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Abstract

This paper studies a model of search engine competition with endogenous obfuscation. Platforms may differ in the quality of their search algorithms. I study the impact of this heterogeneity in consumer surplus, seller profits and platform revenue. I show that the dominant platform will typically induce higher prices but that consumers may benefit from asymmetries. I also show that enabling sellers to price-discriminate across platforms is pro-competitive. I then embed the static model in a dynamic setup, whereby past market shares lead to a better search algorithm. The dynamic consideration is pro-competitive but initial asymmetries are persistent.

JEL-Codes: D430, D830, L130, M370.

Keywords: search engine, platform competition, consumer search.

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Search Engine Competition

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November 29, 2023

1 Introduction

Promoting competition between search engines has long been a priority of regulators on both sides of the Atlantic. In recent years, this regulatory impetus has led to substantial changes in EU law (Digital Markets Act) and an ongoing lawsuit against Google in the US.[1] In its complaint against Google, the DOJ argues that Google's monopolization of the search engine market has led to reduced consumer choice and experience, as well as supra-competitive prices for sponsored search ads. More recently, the strategic partnership between OpenAI and Microsoft/Bing has opened

[1] United States of America v. Google LLC, complaint, In The United States District Court for The District of Columbia (October 20, 2020), https://www.justice.gov/opa/pressrelease/file/1328941/download.

the door to a future with a more fragmented search advertising markets.[2] The aim of this paper is to provide a simple model of search engine competition to explore the impact of such competition on consumers and advertisers in both the short and the long run.

The model revolves around search engines generating profits through sponsored search auctions while exercising control over the design of their search environment. By manipulating the visibility of organic search results, either by direct means (see Armstrong (2006) or De Corniere (2016)) or by informational garbling (Zhong (2023), Nocke and Rey (2023) and Janssen et al (2023)), search engines can increase revenues from sponsored search ads. However, such actions diminish consumer satisfaction and impact their search behavior. As Google own internal communications show, this is the fundamental tradeoff in the design of search engines.[3] I incorporate competition between engines into a conventional Hotelling framework, allowing for heterogeneity in platform information technology.

The model predicts that firms will actively engage in obfuscation as long as competitive pressure, as measured by the horizontal taste parameter, is not too strong.[4] The equilibrium level of obfuscation and the distribution of market shares depend on the shape of the distribution of valuations that each platform generates through its search algorithms. A special case arises when both platforms induce the same (product) demand elasticity but one of the two platforms produces better matches (on average). In this case, the platforms share the market equally and the stronger firm uses the informational advantange to increase the share of consumers who buy from sponsored ads, leading to higher profits. More generally, the more elastic the distribution of valuations, the lower the prices, the higher the search activity and the lower the market share.

Extending this model, I explore two additional dimensions. First, sellers may not always be able to price-discriminate across platforms. If prices are exogenous (determined by offline factors), the equilibrium conditions remain unchanged. If prices are endogenous but firms are forced to offer the same price on both platforms, an externality emerges whereby each platform prefers a slightly higher price

[2] See https://www.theverge.com/23589994/microsoft-ceo-satya-nadella-bing-chatgpt-google-search-ai accessed 03.10.2023

[3] See https://www.justice.gov/d9/2023-09/416692.pdf, accessed on 4.10.2023. Google's own search engine unit argues that some of the proposals coming from the sales unit would make the search experience *unnatural*.

[4] In the view of regulators, the competitive pressure in this market is very weak. As part of their complaint, they accuse Google of engaging in a multitude of practices to reduce consumer switching, such as pre-installation, tying with other products and exclusivity deals with mobile phone suppliers (p. 1 of the complaint). See also the CMA report on search and online advertising (CMA 2020).

to avoid compromising its competitive position. The outcome is higher equilibrium prices and reduced consumer surplus. Second, I investigate information transmission within the platform. I show that informational obfuscation is more efficient than direct obfuscation, as it is targeted to organic sellers. Therefore, we would expect engines to distort the search results of organic outcomes even for moderate levels of competitition.

This static framework suggests a strong relationship between the quality of the search engine's matching algorithm and its market share, even when platforms engage in obfuscation. Conversely, both regulators and industry practitioners agree that there is a strong link between current demand, access to data and the quality of future matches.[5] An alternative hypothesis is that both platforms, knowing the value of data as an input for future profits, have incentives to sacrifice current profits to improve consumer satisfaction. To distinguish between these hypotheses, therefore, I embed the static model in a dynamic model of platform competition. For simplicity, I assume that valuations are uniformly distributed and that platforms differ in the upper bound of the distribution of valuations at some initial period. The key state variable is the difference between these upper bounds, which evolve over time as a function of current market shares and are subject to depreciation. Provided that knowledge depreciates fast enough, market shares are persistent but, eventually, both platforms attain the same technological level and profit. Consumers benefit from this dynamic consideration, as platforms compete more strongly for current consumers. Total platform profits increase in the strength of the dominant firm, while consumers are indifferent. However, if depreciation is slower, market shares diverge and the dominant platform effectively monopolizes the industry, resulting in low consumer satisfaction and lower seller profits.

Although obfuscation has received significant attention, with Ellison and Wolitzky (2012) being a prominent starting point, prior studies focused on sellers deliberately obfuscating their own prices to impede search and bolster profits. In contrast, this study examines obfuscation at the platform level and introduces competition. Similarly, while there is an extensive body of research on product competition in a search engine environment (Athey and Ellison (2011), Chen and He (2011)), these papers assume platform monopolies and concentrate on information transmission, taking the platform's design (search ease) as given. In a recent contribution, Bergemann and Bonatti (2023) study the impact of sponsored advertising on product markets, assuming that search within the platform is costless but some consumers use other

[5] As a former Google CEO acknowledged to the DOJ, "scale is the key. We just have so much scale in terms of the data we can bring to bear" (p. 5 of the 2020 complaint). This is particularly true for *fresh*, location-based and long-tail queries (p. 13).

channels to purchase goods. A few papers include extensions in which they study competition among identical platforms. De Corniere (2016) studies a model of consumer search among sponsored ads (no organic search), whereby the platform sets a per-click fee, which acts as a marginal cost, and firms cannot price discriminate between platforms. In an extension, he explores competition among platforms and allows for asymmetry in their (exogenous) consumer market shares. Since firms cannot price discrimininate and they do not differ in their information technology, consumers are always indifferent between them. Instead, in the model presented in this paper, market shares are endogenenous, search engines must trade off consumer surplus (and thus market share) with firm profits.[6] Dukes and Liu (2016) has a model with obfuscation in which platform's objective is to maximize equilibrium prices (as all consumers buy) and introduces competition as in the current paper. If competitive pressure is strong enough, firms do not obfuscate, while if it is weaker the monopoly outcome arises.[7]

2 Baseline Model

The market consists of two platforms, or search engines, an infinite pool of ex-ante symmetric sellers that are active on both platforms and a mass of consumers (with measure 2). Consumer $i \in [0, 2]$ demands one unit of a consumption good provided by the sellers and derives an unobservable match value x_{ij} from the good of firm j.[8] To access these sellers, every consumer must enter some keywords in one (and only one) of the two search engines, and engage in sequential search. Consumers' are heterogeneous with respect to their preferences over engines ($k = 1, 2$). The utility of consumer i who purchases variety j through search engine k (net of search costs) is:

$$v_{ij}^k = x_{ij} - p_k^j - t(i, k),$$

where $t(i, k) = (\tau/2)((2 - k)i + (k - 1)(2 - i))$ is the (linear) transport cost of consumer i to platform k and p_k^j is the (possibly) platform-specific price of seller j. There is no outside option. Differentiation among platforms may capture the

[6]Dinerstein et al. (2018) studies empirically a related trade-off, whereby a monopolist platform may want to induce active search for prices *instead* of match values, to foster competition. In our framework, search works on both dimensions simultaneously. A crucial difference is that they study an e-commerce platform (Ebay), whose revenue originates in commissions rather than sponsored search advertising.

[7]Mamadehussene (2020) also studies obfuscation in (symmetric) platforms but focuses on price competition among homogenenous producers and thus is silent about information provision and match values.

[8]The population of sellers is fixed. See Eliaz and Spiegler (2011) for a model of a monopolistic engine concerned with entry.

presence of complementary goods (such as virtual assistants, shopping apps, or AI chatbots), pre-installation in different mobile devices, or heterogeneous concern for privacy and the environment.[9]

I assume that match values are independently and identically distributed across sellers according to a continuous distribution $G(x)$. Search engines have access to an (exogenously given) information technology that allows them to make inferences about the valuations of different consumers at different sellers. There is ample evidence that search engines differ in the quality of the matches they generate due to differences in the size of their indices and web-crawling capabilities as well as differences in the algorithm (see CMA (2020)). For simplicity, I assume that these capabilities can be summarized in a binary score z_{ij}^k. Let $F_k(x) = G(x \mid z_{ij}^k = 1)$ represent the conditional distribution of valuations resulting from a high score on platform k. The densities of $F_k(x)$ are denoted as $f_k(x)$ and we let the support of $F_k(x)$ be $(\underline{x}_k, \bar{x}_k)$. In the benchmark model I assume that, for exogenous reasons, engines are committed to discarding those sellers for which $z_{ij}^k = 0$.

In order to discover match values and prices, consumers have to search sequentially. I assume that all consumers first visit the sponsored ad (selected by the platform via a second-price auction) and then engage in random search among non-advertised (organic) sellers. Each search effort costs s but search efforts are successful with probability $1 - \alpha$ resulting in an effective search cost of $s/(1 - \alpha)$. Platforms choose the degree of obfuscation $\alpha \geq 0$ to maximize their profits. [10]

Sellers compete based on prices, and have a constant marginal cost of production normalized to zero. In the baseline model, firms can engage in price discrimination depending on the search engine that directs traffic to them. If the firm is selected to be listed in the sponsored slot in one of the platforms, it has to pay a per-click fee to the platform. Since every consumer clicks, the fee acts as a fixed cost and does not affect equilibrium prices. It follows that one can consider each platform as a stand-alone Wolinsky-type market with a specific distribution of valuations ($F_k(x)$) and search cost, and the only interaction between markets arises through platform competition.

As standard in this literature, given a reservation utility strategy r_k on platform k, there is a unique pricing equilibrium in each platform, with $p_k(r_k) = (1 - F_k(r_k))/f_k(r_k)$, decreasing in r_k due to log-concavity (Anderson and Renault (1999)). Let π_k denote the corresponding profit level per arriving consumer (i.e., $\pi_k = (1 - F_k(r_k))p_k(r_k)$). Since there is an infinite pool of sellers, the zero profit

[9] **?** provides numerous justifications for this assumption in the context of search engines.

[10] Casner (2020) has a similar model in which the monopolist platform includes low-quality sellers as an obfuscation mechanism. Effectively, one can think of α as the measure of low-quality sellers.

condition requires that the equilibrium (second-highest) bid for the privilege of a sponsored position equals π_k. One of those sellers with the highest bid and a high score is then chosen at random.[11]

Let $u_k(\alpha)$ denote the (expected) surplus of a consumer who visits engine k with obfuscation α. Standard arguments show that $u_k(\alpha) = r_k(\alpha) - p_k(r_k(\alpha))$ where $r_k(\alpha)$ solves

$$\int_r^{\bar{x}_k} (x - r) f_k(x) dx = \frac{s}{1 - \alpha}$$

Notice that the search engine only cares about α through its impact on r_k. The problem of a platform is then to choose r to maximize

$$\max_{r \geq \underline{x_k}} \left(1 + \frac{r - p_k(r) - u_{-k}}{\tau} \right) \pi_k(r).$$

The optimization problem is a balancing act between the opposed incentives of consumers and firms. A higher reservation value improves consumer search experience and reduces prices, while reducing the market share of the sponsored seller and its price. It turns out that both effects have the same magnitude and the platform chooses its reservation value to exactly equate the weights it puts on both terms, given its belief about its rival's strategy. This yields a pair of upward-slopping reaction functions $r_k(r_{-k})$. In the following Proposition I describe the unique equilibrium.

Proposition 1. *There exists* $0 < \underline{\tau} < \bar{\tau}$ *such that if* $d \in (\underline{\tau}, \bar{\tau})$ *a unique equilibrium exists with*

$$p_1(r_1^*) + p_2(r_2^*) = 2\tau. \tag{1}$$

and

$$p_1(r_1^*) - p_2(r_2^*) = \frac{2}{3}(r_1^* - r_2^*), \tag{2}$$

and r_k^* *solves* $\int_r (u - r) f_k(u) du = \frac{s}{1-\alpha_k}$. *If instead* $\tau < \underline{\tau}$, *at least one firm chooses* $\alpha_k = 0$ *(no obfuscation) and thus* $r_k = \underline{x_k}$ *and if* $\tau > \bar{\tau}$, *at least one firm chooses* $\alpha = 1$ *(full obfuscation) and thus* $r_k = \bar{x}_k$.

In the interior region, the first condition establishes a link between (average) prices for consumer goods and differentiation among platforms. As platforms become more similar, they tend to reduce their obfuscation to attract more consumers, and sellers end up choosing lower prices. The second condition yields a relation between

[11] I do not attempt here to endogenize the number of sponsored ad slots, as this would require a finite population of bidders. The essential feature is the intra-platform competition between sponsored and non-sponsored sellers.

price dispersion and search activity across platforms. Since market shares depend on $(r_1 - p_1, r_2 - p_2)$, the dominant platform will induce higher consumer prices.

2.1 Information Technologies

In order to make further progress, I need to restrict the heterogeneity in the distribution of valuations generated by these search engines. A particularly convenient way to do this is to use truncations of the distribution $G(x)$, since they preserve quasiconcavity (Bagnoli and Bergstrom (2005)). More precisely, I assume that $F_k(x) = \frac{G(x)-G(\underline{x_k})}{G(\bar{x}_k)-G(\underline{x_k})}$. The parameter $\underline{x_k}$ measures the *relevance* of search results: higher values mean that the algorithm is able to discard less attractive matches.[12] The parameter $\bar{x}_k$ is associated with the ability of the engine to *discover* high-quality matches. Importantly, if $\bar{x}_1 = \bar{x}_2$, then $p_1(x) = p_2(x)$ for all $x > \underline{x_1}$. Instead if $\underline{x_1} = \underline{x_2}$, $p_1(x) > p_2(x)$.[13] I assume throughout that firm 1's distribution of valuations is stronger in the sense that $F_1(x) \leq F_2(x)$.

Consider first the case in which platform algorithms differ in their relevance: i.e., $\bar{x}_1 = \bar{x}_2$, so $p_1(x) = p_2(x)$. It follows that the second equilibrium equation, (2), lies on the 45-degree line on the (r_1, r_2) plane, and so every possible interior equilibrium yields a symmetric distribution of market shares. Hence, $p_1(r^*) = p_2(r^*) = \tau$. This implies that in equilibrium, both firms offer the same utility to consumers, and this remains independent of the search cost or the informativeness of the technology available to each firm within this range. Since the reservation value is the same, total surplus remains constant, resulting in consumer indifference. Notably, a more precise algorithm allows platforms to ensure that the sponsored ads are more relevant, increasing the monetization of the first slot, while increasing obfuscation to maintain prices constant. This leads to lower seller profits and no improvement in consumer surplus, and, therefore, platforms revenue increases by more than social surplus.

In the particular case of differences in relevance, corner solutions are easy to handle. First, if $\tau > p_k(\underline{x})$, competitive pressure is so weak that platforms induce consumers to visit only sponsored ad sellers (as in De Corniere (2016)). Instead if τ is small enough at least one of the two firms strictly prefers not to obfuscate, resulting in asymmetric market shares.

Proposition 2. *Suppose algorithms differ in relevance. If competition is strong enough, the strongest platform obfuscates more and obtains higher profits but both*

[12] Recent contributions by Zhou (2020) and Zhong (2023) discuss in detail the impact of changes to the lower bound of the distribution in models of consumer search. They use the word *selective* to refer to higher values of $\underline{x_k}$. I use *relevance* since it is the term used in the industry (see, e.g., CMA (2020)).

[13] For instance, if $G(x)$ has a linear inverse hazard rate, $a + bx$, then $p_1(x) = p_2(x) + b(\bar{x}_1 - \bar{x}_2)$.

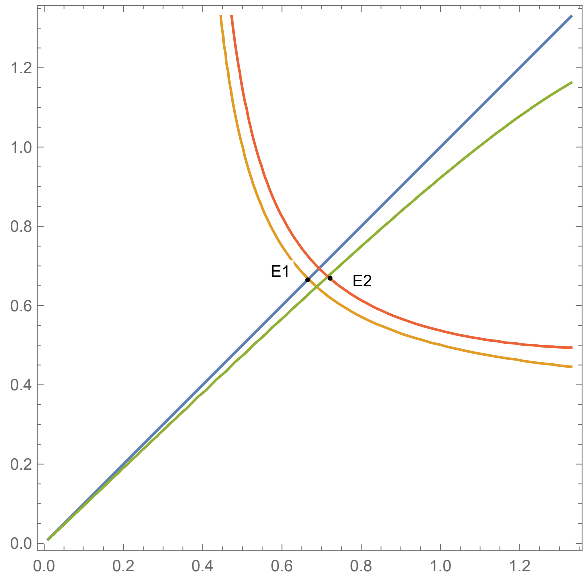

Figure 1: Symmetric and Asymmetric Equilibrium

Notes: E1 represents the symmetric equilibrium and E2 the equilibrium following an improvement in platform 1's discovery.

platforms attract the same number of consumers. If competition is weak, the weakest platform does not obfuscate and the strongest platform is dominant.

If the competitive pressure is strong enough, at least one of the platforms will cease to obfuscate and the platform with the strongest information technology obtains a higher market share. Instead, if competition is sufficiently weak, we have the monopoly outcome described in De Corniere (2016). The interesting case occurs somewhere in the middle, as both firms obfuscate but allow active search. In the remainder of the paper, I focus on this case.[14]

Consider then differences in discovery, which imply that $p_1(x) = p_2(x) + \frac{G(\bar{x}_1)-G(\bar{x}_2)}{g(x)}$. The curve defined by (2) now lies under the 45-degree line while the downslopping curve defined by (1) shifts outwards. The new equilibrium point lies north-east of the original one, so both platforms induce higher reservation values. Platform 1's price increases while platform 2's price falls. This is depicted in Figure 1 for the case of the Weibull distribution with shape parameter 2. The blue and orange lines depict the symmetric case and the green and red lines depict a shift in the technology of platform 1.

Perhaps surprisingly, an improvement in discovery in one platform is akin to a reduction in the differentiation parameter, as the average price curve shifts outwards both with decreases in τ and improvements in discovery. Thus, the impact of this improvement in information technology is positive for both consumers and

[14] A simple condition that guarantees that equilibrium is interior is that $(1/2)p_k(\bar{x}_k) < \tau < p_k(\underline{x_k})$

industry profits. This effect is driven by the endogenous response of obfuscation of the rival firm that experiences a reduction in its market share. The following result summarizes these comparative statics

Proposition 3. *An increase of discovery in platform* 1 *leads to an increase in the reservation value in both platforms, an increase in prices in platform* 1 *and a decrease in prices in platform* 2*. As a result, consumer welfare, search activity and (weighted) average prices increase.*

To conclude this section, consider the case of linear inverse hazard rates, $p_k(r) = a_k - br$. An increase in discovery represents a horizontal shift in the curve, so $a_1 > a_2$. Solving the system of first order conditions yield:

$$p_k = \tau + \frac{a_k - a_{-k}}{2 + 3b}$$

and

$$r_k = \frac{a_k(1 + 3b) + a_{-k} - \tau(2b + 3)}{b(2b + 3)}.$$

The resulting consumer demand of platform 1 is $1 + \frac{a_1 - a_2}{b(2b+3)}$. A horizontal shift in the inverse hazard rate of platform 1 yields a linear increase in consumer surplus, and more consumer search, which (in this case) also benefits sellers.

3 Extensions

3.1 No Price Discrimination

In the baseline model, I have assumed that sellers are able to price discriminate between platforms. In many settings, however, this assumption may be unrealistic. In general, sellers choose prices taking into consideration the elasticity of their demand across different platforms and the offline world. This means that changes in the search environment at a platform is only partially passed-through to consumer prices. For instance, if sellers demand originates exclusively in these two platforms, the derivative of the price with respect to r_1 is half of that in the baseline model. The first order condition in a symmetric interior equilibrium simplifies to

$$p(r) = \tau(1 - \frac{\partial p}{\partial r_1}). \qquad (3)$$

Thus, since $\frac{\partial p}{\partial r_1} \leq 0$, $p(r) \geq \tau$ so $r \leq p^{-1}(\tau)$. Notice that the case of exogenous prices is captured here by $\frac{\partial p}{\partial r_1} = 0$, which leads to the result $\frac{1 - F_k(r)}{f_k(r)} = \tau$ as in the baseline model. In both extreme cases, the platform weights the impact on prices on buyers and sellers symmetrically, inducing the same search intensity. More general, the platform tends to put more weight on the profit level of the ad-slot than in buyers' consumer surplus. It follows that consumers benefit by sellers' ability to price discriminate, as this increases the weight that platforms put on their welfare when choosing obfuscation. Price parity and other similar clauses induce a pricing externality and endogeneously lead to lower surplus and higher prices. The following proposition summarizes these results.

Proposition 4. *Restricting price discrimination harms buyers, and may harm sellers if competition among sellers is sufficiently weak.*

3.2 Information Obfuscation

I now study whether engines do indeed benefit from more information about the match between sellers and consumers, and whether they would use this information to benefit them. More precisely, I extend the model to allow each platform to choose α (direct obfuscation) as well as two parameters $(x_a, x_o) \in [\underline{x}, \underline{x}^*]$ that measure the *relevance* of the matches provided in the sponsored and organic search respectively. The distribution of valuations from the firm that wins the auction in platform k is then $\frac{G(x)-G(x_a)}{1-G(x_a)}$ and likewise for the distribution of valuations of a firm that is listed in the second (and successive slots). Notice that since the number of firms is infinite, the firm may just screen out all firms that do not meet the necessary criterion.[15]

The first observation is that profits are strictly increasing in x_a. To see this notice that x_a does not affect reservation values, but it has a positive impact on the probability that consumers find a good match in the first slot. Therefore, both platforms set $x_a = \underline{x}^*$. The second observation is that, since the distribution of valuations is not stationary, utility equals the surplus in the continuation plus a kickback proportional to the reduction in effective search costs at the ad-slot. More formally,

$$u_k = r_k - p_k(r_k) + \frac{s(G(x_a) - G(x_o))}{(1 - G(x_o))(1 - \alpha)}.$$

This kickback is larger the higher is x_a and (effective) search costs and the lower is x_o. Crucially, the marginal impact of α on this kickback is proportional to $G(x_a) - G(x_o)$, while the marginal impact of α on reservation values is independent of x_a.

[15] I assume throughout that consumers only use one platform, as this is consistent with the evidence (CMA 2020). It should be noted, however, that in this extension consumers would benefit from trying the ads at different platforms, as they may be more informative than the organic results.

Informational obfuscation is more efficient since it specifically targets organic slots while direct obfuscation also reduces consumer satisfaction for the sponsored seller. It follows that if platforms engage in any type of obfuscation, they will always obfuscate informationally ($x_o = \underline{x}^*$ implies $\alpha = 0$) and if they engage in direct obfuscation then they fully obfuscate organic search results ($\alpha > 0$, $x_o = \underline{x}$).

4 Dynamic Model

The preceding analysis suggests that platforms can leverage in technological improvements into a larger market share. In the context of search engines, user data is the main source of those improvements. In particular, search-and-click data generated by current users is a major contributor to future search outcomes (CMA 2020). It follows that firms current market share may be a determinant of future information, establishing a dynamic link. The Justice Department referred to this connection as a *feedback loop* that has entrenched Google's leading position in the search engine market since 2010. As in all antitrust analysis, the question is not whether parties have the ability to use their position to hamper competition but whether they have the incentives to do so.

To answer this question, I study a very dynamic model with exogenous consumer prices.I assume that $1 - F_k^t(x) = \frac{a_k^t - x}{a_k^t}$: that is, the distribution of valuations conditional on receiving the positive signal in platform k is uniform with upper bound a_k^t. I also assume that the future distribution, parametrized by a_k^{t+1}, depends on the current distribution (a_k^t) and the current market share.[16] In particular, assuming that both market shares are strictly positive:

$$a_k^{t+1} = \delta a_k + 1 + \frac{r_k - r_{-k}}{\tau}.$$

Notice that, by construction, the aggregate technological state $a_1^t + a_2^t = \delta(a_1^t + a_2^t) + 1$ and has a steady state at $1/(1-\delta)$. For simplicity, assume that the initial stock is already in steady state. Platforms maximize the present discounted future of their profits, which admit the following Bellman Equation representation:

$$V(a_1, a_2) = \max_{r_1} \pi(r_1, r_2(a_1, a_2), a_1, a_2) + \beta V(a_1', a_2').$$

I guess and verify that $V(a_1, a_2)$ is a quadratic function of the difference between

[16] In Garcia and Shelegia (2018) they explore a model in which previous market shares determine future search behavior, in line with traditional models of switching costs. Here, previous market shares have a direct impact in the information technology that future consumers may enjoy. It is therefore closer to models of learning by doing such as Cabral and Riordan (1994).

technology levels: $V(a_1, a_2) = c_0 + c_1(a_1 - a_2) + c_2(a_1 - a_2)^2$, for some c_0, c_1, c_2. The first order conditions yield:

$$r_1 - r_2 = (a_1 - a_2)\frac{\tau(1 + 4\beta c_2 \delta)}{3d - 4\beta c_2},$$

which is linear in $a_1 - a_2$. Similarly,

$$r_1 + r_2 = \frac{1}{1 - \delta} - 2\tau + 2c_1\beta.$$

Thus, consumers are indifferent about the technological advantage of firm 1 and receive the same surplus in every period, as long as the equilibrium lies in the interior. This yields the following function for the relative technological position in period t:

$$a_1^t - a_2^t = (a_1^0 - a_2^0)\frac{1 + 3\tau\delta}{3\tau - 2c_2\beta}.$$

Solving for the value function we obtain

$$V(a_1 - a_2) = \frac{A_1}{1 - \beta} + \frac{A_2}{1 - \beta\lambda}(a_1 - a_2) + \frac{A_3}{1 - \beta\lambda^2}(a_1 - a_2)^2,$$

which is well defined only if $\beta\lambda^2 < 1$. Consider then the case where either $\beta \approx 0$ or $\delta \approx 0$, ensuring that the problem is well-behaved. Notice that search engines modify the general setup of their search environment only infrequently, so low values of β and δ may be appropriate. We can then solve the system to obtain (c_0, c_1, c_2) as a function of parameters (τ, β, δ). There are three solutions to the system, but only one of them satisfies the necessary conditions (including concavity of the maximization problem) and is also the only solution that converges to the static solution as $\beta \to 0$.

The following proposition summarizes these results.

Proposition 5. *Suppose $\beta < \beta(\delta)$. Then, a MPE exists as characterized above. For β small enough, market shares (and technology) diverge if $\delta > \frac{3\tau-1}{3\tau}$. Furthermore, if $\delta < \frac{3\tau-1}{3\tau}$, an increase in either β or δ lead to an increase in aggregate surplus. Aggregate platform profits are increasing in the initial technological differentiation.*

If $\delta > \frac{3\tau-1}{3\tau}$, either the initial state is symmetric or one of the firms will end up with every consumer, leading to a monopoly outcome. Instead, if δ is sufficiently small, market shares tend to converge. Consumers only care about the current technological level (not its distribution).

To obtain results for larger discount factors I resort to numerical analysis. Figure 2 depicts the equilibrium (average) reservation value for different values of β and

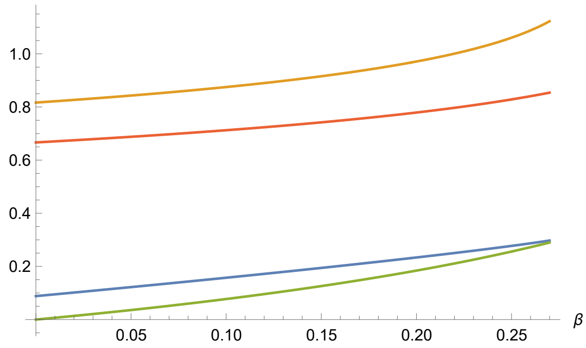

Figure 2: Equilibrium Reservation values and γ

Notes: Constructed for $\tau = 0.5$. The blue and green lines represent the equilibrium reservation values for $\delta = 0.125$ and $\delta = 0$ respectively. The orange and red lines represent the values of γ for $\delta = 0.125$ and $\delta = 0$ respectively.

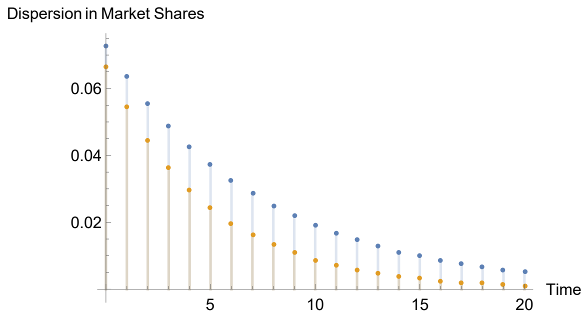

Figure 3: Difference in market shares over time

Notes: Initial difference in technology is 10%. Blue dots correspond to $\beta = 0.1$ and yellow dots to $\beta = 0$. We set $\delta = 0.15$ and $d = 0.5$.

two values of δ (0 and 0.125), as well as the equilibrium value of γ for those parameter configurations. As one can see, the higher the discount factor the higher the utility of the median consumer, but also the larger the dispersion. The dispersion in technology and market shares is also higher the higher the value of δ.

Figure 3 represents the evolution of market shares for a case with convergence and different values of β (0.1 and 0). Higher discount factor induces more persistence in market shares, and therefore, total platform profits increase.

5 Conclusion

This paper presents a simple model of search engine competition, whereby the main strategic variable is obfuscation. Obfuscation allows platforms to effectively choose the intensity of search activity, prices and profits, trading off consumer attractiveness with higher rents. Platforms with a more efficient matching algorithm will tend to have higher market shares, obfuscate more and induce higher prices. Nevertheless, relative to a symmetric outcome, (small) asymmetries may foster competition and welfare. I then embed this static model into a dynamic setting whereby platforms use previous searches to improve their algorithms and show that market shares are persistent but, provided information depreciates fast enough, they tend to converge in the long-run.

From a policy perspective, the model highlights the importance of policies that foster consumer competition in the search engine market. First, increasing consumer willingness-to-switch between platforms should lead to improvements in the quality of search and reductions in equilibrium prices, although these effects may take a while to materialize. Second, the crucial determinant of welfare is the pass-through of technological advantages to consumers, in the form of more efficient search. Our model showcases two examples in which technological improvements by one firm yield completely different outcomes. If one engine improves in discovery, a dominant platform emerges but consumers benefit from the improvements in the search algorithm; if it improves in relevance, instead, the market remains *contested* but neither consumers nor sellers benefit from the improvement. Finally, fostering price competition across platforms (i.e. banning price parity clauses) should not only lead to lower prices but also to a stronger focus on consumer surplus and, thereby, higher welfare.

The model has implications for competition among marketplaces and other similar platforms. The main friction in the model is the preference of the search engine for the sponsored ad seller, which induces obfuscation and higher prices. Since some platforms are vertically integrated and provide goods and services themselves, they have a similar preference for the in-house seller and, therefore, have similar incentives to obfuscate.

References

Anderson, Simon P. and Regis Renault, "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 1999, *30*, 719–735.

Armstrong, Mark, "Competition in two-sided markets," *The RAND Journal of Economics*, 2006, *37* (3), 668–691.

Athey, Susan and Glenn Ellison, "Position auctions with consumer search," *The Quarterly Journal of Economics*, 2011, *126* (3), 1213–1270.

Bagnoli, Mark and Ted Bergstrom, "Log-concave probability and its applications," *Economic Theory*, 2005, *26* (2), 445.

Bergemann, Dirk and Alessandro Bonatti, "Data, competition, and digital platforms," *arXiv preprint arXiv:2304.07653*, 2023.

Cabral, Luis MB and Michael H Riordan, "The learning curve, market dominance, and predatory pricing," *Econometrica: Journal of the Econometric Society*, 1994, pp. 1115–1140.

Casner, Ben, "Seller curation in platforms," *International Journal of Industrial Organization*, 2020, *72*, 102659.

Chen, Yongmin and Chuan He, "Paid placement: Advertising and search on the internet," *The Economic Journal*, 2011, *121* (556), F309–F328.

Corniere, Alexandre De, "Search advertising," *American Economic Journal: Microeconomics*, 2016, *8* (3), 156–188.

Dinerstein, Michael, Liran Einav, Jonathan Levin, and Neel Sundaresan, "Consumer price search and platform design in internet commerce," *American Economic Review*, 2018, *108* (7), 1820–1859.

Dukes, Anthony and Lin Liu, "Online shopping intermediaries: The strategic design of search environments," *Management Science*, 2016, *62* (4), 1064–1077.

Eliaz, Kfir and Ran Spiegler, "A simple model of search engine pricing," *The Economic Journal*, 2011, *121* (556), F329–F339.

Ellison, Glenn and Alexander Wolitzky, "A search cost model of obfuscation," *The RAND Journal of Economics*, 2012, *43* (3), 417–441.

Garcia, Daniel and Sandro Shelegia, "Consumer search with observational learning," *The RAND Journal of Economics*, 2018, *49* (1), 224–253.

Mamadehussene, Samir, "The interplay between obfuscation and prominence in price comparison platforms," *Management Science*, 2020, *66* (10), 4843–4862.

Nocke, Volker and Patrick Rey, "Consumer Search, Steering and Choice Overload," Technical Report, University of Bonn and University of Mannheim, Germany 2023.

Zhong, Zemin, "Platform search design: The roles of precision and price," *Marketing Science*, 2023, *42* (2), 293–313.

Zhou, Jidong, "Improved information in search markets," 2020.

A Omitted Proofs

Proof of Proposition 1. We first derive the equations displayed in the Proposition. Notice that the derivative of consumer surplus with respect to r is simply $1 - p_k'(r)$. By log-concavity, we have that $p_k'(r) < 0$ for both k. The derivative of the profit function with respect to r is

$$\pi_k'(r) = p_k'(r)(1 - F_k(r)) - f_k(r)p_k(r) = (1 - F_k(r))\left(p_k'(r) - \frac{f_k(r)}{1 - F_k(r)}p_k(r)\right).$$

Since $-\frac{f_k(r)}{1-F_k(r)}p_k(r)$ is the product demand elasticity and there is monopolistic competition, we have $\frac{f_k(r)}{1-F_k(r)}p_k(r) = 1$. The first order condition then yields:

$$(1 - F_k(r))(1 - p_k'(r))\left(\frac{p_k(r)}{\tau} - 1 - \frac{u_k(r) - u_{-k}}{\tau}\right) = 0. \tag{4}$$

Since the first two terms of (4) are always strictly positive, $p_k(r) = \tau + u_k(r) - u_{-k}$. Summing up both conditions we obtain $p_k(r) + p_{-k}(r) = \tau$ while taking the difference between them we obtain

$$\begin{aligned} p_1(r_1^*) - p_2(r_2^*) &= 2(u_1(r_1^*) - u_2(r_2^*) \\ &= 2(r_1^* - r_2^* - p_1(r_1^*) + p_2(r^*), \end{aligned}$$

which yields the second condition. I need to verify that there is only one pair of reservation values that satisfies this condition. Notice that the second derivative of the profit function with respect to r equals $2p_k'(r) - 1 < 0$. Hence, the profit function is quasiconcave. I now establish that there is no other interior equilibirum. Suppose (r_1, r_2), (r_1', r_2') are two different equilibria. Substracting (2) in both equilibria we get

$$2(p_1 - p_1') = \frac{4}{3}(r_1 - r_1') - \left(p_2^{-1}(2\tau - p_1(r_1)) - p_2^{-1}(2\tau - p_1(r_1'))\right).$$

Suppose wlog that $p_1 > p_1'$, then it must be that $r_1 < r_1'$ and $p_2^{-1}(2\tau - p_1(r_1)) > p_2^{-1}(2\tau - p_1(r_1')$. But this is a contradiction. Hence $p_1 = p_1'$.

Let $\underline{\tau}$ such that $r_2^* = \underline{x_2}$ and $\bar{\tau}$ such that $r_1^* = \bar{x}_1$. If $\bar{\tau} \geq \tau \geq \underline{\tau}$, the equilibrium conditions guarantee that $1 \geq \alpha_j \geq 0$. If instead $\tau < \underline{\tau}$, $\alpha_2 = 0$, $r_2 = \underline{x_2}$ and $r_1 = \max\{\underline{x_1}, r_1(\underline{x_2})\}$. Analogously, if $\tau > \bar{\tau}$, $\alpha_1 = 1$, $r_1 = \bar{x}_1$ and $r_2 = \min\{\bar{x}_2, r_2(\bar{x}_1)\}$. □

Proof of Proposition 3. Consider a small change in the upper bound of firm 1, so that $p_1(r) = \frac{\Delta}{g(r)} + p_2(r)$ for some $\Delta = G(\bar{x}_1) - G(\bar{x}_2)$. Taking total derivatives with respect to Δ in (1), solve for $p'(r^*)$ as a function of the change in reservation values $(r_1'(a_1), r_2'(a_1))$ and then use (**??**) to conclude that $r_2'(a_1) = \frac{1}{2g(r^*)}\sqrt{9 + 4g(r^*)^2 r_1'(a_1)^2} - \frac{3}{2}$,[17] around the symmetric equilibrium (or everywhere if the inverse hazard rate is linear). Using this back in the formula for $p'(r^*)$, solve for $r_1'(a_1)$ and $r_2'(a_1)$ and obtain,

$$0 < r_1'(a_1) = -\frac{1 - 3p'(r)}{g(r^*)p'(r^*)(2 - 3p'(r^*))} < -1/g(r^*)p'(r^*)$$

and

$$\frac{1}{2g(r^*)}\sqrt{9 + 4\frac{1 - 3p'(r)}{p'(r)(2 - 3p'(r)}} - \frac{3}{2g(r^*)} > 0.$$

Thus, it follows that $p_1(a_1) \geq p^* \geq p_2(a_1)$. To see that consumers benefit notice that platform 1 gains market share even as platform 2 is offering a better search environment and lower prices. □

Proof of Proposition 4. Since $r \leq p^{-1}(\tau)$ and $p \geq \tau$ consumers are harmed. To see the impact on sellers, notice that their surplus is simply $(1 - F(r))F(r)/f(r)$, decreasing in r for r large enough. For the case of linear inverse hazard rates, this condition is satisfied as long as $\tau > \left(\frac{b}{1+b}\right)^b$. On the other hand, platform profits are higher. □

Proof of Proposition 5. Suppose the value function is quadratic. The best response function for firm 1 (analogous for firm 2) is:

$$r_1 = \frac{-2a_1(-\beta c_2 + \beta c_2\delta\tau + \tau) + a_2(2\beta c_2 + 2\beta c_2\delta\tau - \tau) + (d - \beta c_1)(3\tau - 4\beta c_2)}{4\beta c_2 - 3d}$$

Plugging this into the per-period profit function and using the law of motion for the change in the technology, it follows that:

$$V(a_1 - a_2) = \sum_{t=0}^{\infty} \beta^t \left(\theta(a_1 - a_2)\gamma^t + 1\right)\left(\sigma(a_1 - a_2)\gamma^t + \kappa\right),$$

where $\kappa = \frac{(\tau - \beta c_1)(3\tau - 4\beta c_2)}{3\tau - 4\beta c_2}$, $\theta = \frac{4\beta c_2\delta + 1}{3\tau - 4\beta c_2}$, $\gamma = \frac{3\tau\delta + 1}{3\tau - 4\beta c_2}$ and $\sigma = \frac{\tau - 2\beta c_2(\tau\delta + 1)}{3\tau - 4\beta c_2}$. This yields the following expression for the value function:

$$V(a_1 - a_2) = \frac{(a_1 - a_2)^2\theta\sigma}{1 - \beta\gamma^2} + \frac{(a_1 - a_2)(\theta\kappa + \sigma)}{1 - \beta\gamma} + \frac{\kappa}{1 - \beta},$$

[17] This is the only positive root. It is straightforward to show that $r_2'(a_1) \geq 0$.

which is well-defined if $\beta\gamma^2 < 1$. The coefficients c_0, c_1, c_2 can then be obtained by solving a high-order equation. In particular, for $\beta = 0$, $c_1 = 2/3$ and $c_2 = 1/9\tau$. Implicit differentiation with respect to β around $\beta = 0$ yields:

$$\frac{\partial c_2}{\partial \beta} |_{\beta=0} = \frac{(3d\delta + 1)(6c_2\tau + 3\delta\tau + 1)}{81\tau^3} > 0$$

and

$$\frac{\partial c_1}{\partial \beta} |_{\beta=0} = \frac{2(c_2 + 1)(3\tau\delta + 1) - 3c1}{9d} > 0.$$

The derivatives with respect to δ of these parameters vanish as $\beta \to 0$. Thus, $r_1 + r_2$ is increasing in δ (because of lower obselescence), decreasing in d and increasing in β. The dispersion in market shares depends on two components: the technological dispersion and the static incentives. For a fixed technological divergence, $r_1 - r_2$ is increasing in β while the technological dispersion γ is also increasing in β. Hence, the dispersion in market shares increases in β. If $\delta > \frac{3\tau-1}{3\tau}$, market shares diverge and eventually platform 2 is driven out of the market, which leads to lower consumer surplus. Finally, as long as $\delta < \frac{3\tau-1}{3\tau}$, aggregate platform profits are

$$V(a_1 - a_2) + V(a_2 - a_1) = \frac{\kappa}{1-\beta} + 2\frac{(a_1 - a_2)^2\theta\sigma}{1 - \beta\gamma^2}$$

increasing and convex in $a_1 - a_2$. □