CESifo WORKING PAPERS

10966
2024

February 2024

When Should Central Banks Fear Inflation Expectations?

Lucio Gobbi, Ronny Mazzocchi, Roberto Tamborini

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 10966*

When Should Central Banks Fear Inflation Expectations?

Abstract

When inflation picks up, central banks are most concerned that the de-anchoring of inflation expectations and the ignition of wage-price spirals will trigger inflation dynamic instability. However, such scenarios do not materialize in the standard New Keynesian theoretical framework for monetary policy. Using a simulative model, we show that they can materialize upon introducing *in particularly strong doses* boundedly-rational expectations that de-anchor endogenously, as they are updated according to the actual inflation process, with indexed wages, and persistent inflation shocks. In these cases, a more hawkish central-bank stance on inflation expands the stability region of the system, which however remains bounded. On the other hand, the critical combinations of factors that trigger instability can be regarded as extreme in empirical terms, while in "normal times" the system is resilient to shocks and expectation de-anchoring even with more dovish monetary policy.

JEL-Codes: E170, E300, E500.

Keywords: cost-push inflation, New Keynesian models for monetary policy, wage-price spiral, de-anchoring of inflation expectations.

Lucio Gobbi
Department of Economics and Management
University of Trento / Italy
lucio.gobbi@unitn.it

Ronny Mazzocchi
European Parliament Brussels / Belgium
ronny.mazzocchi@europarl.europa.eu

*Roberto Tamborini**
Department of Economics and Management
University of Trento / Italy
roberto.tamborini@unitn.it

*corresponding author

February 2024
Declarations of interest: none
The views expressed in this article are personal and should not be attributed to the European Parliament or its services.

1. Introduction

Since the outbreak of post-pandemic global inflation in the course of 2021, alleged risks of jumping onto an explosive inflationary path figure prominently in the concerns of all major central bankers as a compelling motivation for them to shift to a restrictive stance. The experience of the 1970s is often invoked. Such risks are attributed to the interplay between the persistence of inflation shocks, de-anchored inflation expectations and wage indexation (wage-price spiral) (Bailey 2022, Powell 2022, ECB 2022). History teaches that inflationary processes can indeed spiral out of control. When this happens, inflationary expectations and indexation are often among the culprits (Alvarez et al. 2022, Ari et al. 2023).

There are also dissenting voices: one example is Rudd (2021), who questions both the theoretical and empirical soundness of the overarching importance attributed to expectations in inflationary processes. Regarding the leading macroeconomic framework for monetary policy of the New Keynesian (NK) type, it is somewhat paradoxical that one can hardly find a theoretical characterisation of de-anchored inflation expectations capable by themselves of generating dynamically unstable inflation processes, i.e. processes that deviate from the official target over time. In the standard NK model, all agents are assumed to hold the "one-period-ahead" rational expectation of inflation. By definition, these are a kind of de-anchored expectations, as they anticipate the actual inflation path and are contingent on macroeconomic news relevant to the inflation process (Gürkaynak et al. 2010, ECB 2021, see Section 2 below). Nevertheless, it has been demonstrated that inflation does converge to the target, provided that the central bank abides by the principles of inflation targeting, such as those epitomized by the Taylor rule (e.g., Bullard and Mitra 2002, Galì 2008).

Following Galì and Gertler (1999) and Blanchard et al. (2015), it has become common to partition agents and their expectations into those who are forward-looking and those who are backward-looking. The former may keep their long-run expectations anchored to the central bank's target, or they may seek to anticipate the short-run "one period ahead" inflation. The latter may track a particular series of past realizations of inflation. However, even these variations in the basic framework are not in themselves sufficient to undermine the dynamic stability of inflation (see Section 2 below).

Overall, the NK model economy is an economy that is well shielded against the threats of inflationary instability, unless inflation is driven by exogenous processes that are unstable in themlseves, which is generally ruled out, or the central bank fails grossly on the prescriptions of inflation targeting. Of course, central banks can misbehave and make wrong decisions, but it seems equally (if not more) relevant to investigate whether there are other (less rational?) mechanisms of expectation formation such that the general concern about risks of inflation dynamic instability is justified, orthodox monetary policy notwithstanding.

To this end, we start from the above-mentioned hypothetical partition between long-run forward-looking expectations, which remain anchored to the inflation target, and short-run backward-looking expectations, but, following an idea of Gobbi et al. (2019), we introduce an endogenous expectation formation process in which the fractions of agents with anchored or de-anchored expectations are not predetermined, but change according to the observed realizations of inflation. The attitude of our agents towards the central bank's commitment to keep inflation on target follows the maxim "deeds not words". More formally, our agents update their probabilistic belief about the achievement of the inflation target according to the evidence of the inflation process itself, using a logit-like function. As a result, the larger the deviation of inflation from the target, the larger the proportion of agents that de-anchor their expectation, and the stronger the impulse to the inflation process.

We obtain a complex nonlinear system governed by four key parameters: the reactivity of the de-anchoring mechanism, the share of indexed wages, the inflation coefficient in the central bank's policy rule, and the magnitude of the initial shock. We analyse the system dynamics using simulations based on parameter values taken from the empirical literature, which allow us to identify the stability and instability regions of the system.

Our main finding is that the region of stability is bounded. The inflation process can indeed become dynamically unstable for particularly severe combinations of inflation shocks and their persistence, the endogenous de-anchoring of expectations and the share of wage indexation. The policy message is that when these conditions materialise, an appropriately recalibrated monetary policy, namely a stronger anti-inflationary stance, can keep the system within the stability region, *up to a point*. However, we also argue that the critical combinations of factors that trigger instability can empirically be regarded as extreme, or that the stability region is safely large, even with weaker anti-inflationary stance.

In section 2, we present the baseline model, consisting of the NK three-equation model, which we use to substantiate the above paradox. In Section 3 we elaborate our alternative model of inflation expectation formation, and in Section 4 we also introduce a share of indexed wages that keep pace with inflation but also feed back on it. Section 5 presents a series of simulations in which we explore the parameter space, both globally and pairwise, in order to identify the regions of stability/instability of the system. Section 6 offers some concluding remarks on the implications of our results for monetary policy.

2. Inflation and expectations in the standard New Keynesian model

We conduct our analysis using the reduced form of the standard NK "three-equation" model, which is reproduced in the Appendix. The well-known three equations consist of the IS curve, which determines the market-clearing level of output, the Phillips curve (PC), which determines the level of inflation associated with the level of output, and the central bank's Taylor-type reaction function, which determines the policy rate.

The reduced form is rarely found in the relevant literature, although methodologically it is the most efficient and consistent way to examine the properties of a model, and in particular the partition between exogenous and endogenous variables and the relationships between the former and the latter.

The endogenous variables are the inflation rate π_t, the output y_t and the policy interest rate i_t, all expressed as log-deviations (^) from the respective target (steady-state) values (π^*, y^*, i^*), which for simplicity are assumed to be constant. The exogenous variables, in addition to the targets of the endogenous, are an expectation term of inflation held at time t π^e_t, and an inflation shock $u_{\pi t}$ (e.g. an energy cost shock, see Appendix) in excess of the value of the inflation target. This is the only shock that we consider. The dynamic characteristics of these variables will be discussed later. The model is therefore:

(1) (a) $\hat{\pi}_t = a_1(\pi^e_t - \pi^*) + a_2 u_{\pi t}$

(b) $\hat{y}_t = b_1(\pi^e_t - \pi^*) + b_2 u_{\pi t}$

(c) $\hat{i}_t = c_1(\pi^e_t - \pi^*) + c_2 u_{\pi t}$

The coefficients a_n , b_n , c_n (n = 1, 2) are combinations of the parameters of the three source equations in Appendix. In their standard specification the signs of the coefficients are the following:

$0 < a_1 < 1,\ 0 < a_2 < 1,\ b_1 < 0\ ,\ b_2 < 0,\ c_1 > 0,\ c_2 > 0.$

The transmission of an inflation shock $u_{\pi t} > 0$ to the three endogenous variables goes through an increase in the inflation gap (a_2), which triggers an increase in the policy rate (c_2), which feeds back onto the output gap (b_2). In particular, b2 < 0 is key to the stabilisation of inflation and it necessarily depends on the response of monetary policy to excess inflation, i.e. $\phi_\pi > 0$. Note that $\phi_\pi = 0$ would determine $b_2 = 0$, i.e. inflation shocks would have no effect on aggregate demand and output, as well as $a_2 = 1$, i.e. inflation shocks would have a permanent effect on the inflation gap.

Let us now focus on the role of inflation expectations, i.e. the way in which the variable π^e_t is determined. An important but often neglected distinction is that between long-run and short-run expectations (e.g. Hooper et al. 2019, Rudd 2021). The concept of long-run expectations concerns the (rational) anticipation of whether or not the system will settle down to its steady state, i.e. whether or not inflation will return to its target. Indeed, this seems to be the main task of central banks, according to their repeated statements (Rudd 2021, Lane 2022, Gopinath 2022, Carvalho et al. 2023).

As long as expectations remain anchored, $\pi^e_t = \pi^*$ all t, system (1) shows that the inflation path is dictated by that of the shocks. A common characterization of shock processes in macro-models is by means of a first-order autoregressive process:

$$u_{\pi t} = \theta u_{\pi t\text{-}1} + \varepsilon_{\pi t}$$

where ε_t is a white-noise i.i.d. process and $0 < \theta < 1$. Substituting this process into the system (1), it is easily seen that the parameter value $0 < a_2 < 1$ ($\phi_\pi > 0$) guarantees convergence to the inflation target. It is only if $\theta \geq 1$, i.e. the shock is dynamically unstable in itself, that inflation would never converge to the target no matter how monetary policy is engineered.

One of the fundamental specifications of the NK model, with the PC based on the optimal pricing model by Calvo (1983), instead displays the short-run "one-period-ahead", (rational) expected value of inflation as of time t, $\pi^e_t = \mathrm{E}_t\pi_{t+1}$. This specification introduces an additional dynamic component to system (1).

As demonstrated by several authors (e.g. Bullard and Mitra 2002, Galì 2008), as long as the endogenous variables *and their expected values* can be treated as free variables, and the solution of first-order difference equations can be applied, the parameter value $0 < a_1 < 1$ is also a necessary and sufficient condition for dynamic stability. This condition is satisfied for

(2) $$\phi_\pi > 1 - \frac{(1-\beta)(1+\phi_y\eta)}{\kappa\eta}$$

Two points should be emphasized. First, as also established by Woodford (2003, pp. 253-254), dynamic stability is a property depending on the (ϕ_π, ϕ_y) relationship with the structural parameters of the economy. The (ϕ_π, ϕ_y) relationship (2) can be read as a stability constraint along which the central banks can choose its preferred combination of the two parameters. As can be seen, the so-called Taylor Principle $\phi_\pi > 1$ (raise the interest rate by more than the inflation gap to obtain a higher real interest rate) is sufficient but not necessary. A possible solution may well be $\phi_\pi < 1$, which we may dub "weak" inflation targeting.

Second, expectations modelled in this way are by construction de-anchored from the inflation target. Applying the rational expectations hypothesis to the inflation-gap equation (1)(a), so that the expectation term is the statistical expected value of the data generation process, the result is:

(3) $$\mathrm{E}_t\hat{\pi}_{t+1} = a_1\mathrm{E}_t\hat{\pi}_{t+2} + a_2\mathrm{E}_t u_{\pi t+1}$$

If the shock follows a random walk, then $u_{\pi t}$ is the best predictor of $u_{\pi t+1}$. If the shock process has some predictable pattern this ought to be embodied in the expectation. In any case, the short-run rational expectations would appear to be de-anchored as they follow the common empirical definition of being contingent on macroeconomic news, provided that such news is to some extent relevant to the inflation process (Gürkaynak et al. 2010, ECB 2021).

Nonetheless, iteration and substitution of (3) into (1)-(a) yields

(4) $$\hat{\pi}_t = a_1^n \mathrm{E}_t \hat{\pi}_{t+n} + a_2 (\textstyle\sum_n a_1^n \mathrm{E}_t u_{\pi t+n} + u_{\pi t})$$

which shows that the condition $0 < a_1 < 1$ ensures that, as $n \rightarrow \infty$, the expectation term vanishes and the inflation process is driven only by the (possibly anticipated) sequence of the shocks, similar to the case with anchored expectations. Therefore, the conditions of dynamic stability built into the system (1) also imply that de-anchored expectations are not *per se* a cause for concern.

On the other hand, this property of the system underpins the assumption that long-run expectations remain rationally anchored, while at the same time they are conducive to the dynamic stability of inflation. This mutual consistency can be dubbed as a "good long-run equilibrium".

We may therefore conclude that the standard NK model for monetary policy seems to represent an economy quite resilient to the de-anchoring of inflation expectations. Are there ways whereby expectations are formed such that worries that inflation dynamic stability is impaired are justified? In the subsequent part of the paper we shall seek to answer this question.

3. Modelling the de-anchoring of expectations

The distinction between long-run and short-run expectations cuts across the distinction between forward-looking and backward-looking expectations introduced by Galì and Gertler (1999) (which is equivalent to the distinction between fully rational and boundedly rational expectations). The long-run and short-run types of expectations mentioned above are both forward-looking and fully rational (provided that the stability conditions of the system are satisfied). In the well-established formulation of the PC proposed by Blanchard et al. (2015) (see also Hooper et al. 2019), the expectation term is a weighted average between a measure of a long-run and a short-run component. The former also provides the anchor for the inflation target, while the latter does not and introduces a backward-looking component, namely the average inflation over the previous four quarters.

Let us call λ the weight of long-term expectations anchored to the target, so that $1 - \lambda$ is the weight of short-term de-anchored expectations, and define

$$\hat{\hat{\pi}}(t-4) \equiv \frac{1}{4} \sum_{n=1}^{4} \hat{\pi}_{t-n}$$

Therefore,

(5) $$\pi^e{}_{t+1} - \pi^* = (1 - \lambda)\, \hat{\hat{\pi}}(t-4)$$

so that the inflation-gap equation (1)(a) becomes[2]

(6) $\hat{\pi}_t = a_1(1 - \lambda)\hat{\bar{\pi}}(t-4) + a_2 u_{\pi t}$

Note that this formulation changes the dynamic structure of the system. Since the inflation gap becomes an autoregressive process, the weight of de-anchored expectations comes to play a critical role in the stability condition (see also Hooper 2019). However, as long as $a_1 < 1$, $\lambda < 1$, and $a_1(1 - \lambda) < 1$, such de-anchored expectations certainly contribute to the further amplification of the inflation process, but they are not by themselves a cause of dynamic instability.

As to the fraction of anchored expectations λ, Blanchard et al. (2015) estimated a value of around 0.5 for the US in the high-inflation period 1975-85 and showed that it rose steadily over the following twenty years, before stabilising at around 0.7. This result is confirmed by further estimates (Hooper et al. 2019). The evidence of the time-varying share of anchored (de-anchored) expectations has led us to propose a reformulation of the expectation formation hypothesis (5) where λ is itself *endogenous to the inflation process.*

Following an idea put forward by Gobbi et al. (2019), we interpret λ_t as a measure of the agents' confidence in the central bank's ability (willingness) to drive the inflation-gap process towards zero. The simple and plausible motto of agents might be "deeds not words". In this view, $\lambda_t \in [0, 1]$ can be interpreted as the probabilistic belief that agents hold at time t about whether or not the current inflation gap is transitory and converging to zero. They then inductively revise their belief according to the ongoing evolution of the inflation gap[3]. The fall of λ_t below 1 denotes the de-anchoring of expectations, as it is triggered by, and presents itself as "sensitive" to, current information on the state of the economy. The larger and more persistent the inflation gap, the lower λ_t, and the stronger the path-dependence of inflation in equation (6).

Before developing our model of endogenous λ_t, we want to anticipate two features highlighted by equation (6). First, as λ_t approaches zero, and the shock peters out, the actual path of inflation becomes increasingly dominated by the component of the de-anchored expectations.[4] This mechanism therefore seems to be able to explain central banks' engagement to curb inflation "credibly", which here means quickly enough to prevent a substantial fall in λ_t.[5] On the other hand, if the condition $a_1 < 1$ holds, then

---

[2] Of course equation (5) should be substituted also in equations (1)(b) and (1)(c)

[3] This hypothesis mimics a Bayesian behaviour. Theoretically, we may rely on a broad notion of rationality such that expectations are formed consistently with available knowledge of the generation process of the relevant variable (Evans and Honkapohja 2001, Kurz 1994, 2011, Garcìa-Schmidt and Woodford 2019). Models in the same vein have gained attention during the past decade of low inflation – low output "liquidity traps" (e.g. Arifovic et al. 2017, García-Schmidt and Woodford 2019, Evans and McCough 2018).

[4] The interplay and transmission from short-run to long-run expectations is also examined by Carvalho et al. (2023).

[5] The critical role of the "relative speed" of the policy rate *vis-à-vis* expected inflation can also be found in Woodford (2003, ch. 2).

even in case λ were to fall to zero, fully de-anchored expectations would amplify the inflation process, and make it more persistent, but they would not yet be sufficient to generate its permanent deviation from target.

3.1. The de-anchoring function

For the reasons explained previously, key to our model of λ_t is its relationship with the current state of the economy, i.e.:

(7) $\lambda_t = f(z_t)$

where z_t is an information set of variables observed up t. We restrict this set to the lagged inflation gap as in equation (5), i.e.: $z_t \equiv \hat{\bar{\pi}}(t-4)$.

We want this function to have the following properties:

(i) $f(0) = 1$

(ii) $\lim_{z\to+/-\infty} f(z_t) = 0$

(iii) $f_z(0) = 0$

The first condition states that with zero inflation gap expectations are fully anchored to the inflation target. The second states that as the inflation gap grows unboundedly, λ_t tends to 0 (de-anchoring is maximal). The third condition ensures that λ_t is bounded at 1 when the inflation gap is zero.

A suitable formalisation is provided by logistic maps such as (8) below, which have wide applications in inference problems like that of the agents in our model, that is to say transform observed variables into probabilistic assessments of the occurrence of an event (here the event is the convergence of current inflation to the inflation target)[6]:

(8) $$f(z_t) = A + \frac{BCe^{-\sigma z_t}}{(Ce^{-\sigma z_t}+1)^2}$$

The three required conditions are determined by the following parameter values:

$A = 0$, $B = 4$, $C = 1$

so that we obtain:

(9) $$\lambda_t = \frac{4e^{-\sigma z_t}}{(e^{-\sigma z_t}+1)^2}$$

which we shall call "de-anchoring function" (DA function).

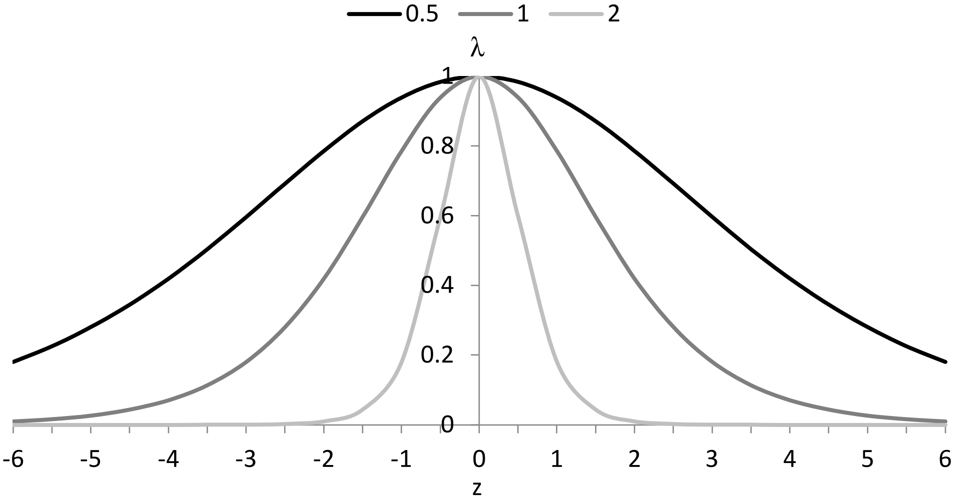
The parameter σ regulates the curvature of the DA function, i.e. the reactivity of λ_t in response to any observed $z_t \neq 0$.[7] Figure 1 portrays the function for increasing values of σ. Thus, high σ may be appropriate when confidence in the central bank is volatile, possibly as a consequence of lack of *past* reputation, whereas high reputation may be reflected in low σ. Note, however, that starting with low σ does not prevent λ_t from falling if inflation gaps are sufficiently large and persistent.

---

[6] The most popular application to binary exclusive events as is our case is the so-called logit model, where z_t is a linear combination of observed variables

[7] It plays a role analogous to the gradient of recursive revisions of estimated parameters in econometric learning models of the data generation process (Evans and Honkapoja 2001).

Figure 1. The DA function with different values of σ



3.2. Simulations

The endogenization of λ_t by way of the DA function creates a truly endogenous mechanism of expectation formation which interacts dynamically with the evolution of inflation. For the reader's convenience we report here our complete system:

(10) (a) $\hat{\pi}_t = a_1(1-\lambda_t)z_t + a_2 u_{\pi t}$

(b) $\hat{y}_t = b_1(1-\lambda_t)z_t + b_2 u_{\pi t}$

(c) $\hat{i}_t = c_1(1-\lambda_t)z_t + c_2 u_{\pi t}$

(d) $\lambda_t = \dfrac{4e^{-\sigma z_t}}{(e^{-\sigma z_t}+1)^2}$

(e) $z_t = \hat{\bar{\pi}}(t-4)$

(f) $u_{\pi t} = \theta_1 t - \theta_2 t^2 + \varepsilon_0$, $t = 0, 1, \ldots$

To explore the properties of such a system it is necessary to resort to numerical simulations, and here we are going to present the most informative results.

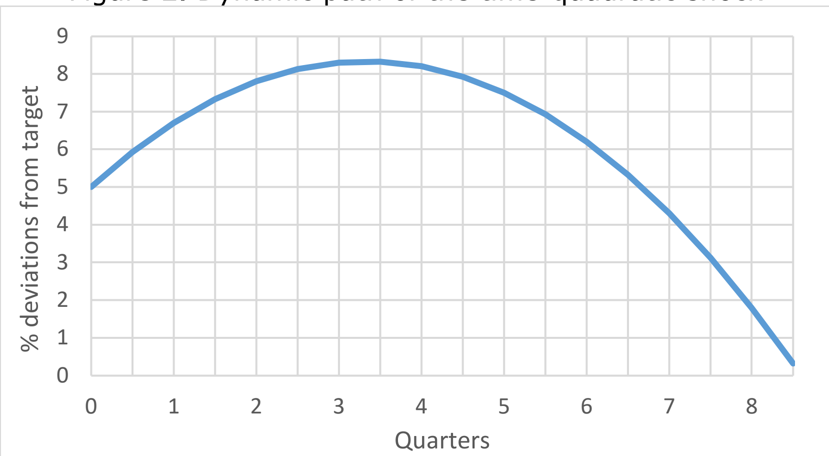
The calibration of the parameters a_n, b_n, c_n $(n = 1, 2)$ by means of consensus empirical values is presented in the Appendix. In the absence of direct evidence om the parameter σ we have resorted to the indirect evidence provided by estimates of λ. To replicate the value of $\lambda_t = 0.5$ at the peak of the inflation process, which is the historically low value of λ associated with historically high inflation estimated by Blanchard et al. (2015), we have found the value σ = 0.16. As mentioned above, the empirical consensus is that the anchoring of expectations in advanced countries has been incresing over time (σ has been decreasing). The constant monitoring of expectations at various time horizons by the central banks in the course of the recent upsurge of inflation has shown a substantial anchorage (De Guindos, 2023; Lane, 2024). We have therefore set the baseline value of σ = 0.1.

For our simulation we have chosen an "unfriendly" shock that mimics the hump-shaped time profile of the energy prices we have been observing since 2021. To this end, we have elaborated a "time-quadratic" process like the following[8]:

(11) $u_{\pi t} = \theta_1 t - \theta_2 t^2 + \varepsilon_0$, $t = 0, 1, \ldots$.

where ε_0 is a random initial shock, and the quadratic map traces the subsequent evolution (Figure 2).The two parameters θ_1 and θ_2 regulate the peak and the length of the process respectively. For the purely illustrative purpose of the simulation, we have chosen $\varepsilon_0 = 5\%$, $\theta_1 = 2$, $\theta_2 = 0.3$ so that the shock peaks at 8.3% (above the inflation target) in three quarters, and peters out in nine quarters.

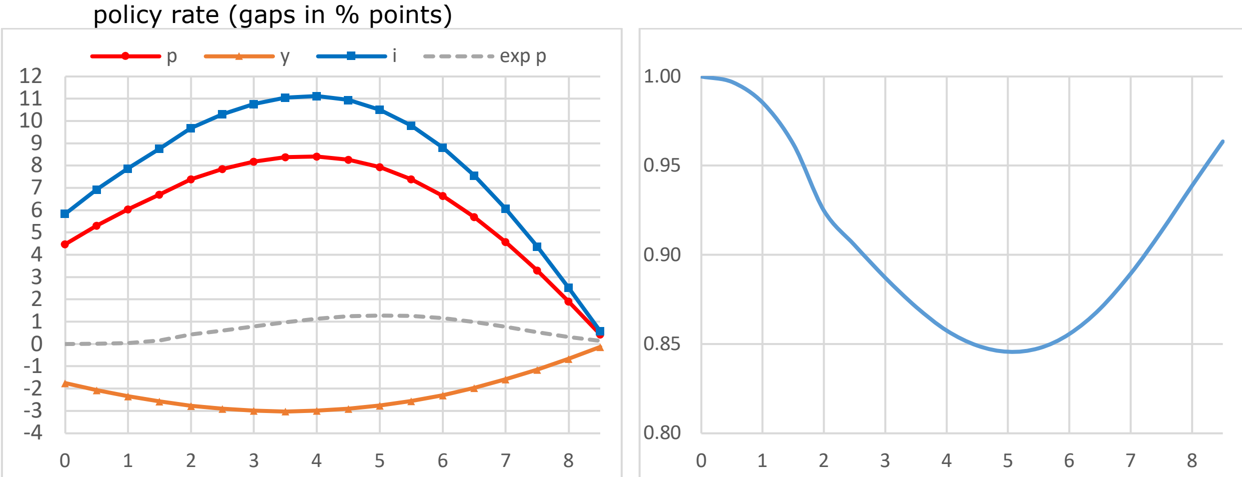
Figure 2. Dynamic path of the time-quadratic shock



Panel (a) of Figure 3 shows the dynamic paths of inflation, expected inflation, output and policy rate as percent gaps from the respective targets under flexible inflation targeting ($\phi_\pi = 1.5$, $\phi_y = 0.5$).

Figure 3. Simulation of system (10) under flexible inflation targeting ($\varepsilon_0 = 5\%$, $\sigma = 0.1$)

(a) Inflation (p), expected inflation, output and policy rate (gaps in % points) (b) Share of anchored expectations



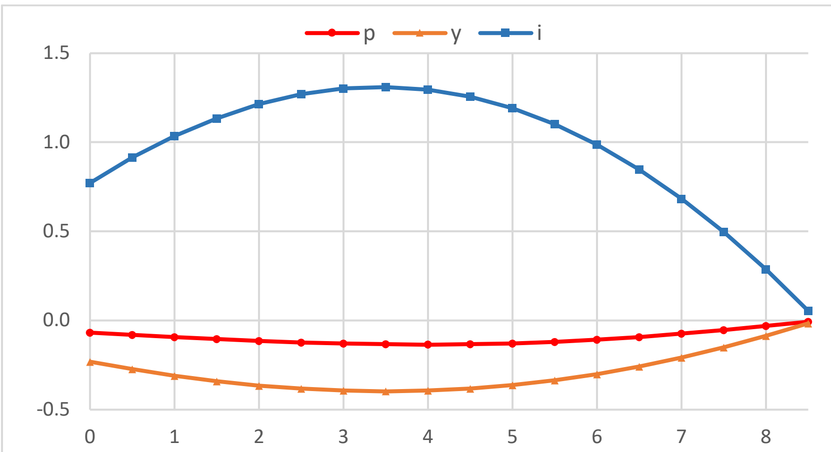
[8] Think of it as the quadratic interpolation of the time series of an energy price index.

The first observation is that the dynamic path of the inflation gap (and of the other variables as well) *is dictated by the hump-shaped path of the shock*. As anticipated in Section 2, this is a general property of this class of models. However, a sizeable amplification of the inflation process is generated by the de-anchored component of expectations, which adds further 1.3 points to reach the peak value of $\hat{\pi}_t$ = 8.4%. Nevertheless, the system remains dynamically stable. In fact, panel (b) shows that λ_t falls to 0.86 after five quarters when z_t = 8.3%, but then it recovers as monetary policy brings the inflation process under control.

What role does monetary policy play? As explained in Section 2, the TR-based monetary policy is key to achieving dynamic stability in the long run, but it should also be stressed that nothing else should be expected on the front of "the fight against inflation". Let us consider the central bank moving from flexible to pure inflation targeting, obtained by setting $\phi_y = 0$. Figure 4 shows the consequent differences in the paths of the inflation, output and policy-rate gaps.

First of all, enhancing the anti-inflation attitude of the central bank *does not change the dynamic profile of inflation*. If the path of the shock is hump-shaped, so will be the path of the inflation process: inflation will return to its target when the shock has dissipated, regardless of the monetary policy stance. The only difference monetary policy can make concerns the trade-off between smaller (larger) inflation gaps and larger (smaller) output gaps along the dynamic path. As can be seen, pure inflation targeting reduces the quarterly average inflation gap by 0.1% while increasing the negative output gap by 0.3%. This is achieved by tightening the policy rate by an additional 1% on average. Indeed, this is nothing other than the inflation-output variability trade-off embedded in the Taylor Rule.

Figure 4. Simulation of system (10) under pure inflation targeting. Differences of % gaps of inflation (p) output and policy rate from flexible inflation targeting (Figure 3)



This simulation, athough it designs an unfriendly environment, confirms what was said above analytically: as long as the TR-based monetary policy ensures that the condition $a_1 < 1$ holds, inflation remains dynamically stable despite the endogenous de-anchoring of expectations and their spiralling with the inflation process.

4. The wage-price spiral

We now wish to introduce the other common source of concern about the dynamic (in)stability of inflation, namely the wage-price spiral that can be triggered when nominal wages are indexed.

4.1. Indexed wages

For the purposes of the simulation, indexation means that nominal wages are linked to the rate of change in prices in order to keep the real wage unchanged. Therefore, in steady state, nominal wages increase at the rate of the target inflation, and the following should be interpreted as marginal increases proportional to inflation gaps. [9]

The most common schemes date back to Taylor's staggered wage contracts (Taylor, 1980), or to extensions of the Calvo firm pricing scheme to wage setting (Galì 2008, 2013). A common result is that only a fraction of nominal wage contracts, say γ, is renewed in line with inflation at each date. For symmetry with the expectation formation mechanism, renewed nominal wages are increased (above trend) by the average inflation gap of the last four quarters.

Wage increases are an additional source of cost push. The effect on the PC goes through the usual change in the marginal costs (see Appendix). If for simplicity we assume that the unit nominal cost of energy and of labour w^{m} have the same weight in the marginal cost function, we can treat

(12) $$\hat{w}_t^{\mathrm{m}} = \gamma\hat{\bar{\pi}}(t-4)$$

as an additional shock to equation (10)-(a)

Introducing both the de-anchored expectations and the wage indexation transforms the inflation-gap equation into the following autoregressive process

(13) $$\hat{\pi}_t = (a_1(1-\lambda_t) + a_2\gamma)\,\hat{\bar{\pi}}(t-4) + a_2 u_{\pi t}$$

As long as the shock process $u_{\pi t}$ is reversible, crucial for dynamic stability is only the condition that the autoregressive coefficient is lower than 1. Now this condition can no longer be taken for granted, but it can be thought of as a "frontier" between λ_t and γ:

(14) $$\lambda_t \geq -\frac{1-a_1}{a_1} + \frac{a_2}{a_1}\gamma$$

---

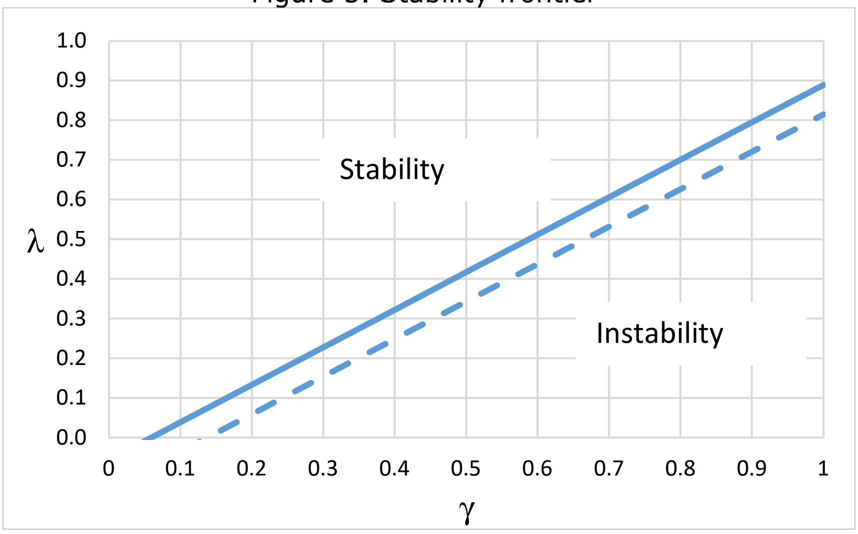
9 Our definition is consistent with the one adopted in empirical analyses such as Alvarez et al. (2022). Theoretically, Lorenzoni and Werning (2023) embed wage-price formation mechanisms in a distributional conflict over the actual real wage paid by firms and earned by workers, and hence they classify wage-price spirals depending on whether they aim at modifying the actual real wage. From this point of view, we are implicitly assuming a wage indexation mechanism the leaves the ruling real wage unchanged, and hence is distributionally neutral.

There are four notable implications. First, what matters for stability is neither the share of de-anchoring nor that of indexed wages *per se* but their relative size. Second, the higher the share of indexed wages, the higher the anchoring of expectations should be; alternatively, lower anchoring requires also lower indexation. Third, as long as the stability condition is met, it remains true that the system can return to the zero-gap equilibrium. Fourth, in this new setup, the monetary policy stance is important for dynamic stability since it affects the condition (14). In fact, the parameters a_1, a_2 (the impact of expectations and the shock on inflation, respectively) depend on the inflation coefficient ϕ_π in the TR in such a way that a higher coefficient *relaxes the stability frontier*. Less anchoring of expectations is necessary for any given wage indexation, or higher wage indexation is allowed for any given anchoring of expectations.

4.2. Simulation

Let us follow the most common NK assumption that the fraction γ of contracts renewed at each date t is $\gamma = 0.25$ as in Galì (2013). On a quarterly basis, this value means that the average duration of contracts is one year, an order of magnitude consistent with the evidence. For the parameter values used for simulations and $\gamma = 0.25$, the stability threshold for λ_t is 0.18: see Figure 5

Figure 5. Stability frontier



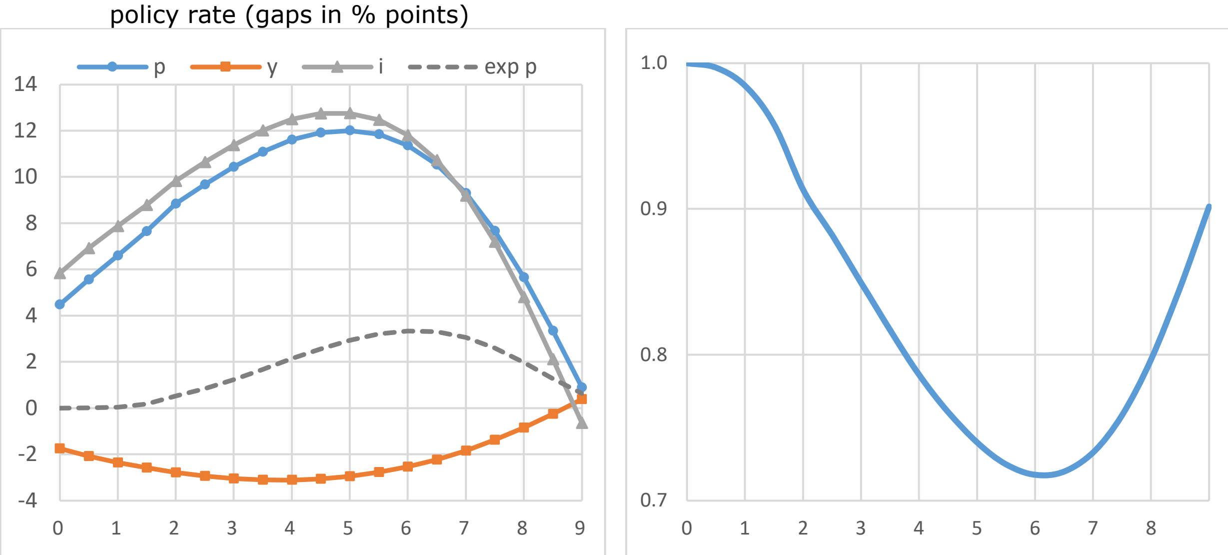
However, as noted above, the stability region can be extended by tightening the TR, i.e. by assigning a higher value to ϕ_π. The dotted line in Figure 5 illustrates this point after an increase of ϕ_π from 1.5 to 2.5.

Figure 6 repeats the simulation of Figure 3 with the standard TR and the addition of indexed wages. It shows that the system remains dynamically stable. The effect of the wage-price spiral is a further amplification of the inflation-gap process, with a higher peak (11.8% vs. 8.4%) and higher average gap per quarter (8.9% vs. 6%) (the same for the other variables). As a result, the de-anchoring of expectations is also greater (λ_t =

0.72 *vs.* 0.85 at the peak value of z_t = 11.8%) but it remains well above the stability threshold.

Figure 6. Simulation of system (10) with wage indexation (γ = 0.25)

(a) Inflation (p), expected inflation, output and policy rate (gaps in % points) (b) Share of anchored expectations



In Section 2 we showed that in the economic system represented by the standard NK model "one-period-ahead" rational expetations of inflation do not jeopardize the stabilization of inflation shocks in the long run, unless the shocks themselves are explosive. Here we have found that a particularly severe combination of shock persistence, strong reactivity of expectations to current inflation and wage indexation, such that condition (14) is violated, can indeed generate dynamic instability of inflation. To provide a more robust assessment of this finding, we now present the results of exploring the parameter space of our system (10)+(12).

5. Exploring the parameter space

The key parameters that regulate our system's dynamics are four: the reactivity of the DA function to inflation gaps σ, the degree of wage indexation γ, the inflation coefficient in the TR ϕ_π, and the initial inflation shock ε_0. Each parameter has been assigned the baseline value already employed above:

$\sigma = 0.1$, $\gamma = 0.25$, $\phi_\pi = 1.5$, $\varepsilon_0 = 5\%$

For each parameter we have also set a range of values containing the baseline one:

$\sigma \in [0, 0.45]$, $\gamma \in [0, 0.45]$, $\phi_\pi \in [0, 2.5]$, $\varepsilon_0 \in [0, 10\%]$

In the first place, given the baseline values of σ, γ, ϕ_π, to what extent can the system absorb shocks? The answer we draw from our simulations is that the extent is limited. However, the maximum initial shock ε_0 that the system can absorb is, empirically, large, i.e. in the neighbourhood of 9% above the target.

We then took the parameters in pairs and simulated the dynamics of the system for all combinations of values of the parameter pairs, with the other parameters left unchanged at their initial values. In the pairwise graphs that follow, the baseline values are indicated by a dot. The final state of each pairwise simulation was classified as "stability" or "instability" depending on whether or not condition (14) was satisfied after 20 runs. Final stability states are coded in yellow, final instability states are coded in blue.[10]

5.1. The role of the reactivity of inflation expectations

The first critical parameter for stability to be examined is the reactivity of the DA function σ. In fact, it regulates the endogenous de-anchoring of expectations λ_t, and if the latter falls rapidly during the inflation process (high σ) it may violate the threshold established by condition (14) for the given share of indexed wages γ.

Figure 7. The role of the reactivity of the DA function

(a) Pairing with entity of shocks (b) Pairing with wage indexation

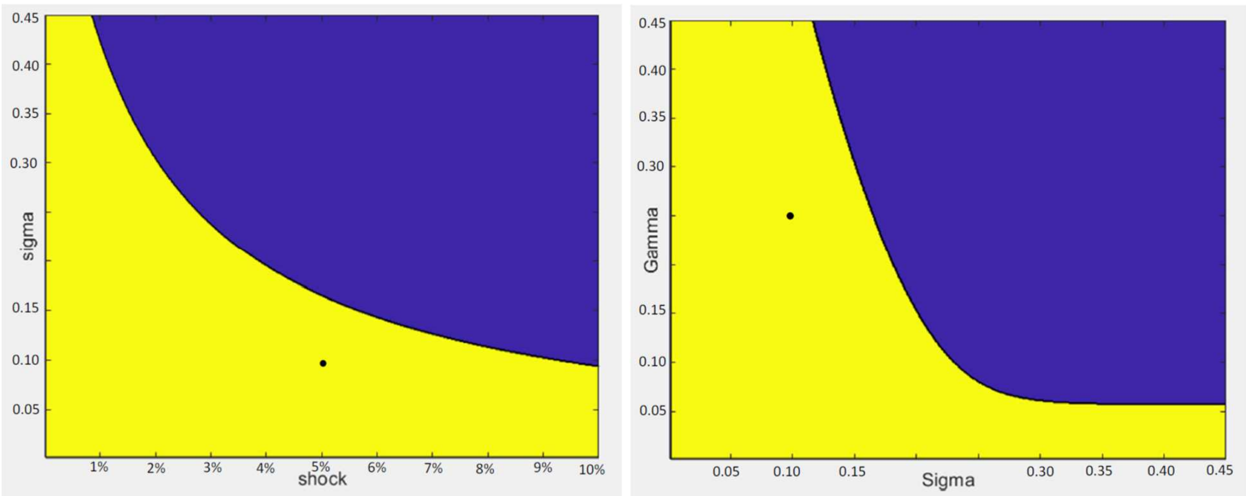


Figure 7(a) shows how stability is affected by different combinations of σ and initial inflation shocks ε_0 . The stability region contains the baseline values of the two parameters, and in fact the simulation reported in Figure 6 shows convergence to the inflation target. The boundary of the stability region draws an inverse relationship between the two parameters: as is intuitive, the system is resilient to larger shocks to the extent that the reactivity of expectations to ongoing inflation is lower. The flatter part of the boundary indicates that σ becomes almost immaterial for stability for values below 0.1. We may say that central banks that enjoy good (past) reputation, and hence low reactivity of agents' expecations to current deviations of inflation from the target, are safely located far from the boundary. On the other hand, the steeper part of the

---

[10] The simulations have been run for discrete steps of 0.001 for the parameters σ and γ and of 0.01 for the parameters ε_0 and ϕ_π.

boundary warns that the stability region shrinks rapidly as σ increases, i.e. smaller and smaller shocks can be accommodated.

A second important mechanism for the dynamic stability of our system is the interaction between σ and the degree of wage indexation γ, reported in Figure 7(b). In fact this exercise indirectly maps how the (λ_t, γ) stability condition (14) shifts in the parameter space. The baseline values $\sigma = 0.1$, $\gamma = 0.25$ lie in the stability region, as confirmed by the simulation reported in Figure 6.

The convex boundary of the stability region is consistent with equation (14): higher σ entails lower λ_t and hence lower γ is necessary for stability. The shape of the stability region indicates that the boundary is reached faster as σ increases for a given value of γ, than the other way round. In other words, high reactivity of expectations to current inflation poses a greater threat to stability than a high wage indexation when expectations are well anchored. On the other hand, the almost horizontal part of the boundary on the lower right-hand side of the map indicates that the system can absorb much greater reactivity of expectations when wage indexation is very low.

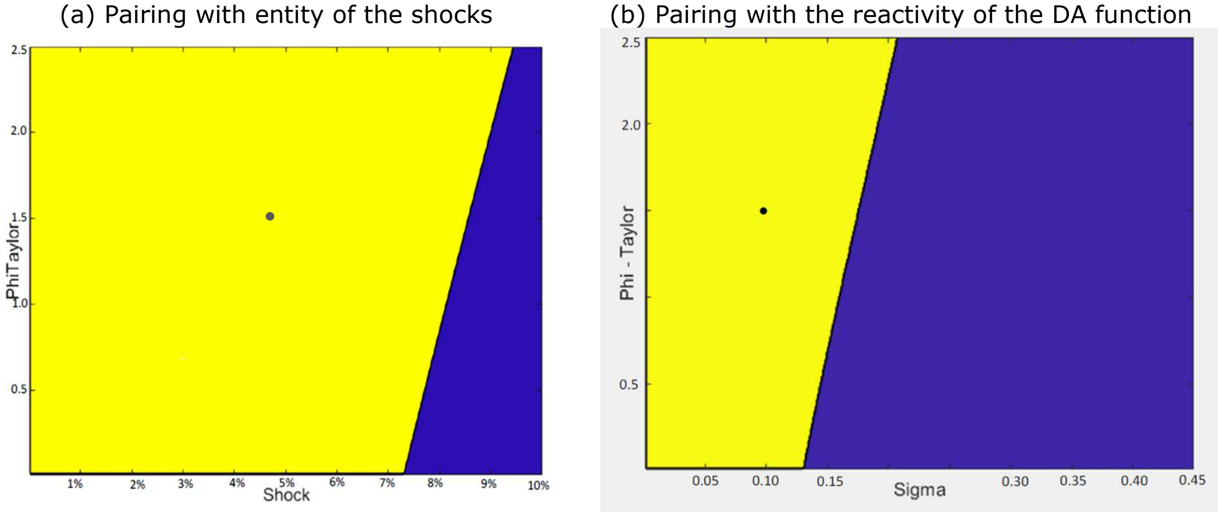
All in all, these exercises confirm that neither the de-anchoring of expectations nor the degree of wage indexation alone pose a major threat to inflation dynamics; rather, the threat comes from the combination of the two in particularly strong doses.

5.2. The role of monetary policy

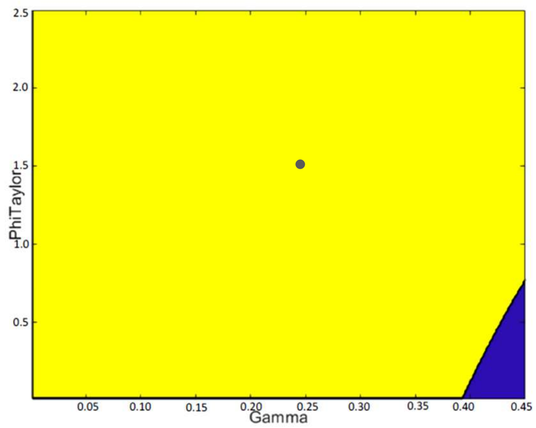
Let us now turn to an assessment of the stabilization capacity of monetary policy. The latter has been codified according to the inflation coefficient ϕ_π in the TR, *vis-à-vis* the other critical parameters that regulate the system's dynamics.

Figure 8. The role of monetary policy

(a) Pairing with entity of the shocks (b) Pairing with the reactivity of the DA function



(c) Pairing with wage indexation



The first result that we present in Figure 8(a) is the mapping of ϕ_π against the entity of the shocks ε_0. In line with our previous remarks, the baseline values of the two parameters ($\phi_\pi = 1.5$, $\varepsilon_0 = 5\%$) fall well within the stability region, and, as said above, to the extent that all the other parameters remain at their baseline values the system can absorb inflation shocks up to about 9% above the target. Larger shocks would require higher values of ϕ_π, i.e. a tighter reaction to inflation, as dictated by the upward-sloping boundary of the stability region.

On the other hand it should be noted that, in relation to lower values of ϕ_π , smaller shocks can be absorbed, but the stability region does not vanish. Recall what was said in section 2: in the standard model the Taylor Principle $\phi_\pi > 1$ is sufficient but not necessary for dynamic stability. This holds true in our model, too, for values of $\phi_\pi < 1$ that we may classify as "weak" inflation targeting.

The second important pairing to be examined is between ϕ_π and the reactivity of the DA function σ (Figure 8(b)). In fact, high σ determines a larger fall in the share of anchored expectations λ_t, but it was shown that higher values of ϕ_π reduce the threshold value of λ_t thereby enhancing the system's stability. This property is confirmed by the map between ϕ_π and σ. The boundary of the stability region shows that with $\phi_\pi = 1.5$ monetary policy can stabilize the initial shock of 5% to the extent that $\sigma < 0.2$. Higher values of σ would require steep increases of ϕ_π , but the same point made above about "weak" inflation targeting ($\phi_\pi < 1$) still applies if σ is sufficiently low.[11]

Finally, the same previous mechanisms can be examined from the side of the degree of wage indexation γ, in that γ could be too high with respect the given value of $\sigma = 0.1$ and the consequent fall of λ_t during the inflation process. Thus, the next map shows how ϕ_π should track γ in order to maintain dynamic stability (Figure 8(c)). The picture is quite different from the previous one. The stability region is much larger and barely

[11] This feature may offer an explanation of the puzzle due to the evidence of weak response to the 2021 upsurge of inflation (too little or no positive real interest rates) put forward by some critics of the Fed and the ECB (e.g. Orphanides 2023, Reis 2023) *vis-à-vis* the substantial stability of long-term inflation expectations.

affected by γ. With the standard value of $\phi_\pi = 1.5$, or even much lower, monetary policy can withstand values of γ much larger than 0.25. This result may suggest that of the two mainconcerns of central banks, the de-anchoring of inflation expectations should prevail over the wage-prices spiral.[12]

6. Concluding remarks

The post-pandemic resurgence of inflation has revived central banks' concerns about scenarios in which the de-anchoring of inflation expectations and the ignition of wage-price spirals trigger inflation dynamic instability, i.e. the inability to bring inflation back to target. We moved from the paradox that the most developed and used theoretical framework for monetary policy, at least in its standard specifications, can hardly replicate such scenarios. This framework represents an economy with remarkable built-in stabilization mechanisms, provided that the central bank adheres to the principles of inflation targeting and that shocks are not dynamically explosive on their own.

We showed that one way of providing a theoretical underpinning for the risks of inflation dynamic instability, and hence better policy guidance, is to reformulate the expectation formation mechanism in an economy where there is also a share of indexed wages. Our agents' attitude towards the central bank's willingness and/or ability to keep inflation on target follows the maxim "deeds not words". They revise their beliefs about the convergence of inflation to the target inductively, according to the evidence as it emerges. The more inflation deviates from the target, the larger the proportion of agents who de-anchor their expectations, and the stronger the boost to the inflation process. The result is a complex nonlinear dynamic system that depends on four key parameters: the reactivity of the de-anchoring mechanism, the share of indexed wages, the inflation coefficient in the central bank's policy rule, and the entity of the initial shock.

We examined the stability/instability regions of the system through simulations over a range of values of each parameter. Our results can be summarized as follows. First, the stability of the system is bounded. That is, there do exist combinations of parameter values such that inflation becomes dynamically unstable. Second, critical are combinations of large and persistent shocks (especially of the inverted-U shape we have been witnessing in the aftermath of the pandemic), high reactivity of the de-anchoring mechanism and a large share of indexed wages. Of the two main concerns of central banks, the de-anchoring of inflation expectations should prevail over the wage-price spiral. Third, in the face of the critical combinations of factors that trigger instability, a

[12] A recent IMF paper by Alvarez et al. (2022) examines the role of wage-price spirals during inflation episodes in a wide set of advanced countries since the 1960s, finding that "only a small minority of such episodes were followed by sustained acceleration in wages and prices. Instead, inflation and nominal wage growth tended to stabilize, leaving real wage growth broadly unchanged" (p. 1)

more hawkish stance on inflation expands the system's stability region, which however remains limited. Fourth, we can also say that these can be regarded as extreme events in the light of consensus empirical evidence, at least for the central banks that enjoy sufficiently high reputation, reflected in sufficiently low reactivity of inflation expectations.

We conclude with a twofold remark. On the one hand, our findings may lend support to the argument that the threats of inflation running out of control posed by the de-anchoring of expectations may be overestimated. First, such threats do not have a clear rationale in the mainstream macroeconomic framework for monetary policy, and they need better theoretical underpinnings beyond (severe) mismanagement of monetary policy. Moreover, our modified NK model still represents an economy that, except for extreme events (magnitude of shocks and/or de-anchoring of expectations), has robust in-built stabilization mechanisms (including "weak" inflation targeting), even in the presence of an endogenous de-anchoring process (plus possibly a wage-price spiral) capable of generating dynamic instability. On the other hand, the alternative hypothesis cannot be excluded that it is the underlying NK apparatus that overestimates the economy's inbuilt stabilization mechanisms – as it turned out to be the case with the Global Financial Crisis – with the consequence that better monetary policy modelling and advice for "hard times" would require deeper theoretical rethinking.

References

Alvarez J., Bluedorn J., Hansen N-J., Huang Y., Pugacheva E., Sollaci A. (2022), "Wage-Price Spirals: What is the Historical Evidence?", *IMF Working Paper*, WP/22/221.

Ari A., Mulas-Granados C., Mylonas V., Ratnovski L., Zhao W. (2023), "One Hundred Inflation Shocks: Seven Stylized Facts", *IMF Working Paper*, n. 190.

Arifovic J., Schmitt-Grohé S., Uribe M. (2017), "Learning to Live in a Liquidity Trap", *NBER Working Paper Series* n. 23725.

Bailey A. (2022), "Bank of England Monetary Policy Report Press Conference. Opening Remarks", May 5.

Bank of Ireland (2014), "Examining the sensitivity of inflation to output gap", *Quarterly Bulletin*, April.

Blanchard O. J., Cerutti E., Summers L. (2015), "Inflation and Activity. Two Explorations and their Monetary Policy Implications" *IMF Working Paper* n. 15/230.

Bullard J., Mitra K. (2002), "Learning about Monetary Policy rules", *Journal of Monetary Economics*, 96, pp. 40-69.

Calvo G. A. (1983), "Staggered Contracts in a Utility-Maximizing Framework", *Journal of Monetary Economics* 12, 383-398.

Carvalho C., Eusepi S., Moench E., Preston B. (2023), "Anchored Inflation Expectations", *American Economic Journal: Macroeconomics* 15, 1–47 https://doi.org/10.1257/mac.20200080

Christiano L., Trabant M., Walentin K. (2010), "DSGE Models for Monetary Policy Analysis", *NBER Working Paper* n. 16074.

Clarida R., Galì J., Gertler M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* 37, 1661-1707.

Clarida R., Galì J., Gertler M. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *Quarterly Journal of Economics* 115, 147-180.

De Guindos L. (2023), "The inflation outlook and monetary policy in the euro area", Keynote speech at the First Annual Conference organised by the Central Bank of Cyprus, 4 October https://www.ecb.europa.eu/press/key/date/2023/html/ecb.sp231004_1~ed2ed840b5.en.html

European Central Bank (2021), "Inflation Expectations and Their Role in Eurosystem Forecasting", *ECB Occasional Paper Series* n. 264.

European Central Bank (2022), Account of the Monetary Policy Meeting of the Governing Council of the European Central Bank held in Frankfurt an Main on Wednesday And Thursday 20-21 July 2022 https://www.ecb.europa.eu/press/accounts/2022/html/ecb.mg220825~162cfabae9.en.html.

Evans G. W., Honkapohja S. (2001), *Learning and Expectations in Macroeconomics*, Princeton, Princeton University Press.

Evans G. W., McGough B. (2018), "Interest Rate Pegs in New Keynesian Models", *Journal of Money Credit and Banking* 50, 939-965.

Galì J. (2008), *Monetary Policy, Inflation, and the Business Cycle. An Introduction to the NK Framework*, Princeton: Princeton University Press.

Galì J. (2013), "Notes for a New Guide to Keynes (I): Aggregate Demand, and Employment", *Journal of the European Economic Association* 11, 973-1003.

Galì J. Gertler M. (1999), "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics* 44, 195-222.

García-Schmidt M., Woodford M. (2019), "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis." *American Economic Review* 109(1), 86-120.

Garnier J., Wilhelmsen B. R. (2005). „The natural rate of interest and the output gap in the euro area. A joint estimation", *European Central Bank Working Paper Series* 546.

Gobbi L., Mazzocchi R., Tamborini R. (2019), "Monetary Policy, De-Anchoring of Inflation Expectations, and the "New Normal", *Journal of Macroeconomics* 61, Special Issue "Macroeconomics, Rationality and Institutions, 1-15

Gopinath G. (2022), "How Will the Pandemic and the War Shape Future Monetary Policy?", Jackson Hole Symposium, August 26. https://www.imf.org/en/News/Articles/2022/08/26/sp-gita-gopinath-remarks-at-the-jackson-hole-symposium.

Gürkaynak R.S., Swanson E., Levin A. (2010), "Does Inflation Targeting Anchor Long-Run Inflation Expectations? Evidence From the U.S., UK, and Sweden", *Journal of the European Economic Association* 8, 1208-1242.

Hooper P., Mishkin F. S., Sufi A. (2019), "Prospects for Inflation in a High Pressure Economy: Is the Phillips Curve Dead or Is It Jus Hibernating?", NBER Working Paper Series n. 25792.

Kurz M. (1994), "On the Structure and Diversity of Rational Beliefs", *Economic Theory*, 4, pp.383-397.

Kurz M. (2011), "Symposium: On the Role of Market Beliefs in Economic Dynamics, an Introduction", *Economic Theory*, 47, pp.189-204.

Lane P. R. (2022), " Monetary Policy in the Euro Area: The Next Phase", Remarks for high-level panel "High Inflation and Other Challenges for Monetary Policy", Annual Meeting 2022 of the Central Bank Research Association (CEBRA), Barcelona.

Lane P. R. (2024), “Disinflation in the euro area”, Speech for the Hutchins Center on Fiscal & Monetary Policy at the Brookings Institution, 8 February.

Laubach T., Williams J. C. (2003). “Measuring the natural rate of interest”. *The Review of Economics and Statistics* 85, 1063–1070

Lorenzoni G., Werning I. (2023), "Wage Price Spirals", MIT, January, mimeo.

Luk P., Vines D (2015), “Optimal Monetary and Fiscal Policy in an Economy with Endogenous Public Debt”, *CEPR Discussion Paper Series* n. 10580.

Oinonen S., Palovitta M. (2014), “Updating the Euro Area Phillips Curve: The slope has increased”, *Bank of Finland Discussion Paper* n. 31.

Orphanides A. (2023), "The Forward Guidance Trap", IMES, Bank of Japan, Discussion Paper, n. 2023-E-6.

Powell J. H. (2022), "Monetary Policy and Price Stability", Speech at the Policy Symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, August 26.

Reis R. (2023), " What Can Keep Euro Area Inflation High?", *Economic Policy*, July, pp. 495-517.

Riggi M. Venditti F. (2014), “Surprise! Euro Area inflation has fallen”, *Bank of Italy Occasional Paper* n. 237.

Rudd J. B. (2021), "Why Do We Think That Inflation Expectations Matter for Inflation? (And Should We?)", Finance and Economics Discussion Series, Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C., n. 2021-062.

Schmidt S., Wieland V. (2013), "The NK Approach to Dynamic General Equilibrium Modeling: Models, Methods, and Macroeconomic Policy Evaluation," in: P.B. Dixon and D.W. Jorgenson (eds.), *Handbook of Computable General Equilibrium Modeling*.

Smets F., Christoffel K., Coenen G., Motto R., Rostagno M. (2010), "DSGE models and their use at the ECB", *Journal of the Spanish Economic Association* 1, 51-65.

Smets F., Wouters R. (2003), “An estimated stochastic dynamic general equilibrium model of the euro area”, *Journal of European Economic Association* 1, 1123–1175.

Taylor J. B. (1980), “Aggregate dynamics and staggered contracts”, *Journal of Political Economy* 88, 1–23.

Taylor J. B. (1993), “Discretion vs. policy rules in theory and practice”, *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Woodford M. (2003), *Interest and Prices*, Princeton, Princeton University Press.

Appendix

The standard NK model for policy analysis consists of three equations determining at each date t, respectively, the level of inflation (Phillips Curve (A1)), the level of output (IS equation (A2)), and the policy rate (Taylor Rule (A3)). The three equations are usually expressed as log-rates of deviation form their SS values (or more commonly as "gaps").[13]

(A1) $\pi_t = \beta\pi^e{}_t + \kappa(y_t - y^*) + u_{\pi t}$

(A2) $y_t = y^* - \eta(i_t - i^*)$

(A3) $i_t = i^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y^*)$

The inflation shock $u_{\pi t}$ can be interpreted as an unanticipated increase in the unit price of any variable input (e.g. energy as well as labour), according to the following considerations. The NK PC is commonly derived as the log-linear combination at date t of the fraction of monopolistically competitive firms that reoptimize their price-quantity decision for any observed shock, and the complementary fraction of those who do not (the latter fraction is thus a measure of the "price stickiness" in the system).

The basis of the optimal pricing decision is the equality between marginal revenue and marginal cost, leading to the standard result of the supply price exceeding the marginal cost by a "mark up" which is a function of the elasticity of demand. Changes in the unit price of any variable input are thus transferred to the supply price taking into account the elasticity of demand and the technical coefficient of the input in the production function.

For precision, therefore, the shocks $u_{\pi t}$ might be weighed by a parameter (as is the case with the output gaps), depending on the specific characteristics of the demand function, the production function, and on the fraction of optimising firms. Disregarding this quantitative detail, however, does not affect our treatment in an essential manner.

The reduced form of the three equations is given by the system (1) in the text, which is reproduced here for convenience

$$\hat{\pi}_t = a_1(\pi^e{}_{t+1} - \pi^*) + a_2 u_{\pi_t}$$
$$\hat{y}_t = b_1(\pi^e{}_{t+1} - \pi^*) + b_2 u_{\pi t}$$
$$\hat{i}_t = c_1(\pi^e{}_{t+1} - \pi^*) + c_2 u_{\pi t}$$

The coefficients a_n, b_n, c_n (n = 1, 2) have the following expressions:

$$a_1 = [\beta(1 + \eta\phi_y) + \eta\kappa]A,\ a_2 = (1 + \eta\phi_y)A$$
$$b_1 = [\eta(1 - \beta\phi_\pi)]A,\ b_2 = -\eta\phi_\pi A$$
$$c_1 = [\eta(\kappa\phi_\pi + \phi_y) + \beta\phi_\pi]A,\ c_2 = \phi_\pi A$$
$$A = [1 + \eta(\kappa\phi_\pi + \phi_y)]^{-1}$$

---

[13] For a reference treatment see Galì (2008).

The parameter values have been set as follows.

- η. Direct econometric estimates of the elasticity of expenditure to the interest-rate gap yield lower values between 0.2 and 0.3 (e.g. Smets and Wouters 2003; Laubach and Williams, 2003; Garnier and Wilhelmsen, 2005). Hence its value has been set at $\eta = 0.3$.
- r^*, β . According to the NK standard model, the equilibrium value of the natural rate is $r^* = 1/\beta - 1$. The consensus value $r^* = 2\%$, dating to the original specification of the Taylor Rule (Taylor, 1993), yields the commonly used value of $\beta = 0.98$.
- κ. Calibration of the slope of the PC κ in NK models yields very low values. For instance, a common order of magnitude of firms not adjusting prices in the face of shocks is around 75% (e.g. Smets and Wouters 2003, Luk and Vines 2015); then, the Calvo equation with $\beta = 0.98$ yields $\kappa = 0.09$. Direct econometric estimates of the slope of the PC equation over the last decades typically provide higher values, in the range of 0.5. However, after Blanchard et al. (2015), various works have produced evidence of "flatter" PC, with κ falling between 0.2 and 0.3. More recent works, mostly based on European data, find a "steepening" of the PC in the aftermath of the Great Recession (e.g. Riggi and Venditti 2014, Bank of Ireland 2014, Oinonen and Paloviita 2014), with the estimated slope around 0.4. A mid value among these estimates has been chosen, i.e. $\kappa = 0.3$.
- ϕ_y, ϕ_π. The Taylor Rule parameters have been set according to the usual benchmark of Taylor's (1993) original empirical model, $\phi_y = 0.5$, $\phi_\pi = 1.5$.