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The Distributional Implications of Pension Benefit Indexation

Abstract

Socio-economic differences in longevity have fuelled a debate whether pension systems have a regressive bias favouring groups with a high life expectancy. We show that the distributional implications of such pooling depend critically on the benefit profile across age/time, which in turn is determined by how benefits are indexed to prices and wages. Choosing indexation scheme involves a choice between a low initial benefit with an increasing profile and a high initial benefit with a flat/decreasing profile, where the former benefits groups with a high life expectancy, and vice versa. We analyse how indexation affects the trade-off between insurance and distribution when groups with different mortality are separated or pooled, and the optimal benefit profile under both standard preferences and temporal risk aversion wrt. the length of life.

JEL-Codes: D140, G220, H550, J180.

Keywords: annuities, differential mortality, distribution, indexation.

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1 Introduction

A seminal result in the pension literature holds that all lifetime savings - absent bequest motives - should be in annuities, see Yaari (1965) and the generalisation in Davidoff et al. (2005). Annuities diversify individual survival risk and removes it from the household consumption smoothing problem. Recent debates have pointed out that these attractive insurance features come at the cost of a regressive bias since groups with short longevity tend to support groups with long longevity, see e.g. Coronado et al. (2011), Auerbach et al. (2017), and Bagchi (2019).

This criticism is based on two observations. First, real-life annuities - both implicit annuities in tax financed public pension schemes and explicit in mandated funded occupational schemes - typically pool mortality risks across different groups. Second, there is a socio-economic gradient in mortality rates, implying that longevity is positively correlated with education and income, see e.g. van Raalte et al. (2014), Cairns et al. (2019), and Bohácek et al. (2021). Annuities based on mortality rates averaged across groups provide insurance for all[1], but the terms are better for groups with a high rather than a low life expectancy.

This seems to leave the dilemma that diversification of mortality risks comes at the cost of a regressive bias. We argue that while this dilemma is inevitable, the extent of this redistributive bias depends critically on the benefit profile of the annuity. The debate and most theoretical work assume that the annuity offers a flat (real) benefit profile. However, annuities based on actuarial principles can support benefit profiles that are constant, declining, or increasing with time and thus age. The benefit profile may thus be low initially but increase with age, or initially high and declining with age. Clearly, the latter shifts consumption towards the "younger" old, benefitting groups with a low life expectancy, and oppositely in the former case. Moreover, age dependencies can easily be implemented via wage or price indexation, having different implications for the benefit profile over time and thus age.

The contribution of this paper is both to understand the distributional implications of the benefit profile, especially when different risk classes are pooled in the same scheme, and to analyse the optimal benefit profile (indexation) when preferences imply risk neutrality (standard preferences) or risk aversion (temporal risk aversion) wrt the length of life.

In actual pension schemes, the benefit profile is determined by indexation to prices and/or wages, but the implications of the specific indexation scheme have not attracted much attention in the literature. This is surprising since indexation of benefits is an important design issue and the fact that there is wide variation in indexation across countries, see e.g. Piggott and Sane (2009), Whitehouse (2009), Checherita-Westphal (2022), and OECD (2021). Some countries index pensions to prices[2] (e.g. Belgium), others to a mix of prices and wages

[1] The return on the annuity is still higher than the return on standard financial products, see Davidoff et al. (2005).

[2] In the Swedish notional defined contribution scheme (in which the implicit return is the

(e.g. Finland with 20% weight to wage inflation and 80% to price inflation), or to wages (e.g. Denmark)[3]. How to index pension benefits can be assessed from two angles. One perspective is that the real value of pensions and hence consumption possibilities are ensured by price indexation, and another that wage indexation maintains the relation between pensions and the income of the active population and thereby prevents (relative) poverty among pensioners (usually defined in terms of incomes relative to average incomes, see e.g. OECD (2022)).

In interpreting the redistribution implied by risk pooling of mortality risk, it is important to ask why there is pooling in the first place. Bernheim (1987) points out that assessing the value of a benefit stream by its actuarial present value using group-specific survival rates is problematic since it implicitly assumes access to a perfect annuity market offering risk-class-specific fair rates of return. If the market does not offer such options, this approach does not take into account the value of the insurance provided by mitigating market failures. Second, it is well known that private information gives rise to adverse selection problems which cause either non-existence of equilibrium or inefficient outcomes, see Rotchschild and Stiglitz (1976) and Mimra and Wambach (2014). This is explicitly considered in Eckstein et al. (1985) in the context of life-annuities in a setting with two risk classes (high and low survival rate) where an adverse selection problem arises since the high survival group is attracted by the higher return on the annuity offered the low-survival group. Eckstein et al. (1985) show that a pooling equilibrium does not exist, but a separating equilibrium exists if the low-survival group is not too large. The separating equilibrium offers the high-survival group the first best contract, but the low-survival group is constrained in the amount of annuities they can buy although the return is actuarially fair. This captures the well-known screening paradox arising under private information that the high-risk group creating the incentive problem exerts a negative effect on the low-risk group (here: a low survival probability), though they are still better off than in the absence of annuities. Eckstein et al. (1985) show that a mandated pooling contract makes both groups better off unambiguously if no equilibrium exists, but it may also be the case when a separating equilibrium exists. Brugiavini (1993) and Andersen (2023) show that households being uncertain about their mortality risk prefer as young to acquire pooled annuity contracts.

Turning to the normative issue, it is well established in the literature that a utilitarian planner maximising expected lifetime utility across a population with different survival probabilities redistributes - other things being equal - from individuals with low life expectancy to individuals with high life expectancy, see e.g. Sheshinski (2008), Simonovits (2006), Cremer et al. (2010), Bommier et

growth in the wage sum), benefits are determined on actuarial terms. The discount rate is chosen such that initial benefits are higher, and hence the growth is lower than the implicit return of the system (growth of the wage sum). This effectively ensures that the real value of the benefit is constant throughout the retirement period, see Pensionsmyndigheten (2021).

[3] Portugal even has a progressive indexation scheme where indexation is declining (in steps) in the level of pensions, see https://eportugal.gov.pt/en/noticias/inflacao-aumento-de-pensoes-e-indexante-dos-apoios-sociais-atualizado-em-2023

al. (2011a,b), Leroux and Ponthiere (2013) and Pestieau and Ponthiere (2016). The intuition is that a utilitarian planner redistributes according to marginal utilities of consumption. A longer life reduces, other things being equal, the consumption flow, which in turn increases the marginal utility of consumption, and this determines the direction of redistribution. This finding has been criticised for relying on standard preferences implying risk neutrality with respect to the length of life, see Bommier (2006). Risk aversion wrt the length of life (temporal risk aversion) can be captured by a social welfare function which is an increasing concave function of individual lifetime utilities[4], see Bommier (2006). Bommier et al. (2011b) consider the implications for pension profiles and find that long-lived individuals should have lower instantaneous consumption than short-lived agents. A number of papers have analysed how pension systems can be designed so as to mitigate the regressive bias[5][6] including the determination of retirement ages, see Bommier et al. (2011a) and Simonovits (2006). The implementation problem arises when the policy maker is unable to observe risk class (the same problem as faced by insurance companies). Some indirect targeting may be accomplished by age-dependent rules, but they easily get complicated. We show that the indexation of benefits introduces such a contingency in the benefit profile and that the optimal benefit profile under a wide set of assumptions can be implemented by the wage/price indexation of prices. For standard preferences the initial benefit level depends on the distribution of longevity, but the benefit profile is determined by the optimal consumption profile known from standard consumption models. When risk classes are pooled, the preferred profile depends on the distribution of longevity, and the potential redistribution across risk classes implies that the risk classes have different views on the optimal profile. Risk aversion wrt the length of life is shown to make the optimal benefit profile depending on the distribution of longevity and generally makes profile flatter or even declining with age. Higher longevity risk (standard deviation also makes the benefit profile flatter and possibly declining.

The paper is organized as follows: The first part presents a positive analysis of how the benefit profile for annuities over time/age influences risk diversification and the ex post redistribution (comparing pooling and separation of risk classes). It is shown how price and wage indexation affect the benefit profile, and therefore can be designed to achieve a wanted time/age profile. The second part turns to the normative issue of the optimal benefit profile in the case of risk neutrality (standard preferences) and risk aversion (temporal risk aversion) wrt the length of life. A concluding section discusses some policy implications.

[4] This can also capture ex post preferences over the distribution of lifetime utility, see Simonovits (2006).

[5] This can also be motivated by a political aversion to multiperiod inequality, see Simonovits (2006) and Bommier et al. (2011a).

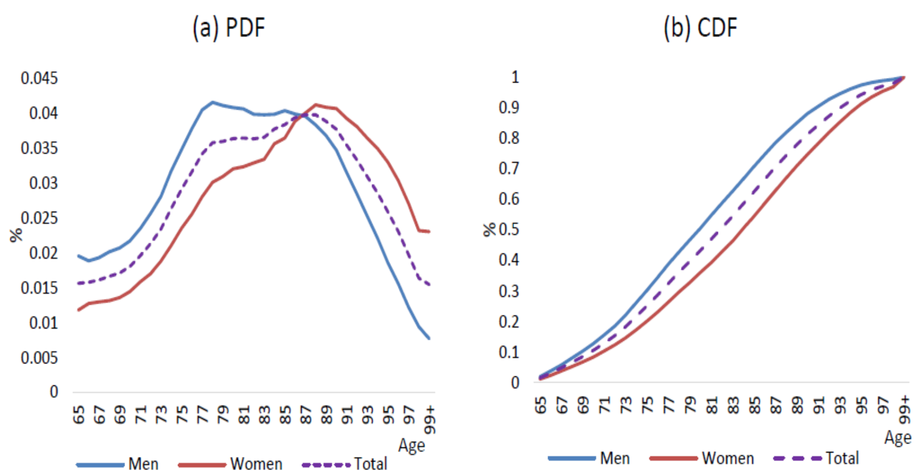
[6] Introducing risk aversion wrt life-length has ambiguous implications for savings, see Pestieau and Ponthiere (2016). They argue, however, that smaller savings are the most likely scenario if agents are highly risk averse - i.e. if the utility function is strongly concave.

2 Annuities, insurance and distribution

The following analyses the basic implications of the benefit structure of annuities for a population with a given density over longevity at retirement (no aggregate longevity risk). We exploit the analytical tractability achieved by specifying mortalities as distributions over longevity rather than in terms of age-conditional survival probabilities (which imply a distribution over longevity). Under the law of large numbers, the density distribution over longevity seen from an individual perspective is the frequency distribution for the relevant population covered by the annuity. The basic mechanisms explored here apply to both DC funded schemes and PAYG schemes where benefits are determined on (quasi) actuarial terms[7].

We use the classic gender difference in longevity as an example to illustrate the results since this gap is well known. However, the results of the paper are not specifically applying to the gender issue, but hold generally in case of pooling across risk classes (e.g., educational groups)[8]. Figure 1 gives for Denmark the density functions at age 65 for men, women, and the total population. The life expectancy for women is larger than for men, but for both groups there is significant dispersion in longevity, implying that some men reach high ages, and some women pass away early. The dispersion in mortality is important when discussing distributional issues. For the particular case depicted in Figure 1, the CDF for women first order stochastically dominates that for men. We consider the density function ($f(\cdot)$) for the total population in the pooling case, and in the separating cases we apply the densities for the two sexes (men: $h(\cdot)$, women: $g(\cdot)$).

Figure 1: Density function - longevity at age 65



Note: Density functions have been smoothed.

[7]Many PAYG semi/NDC schemes have a benefit structure adjustment on (semi) actuarial terms, see OECD (2021) for an overview.

[8]Finkelstein et al. (2009) analyse the consequence of a ban on gender-contingent annuities.

Source: Statistics Denmark.

2.1 Annuities and benefit profiles

Consider a continuous time setting where a given population faces a mortality risk captured by the probability density function $f(L) \geq 0$ defined over longevity $L \in [\underline{L}, \overline{L}]$, $0 \leq \underline{L} \leq \overline{L}$. We interpret this as remaining lifetime at retirement. The density function is common knowledge. Longevity risk is hedged by annuities, either explicitly in e.g. a defined contribution schemes or implicitly in a tax financed public pension schemes. In the following, we interpret the setting as applying to a scheme where a given wealth at retirement is transformed to an annuity; that is, the expected present value of the benefit profile offered by the annuity equals retirement wealth.

Let the benefit $b(a)$ offered by the annuity at a given age a (time) be

$$b(a), \text{ where } b(a) > 0,\ b_a(a) \lesseqgtr 0 \text{ for } \underline{L} \leq a \leq \overline{L}$$

The value of the liability held by the pension fund having committed an annuity with benefits $b(a)$ to members living until age L is

$$A(L) = \int_0^L e^{-ra} b(a) da$$

where r is the relevant discount rate assumed constant and time independent for simplicity. Assuming that the discount rate is the same for the pension fund and individuals, $A(L)$ is also the present value to the individual of the pension benefits received if living until age L. The total liability value of the annuity (A) to the insurance company is thus

$$A = \int_{\underline{L}}^{\overline{L}} A(L) f(L) dL = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a) da \right] f(L) dL$$

Note that expected longevity is $\int_{\underline{L}}^{\overline{L}} f(L)dL$, and hence A can be interpreted as a weighted expected longevity measure.

Financial balance or actuarially fair benefits require

$$W = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a) da \right] f(L) dL \tag{1}$$

where W is the pension wealth at retirement, and (1) determines the set of admissible benefit profiles $\{b(a)\}_{\underline{L}}^{\overline{L}}$. Without loss of generality, the following considers the annuity which 1 unit of pension wealth acquires, $W = 1$.

The risk diversification offered by the annuity essentially works by transferring resources from those deceasing early to those deceasing late. This is also reflected in the liability value of the payment being larger, the longer the longevity

$$A(L_i) > A(L_j) \text{ for } L_i > L_j$$

or

$$\frac{A(L_i)}{A(L_j)} = \frac{\int_0^{L_i} e^{-ra} b(a) da}{\int_0^{L_j} e^{-ra} b(a) da} = 1 + \frac{\int_{L_j}^{L_i} e^{-ra} b(a) da}{\int_0^{L_j} e^{-ra} b(a) da} > 1 \text{ for } L_i > L_j$$

Seen from an individual perspective, annuitization implies ex post that those living longest gain most from the annuity. This holds irrespective of the benefit structure $b(a)$, and it is an implication of diversifying longevity risk, see Yaari (1966) and many others. This effect is stronger, the larger the benefit at higher ages compared to earlier ages, and vice versa.

Multiple benefit structures are consistent with (1), and to gain insight on the possible profiles consider first an age independent benefit structure, $b(a) = b$ for all a. Using (1), the actuarially fair benefit becomes

$$b = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} da \right] f(L) dL}. \tag{2}$$

A simple age dependent profile is captured by the benefit rule[9]

$$b(a) = b(0) e^{\beta a}; \qquad \beta \lesseqgtr 0 \tag{3}$$

where $b(0)$ is the initial benefit level at retirement, and the coefficient β determines the benefit structure over the retirement period; $\beta > 0$ (< 0) implies an increasing (decreasing) benefit profile. Under this benefit structure, it follows that

$$A(L) = b(0) \int_0^L e^{-[r-\beta]a} da$$

which inserted in (1) implies that the initial benefit is

$$b(0) = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-[r-\beta]a} da \right] f(L) dL}$$

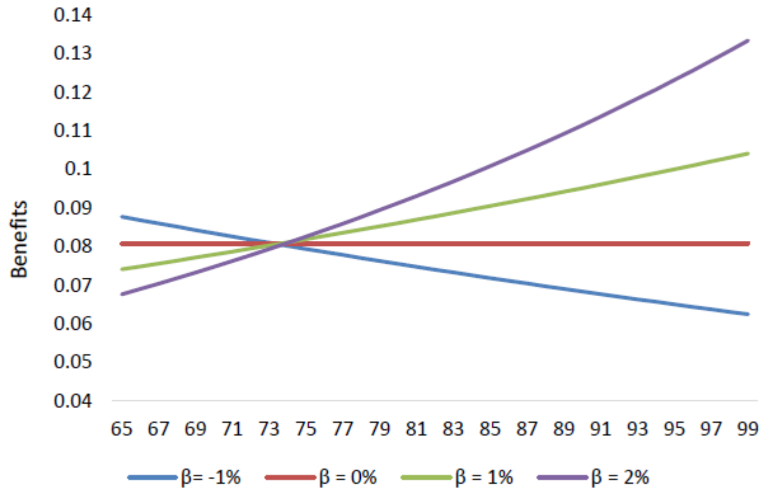
Intuitively, the more the benefit grows with age (β), the lower the initial benefit level (b_o),

$$\frac{\partial b(0)}{\partial \beta} = -\beta b(0)^2 \int_{\underline{L}}^{\overline{L}} \left[\int_0^L a e^{-[r-\beta]a} da \right] f(L) dL < 0$$

Possible benefit profiles are illustrated in Figure 2 as they depend on the age coefficient β.

[9] This implies a monotonic relation between benefits and age, ruling out e.g. a hump-shaped profile, see Bommier et al. (2011b).

Figure 2: Actuarially fair benefit profiles, different age dependencies (β), total population



Note: The numerical illustration is based on the mortality pdf for the entire population (see Figure 1), implying a life expectancy of 81.8 years, and assumes that the market rate of return (r) equals 4%. Wealth is normalized to 1.

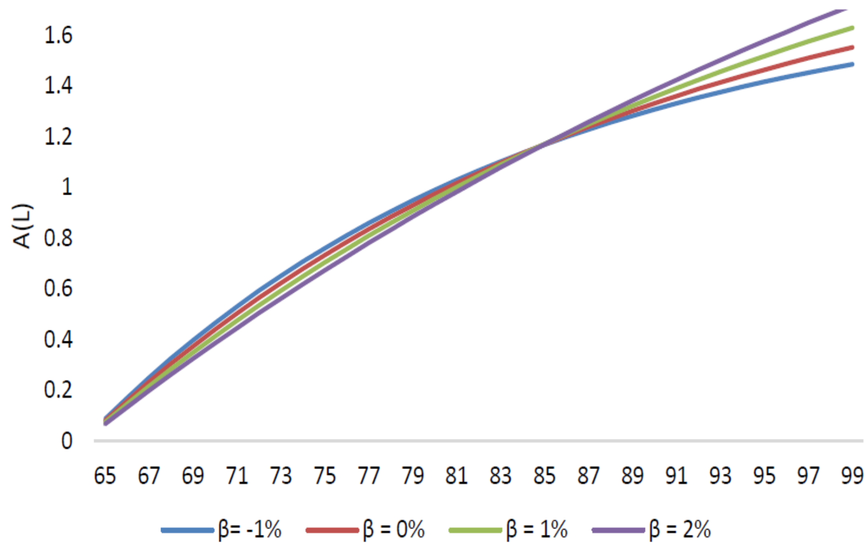
The benefit profile has obvious ex post implications across the population depending on actual longevity. For the age dependent benefit profile (3), a change in the age dependency parameter (β) affects the value of the annuity by

$$\frac{\partial A(L)}{\partial \beta}\frac{1}{A(L)} = \underbrace{\frac{\partial b_o}{\partial \beta}\frac{1}{b(0)}}_{<0} + \underbrace{\frac{\int_0^L ae^{-[r-\beta]a}da}{\int_0^L e^{-[r-\beta]a}da}}_{>0} \lesseqgtr 0$$

where the first term on the LHS is negative and the second is positive. Since the latter is increasing in longevity, there is a critical life length $\widehat{L}$ where those with a short life $L < \widetilde{L}$ are made worse off by a steeper profile (increase in β), and vice versa for those with a long life $L > \widetilde{L}$, see Figure 3[10]. In this sense the annuity becomes more regressive ex post, the more the benefit level increases with age (higher β), and vice versa. The benefit profile thus has distributional implications, and a policy maker unable ex ante to identify those with a short life may indirectly target this group by choosing a declining benefit profile, see discussion in the introduction and below.

[10]From the figure the threshold levels may seem the same for all values of β. They differ, but for the parameterization shown they are so close that it is not detectable in the figure given the chosen scale.

Figure 3: The present value of benefits (A(L)) depending on longevity, total population



Note: See notes for Figure 2.

2.2 Implementing age dependent benefit structure - indexation

Time and hence age dependent benefit profiles can in practice be implemented via indexation of benefits to wages and/or prices. To analyse the implications, assume that prices and wages evolve deterministically[11] according to

$$\begin{aligned} P(a) &= P(0)^{\pi_p a} \\ W(a) &= W(0)e^{\pi_w a} \end{aligned}$$

where π_p is price inflation and π_w wage inflation. A general benefit formula for nominal benefits (B) encompassing price and wage indexation reads

$$B(a) = B(0)e^{[\alpha_p \pi_p + \alpha_w \pi_w]a}, \tag{4}$$

where $\alpha_p \geq 0$ ($\alpha_w \geq 0$) is the weight to price (wage) inflation indexation. Note that a special case of this indexation scheme has $\alpha_p + \alpha_w = 1$, implying that indexation exceeds price indexation ($\alpha_p \pi_p + \alpha_w \pi_w = \pi_p + \alpha_w [\pi_w - \pi_p] > \pi_p$, for $\pi_w > \pi_p$).

Given (4), the nominal annuity factor becomes (where R is the nominal return, $R = r + \pi_p$)

[11] In the presence of risk, indexing to the actual inflation, and thus both expected and unexpected inflation, raises other issues, including scope for diversification of such risks, which are outside the scope of this paper.

$$A(L) = B(0) \int_0^L e^{-[R-\alpha_p \pi_p - \alpha_w \pi_w]a} da$$

implying that the initial benefit conditional on the indexation parameters (α_p, α_w) is

$$B(0) = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-[R-\alpha_p \pi_p - \alpha_w \pi_w]a} da \right] f(L) dL}$$

The following details the cases of full indexation to either price or wage inflation.

Case I: Price indexation ($\alpha_p = 1, \alpha_w = 0$)

If benefits are fully indexed to prices ($\alpha_p = 1, \alpha_w = 0$), the real value of the benefit is constant over the retirement period. Specifically, the real value of the pension is given as (the superscript p refers to price indexation)

$$b^p(a) = \frac{B^p(a)}{P(a)} = \frac{B^p(0)e^{\pi_p a}}{P(0)e^{\pi_p a}} = \frac{B^p(0)}{P(0)} = b^p(0) \text{ for all } a.$$

and thus constant across age/time. This implies an annuity factor

$$A(L_i) = b^p(0) \int_0^{L_i} e^{-ra} da,$$

where

$$b^p(0) = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} da \right] f(L) dL},$$

It is a straightforward implication that the benefit level over time is decreasing relative to wages. This matters if the pension should prevent economic poverty, which is typically measured by a poverty threshold defined as a share of median wages[12]. Specifically, the pension benefit relative to wages develops with age according to

$$\frac{B^p(a)}{W(a)} = \frac{B^p(0)e^{\pi_p a}}{W(0)e^{\pi_w a}} = \frac{B^p(0)}{W(0)} e^{-[\pi_w - \pi_p]a}$$

With price indexation the real value of pensions is constant, but benefits thus fall relative to the wage level due to the growth rate in real wages ($\pi_w - \pi_p > 0$).

Case II: Wage indexation ($\alpha_w = 1$, $\alpha_p = 0$)

If the pension is indexed to the wage level, the benefit formula becomes

$$B^w(a) = B^w(0)e^{\pi_w a}$$

and the annuity factor is

$$A(L) = \int_0^L B^w(0)e^{-[R-\pi_w]a} da = \int_0^L B^w(0)e^{-[r-[\pi_w - \pi_p]]a} da$$

[12] The OECD defines a poverty threshold as 50% of median income, see e.g. OECD (2022), and the EU Commission defines a risk of poverty threshold as 60% of median income, see European Commission (2021).

implying that the future benefits are discounted by the growth corrected real rate of interest $(r-[\pi_w-\pi_p])$. The initial benefit level is

$$B^w(0)=\frac{1}{\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{-[r-[\pi_w-\pi_p]]a}da\right]f(L)dL}$$

It follows straightforwardly that

$$\frac{B^w(0)}{P(0)}<\frac{B^p(0)}{P(0)}\qquad\text{for } \pi_w>\pi_p$$

which is an implication of the general finding above - the more benefits increase with age, the lower the initial value. The real value of the pension is

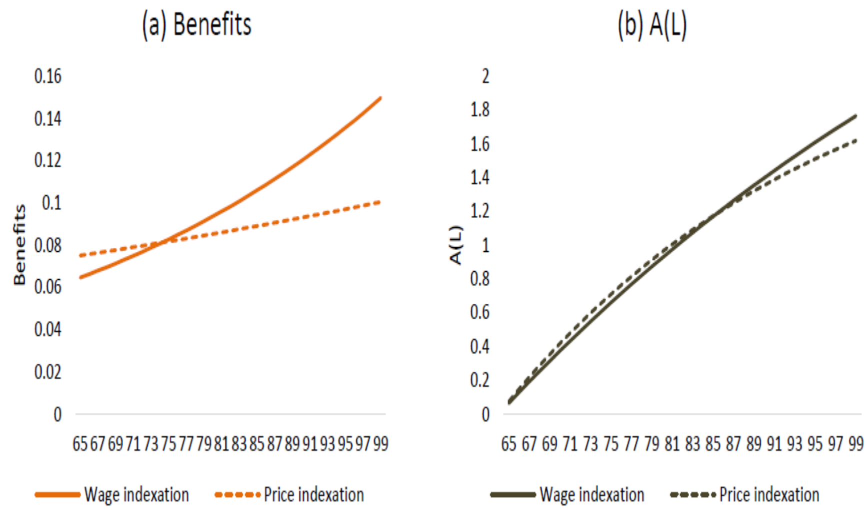
$$\frac{B^w(a)}{P(a)}=\frac{B^w(0)}{P(0)}e^{[\pi_w-\pi_p]a}$$

and real wage growth $(\pi_w-\pi_p>0)$ determines how real benefits grow over time/age. It is implied that the benefit relative to wages is constant throughout retirement

$$\frac{B^w(a)}{W(a)}=\frac{B^w(0)}{W(0)}\text{ for all } a$$

The possible benefit profiles attainable via indexation thus span from a declining profile where the real value of the pension declines by the rate of price inflation in the case of a fixed nominal benefit (for $\alpha_p=\alpha_w=0$) to an increasing profile where real benefits grow by real wage growth due to full wage indexation ($\alpha_p=0$; $\alpha_w=1$). Comparing price and wage indexation points to an important trade-off in deciding how benefits should be indexed. Indexation to prices implies a higher initial level of the pension, which prevents poverty among "younger" pensioners and those turning out to have a short life, see Figure 4a. Indexation to wages implies a lower initial pension level but prevents (relative) poverty among the "older" pensioners since the real value of pensions increases over time. How to index benefits thus involves a trade-off between the interests of those with short and long longevity, see Figure 4b. This comes to the fore if the annuity pools groups with different expected longevity, see below.

Figure 4: Benefit profiles and the present value of pensions ($A(L)$) under wage and price indexation



Note: See notes for Figure 1. Price increases are 0.9% per year, and wage increases 2.5% per year.

2.3 Pooling mortality risk and indexation

Pension schemes often pool risk across different risk classes[13], see introduction. To analyse this in more detail, assume that there are two risk classes with density functions over life length $g(L)$ and $h(L)$, both defined on $\left[\underline{L}, \overline{L}\right]$, respectively, and that the relative size of the former (latter) group is λ $(1-\lambda)$, see Figure 1. The probability density function for the entire population is thus

$$f(L) = \lambda g(L) + [1-\lambda]\, h(L); 0 \leq \lambda \leq 1 \tag{5}$$

and the value of the total liability value of the annuity to the pension fund of a pooled scheme is

$$A = \int_{\underline{L}}^{\overline{L}} A(L) f(L) dL = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a) da \right] f(L) dL$$

Denote the associated cumulative density functions by $G(L)$ and $H(L)$, then $G(L)$ first order stochastically dominates $H(L)$ iff

$$G(L) \leq H(L) \text{ for all } L$$

[13]Here, no distinction is made between objective and subjective density functions. See Heimer et al. (2019), and O´Dea and Sturrock (2021) for analyses of subjective mortality rates.

in which case $E_h(L) \equiv \int_{\underline{L}}^{\overline{L}} Lh(L)dL < E_g(L) \equiv \int_{\underline{L}}^{\overline{L}} Lg(L)dL$. Since $A(L)$ is increasing in L, it follows that the expected value of the arrangement is lower for the h-group than the g-group,

$$A_h = \int_{\underline{L}}^{\overline{L}} A(L)h(L)dL < A_g = \int_{\underline{L}}^{\overline{L}} A(L)g(L)dL$$

and hence the annuity in the aggregate redistributes from the $h-$risk class to the $g-$risk class. This is capturing the essence of the regressive bias discussed in the introduction. For the particular case considered in the numerical illustrations using men (density: $h(\cdot)$) and women (density: $g(\cdot)$) as the two groups, it follows that men prefer a more front-loaded benefit profile than women. Therefore, price indexation is more favourable to men, and wage indexation to women, see Figure 4.

3 Optimal benefit profile

Turn next to the normative issue of the optimal benefit profile. This is considered both for the case where there is risk neutrality (standard preferences) and risk aversion (temporal risk aversion) wrt the length of life. In both cases we compare separation and pooling across groups having different mortality.

3.1 Temporal risk neutrality

Consider standard preferences where utility is specified over consumption at a given age a, $u(c(a))$, and $u(\cdot)$ fulfils all standard assumptions. It is assumed that the only income source is the benefit received from the annuity ($c(a) = b(a)$), thereby other forms of savings are disregarded[14] to focus on how the optimal benefit profiles depend on mortality of those participating in a given pension scheme. Note that b is here interpreted as the real value of the pension benefit (and r the real rate of return), see below.

Ex ante all members of a given population are identical, and they face the same probability density function $f(L)$ over life time. From an ex-ante perspective, they all agree on the objective of choosing a benefit profile maximizing the expected present value of utility. The present value of utility if reaching age L is

$$U(L) = \int_0^L e^{-\delta a} u\left(b(a)\right) da,$$

[14] Note that absent a bequest motive it is optimal to save entirely in annuities, see Yaari (1965) and Davidoff et al. (2005). In the presence of a bequest motive, all savings for old-age consumption should be annuitized.

where δ is the subjective discount rate[15], and expected utility is

$$EU(L) = \int_{\underline{L}}^{\overline{L}} U(L)f(L)dL = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-\delta a} u(b(a))da \right] f(L)dL. \tag{6}$$

Define the maximization problem $\Omega(f(L), f(L))$ as the choice of the benefit profile maximizing expected utility (6) subject to the constraint

$$\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a)da \right] f(L)dL = 1$$

where there is no loss of generality in assuming that wealth is normalized to unity. Note that the definition of the policy problem refers to the PDF relevant for determining expected utility (here: $f(L)$) and the PDF relevant for the budget constraint (here: $f(L)$). This distinction becomes important when comparing pooling and separating risk sharing, see below.

At this level of generality, it is impossible to characterize the optimal benefit profile. Therefore, functional forms are imposed, and the utility function is assumed to belong to the CRRA-family[16], which is often used in the literature, see e.g. Mitchell et al. (1999) and Wettstein et al. (2021),

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}, \sigma > 0. \tag{7}$$

The optimal benefit profile is found within the benefit class,

$$b(a) = b(0)e^{\beta a}; \quad \beta \lesseqqgtr 0 \tag{8}$$

In Appendix A it is shown that the optimal age parameter β is given as

$$\beta = \frac{r - \delta}{\sigma}$$

and the value of the initial payment b_o is

$$b(0) = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} e^{\beta a} da \right] f(L)dL}$$

This is a restatement of a standard result in the literature, see e.g. Attanasio (1999) and Gomes et al. (2021). The optimal benefit (consumption) profile is in general age dependent ($\beta \neq 0$) except in the knife-edge case where the subjective discount factor equals the market rate of return ($\delta = r$). If the discount rate exceeds (falls short of) the subjective discount rate, the optimal benefit is increasing (decreasing) with age. The higher the intertemporal rate of

[15] The evidence on a possible age-gradient in the discount rate is unclear. To explain why many old have large savings, it has been proposed that discount rates decline with age, see e.g. Kureshi et al. (2021).

[16] In the numerical illustrations $\sigma = 0.7$, but all results are similar also for $\sigma > 1$.

substitution (relative risk aversion) (σ), the flatter the optimal benefit profile; that is, a higher intertemporal rate of substitution makes optimal benefits less dependent on age. Importantly, the optimal age parameter β is independent of the distribution of longevity $f(L)$, reflecting that the annuity diversifies the mortality risk, see Yaari (1965). Changes in the distribution and thus expected longevity affect the initial level of the payment but not its age dependency; that is, an increase in longevity causes a parallel shift in the optimal benefit profile, see illustrations below.

We now turn to the possible implementation of the optimal benefit profile. Under indexation the real value of the pension evolves according to

$$\frac{B(a)}{P(a)} = \frac{B(0)e^{[\alpha_p \pi_p + \alpha_w \pi_w]a}}{P(0)e^{\pi_p a}} = \frac{B(0)}{P(0)} e^{[[\alpha_p - 1]\pi_p + \alpha_w \pi_w]a}$$

The optimal profile can be implemented by choosing the indexation such that (see Appendix B)

$$[\alpha_p - 1]\,\pi_p + \alpha_w \pi_w = \beta = \frac{r - \delta}{\sigma}$$

Under the reasonable assumption that $0 \leq \alpha_p \leq 1$, $0 \leq \alpha_w \leq 1$, and $\pi_w > \pi_p$, it is possible to implement the optimal benefit profile by choice of indexation parameters (α_p, α_w) if

$$-\pi_p < \frac{r - \delta}{\sigma} < \pi_w$$

Note that a declining real benefit profile can be implemented by choosing less than full price indexation $(0 \leq \alpha_p < 1, \alpha_w = 0)$. It is an implication that the optimal indexation is depending on the macroeconomic environment (price inflation, wage (productivity) growth, and the market return).

Pooling across risk classes

Turn next to the consequences of pooling mortality risk across different groups. Since the optimal age parameter is independent of the density function for the population participating in the risk pooling, it follows that only the initial benefit varies when considering different possible risk class configurations (for the same utility function). The population density function $(f(L))$ is a weighted average of the density functions for two risk classes, $g(L)$ and $h(L)$, see (5). Given pooling, consider the benefit profile each risk group considers optimal; that is, given that their longevity is determined by either $g(L)$ or $h(L)$, but the benefits are determined based on the pooled density function $f(L)$. Specifically, the policy problem $\Omega(g(L), f(L))$ is to find the benefit profile (β_{gf}) maximizing

$$EU(L) = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-\delta a} u(b(a)) da \right] g(L) dL$$

subject to

$$\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a) da \right] f(L) dL = 1$$

The policy problem $\Omega(h(L), f(L))$ is defined similarly. Assume the utility function (7) and benefit profiles within the class (8), the optimal age parameter, for proof see Appendix C, is

$$\beta_{gf} > \beta_{gg}$$

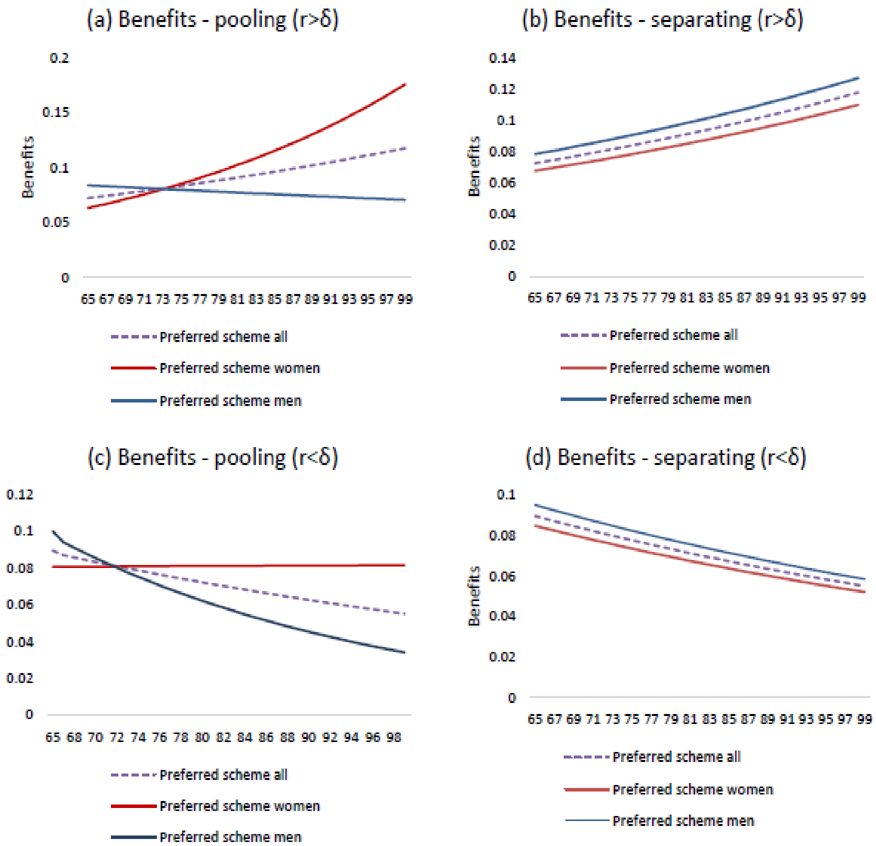
if $F(L)$ first order stochastically dominates $G(L)$. That is, being pooled with a group with a lower longevity implies that the g-risk class wants a steeper benefit profile compared to the separating case. Similarly, if $H(L)$ first order stochastically dominates $F(L)$, the h-risk class wants a less steep benefit profile than in the separating case

$$\beta_{hf} < \beta_{hh}$$

In the case of pooling, the two risk classes thus take more "extreme" positions on the optimal benefit profile than in the respective separating cases. This reflects their respective interest in the cross-subsidisation arising under pooling. More is transferred from the $h-$ to the $g-$risk class if the benefit profile is steeper, and vice versa if it is less steep.

To illustrate the outcomes, Figure 5a plots the optimal profile for the total population (the problem: $\Omega(f(L), f(L))$ - dotted purple line) which has an increasing benefit profile (here: $r > \delta$). The benefit profile that women prefer given the pooling is more steep (the problem: $\Omega(g(L), f(L))$ - the solid red line), and for men ($\Omega(h(L), f(L))$- the solid blue line) it is less steep. This reflects that women have a higher life expectancy than men, and therefore have a stronger interest in the benefit received at higher ages. For comparison, Figure 5b gives the optimal benefit profile in the separating cases, $\Omega(g(L), g(L))$ and $\Omega(h(L), h(L))$, respectively (the dotted purple line is the same in the two figures). The difference between Figure 5a and 5b shows how the interests of the two groups change when they are pooled compared to the separating cases; women want a much steeper profile than in the separating case and vice versa for men. For completeness we show similar graphs in Figure 5c and 5d, where the discount rate is higher than the market rate of interest ($r < \delta$), implying that optimal benefit profiles are declining with age. The same qualitative results apply in this case.

Figure 5: Optimal benefit profile - separating and pooling contract for two risk classes



Note: Figure (a) and (b) give the optimal benefit profiles for $\{\Omega(f,f),\Omega(g,f),\ \Omega(h,g)\}$ and $\{\Omega(f,f),\Omega(g,g),\Omega(h,g)\}$, respectively for $r=0.04$, $\delta= 0.03$, $\sigma= 0.7$.

Figure (c) and (d) for the similar problems as in (a) and (b) but for $r=0.04$, $\delta=0.03$, $\sigma=0.7$.

3.2 Temporal risk aversion

Temporal risk aversion has been proposed in the literature as a way to capture either risk aversion wrt to the length of life[17] (Bommier (2006)) or aversion to multiperiod inequality, see Bommier et al. (2011a) and Simonovits (2006). Temporal risk aversion can be modelled by an increasing concave function $(\phi(\cdot))$

[17]Bommier (2006) defines risk neutrality/aversion with respect to length of life when the individual for any constant flow of consumption profile (discounted utility of consumption) and any current age exhibits risk neutrality/aversion with respect to age at death.

defined over lifetime utility. Specifically, let utility if reaching age L be

$$\widetilde{U}(L) = \phi\left(U(L)\right) = \phi\left(\int_0^L e^{-\delta a} u\left(b(a)\right) da\right),$$

where $\phi_U(\cdot) > 0$ and $\phi_{UU}(\cdot) < 0$, and

$$E\widetilde{U}(L) = \int_{\underline{L}}^{\overline{L}} \widetilde{U}(L) f(L) dL$$

Define the policy problem $\widetilde{\Omega}(f(L), f(L))$ as the choice of the benefit profile maximizing expected lifetime utility $E\widetilde{U}(L)$ subject to the constraint

$$\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a) da\right] f(L) dL = 1$$

As above, we consider benefit profiles in the class (8) given the utility function (7). Comparing the optimal profile $\widetilde{\beta}^{ff}$ to the problem $\widetilde{\Omega}(f(L), f(L))$ with the optimal profile β^{ff} to the problem $\Omega(f(L), f(L))$,we show in Appendix D that

$$\widetilde{\beta}^{ff} < \beta^{ff} \text{ for } \phi_U(\cdot) > 0 \text{ and } \phi_{UU}(\cdot) < 0$$

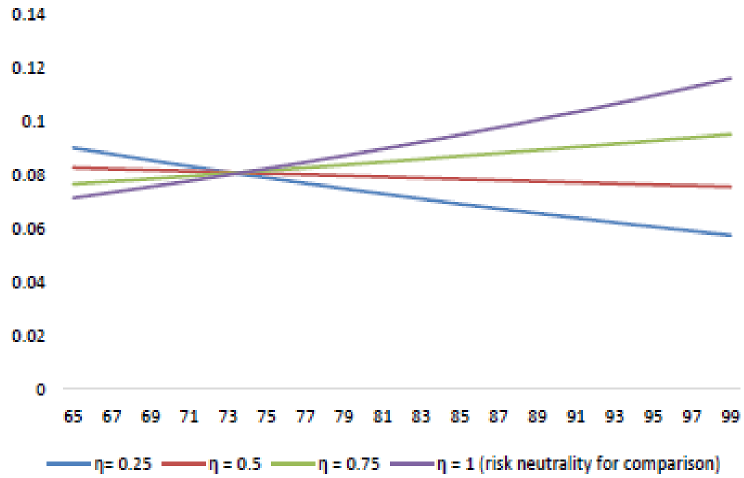
In the presence of risk aversion wrt the length of life, the benefit profile is less steep in age compared to risk neutrality wrt the length of life.

To illustrate the outcome, assume[18] that $\phi(U(L)) = \frac{1}{1-\eta} U(L)^{1-\eta}$, where $\eta \in (0, 1]$. A lower η implies more risk aversion wrt life length. Figure 6 shows the optimal benefit structures for different values of η. The purple line ($\eta = 1$) corresponds to the standard preferences analysed above. The higher the risk aversion wrt life length (lower η), the less benefits increase with age/time, and for sufficiently high risk aversion the benefit profile is declining with age.

[18] Bommier et al. (2011b) use the specification $\Phi(x) = 1 - e^{-x}$, and Bommier et al. (2011a) and Simonovits (2006) the specification $\Phi(x) = \frac{1}{1-\eta} x^{1-\eta}$. The former implies a constant and the latter a declining absolute risk aversion wrt the duration of life,

$$\frac{\Phi_{xx}(x)}{\Phi_x(x)} = \left\{ \begin{array}{ll} -\alpha & \text{for } \Phi(x) = 1 - \alpha e^{-x} \\ -\eta x^{-1} & \text{for } \Phi(x) = \frac{1}{1-\eta} x^{1-\eta} \end{array} \right\}$$

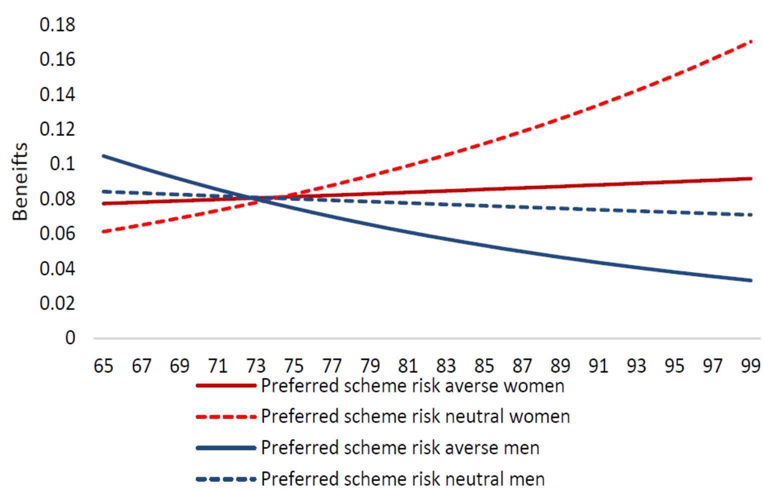
Figure 6: Risk aversion wrt life length and optimal benefit profiles



Note: Figure (a) and (b) give the optimal benefit profiles (β) to the problem $\widetilde{\Omega}(f(L), f(L))$ for different values of η and for $r = 0.04$, δ= 0.03, σ= 0.7.

The conflict of interest arising under risk pooling is seen in Figure 7, showing the optimal profile in the case of risk neutrality and risk aversion wrt the length of life (compare also to Figure 5). Qualitatively the changes are the same as in the case of risk neutrality, but in the presence of risk aversion wrt the length of life women ($g(L)$) prefer an almost flat profile while men ($h(L)$) prefer a clearly declining profile.

Figure 7: Risk aversion and neutrality - pooling

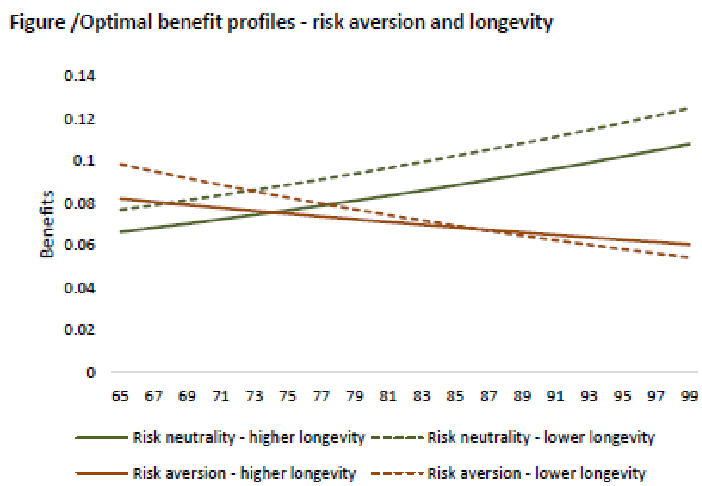


Note: The figure gives the optimal benefit profiles (β) to the problem $\widetilde{\Omega}(g(L), f(L))$ and $\widetilde{\Omega}(h(L), f(L))$ for $\eta = 1$ (risk neutrality) and $\eta = 0.5$ (risk aversion), $r = 0.04$,

δ= 0.03, σ= 0.7.

In Figure 8, we depict the preferred benefit schemes for individuals who are either risk averse or risk neutral for two different density functions over longevity, here women ($g(L)$) and men ($h(L)$), see Figure 1. The risk neutral individuals (green lines) have the same preferred growth rates in both cases, and the difference in initial benefits reflects the difference in longevity, see above. For individuals being risk averse wrt the length of life (orange lines), the benefit profile is declining with age, see above, and both the slope and the intercept change when the distribution of the length of life changes. For the low life expectancy ($h(L)$), risk averse individuals want higher initial payments and steeper declines compared to the group with a high life expectancy.

Figure 8: Optimal benefit profiles - risk aversion and longevity



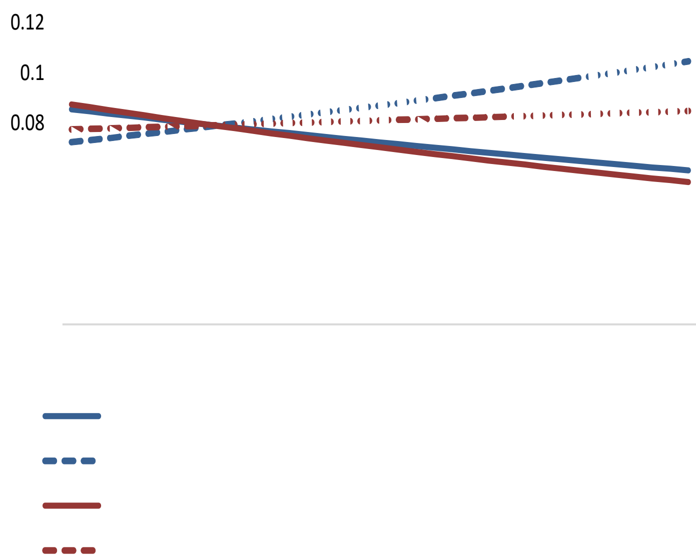
Note: The figure gives the optimal benefit profiles (β) to the problem $\widetilde{\Omega}(g(L), g(L))$ and $\widetilde{\Omega}(h(L), h(L))$ for $\eta = 1$ (risk neutrality) and $\eta = 0.5$ (risk aversion), $r = 0.04$, δ= 0.03, σ= 0.7.

Finally, rather than interpreting the difference between low and high longevity in Figure 8 as two different risk classes at a given point in time it can also be interpreted as a change over time capturing an upward trend in longevity across cohorts (retirement age is unchanged). With temporal risk neutrality an upward shift in longevity implies that the optimal initial benefit level declines, but the profile remains unchanged. In the case of risk aversion, the optimal initial level declines and the profile is less declining with age. The latter reflects that more survive into higher ages and therefore the real value of benefits at higher ages is assigned a higher value.

3.3 Dispersion in mortality

Sasson (2016) has pointed to the problems of comparing groups based on the average life expectancy since that may conceal differences in dispersion of longevity, see also Figure 1. To shed light on the role of dispersion in longevity, Figure 6 shows a case where two groups with the same average life expectancies but different variance are pooled, and the optimal benefit profile is considered under both temporal risk neutrality and risk aversion. We see that the groups who are risk averse wrt longevity prefer frontloading of payments, and temporal risk aversion implies a less steep benefit profile than in the case of temporal risk neutrality all else equal. Thus, the group who wants the highest initial payment is the risk averse group who has low variance in their longevity distribution. However, risk aversion changes the optimal benefit scheme more for the high variance group; that is, a higher dispersion has larger implications in the case of temporal risk aversion.

Figure 9: Longevity risk and optimal benefit structure



Note: Modelled by simulating a normal distribution with $\mu = 82$ and $\sigma = 5$ and 10, respectively. The distribution is conditional on ages 65-100.

4 Concluding remarks

Socio-economic differences in mortality challenge pension system designs due to the regressive bias arising when mortality risk is pooled across different groups. This creates a trade-off between the insurance provided by the annuities and the distributional implications. This paper shows that the profile depends critically on how pension benefits are indexed to prices and/or wages. Indexation implies benefit profiles spanning from real benefits declining by the rate of price inflation (in the case of a fixed nominal benefit) to the real value of the benefit increasing by real wage increases (in the case of wage indexation of benefits).

The choice of indexation of benefits is thus in a simple way by which to implement different benefit profiles and thus the distributional consequences of annuities. The steeper the benefit profile (and hence the lower the initial benefit), the more advantageous it is to groups with a high life expectancy, and vice versa. It is thus straightforward to implement different profiles to affect the distributional profile even if policies can not be made specifically dependent on risk class. These distributional implications also imply that the risk classes participating in a pooling annuity arrangement have different viewpoints on the optimal profile, and they differ from the separating case due to the implications for redistribution across risk classes.

Normatively, preferences displaying aversion to differences in lifetime utility - either due risk aversion wrt the length of life or inequality aversion - imply less steep benefit profiles than under preferences implying risk neutrality wrt the length of life, and it may even be declining for sufficiently high aversion to inequality. When mortality curves change and longevity increases, the optimal adjustment under risk neutrality is a decline in the initial benefit, leaving the profile unchanged, while risk aversion implies a lower initial benefit but also a less declining benefit profile.

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A Optimal benefit profile - standard preference

The optimization problem $\Omega(f(L), f(L))$ is to find the benefit profile $(b(0), \beta)$ maximizing

$$EU(L) = \int_{\underline{L}}^{\overline{L}} U(L)f(L)dL = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-\delta a} \frac{1}{1-\sigma} b(a)^{1-\sigma} da \right] f(L)dL.$$

subject to the constraint that

$$\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-ra} b(a) da \right] f(L)dL = 1$$

Considering policies in the class $b(a) = b(0)e^{\beta a}$, this can be formulated as maximising

$$EU(L) = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \frac{1}{1-\sigma} b(0)^{1-\sigma} e^{[\beta[1-\sigma]-\delta]a} da \right] f(L) dL$$

with $b(0)$ determined by

$$b(0) = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} da \right] f(L) dL}$$

Note that

$$\begin{aligned}
\frac{\partial b(0)}{\partial \beta} &= -\frac{1}{\left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} da \right] f(L) dL \right]^2} \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da \right] f(L) dL \\
&= -b(0)^2 \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da \right] f(L) dL < 0
\end{aligned}$$

The first order condition to this maximization problem reads

$$\frac{\partial EU(L)}{\partial \beta} = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[b(0)^{1-\sigma} e^{[\beta[1-\sigma]-\delta]a} a + b(0)^{-\sigma} \frac{\partial b(0)}{\partial \beta} e^{[\beta[1-\sigma]-\delta]a} \right] da \right] f(L) dL = 0$$

Inserting the expression for $\frac{\partial b(0)}{\partial \beta}$, the condition can be written

$$b(0)^{2-\sigma} \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[\begin{array}{c} \frac{e^{[\beta[1-\sigma]-\delta]a} a}{b(0)} - \\ \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da \right] f(L) dL \right] e^{[\beta[1-\sigma]-\delta]a} \end{array} \right] da \right] f(L) dL = 0$$

or as

$$\begin{aligned}
& \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} da \right] f(L) dL \right] \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta[1-\sigma]-\delta]a} a da \right] f(L) dL \right] \\
= & \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da \right] f(L) dL \right] \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[e^{[\beta[1-\sigma]-\delta]a} \right] da \right] f(L) dL
\end{aligned}$$

It follows straightforwardly that this equation has the solution

$$\begin{aligned}
\beta - r &= \beta [1-\sigma] - \delta \\
\beta &= \frac{r-\delta}{\sigma}
\end{aligned}$$

B Optimal indexation

As an illustration, the following solves for the optimal indexation parameter $\alpha_p + \alpha_w = 1$, capturing many applied schemes, see Introduction. The nominal benefit is thus

$$B(a) = B(0)e^{[\alpha_p \pi_p + [1-\alpha_p]\pi_w]a},$$

where π_p is price inflation, $\pi_w > \pi_p$ wage inflation,

$$\begin{aligned} P(a) &= P(0)e^{\pi_p a} \\ W(a) &= W(0)e^{\pi_w a}, \end{aligned}$$

Consumption depends on the real value of the pension, $b(a) = \frac{B(a)}{P(a)} = \frac{B(0)}{P(0)} e^{[1-\alpha_p][\pi_w - \pi_p]a}$ and hence the expected lifetime utility can be written

$$EU(L) = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{-\delta a} \frac{1}{1-\sigma} \left[\frac{B(0)}{P(0)} e^{[1-\alpha_p][\pi_w - \pi_p]a} \right]^{1-\sigma} da \right] f(L)dL$$

wherewith $b(0)$ determined by

$$\frac{B(0)}{P(0)} = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[[1-\alpha_p][\pi_w - \pi_p] - r]a} da \right] f(L)dL}$$

The problem is now to maximize expected lifetime income wrt α_p, and the associated first order condition

$$\frac{\partial EU(L)}{\partial \alpha_p} = \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[\begin{array}{c} \left[\frac{B(0)}{P(0)}\right]^{1-\sigma} [\pi_w - \pi_p] \, e^{[[1-\alpha_p][\pi_w - \pi_p][1-\sigma] - \delta]a} a \\ + \left[\frac{B(0)}{P(0)}\right]^{-\sigma} \frac{\partial \frac{B(0)}{P(0)}}{\partial \alpha_p} e^{[[1-\alpha_p][\pi_w - \pi_p][1-\sigma] - \delta]a} \end{array} \right] da \right] f(L)dL = 0$$

where

$$\frac{\partial \frac{B(0)}{P(0)}}{\partial \alpha_p} = - \left[\frac{B(0)}{P(0)} \right]^2 [\pi_w - \pi_p] \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[[1-\alpha_p][\pi_w - \pi_p] - r]a} a da \right] f(L)dL$$

Inserting the expression for $\frac{\partial \frac{B(0)}{P(0)}}{\partial \beta}$, the first order condition can be written

$$\int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[\begin{array}{c} \frac{e^{[\beta[1-\sigma]-\delta]a} a}{\frac{B(0)}{P(0)}} - \\ \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[[1-\alpha_p][\pi_w - \pi_p] - r]a} a da \right] f(L)dL \right] e^{[[1-\alpha_p][\pi_w - \pi_p][1-\sigma] - \delta]a} \end{array} \right] da \right] f(L)dL = 0$$

or as

$$\begin{aligned} & \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[[1-\alpha_p][\pi_w - \pi_p] - r]a} da \right] f(L)dL \right] \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[[1-\alpha_p][\pi_w - \pi_p][1-\sigma] - \delta]a} a da \right] f(L)dL \right] \\ = & \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[[1-\alpha_p][\pi_w - \pi_p] - r]a} a da \right] f(L)dL \right] \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[e^{[[1-\alpha_p][\pi_w - \pi_p][1-\sigma] - \delta]a} \right] da \right] f(L)dL \end{aligned}$$

which has the solution

$$[1-\alpha_p]\,[\pi_w-\pi_p]=\alpha_w\,[\pi_w-\pi_p]=\frac{r-\delta}{\sigma}$$

implying $\alpha_w=\frac{1}{\pi_w-\pi_p}\frac{r-\delta}{\sigma}$, where $\alpha_w\in[0,1]$ for $r>\delta$ requires

$$\pi_w-\pi_p>\frac{r-\delta}{\sigma}$$

C Preferred benefit profiles under risk pooling

The optimization problem $\Omega(g(L),f(L))$ is to find the benefit profile $(b_{gg}(0),\beta_{gg})$ which risk group g with a PDF $g(L)$ considers optimal given that they are in a pooling risk sharing arrangement where the terms are determined by the PDF $f(L)$. The optimal benefit profile for risk class g maximizes (subscript g denotes that expected utility is evaluated given $g(L)$)

$$E_gU(L)=\int_{\underline{L}}^{\overline{L}}U(L)g(L)dL=\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{-\delta a}\frac{1}{1-\sigma}\left[b_{gf}(0)e^{\beta_g a}\right]^{1-\sigma}da\right]g(L)dL.$$

subject to the constraint that

$$b_{gf}(0)=\frac{1}{\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta_{gf}-r]a}da\right]f(L)dL}$$

Note that

$$\frac{\partial b(0)}{\partial\beta_{gf}}=-b_{gf}(0)^2\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta_{gf}-r]a}ada\right]f(L)dL<0$$

It follows that

$$\frac{\partial EU(L)}{\partial\beta_{gf}}=\int_{\underline{L}}^{\overline{L}}\left[\int_0^L\left[b_{gf}(0)^{1-\sigma}e^{[\beta_{gf}[1-\sigma]-\delta]a}a+b_{gf}(0)^{-\sigma}\frac{\partial b_{gf}(0)}{\partial\beta}e^{[\beta_{gf}[1-\sigma]-\delta]a}\right]da\right]g(L)dL$$

Inserting the expression for $\frac{\partial b_{gf}(0)}{\partial\beta}$, the condition can be written

$$\begin{aligned}\frac{\partial EU(L)}{\partial\beta_{gf}}&=b_{gf}(0)^{2-\sigma}\int_{\underline{L}}^{\overline{L}}\left[\int_0^L\left[\begin{array}{c}\frac{e^{[\beta[1-\sigma]-\delta]a}a}{b_{gf}(0)}\\-\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta-r]a}ada\right]f(L)dL\right]e^{[\beta[1-\sigma]-\delta]a}\end{array}\right]da\right]g(L)dL\\&=b_{gf}(0)^{2-\sigma}\int_{\underline{L}}^{\overline{L}}\left[\int_0^L\left[\begin{array}{c}e^{[\beta[1-\sigma]-\delta]a}a\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta_{gf}-r]a}da\right]f(L)dL\right]\\-\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta-r]a}ada\right]f(L)dL\right]e^{[\beta[1-\sigma]-\delta]a}\end{array}\right]da\right]g(L)dL\end{aligned}$$

or as

$$\begin{aligned}\frac{\partial EU(L)}{\partial\beta_{gf}} &= b_{gf}(0)^{2-\sigma}\int_{\underline{L}}^{\overline{L}}\left[\begin{array}{c}\left[\int_0^L e^{[\beta[1-\sigma]-\delta]a}ada\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gf}-r\right]a}da\right]f(L)dL\right]\\ -\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta-r]a}ada\right]f(L)dL\right]\left[\int_0^L e^{[\beta[1-\sigma]-\delta]a}da\right]\end{array}\right]g(L)dL\\ &= b_{gf}(0)^{2-\sigma}\left[\begin{array}{c}\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta[1-\sigma]-\delta]a}ada\right]g(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gf}-r\right]a}da\right]f(L)dL\right]\\ -\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta-r]a}ada\right]f(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta[1-\sigma]-\delta]a}da\right]g(L)dL\right]\end{array}\right]\end{aligned}$$

$\widehat{\beta}_{gg}$

$$\begin{aligned}&\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}ada\right]g(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}da\right]g(L)dL\right]\\ =\;&\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}ada\right]g(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}da\right]g(L)dL\right]\end{aligned}$$

$$\left.\frac{\partial\mathcal{L}}{\partial\beta_{gf}}\right|_{\beta_{gg}} = b_{gf}(0)^{2-\sigma}\left[\begin{array}{c}\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}ada\right]g(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}da\right]f(L)dL\right]\\ -\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta gg-r]a}ada\right]f(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta-r]a}da\right]g(L)dL\right]\end{array}\right]=0$$

$sign\ \left.\frac{\partial\mathcal{L}}{\partial\beta_{gf}}\right|_{\beta_{gg}} > 0$

$$\begin{aligned}&\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}ada\right]g(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}da\right]f(L)dL\right]\\ >\;&\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta gg-r]a}ada\right]f(L)dL\right]\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta-r]a}da\right]g(L)dL\right]\end{aligned}$$

or

$$\frac{\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}da\right]f(L)dL\right]}{\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}da\right]g(L)dL\right]} > \frac{\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{[\beta gg-r]a}ada\right]f(L)dL\right]}{\left[\int_{\underline{L}}^{\overline{L}}\left[\int_0^L e^{\left[\beta_{gg}-r\right]a}ada\right]g(L)dL\right]}$$

The following proves that the LHS is larger and the RHS less than one.

Define $A_{gg}(L) \equiv \int_0^L e^{\left[\beta_{gg}-r\right]a}da$, where $\frac{\partial A_{gg}(L)}{\partial L} < 0$ and integrating by parts

$$\begin{aligned}\int_{\underline{L}}^{\overline{L}} A_{gg}(L)f(L)dL &= \left[A_{gg}(L)F(L)\right]_{\underline{L}}^{\overline{L}} - \int_{\underline{L}}^{\overline{L}}\frac{\partial A_{gg}(L)}{\partial L}F(L)dL\\ &= A_{gg}(\overline{L})F(\overline{L}) - A_{gg}(\underline{L})F(\underline{L}) - \int_{\underline{L}}^{\overline{L}}\frac{\partial A_{gg}(L)}{\partial L}F(L)dL\\ &= A(\overline{L}) - \int_{\underline{L}}^{\overline{L}}\frac{\partial A_{gg}(L)}{\partial L}F(L)dL\end{aligned}$$

since $F(\overline{L}) = 1$ and $F(\underline{L}) = 0$. Hence,

$$\begin{aligned}
& \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta_{gg} - r]a} da \right] f(L) dL - \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta_{gg} - r]a} da \right] g(L) dL \\
= & \int_{\underline{L}}^{\overline{L}} A_{gg}(L) f(L) dL - \int_{\underline{L}}^{\overline{L}} A_{gg}(L) g(L) dL \\
= & \int_{\underline{L}}^{\overline{L}} \frac{\partial A_{gg}(L)}{\partial L} G(L) dL - \int_{\underline{L}}^{\overline{L}} \frac{\partial A_{gg}(L)}{\partial L} F(L) dL \\
= & \int_{\underline{L}}^{\overline{L}} \frac{\partial A_{gg}(L)}{\partial L} \left[G(L) - F(L) \right] dL > 0
\end{aligned}$$

iff there is first order stochastic dominance $G(L) \leq F(L)$ for all L since $\frac{\partial A_{gg}(L)}{\partial L} < 0$.

Similarly, $B_{gg}(L) \equiv \int_0^L e^{[\beta_{gg} - r]a} a da$, where $\frac{\partial B_{gg}(L)}{\partial L} > 0$ and integrating by parts

$$\begin{aligned}
\int_{\underline{L}}^{\overline{L}} B_{gg}(L) f(L) dL &= \left[B_{gg}(L) F(L) \right]_{\underline{L}}^{\overline{L}} - \int_{\underline{L}}^{\overline{L}} \frac{\partial B_{gg}(L)}{\partial L} F(L) dL \\
&= B_{gg}(\overline{L}) F(\overline{L}) - B_{gg}(\underline{L}) F(\underline{L}) - \int_{\underline{L}}^{\overline{L}} \frac{\partial B_{gg}(L)}{\partial L} F(L) dL \\
&= B_{gg}(\overline{L}) - \int_{\underline{L}}^{\overline{L}} \frac{\partial B_{gg}(L)}{\partial L} F(L) dL
\end{aligned}$$

and hence

$$\begin{aligned}
& \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta_{gg} - r]a} a da \right] g(L) dL - \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta gg - r]a} a da \right] f(L) dL \\
= & \int_{\underline{L}}^{\overline{L}} B_{gg}(L) g(L) dL - \int_{\underline{L}}^{\overline{L}} B_{gg}(L) f(L) dL \\
= & - \int_{\underline{L}}^{\overline{L}} \frac{\partial B_{gg}(L)}{\partial L} G(L) dL + \int_{\underline{L}}^{\overline{L}} \frac{\partial B_{gg}(L)}{\partial L} F(L) dL \\
= & \int_{\underline{L}}^{\overline{L}} \frac{\partial A_{gg}(L)}{\partial L} \left[F(L) - G(L) \right] dL > 0
\end{aligned}$$

iff there is first order stochastic dominance $G(L) \leq F(L)$ for all L since $\frac{\partial A_{gg}(L)}{\partial L} < 0$.

It is implied that

$$sign \left. \frac{\partial \mathcal{L}}{\partial \beta_{gf}} \right|_{\beta_{gg}} > 0$$

which implies that the $g-type$ in the pooled case prefers a more steep benefit profile than in the separating case

$$\beta_{gf} > \beta_{gg}$$

Proceeding in the same way implies for the h-type that

$$\beta_{hf} < \beta_{hh}$$

iff $F(L) \leq H(L)$ for all L.

D Attitudes towards mortality risk

Utility for longevity L is

$$\widetilde{U}(L) = \Phi\left(\int_0^L e^{-\delta a} u\left(b(a)\right) da\right)$$

and hence expected lifetime utility is

$$E\widetilde{U}(L) = \int_{\underline{L}}^{\overline{L}} \widetilde{U}(L) f(L) dL = \int_{\underline{L}}^{\overline{L}} \Phi\left(\int_0^L e^{-\delta a} u\left(b(a)\right) da\right) f(L) dL$$

and inserting the functional form for the utility function and for the benefit it reads

$$E\widetilde{U}(L) = \int_{\underline{L}}^{\overline{L}} \Phi\left(\int_0^L \frac{1}{1-\sigma} b(0)^{1-\sigma} e^{[\beta[1-\sigma]-\delta]a} da\right) f(L) dL$$

where

$$b(0) = \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} da\right] f(L) dL}$$

It follows straightforwardly that

$$\begin{aligned}
\frac{\partial b(0)}{\partial \beta} &= -\frac{1}{\left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} da\right] f(L) dL\right]^2} \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da\right] f(L) dL \\
&= -b(0)^2 \int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da\right] f(L) dL < 0
\end{aligned}$$

The marginal effect on expected lifetime utility of a change in the benefit parameter β is

$$\begin{aligned}
\frac{\partial E\widetilde{U}(L)}{\partial \beta} &= \int_{\underline{L}}^{\overline{L}} \Phi_{\widetilde{U}}(\cdot) \left[\int_0^L b(0)^{1-\sigma} e^{[\beta[1-\sigma]-\delta]a} a da + b(0)^{-\sigma} \frac{\partial b(0)}{\partial \beta} e^{[\beta[1-\sigma]-\delta]a} da \right] f(L) dL \\
&= \int_{\underline{L}}^{\overline{L}} \Phi_{\widetilde{U}}(\cdot) \left[\begin{array}{c} \int_0^L b(0)^{1-\sigma} e^{[\beta[1-\sigma]-\delta]a} a da \\ -b(0)^{2-\sigma} \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da \right] f(L) dL \right] e^{[\beta[1-\sigma]-\delta]a} da \end{array} \right] f(L) dL \\
&= \int_{\underline{L}}^{\overline{L}} \Phi_{\widetilde{U}}(\cdot) \, b(0)^{1-\sigma} \left[\int_0^L [a da - \Phi] \, e^{[\beta[1-\sigma]-\delta]a} da \right] f(L) dL
\end{aligned}$$

where

$$\Phi \equiv b(0) \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{[\beta-r]a} a da \right] f(L) dL \right]$$

Define $\widehat{\beta}$ as the optimal benefit parameter for standard preferences ($\Phi_{\widetilde{U}}(\cdot) = 1$), which satisfies

$$\widehat{b}(0)^{1-\sigma} \int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[a - \widehat{\Phi} \right] e^{[\beta[1-\sigma]-\delta]a} da \right] f(L) dL = 0$$

where

$$\begin{aligned}
\widehat{\underline{b}} &= \frac{1}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{\left[\widehat{\beta}-r\right]a} da \right] f(L) dL} \\
\widehat{\Phi} &\equiv \widehat{b}(0) \left[\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{\left[\widehat{\beta}-r\right]a} a da \right] f(L) dL \right] \\
&= \frac{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{\left[\widehat{\beta}-r\right]a} a da \right] f(L) dL}{\int_{\underline{L}}^{\overline{L}} \left[\int_0^L e^{\left[\widehat{\beta}-r\right]a} da \right] f(L) dL} > 0
\end{aligned}$$

It follows that

$$\left. \frac{\partial E\widetilde{U}(L)}{\partial \beta} \right|_{\widehat{\beta}} = \widehat{b}(0)^{1-\sigma} \int_{\underline{L}}^{\overline{L}} \Phi_{\widetilde{U}}(\cdot) \left[\int_0^L \left[a - \widehat{\Phi} \right] e^{\left[\widehat{\beta}[1-\sigma]-\delta\right]a} da \right] f(L) dL \qquad (9)$$

It follows that $[a - \Phi]$ is increasing in a, and that $[a - \Phi] \lesseqgtr 0$ for $a \lesseqgtr \Phi$. Moreover,

$$\int_{\underline{L}}^{\overline{L}} \left[\int_0^L \left[a - \widehat{\Phi} \right] e^{[\widehat{\beta}[1-\sigma]-\delta]a} da \right] f(L) dL = 0$$

Since $\Phi_{\widetilde{U}}(\cdot) > 0$, $\Phi_{\widetilde{U}\widetilde{U}}(\cdot) < 0$, and $\widetilde{U}_L(L) > 0$, it follows that the weight to $\left[\int_0^L \left[a - \widehat{\Phi} \right] e^{\left[\widehat{\beta}[1-\sigma]-\delta\right]a} da \right]$ in (9) is declining in L, and hence

$$\left. \frac{\partial E\widetilde{U}(L)}{\partial \beta} \right|_{\widehat{\beta}} = \widehat{b}(0)^{1-\sigma} \int_{\underline{L}}^{\overline{L}} \Phi_{\widetilde{U}}(\cdot) \left[\int_0^L \left[a - \widehat{\Phi} \right] e^{\left[\widehat{\beta}[1-\sigma]-\delta\right]a} da \right] f(L) dL < 0$$

It is implied that $\widetilde{\beta} > \widehat{\beta}$ for $\Phi_{\widetilde{U}}\left(\cdot\right) > 0$, $\Phi_{\widetilde{U}\widetilde{U}}\left(\cdot\right) < 0$.