ (so that her stage-payoff is zero) when the group members hold belief $p^{(m-1)-}$. This implies that $\left|W_0^{\phi^i}(p^{(m-1)-} \mid c_{m-1}) - V_0^{\phi}(p^{(m-1)-} \mid c_{m-1})\right| \leq \delta u_0(\hat{x}_0)$. Therefore, $\overline{\Delta} - \widehat{\Delta} = V_0^{\phi}(p^{(m-1)+} \mid c_{m-1}) - W_0^{\phi^i}(p^{(m-1)+} \mid c_{m-1}) + W_0^{\phi^i}(p^{(m-1)-} \mid c_{m-1}) - V_0^{\phi}(p^{(m-1)-} \mid c_{m-1}) \leq 2\delta u_0(\hat{x}_0)$. We conclude that condition B1 holds whenever $\delta < \overline{\delta} \leq \overline{\delta}_5 \equiv \kappa / \left[2u_0(\hat{x}_0)u'(\hat{x}_0)v_0'(\hat{x}_0)\right]$.

Finally, observe that in any equilibrium φ of a continuation game of Γ that begins under some group member i's dictatorship, she remains a dictator in all future periods—possibly under different procedures. It follows that $V_i^{\varphi}(p; \wp^i) \equiv W_i^{\phi^i}(p)$, and therefore $\beta_i^{\varphi}(p; \wp^i) \equiv \hat{\beta}_i^{\phi^i}(p)$, for every procedure $\wp^i$ under which i is a dictator. This completes the proof of the proposition.

C. Proof of Proposition 2

We begin with some useful observations. First, for every $\lambda \in \Lambda$, let $q_i(\lambda)$ be the probability that group member $i \in N$ proposes first under λ; and let $q \equiv \min\left\{q_i(\lambda) \colon i \in N, \lambda \in \Lambda, q_i(\lambda) > 0\right\}$. Then, there exists a sufficiently small $\hat{\delta}_{6,1} > 0$ such that

$$(1-\delta)q\left[\overline{b} - u\big(y_{K-1}^{+}(\delta)\big)\right]p^0(S_{K-1}^{-}) + \delta\overline{b} < (1-\delta)q\left[\overline{b} - u(x_K)\right],$$

for all $\delta < \hat{\delta}_{6,1}$. Given any equilibrium ϕ, let $\overline{x}^{K-1}$ be defined as in Lemma 2 for $p = p^0$; and observe that $\overline{x}^{K-1} \leq y_{K-1}^{+}(\delta)$ (otherwise, the type-c_{K-1} agent would have a profitable deviation when offered $\overline{x}^{K-1}$). It follows from the inequality above that in any period t, any group member whose period-t benefit is $\overline{b}$ strictly prefers pooling all agent types with certainty to separating those in $\{c_1, \ldots, c_{K-1}\}$ from c_K with a probability greater than or equal to q, regardless of what happens from period $t+1$ onward. By continuity, this also holds for all benefits $b \in (\overline{b} - \varepsilon_1, \overline{b}]$, for some small enough $\varepsilon_1 > 0$. Similarly, there exist sufficiently small $\hat{\delta}_{6,2}, \varepsilon_2 > 0$ such that whenever $\delta < \hat{\delta}_{6,2}$, any group member whose benefit belongs to $[\underline{b}, \underline{b} + \varepsilon_2)$ strictly prefers separating type c_1 from those in $\{c_2, \ldots, c_K\}$ to making the pooling offer, regardless of future play.

Let $\varepsilon \equiv \min\{\varepsilon_1, \varepsilon_2\}$. Moreover, by the same logic as in the proof of Lemma A2, there exists a sufficiently small $\overline{\delta}_6 > 0$, lower than $\min\{\hat{\delta}_{6,1}, \hat{\delta}_{6,2}\}$, such that $\beta^+_{K-1}(\delta) < \overline{b} - \varepsilon$, for all $\delta < \overline{\delta}_6$. Henceforth, we assume that $\delta < \overline{\delta} \leq \overline{\delta}_6$.

Now suppose towards a contradiction that there is an equilibrium ϕ of the extended game in which the sequence of procedures adopted by the group members does not converge almost surely to a dictatorship. In any period t, if $(\lambda^t, \mathcal{D}^t)$ is a dictatorship, then either $(\lambda^{t+1}, \mathcal{D}^{t+1}) = (\lambda^t, \mathcal{D}^t)$, or $(\lambda^{t+1}, \mathcal{D}^{t+1})$ is another dictatorship with the same dictator as in t. Therefore, the set of stochastic sequences of group member benefits, shocks on the agent's types, and proposer sequences for which the group members never adopt a dictatorship in equilibrium constitutes an event that occurs with positive probability. We denote this event by E. Thus, by Proposition 1, at every history in the period-t negotiation phase that is consistent with E, if the belief p^{t-1} is nondegenerate, then the equilibrium pooling cutoff of each group member i, $\beta^\phi_i(p^{t-1}; \wp^t)$, must be be lower than her pooling cutoff when she is a dictator, which we denote by $\hat{\beta}_i(p^{t-1})$.

Let $\mathcal{P}_E$ be the set of procedures that may prevail on paths consistent with E. Our next step is to define for every $(\lambda, \mathcal{D}) \in \mathcal{P}_E$, a lower bound $P(\lambda, \mathcal{D})$ on the probability that the group members adopt a dictatorship as their decision-making procedure if a shock on the agent's type occurs while the ongoing procedure is $(\lambda, \mathcal{D})$. (Markov perfection ensures that this probability only depends on $(\lambda, \mathcal{D})$ and the group members' belief, which must be p^0 after a shock.) For each group member i, let $\overline{\beta}_i(p^0) \equiv \min\left\{\beta^\phi_i(p^0; \lambda, \mathcal{D})\colon (\lambda, \mathcal{D}) \in \mathcal{P}_E\right\}$. Take any $(\lambda, \mathcal{D}) \in \mathcal{P}_E$; pick an arbitrary minimal decisive coalition S_1 in $\mathcal{D}_1$ and a group member i_1 in S_1 who may propose first with positive probability (if such a coalition does not exist, add a first proposer to some minimal decisive coalition); and let F_1 be the positive-probability event: "$b_{i_1} \in \left(\hat{\beta}_{i_1}(p^0), \overline{\beta}_{i_1}(p^0)\right)$, $b_j \in (\overline{b} - \varepsilon, \overline{b}]$ for all $j \in S_1 \setminus \{i_1\}$, and $b_j \in [\underline{b}, \underline{b} + \varepsilon)$ for all $j \in N \setminus S_1$." We claim that at any history (consistent with E) with ongoing procedure $(\lambda, \mathcal{D})$ that ends with a shock on the agent's type, followed by F_1, one of the following procedural changes must occur in equilibrium: either (i) some member of S_1 is made a (formal or informal) dictator; or (ii) some subcoalition of $S_1 \setminus \{i_1\}$ is made minimal decisive; or (iii) some subcoalition of $S_1 \setminus \{i_1\}$ is made blocking, but not decisive, and the first proposer belongs to that subcoalition with probability one. Moreover, the offer made to the agent must be x_K — so that the belief at the start of the next period must still be p^0. To see this, observe first that if i_1 is made a dictator, it will be optimal for her to pool all the agent types, since $b_{i_1} > \hat{\beta}_{i_1}(p^0)$. As $b_j \in (\bar{b} - \varepsilon, \bar{b}]$ for all the other members j of S_1 (and $\delta < \bar{\delta}_6$), this is also their ideal offer, regardless of the prevailing procedure. It follows that in the organizational phase, the only possible outcomes are procedures that induce the pooling offer as the outcome of the ensuing negotiation phase — otherwise, at least one member of the decisive coalition S_1 would have a profitable deviation during the former phase — since making i_1 a dictator guarantees that coalition's ideal outcome. Finally, observe that for offer x_K to be made with certainty in equilibrium of the negotiation phase, one of the following must be true: x_K is the only alternative in the core (leaving the first proposer no other option), i.e., either case (i) or case (ii) above hold; or case (iii) holds, so that x_K belongs to the core and the first proposer always selects it. If some member of S_1 becomes a dictator after F_1, then we set $P(\lambda, \mathcal{D}) \equiv \Pr(F_1) > 0$; otherwise, we denote by $(\lambda_2, \mathcal{D}_2)$ the new ongoing procedure, by S_2 the relevant subcoalition of $S_1 \setminus \{i_1\}$, and we proceed recursively as explained below.

Fix $k = 2, \ldots, |S_1| - 1$. Suppose that we have defined F_ℓ for each $\ell = 1, \ldots, k-1$ (and therefore, S_ℓ for each $\ell = 1, \ldots, k$), but $P(\lambda, \mathcal{D})$ is not yet defined. Fixing $i_k \in S_k$ — when S_k is blocking but not decisive, i_k must be one of the members of S_k who may propose first — we then define the positive-probability event F_k as follows: "events $F_1, \ldots, F_{k-1}$ have successively occurred in the previous $k-1$ periods; $b_{i_k} \in \left(\hat{\beta}_{i_k}(p^0), \overline{\beta}_{i_k}(p^0)\right)$, $b_j \in (\bar{b} - \varepsilon, \bar{b}]$ for all $j \in S_k \setminus \{i_k\}$, and $b_j \in [\underline{b}, \underline{b} + \varepsilon)$ for all $j \in N \setminus S_k$." (Note that by construction, in cases where S_k is not decisive, the first proposer ι_1 must be i_k.) Repeating the same arguments as in the previous paragraph, we obtain that in equilibrium, one of the following procedural changes must occur after F_k: either (i) some member of S_k is made a dictator; or (ii) some subcoalition of $S_k \setminus \{i_k\}$ is made minimal decisive; or (iii) some subcoalition of $S_k \setminus \{i_k\}$ is made blocking, but not decisive, and the first proposer belongs to that subcoalition with probability one. (Note that even when coalition S_k is not decisive, its ideal outcome can still be guaranteed by making i_k a dictator. The coalition being decisive, the pooling offer must belong to the core and be selected by the first proposer, who must be one of its members by construction.) If some member of S_k becomes a dictator after F_k, then we set

$P(\lambda, \mathcal{D}) \equiv \Pr(F_k) > 0$; otherwise, we denote by $(\lambda_{k+1}, \mathcal{D}_{k+1})$ the new ongoing procedure, by S_{k+1} the relevant subcoalition of $S_k \setminus \{i_k\}$, and repeat the same process.

Observe that this process must end with a dictatorship after at most $|S_1|$ iterations. We can then conclude that in event E, the probability that the group members adopt a dictatorship after a shock on the agent's type is bounded from below by $\min\{P(\lambda, \mathcal{D})\colon (\lambda, \mathcal{D}) \in \mathcal{P}_E\} > 0$. As an infinite number of such shocks must occur on any path, this in turn implies that $\Pr(E) = 0$, yielding the desired contradiction.

D. Derivation of Equation (9)

This section derives equation (9). Suppose $c_L < \underline{b} < c_H < \bar{b}$. For every benefits profile $b \in [\underline{b}, \bar{b}]^5$ and rule $\mathcal{D}$, let $\tau^{\text{sep}}(b, \mathcal{D})$ denote the equilibrium probability that the group members offer c_H in period 2, given that they learned that $c = c_H$ in period 1; and let $\tau^{\text{pool}}(b, \mathcal{D})$ denote the probability that they offer c_H in period 2, given that they pooled in period 1. Then, group member i's continuation value from a period-1 separating offer is now

$$W^{\text{sep}}(\mathcal{D}) \equiv p \int_b (b_i - c_L)\, dF(b) + (1-p) \int_b \tau^{\text{sep}}(b, \mathcal{D})(b_i - c_H)\, dF(b) \,,$$

and her continuation value from a period-1 pooling offer is

$$W^{\text{pool}}(\mathcal{D}) \equiv p \int_b \Big[b_i - \tau^{\text{pool}}(b, \mathcal{D}) c_H - \big(1 - \tau^{\text{pool}}(b, \mathcal{D})\big) c_L\Big]\, dF(b) + (1-p) \int_b \tau^{\text{pool}}(b, \mathcal{D})(b_i - c_H)\, dF(b) \,.$$

Hence,

$$\begin{aligned}\Delta(\mathcal{D}) &\equiv W^{\text{sep}}(\mathcal{D}) - W^{\text{pool}}(\mathcal{D}) \\ &= p \int_b \tau^{\text{pool}}(b, \mathcal{D})(c_H - c_L)\, dF(b) + +(1-p) \int_b \big[\tau^{\text{sep}}(b, \mathcal{D}) - \tau^{\text{pool}}(b, \mathcal{D})\big](b_i - c_H)\, dF(b) \,;\end{aligned}$$

and the difference in continuation values, $\Delta(\mathcal{D}) - \Delta(\mathcal{D}^i)$, is equal to

$$\begin{aligned}
&\int_b \big[\tau^{\text{pool}}(b,\mathcal{D}) - \tau^{\text{pool}}(b,\mathcal{D}^i)\big]\big[c_H - pc_L - (1-p)b_i\big]\, dF(b) \\
&\qquad\qquad\qquad + \int_b \big[\tau^{\text{sep}}(b,\mathcal{D}) - \tau^{\text{sep}}(b,\mathcal{D}^i)\big](b_i - c_H)\, dF(b) \\
&= (1-p)\int_b \big[\tau^{\text{pool}}(b,\mathcal{D}^i) - \tau^{\text{pool}}(b,\mathcal{D})\big](b_i - b^*)\, dF(b) \\
&\qquad\qquad\qquad + (1-p)\int_b \big[\tau^{\text{sep}}(b,\mathcal{D}) - \tau^{\text{sep}}(b,\mathcal{D}^i)\big](b_i - c_H)\, dF(b)\ ,
\end{aligned}$$

where the equality follows from the definition of b^*.

Next, we turn to the agent's incentive-compatibility constraints. If the group members choose to separate, then the low-type agent's (binding) constraint under rule $\mathcal{D}$ is

$$\overline{x}^L - c_L + \delta \times 0 = 0 + \delta(c_H - c_L)\int_b \tau^{\text{sep}}(b,\mathcal{D})\, dF(b)\ ,$$

which allows us to define the first-period offer

$$\overline{x}^L(\mathcal{D}) \equiv c_L + \delta(c_H - c_L)\int_b \tau^{\text{sep}}(b,\mathcal{D})\, dF(b)\ .$$

It follows that for any rule $\mathcal{D}$, the net value of separation to group member i is given by

$$\varphi(\mathcal{D}) \equiv p\big[b_i - \overline{x}^L(\mathcal{D})\big] - (b_i - c_H) + \delta\Delta(\mathcal{D})\ .$$

This in turn implies that

$$\begin{aligned}
\varphi(\mathcal{D}) - \varphi(\mathcal{D}^i) &= \delta(1-p)\int_b \big[\tau^{\text{pool}}(b,\mathcal{D}^i) - \tau^{\text{pool}}(b,\mathcal{D})\big](b_i - b^*)\, dF(b) \\
&\quad + \delta(1-p)\int_b \big[\tau^{\text{sep}}(b,\mathcal{D}) - \tau^{\text{sep}}(b,\mathcal{D}^i)\big]\left(b_i - \frac{c_H - pc_L}{1-p}\right) dF(b) \\
&= \delta(1-p)\int_b \big[\tau^{\text{pool}}(b,\mathcal{D}^i) - \tau^{\text{pool}}(b,\mathcal{D})\big](b_i - b^*)\, dF(b) \\
&\quad + \delta(1-p)\int_b \big[\tau^{\text{sep}}(b,\mathcal{D}) - \tau^{\text{sep}}(b,\mathcal{D}^i)\big](b_i - b^*)\, dF(b)
\end{aligned}$$

$$= \delta(1-p) \int_b \Big[\big(\tau^{\text{pool}}(b, \mathcal{D}^i) - \tau^{\text{pool}}(b, \mathcal{D}) \big) + \big(\tau^{\text{sep}}(b, \mathcal{D}) - \tau^{\text{sep}}(b, \mathcal{D}^i) \big) \Big] (b_i - b^*) \, dF(b) \,,$$

as desired.