CESifo WORKING PAPERS

10896
2024

January 2024

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 10896*

CBDC and the Operational Framework of Monetary Policy

Abstract

We analyze the impact of introducing a central bank-issued digital currency (CBDC) on the operational framework of monetary policy and the macroeconomy as a whole. To this end, we develop a New Keynesian model with heterogeneous banks, a frictional interbank market, a central bank with deposit and lending facilities, and household preferences for different liquid assets. The model is calibrated to replicate the main monetary and financial aggregates in the euro area. Our analysis predicts that CBDC adoption implies a roughly equivalent reduction in banks' deposit funding. However, this 'deposit crunch' has a rather small effect on bank lending to the real economy, and hence on aggregate investment and GDP. This result reflects the parallel impact of CBDC on the central bank's operational framework. For relatively moderate CBDC adoption levels, the reduction in deposits is absorbed by an almost one-to-one fall in reserves at the central bank, implying a transition from a 'floor' system –with ample reserves– to a 'corridor' one. For larger CBCD adoption, the loss of bank deposits is compensated by increased recourse to central bank credit, as the corridor system gives way to a 'ceiling' one with scarce reserves.

JEL-Codes: E420, E440, E520, G210.

Keywords: central bank digital currency, interbank market, search and matching frictions, excess reserves.

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This version: January 2024
A previous version of this paper was circulated under the title "Implications of central bank digital currency for the operational framework of monetary policy". The views expressed in this manuscript are those of the authors and do not necessarily represent the views of the BIS, the Banco de España, or the Eurosystem. We would like to thank Lea Bitter, Ben Hemingway, Joël Marbet, Manuel Muñoz, Dirk Niepelt, Frank Smets, Javier Suarez, and Anton van Boxtel, as well as conference and seminar participants at CEMFI Workshop on CBDCs, 2022 CEBRA Annual Meeting, CUNEF, 6th Annual Workshop of the ESCB Research Cluster 3, Bank of England BEAR Conference, 3rd Catalan Economic Society Congress, 8th SEM Annual Conference, 30th AEFIN Finance Forum, EEA-ESEM Annual Congress, 9th Research Workshop of the MPC Task Force on Banking Analysis for Monetary Policy, ECB Conference on Money Markets, Bank of Canada–Sveriges Riksbank 2nd Conference on the Economics of CBDCs, and CEPR–ECB Conference on The Macroeconomic Implications of CBDCs for their comments. All remaining errors are ours. Jorge Abad gratefully acknowledges financial support from the Spanish Ministry of Science and Innovation grant PID2020-114108GB-I00.

1 Introduction

The potential introduction of a central bank digital currency (CBDC) has gained increasing attention in recent years among policymakers and academics. In March 2022, US President Biden's Executive Order on Ensuring Responsible Development of Digital Assets placed "the highest urgency on research and development efforts into the potential design and deployment options of a United States CBDC". Similarly, in October 2023 the European Central Bank (ECB) announced the start of the preparation phase of its 'digital euro' project, aimed at laying foundations for a potential euro-area CBDC.

While the academic literature has thoroughly analyzed the potential implications of CBDC for financial stability and monetary policy transmission, much less attention has been devoted to its impact on the monetary policy implementation framework and how this is likely to shape the macroeconomic effects of CBDC.[1] Nowadays, most central banks in advanced economies operate a "floor system" in which banks' demand for liquidity is satiated with an ample supply of central bank reserves ("excess reserves"), and interbank market rates are effectively controlled by the interest rate on overnight deposits at the central bank.[2] The introduction of a CBDC has the potential to affect the operational framework of monetary policy and the conditions in interbank markets if it brings about a sufficiently large decrease in excess reserves due to the reduction in bank deposits. This, in turn, may have important macroeconomic implications, both in the long run and in the transitional CBDC adoption phase.

This paper analyzes the implications of the introduction of CBDC for the operational framework of monetary policy and for the macroeconomy as a whole. To this end, we introduce CBDC in a tractable New Keynesian model with heterogeneous banks, a frictional interbank market, and central bank standing (deposit and lending) facilities. Our model features banks that differ in the investment opportunities they face, which moti-

[1]See Infante et al. (2022) for a broad revision of the literature on the macroeconomic implications of CBDC.

[2]For instance, the interest rate on reserve balances (IORB) in the case of the US Federal Reserve, or the deposit facility rate (DFR) in the case of the ECB.

vates the existence of an interbank market. Banks with good investment opportunities seek to borrow in the interbank market so as to finance their lending to firms –which use these funds to invest in productive capital–, while those with bad investment opportunities seek to lend in the same market. The interbank market is characterized by search and matching frictions. Every period, lending and borrowing banks search for each other and, upon matching, trade interbank loans, with the central bank's deposit and lending facilities as the outside options. As a result, the equilibrium interbank rate falls inside the interest rate corridor formed by the deposit and lending facility rates. Its actual position within this corridor is determined by the tightness of the interbank market, i.e. by the ratio between demand and supply of interbank funds. Search frictions imply that part of lending banks' liquidity fails to be placed in the interbank market and ends up as reserves in the central bank's deposit facility, whereas part of borrowing banks' funding needs fails to be covered by the interbank market and is satisfied instead by the lending facility.

Demand for CBDC comes from households' preference for holding liquid assets, which in our case are cash, bank deposits, and CBDC. Following recent research, such as Drechsler, Savov, and Schnabl (2017), Di Tella and Kurlat (2021), or Wang (2022), we assume imperfect substitutability between these different assets, which allows for their coexistence despite their potentially different remuneration. Cash and CBDC are issued by the central bank, thus adding to banks' reserve deposits as central bank liabilities. On the asset side, in addition to its lending facility credit, the central bank also holds government bonds.

We calibrate our model to the euro area. We replicate key features of the balance sheet of the Eurosystem and the consolidated commercial banking sector. The core of our analysis is on the long-run effects of introducing non-remunerated CBDC. In particular, we perform a comparative statics exercise in which we vary households' long-run preferences for CBDC, effectively comparing steady states with a different equilibrium demand for this currency. Our analysis predicts that households' demand for non-CBDC liquidity (bank deposits plus cash) falls essentially one-for-one with CBDC demand, but the bulk of the adjustment (about three quarters) falls on bank deposits. Therefore, relatively large levels of CBDC adoption come hand in hand with a 'deposit crunch' on the banking sector. However, the latter does *not* imply a 'credit crunch': even large reductions in deposit funding have rather small effects on bank lending to firms, and therefore on productive investment and GDP. For instance, a level of CBDC adoption equivalent to 14% of GDP reduces bank deposits by 11% of GDP, but this lowers bank lending by less than 0.6% and GDP by barely 0.25%.

At the core of the above result lies the impact that CBDC has in parallel on the central bank's monetary policy operational framework. Our initial (no CBDC) steady state is consistent with the 'floor system' currently implemented by the ECB and other central banks in advanced economies, characterized by an ample supply of central bank reserves and interbank rates pushed against the remuneration of reserve deposits. For long-run levels of CBDC adoption below 4% of GDP, equivalent to CBDC holdings of about €1,900 per adult person,[3] the reduction in bank deposits is essentially absorbed by an almost one-for-one fall in reserve balances at the central bank.[4] This allows the banking sector to preserve most of its lending to the real economy despite the fall in deposits. For that range of CBDC demand, the floor system is preserved. As CBDC adoption goes beyond that level, some banks start borrowing from the central bank lending facility and the floor system is replaced by a 'corridor system', characterized by a low level of central bank reserves and interbank market rates standing around the midpoint of the interest rate corridor. For CBDC adoption levels exceeding 10% of GDP (equivalent to holdings of about €4,800 per adult person), there are no reserves left to absorb the contraction in bank deposits.[5] Instead, banks replace the lost deposits –and thus continue to preserve most of their lending to firms– by increasing their recourse to the central bank's credit

[3]This back-of-the-envelope calculation is the result of multiplying euro area GDP in 2022 (€13.4 tn) by 4% and dividing the resulting amount by the euro area adult population in that year (281.4 million people).

[4]To put the 4% threshold in context, the volume of euro banknotes in circulation as a percentage of euro area GDP by the end of 2019 was 10.5%.

[5]For comparison, a CBDC holding limit of €3,000 per adult person (as suggested e.g. by Bindseil and Panetta, 2020), if binding, would imply a demand for CBDC equivalent to 6.3% of GDP.

facility. At those levels of CBDC demand, the corridor system gives way to a 'ceiling' system, characterized by scarce (in fact, zero) reserves and interbank rates pushed against the lending facility rate. The endogenous response of the central bank, by lowering its policy rate corridor when excess reserves start to become scarce and recourse to its lending facility increases, guarantees that banks are able to substitute their deposit funding with central bank credit without affecting their overall funding costs.

While small compared to its impact on the banking sector, the effect of CBDC on real outcomes is nonetheless far from negligible. In other words, CBDC is not neutral in the sense of Brunnermeier and Niepelt (2019) as it affects prices and macroeconomic aggregates. In our model, the non-neutrality of CBDC is a consequence of two different channels. First, there is a *remuneration of households' savings* channel, by which the lower average return on households' optimal liquidity basket due to the larger share of (non-remunerated) CBDC entails a reduction in households' savings. The reduction in households' savings leads to a decline in investment and physical capital, which reduces output and consumption. These effects are larger the larger the CBDC take-up is. Second, there is an *operational framework* channel, which becomes active when CBDC adoption is such that the operational framework transits to a corridor system. Under a corridor system, banks that borrow from the central bank's lending facility do so at a higher cost than in the interbank market, and banks that lend their liquidity to the deposit facility receive a lower remuneration than in the interbank market. Both factors hurt overall bank profitability and hence bank equity, which in turn impairs bank lending, capital investment and GDP. This channel is not active when the central bank operates either a floor or a ceiling system, because in these cases the facilities are either accessed at market-neutral conditions (e.g. the deposit facility in a floor system) or continue to entail penalized access but are used only marginally (e.g. the lending facility in a floor system).

Our baseline analysis lets the monetary policy operational framework adjust endogenously to different degrees of CBDC adoption. In practice, some major central banks, like the US Federal Reserve, have already announced their intention to continue operating a floor system.[6] Therefore, we also analyze scenarios in which the central bank preserves the pre-CBDC floor system in the long run. This allows the central bank to neutralize the effects associated to the 'operational framework channel' described above. In our model, the central bank may adopt different policies aimed at maintaining the floor system by increasing the amount of reserves.[7] These include (i) an expansion of government bonds purchases, and (ii) targeted lending operations aimed at supplying funds to the banking sector. Targeted lending operations are characterized by an interest rate and an allowance which links the maximum amount of borrowing to the size of each bank's loan portfolio. We quantify the increase in government bond purchases and the size of the targeted lending allowance necessary to maintain excess reserves constant at their level prior to the introduction of CBDC. We find that, for a CBDC adoption above 14% of GDP, the central bank would need to expand its bond holdings or its lending to banks by 10% of GDP relative to the initial (pre-CBDC) equilibrium.

Brunnermeier and Niepelt (2019) analyze the equivalence between public and private money, in the sense that the introduction of CBDC has no macroeconomic impact as the loss in deposits by commercial banks can be compensated by direct lending from the central bank. This result does not hold in our model when CBDC is not remunerated, as discussed above, because the introduction of CBDC changes the average return on the household's optimal liquidity basket. We prove analytically that, if CBDC is remunerated at an interest rate that does not alter households' total savings decisions *and* CBDC adoption is such that the central bank operates either a floor or a ceiling system, then the introduction of CBDC has no impact on long-run prices or real macro aggregates. The equivalence result does not hold if the CBDC-induced reduction in excess reserves is such that the monetary policy framework shifts to a corridor system, because of the 'operational framework channel' described above. However, the macroeconomic

[6] In its March 20, 2019, announcement on "Balance Sheet Normalization Principles and Plans", the Federal Reserve announced its intention to continue to implement monetary policy in a regime with "an ample supply of reserves" (available at: `https://www.federalreserve.gov/newsevents/pressreleases/monetary20190320c.htm`).

[7] We do not discuss the rationale that central banks may have to preserve a floor system, as it goes beyond the scope of the paper.

impact is quantitatively small. Overall, our results suggest that the household savings' remuneration channel is much more important than the operational framework channel at explaining the macroeconomic effects of CBDC in our model.

Finally, we turn to the study of the transitional dynamics. We start with a situation without CBDC and consider the transitions to steady states that differ in the level of demand for CBDC: one such that the central bank continues to operate a floor system, and another one that leads the central bank to adopt instead a corridor system. Both scenarios are characterized by steady declines in aggregate output, for the reasons explained above, which lead to a temporary fall in inflation. Interestingly, this induces a temporary surge in demand for cash: despite the desire to partially substitute cash and deposits by CBDC, households find it optimal to temporarily increase their cash holdings in order to profit from the increase in real returns in a deflationary environment. While the central bank responds by cutting its policy rates in both scenarios, the response is proportionally stronger when the transition involves a shift from a floor to a corridor system, because in that case the central bank also needs to offset the upward movement of its operational target (the interbank rate) within the policy rate corridor.

Related literature. To the best of our knowledge, this is the first paper to analyze quantitatively the implications of CBDC for the operational framework of monetary policy and how this shapes the macroeconomic impact of CBDC. There have been, however, early studies, such as Infante et al. (2022), Meaning et al. (2021), or Malloy et al. (2022), discussing some of the issues raised by us about the effects of CBDC on interbank rates.A related strand of the literature focuses on the consequences of CBDC design for monetary policy and macroeconomic outcomes. Bordo and Levin (2017) argue that an interest-bearing CBDC replacing physical cash could remove the constraints imposed by the effective lower bound on monetary policy rates. Niepelt (forthcoming) studies a two-tiered monetary system with central bank reserves and analyzes the impact of a CBDC on the implicit subsidies for banks derived from liquidity provision. Burlon et al. (forthcoming) characterize the optimal level of CBDC in circulation and explore the welfare effects of different rules for its remuneration. Barrdear and Kumhof (2022) and Jiang and Zhu (2021) also assess the role of CBDC remuneration rules as a monetary policy tool. Assenmacher et al. (2021, 2022) introduce a CBDC in a New Monetarist model and analyze its remuneration, as well as collateral haircuts and quantity constraints. Lamersdorf et al. (2023) also develop a New Monetarist model with banks' demand for reserves as in Poole (1968), and analyze the role of CBDC design features such as remuneration and holding limits on monetary policy implementation. Fraschini et al. (2023) study the links between CBDC and quantitative easing policies in a stylized two-period equilibrium model. Böser and Gersbach (2020) develop a framework in which switching from deposits to CBDC exposes banks to runs and analyze the role of central bank collateral requirements in shaping banks' liquidity management.[8] Implications of CBDC design for international (monetary policy) spillovers are analyzed by Ferrari Minesso, Mehl, and Stracca (2022), Cova et al. (2022), Ikeda (2020, 2022), and Kumhof et al. (2023). Other aspects of CBDC design, such as those regarding privacy, are analyzed by Ahnert, Hoffmann, and Monnet (2022), Garratt and van Oordt (2021), and Agur, Ari, and Dell'Ariccia (2022).

Our paper also relates to the strand of the literature on the effect of CBDC on bank intermediation. Keister and Sanches (2022) show how substitution between CBDC and deposits could raise banks' funding costs and decrease investment, and how CBDC design could compensate for this effect. Whited, Wu, and Xiao (2022) develop a banking industry equilibrium model with imperfect competition in deposit markets. They find that a CBDC-induced decrease in bank deposits does not lead to an equivalent reduction in credit as banks optimally replace deposits with wholesale funding. Andolfatto (2020), Chiu et al. (2023) and Hemingway (2023) also analyze the effect of CBDC on deposit markets characterized by imperfect competition. Piazzesi and Schneider (2022) study the impact of the substitution between CBDC and deposits when banks face complementarities between

[8]The potential of CBDC as a source of runs on bank deposits has also been analyzed in Ahnert et al. (2023), Bindseil (2020), Fernández-Villaverde et al. (2021), Keister and Monnet (2022), Kumhof and Noone (2021), Muñoz and Soons (2023), Schilling et al. (2020), and Williamson (2022a). Kim and Kwon (2023) analyze the interaction between bank runs and the decrease in excess reserves as a result of the introduction of CBDC.

their deposit taking and loan origination activities. Williamson (2022b) compares CBDC and bank deposits as means of payments, their role as safe assets, and their implications for banks' incentive problems.

Finally, our paper is also related to the literature analyzing the operational framework of monetary policy in models with search-frictional interbank markets, such as Afonso and Lagos (2015), Armenter and Lester (2017), Bianchi and Bigio (2022) or Bigio and Sannikov (2021). In particular, we model the interbank market as in Arce, Nuño, Thaler, and Thomas (2020).

2 Model

Time is discrete. The economy is composed of households, non-financial firms (intermediate-good firms, final-good producers and retailers), banks, the central bank and the government. Figure 1 depicts the balance sheets of the different consolidated sectors of the economy.

2.1 Households

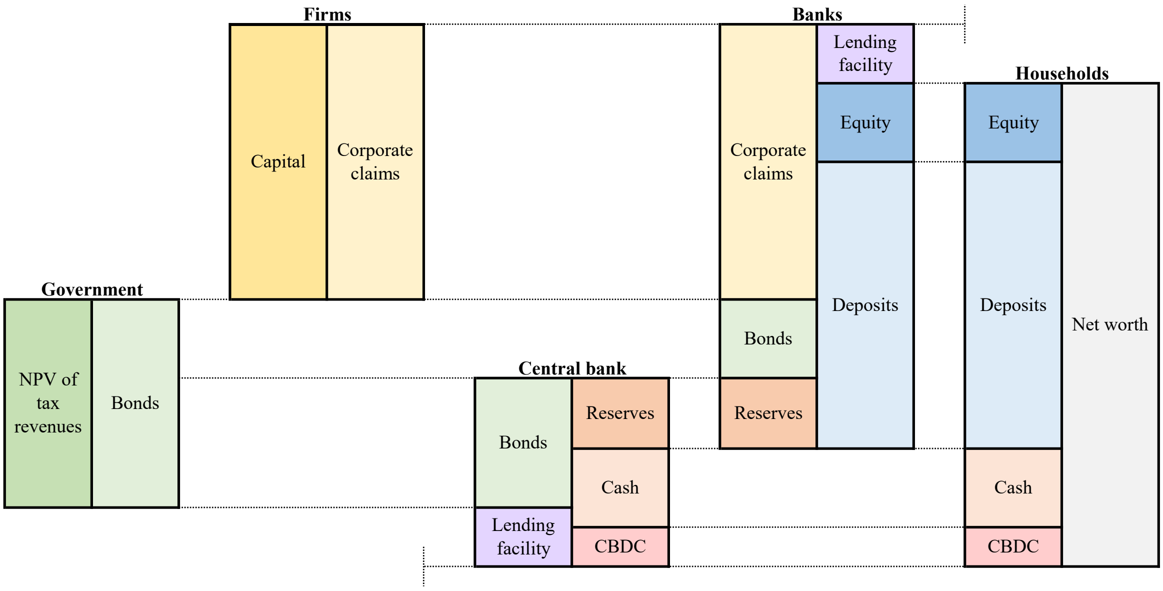
The representative household's utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) + v(L_t) - g(H_t) \right],$$

where C_t is consumption, L_t is a CES aggregator over liquid assets, H_t is labor supply and β is the household's discount factor. Households can save in the form of bank deposits, the real value of which is denoted by D_t, in the form of cash, with *real* value M_t, and in the form of *central bank-issued digital currency* (CBDC), the real value of which is denoted by D_t^{DC}. They also build new capital goods K_t using the technology

$$K_t = \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t + (1-\delta)\, \Omega_{t-1} K_{t-1},$$

Figure 1: Balance sheets of the different consolidated sectors of the model economy.



where I_t are final goods used for investment purposes, and $(1-\delta)\,\Omega_{t-1}K_{t-1}$ is depreciated effective capital repurchased from firms after production in period t; in the latter term, δ is the depreciation rate and Ω_{t-1} is an effective capital index, to be defined below, which the household takes as given. The function S satisfies $S(1) = S'(1) = 0$ and $S''(1) \equiv \zeta > 0$. Liquid assets (deposits, cash, and CBDC) are assumed to be imperfect substitutes, and enter in the household's preferences through a CES aggregator:

$$L_t = \left[(D_t)^{\frac{\varepsilon-1}{\varepsilon}} + \eta_M \, (M_t)^{\frac{\varepsilon-1}{\varepsilon}} + \eta_{DC} \left(D_t^{DC}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with $\eta_M, \eta_{DC} \geq 0$, and $\varepsilon > 1$.[9] The budget constraint of the household is

$$\begin{aligned} C_t + I_t + D_t + M_t + D_t^{DC} \;&=\; W_t H_t + \tfrac{R_{t-1}^D}{P_t/P_{t-1}} D_{t-1} + \tfrac{1}{P_t/P_{t-1}} M_{t-1} + \tfrac{R_{t-1}^{DC}}{P_t/P_{t-1}} D_{t-1}^{DC} \\ &\quad + Q_t^K \left[1 - S\left(\tfrac{I_t}{I_{t-1}}\right)\right] I_t + \textstyle\sum_{s=R,B} \Pi_t^s - T_t, \end{aligned} \tag{1}$$

where P_t is the aggregate price level, R_{t-1}^D is the gross nominal deposit rate, R_{t-1}^{DC} is the gross nominal remuneration on CBDC holdings, W_t is the real wage, Q_t^K is the real price of capital goods, $\{\Pi_t^s\}_{s=R,B}$ are lump-sum real dividend payments from the household's ownership of retailers ($s = R$) and banks ($s = B$), and T_t are lump-sum taxes. The first order conditions (FOCs) for deposits, cash and CBDC are given respectively by:

$$1 - \frac{v'(L_t)}{u'(C_t)} \frac{\partial L_t}{\partial D_t} = \mathbb{E}_t \Lambda_{t,t+1} \frac{R_t^D}{1 + \pi_{t+1}}, \tag{2}$$

$$1 - \frac{v'(L_t)}{u'(C_t)} \frac{\partial L_t}{\partial M_t} = \mathbb{E}_t \Lambda_{t,t+1} \frac{1}{1 + \pi_{t+1}}, \tag{3}$$

$$1 - \frac{v'(L_t)}{u'(C_t)} \frac{\partial L_t}{\partial D_t^{DC}} = \mathbb{E}_t \Lambda_{t,t+1} \frac{R_t^{DC}}{1 + \pi_{t+1}}, \tag{4}$$

where $\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor and $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate. The FOCs for labor supply and investment are standard (see Appendix B).

2.2 Intermediate good firms

We assume that intermediate good firms (and banks) are segmented across a continuum of 'islands', indexed by $j \in [0, 1]$. The representative firm on island j is perfectly competitive

[9] Similar preferences over liquid assets with imperfect degree of substitutability have been used by Drechsler et al. (2017), Di Tella and Kurlat (2021), and Wang (2022), among others. Imperfect substitution between CBDC and other forms of money can arise from heterogeneous preferences over anonymity and security, and from network effects, as in Agur, Ari, and Dell'Ariccia (2022). We think about imperfect substitutability as capturing heterogeneous preferences for the different types of liquid assets across households.

and produces units of the intermediate good, Y_t^j, according to a Cobb-Douglas technology,

$$Y_t^j = Z_t(\omega_{t-1}^j K_{t-1}^j)^\alpha (L_t^j)^{1-\alpha}, \tag{5}$$

where Z_t is an exogenous aggregate total factor productivity (TFP) process, L_t^j is labor, K_{t-1}^j is the pre-determined stock of installed capital, and ω_{t-1}^j is an island-specific shock to effective capital.

The timing is as follows: At the end of period $t-1$ each firm j learns the realization of the shock to next period's effective capital, ω_{t-1}^j. These shocks are iid over time and across islands, and have cumulative distribution function $F(\omega)$. At this point each firm needs to install capital on its island, which it buys from the household at unit price Q_{t-1}^K. In order to finance this purchase, the firm must obtain funding from its local bank. As in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), we assume that the firm sells to the bank one unit of equity A_{t-1}^j per unit of capital acquired: $A_{t-1}^j = K_{t-1}^j$. Equity is a perfectly state-contingent claim on the future return from one unit of capital and is traded at price $Q_{t-1}^{A,j}$. By perfect competition, the price of the capital good and of equity coincide ($Q_{t-1}^K = Q_{t-1}^{A,j}$), and therefore $Q_{t-1}^K K_{t-1}^j = Q_{t-1}^K A_{t-1}^j$. Finally, at the beginning of period t, the firm hires labor and produces.

Each firm j chooses labor in order to maximize operating profits, $P_t^Y Y_t^j - P_t W_t L_t^j$, subject to (5), where P_t^Y is the nominal price of the intermediate good. The first order condition with respect to labor implies that the effective capital-labor ratio is equalized across islands,

$$\frac{\omega_{t-1}^j K_{t-1}^j}{L_t^j} = \left(\frac{W_t}{MC_t (1-\alpha) Z_t} \right)^{1/\alpha}, \tag{6}$$

for all j, where $MC_t \equiv P_t^Y / P_t$ is the inverse of the average gross markup of final goods prices over the intermediate good price, as explained below. The firm's nominal profits then equal $P_t^Y Y_t^j - P_t W_t L_t^j = P_t R_t^k \omega_{t-1}^j K_{t-1}^j$, where

$$R_t^k \equiv \alpha MC_t Z_t \left[\frac{(1-\alpha) MC_t Z_t}{W_t} \right]^{(1-\alpha)/\alpha}$$

is the common real return on effective capital. After production, the firm sells the depreciated effective capital $(1-\delta)\,\omega_{t-1}^{j}K_{t-1}^{j}$ to households at unit price Q_t^K. The total real cash flow from the firm's investment project equals the sum of operating profits and proceeds from the sale of depreciated capital,

$$R_t^k \omega_{t-1}^j K_{t-1}^j + (1-\delta)\, Q_t^K \omega_{t-1}^j K_{t-1}^j. \tag{7}$$

Since capital is financed entirely by equity, the cash flow in (7) is paid off entirely to the lending bank.

2.3 Banks

On each island there exists a representative bank. Only the bank on island j has the technology to obtain perfect information about firms on that island, monitor them, and enforce their contractual obligations.[10] This effectively precludes firms from obtaining funding from other sources, including households or other banks. As indicated before, banks finance firms' investment in the form of perfectly state-contingent debt, A_t^j. After production in period $t+1$, island j's firm pays the bank the entire cash flow from the investment project,

$$\left[R_{t+1}^k + (1-\delta)\, Q_{t+1}^K\right] \omega_t^j A_t^j = \frac{R_{t+1}^k + (1-\delta)\, Q_{t+1}^K}{Q_t^K} \omega_t^j Q_t^K A_t^j.$$

The gross return on the bank's investment in real assets $(Q_t^K A_t^j)$ is thus the product of an aggregate component,

$$R_{t+1}^A \equiv \frac{R_{t+1}^k + (1-\delta)\, Q_{t+1}^K}{Q_t^K},$$

and an island-specific component, ω_t^j. Besides investing in the local firm, the bank may borrow or lend funds in the *interbank market* by means of one-period nominal loans. Because the interbank market is frictional, each bank will generally not be able to borrow

[10]The costs of these activities for the bank are assumed to be negligible.

or lend as much as desired. Let $B_t^{+,j}$ and $B_t^{-,j}$ denote the real amount of *desired* borrowing and lending on the interbank market, respectively, by island j's bank at time t, with $B_t^{+,j}, B_t^{-,j} \geq 0$. For each unit of desired lending the bank receives a noncontingent gross nominal return R_t^L at the beginning of period $t+1$, whereas each unit of desired borrowing costs the bank the noncontingent gross nominal rate R_t^B at the beginning of $t+1$. Both rates are taken as given by the bank. Later we will see how they are determined.[11] As of now it suffices to know that in equilibrium $R_t^B \geq R_t^L$. The bank can also purchase *nominal Treasury bonds*, with nominal return R_{t+1}^G. We denote by $B_t^{G,j}$ the real market value of the bank's government bond portfolio at the end of period t. Finally, the bank takes a real amount D_t^j of *deposits* from the household, which as mentioned before pay a gross nominal return R_t^D.

Combining all these elements, the bank's real net earnings at the start of the following period, denoted by E_{t+1}^j, are given by

$$E_{t+1}^j = R_{t+1}^A \omega_t^j Q_t^K A_t^j + \frac{R_t^L B_t^{-,j} - R_t^B B_t^{+,j}}{1+\pi_{t+1}} + \frac{R_{t+1}^G}{1+\pi_{t+1}} B_t^{G,j} - \frac{R_t^D}{1+\pi_{t+1}} D_t^j. \qquad (8)$$

In each period t the sequence of events is as follows. The bank starts the period with net earnings E_t^j. We assume that the bank pays a fraction $1 - \varsigma \in (0,1)$ of its earnings to households as dividends. The remaining fraction ς is retained as post-dividend equity, denoted by $N_t^j = \varsigma E_t^j$.[12] Following the dividend payment, but *before* learning the shock to the local firm's capital productivity in the next period (ω_t^j), the bank takes deposits D_t^j from households. The deposits market then closes, after which the island-specific shock ω_t^j is realized. Upon observing it, the bank then chooses how much to invest in the local firm ($Q_t^K A_t^j$) and in government bonds ($B_t^{G,j}$), and how much to borrow or lend in the

[11] In particular, they are both a function of the central bank's deposit and lending facility rates, and of the actual interbank market rate.

[12] In equilibrium, this specification is equivalent to assuming that banks do not pay dividends but each period a constant fraction $1 - \varsigma$ of randomly selected banks close for exogenous reasons and pay their accumulated net worth to the household as dividends. For models using specifications similar to the latter, see e.g. Gertler and Karadi (2011) and Nuño and Thomas (2017).

interbank market $(B_t^{+,j}, B_t^{-,j})$, subject to its balance sheet constraint,

$$Q_t^K A_t^j + B_t^{-,j} + B_t^{G,j} = N_t^j + D_t^j + B_t^{+,j}. \tag{9}$$

Finally, banks face an exogenous leverage constraint,

$$Q_t^K A_t^j \leq \phi N_t^j, \tag{10}$$

with $\phi > 1$;[13] and they can not short-sell assets $(A_t^j, B_t^{+,j}, B_t^{G,j} \geq 0)$ or lend negative amounts $(B_t^{-,j} \geq 0)$.

The bank maximizes the expected discounted stream of dividends, $\mathbb{E}_t \sum_{t=1}^{\infty} \Lambda_{t,t+s}(1-\varsigma)E_{t+s}^j$. The problem can be expressed recursively as a two-stage problem within each period, whereby the bank first chooses deposits and then, after the realization of the idiosyncratic shock, chooses the remaining balance-sheet items,

$$V_t(N_t^j) = \max_{D_t^j \geq 0} \int \bar{V}_t(N_t^j, D_t^j, \omega) dF(\omega),$$

$$\bar{V}_t(N_t^j, D_t^j, \omega_t^j) = \max_{A_t^j \geq 0, B_t^{G,j} \geq 0, B_t^{+,j} \geq 0, B_t^{-,j} \geq 0} \mathbb{E}_t \Lambda_{t+1} \left[(1-\varsigma) E_{t+1}^j + V_{t+1}(\varsigma E_{t+1}^j) \right],$$

subject to equations (8), (9) and (10).

Next we assume that parameters are such that the following inequality holds in equilibrium for all t: $D_t \leq (\phi - 1) N_t$, which ensures that in equilibrium the interbank market will be active. This condition simplifies the solution of the banks problem, since it avoids additional case distinctions. Given these assumptions, the solution of the bank's problem

[13] We are assuming that government bonds or interbank lending do not enter the leverage constraint in equation (10). This is completely inconsequential. As we show below, in equilibrium the banks for which the leverage constraint binds choose *not* to invest in bonds or interbank loans. Conversely, the leverage constraint is slack for those banks which choose to invest in bonds or interbank loans.

is given by an investment policy,[14]

$$A_t^j = \begin{cases} \phi N_t^j / Q_t^K, & \text{if } \omega_t^j > \omega_t^B, \\ \left(N_t^j + D_t^j\right) / Q_t^K, & \text{if } \omega_t^L \leq \omega_t^j \leq \omega_t^B, \\ 0, & \text{if } \omega_t^j < \omega_t^L, \end{cases} \tag{11}$$

and a demand policy for interbank borrowing,

$$B_t^{+,j} = \begin{cases} (\phi - 1) N_t^j - D_t^j, & \text{if } \omega_t^j \geq \omega_t^B, \\ 0, & \text{if } \omega_t^j < \omega_t^B. \end{cases} \tag{12}$$

where

$$\omega_t^B \equiv \frac{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_t^B / (1 + \pi_{t+1})\right]}{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_{t+1}^A\right]}, \qquad \omega_t^L \equiv \frac{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_t^L / (1 + \pi_{t+1})\right]}{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_{t+1}^A\right]}, \tag{13}$$

$\tilde{\Lambda}_{t,t+1} \equiv \Lambda_{t,t+1} \left(1 - \varsigma + \varsigma \lambda_{t+1}^N\right)$ is the adjusted discount factor, and λ_t^N is the marginal value of equity. Demand for government bonds and interbank lending satisfies

$$B_t^{G,j} = B_t^{-,j} = 0, \quad \text{if } \omega_t^j \geq \omega_t^L, \tag{14}$$

$$B_t^{G,j} + B_t^{-,j} = N_t^j + D_t^j, \quad (B_t^{G,j}, B_t^{-,j}) \geq 0, \ \text{if } \omega_t^j < \omega_t^L. \tag{15}$$

Banks' individual demand for deposits satisfies:

$$D_t^j \in \left[0, (\phi - 1) N_t^j\right].$$

The ex-ante return on government bonds and the return on interbank lending satisfy a no-arbitrage condition,

$$\mathbb{E}_t \left(\tilde{\Lambda}_{t,t+1} \frac{R_{t+1}^G}{1 + \pi_{t+1}}\right) = \mathbb{E}_t \left(\tilde{\Lambda}_{t,t+1} \frac{R_t^L}{1 + \pi_{t+1}}\right). \tag{16}$$

[14] A derivation of the solution can be found in Appendix A.1.

Finally, the nominal deposit rate equals

$$\begin{aligned} R_t^D &= \left[1 - F\left(\omega_t^B\right)\right] R_t^B + F\left(\omega_t^L\right) R_t^L \\ &\quad + \left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right] \frac{\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^{CB}\right) \mathbb{E}_t\left[\tilde{\Lambda}_{t,t+1} R_{t+1}^A\right]}{\mathbb{E}_t\left[\tilde{\Lambda}_{t,t+1} / \left(1 + \pi_{t+1}\right)\right]}. \end{aligned} \tag{17}$$

In summary, according to their island-specific return realization ω_t^j, banks endogenously split into the following three groups:

- On islands where the local firm draws an idiosyncratic shock above the *borrowing threshold* ω_t^B, the local bank borrows from the interbank market so as to invest in the firm up to the leverage constraint.
- On islands where the local firm draws an idiosyncratic shock below the *borrowing threshold* ω_t^B but above the *lending threshold* ω_t^L, the local bank does not borrow or lend in the interbank market, and invests its equity, deposits and central bank loans in the local firm.
- On islands where the local firm draws an idiosyncratic shock below the *lending threshold* ω_t^L, the local bank lends its resources (equity and deposits) in the interbank market and to the government, with both investments offering the same *ex ante* return according to equation (16).[15]

This implies that the leverage constraint is always binding for the more productive banks, while it is slack for the less productive ones.

Notice also that, according to equation (17), the unit cost of taking deposits at the beginning of the period – i.e. the deposit rate – equals the expected benefit across realizations of ω_t^j. For high-profitability banks ($\omega_t^j > \omega_t^B$) that are leverage-constrained, an additional unit of deposits allows them to reduce their interbank borrowing, thus

[15] Notice that, for these banks, demand for government bonds $B_t^{G,j}$ versus interbank lending $B_t^{-,j}$ is undetermined at the individual level, as both assets are equally profitable *ex ante*. However, it *will* be determined at the aggregate level as explained later on.

saving $\frac{R_t^B}{1+\pi_{t+1}}$. For low-profitability banks ($\omega_t^j < \omega_t^L$), each additional unit of deposits is invested in interbank lending or government bonds, which yields $\frac{R_t^L}{1+\pi_{t+1}}$. For intermediate-profitability banks ($\omega_t^L \leq \omega_t^j \leq \omega_t^B$), each additional unit of deposits is invested in the local firm, with an average idiosyncratic return of $\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B\right)$.[16]

2.4 The interbank market

We model the interbank market as a decentralized, over-the-counter (OTC) market subject to search frictions, in the spirit of Afonso and Lagos (2015), Armenter and Lester (2017), or Bianchi and Bigio (2022), among others. Our modeling of the interbank market follows Arce et al. (2020) closely. Search frictions imply that the market does not automatically clear. Rather, borrowing and lending orders engage in directed search.

As shown in equation (12), banks with $\omega_t^j > \omega_t^B$ borrow in the amount $B_t^{+,j} = (\phi - 1) N_t^j - D_t^j \geq 0$, whereas according to equation (15) those with $\omega_t^j < \omega_t^L$ lend in the amount $B_t^{-,j} = (N_t^j + D_t^j) - B_t^{G,j} \geq 0$. The mass of borrowing and lending orders are thus given respectively by

$$\Phi_t^B \equiv \int_0^1 B_t^{+,j} dj = \int_{j:\omega_t^j > \omega_t^B} \left[(\phi - 1) N_t^j - D_t^j\right] dj = \left[1 - F\left(\omega_t^B\right)\right] \left[(\phi - 1) N_t - D_t\right], \tag{18}$$

$$\Phi_t^L \equiv \int_0^1 B_t^{-,j} dj = \int_{j:\omega_t^j < \omega_t^L} \left[(N_t^j + D_t^j) - B_t^{G,j}\right] dj = F\left(\omega_t^L\right)(N_t + D_t) - B_t^G, \tag{19}$$

where $N_t \equiv \int_0^1 N_t^j dj$ is aggregate bank equity, $B_t^G \equiv \int_{j:\omega_t^j < \omega_t^L} B_t^{G,j} dj$ are aggregate bank holdings of government bonds, and in last equality of each equation we have used the fact that ω_t^j is distributed independently from N_t^j and D_t^j.

Borrowing and lending orders are matched according to a matching function $\Upsilon\left(\Phi_t^L, \Phi_t^B\right)$. We assume that Υ is $C^1\left(\mathbb{R}_+^2\right)$, weakly increasing and concave in both arguments. We also assume that it satisfies $0 \leq \Upsilon(x, y) \leq \min(x, y)$, and that it has constant returns to

[16] Since the bank's problem is locally linear in deposits D_t^j, the banks optimal conditions do not pin down the individual amount of deposit taking but instead the equilibrium deposit rate: By equation (17) in equilibrium the bank breaks even *ex ante*, so it is indifferent between taking one more unit of deposits or not. The only requirement is that all banks satisfy $0 \leq D_t^j \leq (\phi - 1) N_t^j$.

scale. Given constant returns to scale, each lending order finds a borrowing order with probability

$$\frac{\Upsilon\left(\Phi_t^L,\Phi_t^B\right)}{\Phi_t^L}=\Upsilon\left(1,\frac{\Phi_t^B}{\Phi_t^L}\right)\equiv\Gamma^L\left(\frac{\Phi_t^B}{\Phi_t^L}\right), \tag{20}$$

in which case it earns the interest rate R_t^{IB}; otherwise the unit of funds is deposited at the central bank and earns the *deposit facility rate,* R_t^{DF}. Similarly, each borrowing order finds a lending order with probability

$$\frac{\Upsilon\left(\Phi_t^L,\Phi_t^B\right)}{\Phi_t^B}=\Upsilon\left(\frac{1}{\Phi_t^B/\Phi_t^L},1\right)\equiv\Gamma^B\left(\frac{\Phi_t^B}{\Phi_t^L}\right), \tag{21}$$

in which case it pays the interest rate R_t^{IB}; otherwise the unit of funds must be borrowed from the central bank at the *lending facility rate,* R_t^{LF}, with $R_t^{LF}>R_t^{DF}$. Let $\theta_t\equiv\Phi_t^B/\Phi_t^L$ denote the ratio of borrowing to lending, which we henceforth refer to as interbank market *tightness.* Thus, the matching probability for lending (borrowing) orders Γ^L (Γ^B) is increasing (decreasing) in market tightness.

Given the above matching probabilities, the expected return on each lending and borrowing order is given respectively by

$$\Gamma^L(\theta_t)R_t^{IB}+(1-\Gamma^L(\theta_t))R_t^{DF}\equiv R_t^L, \tag{22}$$

$$\Gamma^B(\theta_t)R_t^{IB}+(1-\Gamma^B(\theta_t))R_t^{LF}\equiv R_t^B. \tag{23}$$

We assume competitive search in the interbank market. This assumption allows the model to deliver a natural explanation for the relationship observed in the euro area and other advanced economies between excess reserves and the spread between short-term interbank rates and the interest on reserves. As shown in Appendix A.2, under competitive search the equilibrium interbank interest rate is given by

$$R_t^{IB}=\varphi\left(\theta_t\right)R_t^{DF}+\left(1-\varphi\left(\theta_t\right)\right)R_t^{LF}, \tag{24}$$

where

$$\varphi\left(\theta_t\right) \equiv \frac{d\Gamma^L\left(\theta_t\right)}{d\theta} \frac{\theta_t}{\Gamma^L\left(\theta_t\right)} = \frac{\partial \Upsilon\left(\Phi_t^L, \Phi_t^B\right)}{\partial \Phi_t^B} \frac{\Phi_t^B}{\Upsilon\left(\Phi_t^L, \Phi_t^B\right)} \in (0,1), \tag{25}$$

is the elasticity of the matching probability for lending orders with respect to market tightness –which in turn equals the elasticity of the matching function with respect to the number of borrowing orders.

The equilibrium interest rate for matched orders is a weighted average of the respective outside return/cost: the deposit facility rate R_t^{DF} and the lending facility rate R_t^{LF}. The weight on the former is given by the elasticity $\varphi\left(\theta_t\right)$. Under an appropriately specified matching function, this weight *decreases* with the tightness of the interbank market. Intuitively, as the ratio between borrowing and lending orders increases and the interbank market becomes tighter, it becomes harder for borrowers to find lenders, so the former must offer rates that are higher and hence closer to the lending facility rate. Conversely, in a slack interbank market with abundant lending orders, lenders must accept rates that are lower and hence closer to the deposit facility rate. Since excess reserves effectively are a measure of interbank market slackness, this setup provides a simple explanation for the downward-sloping relationship between excess reserves and the spread between the interbank rate and the interest on reserves observed in the euro area and other major advanced economies.

2.5 Final good producers

A competitive representative final good producer aggregates a continuum of differentiated retail goods indexed by $i \in [0,1]$ using a Dixit-Stiglitz technology with elasticity of substitution $\epsilon > 1$ across retail goods. Cost minimization implies

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t \equiv Y_t^d\left(P_{i,t}\right), \tag{26}$$

where $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{1/(1-\epsilon)}$ is a price index. Total spending in intermediate inputs then equals $\int_0^1 P_{i,t} Y_{i,t} di = P_t Y_t$. Free entry implies zero profits, such that the equilibrium price of the final good is exactly P_t.

2.6 Retail goods producers

We assume that the monopolistic competition occurs at the retail level. Retailers purchase units of the intermediate good, transform them one-for-one into retail good varieties, and sell these to final good producers. Each retailer i sets a price $P_{i,t}$ as in the sticky price model of Calvo (1983) taking as given the demand curve $Y_t^d(P_{i,t})$ and the price of the intermediate good, P_t^y . Specifically, during each period a fraction of firms $(1-\theta)$ are allowed to change prices, whereas the other fraction, θ, do not change. Retailers that are able to change prices in period t choose a new optimal price in order to maximize its expected discounted stream of profits,

$$\max_{P_{i,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \left(\frac{P_{i,t}}{P_{t+k}} - MC_{t+k} \right) \left(\frac{P_{i,t}}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \right]. \tag{27}$$

The first-order condition is standard, with all time-t price-setters choosing a common price P_t^*. The price level P_t evolves according to $P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}$.

2.7 Central Bank

Interest rate policy. The central bank sets three nominal policy rates (all expressed in gross terms): the deposit facility rate R_t^{DF}, the lending facility rate R_t^{LF}, and (once CBDC is introduced) the CBDC remuneration rate R_t^{DC}. We assume that the policy rates are set such that: (i) a constant corridor of width $\chi > 0$ is maintained between the deposit facility rate and the lending facility rate, i.e.

$$R_t^{LF} = R_t^{DF} + \chi, \tag{28}$$

(ii) CBDC is remunerated at a rate of 0, and (iv) the central bank's operational target, which we assume to be the interbank rate, achieves a certain target level. This target level is described by a conventional Taylor rule,

$$R_t^{IB} = \rho R_{t-1}^{IB} + (1-\rho)\left(\bar{R}_{ss} + \upsilon \pi_t\right), \tag{29}$$

where $\bar{R}_{ss}$ is the steady-state nominal interbank rate, $\rho \in (0,1)$ is the interest-rate smoothing parameter, and $\upsilon > 1$ determines the response to deviations in net inflation from target (assumed to be zero). Combining equation (24) and (28), we obtain the following relationship between the operational target and the deposit facility rate: $R_t^{IB} = R_t^{DF} + (1-\varphi_t)\chi$, where $\varphi_t \equiv \varphi(\theta_t)$. Using this and the Taylor rule (29), we can then find the deposit facility rate that implements the desired level for the operational target,

$$R_t^{DF} = \rho\left[R_{t-1}^{DF} + (1-\varphi_{t-1})\chi\right] + (1-\rho)\left(\bar{R}_{ss} + \upsilon\pi_t\right) - (1-\varphi_t)\chi. \tag{30}$$

Balance sheet policy. The central bank also chooses the real market value of its government bond holdings, $B_t^{G,CB}$. We assume that it is a constant fraction of the ratio of total government bonds outstanding to steady-state GDP

$$B_t^{G,CB} = \varrho \overline{B}_t, \tag{31}$$

where $\overline{B}_t$ is the real market value of government debt outstanding.

The central bank's assets are government bonds, $B_t^{G,CB}$, and loans to banks extended by its marginal lending facility, i.e. the mass of borrowing orders that did not find matches in the interbank market: $\Phi_t^B\left(1-\Gamma_t^B\right)$. Its liabilities are households' cash and digital currency holdings, M_t and D_t^{DC} respectively, and banks' reserves at its deposit facility, i.e. the mass of interbank lending orders that did not find a match: $\Phi_t^L\left(1-\Gamma_t^L\right)$. We assume that the central bank accumulates no equity and pays all profits to the government.[17]

[17]In case of central bank losses, these are assumed to be covered by the Treasury.

The central bank's *balance sheet*, expressed in real terms, is therefore

$$B_t^{G,CB} + \Phi_t^B \left(1 - \Gamma_t^B\right) = \Phi_t^L \left(1 - \Gamma_t^L\right) + M_t + D_t^{DC}. \tag{32}$$

Finally, the central bank's real profits are

$$\begin{aligned} \Pi_t^{CB} \quad = \quad & \tfrac{R_t^G}{1+\pi_t} B_{t-1}^{G,CB} + \tfrac{R_{t-1}^{LF}}{1+\pi_t} \Phi_{t-1}^B \left(1 - \Gamma_{t-1}^B\right) \\ & - \tfrac{R_{t-1}^{DF}}{1+\pi_t} \Phi_{t-1}^L \left(1 - \Gamma_{t-1}^L\right) - \tfrac{1}{1+\pi_t} M_{t-1} - \tfrac{R_{t-1}^{DC}}{1+\pi_t} D_{t-1}^{DC}. \end{aligned} \tag{33}$$

2.8 Government

The budget constraint of the government expressed in real terms is given by

$$\overline{B}_{t-1} \frac{R_t^G}{1+\pi_t} = \overline{B}_t + T_t + \Pi_t^{CB}.$$

Without loss of generality, the debt-to-GDP ratio is assumed to be held constant at a certain level: $\overline{B}_t / Y_t = \overline{b}$.[18]

2.9 Aggregation, market clearing and equilibrium

An equilibrium in this model is defined as a set of state-contingent functions for prices and quantities such that all agents' optimization problems are solved and markets clear. Appendix A.3 derives the aggregation and market clearing conditions. Appendix B lists the complete set of conditions that have to hold in equilibrium for aggregate variables.

3 Monetary policy implementation frameworks

In this section we compare the properties of a corridor system, in which the interbank rate lies in the middle of the corridor formed by the interest rates of the central bank's

[18]By assuming that the debt-to-GDP ratio is always constant, we abstract from any fiscal policy impact associated with the introduction of a CBDC.

standing facilities, with those of a floor (ceiling) system, in which the interbank rate is pushed against the floor (ceiling) of such corridor.

3.1 Floor and ceiling systems

A *floor system* is characterized by an interbank rate that sits at the floor of the policy rates corridor, i.e., it is equal or close to the deposit facility rate, $R_t^{IB} \approx R_t^{DF}$. From equation (24), this is the case when $\varphi(\theta_t) \to 1$, which occurs when $\theta_t \to 0$, i.e. when the interbank market becomes arbitrarily *slack*, such that the amount of lending orders is large compared to the amount of borrowing orders. From equations (20) and (21), this implies $\Gamma^B(\theta_t) \to 1$ and $\Gamma^L(\theta_t) \to 0$, i.e. all borrowing orders are matched with lending ones, while most lending orders fail to be matched. Lending orders in excess of the total volume of borrowing orders end up at the central bank's deposit facility as reserves. This is a regime characterized by a structural surplus of bank reserves at the central bank.

Conversely, a *ceiling system* is characterized by an interbank rate that hits the ceiling of the policy rates corridor, i.e. it is equal or close to the lending facility rate, $R_t^{IB} \approx R_t^{LF}$. This is the case when $\varphi(\theta_t) \to 0$, which occurs when $\theta_t \to \infty$, i.e., when the interbank market becomes arbitrarily *tight*. This implies $\Gamma^L(\theta_t) \to 1$ and $\Gamma^B(\theta_t) \to 0$, i.e. all lending orders are matched with borrowing ones –such that there are no bank reserves at the deposit facility– while most borrowing orders fail to be matched. Borrowing needs in excess of the total volume of lending orders are met by the central bank through its lending facility. This is a regime characterized by a structural deficit of bank liquidity, in which the banking sector as a whole obtains funding from the central bank but holds no reserves against it.

A corollary of this is that, both in a floor and ceiling system, all interbank lending (borrowing) orders –whether matched or not– end up earning (costing) the interbank rate R_t^{IB}. Therefore, recourse to central bank standing facilities implies enjoying neutral lending or borrowing conditions *vis-à-vis* interbank market conditions.

3.2 Corridor system

A *corridor system* is characterized by an interbank market rate that trades around the middle of the central bank's standing facility rates, i.e. $R_t^{IB} \approx \frac{R_t^{DF}+R_t^{LF}}{2}$. This is the case when $\varphi(\theta_t) \approx \frac{1}{2}$, which in turn requires the central bank's balance sheet to be relatively 'lean'. To see this, assume that central bank bond holdings are just large enough to support its cash and (once in place) CBDC liabilities: $B_t^{G,CB} = M_t + D_t^{DC}$. From the central bank's balance sheet constraint, equation (32), outstanding amounts in both standing facilities must then be the same: $\Phi_t^B\left(1-\Gamma_t^B\right) = \Phi_t^L\left(1-\Gamma_t^L\right)$. Market clearing in the interbank market requires $\Phi_t^B\Gamma_t^B = \Phi_t^L\Gamma_t^L$, implying $\Phi_t^B = \Phi_t^L$, or equivalently $\theta_t = 1$, i.e. perfectly balanced interbank borrowing and lending orders. Under the natural assumption that the matching function satisfies $\varphi(1) = \frac{1}{2}$,[19] or at least $\varphi(1) \approx \frac{1}{2}$, this lean balance sheet regime delivers a corridor system.

In turn, $\theta_t = 1$ implies the following matching probabilities: $\Gamma_t^L = \Gamma_t^B = \Upsilon(1,1)$, the value of which depends on the assumed matching function. Arce et al. (2020) define a matching technology as *match-efficient* if it satisfies $\Upsilon(x,x) = x$, such that if both sides of the market are equally sized, then all searchers are matched to trading partners. Under our assumption that Υ has constant returns to scale, match-efficiency is equivalently defined as $\Upsilon(1,1) = 1$. Therefore, in the special case of match-efficiency, $\Gamma_t^L = \Gamma_t^B = 1$, such that all interbank borrowing and lending orders are matched, and no recourse is made to either the deposit or lending facility.

More generally, matching technologies that are not match-efficient imply matching probabilities lower than 1, i.e. some trading orders on both sides of the interbank market fail to find a counterpart, such that there is recourse to both central bank facilities in equilibrium. Since in the corridor system the interbank rate lies in the midpoint of the rate corridor, non-matched lending orders deposited at the central bank earn a lower return than the interbank rate, and non-matched liquidity needs satisfied by lending facility credit cost more than the interbank rate. This hurts the profitability of the banking

[19] This will be the case in our numerical analysis.

sector as a whole, which is effectively taxed when accessing the central bank standing facilities under a corridor system.

4 Calibration

We calibrate the model to the euro area. In particular, we calibrate the model's initial (pre-CBDC) equilibrium in order to broadly replicate the monetary conditions expected to prevail around the end of this decade.[20] As will be shown later, current monetary analysts' expectations on the size of the Eurosystem's asset portfolio for the coming years imply that, in the initial equilibrium, the ECB continues to operate under a 'floor system', in which interbank rates, R_t^{IB}, are pegged to the deposit facility rate, R_t^{DF}. In particular, we target a central bank balance sheet that is smaller than the current size (as the Eurosystem is expected to continue running down its monetary policy portfolio of bonds) but larger than in a 'corridor system'. We assume a quarterly time frequency.

We assume standard preferences over consumption, liquidity, and labor: $u(C_t) = \log(C_t)$, $v(L_t) = \vartheta \log(L_t)$,and $g(H_t) = H_t^{1+\kappa}/(1+\kappa)$. We also use a standard quadratic specification for investment adjustment costs: $S(x) = \frac{\iota}{2}(x-1)^2$, where ι is a scale parameter. Idiosyncratic shocks ω are assumed to be log-normally distributed with parameters μ and σ. The matching function is as in den Haan et al. (2000),

$$\Upsilon\left(\Phi_t^L, \Phi_t^B\right) = \frac{\Phi_t^L \Phi_t^B}{\left(\left(\Phi_t^L\right)^\lambda + \left(\Phi_t^B\right)^\lambda\right)^{1/\lambda}}.$$

The technology parameters (α, δ, ι), the preference parameters not related to liquid assets (β, κ), the New Keynesian parameters $(\theta, \epsilon, \upsilon, \rho)$, and banks' dividend ratio (ς) are all taken from Gertler and Karadi (2011). The elasticity of substitution between the different types of liquid assets held by the household (ε) is taken from Di Tella and Kurlat (2021).

[20]This way, we isolate our analysis from the effect of recent shocks (pandemic, energy crisis) on current euro area monetary conditions (policy interest rates, Eurosystem balance-sheet size, etc.)

Table 1: Calibrated parameter values

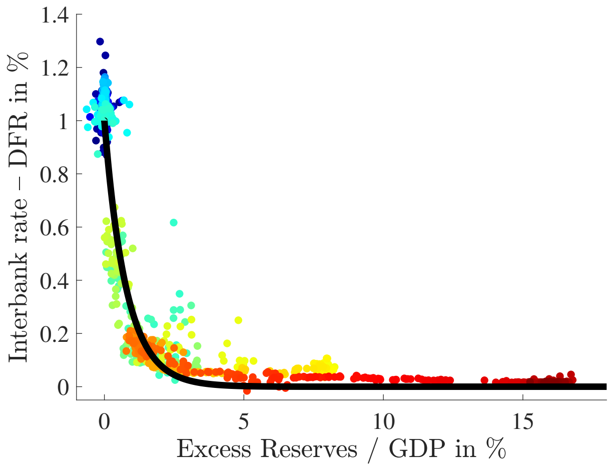
Parameter		Value	Source/Target
α	Capital share	0.33	
δ	Depreciation	0.025	
β	Discount factor	0.995	
κ	Inverse Frisch elasticity	0.276	
θ	Calvo frequency parameter	0.779	Gertler and Karadi (2011)
ϵ	Markup	4.167	
ι	Investment adjustment costs	1.728	
υ	Taylor rule inflation	1.5	
ρ	Taylor rule persistence	0.8	
ς	Bank dividend ratio	0.975	
ε	Liquidity elasticity of substitution	6.6	Di Tella and Kurlat (2021)
μ	Mean of idiosyncratic shocks	-0.0022	Normalize $\overline{\Omega} = 1$
σ	Std of idiosyncratic shocks	0.0032	Share of interbank claims (18.8% of total assets)
ϕ	Leverage constraint	14.5	Steady-state equity ratio (7.9% of total assets)
λ	Interbank matching function	76	Elasticity of DFR–IB spread to excess reserves
ϑ	Household liquidity preference	0.032	Steady-state DFR (1% annualized)
ϱ	Government debt held by CB	0.2567	CB steady-state bond holdings (16% of GDP)
χ	Policy rates wedge	0.25%	Corridor width (1% annualized)
$\overline{b}$	Government debt ratio	2.49	Government debt over GDP (62.3% of GDP)
η_M	Relative weight of cash	1.246	Banknotes in circulations (10.5% of GDP)
η_{DC}	Relative weight of CBDC	0	No CBDC in baseline

The remaining parameters are jointly set to match a number of targets. Nonetheless, each parameter can be shown to be especially important in matching a particular empirical target. For this reason, in what follows we relate individual parameters to specific targets, as described in Table 4. The mean of the iid shocks to island specific capital efficiency μ is set such that the steady state capital efficiency Ω_{ss} is normalized to 1. The matching function parameter λ is calibrated such that the model broadly reproduces the historical relationship between excess reserves over GDP and the interbank-deposit facility rate spread, as shown in Figure 2.[21]

We choose the parameters ϑ and ϱ (respectively, the parameter determining households' preference for liquidity and the fixed share of government bonds held by the central

[21] In particular, we compute the steady-state spread and the steady-state excess reserves to GDP ratio for different values of ϱ (the parameter determining the share of government bonds held by the central bank). We then choose the parameter λ that minimizes the weighted mean absolute error between the data (the dots in Figure 2) and the model prediction across those different steady states (the solid line in the same figure).

Figure 2: Relationship between excess reserves and interbank rate spread in the model and in the data



Note: The figure shows the relationship between the Interbank rate-DFR spread (vertical axis) and the volume of excess reserves over GDP. The solid black line displays the steady-state relationship between both variables in the model for different values of the parameter determining the share of government bonds held by the central bank (ϱ). The dots display weekly Euro area data (colours indicate different time periods, ranging from 1999 in dark blue to 2019 in dark red) where the interbank rate is the EUREPO. Since the shortest available maturity for the EUREPO is 4 weeks, we approximate the expected DFR over the next 4 weeks by the materialized DFR.

bank) to match the level of the deposit facility rate (1%) and the size of the ECB asset purchases programs (16% of GDP) expected to prevail at the end of this decade, according to the April 2022 ECB Survey of Monetary Analysts.[22] The parameter defining the corridor width χ is set to 0.25% per quarter, which implies an annualized corridor width of one percentage point. The parameter $\bar{b}$ is set to match the outstanding level of government debt as a percentage of GDP (62.3%).[23]

The volatility of i.i.d. shocks σ and the leverage constraint parameter ϕ are set to match, respectively, the share of interbank claims over total assets (18.8%) and the bank equity to assets ratio (7.9%) of the euro area commercial banking sector by the end of

[22]In particular, we calibrate the steady-state deposit facility rate, R_{ss}^{DF}, to the median expectation (across SMA respondents) of the long-run (from 2029 onwards) value of the DFR; and the steady-state ratio of central bank bond holdings to GDP, $B_{ss}^{G,CB}/Y_{ss}$, to the median expectation of the sum of the APP and PEPP portfolios in 2031 divided by a projection of nominal euro area GDP in the same year. We project nominal euro area GDP using median expectation across SMA respondents for real GDP growth and HICP inflation rates, where the latter is used as a reasonable proxy for projections of GDP deflator inflation up to 2031 (which are not available in the SMA).

[23]Our model's government debt to GDP ratio ($\bar{b}$) must be interpreted as reflecting only the debt held by the banks and the central bank (as we abstract from holdings by other agents, e.g. households). We use the projections for the ratio for total euro area general government debt over GDP in 2031 in the 2022 European Commision's Debt Sustainability Monitor. We then assume that the share of government debt held by banks and the central bank in 2031 will be the same as in the latest observation available.

Table 2: Aggregate commercial banking sector balance sheet

Assets		**Liabilities**	
Claims on non-financial firms	64.9% (206.9%)	Deposits	73.3% (233.5%)
Government bonds	14.5% (46.3%)	Equity	7.9% (25.1%)
Interbank claims	18.8% (60.0%)	Interbank liabilities	18.8% (60.0%)
Central bank reserves	1.7% (5.5%)	Central bank loans	0.0% (0.0%)
Total Assets	100% (318.7%)	Total liabilities	100% (318.7%)

Note: Numbers between brackets are in percentage of GDP.

2019 according to ECB data.[24] The relative weight on cash η_M in households' liquidity preferences is set to match the value of cash in circulation as a percentage of GDP (10.5%) at the end of 2019.[25] We also assume a baseline value of η_{DC} of zero, so that households hold no CBDC in the initial steady state.

Tables 2 and 3 display the balance sheet of the aggregate (non-consolidated) commercial banking sector and the central bank in the model. Our calibration implies that, in the initial steady state, central bank reserves amount to 5.5% of GDP. As shown in Figure (2), this level of excess reserves implies that the central bank continues to operate a floor system –with the interbank rate equal to the deposit facility rate– right before the introduction of CBDC.

Table 3: Central bank balance sheet

Assets		**Liabilities**	
Government bonds	100% (16.0%)	Cash	65.9% (10.5%)
Lending to banks		Reserves	34.1% (5.5%)
Total Assets	100% (16.0%)	Total liabilities	100% (16.0%)

Note: Numbers between brackets are in percentage of GDP.

[24]ECB MFI aggregated balance sheet data (BSI - MFI Balance Sheet Items). Available at: `https://sdw.ecb.europa.eu/browse.do?node=9691115`.

[25]Notice that, in calibrating the latter three parameters, we do not use end-of-decade projections (like those used for other parameters) but rather observed ratios as of 2019. This is because we lack reliable long-run projections for those ratios. Therefore, we simply assume that the 2019 ratios are a good proxy for the values expected to prevail at the end of this decade.

5 Long-run implications of CBDC

This section analyzes the long-run economic implications of introducing CBDC under different scenarios. Given the uncertainty about the future take-up of CBDC, we consider a wide range of values of the parameter η_{DC}, which determines the households' preferences for CBDC holdings and, in turn, their equilibrium demand.

5.1 Baseline analysis: non-remunerated CBDC and endogenous adjustment of the operational framework

Our main analysis focuses on the long-run (steady-state) effects of introducing a non-remunerated CBDC: $R_t^{DC} = 1$. This represents the case in which CBDC and cash earn the same nominal return (zero), which we consider to be a plausible benchmark. Also, we let the central bank's monetary policy operational framework adjust endogenously as we vary the level of CBDC demand.

Scenarios. Figure 3 depicts the long-run values of selected variables for different long-run levels of CBDC adoption (as a percentage of GDP). Higher demand for CBDC results in a reduction in households' demand for cash and deposits (panel a). The reason is that cash, deposits, and CBDC are partial substitutes, and the increase in the demand for one of them implies a relative reduction in the demand for the others. To see this, consider the steady-state version of the Euler equations (2-4):

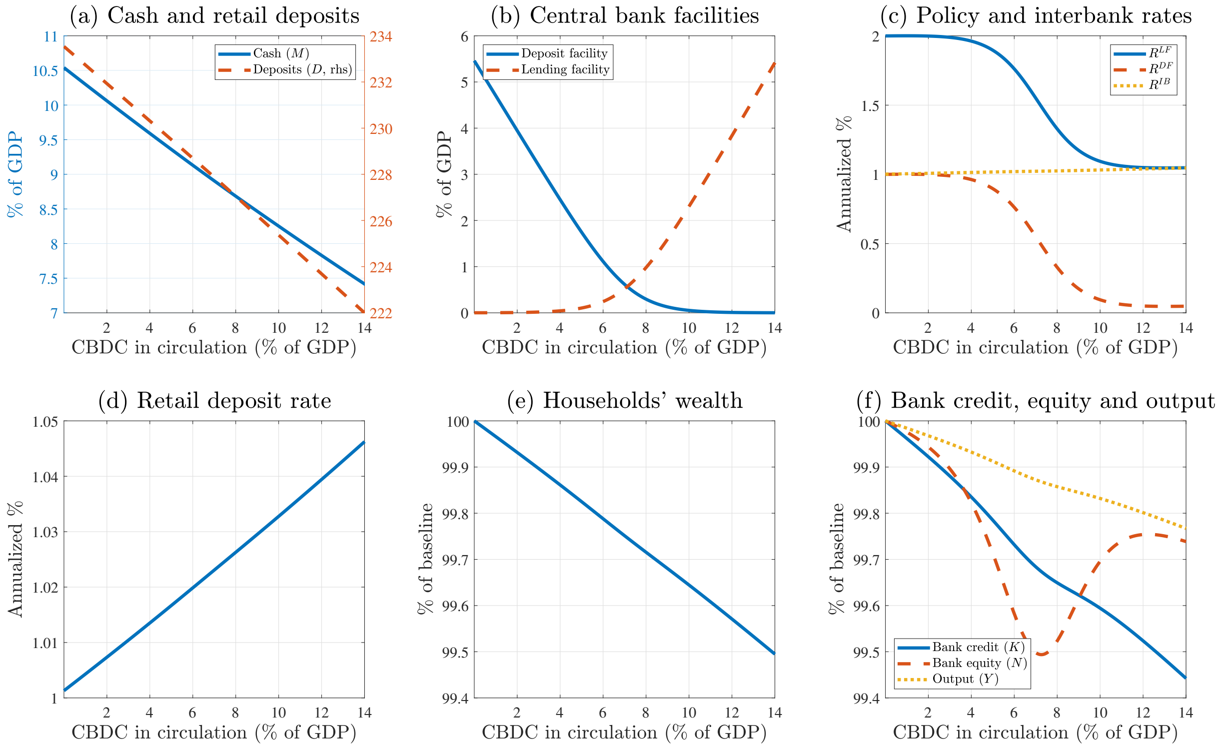
$$1-\frac{v'(L)}{u'(C)}\left(L/D\right)^{\frac{1}{\varepsilon}} = \beta R^D,\ 1-\frac{v'(L)}{u'(C)}\eta_M\left(L/M\right)^{\frac{1}{\varepsilon}} = \beta,\ 1-\frac{v'(L)}{u'(C)}\eta_{DC}\left(L/D^{DC}\right)^{\frac{1}{\varepsilon}} = \beta, \quad (34)$$

which we can combine to obtain

$$\frac{D^{DC}}{M} = \left(\frac{\eta_{DC}}{\eta_M}\right)^{\varepsilon},\ \frac{D^{DC}}{D} = \left(\frac{\left(1-\beta R^D\right)\eta_{DC}}{1-\beta}\right)^{\varepsilon}.$$

The first equation implies that an increase in η_{DC} translates directly into an increase in

Figure 3: Steady-state endogenous variables as a function of the demand for CBDC



Note: Demand for CBDC is varied by changing the parameter η_{DC} which determines the household's preferences for CBDC holdings. Variables presented as "annualized %" refer to annualized percentage points; those presented as "% of GDP" refer to percentages of annualized output; and those presented as "% of baseline" refer to percentages of the corresponding value in the baseline model without CBDC.

the ratio of CBDC over cash, with a (log) slope equal to the elasticity of substitution between liquid assets (ε). The second equation offers a similar result for the ratio of CBDC over deposits, with the particularity that, in this case, the return on deposits R^D operates in the opposite direction. As shown by the figure, the bulk of the adjustment falls on bank deposits, in a proportion of about 3 to 4. For instance, CBCD adoption amounting to 14% of GDP is accompanied by reductions in deposits and cash holdings of about 11% and 3% of GDP, respectively.

Excess reserves held by the banking sector fall linearly from 5.5% of GDP to around 1% when the level of CBDC adoption reaches 6% of GDP (solid blue line, panel b). According to our calibration, when the volume of excess reserves falls below 2.5% of GDP (which happens for a CBDC take-up of around 4% of GDP), the conditions in the interbank market change: banks are not satiated' in reserves anymore and, some of them start borrowing from the central bank's lending facility (dashed red line, panel b), and the interbank rate starts lifting off from the deposit facility rate. Beyond this point the central bank is forced to shift its policy-rate corridor down in order to keep its operational target (the interbank rate) at the level prescribed by the Taylor rule (panel c). For CBDC take-up of about 7%, reserves become scarce enough that the operational target lies right in the middle of the policy rate corridor; i.e. central bank transitions to a 'corridor' system.

For levels of CBDC demand larger than a certain threshold (around 10% of GDP), there are no more reserves left to absorb the decline in deposit funding. As CBDC demand grows beyond that point, banks' recourse to the lending facility continues increasing and the interbank market becomes tighter and tighter. As a result, the interbank rate is pushed against the ceiling of the corridor, such that the lending facility rate becomes the relevant policy rate (solid blue line, panel c). The operational framework then becomes a 'ceiling system', in which there is a structural lack of liquidity in interbank markets. According to our estimates, and absent any other policy intervention, the transition from a corridor to a ceiling regime happens for a CBDC take-up larger than 12% of GDP.

The macroeconomic implications of the introduction of CBDC can be explained by analyzing the effect on the wealth of the different agents in the economy. Figure 1 illustrates that the ultimate sources of funds in the economy are comprised of the assets owned by the representative household. These are households' liquid asset holdings (physical cash, M; bank deposits, D; and CBDC, D^{DC}), and equity accumulated and managed by banks (N), which is ultimately owned by households too. In what follows we discuss how the introduction of CBDC affects the accumulation of both forms of wealth, and how this translates into macroeconomic effects. In doing so, we highlight two channels, which we refer to as the *remuneration of households' savings channel* and the *operational framework channel*.

The remuneration of households' savings channel. As shown in panel e, the total volume of households' liquid assets, $\mathcal{W} = M + D + D^{DC}$, decreases almost linearly, by up to 0.5% of GDP as CBDC demand reaches 14%. In order to understand this effect, notice that the household's budget constraint (1) can be expressed as

$$C + \mathcal{W} \;\; = \;\; WH + R^{\mathcal{W}}\mathcal{W} + \textstyle\sum_{s=R,B.} \Pi^s - T,$$

where $R^{\mathcal{W}} = R^D D/\mathcal{W} + M/\mathcal{W} + D^{DC}/\mathcal{W}$ is the *(weighted) average return on liquidity.* As the share of D^{DC} over total liquid assets increases, and given that its remuneration is zero, the return on liquidity would decrease, unless the return on deposits, R^D, increases enough to compensate for this. As shown in panel d of Figure 3, the deposit rate increases, for reasons explained below, but this increase is tiny (less than 5 basis points for a CBDC demand of 14% of GDP) compared with the decline in the share of deposits over liquid assets (which falls by 5 pp, from around 96% in the initial pre-CBDC steady state). The decline in the return on liquidity explains why households save less on the aggregate, and hence the decline in total household's liquid assets $\mathcal{W}$.

The operational framework channel. As shown in panel f, bank equity follows an inverse hump-shape behavior around the region in which the interbank rate lies in the middle of the policy rate corridor. This is because, when the central bank operates a corridor system, those banks that fail to find a match in the interbank market are forced to resort to the central bank facilities, where borrowing is more expensive (the lending facility rate is above the interbank rate) and deposits offer a lower remuneration (the deposit facility rate is below the interbank rate). This hurts banks' profitability and depresses the aggregate level of bank equity, which can only be accumulated via retained earnings. This does not happen when the central bank operates a floor (ceiling) system, in which all lending (borrowing) banks find a partner in the interbank market and all borrowing (lending) banks that trade with the central bank do so at the same rate that prevails in the interbank market. The inverse hump-shape in bank lending and output when the central bank moves to a corridor system stems from the fact that banks are leverage constrained and, thus, credit is linked to the total amount of bank equity available. It is, however, not as pronounced as in the case of bank equity, since it is partly compensated by a fall in ω^L (i.e. the return threshold below which banks decide to lend their funds in the interbank market instead of investing in productive firms), reflecting the lower remuneration for lending orders that fail to find a match and thus end up at the central bank's deposit facility.

Total effect. The volume of household's liquid assets and bank equity (which constitute the liability side of the consolidated balance sheet of the financial sector, including the central bank) is ultimately linked to the stock of physical capital operated by firms and the stock of outstanding government debt. The consolidated (steady-state) balance sheet of the financial sector, including the central bank, is[26]

$$K + \overline{B} = \mathcal{W} + N. \tag{35}$$

Ceteris paribus, the reduction of the economy's wealth implies a reduction of the stock of physical capital K and therefore in aggregate output.[27] The reduction in capital, given its decreasing marginal product in the aggregate production function, leads to an increase in its return, which in turn, lifts the deposit and interbank interest rates (panels c and d) in an almost linear fashion. As stated above, the increase in the deposit rate is too small to compensate for the fall in the average return on household savings. Finally, the lower stock of physical capital brings about a reduction in output (panel f), which decreases almost linearly, by up to 0.25% when demand for CBDC reaches 14% of GDP.[28]

In Sections 5.2 and 5.3 we separately analyze the central bank policies necessary to switch off each of these two channels. First, central bank policies aimed at preserving the aggregate level of excess reserves prior to the introduction of CBDC allow the central bank to continue operating a floor system and undo the negative effects on bank profitability

[26]Notice that in the steady state the price of corporate claims equals $Q = 1$, such that bank holdings of those claims are $QK = K$.

[27]Given that the stock of outstanding government debt is assumed to be equal to a constant fraction of output, the fall in output also implies a reduction in $\overline{B}$.

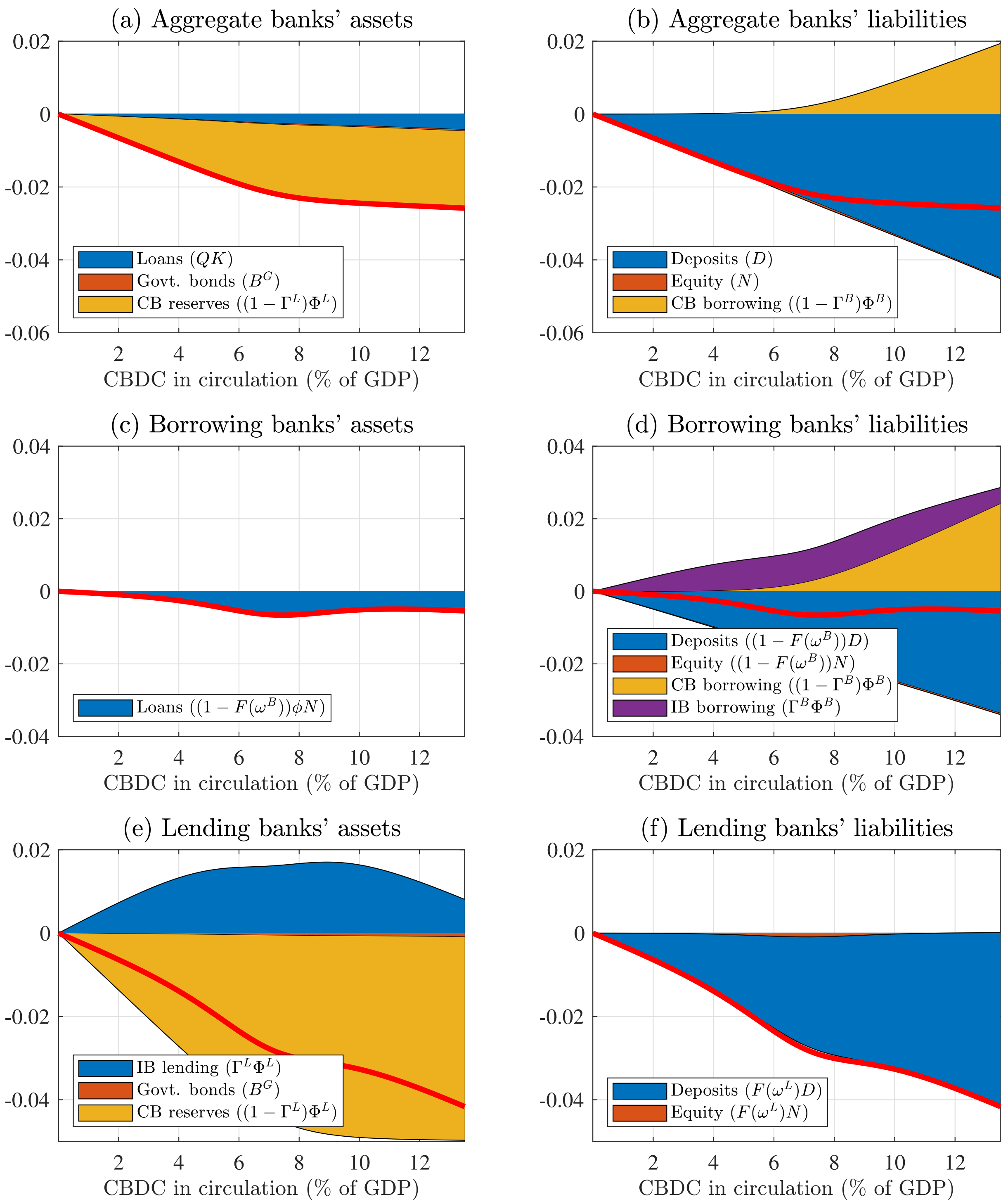
[28]At the same time, labor input H_t also decreases, although to a lesser extent, by 0.07%.

associated to a shift to a corridor system in the baseline scenario (i.e. the *operational framework channel* is switched off). Second, we characterize a particular remuneration rate of CBDC that keeps the average return on households' liquid assets unchanged, which allows to undo the negative effects on households' saving incentives (i.e., the *remuneration of households' savings channel* is deactivated). Finally, we prove analytically that these two policies combined can undo the long-run macroeconomic effects of the introduction of CBDC and render it neutral from the point of view of prices and allocations.

Impact on the banking system. To further understand the effects of the introduction of CBDC on *bank intermediation*, Figure 4 depicts the response of the different components of banks' balance sheet. Panels a and b do so for the consolidated banking sector as a whole. For intermediate levels of CBDC adoption (of up to 6% of GDP), the fall in deposit liabilities is absorbed by an almost one-for-one reduction in reserves at the central bank. Crucially, this allows the banking system to preserve most of its lending to firms. For adoption levels above 8% of GDP, further decreases in deposit liabilities are matched one-for-one with increased recourse to the central bank's lending facility. Again, this allows banks to limit the impact of CBDC on their lending to the real economy.

The response in consolidated assets and liabilities, however, masks differing responses between interbank-borrowing and interbank-lending banks. Having no reserves to begin with, borrowing banks compensate their loss of deposits by borrowing more in the interbank market and, for sufficiently large CBDC adoption, also by borrowing more from the central bank (panel d). This allows them to preserve most of their lending to firms (panel c). By contrast, lending banks respond to their deposit loss (panel f) by reducing their central bank reserves; in fact, they do so by *more* than the actual fall in deposits, as they use part of their liquidity to increase their lending in the interbank market (panel e). For sufficiently large demand for CBDC, however, lending banks run out reserves, and additional deposit outflows are met with a cutback in interbank lending. It is at this point that borrowing banks start borrowing from the central bank lending facility, and that the tightening in the interbank market drives the transition from the corridor to the

Figure 4: Banks' balance sheet variables as a function of the demand for CBDC



Note: Demand for CBDC is varied by changing the parameter η_{DC} which determines the household's preferences for CBDC holdings. Units in the vertical axes are relative to total balance sheet size of each of the groups of banks in the baseline scenario without CBDC.

ceiling system.

5.2 Central bank policies to maintain a floor system

We next analyze the implications of different central bank policies aimed at maintaining a floor system. We do not discuss the rationale that central banks may have to preserve the operations of a floor system, as it goes beyond the scope of the paper. This exercise, however, allows us to illustrate the effects of switching off the impact of CBDC introduction associated to the shift from a floor to a corridor system (i.e., the *operational framework channel*). We will focus on two different policies: (i) an expansion of asset purchases; and (ii) targeted loans to banks. Both policies aim at maintaining a sufficiently high level of reserves held by banks.

Asset purchases. The first policy, an expansion of *asset purchases*, consists of finding, for each value of η_{DC}, the value of ϱ (the fraction of government debt held by the central bank) that keeps the level of aggregate reserves constant at their pre-CBDC level.

Loans to banks. The second policy consists of introducing *targeted lending to banks* at an interest rate $R_t^{CB} \leq R_t^{DF}$.[29] Banks can borrow up to a maximum allowance assumed to equal a constant fraction ψ of each bank's lending to firms. Therefore, the more a bank lends to the real economy, the more funding on advantageous terms it can obtain from the central bank, hence the targeted nature of these loans.[30] In equilibrium, only banks with $\omega \geq \omega_t^L$ demand targeted central bank loans, and they do so up to the maximum allowance:

$$B_t^{CB,j} = \begin{cases} \psi Q_t^K A_t^j, & \text{if } \omega_t^j \geq \omega_t^L, \\ 0, & \text{if } \omega_t^j < \omega_t^L. \end{cases} \tag{36}$$

[29]In a floor system, in which all markets rates are pushed against the deposit facility rate, if $R_t^{CB} > R_t^{DF}$ then banks' demand for targeted lending would be zero, since it would be cheaper for them to rely on other sources of funding, including interbank borrowing and retail deposits.

[30]The introduction of this new liability in banks' balance sheets requires recomputing the optimal banking problem laid out in Section 2. We have done so and the complete set of equations is included in Appendix B.

With targeted loans, and focusing again on the case of non-remunerated CBDC ($R_t^{DC} = 1$), the central bank's balance sheet identity and profits become, respectively,

$$B_t^{CB} + B_t^{G,CB} + \Phi_t^B \left(1 - \Gamma_t^B\right) = \Phi_t^L \left(1 - \Gamma_t^L\right) + M_t + D_t^{DC}, \tag{37}$$

$$\begin{aligned} \Pi_t^{CB} \quad = \quad & \tfrac{R_t^G}{1+\pi_t} B_{t-1}^{G,CB} + \tfrac{R_{t-1}^{LF}}{1+\pi_t} \Phi_{t-1}^B \left(1 - \Gamma_{t-1}^B\right) + \tfrac{R_{t-1}^{CB}}{1+\pi_t} B_{t-1}^{CB} \\ & - \tfrac{R_{t-1}^{DF}}{1+\pi_t} \Phi_{t-1}^L \left(1 - \Gamma_{t-1}^L\right) - \tfrac{1}{1+\pi_t} M_{t-1} - \tfrac{1}{1+\pi_t} D_{t-1}^{DC}. \end{aligned} \tag{38}$$

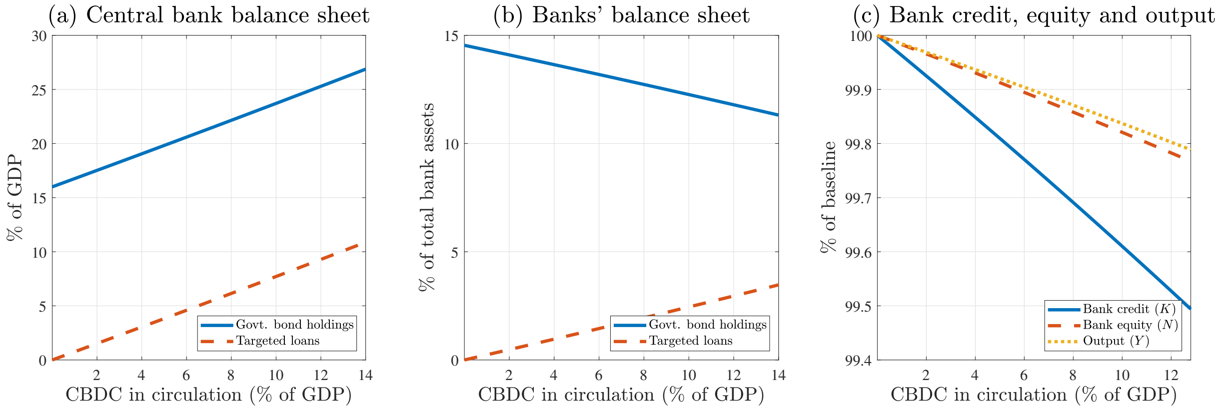
where B_t^{CB} is total targeted lending. In what follows we assume that $R_t^{CB} = R_t^{DF}$. Since in a floor system the interbank market rate equals the deposit facility rate, (floor-preserving) targeted loans are therefore offered on *market-neutral* terms. For each value of the CBDC preference parameter η_{DC}, we then find the value of the allowance parameter ψ that keeps the level of aggregate reserves constant at their pre-CBDC level.

Results. Figure 5 depicts the size of both policies necessary to keep reserves constant at their pre-CBDC level (5.5% of GDP). When CBDC demand goes from 0 to 14% of GDP, central bank holdings of government bonds as a fraction of GDP need to increase by more than 10 percentage points in order to keep excess reserves constant (solid blue line panel a). This means that the central bank bond holdings need to rise from 25% to around 43% of the total stock of outstanding government debt. This highlights a limitation of this policy: its potential to preserve a floor system in an environment of high CBDC demand is constrained by the total supply of government bonds, and especially by institutional limits on the share of eligible government bonds that can be held by the central bank.[31] In parallel, bond holdings by banks drop from 15% to 11% of their total assets (panel b).

The necessary increase in central bank targeted loans as a percentage of GDP is of the same size (above 10 pp for a CBDC demand of 14% of GDP, dashed red line in panel a) as the necessary asset purchase expansion when the latter is the chosen floor-preserving policy, since one additional unit of targeted loans and one additional unit of government

[31] For instance, under the public sector purchase program (PSPP), the Eurosystem is restricted not to exceed an issuer share limit of 33%.

Figure 5: Policies aimed at keeping the level of excess reserves constant



Note: Demand for CBDC is varied by changing the parameter η_{DC} which determines the household's preferences for CBDC holdings. The size of the policies presented above is the one that, for a given demand for CBDC, keep the level of excess reserves constant at their pre-CBDC level.

bonds holdings both result in the same increase in reserves on the liabilities side of the central bank's balance sheet. As a share of total bank assets, targeted loans would amount to about 4% when demand for CBDC reaches 14% of GDP (panel b).

Panel c and its comparison to panel f in Figure 3 allows to gauge the differential effect of the operational framework channel. In Figure 3, the fall in bank equity is around 0.5% of its pre-CBDC level at the trough of the inverse-hump shape followed by bank equity. This happens when the equilibrium amount of CBDC in circulation equals 7% of GDP and the interbank rate is at the middle of the policy rates corridor. By contrast, in Figure 5, for the same equilibrium demand for CBDC, this fall is only around 0.12%. However, the fall in capital and output are essentially the same as in Figure 3. Therefore, the 'remuneration of households' savings channel' is far more important than the 'operational framework channel' at explaining the macroeconomic effects of CBDC in our model.

5.3 CBDC remuneration and the equivalence result

Brunnermeier and Niepelt (2019) make an important contribution by showing how the introduction of CBDC can be neutral, in the sense that it does not affect real macroeconomic aggregates and prices. They refer to it as "equivalence of private and public money". The intuition provided by Brunnermeier and Niepelt (2019) is that the central bank can substitute the loss in commercial banks' deposits due to CBDC with direct loans to banks, in what they refer to as "making central bank's implicit lender-of-last-resort guarantee explicit". As we have seen in the previous section, floor-preserving central bank loans are not enough to guarantee the neutrality of CBDC in our model. Brunnermeier and Niepelt (2019)'s equivalence result hinges on "wealth neutrality", that is, it requires that the introduction of CBDC does not change the distribution of wealth across different agents and does not tighten or relax means-of-payment constraints. In our model, this assumption is violated in the case of an non-remunerated CBDC, for the reasons exposed in Section 5.1.

CBDC remuneration. We can, however, demonstrate that there exists a particular remuneration rate of CBDC that does not distort households' savings decisions and thus does not change households' aggregate wealth, *as long as the central bank operates a floor or a ceiling system*. As we will see, this wealth-neutral rate is the one that keeps constant the return on households' savings, that is, it switches off the *remuneration of households' savings channel*. More precisely, let X and X' be the steady-state values of variable X_t before and after CBDC is introduced, respectively. Then, the wealth-neutral remuneration rate of CBDC, denoted by $\bar{R}^{DC}$, is the one that keeps the average return on liquid wealth, $R^{\mathcal{W}}$, unchanged at its pre-CBDC level:

$$\frac{R^D D + M}{\mathcal{W}} = \frac{R^D D' + M' + \bar{R}^{DC} D^{DC}}{\mathcal{W}'}.$$

Note that R^D appears on both sides of the equation since, by definition, the wealth-neutral remuneration of CBDC is the one that does not change real prices (including the real return on deposits, $R^D/(1+\pi) = R^D$) and allocations. Using the fact that $\mathcal{W} = \mathcal{W}'$ under wealth neutrality, and rearranging, we obtain expression (39) below for the wealth-neutral CBDC remuneration, where $\Delta X \equiv X' - X$ for any variable X. For that particular remuneration of CBDC, and provided the interbank market matching technology is *match-efficient* as defined in Section 3.2, we are able to obtain algebraically the following neutrality result:

Proposition 1 (Wealth-neutral CBDC remuneration) *Let the matching technology in the interbank market be match-efficient, such that $\Upsilon(x, x) = x$. In this case, if the central bank operates a floor or a ceiling system, and CBDC is remunerated at a rate*

$$\bar{R}^{DC} = \frac{R^D \Delta D + \Delta M}{\Delta D + \Delta M}, \tag{39}$$

then all real macroeconomic variables and prices remain invariant after the introduction of a CBDC.

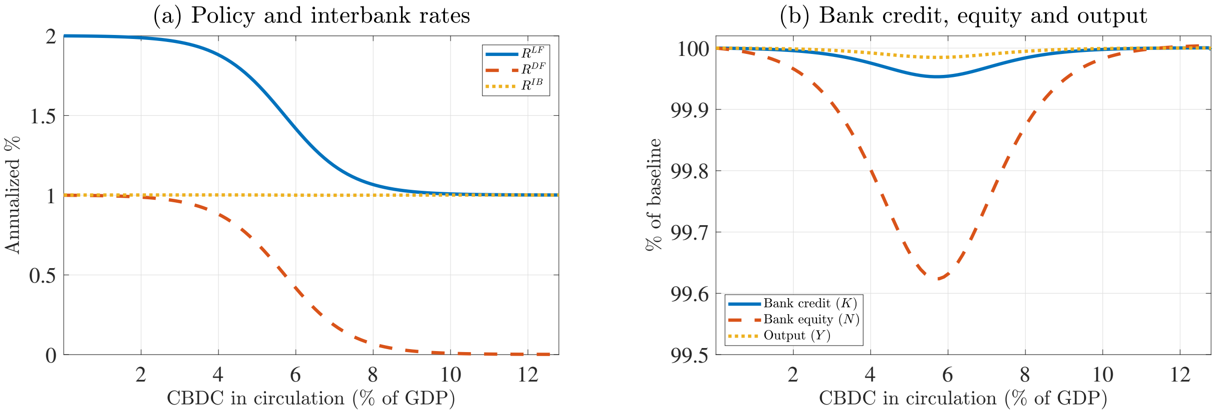
The proof can be found in Appendix A.4. When CBDC is remunerated at the rate $\bar{R}^{DC}$, an increase in the demand for CBDC does not have any long-run effect on prices and allocations, and simply results in a swap between the assets and liabilities held by the different agents in the economy.[32] CBDC demand reduces retail deposits and cash holdings by households. The reduction in deposits on the liability side of the banking sector is matched by an equal reduction in reserves. Since both deposits and reserves are remunerated at the same rate in equilibrium, the effect on bank profits is neutral.

Notice that, in the absence of match-efficiency in the interbank market, as it is the case in our calibration, the neutrality result can still hold numerically as long as the central bank preserves the floor system in the new steady state with CBDC since, in this case, a sufficiently high level of excess reserves makes both the spread between the interbank market rate and the deposit facility rate and recourse to the central bank's lending facility arbitrarily close to zero. If the floor is abandoned, the return on bonds and deposits will differ from the DFR, and there is a non-zero recourse to the central bank's lending facility, hurting banks' profits and distorting their lending decisions relative to the pre-CBDC steady state, in what we referred to above as the 'operational framework channel'.

[32] Differently from Brunnermeier and Niepelt (2019), however, our result does not imply imply neutrality from the point of view of *social welfare*, given our assumption of non-linearity in the preferences for liquid asset holdings in the instantaneous utility function of the representative household. We do not analyze changes in welfare in our comparative statics exercises because preferences are not constant across different steady states, and hence we limit our analysis to positive considerations only.

The neutrality result also goes through if the central bank operates a *ceiling* system, in which all market rates are pushed against the lending facility rate. This is because, in this regime, banks compensate the reduction in deposits with an increase in their recourse to the central bank's lending facility which, in a ceiling system, are remunerated at the same interest rate. Alternatively, the result would also hold if the width of the corridor is zero ($\chi = 0$) and the interest rate on both central bank facilities is therefore the same.

Figure 6: Steady-state endogenous variables as a function of the demand for CBDC with a neutral rate CBDC



Note: Demand for CBDC is varied by changing the parameter η_{DC} which determines the household's preferences for CBDC holdings. Variables presented as "annualized p.p." refer to annualized percentage points, and those presented as "% of baseline" refer to percentages of the corresponding value in the baseline model without CBDC.

By contrast, the proposed wealth-neutral CBDC remuneration rate fails to achieve macroeconomic neutrality if CBDC adoption falls in the intermediate range that implies a *corridor* system. The reason, as discussed above, is the 'operational framework channel': in that case, banks that borrow from the central bank's lending facility do so at a higher cost than in the interbank market, and banks that lend their liquidity to the deposit facility receive a lower remuneration than in the interbank market. Both factors hurt overall bank profitability and hence bank equity, which in turn impairs bank lending, capital investment and GDP. To see this, Figure 6 shows how, in the region in which CBDC take-up ranges from 2 to 9% of GDP, the interbank rate becomes detached from the two policy rates (panel a). This implies a decrease in bank equity, bank loans, and output (panel b), which is larger the closer the interbank rate is to the middle of the corridor, although the effects are rather small, especially for lending and output.

6 Transitional dynamics

This section analyzes the transitional dynamics following the introduction of CBDC. As in the baseline long-run analysis, we focus on the case of non-remunerated CBDC. We analyze two different long run scenarios, characterized by a steady-state take-up of CBDC of 1% and 4% of GDP, which imply a shift from a floor system to a corridor system only in the latter case.

The economy is initially at the steady-state without CBDC, outlined in the calibration section . In this section, we assume that the weight on CBDC in household preferences for liquid is actually time-varying: $\eta_{DC,t}$. In particular, in period one the introduction of CBDC is announced and, from then on, $\eta_{DC,t}$ evolves according to the following law of motion

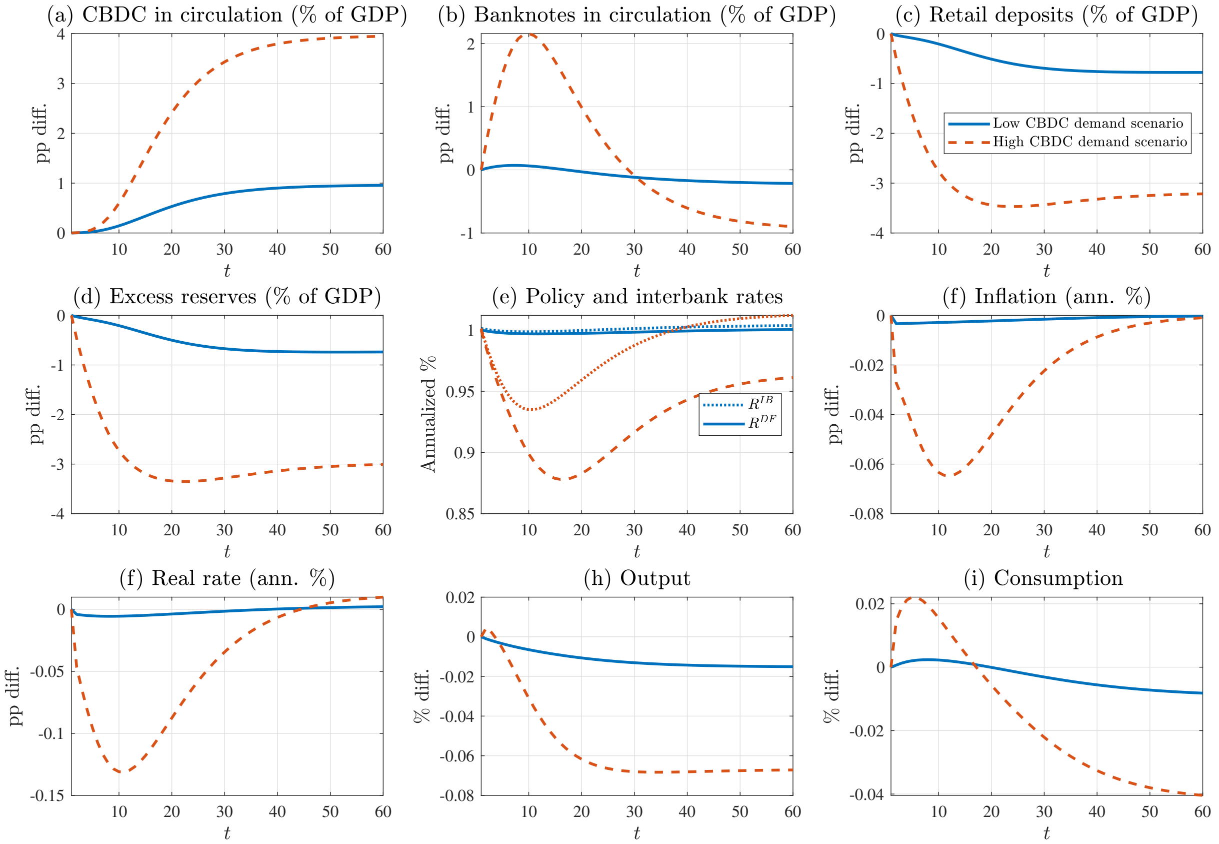
$$\eta_{DC,t} = \rho_{DC}\eta_{DC,t-1} + (1 - \rho_{DC})\,\bar{\eta}_{DC},$$

where $\bar{\eta}_{DC}$ is the value in the terminal steady state, and $\rho_{DC} \in [0, 1)$ is the persistence of the preference parameter. We set $\rho_{DC} = 0.9$, so that the transition to the terminal steady state takes around 60 quarters (15 years).

Figure 7 displays the transition to the new steady state. As explained in Section 5.1, the introduction of non-remunerated CBDC (panel a) reduces the size of the banking sector, implying a small, though still non-negligible, impact on bank lending to firms and capital investment. This implies a reduction in aggregate output, which leads to a transitory fall in inflation (panel f). This forces the central bank to temporarily reduce its policy rates (panel e).

The decline in inflation and nominal rates interacts with the adoption of CBDC along the transition path. In particular, the decline in inflation increases the real return on cash (and CBDC), while in the case of deposits, this effect is muted by the fall in nominal deposit rates. This leads to a temporary surge in the demand for cash (panel b) during

Figure 7: Transition to a new steady state



Note: Time (in quarters) is represented in the horizontal axis.

the first years after the introduction of CBDC. As time goes by, the return of inflation to its target and the increase in the preferences towards CBDC reverse the initial surge in cash, and the latter declines below its initial volume towards its long-run equilibrium. Deposits, however, decline over the whole period (panel c).

As regards real aggregates, the transitional dynamics also yield interesting insights. Despite the long-run decline in cash and deposits, and the negative long-run effects on output and consumption (panels h and i), consumption increases during the first years of CBDC circulation due to the deflationary impact of the CBDC announcement. This deflationary impact forces the central bank, following the Taylor rule, to decrease its policy rates in a way that eventually lowers its operational target (the interbank rate) more than proportionally to the fall in inflation.[33] The fall in the interbank rate carries

[33]Note that our Taylor rule assumes a gradual adjustment of the interbank rate to inflation developments, such that the Taylor principle (by which the policy rate must adjust more than one-for-one with

over to the household deposit rate, thus depressing long-run real rates and stimulating consumption in the first years of the transition (panel i).

While the response of real variables differs across both transitions only in the magnitude of the responses, the decline in excess reserves (panel d) in the low demand scenario is small enough so that the spread between the deposit facility rate and the interbank rate barely changes, as the central bank continues to operate a floor system (panel e). In the high demand scenario, however, the central bank is forced to decrease its nominal policy rate proportionally more since, at the same time, the reduction in excess reserves is such that the interbank rate goes up relative to its previous position within the policy rates corridor, as the central bank shifts its operational framework to a corridor system.

7 Conclusions

This paper studies the impact of CBDC on the operational framework of monetary policy and the macroeconomy as whole. It shows how CBDC adoption implies a roughly equivalent reduction in banks' deposit funding. However, this 'deposit crunch' has a rather small effect on bank lending to the real economy, and hence on aggregate investment and GDP. This result reflects the parallel impact of CBDC on the central bank's operational framework. The CBDC-induced deposit crunch is almost fully absorbed, first, by banks' excess reserves –implying the shift from a floor to a corridor system– and, for sufficiently high long-run CBDC demand, by increased recourse to the central bank's lending facility –such that the corridor system gives way to a ceiling one.

Given the uncertainty about the reasons to adopt CBDC, we have directly assumed that CBDC will enter household preferences for "liquidity services", together with cash and bank deposits. One natural extension would be to provide microfoundations for money demand in the spirit of Lagos and Wright (2005), as in Keister and Sanches (2022) and Keister and Monnet (2022), so that CBDC adoption becomes endogenous.[34] We leave

inflation in order to stabilize it) materializes only gradually over time.

[34]Marbet (2023) develops an heterogeneous agents quantitative model which combines New Monetarist

this analysis for future research.

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and New Keynesian elements in which the role of money as medium of exchange breaks monetary super-neutrality, and discusses how the introduction of a CBDC could bring long-run monetary neutrality back.

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Appendix

A. Derivations

A.1. Solution to the bank's problem

Bank j's problem at the beginning of period t is the following,

$$V_t(N_t^j) = \max_{D_t^j} \int \bar{V}_t(N_t^j, D_t^j, \omega) dF(\omega),$$

$$\bar{V}_t(N_t^j, D_t^j, \omega_t^j) = \max_{A_t^j \geq 0, B_t^{G,j} \geq 0, B_t^{+,j} \geq 0, B_t^{-,j} \geq 0} \mathbb{E}_t \Lambda_{t,t+1} \left[(1-\varsigma) E_{t+1}^j + V_{t+1}(\varsigma E_{t+1}^j) \right],$$

subject to

$$Q_t^K A_t^j + b_t^{G,j} + B_t^{-,j} = N_t^j + B_t^{+,j} + D_t^j, \tag{40}$$

$$Q_t^K A_t^j \leq \phi N_t^j, \tag{41}$$

where

$$E_{t+1}^j = R_{t+1}^A \omega_t^j Q_t^K A_t^j + \frac{R_{t+1}^G B_t^{G,j} + R_t^L B_t^{-,j}}{1+\pi_{t+1}} - \frac{R_t^D D_t^j + R_t^B B_t^{+,j}}{1+\pi_{t+1}}. \tag{42}$$

We use (40) to substitute for $B_t^{j,+}$ in the above problem. Let $\lambda_{At}^j, \lambda_{Gt}^j, \lambda_{Bt}^{+,j}, \lambda_{Bt}^{-,j}, \lambda_{\phi t}^j$ denote the Lagrange multipliers associated to $A_t^j \geq 0, B_t^{G,j} \geq 0, B_t^{+,j} \geq 0, B_t^{-,j} \geq 0$ and the leverage constraint (41), respectively. A solution to the banks problem must satisfy both the FOC with respect to $D_t^j, A_t^j, B_t^{G,j}, B_t^{-,j}$, given respectively by

$$\int \frac{\partial \bar{V}_t}{\partial D_t^j}(N_t^j, D_t^j, \omega) dF(\omega) = 0, \tag{43}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left[1-\varsigma+\varsigma V'_{t+1}(N_{t+1}^j) \right] \left(R_{t+1}^A \omega_t^j - \frac{R_t^B}{1+\pi_{t+1}} \right) + \frac{\lambda_{At}^j}{Q_t^K} + \lambda_{Bt}^{+,j} - \lambda_{\phi t}^j = 0, \tag{44}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left[1-\varsigma+\varsigma V'_{t+1}(N_{t+1}^j) \right] \left(\frac{R_{t+1}^G - R_t^B}{1+\pi_{t+1}} \right) + \lambda_{Gt}^j + \lambda_{Bt}^{+,j} = 0, \tag{45}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left[1-\varsigma+\varsigma V'_{t+1}(N_{t+1}^j) \right] \left(\frac{R_t^L - R_t^B}{1+\pi_{t+1}} \right) + \lambda_{Bt}^{-,j} + \lambda_{Bt}^{+,j} = 0 \tag{46}$$

and the Kuhn Tucker conditions

$$\min\left(A_t^j\ ,\ \lambda_{At}^j\right) = 0, \tag{47}$$

$$\min\left(B_t^{G,j}\ ,\ \lambda_{Gt}^j\right) = 0, \tag{48}$$

$$\min\left(B_t^{-,j}\ ,\ \lambda_{Bt}^{+,j}\right) = 0 \text{ where } B_t^{-,j} = Q_t^K A_t^j + b_t^{G,j} + B_t^{-,j} - N_t^j - D_t^j, \tag{49}$$

$$\min\left(B_t^{-,j}\ ,\ \lambda_{Bt}^{-,j}\right) = 0, \tag{50}$$

$$\min\left(\phi N_t^j - Q_t^K A_t^j\ ,\ \lambda_{\phi t}^j\right) = 0. \tag{51}$$

Using the envelope condition

$$\frac{\partial \bar{V}_t}{\partial D_t^j}(N_t^j, D_t^j, \omega_t^j) = \mathbb{E}_t \Lambda_{t,t+1}\left[1 - \varsigma + \varsigma V'_{t+1}(N_{t+1}^j)\right]\left(\frac{R_t^B - R_t^D}{1+\pi_{t+1}}\right) - \lambda_{Bt}^{+,j} \tag{52}$$

using $N_{t+1}^j = \varsigma E_{t+1}^j$ we can express the FOC with respect to deposits (43) as[35]

$$\int \mathbb{E}_t \Lambda_{t,t+1}\left[1 - \varsigma + \varsigma V'_{t+1}(N_{t+1}^j)\right]\left(\frac{R_t^B - R_t^D}{1+\pi_{t+1}}\right) dF(\omega) - \int \lambda_{Bt}^{+,j} dF(\omega) = 0. \tag{53}$$

The marginal value of equity is given by the envelope condition

$$V'_t(N_t^j) = \int \frac{\partial \bar{V}_t}{\partial N_t^j}(N_t^j, D_t^j, \omega) dF(\omega), \tag{54}$$

where

$$\frac{\partial \bar{V}_t}{\partial N_t^j}(N_t^j, D_t^j, \omega_t^j) = \mathbb{E}_t \Lambda_{t,t+1}\left[1 - \varsigma + \varsigma V'_{t+1}(N_{t+1}^j)\right]\frac{R_t^B}{1+\pi_{t+1}} - \lambda_{Bt}^{+,j} + \lambda_{\phi t}^j \phi. \tag{55}$$

We guess that in equilibrium $V'_t(N_t^j) \equiv \lambda_t^N$ is equalized across banks. Let

$$\tilde{\Lambda}_{t,t+1} \equiv \Lambda_{t,t+1}\left(1 - \varsigma + \varsigma \lambda_{t+1}^N\right) \tag{56}$$

[35]Notice that the first integrand in equation (53) depends on ω_t^j through the term $N_{t+1}^j = \varsigma E_{t+1}^j$, where in turn E_{t+1}^j is given by equation (42).

. We also make use of the fact that in equilibrium $R_t^B \geq R_t^L$.

Conjectured solution. We conjecture the following solution for the bank's problem. For some thresholds ω_t^B, ω_t^L to be derived below:

- Banks with $\omega_t^j > \omega_t^B$ borrow in the interbank market up to the leverage constraint,

$$A_t^j = \phi N_t^j / Q_t^K,$$

$$B_t^{+,j} = (\phi - 1) N_t^j - D_t^j,$$

$$B_t^{G,j} = B_t^{-,j} = 0,$$

  together with $\lambda_{At}^j = \lambda_{Bt}^{+,j} = 0 < \lambda_{\phi t}^j$, and $\lambda_{Gt}^j, \lambda_{Bt}^{-,j} \geq 0$;

- Banks with $\omega_t^j \in [\omega_t^L, \omega_t^B]$ invest their equity and deposits in real assets,

$$A_t^j = (N_t^j + D_t^j)/Q_t^K \leq \phi N_t^j / Q_t^K,$$

$$B_t^{G,j} = B_t^{-,j} = B_t^{+,j} = 0,$$

  together with $\lambda_{At}^j = \lambda_{\phi t}^j = 0 \leq \lambda_{Bt}^{-,j}, \lambda_{Gt}^j, \lambda_{Bt}^{+,j}$, the latter with strict inequality if $\omega_t^j \in (\omega_t^L, \omega_t^B)$;

- Banks with $\omega_t^j < \omega_t^L$ invest their equity and deposits in the interbank and government bond markets,

$$A_t^j = B_t^{+,j} = 0,$$

$$B_t^{G,j} + B_t^{-,j} = N_t^j + D_t^j,$$

  together with $\lambda_{Gt}^j = \lambda_{Bt}^{-,j} = \lambda_{\phi t}^j = 0 < \lambda_{At}^j$ and $\lambda_{Bt}^{+,j} \geq 0$.

Also, each bank's deposits D_t^j are not determined but are only required to be in the range $[0, (\phi - 1) N_t^j]$.

Verifying the conjecture. We now use our conjectured solution to evaluate the FOCs conditional on ω_t^j:

- FOC with respect to A_t^j:
  - Case $\omega_t^j > \omega_t^B$:

$$\lambda_{\phi t}^j = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{t+1}^A \omega_t^j - \frac{R_t^B}{1+\pi_{t+1}} \right) \right] > 0 \Leftrightarrow \omega_t^j > \frac{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B}{1+\pi_{t+1}} \right]}{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_{t+1}^A \right]} \equiv \omega_t^B. \tag{57}$$

  - Case $\omega_t^L \leq \omega_t^j \leq \omega_t^B$:

$$\lambda_{Bt}^{+,j} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(\frac{R_t^B}{1+\pi_{t+1}} - R_{t+1}^A \omega_t^j \right) \right] \geq \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(\frac{R_t^B}{1+\pi_{t+1}} - R_{t+1}^A \omega_t^B \right) \right] = 0. \tag{58}$$

  - Case $\omega_t^j < \omega_t^L$:

$$\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{t+1}^A \omega_t^j - \frac{R_t^B}{1+\pi_{t+1}} \right) \right] + \frac{\lambda_{At}^j}{Q_t^K} + \lambda_{Bt}^{+,j} = 0. \tag{59}$$

- FOC with respect to $B_t^{G,j}$:
  - Case $\omega_t^j > \omega_t^B$:

$$\lambda_{Gt}^j = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B - R_{t+1}^G}{1+\pi_{t+1}} \right] \geq 0 \tag{60}$$

  - Case $\omega_t^L \leq \omega_t^j \leq \omega_t^B$:

$$\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_{t+1}^G - R_t^B}{1+\pi_{t+1}} \right] + \lambda_{Gt}^j + \lambda_{Bt}^{+,j} = 0 \tag{61}$$

  - Case $\omega_t^j < \omega_t^L$:

$$\lambda_{Bt}^{+,j} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B - R_{t+1}^G}{1+\pi_{t+1}} \right] \geq 0, \tag{62}$$

    where in (60) and (62) we conjecture (and verify below) that

$$\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B}{1+\pi_{t+1}} \right] \geq \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_{t+1}^G}{1+\pi_{t+1}} \right].$$

- FOC with respect to $B_t^{-,j}$:

– Case $\omega_t^j > \omega_t^B$:

$$\lambda_{Bt}^{-,j} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B - R_t^L}{1+\pi_{t+1}} \right] \geq 0, \tag{63}$$

– Case $\omega_t^L \leq \omega_t^j \leq \omega_t^B$:

$$\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^L - R_t^B}{1+\pi_{t+1}} \right] + \lambda_{Bt}^{-,j} + \lambda_{Bt}^{+,j} = 0, \tag{64}$$

– Case $\omega_t^j < \omega_t^L$:

$$\lambda_{Bt}^{+,j} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B - R_t^L}{1+\pi_{t+1}} \right] \geq 0. \tag{65}$$

- The Kuhn Tucker conditions (47), (48), (50), (51) are obviously satisfied as well. 49 obviously holds for $\omega_t^j \leq \omega_t^B$. For $\omega_t^j > \omega_t^B$ this condition holds since we conjectured $B_t^{-,j} = (\phi - 1) N_t^j - D_t^j$ and $D_t^j \in [0, (\phi-1) N_t^j]$.

Equations (62) and (65) imply

$$\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^L}{1+\pi_{t+1}} \right] = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_{t+1}^G}{1+\pi_{t+1}} \right]$$

i.e. the (expected risk-adjusted real) return on government bonds equals the (expected risk-adjusted real) effective lending rate R_t^L. The latter condition, together with the equilibrium relationship $R_t^B \geq R_t^L$, verifies our conjecture that $\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^B}{1+\pi_{t+1}} \right] \geq \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_{t+1}^G}{1+\pi_{t+1}} \right]$. Using (65) to substitute for $\lambda_{Bt}^{+,j}$ in (59) yields

$$\frac{\lambda_{At}^j}{Q_t^K} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(\frac{R_t^L}{1+\pi_{t+1}} - R_{t+1}^A \omega_t^j \right) \right] > 0 \Leftrightarrow \omega_t^j < \frac{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \frac{R_t^L}{1+\pi_{t+1}} \right]}{\mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_{t+1}^A \right]} \equiv \omega_t^L. \tag{66}$$

Thus, the threshold definitions (57) and (66), together with the equilibrium relationship $R_t^B \geq R_t^L$, imply

$$\omega_t^L \geq \omega_t^B.$$

Using (58) to substitute for $\lambda_{Bt}^{+,j}$ in (64) and (61) yields, respectively,

$$\lambda_{Bt}^{-,j} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{t+1}^A \omega_t^j - \frac{R_t^L}{1+\pi_{t+1}} \right) \right] \geq \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{t+1}^A \omega_t^L - \frac{R_t^L}{1+\pi_{t+1}} \right) \right] = 0,$$

$$\lambda_{Gt}^{j} = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{t+1}^A \omega_t^j - \frac{R_{t+1}^G}{1+\pi_{t+1}} \right) \right] = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} \left(R_{t+1}^A \omega_t^j - \frac{R_t^L}{1+\pi_{t+1}} \right) \right] \geq 0.$$

Equilibrium deposit rate. We can write (53) as

$$\mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1} R_t^D}{1+\pi_{t+1}} \right] = \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1} R_t^B}{1+\pi_{t+1}} \right] - \int \lambda_{Bt}^{+,j} dF(\omega).$$

Using the equilibrium values of $\lambda_{Bt}^{+,j}$ in equations (58) for $\omega_t^j \in [\omega_t^L, \omega_t^B]$ and (65) for $\omega_t^j < \omega_t^L$, as well as the fact that $\lambda_{Bt}^{+,j} = 0$ for $\omega_t^j > \omega_t^B$, we finally obtain

$$\begin{aligned}
\mathbb{E}_t \frac{\tilde{\Lambda}_{t,t+1} R_t^D}{1+\pi_{t+1}} &= \mathbb{E}_t \frac{\tilde{\Lambda}_{t,t+1} R_t^B}{1+\pi_{t+1}} - \mathbb{E}_t \frac{\tilde{\Lambda}_{t,t+1} \left(R_t^B - R_t^L \right)}{1+\pi_{t+1}} F\left(\omega_t^L\right) \\
&\quad - \int_{\omega_t^L}^{\omega_t^B} \mathbb{E}_t \tilde{\Lambda}_{t,t+1} \left(\frac{R_t^B}{1+\pi_{t+1}} - R_{t+1}^A \omega_t^j \right) dF(\omega) \\
&= \left[1 - F\left(\omega_t^B\right)\right] \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1} R_t^B}{1+\pi_{t+1}} \right] + F\left(\omega_t^L\right) \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1} R_t^L}{1+\pi_{t+1}} \right] \\
&\quad + \left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right) \right] \mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B \right) \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} R_{t+1}^A \right], \quad (67)
\end{aligned}$$

where $\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B \right) \equiv \left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right) \right]^{-1} \int_{\omega_t^L}^{\omega_t^B} \omega dF(\omega)$. Therefore, the (expected risk-adjusted real) marginal cost of deposits, $R_t^D \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1}}{1+\pi_{t+1}} \right]$, must equal the (expected risk-adjusted real) marginal benefit across realizations of ω_t^j after the closing of the deposits market. Conditional on being a high-profitability bank that is leveraged up to the maximum ($\omega_t^j \geq \omega_t^B$), an additional unit of deposits will allow it to reduce its interbank funding needs by one unit, thus saving $R_t^B \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1}}{1+\pi_{t+1}} \right]$ in expected real risk-adjusted terms. Conditional on being a low-profitability bank ($\omega_t^j \leq \omega_t^L$), each additional unit of deposits will be invested in interbank lending or government bonds, which yields $R_t^L \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1}}{1+\pi_{t+1}} \right]$ ($= \mathbb{E}_t \left[\frac{\tilde{\Lambda}_{t,t+1}}{1+\pi_{t+1}} R_{t+1}^G \right]$). For intermediate-profitability banks

($\omega_t^L \leq \omega_t^j \leq \omega_t^B$), each additional unit of deposits will be invested in real firm assets, which yields $\mathbb{E}_t\left[\tilde{\Lambda}_{t,t+1}R_{t+1}^A\right]\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B\right)$ on average.

To prove that $R_t^D \in [R_t^L, R_t^B]$, notice that, using the definition of the borrowing threshold ω_t^B (see eq. 57), we can express (67) as

$$\begin{aligned} R_t^D &= \left[1-F\left(\omega_t^B\right)\right]R_t^B + F\left(\omega_t^L\right)R_t^L + \left[F\left(\omega_t^B\right)-F\left(\omega_t^L\right)\right]\frac{\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B\right)}{\omega_t^B}R_t^B \\ &\leq \left[1-F\left(\omega_t^B\right)\right]R_t^B + F\left(\omega_t^L\right)R_t^B + \left[F\left(\omega_t^B\right)-F\left(\omega_t^L\right)\right]R_t^B = R_t^B, \end{aligned}$$

where the inequality uses both $\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B\right) \leq \omega_t^B$ and the fact that in equilibrium $R_t^L \leq R_t^B$. Using instead in equation (67) the definition of the lending threshold ω_t^L (eq. 57) and the fact that $\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega \leq \omega_t^B\right) \geq \omega_t^L$, one can analogously show that $R_t^D \geq R_t^L$. Therefore, $R_t^L \leq R_t^D \leq R_t^B$.

Value of net worth. From (52) and (55), we learn that

$$\frac{\partial \bar{V}_t}{\partial N_t^j}(N_t^j, D_t^j, \omega_t^j) = \frac{\partial \bar{V}_t}{\partial D_t^j}(N_t^j, D_t^j, \omega_t^j) + \mathbb{E}_t\left[\tilde{\Lambda}_{t,t+1}\frac{R_t^D}{1+\pi_{t+1}}\right] + \lambda_{\phi t}^j \phi.$$

Averaging across realizations of ω_t^j after the closure of the deposits market, and using (54), we obtain the marginal value of real net worth,

$$\begin{aligned} \lambda_t^N &= \int \frac{\partial \bar{V}_t}{\partial D_t^j}(N_t^j, D_t^j, \omega_t^j)dF(\omega_t^j) + \mathbb{E}_t\left[\tilde{\Lambda}_{t,t+1}\frac{R_t^D}{1+\pi_{t+1}}\right] + \phi\int \lambda_{\phi t}^j dF(\omega_t^j) \\ &= \mathbb{E}_t\tilde{\Lambda}_{t,t+1}\left[\frac{R_t^D}{1+\pi_{t+1}} + \phi\int_{\omega_t^B}\left(R_{t+1}^A\omega - \frac{R_t^B}{1+\pi_{t+1}}\right)dF(\omega)\right] > 0. \end{aligned}$$

where in the second equality we have used (43), together with (57) and the fact $\lambda_{\phi t}^j = 0$ for $\omega_t^j \leq \omega_t^B$. Additional equity allows all banks – regardless of their subsequent realization of ω_t^j – to economize on deposit financing, which has a unit nominal cost of R_t^D. Moreover, equity has an additional marginal benefit for banks that draw $\omega_t^j \geq \omega_t^B$ later in the period, because it relaxes their leverage constraint. Notice finally that, since ω_t^j is iid, λ_t^N is indeed equalized across banks, which verifies our earlier conjecture.

Deposit allocation across banks. A final note is in order. Equation (67) implies that banks break even *ex ante* when taking deposits at the beginning of the period, so they are indifferent between taking one more units of deposits or not. Therefore, as mentioned earlier, individual deposit-taking by each bank is not pinned down[36] – although it *will* be pinned down in the aggregate in general equilibrium by the households deposit supply. The only requirement, implicitly assumed in the above conjectured (and verified) solution, is that no bank takes more deposits than

$$D_t^j \leq (\phi - 1)\, N_t^j.$$

For banks that draw $\omega_t^j > \omega_t^B$ after the closure of the deposits market, the latter inequality guarantees that $B_t^{j+} \geq 0$, i.e. they effectively need to borrow in the interbank market so as to finance their investment in the local firm. For those that draw $\omega_t^j \in \left[\omega_t^B, \omega_t^B\right]$, it guarantees that $Q_t^K A_t^j \leq \phi N_t^j$, i.e. they do not find themselves with more funds than they can invest in the local firm while still respecting the leverage constraint. The above condition can only hold for each individual bank if it holds in aggregate:

$$D_t \leq (\phi - 1)\, N_t$$

This assumption makes sure parameters are such that the latter condition holds and our conjecture indeed is a solution.

A.2. Determination of the interbank rate

Consider a bank with equity N_t^j, deposits D_t^j, and an island-specific return ω_t^j for the next period, that accesses the interbank market in period t after making its optimal portfolio decision as per equation (11). We denote the latter portfolio by $A_t^{j*}, b_t^{G,j*}, B_t^{+,j*}, B_t^{-,j*}$. According to equation (14), banks that draw $\omega_t^j > \omega_t^B$ choose $B_t^{G,j*} = B_t^{-,j*} = 0$ and

[36] Note that the distribution of deposits across banks is irrelevant for aggregate variables since banks are atomistic and the idiosyncratic shock ω_t^j is iid.

borrow in the interbank market in the amount $B_t^{+,j*} = (\phi - 1) N_t^j - D_t^j$. Borrowing (and lending) orders are made on a per-unit basis. Let us assume that the interbank market is divided into many different 'submarkets', each of them consisting of borrowers and lenders searching for each other. The borrowing bank send its orders to a submarket offering a combination $\left(R_t^B, \theta_t\right)$ of interest rate and (sub)market tightness. A fraction $\Gamma^B(\theta_t)$ of orders will be matched to lending orders, in which case each of them pays the rate R_t^B; the remaining fraction fail to be matched and the bank must borrow instead from the lending facility at rate R_t^{LF}. The value of a borrowing bank at the time of accessing the interbank market can then be written as

$$\begin{aligned}
\bar{V}_t^B(N_t^j, D_t^j, \omega_t^j) &= \mathbb{E}_t \Lambda_{t,t+1} \left[(1-\varsigma) E_{t+1}^j + V_{t+1}(\varsigma E_{t+1}^j) \right], \\
\text{where } \; E_{t+1}^j &= R_{t+1}^A \omega_t^j Q_t^K A_t^{j*} - \frac{R_t^D D_t^j}{1+\pi_{t+1}} - \frac{B_t^{+,j*}}{1+\pi_{t+1}} \left[\Gamma^B(\theta_t) R_t^{IB} + \left(1 - \Gamma^B(\theta_t)\right) R_t^{LF} \right].
\end{aligned} \tag{68}$$

Likewise, banks that draw $\omega_t^j < \omega_t^L$ choose $B_t^{+,j*} = 0$ and lend in the interbank market. For a bank sending its lending orders to the submarket with interest rate-tightness pair $\left(R_t^B, \theta_t\right)$, a fraction $\Gamma^L(\theta_t)$ of them will be matched to borrowing orders; the remaining fraction will not and those funds will be lent to the deposit facility at rate R_t^{DF}. Their value at the time of accessing the interbank market can then be again written as

$$\begin{aligned}
\bar{V}_t^L(N_t^j, D_t^j, \omega_t^j) &= \mathbb{E}_t \Lambda_{t,t+1} \left[(1-\varsigma) E_{t+1}^j + V_{t+1}(\varsigma E_{t+1}^j) \right], \\
\text{where } \; E_{t+1}^j &= R_{t+1}^A \omega_t^j Q_t^K A_t^{j*} + \frac{R_{t+1}^G B_t^{G,j*} - R_t^D D_t^j}{1+\pi_{t+1}} \\
&\quad + \frac{B_t^{-,j*}}{1+\pi_{t+1}} \left[\Gamma^L(\theta_t) R_t^{IB} + \left(1 - \Gamma^L(\theta_t)\right) R_t^{DF} \right].
\end{aligned} \tag{69}$$

Both lending and borrowing banks choose the submarket that offers them the highest value. Before solving the latter problem, we first express value functions in a more convenient way. In Appendix A.1 we showed that the (beginning-of-period) value function is linear in equity N_t^j: $V_{t+1}(N_{t+1}^j) = \lambda_{t+1}^N N_{t+1}^j$, where λ_{t+1}^N is the common marginal value of equity at time $t+1$ across banks. Defining $\tilde{\Lambda}_{t,t+1} \equiv \Lambda_{t,t+1}\left(1 - \varsigma + \varsigma \lambda_{t+1}^N\right)$ as in equation

(56), we can express (68) and (69) as

$$\bar{V}_t^B(\cdot) = \mathbb{E}_t\tilde{\Lambda}_{t,t+1}\left\{R_{t+1}^A\omega_t^j Q_t^K A_t^{j*} - \frac{R_t^D D_t^j}{1+\pi_{t+1}} - \frac{B_t^{+,j*}}{1+\pi_{t+1}}\left[\Gamma^B(\theta_t)R_t^{IB} + \left(1-\Gamma^B(\theta_t)\right)R_t^{LF}\right]\right\}, \tag{70}$$

$$\begin{aligned}\bar{V}_t^L(\cdot) \;=\; \mathbb{E}_t\tilde{\Lambda}_{t,t+1}\Big\{ & R_{t+1}^A\omega_t^j Q_t^K A_t^{j*} + \tfrac{R_{t+1}^G B_t^{G,j*} - R_t^D D_t^j}{1+\pi_{t+1}} \\ & + \tfrac{B_t^{-,j*}}{1+\pi_{t+1}}\left[\Gamma^L(\theta_t)R_t^{IB} + \left(1-\Gamma^L(\theta_t)\right)R_t^{DF}\right]\Big\},\end{aligned} \tag{71}$$

respectively. Since the returns to search activity in the interbank market (the terms in square brackets in equations 70 and 71) are deterministic from the point of view of period t, it follows that value maximization with respect to $\left(R_t^B, \theta_t\right)$ is equivalent to *minimization* of

$$\Gamma^B(\theta_t)R_t^{IB} + \left(1-\Gamma^B(\theta_t)\right)R_t^{LF} \equiv R_t^B$$

in the case of borrowers, and *maximization* of

$$\Gamma^L(\theta_t)R_t^{IB} + \left(1-\Gamma^L(\theta_t)\right)R_t^{DF} \equiv R_t^L$$

in the case of lenders. Let R_t^{L*} denote the maximum average return that lenders can obtain. In order to attract lenders, any submarket must therefore offer them an average return R_t^{L*}. Subject to this, borrowers choose the combination $\left(R_t^{IB}, \theta_t\right)$ that minimizes their own average borrowing cost, i.e. they solve

$$\begin{aligned}&\min_{R_t^{IB},\theta_t} \Gamma^B(\theta_t)R_t^{IB} + \left(1-\Gamma^B(\theta_t)\right)R_t^{LF} \\ &\quad s.t.\ \Gamma^L(\theta_t)R_t^{IB} + \left(1-\Gamma^L(\theta_t)\right)R_t^{DF} = R_t^{L*}\end{aligned}$$

The first-order conditions of this problem are

$$\Gamma^B(\theta_t) + \lambda_t^{L*}\Gamma^L(\theta_t) = 0,$$

$$\frac{d\Gamma^B}{d\theta}\left(R_t^{IB}-R_t^{LF}\right)+\lambda_t^{L*}\frac{d\Gamma^L}{d\theta}\left(R_t^{IB}-R_t^{DF}\right)=0.$$

Combining the latter two, and using the fact that $\Gamma^L\left(\theta_t\right) = \Gamma^B\left(\theta_t\right)\theta_t$ and therefore $\frac{d\Gamma^L}{d\theta}=\frac{d\Gamma^B}{d\theta}\theta_t+\Gamma^B$, we obtain

$$\left(1-\frac{\frac{d\Gamma^L}{d\theta}\theta_t}{\Gamma^L\left(\theta_t\right)}\right)\left(R_t^{LF}-R_t^{IB}\right)=\frac{\frac{d\Gamma^L}{d\theta}\theta_t}{\Gamma^L\left(\theta_t\right)}\left(R_t^{IB}-R_t^{DF}\right).$$

Letting $\frac{d\Gamma^L(\theta_t)}{d\theta}\frac{\theta_t}{\Gamma^L(\theta_t)}\equiv\varphi\left(\theta_t\right)$ denote the elasticity of lender's matching probability with respect to tightness, we obtain

$$R_t^{IB}=\varphi\left(\theta_t\right)R_t^{DF}+\left(1-\varphi\left(\theta_t\right)\right)R_t^{LF}.$$

Finally, using $\Gamma^L\left(\theta_t\right)=\frac{\Upsilon(\Phi_t^L,\Phi_t^B)}{\Phi_t^L}=\Upsilon\left(1,\theta_t\right)$, we can also express $\varphi\left(\theta_t\right)$ as

$$\varphi\left(\theta_t\right)=\frac{\partial\Upsilon}{\partial\Phi_t^B}\left(1,\theta_t\right)\frac{\Phi_t^B/\Phi_t^L}{\Upsilon\left(\Phi_t^L,\Phi_t^B\right)/\Phi_t^L}=\frac{\partial\Upsilon}{\partial\Phi_t^B}\left(\Phi_t^L,\Phi_t^B\right)\frac{\Phi_t^B}{\Upsilon\left(\Phi_t^L,\Phi_t^B\right)},$$

where the second equality uses the fact that, for any function $\Upsilon\left(x,y\right)$ with constant returns to scale, $\Upsilon_y\left(x,y\right)=\Upsilon_y\left(1,y/x\right)$. Therefore, $\varphi\left(\theta_t\right)$ represents the elasticity of the function function with respect to borrowing orders.

It only remains to show that $\varphi\left(\theta_t\right)\in[0,1]$. Let $(x,y)\equiv\left(\Phi_t^L,\Phi_t^B\right)$ for ease of notation. Constant returns to scale implies $\Upsilon\left(x,y\right)=x\Upsilon\left(1,y/x\right)$. Differentiating with respect to x, we get

$$\frac{\partial\Upsilon}{\partial x}\left(x,y\right)=\Upsilon\left(1,\frac{y}{x}\right)-\frac{\partial\Upsilon}{\partial y}\left(1,\frac{y}{x}\right)\frac{y}{x}.$$

Multiplying both sides by x, using the fact that $\frac{\partial\Upsilon}{\partial y}\left(1,\frac{y}{x}\right)=\frac{\partial\Upsilon}{\partial y}\left(x,y\right)$, and rearranging, we obtain $\frac{\partial\Upsilon}{\partial x}\left(x,y\right)x+\frac{\partial\Upsilon}{\partial y}\left(x,y\right)y=\Upsilon\left(x,y\right)$, or equivalently

$$\frac{\partial\Upsilon}{\partial x}\left(x,y\right)\frac{x}{\Upsilon\left(x,y\right)}+\frac{\partial\Upsilon}{\partial y}\left(x,y\right)\frac{y}{\Upsilon\left(x,y\right)}=1.$$

Therefore, the two elasticities with respect to each argument add up to one. Since both of them must be positive, by virtue of $\frac{\partial \Upsilon}{\partial x}, \frac{\partial \Upsilon}{\partial y}, x, y, \Upsilon \geq 0$, it follows that each of them must be less than one. In particular, $\frac{\partial \Upsilon}{\partial y}(x,y)\frac{y}{\Upsilon(x,y)} \equiv \varphi\left(\frac{y}{x}\right) \leq 1$. We thus have $\varphi\left(\frac{y}{x}\right) \in [0,1]$.

A.3. Aggregation, market clearing and equilibrium

Market clearing for capital requires that total supply by households, K_t, equals total demand by intermediate firms, $\int_0^1 K_t^j dj$. Since $K_t^j = A_t^j$ on each island j the capital stock K_t equals total demand for firms' assets by banks, $\int_0^1 A_t^j dj$. We obtain

$$\begin{aligned} K_t &= \int_{j:\omega_t^j > \omega_t^B} \frac{\phi N_t^j}{Q_t^K} dj + \int_{j:\omega_t^j \in [\omega_t^L, \omega_t^B]} \frac{N_t^j + D_t^j}{(1-\psi)Q_t^K} dj \\ &= \frac{\phi\left[1-F\left(\omega_t^B\right)\right]N_t + \left[F\left(\omega_t^B\right)-F\left(\omega_t^L\right)\right](N_t+D_t)/(1-\psi)}{Q_t^K}, \end{aligned} \tag{72}$$

where in the second equality we have used the fact that ω_t^j is independently distributed from N_t^j and D_t^j.

Labor market clearing requires that household's labor supply L_t equals firms' total labor demand, $\int_0^1 L_t^j dj$. To calculate the latter, we start by using (6) to solve for individual labor demand L_t^j and we then aggregate across firms: $\int_0^1 L_t^j dj = \left(\frac{(1-\alpha)Z_t MC_t}{W_t}\right)^{1/\alpha} \int_0^1 \omega_{t-1}^j K_{t-1}^j dj$. To solve for $\int_0^1 \omega_{t-1}^j K_{t-1}^j dj$, we use equation (11) and $K_t^j = A_t^j$ to obtain

$$\begin{aligned} \int_0^1 \omega_t^j K_t^j dj &= \frac{\phi N_t}{Q_t^K} \int_{\omega_t^B} \omega dF(\omega) + \frac{N_t + D_t}{(1-\psi)Q_t^K} \int_{\omega_t^L}^{\omega_t^B} \omega dF(\omega) \\ &= \frac{\phi N_t}{Q_t^K}\left[1 - F\left(\omega_t^B\right)\right] \mathbb{E}\left(\omega \mid \omega \geq \omega_t^B\right) \\ &\quad + \frac{N_t + D_t}{(1-\psi)Q_t^K}\left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right] \mathbb{E}\left(\omega \mid \omega_t^L \leq \omega < \omega_t^B\right), \end{aligned}$$

where we have used again the fact that ω_t^j is independently distributed from N_t^j, D_t^j. Using (72), we can express the above equation more compactly as

$$\int_0^1 \omega_t^j K_t^j dj = \Omega_t K_t, \tag{73}$$

where

$$\Omega_t \equiv \frac{\phi[1-F(\omega_t^B)]\mathbb{E}(\omega|\omega\geq\omega_t^B)}{\phi[1-F(\omega_t^B)]+\frac{N_t+D_t}{(1-\psi)N_t}[F(\omega_t^B)-F(\omega_t^L)]} + \frac{\frac{N_t+D_t}{(1-\psi)N_t}[F(\omega_t^B)-F(\omega_t^L)]\mathbb{E}(\omega|\omega_t^L\leq\omega<\omega_t^B)}{\phi[1-F(\omega_t^B)]+\frac{N_t+D_t}{(1-\psi)N_t}[F(\omega_t^B)-F(\omega_t^L)]} \quad (74)$$

is an index of capital efficiency.[37] Labor market clearing then requires

$$L_t = \left(\frac{(1-\alpha)\,Z_t MC_t}{W_t}\right)^{1/\alpha} \Omega_{t-1}K_{t-1}. \quad (75)$$

Aggregate supply of the intermediate good equals $\int_0^1 Y_t^j dj$. Equations (6) and (75) imply that the effective capital-labor ratio $\omega_{t-1}^j K_{t-1}^j / L_t^j$ equals $\Omega_{t-1}K_{t-1}/L_t$ for all firms. From equation (5), we then have

$$\int_0^1 Y_t^j dj = Z_t \left(\frac{L_t}{\Omega_{t-1}K_{t-1}}\right)^{1-\alpha} \int_0^1 \omega_{t-1}^j K_{t-1}^j dj = Z_t L_t^{1-\alpha}\left(\Omega_{t-1}K_{t-1}\right)^{\alpha},$$

where in the second equality we have used (73). Using (26), aggregate demand of the intermediate good equals $\int_0^1 Y_{i,t}di = Y_t\int_0^1\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon}di = Y_t\Delta_t$, where $\Delta_t \equiv \int_0^1 \left(P_{i,t}/P_t\right)^{-\epsilon}di$ is an index of relative price dispersion. Market clearing for the intermediate good therefore requires

$$Y_t = \frac{Z_t}{\Delta_t} L_t^{1-\alpha}\left(\Omega_{t-1}K_{t-1}\right)^{\alpha}.$$

Aggregate supply of the final good must equal consumption and investment demand by households,

$$Y_t = C_t + I_t.$$

Market clearing for government bonds requires supply to equal demand by private banks and the central bank,[38]

$$\overline{B}_t = B_t^G + B_t^{G,CB}.$$

Finally, we can aggregate equation (8) across banks and use $N_t^i = \varsigma E_t^j$ to find an expression

[37] In the limiting case in which $\omega_{t-1}^B = \omega_{t-1}^L \equiv \bar{\omega}_{t-1}$, Ω_t collapses to $\mathbb{E}\left(\omega \mid \omega \geq \bar{\omega}_{t-1}\right)$.

[38] Notice that we have implicitly assumed that the household cannot hold government bonds. This assumption is innocuous, since in equilibrium the household will always prefer deposits over bonds.

for aggregate bank equity,

$$\frac{N_t}{\varsigma} = R_t^A \Omega_{t-1} Q_{t-1}^K K_{t-1} + \frac{R_{t-1}^L}{1+\pi_t} \Phi_{t-1}^L + \frac{R_t^G}{1+\pi_t} B_{t-1}^G - \frac{R_{t-1}^D}{1+\pi_t} D_{t-1} - \frac{R_{t-1}^{CB}}{1+\pi_t} B_{t-1}^{CB} - \frac{R_{t-1}^B}{1+\pi_t} \Phi_{t-1}^B,$$

where we have used (73) and $A_{t-1}^j = K_{t-1}^j$ to substitute for $\int_0^1 \omega_{t-1}^j A_{t-1}^j dj$ $(= \Omega_{t-1} K_{t-1})$.

We define an equilibrium in this model as a set of state-contingent functions for prices and quantities such that all agents' optimization problems are solved and markets clear. Appendix B.1 lists the conditions that have to hold in equilibrium for aggregate variables.

A.4. Proof of Proposition 1

In this proof we show how, with a match-efficient matching technology, the steady state values of aggregate real macro variables and prices remain invariant once we introduce CBDC as long as CBDC is remunerated at rate given by equation (39) and the central bank operates a floor or a ceiling system. If a matching technology is match-efficient, then $\Upsilon(x, y) = \min\{x, y\}$,meaning that all orders on the short side of the market find a partner in the interbank market, while those on the other side that do not find a partner trade with the central bank. Without loss of generality, this implies that, when the volume of lending orders is larger than the volume of borrowing orders ($\Phi^L > \Phi^B$), the elasticity of the matching function with respect to the volume of borrowing orders equals one ($\varphi(\theta) = 1$) and, from equation (24) the interbank rate equals the deposit facility rate ($R^{IB} = R^{DF}$).[39]

Then, from equations (22) and (23), we obtain $R^L = R^B = R^{IB}$ and, from (13), $\omega^L = \omega^B$, which, together with equations (17) and (100), in turn implies that

$$R^D = R^G = R^L = R^B = R^{IB}. \tag{76}$$

From here, together with equation (74), $\Omega = \mathbb{E}\left(\omega \mid \omega \geq \omega^B\right)$ and, from (13), $R^A = \omega^B R^B$.

[39] Alternatively, if the volume of borrowing orders is larger than the volume of lending orders ($\Phi^B > \Phi^L$), the elasticity of the matching function with respect to the volume of borrowing orders equals zero ($\varphi(\theta) = 0$) and, from equation (24) the interbank rate equals the lending facility rate ($R^{IB} = R^{LF}$). Thus, the rest of the proof goes through in the opposite case to the one described above.

The remaining of the proof follows a guess and verify strategy. It proceeds in two steps. First,we start by guessing that the return on deposits R^D remains constant across steady states. We show that, if that is true, then all real allocations remain constant across steady states. We then verify that indeed deposit rates remain constant if the previous conditions are met.

We analyze first the dynamics of bank equity

$$N = \varsigma \left[R^A \Omega K - R^B \Phi^B + R^L \Phi^L + R^G B^G - R^D D \right],$$

where we substitute $\Phi^B = [N(\phi - 1) - D] \left(1 - F\left(\omega^B\right)\right)$, from equation (18), and $\Phi^L = (N + D)F\left(\omega^L\right) - B^G$, from equation (19), to get

$$\begin{aligned} N &= \varsigma \Big[\quad R^A \Omega K - R^B [N(\phi - 1) - D] \left(1 - F\left(\omega^B\right)\right) \\ &\qquad + R^L (N + D) F\left(\omega^L\right) - B^G + R^G B^G - R^D D \Big] \\ &= \varsigma \Big[\quad R^A \Omega K - R^D N(\phi - 1) \left(1 - F\left(\omega^B\right)\right) \\ &\qquad + R^D N F\left(\omega^B\right) + (R^D - 1) B^G \Big], \end{aligned}$$

which combined with

$$K = \phi \left[1 - F\left(\omega^B\right)\right] N + \frac{N + D}{1 - \psi} \left[F\left(\omega^B\right) - F\left(\omega^L\right)\right] = \phi \left[1 - F\left(\omega^B\right)\right] N,$$

from equation (72), implies that N only depends on variables constant across steady states, and thus it also remains invariant, and so does K, as long as $B^G = (1 - \varrho)\overline{B}$ is also constant. Since the government debt to GDP ratio $\overline{B}/Y$ is constant and equal to $\overline{b}$, it suffices to show that the previous conditions imply that output Y remains constant too. The law of motion of capital, $I = K[1 - (1 - \delta)\Omega]$, allows us to prove that investment is also constant, and so is the return on capital $R^k = R^A - (1 - \delta)$, wages $\left(R^k = \alpha Z \frac{\epsilon}{\epsilon - 1} \left[\frac{(1-\alpha)(\epsilon - 1) Z}{W \epsilon} \right]^{(1-\alpha)/\alpha} \right)$, labor supply $\left(H = \left(\frac{(1-\alpha) Z (\epsilon - 1)}{W \epsilon} \right)^{1/\alpha} \Omega K \right)$, consumption $(g'(H) = W u'(C))$ and then output $Y = I + C$ is constant.

Finally, from the consolidated balance sheet of the financial sector (2), $K+\overline{B}=\mathcal{W}+N$, aggregate household wealth $\mathcal{W}$ is also constant across steady states. Thus, all that remains to show is R^D indeed remains invariant after the introduction of CBDC.

The second part of the proof shows how the return on deposits only remains invariant if CBDC is remunerated at rate given by equation (39). We first analyze the steady-state household's Euler equations (2-4):

$$1-\frac{v'(L)}{u'(C)}\left(L/D\right)^{\frac{1}{\varepsilon}}=\beta R^D,\ 1-\frac{v'(L)}{u'(C)}\eta_M\left(L/M\right)^{\frac{1}{\varepsilon}}=\beta,\ 1-\frac{v'(L)}{u'(C)}\eta_{DC}\left(L/D^{DC}\right)^{\frac{1}{\varepsilon}}=\beta R^{DC}, \tag{77}$$

which we can combine to obtain

$$M=\left(\frac{(1-\beta)}{\left(1-\beta R^{DC}\right)}\frac{\eta_{DC}}{\eta_M}\right)^{-\varepsilon}D^{DC},\ D=\left(\frac{\left(1-\beta R^{D}\right)}{1-\beta R^{DC}}\eta_{DC}\right)^{-\varepsilon}D^{DC},\ D=\left(\frac{\left(1-\beta R^{D}\right)\eta_M}{1-\beta}\right)^{-\varepsilon}M.$$

Liquidity is

$$\begin{aligned}
L &= \left[(D)^{\frac{\varepsilon-1}{\varepsilon}}+\eta_M\left(M\right)^{\frac{\varepsilon-1}{\varepsilon}}+\eta_{DC}\left(D^{DC}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \\
&= \left[\left(\left(\frac{\left(1-\beta R^{D}\right)}{1-\beta R^{DC}}\eta_{DC}\right)^{-\varepsilon}D^{DC}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\eta_M\left(\frac{(1-\beta)}{\left(1-\beta R^{DC}\right)}\frac{\eta_{DC}}{\eta_M}D^{DC}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\eta_{DC}\left(D^{DC}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= D^{DC}\underbrace{\left[\left(\frac{\left(1-\beta R^{D}\right)\eta_{DC}}{1-\beta R^{DC}}\right)^{1-\varepsilon}+\eta_M\left(\frac{(1-\beta)}{\left(1-\beta R^{DC}\right)}\frac{\eta_{DC}}{\eta_M}\right)^{1-\varepsilon}+\eta_{DC}\right]^{\frac{\varepsilon}{\varepsilon-1}}}_{\Psi}.
\end{aligned}$$

Then we can express the Euler equation for cash as

$$1-\frac{v'(D^{DC}\Psi)}{u'(C)}\eta_M\left(\left(\frac{\eta_{DC}}{\eta_M}\right)^{\varepsilon}\Psi\right)^{\frac{1}{\varepsilon}}=\beta,$$

and, rearranging terms,

$$1-\beta=\frac{v'(D^{DC}\Psi)}{u'(C)}\eta_{DC}\left(\Psi\right)^{\frac{1}{\varepsilon}}.$$

If we replace the functional form for the utility, we get

$$1-\beta=\frac{C}{D^{DC}}\vartheta\eta_{DC}\left(\Psi\right)^{\frac{1}{\varepsilon}-1},$$

or, equivalently,

$$\begin{aligned}
C &= (1-\beta)\,D^{DC}\frac{\left(\frac{(1-\beta R^{D})\eta_{DC}}{1-\beta R^{DC}}\right)^{1-\varepsilon}+\eta_M\left(\frac{(1-\beta)}{(1-\beta R^{DC})}\frac{\eta_{DC}}{\eta_M}\right)^{1-\varepsilon}+\eta_{DC}}{\vartheta\eta_{DC}}\\
&= (1-\beta)\,D^{DC}\frac{\left(\frac{(1-\beta R^{D})}{1-\beta R^{DC}}\right)^{1-\varepsilon}\eta_{DC}^{-\varepsilon}+\left(\frac{(1-\beta)}{(1-\beta R^{DC})}\right)^{1-\varepsilon}\eta_M^{\varepsilon}\eta_{DC}^{-\varepsilon}+1}{}\\
&= \frac{(1-\beta)}{\vartheta\left(1-\beta R^{DC}\right)}\left[\left(1-\beta R^{D}\right)D+(1-\beta)\,M+\left(1-\beta R^{DC}\right)D^{DC}\right]
\end{aligned}$$

Now consider two steady states, characterized by different values of the parameter η_{DC}. The first steady state is corresponds to the case in which there is no demand for CBDC $\left(D^{DC}=0\right)$:

$$C=\frac{(1-\beta)}{\vartheta\left(1-\beta R^{DC}\right)}\left[\left(1-\beta R^{D}\right)D+(1-\beta)\,M\right].$$

The second steady state is characterized by a positive take-up $\left(D^{DC}>0\right)$:

$$C' = \frac{(1-\beta)}{\vartheta\left(1-\beta R^{DC}\right)}\left[\left(1-\beta R^{D'}\right)D'+(1-\beta)\,M'+\left(1-\beta R^{DC}\right)D^{DC}\right].$$

Notice that we allow the return on deposits to vary between these two steady states.

We showed above that consumption remains constant across steady states:

$$\frac{C'}{C}=\frac{\left(1-\beta R^{D'}\right)D'+(1-\beta)\,M'+\left(1-\beta R^{DC}\right)D^{DC}}{\left(1-\beta R^{D}\right)D+(1-\beta)\,M}=1.$$

Rearranging terms:

$$\left(1-\beta R^{D'}\right)D'+(1-\beta)\,M'+\left(1-\beta R^{DC}\right)D^{DC}=\left(1-\beta R^{D}\right)D+(1-\beta)\,M.$$

As total wealth $\mathcal{W}$ does not change either, then

$$D' + M' + D^{DC} = D + M,$$

and the expression simplifies to

$$R^{D'}D' + M' + R^{DC}D^{DC} = R^{D}D + M.$$

We substitute the value of the remuneration of CBDC (39):

$$\bar{R}^{DC} = \frac{R^{D}\left(D' - D\right) + \left(M' - M\right)}{\left(D' - D\right) + M' - M} = \frac{R^{D}\left(D - D'\right) + M - M'}{D^{DC}}, so$$

that

$$R^{D'}D' + M' + R^{D}\left(D - D'\right) + M - M' = R^{D}D + M,$$

which simplifies to

$$R^{D'} = R^{D}.$$

This proves that the return on deposits is invariant as long as the CBDC is remunerated at rate (39), and thus that rate guarantees neutrality. This concludes the proof.

B. Complete set of equations

We display below the complete set of equations of the model. We define $p_t^* \equiv P_t^*/P_t$.

B.1. Transitional dynamics

- Households

$$1-\frac{v'(L_t)}{u'(C_t)}\frac{\partial L_t}{\partial D_t} = \Lambda_{t,t+1}\frac{R_t^D}{1+\pi_{t+1}}, \tag{78}$$

$$1-\frac{v'(L_t)}{u'(C_t)}\frac{\partial L_t}{\partial M_t} = \Lambda_{t,t+1}\frac{1}{1+\pi_{t+1}}, \tag{79}$$

$$1-\frac{v'(L_t)}{u'(C_t)}\frac{\partial L_t}{\partial D_t^{DC}} = \Lambda_{t,t+1}\frac{R_t^{DC}}{1+\pi_{t+1}}, \tag{80}$$

$$W_t = \frac{g'(H_t)}{u'(C_t)}, \tag{81}$$

$$\Lambda_{t,t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)} \tag{82}$$

$$\begin{aligned}1 = {} & Q_t^K\left[1-S\left(\tfrac{I_t}{I_{t-1}}\right)-S'\left(\tfrac{I_t}{I_{t-1}}\right)\tfrac{I_t}{I_{t-1}}\right] \\ & +\Lambda_{t,t+1}Q_{t+1}^K S'\left(\tfrac{I_{t+1}}{I_t}\right)\left(\tfrac{I_{t+1}}{I_t}\right)^2,\end{aligned} \tag{83}$$

$$K_t = \left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t+(1-\delta)\,\Omega_{t-1}K_{t-1} \tag{84}$$

$$L_t = \left[(D_t)^{\frac{\varepsilon-1}{\varepsilon}}+\eta_M\,(M_t)^{\frac{\varepsilon-1}{\varepsilon}}+\eta_{DC}\left(D_t^{DC}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{85}$$

- Firms

$$Y_t = \frac{Z_t}{\Delta_t}H_t^{1-\alpha}\left(\Omega_{t-1}K_{t-1}\right)^{\alpha}, \tag{86}$$

$$1 = \theta\,(1+\pi_t)^{\epsilon-1}+(1-\theta)\,(p_t^*)^{1-\epsilon}, \tag{87}$$

$$p_t^* = \frac{\Xi_t^1}{\Xi_t^2}, \tag{88}$$

$$\Xi_t^1 = \frac{\epsilon}{\epsilon-1}X_tY_t+\theta\mathbb{E}_t\Lambda_{t,t+1}\,(1+\pi_{t+1})^{\epsilon}\,\Xi_{t+1}^1, \tag{89}$$

$$\Xi_t^2 = Y_t+\theta\mathbb{E}_t\Lambda_{t,t+1}\,(1+\pi_{t+1})^{\epsilon-1}\,\Xi_{t+1}^2, \tag{90}$$

$$\Delta_t = (1-\theta)\,(p_t^*)^{-\epsilon}+\theta\,(1+\pi_t)^{\epsilon}\,\Delta_{t-1}, \tag{91}$$

$$R_t^A = \frac{R_t^k+(1-\delta)\,Q_t^K}{Q_{t-1}^K}, \tag{92}$$

$$R_t^k = \alpha X_tZ_t\left[\frac{(1-\alpha)\,X_tZ_t}{W_t}\right]^{(1-\alpha)/\alpha}, \tag{93}$$

$$H_t = \left(\frac{(1-\alpha)\,Z_tX_t}{W_t}\right)^{1/\alpha}\Omega_{t-1}K_{t-1}. \tag{94}$$

- Banks

$$Q_t^K K_t = \left\{ \phi N_t \left[1 - F\left(\omega_t^B\right)\right] + \frac{N_t + D_t}{1 - \psi} \left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right] \right\}, \tag{95}$$

$$B_t^{CB} = \left\{ \psi \phi N_t \left[1 - F\left(\omega_t^B\right)\right] + \frac{\psi}{1 - \psi} \left(N_t + D_t\right) \left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right] \right\} \tag{96}$$

$$N_t = \varsigma \left[\begin{array}{c} R_t^A Q_{t-1}^K \Omega_{t-1} K_{t-1} - \frac{R_{t-1}^B}{1+\pi_t} \Phi_{t-1}^B - \frac{R_{t-1}^{CB}}{1+\pi_t} B_{t-1}^{CB} + \\ \frac{R_{t-1}^L}{1+\pi_t} \Phi_{t-1}^L + \frac{R_t^G}{(1+\pi_t)} B_{t-1}^G - \frac{R_{t-1}^D}{(1+\pi_t)} D_{t-1} \end{array} \right], \tag{97}$$

$$\omega_t^B = \frac{R_t^B}{R_{t+1}^A \left(1 + \pi_{t+1}\right)}, \tag{98}$$

$$\omega_t^L = \frac{R_t^L}{R_{t+1}^A \left(1 + \pi_{t+1}\right)}, \tag{99}$$

$$R_{t+1}^G = R_t^L. \tag{100}$$

$$R_t^D = \begin{array}{c} \left[1 - F\left(\omega_t^B\right)\right] R_t^B + F\left(\omega_t^L\right) R_t^L + \\ \left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right] R_{t+1}^A \left(1 + \pi_{t+1}\right) \mathbb{E}\left[\omega_t | \omega_t^B > \omega_t > \omega_t^L\right]. \end{array} \tag{101}$$

- Interbank market

$$\Phi_t^B = \left[N_t \left(\phi(1 - \psi) - 1\right) - D_t\right] \left[1 - F\left(\omega_t^B\right)\right], \tag{102}$$

$$\Phi_t^L = \left(N_t + D_t\right) F\left(\omega_t^L\right) - B_t^G, \tag{103}$$

$$\Gamma_t^B = \Upsilon\left(\frac{\Phi_t^L}{\Phi_t^B}, 1\right), \tag{104}$$

$$\Gamma_t^L = \Upsilon\left(1, \frac{\Phi_t^B}{\Phi_t^L}\right), \tag{105}$$

$$R_t^B = \varphi_t \Gamma_t^B R_t^{DF} + \left[1 - \varphi_t \Gamma_t^B\right] R_t^{LF}, \tag{106}$$

$$R_t^L = \left(1 - \varphi_t\right) \Gamma_t^L R_t^{LF} + \left(1 - (1 - \varphi_t)\Gamma_t^L\right) R_t^{DF}, \tag{107}$$

$$\varphi_t = \frac{1}{\left(\Phi_t^B / \Phi_t^L\right)^\lambda + 1} \tag{108}$$

- Central bank

$$R_t^{LF} = R_t^{DF} + \chi \quad (109)$$

$$R_t^{DF} = \rho(R_{t-1}^{DF}) + (1-\rho)\left[\bar{R} + \upsilon\left(\pi_t - \bar{\pi}\right)\right], \quad (110)$$

$$R_t^{CB} = R_t^{DF} - \chi^{CB} \quad (111)$$

$$R_t^{DC} = R_t^{DF} + \chi^{DC} \quad (112)$$

$$B_t^{G,CB} + B_t^{CB} + \Phi_t^B\left(1 - \Gamma_t^B\right) = \Phi_t^L\left(1 - \Gamma_t^L\right) + M_t + D_t^{DC}, \quad (113)$$

$$B_t^{G,CB} = \varrho \overline{B}_t, \quad (114)$$

- Government

$$\overline{B}_t = B_t^{G,CB} + B_t^G, \quad (115)$$

$$\overline{B}_t / Y_t = \bar{b}. \quad (116)$$

- Aggregate constraint

$$\Omega_t \equiv \frac{\phi\left[1 - F\left(\omega_t^B\right)\right]\mathbb{E}\left(\omega \mid \omega \geq \omega_t^B\right) + \frac{N_t + D_t}{(1-\psi)N_t}\left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right]\mathbb{E}\left(\omega \mid \omega_t^L \leq \omega < \omega_t^B\right)}{\phi\left[1 - F\left(\omega_t^B\right)\right] + \frac{N_t + D_t}{(1-\psi)N_t}\left[F\left(\omega_t^B\right) - F\left(\omega_t^L\right)\right]} \quad (117)$$

$$Y_t = C_t + I_t. \quad (118)$$

There are 42 equations and 42 endogenous variables: Y_t, Q_t^K, I_t, C_t, K_t, N_t, W_t, H_t, $\Lambda_{t,t+1}$,X_t, π_t, p_t^*, Ξ_t^1, Ξ_t^2, Δ_t, R_t^A, R_t^k, R_t^L, R_t^B, R_t^{DF}, R_t^{LF}, R_t^G, R_t^D, Γ_t^B, Γ_t^L, Φ_t^L, Φ_t^B,φ_t, ω_t^B, ω_t^L, $B_t^{G,CB}$, B_t^G, $\overline{B}_t$, D_t, Ω_t, L_t, M_t, D_t^{DC}, R_t^{DC}, ω_t^{CB},B_t^{BC}, R_t^{BC}.

B.2. Steady-state with zero inflation

- Households

$$
\begin{aligned}
\beta R^{D} &= 1 - \frac{v'(L)}{u'(C)} \frac{\partial L}{\partial D}, \\
\beta &= 1 - \frac{v'(L)}{u'(C)} \frac{\partial L}{\partial M}, \\
\beta R^{DC} &= 1 - \frac{v'(L)}{u'(C)} \frac{\partial L}{\partial D^{DC}}, \\
\Lambda &= \beta, \\
W &= \frac{g'(H)}{u'(C)}, \\
Q &= 1, \\
I &= K \left[1 - (1 - \delta) \Omega\right].
\end{aligned}
$$

- Firms

$$
\begin{aligned}
Y_t &= (\Omega K)^{\alpha} H^{1-\alpha}, \\
\Delta &= 1, \\
p^* &= 1, \\
\Xi^1 &= \frac{\epsilon}{(\epsilon - 1)(1 - \theta\beta)} XY, \\
\Xi^2 &= \frac{Y}{(1 - \theta\beta)}, \\
X &= \frac{(\epsilon - 1)}{\epsilon}, \\
R^k &= \alpha X Z \left[\frac{(1 - \alpha)(\epsilon - 1) Z}{W\epsilon}\right]^{(1-\alpha)/\alpha}, \\
R^A &= R^k + (1 - \delta), \\
H &= \left(\frac{(1 - \alpha) Z (\epsilon - 1)}{W\epsilon}\right)^{1/\alpha} \Omega K.
\end{aligned}
$$

- Banks

$$
\begin{aligned}
K &= \left\{\phi N\left[1-F\left(\omega^{B}\right)\right]+\frac{N+D}{1-\psi}\left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]\right\}, \\
B^{CB} &= \left\{\psi\phi N\left[1-F\left(\omega^{B}\right)\right]+\frac{\psi}{1-\psi}(N+D)\left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]\right\} \\
N &= \varsigma\left[\begin{array}{c} R^{A}\Omega K-R^{B}\Phi^{B}-R^{CB}B^{CB}+ \\ R^{L}\Phi^{L}+R^{G}B^{G}-R^{D}D \end{array}\right], \\
\omega^{B} &= \frac{R^{B}}{R^{A}}, \\
\omega^{L} &= \frac{R^{L}}{R^{A}}, \\
R^{G} &= R^{L}, \\
R^{D} &= \begin{array}{c} \left[1-F\left(\omega^{B}\right)\right]R^{B}+F\left(\omega^{L}\right)R^{L}+ \\ \left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]R^{A}\mathbb{E}\left[\omega|\omega^{B}>\omega>\omega^{L}\right]. \end{array}
\end{aligned}
$$

- Interbank market

$$
\begin{aligned}
\Phi^{B} &= \left[N\left(\phi(1-\psi)-1\right)-D\right]\left(1-F\left(\omega^{B}\right)\right), \\
\Phi^{L} &= (N+D)F\left(\omega^{L}\right)-B^{G} \\
\Gamma^{B} &= \Upsilon\left(\frac{\Phi^{L}}{\Phi^{B}},1\right), \\
\Gamma^{L} &= \Upsilon\left(1,\frac{\Phi^{B}}{\Phi^{L}}\right), \\
R^{B} &= \bar{R}-\Gamma^{B}\varphi\chi, \\
R^{L} &= \bar{R}-\left(1-(1-\varphi)\Gamma^{L}\right)\chi, \\
\varphi &= \frac{1}{\left(\Phi^{B}/\Phi^{L}\right)^{\lambda}+1}
\end{aligned}
$$

- Central bank

$$\begin{aligned}
R^{LF} &= \bar{R}, \\
R^{DF} &= \bar{R} - \chi, \\
R^{CB} &= R^{DF} - \chi^{CB} \\
R^{DC} &= R^{DF} + \chi^{DC} \\
b^{G,CB} + B^{CB} + \Phi^{B}\left(1 - \Gamma^{B}\right) &= \Phi^{L}\left(1 - \Gamma^{L}\right) + M + D^{DC}, \\
B^{G,CB} &= \varrho \overline{B}.
\end{aligned}$$

- Government

$$\begin{aligned}
\overline{B} &= B^{G,CB} + B^{G}, \\
\overline{B}/Y &= \bar{b}.
\end{aligned}$$

- Aggregate constraint

$$\begin{aligned}
\Omega &= \frac{\phi\left[1 - F\left(\omega^{B}\right)\right] \mathbb{E}\left(\omega \mid \omega \geq \omega^{B}\right) + \frac{N+D}{N(1-\psi)}\left[F\left(\omega^{B}\right) - F\left(\omega^{L}\right)\right] \mathbb{E}\left(\omega \mid \omega^{L} \leq \omega < \omega^{B}\right)}{\phi\left[1 - F\left(\omega^{B}\right)\right] + \frac{N+D}{N(1-\psi)}\left[F\left(\omega^{B}\right) - F\left(\omega^{L}\right)\right]}, \\
Y &= C + I.
\end{aligned}$$