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Hans Gersbach, Fikri Pitsuwan, Giovanni Valvassori Bolgè

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Abstract

We examine democratic public-good provision with heterogeneous legislators. Decisions are taken by majority rule and an agenda-setter proposes a level of the public good, taxes, and subsidies. Members are heterogeneous with respect to their benefits from the public good. We find that, depending on the status quo public-good level, the agenda-setter will form a coalition with the agents who most desire, or least desire, the public good, and we may observe 'strange bedfellow' coalitions. Moreover, public-good provision is a non-monotonic function of the status quo public-good level. In the dynamic setting, public-good provision fluctuates endogenously, even if the agenda-setter stays the same over time. Moreover, the more polarized the legislature is, the higher is the volatility of public-good provision and the longer it may take for a society to recover from negative shocks to public-good provision. We illustrate these findings for a two-party system with polarized parties.

JEL-Codes: C730, D720, H500.

Keywords: legislative bargaining, coalition, public goods, polarization, resilience.

Hans Gersbach
KOF Swiss Economic Institute
ETH Zurich / Switzerland
hgersbach@ethz.ch

Fikri Pitsuwan
Center of Economic Research at
ETH Zurich / Switzerland
fpitsuwan@ethz.ch

Giovanni Valvassori Bolgè
Center of Economic Research at
ETH Zurich / Switzerland
gvalvassori@ethz.ch

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1 Introduction

In this paper we examine public-good provision in representative democracies with heterogeneous agents. The legislature operates under standard democratic governance rules. First, some members of the legislature are recognized as agenda-setters, and decisions are taken by simple or supermajority rule.[1] Second, the parliament can levy taxes and, third, it can grant subsidies. Regarding taxation, we adopt the principle of 'horizontal equity' present in most constitutions, according to which equal income should be taxed equally.[2] Regarding subsidization, most constitutions allow for transfers that target a subset of the population, while excluding other parts of the population.[3]

We use a simple model of legislative decision-making for the provision of a public good, subject to the three aforementioned institutional features. Parliamentary members (henceforth simply called 'agents') are heterogeneous with respect to their benefits from the public good (henceforth referred to as agents' 'values' for the public good). One agent is selected to be the agenda-setter. S/he makes a take-it-or-leave-it policy proposal consisting of a level of the public good, a tax rate, and an array of subsidies to agents. For the policy to be implemented, the agenda-setter needs the approval of a simple majority of agents. If the proposal is rejected, the status quo is implemented.

We consider the static and dynamic versions of the legislative game and characterize the equilibrium levels of public-good provision, taxes, and subsidies as a function of the status quo public-good level. We focus on how heterogeneity affects public-good provision. In particular, we address how public-good provision evolves in two-party systems when party polarization increases. Moreover, we investigate whether more polarized legislatures are more or less resilient after shocks to public-good provision.

Our main insights are as follows: In the static version, there exists a unique proposal made by the agenda-setter, in which s/he proposes a level of the public good, a tax rate, and an array of subsidies to form a coalition that subsequently adopts his/her policy proposal. Each status quo public-good level partitions potential members of the winning coalition into three sets: those who can be convinced by an increase in public-good provision; those who require a strictly positive transfer; and those who will accept any

[1] See Chapter 5 in Schwartzberg (2013) for a thorough discussion on majority and supermajority rules procedures in parliaments.

[2] "To Equall Justice, appertaineth also the Equall imposition of Taxes" (Thomas Hobbes, 1651) as cited in Elkins (2006); "[t]axation shall be equal and uniform" (The Texas Constitution, Article 8, Sec. 1(a)), as cited in Gersbach et al. (2013).

[3] On the one hand, this applies at the institutional level, as in the case of subnational entities, such as states in federal countries (for a comparative analysis, see Ter-Minassian (1997)). On the other hand, this can also be true at the individual level, making a subset of the population eligible for a particular transfer or subsidy (e.g., environmental subsidies; see, *inter alia*, Gillingham et al. (2006)).

proposal.

Intuitively, starting from a low public-good level, marginal increases will yield high marginal returns in terms of utility and suffice to convince other agents. However, these returns are diminishing; hence, when the status quo public-good level is higher, the agenda-setter needs to start paying strictly positive transfers in order to win their support. Once the status quo public-good level is sufficiently high, all members of the coalition are convinced through strictly positive transfers, and the public-good level equals the sum of the marginal benefits of the members of the coalition.

Depending on the status quo public-good level, the agenda-setter will form a coalition with the agents who most desire, or least desire, the public good, and we may observe 'strange bedfellow' coalitions. An agenda-setter with a strong (weak) desire for the public good may form a coalition with agents who have a weak (strong) desire for it. In fact, there exists a cutoff value for the status quo public-good level at which the agenda-setter's choice changes discontinuously. When the status quo public-good level is higher than this cutoff, it is optimal to form a coalition with the agents having the lowest values for the public good (henceforth called 'lowest' coalition). When the level is lower than the cutoff, it is optimal to form a coalition with the agents having the highest values for the public good (henceforth called 'highest' coalition). Therefore, the public-good level chosen by any agenda-setter is a non-monotonic function of the status quo public-good level.

We then consider the same model in a simple dynamic setting, where the public-good level provided at time $t-1$ becomes the status quo at time t. Agents are assumed to care only about the payoffs in the current period. This reveals a variety of further insights. First, public-good provision fluctuates. Once the public-good level is sufficiently high, public-good provision oscillates between the sum of the marginal benefits of the highest and lowest coalitions. This result does not hinge on the agenda-setter's preference for the public good, it is an endogenous feature of the coalition formation problem s/he faces. In other words, democratic public-good provision creates *endogenous volatility* of public-good levels.

We then apply our model to party polarization and assume that agents' values for the public good can only be either high or low. The difference between these two levels indicates the degree of polarization. We show that the volatility of public-good provision becomes more pronounced when polarization is higher. Moreover, public-good provision is less resilient in the following sense: Suppose that a shock (e.g., a pandemic, a natural catastrophe, or an economic crisis) lowers the public-good level. Then, higher party polarization results in slower recovery. In other words, it takes longer in polarized parliaments for public-good provision to return to pre-shock levels. We illustrate these findings for a two-party system with polarized parties and constant fluctuations of agenda-setting power.

The rest of the paper is structured as follows: Section 2 discusses the related literature. Section 3 introduces the model. The main results are presented in Section 4. Section 5 demonstrates some implications of the model. Section 5.2 considers the application to a two-party system with an alternating agenda-setter. Section 6 discusses further directions and concludes.

2 Related Literature

This paper contributes to three branches of the literature on political economy. The first branch develops a positive theory of public-good provision in a legislative bargaining framework, as in the seminal work by Baron (1996), Persson et al. (2000), Battaglini and Coate (2007) and Battaglini and Coate (2008).

We contribute to this literature by introducing heterogeneity from benefits in public-good consumption. This heterogeneity significantly complicates the analysis, precluding the use of recursive techniques to solve for the equilibrium proposal. Yet, adding heterogeneity yields new and interesting insights into the coalition formation problem and public-good provision in democracy. The dynamic version of our model with a changing status quo level pinpoints phenomena like endogenous volatility and differing degrees of resilience.

The second branch we contribute to is distributive politics and legislative bargaining. Following the pioneering work by Baron and Ferejohn (1989), models of multilateral bargaining have been used to analyze many different institutions: *inter alia*, bicameralism in Diermeier and Myerson (1999); government formation in Bassi (2013); bankruptcy in Eraslan (2008); and public goods in Volden and Wiseman (2007).[4] These models study the strategic interaction of players who have to agree, though not necessarily unanimously, on a proposal made by an agenda-setter. Some more recent literature considers legislative bargaining with an endogenous status quo (for an extensive survey, see Eraslan et al. (2022)). The above contributions study how the endogeneity of the status quo affects proposal-making and allocations from a dynamic perspective. Whereas we consider a simplified setting in which agents are assumed to care only about the current public-good level, we introduce heterogeneity regarding their benefits from public-good consumption in legislative bargaining, which entails the phenomena discussed above. Our paper is complementary to Drazen and Ilzetzki (2023), who show that pork from "favors" in the

[4] For an extensive treatment, see Eraslan and Evdokimov (2019).

presence of heterogeneous benefits from public goods enables better-informed legislative agenda-setters to convince less well-informed legislators of policy changes.

Lastly, we contribute to a large body of research on the effects of polarization. In the US, rising polarization has been a matter for concern among policymakers and researchers for several decades.[5] We contribute to this literature by examining how polarization regarding the desire for public goods affects public-good provision. We also tailor a specific application of our model to a polarized two-party system to explore the static and dynamic implications of an increase in party polarization. This may be particularly relevant to analyzing the situation in the US, where there is some consensus that polarization is a phenomenon mainly observed in Congress and less so in the electorate.[6]

3 Model

Consider a simple model of legislative decision-making with $n \geq 3$ (odd) members of parliaments (henceforth, 'agents') indexed by $i \in N = \{1, \dots, n\}$. Each agent is endowed with one unit of income that the government taxes to raise its revenue. The revenue can be used for two purposes: (i) to finance public-good provision, and (ii) to provide district-specific subsidies. The government policy is then described by a vector $(\tau, g, \boldsymbol{s})$, where $\tau \in [0, 1]$ denotes the tax rate, $g \geq 0$ denotes the level of public good, and $\boldsymbol{s} = (s_1, \dots, s_n) \geq 0$ denotes the vector of district-specific subsidies. For feasibility, a policy vector $(\tau, g, \boldsymbol{s})$ must also satisfy the resource constraint $g + \sum_{i \in N} s_i \leq n\tau$.

Agents have quasi-linear preferences over income and the public good, but are potentially heterogeneous with respect to the value they attach to the public good. Specifically, given a policy $(\tau, g, \boldsymbol{s})$, the payoff of agent i is given by

$$u(\tau, g, \boldsymbol{s}; \theta_i) = (1 - \tau) + s_i + \theta_i \ln g, \tag{1}$$

where $\theta_i \in (0, 1)$ denotes agent i's value for the public good.[7] As we are considering a legislative setting, θ_i can be thought of as the average value of the members of the constituency represented by agent i. Without loss of generality, the agents are indexed in non-decreasing order of their value for the public good, i.e., $\theta_1 \leq \theta_2 \leq \dots \leq \theta_n$.

The government's policy of tax rate, level of the public-good, and subsidies is determined by a legislative decision-making process consisting of the following sequence of events:

[5] See, *inter alia*, Poole and Rosenthal (1984), Binder (1999), Jones (2001), Fiorina et al. (2005), McCarty et al. (2006).

[6] See the discussion in Gersbach et al. (2021).

[7] We adopt the convention that $\ln 0 = -\infty$.

1. One of the agents, $a \in N$, is the agenda-setter.

2. The agenda-setter a makes a take-it-or-leave-it policy proposal, $(\tau, g, \boldsymbol{s})$.

3. The legislature votes on the proposal. If at least a simple majority of the legislators votes in favor, the policy $(\tau, g, \boldsymbol{s})$ is implemented. If not, a default outcome $(g^\circ/n, g^\circ, \mathbf{0})$ for some status quo public-good level $0 \leq g^\circ \leq n$ is implemented.

The agenda-setter's problem, which we now derive, is to propose a policy that maximizes his/her payoff subject to the voting decisions of the legislature. Consider first the voting decisions of the other agents, given a proposal $(\tau, g, \boldsymbol{s})$ and a status quo public-good level g°. Each agent $i \neq a$ votes in favor of the proposal if his/her payoff from the proposed policy is no lower than the payoff from the status quo:

$$(1 - \tau) + s_i + \theta_i \ln g \geq (1 - g^\circ/n) + \theta_i \ln g^\circ. \tag{2}$$

This is the *incentive compatibility* constraint of agent i. The agenda-setter's problem is then to propose a policy $(\tau, g, \boldsymbol{s})$ to maximize his/her payoff, subject to the feasibility constraints and subject to having the incentive compatibility constraints hold for at least $(n-1)/2$ other agents.[8] Because of heterogeneity, however, this can be modeled as if the agenda-setter were choosing both the policy and a set of agents, a minimal winning coalition, for which the incentive compatibility constraints are satisfied.
Formally, let $q \equiv (n+1)/2$, and define

$$\mathcal{Q} \equiv \{Q \subseteq N : a \in Q \text{ and } |Q| = q\}$$

as the set of all minimal winning coalitions that include a. The agenda-setter's problem $\mathcal{P}(g^\circ)$ for a given status quo public-good level $g^\circ \in [0, n]$ is

$$\begin{aligned} v_a(g^\circ) = \max_{(\tau, g, \boldsymbol{s}, Q)} & \; (1 - \tau) + s_a + \theta_a \ln g \\ \text{subject to } \; & g + \sum_{i \in N} s_i \leq n\tau, \; \boldsymbol{s} \geq 0, \; \tau \in [0,1], \; g \geq 0, \\ & \text{and for some } Q \in \mathcal{Q}, \\ (1 - \tau) + s_i + \theta_i \ln g \geq & \; (1 - g^\circ/n) + \theta_i \ln g^\circ \quad \text{for all } i \in Q \setminus \{a\}. \end{aligned} \tag{$\mathcal{P}(g^\circ)$}$$

Some remarks on notation are in order. For any subset of agents $A \subseteq N$, let $\Theta(A) = \sum_{i \in A} \theta_i$ be A's (aggregate) value for the public good. The agenda-setter's choice of

[8]Note that since the status quo public-good level is feasible, the agenda-setter's incentive compatibility constraint will be satisfied. In other words, s/he can do no worse than the default outcome.

$Q \in \mathcal{Q}$ turns out to crucially depend on $\Theta(Q)$, coalition Q's value for the public good. In particular, the coalitions with extreme values for the public good will play a major role in the remainder of the paper. To this end, define

$$\underline{Q} \equiv \arg\min_{Q \in \mathcal{Q}} \Theta(Q) \quad \text{and} \quad \overline{Q} \equiv \arg\max_{Q \in \mathcal{Q}} \Theta(Q),$$

to be, respectively, the 'lowest' and 'highest' coalitions as measured by their value for the public good.

4 The Agenda-Setter's Problem

In this section, we solve the agenda-setter's problem $\mathcal{P}(g^\circ)$ in the following four main steps, denoting the solution by $(\tau^*, g^*, \boldsymbol{s}^*, Q^*)$.[9]

1. Proposition 1 shows that the tax rate must be maximal, so the problem simplifies to choosing $(g^*, \boldsymbol{s}^*, Q^*)$.

2. We define the auxiliary problem $\mathcal{P}^Q(g^\circ)$ as the agenda-setter's problem constrained to a fixed coalition Q and fully characterize its solution, denoted by $(g^Q, \boldsymbol{s}^Q)$, in Theorem 1.

3. We then show in Proposition 4 that an agenda-setter's optimal choice of Q for the full problem is either $\underline{Q}$ or $\overline{Q}$.

4. The above results taken together imply that $(\tau^*, g^*, \boldsymbol{s}^*, Q^*)$ is either

$$(1, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q}) \quad \text{or} \quad (1, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q}).$$

   We then show in Theorem 2 that there exists a unique status quo public-good threshold value, $\hat{g}$, such that $(1, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q})$ is a solution if the status quo is lower than $\hat{g}$, whereas $(1, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q})$ is a solution if the status quo is higher than $\hat{g}$. This completes the characterization.

We begin by showing the existence of a solution to the agenda-setter's problem and some preliminary observations on the solution.

Proposition 1. *A solution to $\mathcal{P}(g^\circ)$ exists and for any solution $(\tau^*, g^*, \boldsymbol{s}^*, Q^*)$, the following holds: (i) $g^* > 0$, (ii) $g^* + \sum_{i \in N} s_i^* = n\tau^*$, and (iii) $\tau^* = 1$.*

[9] Of course, the solution depends on g°: $\tau^*(g^\circ), g^*(g^\circ), \boldsymbol{s}^*(g^\circ), Q^*(g^\circ)$. When there is no risk of confusion, we suppress explicit dependence on g°.

Existence follows from standard arguments, since the objective function is continuous on the relevant domain, which is compact. Observations (i) and (ii) of Proposition 1 hold because the agenda-setter's utility is logarithmic in g and is strictly increasing in g and s_a. For (iii), the agenda-setter will find it optimal to propose the maximum tax rate because the tax is levied on all n agents, whereas the revenue is distributed among $(n+1)/2$ agents at the most.[10] The problem then simplifies to choosing $(g, \boldsymbol{s}, Q)$.

4.1 The Auxiliary Problem

The next step is to solve an auxiliary problem in which the agenda-setter is constrained to a fixed coalition, i.e., the incentive compatibility constraints for all agents in a *given* $Q \in \mathcal{Q}$ need to hold. Denote $R \equiv Q \setminus \{a\}$ as the set of agents in Q other than the agenda-setter, and let $r \equiv q - 1 = (n-1)/2$. Since agents are indexed by their values for the public good, any R will have the structure

$$R = \{i_1, i_2, \ldots, i_r\} \quad \text{with} \quad \theta_{i_1} \leq \cdots \leq \theta_{i_r}.$$

Formally, for a given coalition $Q \in \mathcal{Q}$ and a status quo public-good level $g^\circ \in [0, n]$, the auxiliary problem $\mathcal{P}^Q(g^\circ)$ is

$$\begin{aligned} v_a^Q(g^\circ) = \max_{(g,\boldsymbol{s})} \; & s_a + \theta_a \ln g \\ \text{subject to } \; & g + \sum_{i \in N} s_i = n, \; \boldsymbol{s} \geq 0, \; g \geq 0 \\ & \text{and} \\ & s_i + \theta_i \ln g \geq (1 - g^\circ/n) + \theta_i \ln g^\circ \quad \text{for all } i \in R. \end{aligned} \tag{$\mathcal{P}^Q(g^\circ)$}$$

Denote a solution of $\mathcal{P}^Q(g^\circ)$ by $(g^Q, \boldsymbol{s}^Q)$. The first step in solving the auxiliary problem is to note that $(g^Q, \boldsymbol{s}^Q)$ induces a partition of R into three sets:

$$R = K^Q \cup L^Q \cup M^Q.$$

The interpretation of these sets is as follows. The agenda-setter has two options for obtaining majority support: either through direct subsidies (s_i) or via an increase in the public-good level (g). The first set, K^Q, consists of the agents whom the agenda-setter compensates via direct subsidies. The second set, L^Q, is composed of those whom

[10] Our results still apply if: (i) there is an upper limit on the tax rate, $\bar{\tau} < 1$, which might be enshrined in the constitution (for a discussion on constitutional bounds to taxation, see Gersbach et al. (2019); (ii) taxation is distortionary; for each dollar raised through income tax, only a fraction $\gamma < 1$ can be spent on public-good provision and transfers. More is discussed in Section 6.

the agenda-setter convinces by increasing the public-good level. The third set, M^Q, consists of the agents who are satisfied with the proposal without being compensated, i.e., needing neither direct subsidies nor any increase in the public-good level. The following proposition characterizes the structure of these sets:

Proposition 2. *For some $k, l \in \{0, 1, \ldots, r\}$ with $k \leq l$,*

$$K^Q(g^\circ) = \{i_1, \ldots, i_k\},$$
$$L^Q(g^\circ) = \{i_{k+1}, \ldots, i_l\},$$
$$M^Q(g^\circ) = \{i_{l+1}, \ldots, i_r\},$$

with the convention that $i_0 = 0$ and $\{i_x, i_y\} = \varnothing$ if $x > y$. The indices k and l are increasing in g°. Moreover, agents in $L^Q(g^\circ)$ have the same valuation. If the values are all distinct, then $L^Q(g^\circ)$ has at most one member.

Proposition 2 shows that the size and composition of these sets depend on the status quo g°. In particular, it is a consequence of which incentive compatibility constraints in $\mathcal{P}^Q(g^\circ)$ bind. Intuitively, suppose first that the status quo public-good level is undesirably low. Since utility functions are logarithmic in the public good, the implementation of the status quo will yield a very low utility to the agents. Therefore, in this case the agenda-setter is essentially solving a 'dictator' problem, where none of the incentive compatibility constraints matter. Hence, the agenda-setter proposes his/her individually optimal public-good level $g^Q = \theta_a$ and all other agents will accept it without requiring subsidies. That is, all agents are in set M^Q.

For this to be a solution, g° must be such that the incentive compatibility constraints of all agents are slack. In particular, this must hold for the agent with the lowest value, agent i_1. As g° increases, there will be a value of the status quo public-good level such that the incentive compatibility constraint of agent i_1 will be just binding:

$$\theta_{i_1} \ln \theta_a = (1 - g^\circ/n) + \theta_{i_1} \ln g^\circ. \tag{3}$$

At this status quo public-good level, agent i_1 is now in set L, whereas all other agents are still in set M^Q. An analogous argument explains the 'transition' from set L^Q to set K^Q. Suppose now that, for some values of g^Q and g°, with $g^Q \geq g^\circ$, the incentive compatibility constraint of agent i_j is binding:

$$\theta_{i_j} \ln g^Q = (1 - g^\circ/n) + \theta_{i_j} \ln g^\circ. \tag{4}$$

Then, agent θ_{i_k}, with $\theta_{i_k} < \theta_{i_j}$ can be neither in L^Q nor in M^Q, otherwise his/her incentive compatibility constraint would be violated. Therefore, it must be the case that $s_{i_k} > 0$ and s/he is in set K^Q.

Proposition 2 shows the qualitative structure of the sets. In particular, it demonstrates that the 'transition' between sets is ordered: as g° increases, agent i_1 will be the first to move from set M^Q to set L^Q, and then from L^Q to K^Q. As g° increases further, the other agents will follow. The next step is to investigate the quantitative structure of these sets for any g°; i.e., to derive the indices $k = k^Q(g^\circ)$ and $l = l^Q(g^\circ)$. To this end, define $\bar{g}(\Theta; \theta_i)$ as the status quo public-good level which makes agent i indifferent with some public-good level $\Theta \in (0, n)$. That is, $\bar{g}(\Theta; \theta_i)$ is the unique g° that solves $\theta_i \ln \Theta = (1 - g^\circ/n) + \theta_i \ln g^\circ$.

The next proposition pins down the structure of K^Q, L^Q, and M^Q as functions of g°.

Proposition 3. *Suppose the values for the public good are all distinct. Then the indices $k = k^Q(g^\circ)$ and $l = l^Q(g^\circ)$ are given by*

$$k^Q(g^\circ) = \begin{cases} 0 & \text{if} \quad 0 \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_1}) \\ 1 & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_1}) \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1} + \theta_{i_2}; \theta_{i_2}) \\ 2 & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1} + \theta_{i_2}; \theta_{i_2}) \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1} + \theta_{i_2} + \theta_{i_3}; \theta_{i_3}) \\ \vdots & \text{if} \quad \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ r & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_r}; \theta_{i_r}) \leq g^\circ \leq n \end{cases} \tag{5}$$

and

$$l^Q(g^\circ) = \begin{cases} 0 & \text{if} \quad 0 \leq g^\circ < \bar{g}(\theta_a; \theta_{i_1}) \\ 1 & \text{if} \quad \bar{g}(\theta_a; \theta_{i_1}) \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_2}) \\ 2 & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_2}) \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1} + \theta_{i_2}; \theta_{i_3}) \\ \vdots & \text{if} \quad \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ r & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_{r-1}}; \theta_{i_r}) \leq g^\circ \leq n. \end{cases} \tag{6}$$

Given $g^\circ \in [0, n]$, Proposition 2 and Proposition 3 determine the exact structures of $K^Q(g^\circ)$, $L^Q(g^\circ)$, and $M^Q(g^\circ)$. That is, for each $g^\circ \in [0, n]$ and each $i \in R$, the two propositions determine whether i receives a strictly positive subsidy and whether i's incentive compatibility constraint binds at the solution $(g^Q, \boldsymbol{s}^Q)$. The characterization of the solution to the auxiliary problem follows.

Theorem 1. *The solution to $\mathcal{P}^Q(g^\circ)$ is $(g^Q, \boldsymbol{s}^Q)$, where g^Q is given by*

$$g^Q = \begin{cases} \theta_a + \sum_{i \in K^Q(g^\circ)} \theta_i & \text{if} \quad L^Q(g^\circ) = \varnothing \\ G(g^\circ; \theta_{i_l}) & \text{if} \quad L^Q(g^\circ) \neq \varnothing, \end{cases}$$

with $G(g^\circ;\theta_i) \equiv g^\circ \exp\left\{\frac{1-g^\circ/n}{\theta_i}\right\}$, *and* $\boldsymbol{s}^Q$ *is given by*

$$s_i^Q = \begin{cases} (1-g^\circ/n) + \theta_i\left(\ln g^\circ - \ln g^Q\right) & \text{if} \quad i \in K^Q(g^\circ) \\ 0 & \text{if} \quad i \notin K^Q(g^\circ) \end{cases}$$

and $s_a^Q = n - g^Q - \sum_{i\in K^Q(g^\circ)} s_i^Q$. *The solution is unique if the values are distinct.*

Some remarks are in order. First, the public-good level g^Q increases with the status quo, alternating between constant and strictly increasing regimes. When $L^Q(g^\circ) = \varnothing$, the public-good level is constant and equal to the sum of the values of the agenda-setter and the agents in set $K^Q(g^\circ)$. When $L^Q(g^\circ) \neq \varnothing$, the public-good level adjusts such that the incentive compatibility constraint of the agent in $L^Q(g^\circ)$ is just binding.

Second, as the status quo public-good level increases, the set of agents receiving strictly positive subsidies expands in an ordered way. In fact, according to Proposition 2, an agent with a particular value for the public good will only receive a strictly positive transfer when an agent with a lower value does the same. The following example illustrates these features.

Example 1. Consider the auxiliary problem with $Q = \{1,2,3\}$ and $a = 1$ as the agenda-setter. Moreover, assume $\theta_1 < \theta_2 < \theta_3$. Figure 1 and Figure 2 illustrate the equilibrium public-good provision and transfers, respectively.[11]

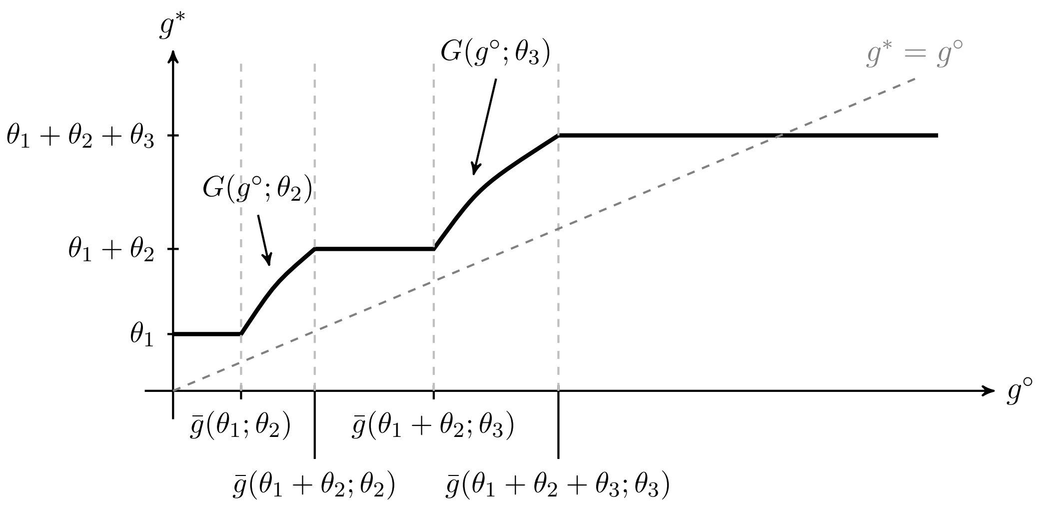

Figure 1: g^* for $Q = \{1,2,3\}$ example.

□

[11] The details can be found in Appendix B.1.

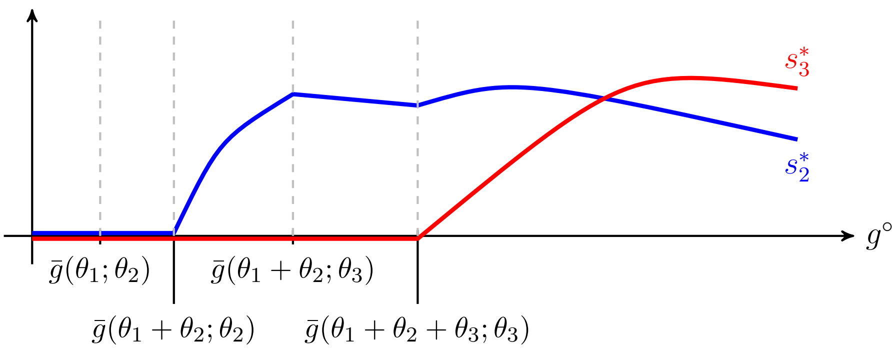

Figure 2: s_2^* and s_3^* for $Q = \{1, 2, 3\}$ example.

4.2 Solution to the Agenda-Setter's Problem

Having solved the auxiliary problem for $(g^Q, \boldsymbol{s}^Q)$ for a given coalition Q, we are left with the task of determining the optimal choice of the coalition. The first step is to note that there is always a solution in which the agenda-setter chooses either the highest or the lowest coalition.

Proposition 4. *Consider a solution $(\tau^*, g^*, \boldsymbol{s}^*, Q^*)$ of $\mathcal{P}(g^\circ)$. If $g^* \geq g^\circ$, then $(\tau^*, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q})$ is also a solution. If $g^* < g^\circ$, then $(\tau^*, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q})$ is also a solution.*

Proposition 4 does not directly provide a characterization of the solution. It implies rather that it is sufficient to consider only the lowest and the highest coalitions. In other words, either $(1, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q})$ or $(1, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q})$ is a solution to the agenda-setter's problem $\mathcal{P}(g^\circ)$. In light of this and the previous results, we are now ready to state Theorem 2, which fully characterizes the solution of the agenda-setter's problem.

Theorem 2. *There exists a unique status quo $\hat{g} \in \left(\Theta\left(\underline{Q}\right), \Theta(\overline{Q})\right]$ such that a solution to $\mathcal{P}(g^\circ)$ is*

$$(\tau^*, g^*, \boldsymbol{s}^*, Q^*) = \begin{cases} (1, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q}) & \text{if} \quad g^\circ \leq \hat{g} \\ (1, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q}) & \text{if} \quad g^\circ > \hat{g}. \end{cases}$$

Theorem 2 asserts that when the status quo public-good level is lower than a threshold value, $\hat{g}$, it is optimal for the agenda-setter to form a coalition with those agents who most value the public good. On the other hand, when the status quo public-good level is higher than $\hat{g}$, it is optimal to form a coalition with those agents who least value the public good. This result has two important implications.

First, the equilibrium public-good level is a non-monotonic function of the status quo. Second, we may observe 'strange bedfellow' coalitions. For example, if $a = 1$, and the status quo is low, 1 would form a coalition with $\{(n+3)/2, \ldots, n\}$, and $Q^* =$

$\{1, (n+3)/2, \ldots, n\}$—a strange bedfellow coalition. The following example illustrates these features.

Example 2. Consider $N = \{1, 2, 3\}$ with $a = 1$ as the agenda-setter. Moreover, assume $\theta_1 < \theta_2 < \theta_3$. Figure 3 and Figure 4 display the equilibrium public-good provision and subsidies, respectively.[12]

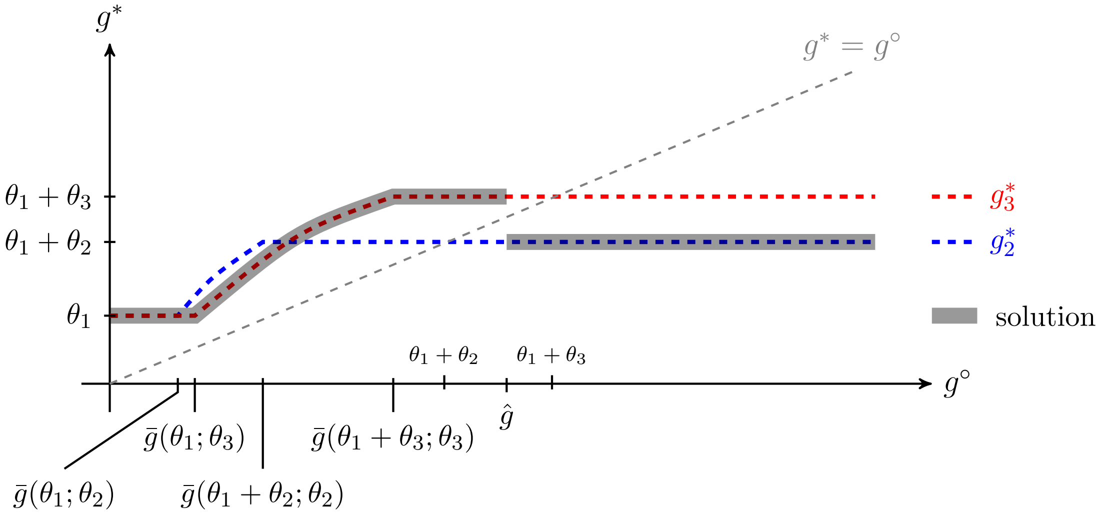

Figure 3: g^* for $N = \{1, 2, 3\}$ example.

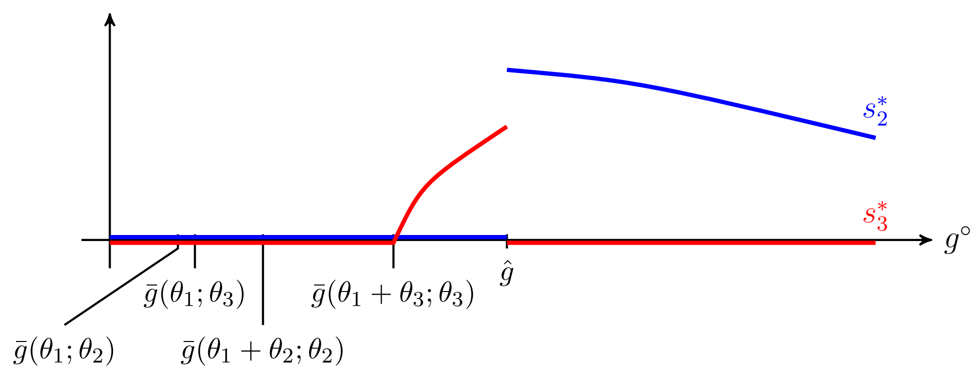

Figure 4: s_2^* and s_3^* for $N = \{1, 2, 3\}$ example.

When $g^\circ \geq \hat{g}$, equilibrium public-good provision drops discontinuously from $\theta_1 + \theta_3$ to $\theta_1 + \theta_2$. Moreover, when $g^\circ \leq \hat{g}$, agent θ_1 will form a coalition with agent θ_3, even though the latter has the highest value for the public good. Interestingly, the switch between coalitions at $\hat{g}$ occurs even though including agent θ_2 means increasing transfers.

[12]The details can be found in Appendix B.2.

4.3 Homogeneous Agents

One of the interesting features from Theorem 2, namely the formation of strange bedfellow coalitions, is a direct consequence of the heterogeneity in the values for the public good among agents. To see this more clearly, consider the special case of a legislature characterized by a homogeneous value for the public good; i.e., $\theta_i = \theta$ for all $i \in N$. Then, the solution to the agenda-setter's problem simplifies significantly, because the choice of coalition becomes irrelevant. In fact, we have that $(1, g^*, \boldsymbol{s}^*, Q^*)$ is a solution to the agenda-setter's problem $\mathcal{P}(g^\circ)$ for any $Q^* \in \mathcal{Q}$ with

$$g^* = \begin{cases} \theta & \text{if} \quad 0 \le g^\circ < \bar{g}(\theta;\theta) \\ G(g^\circ;\theta) & \text{if} \quad \bar{g}(\theta;\theta) \le g^\circ < \bar{g}(q\theta;\theta) \\ q\theta & \text{if} \quad \bar{g}(q\theta;\theta) \le g^\circ < n \end{cases}$$

and

$$s^*_{-a} = \begin{cases} 0 & \text{if} \quad 0 \le g^\circ < \bar{g}(q\theta;\theta) \\ 1 - g^\circ/n + \theta(\ln g^\circ - \ln(q\theta)) & \text{if} \quad \bar{g}(q\theta;\theta) \le g^\circ \le n. \end{cases}$$

Figure 5 and Figure 6 illustrate the solution. As already shown in Examples 1 and 2, the equilibrium public-good provision displays a constant and an increasing regime, as a function of g°. When $0 < g^\circ \le \bar{g}(q\theta;\theta)$, the agenda-setter is able to implement the 'dictator' solution: $g^* = \theta$ and no strictly positive transfers are paid. When $\bar{g}(\theta;\theta) < g^\circ \le \bar{g}(q\theta;\theta)$, the equilibrium public-good level adjusts so as to satisfy the incentive compatibility constraint of every individual. Here again, no transfers are paid. However, whenever $g^\circ > \bar{g}(q\theta;\theta)$, the equilibrium public-good provision is constant at $q\theta$ and strictly positive, and equal transfers are paid to agents. Therefore, unlike Example 2, public-good provision is (weakly) monotonically increasing at the status quo public-good level. This follows from the fact that any coalition will have the same value; hence, Theorem 2 has no 'bite', since any coalition is optimal.

5 Implications

We now consider two implications of the model. First, we discuss the volatility of public-good provision, a feature which emerges endogenously in our model. Then we consider a polarized two-party system and examine how changes in polarization impacts public-good provision.

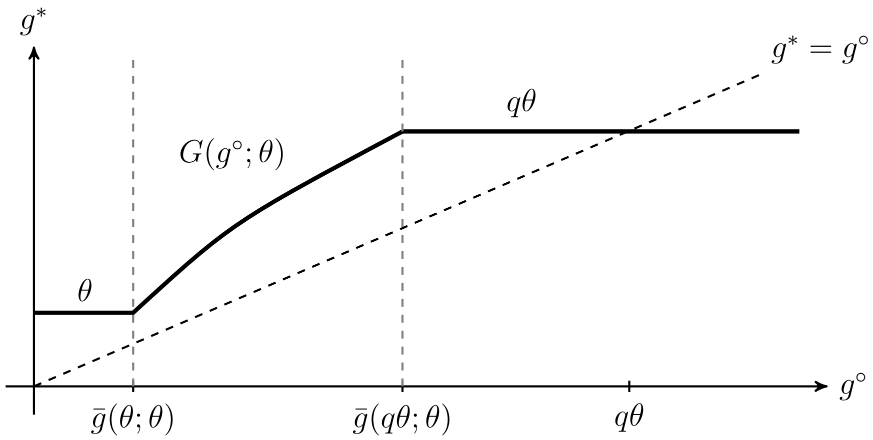

Figure 5: g^* for homogeneous agents.

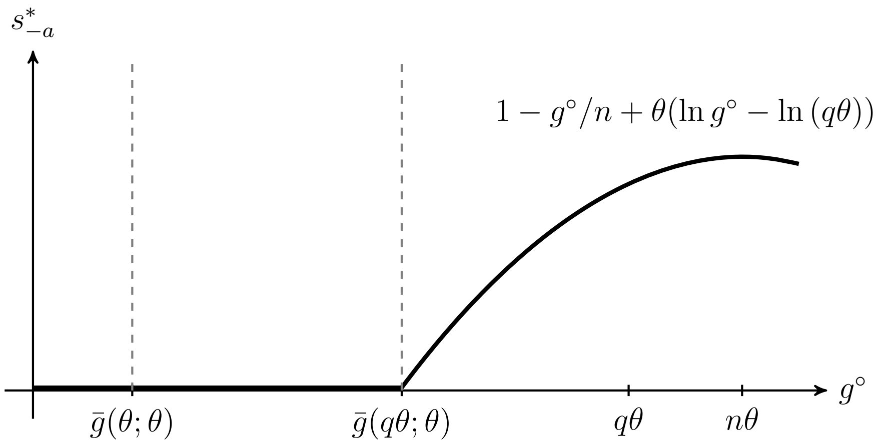

Figure 6: s^*_{-a} for homogeneous agents.

5.1 Endogenous Volatility

Consider a dynamic setting in which the legislative decision-making process repeats over T periods, where T could be finite or infinite. The identity of the agenda-setter, $a \in N$, is kept fixed and the status quo level of public good is endogenously determined: the public-good level provided at time $t-1$ becomes the status quo public-good level at time t. At the beginning of each period, the agenda-setter makes a policy proposal, and the legislature votes on it. Let us suppose for simplicity that agents only care about the outcome of the collective decision in the current period. This assumption is made for tractability, but it may be quite plausible in cases of legislative decision-making where a period is interpreted as a term.

An interesting implication of Theorem 2 is the endogenous volatility of public-good provision, i.e., the public-good level oscillates over time, even if the agenda-setter remains the same. Crucially, this oscillation is an intrinsic property of the solution to the agenda-setter's problem and does not hinge on any stochastic element.

To illustrate this in the simplest setting, let us consider a low level of g°. Here, in each period, the agenda-setter will choose a higher public-good level than in the previous one. This trend continues until the public-good level is equal to $\Theta(\overline{Q})$, i.e., the sum of marginal benefits of the individuals in the highest coalition.

Now consider what happens in the next period, if this public-good level is the status quo, $g^\circ = g^{\overline{Q}} = \Theta(\overline{Q})$. If the agenda-setter chooses the same public-good level, then transfers are simply given by: $s_i = (1 - g^\circ/n)$ for each $i \in \overline{Q} \setminus \{a\}$.[13] In particular, the utility of agenda-setter a is given by

$$\begin{aligned} v_a(g^{\overline{Q}}) &= \theta_a \ln g^{\overline{Q}} + s_a^{\overline{Q}} \\ &= \theta_a \ln g^{\overline{Q}} + \left[n - g^{\overline{Q}} - (q-1)(1 - g^{\overline{Q}}/n) \right]. \end{aligned}$$

Can the agenda-setter do better by reducing the public-good level and/or changing the coalition?[14] It turns out that the agenda-setter can achieve higher utility by switching to the lowest coalition and proposing a lower public-good level, although this amounts to *increasing* transfers. To see this, suppose that the agenda-setter chooses some coalition Q and slightly lowers the public-good level: $g^* = g^{\overline{Q}} - \epsilon$. The variation in the value is given by

$$\frac{\partial}{\partial \epsilon} v_a^Q(g^\circ)\Big|_{\epsilon=0} = -\frac{\theta_a}{g^{\overline{Q}}} + 1 - \frac{\Theta(Q)}{g^{\overline{Q}}} = \frac{\Theta(\overline{Q}) - \Theta(Q)}{\Theta(\overline{Q})} \geq 0$$

with strict inequality for any Q other than $\overline{Q}$.

Theorem 2 states that a will choose $\underline{Q}$.[15] Therefore, although decreasing the public-good level amounts to increasing transfers, the value of the agenda-setter still increases: The decrease in the cost of public-good provision offsets the utility losses suffered by members of the coalition. This stands in sharp contrast to the case where $g^\circ < g^{\overline{Q}}$.

In fact, for the agenda-setter it is then always optimal to increase the public-good level in order to win the support of the $q-1$ members of the coalition $\overline{Q}$. Intuitively, there are decreasing marginal returns to these further increases, and the agents with the highest values will be the ones most willing to support them. However, once the threshold $g^{\overline{Q}}$ is reached, increasing the public-good level further becomes too costly. It is now optimal for the agenda-setter to slightly decrease the public-good level and win the support of the agents with the lowest values, as they are the individuals who suffer least from the decrease in public-good provision.

[13]In this case, transfers do not depend on the choice of the coalition; hence, any coalition will yield the same utility to agenda-setter a.

[14]Increasing the public-good level further would violate the first-order condition for g; hence, it cannot be optimal.

[15]The variation in value in the above expression is maximized at $Q = \underline{Q}$.

Since public-good provision oscillates between $g^{\underline{Q}}$ and $g^{\overline{Q}}$, we refer to these two values as the steady-state values.They are characterized by the fact that one is the optimal public-good provision when the other is the status quo public-good level.[16]

We note that our main conclusion regarding endogenous volatility of public-good provision is also likely to hold when agents are far-sighted. This can be demonstrated in the simplest two-period setting.

Example 3. As in Example 2, consider $N = \{1, 2, 3\}$ and assume $\theta_1 < \theta_2 < \theta_3$. However, there are now two periods, all agents are far-sighted, and $a = 1$ is the agenda-setter in both periods. In Appendix B.3 we show that the agenda-setter will choose different public-good levels across the two periods. The main idea is the following: In the second period, the agenda-setter solves a problem equivalent to $\mathcal{P}(g^\circ)$. Therefore, for a given g^1 chosen in the first period, it is never optimal to choose the status quo public-good level.

When there are more than two periods and $g^\circ \neq 0$, the logic behind the example is the same: Working backwards in a finite time set-up with a fixed agenda-setter produces fluctuating public-good levels. However, a comprehensive analysis is significantly more complex and it is left for future research.

5.2 Application: A Polarized Two-party System

We next apply our model to a two-party system with polarized parties in a legislature. Suppose that agents' values for public good can either be high (H) or low (L), and denote them as θ_H and θ_L, respectively. Assume further that $\theta_L < \theta_H$, and suppose that $(n-1)/2$ individuals belong to each party. In addition, there is an agenda-setter, a, who can belong to either party.[17]

The degree of polarization is defined as $\Delta \equiv \theta_H - \theta_L$, and an increase in polarization is defined by two values $\theta'_H > \theta_H$ and $\theta'_L < \theta_L$, with $\theta'_H + \theta'_L = \theta_H + \theta_L$. We investigate how the degree of polarization affects equilibrium public-good provision, both in the static and the dynamic version of the model.

5.2.1 Static Public-Good Provision

From Theorem 1 and Theorem 2 we have the following

[16] This is not the only possible scenario. When the agenda-setter switches from $g^{\overline{Q}}$ to $g^{\underline{Q}}$, it may take more than one step to reach $g^{\overline{Q}}$ (see Figure 3).

[17] Recall that n is odd.

Corollary 1. *There exists a unique status quo* $\hat{g} \in \left(\Theta(\underline{Q_a}), \Theta(\overline{Q_a})\right]$*, such that a solution to* $\mathcal{P}(g^\circ)$ *is*

$$\tau^* = 1 \quad \textit{and} \quad (g^{Q_a}, \boldsymbol{s}^{Q_a}, Q^*) = \begin{cases} (g^{\overline{Q_a}}, \boldsymbol{s}^{\overline{Q_a}}, \overline{Q_a}) & \textit{if} \quad g^\circ \leq \hat{g} \\ (g^{\underline{Q_a}}, \boldsymbol{s}^{\underline{Q_a}}, \underline{Q_a}) & \textit{if} \quad g^\circ > \hat{g}, \end{cases}$$

where $\overline{Q_a}$ *and* $\underline{Q_a}$ *are defined as*

$$\overline{Q_a} = \{\theta_a, \underbrace{\theta_H, ..., \theta_H}_{(n-1)/2}\} \quad \textit{and} \quad \underline{Q_a} = \{\theta_a, \underbrace{\theta_L, ..., \theta_L}_{(n-1)/2}\}$$

for $\theta_a \in \{\theta_L, \theta_H\}$*.* g^{Q_a} *and* $\boldsymbol{s}^{Q_a}$ *are defined as*

$$g^{Q_a} = \begin{cases} \theta_a + \sum_{i \in K^{Q_a}(g^\circ)} \theta_i & \textit{if} \quad L^{Q_a}(g^\circ) = \varnothing \\ G(g^\circ; \theta_H) & \textit{if} \quad L^{Q_a}(g^\circ) \neq \varnothing, \end{cases}$$

$$s_i^{Q_a} = \begin{cases} (1 - g^\circ/n) + \theta_i \left(\ln g^\circ - \ln g^{Q_a}\right) & \textit{if} \quad i \in K^{Q_a}(g^\circ) \\ 0 & \textit{if} \quad i \notin K^{Q_a}(g^\circ) \end{cases}$$

and $s_a^{Q_a} = n - g^{Q_a} - \sum_{i \in K^{Q_a}(g^\circ)} s_i^{Q_a}$*.*

Figure 7 displays the solution for each agenda-setter's type. In the upper part of the figure, the black lines represent the equilibrium public-good provision when the agenda-setter has value θ_H; in the lower part of the figure, s/he has value θ_L. The red lines represent the solution when the degree of polarization is higher.

An increase in polarization has several effects on equilibrium public-good provision. First, it increases $g^{\overline{Q_a}}$ and decreases $g^{\underline{Q_a}}$; in both cases, we have $g^{\overline{Q}_{\theta'_H}} > g^{\overline{Q}_{\theta_H}}$ and $g^{\underline{Q}_{\theta'_L}} < g^{\underline{Q}_{\theta_L}}$ in the flat parts of the curves. Second, since $\theta'_H > \theta_H$ and $\theta'_L < \theta_L$, when g° is low the solution will entail a higher (lower) public-good level when the proposer has a high (low) value for the public good. Hence, higher polarization generates higher volatility of public-good provision.

5.2.2 Dynamic Public-Good Provision

Next, we illustrate what happens in a repeated setting. Let us consider a time horizon T, which can be finite or infinite. We first consider the scenario where the same agent holds the agenda-setting power in every period. From Corollary 1 and the graphical illustration in Figure 7 we obtain two implications for the dynamic setting.

First, regardless of the agenda-setter's value, public-good provision will take both higher and lower values when polarization is higher. Hence, an increase in polarization

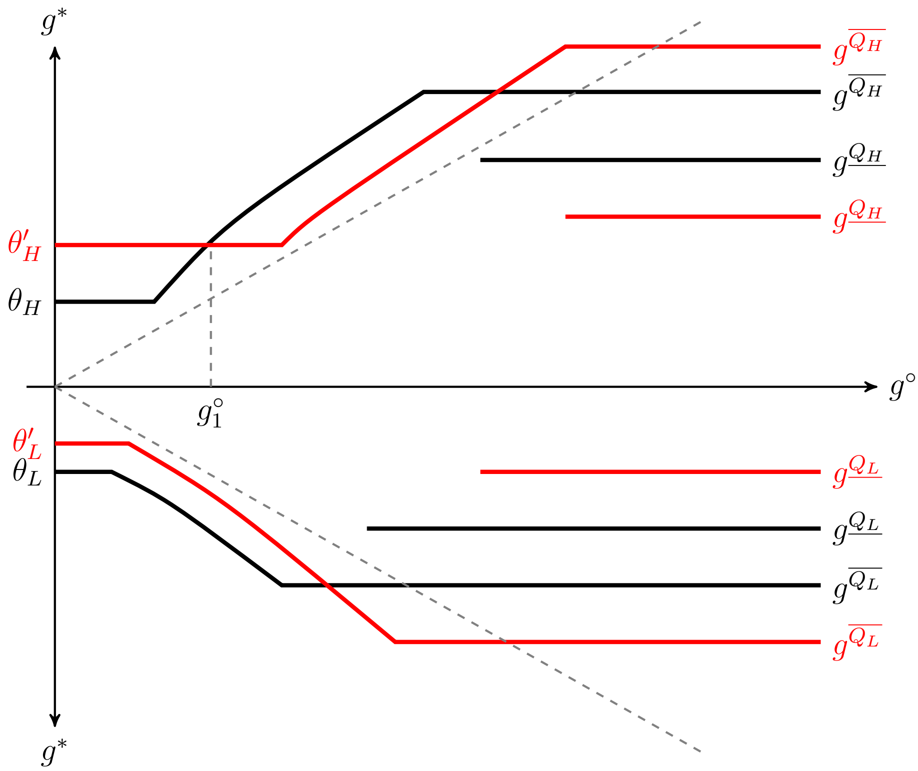

Figure 7: g^* with polarization.

enhances the fluctuations of public-good provision, even if the same agent is in power in each period.

Second, suppose that the agenda-setter has a high value for the public good and suppose that at $t = 0$ public-good provision is either at $g^{\overline{Q}_{\theta_H}}$ or $g^{\underline{Q}_{\theta_H}}$; i.e., the steady-state values. Suppose that at $t = 1$ a shock occurs, drastically reducing the public-good level to a value less than or equal to g_1°. We observe that the recovery to the original level, $g^{\overline{Q}_{\theta_H}}$ or $g^{\underline{Q}_{\theta_H}}$, will occur faster when the agenda-setter has value θ_H than when the value is θ'_H.

This is evident from the upper part of Figure 7, as the red curve is closer to the 45-degree line than the black curve. The reason is that agents in $\overline{Q}_{\theta'_H}$ have a higher value for the public good. This enables the agenda-setter to increase public-good provision by a smaller amount and still obtain their support.

As a result, the time taken to recover from a negative shock to public-good provision is longer when polarization is higher. In other words, the resilience of public-good provision is lower when polarization in the legislature is higher. However, the agenda-setter also has a higher value for the public-good. In extreme scenarios where the shock reduces the public-good level below g_1°, this effect will dominate and public-good provision will initially be higher when polarization is higher.

An even starker result applies when the agenda-setter has a low value for the public good. A shock to public-good provision of *any* magnitude implies lower resilience when polarization is higher. This is evident from the lower part of Figure 7.

In the second scenario, we consider alternating agenda-setting. Suppose that the value of the agenda-setter alternates between θ_L and θ_H in every period. Hence, the majority will change in every period: at time t, a majority of agents will have a high (low) value for the public good, while at time $t+1$, a majority of agents will have a low (high) value for the public good.

We observe that the conclusion for a fixed agenda-setter can be readily extended to such scenarios. When polarization is higher, volatility increases. Moreover, if g° drops to a value above g_1°, then resilience will increase, as can be seen in Figure 7. The calculation of the increase in volatility is more complex as four different values of public-good provision have to be taken into account.

We summarize all these observations in the following corollary:

Corollary 2. *In a polarized legislature with fixed or alternating agenda-setting, higher levels of polarization will increase the volatility of public-good provision and lower its resilience.*

6 Discussion and Conclusions

We have derived a set of properties for public-good provision in a legislature with heterogeneous agents using the simplest possible model. Numerous extensions could be pursued. The most straightforward is the introduction of tax distortions.

Our results are robust to the simple case of linear tax distortions. For any dollar of revenue raised from taxation, only a fraction $\gamma \leq 1$ can be spent on public-good provision.[18] However, in the presence of more general tax distortions, the equilibrium tax rate would not necessarily be maximum, but would enter the picture as another choice variable in the policy proposal, along with the public-good level and the transfers to coalition members.

Moreover, a variety of additional applications to public-good provision in legislatures consisting of two (or more) parties can be envisioned; for instance, by introducing stochastic changes of agenda-setting power and the possibility of forming factions. Another direction for future research would entail further investigation of how more farsighted agenda-setters will affect current public-good provision.

Finally, we have performed a positive analysis on democratic public-good provision. An interesting avenue for future research would be to engage into a normative analysis,

[18] It is straightforward to show that, provided $q/n \leq \gamma \leq 1$, all our results still hold.

inquiring whether the standard institutional environment that we have adopted is optimal for public-good provision.

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A Proofs

Proof of Proposition 1. First, the proposed public good is strictly positive because the marginal utility of g near 0 is unbounded. Second, the resource constraint is binding since the agenda-setter's utility is strictly increasing in g and s_a. Third, it must hold that $\tau^* > 0$. If not, it would contradict the first two observations. Moreover, suppose there exists a solution $(\tau, g, \boldsymbol{s}, Q)$ such that $\tau < 1$. Consider an alternative policy $(\tau^\epsilon, g, \boldsymbol{s}^\epsilon, Q)$ for some $\epsilon > 0$ such that $\tau^\epsilon \equiv \tau + \epsilon \leq 1$. We will show that the utility of the agenda-setter is strictly higher under this alternative policy. For some $\alpha \in \left(\frac{n-1}{n}, 1\right)$, define

$$s_a^\epsilon = s_a + (1-\alpha)n\epsilon \qquad \text{and} \qquad s_i^\epsilon = \begin{cases} s_i + \alpha \frac{n\epsilon}{(n-1)/2} & \text{if} \quad i \in Q \setminus \{a\} \\ s_i & \text{if} \quad i \notin Q. \end{cases}$$

Such a policy is feasible since the non-negativity constraints hold and

$$g + s_a^\epsilon + \sum_{i \in N \setminus \{a\}} s_i^\epsilon = g + s_a + \sum_{i \in N \setminus \{a\}} s_i + n\epsilon \leq n\tau + n\epsilon = n\tau^\epsilon.$$

Also, for all $i \in Q \setminus \{a\}$,

$$(1-\tau^\epsilon) + \theta_i \ln g + s_i^\epsilon = (1-\tau) + \theta_i \ln g + s_i + \frac{\alpha n - (n-1)/2}{(n-1)/2}\epsilon > (1 - g^\circ/n) + \theta_i \ln g^\circ.$$

Since the original solution is feasible,

$$(1-\tau^\epsilon) + \theta_i \ln g + s_i^\epsilon = (1-\tau) + \theta_i \ln g + s_i + \frac{\alpha n - (n-1)/2}{(n-1)/2}\epsilon > (1-\tau) + \theta_i \ln g + s_i,$$

which contradicts the optimality of the solution. Hence, in any solution, $\tau^* = 1$. □

Proof of Proposition 2. The proof proceeds in several steps. First, we set up the Lagrangian for the auxiliary problem and state the Karush–Kuhn–Tucker (KKT) conditions. Second, we rule out the possibility of the agenda-setter receiving no transfers. Third, we use the conditions to characterize the general structures of K, L, and M. Fourth, we determine $k(g^\circ)$ and $l(g^\circ)$ to pin down the specific structures of K, L, M for each g°.

Step 1. The Lagrangian of the problem $\mathcal{P}^Q(g^\circ)$ is

$$\mathcal{L} = \theta_a \ln g + s_a + \lambda \left(n - s_a - \sum_{i \in R} s_i - g \right) + \sum_{i \in R} \mu_i \left[s_i - (1 - g^\circ/n) - \theta_i \left(\ln g^\circ - \ln g \right) \right] + \nu_a s_a + \sum_{i \in R} \nu_i s_i,$$

where λ, μ_i's, ν_a, ν_i's are the multipliers. The KKT conditions are as follows:

$$\begin{aligned}
\frac{\theta_a}{g} - \lambda + \frac{\sum_{i \in R} \mu_i \theta_i}{g} &= 0 && [g] \\
1 - \lambda + \nu_a &= 0 && [s_a] \\
-\lambda + \mu_i + \nu_i &= 0 && [s_i, i \in R] \\
\nu_a &\geq 0 && [\text{non-neg for } a] \\
\mu_i, \nu_i &\geq 0 && [\text{non-neg for } i \in R] \\
\mu_i \left[s_i - (1 - g^\circ/n) - \theta_i \left(\ln g^\circ - \ln g \right) \right] &= 0 && [\text{CS for incentive compatibility}] \\
\nu_a s_a &= 0 && [\text{CS for } s_a] \\
\nu_i s_i &= 0 && [\text{CS for } s_i\text{'s}] \\
n - s_a - \sum_{i \in R} s_i - g &= 0 && [\text{resource}] \\
s_i - (1 - g^\circ/n) - \theta_i \left(\ln g^\circ - \ln g \right) &\geq 0 && [\text{incentive compatibility for } i \in R].
\end{aligned}$$

The variables g, s_a, s_i's and multipliers λ, μ_i's, ν_a, ν_i's that satisfy the KKT conditions are sufficient for optimality. It is straightforward to see that

(i) if $s_a > 0$ then $\nu_a = 0$ and $\lambda = 1$,

(ii) if $s_i > 0$ then $\nu_i = 0$ and $\mu_i = \lambda \geq 1 > 0$,

(iii) if $\mu_i = 0$ then $\nu_i = \lambda \geq 1$ and $s_i = 0$.

Step 2. We first show that $s_a > 0$ and as a result $\lambda = 1$.

Suppose on the contrary that $s_a = 0$ and thus $\lambda \geq 1$. There are two possibilities:

– If $K = \varnothing$, then $s_i = 0$ for all $i \in R$. This would imply that $g = n$ by the resource constraint. However, this leads to a contradiction since

$$g = \frac{\theta_a + \sum_{i\in R}\mu_i\theta_i}{\lambda} \le \frac{\theta_a + \lambda\sum_{i\in R}\theta_i}{\lambda} < \frac{1+\lambda(q-1)}{\lambda} = \frac{1}{\lambda} + q - 1 \le q < n.$$

– If $K \neq \varnothing$, then $s_i = (1 - g^\circ/n) + \theta_i(\ln g^\circ - \ln g) > 0$ and $\mu_i = \lambda \ge 1$ for $i \in K$. The resource constraint demands

$$n - g - k(1 - g^\circ/n) - \sum_{i\in K}\theta_i(\ln g^\circ - \ln g) = 0, \tag{7}$$

while the FOC for the public good is

$$g = \frac{\theta_a}{\lambda} + \sum_{i\in K}\theta_i + \sum_{i\in L}\frac{\mu_i}{\lambda}\theta_i. \tag{8}$$

Because $\lambda \ge 1$, it holds that $\sum_{i\in K}\theta_i < g \le \theta_a + \sum_{i\in K,L}\theta_i$. Then, for some $\lambda \ge 1$ and some μ_i's, for $i \in L$, there exists a g solving (8) such that (7) holds with equality. Moreover, since for a constant C the function $f(x) = C\ln x - x$ is decreasing for $x \ge C$, it follows that

$$n - \underbrace{\left(\theta_a + \sum_{i\in K,L}\theta_i\right)}_{g} - k(1 - g^\circ/n) - \sum_{i\in K}\theta_i\left[\ln g^\circ - \ln\underbrace{\left(\theta_a + \sum_{i\in K,L}\theta_i\right)}_{g}\right] \le 0. \tag{9}$$

Applying Lemma 1 to the set $\{a\} \cup K \cup L$ shows that this is impossible. In other words, there are no λ, μ_i's, and consequently no g, such that (7) and (8) are satisfied for any set K.

Lemma 1. *For any $A \subseteq N$, such that $a \in A$ and $|A| \le (n+1)/2$,*

$$n - \Theta(A) - (|A| - 1)(1 - g^\circ/n) - \Theta(A \setminus \{a\})\left[\ln g^\circ - \ln(\Theta(A))\right] > 0.$$

Proof of Lemma 1. Suppose first that $g^\circ > \Theta(A)$. Then, using the fact that

$\ln x \le x - 1$ for $x > 0$,

$$\begin{aligned}\Theta(A \setminus \{a\}) \ln\left(\frac{g^\circ}{\Theta(A)}\right) &\le \Theta(A \setminus \{a\}) \left[\frac{g^\circ}{\Theta(A)} - 1\right] \\ &= \frac{\Theta(A \setminus \{a\})}{\Theta(A)} \left[g^\circ - \Theta(A)\right] \\ &< g^\circ - \Theta(A) \\ &\le n - \Theta(A) - (q-1)(1 - g^\circ/n).\end{aligned}$$

The last inequality follows from $(q-1)(1 - g^\circ/n) \le n(1 - g^\circ/n)$. Rewriting the inequality yields the result.

Suppose now that $g^\circ \le \Theta(A)$. Then,

$$\begin{aligned}&n - \Theta(A) - (|A| - 1)(1 - g^\circ/n) - \Theta(A \setminus \{a\}) \left[\ln g^\circ - \ln\left(\Theta(A)\right)\right] \\ &\qquad = n - \Theta(A) - (|A| - 1) + (|A| - 1)g^\circ/n - \Theta(A \setminus \{a\}) \left[\ln g^\circ - \ln\left(\Theta(A)\right)\right] \\ &\qquad \ge n - \Theta(A) - (|A| - 1) \\ &\qquad > n - |A| - (|A| - 1) \\ &\qquad = n + 1 - 2|A| \ge 0.\end{aligned}$$

$\square$

Step 3. Observations (ii), (iii), and the complementary slackness conditions imply that

$$\begin{aligned}K &= \{i \in R : s_i > 0 \text{ and } \mu_i = \lambda = 1\} \\ L &= \{i \in R : s_i = 0 \text{ and } \mu_i > 0\} \\ M &= \{i \in R : s_i = 0 \text{ and } \mu_i = 0\} \\ O &= \varnothing.\end{aligned}$$

We now show that s_i's and μ_i's that satisfy the KKT conditions structure K, L, and M as claimed. Since K, L, and M are exhaustive, it suffices to show that for any $k \in K$,

$l \in L$, and $m \in M$ (whenever these elements exist), $\theta_k \leq \theta_l \leq \theta_m$. There are several possibilities:

- If all three sets are non-empty, consider $k \in K$, $l \in L$, and $m \in M$. The conditions defining their transfers yield the following chain of inequalities:

$$\begin{aligned}
\underbrace{(1 - g^\circ/n) + \theta_k (\ln g^\circ - \ln g)}_{s_k} &> 0 \\
&= \underbrace{(1 - g^\circ/n) + \theta_l (\ln g^\circ - \ln g)}_{s_l} \\
&= 0 \\
&\geq (1 - g^\circ/n) + \theta_m (\ln g^\circ - \ln g) .
\end{aligned}$$

  Because $g^\circ \leq n$, the last inequality implies that $g^\circ \leq g$. If $g = g^\circ$, then it must hold that $g^\circ = g = n$, which would violate the first inequality.[19] Thus, $g^\circ < g$ and the chain of inequalities implies that $\theta_k < \theta_l \leq \theta_m$.

- If only two of the three sets are non-empty, then a similar argument respectively establishes that $\theta_k < \theta_l$ or $\theta_l \leq \theta_m$ or $\theta_k < \theta_l$.

- If only one of K, L, M is non-empty, then such a set is R, and we are done.

We have established that there exist $k \leq l \in \{0, 1, \ldots, r\}$ such that

$$R = \underbrace{\{i_1, \ldots, i_k\}}_{K} \cup \underbrace{\{i_{k+1}, \ldots, i_l\}}_{L} \cup \underbrace{\{i_{l+1}, \ldots, i_r\}}_{M},$$

with the convention that $i_0 = 0$ and $\{i_x, i_y\} = \varnothing$ if $x > y$. Now, for any two elements $l, l' \in L$, a similar argument shows that $\theta_l = \theta_{l'}$.

□

We begin with the following useful

[19] Note that this argument rests on the non-emptiness of the sets. As we will see, $g = g^\circ$ is possible when $L = M = \varnothing$.

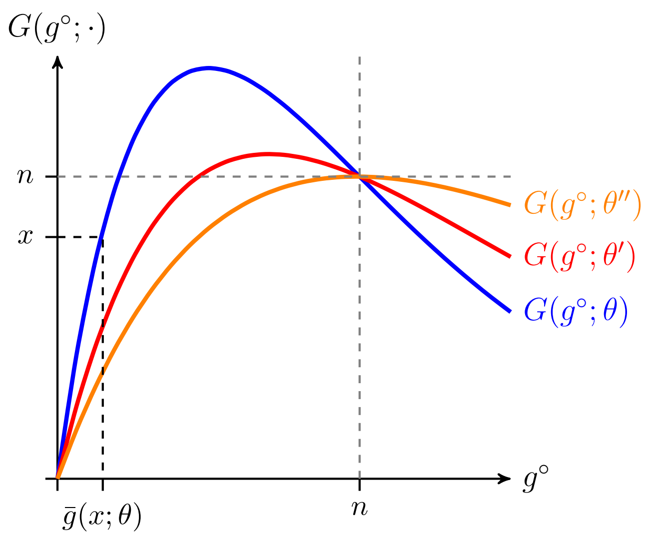

Figure 8: $G(g^\circ;\theta_i)$ for $\theta < \theta' < \theta''$.

Lemma 2. *For $\theta \in (0,1)$ and $x \in [0,n]$, there exists a unique $\bar{g}(x;\theta)$ such that*

$$G(\bar{g}(x;\theta);\theta) = x.$$

In addition, $\bar{g}(x;\theta)$ is strictly increasing in both x and θ.

Proof of Lemma 2. It can be readily established that $G(g^\circ;\theta_i)$ has the following properties:

(i) For $\theta < \theta'$, $G(g^\circ;\theta) \geq G(g^\circ;\theta') \geq g^\circ$ on its domain. Equality holds if and only if $g^\circ = 0$ or $g^\circ = n$.

(ii) $G(g^\circ;\theta_i)$ is strictly increasing for $g^\circ \in [0, n\theta_i)$ and strictly decreasing for $g^\circ \in (n\theta_i, n]$. It attains a unique maximum at $n\theta_i$ with the value $n\theta_i e^{\frac{1-\theta_i}{\theta_i}}$.

These properties imply the result because for the range $[0,n]$, $G(g^\circ;\theta)$ is monotone increasing in g°. $\square$

Figure 8 plots $G(g^\circ;\theta_i)$ as a function of the status quo g° for various values of θ_i's and shows that for $\theta < \theta'$, $G(g^\circ;\theta) \geq G(g^\circ;\theta') \geq g^\circ$ and that $G(g^\circ;\theta_i)$ is strictly increasing for $g^\circ \in [0, n\theta_i)$ and strictly decreasing for $g^\circ \in (n\theta_i, n]$.

Proof of Proposition 3.

We now go through the cases to determine $k \equiv k(g^\circ)$ and $l \equiv l(g^\circ)$.

- $k = l = 0$ ($K = L = \varnothing$ and $M = R$).

  We have $\mu_i = 0$ for all j, and this implies $s_i = 0$ for all i and $g = \theta_a$ by the FOC for public good. For this to hold, we need $0 \geq (1 - g^\circ/n) + \theta_i (\ln g^\circ - \ln g)$ to hold for all $i \in R$. The first term on the RHS is non-negative, so it must be the case that $g^\circ \leq g$, which implies that the inequality must hold, in particular, for the agents with the lowest valuation, namely θ_{i_1}. Substituting $g = \theta_a$ yields the condition $\theta_{i_1} \ln \theta_a \geq (1 - g^\circ/n) + \theta_{i_1} \ln g^\circ$. Thus, we need

$$0 < g^\circ \leq \bar{g}(\theta_a; \theta_{i_1}).$$

- $0 = k < l \leq r$ ($K = \varnothing$ and $L \neq \varnothing$).

  The public good is $g = \theta_a + \sum_{i \in L} \mu_i \theta_i$. Consider $i \in L$. Then $0 = (1 - g^\circ/n) + \theta_i (\ln g^\circ - \ln g)$ implies that $g^\circ < g$. This means that L only consists of the agent(s) with the lowest valuation, $\theta_{i_1} = \cdots = \theta_{i_l}$. Rearranging the condition yields $\theta_{i_1} \ln g = (1 - g^\circ/n) + \theta_{i_1} \ln g^\circ$. Moreover, $0 < \mu_i \leq 1$, so $\theta_a < g \leq \theta_a + \underbrace{\theta_{i_1} + \cdots + \theta_{i_l}}_{= l\theta_{i_l}}$ yields the following condition:

$$\bar{g}(\theta_a; \theta_{i_1}) < g^\circ \leq \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_l}; \theta_{i_1}).$$

- $0 < k = l < r$ ($K \neq \varnothing$ and $L = \varnothing$).

  The public good is $g = \theta_a + \sum_{i \in K} \theta_i$. For all $i \in K$, $s_i = (1 - g^\circ/n) + \theta_i (\ln g^\circ - \ln g) > 0$. The inequality must hold, in particular, for i_k, which gives the condition: $\theta_{i_k} \ln g < (1 - g^\circ/n) + \theta_{i_k} \ln g^\circ$. On the other hand, for all $i \in M$, $0 \geq (1 - g^\circ/n) + \theta_i (\ln g^\circ - \ln g)$. In particular, this must hold for i_{k+1}, i.e. $\theta_{i_{k+1}} \ln g \geq (1 - g^\circ/n) + \theta_{i_{k+1}} \ln g^\circ$. Accordingly, we arrive at the following condition:

$$\bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_k}; \theta_{i_k}) < g^\circ \leq \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_k}; \theta_{i_{k+1}}).$$

- $0 < k < l \leq r$ ($K \neq \varnothing$ and $L \neq \varnothing$).

  The public good is $g = \theta_a + \sum_{i \in K} \theta_i + \sum_{i \in L} \mu_i \theta_i$. For $i \in L$, $0 = (1 - g^\circ/n) + \theta_i (\ln g^\circ - \ln g)$ implies that $g^\circ < g$. Again, this means that L only consists of the agent (possibly, agents) with the lowest valuation above θ_{i_k}, which in this case is $\theta_{i_{k+1}} = \cdots = \theta_{i_l}$. Rearranging the condition gives $\theta_{i_l} \ln g = (1 - g^\circ/n) + \theta_{i_l} \ln g^\circ$. Together with the expression of g in terms of the values for the public good and the fact that $0 < \mu_i \leq 1$, we have the following condition:

$$\bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_k}; \theta_{i_l}) < g^\circ \leq \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_k} + \underbrace{\theta_{i_{k+1}} \cdots + \theta_{i_l}}_{=(l-k)\theta_{i_l}}; \theta_{i_l}).$$

- $k = l = r$ ($K = R$ and $L = M = \varnothing$).

  The public good is $g = \theta_a + \sum_{i \in R} \theta_i$. The conditions on the transfers demand that $\theta_{i_r} \ln g < (1 - g^\circ/n) + \theta_{i_r} \ln g^\circ$. Thus

$$\bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_r}; \theta_{i_r}) < g^\circ \leq n.$$

In conclusion, we have

$$k(g^\circ) = \begin{cases} 0 & \text{if} \quad 0 \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_1}) \\ 1 & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_1}) \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1} + \theta_{i_2}; \theta_{i_2}) \\ \vdots & \text{if} \quad \vdots \\ r & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_{r-1}}; \theta_{i_r}) \leq g^\circ \leq n \end{cases} \tag{10}$$

and

$$l(g^\circ) = \begin{cases} 0 & \text{if} \quad 0 \leq g^\circ < \bar{g}(\theta_a; \theta_{i_1}) \\ 1 & \text{if} \quad \bar{g}(\theta_a; \theta_{i_1}) \leq g^\circ < \bar{g}(\theta_a + \theta_{i_1}; \theta_{i_2}) \\ \vdots & \text{if} \quad \vdots \\ r & \text{if} \quad \bar{g}(\theta_a + \theta_{i_1} + \cdots + \theta_{i_r}; \theta_{i_r}) \leq g^\circ \leq n. \end{cases} \tag{11}$$

□

Proof of Theorem 1. Proposition 2 gives the structures of $K(g^\circ)$, $L(g^\circ)$, and $M(g^\circ)$.

Let g^Q and $\boldsymbol{s}^Q$ solve the KKT conditions. By the definition of K, $s_i^Q = (1 - g^\circ/n) + \theta_i(\ln g^\circ - \ln g^Q)$ for $i \in K$. Otherwise, $s_i^Q = 0$.

For g^Q, we distinguish two cases.

(i) If L is empty, then $R = K \cup M$. By the FOC for public goods,

$$g^Q = \frac{\theta_a + \sum_{i\in R} \mu_i \theta_i}{\lambda}.$$

At the optimal solution, $\lambda = 1$, $\mu_i = 1$ for $i \in K$, and $\mu_i = 0$ otherwise. Thus,

$$g^Q = \theta_a + \sum_{i\in K} \theta_i.$$

(ii) If L is non-empty, then for $l \in L$, $0 = (1 - g^\circ/n) + \theta_l \left(\ln g^\circ - \ln g^Q\right)$. That is,

$$g^Q = G(g^\circ; \theta_l).$$

Lastly, because the budget constraint is binding, $s_a^Q = n - g^Q - \sum_{i\in R} s_i^Q$. $\square$

Proof of Proposition 4. Suppose $(\tau^*, g^*, \boldsymbol{s}^*, Q^*)$ is a solution to $\mathcal{P}(g^\circ)$, but $Q^* \notin \{\underline{Q}, \overline{Q}\}$, we show that either $(\tau^*, g^*, \underline{\boldsymbol{s}}, \underline{Q})$ or $(\tau^*, g^*, \overline{\boldsymbol{s}}, \overline{Q})$ is also a solution, where $\underline{\boldsymbol{s}}$ and $\overline{\boldsymbol{s}}$ need to be specified. The idea is that the agenda-setter can replace an agent in Q^* by an agent in $\overline{Q}$ or $\underline{Q}$ while still obeying the constraints, keeping the same level of public good and total transfers, and thus maintaining the same utility.

There are two possibilities, either $g^* \geq g^\circ$ or $g^* < g^\circ$. We show that if $g^* \geq g^\circ$, then $(\tau^*, g^*, \overline{\boldsymbol{s}}, \overline{Q})$ is also a solution. The idea is to match a voter in the old coalition and a voter in the new coalition and construct $\overline{\boldsymbol{s}}$ by switching their transfers. Define $\overline{\boldsymbol{s}}$ to be the same as $\boldsymbol{s}^*$, except for the coordinates corresponding to agents in $Q^* \setminus \overline{Q}$ and $\overline{Q} \setminus Q^*$. For two agents with indices $i \in Q^* \setminus \overline{Q}$ and $j \in \overline{Q} \setminus Q^*$, let

$$\overline{s}_i = 0 \quad \text{and} \quad \overline{s}_j = s_i^*.$$

Since $g^* \geq g^\circ$ and $\theta_j \geq \theta_i$ by the definition of $\overline{Q}$,

$$\bar{s}_j = s_i^* \geq (\tau^* - g^\circ/n) + \theta_i (\ln g^\circ - \ln g^*) \geq (\tau^* - g^\circ/n) + \theta_j (\ln g^\circ - \ln g^*).$$

The new transfer satisfies the incentive compatibility constraint for j. To construct $\bar{\boldsymbol{s}}$, we do this for every pair of agents between $Q^* \setminus \overline{Q}$ and $\overline{Q} \setminus Q^*$. Since we are only switching transfers, the total transfer stays the same. Thus, $(\tau^*, g^*, \bar{\boldsymbol{s}}, \overline{Q})$ attains the same utility and hence is also a solution to $\mathcal{P}(g^\circ)$. But since there is a solution with $\overline{Q}$ as the minimal winning coalition, it must be the case that the auxiliary solution for $\overline{Q}$ is also a solution. Therefore, $(\tau^*, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q})$ is a solution.

An analogous argument shows that if $g^* < g^\circ$, then $(\tau^*, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q})$ is also a solution to $\mathcal{P}(g^\circ)$. □

A.1 Proof of Theorem 2

In light of the above lemmata and propositions, it suffices to determine the values of g° for which $(1, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q})$ and $(1, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q})$ are solutions. We assume throughout the proof that $\Theta(\underline{Q}) < \Theta(\overline{Q})$.

From the solution to the auxiliary problem, we derive the following observation:

Corollary 3. *For any Q,*

(i) $g^Q \geq g^\circ$ *for* $0 \leq g^\circ \leq \Theta(Q)$,

(ii) $g^Q = \Theta(Q)$ *for* $g^\circ > \Theta(Q)$.

It follows that, for $0 \leq g^\circ \leq \Theta(\underline{Q})$, both $g^{\overline{Q}}$ and $g^{\underline{Q}}$ are greater than or equal to g°. This means that g^*, which takes one of these two values, is also greater than or equal to g°. By Proposition 4, $(1, g^{\overline{Q}}, \boldsymbol{s}^{\overline{Q}}, \overline{Q})$ is then a solution to $\mathcal{P}(g^\circ)$ for $g^\circ \in \big[0, \Theta(\underline{Q})\big]$. On the other hand, for $g^\circ > \Theta(\overline{Q})$, $g^* < g^\circ$, so by Proposition 4 $(1, g^{\underline{Q}}, \boldsymbol{s}^{\underline{Q}}, \underline{Q})$ is a solution for such g°.

It remains to determine the case for $\Theta(\underline{Q}) < g^\circ \leq \Theta(\overline{Q})$. Let $\overline{R} = \overline{Q} \setminus \{a\}$ and $\underline{R} = \underline{Q} \setminus \{a\}$. For such g°, $K^{\underline{Q}} = \underline{R}$, which means that $g^{\underline{Q}} = \Theta(\underline{Q})$ and for all $i \in \underline{R}$,

$s_i = (1 - g^\circ/n) + \theta_i(\ln g^\circ - \ln g^{\underline{Q}})$. It follows that

$$v_a^{\underline{Q}}(g^\circ) = \theta_a \ln g^{\underline{Q}} + s_a^{\underline{Q}} = f\left(g^\circ, \Theta(\underline{Q})\right), \tag{12}$$

where

$$f(g^\circ, x) \equiv n - x - (q-1)(1 - g^\circ/n) - (x - \theta_a)\ln g^\circ + x \ln x.$$

The expression for $v_a^{\overline{Q}}(g^\circ)$, however, depends on the exact value of g°.

Lemma 3. *For $g^\circ \in \left(\Theta(\underline{Q}), \Theta(\overline{Q})\right]$, $g^{\underline{Q}} < g^{\overline{Q}}$.*

Proof. By the first part of Corollary 3, we have $g^{\overline{Q}} \geq g^\circ$ in this range. Moreover, setting $Q = \underline{Q}$ in the second part of Corollary 3 yields $g^{\underline{Q}} = \Theta(\underline{Q}) < g^\circ$. □

Lemma 4. *For $g^\circ \in \left(\Theta(\underline{Q}), \Theta(\overline{Q})\right]$, the difference*

$$v_a^{\overline{Q}}(g^\circ) - v_a^{\underline{Q}}(g^\circ) \tag{13}$$

is strictly decreasing.

Proof. The above difference is equal to

$$v_a^{\overline{Q}}(g^\circ) - v_a^{\underline{Q}}(g^\circ) = g^{\overline{Q}} \ln g^{\overline{Q}} - g^{\underline{Q}} \ln g^{\underline{Q}} - (g^{\overline{Q}} - g^{\underline{Q}}) \ln g^\circ - (k-q)(1 - g^\circ/n) - (g^{\overline{Q}} - g^{\underline{Q}}).$$

If $k = q$, then the expression above only depends on $-(g^{\overline{Q}} - g^{\underline{Q}}) \ln g^\circ$, which is strictly decreasing by Lemma 3. If $k < q$, then the first-order derivative of (13) is equal to

$$-\frac{g^{\overline{Q}} - g^{\underline{Q}}}{g^\circ} + \frac{k-q}{n} < 0$$

since it is the sum of two negative terms. □

Thus, (13) has a single crossing defined to be at $\hat{g}$.

B Examples

B.1 Details of Example 1

Using Proposition 2, Proposition 3, and Theorem 1, the partition of $R = \{2, 3\}$ and the solution $(g^Q, s_1^Q, s_2^Q, s_3^Q)$ as a function of g° are as follows:

- If $0 \le g^\circ < \bar{g}(\theta_1; \theta_2)$, then $K = L = \varnothing$ and $M = \{2, 3\}$, and

$$g^Q = \theta_1 \qquad s_2^Q = 0 \qquad s_3^Q = 0.$$

- If $\bar{g}(\theta_1; \theta_2) \le g^\circ < \bar{g}(\theta_1 + \theta_2; \theta_2)$, then $K = \varnothing$, $L = \{2\}$, and $M = \{3\}$, and

$$g^Q = G(g^\circ; \theta_2) \qquad s_2^Q = 0 \qquad s_3^Q = 0.$$

- If $\bar{g}(\theta_1 + \theta_2; \theta_2) \le g^\circ < \bar{g}(\theta_1 + \theta_2; \theta_3)$, then $K = \{2\}$, $L = \varnothing$, and $M = \{3\}$, and

$$g^Q = \theta_1 + \theta_2 \qquad s_2^Q = 1 - g^\circ/n + \theta_2(\ln g^\circ - \ln g^Q) \qquad s_3^Q = 0.$$

- If $\bar{g}(\theta_1 + \theta_2; \theta_3) \le g^\circ < \bar{g}(\theta_1 + \theta_2 + \theta_3; \theta_3)$, then $K = \{2\}$, $L = \{3\}$, and $M = \varnothing$, and

$$g^Q = G(g^\circ; \theta_3) \qquad s_2^Q = 1 - g^\circ/n + \theta_2(\ln g^\circ - \ln g^Q) \qquad s_3^Q = 0.$$

- If $\bar{g}(\theta_1 + \theta_2 + \theta_3; \theta_3) \le g^\circ \le n$, then $K = \{2, 3\}$, $L = M = \varnothing$, and

$$g^Q = \theta_1 + \theta_2 + \theta_3 \qquad s_2^Q = 1 - g^\circ/n + \theta_2(\ln g^\circ - \ln g^Q) \qquad s_3^Q = 1 - g^\circ/n + \theta_3(\ln g^\circ - \ln g^Q).$$

B.2 Details of Example 2

Using Proposition 2 and Theorem 1, we look at the solution for the auxiliary problems for $\overline{Q} = \{1, 3\}$ and $\underline{Q} = \{1, 2\}$, respectively.

- The auxiliary solution for $\overline{Q} = \{1, 3\}$ is:

– If $0 \le g^\circ < \bar{g}(\theta_1; \theta_3)$, then

$$g^{\overline{Q}} = \theta_1 \qquad s_3^{\overline{Q}} = 0.$$

– If $\bar{g}(\theta_1; \theta_3) \le g^\circ < \bar{g}(\theta_1 + \theta_3; \theta_3)$, then

$$g^{\overline{Q}} = G(g^\circ; \theta_3) \qquad s_3^{\overline{Q}} = 0.$$

– If $\bar{g}(\theta_1 + \theta_3; \theta_3) \le g^\circ \le 3$, then

$$g^{\overline{Q}} = \theta_1 + \theta_3 \qquad s_3^{\overline{Q}} = 1 - g^\circ/3 + \theta_3(\ln g^\circ - \ln g^{\overline{Q}}).$$

In all cases, $s_2^{\overline{Q}} = 0$ and $s_1^{\overline{Q}} = 3 - g^{\overline{Q}} - s_3^{\overline{Q}}$.

- The auxiliary solution for $\underline{Q} = \{1, 2\}$ is:

– If $0 \le g^\circ < \bar{g}(\theta_1; \theta_2)$, then

$$g^{\underline{Q}} = \theta_1 \qquad s_2^{\underline{Q}} = 0.$$

– If $\bar{g}(\theta_1; \theta_2) \le g^\circ < \bar{g}(\theta_1 + \theta_2; \theta_2)$, then

$$g^{\underline{Q}} = G(g^\circ; \theta_2) \qquad s_2^{\underline{Q}} = 0.$$

– If $\bar{g}(\theta_1 + \theta_2; \theta_2) \le g^\circ \le 3$, then

$$g^{\underline{Q}} = \theta_1 + \theta_2 \qquad s_2^{\underline{Q}} = 1 - g^\circ/n + \theta_2(\ln g^\circ - \ln g^{\underline{Q}}).$$

In all cases, $s_3^{\underline{Q}} = 0$ and $s_1^{\underline{Q}} = 3 - g^{\underline{Q}} - s_2^{\underline{Q}}$.

For the full solution, observe first that for $0 < g^\circ \le \theta_1 + \theta_2$, both $g^{\overline{Q}}$ and $g^{\underline{Q}}$ are larger than g°. This means that $g^* \ge g^\circ$ and by Proposition 4 $\overline{Q}$ is optimal for $g^\circ \in (0, \theta_1 + \theta_2]$.

Observe further that for $\theta_1 + \theta_3 < g^\circ$, both $g^{\overline{Q}}$ and $g^{\underline{Q}}$ are smaller than g°. This again means that $\underline{Q}$ is optimal for $g^\circ \in (\theta_1 + \theta_3, 3]$.

Therefore, it remains to determine which of $\overline{Q}$ and $\underline{Q}$ is optimal for the range $\theta_1 + \theta_2 < g^\circ \le \theta_1 + \theta_3$. This can be done by directly comparing $v_1^{\overline{Q}}(g^\circ)$ and $v_1^{\underline{Q}}(g^\circ)$, the values to the agenda-setter of the respective auxiliary problems.

For $\theta_1 + \theta_2 < g^\circ \le \theta_1 + \theta_3$,

$$v_1^{\underline{Q}}(g^\circ) = 3 - (\theta_1 + \theta_2) - (1 - g^\circ/3) - \theta_2 \ln g^\circ + (\theta_1 + \theta_2) \ln (\theta_1 + \theta_2).$$

However, there are two possibilities for $v_1^{\overline{Q}}(g^\circ)$, depending on g°. First,

$$v_1^{\overline{Q}}(g^\circ) = 3 - G(g^\circ; \theta_3) + \theta_1 \ln G(g^\circ; \theta_3)$$

if $\theta_1 + \theta_2 < g^\circ \le \bar{g}(\theta_1 + \theta_3; \theta_3)$. Second,

$$v_1^{\overline{Q}}(g^\circ) = 3 - (\theta_1 + \theta_3) - (1 - g^\circ/3) - \theta_3 \ln g^\circ + (\theta_1 + \theta_3) \ln (\theta_1 + \theta_3)$$

if $\bar{g}(\theta_1 + \theta_3; \theta_3) < g^\circ \le \theta_1 + \theta_3$. Comparing these yields the unique $\hat{g}$, such that $\overline{Q}$ is optimal if $g^\circ \le \hat{g}$. Otherwise $\underline{Q}$ is optimal.

B.3 Details of Example 3

If agents are far-sighted, the agenda-setter now solves[20]

$$\max_{(\tau,g^1,g^2,\boldsymbol{s}^1,\boldsymbol{s}^2)} (1-\tau) + s_1^1 + \theta_1 \ln g^1 + \delta\left[\theta_1 \ln g^2(g^1) + s_1^2(g^1)\right]$$

$$\text{subject to } \; g^1 + s_1^1 + s_2^1 + s_3^1 \leq 3,$$

$$g^2 + s_1^2 + s_2^2 + s_3^2 \leq 3,$$

$$\text{and either}$$

$$\begin{gathered}(1-\tau) + s_2^1 + \theta_2 \ln g^1 + \delta\left[\theta_2 \ln g^2(g^1) + s_2^2(g^1)\right] \\ \geq \\ (1-g^\circ/3) + \theta_2 \ln g^\circ + \delta\left[\theta_2 \ln g^2(g^\circ) + s_2^2(g^\circ)\right] \\ \text{or} \\ (1-\tau) + s_3^1 + \theta_3 \ln g^1 + \delta\left[\theta_3 \ln g^2(g^1) + s_3^2(g^1)\right] \\ \geq \\ (1-g^\circ/3) + \theta_3 \ln g^\circ + \delta\left[\theta_3 \ln g^2(g^\circ) + s_3^2(g^\circ)\right].\end{gathered} \tag{$\mathcal{P}(g^1)$}$$

In the first period, agents also take into account the payoffs in the second period: hence the extra term in square brackets represents the continuation payoff in the second period, discounted to the first period by a factor $\delta \in [0,1]$.

The agenda-setter selects g^1 in the first period. This public-good level will then become the outside option in the second period. For a given g^1, the equilibrium public-good provision in the second period will necessarily be in accordance with Theorem 1 and Theorem 2, as this is the last period.

As a simple example, consider $g^\circ = 0$. It is straightforward to verify that $\mathcal{P}(g^1)$ reduces to

$$\max_{(g^1,g^2,\boldsymbol{s}^2)} \theta_i \ln g^1 + 3 - g^1 + \delta\left[\theta_i \ln g^2(g^1) + s_i^2(g^1)\right]$$

$$\text{subject to } \; g^2 + s_a^2 + s_i^2 \leq 3.$$

with $\theta_i \in \{\theta_2, \theta_3\}$. Once g^1 is chosen, the problem in the second period will be equivalent

[20] The superscripts 1 and 2 refer to variables in the first and second period, respectively.

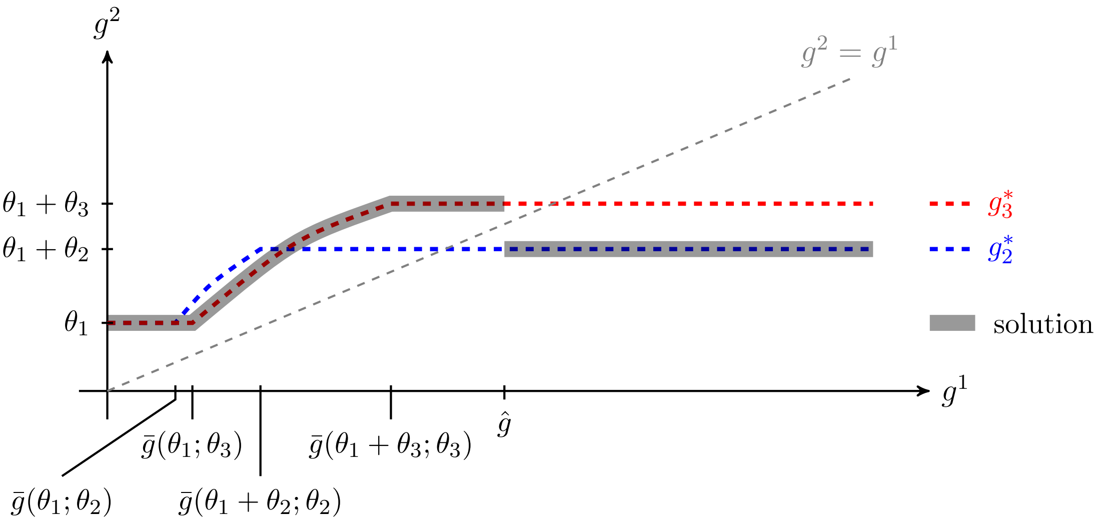

Figure 9: g^2 for $N = \{1, 2, 3\}$ example.

to $\mathcal{P}(g^\circ)$. Hence, by selecting g^1, the agenda-setter is solving the following piecewise maximization problem

$$\max \left\{ \max_{g^1 \in [0, \bar{g}(\theta_1;\theta_3)]} \theta_1 \ln g^1 + 3 - g^1 + \theta_1 \ln \theta_1 + 3 - \theta_1, \right.$$

$$\max_{g^1 \in [\bar{g}(\theta_1;\theta_3), \bar{g}(\theta_1+\theta_3;\theta_3)]} \theta_1 \ln g^1 + 3 - g^1 + \theta_1 \ln \left\{ g^1 \exp \frac{1 - g^1/3}{\theta_3} \right\} + 3 - g^1 \exp \frac{1 - g^1/3}{\theta_3},$$

$$\max_{g^1 \in [\bar{g}(\theta_1+\theta_3;\theta_3), n]} \theta_1 \ln g^1 + 3 - g^1 + \theta_1 \ln (\theta_1 + \theta_i) +$$

$$\left. + 3 - (\theta_a + \theta_i) - \left(1 - \frac{g^1}{3}\right) - \theta_i \left[\ln g^1 - \ln (\theta_a + \theta_i) \right] \right\}.$$

Depending on the choice of g^1, g^2 will be in one of the three regimes, $[0, \bar{g}(\theta_1;\theta_3)]$, $[\bar{g}(\theta_1;\theta_3), \bar{g}(\theta_1 + \theta_3;\theta_3)]$ or $[\bar{g}(\theta_1 + \theta_3;\theta_3), n]$, as shown in Figure 9. For each regime, the agenda-setter will calculate the utility-maximizing g^1. Given these values, the agenda-setter will then choose the regime that yields the highest utility.

Note that the equilibrium public-good provision never crosses the 45-degree line. Hence, it can never be the case that the same public-good level is proposed across the two periods.