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Abstract

We consider firms' choices between a clean technology that benefits, and a dirty technology that harms, the environment. Green firms are more suited to the clean, and brown firms are more suited to the dirty technology. We use a model derived from complexity theory that takes account of true uncertainty and increasing returns to technology adoption. We examine theoretically, the properties of the long-run equilibrium, and provide simulated time paths of technology adoption, using plausible dynamics. The long-run outcome is an 'emergent property' of the system, and it unpredictable despite there being no external technological or preference shocks. We describe the role of taxes and subsidies in facilitating adoption of the clean technology; the conflict between optimal Pigouvian taxes and adoption of clean technologies; the optimal temporal profile of subsidies; and the desirability of an international fund to provide technology assistance to poorer countries. Finally, we extend our model to stochastic dynamics in which firms experiment with technological alternatives, and demonstrate the existence of punctuated equilibria.

JEL-Codes: D010, D210, D900, H320.

Keywords: technology choice, climate change, complexity, lock-in effects, increasing returns, green subsidies, public policy, Pigouvian taxes, stochastic dynamics.

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1 Introduction

There is growing awareness of the dangers of climate change and the need for action using 'non-standard' approaches, relative to the traditional approaches used in economics (Nordhaus, 2019; Stern, 2022; Stern and Stiglitz, 2022).[1]

The following three key features provide the basis for a 'non-standard' approach as highlighted in Stern and Stiglitz (2022). First, there are strong scale economies of adoption, and increasing returns, in green technologies. Second, the pervasiveness of *true uncertainty*, or Knightian uncertainty, or radical uncertainty (Knight, 1921; King and Kay, 2020). This makes the calculation of optimal choices difficult, if not impossible, with the use of standard social welfare functions.[2] Thus, we need a 'guardrail approach' with broad objectives/aspirations/targets such as limiting the increase of global temperatures at 1.5 degrees, based on the best available science, and not on maximizing a standard social welfare function.[3] Third, the presence of complex adaptive dynamics characterized by decision feedbacks, with the existence of tipping points that lead to large changes arising from small accidents of history. Stern and Stiglitz (2022) lament that none of these features, individually, or as a group, has been satisfactorily incorporated in the standard economic analysis of the problem of climate change.

In actual practice, an important aspect of the problem is that firms have a variety of technological choices to make. These choices typically trade off a *clean technology* that is good for the environment against a *dirty technology* that produces some environment bads.[4]

We consider the technology adoption decision of firms (in terms of clean vs dirty technology) and appropriate public policy, while taking account of the three key non-standard features of interest outlined above.[5] We are interested not just in the long run equilibrium but also in the

[1]Similar observations are made in successive IPCC reports. The implications of climate change are profound (Stern, 2022, p.1261): "...fires associated with heat and drought; severe flooding; hurricanes and typhoons; storm surges; sea level rises; local temperatures at levels dangerous to human life..... Many parts of the world could become uninhabitable...Hundreds of millions, possibly billions, would have to move, likely resulting in severe and extended conflict."

[2]A separate, but not less serious problem is that the social welfare functions might not even be defined in the presence of catastrophic climate change risks, such as deaths/displacement of several billion people. The utility function in this case is likely to tend to minus infinity in the worst states of the world, precluding any meaningful analysis of risk based on, say, expected utility theory (Weitzman, 2009).

[3]Stern (2022, p. 1277) is explicit about this point: "...the consensus across more than 190 countries embodied in the Paris 2015 UNFCCC agreement did not require full agreement on the utility function to be maximized, the correct damage function, discounting or the probabilities of outcomes. Instead, as it became clear, and broadly accepted, that with temperature rise over 2^0C there was a significant probability of extremely bad outcomes, and that those outcomes could be avoided at moderate costs, there emerged consensus that we should act strongly to try to avoid them. It should be noted that it is this understanding that has led the international community to focus on achieving net zero emissions by around 2050, not the recommendations of economists based on IAMs [integrated assessment models]." Other examples include pledges to phase out incandescent bulbs and internal combustion engines by a cutoff date, despite the, then, non-existence of viable alternatives. Yet, once these announcements were made, the alternatives were quickly developed, exhibited increasing returns, and were adopted relatively quickly.

[4]Not all of these technology choices are in terms of plants and equipment. Some of these technology choices are likely to involve a "green package" comprising several useful technologies. For instance, the installation of solar panels for power; use of cloud computing or green data centers that reduce environmental costs of less efficient in-house servers; remote work technologies to save environmental commuting costs; installation of energy efficient devices to monitor energy use; hybrid or EV company vehicles; and green packaging and shipping material.

[5]For this reason, our paper is not related to the literature on *integrated assessment models* (IAMs). This literature was pioneered by Nordhaus (1991, 2019) and was further developed in several directions; see Nordhaus, (2017). For an extensive critique of the non-suitability of IAMs to some of the key issues of climate change and

transition paths to equilibrium.

1.1 Essentials of our model

In our model, a large number of firms make sequential technology adoption choices between a 'clean technology' that produces no emissions and a 'dirty technology' that produces harmful emissions.[6] We consider the case of increasing returns to scale in the sense that greater adoption of a technology lowers its costs and/or increases its benefits.[7] Technically, the decision mechanism of our model merges and extends two models of technology competition with increasing returns to scale, Arthur et al. (1987) and Arthur (1989).

The clean technology has relatively 'stronger' returns to scale from adoption (higher benefits or lower costs as more firms adopt it) as compared to the dirty technology.[8] Firms are of two types, G (green) and B (brown). Type G firms are relatively more 'suited' to adopting the clean technology because their net benefits from adopting it are higher.[9] Type B firms are relatively more 'suited' to adopting the dirty technology because their net benefits from adopting it are higher.

In a nutshell, we have *technology-contingent differences* (clean technology offers relatively greater returns to scale from adoption) and *type contingent differences* (type G firms are relatively more suited to adopting the clean technology). None of the existing theoretical models simultaneously allow for these two differences, which have strong empirical grounds.

In each time period, nature picks one of the firms randomly to make a technology choice. The probability that firms of type G and B are picked to make a choice depends on their respective proportions in the population of firms. We are interested in the long-run equilibrium and the dynamic properties of this system. We show that our results are robust to including uncertainty, time discounting, switching costs, allowing multiple firms to make a simultaneous technology choice, and to more than two types of technologies. We also extend our model to

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for the relevant references, see Stern (2022) and Stern and Stiglitz (2022). We do not contribute to this debate in our paper.

[6] It would perhaps be more appropriate to label the clean and dirty technologies as, respectively, 'green' and 'brown' technologies. However, we reserve green and brown for the types of firms in our model.

[7] This is a reduced form way of capturing several possible benefits to the firms from adopting the clean technology. For instance, a reduction in technological/organizational costs of production on account of learning by doing; an increase in reputational benefits from adopting the clean technology over time; network externality effects (Katz and Shapiro, 1985); establishment of common standards that reduce costs; social interaction effects (Young, 2009); and strategic complementarities that reduce costs with greater adoption. In our paper, we are agnostic as to the exact transmission channel.

[8] Any technology improves with greater usage on account of learning by doing that results in improvements in the technology and/or in the organizational use of that technology. Indeed, the literature on learning by doing, and the associated empirical evidence, was developed in the context of dirty technologies. Hence, it would be undesirable to assume that dirty technologies necessarily become more expensive as more firms use them. However, the evidence does strongly indicate that the rate at which clean technologies become relatively cheaper with greater adoption exceeds that of dirty technologies; for a review see Stern (2022). This is the assumption we make. It might, however, be the case that in periods of rapid adoption, there could be scarcity of raw materials that creates bottlenecks for both types of technologies. We abstract from such issues.

[9] For instance, firms that have a higher share of energy costs might benefit more from energy-saving innovations; firms that have more funds could be in a better financial position to invest in cleaner technologies; or the existing technological/organizational features within a firm may have greater strategic complementarities with cleaner technologies. These type-contingent differences among firms in the relative benefits and costs of adopting cleaner technologies are well supported by the empirical evidence (Arvanitis et al., 2017; Hottenrott et al., 2016; Stucki, 2019).

stochastic technology dynamics; see Young (1993, 1998, 2006) and Dhami (2020, Vol. 6) for an introduction to this literature and methods.

1.2 Main results

Our framework is reasonably simple, yet offers powerful results. We discuss the results below under two main headings.

1. *System dynamics*: Our system dynamics have some of the features of complex adaptive systems.[10] (i) Ex-ante, the long-run outcome is unpredictable despite there being no fundamental uncertainty related to technology or preference shocks in the model. For instance, in the long-run, we could either have all firms eventually choosing the dirty technology (state $S = d$), or all firms choosing the clean technology (state $S = c$). We may observe lock-in effects into a particular technology after certain tipping points are reached. (ii) The long-run outcome is history dependent and the dynamic system is not ergodic. (iii) Small accidents of history can alter the eventual outcomes. These effects are predicted by our general model that we illustrate with several simulation exercises.

   When we introduce stochastic technology dynamics in Section 6 we show that there might be 'punctuated equilibria' and we give simple conditions that show which of the two states, $S = d$ or $S = c$, is stochastically stable. We also show why evolutionary dynamics are not appropriate in our framework.

2. *Public policy*: The final outcome is unpredictable, yet at the beginning of time, and prior to any technology adoption decisions, governments need to formulate and implement tax and subsidy policies towards technology adoption. Since the dynamic paths are unpredictable, a time-varying policy will create considerable temporal policy uncertainty for the government and would appear to be impractical and unpopular. Hence, we assume that the government credibly announces all taxes and subsidies at the beginning of time, before any technology adoption decisions have been made.[11]

   In the face of extreme uncertainty, the only reasonable basis for choosing the levels of taxes and subsidies on the costs and benefits of alternative technologies would appear to be ex-ante expectations of the long-run outcomes. For the reasons mentioned above, the government does not maximize some well defined social welfare function. It follows a 'guardrail' approach suggested in Stern (2022), Stern and Stiglitz (2022). For instance, the government's stated aspiration/objective could be that in the long-run it wishes all firms to adopt the clean technology.[12]

[10] For an introduction to complex adaptive systems, the reader can consult Arthur (2015), Dhami (2020, Vol. 6), Hommes (2021) and follow up the extensive references therein.

[11] In the real world, one might imagine periodic reviews of such policies and occasional revisions of the initially announced taxes and subsidies. Although we do not introduce such considerations in the main, we consider their implications in a separate subsection.

[12] Several examples can be given from the real world. For instance, banning diesel cars/incandescent light bulbs/ozone-depleting CFCs by a certain cutoff date in different countries. These objectives are not derived from sophisticated cost-benefit considerations based on an underlying social welfare function (Stern and Stiglitz, 2022). Indeed, such calculations are not feasible in the presence of true uncertainty. Rather, they are based on some sort of holistic public judgement that draws on the best available science, and perhaps a desire to avoid

Furthermore, political/legal/constitutional/fairness constraints on the imposition of taxes and subsidies are pervasive in the real world and this sets 'feasible bounds' on these fiscal instruments. We show the following results.

(i) Implementing optimal Pigouvian taxes on emissions from dirty technology might be inconsistent with the objective of ensuring the long-run adoption of green technologies. The reason is that Pigouvian taxes that simultaneously ensure long-run clean technology adoption might violate the feasible bounds on fiscal instruments. In that case, the government will need to abandon one of the two objectives (optimal Pigouvian taxes or the stated objectives on long-term clean technology adoption).

(ii) In the presence of increasing returns to scale from adoption, the government can announce a time path of subsides that decreases over time.[13] Once enough firms have adopted the clean technology, the subsidies can be completely withdrawn, and yet the dynamic system may lock-into the clean technology.

(iii) The level of subsidies needed to ensure adoption of the clean technology might be unaffordable for poor countries and lead to the proliferation of global environmental bads. This will require the formation of an international fund to subsidize clean technology adoption in poor countries.

1.3 Related literature

The closest paper is the seminal work of Arthur (1989) but there are important differences. First, he allows only for type-contingent differences in returns to scale, while we allow for type-contingent differences in the relative benefits from different technologies and technology-contingent differences in returns to scale. In one of our numerical simulations, we consider the special case in Arthur (1989). Second, his focus is not on the role of fiscal policy, which plays an important role in our analysis. Our paper is also different from Zeppini (2015) where there are representative firms, all of the same type, so there are no type-contingent and technology-contingent differences.

The endogenous growth theory literature is well developed in economics (Aghion and Howitt, 1997) as is the literature on the economics of innovation (Bloom et al., 2019). Neither of these literatures deal with issues that are central to our paper (Stern and Valero, 2020), such as type-contingent and technology-contingent net benefits from alternative technologies under increasing returns to scale from adoption, nor does it lead to the set of results obtained in our model (endogenous fluctuations, lock-in effects, history-dependent small accidents that have large effects, and emergent phenomena).

the worse possible states of the world– a form of robust control. We are agnostic about the exact transmission mechanism.

[13] Examples of subsidies to cleaner technologies abound. In the UK, in 2021, within 6 months of the Prime Minister's 10 Point Plan for a Green Industrial Revolution, a £166.5 million cash injection was announced. In 2016, subsidies towards renewable energy technologies in the US amounted to US$140 billion. In China, over the period 2016–2018, government spending on ecological/environmental protection was CNY 2.451 trillion. Similar examples can be given for many of the richer OECD countries. In some cases, tax rebates on the use of clean technology also serve to subsidize its usage.

1.4 Plan of the paper

Section 2 outlines the model. Section 3 considers the optimization decision of each type of firm; the equilibrium in the model; and its main features. Section 4 contains the dynamical version of the model, and presents simulation of time series and distribution of long-run outcomes. Section 5 considers policy applications for Pigouvian taxes, the dynamic path of subsidies, and arguments for a fiscal fund for poorer countries to afford green technologies. Section 6 gives extensions of the model to stochastic technology choices, and relaxes several assumptions in the model.

2 Model

There are $i = 1, ..., N$ firms within a country, where N is countably infinite. Each firm produces 1 unit of output. There are two types of technologies. A "clean technology" that produces no emissions and a "dirty technology" that produces a fixed amount of emissions with marginal social costs given by $C_s > 0$. For our purposes, we only require C_s to be the best current estimate of the marginal social cost by a policymaker, even if there is true uncertainty.

Time $t = 0, 1, 2, ...$ is discrete. In each time period, t, a randomly chosen firm must make a mandatory technology adoption decision between the clean and the dirty technologies, while all other firms are passive in that period; we relax this assumption in Section 6. At the beginning of time t, before any technology choice decisions have been made, t_c firms in the past have chosen the clean technology, and t_d have chosen the dirty technology.

There are two types of firms, a 'green type' G and a 'brown type' B. At the beginning of time $t = 0$, before any technology adoption decisions have been made, a type G firm is relatively better suited to the clean technology, while a a type B firm is relatively better suited to the dirty technology (type-contingent differences). The notion of 'better suited' is formalized below. At the beginning of time, a share $0 < \lambda_G < 1$ of the firms are type G and the remaining share $\lambda_B = 1 - \lambda_G$ is type B firms. The proportion of brown firms is relatively greater so that

$$\lambda_G < \lambda_B. \tag{2.1}$$

Reflecting the proportions of each type of firm in the population, in each time period $t = 0, 1, 2, ...$, nature randomly picks a type G firm with probability λ_G and a type B firm with probability λ_B to make the technology choice decision.

The government can levy taxes on benefits from any of the technologies and give subsidies on costs; we observe both kinds of instruments in the real world. For reasons discussed in the introduction, the government does not choose its taxes and subsidies in the classical manner to optimize a social welfare function. Instead, it follows a guardrail approach and sets broad targets and aspirations, such as the adoption of the clean technology by a certain time period.

We use the following notation. The respective variables pertaining to the benefits/costs of the clean technology and the dirty technology, respectively, are subscripted with a lowercase 'c' and a lowercase 'd.' Variables pertaining to the 'type' of the firm are superscripted with an uppercase $j = G, B$; the only exception is our usage of subscripts in the case of λ_G and λ_B.

2.1 Net type-contingent benefits of each technology

The net benefits from each of the technologies, for each type of firm, are calculated as follows. We first define the *time-invariant* and *type-continent* net benefit, B_c^j, of the clean technology to a firm of type $j = G, B$.

$$B_c^j(\tau_c, s_c) = b_c^j(1 - \tau_c) - c_c^j(1 - s_c);\ j = G, B. \tag{2.2}$$

In (2.2), b_c^j and c_c^j are, respectively, the gross benefits and costs from adoption of the clean technology to a type $j = G, B$ firm.[14] The tax rate on benefits, and the subsidy rate on cost, for the clean technology, are respectively given by $\tau_c, s_c \in [0, 1]$. For a given technology, the fiscal parameters cannot be differentiated by the type of the firm due to, say, legal/fairness/informational reasons. We have that

$$B_c^G(0, 0) > B_d^G(0, 0)\,; \tag{2.3}$$

so, in the absence of any government policy, the clean technology offers a relatively higher time-invariant benefit to type G firms relative to the dirty technology. This is the sense in which firms of type G are relatively 'better suited' to the clean technology.[15]

The clean technology gives the following utility to a firm of type $j = G, B$ when t_c firms have already chosen the clean technology:

$$U_c^j = B_c^j(\tau_c, s_c) + r_c t_c;\ j = G, B,\ r_c > 0,\ s_c, \tau_c \in [0, 1]\,. \tag{2.4}$$

The first component on the RHS of (2.4) is the net *time-invariant* benefit of the clean technology, defined in (2.2). The second component on the RHS of (2.4), captures the *time-dependent* variable return $r_c t_c$ from the clean technology, which depends on the number of other firms, t_c, that have already adopted the clean technology. The parameter $r_c > 0$ captures the effects of returns to scale from the adoption of the clean technology. For every clean technology adoption by a firm, the extra marginal benefit to a subsequent adoptor of the clean technology is r_c.

Define

$$B_d^j(\tau_d, s_d) = b_d^j(1 - \tau_d) - c_d^j(1 - s_d)\, j = G, B, \tag{2.5}$$

as the *time-invariant* and *type-contingent* net benefit of the dirty technology to a firm of type j. b_d^j and c_d^j are the respective gross benefits and costs, and τ_d, s_d are the corresponding tax and subsidy rates on, respectively, the benefits and costs of the dirty technology. In the absence of any government intervention, the dirty technology offers relatively greater net benefits to type B firms, so

$$B_c^B(0, 0) < B_d^B(0, 0)\,. \tag{2.6}$$

[14] The term b_c^j captures several potential benefits to the firm, including the classical gross revenues to the firm from selling its output. Our interest does not lie in studying the effects of market structure on the benefits to the firm, so we abstract from these issue. Similar comments apply to the benefits to firms adopting the dirty technology.

[15] Recall from the introduction, the discussion on this issue and the empirical evidence (Arvanitis et al., 2017; Hottenrott et al., 2016; Stucki, 2019). Some of the factors conducive to making some types of firms more suited to the clean technology include the share of their energy costs; complementarity of their existing technology and organizational structure with the new clean technology; type of output produced; availability of funds for green investment; and the stated corporate social responsibility of the firm.

The inequality in (2.6) formalizes the sense in which the type B firms are relatively 'better suited' to the dirty technology.

The utility from the adoption of the dirty technology to a firm of type $j = G, B$ is:

$$U_d^j = B_d^j(\tau_d, s_d) + r_d t_d;\ j = G, B,\ r_d > 0,\ s_d, \tau_d \in [0, 1], \tag{2.7}$$

The first component on the RHS of (2.7) is the net type-contingent benefit from adoption of the dirty technology. The second component on the RHS of (2.7), $r_d t_d$, is the time-dependent benefit from adoption of the dirty technology. It depends on the number of firms, t_d, that have already adopted the dirty technology. The parameter r_d gives the returns to scale from adoption of the dirty technology.

We summarize the relevant information in this subsection in Table 1.

Net Benefits	clean technology	dirty technology
Type G firm	$B_c^G(\tau_c, s_c) + r_c t_c$	$B_d^G(\tau_d, s_d) + r_d t_d$
Type B firm	$B_c^B(\tau_c, s_c) + r_c t_c$	$B_d^B(\tau_d, s_d) + r_d t_d$

Table 1: Type and technology-contingent net benefits from technology adoption.

2.2 The robustness and generality of our framework

Our simple model nests several possible extensions such as uncertainty over benefits and costs, switching costs, non-linear net benefits, and time discounting. We show below that this requires a simple relabelling of our variables. We only consider the net benefits from the adoption of the clean technology (analogous results can be readily stated for the benefits from the dirty technology, B_d^j).[16]

1. Risk and uncertainty: Suppose that the benefits from investing in the clean technology are uncertain and there is a distribution of benefits B_c^j over the interval $[\underline{b}, \overline{b}]$ with an underlying distribution function F_g; such a distribution can be an objective distribution (risk) or a subjective distribution (uncertainty). Redefine (2.5) as

$$B_c^j(\tau_c, s_c) = (1 - \tau_c)\widehat{b}_c^j - c_c^j(1 - s_c),$$

where $\widehat{b}_c^j = \int_{\underline{b}}^{\overline{b}} b_c^j dF_g$, and the entire analysis carries over unchanged. One can similarly accommodate uncertain costs.

2. Time-varying net benefits and discounting: Suppose that a firm making a decision to adopt the clean technology at time $t = k$ receives time-varying benefits in the future,

[16] In fact the extension of our model goes much beyond uncertainty about benefits or costs. In Section 4, we consider explicitly the dynamics of technology adoption, where we provide an extension of Arthur (1989). In Section 4, in addition to the random choice by nature of firms to make a technology choice in each period, we consider a stochastic process by which one of the two technologies is chosen, as in Zeppini and van den Bergh (2020). A further innovation with respect to Zeppini and van den Bergh (2020) is that we consider the convolution of two probability distributions of technology adoption to take account of the two types of firms in our model.

$b_c^j(t)$, $t = k, k+1, ..., k+\widetilde{T}$, where $b_c^j(t)$ is the time t benefit and $k+\widetilde{T}$ is some terminal date after which benefits cease. Redefine (2.5) as follows

$$B_c^j(\tau_c, s_c) = (1-\tau_c)\widetilde{b}_c^j - c_c^j(1-s_c), t \geq k,$$

where $\widetilde{b}_c^j = \sum_{t=k}^{t=k+\widetilde{T}} \delta^{t-k} b_c^j(t)$, and $0 < \delta < 1$ is the discount factor.

3. Switching costs from adopting the new technology: Let $\psi_{c \to d}^j$ be the fixed cost of switching from the clean to a dirty technology for a firm of type $j = G, B$. Similarly, let $\psi_{d \to c}^j$ be the fixed cost of switching from the dirty to a clean technology for a firm of type $j = G, B$. Consider a firm of type j that originally uses a dirty technology and it wishes to switch to a clean technology. The analysis is unchanged by defining

$$B_c^j(\tau_c, s_c) = b_c^j(1-\tau_c) - c_c^j(1-s_c) - \psi_{d \to c}^j;\ j = G, B.$$

We can analogously define the net benefits from switching from a dirty to a clean technology.[17]

4. Non-linear utility: Suppose that the utility from net benefits is non-linear, and $B_c^j \subset X$. We can now introduce a utility function $V : X \to R$ and restate all our results in terms of $V(B_c^j)$, without changing any insights.

We can also combine uncertainty and time discounting by utilizing, say, expected utility and exponential discounting, without altering any insights. Our model is set up in such a way that any firm that is faced with a technology adoption decision must choose either the clean technology or the dirty technology. It cannot continue with the status-quo, i.e., choose neither. One can relax this feature, but it does not alter our core insights.

We consider the following extensions of the model in Section 6. Our results are robust to extending our model to allow for several firms simultaneously making the technology adoption decision at time $t = 0, 1, 2, ...$ The model can also be easily extended to more than 2 technologies. We also extend our model to stochastic technology dynamics where each firm chooses its optimal technology with some probability $0 < \varepsilon < 1$ and engages in some 'experimentation' by randomizing over both technologies with some small probability $1 - \varepsilon$.

2.3 Returns to scale from technology adoption

As noted in the introduction, the empirical evidence suggests that cleaner technologies, because they are newer and developed within the confines of the current technological environment, confer relatively larger economies of scale from adoption.[18] By contrast, dirty technologies are

[17] If switching costs to a clean technology, $\psi_{d \to c}^j$, are too high, so that $B_c^j < 0$ then it might be the case that the clean technology is not adopted. However, in that case, the policymakers might wish to alter the taxes and subsidies to ensure that B_c^j is not too low, so that the utility from clean technology adoption U_c^j is high enough for it to be adopted. These are the usual policy tradeoffs that one must expect in such models. Arthur (1989, fn. 3) motivates the absence of switching costs in his model by assuming that "adopters need to replace an obsolete technology that breaks down..."

[18] Consumers would not have failed to notice the massive drops in the prices of solar panels, led bulbs, and electric vehicles. Similar, and rapid, drops in prices have also been observed for technological inputs purchased

older and additional improvements in returns to scale arising from technological/organizational improvements are more limited. Hence, we assume

$$0 < r_d < r_c. \tag{2.8}$$

2.4 Set of Fiscal instruments

Suppose that there are political/legal/constitutional/fairness constraints on the imposition of taxes and subsidies.[19] The government fiscal policy instruments are the technology-contingent taxes on benefits, and the subsidies on costs, $(\tau_c, \tau_d) \times (s_c, s_d) \in T$, where the set of all taxes and subsidies that respects the relevant constraints on taxes and subsidies is $T \subset R^4$.

As suggested in Stern and Stiglitz (2022), the fiscal instruments are not chosen by maximizing a social welfare function, but by using a guard-rail approach which has broad objectives based on the best available science. We assume that fiscal policy does not alter the intrinsic type-contingent pre-tax relative advantages of the two technologies to the two types of firms in (2.3), (2.6). Thus, post-tax, the type G firms continue to be relatively more suited to the clean technology and type B firms relatively better suited to the dirty technology.[20]

Thus, the set of feasible fiscal instruments lie in the following set.

$$\Gamma = \left\{ (\tau_c, \tau_d) \times (s_c, s_d) \in T : B_c^G(\tau_c, s_c) > B_d^G(\tau_d, s_d), \ B_c^B(\tau_c, s_c) < B_d^B(\tau_d, s_d) \right\}, \tag{2.9}$$

where $B_c^j(\tau_c, s_c)$ and $B_d^j(\tau_d, s_d)$, $j = G, B$, are defined in (2.2) and (2.5).

The set Γ specifies upper and lower feasible bounds, between 0 and 1, on technology-specific taxes and subsidies, thus:

$$\tau_i \in [\underline{\tau}_i, \overline{\tau}_i]; \ s_i \in [\underline{s}_i, \overline{s}_i], \ i = c, d. \tag{2.10}$$

by firms. Costs of electricity from utility-scale solar photovoltaics (PV) alone fell 85% between 2010 and 2020. The United Nations Climate Change press release dated 14 July 2022 points out that over just a period of about an year, the cost of electricity from onshore wind fell by 15%; and offshore wind and solar PV fell by 13%. The report goes on to say that "almost two-thirds or 163 gigawatts (GW) of newly installed renewable power in 2021 had lower costs than the world's cheapest coal-fired option in the G20... given the current high fossil fuel prices, the renewable power added in 2021 saves around USD 55 billion from global energy generation costs in 2022."

[19] Several examples of such constraints can be given. For instance, an entrenched dirty technology lobby might prevent variations in τ_d, s_d, which might force governments to rely more on the use of taxes/subsidies on the clean technology. The state of government finances might not allow subsidies beyond a certain limit. Electoral concerns might make particular taxes either more or less popular. There are several legal constraints on how high certain taxes can be. Taxing two activities that are broadly similar might invite charges of unfairness, and possibly a legal challenge, or have political consequences.

[20] We do not allow fiscal policy to discriminate between the types of firms, as this is likely to legally unfeasible. Can initially high subsidies to clean technology induce type B firms to adopt the clean technology? However, we believe that this is not an empirically interesting case. If the fiscal solution to the adoption of clean technologies was sufficient, there would be no problem of clean technology adoption anywhere; yet, this is not the case. As noted, the main reason for this is the political/legal/constitutional/fairness constraints on the imposition of taxes and subsidies, which set bounds on taxes and subsidies. Indeed such constraints are likely to be severe; taxes are unpopular and subsidies are extremely difficult to afford (particularly in most of the developing world). Consider the example of the widely publicized move to get people to adopt electric vehicles. Despite extensive subsidies on electric vehicles, the first adoptors of electric vehicles were those who already had fairly pro-environmental attitudes and could afford to buy the electric cars (analogous to type G in our model). Those who did not have such strong pro-environmental attitudes (the analogue of type B in our model) did not buy, despite the subsidies on offer. Indeed, such people are likely to be the last to buy the electric vehicles.

3 Optimal technology choice and equilibrium

Consider the technology adoption decision at time $t = 0, 1, 2, ...$ when the fiscal instruments belong to the set Γ defined in (2.9). Given our assumptions, one of the firms is randomly chosen to make a technology choice decision; with probability λ_G this is a type G firm and with probability λ_B this is a type B firm. In this section, we consider a type $j = G, B$ firm making a technology choice decision, at time t, conditional on t_c (respectively, t_d) other firms having made a decision to choose the clean (respectively, dirty) technology in the past.

3.1 Type G firm

Using the first row of Table 1, a firm of type G chooses the clean technology over the dirty technology if

$$\left(B_c^G - B_d^G\right) \geq r_d t_d - r_c t_c. \tag{3.1}$$

Factors conducive in the adoption of the clean technology are:[21] (1) Relatively higher net time-invariant benefits from the clean technology relative to the dirty technology, $B_c^G - B_d^G$. (2) Relatively higher returns to scale from the adoption of the clean technology, r_c, and higher number of previous adoptions of the clean technology, t_c. (3) Relatively lower returns to scale from the adoption of the dirty technology, r_d, and lower number of previous adoptions of the dirty technology, t_d. A firm of type G picks the dirty technology if $\left(B_c^G - B_d^G\right) < r_d t_d - r_c t_c$. It is more convenient to rewrite (3.1) as

$$\left(B_d^G - B_c^G\right) \leq r_c t_c - r_d t_d. \tag{3.2}$$

3.2 Type B firm

Using the second row of Table 1, a type B firm chooses the clean technology over the dirty technology if

$$r_c t_c - r_d t_d \geq \left(B_d^B - B_c^B\right), \tag{3.3}$$

otherwise the firm chooses the dirty technology. The conditions conducive to this choice for a type B firm are identical to those of a type G firm. Note that the RHS of (3.2) and the LHS of (3.3) are identical.

3.3 Equilibrium outcomes

Define an upper barrier, $\overline{B}$, and a lower barrier, $\underline{B}$, as follows

$$\overline{B} \equiv B_d^B - B_c^B > 0; \; \underline{B} \equiv B_d^G - B_c^G < 0. \tag{3.4}$$

The signs in (3.4) follow from (2.9) and capture the feature that type G firms find the clean technology relatively more attractive and type B firms find the dirty technology relatively more attractive. Using (2.2), (2.5), we can write $\underline{B}, \overline{B}$ in full

$$\underline{B} = B_d^G - B_c^G = \left[B_d^G(1-\tau_d) - B_c^G(1-\tau_c)\right] - \left[c_d^G(1-s_d) - c_c^G(1-s_c)\right]. \tag{3.5}$$

[21] For pedagogical convenience we use the tie-breaking rule that when the net benefits from the two technologies are identical, firms choose the clean technology.

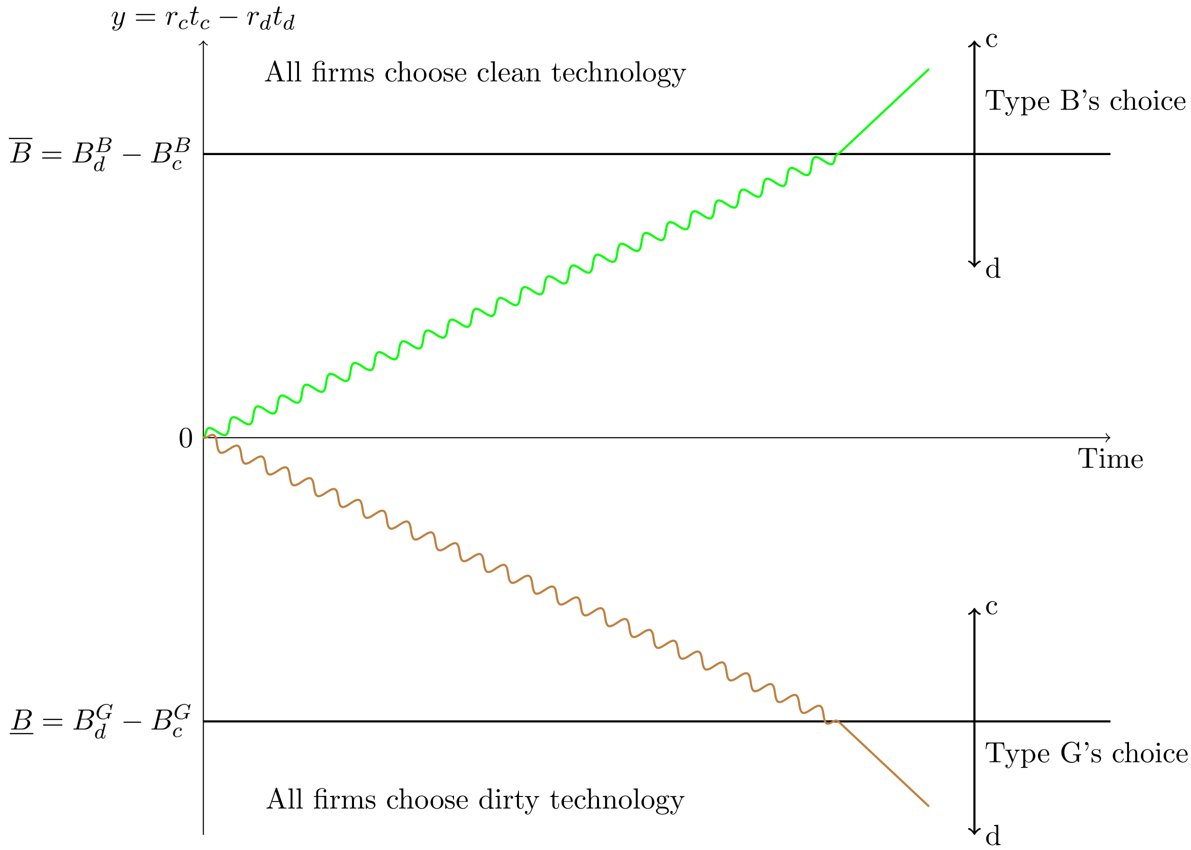

Figure 1: The dynamics of technology adoption.

$$\overline{B} = B_d^B - B_c^B = \left[B_d^B(1-\tau_d) - B_c^B(1-\tau_c) \right] - \left[c_b^B(1-s_d) - c_g^B(1-s_c) \right]. \qquad (3.6)$$

Figure 1 shows the dynamics of the system in time periods $t = 0, 1, 2, ...$; two squiggly hypothetical dynamic paths are shown in continuous time for pedagogical simplicity.[22] We measure our state variable $y = r_c t_c - r_d t_d$ along the vertical axis, and time along the horizontal axis. A single technology adoption decision is made at each time, which either moves the path up a bit, or down a bit, depending on which technology has been adopted, as we explain below; and this accounts for the sqiggles in the hypothetical paths. The upper and the lower barriers, $\overline{B}$ and $\underline{B}$, defined in (3.4), are also shown.

Using the technology adoption decisions of the two types of firms in (3.2), (3.3) it is easy to see the following. On and above the upper barrier $\overline{B}$, both types of firms choose the clean technology. On and below the lower barrier $\underline{B}$, both types of firms choose the dirty technology. In between the two barriers, $\overline{B}, \underline{B}$, type G firms choose the clean technology and type B firms choose the dirty technology (these choices are indicated in Figure 1 with vertical arrows for each type).

It follows that the two barriers are *absorbing barriers*, in the sense that once a dynamic path reaches one of the two barriers (if it does), the technology choice of all firms becomes identical, or locked-in, for all times to come– clean on and above the upper barrier, $\overline{B}$, and dirty on or below the lower barrier, $\underline{B}$. A government with a stated guardrail aim of universal adoption

[22]Actual dynamic paths are shown in the numerical simulations in Section 4 below.

of clean technology by a cutoff year would be particularly interested in paths approaching the upper barrier.

Consider any dynamic path within the two barriers, $\overline{B}$ and $\underline{B}$, and some time period t. There is a an ex-ante probability λ_G that a type G firm gets an opportunity to make a technology choice and it chooses the clean technology (see (3.2)). With the probability $\lambda_B = 1 - \lambda_G$, a type B firm is chosen to make a technology choice and it picks the dirty technology (see (3.3)). Recall that the vertical axis in Figure 1 measures the state variable $y \equiv r_c t_c - r_d t_d$. Thus, ex-ante, before any technology choice is made in any time period, there is a probability that the dynamic path will move down by $\lambda_B r_d > 0$ and up by $\lambda_G r_c > 0$ units.

Ex-ante, it is impossible to predict whether a given dynamic path will reach the upper barrier first or the lower barrier first. Indeed, some dynamic paths might never actually converge, or even be expected to converge to one of the two barriers in an ex-ante sense (see Section 4 for illustrations). In Figure 1, for illustrative purposes, we show two hypothetical paths– an upward sloping path that hits the upper barrier, following which the dynamic system is forever locked-into the clean technology, and a downward sloping path that hits the lower barrier, following which the dynamic system is forever locked-into the dirty technology.[23]

The only uncertainty in our model in this section is which of the two types of firms gets picked in any time period to make a technology choice.[24] However, there is no inherent 'fundamental uncertainty' in our model in the macroeconomic sense, i.e., there are no taste shocks or technology shocks. By contrast, in business cycle models and models in the New Keynesian Synthesis, technology shocks and demand shocks, respectively, give rise to economic fluctuations. The final outcome of the state variable in our model, e.g. the fraction of technologies adoption choices (all-clean, all-dirty, or convergence to a segmented, or polymorphic, equilibrium), is an *emergent property* of the complex system. In Section 4 we explore a wide spectrum of these different emergent outcomes of the model.

Different histories are likely to produce very different outcomes, i.e., the system is not ergodic. This is not limited to different starting points (initial conditions) of the state variable of the model, but also to early adoption events, in the spirit of Arthur (1989). Accidents of history may lead a dynamic path to hit the upper barrier rather than the lower barrier, or no barrier. Accordingly a structure emerges in the distribution of technologies adoption choices (see the Monte Carlo simulations at the end of Section 4). These are standard, and well known, properties of complex systems. We illustrate some of these properties in the simple model below.

Example 1 *: Suppose, and purely for illustrative purposes, that* $\lambda_G = 0.3$, $\lambda_B = 0.7$, $r_c = 1$, $r_d = 0.4$, $\overline{B} \equiv B_d^B - B_c^B = 2$, $\underline{B} \equiv B_d^G - B_c^G = -2$, *and there are* 30 *time periods. Hence, in each period, nature picks a type G firm with a* 30% *probability and a type B firm with a* 70% *probability. Suppose that at the end of 30 time periods, nature has, purely by chance, picked the*

[23] An infinite number of other shapes for such dynamic paths are possible and some paths could get close to the upper barrier, yet turn around and then head for the lower barrier and vice-versa. The shape is unpredictable. Our two hypothetical paths are simply shown for illustration purposes and an analysis with more realistic paths is possible; see Section 4.

[24] As already noted, we consider further sources of uncertainty related to stochastic process for technology adoption in Section 4.

type G firms 10 *times and the type B firms* 20 *times, so* $t_c = 10$ *and* $t_d = 20$*. Then, ex-post,* $y = r_c t_c - r_d t_d = 10 - 8 = 2$*. Thus, ex-post, the dynamic path hits the upper barrier,* $\overline{B}$*, which is an absorbing barrier. All subsequent adoptions of the technology are now clean technologies; the system forever locks-into the clean technology.*
Now suppose, due to an accident of history, that nature had picked out the type G firms $t_c = 7$ *times and the type B firms* $t_d = 23$ *times. Then,* $y = 7 - 9.2 = -2.2$*. In this case, the dynamic path reaches the lower barrier,* $\underline{B}$*, and all firms are forever locked into the dirty technology. Thus, the dynamic system is sensitive to historical accidents. However, in both cases, the final outcome is unpredictable from the perspective of time period* $t = 0$ *before any technology adoption decisions have been made, yet there is no fundamental macroeconomic uncertainty in the model.*

There are at least three sets of important questions to ask, based on the analysis above. First, what are the transition paths of technology adoption when we employ plausible system dynamics. We consider this in Section 4 below. A second set of questions relate to the 'expected' long-run properties of the model. This question is important for a government that must announce tax and subsidy policies at time $t = 0$, while subjected to the sort of extreme future uncertainties, our model predicts. We deal with some of these questions in Section 5 below. The third set of questions relates to stochastic technology dynamics that might lead to punctuated equilibria. We address this in Section 6.1 below.

4 Dynamic microfoundations

In this section, we show how the equilibrium analysis in Section 3 can be supported by plausible dynamics and what sort of transition dynamics to expect under different parameter constellations. In order to model the system dynamics, it is convenient to define the time dated state variable, x_t, which is the fraction of clean technology choices at time t arising out of all past technology choice decisions. We are interested in the time evolution at times $t = 0, 1, 2, ...$ of the variable x_t, which is then defined as $x_t \equiv \frac{t_c}{t}$. In Section 3, we have found it convenient to use the state variable $y \equiv r_c t_c - r_d t_d$ (see, e.g, the vertical axis of Figure 1), which captures the difference in returns to scale, weighted by the number of adoptions of the two technologies in 'absolute terms', while technology adoption decisions are made in a sequential temporal manner. However, in analyzing the dynamics it is more convenient to consider the proportion of clean technology choices at time t, given by x_t, as this variable ranges in the interval $[0, 1]$.[25] We only use the state variable x_t in this section, but in all other sections of the paper, we use the state variable y.

The variable x_t follows a discrete stochastic process that describes the sequential technology choice decision of firms. Following Arthur et al. (1987), such discrete choice is modeled through

[25]Consider the following useful analogy from evolutionary game theory. The general conditions for an 'evolutionary stable state' require only the equilibrium conditions (primary and secondary criteria) stated in 'absolute terms.' However, when analyzing dynamic paths, e.g., replicator dynamics, it is convenient to define a state variable x_t (the fraction of the population following a particular strategy at time t).

a binary variable $\alpha : [0,1] \to \{0,1\}$ that depends on the fraction x_t as follows.

$$\alpha(x_t) = \begin{cases} 1 & \text{with probability} \quad f(x_t) \\ 0 & \text{with probability} \quad 1 - f(x_t). \end{cases} \tag{4.1}$$

The 'endogenous' probability $f(x_t)$, referred to as *allocation function*, depends on the decision environment at time t (represented by the state variable x_t). In words: $f(x_t)$ is the state-dependent endogenous probability that a firm making a technology choice at time t, chooses the clean technology; with the complementary probability $1 - f(x_t)$, the dirty technology is chosen. The interpretation of the binary variable α is as follows. If the firm deciding at time t (of either type G or type B) ends up adopting the clean technology then $\alpha = 1$ and if it ends up adopting the dirty technology, then $\alpha = 0$. The 'actual' realization of α, at any time t, dictates the discrete evolution of the fraction of firms, x_t, that have adopted the clean technology at time t, as follows:

$$x_{t+1} = x_t + \frac{1}{w+t}\left[\alpha(x_t) - x_t\right]; t = 0, 1, 2, ... \tag{4.2}$$

In (4.2), w is a parameter related to the initial time $t = 0$ conditions on the number of technology adoption choices already made.[26]

If the firm making the technology choice decision at time t adopts the clean technology ($\alpha = 1$), then x_t increases to a new value x_{t+1} at time $t+1$, and it decreases to a new value if the firm adopts the dirty technology. The conditional expectation of x_{t+1} is[27]

$$E[x_{t+1}|x_t] = x_t + \frac{1}{w+t}\left[f(x_t) - x_t\right]. \tag{4.3}$$

Equation (4.3) represents a dynamical system, whose equilibria, if they exist, are the fixed points of $f(x_t)$.[28] This function governs the distribution of technology choices, and allows us to model the specific decision problem of our paper.

The function $f(x_t)$ is the probability that a firm (of either type G or type B), which has an opportunity to choose between the competing technologies at time t, adopts the clean technology. In our model, this becomes the joint probability that a firm belongs to a given type and that a firm of that type adopts the clean technology. Since Green firms have mass λ_G, and Brown firms a mass $1 - \lambda_G$, this probability is given by:

$$\begin{aligned} &f(x_t) = Prob\{\text{A firm adopts clean technology}\} = \\ &\lambda_G \cdot Prob\{\text{Green adopts clean technology}\} + (1-\lambda_G) \cdot Prob\{\text{Brown adopts dirty technology}\}. \end{aligned} \tag{4.4}$$

Following Zeppini and Van Den Bergh (2020), we model the technology adoption choice within the framework of discrete choice theory (McFadden, 1981; Brock and Durlauf, 2001), with a

[26] Equation (4.2) with the parameter w follows Ermoliev and Kaniovski (1987). In the probabilistic mathematical framework of stochastic Polya processes, w is the initial number of balls contained in the urn where extraction takes place. In technology competition a' la Arthur it is the number of adoption decisions that have been made before the initial time $t = 0$. From a computational point of view, one needs to choose a value for w along with with the other parameters in order to be able to explore a wide range of dynamic scenarios.

[27] We use $E[\alpha(x_t)|x_t] = 1.f(x_t) + 0.(1 - f(x_t)) = f(x_t)$.

[28] If x^* is an equilibrium, then $x_t = x_{t+1} = ... = x^*$. From (4.3) this implies that $E[x_{t+1}|x^*] \equiv x^* = x^* + \frac{1}{w+t}[f(x^*) - x^*]$, or $f(x^*) - x^* = 0$, or $f(x^*) = x^*$, so x^* is a fixed point of f.

logistic distribution. One novelty of our model is that we have two discrete choice decisions, one for each type of firms. For Green firms, the probability of adoption of the clean technology is

$$Prob\{\text{Green adopts clean technology}\} = \frac{e^{\beta U_c^G}}{e^{\beta U_c^G} + e^{\beta U_d^G}} = \frac{1}{1 + e^{\beta(U_d^G - U_c^G)}}, \tag{4.5}$$

where U_c^G and U_d^G are the utilities for a type G firm from adopting the clean and the dirty technologies. β is a parameter of the logistic distribution, which captures the underlying noise in the choice in the optimal technology. As $\beta \to 0$, the firm randomizes equally between the clean and dirty technologies, so the RHS of (4.5) equals 0.5. At the other extreme, as $\beta \to \infty$, the optimal choice is played with probability 1, so for instance, if $U_c^G > U_d^G$, then a type G firm chooses the clean technology with probability 1. Similarly, for Brown firms the probability of adoption of the clean technology is

$$Prob\{\text{Brown adopts clean technology}\} = \frac{e^{\beta U_c^B}}{e^{\beta U_c^B} + e^{\beta U_d^B}} = \frac{1}{1 + e^{\beta(U_d^B - U_c^B)}}, \tag{4.6}$$

with U_c^B and U_d^B are the utilities for a type B firm from adopting the clean and the dirty technologies.

In Section 3, above, we defined U_c^j and U_d^j, $j = G, B$, respectively in (2.4) and (2.7), in terms of 'absolute levels.' In this Section, we are interested in the time path of, x_t, the fraction of firms choosing the clean technology. A natural adaptation of (2.4) and (2.7) in terms of x_t for type G and B firms is given by

$$U^G = \begin{cases} U_c^G = B_c^G + r_c x_t & \text{if Green adopts clean technology} \\ U_d^G = B_d^G + r_d(1 - x_t) & \text{if Green adopts dirty technology.} \end{cases} \tag{4.7}$$

$$U^B = \begin{cases} U_c^B = B_c^B + r_c x_t & \text{if Green adopts clean technology} \\ U_d^B = B_d^B + r_d(1 - x_t) & \text{if Green adopts dirty technology.} \end{cases} \tag{4.8}$$

The only difference between (2.4), (4.7), and (2.7), (4.8), respectively, is in the last terms on the RHS in each of the expressions. For instance, comparing the first row of (2.4) and (4.7), the last terms are $r_c t_c$ and $r_c x_t$, stated respectively in terms of levels (t_c) and proportions (x_t), where $x_t = \frac{t_c}{t}$. The 'level' and 'proportion' terms are both increasing in the number of previous technology adoptions of the clean technology, t_c, and differ only by a normalizing factor $\frac{1}{t}$.

A Green firm is indifferent between the clean and the dirty technologies when $B_c^G + r_c x_t = B_d^G + r_d(1 - x_t)$. This condition defines the following indifference value of the fraction of technology adoption choices for Green firms:

$$\tilde{x}^G = \frac{B_d^G - B_c^G + r_d}{r_c + r_d}. \tag{4.9}$$

A Brown firm is indifferent between the clean and the dirty technologies when $B_c^B + r_c x_t = B_d^B + r_d(1 - x_t)$. Thus, the indifference value of x_t for Brown firms is

$$\tilde{x}^B = \frac{B_d^B - B_c^B + r_d}{r_c + r_d}. \tag{4.10}$$

From (2.9), $B_c^G > B_d^G$ and $B_d^B > B_c^B$, so $\tilde{x}^G < \tilde{x}^B$. It follows that a Green firm prefers the clean technology if $x_t > \tilde{x}^G$, while a Brown firm prefers the clean technology if $x_t > \tilde{x}^B$. In particular, if $x_t > \tilde{x}^B$ both types prefer the clean technology, while if $x_t < \tilde{x}^G$ then both types prefer the dirty technology. When $\tilde{x}^G < x_t < \tilde{x}^B$ a Green firm prefers the clean technology, while a Brown firm prefers the dirty one. Such state dependent preferences define three regions in the x_t space, separated by two *barriers*, as depicted in Figure 2.

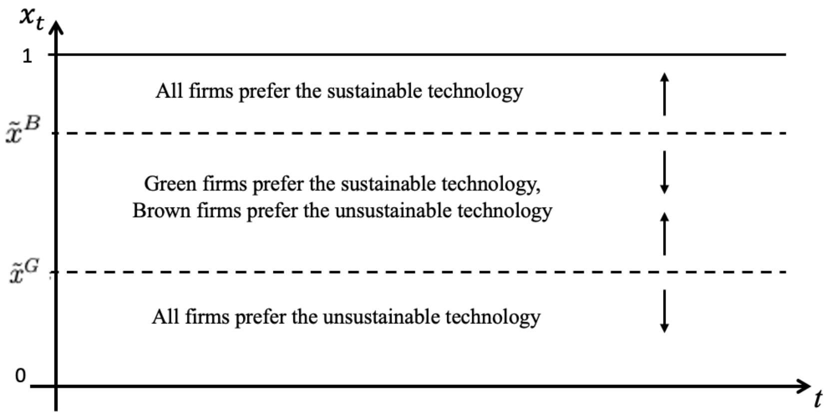

Figure 2: Representation of technology adoption preferences for Green and Brown firms in the x_t space (fraction of clean technology choices). The indifference levels $\tilde{x}^G$ and $\tilde{x}^B$ are barriers that separate the regions, and the arrows indicate the direction of change of the state variable x_t.

As the dynamics of technology adoption choices unfold, if x_t surpasses one of the two barriers, $\tilde{x}^G, \tilde{x}^B$, it gets attracted into an equilibrium where one technology is prevalent, converging either to $x = 1$ or to $x = 0$. If this event never happens, x_t converges to a segmented or polymorphic equilibrium where both technologies are used by firms. If returns on adoption rates are equal, with $r_c = r_d = r$, we have that $\tilde{x}^G = \frac{1}{2} - \frac{B_c^G - B_d^G}{2r} < \frac{1}{2}$, while $\tilde{x}^B = \frac{1}{2} + \frac{B_d^B - B_c^B}{2r} > \frac{1}{2}$. If $B_c^G = B_d^G$ and $B_c^B = B_d^B$ the two barriers collapse into a single barrier located at $x = \frac{1}{2}$, such that above this barrier, all firms prefer the clean technology and below it all prefer the dirty technology.[29]

Each of the three regions in the x_t space described above is a basin of attraction of an equilibrium of x_t that, as noted above, is a fixed point of the probability function f. We now rewrite this probability as a function of x_t by substituting (4.5), (4.6) in (4.4).

$$
\begin{aligned}
f(x_t) &= \lambda_G \frac{e^{\beta U_c^G}}{e^{\beta U_c^G} + e^{\beta U_d^G}} + (1-\lambda_G)\frac{e^{\beta U_s^B}}{e^{\beta U_s^B} + e^{\beta U_d^B}} \\
&= \frac{\lambda_G}{1 + e^{\beta(U_d^G - U_c^G)}} + \frac{1-\lambda_G}{1 + e^{\beta(U_d^B - U_s^B)}} \\
&= \frac{\lambda_G}{1 + e^{\beta[-\Delta B^G + r_d - (r_c + r_d)x_t]}} + \frac{1-\lambda_G}{1 + e^{\beta[\Delta B^B + r_d - (r_c + r_d)x_t]}},
\end{aligned} \tag{4.11}
$$

where $\Delta B^G = B_s^G - B_d^G > 0$ and $\Delta B^B = B_d^B - B_c^B > 0$. The function f is continuously differentiable and monotonic increasing, as $f'(x) > 0 \; \forall x \in [0, 1]$. It is characterized by intervals where it is concave or convex, with $f''(x) < 0$ and $f''(x) > 0$ respectively, separated by the

[29]We have assumed positive returns to scale, with $r_c, r_d > 0$. In the opposite case of negative returns to scale we obtain repelling barriers, that keep x_t confined in the region of its initial condition. In this case, under certain conditions, the dynamic system of Equation (4.3) could present negative feedback and possibly chaotic dynamics.

indifference values $\tilde{x}^G$ and $\tilde{x}^B$, which are the points of inflection, $f''(\tilde{x}^G) = f''(\tilde{x}^B) = 0$. The following Proposition states a result on the number of stable equilibria of the sequential decision system in (4.3):

Proposition 1 *The fraction of technology choices x_t described by the process (4.3) has at least one stable steady state (unique equilibrium) and at most three stable steady states (three equilibria).*

This result can be proved by using the mean value theorem, and the fact that $f(x_t)$ has 'at most' two points of inflection. This means that f can cross the 45^o line 'at most' in five points, and 'at most' three points are characterized by $f' < 1$, the condition for a stable steady state. Such stable fixed points are separated by fixed points that are unstable steady states. If $\beta \to \infty$ the unstable fixed points of $f(x_t)$ coincide with the indifference levels of discrete choice utility.

4.1 Time series simulations

We simulate the model by considering five different settings that showcase the most meaningful scenarios, labelled from A to F.[30] Table 2 summarizes the scenarios with their characteristics and outcomes. The second column describes situations where there is symmetry in terms of the benefits and relative proportions of the two types of firms, and the returns to scale for the two technologies (i.e., $\lambda_G = 0.5, \Delta B^G = \Delta B^B$, and $r_c = r_d = r$); by varying the values of ΔB^G and r_c we get the three different symmetric cases A, B, C in Table 2. The remaining cases, D, E, F, are asymmetric. The third column reports the location of barriers, $\tilde{x}^G$ and $\tilde{x}^B$, that separate the middle from the two extreme equilibria ($x^* = 0$ and $x^* = 1$) of the fraction of sustainable technology choice. The fourth column reports the values of the three stable equilibria, where $f'(x^*) < 1$. Finally, the fifth column contains a summary of the distribution of values of the fraction of times each of the stable equilibria ($x^* = 0$, $0 < x^* < 1$, $x^* = 1$) arise from 20 simulation runs.

Scenario	symmetry	barriers	equilibria	outcomes
A	symmetric ($\Delta B^G = \Delta B^B = 1; r = 3$)	0.36, 0.63	$(0, 0.5, 1)$	$[4, 11, 5]$
B	symmetric ($\Delta B^G = \Delta B^B = 1.5; r = 3$)	0.25, 0.75	$(0, 0.5, 1)$	$[0, 19, 1]$
C	symmetric ($\Delta B^G = \Delta B^B = 1; r = 6$)	0.42, 0.58	$(0, 0.5, 1)$	$[10, 0, 10]$
D	asymmetric ($\lambda_G = 0.4$)	0.25, 0.75	$(0, 0.4, 1)$	$[4, 16, 0]$
E	asymmetric ($\Delta B^G = 1$, $\Delta B^B = 2$)	0.36, 0.86	$(0, 0.5, 1)$	$[6, 14, 0]$
F	asymmetric ($r_c = 4$, $r_d = 3$)	0.21, 0.64	$(0, 0.5, 1)$	$[0, 16, 4]$

Table 2: Summary of possible outcomes induced by varying the model parameters.

In all settings we have used a distribution parameter $\beta = 5$ (see (4.11)) and a 'history' parameter $w = 100$ (number of previous adoptions; see (4.3)). These parameter values ensure that the basins of attraction of the different equilibria are well separated, given the variability of changes in x_t. In particular, the indifference points $\tilde{x}^G$ and $\tilde{x}^B$, and unstable fixed point are

[30] The model is implemented in Matlab, and the code is available on request from the authors.

relatively close. This allows us to evaluate and illustrate different outcomes in terms of long run dynamics, for different parameter values.

In what follows, for each scenario, we plot in the right panel of each of the figures, the time series x_t representing the relative size of clean technology choices in 20 simulation runs, and in the left panels the probability distribution $f(x_t)$ of the stochastic process x_t (equations (4.1) and (4.2)). We always start with an initial condition $x_1 = 0.5$, i.e., equal proportion of technologies.

Scenario A:

This scenario presents the perfectly symmetric situation, with an equal proportion of firms of both types and identical type-invariant relative benefits from the technologies ($\Delta B^G = \Delta B^B = 1$) and $r_c = r_d = 3$. Using (4.9), (4.10) we have relatively narrow indifference barriers, with $\tilde{x}^G = 0.36$ and $\tilde{x}^B = 0.63$.

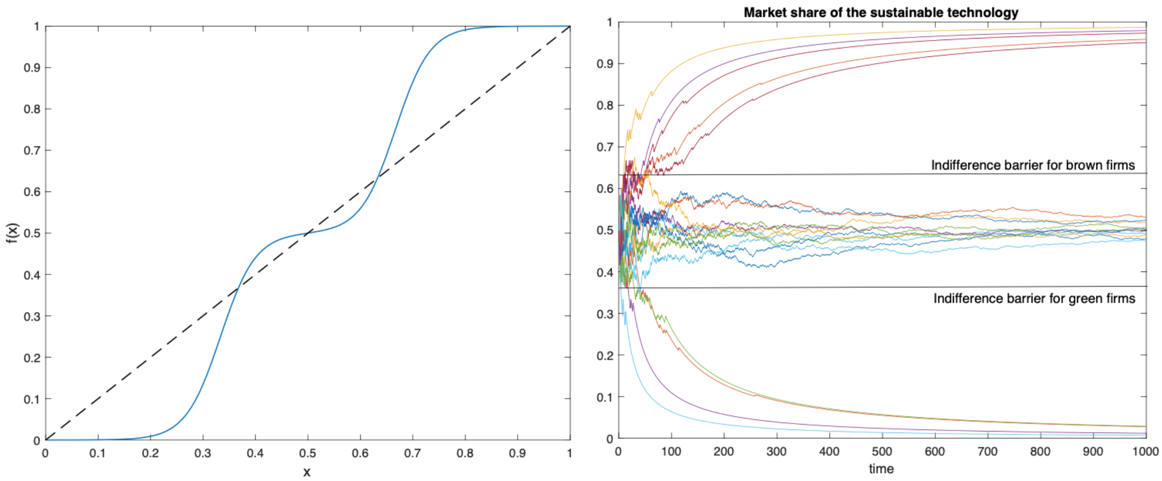

Figure 3: Scenario A: Symmetric barriers $\tilde{x}^G = 0.36$ and $\tilde{x}^B = 0.63$ and equal number of Green and Brown firms, with $\lambda_G = 0.5$; returns on adoption of technologies $r_c = 3$, $r_d = 3$; difference of time invariant benefits for Green firms $\Delta B^G = B^G_s - B^G_d = 1$ and for Brown firms $\Delta B^B = B^B_c - B^B_d = 1$. Left: probability distribution of choices; Right: 20 simulation runs of time series of the fraction of sustainable technology choices.

In this case, the barriers are close enough for early fluctuations of x_t to surpass them, either on the lower or the upper side. As a result, the population of firms gets locked-in any of the three stable equilibria $x^* = (0, 0.5, 1)$, the middle equilibrium of nearly equal adoption sizes ($x^* = 0.5$), the lower equilibrium of basically no sustainable technology adoption ($x^* = 0$), and the upper equilibrium of only clean technology adoption choices ($x^* = 1$). Out of the 20 simulations, four get locked in the lower and five in the upper equilibrium, while eleven converge to the middle equilibrium (this information can also be read off from the last column of Table 2).

Scenario B:

This is again a symmetric setting with symmetric benefit differences, $\Delta B^G = \Delta B^B = 1.5$, and symmetric returns to scale from the two technologies $r_c = r_d = 3$. Substituting these values in (4.9), (4.10) we find that the indifference barriers are further apart relative to scenario A, with

$\tilde{x}^G = 0.25$ and $\tilde{x}^B = 0.75$ (the indifference values and the unstable equilibria almost coincide in this example).

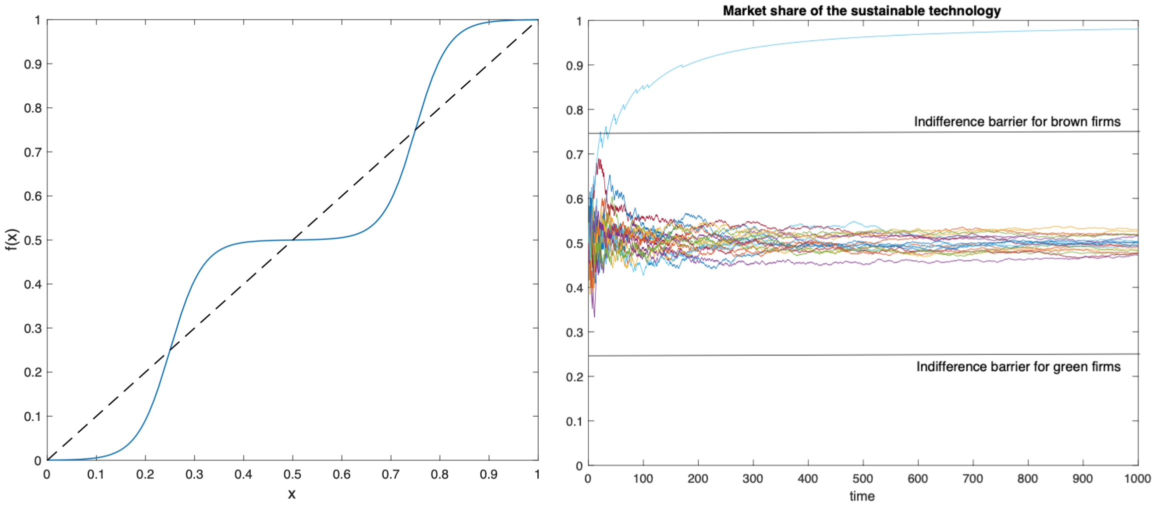

Figure 4: Scenario B: Symmetric barriers $\tilde{x}^G = 0.25$ and $\tilde{x}^B = 0.75$ (more far apart) and equal number of Green and Brown firms, with $\lambda_G = 0.5$; returns on adoption of technologies $r_c = r_d = 3$; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1.5$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1.5$. Left: probability distribution of choices; Right: 20 simulation runs of time series of the fraction of sustainable technology choices.

Since the barriers are more far apart, it is less likely for dynamic paths to hit the barriers, and most simulation runs get attracted into the middle equilibrium ($x^* = 0.5$). Only for one simulation run does the fraction x_t pass the upper barrier $\tilde{x}^B$ and get attracted into the upper equilibrium ($x^* = 1$).

Scenario C:

In this symmetric case, $\Delta B^G = \Delta B^B = 1$ and $r_c = r_d = 6$. Using (4.9), (4.10), the indifference barriers are relatively close to each other, with $\tilde{x}^G \simeq 0.42$ and $\tilde{x}^B = 0.58$.

In this case, the basin of attraction of the middle equilibrium (area between the two barriers) is relatively small. The variability of sequential adoption decision is such that it is very likely for x_t to reach one of the two barriers, and become locked into a convergence pattern towards one of the two extreme equilibria, either $x^* = 0$ (dominance of the dirty technology) or $x^* = 1$ (dominance of the clean technology). We can observe that for one of the simulation runs, x_t remained in between the two barriers for a relatively long time (about 300 time periods); but as soon as the lower barrier is reached, a regular convergence pattern towards the lower equilibrium ensues. This scenario resembles closely the classic outcome of technology competition with homogeneous firms of Arthur et al. (1987): when barriers are so close and easy to reach, the result is lock-in into one or the other technology ($x^* = 0$ or $x^* = 1$), with equal probability.[31]

Scenario D:

[31] This example shows that attracting barriers are given by the indifference values of utility $\tilde{x}^G$ and $\tilde{x}^B$, and not by the unstable fixed points of $f(x)$. The indifference values divide regions of x where the two types of firms adopt with higher probability the same technology or a different technology, while the unstable fixed points do not play any role.

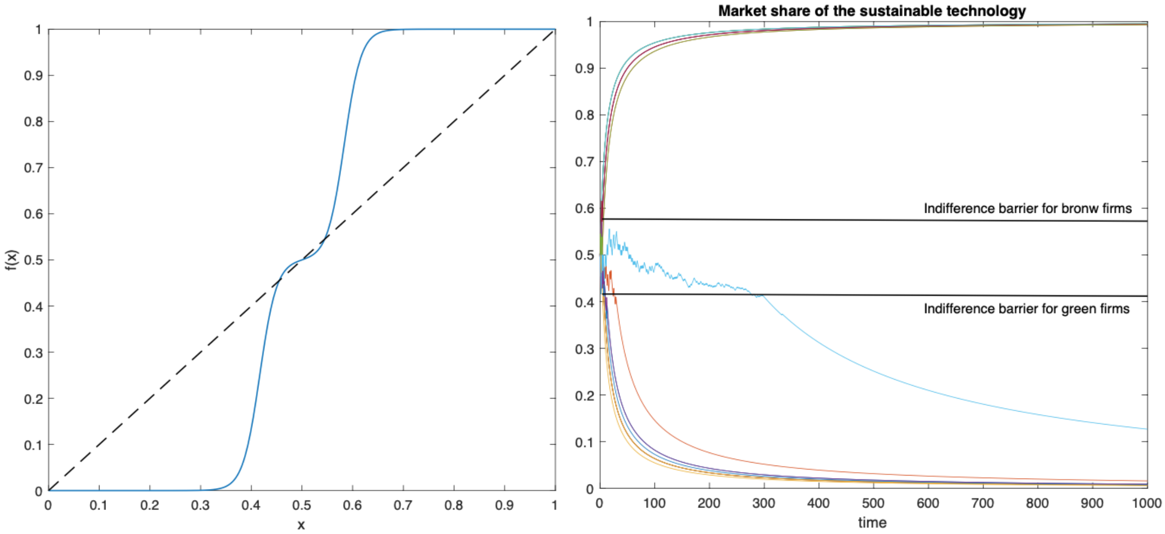

Figure 5: Scenario C: Symmetric barriers $\tilde{x}^G \simeq 0.42$ and $\tilde{x}^B = 0.58$ (closer to each other) and equal number of Green and Brown firms, with $\lambda_G = 0.5$; returns on adoption of technologies $r_c = r_d = 6$; difference of time invariant benefits for Green firms $\Delta B^G = B^G_s - B^G_d = 1$ and for Brown firms $\Delta B^B = B^B_c - B^B_d = 1$. Left: probability distribution of choices; Right: 20 simulation runs of time series of the fraction of sustainable technology choices.

Consider an unequal proportion (40/60) of Green and Brown firms, so that $\lambda_G = 0.4$. The remaining settings are as in scenario B. Using (4.9), (4.10), the indifference barriers are $\tilde{x}^G = 0.25$ and $\tilde{x}^B = 0.75$.

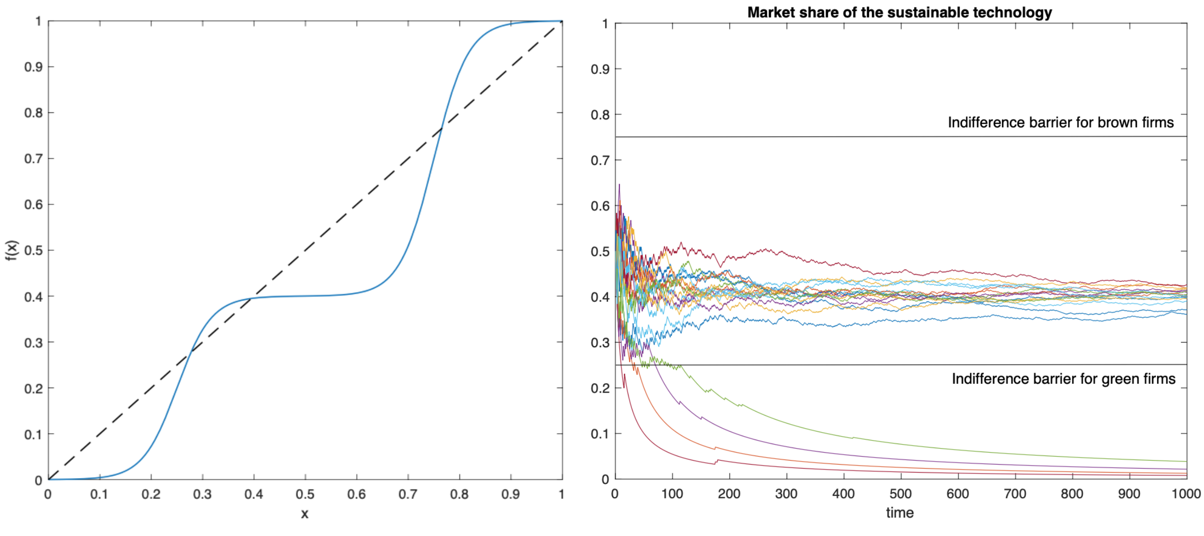

Figure 6: Scenario D: Symmetric barriers $\tilde{x}^G = 0.25$ and $\tilde{x}^B = 0.75$ and prevalence of Brown firms, with $\lambda_G = 0.4$; returns on adoption of technologies $r_c = 3$, $r_d = 3$; difference of time invariant benefits for Green firms $\Delta B^G = B^G_s - B^G_d = 1.5$ and for Brown firms $\Delta B^B = B^B_c - B^B_d = 1.5$. Left: probability distribution of choices; Right: 20 simulation runs of time series of the fraction of sustainable technology choices.

Relative to scenario B, the middle equilibrium is lower, and approximately equal to $x^* = 0.4$. Since the fraction x_t starts close to this level at time 0, it is more likely for it to reach the lower barrier $\tilde{x}^G = 0.25$ than the upper one, $\tilde{x}^B = 0.75$. Consequently, the basin of attraction of the lower equilibrium is larger, and 4/20 times, x_t ends up converging to the lower equilibrium, and never to the upper equilibrium. All the remaining simulations in this scenario are characterized by a convergence to the middle equilibrium 0.4.

Scenario E:

Consider an asymmetric setting, with a difference in time invariant benefits equal to $\Delta B^G = B_s^G - B_d^G = 1$ for Green firms and $\Delta B^B = B_c^B - B_d^B = 2$ for Brown firms. The returns to scale are symmetric, $r_c = 3, r_d = 3$. Using (4.9), (4.10), the indifference barriers are $\tilde{x}^G = 1/3$ and $\tilde{x}^B = 5/6$.

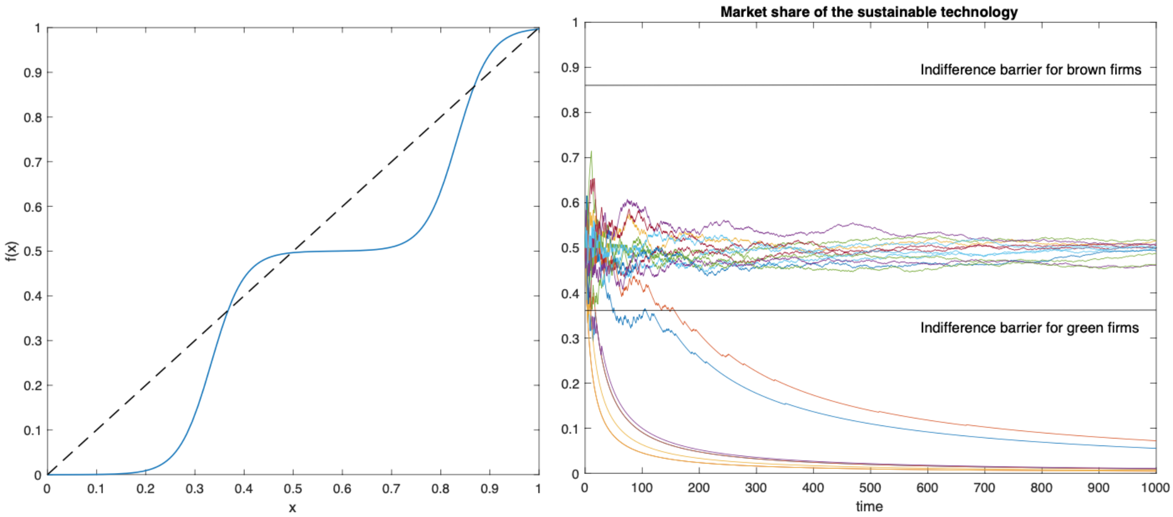

Figure 7: Scenario E: Asymmetric and relatively high barriers $\tilde{x}^G = 1/3$ and $\tilde{x}^B = 5/6$, and equal number of Green and Brown firms, with $\lambda_G = 0.5$: difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 2$; returns on adoption of technologies $r_c = 3$, $r_d = 3$. Left: probability distribution of choices; Right: 20 simulation runs of time series of the fraction of sustainable technology choices.

Despite starting from an initial condition $x_0 = \lambda_G = 0.5$, the barrier for Green firms, $\tilde{x}^G = 1/3$, is relatively easy to cross for x_t. Out of our 20 simulation runs, this occurs six times, when the time series of x_t gets attracted into the lower equilibrium ($x^* = 0$), while the barrier for Brown firms, $\tilde{x}^B = 5/6$, is never reached, and the upper equilibrium ($x^* = 1$) is never attained.

Scenario F:

This is an asymmetric setting where the asymmetry originates from different returns on adoption, $r_c = 4$ and $r_d = 3$, for the clean and dirty technologies, respectively. The remaining choice of parameter values for this setting is symmetric ($\Delta B^G = \Delta B^B = 1.5$ and $\lambda_G = 0.5$). Using (4.9), (4.10), this translates into indifference values $\tilde{x}^G \simeq 0.21$ and $\tilde{x}^B = 0.64$.

The two barriers are now relatively low, with fixed points of f equal to about 0.21 and 0.61. The lower barrier is never reached by the time series x_t in this example, while the upper barrier is reached in 4/20 simulation runs, and consequently the fraction of clean technology adoptions converges to the upper equilibrium four times. In the other 16/20 runs the dynamic path converges to the middle equilibrium with an equal share of adoption choices.

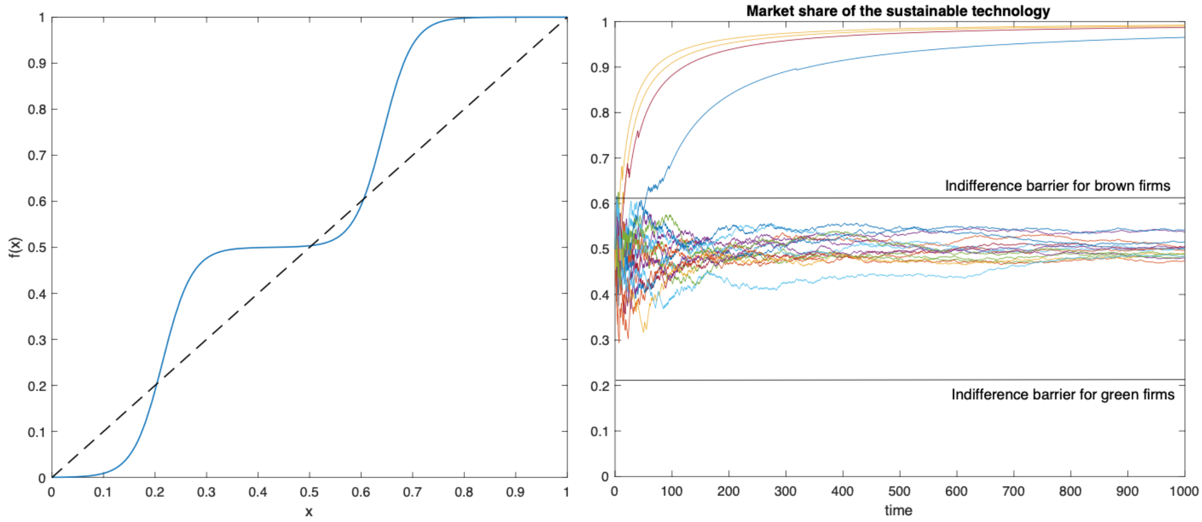

Figure 8: Scenario F: Asymmetric and relatively low barriers $\tilde{x}^G \simeq 0.21$ and $\tilde{x}^B = 0.64$, and equal number of Green and Brown firms, with $\lambda_G = 0.5$; returns on adoption are $r_c = 4$ and $r_d = 3$, for the sustainable and unsustainable technologies, respectively; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1.5$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1.5$. Left: probability distribution of choices; Right: 20 simulation runs of time series of the fraction of sustainable technology choices.

4.2 Monte Carlo simulations: Distribution of long run outcomes

Complementing the analysis in Section 4.1, in this section we provide a more extensive and systematic numerical analysis through a Monte Carlo approach, by running a large number of simulations (10000 simulation runs). We report the distribution of long run outcomes across these simulation runs for the value of the fraction of clean technology adoption choices. The long run is represented by the end of the time horizon considered, $T = 1000$ time periods. As we have observed in the simulation of time series of x_t above, this time horizon is long enough for our model to converge to long-run equilibria, within all scenarios considered.[32]

We perform the Monte Carlo simulations for each of the scenarios considered above in Table 2. The results are reported in Figure 9. These numerical experiments with a large number of simulations also give an insight into equilibrium selection.

In Scenario A, all three equilibria get selected, with a prevalence of the middle equilibrium where competing technologies are adopted in equal proportions. In Scenario B, the barriers are spaced relatively far apart, as a result of a relatively large difference in time invariant benefits of the clean and the dirty technologies for each type of firm, and consequently the dominance of one technology happens very rarely (it does occur though, as one can see few counts in the first and in the last bins). This is a scenario where each firm adopts its most preferred technology. The opposite outcome occurs in Scenario C where the barriers are very close to the center of the distribution, because of relatively strong returns on adoption. So, it is very likely for the process x_t to reach one of them and get attracted towards one of the two extreme equilibria ($x^* = 0$ or $x^* = 1$). Due to the symmetry of the setting, this is equally likely for the lower

[32] The factor $\frac{1}{w+t}$ in Equation (4.3) favours convergence. It is then sufficient to consider a time horizon long enough for equilibrium selection to occur.

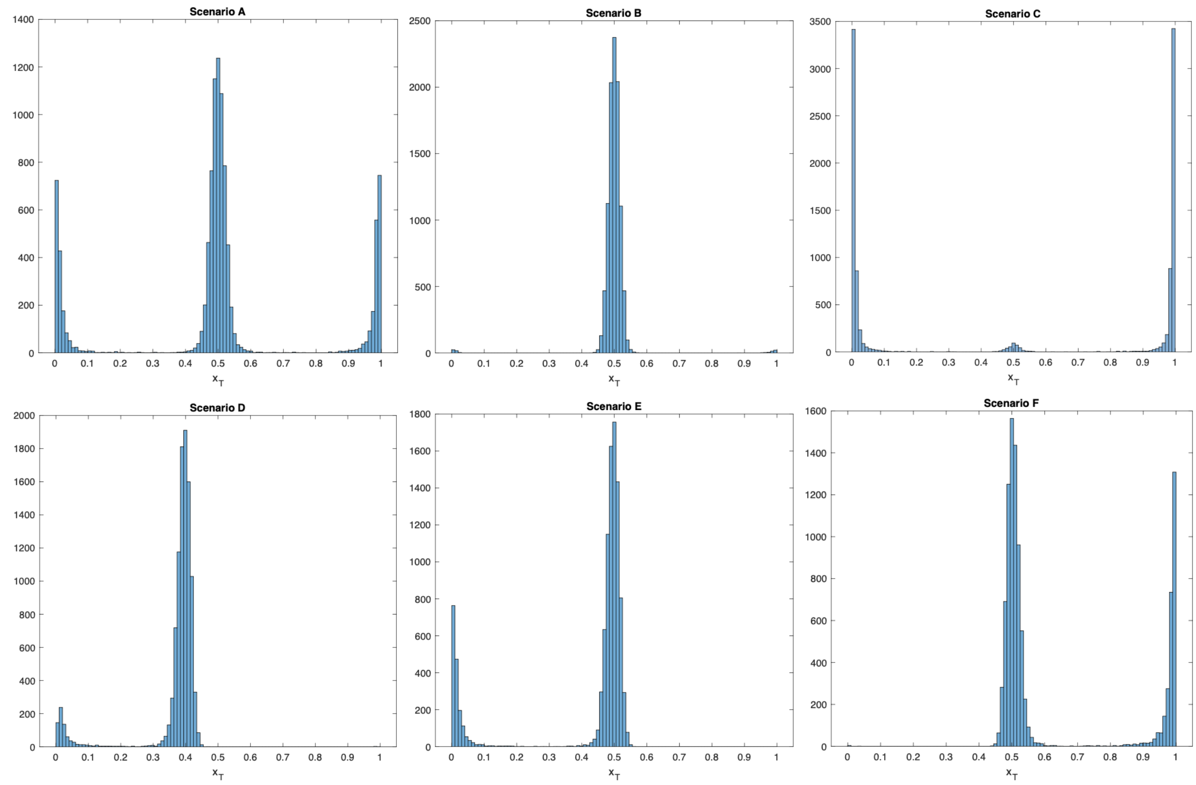

Figure 9: Monte Carlo simulations of the fraction of clean technology adoption x_t. Histograms represent the distribution of values of the fraction of clean technology choices after $T = 10,000$ time periods. There are 100 bins for x_T, each with a width of 0.01, covering the range $[0, 1]$. Scenario A (top-left) has equal firm mass $\lambda_G = 0.5$; returns on adoption of technologies $r_c = 3$, $r_d = 3$; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1$. Scenario B (top-center) has firms mass $\lambda_G = 0.5$; returns on adoption of technologies $r_c = 3$, $r_d = 3$; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1.5$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1.5$. Scenario C (top-right) has firms mass $\lambda_G = 0.5$; returns on adoption of technologies $r_c = r_d = 6$; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1$. Scenario D (bottom-left) has a prevalence of Brown firms, with $\lambda_G = 0.4$; returns on adoption of technologies $r_c = r_d = 3$; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1.5$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1.5$, Scenario E (bottom-center) has an equal mass of Green and Brown firms, with $\lambda_G = 0.5$; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 2$; returns on adoption of technologies $r_c = r_d = 3$. Scenario F (bottom-right) has equal mass of Green and Brown firms, with $\lambda_{GG} = 0.5$; returns on adoption are $r_c = 4$ for clean and $r_d = 3$, for dirty technologies; difference of time invariant benefits for Green firms $\Delta B^G = B_s^G - B_d^G = 1.5$ and for Brown firms $\Delta B^B = B_c^B - B_d^B = 1.5$. In all scenarios $\beta = 5$, $w = 10$ and $x_0 = 0.5$.

equilibrium ($x^* = 0$) and for the upper equilibrium ($x^* = 1$). This scenario resembles closely the classic case of technological lock-in in Arthur (1989). Nevertheless, in a few simulation runs, x_t converges to the middle equilibrium $x^* = 0.5$, where each type selects 'its' preferred technology.

The scenarios reported in the bottom panels of Figure 9 are all characterized by various sorts of asymmetries. In Scenario D, Brown firms are in a majority (60%) while Green firms are in a minority (40%). Despite the indifference barriers being symmetric, and despite an initial condition where there is an equal proportion of both technologies with $x_0 = 0.5$, the lower barrier is easier to reach, and in several cases the dirty technology is selected, while the clean technology is never selected. A similar distribution obtains in Scenario E, this time because of an asymmetry in the values of time invariant benefits: Brown firms derive relatively higher benefits from the dirty technology, compared to the benefits of the Green firms from the clean technology. As a result, the lower barrier is closer to the center of the support of x_t, and easier to reach, and convergence to the lower equilibrium ($x^* = 0$) occurs relatively often, while there is never a convergence to the upper equilibrium ($x^* = 1$). Finally, in Scenario F, the clean technology benefits from a relatively higher rate of return on adoption ($r_c = 4$, compared to $r_d = 3$), which sets the upper barrier closer to the center. Hence, x_t reaches it and converges to the upper equilibrium ($x^* = 1$) in several simulation runs.

5 Economic policy in a uncertain world

Real world policymaking must take account of future uncertainty, including true uncertainty. As noted in the previous sections, the future outcome is inherently uncertain. In particular, in Section 4, we saw that under several parameter configurations, all three cases (all firms adopt clean technology; all firms adopt dirty technology; and a polymorphic equilibrium where both technologies are present in the long run). Yet, the government, must announce at time $t = 0$, the fiscal policy quadruple $(\tau_c, s_c, \tau_d, s_d)$ before any of this information is available. This is a very challenging problem and a traditional analysis based on social welfare functions is unlikely to be suitable; and for this reason, as noted, Stern (2022) and Stern and Stiglitz (2022) propose a guardrail approach.

A continually adjusting economic policy that responds to every twist and turn in the dynamic path is undesirable; it will require a new configuration for $(\tau_c, s_c, \tau_d, s_d)$ at each instant in time. On the other hand, a pragmatic policy that is credibly announced at time $t = 0$ will have to trade-off the policy uncertainty created for the private sector on account of frequent policy changes, versus the flexibility that such a policy offers. The best that a government can do at time $t = 0$ is to base its announced policy on its best ex-ante guess of the long-term expected outcome before any technology adoption decisions have been made. Thus, we assume that the government wishes to make a credible announcement of the choice of the fiscal policy quadruple $(\tau_c, s_c, \tau_d, s_d)$ from the set Γ in (2.9), using as inputs the expected long-run outcomes.

5.1 Expected long-run outcomes

We now conduct the analysis from the vantage point of time $t = 0$, in terms of expectations about long-run outcomes. Given the nature of uncertainty, these expectations may or may not be realized in the long-run (see Sections 3, 4 above).

In each time period, there is an ex-ante probability λ_G that a type G firm will make a technology choice. Hence, ex-ante, we expect that, within the barriers $\overline{B}$ and $\underline{B}$, the long-run number of expected technology adoptions of the clean technology to be $\underset{t\to\infty}{lim}\, t_c = \lambda_G N$, where N is the number of firms. The ex-ante probability that a type B firm is chosen to make a technology choice is $\lambda_B = 1 - \lambda_G$, hence, ex-ante, within the barriers $\overline{B}$ and $\underline{B}$, the long-run expected number of technology choices of the dirty technology is $\underset{t\to\infty}{lim}\, t_d = \lambda_G N$. Recalling that $y = r_c t_c - r_d t_d$, and we measure it along the vertical axis in Figure 1, the ex-ante long-run expected value is

$$\underset{t\to\infty}{lim}\, y = \underset{t\to\infty}{lim}\,(r_c t_c - r_d t_d) = r_c \lambda_G N - r_d \lambda_B N. \tag{5.1}$$

It follows that

$$\underset{t\to\infty}{lim}\, y < 0 \Leftrightarrow \frac{r_c}{r_d} < \frac{\lambda_B}{\lambda_G}. \tag{5.2}$$

If (5.2) holds, then, from an ex-ante perspective, the economy is never expected to hit the upper barrier. Hence, the time $t = 0$ expectations are that the economy will never completely adopt the clean technology at the current levels of taxes and subsidies that are chosen by the government. Furthermore, the condition

$$\underset{t\to\infty}{lim}\, y < \underline{B} \equiv b_d^G - b_c^G \tag{5.3}$$

is sufficient to ensure, from a time $t = 0$ perspective, that in the long-run, all firms in the economy are expected to adopt the dirty technology. If (5.3) holds, then the fiscal instruments currently employed, are 'expected' to be ineffective in persuading firms to adopt the clean technology in the long-run, which might be an important guardrail objective. This might call for a revision of the current long-run public policy and persuade governments to alter the mix of taxes and subsidies.

The conditions that make the perverse outcome in (5.2) 'less likely' are: (1) A higher ratio of clean to dirty technology returns to scale from adoption, $\frac{r_c}{r_d}$, and (2) a lower ratio of type B to type G firms, $\frac{\lambda_B}{\lambda_G}$. These conditions might hold for some types of technologies, but not others.

Consider now the opposite case to the one shown in (5.2), namely

$$\underset{t\to\infty}{lim}\, y > 0 \Leftrightarrow \frac{r_c}{r_d} > \frac{\lambda_B}{\lambda_G}. \tag{5.4}$$

In this case, from a time $t = 0$ perspective, the long-run expected value of the dynamic path is positive, so all type G firms are expected in the long-run to choose the clean technology.

However, even if (5.4) holds, the long-run expected value of y might not be high enough to cross the upper barrier, $\overline{B}$, where all firms adopt the clean technology. In this case the dynamic path is not expected to converge to the barrier $\overline{B}$ (not shown in Figure 1; but see several examples of simulated paths in Section 4). This would be 'expected' to occur when $\underset{t\to\infty}{lim}\, y < \overline{B} \equiv B_d^B - B_c^B$, where $\overline{B}$ is defined in (3.6).

From the perspective of time $t = 0$, a sufficient condition under which all firms are 'expected' to adopt the clean technology in the long-run is

$$\underset{t\to\infty}{lim}\, y \geq \overline{B} \Leftrightarrow \underset{t\to\infty}{lim}\,(r_c t_c - r_d t_d) \geq \overline{B}. \tag{5.5}$$

Using (3.6), (5.1) we get that $\underset{t\to\infty}{lim}\, y \geq \overline{B} \Leftrightarrow$

$$N\left[r_c \lambda_G - r_d \lambda_B\right] \geq B_d^B(\tau_d, s_d) - B_c^B(\tau_c, s_c), \tag{5.6}$$

where

$$B_d^B(\tau_d, s_d) - B_c^B(\tau_c, s_c) = \left[b_d^B(1-\tau_d) - b_c^B(1-\tau_c)\right] - \left[c_b^B(1-s_d) - c_g^B(1-s_c)\right]. \tag{5.7}$$

If condition (5.6) holds then, from the perspective of time $t = 0$, in the long-run all firms are 'expected' to adopt the clean technology, even if ex-post these expectations are not fulfilled.[33] A guardrail approach to policy where the aspiration is to have all firms adopt the clean technology in the long-run must then, at a minimum, require, at time $t = 0$, announcing the fiscal policy quadruple $(\tau_c, s_c, \tau_d, s_d)$ to satisfy (5.6). Ex-post, such a policy might not eventually deliver the expected outcomes because of true uncertainty about the shape of the possible dynamic paths. However, at the time of making the policy announcement, it is the best that any government can do.

5.2 Periodic short-run evaluations

In Subsection 5.1 we considered the 'expected' long run outcome from the perspective of time $t = 0$. This is the only basis on which to base the originally announced policy. However, as noted above, the future is truly unpredictable in our model, and expectations may not turn out to be an accurate guide. In such cases, public policy might have to be periodically evaluated. However, frequent evaluations and policy changes in taxes and subsidies are unlikely to be acceptable because they would create considerable policy uncertainty.

Under true uncertainty, one might not even to be able to imagine all the possible shapes of the dynamic paths in Figure 1. Hence, it is not possible to offer general policy insights. However, occasional path corrections might take the form of public policy revisions if, say, a dynamic path comes within a 'critical' distance d of the lower barrier, $\underline{B}$. Economic theory does not provide firm guidance on the size of d. This must then depend on factors such as the government's financial situation, political economy concerns, political willpower, or just plain gut feelings of the policymakers.

Suppose, for the sake of argument, that there is an agreement within the government which sets a specific size of $d = d^*$. In this case, the obvious intervention, if a dynamic path comes within a distance d^* of the lower barrier $\underline{B}$, is to lower the barrier in order to prevent a situation where all firms will choose the dirty technology. From (3.5), the barrier $\underline{B}$ can be lowered using fiscal policy by either increasing τ_d, s_c (taxes on benefits of dirty technology and subsidies on costs of clean technology) and/or decreasing τ_c, s_d (taxes on benefits from clean technology and subsidies on costs of dirty technology). From (3.6), such a policy will also simultaneously lower the upper barrier, $\overline{B}$.

There is no guarantee that any particular periodic review will ensure that in the future, the dynamic path hits the upper barrier, $\overline{B}$. Starting from any history of play t_c, t_d, at the start of the periodic review, a change in the position of the barriers, arising from the review, restarts the entire dynamic process once again, perhaps from a more favourable position, but the future remains uncertain.

We have already noted in Section 2.4 that there are likely to be bounds on the maximum and minimum levels of the technology-contingent taxes and the subsidies that can be imposed,

[33] Several factors are conducive to the satisfaction of this condition. (1) A relatively greater fraction of green firms relative to brown firms, (2) relatively greater returns to scale from the adoption of the clean technology relative to the dirty technology, (3) lower taxes and higher subsidies on the clean technology, and (4) higher taxes and lower subsidies on the dirty technology.

due to a variety of constraints. Furthermore, the deadweight loss of a tax increases in the square of the tax rate. Hence, the government might need to use a combination of all available fiscal instruments to efficiently achieve feasible movements in the two barriers, $\overline{B}$ and $\underline{B}$, in order to implement desirable outcomes.

5.3 Some policy implications

In this section, we examine some policy implications arising from our analysis. Recall that the fiscal policy quadruple chosen by the government at time $t = 0$ is $(\tau_c, s_c, \tau_d, s_d) \in \Gamma$, where Γ is defined in (2.9) and the bounds on the taxes and subsidies are given in (2.8).

5.3.1 Pigouvian taxes

Recall that the clean technology produces no emissions. A firm that adopts the dirty technology produces emissions with a constant marginal social costs on society equal to $C_s > 0$. In such a situation, economists would advocate that, in addition to the existing fiscal policy, $(\tau_c, s_c, \tau_d, s_d)$, the government must levy Pigouvian taxes at the socially optimal level, $\tau_P = C_s > 0$, on each firm that chooses the dirty technology. In this subsection, we study the implications of such Pigouvian taxes under our model, and take as given the other policy instruments $(\tau_c, s_c, \tau_d, s_d) \in \Gamma$. We assume that the Pigouvian taxes on the dirty technology must also obey the bounds for taxes on dirty technology specified in (2.10).[34]

Since all taxes are determined at time $t = 0$, the government uses its ex-ante expectations in order to choose the tax/subsidy rates consistent with some stated 'guardrail' objective. Suppose that the stated objective is to get all firms to adopt the clean technology in the long-run.

In the presence of Pigouvian taxes, τ_P, on firms adopting the dirty technology, we can rewrite (5.6), which ensures that, from the perspective of time $t = 0$, in the long-run all firms are 'expected' to adopt the clean technology, as follows

$$N\left[r_c\lambda_G - r_d\lambda_B\right] \geq \left[B_d^B(\tau_d, s_d) - \tau_P\right] - B_c^B(\tau_c, s_c), \tag{5.8}$$

where $B_d^B(\tau_d, s_d) - B_c^B(\tau_c, s_c)$ is defined in (5.7). Rewriting (5.8), we get

$$\tau_P \geq \widehat{\tau} = B_d^B(\tau_d, s_d) - B_c^B(\tau_c, s_c) - N\left[r_c\lambda_G - r_d\lambda_B\right]. \tag{5.9}$$

From (5.9) we get the required lower bound on Pigouvian taxes on firms adopting the dirty technology such that the guardrail objective of long-run clean technology adoption is met, in an ex-ante sense, at time $t = 0$. From (2.9) and (2.8) fiscal constraints that require taxes and subsidies to lie within feasible bounds, may lead to a violation of (5.9) if $\widehat{\tau} > \overline{\tau}_d$, where $\overline{\tau}_d$ is the maximum possible tax that can be levied on the dirty technology. A sufficient condition for such a violation is that for the smallest possible value of $\widehat{\tau}$, conditional on the fiscal policy parameters, we still have that $\widehat{\tau} > \overline{\tau}_d$, i.e., if

$$\widehat{\tau} = B_d^B(\overline{\tau}_d, \underline{s}_d) - B_c^B(\underline{\tau}_c, \overline{s}_c) - N\left[r_c\lambda_G - r_d\lambda_B\right] \geq \overline{\tau}_d, \tag{5.10}$$

[34] This is not restrictive. Our analysis goes through if there are other kinds of bounds based on fairness, legal, and implementation concerns that are specific to Pigouvian taxes. The important consideration is that such bounds exists.

where the bounds $\overline{\tau}_d, \underline{s}_d, \underline{\tau}_c, \overline{s}_c$ are defined in (2.10). We now make two main points.

1. The condition in (5.10) is sufficient to imply the impossibility of simultaneously levying optimal Pigouvian taxes, and ensuring long-run adoption of green technological choices in an expected ex-ante sense. A credible government policy at time $t = 0$ must then forego one of the two objectives (either imposing optimal Pigouvian taxes, or long-run adoption of clean technology). Thus, the standard resolution of advocating Pigouvian taxes under true uncertainty needs to be carefully thought of.

2. Suppose that the sufficient condition in (5.10) does not hold, but at the existing feasible fiscal policy, we nevertheless have $\widehat{\tau} > \overline{\tau}_d$. Then, from (5.7) and (5.9), higher taxes and lower subsidies on the dirty technology (higher τ_d, lower s_d), and lower taxes and higher subsidies on the clean technology (lower τ_c, higher s_c), if feasible, will reduce $\widehat{\tau}$ and may ensure that Pigouvian taxes become feasible, i.e., $\widehat{\tau} < \overline{\tau}_d$. This is an example of unexpected complementarities between the different fiscal instruments in ensuring the simultaneous feasibility of the long-run adoption of the clean technology and optimal Pigouvian taxes; these considerations may not arise in the standard analysis. This also illustrates the importance of considering the full set of policy instruments.

5.3.2 Dynamic pattern of subsidies

Since $r_c > 0$ (increasing returns to scale from adoption), after every adoption of the clean technology, subsequent adoptions of the clean technology become less expensive. In the real world, initial adoptions of the clean technology often require subsidies to kickstart adoption (e.g., in buying clean cars, or installing domestic solar panels, or installing pollution reduction equipment in firms). Yet subsidies are expensive, create distortions elsewhere, and their precise levels may be politically controversial. However, as clean technology adoptions increase, its reduced costs, on account of $r_c > 0$, can be used to leverage a fall in subsidies, and ultimately their complete withdrawal.

Suppose that at time $t = 0$, the government credibly announces a time path of subsidies contingent on the subsequent technology adoption decisions of the firms. Suppose that the explicitly declared 'ex-ante' guardrail objectives of the government (Stern and Stiglitz, 2022) are that (i) it aims for all firms to eventually adopt the clean technology, and that (ii) the temporal costs of the subsidy will be minimized. For pedagogical simplicity, we assume that the only fiscal instrument used is the subsidy on the adoption of the clean technology, $s_c > 0$, but the remaining instruments are set equal to zero, $\tau_d = \tau_c = s_d = 0$. The insights below are easily extended to the case $\tau_d, \tau_c, s_d \geq 0$.

The reader should imagine that the barriers $\overline{B}, \underline{B}$ in Figure 1 are now drawn for this special case. Consider dynamic paths that lie in between the two barriers $\overline{B}$ and $\underline{B}$. Within these barriers, type B firms always choose the dirty technology and type G firms always choose the clean technology. Thus, we can rewrite (3.2), the condition necessary for a type G firm to adopt the clean technology as

$$\left[b_d^G - b_c^G\right] - \left[c_d^G - c_c^G(1 - s_c)\right] \leq r_c t_c - r_d t_d. \tag{5.11}$$

Since subsidies are expensive, and the objective is to minimize the subsidy costs, conditional on t_c adoptions of the clean technology in the past, s_c solves (5.11) at equality, thus

$$s_c(t_c) = \frac{1}{c_c^G}\left[(r_d t_d - r_c t_c) + \left((b_d^G - b_c^G) - (c_d^G - c_c^G)\right)\right]. \tag{5.12}$$

We have the following results.

1. The dynamic path for subsidies, so long as the system stays within the barriers $\overline{B}, \underline{B}$ in Figure 1, is given by (5.12), as the number of adoptions of the clean technology $t_c = 0, 1, 2, ...$ Every time a type G firm adopts the clean technology, from (5.12), the government can reduce the subsidy by an amount Δs_c such that

$$\Delta s_c = -\frac{r_c}{c_c^G}, \tag{5.13}$$

   and still ensure that subsequent choices by type G firms will be to adopt the clean technology within the barriers $\overline{B}, \underline{B}$. Conditional on t_c adoptions having taken place, the level of subsidies in (5.12) is lower if (i) the relative benefits of the clean technology are higher but its relative costs are lower, and (ii) the dirty technology has low returns to scale from adoption (low r_d) while the returns to scale from the adoption of the clean technology, r_c, are high.

2. A decreasing profile of subsidies, consistent with (5.12), will ensure that whenever a type G firm is offered a technology choice, it will adopt the clean technology (because (5.11) holds). This contributes to upward movements of the dynamic path in Figure 1 within the barriers $\overline{B}, \underline{B}$. If the upper barrier, $\overline{B}$, is hit then both types of firms will find it in their self interest to adopt the clean technology (recall that $\overline{B}$ is an absorbing barrier that locks-in all firms into the clean technology). The subsidy can be gradually entirely removed with increased adoptions, so eventually $s_c = 0$.[35]

5.3.3 International fund for the adoption of green technologies

The level of subsidies required in (5.12) may be difficult to implement for poor countries due to severe resource constraints, such as poor tax collections and low incomes, as well as the presence of political/institutional constraints. Suppose that the maximum subsidies on the clean technology that a poor country can offer equal $0 \leq \overline{s}_c < 1$. A policymaker in such a country who can calculate the required dynamic path for subsidies in (5.12) might find that $s_c(0) > \overline{s}_c$, so that subsidies to support the very first adoption of the clean technology might not be feasible. Suppose that, in some time period, t_d adoptions of the dirty technology have taken place but no clean technology adoptions have occurred yet. Then, the problem can be illustrated by setting $t_c = 0$ in (5.12) to get:

$$s_c(0) = \frac{1}{c_c^G}\left[r_d t_d + \left((b_d^G - b_c^G) - (c_d^G - c_c^G)\right)\right] > \overline{s}_c. \tag{5.14}$$

[35] Setting $s_c = 0$ in (5.12) and solving out for t_c gives the number of technology adoptions of the clean technology after which the subsidy can be entirely removed.

If (5.14) holds, then clean technology adoption never takes off in a resource-poor country, in the absence of external intervention equal to the gap $s_c(0) - \bar{s}_c$. From (5.14), this gap is increasing in the number of existing dirty technology adoptions, t_d, because an increase in t_d increases the relative benefits from adopting the dirty technology for subsequent adopters. Hence, the longer is such external intervention delayed, the larger is the deficit $s_c(0) - \bar{s}_c$ that needs to be addressed in order to induce the first clean technology adoption in the poor country.

Since several forms of environmental bads are global public bads, there might be no other option but to form an international fund, financed by the richer countries, to subsidize the worldwide adoption of green technologies. Such subsidies may also take the form of direct supplies of the clean technology at affordable terms to poorer countries.

6 Extensions of the model

This section covers several different extensions to the basic model. We extend the model to stochastic technology dynamics (Subsection 6.1); to the case of simultaneous technology adoptions and more than two kinds of technologies (Subsection 6.2); and to a brief consideration of other types of dynamics (Subsection 6.3).

6.1 Stochastic technological dynamics

In this subsection, we use the insights from the 'stochastic social dynamics' framework surveyed in Young (1998, 2006) and Dhami (2020, Vol. 6), and apply it to our model of technology adoption.

Suppose that any firm that is faced with an opportunity to choose a technology makes its technology choice in the following manner. With a high probability $\varepsilon > 0$, the firm chooses the optimal technology (the one that maximizes net benefits, as in Sections 3.1, 3.2), but with the low complementary probability, $1 - \epsilon$, the firm simply randomizes equally between the two technologies. This is a form of "experimentation" that can be formalized in several alternative ways in the theoretical model and can be justified by using different behavioral features (Dhami, 2020, Vol. 6).

Hence, in effect, firms pick their optimal technology with a probability $\varepsilon + \frac{1-\varepsilon}{2} = \frac{1+\varepsilon}{2}$ and the non-optimal technology with a probability $\frac{1-\varepsilon}{2}$. Defining $\rho = \frac{1+\varepsilon}{2}$, it follows that each firm chooses its optimal technology with a high probability ρ and the non-optimal choice with the low complementary probability $1 - \rho$. We call $1 - \rho$ as the probability of experimentation, and the sense in which it is 'low' is specified in the following condition.

$$\frac{\rho}{(1-\rho)} > \frac{\lambda_B}{\lambda_G}. \tag{6.1}$$

The inequality in (6.1) is always satisfied if $1 - \rho$ is low enough.

As shown above in Section 3, in the absence of experimentation ($\rho = 1$), there are at least two asymptotically stable states – all firms choose the clean technology (state $S = c$) and all firms choose the dirty technology (state $S = d$). In the presence of experimentation ($\rho < 1$), the key equilibrium concept is that of a *stochastically stable state*; it is the state whose basin of attraction is most difficult to escape from in the presence of persistent randomness arising from experimentation. We now consider this situation below.

Essentially, we have a Markov process in which a firm only needs knowledge of the history in the previous period to make its technology adoption decision. A history here is simply a pair of numbers (t_c, t_d) that specify the number of firms that have adopted the clean and the dirty technologies in the past. When the dynamic system is able to move between the basins of attractions of more than one asymptotically stable equilibria ($S = c$ and $S = c$) that exist in the absence of experimentation ($\rho = 1$), then we have an irreducible Markov process.[36] Our model can be formally recast in these terms. But this is a standard, well known, framework, that does not add new insights to our discussion; for a formal exposition, see Young (1998) and Dhami (2020, Vol. 6). However, our slightly more informal and accessible approach uses the same methods to find the stochastically stable states, and is presented here below.

Suppose that the dynamic path lies strictly within the two barriers, $\overline{B}$ and $\underline{B}$, in Figure 1, where a firm of type G finds it optimal to choose the clean technology (see (3.1)), and a firm of type B finds it optimal to choose the dirty technology (see (3.3)). It follows that, in the presence of experimentation, (i) type G firms choose the clean technology with a probability ρ and the dirty technology with a probability $1 - \rho$, (ii) type B firms choose the dirty technology with a probability ρ and the clean technology with a probability $1 - \rho$.

Recall that the vertical axis in Figure 1 measures the state variable $y = r_c t_c - r_d t_d$ and nature chooses a type G firm and a type B firm to make a technology choice with respective probabilities λ_G and λ_B. Thus, before the time t technology choice is made, the expected outcome depends on the type of the firms that makes a technology choice; we consider the two possibilities below. The relevant situation is described in Figure 10.

1. A type G firm is chosen to make the time t technology choice: At the beginning of time t, there is a probability λ_G that a type G firm is chosen and a probability ρ that it makes its optimal choice of a clean technology. Thus, there is an ex-ante probability $\rho\lambda_G$ that the dynamic path (whose value is given by the vertical axis $y = r_c t_c - r_d t_d$ in Figure 10) will move up by r_c; and a probability $(1 - \rho)\lambda_G$ that it will move down by r_d (when the type G firm chooses the non-optimal dirty technology).

2. A type B firm is chosen to make the time t technology choice: At the beginning of time t, there is a probability λ_B that a type B firm is chosen and a probability ρ that it makes its optimal choice of a dirty technology. Thus, there is an ex-ante probability $\rho\lambda_B$ that the dynamic path (whose value is given by the vertical axis $y = r_c t_c - r_d t_d$ in Figure 10) will move down by r_d; and a probability $(1 - \rho)\lambda_B$ that it will move up by r_c (when the type B firm chooses the non-optimal clean technology).

At the beginning of time $t = 1$, one cannot predict which of the two barriers $\overline{B}$ or $\underline{B}$ will be eventually reached. In the absence of any experimentation by the players (i.e., $\rho = 1$, as in Section 3), each of the barriers is an absorbing barrier. However, in the presence of

[36] In the absence of experimentation, Column 4 of Table 2 lists the asymptotically stable states for the various cases, A–F, considered in our numerical exercise.

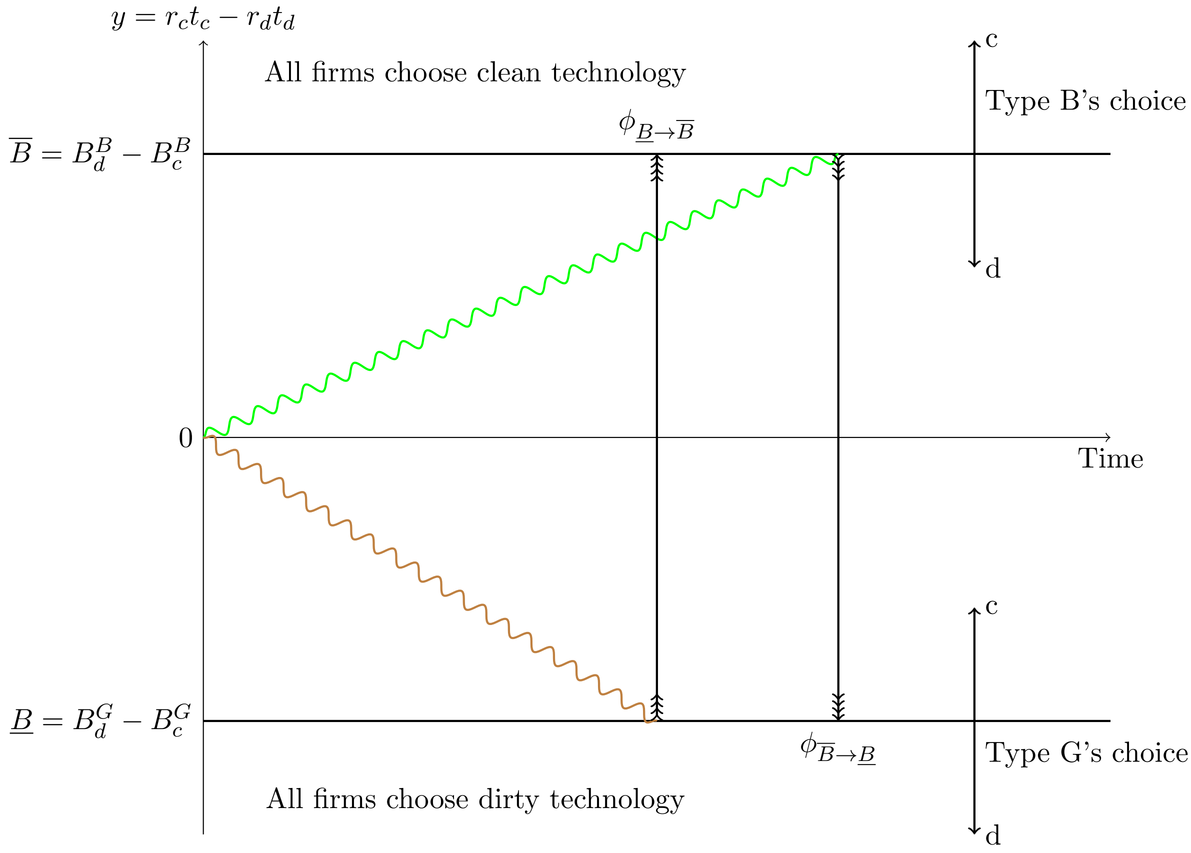

Figure 10: The dynamics of technology adoption in the presence of experimentation.

experimentation, $0 < \rho < 1$, once a dynamic path reaches any of the barriers, it is not guaranteed to stay there forever. Starting at a point on or within any of the two barriers (above and on $\overline{B}$; or below and on $\underline{B}$), experimentation by the two types of firms may push the dynamic path back in between the two barriers $\overline{B}$ and $\underline{B}$. Once a dynamic path reaches between the two barriers, there is no guarantee that it will not eventually reach the opposite barrier. Thus, we may get *punctuated equilibria*, in which the dynamic path escapes the basin of attraction of one of the two states ($S = c$ and $S = d$) and moves into the basin of attraction of the other state.

We now formally check for stochastic stability. The relevant situation is outlined in Figure 10.

Consider a dynamic path that has just hit the upper barrier $\overline{B}$, which corresponds to the state $S = c$. We now ask, starting at a point on the barrier $\overline{B}$, what is the minimum number of successive adoptions of the dirty technology that will allow the dynamic path to escape the basin of attraction of the state $S = c$ and arrive at the lower barrier $\underline{B}$, which corresponds to the state $S = d$?

The vertical distance between the two barriers is given by $\overline{B} - \underline{B}$ where $\underline{B}, \overline{B}$ are 'constants' that are defined in (3.5), (3.6). Hence, the shortest distance that any dynamic path starting from $\overline{B}$ must travel to reach $\underline{B}$ is the vertical distance, equal to $\overline{B} - \underline{B}$. We now calculate the total expected number of technology adoptions that will allow a *path of least resistance* (described below) to travel the distance $\overline{B} - \underline{B}$. The relevant path is shown in Figure 10 with arrows pointing downwards.

The vertical axis of Figure 10 measures $y = r_c t_c - r_d t_d$. To move downward from a given point on $\overline{B}$, conditional on the history of technology adoptions, t_c, t_d, one of the two types of firms must first choose a non-optimal action and pick the dirty technology on account of experimentation.[37]

Starting from the upper barrier $\overline{B}$, once the dynamic path enters any point within the barriers, $\overline{B}, \underline{B}$, there are two possibilities.

(i) With probability λ_G, nature chooses a type G firm to make a technology choice. A type G firm will choose the non-optimal dirty technology with a probability $1-\rho$. Every time a type G firm chooses the dirty technology, the term $r_c t_c - r_d t_d$ moves down by r_d. Thus, ex-ante, the value on the vertical axis, $r_c t_c - r_d t_d$, moves down by the following expected value

$$\lambda_G(1-\rho)r_d. \tag{6.2}$$

(ii) With probability λ_B, nature chooses a type B firm, which picks its optimal dirty technology with probability ρ. In this case, the value on the vertical axis, $r_c t_c - r_d t_d$, moves down by the following expected value

$$\lambda_B \rho r_d. \tag{6.3}$$

In order to find the *path of least resistance* from a point on the barrier $\overline{B}$ to the nearest point on the barrier $\underline{B}$, we first need to determine which of the two expressions in (6.2), (6.3) is bigger. From (2.1), brown firms are relatively more numerous, $\lambda_B > \lambda_G$; the probability of experimentation is relatively small, so $\rho > (1-\rho)$; and since $r_d > 0$, we have that

$$\lambda_B \rho r_d > \lambda_G(1-\rho)r_d. \tag{6.4}$$

Thus, the path of least resistance (or the ex-ante fastest route), starting from the barrier $\overline{B}$, will require k successive (optimal) technology choices by type B firms of the dirty technology to arrive at the barrier $\underline{B}$. We can find the value of k by solving out for $k\lambda_B\rho r_d = \overline{B} - \underline{B}$, so

$$k = \frac{\overline{B} - \underline{B}}{\lambda_B \rho r_d}. \tag{6.5}$$

The number k is known as the 'stochastic potential' of the path of least resistance from $\overline{B}$ to $\underline{B}$ and we denote it by $\phi_{\overline{B}\to\underline{B}}$. The path of least resistance is shown in Figure 10 as the vertical line starting from the barrier $\overline{B}$ and extending to the barrier $\underline{B}$ with downward pointing arrows; for ease of reference, we terminate this line with the stochastic potential $\phi_{\overline{B}\to\underline{B}}$.

Next, we wish to calculate the analogous stochastic potential, $\phi_{\underline{B}\to\overline{B}}$, of the path of least resistance from the all-dirty technology state ($S = d$) to the all-clean technology state ($S = c$). This path of least resistance is shown in Figure 10 as the vertical line starting from the barrier $\underline{B}$ and extending to the barrier $\overline{B}$ with upward pointing arrows; for ease of reference we terminate this line with the corresponding stochastic potential $\phi_{\underline{B}\to\overline{B}}$ that we determine below.

Suppose that we begin at the lower barrier $\underline{B}$, conditional on some history of the game, t_c, t_d. We now wish to find the least number of technology adoption choices of the clean technology

[37] Recall also that on or above the barrier $\overline{B}$, both firms find it optimal to choose the clean technology. We ignore this single initial move, on account of experimentation, in calculating the 'stochastic potential' (see below) in moving between the two barriers in both directions. So this single move cancels out in both directions.

that will move a dynamic path vertically upwards to the nearest point on the barrier, $\overline{B}$. This follows a single initial non-optimal choice by any of the two types of firms (on account of experimentation) which induces them to choose the clean technology.[38]

There are two possible ways in which the clean technology may be adopted in between the barriers $\underline{B}$ and $\overline{B}$.

(i) A type G firm will choose its optimal clean technology within the barriers $\overline{B}, \underline{B}$ with a probability ρ whenever it is making a technology adoption decision, an event that occurs with probability λ_G. Hence, every time a type G firm chooses the clean technology, the term $r_c t_c - r_d t_d$ moves up by r_c. Thus, ex-ante, before the technology adoption decision, the value, $r_c t_c - r_d t_d$, on the vertical axis of Figure 10, moves up by the following expected value

$$\lambda_G \rho r_c. \tag{6.6}$$

(ii) With probability λ_B, a type B firms makes a technology adoption decision. It chooses its non-optimal clean technology with a probability $(1-\rho)$ on account of experimentation. Thus, within the barriers $\overline{B}, \underline{B}$, the value on the vertical axis, $r_c t_c - r_d t_d$, moves up by the following expected value

$$\lambda_B (1-\rho) r_c. \tag{6.7}$$

Using (6.1), we have $\lambda_G \rho r_c > \lambda_B (1-\rho) r_c$. Hence, in order to calculate the path of least resistance from the barrier $\underline{B}$ to the barrier $\overline{B}$, we ask the following question. Starting from the barrier $\underline{B}$, how many successive technology adoptions, $\widehat{k}$, of the clean technology by type G firms do we need in order to reach the nearest point on the barrier, $\overline{B}$. Each of these technology adoptions moves the dynamic path upwards by an, ex-ante, expected value $\lambda_G \rho r_c$. Thus, in order to determine $\widehat{k}$, we need to solve $\widehat{k} \lambda_G \rho r_c = \overline{B} - \underline{B}$, so

$$\widehat{k} = \frac{\overline{B} - \underline{B}}{\lambda_G \rho r_c}. \tag{6.8}$$

It follows that the relevant stochastic potential of the path of least resistance is $\phi_{\underline{B} \to \overline{B}} = \widehat{k}$. Summarizing our discussion above

$$\phi_{\overline{B} \to \underline{B}} = k; \; \phi_{\underline{B} \to \overline{B}} = \widehat{k},$$

where k is defined in (6.5) and $\widehat{k}$ is defined in (6.8).

The state $S = c$ is the stochastically stable state if $\phi_{\overline{B} \to \underline{B}} > \phi_{\underline{B} \to \overline{B}}$, because (along the path of least resistance) it requires a relatively greater number of successive mutations in order for the dynamic path to escape from its basin of attraction. Conversely, if $\phi_{\overline{B} \to \underline{B}} < \phi_{\underline{B} \to \overline{B}}$, then the state $S = d$ is the stochastically stable state. We have that

$$\phi_{\overline{B} \to \underline{B}} \gtreqless \phi_{\underline{B} \to \overline{B}} \Leftrightarrow \frac{r_c}{r_d} \gtreqless \frac{\lambda_B}{\lambda_G}. \tag{6.9}$$

From (6.9), the state $S = c$ is more likely to be the stochastically stable state if $\frac{r_c}{r_d} > \frac{\lambda_B}{\lambda_G}$, i.e., if the returns to scale of the clean technology are relatively high and the proportion of type

[38] As noted above, we ignore this single initial move in our calculations in both directions.

B firms to type G firms is not too high. This might be true for some technologies/countries but not others. Furthermore, any government policy that potentially increases r_c relative to r_d is helpful, for instance, greater information dissemination about the benefits of the clean technology through, say, government sponsored technology fairs; nudges; and the provision of requisite public infrastructure and assistance.[39]

In summary, we have shown that in the presence of stochastic technology dynamics, we have *punctuated equilibria*. The equilibrium switches between the clean and the dirty technologies. We have also derived the conditions for the all-clean technology adoption state to be the stochastically stable state so that the dynamic system spends most of its time in the state $S = c$. From (6.9), the stochastically stable state does not depend on the fiscal parameters or on the relative benefits and costs of the technologies to the firm, unless the fiscal system can somehow directly target the returns to scale, r_c, r_d.

6.2 Simultaneous technology choice and several technologies

In our model, outlined in Section 2, in each time period, t, a single randomly chosen firm makes a technology adoption decision between the clean and the dirty technologies, conditional on the history of technology adoptions t_c and t_d. Our model is readily extended to the case of $n = 2, 3..., m$ firms simultaneously making a technology choice decision at any time t, conditional on the history of technology adoptions t_c and t_d.

Concurrent technology adoptions by several firms do not influence the time-dependent variable returns, $r_c t_c$, $r_d t_d$, from technology adoption. Such returns only depend on the history of past adoptions. Thus, any firm making the technology adoption decision continues to face the same payoff matrix as given in Table 1, irrespective of the number of other firms concurrently making the technology adoption decision. Hence, each firm makes the same decision as in Section 2 (type G firms choose according to (3.1), and type B firms choose according to (3.3)). However, ex-ante, a fraction λ_G of the n firms are expected to be type G and if (3.1) holds, they will all choose the clean technology, otherwise they choose the dirty technology. Similarly, ex-ante, a fraction λ_B of the n firms are expected to be type B and if (3.3) holds, they will all choose the clean technology, otherwise they choose the dirty technology.

This extension only makes an "accounting difference" to the way the dynamic path moves up or down in Figure 1. Suppose that we are in between the two barriers $\overline{B}$ and $\underline{B}$ in Figure 1. At the end of any time period t, we update the history of technology adoptions t_c and t_d by taking account of the decisions of all n firms. Suppose that in time period t, we observe that k firms have chosen the clean technology and $n - k$ firms have chosen the dirty technology in the past. Then, in the next time period, we update t_c to $t_c + k$ and t_d to $t_d + (n - k)$. Otherwise, the model is unchanged.

The model is easily extended to more than 2 technologies. This requires us to check more

[39] Referee 1 notes the similarity of the condition in (6.9) with the condition in (5.4) which is $\underset{t\to\infty}{lim}\, y > 0 \Leftrightarrow \frac{r_c}{r_d} > \frac{\lambda_B}{\lambda_G}$. However, the statements on the LHS of the two conditions are different. In particular, the condition in (5.4) ensures that, from a time $t = 0$ perspective, the long-run expected value of the dynamic path is positive. While it is interesting that one side of the implication in these two conditions is identical, they are derived from very different underlying considerations.

inequalities of the form (3.1) and (3.3) and the dynamic analysis in Section 4 will require solving a system of equations, but the basic analysis and insights are unchanged.[40]

6.3 Other dynamics

Evolutionary dynamics require a well-specified strategic game to be played in each time period $t = 1, 2, ...$ One can then apply the relevant equilibrium concepts (e.g., an evolutionary stable equilibrium) and study the properties of the appropriate dynamics of the system (e.g. replicator dynamics). However, we do not have such a strategic game being played. As in Arthur (1989), we have a sequence of technology choices being made in each time period, conditional on the history of technology adoptions so far. Hence, this is more akin to a game against nature, and there are no relevant evolutionary dynamics to consider.

7 Conclusions

We present a simple model of technology choice by heterogeneous firms where clean technologies have relatively higher returns to scale compared to dirty technologies and there are type-contingent differences in the suitability of the technologies. Our approach closely follows the recent suggestions of Stern (2022) and Stern and Stiglitz (2022) in modeling key issues of climate change and the nature of public policy. We show that our simple model nests a large number of important cases and is robust to reasonable extensions of the model. We illustrate the extreme unpredictability of the final outcome, outline plausible dynamics, and consider the role of public policy in the form of taxes and subsidies in influencing the long-run expected outcome. Our model is fairly parsimonious, yet it has the ability to provide reasonably powerful insights into the nature of clean technology adoption and the design of public policy. It also highlights the challenges and limitations of public policy in such scenarios.

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[40] For instance, suppose that we had three technologies. In terms of Figure 1, the relevant change is that we have a three dimensional box, the relevant barriers are the boundaries of the box, and dynamic paths are now in three dimensional space.

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