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Sophie Zhou, Frederick van der Ploeg

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Abstract

We study climate change in a model with a carbon-intensive and a green sector, each subject to stochastic sectoral productivity shocks, and show how the underlying economic structure affects the risk-adjusted discount rate and the climate risk premium in the social cost of carbon (SCC). Consumption growth, aggregate consumption volatility, and the climate beta are all affected by the elasticity of substitution between the two sectors and the relative size of the sectors, and vary as the green transition progresses. The climate risk premium is hump-shaped during the green transition, with the climate beta playing a dominant role in its magnitude. For sufficiently strong substitutability between the two sectors and sufficiently low correlation between the sectoral shocks, decarbonisation can temporarily reduce aggregate consumption risk, as the climate beta becomes negative in the mid phase of the transition. The risk-adjusted discount rate first falls then rises during the green transition, leading to a SCC to GDP ratio that rises then falls as the green sector grows. We illustrate our analytical results numerically.

JEL-Codes: E600, G120, H230, O410, Q540.

Keywords: social cost of carbon, climate beta, carbon risk premium, two-sector model, asset pricing.

Sophie Zhou
Deutsche Bundesbank Research Centre
Wilhelm-Epstein-Straße 14
Germany – 60431 Frankfurt am Main
sophie.zhou@bundesbank.de

Frederick van der Ploeg
Department of Economics
University of Oxford
United Kingdom – Oxford, OX1 3UQ
rick.vanderploeg@economics.ox.ac.uk

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1 Introduction

There is growing interest on how macroeconomic uncertainty affects the social discount rate and the social cost of carbon (SCC) (e.g., Lemoine, 2021; van den Bremer and van der Ploeg, 2021). Existing studies mainly focus on the SCC at the present time because in these one-sector models the SCC tends to grow in tandem with the economy if damages are proportional to economic activity. These studies, however, cannot address the impact of a changing economic structure during the green transition. Our aim is thus to investigate, analytically and quantitatively, how a changing economic structure during the green transition interacts with macroeconomic uncertainty and affects the social discount rate and the SCC over time.

We focus on the changing composition of the climate risk premium, i.e., the factor by which the deterministic SCC is multiplied to take account of uncertainty,[1] during the green transition. We show that the climate risk premium consists of a positive precautionary term and an insurance term, which has the opposite sign to the climate beta. Here the climate beta is the elasticity of marginal global warming damages with respect to aggregate output, so that it is positive (negative) if marginal damages from global warming increase (curb) aggregate economic risk and thus the insurance term in the climate risk premium is negative (positive).[2] Because aggregate consumption risk depends on the composition of the economy, whether climate change mitigation provides an insurance value to aggregate consumption and how large the insurance value is depends on the structure of the economy.

We extend the typical one-sector framework of integrated assessment models (IAMs) to a two-sector setting. While both sectors contribute to aggregate output, only the carbon-based (or "dirty") sector contributes to carbon emissions and global warming. We use a simple two-period model with exogenous stochastic sectoral productivity shocks for our analytical analysis, but extend the model to infinite horizon for our quantitative assessment.

[1] This reflects opposite changes in the social discount rate.

[2] Insurance value is linked to the "consumption beta" in the Consumption-based Capital Asset Pricing Model (CCAPM) (Lucas, 1978; Breeden, 1979). According to the CCAPM, the risk premium should be proportional to the beta of the project, defined as the elasticity of the project's net benefit with respect to aggregate consumption. Since a project with a negative beta dampens aggregate fluctuations, it offers insurance value for the overall portfolio and, thus, future profits from the project should be discounted at a lower rate (and the value of the project will be higher).

We show that substitutability and the relative size of the two sectors, and the correlation and relative size of the sectoral shocks, are crucially important for the climate risk premium, the social discount rate, and the SCC. Starting from an initially small clean sector, aggregate consumption growth first falls then rises, as the clean sector grows during the green transition. The less substitutable the two sectors are, the larger is the magnitude of change. Further, both the precautionary and the insurance motives change during the green transition, since volatility of aggregate consumption growth is affected by the varying composition of volatilities from the two sectors over time. The insurance value is further affected by the green transition through its effect on the climate beta, since the sign and size of the climate beta depend on the substitutability and relative size of the two sectors. Starting from a relatively large dirty sector, if outputs from the two sectors are sufficiently substitutable, as the clean sector becomes larger during the green transition, the climate beta can become negative, leading to a positive insurance value for climate change mitigation. However, as the clean sector eventually dominates the economy, substitution between sectors matters less. The climate beta eventually approaches a positive constant and the insurance value again becomes negative. The more negatively correlated the sectoral shocks, the smaller the relative size of the clean sector shock, and the higher the substitutability between the sectors, the more sizeable are changes in the climate beta during the green transition.

We seek to understand how the precautionary and insurance components of the climate risk premium change during the green transition. By demonstrating that consumption growth, aggregate volatility, and the climate beta depend on the elasticity of substitution and the relative size of the green sectors, we shed light on the importance of the economic structure for carbon pricing and climate policy. Three key insights emerge from our analysis. First, the climate risk premium is hump-shaped during the green transition. This highlights the importance of economic structure for the role of uncertainty. As the climate risk premium is mostly dominated by the insurance value, structural change during the green transition is particularly important for the climate beta and the insurance value. Second, unlike in a single-sector model, the risk-adjusted discount rate in a two-sector economy is non-monotonic during the green transition. As a result, the SCC to GDP ratio also varies over time: it first rises then falls, as the clean sector grows during the green transition. Third, the effect of decarbonization on aggregate consumption risk varies during the green transition. This suggests that the relation between climate policy and other policy objectives (such as promoting growth and mitigating economic recessions) also change over time. Climate policy may initially lower both economic growth and aggregate uncertainty, while raising both later on.

Our paper contributes to a strand of the literature that analyses the effect of uncertainty on the SCC (e.g., Ackerman et al., 2013; Jensen and Traeger, 2014; Bretschger and Vinogradova, 2018; Cai and Lontzek, 2019; van den Bremer and van der Ploeg, 2021; and Lemoine, 2021). Like Hambel et al. (2021), who use a continuous-time framework to study the interplay between asset diversification motives and climate change mitigation, we use a two-sector model but our focus is on the effect of a changing economic structure on the precautionary and insurance components of the climate risk premium. In particular, we highlight how the substitutability between sectors and the relative size of the green sector share, affect climate premium risk during the green transition.

Our paper also relates to Gollier (2021) who uses a one-sector model to demonstrate that ignoring the effect of the climate beta on the social discount rate can lead to large welfare losses.[3] Lemoine (2021) criticizes that "conventional models of climate change imply that worlds with high emissions are also worlds in which marginally reducing emissions avoids relatively less warming".[4] While these studies all focus on different specifications of the climate damage function, our paper brings in the economic structure as a new consideration that affects the climate beta, which further allows us to study how the climate beta may change over time.

Section 2 sets up a simple two-period IAM with exogenous stochastic sectoral productivity shocks. Section 3 analyzes the SCC and the risk-adjusted discount rate, while Section 4 shows how the risk-adjusted discount rate, through the variance of consumption growth rate and the climate beta, changes with the economic structure.

[3]If the main source of uncertainty is in the climate system or the damages from warming, there is a positive insurance value of climate change mitigation and thus a negative climate beta (e.g., Sandsmark and Vennemo, 2007; Daniel et al., 2019). However, if multiple sources of uncertainty are considered, Nordhaus (2011) deduces from his calibrated IAM that productivity uncertainty outweighs the uncertainties in the climate system and the damage function, leading to a *positive* correlation between climate damage and aggregate consumption and thus a positive climate beta. The use of multiplicative damages in most IAMs constitutes a built-in mechanism for a *positive* climate beta (close to one), as "doubling income also doubles absolute climate damages, all else being equal" (Dietz et al., 2018).

[4]If warming is concave in emissions, he shows that there is positive insurance value of climate change mitigation even with a multiplicative damage function.

Section 5 calibrates an infinite-horizon version of the model, and Section 6 presents the quantitative results. Section 7 concludes.

2 Two-period IAM with sectoral productivity shocks

We set up a simple analytical IAM with a production structure similar to that of Acemoglu et al. (2012) and a carbon-climate module following Matthews et al. (2009), van der Ploeg (2018), and Dietz and Venmans (2019). There are two time periods. In period 0, agents produce with exogenous technologies, decide on technology investment for period 1, and consume. In period 1, each of the two sectors is subject to a sectoral productivity shock. Production in period 1 occurs after sectoral shocks hit, but period 1 productivity is a priori uncertain.

2.1 Production

Final goods Final goods Y_t are produced by perfectly competitive firms using two sectoral aggregates $Y_{c,t}$ (clean goods) and $Y_{d,t}$ (dirty goods):

$$Y_t = \left[\theta_c^{\frac{1}{\epsilon}} Y_{c,t}^{\frac{\epsilon-1}{\epsilon}} + \theta_d^{\frac{1}{\epsilon}} Y_{d,t}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where $\epsilon > 0$ is the elasticity of substitution between the two sectors, θ_c and θ_d are the sectoral weights of clean and dirty goods, respectively, and $\theta_c + \theta_d = 1$. The price of final goods is denoted by P_t.

Sectoral aggregates The sectoral aggregate $Y_{j,t}$ is produced by perfectly competitive firms using labour $L_{j,t}$ and a continuum of intermediate goods $x_{j,i,t}$:

$$Y_{j,t} = L_{j,t}^{1-\alpha} \int_{i=0}^{1} A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha} di, \tag{2}$$

where $A_{j,i,t}$ denotes the productivity of intermediate good i of sector j and $0 < \alpha < 1$ is the share of intermediates in the production of the sectoral aggregate.

Intermediate goods Each intermediate good is produced by a monopolist,

$$x_{j,i,t} = I_{j,i,t}^{1-\nu} E_{j,i,t}^{\nu}, \tag{3}$$

where $I_{j,i,t}$ is a "machine" that depreciates immediately after use and $E_{j,i,t}$ is energy use. Each machine costs ψ units of final goods to produce[5], while energy in sector j costs $\psi_{E,j}$ units of final goods. Intermediate good producers thus maximize profits

$$\max_{x_{j,i,t}} \pi_{j,i,t} = P_{j,i,t}x_{j,i,t} - \psi P_t I_{j,i,t} - \psi_{E,j} P_t E_{j,i,t} \tag{4}$$

subject to their production function (3) and demand for their goods.

Labour supply Labour is mobile between the two sectors. Total labour supply is inelastic and normalized to 1, so labour market equilibrium is given by

$$L_{c,t} + L_{d,t} = 1. \tag{5}$$

2.2 Consumption and welfare

Households derive utility from consuming final goods in period 0 and in period 1, denoted by C_0 and C_1, respectively. Intertemporal welfare is

$$W = u(C_0) + \delta \mathbb{E}_0 \left[u(C_1) \right], \tag{6}$$

where δ denotes the discount factor. We assume power utility

$$u(C_t) = \frac{C_t^{1-\eta}}{1-\eta} \qquad \text{with} \qquad C_t = (1-d_t)\tilde{C}_t \tag{7}$$

for $t = 0, 1$, where C_t is aggregate consumption, which is also our measure of GDP, $\tilde{C}_t$ is pre-damage consumption, d_t the climate damage ratio, and $\eta > 0$ the coefficient of relative risk aversion (or inverse of the elasticity of intertemporal substitution).

[5] We set $\psi = (1-\nu)(\alpha^2\nu^\nu)^{1/(1-\nu)}$ to ease notation as in Acemoglu et al. (2012) (see Appendix A).

2.3 Carbon emissions, temperature, and climate damage

Global mean temperature is approximately linear in cumulative carbon emissions and the warming response to emissions is virtually immediate and remains at that constant level thereafter (e.g., Matthews and Caldeira, 2008; Matthews et al., 2009). Both the IPCC and economists used such a temperature model (e.g., van der Ploeg, 2018; Dietz and Venmans, 2019) and so will we. The dirty sector generates carbon emissions as by-product of production: $M_t = \omega_2 Y_{d,t}$, where $\omega_2 > 0$ is the emission intensity (emissions per unit of output of dirty goods). The change in global mean temperature, denoted by T_t, is thus proportional to output of dirty goods:

$$T_t = T_{t-1} + \omega_1 M_t = T_{t-1} + \omega_1 \omega_2 Y_{d,t}, \tag{8}$$

where $\omega_1 > 0$ is the transient climate response to cumulative emission (or TCRE) or the marginal effect of cumulative emissions on temperature.

The damage ratio d_t and total damages TD_t increase with global warming and pre-damage output, so that

$$d_t = aT_t^{\kappa} Y_t^{\zeta-1} \text{ and } TD_t = d_t \tilde{C}_t = aT_t^{\kappa} Y_t^{\zeta-1} \tilde{C}_t, \tag{9}$$

where $a > 0, \kappa > 0$, and $\zeta > 1$. In the DICE model developed by Nordhaus and Moffat (2017) damages are proportional to temperature squared and aggregate output, corresponding to $\kappa = 2$ and $\zeta = 1$.[6] We use this case in Sections 5 and 6.

2.4 Static equilibrium conditions

Profit maximization and labour and goods market equilibrium give (Appendix A.1):

$$Y_t = A_t, \tag{10}$$

$$Y_{j,t} = \tilde{\theta}_j \psi_{E,j}^{-\nu\alpha} A_{j,t}^{(1-\alpha)\epsilon} A_t^{1-(1-\alpha)\epsilon}, \tag{11}$$

$$L_{j,t} = \tilde{\theta}_j A_{j,t}^{-\phi} A_t^{\phi}, \tag{12}$$

$$E_{j,t} = \nu\alpha^2 \tilde{\theta}_j \psi_{E,j}^{-1} A_j^{-\phi} A_t^{1+\phi}, \tag{13}$$

[6] This damage ratio nests Dietz et al. (2018) as a special case, where climate damage in each period depends only on the additional warming of that period relative to the previous period. Setting $\zeta = 1$ and replacing T_t by $T_t - T_{t-1}$ in (9) gives the damage ratio used in Dietz et al. (2018).

where $\phi \equiv (1-\epsilon)(1-\alpha)$ and $\tilde{\theta}_j \equiv \theta_j \psi_{E,j}^{\nu\alpha(1-\epsilon)}$. Here $A_{j,t} \equiv \int_{i=0}^{1} A_{j,i,t} di$ is aggregate sectoral productivity and $E_{j,t} \equiv \int_{i=0}^{1} E_{j,i,t} di$ is aggregate energy use in sector j, while

$$A_t \equiv \left[\tilde{\theta}_c A_{c,t}^{-\phi} + \tilde{\theta}_d A_{d,t}^{-\phi}\right]^{-\frac{1}{\phi}} \tag{14}$$

defines aggregate total factor productivity.

Market clearing for final goods implies that pre-damage consumption is output net of final goods used in intermediate goods production. Let $k_{j,i,t}$ denote the unit production cost of $x_{j,i,t}$ in units of final goods. We then have

$$\tilde{C}_t = Y_t - \int_{i=0}^{1} k_{c,i,t} x_{c,i,t} di - \int_{i=0}^{1} k_{d,i,t} x_{d,i,t} di = (1-\alpha^2) Y_t, \tag{15}$$

where the second equality follows from the equilibrium levels of $x_{j,i,t}$ and $k_{j,i,t}$ (see Appendix A.1). Hence, aggregate consumption (and thus GDP in our model) equals

$$C_t = (1-\alpha^2)(1-d_t) A_t. \tag{16}$$

For future reference, we define the marginal damage caused to consumption in period t of emitting one unit in period 0 by $D_t \equiv -\partial C_t / \partial M_0$. Combining equations (8)-(9) and (15), we establish that the marginal damage of carbon emission equals

$$D_t = h T_t^{\kappa-1} A_t^{\zeta}, \tag{17}$$

where $h \equiv a\omega_1 \kappa (1-\alpha^2) > 0$ is a constant.

2.5 Stochastic shocks to sectoral productivities

Production of intermediate goods in period 1 occurs after the realization of a sector-specific productivity shock. This shock is a priori uncertain. For now, we do not specify the exact distribution of the sector-specific productivity shocks, but assume that the resulting aggregate consumption in period 1 is log-normally distributed such that $\ln(C_1/C_0) \sim \mathcal{N}(\mu, \sigma^2)$, where μ and σ denote the mean and standard deviation.

3 The SCC and the climate risk premium

3.1 The social cost of carbon

The SCC is defined as the expected net present value of the marginal damages caused by one unit of emissions in period 0, evaluated in units of consumption in period 0:

$$\text{SCC} \equiv \mathbb{E}_0\left[\sum_{t=0}^{t=1}\delta^t \frac{u'(C_t)}{u'(C_0)}D_t\right], \tag{18}$$

where $\delta^t u'(C_t)/u'(C_0)$ is the stochastic discount factor. We write the SCC as

$$\text{SCC} = D_0 + \left(\frac{\delta u'\left(\mathbb{E}_0[C_1]\right)\mathbb{E}_0\left[D_1\right]}{u'(C_0)}\right)(1+\Pi), \tag{19}$$

where $\Pi \equiv \mathbb{E}_0\left[u'(C_1)D_1\right]/\left[u'\left(\mathbb{E}_0[C_1]\right)\mathbb{E}_0\left[D_1\right]\right] - 1$ is the climate risk premium. This premium is the markup of the first-period SCC on its certainty-equivalent value (or deterministic first-period SCC for short), $\frac{\delta u'(\mathbb{E}_0[C_1])\mathbb{E}_0[D_1]}{u'(C_0)}$. We then write the SCC as the present discounted value of expected marginal damages:

$$\text{SCC} = \sum_{t=0}^{t=1} e^{-rt}\mathbb{E}_0[D_t] = D_0 + e^{-r}\mathbb{E}_0[D_1]. \tag{20}$$

Here the risk-adjusted discount rate for climate mitigation follows from (19):

$$r = \rho - \ln\left[u'\left(\mathbb{E}_0[C_1]\right)/u'(C_0)\right] - \Pi, \tag{21}$$

where $\rho \equiv -\ln\delta$ denotes the rate of pure time preference.[7]

3.2 The climate risk premium

The climate risk premium (see Appendix A.3) for the power utility function (7) is

$$\Pi \approx \frac{1}{2}\eta(1+\eta)\sigma^2 - \beta\eta\sigma^2. \tag{22}$$

[7] This uses the expression $r = \rho - \ln\left[u'\left(\mathbb{E}_0[C_1]\right)/u'(C_0)\right] - \ln(1+\Pi)$ and $\ln(1+\Pi) \approx \Pi$ for small Π.

Here the relative variance (the square of the coefficient of variation) of consumption is $\sigma^2 \equiv \frac{\mathrm{Var}_0[C_1]}{(\mathbb{E}_0[C_1])^2}$, the coefficient of relative risk aversion is $\eta = -\frac{\mathbb{E}_0[C_1]u''(\mathbb{E}_0[C_1])}{u'(\mathbb{E}_0[C_1])}$, the coefficient of relative prudence is $1 + \eta = -\frac{\mathbb{E}_0[C_1]u'''(\mathbb{E}_0[C_1])}{u''(\mathbb{E}_0[C_1])}$, and the climate beta is $\beta \equiv \frac{\partial D_1}{\partial C_1}\frac{C_1}{D_1}\Big|_{\mathbb{E}_0[C_1]}$. The climate beta β is defined as the elasticity of the marginal damage of carbon emissions with respect to aggregate consumption, evaluated at the expected value of period 1 consumption. It corresponds to the "consumption beta" associated with emission reductions (cf. Dietz et al., 2018; Gollier, 2021).

The first term in the expression for the climate risk premium (22) is the prudence term and captures the risk aversion and precautionary motives of households. It increases in the coefficient of relative risk aversion η, the coefficient of relative prudence $1 + \eta$, and the coefficient of relative variation of future consumption, $\frac{\mathrm{Var}_0[C_1]}{\mathbb{E}_0[C_1]^2} > 0$. This term is positive and pushes up the SCC.

The second term in the climate risk premium (22) is the insurance term. It depends on the sign of the climate beta and its magnitude increases in the coefficient of relative risk aversion and the coefficient of relative variation of future consumption. If the marginal damage from global warming and future consumption are positively correlated, $\beta > 0$ so there is a negative insurance value. In that case, the climate risk premium is adjusted downwards. Conversely, if the future marginal damage from global warming and consumption are negatively correlated, $\beta < 0$ and there is positive insurance value so the climate risk premium is adjusted upwards.

Using equations (21) and (22), the risk-adjusted discount rate becomes

$$r = \rho + \eta g - \frac{1}{2}\eta(1 + \eta)\sigma^2 + \beta\eta\sigma^2, \tag{23}$$

where $g \equiv \ln[\mathbb{E}_0[C_1]/C_0]$ is expected consumption growth, $\rho + \eta g$ the deterministic discount rate, and the last two terms the inverse of the climate risk premium, $-\Pi$.

3.3 Extension to infinite horizons

Our analysis can be easily extended to an infinite-horizon setting if shocks to consumption growth are serially uncorrelated. Equation (6) for intertemporal welfare becomes $W = u(C_0) + \mathbb{E}_0\left[\sum_{t=1}^{\infty} \delta^t u(C_t)\right]$. Instead of (20), the SCC thus becomes

$$\text{SCC} = D_0 + \mathbb{E}_0\left[\sum_{t=1}^{\infty} e^{-\sum_{s=1}^{t} r_s} D_t\right] \tag{24}$$

with r_s the discount rate for climate mitigation projects corresponding to period s.

4 Economic structure and effect of uncertainty

To evaluate the component of the climate risk premium (22), we need to derive the variance of aggregate consumption growth, σ^2, and the climate beta, β, in terms of the characteristics of the underlying stochastic shocks to sectoral factor productivities. We illustrate this for three scenarios: (1) only shocks to productivity in the clean sector, (2) only shocks in the dirty sector, and (3) shocks to both sectors. The results are given in three propositions with proofs relegated to Appendix B.

We denote the mean and standard deviation of $A_{j,1}$ by by μ_{Aj} and σ_{Aj}, respectively, the correlation coefficient between $A_{c,1}$ and $A_{d,1}$ by $\rho_{Ac,Ad}$, and the mean and standard deviation of aggregate productivity A_1 by μ_A and σ_A, respectively.

4.1 Only productivity shocks in clean sector

Proposition 1. *Suppose that the expected damage ratio $\mathbb{E}_0[d_1]$ is low, and there are only productivity shocks in the clean sector. The climate risk premium is (22) with the variance of consumption growth rate and the climate beta given by*

$$\sigma^2 = \sigma_{g,c}^2 \equiv \left(\tilde{\theta}_c \mu_{Ac}^{-\phi} / \mu_A^{-\phi}\right)^2 (\sigma_{Ac}/\mu_{Ac})^2, \tag{25}$$

$$\beta_c = \zeta + \tilde{\kappa}\left[1 - (1-\alpha)\epsilon\right], \tag{26}$$

where $\tilde{\theta}_c \mu_{Ac}^{-\phi}/\mu_A^{-\phi}$ *is the clean sector share and* $\tilde{\kappa} \equiv d \ln D_1 / d \ln Y_{d,1} = (\kappa - 1)(1 - T_0/\mathbb{E}_0[T_1])$ *is the elasticity of marginal damage with respect to dirty sector production.*

The variance of consumption growth increases with the clean sector share and the coefficient of relative variation of clean sector productivity. The latter isconstant if productivity shocks are serially uncorrelated. The variance of consumption growth increases over time, as the clean sector becomes more dominant during the green transition. While initially substitution between the two sectors helps to reduce the variance of consumption growth, this effect diminishes as the dirty sector shrinks during the green transition.

As in a one-sector model (e.g. Dietz et al., 2018), the climate beta depends on the elasticities of climate damages with respect to global mean temperature, κ, and pre-damage output, ζ (cf. equation (17)). However, in our two-sector framework, the climate beta also depends on the elasticity of substitution between the outputs of the clean and dirty sectors, ϵ (see equation (1)). This elasticity affects how essential carbon emissions are as a production input.

Damages are typically assumed to be a convex function of temperature so $\kappa > 1$. For example, in the DICE model damages are a quadratic function of temperature and thus $\kappa = 2$ (Nordhaus, 2011). Given that future temperature is expected to rise, it follows that $\tilde{\kappa} > 0$. Furthermore, damages are typically taken to be proportional to pre-damage output so that $\zeta = 1$. We take these two values as base values with $\beta_c = 1 + (1 - T_0/\mathbb{E}_0[T_1])\,[1 - (1 - \alpha)\epsilon]$.

Consider the case of a Leontief production function for the final good ($\epsilon = 0$). The climate beta for our base values thus exceeds one, $1 < \beta_c = 2 - T_0/\mathbb{E}_0[T_1] < 2$. In the medium and longer run substitution is feasible ($\epsilon > 0$). This depresses the climate beta β_c and leads to a higher climate risk premium (22).

If the final goods production function is Cobb-Douglas ($\epsilon = 1$), the climate beta is $\beta_c = 1 + \alpha(1 - T_0/\mathbb{E}_0[T_1])$. A higher value of the share of intermediate goods in the production of the sectoral aggregate (α) thus increases the climate beta and the risk-adjusted discount rate but depresses the climate risk premium. If sectoral aggregates are perfect substitutes ($\epsilon \to \infty$), the climate beta is negative if temperature is still rising ($\tilde{\kappa} > 0$). The positive insurance value increases the climate risk premium.

Over time, the damage elasticity of dirty production $\tilde{\kappa}$ falls and the substitution elasticity ϵ becomes less important, particularly if the dirty sector shrinks fast ($T_0/\mathbb{E}_0[T_1]$ large). If at the end of the green transition carbon-intensive production ceases completely and temperature no longer changes, $\tilde{\kappa}$ becomes zero. We then have $\beta_c = 1$ (independent of ϵ) as the economy collapses into a single sector economy.

Departing from the base values, the climate beta also increases in the elasticity of damages with respect to temperature (κ), if goods substitutability is low $\epsilon < 1/(1-\alpha)$, and vice versa.[8] Also, if damages are less than proportional to pre-damage output (lower ζ), the climate beta is lower and the climate risk premium is higher. In general, with damages convex in temperature ($\kappa > 1$), a high elasticity of damages with respect to output (ζ), and a low elasticity of substitution between clean and dirty outputs (ϵ), the climate beta will be positive. The insurance term in the climate risk premium (22) is then negative, which pushes the SCC down. However, if the elasticity of substitution is large enough ($\epsilon > 1/(1-\alpha)$), the climate beta can be negative, in which case the SCC is higher.

4.2 Only productivity shocks in dirty sector

Proposition 2. *Suppose that the expected damage ratio $\mathbb{E}_0[d_1]$ is low and there are only productivity shocks in the dirty sector. The climate risk premium is (22) with variance of consumption growth rate and the climate beta given by*

$$\sigma^2 = \sigma_{g,d}^2 \equiv \left(\tilde{\theta}_d \mu_{Ad}^{-\phi}/\mu_A^{-\phi}\right)^2 (\sigma_{Ad}/\mu_{Ad})^2 \tag{27}$$

$$\beta_d = \beta_c + \tilde{\kappa}(1-\alpha)\epsilon/\xi_d, \tag{28}$$

where $\tilde{\theta}_d \mu_{Ad}^{-\phi}/\mu_A^{-\phi}$ is the dirty sector share and $\xi_d \equiv \partial \ln A_1/\partial \ln A_{d,1} = \tilde{\theta}_d \mu_{Ad}^{-\phi}/\mu_A^{-\phi}$ is the elasticity of aggregate productivity with respect to dirty technology.

The variance of consumption growth now increases with the dirty sector share and the coefficient of relative variation of the dirty sector productivity. If climate policy induces an increasing clean sector share over time, the economy gradually shifts away from the risky sector and aggregate risk now falls over time.

[8] Higher κ magnifies the effect of the elasticity of substitution between clean and dirty goods and the effect of a shrinking dirty sector on the climate beta.

As for the climate beta β_d, its sign and that of the insurance term depends on the productivity of the dirty sector relative to that of the clean sector (since $\tilde{\theta}_d \mu_{Ad}^{-\phi}/\mu_A^{-\phi} = 1/(1+(\tilde{\theta}_c/\tilde{\theta}_d)(\mu_{Ac}/\mu_{Ad})^{-\phi}))$. If carbon-intensive and carbon-free goods are gross substitutes ($\epsilon > 1$), $\phi = (1-\alpha)(1-\epsilon) < 0$. If over time carbon pricing or other policies lead to directed technical progress favouring carbon-free methods of production ($\mu_{Ac} \uparrow$ and $\xi_d \downarrow$), this leads to a rising risk-adjusted discount rate and falling climate risk premium. Clean innovation and emission reduction are then strategic substitutes.

The difference between Propositions 1 and 2 illustrates that it matters for the climate beta and the climate risk premium where the productivity shocks come from. This emphasizes that a changing economic structure matters for the SCC.

4.3 Productivity shocks in clean and dirty sectors

Proposition 3. *Suppose that the expected damage ratio $\mathbb{E}_0[d_1]$ is low and that there are productivity shocks in both sectors. The climate risk premium is then (22) with variance of the consumption growth rate and the climate beta given by*

$$\sigma^2 = \sigma_{g,c}^2 + \sigma_{g,d}^2 + 2\rho_{Ac,Ad}\sigma_{g,c}\sigma_{g,d} \tag{29}$$

$$\beta = (1-\lambda_d)\beta_c + \lambda_d \beta_d \tag{30}$$

where the $\sigma_{g,j}$ are given by (25) and (27), the β_j are given by (26) and (28), $\rho_{Ac,Ad}$ is the correlation coefficient between $A_{c,1}$ and $A_{d,1}$, and the dirty sector weight λ_d is

$$\lambda_d \equiv \frac{\tilde{\theta}_d^2 \mu_{Ad}^{-2(1+\phi)}\sigma_{Ad}^2 + \tilde{\theta}_c\tilde{\theta}_d(\mu_{Ac}\mu_{Ad})^{-(1+\phi)}\rho_{Ac,Ad}\sigma_{Ac}\sigma_{Ad}}{\tilde{\theta}_c^2 \mu_{Ac}^{-2(1+\phi)}\sigma_{Ac}^2 + \tilde{\theta}_d^2 \mu_{Ad}^{-2(1+\phi)}\sigma_{Ad}^2 + 2\tilde{\theta}_c\tilde{\theta}_d(\mu_{Ac}\mu_{Ad})^{-(1+\phi)}\rho_{Ac,Ad}\sigma_{Ac}\sigma_{Ad}}. \tag{31}$$

If there are multiple sources of economic uncertainty, which sector contributes more to aggregate uncertainty depends on the relative size of the two sectors. In addition, the aggregate risk depends on how the shocks in the two sectors are correlated. A negative correlation $\rho_{Ac,Ad}$ reduces volatility of aggregate consumption, while a positive $\rho_{Ac,Ad}$ exacerbates aggregate uncertainty.

The climate beta is now the weighted sum of the climate betas for when there are shocks to only the clean or dirty sectors, respectively. The sectoral weights depend on both the elasticity of substitution between clean and dirty goods (ϵ) and the coefficient of relative variation of sectoral productivities (μ_{Aj}/σ_{Aj}). Keeping μ_{Aj}/σ_{Aj} constant, if the two goods are gross substitutes ($\epsilon > 1$ and $\phi < 0$), the higher the productivity of a sector is relative to that of the other sector, the higher is the weight given to that sector. On the other hand, keeping the expected productivity of the sectors μ_{Aj} constant, the higher the relative variation of a sector's productivity relative to the other sector, the higher is the weight given to that sector. Equation (30) for the climate beta can be rewritten as

$$\beta = \zeta + \tilde{\kappa}\left[1 - (1-\alpha)\epsilon(1 - \lambda_d/\xi_d)\right]. \tag{32}$$

For our base values ($\zeta = 1$ and $\kappa = 2$), the climate beta boils down to $\beta = 1 + (1 - T_0/\mathbb{E}_0[T_1])\left[1 - (1-\alpha)\epsilon(1 - \lambda_d/\xi_d)\right]$.[9] Since $\tilde{\kappa}$ and λ_d/ξ_d both depend on the relative size of the two sectors, the climate beta β changes over time during the green transition, as we show in the next proposition.

Proposition 4. *Suppose $\zeta + \tilde{\kappa} > 0$. There exist a unique $\tilde{\mu} \in [0, \infty)$ and a unique $\tilde{\tilde{\mu}} \geq \tilde{\mu}$ such that the climate beta is always positive if the clean sector share is lower than $\tilde{\mu}$ or higher than $\tilde{\tilde{\mu}}$. For the climate beta to be negative, the clean sector share must be between $\tilde{\mu}$ and $\tilde{\tilde{\mu}}$. The higher the elasticity of substitution ϵ, the larger is the range of the clean sector share that gives rise to a negative climate beta.*

Hence, the climate beta can evolve in a non-monotonic manner through the green transition. Starting from an initially small clean sector, decarbonization increases aggregate consumption risk as it forces the economy to use a less productive sector. Once the size of the clean sector reaches a certain threshold, further decarbonization may reduce aggregate consumption risk and offer positive insurance value. As the clean sector grows, the insurance value of further decarbonization may once again turn negative so further decarbonization increases aggregate consumption risk.

Proposition 4 suggests that while decarbonization may offer diversification benefits, this benefit is temporary. This result is akin to that of Hambel et al. (2021), who assume perfect substitution between clean and dirty sectors. They find that the diversification motive initially speeds up the optimal green transition when the clean sector is small, but eventually prevents the dirty sector from being fully eliminated

[9] If damages are linear in temperature, $\kappa = 1$, the climate beta boils down to $\beta = 1$ so the structural properties of the economy only affect the climate beta if damages are convex in temperature.

once the economy becomes sufficiently green. With lower degrees of substitution and the resulting slower market dynamics, our result indicates that the diversification benefits may only appear after the clean sector reaches a certain size.

Since both the climate beta and aggregate consumption risk change over time, so does the climate risk premium. If the climate beta is sufficiently small ($\beta < \frac{1}{2}(1+\eta)$), the climate risk premium will be positive and increasing in aggregate consumption risk σ^2. The risk-adjusted discount rate is further affected by aggregate consumption growth, which, unless technologies in both sectors grow at the same rate and there is distortionary policy, should also vary over time.

5 Calibration of integrated assessment model

For our quantitative assessments, we extend our two-period to an infinite-horizon model with each time step a decade. The static equilibrium conditions in Section 2.4 are unaffected. Provided shocks to productivity growth are serially uncorrelated, the derivations of the climate risk premium and the risk-adjusted discounted rate remain valid. The expression for the SCC now corresponds to the sum of discounted marginal damages over an infinite horizon as can be seen from equation (24).

In line with our model in Section 2, we assume that there are no stochastic shocks in the first decade 2020-2029 (period 0), while all subsequent periods are subject to productivity shocks (cf. Gollier, 2021). We first calibrate the parameters of the model to annual data, and then convert them for a decadal model.

5.1 Production

We set the share of labour in the production of clean goods and of dirty goods to 2/3, so the share of intermediates is $\alpha = 1/3$. Calibrating the energy expenditure share in GDP, $(\psi_{E,c}E_{c,0} + \psi_{E,d}E_{d,0})/C_0 = \nu\alpha^2/((1-\alpha^2)(1-d_0))$ (see (A.21) in Appendix A.1), to 8% as in Grubb et al. (2018) and taking an initial damage ratio of 2% gives an energy share in production of intermediates of $\nu = 0.63$. A large range of values has been used for the elasticity of substitution between clean and dirty products in

Parameter	Value	Moment/Source
Unchanged for all scenarios		
ρ (time preference)	1%/year	Standard
η (RRA and inverse of EIS)	2	Standard
α (share of intermediates)	1/3	Standard
ν (energy share in intermediates)	0.63	Energy expenditure share: 8% (Grubb et al., 2018)
θ_d (share parameter dirty goods)	0.5	Assumption
θ_c (share parameter clean goods)	0.5	Assumption
$\psi_{E,c}/\psi_{E,d}$ (energy cost ratio)	1.5	Abiry et al. (2021)
$\psi_{E,d}$ (cost of dirty energy)	0.0112 USD/MJ	Fossil energy expenditure
A_0^{annual} (pre-damage GDP)	100.5 trillion USD	2019 global GDP (World Bank)
ω_1 (TCRE)	1.5 °C/TtC	Matthews et al. (2009)
κ (temperature elasticity of damage)	2	Hsiang et al. (2017)
Baseline values		
ϵ (elasticity of substitution)	3	Base value
$A_{c,0}^{\text{annual}}$ (clean productivity)	14.99	2019 clean energy use: 91 exajoules (BP, 2021)
$A_{d,0}^{\text{annual}}$ (dirty productivity)	34.30	2019 fossil energy use: 490 exajoules (BP, 2021)
ω_2 (emission intensity)	0.1 ktC/USD	2019 carbon emission: 38 GtCO2 (EU Science Hub)
$\rho_{c,d}$ (correlation of prod. shocks)	0	Base value
σ_c/σ_d (relative risk)	1	Base value
μ_c (mean of clean prod. shock)	0.0285	Jointly calibrated to average growth rate clean (fossil) energy consumption 2000-2019: 3.2% (2.2%) (BP, 2021), avg. GDP growth rate: 2%
σ_c (sd of clean sector shock)	0.0227	
μ_d (mean of dirty sector shock)	0.0208	
σ_d (sd of dirty sector shock)	0.0227	
ζ (income elasticity of damage)	1	Assumption
a (damage scaling)	0.02	Initial damage ratio: 2% (Nordhaus and Moffat, 2017)

Table 1: Calibration of parameters on annual basis

final goods production (e.g., 2 in Papageorgiou et al., 2017, 5 in Benmir and Roman, 2020, and 26 in Abiry et al., 2021). We set $\epsilon = 3$ but also consider values up to 10.

We set initial annual aggregate consumption to the 2019 value of world GDP, 87.55 trillion USD (World Bank, 2021). Using pre-damage output as numeraire ($P_t = 1$ for all $t \geq 0$) and a damage ratio of 2% at 1 degree warming at the start of the first decade, (16) sets initial, pre-damage output (A_0^{annual}) to 100.5 trillion USD per year.

For the size of the two sectors, we set $\theta_c = \theta_d = 0.5$ so that the output ratio of the two sectors is determined by the relative price alone. We set the relative cost of clean energy to that of dirty energy ($\psi_{E,c}/\psi_{E,d}$) to 1.5 following Abiry et al. (2021) and Hambel et al. (2021). The dirty energy cost parameter $\psi_{E,d}$ then follows from

Period 0 (2020-2029)	Value
A_0 (initial pre-damage decadal gross output)	1117 trillion USD
$A_{c,0}$ (clean productivity)	171.5
$A_{d,0}$ (dirty productivity)	378.1

Note: corresponding to the baseline example of $\epsilon = 3$, $\rho = 0$, $\sigma_c/\sigma_d = 1$, $\zeta = 1$.

Table 2: Output and productivity in initial decade

the energy expenditure share ($(\psi_{E,c}E_{c,0}^{\text{annual}} + \psi_{E,d}E_{d,0}^{\text{annual}})/C_0^{\text{annual}} = 8\%$), and is set to 0.0112 USD per megajoule (or 68.8 USD per barrel of oil equivalent).

Initial annual productivity levels are derived from calibrating initial annual clean and dirty energy use ($E_{c,0}^{\text{annual}}$, $E_{d,0}^{\text{annual}}$) to the 2019 clean and fossil energy consumption in primary energy: 91.4 exajoules for clean energy and 490.1 exajoules for fossil energy (BP, 2021). The resulting productivity levels depends on the elasticity of substitution ϵ. For $\epsilon = 3$, we obtain $A_{c,0}^{\text{annual}} = 14.99$ and $A_{d,0}^{\text{annual}} = 34.30$.

5.2 Probability distributions of productivity shocks

In Section 2 we were agnostic about the distribution of the sectoral productivity shocks, requiring only that they generate log-normally distributed aggregate consumption. We now assume sectoral productivities follow from $A_{j,i,1} = e^{z_j} A_{j,i,0}$, where $z_j \sim \mathcal{N}(\mu_j, \sigma_j)$ is a normally distributed sectoral productivity shock.[10]

We consider a range of possible values for the correlation coefficient $\rho_{c,d}$ between the sectoral productivity shocks and the ratio of the standard deviations of the two shocks (σ_c/σ_d). To calibrate the mean and standard deviation of the sectoral shocks, we first compute the average growth rate of sectoral productivities (i.e., growth rate of A_c and A_d) from (13), which implies that the growth rate of energy consumption depends on the growth rate of sectoral productivity and GDP ($g_{E_j} = -\phi g_{A_j} + (1+\phi)g_A$). Using the average growth rate of energy consumption from 2002 to 2021 for clean (3.23%) and fossil energy (2.17%) (BP, 2021) and an average GDP growth rate of 2% per year, we derive an average growth rate of 2.92% per year for clean technology and of 2.12% per year for dirty technology. Given $\rho_{c,d}$

[10] Note that due to CES production function for the final goods (equation 1), the distribution of aggregate consumption can only be approximately log-normal, see Schwartz and Yeh (1982). This approximation is however very good as Appendix OA3 shows.

and σ_c/σ_d, we then jointly calibrate the means and standard deviations of the two sectoral productivity shocks to match the average technology growth rate of the two sectors and the average GDP growth rate per year. For example, if $\rho_{c,d} = 0$ and $\sigma_c/\sigma_d = 1$, we obtain the means $\mu_c = 0.0285$ and $\mu_d = 0.0208$, and the standard deviations of sectoral productivity growth, $\sigma_c = \sigma_d = 0.0227$. Note that the average growth rate of clean technology then equals $\mu_c + \frac{1}{2}\sigma_c^2 = 2.85\%$ per year, and similarly for the average growth rate of dirty technology.

5.3 Global warming and climate damages

The transient climate response to cumulative emissions (TCRE) in equation (8) is set to $\omega_1 = 1.5°$C per trillion tonnes of carbon (TtC) (cf. Matthews et al., 2009). We set the temperature elasticity for global warming damages to $\kappa = 2$ (cf. Nordhaus and Moffat, 2017; Hsiang et al., 2017). The emission intensity parameter ω_2 follows from annual global carbon emissions (M_0^{annual}) in 2019, i.e., 10.4 gigaton of carbon (GtC) or 38 GtCO2, and initial output of carbon-intensive goods in 2019. The result depends on the elasticity of substitution between clean and dirty goods, ϵ. For example, with $\epsilon = 3$ we obtain $\omega_2 = 0.106$ kilotons of carbon per USD. We assume that global warming damages are proportional to pre-damage GDP, so that $\zeta = 1$. Finally, the damage scaling parameter a in equation (9) is derived from calibrating the initial damage ratio at 1°C to a common value of 2 % (e.g. Nordhaus and Moffat, 2017). Using a global mean temperature of 1°C above pre-industrial level in 2019, $T_0^{\text{annual}} = 1$, and given $\zeta = 1$, we obtain $a = 0.02$.

5.4 Reformulation as a decadal model

Using the average growth rate of clean technology (2.92%) and of dirty technology (2.12%), we derive $A_{c,0} = 171.5$ and $A_{d,0} = 378.1$. Using equation (14), we obtain decadal GDP for the period 2020-2029, i.e, $A_0 = 1117$ trillion USD.

For every decade after the initial decade (provided that productivity shocks are serially uncorrelated) the change in log sectoral productivity in each decade corresponds to the sum of 10 independent draws of an annual growth rate (cf. Gollier, 2021), so that

$$z_j^{10} \equiv \ln \frac{A_{j,1}}{A_{j,0}} = \sum_{i=1}^{10} z_{j,i}, \qquad z_{j,i} \sim \mathcal{N}(\mu_j, \sigma_{z,j}). \tag{33}$$

Hence, $z_j^{10} \sim \mathcal{N}(10\mu_j, \sqrt{10}\sigma_j)$, and $Cov(z_c^{10}, z_d^{10}) = 10Cov(z_c, z_d)$. The decadal values of the mean and standard deviation (μ_j and σ_j) and of the correlation coefficient ($\rho_{c,d}$) between the two sectoral productivity shocks can then be derived.

6 Quantitative assessment

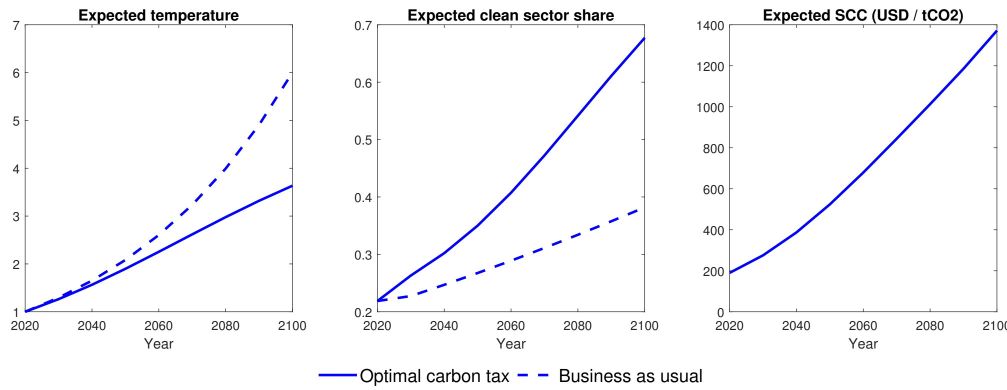
6.1 Optimal and business-as-usual simulation for base values

Figure 1 plots expected temperature, the clean sector share, and the SCC from 2020 to 2100 under the optimal and business-as-usual (BAU) scenarios.[11] Since productivity shocks are calibrated to generate higher expected growth rate for the clean sector, the clean sector share grows even under BAU. However, under BAU the clean sector rises only to about 38% by 2100, while expected temperature rises to 6°C above the pre-industrial level. In the optimal scenario, the expected clean sector share rises to almost 70% and expected temperature rises to 3.6°C at the end of the century.[12] The emissions reductions are attained by implementing a carbon price that rises monotonically from \$190/tCO2 in 2020 to \$1371/tCO2 in 2100.[13]

[11] We solve the social planner's solution numerically for 28 decades from year 2030 to 2300 using Monte-Carlo integration. Using the sectoral shock parameters, we generate 300,000 sets of sample paths. Each set has two sample paths, one for clean and one for dirty technology. Since our model has an asymptotic steady state where the clean sector share approaches 100%, we assume that the economy is in steady state after 2300 with negligible dirty production. One may also use an iterative procedure, which guesses the start of the steady state (say year 2200), solves for optimal allocation during the green transition, compares the clean sector share at the initial guess to 100%, updates the guess, and iterates until the clean sector share indeed reaches approximately 100% at the beginning of the steady state. Since in all our simulations the clean sector share approaches 100% by 2300, imposing a steady state from 2300 onwards is a sufficiently good approximation.

[12] This relatively high temperature reflects the relatively low damage ratio in the calibration in line with Nordhaus and Moffat (2017).

[13] Our initial SCC is very close to the preferred uncertainty-adjusted estimate of \$185/tCO2 (also 2020 US dollars) of Rennert et al. (2022), which is 3.6 times higher than the US government's current value of \$51 per tCO2.



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure 1: Expected temperature, clean sector share, and social cost of carbon under optimal carbon pricing and business as usual scenarios

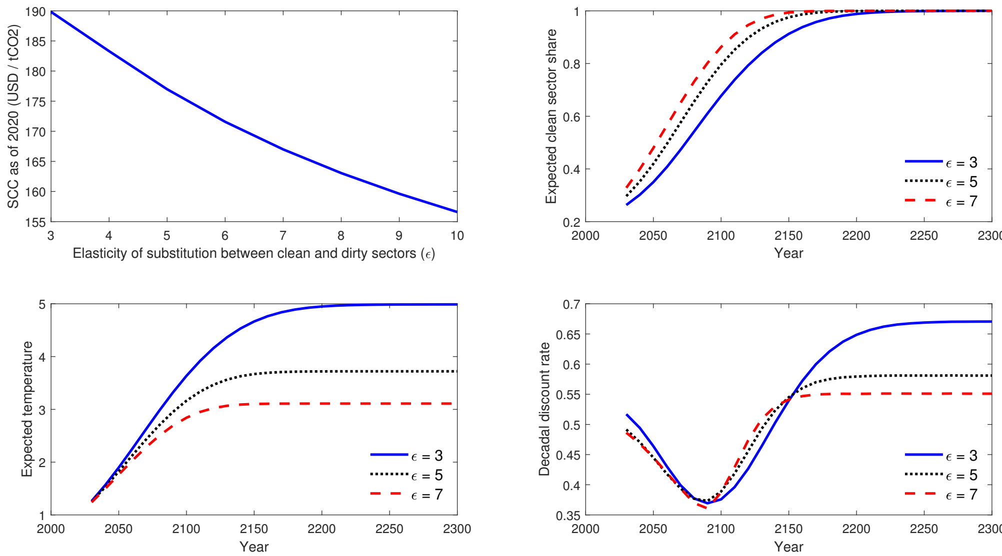
6.2 The social cost of carbon: comparative statics

We show how the initial SCC depends on the substitutability between the two sectors, the clean sector share at the start of period 0, the correlation between sectoral shocks, and the relative size of the sectoral shocks. We do this by varying each of these four parameters while keeping the other three at their baseline value.[14]

The top left panel of Figure 2 shows that the initial SCC declines with the elasticity of substitution between the two sectors. This is intuitive, since if the elasticity of substitution is high, the economy can more readily substitute away from dirty production. The top right and lower left panels indicate that as a result, with a higher elasticity of substitution, the clean sector rise more rapidly and temperature rises less, leading to a lower climate damage ratio. Interestingly, the discount rate in the bottom right panel first falls and then rises during the green transition. However, even though with lower substitutability the discount rate is higher both initially and in the long run, in our simulations the effect of the higher damage ratio outweighs the effect of discounting on the SCC.

Figure 3 plots the effects of varying initial clean sector productivity $A_{c,0}^{\text{annual}}$ so that the initial clean sector share increases from 0.5 to 1.5 times its baseline value (from 10.9% to 32.8%). The top left panel indicates that the SCC decreases with the initial clean sector share. Again, this is intuitive since the larger the clean sector is,

[14]We recalibrate the entire set of parameters for each elasticity of substitution, while the comparative statics for the other three parameters are done without recalibrating the other parameters.



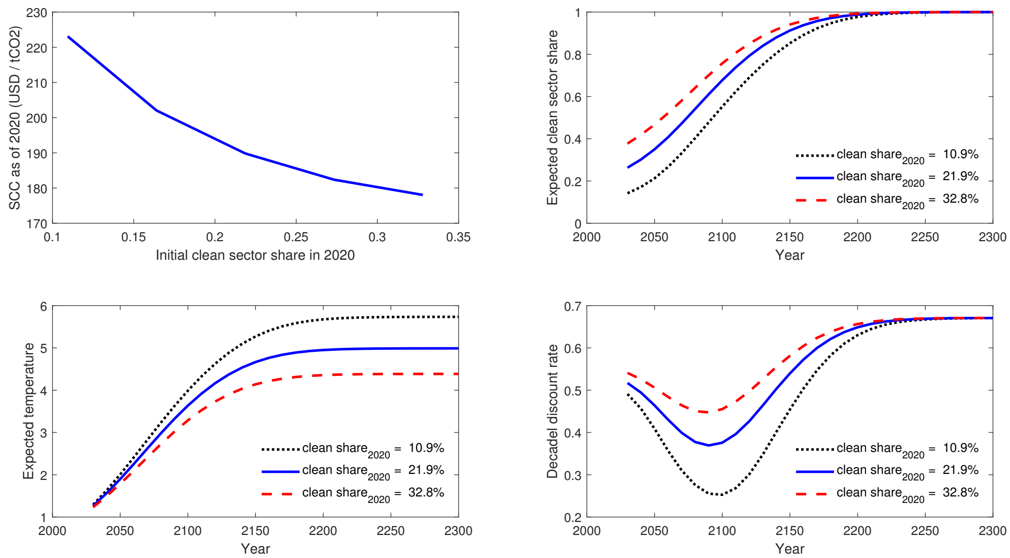
Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$, initial clean sector share = 21.9%

Figure 2: The effect of elasticity of substitution

the lower is dirty output and thus less global warming damages must be internalized. The top right and lower left panels indiate that a smaller initial clean sector share implies a longer green transition and higher temperatures. Finally, a smaller initial clean sector share implies a lower discount rate. Both the higher climate damage ratio and the smaller average discount rate contribute to a higher SCC.

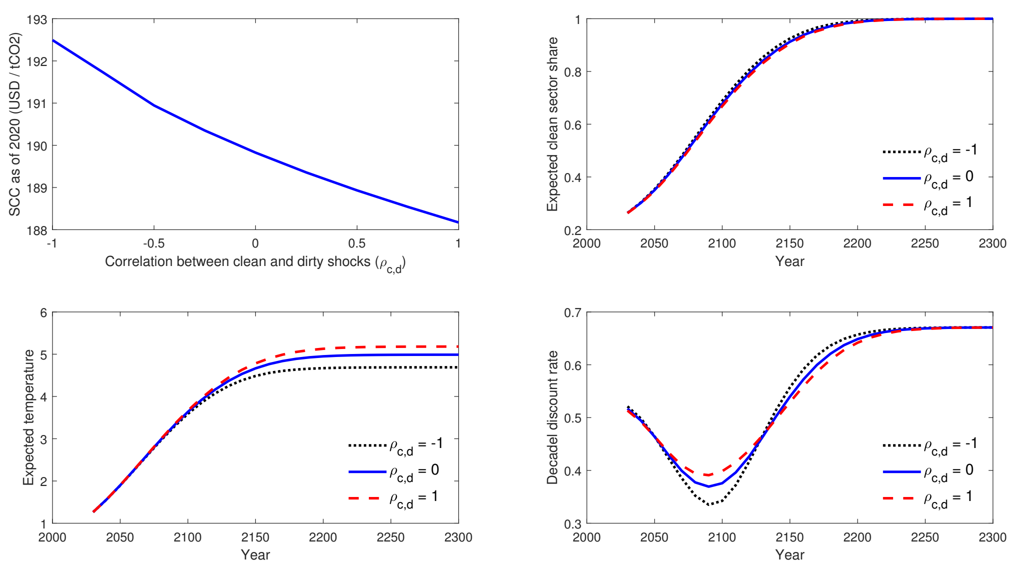
Figure 4 shows the effects of varying the correlation coefficient of the sectoral productivity shocks $\rho_{c,d}$ from -1 to 1 on the SCC. The top left panel indicates that the SCC declines with this correlation coefficient. Although more negatively correlated sectoral shocks correspond to a slightly higher clean sector share (top right panel) and a slightly lower steady-state temperature (lower left panel), they also lead to a lower discount rate from mid-century till around 2150 (lower right panel). These lower discount rates at this rather early stage of the green transition contribute to a higher SCC during this period. Intuitively, the more negatively correlated the sectoral shocks are, the more insurance benefit clean investment provides and the lower should its discount rate be. This leads to a higher value of clean investment and consequently a higher SCC and more ambitious climate policy.

Finally, Figure 5 shows the effects of increasing the standard deviation of the clean productivity shock σ_c from 0.25 to 2 times of its baseline value so that σ_c/σ_d increases



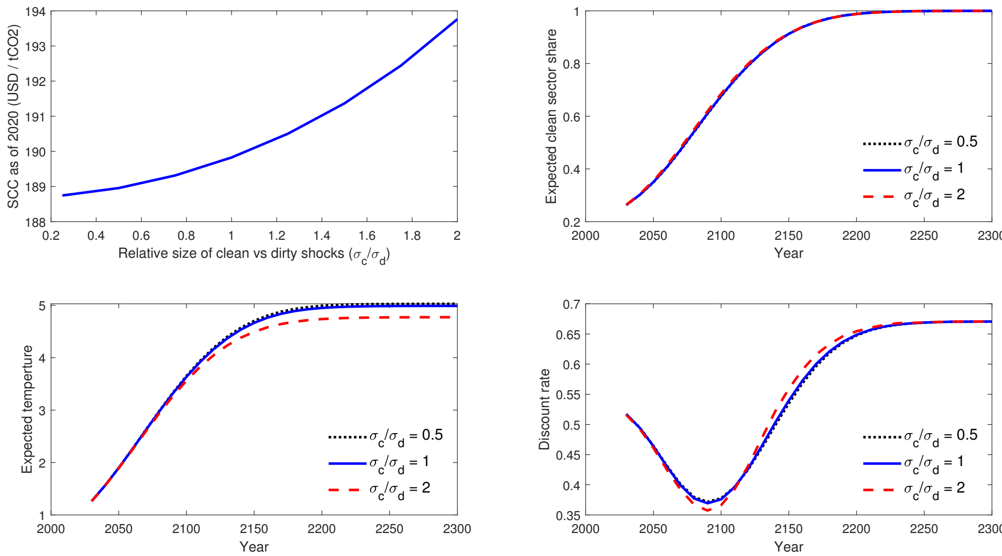
Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$, initial clean sector share $= 21.9\%$

Figure 3: The effect of initial clean sector share



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$, initial clean sector share $= 21.9\%$

Figure 4: The effect of correlation between sectoral shocks



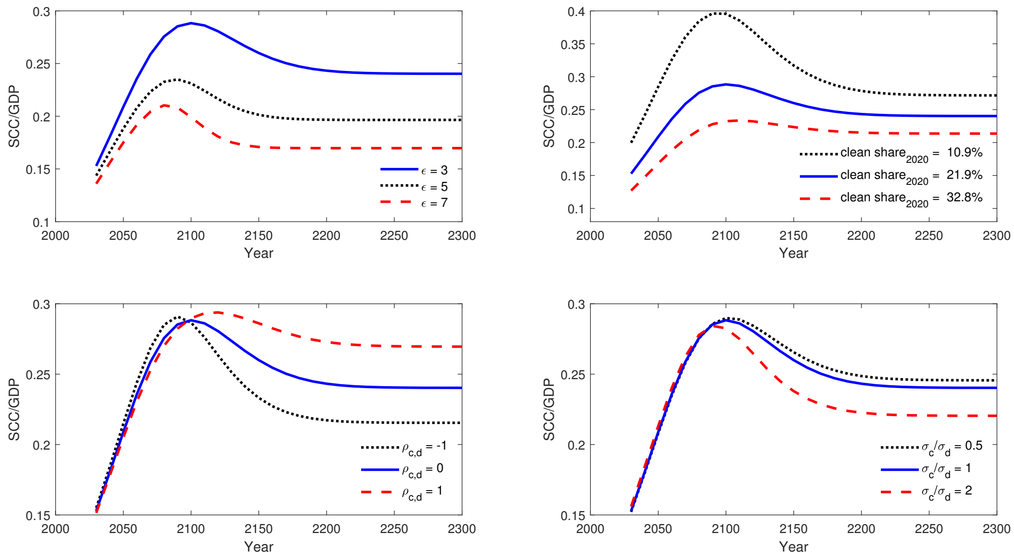
Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$, initial clean sector share = 21.9%

Figure 5: The effect of the relative size of the sectoral shocks

from 0.25 to 2. The top left panel indicates that the initial SCC increases in the relative size of the clean sector shock. While varying relative size of the shocks has minimal effect on the evolution of the clean sector share over time (top right panel), a relatively larger clean shock corresponds to a lower steady-state temperature (lower left panel) and thus a lower climate damage ratio. However, a relatively larger clean shock also means a lower discount rate early on, which raises the SCC. Intuitively, a larger clean sector shock induces a stronger precautionary motive and a higher SCC.

6.3 Does the SCC grow in line with GDP?

The comparative statics exercises have shown that the underlying economic structure and the characteristics of the productivity shocks affect the level of the SCC. We are also interested in how the SCC changes over time. In single-sector models, the SCC tends to grow in tandem with GDP (e.g., Golosov et al., 2014). However, Figure 6 shows that SCC/GDP in our two-sector model is not constant over time. Instead, during the green transition SCC/GDP first grows over time until around 2100, before slowly declining to its steady state value. One reason for the initially rising SCC/GDP is that consumption growth falls in the initial and mid phases of the green transition, as the economy switches to the less productive clean sector. In



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$, initial clean sector share = 21.9%

Figure 6: SCC/GDP ratio over time

addition, as the clean sector grows, it also starts to offer an increasingly large, positive insurance value in the mid phase of the green transition, leading to a rising climate risk premium (more details in Section 6.4). As a result, the discount rate falls (cf. equation (23) and lower right panels of Figures 2-5) and the SCC grows more than consumption. Over time, as the clean sector catches up, consumption growth picks up again while the insurance value and the climate risk premium fall, resulting in a falling SCC/GDP. Eventually, as the clean sector dominates, the economy approaches a single-sector economy and SCC/GDP approaches a constant. Deviating from the baseline values, we see that while the level and curvature of the SCC/GDP path vary across different parameter combinations, the basic hump-shaped pattern remains the same. Finally, the characteristics of the productivity shocks affect the SCC/GDP ratio differently over time. While a negative correlation between the shocks and a higher relative size of the clean shock correspond to a higher SCC/GDP early on, they lead to a lower SCC/GDP once the clean sector is sufficiently large. This reflects the changing climate risk premium during the green transition, as we discuss in Section 6.4.

Note that aggregate consumption growth depends on the relative size of the sectors and sectoral growth (see Figure OA.5 in the online appendix). Starting from an initially less productive clean sector, decarbonization first dampens the rate of economic growth. As the clean sector grows and becomes more productive, decarbonization increases consumption growth. The time path of the SCC is driven by two factors: (i) first rising, then falling SCC/GDP; and (ii) first falling, then rising rate of economic growth. On balance, these two effects approximately offset each other which is why the time path for the SCC rises monotonically (see Figure 1).

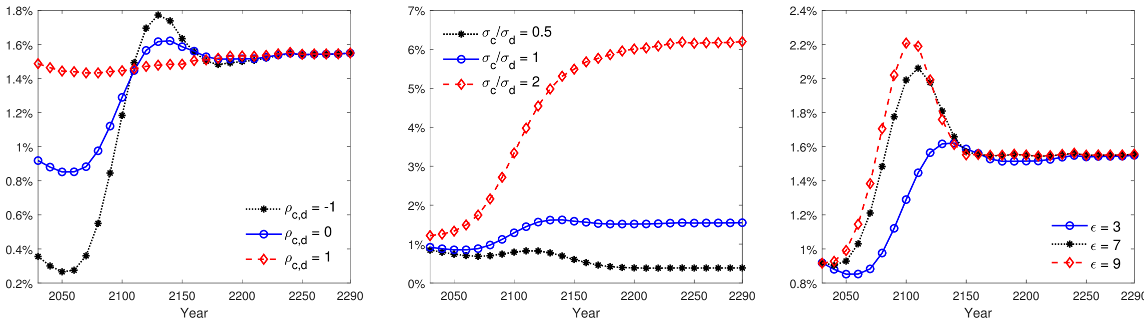
6.4 The climate risk premium

To examine how the changing economic structure during the green transition interacts with uncertainty, we compute the climate risk premium Π_t using (21) and decompose it into the prudence and insurance terms according to (22) (see Online Appendix for more details).

6.4.1 The prudence component of the climate risk premium

Figure 7 shows how the prudence term of the climate risk premium, which is proportional to the relative variance (the square of the coefficient of variation) of consumption, varies over time for various parameter combinations. We see that the prudence component is always positive, suggesting that the precautionary motive always increases the climate risk premium. During the green transition, the prudence term first falls, then rises, before falling towards its long-run level. The reason for this is the varying composition of aggregate volatility during the green transition, as shown in Proposition 3. In the initial phase of the green transition, the dirty sector dominates and consumption volatility mainly comes from dirty technology shocks. Decarbonization thus lowers aggregate volatility early on. A growing clean sector gradually contributes more to aggregate volatility and it grows in tandem. Eventually, the falling dirty sector starts to reduce aggregate volatility. In the long run, the dirty sector is negligible and aggregate consumption volatility comes entirely from the clean sector so it approaches the variance of the clean sector shock (σ_c^2).

Comparing the time paths for different correlation coefficients (see left panel), we see that the pattern of change is more muted, the more positively correlated the sectoral shocks are. This makes sense since with positively correlated shocks it matters less which sector dominates and the composition is less important. The



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure 7: The prudence term of the climate risk premium, $\eta(1+\eta)\sigma^2/2$ (%/year)

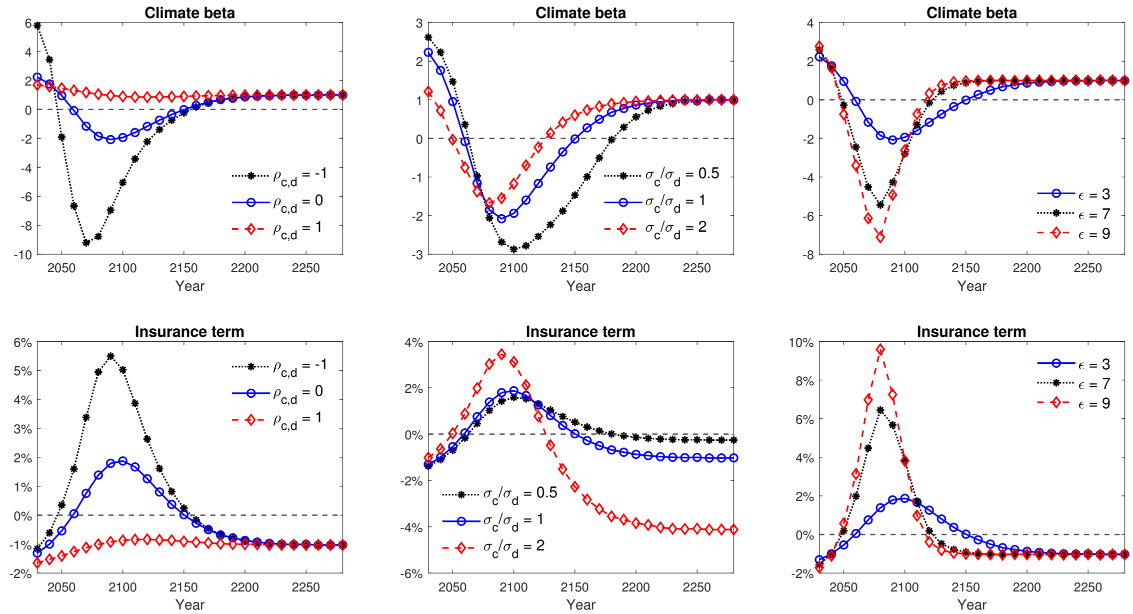
middle panel shows that the larger the clean sector shocks, the more a growing clean sector drives up aggregate volatility, and the less impact a falling dirty sector has. A higher elasticity of substitution speeds up changes in aggregate volatility (see right panel), since the green transition happens faster with a faster market dynamics.

6.4.2 The insurance component of the climate risk premium

Figure 8 shows how the climate beta and the insurance component of the climate risk premium vary over time for various parameter combinations. From the three top panels, we see that the climate beta in a two-sector economy can differ substantially from its counterpart in a single-sector economy. In a single-sector economy, the climate beta corresponds to the elasticity of the marginal damage with respect to warming. With a standard quadratic damage function, the climate beta in a single-sector economy is always 1. With two sectors, however, the climate beta can differ substantially from 1, ranging from nearly -10 to 6 in the top left panel.

Over time, the climate beta follows a non-monotonic time path. In line with Proposition 4, the climate beta can only be negative in the mid phase of the green transition when the clean sector share is neither too low nor too high. In the long run, as the clean sector dominates and the economy approaches a single-sector economy, the climate beta approaches 1.

The non-monotonic time path of the climate beta reflects two driving forces (cf. equation (32)). On the one hand, as the clean sector grows during the green transition, the elasticity of marginal global warming damages with respect to dirty sector production, $\tilde{\kappa}$, falls, which increases the climate beta. On the other hand, once the



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure 8: The climate beta and insurance term of the climate risk premium, $-\beta\eta\sigma^2$

clean sector share has grown sufficiently large, the ratio of the dirty sector weight to the productivity elasticity of dirty technology, λ_d/ξ_d, falls so that the climate beta is driven more by the clean sector productivity shock. Since the marginal global warming damages tend to be negatively correlated with clean sector productivity, when the clean sector productivity shock plays a bigger role, the climate beta tends to be lower and possibly negative.

The time path of the climate beta varies significantly across different coefficients of correlation between the sectoral shocks, across different relative sizes of the sectoral shocks, and across different elasticities of substitution between the two sectors. While with perfectly correlated shocks the climate beta is still relatively close to 1, this is no longer the case if there is no correlation or a negative correlation (see top left panel). In the top middle panel, we see that a lower clean sector shock leads to a higher climate beta early on but a lower climate beta when the clean sector is sufficiently large. This again reflects that as the clean sector grows, its role in risk diversification changes. The larger the clean sector shocks, the larger is the variation of the climate beta over time. Further, the top right panel indicates that a higher elasticity of substitution speeds up the dynamics in the climate beta, again due to the faster market dynamics.

From the three lower panels in Figure 8, we see that the insurance term of the climate risk premium is hump-shaped, reflecting (mostly) the changing magnitude of the climate beta. In the mid phase of the green transition, decarbonization offers a positive insurance value.

6.4.3 The combined effect

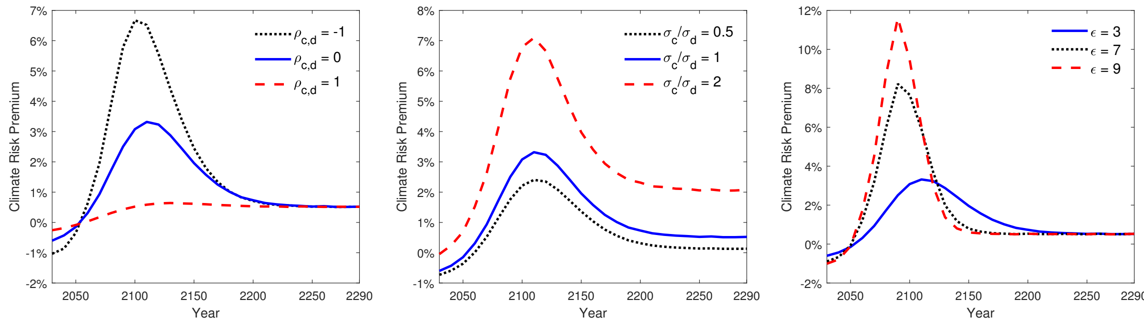
Figure 9 shows the climate beta as the combined effect of the prudence and insurance terms. In a two-sector economy, the climate risk premium is not constant over time but hump-shaped. In the initial phase of the green transition, the climate risk premium can be temporarily negative due to a relatively large, positive climate beta. As the clean sector grows, the insurance term becomes positive and grows simultaneously as the prudence term, resulting in a growing climate risk premium until it reaches its peak around 2100. As the clean sector starts to dominate, both the prudence and insurance terms become smaller and the climate risk premium falls. As the economy approaches a single-sector economy, the climate risk premium reaches its constant, long-run level.

The hump-shaped climate risk premium contributes to the hump-shaped time path of SCC/GDP. It suggests that uncertainty plays a particularly important role in the composition of the SCC during the mid phase of the green transition, and also highlights the importance of economic structure for the role of uncertainty.

While both the precautionary and self-insurance motives contribute to the climate risk premium, comparing Figures 7-9 we see that the climate risk premium is mostly dominated by the insurance term. This points out the importance of the climate beta, in the same vein as in Gollier (2021). More importantly, it suggests that structural change during the green transition is particularly important for the climate beta and the insurance value.

7 Conclusions

The underlying structure of the economy matters for the climate risk premium, the risk-adjusted discount rate, and the SCC. Our first key insight in our two-sector



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure 9: The climate risk premium

framework is that the climate risk premium is hump-shaped during the green transition, with its magnitude dominated by the insurance value. This suggests that structural change during the green transition indeed alters how uncertainty affects the SCC, particularly by changing the insurance value.

Our second key insight is that the risk-adjusted discount rate is non-monotonic during the green transition. It first falls then rises, as the clean sector grows over time. Consequently, the result of many one-sector integrated assessment models that the SCC is proportional to aggregate output (e.g., Golosov et al., 2014) no longer holds. Instead, the ratio of SCC to GDP first grows and then falls during the green transition, despite both rising over time.

The reason for this non-monotonicity is due to the changing effect of climate mitigation on consumption growth rate and the climate risk premium. In the initial phase of the green transition, the economy switches to a less productive sector, resulting in a lower consumption growth rate, which depresses the discount rate. This trend continues in the mid phase of the transition, when the positive insurance value further leads to an increasingly larger climate risk premium, which also depresses the discount rate. As the clean sector grows larger and more productive, however, consumption growth rate picks up, while the climate risk premium starts to fall. The risk-adjusted discount rate rises as a result.

Our third key insight is that the effect of decarbonization on aggregate uncertainty also varies over time. This is captured by the varying climate beta during the green transition. Initially, decarbonization increases aggregate consumption risk as it forces the economy to use a less productive sector. Once the size of the clean sector reaches a certain threshold, further decarbonization may reduce aggregate consumption risk and offer positive insurance value, particularly when sectoral shocks are negatively correlated, the clean sector shock is relatively small, or the substitutability between the two sectors is strong. As the clean sector becomes dominant, the insurance value of further decarbonization once again turns negative and further decarbonization increases aggregate consumption risk. The diversification benefits only appear after the clean sector has reached a certain size.

We have further shown that the SCC is lower if it is easier to substitute between the carbon-intensive and clean goods in which case the clean sector rises more rapidly, and temperature rises less. The SCC is also lower if the initial clean energy share is high. If shocks in the two sectors are more negatively correlated, the insurance benefit of clean investment is higher and thus the SCC is higher.

Our results have two important policy implications. First, optimal climate policy depends on the underlying economic structure of an economy, and it thus should differ across countries and over time. Second, as the effect of decarbonization on aggregate consumption risk varies during the green transition, so does the relation between climate policy and other policy objectives such as promoting growth and mitigating economic recessions. Climate policy may initially lower both economic growth and aggregate uncertainty, while raising both later on.[15]

Further work is needed on the empirics of our insights. To do this, we could allow for the risk of rare macroeconomic disasters to help explain asset pricing puzzles, where the frequency of some disasters (e.g., due to extreme weather events) will increase with global warming. This will lead to extra precautionary savings and insurance terms which can also lead to non-monotonic behaviour of the risk-adjusted discounted rate and the SCC to GDP ratio. To the extent that the intensity of disasters increases with temperature, there is an extra externality to be internalized which requires an addiitonal term in the SCC. We might allow for stochastic shocks to damages or to the climate sensitivity (cf. van den Bremer and van der Ploeg, 2021) which would lead to additional terms in the climate risk premium. These terms will

[15] Our results also have implications for business cycle volatility. At the macro level, the correlation between carbon emissions and real GDP should change over time in a non-monotonic fashion as the economy moves away from polluting technologies during the green transition. At the micro level, the climate risk premium is neither uniform across countries nor stationary over time. Furthermore, the importance of low-carbon projects in security portfolios will change over time, as the correlation between carbon emissions and aggregate outcomes changes.

be larger the more skewed these shocks are. Finally, we may use our framework to gain better understanding of the crucial phenomenon of transition risk and how this affects the climate beta and the SCC.

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Appendices

A Model solutions

We solve for the decentralized equilibrium and the social planner's solution.

A.1 Decentralized equilibrium

Profit maximization of the final good producers leads to the output ratio

$$\frac{Y_{j,t}}{Y_t} = \theta_j \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon}, \tag{A.1}$$

where

$$P_t \equiv \left[\theta_c P_{c,t}^{1-\epsilon} + \theta_d P_{d,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{A.2}$$

is the (ideal) price index. Profit maximization of the sector aggregate producers gives

$$w_t = (1-\alpha)\frac{P_{j,t}Y_{j,t}}{L_{j,t}}, \tag{A.3}$$

$$P_{j,i,t} = \alpha P_{j,t} L_{j,t}^{1-\alpha} A_{j,i,t}^{1-\alpha} x_{j,i,t}^{\alpha-1}. \tag{A.4}$$

Intermediate goods producers maximize their profits subject to (A.4) and (3), so

$$\psi_{E,j} = \nu \frac{k_j x_{j,i,t}}{E_{j,i,t}}, \tag{A.5}$$

$$\psi = (1-\nu)\frac{k_j x_{j,i,t}}{I_{j,i,t}}, \tag{A.6}$$

$$P_{j,i,t} = \frac{k_j}{\alpha} P_t, \tag{A.7}$$

$$k_j \equiv \left(\frac{\psi}{1-\nu}\right)^{1-\nu} \left(\frac{\psi_{E,j}}{\nu}\right)^{\nu} = k\psi_{E,j}^{\nu}, \tag{A.8}$$

where $k \equiv (\psi/(1-\nu))^{1-\nu}\nu^{-\nu}$ and k_j is the unit production cost of $x_{j,i,t}$ in units of final goods. Without loss of generality, we set $\psi = (1-\nu)(\alpha^2\nu^\nu)^{1/(1-\nu)}$ so that $k = \alpha^2$.

Combining equations (A.6), (A.4),(A.8), and (A.5), we further have

$$x_{j,i,t} = \psi_{E,j}^{-\frac{\nu}{1-\alpha}} \left(\frac{P_{j,t}}{P_t}\right)^{\frac{1}{1-\alpha}} A_{j,i,t} L_{j,t}, \tag{A.9}$$

$$E_{j,i,t} = \nu k \psi_{E,j}^{-\frac{\nu\alpha}{1-\alpha}-1} \left(\frac{P_{j,t}}{P_t}\right)^{\frac{1}{1-\alpha}} A_{j,i,t} L_{j,t}. \tag{A.10}$$

Consequently, we can write sectoral output as

$$Y_{j,t} = \psi_{E,j}^{-\frac{\nu\alpha}{1-\alpha}} \left(\frac{P_{j,t}}{P_t}\right)^{\frac{\alpha}{1-\alpha}} A_{j,t} L_{j,t}, \tag{A.11}$$

where $A_{j,t} \equiv \int_{i=0}^{1} A_{j,i,t} di$ is aggregate sectoral productivity. Combining equations (A.3), (A.11), and (A.1), we obtain

$$\frac{P_{c,t}}{P_{d,t}} = \left(\frac{\psi_{E,c}}{\psi_{E,d}}\right)^{\nu\alpha} \left(\frac{A_{c,t}}{A_{d,t}}\right)^{-(1-\alpha)}, \tag{A.12}$$

$$\frac{L_{c,t}}{L_{d,t}} = \frac{\theta_c}{\theta_d} \left(\frac{\psi_{E,c}}{\psi_{E,d}}\right)^{\nu\alpha(1-\epsilon)} \left(\frac{A_{c,t}}{A_{d,t}}\right)^{-\phi}, \tag{A.13}$$

where $\phi \equiv (1-\epsilon)(1-\alpha)$. Combining equations (A.2) and (A.12) gives

$$\frac{P_{j,t}}{P_t} = \psi_{E,j}^{\nu\alpha} A_{j,t}^{-(1-\alpha)} A_t^{1-\alpha}, \tag{A.14}$$

where $A_t \equiv \left[\tilde{\theta}_c A_{c,t}^{-\phi} + \tilde{\theta}_d A_{d,t}^{-\phi}\right]^{-1/\phi}$ and $\tilde{\theta}_j \equiv \theta_j \psi_{E,j}^{\nu\alpha(1-\epsilon)}$.

Combining equations (5) and (A.13), we obtain

$$L_{j,t} = \tilde{\theta}_j A_{j,t}^{-\phi} A_t^{\phi}. \tag{A.15}$$

Substituting equations (A.14) and (A.15) into equation (A.10) and aggregating across all firms of a sector, we find that the energy use of sector j is

$$E_{j,t} = \nu k \theta_j \psi_{E,j}^{-1} A_j^{-\phi} A_t^{1+\phi}. \tag{A.16}$$

Plugging equations (A.14) into equation (A.1) gives

$$Y_{j,t} = \theta_j \psi_{E,j}^{-\nu\alpha\epsilon} \left(A_{j,t}/A_t\right)^{(1-\alpha)\epsilon} Y_t. \tag{A.17}$$

Entering equations(A.14) and (A.15) into equation (A.11) gives

$$Y_{j,t} = \theta_j \psi_{E,j}^{-\nu\alpha\epsilon} A_{j,t}^{1-\alpha-\phi} A_t^{\alpha+\phi} = \theta_j \psi_{E,j}^{-\nu\alpha\epsilon} A_{j,t}^{(1-\alpha)\epsilon} A_t^{1-(1-\alpha)\epsilon}. \tag{A.18}$$

Equating equations (A.17) and (A.18) then gives

$$Y_t = A_t. \tag{A.19}$$

Pre-damage consumption is final output net of input use of intermediate goods,

$$\tilde{C}_t = Y_t - kA_t = \left(1 - \alpha^2\right) A_t, \tag{A.20}$$

so that aggregate consumption (or GDP) is $C_t = (1 - d_t)(1 - \alpha^2)A_t$. Using (A.16), the share of energy expenditure in GDP is then

$$\frac{\psi_{E,c}E_{c,t} + \psi_{E,d}E_{d,t}}{C_t} = \frac{\nu\alpha^2 A_t^{1+\phi}\left(\tilde{\theta}_c A_{c,t}^{-\phi} + \tilde{\theta}_d A_{d,t}^{-\phi}\right)}{(1-\alpha^2)(1-d_t)A_t} = \frac{\nu\alpha^2}{(1-\alpha^2)(1-d_t)}. \tag{A.21}$$

A.2 The social optimum

Since static allocation is made after the productivity shock is realized in each period, there is no productivity uncertainty for that period at the time of production and consumption decisions. However, since the allocation decision in period 0 affect global warming damages in period 1, the planner needs to take productivity uncertainty of period 1 into account when making the allocation decision in period 0.

Thus at each period t, the social planner maximizes the Lagrangian function

$$
\begin{aligned}
\mathcal{L}_t = \mathbb{E}_t \sum_{s=t}^{s=1} \delta^{s-t} \Bigg\{ & \frac{C_s^{1-\eta}}{1-\eta} + \lambda_{C,s} \left[(1-d_s) \left(Y_s - \int_{i=0}^{1} \psi I_{c,i,s} di - \int_{i=0}^{1} \psi I_{d,i,s} di \right.\right. \\
& \left.\left. - \int_{i=0}^{1} \psi_{E,c} E_{c,i,s} di - \int_{i=0}^{1} \psi_{E,d} E_{d,i,s} di \right) - C_s \right] + \lambda_{Y,s} \left[\left(\theta_c^{\frac{1}{\epsilon}} Y_{c,s}^{\frac{\epsilon-1}{\epsilon}} + \theta_d^{\frac{1}{\epsilon}} Y_{d,s}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} - Y_s \right] \\
& + \lambda_{Yj,s} \left[L_{j,s}^{1-\alpha} \int_{i=0}^{1} A_{j,s}^{1-\alpha} x_{j,i,s}^{\alpha} di - Y_{j,s} \right] + \lambda_{L,s} \left(1 - L_{c,s} - L_{d,s} \right) \\
& + \sum_{j \in \{c,d\}} \int_{i=0}^{i=1} \lambda_{xji,s} \left(I_{j,i,s}^{1-\nu} E_{j,i,s}^{\nu} - x_{j,i,s} \right) di - \lambda_{M,s} \left(\omega_2 Y_{d,s} - M_s \right) \\
& - \lambda_{T,s} \left(\omega_1 M_s + T_{s-1} - T_s \right) - \lambda_{d,s} \left[a T_t^{\kappa} Y_s^{\zeta-1} - d_s \right] \Bigg\}.
\end{aligned}
\tag{A.22}
$$

For $t \in \{0, 1\}$, the first-order optimality conditions are

$$C_t: \quad \mathbb{E}_t \left[C_t^{-\eta} - \lambda_{C,t} \right] = 0, \tag{A.23}$$

$$d_t: \quad \mathbb{E}_t \left[\lambda_{C,t} \frac{C_t}{1-d_t} - \lambda_{d,t} \right] = 0, \tag{A.24}$$

$$Y_t: \quad \mathbb{E}_t \left[\lambda_{Y,t} - (1-d_t)\lambda_{C,t} + (\zeta-1)\frac{d_t}{Y_t}\lambda_{d,t} \right] = 0, \tag{A.25}$$

$$M_t: \quad \mathbb{E}_t \left[\lambda_{M,t} - \omega_1 \lambda_{T,t} \right] = 0, \tag{A.26}$$

$$T_1: \quad \mathbb{E}_1 \left[\lambda_{T,1} - \kappa \frac{d_1}{T_1} \lambda_{d,1} \right] = 0, \tag{A.27}$$

$$T_0: \quad \mathbb{E}_0 \left[\lambda_{T,0} - \kappa \frac{d_0}{T_0} \lambda_{d,0} - \delta \lambda_{T,1} \right] = 0, \tag{A.28}$$

$$x_{j,i,t}: \quad \mathbb{E}_t \left[\lambda_{xji,t} - \alpha \lambda_{Yj,t} L_j^{1-\alpha} A_{j,t}^{1-\alpha} x_{j,i,t}^{\alpha-1} \right] = 0, \tag{A.29}$$

$$I_{j,i,t}: \quad \mathbb{E}_t \left[\lambda_{C,t}(1-d_t)\psi - (1-\nu)\lambda_{xji,t} \frac{x_{j,i,t}}{I_{j,i,t}} \right] = 0, \tag{A.30}$$

$$E_{j,i,t}: \quad \mathbb{E}_t \left[\lambda_{C,t}(1-d_t)\psi_{E,j} - \nu \lambda_{xji,t} \frac{x_{j,i,t}}{E_{j,i,t}} \right] = 0, \tag{A.31}$$

$$L_{j,t}: \quad \mathbb{E}_t \left[\lambda_{L,t} - (1-\alpha)\lambda_{Yj,t} \frac{Y_{j,t}}{L_{j,t}} \right] = 0, \tag{A.32}$$

$$Y_{c,t}: \quad \mathbb{E}_t \left[\lambda_{Y,t} \theta_c^{1/\epsilon} \left(Y_{c,t}/Y_t \right)^{-1/\epsilon} - \lambda_{Yc,t} \right] = 0, \tag{A.33}$$

$$Y_{d,t}: \quad \mathbb{E}_t \left[\lambda_{Y,t} \theta_d^{1/\epsilon} \left(Y_{d,t}/Y_t \right)^{-1/\epsilon} - \lambda_{Yd,t} - \omega_2 \lambda_{M,t} \right] = 0. \tag{A.34}$$

Since in each period $E_t[X_t] = X_t$ for any X_t, combining (A.29)-(A.10), we obtain

$$x_{j,i,t} = \left(\frac{\alpha}{k_j}\frac{\lambda_{Yj,t}}{(1-d_t)\lambda_{C,t}}\right)^{\frac{1}{1-\alpha}} A_{j,i,t}L_{j,t}, \tag{A.35}$$

$$Y_{j,t} = \left(\frac{\alpha}{k_j}\frac{\lambda_{Yj,t}}{(1-d_t)\lambda_{C,t}}\right)^{\frac{\alpha}{1-\alpha}} A_{j,t}L_{j,t}, \tag{A.36}$$

$$\frac{Y_{c,t}}{Y_{d,t}} = \left(\frac{\psi_{E,c}}{\psi_{E,d}}\right)^{-\frac{\alpha\nu}{1-\alpha}} \left(\frac{\lambda_{Yc,t}}{\lambda_{Yd,t}}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{c,t}}{A_{d,t}}\frac{L_{c,t}}{L_{d,t}}, \tag{A.37}$$

where recalling from (A.8) that $k_j \equiv \left(\frac{\psi}{1-\nu}\right)^{1-\nu}\left(\frac{\psi_{E,j}}{\nu}\right)^{\nu}$ is the unit production cost of the intermediate goods in units of the final goods.

Combining equations (A.32) and (A.36), we obtain

$$\frac{\lambda_{Yc,t}}{\lambda_{Yd,t}} = \left(\frac{\psi_{E,c}}{\psi_{E,d}}\right)^{\alpha\nu} \left(\frac{A_{c,t}}{A_{d,t}}\right)^{-(1-\alpha)}. \tag{A.38}$$

From equations (A.33), (A.34) and (A.38), we obtain

$$Y_{c,t} = \theta_c \left(\frac{\lambda_{Yc,t}}{\lambda_{Y,t}}\right)^{-\epsilon} Y_t, \tag{A.39}$$

$$Y_{d,t} = \theta_d \left(\frac{\lambda_{Yd,t}}{\lambda_{Y,t}}\right)^{-\epsilon} \Lambda_t^{-\epsilon} Y_t, \tag{A.40}$$

$$\frac{Y_{c,t}}{Y_{d,t}} = \frac{\theta_c}{\theta_d}\left(\frac{\psi_{E,c}}{\psi_{E,d}}\right)^{-\alpha\nu\epsilon} \left(\frac{A_{c,t}}{A_{d,t}}\right)^{\epsilon(1-\alpha)} \Lambda_t^{\epsilon}, \tag{A.41}$$

where $\Lambda_t \equiv 1 + \omega_2 \frac{\lambda_{M,t}}{\lambda_{Yd,t}}$.

Combining equations (A.37), (A.38), (A.41), and (5), we obtain

$$L_{c,t} = \tilde{\theta}_c A_{c,t}^{-\phi} A_{L,t}^{\phi}, \tag{A.42}$$

$$L_{d,t} = \tilde{\theta}_d A_{d,t}^{-\phi} \Lambda_t^{-\epsilon} A_{L,t}^{\phi}, \tag{A.43}$$

$$A_{L,t} \equiv \left[\tilde{\theta}_c A_{c,t}^{-\phi} + \tilde{\theta}_d A_{d,t}^{-\phi} \Lambda_t^{-\epsilon}\right]^{-\frac{1}{\phi}}, \tag{A.44}$$

where $\tilde{\theta}_j = \theta_j \psi_{E,j}^{\alpha\nu(1-\epsilon)}$.

Substituting equations (A.39) and (A.40) into equation (1) gives

$$\frac{\lambda_{Yj,t}}{\lambda_{Y,t}} = \psi_{E,j}^{\alpha\nu} A_{j,t}^{-(1-\alpha)} A_{Y,t}^{1-\alpha}, \tag{A.45}$$

$$A_{Y,t} \equiv \left[\tilde{\theta}_c A_{c,t}^{-\phi} + \tilde{\theta}_d A_{d,t}^{-\phi} \Lambda_t^{1-\epsilon}\right]^{-\frac{1}{\phi}}. \tag{A.46}$$

Plugging equations (A.45) into equations (A.39) and (A.40), and plugging equation (A.42) into equation (A.36), we further obtain

$$Y_{c,t} = \theta_c \psi_{E,c}^{-\alpha\nu\epsilon} A_{c,t}^{\epsilon(1-\alpha)} A_{Y,t}^{-\epsilon(1-\alpha)} Y_t, \tag{A.47}$$

$$Y_{d,t} = \theta_d \psi_{E,d}^{-\alpha\nu\epsilon} A_{d,t}^{\epsilon(1-\alpha)} A_{Y,t}^{-\epsilon(1-\alpha)} \Lambda_t^{-\epsilon} Y_t, \tag{A.48}$$

$$Y_t = \left(\frac{\alpha}{k}\chi_t\right)^{\frac{\alpha}{1-\alpha}} A_{Y,t} \left(\frac{A_{Y,t}}{A_{L,t}}\right)^{-\phi}, \tag{A.49}$$

where $\chi_t \equiv \lambda_{Y,t}/((1-d_t)\lambda_{C,t})$ and $k \equiv k_j/\psi_{E,j}^{\nu} = (\psi/(1-\nu))^{1-\nu}\nu^{\nu}$. From the aggregate resource constraint, we obtain

$$C_t = (1-d_t)\left(1 - \alpha \left(\frac{A_{Y,t}}{A_{L,t}}\right)^{\phi} \chi_t\right) Y_t. \tag{A.50}$$

Combining the definition of χ_t with equations (A.24), (A.25) and (A.50), we finally obtain

$$\chi_t = \left[1 + \left(1 - \alpha\left(\frac{A_{Y,t}}{A_{L,t}}\right)^{\phi}\right) \frac{(\zeta-1)d_t}{1-\zeta d_t}\right]^{-1}, \tag{A.51}$$

where using equation (9) we have

$$d_t = a\left(T_{t-1} + \omega_1\omega_2 Y_{d,t}\right)^{\kappa} Y_t^{\zeta-1}. \tag{A.52}$$

Substituting equations (A.48) and (A.49) into equation (9) gives us an expression of d_t as an implicit function of Λ_t, i.e.,

$$d_t = h_{b,t} Y_{d,t}^{\kappa} Y_t^{\zeta-1}, \tag{A.53}$$

where $h_{b,t} \equiv a(\omega_1\omega_2)^{\kappa} T_{t-1}^{\kappa_0}$.

Substituting equations (A.51), (A.49), and (A.48) into equation (A.52) gives d_t as an implicit function of Λ_t.[16] Entering equations (A.24)-(A.28) into the definition of Λ_t, we derive an expression of Λ_t as an implicit function of d_t. In each period, we thus have a two non-linear equations with Λ_t and d_t being the two unknowns. The two-period model can then be solved through backward induction. In period 1, conditional on T_0, we can solve for Λ_1 and d_1 in period 1. Going back to period 0, the two equations together with the expectation of period 1 equilibrium outcomes allow us to solve for Λ_0 and d_0. This solves the model.

Using equation (A.24) and equations (A.26)-(A.28), the optimal SCC is given by

$$\text{SCC} = \frac{\lambda_{M,0}}{\lambda_{C,0}} = D_0^s + \mathbb{E}_0 \left[\delta \left(\frac{C_1}{C_0} \right)^{-\eta} D_1^s \right] \tag{A.54}$$

where the marginal damage in period t from a unit of period 0 emission in the planner's solution is given by

$$D_t^s = a\omega_1 \kappa T_t^{\kappa-1} Y_t^{\zeta} \frac{C_t}{(1-d_t)Y_t}. \tag{A.55}$$

While equation (A.54) is similar to the SCC in the decentralized economy (cf. equation (18)), comparing equation (A.55) to equation (17) we see a few important differences. First, in the decentralized equilibrium, the post-damage-consumption-to-output ratio is a constant: $C_t/[(1-d_t)Y_t] = 1-\alpha^2$. However, for a social planner, this ratio should vary according to climate damage and is given by

$$\frac{C_t}{(1-d_t)Y_t} = 1 - \alpha \left(\frac{A_{Y,t}}{A_{L,t}} \right)^{\phi} \chi_t. \tag{A.56}$$

Second, output Y_t in each period differs in the planner's solution from that of the decentralized equilibrium. Comparing equation (A.49) with equation (10), we see that the social planner will correct the underproduction caused by monopoly power, as reflected by the ratio $\alpha/\Psi = 1/\alpha$. On the other hand, the social planner takes into account that output not only contributes to consumption but also to higher global warming damages, and will thus lower output, reflected by $\chi_t < 1$. Finally,

[16] Given that Λ_t, $A_{Y,t}$ and $A_{L,t}$ are uniquely determined. From equation (A.51), $\partial \chi_t / \partial d_t < 0$ follows. It thus follows from equation (A.49) that $\partial Y_t / \partial d_t < 0$ and from equation (A.48) that $\partial Y_{d,t} / \partial d_t < 0$. Thus, given Λ_t, the right-hand side of (A.52) decreases in d_t while the left-hand side increases in d_t. It follows that given Λ_t, there exists a unique d_t and a unique χ_t.

due to the contribution of dirty sectoral output to emissions and to global warming damages ($\Lambda_t > 1$), we have that $A_{Y,t}/A_{L,t} \neq 1$.

Since equation (A.54) is qualitatively similar to equation(18), the results in Section 3 apply also to the optimal SCC given in equation (A.54) with the qualification that the expected value and standard deviation of the consumption growth rate in the planner's solution (g^s and σ^s) may differ from the values in the decentralized equilibrium.

As for the climate beta, we follow the same procedure as in Section 4. Note that the dependency of both C_1 and D_1^s on $A_{j,1}$ is more complicated than in the decentralized equilibrium, since $A_{j,1}$ now affects $A_{Y,1}$, $A_{L,1}$, d_1, χ_1, and Λ_1. However, with $\mathbb{E}_0[d_1/C_1] \approx 0$, it is easily verified that $\mathbb{E}_0[\Lambda_1] \approx 1$, $\mathbb{E}_0[\chi_1] \approx 1$, $\partial d_1/\partial A_{j,1} \approx 0$, $\partial(A_{Y,1}/A_{L,1})/\partial A_{j,1} \approx 0$ and $\partial\chi/\partial A_{j,1} \approx 0$. Similar to equations (B.4)-(B.7), we thus have

$$\Omega^*_{D,c} \approx \tilde{\theta}_c \left(\frac{\mathbb{E}_0[A_{Y,1}]}{\mathbb{E}_0[A_{c,1}]}\right)^{1+\phi} \frac{\mathbb{E}_0[D_1^s]}{\mathbb{E}_0[A_{Y,1}]}\beta_c, \tag{A.57}$$

$$\Omega^*_{C,c} \approx \tilde{\theta}_c \left(\frac{\mathbb{E}_0[A_{Y,1}]}{\mathbb{E}_0[A_{c,1}]}\right)^{1+\phi} \frac{\mathbb{E}_0[C_1]}{\mathbb{E}_0[A_{Y,1}]}, \tag{A.58}$$

$$\Omega^*_{D,d} \approx \tilde{\theta}_d \left(\frac{\mathbb{E}_0[A_{Y,1}]}{\mathbb{E}_0[A_{d,1}]}\right)^{1+\phi} \frac{\mathbb{E}_0[D_1^s]}{\mathbb{E}_0[A_{Y,1}]} \left[\beta_c + (\kappa-1)(1-T_0/\mathbb{E}_0[T_1])(1-\alpha)\epsilon\tilde{\theta}_d^{-1}\left(\frac{\mathbb{E}_0[A_{Y,1}]}{\mathbb{E}_0[A_{d,1}]}\right)^{-\phi}\right], \tag{A.59}$$

$$\Omega^*_{C,d} \approx \tilde{\theta}_d \left(\frac{\mathbb{E}_0[A_{Y,1}]}{\mathbb{E}_0[A_{d,1}]}\right)^{1+\phi} \frac{\mathbb{E}_0[C_1]}{\mathbb{E}_0[A_{Y,1}]}, \tag{A.60}$$

where we approximate $\mathbb{E}_0[A_{Y,1}]$ by $\mathbb{E}_0[A_1]$. Consequently, the results in Section 4 also hold for the planner's solution with the qualification that, again, the expected value and standard deviation of the consumption growth rate in the planner's solution (g^s and σ^s) may differ from the values in the decentralized equilibrium.

A.3 Derivation of the climate risk premium

Taking a second-order Taylor expansion of $\mathbb{E}_0\left[u'(C_1)D_1\right]$ around $u'\left(\mathbb{E}_0[C_1]\right)\mathbb{E}_0\left[D_1\right]$ gives

$$\Pi \approx \frac{1}{2}\frac{u'''\left(\mathbb{E}_0[C_1]\right)}{u'\left(\mathbb{E}_0[C_1]\right)}\text{Var}_0[C_1] + \frac{u''\left(\mathbb{E}_0[C_1]\right)}{u'\left(\mathbb{E}_0[C_1]\right)\mathbb{E}_0\left[D_1\right]}\text{Cov}_0[C_1, D_1]. \tag{A.61}$$

Taking a first-order Taylor expansion of D_1 around $\mathbb{E}_0[C_1]$, we obtain

$$D_1 = D_1|_{\mathbb{E}_0[C_1]} + \left.\frac{\partial D_1}{\partial C_1}\right|_{\mathbb{E}_0[C_1]}\left(C_1 - \mathbb{E}_0[C_1]\right), \tag{A.62}$$

which implies $\mathbb{E}_0[D_1] \approx D_1|_{\mathbb{E}_0[C_1]}$ and thus

$$D_1 \approx \mathbb{E}_0[D_1] + \left.\frac{\partial D_1}{\partial C_1}\right|_{\mathbb{E}_0[C_1]}\left(C_1 - \mathbb{E}_0[C_1]\right). \tag{A.63}$$

We thus obtain

$$\text{Cov}_0[C_1, D_1] = \left(\mathbb{E}_0[D_1]/\mathbb{E}_0[C_1]\right)\beta\text{Var}_0[C_1]. \tag{A.64}$$

The climate risk premium then follows immediately as

$$\Pi \approx \frac{1}{2}\eta(1+\eta)\frac{\text{Var}_0[C_1]}{\left(\mathbb{E}_0[C_1]\right)^2} - \beta\eta\frac{\text{Var}_0[C_1]}{\left(\mathbb{E}_0[C_1]\right)^2}. \tag{A.65}$$

B Proofs

B.1 Preliminaries: climate beta and variance of consumption

Recall that by linearizing D_1 around $\mathbb{E}_0[C_1]$, we have $\text{Cov}_0[C_1, D1] = \beta\frac{\mathbb{E}_0[D_1]}{\mathbb{E}_0[C_1]}\text{Var}_0[C_1]$ (see Section A.3), which gives

$$\beta = \frac{\text{Cov}_0[C_1, D_1]}{\text{Var}_0[C_1]}\frac{\mathbb{E}_0[C_1]}{\mathbb{E}_0[D_1]}. \tag{B.1}$$

We note from equations (14),(16) and (17) that C_1 and D_1 are essentially non-linear functions of the sectoral productivities $A_{c,1}$ and $A_{d,1}$. Linearizing C_1 and D_1 around the expected sectoral productivities $(E[A_{c,1}], E[A_{d,1}])$, we have $C_1 \approx \mathbb{E}_0[C_1] + \Omega_{C,c}(A_{c,1} - E[A_{c,1}]) + \Omega_{C,d}(A_{d,1} - E[A_{d,1}])$ and $D_1 \approx \mathbb{E}_0[D_1] + \Omega_{D,c}(A_{c,1} - E[A_{c,1}]) + \Omega_{D,d}(A_{d,1} - E[A_{d,1}])$, where $\Omega_{D,j} \equiv \partial D_1/\partial A_{j,1}$ and $\Omega_{C,j} \equiv \partial C_1/\partial A_{j,1}$ $(j \in \{c,d\})$ are the partial derivatives of D_1 and C_1 with respect to the sectoral productivities, evaluated at expected sectoral productivities.

The variance of consumption in period 1 can thus be approximated by

$$\text{Var}_0[C_1] = (\Omega_{C,c})^2 \sigma_{Ac}^2 + (\Omega_{C,d})^2 \sigma_{Ad}^2 + 2\Omega_{C,c}\Omega_{C,d}\rho_{Ac,Ad}\sigma_{Ac}\sigma_{Ad}, \tag{B.2}$$

where σ_{Aj} denotes the standard deviation of productivity in sector j, and $\rho_{Ac,Ad}$ denotes the correlation coefficient between the two sectoral productivities.

Similarly, we can write the covariance between period 1 consumption and marginal damage as[17]

$$\text{Cov}_0[C_1, D_1] = \Omega_{C,c}\Omega_{D,c}\sigma_{Ac}^2 + \Omega_{C,d}\Omega_{D,d}\sigma_{Ad}^2 + (\Omega_{C,c}\Omega_{D,d} + \Omega_{C,d}\Omega_{D,c})\rho_{A_c,A_d}\sigma_{Ac}\sigma_{Ad}. \tag{B.3}$$

B.2 Proof of Propositions 1 to 3

Using equations (16) and (17), we obtain

$$\frac{\partial D_1}{\partial A_{c,1}} = \tilde{\theta}_c \left(\frac{A_1}{A_{c,1}}\right)^{1+\phi} \frac{D_1}{A_1} \left[\zeta + (\kappa - 1)(1 - T_0/T_1)\left(1 - (1-\alpha)\epsilon\right)\right],$$

$$\frac{\partial C_1}{\partial A_{c,1}} = \tilde{\theta}_c \left(\frac{A_1}{A_{c,1}}\right)^{1+\phi} \frac{C_1}{A_1} \left[1 - \frac{d_1}{1-d_1}\left(\zeta - 1 + \kappa(1 - T_0/T_1)\left(1 - (1-\alpha)\epsilon\right)\right)\right],$$

$$\frac{\partial D_1}{\partial A_{d,1}} = \tilde{\theta}_d \left(\frac{A_1}{A_{d,1}}\right)^{1+\phi} \frac{D_1}{A_1} \Bigg\{\zeta + (\kappa - 1)(1 - T_0/T_1) \times \left[(1-\alpha)\epsilon\left(1 + \frac{\tilde{\theta}_c}{\tilde{\theta}_d}\left(\frac{A_{c,1}}{A_{d,1}}\right)^{-\phi}\right) + 1 - (1-\alpha)\epsilon\right]\Bigg\},$$

[17] Given the log-normal distributions of $A_{c,1}$ and $A_{d,1}$, the correlation coefficient between the two is given by $\rho_{A_c,A_d} = \left(e^{\rho_{Ac,Ad}\sigma_{Ac}\sigma_{Ad}} - 1\right) / \left((e^{\sigma_{Ac}^2} - 1)(e^{\sigma_{Ad}^2} - 1)\right)^{1/2}$, where $\rho_{Ac,Ad}$ is the correlation coefficient between $\ln A_{c,1}$ and $\ln A_{d,1}$.

$$\frac{\partial C_1}{\partial A_{d,1}} = \tilde{\theta}_d \left(\frac{A_1}{A_{d,1}}\right)^{1+\phi} \frac{C_1}{A_1}\Bigg\{1 - \frac{d_1}{1-d_1}\Bigg[\zeta - 1 + \kappa(1 - T_0/T_1) \times \left((1-\alpha)\epsilon\tilde{\theta}_d^{-1}\left(\frac{A_1}{A_{d,1}}\right)^{-\phi} + 1 - (1-\alpha)\epsilon\right)\Bigg]\Bigg\}.$$

To obtain $\Omega_{C,j}$ and $\Omega_{D,j}$ ($j \in \{c, d\}$), we linearize all variables around $(E[A_{c,1}], E[A_{d,1}])$ in the spirit of equation (B.1). It follows that in the neighbourhood of $(E[A_{c,1}], E[A_{d,1}])$, $\mathbb{E}_0[A_1] \approx A_1|_{(E[A_{c,1}],E[A_{d,1}])}$, $\mathbb{E}_0[d_1] \approx d_1|_{(E[A_{c,1}],E[A_{d,1}])}$. Since $E[d_1]$ is small, we have

$$\Omega_{D,c} \approx \tilde{\theta}_c \left(\frac{\mu_A}{\mu_{Ac}}\right)^{1+\phi} \frac{\mathbb{E}_0[D_1]}{\mu_A}\beta_c, \tag{B.4}$$

$$\Omega_{C,c} \approx \tilde{\theta}_c \left(\frac{\mu_A}{\mu_{Ac}}\right)^{1+\phi} \frac{\mathbb{E}_0[C_1]}{\mu_A}, \tag{B.5}$$

$$\Omega_{D,d} \approx \tilde{\theta}_d \left(\frac{\mu_A}{\mu_{Ad}}\right)^{1+\phi} \frac{\mathbb{E}_0[D_1]}{\mu_A}\left\{\beta_c + (\kappa - 1)(1 - T_0/T_1)(1-\alpha)\epsilon\left(1 + \frac{\tilde{\theta}_c}{\tilde{\theta}_d}\left(\frac{\mu_{Ac}}{\mu_{Ad}}\right)^{-\phi}\right)\right\}, \tag{B.6}$$

$$\Omega_{C,d} \approx \tilde{\theta}_d \left(\frac{\mu_A}{\mu_{Ad}}\right)^{1+\phi} \frac{\mathbb{E}_0[C_1]}{\mu_A}, \tag{B.7}$$

where β_c is given by equation (26).

With uncertainty only in the clean sectoral productivity, $\text{Var}_0[C_1] = (\Omega_{C,c})^2\sigma_{Ac}^2$ and $\beta_c = (\Omega_{D,c}\mathbb{E}_0[C_1])/(\Omega_{C,c}\mathbb{E}_0[D_1])$. Plugging in equations (B.4) and (B.5) gives equations (25) and (26).

With only productivity shocks in the dirty sector, $\text{Var}_0[C_1] = (\Omega_{C,d})^2\sigma_{Ad}^2$ and $\beta_d = (\Omega_{D,d}\mathbb{E}_0[C_1])/(\Omega_{C,d}\mathbb{E}_0[B_1])$. Plugging in (B.6) and (B.7) gives (27) and (28).

Finally, with uncertainties from both sectors, combining (B.4)-(B.7) with (B.1)-(B.3) gives (29) and (30).

B.3 Proof of Proposition 4

We first state and prove two lemmas.

Lemma B.1. *The marginal damage elasticity with respect to dirty sector production, $\tilde{\kappa}$, is decreasing in the expected clean sector share, and $\tilde{\kappa} \to 0$ as the expected clean sector share approaches 1.*

Proof of Lemma B.1. Since $\tilde{\kappa}$ is proportional to $1/(1 + T_0/(\omega_1\omega_2\mathbb{E}_0[Y_{d,1}]))$ and the expected dirty sector production $\mathbb{E}_0[Y_{d,1}]$ is decreasing in the expected clean sector share, $\tilde{\kappa}$ is decreasing in the expected clean sector share. As the clean sector share approaches 1, $\mathbb{E}_0 Y_{d,1} \to 0$ so $\tilde{\kappa} \to 0$.

Note that in a dynamic setting, $\tilde{\kappa}_t = (\kappa - 1)/(1 + T_{t-1}/(\omega_1\omega_2\mathbb{E}_{t-1}[Y_{d,t}]))$. So T_{t-1} grows over time, which also decreases $\tilde{\kappa}_t$ over time.

□

Lemma B.2. *Suppose $\zeta + \tilde{\kappa} > 0$. Denote $\mu_r = \tilde{\theta}_c/\tilde{\theta}_d(\mu_{Ac}/\mu_{Ad})^{-\phi}$ and $\sigma_r = \frac{\sigma_{Ac}/\mu_{Ac}}{\sigma_{Ad}/\mu_{Ad}}$. The following holds for the ratio λ_d/ξ_d:*

1. *If $\rho_{A_c,A_d} \geq 0$, there exists a unique $\underline{\mu} \geq 0$, such that $\lambda_d/\xi_d \geq 1$ if $\mu_r \leq \underline{\mu}$, and $\lambda_d/\xi_d \in (0,1)$ if $\mu_r > \underline{\mu}$;*
2. *If $\rho_{A_c,A_d} < 0$, there exist $\bar{\mu} \equiv -1/(\rho_{A_c,A_d}\sigma_r)$ and a unique $\underline{\mu} \in (-\rho_{A_c,A_d}/\sigma_r, \bar{\mu})$ such that $\lambda_d/\xi_d \geq 1$ if $\mu_r \leq \underline{\mu}$, $\lambda_d/\xi_d \in (0,1)$ if $\mu_r \in (\underline{\mu}, \bar{\mu})$, and $\lambda_d/\xi_d \in (\rho_{A_c,A_d}/\sigma_r, 0)$ if $\mu_r > \bar{\mu}$.*

Proof of Lemma B.2. Denote $\mu_r = \tilde{\theta}_c/\tilde{\theta}_d(\mu_{Ac}/\mu_{Ad})^{-\phi}$ and $\sigma_r = \frac{\sigma_{Ac}/\mu_{Ac}}{\sigma_{Ad}/\mu_{Ad}}$. From the definition of λ_d and ξ_d, we have

$$\lambda_d/\xi_d = \frac{1 + \mu_r}{1 + \mu_r \frac{\rho_{A_c,A_d}\sigma_r + \mu_r\sigma_r^2}{1 + \rho_{A_c,A_d}\mu_r\sigma_r}}. \tag{B.8}$$

If $\rho_{A_c,A_d} \geq 0$, $\lambda_d/\xi_d > 0$ for all μ_r. It is easily verified that $(\rho_{A_c,A_d}\sigma_r + \mu_r\sigma_r^2)/(1 + \rho_{A_c,A_d}\mu_r\sigma_r)$ is non-decreasing in μ_r and its value is between $\rho_{A_c,A_d}\sigma_r$ and σ_r. If $\rho_{A_c,A_d}\sigma_r < 1 < \sigma_r$, then there exists $\underline{\mu} \in (0, \infty)$ such that $\lambda_d\xi_d > 1$ if $\mu_r < \underline{\mu}$ and $\lambda_d\xi_d < 1$ if $\mu_r > \underline{\mu}$. If $\sigma_r \leq 1$, $\lambda_d\xi_d \geq 1$ always holds and $\underline{\mu} = 0$; if $\rho_{Ac,Ad}\sigma_r \geq 1$, then $\lambda_d\xi_d \leq 1$ always holds and $\underline{\mu} \to \infty$. This proves the first point of the lemma.

If $\rho_{A_c,A_d} < 0$, the expression $(\rho_{A_c,A_d}\sigma_r + \mu_r\sigma_r^2)/(1 + \rho_{A_c,A_d}\mu_r\sigma_r)$ is discontinuous at $\mu_r = \bar{\mu}$. In particular, it approaches $+\infty$ $(-\infty)$ as μ_r approaches $\bar{\mu}$ from the left (right). However, it is still non-decreasing in μ_r on either sides of the discontinuity. We find that $(\rho_{A_c,A_d}\sigma_r + \mu_r\sigma_r^2)/(1 + \rho_{A_c,A_d}\mu_r\sigma_r) \in [\rho_{Ac,Ad}\sigma_r, 0]$ if $\mu_r \leq -\rho_{A_c,A_d}/\sigma_r$, $\in (0, \infty)$ if $\mu_r \in (-\rho_{A_c,A_d}/\sigma_r, \bar{\mu})$, and $\in (-\infty, \sigma_r/\rho_{A_c,A_d})$ if $\mu_r > \bar{\mu}$.

Let $\underline{\mu}$ be defined by the value of μ_r that set $(\rho_{A_c,A_d}\sigma_r + \mu_r\sigma_r^2)/(1 + \rho_{A_c,A_d}\mu_r\sigma_r) = 1$. Clearly, $\underline{\mu} \in (-\rho_{A_c,A_d}/\sigma_r, \bar{\mu})$. We find $\lambda_d/\xi_d \geq 1$ for $\mu_r \leq \underline{\mu}$, and $\lambda_d/\xi_d \in (0, 1)$ if $\mu_r \in (\underline{\mu}, \bar{\mu})$. Further, if $\mu_r > \bar{\mu}$, $\lambda_d/\xi_d \in (\rho_{A_c,A_d}/\sigma_r, 0)$. Finally, $\lambda_d/\xi_d \to 0$ as $\mu_r \to \bar{\mu}$ and $\lambda_d/\xi_d \to \rho_{A_c,A_d}/\sigma_r$ as $\mu_r \to \infty$. This proves the second point of the lemma.

□

Combing (32) and the two lemmas, we see that $\beta > 0$ always holds whenever $\mu_r \leq \underline{\mu}$. Further, as $\mu_r \to \infty$, $\tilde{\kappa} \to 0$ and $\lambda_d/\xi_d \to \rho_{A_c,A_d}/\sigma_r$, thus $\beta \to \zeta > 0$. Since the expected clean sector share $(\tilde{\theta}_c\mu_{Ac}^{-\phi}/\mu_A^{-\phi})$ is monotonically increasing in μ_r, this means that $\beta > 0$ holds when the clean sector share is either too small or too large. We can thus find $\tilde{\mu} \in [0, \infty]$ and $\tilde{\tilde{\mu}} \geq \tilde{\mu}$ such that $\beta > 0$ holds when the clean sector share is no greater than $\tilde{\mu}$ or greater than $\tilde{\tilde{m}}u$.

Finally, in order for β to be negative, we must have

$$\lambda_d/\xi_d < 1 - \frac{\zeta/\tilde{\kappa} + 1}{(1 - \alpha)\epsilon}. \tag{B.9}$$

This requires λ_d/ξ_d to be sufficiently low, which requires $\mu_r > \underline{\mu}$, and $\tilde{\kappa}$ to be sufficiently high, which requires μ_r to be sufficiently low. This condition can thus only be fulfilled if the expected clean sector share is between $\tilde{\mu}$ and $\tilde{\tilde{\mu}}$. Further, the higher ϵ, the easier it is for this condition to be met. This proves the proposition.

OA Online Appendix

OA1 Risk-adjusted and risk-free discount rates

The risk-adjusted discount rate (23) reads $r = \rho + \eta g - \frac{1}{2}\eta(1+\eta)\sigma^2 + \beta\eta\sigma^2$, where $g \equiv \ln[\mathbb{E}_0[C_1]/C_0]$ defines expected growth of consumption[1], and the difference between the deterministic discount rate $\rho + \eta g$ and the risk-adjusted discount rate r is the climate risk premium Π given in (22).

The first two terms of equation (23) correspond to the deterministic Keynes-Ramsey rule. The first term ρ is the time impatience effect, which indicates that with more impatience investors demand higher returns and thus the SCC is lower. The second term ηg is the affluence effect, which indicates that with higher growth and more intertemporal inequality aversion (i.e., lower elasticity of intertemporal substitution) investors require higher returns and this implies a lower SCC. Effectively, if society is richer in the future, it will do less today to combat climate change. The third term on the right-hand side of equation (23) is the prudence term. This term always depresses the discount rate and pushes up the SCC. This prudence effect increases in the coefficient of relative risk aversion η, the coefficient of relative prudence $1 + \eta$, and the variance of consumption growth. The fourth term in (23) is the insurance term. If future marginal damages are positively correlated with future consumption, i.e., $\beta > 0$, there is potential for self-insurance so that the return and the discount rate are higher, and thus the SCC is lower. If the climate beta is negative, $\beta < 0$, the SCC will be higher and climate policy must be more ambitious.

Finally, we can write equation (23) in the standard CCAPM form as

$$r = r_f + \beta\eta\sigma^2,$$

where $r_f \equiv \rho + \eta g - \frac{1}{2}\eta(1+\eta)\sigma^2$ is the risk-free discount rate and $\beta\eta\sigma^2$ the risk premium. The prudence effect depresses the risk-free rate as the additional precautionary saving resulting from macroeconomic uncertainty implies lower returns.

[1] Recall from Section 2.5 that consumption growth $\ln(C_1/C_0)$ is normally distributed with mean μ and standard deviation σ. Hence, $\mathbb{E}_0[C_1/C_0] = e^{\mu+\sigma^2/2}$ and $\mathrm{Var}_0[C_1/C_0] = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$.

OA2 Computing the climate risk premium

We derive in this appendix the one-period-ahead risk-adjusted discount rate and climate risk premium in an infinite-horizon model. With power utility, the SCC at time t is given by

$$\begin{aligned} SCC_t &= \mathbb{E}_t\left[\sum_{s=t}^{\infty} e^{-\rho(s-t)}\frac{D_s u'(C_s)}{u'(C_t)}\right] = \sum_{s=t}^{\infty} e^{-\rho(s-t)}\mathbb{E}_t\left[\frac{D_s u'(C_s)}{u'(C_t)}\right] \\ &= \sum_{s=t}^{\infty} e^{-\rho(s-t)+\ln(1+\pi_{t,s})}\frac{\mathbb{E}_t(D_s)u'(\mathbb{E}_t(C_s))}{u'(C_t)} \\ &= \sum_{s=t}^{\infty} e^{-(\rho(s-t)+\eta g_{t,s}-\ln(1+\pi_{t,s}))}\mathbb{E}_t(D_s) \\ &= \sum_{s=t}^{\infty} e^{-r_{t,s}}\mathbb{E}_t(D_s), \end{aligned} \tag{OA.1}$$

where

$$\pi_{t,s} \equiv \frac{\mathbb{E}_t\left(D_s u'(C_s)\right)}{\mathbb{E}_t(D_s)u'(\mathbb{E}_t(C_s))} - 1 \approx \frac{1}{2}\eta(1+\eta)\frac{Var_t(C_s)}{(\mathbb{E}_t(C_s))^2} - \eta\beta_{t,s}\frac{Var_t(C_s)}{(\mathbb{E}_t(C_s))^2} \tag{OA.2}$$

$$\beta_{t,s} = \frac{Cov_t(C_s, D_s)}{Var_t(C_s)}\frac{\mathbb{E}_t(C_s)}{\mathbb{E}_t(D_s)} \tag{OA.3}$$

$$g_{t,s} = \ln\left(\frac{\mathbb{E}_t(C_s)}{C_t}\right) \tag{OA.4}$$

$$r_{t,s} = \rho(s-t) + \eta g_{t,s} - \ln(1+\pi_{t,s}) \approx \rho(s-t) + \eta g_{t,s} - \pi_{t,s}. \tag{OA.5}$$

That is, $\pi_{t,s}$ is the cumulative climate risk premium from period t to period s, $\beta_{t,s}$ is the elasticity of a marginal damage in period s w.r.t. a unit increase of consumption in period t, $g_{t,s}$ is the growth rate of consumption in period s relative to period t, and $r_{t,s}$ is the risk-adjusted discount rate for damage in period s to period t.

To decompose the cumulative risk-adjusted discount rate to one-period-ahead discount rate, note that

$$SCC_t = D_t + e^{-r_{t,t+1}}\sum_{s=t+1}^{\infty} e^{-r_{t+1,s}}\mathbb{E}_t(D_s) = D_t + e^{-r_{t,t+1}}\mathbb{E}_t(SCC_{t+1}) \tag{OA.6}$$

Continuing this process we find

$$SCC_t = D_t + \sum_{s=t+1}^{\infty} e^{-\sum_{v=t}^{s} r_{v,v+1}} \mathbb{E}(D_s). \tag{OA.7}$$

Equating (OA.7) and (18) allows us to find $r_{s,s+1}$ for each period s: $r_{s,s+1} = r_{t,s+1} - r_{t,s}$, which is the one-period-ahead risk-adjusted discount rate conditional on information at t. Since

$$r_{s,s+1} = r_{t,s+1} - r_{t,s} = \rho + \eta \ln \left[\frac{\mathbb{E}_t(C_{s+1})}{\mathbb{E}_t(C_s)} \right] - (\pi_{t,s+1} - \pi_{t,s}), \tag{OA.8}$$

the one-period-ahead climate risk premium (conditional on information at t) can be computed either as $\pi_{s,s+1} = \rho + \eta \ln(\mathbb{E}_t(C_{s+1})/\mathbb{E}_t(C_s)) - r_{s,s+1}$ or $\pi_{s,s+1} = \pi_{t,s+1} - \pi_{t,s}$.

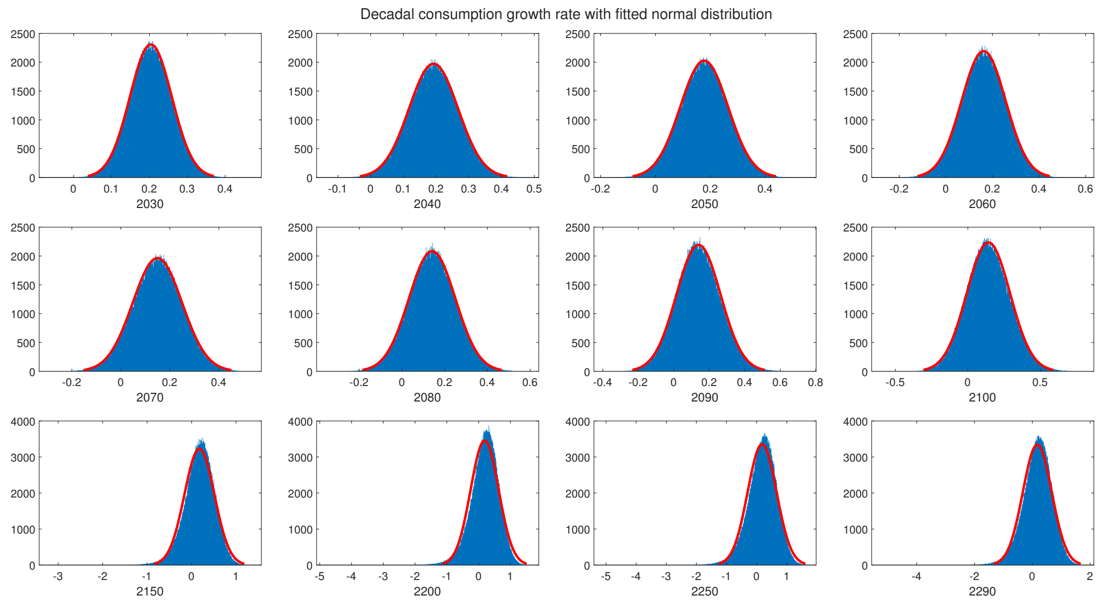
We next decompose the climate risk premium into the prudence and insurance term using

$$\pi_{s,s+1} = \frac{1}{2}(1+\eta)\sigma^2_{s,s+1} - \beta_{s,s+1}\eta\sigma^2_{s,s+1} \tag{OA.9}$$

where $\sigma^2_{s,s+1} = \frac{Var_s(C_{s+1})}{(\mathbb{E}_s(C_{s+1}))^2}$. Since consumption can be very well approximated by a log-normal distribution (see Appendix OA3) and since consumption volatility is small, we can approximate $\sigma^2_{s,s+1}$ by $\sigma^2_{s,s+1} \approx Var_t(\ln(C_{s+1}/C_s))$. We derive the one-period-ahead climate beta as a residual from (OA.9), and compute the prudence and insurance terms accordingly.

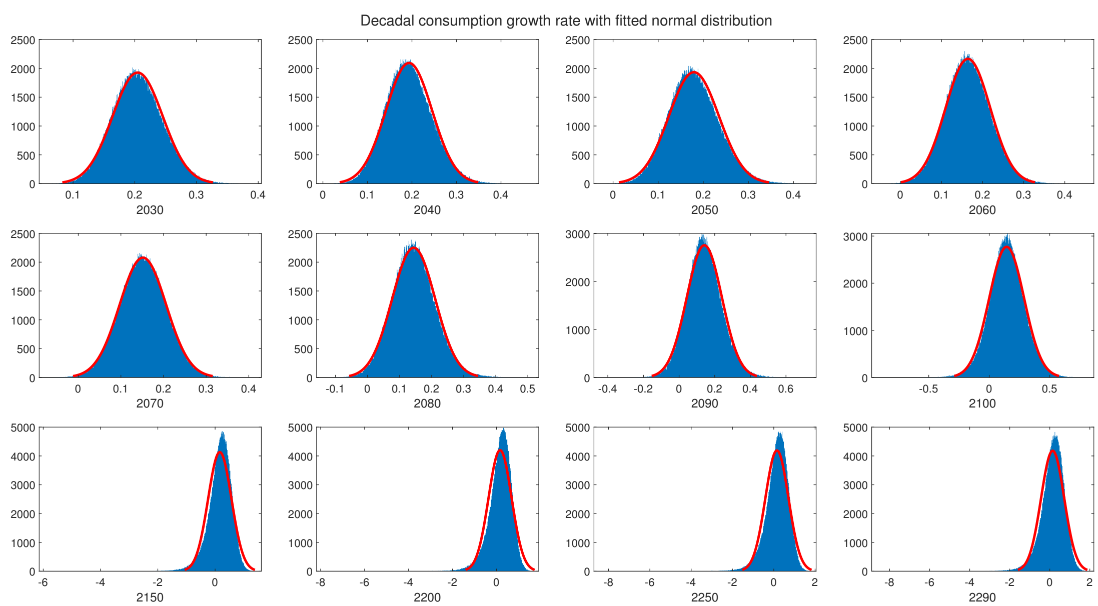
OA3 Distribution of consumption growth rate

Figures OA.1-OA.4 illustrated the histograms (in blue) and fitted normal distributions (in red) for the decadal consumption growth rates from the simulated data used in the quantitative assessment. The distribution of consumption growth rates can be very well approximated by a normal distribution for almost all correlation coefficients and for almost all periods.



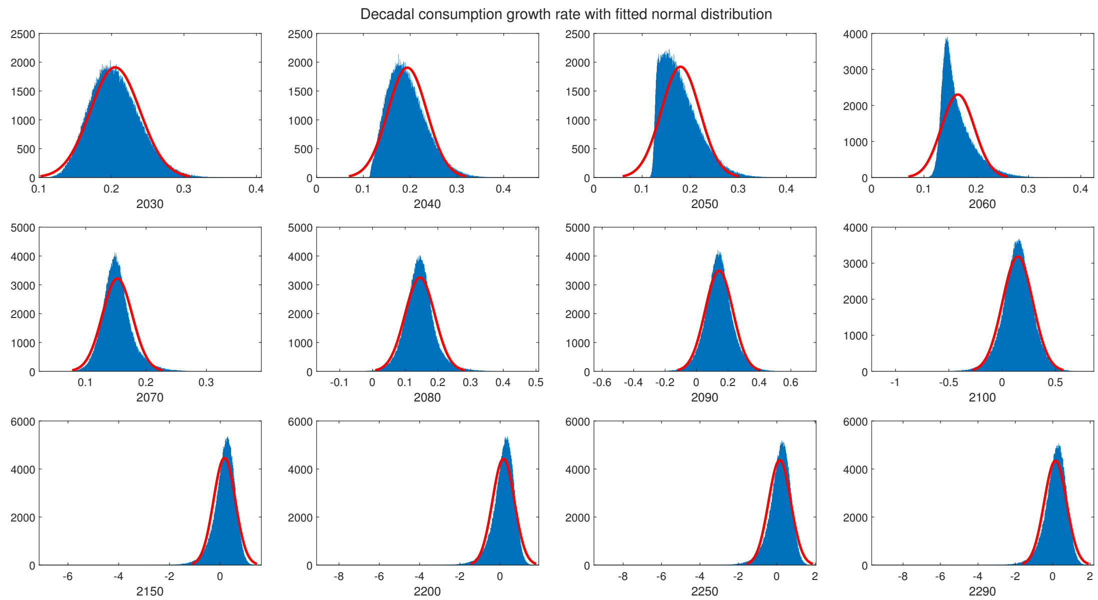
Parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure OA.1: Distribution of decadal consumption growth rate with $\rho_{c,d} = 0$



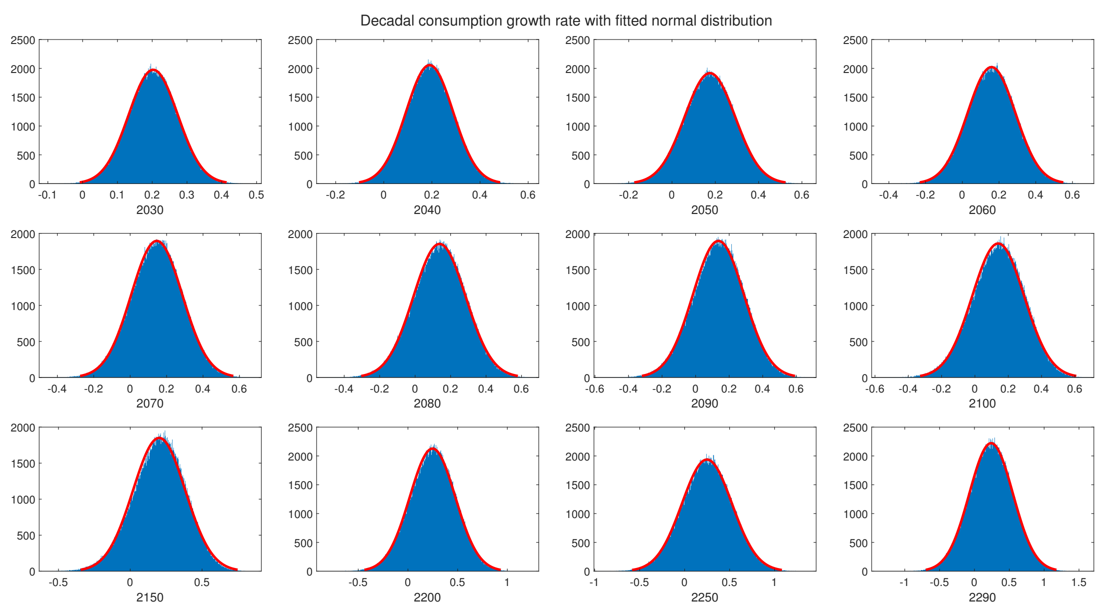
Parameters: $\epsilon = 3$, $\rho_{c,d} = -0.75$, $\sigma_c/\sigma_d = 1$

Figure OA.2: Distribution of decadal consumption growth rate with $\rho_{c,d} = -0.75$



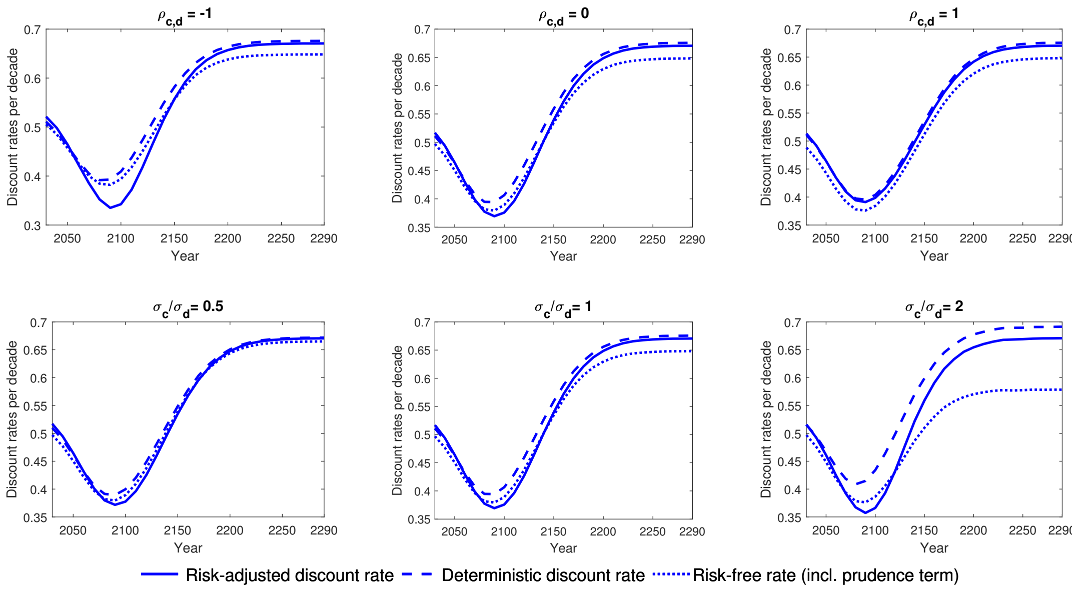
Parameters: $\epsilon = 3$, $\rho_{c,d} = -1$, $\sigma_c/\sigma_d = 1$

Figure OA.3: Distribution of decadal consumption growth rate with $\rho_{c,d} = -1$



Parameters: $\epsilon = 3$, $\rho_{c,d} = 1$, $\sigma_c/\sigma_d = 1$

Figure OA.4: Distribution of decadal consumption growth rate with $\rho_{c,d} = 1$

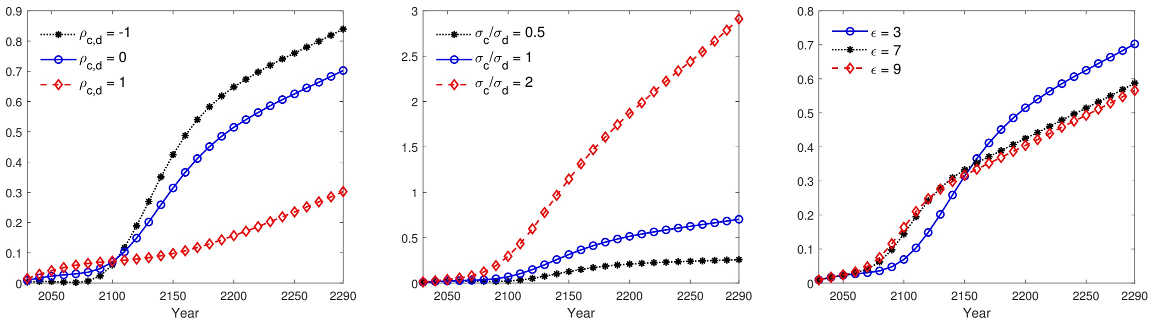


Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure OA.5: Decomposition of discount rates for different correlation coefficients and different sizes of correlation and relative shocks

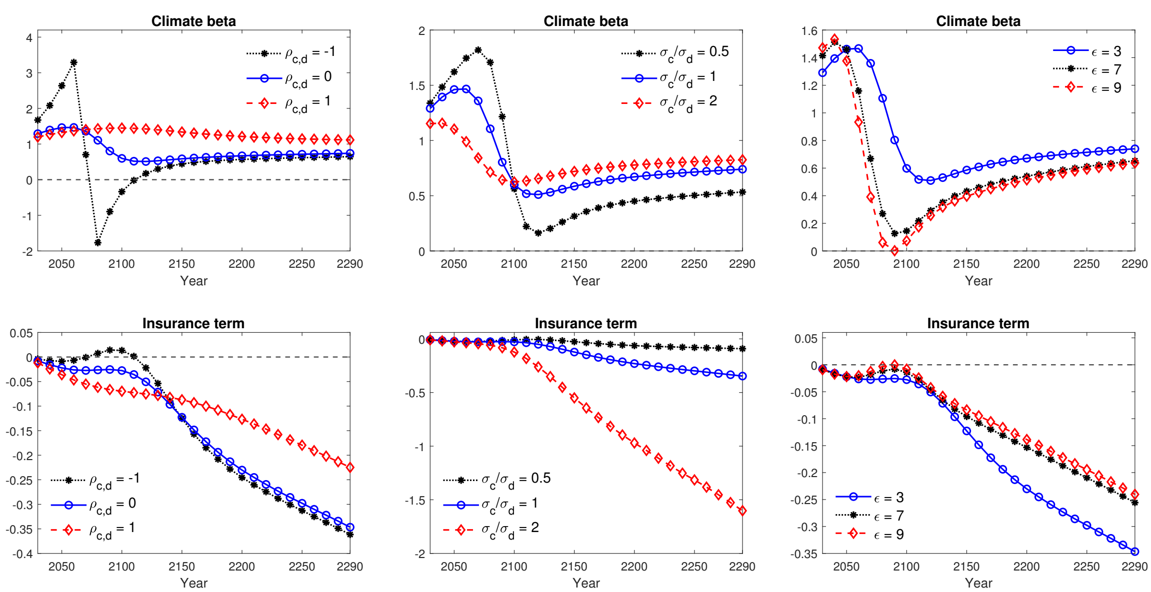
OA4 Additional graphs

Figure OA.5 illustrates the deterministic rate ($\rho+\eta g$), the risk-free rate, and the risk-adjusted discount rate over time. The gap between the risk-free and the deterministic rate is the prudence term and the gap between the risk-adjusted discount rate and the risk-free rate is the insurance term. The dashed lines illustrated the deterministic discount rate, which is proportional to expected consumption growth. The dashed lines are always above the dotted lines, suggesting that the prudence term always lowers the discount rate. Comparing the dashed and solid lines, we see that the insurance value is positive in the mid phase of the transition. Figures OA.6 and OA.7 illustrate the cumulative prudence and insurance terms. The sum of the two correspond to the cumulative climate risk premium $\pi_{t,s}$ in (OA.2). Time-differencing the cumulative prudence and insurance terms will give us the one-period-ahead prudence and insurance terms in Figures 7 and 8.



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure OA.6: The cumulative prudence term



Baseline parameters: $\epsilon = 3$, $\rho_{c,d} = 0$, $\sigma_c/\sigma_d = 1$

Figure OA.7: The cumulative insurance term