

# Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity

*Nicolas Caramp, Dejanir H. Silva*

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Poschingerstr. 5, 81679 Munich, Germany

Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email [office@cesifo.de](mailto:office@cesifo.de)

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# Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity

## Abstract

We study the role of asset revaluation in the monetary transmission mechanism. We build an analytical heterogeneous-agents model with two main ingredients: i) rare disasters; ii) heterogeneous beliefs. The model captures time-varying risk premia and precautionary savings in a setting that nests the textbook New Keynesian model. The model generates large movements in asset prices after a monetary shock but these movements can be neutral on real variables. Real effects depend on the redistribution among agents with heterogeneous precautionary motives. In a calibrated exercise, we find that this channel accounts for the majority of the transmission to output.

JEL-Codes: E210, E440, E520, G120.

Keywords: monetary policy, wealth effects, asset prices, aggregate risk, heterogeneity beliefs.

*Nicolas Caramp*  
*UC Davis / CA / USA*  
*ncaramp@ucdavis.edu*

*Dejanir H. Silva*  
*Krannert School of Management*  
*Purdue University, West Lafayette / IN / USA*  
*dejanir@purdue.edu*

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# 1 Introduction

A long tradition in monetary economics emphasizes the role of the revaluation of real and financial assets in shaping the economy's response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists.<sup>1</sup> Keynes himself described the effects of interest rate changes as follows:

Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, *depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets.*

- John M. Keynes, *The General Theory of Employment, Interest, and Money* (emphasis added).

These revaluation effects caused by monetary policy have been documented by an extensive empirical literature. [Bernanke and Kuttner \(2005\)](#) study the effects of monetary shocks on stock prices. [Gertler and Karadi \(2015\)](#) and [Hanson and Stein \(2015\)](#) consider the effects on bonds. A robust finding of this literature is that changes in asset prices are explained mainly by fluctuations in future excess returns, related to changes in the risk premia, rather than changes in the risk-free rate.<sup>2</sup>

The extent to which changes in asset prices play a relevant role in the transmission of monetary policy to the real economy, however, has been controversial. One view highlights the importance of *wealth effects*. [Cieslak and Vissing-Jorgensen \(2020\)](#) show that policymakers track the behavior of stock markets because of their impact on households' consumption, while [Chodorow-Reich, Nenov and Simsek \(2021\)](#) study the importance of this channel empirically. An alternative view defends that changes in asset valuations have no real implications. [Cochrane \(2020\)](#) and [Krugman \(2021\)](#) argue that movements in discount rates lead to changes in "paper wealth," without an impact on consumption.

In this paper, we study how monetary policy affects the real economy through changes

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<sup>1</sup>The revaluation of government liabilities was central to [Pigou \(1943\)](#) and [Patinkin \(1965\)](#), while [Metzler \(1951\)](#) considered stocks and money. [Tobin \(1969\)](#) focused on the revaluation of real assets.

<sup>2</sup>For a recent review of this work, see [Bauer and Swanson \(2023\)](#).

in asset prices in a New Keynesian setting. We provide a new framework that generates rich asset-pricing dynamics and heterogeneous portfolios while preserving the simplicity of the textbook model. In particular, we propose a new solution technique that delivers time-varying risk premium and precautionary savings motive without having to resort to higher-order approximations.<sup>3</sup> We derive necessary conditions for changes in risk premia to affect the real economy. Under special conditions, we obtain a *risk-premium neutrality result*, where changes in risk premia caused by monetary shocks affect asset prices, but they have no effect on output and inflation. We identify the redistribution generated by heterogeneous portfolios revaluations among agents with different precautionary motives as the main channel through which risk premia affect the real economy. We assess quantitatively the importance of this channel and find that changes in risk premia account for a large fraction of the response of output and inflation to changes in monetary policy.

We consider an economy populated by workers and savers with two main ingredients: i) rare disasters, and ii) heterogeneous beliefs. Rare disasters enable us to capture both a precautionary savings motive and realistic risk premia.<sup>4</sup> Savers invest in stocks, long-term government bonds, and short-term debt, and have heterogeneous beliefs, as in Caballero and Simsek (2020).<sup>5</sup> This has two consequences. First, they hold heterogeneous portfolios in equilibrium. Second, they have heterogeneous marginal propensities to consume (MPCs) out of changes in wealth due to different precautionary motives. This generates *time-variation* in risk premia in response to monetary shocks. Despite being stylized, the model captures quantitatively central aspects of the monetary transmission mechanism, including the term premium, the equity premium, and corporate spreads.

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<sup>3</sup>As shown in e.g. Schmitt-Grohé and Uribe (2004), a standard perturbation around the non-stochastic steady state can only generate time-varying risk premia with at least a third-order approximation.

<sup>4</sup>Rare disasters have been widely used to explain a range of asset-pricing “puzzles”; see Tsai and Wachter (2015) for a review.

<sup>5</sup>For recent evidence from bond returns consistent with belief heterogeneity, see Bauer and Chernov (2023). A large literature on asset pricing studies models with heterogeneous beliefs, see e.g. Detemple and Murthy (1994), Basak (2005), and Atmaz and Basak (2018).

Our first contribution is methodological and consists of an aggregation result. Given investor heterogeneity, we must characterize not only the dynamics of aggregate output and inflation, but also the behavior of portfolios, asset prices, and individual consumption. This increases the dimensionality of the problem and typically makes deriving analytical results infeasible. We show that our economy satisfies an *as if* result: the economy with heterogeneous savers behaves as an economy with a representative saver, but the probability of disaster, as implied by market prices, is time-varying and responds to monetary policy. This *market-implied disaster probability* is a key determinant of asset prices, and it is the main channel through which investor heterogeneity affects the real economy.

Our second contribution identifies conditions under which time-varying risk premia plays a role in the monetary transmission mechanism. Consistent with the evidence, a contractionary monetary shock leads to an increase in risk premia and a reduction in the price of risky assets. One could then conclude that this reduction in households' wealth leads to a reduction in consumption. However, as the discount rate increases, the amount of wealth required to finance the same amount of consumption also decreases. The net effect of changes in risk premium is ambiguous and depends on whether households are net buyers or net sellers of risky assets. As recently articulated by [Cochrane \(2020\)](#) and [Krugman \(2021\)](#), a household who consumes the dividends from their financial assets can still afford the same level of consumption after a change in discount rates.

Formally, we show that the aggregate wealth effect corresponds to the sum of all households' wealth *net* of the change in the cost of the original consumption bundle. Interestingly, the aggregate wealth effect does not depend on the equity premium. Movements in equity prices redistribute wealth among investors but do not generate gains or losses for the household sector as a whole. In a closed economy, the government is the only counterpart to the household sector, so the aggregate wealth effect depends on the revaluation of government bonds and the amount of trading in these bonds.

Risk also affects the households' precautionary motive, given the redistribution among

savers after a monetary shock. Because optimists hold a larger fraction of their wealth in risky assets, an increase in the interest rate disproportionately reduces their wealth. Holding the *aggregate* wealth effect constant, this redistribution of wealth is then reflected in the market-implied probability of disaster, which increases after the monetary shock. This is the “as-if” result in action: redistribution between optimists and pessimists is akin to an increase in the “objective” probability of disaster risk in a representative-agent model.

We consider next the quantitative importance of risk and heterogeneity for the transmission of monetary shocks to the real economy. We find that the time-varying precautionary motive accounts for roughly 60% of the response of output on impact, while the response coming from the aggregate wealth effect accounts for roughly 30% of the overall output response. The intertemporal-substitution effect accounts for less than 10% of the response of output on impact. Heterogeneous beliefs are crucial for this result. The response of output in the economy with heterogeneous beliefs is more than three times larger than in the economy with homogeneous beliefs. We introduce long-term defaultable household debt and find that it amplifies the response of output. Hence, risk and heterogeneity play a large role in how monetary policy affects the real economy.

**Literature review.** Wealth effects have a long tradition in monetary economics. [Pigou \(1943\)](#) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. [Kalecki \(1944\)](#) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while Tobin highlighted the role of private assets and high-MPC borrowers. Recently, wealth effects have regained relevance. [Kaplan, Moll and Violante \(2018\)](#) build a quantitative HANK model and find only a minor role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects. Much of the literature has focused on the role of heterogeneous marginal propensities to consume (MPCs) in settings with idiosyncratic income risk. Instead, we focus on aggregate risk and heterogeneous portfolios.

Our work is closely related to two strands of literature. First, it is related to work on the interaction between monetary policy and changes in asset prices, including models with sticky prices, such as [Caballero and Simsek \(2020\)](#), and models with financial frictions, such as [Brunnermeier and Sannikov \(2016\)](#) and [Drechsler, Savov and Schnabl \(2018\)](#).<sup>6</sup> In a recent contribution, [Kekre and Lenel \(2022\)](#) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the economy to monetary policy. [Kekre, Lenel and Mainardi \(2023\)](#) consider the role of market segmentation in the determination of the term premium. We contribute to this literature by presenting an analytical framework that features aggregate risk and generates a sizable time-varying risk premium while preserving the tractability of standard New Keynesian models. Also related is [Campbell, Pflueger and Viceira \(2020\)](#) and [Pflueger and Rinaldi \(2022\)](#), which use a habit model to study the role of monetary policy in determining bond and equity premia. Their models generate an exact log-linear Euler equation that is independent of risk, which implies that output and inflation are independent of risk, consistent with our risk-premium neutrality result. In contrast, aggregate risk, through the precautionary motives they generate, are a crucial channel of transmission in our model.

The paper is also closely related to the analytical HANK literature, such as [Werning \(2015\)](#) and [Debortoli and Galí \(2017\)](#). While this literature focuses primarily on how the cyclicalities of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. As e.g. [Eggertsson and Krugman \(2012\)](#), we consider the role of household debt, but they abstract from risk and focus instead on the implications of deleveraging. [Iacoviello \(2005\)](#) considers a monetary economy with private debt but focuses instead on housing as collateral. Our work is also related to [Auclert \(2019\)](#), which studies the redistribution channel of monetary policy arising from portfolio heterogeneity. Our paper emphasizes the redistribution channel in

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<sup>6</sup>Our work is also related to the literature on unconventional monetary policy and asset prices, see e.g. [Silva \(2020\)](#), [Caballero and Simsek \(2021\)](#), and [Corhay, Kind, Kung and Morales \(2023\)](#).



the context of a general equilibrium setting with aggregate risk.

Finally, a literature studies rare disasters and business cycles. [Gabaix \(2011\)](#) and [Gou-rio \(2012\)](#) consider a real business cycle model with rare disasters, while [Andreasen \(2012\)](#) and [Isoré and Szczerbowicz \(2017\)](#) allow for sticky prices. They focus on changes in disaster probability while we study monetary shocks in a heterogeneous-agent model.

## 2 D-HANK: A Rare Disasters Analytical HANK Model

In this section, we consider an analytical HANK model with two main ingredients: i) the possibility of rare disasters, and ii) heterogeneous beliefs.

### 2.1 The Model

**Environment.** Time is continuous and denoted by  $t \in \mathbb{R}_+$ . The economy is populated by households, firms, and a government. There is a continuum of households that can be of three types: *workers*, *optimistic savers*, and *pessimistic savers* (denoted by  $w$ ,  $o$  and  $p$ , respectively), who differ in their discount rates and beliefs about the probability of the economy being hit by an aggregate shock. We let  $\mu_j \geq 0$  denote the mass of households of type  $j \in \{w, o, p\}$ , where  $\mu_w + \mu_o + \mu_p = 1$ . Households can borrow or lend at a riskless rate subject to a borrowing constraint, and they can save on long-term nominal government bonds and corporate equity. In this section, we assume that the borrowing limit is zero. We study the case of a positive borrowing limit and defaultable long-term household debt in Section 5. Workers are the only ones who supply labor, and they are relatively impatient, so their borrowing constraint is binding in equilibrium.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity  $\epsilon > 1$ . Intermediate-goods producers use labor as their only input and face quadratic ([Rotem-](#)

berg, 1982) pricing adjustment costs. Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity  $\bar{\lambda} \geq 0$ , their productivity is permanently reduced. This shock captures the possibility of rare disasters: low-probability, large drops in productivity and output, as in the work of Barro (2006, 2009). Periods that predate the realization of the shock are in the *no-disaster state*, and periods that follow the shock are in the *disaster state*. The disaster state is absorbing, and there are no further shocks after the disaster is realized.<sup>7</sup>

The government sets fiscal policy, comprising of transfers to workers and savers, and monetary policy, specified by an interest rate rule subject to monetary shocks.

**Savers' problem.** Savers face a portfolio problem where they choose how much to invest in short-term bonds, long-term bonds, and corporate equity.

A long-term bond issued in period  $t$  trades at a nominal market price  $Q_{L,t}$  in the no-disaster state and promises to pay coupons  $e^{-\psi_L(s-t)}$  at all dates  $s \geq t$ . Because of the structure of the coupon payments, the prices of the bonds issued at previous dates are proportional to new issues, i.e. a bond issued in  $t - z$  trades at  $Q_{L,t}e^{-\psi_L z}$  in period  $t$ . The rate of decay  $\psi_L$  is inversely related to the bond's duration, where a consol corresponds to  $\psi_L = 0$  and the limit  $\psi_L \rightarrow \infty$  corresponds to the case of short-term bonds. We denote by  $Q_{L,t}^*$  the price of the bond in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state. Then, the nominal return on the long-term bond is given by

$$dR_{L,t} = \left[ \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L \right] dt + \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where  $\mathcal{N}_t$  is a Poisson process with arrival rate  $\bar{\lambda}$  (under the objective measure).

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<sup>7</sup>Assuming an absorbing disaster state simplifies the presentation, but it is not essential for our results. Allowing for partial recovery, as in e.g. Barro, Nakamura, Steinsson and Ursúa (2013), introduces dynamics in the disaster state, but it does not change the implications for the no-disaster state, which is our focus.

The price of a claim on real aggregate corporate profits is denoted by  $Q_{E,t}$  and the real return on equities evolves according to

$$dR_{E,t} = \left[ \frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} \right] dt + \frac{Q_{E,t}^* - Q_{E,t}}{Q_{E,t}} d\mathcal{N}_t,$$

where  $\Pi_t$  denotes real profits and  $Q_{E,t}^*$  is the equity price in the disaster state.

Savers have heterogeneous beliefs regarding the probability of a disaster. Subjective beliefs about the arrival rate of the aggregate productivity shock are given by  $\lambda_j$ , for  $j \in \{o, p\}$ , where  $\lambda_o \leq \lambda_p$ . We follow [Chen, Joslin and Tran \(2012\)](#) and assume that savers are dogmatic in their beliefs about disaster risk, so we abstract from any learning process.

Savers' subjective discount rate is a function of their consumption share,  $\rho_{j,t} = \rho_j \left( \frac{C_{j,t}}{C_{s,t}} \right)$ , where  $C_{s,t} = \frac{\mu_o}{\mu_o + \mu_p} C_{o,t} + \frac{\mu_p}{\mu_o + \mu_p} C_{p,t}$  denotes savers' aggregate consumption. Following [Schmitt-Grohé and Uribe \(2003\)](#), we assume that  $\rho_j(\cdot)$  depends on the average consumption of type- $j$  savers, so it is taken as given by any individual saver. This formulation, a form of [Uzawa \(1968\)](#) preferences, implies that there is a unique stationary wealth distribution, but it is otherwise not central to our results.

Let  $B_{j,t} = B_{j,t}^S + B_{j,t}^L + B_{j,t}^E$  denote the net worth of a type- $j$  saver, the sum of short-term bonds  $B_{j,t}^S$ , long-term bonds  $B_{j,t}^L$ , and equity holdings  $B_{j,t}^E$ . A type- $j$  saver chooses consumption  $C_{j,t}$ , long-term bonds  $B_{j,t}^L$ , and equity holdings  $B_{j,t}^E$ , given an initial net worth  $B_{j,t} > 0$ , to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{\{C_{j,z}, B_{j,z}^L, B_{j,z}^E\}_{z \geq t}} \mathbb{E}_{j,t} \left[ \int_t^{t^*} e^{-\int_t^z \rho_{j,u} du} \frac{C_{j,z}^{1-\sigma}}{1-\sigma} dz + e^{-\int_t^{t^*} \rho_{j,u} du} V_{j,t^*}^*(B_{j,t^*}^*) \right],$$

subject to the flow budget constraint

$$dB_{j,t} = \left[ (i_t - \pi_t) B_{j,t} + r_{L,t} B_{j,t}^L + r_{E,t} B_{j,t}^E + T_{j,t} - C_{j,t} \right] dt + \left[ B_{j,t}^* - B_{j,t} \right] d\mathcal{N}_t,$$

and borrowing constraint  $B_{j,t} \geq 0$ , given  $B_{j,0} > 0$ , where  $B_{j,t}^* = B_{j,t} + B_{j,t}^L \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} + B_{j,t}^E \frac{Q_{E,t}^* - Q_{E,t}}{Q_{E,t}}$  denotes savers' net worth after the disaster is realized,  $i_t$  is the nominal interest rate,  $\pi_t$  is the inflation rate,  $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t$  is the excess return on long-term bonds conditional on no disasters,  $r_{E,t} \equiv \frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} - (i_t - \pi_t)$  is the excess return on equities conditional on no disasters, and  $T_{j,t}$  denotes government transfers. The random arrival time  $t^*$  represents the period in which the aggregate shock hits the economy.  $V_{j,t^*}^*$  denotes the value function in the disaster state. The savers' problem in the disaster state corresponds to a deterministic version of the problem above. The non-negativity constraint on  $B_{j,t}$  captures the assumption that households cannot borrow on net.

The savers' Euler equation for short-term bonds is given by

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho_{j,t}) + \frac{\lambda_j}{\sigma} \left[ \left( \frac{C_{j,t}}{C_{j,t}^*} \right)^\sigma - 1 \right], \quad (1)$$

where  $C_{j,t}^*$  is the consumption of a type- $j$  saver in the disaster state. The first term captures the usual intertemporal-substitution force present in RANK models. The second term captures the *precautionary savings motive* generated by the disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

The Euler equation for long-term bonds is given by

$$r_{L,t} = \underbrace{\lambda_j \left( \frac{C_{j,t}}{C_{j,t}^*} \right)^\sigma}_{\text{price of disaster risk}} \underbrace{\frac{Q_{L,t} - Q_{L,t}^*}{Q_{L,t}}}_{\text{quantity of risk}}. \quad (2)$$

This expression captures a risk premium on long-term bonds, which pins down long-term interest rates in equilibrium. The premium on long-term bonds is given by the product of the *price of disaster risk*, the compensation for a unit exposure to the risk factor, and the *quantity of risk*, the loss the asset suffers conditional on switching to the disaster state.

Similarly, the Euler equation for equities is given by

$$r_{E,t} = \lambda_j \left( \frac{C_{j,t}}{C_{j,t}^*} \right)^\sigma \frac{Q_{E,t} - Q_{E,t}^*}{Q_{E,t}}. \quad (3)$$

The expression above pins down the (conditional) equity premium. Note that differences in the quantity of risk drive the differences in expected returns between stocks and bonds.

**Workers' problem.** Workers supply labor and have GHH preferences (Greenwood, Hercowitz and Huffman, 1988) over consumption and labor. Their problem is given by

$$V_{w,t}(B_{w,t}) = \max_{[C_{w,z}, N_{w,z}]_{z \geq t}} \mathbb{E}_{w,t} \left[ \int_t^{t^*} \frac{e^{-\rho_w(z-t)}}{1-\sigma} \left( C_{w,z} - \frac{N_{w,z}^{1+\phi}}{1+\phi} \right)^{1-\sigma} dz + e^{-\rho_w(t^*-t)} V_{w,t^*}^*(B_{w,t^*}) \right],$$

subject to the flow budget constraint  $dB_{w,t} = \left[ (i_t - \pi_t) B_{w,t} + \frac{W_t}{P_t} N_{w,t} + T_{w,t} - C_{w,t} \right] dt$ , and the borrowing constraint  $B_{w,t} \geq 0$ , where  $W_t$  is the nominal wage,  $P_t$  is the price level, and  $T_{w,t}$  denotes fiscal transfers to workers.

We focus on the case where the initial condition is  $B_{w,0} = 0$  and  $\rho_b$  is sufficiently large, so workers are constrained at all periods. As workers are constrained, their beliefs about the disaster probability play no role in the determination of equilibrium. The labor supply is determined by the condition  $\frac{W_t}{P_t} = N_{w,t}^\phi$ . GHH preferences imply that there is no income effect on labor supply, roughly in line with the evidence (see e.g. Auclert, Bardóczy and Rognlie, 2021), and simplifies the model aggregation.<sup>8</sup>

**Market-implied probabilities and the SDF.** From equations (2) and (3), we can see that, even though savers disagree on the probability of a disaster, they agree on the *value* of a

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<sup>8</sup>GHH preferences avoid the counterfactual implications caused by income effects on labor supply in sticky-price heterogeneous-agent models emphasized by Broer, Harbo Hansen, Krusell and Öberg (2020).

unit of consumption in that state.<sup>9</sup> We can then price any cash flow using the beliefs and marginal utility of either optimistic or pessimistic savers. Instead of using the beliefs of a specific saver, it is convenient to define the economy's stochastic discount factor (SDF) using the aggregate consumption of savers, and the corresponding disaster probability implied by asset prices, as shown in Proposition 1.

**Proposition 1** (Market-implied disaster probability). *Define the market-implied disaster probability  $\lambda_t$  as follows:*

$$\lambda_t \equiv \left[ \frac{\mu_o C_{o,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_{p,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \lambda_p^{\frac{1}{\sigma}} \right]^\sigma, \quad (4)$$

and let  $\mathbb{E}_t[\cdot]$  denote the expectation operator associated with the arrival rate  $\lambda_t$  for the disaster shock. Then,  $\eta_t = e^{-\int_0^t \rho_{s,z} dz} C_{s,t}^{-\sigma}$  is a valid stochastic discount factor, i.e.,  $\eta_t$  correctly prices all tradeable assets given the disaster probability  $\lambda_t$  and an appropriately chosen process for  $\rho_{s,t}$ .

*Proof.* To ensure that  $\eta_t$  correctly prices long-term bonds and equities, consistent with equations (2) and (3), the market-implied disaster probability must satisfy the condition  $\lambda_t \left( \frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma = \lambda_j \left( \frac{C_{j,t}}{C_{j,t}^*} \right)^\sigma \Rightarrow C_{j,t}^* = \left( \frac{\lambda_j}{\lambda_t} \right)^{\frac{1}{\sigma}} \frac{C_{s,t}^*}{C_{s,t}} C_{j,t}$ . Plugging  $C_{j,t}^*$  into the definition of savers' average consumption in the disaster state,  $C_{s,t}^* \equiv \frac{\mu_o}{\mu_o + \mu_p} C_{o,t}^* + \frac{\mu_p}{\mu_o + \mu_p} C_{p,t}^*$ , and rearranging gives equation (4). By setting  $\rho_{s,t} \equiv \sum_{j \in \{o,p\}} \frac{\mu_j C_{j,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} (\rho_{j,t} + \lambda_j) - \lambda_t$ , we ensure that  $\eta_t$  correctly prices risk-free bonds, i.e.,  $\mathbb{E}_t[d\eta_t]/\eta_t = -(i_t - \pi_t)dt$ .  $\square$

The market-implied probability  $\lambda_t$  is a CES aggregator of individual probabilities, weighted by the corresponding consumption share. Expression (4) is reminiscent of the complete-markets formula with heterogeneous beliefs in [Varian \(1985\)](#). In our setting, consumption shares can potentially move over time, which leads to endogenous time-variation in the perceived probability of a disaster. We can then price assets as-if the

<sup>9</sup>The value of a consumption unit in the disaster state for saver  $j$  is  $\lambda_j (C_{j,t}^*/C_{j,t})^{-\sigma}$ , the continuous-time version of the standard expression for state prices, which is equalized for all savers from equations (2)-(3).

economy has a representative saver with (endogenous) time-varying beliefs.

**Firms' problem.** Intermediate-goods producers are indexed by  $i \in [0, 1]$  and operate in monopolistically competitive markets. Final good producers are price takers and combine intermediate goods to produce the final good. Their demand for variety  $i$  is given by  $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ , and the equilibrium price level is given by  $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ .

Intermediate-goods producers operate the linear technology  $Y_{i,t} = A_t N_{i,t}$ . Productivity in the no-disaster state is given by  $A_t = A$ , and productivity in the disaster state is given by  $A_t = A^*$ , where  $0 < A^* < A$ . Intermediate-goods producers choose the rate-of-change of prices  $\pi_{i,t} = \dot{P}_{i,t}/P_{i,t}$ , given the initial price  $P_{i,0}$ , to maximize the expected discounted value of real profits subject to Rotemberg quadratic adjustment costs. These costs are rebated back to shareholders, so they do not represent real resource costs. The optimality condition for the firms' problem delivers the non-linear New Keynesian Phillips curve (NKPC):

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right) \pi_t - \frac{\epsilon}{\varphi A} \left(\frac{W_t}{P_t} - (1 - \epsilon^{-1})A\right) Y_t, \quad (5)$$

assuming a symmetric initial condition  $P_{i,0} = P_0$ , for all  $i \in [0, 1]$ , and  $\pi_{i,t}^* = 0$ .

**Government.** The government is subject to a flow budget constraint

$$\dot{D}_{G,t} = (i_t - \pi_t + r_{L,t})D_{G,t} + \sum_{j \in \{w,o,p\}} \mu_j T_{j,t},$$

and a No-Ponzi condition  $\lim_{t \rightarrow \infty} \mathbb{E}_0[\eta_t D_{G,t}] \leq 0$ , where  $D_{G,t}$  denotes the real value of government debt,  $D_{G,0} = D_G$  is given, and analogous conditions hold in the disaster state. Transfers to workers are given by the policy rule  $T_{w,t} = T_w(Y_t)$ . We assume  $T_{o,t} = T_{p,t}$ , and the government adjusts transfers to savers such that the No-Ponzi condition is satisfied.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \phi_\pi \pi_t + u_t, \quad (6)$$

where  $\phi_\pi > 1$ ,  $u_t$  is a monetary shock, and  $r_n$  denotes the real rate when  $\pi_t = u_t = 0$  at all periods. We assume that in the disaster state there are no monetary shocks, that is,  $i_t^* = r_n^* + \phi_\pi \pi_t^*$ . By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during “normal times.”

**Market clearing.** The market-clearing conditions are given by

$$\sum_{j \in \{w, o, p\}} \mu_j C_{j,t} = Y_t, \quad \sum_{j \in \{w, o, p\}} \mu_j B_{j,t}^S = 0, \quad \sum_{j \in \{w, o, p\}} \mu_j B_{j,t}^L = D_{G,t}, \quad \sum_{j \in \{w, o, p\}} \mu_j B_{j,t}^E = Q_{E,t},$$

and  $\mu_w N_{w,t} = N_t$ , where  $Y_t = \left( \int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  and  $N_t = \int_0^1 N_{i,t} di$ .

## 2.2 Equilibrium dynamics

**Stationary equilibrium.** We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. The economy will be in a stationary equilibrium in the absence of monetary shocks, that is,  $u_t = 0$  for all  $t \geq 0$ . Since variables are constant in each state, we drop time subscripts and write, for instance,  $C_{j,t} = C_j$  and  $C_{j,t}^* = C_j^*$ . For ease of exposition, we follow [Bilbiie \(2018\)](#) and assume that  $T_w$  implements  $C_w = Y$  and  $C_w^* = Y^*$ , and a symmetric allocation in the disaster state:  $C_w^* = C_o^* = C_p^*$ . We discuss a more general case in [Appendix A](#).

The natural interest rate, the real rate in the stationary equilibrium, is given by

$$r_n = \rho_s - \lambda \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right],$$



where  $\rho_s$  and  $\lambda$  are the values of  $\rho_{s,t}$  and  $\lambda_t$  in the stationary equilibrium, and  $0 < C_s^* < C_s$ . We assume that the natural rate is positive,  $r_n > 0$ . The precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy.

In a stationary equilibrium where both types of savers are unconstrained, the following condition must hold  $\rho_o + \lambda_o = \rho_p + \lambda_p$ . As  $\rho_j$  depends on the consumption share, this condition pins down the stationary-equilibrium consumption and wealth distributions. For simplicity, we assume that this equality holds when both types have the same net worth, i.e.,  $B_o = B_p$ , which implies  $C_o > C_p$ .

From equation (2), we can pin down the term spread, the difference between the yield on the long-term bond and the short-term rate, which is given by  $r_L = \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L}$ , and  $Q_L^* < Q_L$ . It can be shown that  $r_L = i_L - r_n$ , where  $i_L = Q_L^{-1} - \psi_L$  is the yield on the long-term bond. Thus, our model generates an upward-sloping yield curve, where long-term yields exceed the short rate, consistent with the data.<sup>10</sup> Similarly, the equity premium (conditional on no-disaster) is given by  $r_E = \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_E - Q_E^*}{Q_E}$ , and  $Q_E^* < Q_E$ .<sup>11</sup> Therefore, the equity premium is positive in the stationary equilibrium.

Households have heterogeneous portfolios in equilibrium. Workers are against the borrowing constraint and hold no equities or long-term bonds. Optimistic savers are more exposed to disaster risk than pessimist investors. The exact composition of their portfolio is indeterminate, as we have one redundant asset. For concreteness, we focus on the case  $B_o^E = B_p^E$ , so optimists hold more long-term bonds, i.e.  $B_o^L > B_p^L$ . This leads to a simpler presentation in the analysis that follows.

**Log-linear dynamics.** We focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize

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<sup>10</sup>The upward-sloping yield curve is caused by the lack of precautionary savings in the disaster state. We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).

<sup>11</sup>The unconditional equity premium equals  $r_E$  minus the expected loss on a disaster. Using  $\lambda$  to compute the expected loss, the (unconditional) equity premium would be given by  $\lambda [(C_s/C_s^*)^\sigma - 1] (Q_E - Q_E^*)/Q_E$ .

the equilibrium conditions around the (stochastic) stationary equilibrium described above. Formally, we perturb the allocation around the economy where  $u_t = 0$  and  $\lambda > 0$ , while the standard approach would perturb around the economy where  $u_t = \lambda_t = 0$ . This enables us to capture the effects of (time-varying) precautionary savings and risk premia in a linear setting, as shown below.<sup>12</sup>

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g.,  $y_t \equiv \log Y_t/Y$  and  $c_{w,t} \equiv \log C_{w,t}/C_w$ . Workers' consumption is given by

$$c_{w,t} = \frac{WN_w}{PY} (w_t - p_t + n_{w,t}) + T'_w(Y)y_t \Rightarrow c_{w,t} = \chi_y y_t, \quad (7)$$

using  $w_t - p_t = \phi y_t$  and  $n_{w,t} = y_t$ , where  $\chi_y \equiv \frac{WN_w}{PY} (1 + \phi) + T'_w(Y)$ . The coefficient  $\chi_y$  controls the cyclicalty of income inequality among workers and savers. We focus on the case  $0 < \chi_y < \mu_w^{-1}$ , such that the consumption of savers, which is given by  $c_{s,t} = \frac{1 - \mu_w \chi_y}{1 - \mu_w} y_t$  from the market clearing condition for goods, is also increasing in  $y_t$ .

Linearizing equation (1) and aggregating across savers, we obtain

$$\dot{c}_{s,t} = \sigma^{-1} (i_t - \pi_t - r_n) + \frac{\lambda}{\sigma} \left( \frac{C_s}{C_s^*} \right)^\sigma p_{d,t}, \quad (8)$$

where

$$p_{d,t} \equiv \sigma (c_{s,t} - c_{s,t}^*) + \hat{\lambda}_t \quad (9)$$

denotes the price of (disaster) risk,  $\hat{\lambda}_t \equiv \log \frac{\lambda_t}{\lambda}$ , and we used the linearized discount-rate function:  $\rho_{j,t} = \rho_j + \sigma \zeta (c_{j,t} - c_{s,t})$ .<sup>13</sup> The expression for the price of risk has two terms. The first term captures the change in the savers' marginal utility of consumption if the disaster shock is realized. The second term represents the change in the market-implied

<sup>12</sup>This method differs from the procedure considered by [Coerdacier, Rey and Winant \(2011\)](#) or [Fernández-Villaverde and Levintal \(2018\)](#), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks.

<sup>13</sup>Uzawa preferences correspond to the case  $\zeta > 0$  and constant discount rates correspond to  $\zeta = 0$ . To simplify the model's aggregation, we assume that the slope coefficient  $\sigma \zeta$  is the same for both types.

disaster probability after a monetary shock.

Combining condition (7) for borrowers' consumption, equation (8) for savers' Euler equation, and the market-clearing condition for goods, we obtain the evolution of aggregate output. Proposition 2 characterizes the dynamics of aggregate output and inflation, given the paths of  $i_t$  and  $p_{d,t}$ . Proofs omitted in the text are provided in the appendix.

**Proposition 2** (Aggregate dynamics). *Given  $[i_t, p_{d,t}]_{t \geq 0}$ , the dynamics of output and inflation is described by the conditions:*

*i. Aggregate Euler equation:*

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \chi_{p_d} p_{d,t}, \quad (10)$$

$$\text{where } \tilde{\sigma}^{-1} \equiv \frac{1-\mu_w}{1-\mu_w\chi_y} \sigma^{-1} \text{ and } \chi_{p_d} \equiv \frac{\lambda}{\tilde{\sigma}} \left( \frac{C_s}{C_s^*} \right)^\sigma.$$

*ii. New Keynesian Phillips curve:*

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \quad (11)$$

$$\text{where } \rho \equiv \rho_s + \lambda \text{ and } \kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y.$$

Condition (10) represents the *aggregate Euler equation*. This equation has two terms, capturing the effects of heterogeneous MPCs, aggregate risk, and heterogeneous beliefs. The first term is the product of the aggregate elasticity of intertemporal substitution (EIS),  $\tilde{\sigma}^{-1}$ , and the real interest rate. The aggregate EIS depends on the cyclical inequality among workers and savers, as captured by  $\chi_y$ . As in the work of [Werning \(2015\)](#) and [Bilbiie \(2019\)](#), heterogeneous MPCs amplify the effect of changes in interest rates if workers' consumption share is procyclical (i.e.,  $\chi_y > 1$ ), as it implies that  $\tilde{\sigma}^{-1} > \sigma^{-1}$ .

The second term,  $\chi_{p_d} p_{d,t}$ , captures the effect of aggregate risk. To understand the economic forces behind this expression, it is useful to rewrite equation (9) as  $p_{d,t} = \tilde{\sigma} y_t +$

$\hat{\lambda}_t$  where we used that  $y_t^* = 0$ . Then, the aggregate Euler equation can be written as

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + \chi_{p_d} \hat{\lambda}_t,$$

where  $\delta \equiv \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma$ . In the absence of belief heterogeneity, so  $\hat{\lambda}_t = 0$ , we can write output as  $y_t = -\tilde{\sigma}^{-1} \int_t^\infty e^{-\delta(s-t)} (i_s - \pi_s - r_n) ds$ . Hence, a positive  $\delta$  dampens the effect of future real interest rates, as in the discounted Euler equation of [McKay, Nakamura and Steinsson \(2017\)](#). In our setting, this is the result of a precautionary motive in response to aggregate disaster risk instead of idiosyncratic income risk. The last term,  $\chi_{p_d} \hat{\lambda}_t$ , captures the effect of heterogeneous beliefs. An increase in the market-implied disaster probability implies that pessimistic investors have a higher consumption share, as shown in [Proposition 1](#). This increase in pessimism triggers a stronger precautionary motive in the aggregate.

Finally, [Proposition 2](#) derives the NKPC. As in a textbook New Keynesian model, inflation is given by the present discounted value of future output gaps,  $\pi_t = \kappa \int_t^\infty e^{-\rho(s-t)} y_s ds$ .

**Fiscal backing.** The log-linearized government's flow budget constraint is given by

$$\bar{d}_G \dot{d}_{G,t} = i_L \bar{d}_G d_{G,t} + \bar{d}_G (i_t - \pi_t + r_{L,t} - i_L) - (\chi_\tau y_t + \tau_t), \quad (12)$$

where  $\bar{d}_G \equiv \frac{D_G}{Y}$ , and  $\chi_\tau y_t + \tau_t$  denotes the primary surplus. The coefficient  $\chi_\tau \equiv -\mu_w T'_w(Y)$  captures the elasticity of tax revenues to output and  $\tau_t \equiv -\sum_{j \in \{o,p\}} \mu_j \frac{T_{j,t} - T_j}{Y}$  represents taxes on savers. As the government adjusts  $\tau_t$  to ensure the No-Ponzi condition is satisfied, we refer to  $\tau_t$  as the *fiscal backing* to the monetary shock.

## 2.3 Monetary policy and risk premia

**Asset prices.** The response of asset prices to monetary policy depends crucially on the behavior of the price of disaster risk, as shown in [equations \(2\) and \(3\)](#). Given the (lin-

earized) price of risk in equation (9), we can price any financial asset in this economy. For example, the price of the long-term bond in period zero is given by

$$q_{L,0} = - \underbrace{\int_0^\infty e^{-(\rho+\psi_L)t} (i_t - r_n) dt}_{\text{path of nominal interest rates}} - \underbrace{\int_0^\infty e^{-(\rho+\psi_L)t} r_L p_{d,t} dt}_{\text{term premium}}. \quad (13)$$

The yield on the long-term bond, expressed as deviations from the stationary equilibrium, is given by  $-Q_L^{-1}q_{L,0}$ , which can be decomposed into two terms: the path of nominal interest rates, as in the expectations hypothesis, and a *term premium*, capturing variations in the compensation for holding long-term bonds. The term premium depends on the price of risk,  $p_{d,t}$ , and the asset-specific loading  $r_L$ . Because the term premium responds to monetary shocks, the expectation hypothesis does not hold in this economy.

The pricing condition for equities is analogous to the one for long-term bonds:

$$q_{E,0} = \underbrace{\frac{Y}{Q_E} \int_0^\infty e^{-\rho t} \hat{\Pi}_t dt}_{\text{dividends}} - \underbrace{\int_0^\infty e^{-\rho t} [i_t - \pi_t - r_n + r_E p_{d,t}] dt}_{\text{discount rate}}, \quad (14)$$

where  $\hat{\Pi}_t = y_t - \frac{WN}{PY}(w_t - p_t + n_t)$ . Equity prices respond to changes in monetary policy through two channels: a *dividend channel*, capturing changes in firms' profits, and a *discount rate channel*, capturing changes in real interest rates and risk premia. Risk premia depends on the price of risk,  $p_{d,t}$ , and the asset-specific loading  $r_E$ .

**Market-implied disaster probability.** Recall that the price of risk depends on  $y_t$  and  $\hat{\lambda}_t$ . We now characterize  $\hat{\lambda}_t$ . Log-linearizing equation (4), we obtain

$$\frac{1}{\sigma} \lambda^{\frac{1}{\sigma}} \hat{\lambda}_t = \mu_{c,o} \mu_{c,p} \left( \lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}} \right) [c_{p,t} - c_{o,t}], \quad (15)$$

where  $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p}$ , for  $j \in \{o, p\}$ . The market-implied disaster probability increases when the monetary shock redistributes wealth towards pessimistic savers. As shown in Appendix A.3, the relative consumption of the two types of savers evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\bar{\zeta}(c_{p,t} - c_{o,t}), \quad (16)$$

and the law of motion of relative net worth  $b_{p,t} - b_{o,t}$  is given by

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \rho(b_{p,t} - b_{o,t}) - \chi_{b,c}(c_{p,t} - c_{o,t}) + \chi_{b,c_s} c_{s,t},$$

where the coefficients  $\chi_{b,c}$  and  $\chi_{b,c_s}$  are functions of portfolios and returns in the stationary equilibrium. Given that the evolution of relative net worth depends on  $c_{s,t}$ , and  $c_{s,t}$  depends on  $y_t$ , we must simultaneously solve for  $[c_{p,t} - c_{o,t}, b_{p,t} - b_{o,t}]_0^\infty$  and  $[i_t, y_t, \pi_t]_0^\infty$ . In this case, obtaining analytical results would likely be infeasible. We show next that this system satisfies an *approximate block recursivity* property, where we can solve for  $c_{p,t} - c_{o,t}$  and  $b_{p,t} - b_{o,t}$  independently of  $(y_t, \pi_t)$ , provided the effect of  $c_{s,t}$  on risk premia is small.

**Proposition 3** (Approximate block recursivity). *Suppose  $r_k \sigma c_{s,t}$  is small for  $k \in \{L, E\}$ , i.e.  $r_k \sigma c_{s,t} = \mathcal{O}(\|i_t - r_n\|^2)$ . Then, the market-implied probability of disaster  $\hat{\lambda}_t$  and relative net worth  $b_{p,t} - b_{o,t}$  can be solved independently of  $(y_t, \pi_t)$ , and they are given by*

$$\hat{\lambda}_t = e^{-\psi_\lambda t} \hat{\lambda}_0, \quad (17)$$

$b_{p,t} - b_{o,t} = e^{-\psi_\lambda t} (b_{p,0} - b_{o,0})$ , and  $\psi_\lambda = \bar{\zeta}$ . If  $i_t - r_n = e^{-\psi_m t} (i_0 - r_n)$ , then  $\hat{\lambda}_0$  is given by

$$\hat{\lambda}_0 = \epsilon_\lambda (i_0 - r_n), \quad (18)$$

where  $\epsilon_\lambda \geq 0$  and the inequality is strict if and only if  $\lambda_p > \lambda_o$ .

Proposition 3 shows that we can solve for  $\hat{\lambda}_t$  and  $b_{p,t} - b_{o,t}$  independently of output and inflation if  $r_k \sigma c_{s,t}$  is small. If  $r_k \sigma c_{s,t}$  is second-order on the size of the monetary shock, its first-order impact on risk premia is negligible. In this case, we can solve for  $\hat{\lambda}_t$  and  $b_{p,t} - b_{o,t}$  independently of  $(y_t, \pi_t)$ . As the dynamics of  $(y_t, \pi_t)$  depends on  $\hat{\lambda}_t$ , but  $\hat{\lambda}_t$  does not depend on  $(y_t, \pi_t)$ , we say the system is (approximately) block recursive. In Appendix A.4, we assess the quantitative importance of the term  $r_k \sigma c_{s,t}$ . For our calibrated parameters, we find that risk premium effects on stocks and bonds when we include the term  $r_k \sigma c_{s,t}$  are nearly identical to the solution when these terms are omitted.

Uzawa preferences ensure that the effects of the monetary shock on the price of risk are transitory. If  $\xi = 0$ , so subjective discount rates are constant, then  $\psi_\lambda = 0$  and a temporary monetary shock has a permanent effect on  $\hat{\lambda}_t$ . The reason is that a monetary policy surprise leads to permanent changes in relative net worth and relative consumption in this case. With Uzawa preferences, savers' net worth eventually converge to their stationary-equilibrium level, so the effect on  $\hat{\lambda}_t$  is transitory.

An important implication of equation (18) is that the price of risk increases after a contractionary monetary shock. A monetary tightening redistributes wealth away from optimistic investors, as they are more exposed to risky assets. The economy becomes on average more pessimistic, which raises the required compensation for holding risky assets. The increase in risk premia in response to contractionary monetary shocks is consistent with the evidence in, e.g., Gertler and Karadi (2015) and Hanson and Stein (2015). Notice that investor heterogeneity is necessary for this result, as  $\hat{\lambda}_t = 0$  when  $\lambda_o = \lambda_p$ .

**The four-equation system.** Proposition 3 allows us to write the price of risk as follows:

$$p_{d,t} = \tilde{\sigma} y_t + e^{-\psi_\lambda t} \hat{\lambda}_0, \quad (19)$$

where  $\hat{\lambda}_0$  is a function of the path of nominal interest rates. Combining the expression above for the price of risk with the interest rate rule (6), the aggregate Euler equation (10), and the NKPC (11), we obtain a four-equation system describing the economy's aggregate dynamics. The system is similar to the textbook three-equation model (see, e.g., Galí, 2015). The interest rate rule and the NKPC are isomorphic to the ones in the simple model. Equation (10) is analogous to the standard Euler equation but features an additional term that depends on the price of risk,  $p_{d,t}$ . It is this term that connects aggregate risk, asset prices, and macroeconomic variables. Finally, equation (19) characterizes how the price of risk depends on aggregate output and changes in monetary policy.

The approximate block-recursivity is crucial to allow us to write the system in terms of aggregate variables, without having to simultaneously solve for the dynamics of individual balance sheets. The portfolio dynamics is summarized by two coefficients:  $\epsilon_\lambda$ , which captures the pass-through of nominal rates to the initial price of risk, and  $\psi_\lambda$ , which controls the persistence of the price of risk. Both coefficients depend on investors' beliefs and their portfolio holdings in the stationary equilibrium.

### 3 Monetary Policy and Wealth Effects

We considered so far how monetary policy affects risk premia and asset prices through their impact on the price of risk,  $p_{d,t}$ , and the market-implied disaster probability,  $\hat{\lambda}_t$ . We study next how the revaluation of real and financial assets affects the real economy.

#### 3.1 Wealth effects and asset revaluations

Asset revaluations caused by monetary policy have received significant attention recently. For instance, Cieslak and Vissing-Jorgensen (2020) show that policymakers pay attention to the stock market due to its potential (consumption) wealth effect. In contrast, Cochrane



(2020) and Krugman (2021) argue that wealth gains on “paper” are not relevant for households who simply consume their dividends. To understand how changes in wealth ultimately affect the real economy, we proceed by first providing a formal definition of wealth effects and then showing how wealth effects shape households’ consumption behavior.

**Wealth effects.** Define the *wealth effect* of household  $j \in \{w, o, p\}$  as (minus) the total compensation required for the household’s initial consumption bundle to be just affordable. Thus, a monetary policy shock generates a negative wealth effect if a positive compensation is required for a household to afford her pre-shock consumption level. Formally, we define the wealth effect, normalized by the initial consumption level, as follows:

$$\Omega_{j,0} \equiv -\frac{1}{C_j} \left( \mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} \bar{C}_{j,t} dt \right] - \mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} C_{j,t} dt \right] \right). \quad (20)$$

where  $\bar{C}_{j,t}$  denotes consumption in the stationary equilibrium, i.e.  $\bar{C}_{j,t} = C_j$  in the no-disaster state and  $\bar{C}_{j,t} = C_j^*$  in the disaster state. The first term inside parenthesis corresponds to the present value of the consumption bundle in the stationary equilibrium discounted by the after-shock SDF, and the second term corresponds to the present discounted value of the consumption bundle in the economy with a monetary shock. The difference between the two equals the additional amount of wealth required for the household to afford the stationary-equilibrium consumption bundle under the new prices. This definition corresponds to (minus) the Slutsky wealth compensation, as defined in Mas-Colell, Whinston and Green (1995), which justifies referring to  $\Omega_{j,0}$  as a wealth effect.<sup>14</sup>

Linearizing equation (20), we obtain

$$\Omega_{j,0} = \int_0^\infty e^{-\rho t} \left( c_{j,t} + \chi_{c_j^*} c_{j,t}^* \right) dt,$$

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<sup>14</sup>Mas-Colell et al. (1995) also proposed an alternative wealth compensation, the so-called Hicksian wealth compensation. We show in Appendix B.2 that the two definitions are equivalent up to first order.

where  $\chi_{c_j^*} \equiv \frac{\delta}{r_n^*} \frac{C_j^*}{C_j}$ . The wealth effect determines the present discounted value of consumption across the two states. Therefore, a monetary shock must generate a positive wealth effect to stimulate consumption in all dates and states. In the absence of a wealth effect, monetary policy can only shift demand over time or across states.

**Asset revaluation.** In equilibrium, the wealth effect depends on the revaluation of real and financial assets. To show this connection, consider the intertemporal budget constraint (IBC) for saver  $j \in \{o, p\}$ . From the flow budget constraint and transversality condition, we obtain:

$$\mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} C_{j,t} dt \right] = B_{j,0} + \mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} T_{j,t} dt \right].$$

The left-hand side corresponds to the value of a claim on consumption, which we denote by  $Q_{C_j,t} \equiv \mathbb{E}_t \left[ \int_t^\infty \frac{\eta_z}{\eta_t} C_{j,z} dz \right]$ . The right-hand side corresponds to saver's net worth  $B_{j,0}$ , the value of stocks and bonds, and a claim on fiscal transfers, denoted by  $Q_{T_j,t} \equiv \mathbb{E}_t \left[ \int_t^\infty \frac{\eta_z}{\eta_t} T_{j,t} dz \right]$ . The linearized intertemporal budget constraint is given by:

$$Q_{C_j} q_{C_j,0} = B_j^L q_{L,0} + B_j^E q_{E,0} + Q_{T_j} q_{T_j,0},$$

where  $q_{C_j,0} \equiv \log Q_{C_j,0} / Q_{C_j}$  and  $q_{T_j} = \log Q_{T_j,0} / Q_{T_j}$ .

We can price the consumption and transfer claims in the same way as we priced stocks and bonds (see equations 13 and 14). For instance, the price of the consumption claim is

$$q_{C_j,0} = \frac{C_j}{Q_{C_j}} \int_0^\infty e^{-\rho t} (c_{j,t} + \chi_{c_j^*} c_{j,t}^*) dt - \int_0^\infty e^{-\rho t} (i_t - \pi_t - r_n + r_{C_j} p_{d,t}) dt, \quad (21)$$

where  $r_{C_j} \equiv \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_{C_j} - Q_{C_j}^*}{Q_{C_j}}$ .

Combining the pricing condition for consumption and the linearized IBC, we obtain

$$\underbrace{\Omega_{j,0}}_{\text{wealth effect}} = \underbrace{\frac{1}{C_j} \left[ B_j^L q_{L,0} + B_j^E q_{E,0} + Q_{T_j} q_{T_j,0} \right]}_{\text{asset-revaluation effect}} + \underbrace{\frac{Q_{C_j}}{C_j} \int_0^\infty e^{-\rho t} (i_t - \pi_t - r_n + r_{C_j} p_{d,t}) dt}_{\text{consumption's discount-rate effect}}.$$

The wealth effect induced by monetary policy has two components. The first component corresponds to the *asset-revaluation effect*, i.e., the change in the value of stocks, bonds, and fiscal transfers. Intuitively, an increase in interest rates reduces the value of stocks and bonds making the household poorer. The second component corresponds to the *consumption's discount-rate effect*, i.e., the change in the value of the consumption claim due to changes in discount rates. An increase in interest rates reduces the value of the consumption claim, everything else constant, so less wealth is required to finance the same consumption bundle. The net effect depends on the sensitivity of households' assets to changes in discount rates relative to the sensitivity of the consumption claim.

**Cash flows vs. discount rates.** Using the pricing condition for bonds, equities, and the transfers claim, we can write the wealth effect as follows:

$$\begin{aligned} \Omega_{j,0} = & -\frac{B_j^L}{C_j} \int_0^\infty e^{-\rho t} \pi_t dt + \frac{Y}{C_j} \int_0^\infty e^{-\rho t} (B_j^E \hat{\Pi}_t + Q_{T_j} \hat{T}_{j,t}) dt \\ & + \int_0^\infty e^{-\rho t} \frac{B_j^S}{C_j} (i_t - \pi_t - r_n) dt - \int_0^\infty e^{-\rho t} \frac{\psi_L B_j^L}{C_j} q_{L,t} dt, \end{aligned} \quad (22)$$

using  $Q_{C_j} = B_j^S + B_j^E + B_j^L + Q_{T_j}$  and  $Q_{C_j} r_{C_j} = B_j^E r_E + B_j^L r_L + Q_{T_j} r_{T_j}$ , and where  $\hat{T}_{j,t} \equiv \frac{T_{j,t} - T_j}{Y}$ .

The first line in the expression above captures the (real) cash-flow effect for long-term bonds, stocks, and fiscal transfers. Naturally, a household is better off if inflation is lower or if profits and transfers are higher, everything else constant. The second line captures

the effect of changes in discount rates. An important implication of equation (22) is that the discount-rate effect depends on the present discounted value of profits from buying and selling assets. In the stationary equilibrium, savers buy-and-hold stocks, so there is no trade in stocks. Expression (22) shows that, absent changes in dividends, movements in stock prices do not generate a wealth effect. In contrast, because coupons decay at rate  $\psi_L$ , investors must purchase  $\psi_L B_j^L$  units of the long-term bond to maintain a constant amount invested in those bonds in the stationary equilibrium. The wealth effect then depends on the net purchase of bonds,  $\psi_L B_j^L$ , and the bond revaluation  $q_{L,t}$ . Finally, as short-term bonds mature instantaneously, investors must purchase the whole amount  $B_j^S$  at every moment, so the wealth effect depends on  $B_j^S$  and the change in the interest rate in these bonds.

To understand the economics behind the wealth effect, a particularly illustrative case corresponds to the situation where an investor holds no bonds, so  $B_j^L = B_j^S = 0$ , and there are no changes in cash flows,  $\hat{\Pi}_t = \hat{T}_{j,t} = 0$ . In this case, the wealth effect is equal to zero, despite a potentially large revaluation effect caused by the drop in equity prices. How is it possible that a household's financial wealth suffers a large drop, while the wealth effect is zero? The reason is that an investor who buy-and-hold stocks can still afford the initial consumption bundle after the shock, as long as the investor does not sell the stocks. Therefore, the wealth effect is zero in this case.

A similar point emerges in the discussion of capital-gains taxation. Discussing the impact of a drop in interest rates for an investor (Bob) whose consumption equals dividends every period, [Cochrane \(2020\)](#) says

"When the interest rate goes down, it takes more wealth to finance the same consumption stream. The present value of liabilities – consumption – rises just as much as the present value of assets, so on a net basis Bob is not at all better."

In our terms, the increase in financial wealth does not translate into a positive wealth

effect, as the increase in the price of stocks exactly cancels out the increase in the value of the consumption claim after a drop in interest rates when consumption equals dividends.

Wealth effects are also a relevant measure of how monetary shocks affect welfare. We show in Appendix B.3 that the change in welfare relative to the stationary equilibrium is given by

$$\Delta V_j = C_j^{1-\sigma} \Omega_{j,0}.$$

Moreover,  $C_j \Omega_{j,0}$  coincides, up to first order, with the compensating variation (CV) and equivalent variation (EV) associated with the policy change. Expression (22), for the special case without cash-flow effects, coincides with the welfare metric in Fagereng, Gomez, Gouin-Bonenfant, Holm, Moll and Natvik (2022). They show that the present discounted value of trading profits captures the welfare effects of discount-rate changes. An important implication of this result is that the wealth effect, and ultimately welfare, depends on how much investors *trade* financial assets instead of how much they *hold* of these assets.

### 3.2 Risk-premium neutrality

Define the aggregate wealth effect as  $\Omega_0 \equiv \sum_{j \in \{w,o,p\}} \frac{\mu_j C_j}{Y} \Omega_{j,0}$ . The aggregate wealth effect determines the average level of aggregate consumption in the no-disaster state,  $\Omega_0 = \int_0^\infty e^{-\rho t} c_t dt$ , as  $c_t^* = 0$ . Hence,  $\Omega_0$  plays an important role in how monetary shocks affect the real economy. The next lemma provides a characterization of  $\Omega_0$ .

**Lemma 1.** *The aggregate wealth effect  $\Omega_0$  is given by*

$$\Omega_0 = \underbrace{\int_0^\infty e^{-\rho t} \left[ (1 - \chi_\tau) y_t - \tau_t - e^{-\psi_L t} \bar{d}_G \pi_t \right] dt}_{\text{cash-flow effect}} + \underbrace{\int_0^\infty e^{-\rho t} (1 - e^{-\psi_L t}) \bar{d}_G (i_t - \pi_t - r_n + r_L p_{d,t}) dt}_{\text{net discount-rate effect}}, \quad (23)$$

where  $\chi_\tau \equiv -\mu_w T'_w(Y)$ .

Analogous to the individual wealth effect, the aggregate wealth effect has two compo-

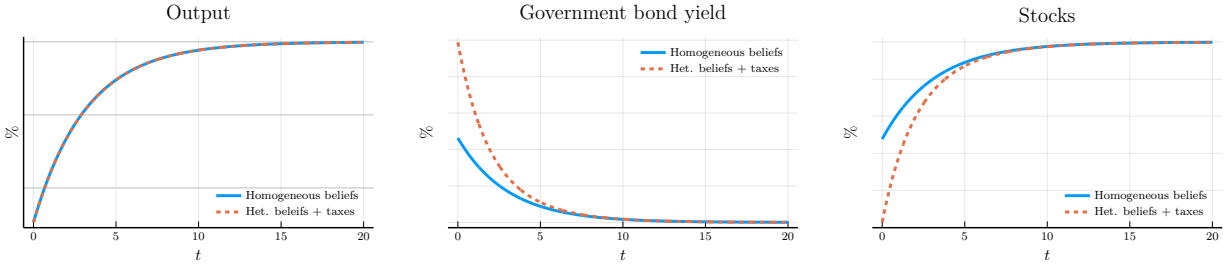
nents. The cash-flow effect, which captures changes in income, taxes, and real coupons on government bonds, and the net discount-rate effect, which depends on the net purchases of government bonds by the household sector and the real return on those bonds. The net discount-rate effect is independent of the conditional equity premium. The household sector as a whole is neither a net buyer or net seller of stocks. Hence, changes in stock prices redistribute across households without affecting the aggregate wealth effect.<sup>15</sup>

**Risk-premium neutrality.** We are ready to state the main result of this section. Proposition 4 shows that, under certain conditions, two economies can have different asset prices, driven by differences in risk premia, but exactly the same path of output and inflation.

**Proposition 4** (Risk-premium neutrality). *Suppose the government uses a consumption tax to neutralize the precautionary motive induced by  $\hat{\lambda}_t$ , that is, consider  $\tau_t^c$  satisfying  $\hat{\tau}_t^c = \lambda \left( \frac{C_s}{C_s^*} \right) \hat{\lambda}_t$ , where  $\hat{\tau}_t^c \equiv \log(1 + \tau_t^c)$ ,  $\tau_t^c = \tau_t^{c,*}$ , and the revenue is rebated back to households such that it is budget neutral for them. Then,  $[y_t, \pi_t]_0^\infty$  is independent of  $\hat{\lambda}_t$ . Moreover, the fiscal backing  $\tau_t$  is independent of  $\hat{\lambda}_t$  if one of the following conditions are satisfied: i)  $\bar{d}_G = 0$ ; ii)  $\bar{d}_G > 0$  and  $\psi_L = \infty$ ; iii)  $\bar{d}_G > 0$  and  $\psi_L = 0$ .*

*Proof.* Savers' Euler equation for the riskless bond is now given by  $\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n - \hat{\tau}_t^c) + \frac{\lambda}{\sigma} \left( \frac{C_s}{C_s^*} \right)^\sigma [\hat{\lambda}_t + \sigma c_{s,t}]$ , which is independent of  $\hat{\lambda}_t$  if  $\hat{\tau}_t^c = \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \hat{\lambda}_t$ . As  $\tau_t^c = \tau_t^{c,*}$ , Euler equations for risky assets are not affected. As the revenue is rebated back to households, workers are not affected. The aggregate Euler equation then takes the same form as in equation (10), but with  $\chi_{p_d} = 0$ . Combining it with equations (6) and (11), we obtain  $[y_t, \pi_t, i_t]$  independently of  $\hat{\lambda}_t$ . The fiscal backing  $\tau_t$  will also be independent of  $\hat{\lambda}_t$  if  $(1 - e^{-\psi_L t})\bar{d}_G r_L = 0$  in equation (23). We have that  $(1 - e^{-\psi_L t})\bar{d}_G$  is zero if  $\bar{d}_G = 0$  or  $\psi_L = 0$ . The excess return on government bonds  $r_L$  is zero if  $\psi_L = \infty$ .  $\square$

<sup>15</sup>In our baseline model, savers buy-and-hold stocks in the stationary equilibrium. In Appendix B.1, we consider an extension where savers actively trade stocks. We show that even when individual investor trade stocks, there is no associated wealth effect in the aggregate, only redistribution across savers.



**Figure 1:** Output and asset prices response

Proposition 4 shows how asset revaluations caused by monetary policy can have no real effects. The proposition provides conditions under which the price of risk does not impact the monetary transmission mechanism. Under such conditions, heterogeneity in portfolios among savers may help improve the model's asset-pricing implications, but they have no bearing on how monetary shocks ultimately affect the real economy. In particular, output and inflation are independent of  $\lambda_p - \lambda_o$ . Due to the increase in the risk premium, an economy with heterogeneous beliefs would have a larger drop in asset prices after a monetary contraction than an economy where  $\lambda_p = \lambda_o$ . Despite the larger decline in the value of stocks and bonds, the response of output and inflation would be the same as in the economy without belief heterogeneity. Figure ?? illustrates this result in a numerical example, which shows output and asset prices in two economies, with and without belief heterogeneity. Despite the large differences in the price of stocks and bonds, the response of output to the monetary shock is the same in both economies.

But why do households in the economy that suffered a larger drop in asset prices consume the same as households in the economy where asset prices did not drop as much? Take for instance the case  $\bar{d}_G = 0$ , so savers only hold stocks in equilibrium. One could expect that, as stock prices fall more sharply in the economy with  $\hat{\lambda}_t > 0$ , households would feel poorer and cut consumption relative to the economy with  $\hat{\lambda}_t = 0$ . However, this intuition does not take into account the fact that households can afford the same level of consumption with less wealth now. As households do not need more resources to af-

ford their initial consumption bundle, since the return on their savings has increased, this decline in asset prices does not create a negative wealth effect. This provides a precise sense in which changes in wealth may reflect only “paper wealth.”<sup>16</sup>

Finally, Proposition 4 also establishes the conditions under which the taxpayer is not exposed to movements in risk premia. If there is no government debt, i.e.,  $\bar{d}_G = 0$ , then the fiscal backing will be trivially independent of asset prices and, therefore, of risk premia. If government debt is positive, then there are two cases in which the fiscal backing is independent of movements in the risk premia. When government debt is a consol, debt repayments are constant every period, independently of the fluctuations in asset prices. In this case, bond holders bear all the repricing risk from the disaster. In the opposite extreme case of short-term debt, government debt is completely safe so it does not carry a risk premium. This is an implicit insurance contract from taxpayers to bondholders: bondholders accept a relatively low return in normal times for the promise of a safe payoff in the case of a disaster. This result is related to the analysis in [Jiang, Lustig, Stanford, Van Nieuwerburgh and Xiaolan \(2022\)](#), who show that there is a trade-off between exposing the taxpayers or bondholders to aggregate risk. They state their results in terms of the risk properties of the fiscal surplus, while we take the complementary approach and focus on the properties of government debt instead.

**Redistribution and iMPCs.** In our economy, risk-premium neutrality holds only if the government offsets the precautionary motive using taxes. [Campbell et al. \(2020\)](#) and [Bianchi, Lettau and Ludvigson \(2022\)](#) consider economies that feature a risk-premium neutrality result in the absence of government intervention, as the price of risk does not enter the Euler equation of those models.<sup>17</sup> As a result, risk affects asset prices but not

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<sup>16</sup>For instance, [Fagereng et al. \(2022\)](#) says “For such an individual [who only consumes dividends], rising asset prices are merely “paper gains,” with no corresponding welfare implications.”

<sup>17</sup>[Campbell et al. \(2020\)](#) assume a habit process that neutralizes the precautionary motive present in our model. In [Bianchi et al. \(2022\)](#), portfolio decisions are limited to a small set of sophisticated investors and



the real economy. In our model, in the absence of such tax changes, risk and belief heterogeneity affect how aggregate output responds to monetary policy. The next result shows that intertemporal MPCs (iMPCs) in the no-disaster and disaster states, that is, how much consumption at date  $t$  in a given state responds to changes in wealth in period 0, are different for optimistic and pessimistic savers.<sup>18</sup>

**Lemma 2** (Intertemporal MPCs). *The iMPC at time  $t$  for saver  $j \in \{o, p\}$  is given by*

$$\mathcal{M}_{j,t} \equiv \frac{1}{C_j} \frac{\partial C_{j,t}}{\partial \Omega_{j,0}} = \frac{(\rho + \xi)}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} e^{-\xi t}, \quad \mathcal{M}_{j,t}^* \equiv \frac{1}{C_j} \frac{\partial C_{j,t}^*}{\partial \Omega_{j,0}} = \frac{(\rho + \xi) \bar{\chi}^* \lambda_j^{\frac{1}{\sigma}}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} e^{-\xi t}.$$

Moreover, iMPCs satisfy the following condition:

$$\int_0^{\infty} e^{-\rho t} \left[ \mathcal{M}_{j,t} + \frac{\delta}{r_n^*} \mathcal{M}_{j,t}^* \right] dt = 1. \quad (24)$$

Lemma 2 shows that optimistic savers have a higher iMPC than pessimistic investors in the no-disaster state, while pessimistic investors have a higher iMPC than optimistic investors in the disaster state. Equation (24) shows that the average iMPC is the same for both investors. Intuitively, optimistic investors buy more consumption goods in the no-disaster state after an increase in wealth than pessimistic investors. These results complement the findings in the HANK literature, where the focus is on the heterogeneity of MPCs generated by the presence of borrowing constraints. Instead, our analysis shows that heterogeneous precautionary motives can generate similar results.

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macro variables are independent of movements in risk premia.

<sup>18</sup>For a discussion of iMPCs in the context of HANK models, see [Auclert, Rognlie and Straub \(2018\)](#). [Auclert \(2019\)](#) analyzes the redistribution channel of monetary policy in a model without aggregate risk.

### 3.3 Intertemporal substitution, risk, and wealth effect

We consider next the general equilibrium response of output and inflation to changes in monetary policy. Consider the system of differential equations in Proposition 2:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix},$$

where we have substituted  $p_{d,t}$  with the expression in equation (9), and  $v_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + \chi_{p_d} \hat{\lambda}_t$  depends only on the path of nominal interest rates. The eigenvalues of the system are given by

$$\bar{\omega} = \frac{\rho + \delta + \sqrt{(\rho + \delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}, \quad \underline{\omega} = \frac{\rho + \delta - \sqrt{(\rho + \delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}.$$

The following assumption, which we assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and have opposite signs, i.e.,  $\bar{\omega} > 0$  and  $\underline{\omega} < 0$ .

**Assumption 1.** *The following condition holds:  $\tilde{\sigma}^{-1}\kappa > \rho\delta$ .*

This assumption implies that local uniqueness of the equilibrium requires a positive coefficient on inflation in the Taylor rule. We show in Section 3.5 that the equilibrium is locally unique if  $\phi_\pi \geq 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \bar{\phi}_\pi$ . Assumption 1 ensures that  $\bar{\phi}_\pi > 0$ .

**Output.** We characterize next the output response to a monetary shock. We extend the analysis in Caramp and Silva (2023), which decomposes the equilibrium path of output into an *intertemporal substitution effect* (ISE) and a *wealth effect*, to our setting with aggregate risk. For ease of exposition, we focus on the case in which the monetary shock induces an exponentially decaying path for the nominal interest rates; that is, we assume  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ , where  $\psi_m$  determines the persistence of interest rates.<sup>19</sup>

<sup>19</sup>The proof of the proposition contains the general case.

**Proposition 5** (Aggregate output in D-HANK). *Suppose that  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$  and  $\psi_k \neq -\underline{\omega}$ , for  $k \in \{m, \lambda\}$ . The path of aggregate output is then given by*

$$y_t = \underbrace{\tilde{\sigma}^{-1} \hat{y}_{m,t}}_{\text{ISE}} + \underbrace{\chi_\lambda \hat{y}_{\lambda,t}}_{\substack{\text{time-varying} \\ \text{precautionary motive}}} + \underbrace{(\rho - \underline{\omega}) e^{\underline{\omega} t} \Omega_0}_{\substack{\text{GE factor} \times \\ \text{aggregate wealth effect}}}, \quad (25)$$

where  $\chi_\lambda \equiv \chi_{p_d} \epsilon_\lambda$ ,  $\hat{y}_{k,t}$  is given by

$$\hat{y}_{k,t} = \frac{(\rho - \underline{\omega}) e^{\underline{\omega} t} - (\rho + \psi_k) e^{-\psi_k t}}{(\bar{\omega} + \psi_k)(\underline{\omega} + \psi_k)} (i_0 - r_n), \quad (26)$$

and satisfies  $\int_0^\infty e^{-\rho t} \hat{y}_{k,t} dt = 0$ ,  $\frac{\partial \hat{y}_{k,0}}{\partial i_0} < 0$ , for  $k \in \{m, \lambda\}$ .

Proposition 5 shows that output can be decomposed into three terms: an intertemporal-substitution effect (ISE), a time-varying precautionary motive, and the aggregate wealth effect. The first two terms correspond to the effects of monetary policy that are not mediated by a change in the aggregate wealth effect. The third term reflects the general equilibrium effects of the wealth effect.

The first term captures the standard intertemporal substitution channel present in RANK models. It depends on the aggregate EIS,  $\tilde{\sigma}^{-1} = \frac{1 - \mu_w}{1 - \mu_w \chi_y} \sigma^{-1}$ , and  $\hat{y}_{m,t}$  given in (26). Notice that, even though only a fraction  $1 - \mu_w$  of agents substitute consumption intertemporally, the ISE does not necessarily get weaker as we reduce the mass of savers in the economy. As we reduce  $1 - \mu_w$ , less agents are capable of intertemporal substitution, but the amplification from hand-to-mouth agents gets stronger. The two effects exactly cancel out when  $\chi_y = 1$ . Importantly, the ISE is equal to zero on average, i.e.  $\int_0^\infty e^{-\rho t} \hat{y}_{m,t} dt = 0$ . An increase in interest rates shifts demand from the present to the future, but by itself it does not change the overall level of aggregate demand.

The second term captures the effect of the time-varying precautionary motive. This

term is equal to zero in the absence of belief heterogeneity. In this case, the model behaves as a TANK model with zero liquidity (see e.g. [Bilbiie 2019](#) and [Broer et al. 2020](#)). With belief heterogeneity, savers have heterogeneous MPCs. The redistribution towards pessimistic investors after a contractionary monetary policy depresses aggregate output on impact. It also triggers movements in the price of risk, which affects the magnitude of the precautionary motive. As with the EIS, the precautionary motive shifts demand from the present to the future without changing its overall level, that is,  $\int_0^\infty e^{-\rho t} \hat{y}_{\lambda,t} dt = 0$ . The persistence of the precautionary effects is controlled by  $\psi_\lambda$ , as it depends on the rate at which the balance sheet of optimistic investors recover after a contractionary shock.

The third term in expression (25) plays an important role, as the aggregate wealth effect determines the average response of output to the monetary shock. The GE factor shifts the impact of the wealth effect over time, as we have that  $(\rho - \underline{\omega}) \int_0^\infty e^{-(\rho - \underline{\omega})t} dt = 1$ . Everything else constant, an increase in  $\Omega_0$  would tend to raise output in all periods by  $\rho\Omega_0$ , creating a parallel shift in output over time. In general equilibrium, a positive aggregate wealth effect leads to inflation on impact, which reduces the real rate and shifts consumption to the present. The GE factor shows that the effect of  $\Omega_0$  on  $y_0$  exceeds the effect on average consumption,  $\rho\Omega_0$ , by the factor  $\frac{\rho - \underline{\omega}}{\rho} > 1$ .

**Inflation.** The next proposition characterizes the behavior of inflation.

**Proposition 6** (Inflation in D-HANK). *Suppose  $i_t - r_n = e^{-\psi_m t} (i_0 - r_n)$  and  $\psi_k \neq -\underline{\omega}$  for  $k \in \{m, \lambda\}$ . The path of inflation is given by*

$$\pi_t = \tilde{\sigma}^{-1} \hat{\pi}_{m,t} + \chi_\lambda \hat{\pi}_{\lambda,t} + \kappa e^{\underline{\omega} t} \Omega_0, \quad (27)$$

where  $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega} t} - e^{-\psi_k t})}{(\underline{\omega} + \psi_k)(\bar{\omega} + \psi_k)} (i_0 - r_n)$ ,  $\hat{\pi}_{k,0} = 0$  and  $\frac{\partial \hat{\pi}_{k,t}}{\partial i_0} \geq 0$ , for  $k \in \{m, \lambda\}$ .

Inflation can be analogously decomposed into three terms. The first two terms capture the impact of the ISE and time-varying precautionary motive, while the last term captures

the impact of the aggregate wealth effect. Because  $\hat{\pi}_{k,0} = 0$ , the first two terms are initially zero. Initial inflation is then entirely determined by the aggregate wealth effect.

### 3.4 The aggregate wealth effect

Propositions 5 and 6 provide a characterization of output and inflation in terms of nominal interest rates and the aggregate wealth effect  $\Omega_0$ . We show next that the aggregate wealth effect can be expressed in terms of policy variables, namely the nominal interest rate  $i_t$  and the fiscal backing  $\tau_t$ .

**Proposition 7.** *Suppose  $\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}} > 0$ . Then,  $\Omega_0$  is a function of  $[i_t, \tau_t]_0^\infty$  given by*

$$\Omega_0 = \frac{\rho - \underline{\omega}}{(\rho - \underline{\omega})\chi_\tau + \bar{d}_G \kappa} \left[ - \int_0^\infty e^{-\rho t} \tau_t dt + \left( -\bar{d}_G \int_0^\infty e^{-\rho t} \hat{\pi}_t dt + \int_0^\infty e^{-\rho t} \Delta B_t^L (i_t - r_n + r_L \hat{\lambda}_t) dt \right) \right], \quad (28)$$

where  $\hat{\pi}_t \equiv \tilde{\sigma}^{-1} \hat{\pi}_{m,t} + \chi_\lambda \hat{\pi}_{\lambda,t}$  is a function of  $[i_t]_0^\infty$ .

*Proof.* Using  $\int_0^\infty e^{-\rho t} y_t dt = \Omega_0$  and  $\int_0^\infty e^{-\rho t} \pi_t dt = \int_0^\infty e^{-\rho t} \hat{\pi}_t dt + \frac{\kappa}{\rho - \underline{\omega}} \Omega_0$ , we obtain

$$\left( \chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}} \right) \Omega_0 = - \int_0^\infty e^{-\rho t} \tau_t dt - \bar{d}_G \int_0^\infty e^{-\rho t} \hat{\pi}_t dt + \int_0^\infty e^{-\rho t} \Delta B_t^L (i_t - r_n + r_L p_{d,t}) dt,$$

after rearranging equation (23). Given our assumption, we can divide both sides by  $\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}}$ . This gives equation (28), using the fact that  $r_L p_{d,t} = r_L \hat{\lambda}_t$  up to first order.  $\square$

Proposition 7 shows that  $\Omega_0$  is uniquely pin down by  $[i_t, \tau_t]_0^\infty$ , given  $\chi_\tau + \frac{\bar{d}_G \kappa}{\rho - \underline{\omega}} > 0$ . This assumption states that monetary policy affects the fiscal authority either through tax revenues or through the cost of servicing the debt (or both). This proposition has an important implication: there are only *two* ways through which monetary policy impacts the aggregate wealth effect. First, monetary policy affects  $\Omega_0$  through its fiscal backing. Second, monetary policy affects  $\Omega_0$  through the revaluation and net discount rate effect

on long-term bonds. As discussed in the context of Proposition 4, the discount-rate effect for equities cancels out at the aggregate, given it only redistributes among investors.

### 3.5 Determinacy and implementability

We derive next the conditions for local determinacy in our D-HANK model. We also show that any path of the nominal interest rate and the fiscal backing can be obtained with the monetary rule (6) and an appropriately chosen path of the monetary shock,  $[u_t]_0^\infty$ .

**Proposition 8** (Determinacy and implementability). *Consider a given monetary shock  $[u_t]_0^\infty$ .*

- i. (Determinacy) *If  $\phi_\pi \geq \bar{\phi}_\pi \equiv 1 - \frac{\rho\delta}{\sigma-1\kappa}$ , then there exists a unique bounded solution to the system comprised of the Taylor rule (6), the aggregate Euler equation (10), the NKPC (11), the market-implied disaster probability (15), and the law of motion of relative consumption (16) and relative net worth (17). We denote this solution by  $[i_t^*, y_t^*, \pi_t^*, \hat{\lambda}_t^*, c_{p,t}^* - c_{o,t}^*, b_{p,t}^* - b_{o,t}^*]$  and the associated path of taxes by  $\tau_t^*$ .*
- ii. (Implementability) *For a given path of nominal interest rates  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ ,  $\psi_m \neq -\underline{\omega}$ , and fiscal backing  $\int_0^\infty e^{-\rho t} \tau_t dt$ , let  $\hat{\lambda}_t$  be given by (17),  $y_t$  be given by (25), and  $\pi_t$  be given by (27), where  $\Omega_0$  is given by (28). If the monetary shock  $u_t$  is given by*

$$u_t = i_t - r_n - \phi_\pi \pi_t, \tag{29}$$

*then  $i_t^* = i_t$  and  $\int_0^\infty e^{-\rho t} \tau_t^* dt = \int_0^\infty e^{-\rho t} \tau_t dt$ . Moreover,  $y_t^* = y_t$ ,  $\pi_t^* = \pi_t$ , and  $\hat{\lambda}_t^* = \hat{\lambda}_t$ .*

The first part of Proposition 8 shows that there is a unique bounded equilibrium if  $\phi_\pi \geq \bar{\phi}_\pi$ . As in Acharya and Dogra (2020), the threshold for determinacy satisfies  $\bar{\phi}_\pi < 1$ , so uniqueness is obtained under a weaker condition than in the textbook model.

The second part of Proposition 8 shows how to implement any given path of policy variables by appropriately choosing the monetary shock  $u_t$ . Combined with Propositions

5-7, this provides a complete characterization of how output and inflation respond to monetary policy. In our quantitative analysis, we consider two approaches to discipline the monetary shocks. First, we estimate the fiscal backing directly from the data and find the monetary shock that implements the empirically estimated fiscal backing. Second, we consider the monetary shock that implements the *minimum state-variable (MSV)* solution (see [McCallum 1999](#)). This corresponds to the standard method used to compute the solution of the textbook NK model. The MSV corresponds to the unique solution where output and inflation are linear functions of contemporaneous values of  $i_t$  and  $\hat{\lambda}_t$ .

## 4 The Quantitative Importance of Wealth Effects

In this section, we study the quantitative importance of risk and wealth effects in the transmission of monetary shocks.

### 4.1 Calibration

The parameter values are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of  $r_n = 1\%$ . We assume a Frisch elasticity of one,  $\phi = 1$ , and set the elasticity of substitution between intermediate goods to  $\epsilon = 6$ , common values adopted in the literature. The fraction of workers is set to  $\mu_w = 30\%$ , consistent with the fraction of (poor and wealthy) hand-to-mouth agents in the U.S. estimated by [Kaplan, Violante and Weidner \(2014\)](#). The parameter  $\bar{d}_G$  is chosen to match a ratio of the market value of public debt in the hands of the private sector to GDP of 28% and  $\psi_L$  is chosen to match a duration of five years, roughly in line with the historical average between 1962 and 2007 for the United States ([Hall and Sargent 2011](#)). The parameter  $T'_w(Y)$  is chosen such that  $\chi_y = 1$ , which requires countercyclical transfers to balance the procyclical wage income. A value of  $\chi_y = 1$  is consistent with the evidence in [Cloyne, Ferreira and Surico](#)

(2020) on the monetary policy impact on the income of borrowers (proxied for hand-to-mouth agents) and savers, where they show that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks.

To calibrate the disaster risk parameters, we follow closely Barro (2006). We set  $\lambda$  (the steady-state disaster intensity) to match an annual disaster probability of 1.7%. To better map the model to the data, we consider an extension where the magnitude of the drop in productivity,  $\zeta_A \equiv 1 - \frac{A^*}{A}$ , is stochastic and draw from a given distribution known by all agents. We adopt the empirical distribution estimated by Barro (2006), where  $\zeta_A$  ranges from 15% to 64%, with an average of 29%. Introducing a random disaster size has only a minor effect on the analytical expressions, with the term  $(C_s^*)^{-\sigma}$  being typically replaced by  $\mathbb{E}[(C_s^*)^{-\sigma}]$ , where the expectation is taken over the disaster size  $\zeta_A$ .<sup>20</sup>

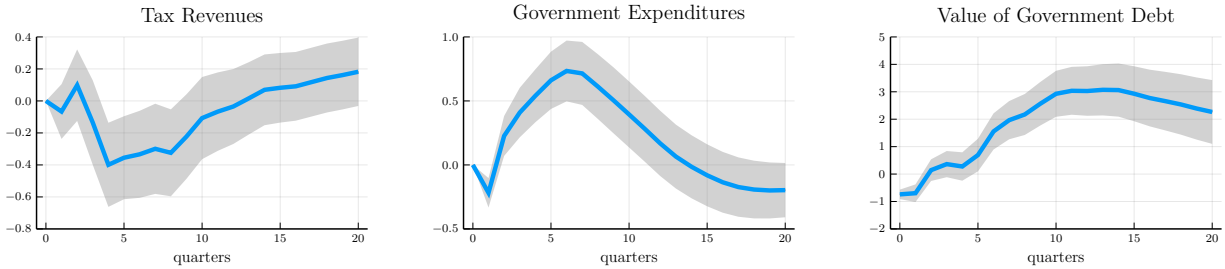
The risk-aversion coefficient is set to  $\sigma = 4$ , a value within the range of reasonable values according to Mehra and Prescott (1985), but substantially larger than  $\sigma = 1$ , a value often adopted in macroeconomic models. Our calibration implies an equity premium in the stationary equilibrium of 7.0%, in line with the observed equity premium (Campbell 2003). Moreover, by setting  $\sigma = 4$  we obtain a micro EIS of  $\sigma^{-1} = 0.25$ , in the ballpark of an EIS of 0.1 as recently estimated by Best, Cloyne, Ilzetzki and Kleven (2020), and in line with the estimates for asset holders by Havránek (2015) of 0.3. The pricing cost parameter  $\varphi$  is chosen to match a slope of the Phillips curve of  $\kappa = 0.30$ , which is the value for  $\kappa$  in the textbook model with an average price duration of three quarters and  $\sigma = 4$ .

For the policy variables, we follow Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2019) and estimate a standard VAR augmented with fiscal variables and compute empirical IRFs applying the recursiveness assumption. We provide the details of the estimation in Appendix D. Figure 2 shows the dynamics of fiscal variables in the estimated VAR in response to a contractionary monetary shock that increases the policy rate by 100 bps on

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<sup>20</sup>With a risk aversion of  $\sigma = 4$  and the estimated distribution of disaster sizes, the expected change in marginal utility conditional on a disaster is given by  $\mathbb{E}[(1 - \zeta_A)^{-\sigma}] = 7.69$ .





**Figure 2:** Estimated fiscal response to a monetary policy shock

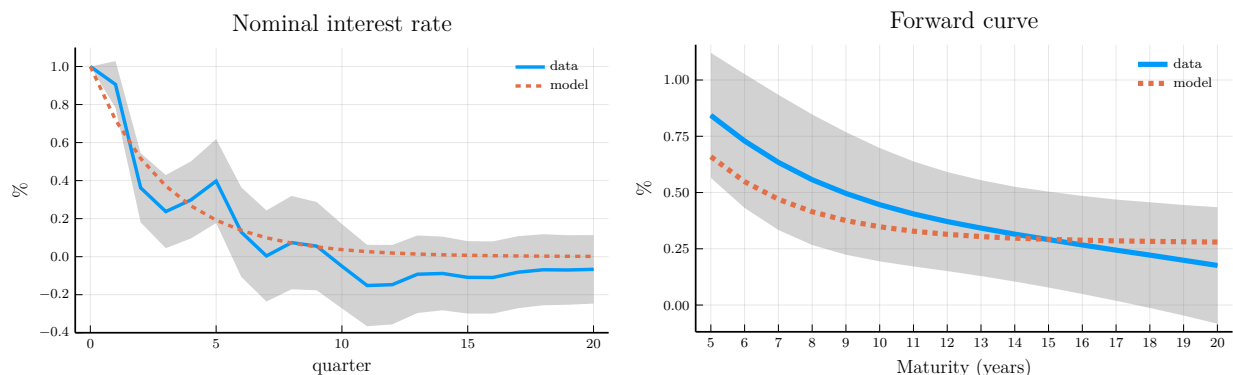
Note: IRFs computed from a VAR identified by a recursiveness assumption. Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures over GDP, federal funds rate, 5-year constant maturity rate and the real value of government debt over GDP. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 68% confidence bands.

impact. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 36 bps and the initial value of government debt drops by 18 bps.<sup>21</sup> The present value of primary surpluses increases by just 9 bps.

## 4.2 Asset-pricing implications of D-HANK

We focus on a monetary shock that generates a path for the nominal interest rate that can be represented by  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ . We set  $\psi_m = 0.33$ , which gives a half-life of roughly two quarters, so the path of nominal interest rates closely matches the impulse-response of the Federal Funds rate from the VAR, as shown in the left panel of Figure 3. To obtain  $\hat{\lambda}_t$ , we need to calibrate  $\epsilon_\lambda$ , which determines the elasticity of asset prices to monetary shocks, and  $\psi_\lambda$ , which captures the persistence of changes in risk premia. We

<sup>21</sup>The present discounted value of interest payments is calculated as  $\sum_{t=0}^{\mathcal{T}} \left( \frac{1+g}{1+i_L} \right)^t \left[ \bar{a}_t^g (\hat{i}_{L,t} - \hat{\pi}_t) \right]$ , and similarly for other variables, where  $\mathcal{T}$  is the truncation period,  $\hat{i}_{L,t}$  is the IRF of the 5-year rate estimated in the data, and  $\hat{\pi}_t$  is the IRF of inflation. We set  $g = 0.02$  and  $i_L = 0.043$ . We choose  $\mathcal{T} = 60$  quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values.



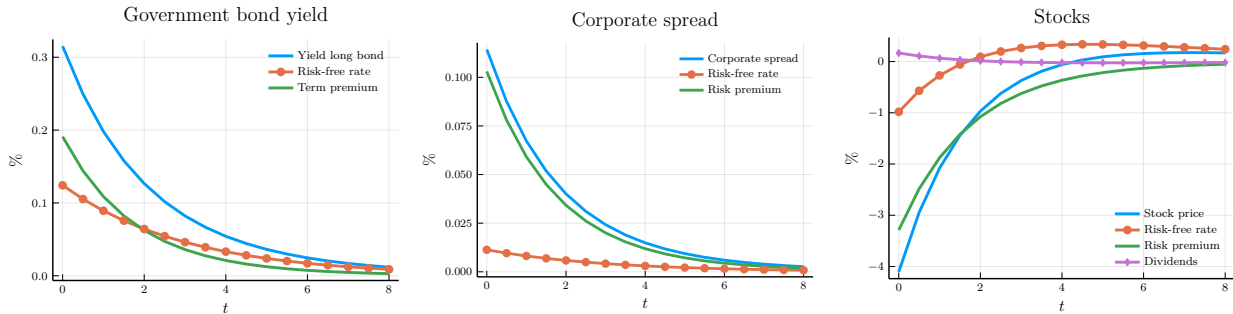
**Figure 3:** Nominal interest rate and forward curve.

Note: The left panel shows the IRF for the Federal Funds rate in the VAR and the path of nominal interest rates in the model. The right panel shows the response of forward rates to a 100 bps change in the two-year yield, as estimated by [Hanson and Stein \(2015\)](#), and the corresponding forward curve in the model when the monetary shock is scaled such that the two-year yield increases by 100 bps. Grey areas are confidence bands.

calibrate these parameters to match two sets of moments. First, the initial response of the 5-year yield on government bonds to a monetary shock. We find that a 100 bps increase in the nominal interest rate leads to a 32 bps increase in the 5-year yield. Second, the response of the entire *forward curve* around FOMC meetings, as estimated by [Hanson and Stein \(2015\)](#). The solid line in the right panel of [Figure 3](#) shows their estimates of the response of forwards rates to a 100 bps change in the two-year yield, while the dashed line shows the corresponding response of forward rates in the model.<sup>22</sup> A striking feature of [Hanson and Stein’s \(2015\)](#) results was that monetary shocks affected forward rates in the far distant future, a fact at odds with standard models. In contrast, [Figure 3](#) shows that our model is able to closely match their evidence.

The procedure above gives a value of 0.57 to  $\psi_\lambda$ , implying a half-life of roughly 4 months. The value of  $\epsilon_\lambda$  is 315, which implies a change of 33 bps in the probability of disaster in response to a 25 bps monetary shock. Given that monetary shocks are typically small in the data, this implies a variability in the market-implied disaster probability in-

<sup>22</sup>Appendix C contains the derivation of the partial differential equation (PDE) describing the evolution of forward rates and the procedure we used to numerically solve it.



**Figure 4:** Asset-pricing response to monetary shocks.

duced by monetary shocks that is only a small fraction of the overall volatility in the disaster probability of 114 bps, as estimated by [Wachter \(2013\)](#).

Figure 4 shows the response of the yield on the long-term bond and the contributions of the path of future interest rates and of the term premium. The bulk of the reaction of the yield reflects movements in the term premium, consistent with the findings of e.g. [Gertler and Karadi \(2015\)](#). The model also captures the responses of asset prices that were not directly targeted in the calibration. Consider first the *corporate spread*, the difference between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This is consistent with the way the GZ spread is computed in the data by [Gilchrist and Zakrajšek \(2012\)](#). Let  $e^{-\psi_F t}$  denote the coupon paid by the bond issued by the representative firm. We assume that the monetary shock is too small to trigger a default, but corporate bonds default if a disaster occurs, where lenders recover the fraction  $1 - \zeta_F$  of promised coupons. We calibrate  $\psi_F$  and  $\zeta_F$  to match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, consistent with the estimates reported by [Gilchrist and Zakrajšek \(2012\)](#). Note that the calibration targets the *unconditional* level of the credit spread. We evaluate the model on its ability to generate an empirically plausible *conditional* response to monetary shocks.

Figure 4 shows that the corporate spread responds to monetary shocks by 11 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of

6.5 bps with a standard-error of 3.1 bps, roughly consistent with the model's prediction. Thus, the model produces quantitatively plausible movements in the corporate spread.

Another untargeted moment is the response of equity prices. The model generates a decline in stocks of 4.0% in response to a 100 bps increase in interest rates, which coincides with the point estimate of [Bernanke and Kuttner \(2005\)](#).<sup>23</sup> Consistent with their findings, the response of stocks is explained mostly by movements in the risk premium. Notice the price-dividend ratio falls after a contractionary shock, despite a low EIS. In contrast, [Barro \(2009\)](#) finds that the price-dividend ratio in the endowment disaster model with separable utility *increases* with the probability of disaster when the EIS is less than one. This motivates the adoption of a high EIS in an Epstein-Zin setting.<sup>24</sup> Sticky prices is crucial to avoid counterfactual movements in equity prices in our CRRA setting, as changes in disaster probability would have the opposite effect on stock prices in the flexible-price version of the model. Dividends are roughly acyclical. Due to the assumption of GHH preferences, we avoid the strongly countercyclical profits typical of sticky-prices models.

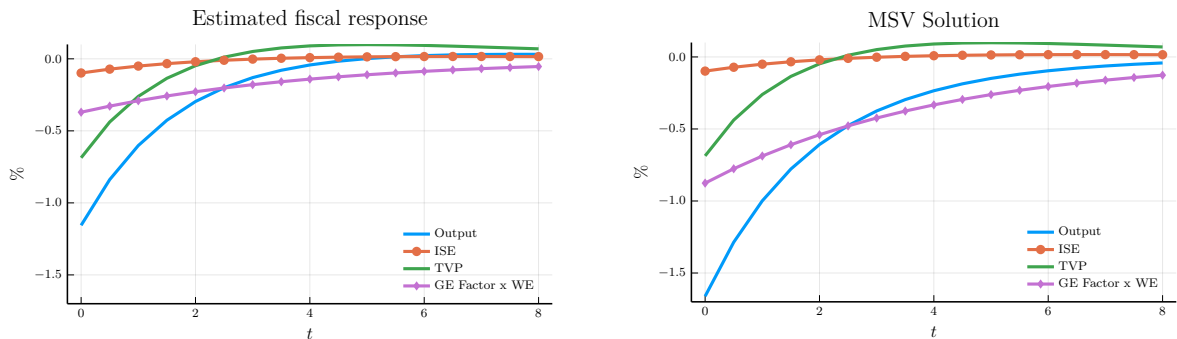
### 4.3 Wealth effects in the monetary transmission mechanism

Figure 5 presents the response to a monetary shock of output and its components. The left panel shows the solution when the fiscal backing matches the empirical estimates of Section 4.1, while the right panel shows the conventional MSV solution. In the case with the estimated fiscal backing, output drops on impact by 1.15% in response to an increase of 100 bp in the nominal interest rate, roughly in line with the estimates by [Miranda-Agrippino and Ricco \(2021\)](#). The time-varying precautionary motive (TVP) accounts for 60% of the initial output response, while the aggregate wealth effect (adjusted by the GE factor) accounts for 30%. The ISE accounts for less than 10% of the initial output response,

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<sup>23</sup>We follow standard practice in the asset-pricing literature and report the response of a levered claim on firms' profits, using a debt-to-equity ratio of 0.5, as in [Barro \(2006\)](#).

<sup>24</sup>For a similar reason, a high EIS is adopted in long-run risk models, see e.g. [Bansal and Yaron \(2004\)](#).



**Figure 5: Output in D-HANK.**

Note: In both plots, the path of the nominal interest rate is given by  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ , where  $i_0 - r_n$  equals 100 bps. The left panel shows the solution with the estimated fiscal backing, while the right panel shows the MSV solution.

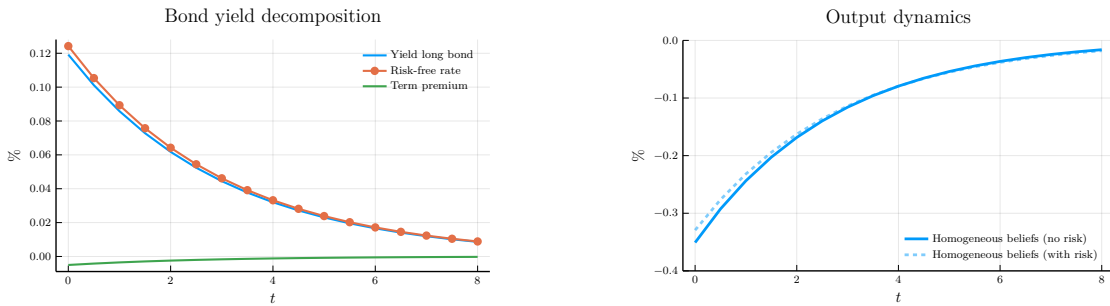
indicating that intertemporal substitution plays only a minor role in our model.

We find stronger real effects with the MSV solution, where output drops by 1.66% on impact. The difference is entirely driven by the aggregate wealth effect, which now explains more than the 50% of the overall effect, with the ISE and TVP being numerically the same as in the case with the estimated fiscal backing. The stronger impact on output, however, requires an increase in the present value of primary surpluses of more than 220 bps, which is more than twenty times bigger than what we estimate in the data.

#### 4.4 The limitations of the homogeneous-beliefs model

The model delivers a substantial response of output, despite a relatively weak intertemporal substitution channel. But is this the result of introducing disaster risk or is it due to heterogeneous beliefs? To answer this question, we consider the behavior of asset prices and output in an economy with homogeneous beliefs (i.e.  $\lambda > 0$  but  $\epsilon_\lambda = 0$ ).

Figure 6 (left) shows that the yield on the long bond increases by only 12 bps, less than half of the response estimated by the VAR in Section 4.1. Moreover, the term premium is essentially zero. In this case, stocks would also be mostly driven by interest rates instead



**Figure 6:** Long-term bond yields and output for economies with and without risk.

of risk premia, inconsistent with the evidence in [Bernanke and Kuttner \(2005\)](#).

Figure 6 (right) shows the response of output for an economy with disaster risk and homogeneous beliefs (solid line) and an economy without disaster risk (dashed line). In both cases, we consider the solution that matches the estimated fiscal backing. In the absence of belief heterogeneity, the impact on output of a monetary shock is substantially weaker, with a drop in output of roughly 0.35%. This is more than three times smaller than the impact on output in the case with belief heterogeneity. Moreover, the solution with disaster risk and homogeneous beliefs is almost identical to the one without disasters.

Introducing disaster risk allows the model to capture important *unconditional* asset-pricing moments, such as the equity premium or an upward-sloping yield curve, but the model is unable to match key *conditional* moments, such as the response of asset prices to monetary policy, with affects how monetary policy impacts the real economy.

## 5 The Effect of Risk and Maturity of Household Debt

We have focused so far on how monetary policy affects the value of households' assets, such as stocks and bonds. However, movements in risk premia can also affect the real economy through its impact on household debt. In this section, we extend the baseline model to allow workers to borrow a positive amount using long-term risky debt.

## 5.1 The model with long-term risky household debt

Workers issue long-term debt that promises to pay exponentially decaying coupons given by  $e^{-\psi_P t}$  at period  $t \geq 0$ , where  $\psi_P \geq 0$ . In response to a large shock, i.e., the occurrence of a disaster, workers default and lenders receive a fraction  $1 - \zeta_P$  of the promised coupons, where  $0 \leq \zeta_P \leq 1$ . Fluctuations in the no-disaster state are small enough such that they do not trigger a default. Thus, workers default only in the disaster state.

Workers can borrow up to  $\bar{D}_{P,t} = Q_{P,t} \bar{F}$ , which effectively puts a limit on the face value of private debt  $\bar{F}$ .<sup>25</sup> The (log-linearized) consumption of workers is given by

$$c_{w,t} = \chi_y y_t - \left( \frac{\psi_P}{i_P + \psi_P} (i_{P,t} - i_P) - \pi_t \right) \bar{d}_P, \quad (30)$$

where  $\bar{d}_P \equiv \frac{\bar{D}_P}{Y}$  denotes the debt-to-income ratio in the stationary equilibrium, and  $i_{P,t} = \frac{1}{Q_{P,t}} - \psi_P$  is the yield on household debt. Equation (30) generalizes the expression for workers' consumption given in Section 2. When debt is short-term,  $\psi_P \rightarrow \infty$ , and riskless,  $\zeta_P = 0$ , we obtain  $i_{P,t} = i_t$ . With a consol,  $\psi_P = 0$ , households simply pay the coupon every period and there is no need to issue new debt. Therefore, they are insulated from movements in nominal rates. For intermediate values of maturity and risk, monetary policy affects workers through changes in the nominal interest rate  $i_t$  and the spread  $r_{P,t}$ .

**Proposition 9** (Aggregate output with long-term risky household debt). *Suppose that  $i_t - r_n = e^{-\psi_m t} (i_0 - r_n)$  and  $r_P \sigma_{c_s,t} = \mathcal{O}(\|i_t - r_n\|^2)$ . Aggregate output is then given by*

$$y_t = \underbrace{\tilde{\sigma}^{-1} \hat{y}_{m,t}}_{\text{ISE}} + \underbrace{\chi_\lambda \hat{y}_{\lambda,t}}_{\substack{\text{time-varying} \\ \text{precautionary motive}}} + \underbrace{\frac{\mu_w \bar{d}_P \psi_P}{1 - \mu_w \chi_y} \left[ \frac{\tilde{\psi}_m \hat{y}_{m,t}}{\rho + \psi_P + \psi_m} + \frac{r_P \epsilon_\lambda \tilde{\psi}_\lambda \hat{y}_{\lambda,t}}{\rho + \psi_P + \psi_\lambda} \right]}_{\text{household-debt effect}} + \underbrace{(\rho - \omega) e^{\omega t} \Omega_0}_{\substack{\text{GE factor} \times \\ \text{aggregate wealth effect}}}$$

<sup>25</sup>This formulation guarantees that, after an increase in nominal rates, the value of household debt and the borrowing limit decline by the same amount. This specification of the borrowing constraint, combined with the assumption of impatient borrowers, guarantees that borrowers are constrained at all periods.

where  $\tilde{\psi}_k = \psi_k + \rho - r_n$  for  $k \in \{m, \lambda\}$ .

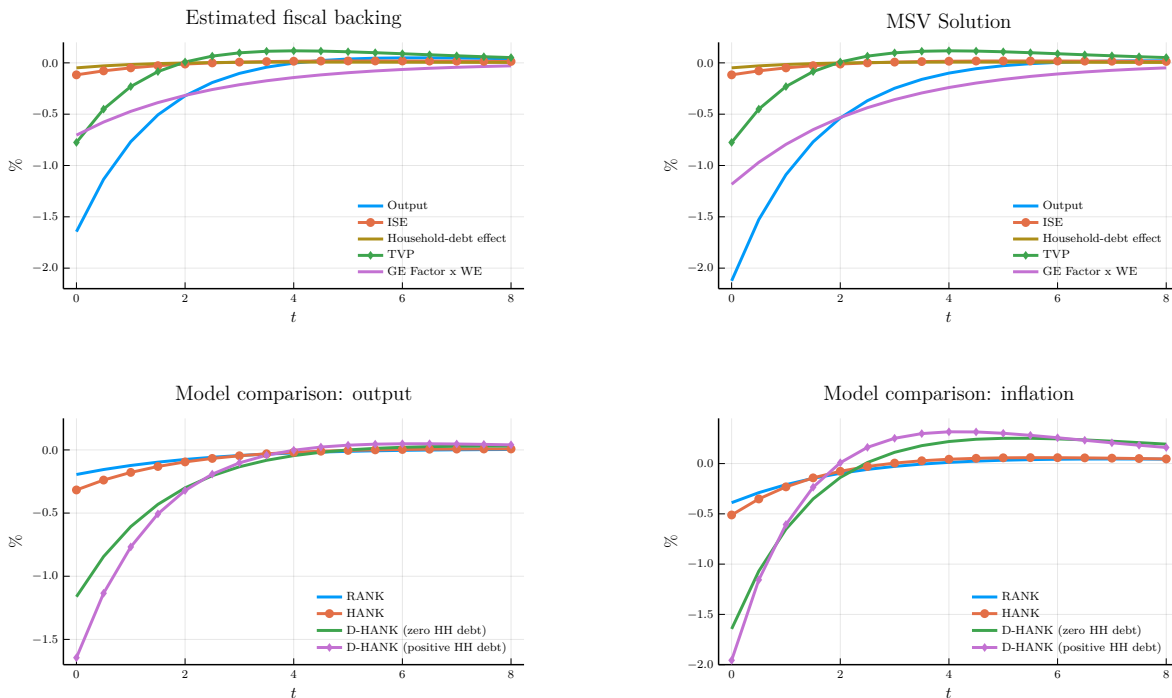
Proposition 9 extends the decomposition in Proposition 5 to the case of long-term risky household debt. Household debt effectively amplifies the ISE and the time-varying precautionary motive effect. If household debt is safe and short term (i.e,  $\zeta_P = 0$  and  $\psi_P \rightarrow \infty$ ), then the household-debt effect loads only on  $\hat{y}_{m,t}$ , amplifying the ISE. When debt is long-term or when households can default, then  $r_P > 0$  and the household-debt effect also loads on  $\hat{y}_{\lambda,t}$ , amplifying the precautionary motive effect.

## 5.2 Quantitative implications

We consider next the quantitative effects of introducing household debt. We calibrate  $\bar{d}_P$  to match a debt service payment to disposable personal income of 10%. We choose  $\psi_P$  to match a duration of 5 years, consistent with the mortgage duration estimated by [Greenwald, Leombroni, Lustig and Van Nieuwerburgh \(2021\)](#) of 5.2 years. We choose  $\zeta_P$  to match a spread of 2% in a stationary equilibrium relative to the riskless bond with the same promised coupons. Figure 7 shows the role of household debt in the transmission of monetary policy to the economy. The top left panel shows the output decomposition with the estimated fiscal backing. Output on impact drops by 1.6% in response to a 100 bp increase in nominal rates, where the TVP channel accounts for roughly half of the overall response and the aggregate wealth effect (adjusted by the GE factor) accounts for roughly 40%. The top right panel shows the decomposition for the MSV solution. In this case, the drop in output is nearly 50 bp larger than the one with the estimated fiscal backing. However, this requires a present value of primary surplus that is ten times larger than the one we estimated.

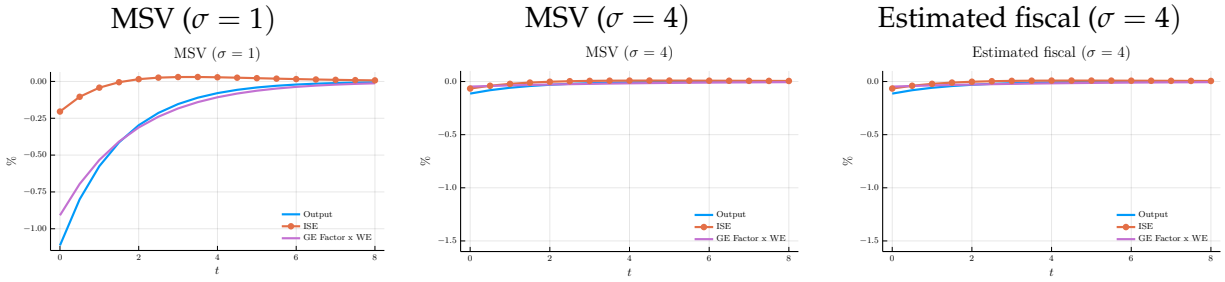
The bottom left panel of Figure 7 shows the impact on output for a range of special cases nested by our model. In all cases, we focus on the solution that matches the estimated fiscal backing. The line denoted by RANK corresponds to the solution without





**Figure 7:** The role of household debt: output decomposition and model comparison

disaster risk and zero household debt, which aggregates to the textbook model. The line denoted by HANK corresponds to the solution with positive debt, which given the heterogeneous MPCs between workers and savers captures an important channel of typical HANK models. We also consider two versions of the model with heterogeneous beliefs (D-HANK), with and without household debt. The output response in HANK is 12 bp larger than in RANK. However, the impact on output in HANK is substantially smaller than in either version of D-HANK. Introducing household debt in D-HANK raises the impact on output by 48 bp. Hence, household debt interacts in important ways with disaster risk. The bottom right panel shows the impact on inflation. A similar pattern emerges: we obtain a larger response of inflation under HANK than under RANK, but it is substantially weaker than the inflation response under D-HANK.



**Figure 8:** The role of the EIS in RANK’s quantitative performance.

### 5.3 The role of the EIS

We have seen that the real effects of monetary shocks are significantly weaker when we shut down risk and heterogeneity. This appears to be in contrast with standard results from the textbook model, which typically generates large real effects. Figure 8 shows that the calibration of the EIS plays an important role for this result. The left panel shows the MSV solution of the RANK model when we set  $\sigma = 1$  and use the persistence of monetary shocks from Galí (2015). Output drops by 1.1% in response to a 100 bp increase in nominal rates, a substantial effect. The aggregate wealth effect, adjusted by the GE factor, accounts for the majority of the output response. The middle panels shows the MSV solution of the RANK model for  $\sigma = 4$ , as in our baseline calibration. We keep all the other parameters fixed, including the slope of the Phillips curve  $\kappa$ . The response of output is now ten times smaller. The right panel shows the solution that matches the estimated fiscal backing with  $\sigma = 4$ , which is nearly the same as the MSV solution with  $\sigma = 4$ .

These results indicate that the quantitative performance of the standard RANK model relies on a counterfactually strong intertemporal-substitution effect, which ends up being amplified in general equilibrium by a large wealth effect. When the model is calibrated to match the observed levels of public debt, this strong wealth effect requires an implied fiscal backing that is too large relative to empirical estimates. This shows that the standard model lacks realistic mechanisms to generate large real effects of monetary policy.

Introducing heterogeneous MPCs and household debt improves the model performance, but effects are still not large enough, in particular when debt is long term. We have seen that risk and belief heterogeneity provide a powerful mechanism to generate the strong real effects of monetary shocks observed in the data.

## 6 Conclusion

In this paper, we provide a novel unified framework to analyze the role of risk and heterogeneity in a tractable New Keynesian model. The methods introduced in this paper can be applied in other settings. For instance, they can be used to introduce time-varying risk premia in a full quantitative HANK model with idiosyncratic risk. One could also introduce a richer capital structure for firms and study the pass-through of monetary policy to households and firms. These methods may enable us to bridge the gap between the existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.

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## Appendix: Proofs

*Proof of Proposition 2.* Consider the New Keynesian Phillips curve  $\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right) \pi_t - \frac{\epsilon}{\varphi A} \left(\frac{W}{P} e^{w_t - p_t} - (1 - \epsilon^{-1})A\right) Y e^{y_t}$ . Linearizing the above expression, and using  $\frac{W}{P} = (1 - \epsilon^{-1})A$ , we obtain  $\dot{\pi}_t = \left(r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma\right) \pi_t - \varphi^{-1}(\epsilon - 1)Y(w_t - p_t)$ . Using the fact that  $w_t - p_t = \phi y_t$ , we obtain  $\dot{\pi}_t = (\rho_s + \lambda) \pi_t - \kappa y_t$ , where  $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$  and we used that  $r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma = \rho_s + \lambda$ .

Consider next the generalized Euler equation. From the market-clearing condition for goods and workers' consumption, we obtain  $c_{s,t} = \frac{1 - \mu_w \chi_y}{1 - \mu_w} y_t$ . Combining this condition with the Phillips Curve and savers' Euler equation, and using the fact that  $r_n = \rho - \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma$ , we obtain  $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi - r_n) + \delta y_t + \chi_\lambda \hat{\lambda}_t$ , where the constants  $\tilde{\sigma}^{-1}$ ,  $\delta$ , and  $\chi_\lambda$  are defined in the proposition. □

*Proof of Proposition 3.* The linearized Euler equation for saver  $j$  is given by  $\dot{c}_{j,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma} \left(\frac{C_s}{C_s^*}\right)^\sigma (\hat{\lambda}_t + \sigma c_{s,t}) - \zeta(c_{j,t} - c_{s,t})$ . Taking the difference of the Euler equation for the two types, we obtain  $\dot{c}_{p,t} - \dot{c}_{o,t} = -\zeta(c_{p,t} - c_{o,t})$ . Linearizing the savers' flow budget constraint, we obtain  $\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k \in \{L, E\}} r_k \left[ \hat{r}_{k,t} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) + \frac{B_p^k}{B_p} b_{p,t}^k - \frac{B_o^k}{B_o} b_{o,t}^k \right] - \left(\frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t}\right) + r_n(b_{p,t} - b_{o,t})$  where  $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_k^*}{Q_k - Q_k^*} q_{k,t}$ . The relative net worth in the disaster state at  $t = t^*$  is given by  $\frac{B_p^*}{B_p} b_{p,t^*}^* - \frac{B_o^*}{B_o} b_{o,t^*}^* = b_{p,t^*} - b_{o,t^*} - \sum_{k \in \{L, E\}} \left[ \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) \frac{Q_k^*}{Q_k} q_{k,t^*} + \frac{Q_k - Q_k^*}{Q_k} \left(\frac{B_p^k}{B_p} b_{p,t^*}^k - \frac{B_o^k}{B_o} b_{o,t^*}^k\right) \right]$ .

From the revaluation of net worth in the disaster state, shown above, we can solve for the difference in portfolios  $\frac{B_p^k}{B_p} b_{p,t^*}^k - \frac{B_o^k}{B_o} b_{o,t^*}^k$ . From the optimality condition for risky assets, we obtain  $c_{p,t} - c_{o,t} = c_{p,t}^* - c_{o,t}^*$ . Savers' consumption in the disaster state is given by  $c_{j,t}^* = \frac{r_n^* B_j^*}{C_s^*} b_{j,t}^*$ . Combining these expressions, we obtain the relative net worth in the disaster state. We can then solve for the dynamics of relative net worth in the no-disaster state:  $\dot{b}_{p,t} - \dot{b}_{o,t} = \rho(b_{p,t} - b_{o,t}) - \chi_{b,c}(c_{p,t} - c_{o,t}) + \chi_{b,c_s} c_{s,t}$ , where  $\chi_{b,c_s} \equiv (\sigma -$

1)  $\sum_{k \in \{L, E\}} r_k \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right)$ , and  $\chi_{b,c} \equiv \sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \sum_{k \in \{L, E\}} r_k \left( \frac{B_o^k}{B_o} - \frac{B_p^k}{B_p} \right) + \mu_{c,p} \frac{C_o}{B_o} + \mu_{c,o} \frac{C_p}{B_p} + \frac{C_s^* (\rho - r_n)}{r_n^* B_s}$ , where  $\chi_{b,c} > 0$ . Assuming  $\sigma r_k c_s = \mathcal{O}(\|i_t - r_n\|^2)$ , the term involving  $c_{s,t}$  can be ignored up to first order. We then obtain a dynamic system in  $c_{p,t} - c_{o,t}$  and  $b_{p,t} - b_{o,t}$ , which has a positive and a negative eigenvalue, so there is a unique bounded solution given by  $c_{p,t} - c_{o,t} = \frac{\rho + \xi}{\chi_{b,c}} e^{-\psi_\lambda t} (b_{p,0} - b_{o,0})$  and  $b_{p,t} - b_{o,t} = e^{-\psi_\lambda t} (b_{p,0} - b_{o,0})$ , where  $\psi_\lambda = \xi$ . We can then write the market-implied disaster probability as  $\hat{\lambda}_t = e^{-\psi_\lambda t} \chi_{\lambda,c} \frac{\rho + \xi}{\chi_{b,c}} (b_{p,0} - b_{o,0})$ , where  $\chi_{\lambda,c} = \sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right)$ . The revaluation of the relative net worth is given by  $b_{p,0} - b_{o,0} = \left( \frac{B_p^L}{B_p} - \frac{B_o^L}{B_o} \right) q_{L,0}$ , using the assumption that  $B_o^E = B_p^E$ . The price of the long-term bond is given by  $q_{L,0} = -\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} - \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}$ . Combining the expressions for  $\hat{\lambda}_t$ , relative net worth, and bond prices, we obtain  $\hat{\lambda}_t = e^{-\psi_\lambda t} \epsilon_\lambda (i_0 - r)$ , where  $\epsilon_\lambda$  is given by  $\epsilon_\lambda \equiv \left[ 1 - \chi_{\lambda,c} \frac{\rho + \xi}{\chi_{b,c}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{r_L}{\rho + \psi_L + \psi_\lambda} \right]^{-1} \left[ \chi_{\lambda,c} \frac{\rho + \xi}{\chi_{b,c}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{1}{\rho + \psi_L + \psi_m} \right]$ .  $\square$

*Proof of Lemma 1.* Linearizing the aggregate intertemporal budget constraint, we obtain  $Q_C q_{c,0} = D_G q_{L,0} + Q_E q_{E,0} + Q_H q_{H,0}$ , where  $Q_{H,t}$  is the present discounted value of wages plus transfers. Using the pricing condition for  $q_{k,0}$ ,  $k \in \{C, H, E\}$ , we obtain

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y} \int_0^\infty e^{-\rho t} [i_t - \pi_t - r_n + r_C p_{d,t}] dt = \int_0^\infty e^{-\rho t} \left[ \hat{\Pi}_t + \frac{WN}{PY} (w_t - p_t + n_t) + \hat{T}_t \right] dt - \frac{Q_H + Q_E}{Y} \int_0^\infty e^{-\rho t} [i_t - \pi_t - r_n] dt - \left[ \frac{Q_H}{Y} r_H + \frac{Q_E}{Y} r_E \right] \int_0^\infty e^{-\rho t} p_{d,t} dt + \frac{D_G}{Y} q_{L,0}.$$

Using the fact that  $Q_C = D_G + Q_E + Q_H$  and  $Q_C^* = D_G \frac{Q_L^*}{Q_L} + Q_E^* + Q_H^*$ , we obtain  $\frac{Q_C}{Y} - \frac{Q_H + Q_E}{Y} = \frac{D_G}{Y} \equiv \bar{d}_G$  and  $\frac{Q_C}{Y} r_C - \frac{Q_H r_H + Q_E r_E}{Y} = \bar{d}_G r_L$ , given  $r_k = \lambda \left( \frac{C_s^*}{C_s} \right)^\sigma \frac{Q_k - Q_k^*}{Q_k}$ . Combining these expressions with the equation above, we obtain (23) after some rearrangement, and using the fact that  $\hat{\Pi}_t + \frac{WN}{PY} (w_t - p_t + n_t) + \hat{T}_t = y_t - (\chi_\tau y_t + \tau_t)$ .  $\square$

*Proof of Lemma 2.* Consumption of a type- $j$  saver satisfies the conditions:  $\int_0^\infty e^{-\rho t} (c_{j,t} +$

$\chi c_j^* c_{j,t}^*$ ,  $\dot{c}_{j,t} = \dot{c}_{s,t} - \xi(c_{j,t} - c_{s,t})$ , and  $\sigma(c_{j,t} - c_{s,t}^*) = \hat{\lambda}_t + \sigma(c_{s,t} - c_{s,t}^*)$ . Combining these conditions, we obtain  $c_{j,t} = c_{s,t} + \frac{(\rho+\xi)e^{-\xi t}}{1+\bar{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0} - \Omega_{s,0}) + \frac{\bar{\chi}\lambda_j^{\frac{1}{\sigma}}}{1+\bar{\chi}\lambda_j^{\frac{1}{\sigma}}}\frac{\hat{\lambda}_t}{\sigma}$  and  $c_{j,t}^* = \frac{(\rho+\xi)e^{-\xi t}}{1+\bar{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0} - \Omega_{s,0}) - \frac{1}{1+\bar{\chi}\lambda_j^{\frac{1}{\sigma}}}\frac{\hat{\lambda}_t}{\sigma}$ , using  $\chi c_j^* = \bar{\chi}\lambda_j^{\frac{1}{\sigma}}$ , where  $\bar{\chi} \equiv \frac{\delta}{r_n^*}\bar{\chi}^*$  and  $\bar{\chi}^* \equiv \lambda^{-\frac{1}{\sigma}}\frac{C_s^*}{C_s}$ . From these expressions, we obtain the iMPCs. From  $\mathcal{M}_{j,t} + \frac{\delta}{r_n^*}\mathcal{M}_{j,t}^* = (\rho + \xi)e^{-\xi t}$ , we obtain (24).  $\square$

*Proof of Propositions 5 and 6.* We can write dynamic system in matrix form as  $\dot{Z}_t = AZ_t + Bv_t$ , where  $B = [1, 0]'$ . Applying the eigendecomposition to matrix  $A$ , we obtain  $A = V\Omega V^{-1}$  where  $V = \begin{bmatrix} \frac{\rho-\bar{\omega}}{\kappa} & \frac{\rho-\omega}{\kappa} \\ 1 & 1 \end{bmatrix}$ ,  $V^{-1} = \frac{\kappa}{\bar{\omega}-\omega} \begin{bmatrix} -1 & \frac{\rho-\omega}{\kappa} \\ 1 & -\frac{\rho-\bar{\omega}}{\kappa} \end{bmatrix}$ , and  $\Omega = \begin{bmatrix} \bar{\omega} & 0 \\ 0 & \omega \end{bmatrix}$ . Decoupling the system, we obtain  $\dot{z}_t = \Omega z_t + b v_t$ , where  $z_t = V^{-1}Z_t$  and  $b = V^{-1}B$ .

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$\begin{aligned} y_t &= V_{12} \left( V^{21}y_0 + V^{22}\pi_0 \right) e^{\omega t} - V_{11}V^{11} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + V_{12}V^{21} \int_0^t e^{\omega(t-z)} v_z dz \\ \pi_t &= V_{22} \left( V^{21}y_0 + V^{22}\pi_0 \right) e^{\omega t} - V_{21}V^{11} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + V_{22}V^{21} \int_0^t e^{\omega(t-z)} v_z dz, \end{aligned}$$

where  $V^{i,j}$  is the  $(i, j)$  entry of matrix  $V^{-1}$ . Integrating  $e^{-\rho t}y_t$ , we obtain  $\Omega_0 = V_{12} (V^{21}y_0 + V^{22}\pi_0) \frac{1}{\rho-\omega} - \frac{1}{\rho-\bar{\omega}} V_{11}V^{11} \int_0^\infty (e^{-\bar{\omega}t} - e^{-\rho t}) v_t dt + \frac{1}{\rho-\omega} V_{12}V^{21} \int_0^\infty e^{-\rho t} v_t dt$ . Rearranging the above expression, we obtain  $V_{12} (V^{21}y_0 + V^{22}\pi_0) = (\rho - \omega)\Omega_0 + \frac{\rho-\omega}{\rho-\bar{\omega}} V_{11}V^{11} \int_0^\infty e^{-\bar{\omega}t} v_t dt$ , where we used the fact  $\frac{V_{11}V^{11}}{\rho-\bar{\omega}} + \frac{V_{12}V^{21}}{\rho-\omega} = 0$ . Output is then given by  $y_t = \tilde{y}_t + (\rho - \omega)e^{\omega t}\Omega_0$ , where  $\tilde{y}_t = -\frac{\bar{\omega}-\rho}{\bar{\omega}-\omega} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + \frac{\bar{\omega}-\delta}{\bar{\omega}-\omega} \int_0^t e^{\omega(t-z)} v_z dz - \frac{\rho-\omega}{\bar{\omega}-\omega} e^{\omega t} \int_0^\infty e^{-\bar{\omega}z} v_z dz$ . Inflation is given by  $\pi_t = \tilde{\pi}_t + \kappa e^{\omega t}\Omega_0$ , where  $\tilde{\pi}_t = \frac{\kappa}{\bar{\omega}-\omega} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + \frac{\kappa}{\bar{\omega}-\omega} \int_0^t e^{\omega(t-z)} v_z dz - \frac{\kappa}{\bar{\omega}-\omega} e^{\omega t} \int_0^\infty e^{-\bar{\omega}z} v_z dz$ .

If  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ , then  $v_t = \tilde{\sigma}^{-1}e^{-\psi_m t}(i_0 - r_n) + \chi_{p_d}\epsilon_\lambda e^{-\psi_\lambda t}(i_0 - r_n)$ . Then,  $\tilde{y}_t = \tilde{\sigma}^{-1}\hat{y}_{m,t} + \chi_\lambda \hat{y}_{\lambda,t}$  and  $\tilde{\pi}_t = \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_\lambda \hat{\pi}_{\lambda,t}$ , where  $\chi_\lambda \equiv \chi_{p_d}\epsilon_\lambda$ ,  $\hat{y}_{k,t} = \frac{(\rho-\omega)e^{\omega t} - (\rho+\psi_k)e^{-\psi_k t}}{(\psi_k+\bar{\omega})(\psi_k+\omega)}(i_0 - r_n)$ , and  $\hat{\pi}_{k,t} = \frac{\kappa(e^{\omega t} - e^{-\psi_k t})}{(\omega+\psi_k)(\bar{\omega}+\psi_k)}(i_0 - r_n)$ . Note that  $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$ ,  $\frac{\partial \hat{y}_{k,0}}{\partial i_0} = -\frac{1}{\psi_k+\bar{\omega}} < 0$ , and

$\lim_{t \rightarrow \infty} \hat{y}_{k,t} = 0$ . Moreover,  $\hat{\pi}_0 = 0$ ,  $\frac{\partial \hat{\pi}_{k,t}}{\partial i_0} \geq 0$  with strict inequality if  $t > 0$ .

□

*Proof of Proposition 8.* Combining the aggregate Euler equation (10), the NKPC (11), and the system (A.54), we obtain a dynamic system in the variables  $[y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}]$ :

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \\ \dot{\hat{\lambda}}_t \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} \delta & \tilde{\sigma}^{-1}(\phi_\pi - 1) & \chi_{p_d} & 0 \\ -\kappa & \rho & 0 & 0 \\ 0 & 0 & -\xi & 0 \\ 0 & 0 & -\chi_{b,\lambda} & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ \hat{\lambda}_t \\ b_{p,t} - b_{o,t} \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u_t,$$

where  $\hat{\lambda}_t = \chi_{\lambda,c}(c_{p,t} - c_{o,t})$  and  $\chi_{b,\lambda} = \chi_{b,c}/\chi_{\lambda,c}$ , given the boundary condition  $b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right) \int_0^\infty e^{-(\rho+\psi_L)t} (\phi_\pi \pi_t + u_t + r_L \hat{\lambda}_t) dt$ .

Denote the matrix of coefficients by  $A$  and consider the eigendecomposition of the matrix  $A = V\Omega V^{-1}$ , where  $\Omega$  is the diagonal matrix of eigenvalues and  $V$  the matrix of eigenvectors. The eigenvalues are given by

$$\omega_1 = \frac{\rho + \delta + \sqrt{(\rho + \delta)^2 - 4(\tilde{\sigma}^{-1}(\phi_\pi - 1)\kappa + \rho\delta)}}{2}, \omega_2 = \frac{\rho + \delta - \sqrt{(\rho + \delta)^2 - 4(\tilde{\sigma}^{-1}(\phi_\pi - 1)\kappa + \rho\delta)}}{2},$$

$\omega_3 = \rho$ , and  $\omega_4 = -\xi$ . The system has a unique bounded solution if three eigenvalues have positive real parts and one eigenvalue has a negative real part. This will be the case if  $\phi_\pi > 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \bar{\phi}_\pi$ .

**Solution of the dynamic system.** In matrix form, the dynamic system is given by  $\dot{Z}_t = AZ_t + Bu_t$ , where  $Z_t = [y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}]'$  and  $B = [\tilde{\sigma}^{-1}, 0, 0, 0]'$ . Let  $z_t = V^{-1}Z_t$  and  $b = V^{-1}B$ , which gives the system  $\dot{z}_t = \Omega z_t + bu_t$ . For  $i = 1, 2, 3$ , we can solve each equation forward,  $z_{i,t} = -b_i \int_t^\infty e^{-\omega_i(s-t)} u_s ds$ , and for  $i = 4$  we solve it backwards:  $z_{4,t} = e^{\omega_4 t} z_{4,0} + b_4 \int_0^t e^{\omega_4(t-s)} u_s ds$ . Rotating back to the original coordinates, we obtain  $Z_t = v_4 e^{\omega_4 t} z_{4,0} - \sum_{i=1}^3 v_i b_i \int_t^\infty e^{-\omega_i(s-t)} u_s ds + v_4 b_4 \int_0^t e^{\omega_4(t-s)} u_s ds$ , where  $v_i$  denotes the  $i$ th

eigenvector, which are given by  $v_1 = [(\rho - \omega_1)/\kappa, 1, 0, 0]'$ ,  $v_2 = [(\rho - \omega_2)/\kappa, 1, 0, 0]'$ ,  $v_3 = [0, 0, 0, 1]'$ , and  $v_4 = \left[ v_{4,1}, \frac{\kappa v_{4,1}}{\rho + \xi}, \frac{\rho + \xi}{\chi_{b,\lambda}}, 1 \right]'$ , where  $v_{4,1} = -\frac{(\rho + \xi)^2 \chi_{p_d} / \chi_{b,\lambda}}{(\delta + \xi)(\rho + \xi) + \kappa \tilde{\sigma}^{-1}(\phi_\pi - 1)}$ , and  $b = \left[ -\frac{\kappa \tilde{\sigma}^{-1}}{\omega_1 - \omega_2}, \frac{\kappa \tilde{\sigma}^{-1}}{\omega_1 - \omega_2}, 0, 0 \right]'$ . Using the fact that  $\psi_\lambda = -\omega_4$ , we obtain  $b_{p,t} - b_{o,t} = e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$  and  $\hat{\lambda}_t = \frac{\rho + \xi}{\chi_{b,\lambda}} e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$ , which coincides with the results from Proposition 3.  $y_t$  and  $\pi_t$  are given by

$$y_t = \sum_{i=1}^2 (-1)^i \frac{\tilde{\sigma}^{-1}(\omega_i - \rho)}{\omega_1 - \omega_2} \int_t^\infty e^{-\omega_i(s-t)} u_s ds - \frac{(\rho + \psi_\lambda) \chi_{p_d}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t$$

$$\pi_t = \sum_{i=1}^2 (-1)^{i-1} \frac{\kappa \tilde{\sigma}^{-1}}{\omega_1 - \omega_2} \int_t^\infty e^{-\omega_i(s-t)} u_s ds - \frac{\kappa \chi_{p_d}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t.$$

If  $u_t = \sum_{k=1}^K \varphi_k u_{k,t}$ , where  $u_{k,t} = e^{-\psi_k t} u_{k,0}$ ,  $\psi_k \geq 0$ , then

$$y_t = - \sum_{k=1}^K \varphi_k \frac{\rho + \psi_k}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{\chi_{p_d}(\rho + \psi_\lambda)}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t$$

$$\pi_t = - \sum_{k=1}^K \varphi_k \frac{\kappa}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{\kappa \chi_{p_d}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t.$$

The nominal interest rate is given by  $i_t - r_n = \sum_{k=1}^K \varphi_k \frac{(\delta + \psi_k)(\rho + \psi_k) - \tilde{\sigma}^{-1} \kappa}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} u_{k,t} - \frac{\phi_\pi \kappa \chi_{p_d}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t$ .

The initial value of  $\hat{\lambda}_0$  satisfies the condition  $\hat{\lambda}_0 = -\frac{\rho + \xi}{\chi_{b,\lambda}} \left( \frac{B_p^L}{B_p} - \frac{B_o^L}{B_o} \right) \left[ \int_0^\infty e^{-(\rho + \psi_L)t} (i_t - r_n) dt + \frac{r_L}{\rho + \psi_L + \psi_\lambda} \hat{\lambda}_0 \right]$ , and solving for  $\hat{\lambda}_0$ , we obtain  $\hat{\lambda}_0 = \frac{\frac{\rho + \xi}{\chi_{b,\lambda}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right)}{1 - \frac{\rho + \xi}{\chi_{b,\lambda}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{r_L}{\rho + \psi_L + \psi_\lambda}} \int_0^\infty e^{-(\rho + \psi_L)t} (i_t - r_n) dt$ . Combining the expression above with the expression for  $i_t$ , we obtain

$$\hat{\lambda}_0 = \frac{\frac{\rho + \xi}{\chi_{b,\lambda}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \sum_{k=1}^K \varphi_k \frac{(\delta + \psi_k)(\rho + \psi_k) - \tilde{\sigma}^{-1} \kappa}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} \frac{u_{k,0}}{\rho + \psi_L + \psi_k}}{1 + \frac{\rho + \xi}{\chi_{b,\lambda}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \left[ \frac{\phi_\pi \kappa \chi_{p_d}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \frac{1}{\rho + \psi_L + \psi_\lambda} - \frac{r_L}{\rho + \psi_L + \psi_\lambda} \right]}.$$

**Implementability condition.** Take  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$  and  $\int_0^\infty e^{-\rho t} \tau_t dt$  as given, let  $\Omega_0$  be given by (28),  $y_t$  be given by (25) and  $\pi_t$  be given by (27). Define  $u_t$  as follows:

$$u_t = i_t - r_n - \phi_\pi \pi_t. \quad (31)$$

Let  $[y_t^*, \pi_t^*, \hat{\lambda}_t^*, b_{p,t}^* - b_{o,t}^*]_0^\infty$  be the solution to the four-dimensional dynamic system discussed above and  $[i_t^*, \tau_t^*]$  the associated interest rate and fiscal backing. We show next that  $y_t^* = y_t$ ,  $\pi_t^* = \pi_t$ ,  $\hat{\lambda}_t^* = \hat{\lambda}_t$  and  $i_t^* = i_t$ . First, notice that  $u_t = \sum_{k=1}^3 \varphi_k e^{-\psi_k t} u_{k,0}$ , where  $u_{k,0} = i_0 - r_n$ , and  $\varphi_k$  and  $\psi_k$  are given by

$$\begin{aligned} \varphi_1 &= 1 + \frac{\phi_\pi \tilde{\sigma}^{-1} \kappa}{(\underline{\omega} + \psi_m)(\bar{\omega} + \psi_m)}, & \psi_1 &= \psi_m, & \varphi_2 &= \frac{\kappa \phi_\pi \chi_\lambda}{(\underline{\omega} + \psi_\lambda)(\bar{\omega} + \psi_\lambda)}, & \psi_2 &= \psi_\lambda, \\ \varphi_3 &= -\kappa \phi_\pi \left[ \frac{\tilde{\sigma}^{-1}}{(\underline{\omega} + \psi_m)(\bar{\omega} + \psi_m)} + \frac{\chi_\lambda}{(\underline{\omega} + \psi_\lambda)(\bar{\omega} + \psi_\lambda)} + \frac{\Omega_0}{i_0 - r_n} \right], & \psi_3 &= -\underline{\omega}. \end{aligned}$$

The values of  $(\varphi_1, \varphi_2)$  and  $(\psi_1, \psi_2)$  ensure that  $i_t^* - r_n = e^{-\psi_m t}(i_0 - r_n)$ . Notice that, as  $\psi_3 = -\underline{\omega}$  and  $(\delta - \underline{\omega})(\rho - \underline{\omega}) - \tilde{\sigma}^{-1} \kappa = 0$ , then  $i_t^* - r_n$  is independent of  $u_{3,t}$ . The value of  $\varphi_3$  is chosen such that  $\int_0^\infty e^{-\rho t} y_t^* dt = \Omega_0$ . As  $u_{k,0}$  is proportional to  $i_0 - r_n$ , for  $k = 1, 2, 3$ , and  $\hat{\lambda}_0^*$  is proportional to a linear combination of the  $u_{k,0}$ , then  $\hat{\lambda}_t^* = \epsilon_\lambda^* e^{-\psi_\lambda t}(i_0 - r_n) = \epsilon_\lambda^* u_{2,t}$ , for some constant  $\epsilon_\lambda^*$ . If  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ , then  $\epsilon_\lambda^* = \epsilon_\lambda$ . From the Taylor rule we have that  $u_t = i_t - r_n - \phi_\pi \pi_t = i_t^* - r_n - \phi_\pi \pi_t^*$ , so  $\pi_t^* = \pi_t$ . If the nominal interest rate and  $\hat{\lambda}_t$  coincide in the two equilibria, then we must have  $b_{p,t}^* - b_{o,t}^* = b_{p,t} - b_{o,t}$ . From the aggregate Euler equation, we obtain

$$y_t^* = - \int_0^\infty e^{-\delta(s-t)} (i_s^* - r_n - \pi_s^* + \chi_\lambda \hat{\lambda}_s^*) ds = - \int_0^\infty e^{-\delta(s-t)} (i_s - r_n - \pi_s + \chi_\lambda \hat{\lambda}_s) ds = y_t,$$

so  $y_t^* = y_t$ . Finally, if output, inflation, nominal interest rates, and the market-implied disaster probability coincide in the two equilibria, from the intertemporal budget constraint we must have  $\int_0^\infty e^{-\rho t} \tau_t^* dt = \int_0^\infty e^{-\rho t} \tau_t dt$ .  $\square$

*Proof of Proposition 9.* The workers' financial wealth in the no-disaster state evolves according to  $\dot{B}_{w,t} = (i_t - \pi_t + r_{P,t})B_{w,t} + W_t N_{w,t} + T_{w,t} - C_{w,t}$ . Using the fact that  $B_{w,t} = -Q_{P,t}\bar{F}$  and  $q_{P,t} = -\frac{i_{P,t} - i_P}{i_P + \psi_P}$ , we obtain equation (30). From the market clearing condition for goods, we obtain savers' consumption:  $c_{s,t} = \frac{1 - \mu_w \chi_y}{1 - \mu_w} y_t + \frac{\mu_w \bar{d}_P}{1 - \mu_w} \left( \frac{\psi_P}{i_P + \psi_P} (i_{P,t} - i_P) - \pi_t \right)$ . Assuming exponentially decaying interest rates, and using the yield on the private bond  $i_{P,t} - i_P = \frac{i_P + \psi_P}{\rho + \psi_P + \psi_m} (i_t - r_n) + \frac{i_P + \psi_P}{\rho + \psi_P + \psi_\lambda} r_P \hat{\lambda}_t$ , we can write savers' consumption as follows

$$c_{s,t} = \frac{1 - \mu_w \chi_y}{1 - \mu_w} y_t + \frac{\mu_w \bar{d}_P}{1 - \mu_w} \left[ \frac{\psi_P}{\rho + \psi_P + \psi_m} (i_t - r_n) + \frac{\psi_P r_P}{\rho + \psi_P + \psi_\lambda} \hat{\lambda}_t - \pi_t \right]. \quad (32)$$

The Euler equation for savers can be written as

$$\dot{c}_{s,t} = \sigma^{-1} (i_t - \pi_t - r_n) + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma [c_{s,t} + \sigma^{-1} \hat{\lambda}_t]. \quad (33)$$

Combining equations (32) and (33), we obtain

$$\begin{aligned} \dot{y}_t &= \left[ \tilde{\sigma}^{-1} - \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} r_n \right] (i_t - \pi_t - r_n) + \left[ \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma - \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} \kappa \right] y_t \\ &+ \left[ \chi_{p_d} + \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} \frac{\psi_P r_P (\rho - r_n + \psi_\lambda)}{\rho + \psi_P + \psi_\lambda} \right] \hat{\lambda}_t + \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} \left[ r_n + \frac{\psi_P (\rho - r_n + \psi_m)}{\rho + \psi_P + \psi_m} \right] (i_t - r_n). \end{aligned}$$

The aggregate Euler equation is given by  $\dot{y}_t = -\hat{\sigma}^{-1} \pi_t + \hat{\delta} y_t + \hat{v}_t$ , where  $\hat{\sigma}^{-1} \equiv \tilde{\sigma}^{-1} - \frac{\mu_w \bar{d}_P r_n}{1 - \mu_w \chi_y}$ ,  $\hat{\delta} \equiv \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma - \frac{\mu_w \bar{d}_P \kappa}{1 - \mu_w \chi_y}$ , and  $\hat{v}_t \equiv \left[ \chi_{p_d} + \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} \frac{\psi_P r_P \psi_\lambda}{\rho + \psi_P + \psi_\lambda} \right] \hat{\lambda}_t + \left[ \tilde{\sigma}^{-1} + \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} \frac{\psi_P \hat{\psi}_m}{\rho + \psi_P + \psi_m} \right] (i_t - r_n)$ , where  $\hat{\psi}_k \equiv \psi_k + \rho - r_n$  for  $k \in \{m, \lambda\}$ . Therefore, following a derivation analogous to the one in Proposition 5, output is given by  $y_t = \tilde{\sigma}^{-1} \hat{y}_{m,t} + \chi_\lambda \hat{y}_{\lambda,t} + \frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y} \left[ \frac{\psi_P \hat{\psi}_m \hat{y}_{m,t}}{\rho + \psi_P + \psi_m} + \frac{r_P \epsilon_\lambda \hat{\psi}_\lambda \hat{y}_{\lambda,t}}{\rho + \psi_P + \psi_\lambda} \right] + (\rho - \underline{\omega}) e^{\underline{\omega} t} \Omega_0$ , where the eigenvalues are given by  $\bar{\omega} = \frac{\rho + \hat{\delta} + \sqrt{(\rho + \hat{\delta})^2 + 4(\tilde{\sigma}^{-1} \kappa - \rho \hat{\delta})}}{2}$  and  $\underline{\omega} = \frac{\rho + \hat{\delta} - \sqrt{(\rho + \hat{\delta})^2 + 4(\tilde{\sigma}^{-1} \kappa - \rho \hat{\delta})}}{2}$ . □

# Internet Appendix

## A Derivations for Section 2

### A.1 The non-linear model

**Savers' problem.** The HJB for the savers' problem is given by

$$\rho_{j,t} V_{j,t} = \max_{C_{j,t}, B_{j,t}^L, B_{j,t}^E} \frac{C_{j,t}^{1-\sigma}}{1-\sigma} + \frac{\partial V_{j,t}}{\partial t} + \lambda_j [V_{j,t}^* - V_{j,t}] + \frac{\partial V_{j,t}}{\partial B_{j,t}} [(i_t - \pi_t) B_{j,t} + r_{L,t} B_{j,t}^L + r_{E,t} B_{j,t}^E + T_{j,t} - C_{j,t}]. \quad (\text{A.1})$$

where  $V_{j,t}^*$  is evaluated at  $B_{j,t}^* = B_{j,t} + B_{j,t}^L \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} + B_{j,t}^E \frac{Q_{E,t}^* - Q_{E,t}}{Q_{E,t}}$  and  $B_{j,t} > 0$ .

The corresponding HJB in the disaster state is given by

$$\rho_{j,t}^* V_{j,t}^* = \max_{C_{j,t}^*, B_{j,t}^{L,*}, B_{j,t}^{E,*}} \frac{(C_{j,t}^*)^{1-\sigma}}{1-\sigma} + \frac{\partial V_{j,t}^*}{\partial t} + \frac{\partial V_{j,t}^*}{\partial B_{j,t}^*} [(i_t^* - \pi_t^*) B_{j,t} + T_{j,t}^* - C_{j,t}^*],$$

where we imposed that  $r_{L,t}^* = r_{E,t}^* = 0$ , as there is no risk in the disaster state.

The first-order conditions are given by<sup>1</sup>

$$C_{j,t}^{-\sigma} = \frac{\partial V_{j,t}}{\partial B_{j,t}}, \quad \frac{\partial V_{j,t}}{\partial B_{j,t}} r_{k,t} = \frac{\partial V_{j,t}^*}{\partial B_{j,t}^*} \frac{Q_{k,t} - Q_{k,t}^*}{Q_{k,t}}, \quad (C_{j,t}^*)^{-\sigma} = \frac{\partial V_{j,t}^*}{\partial B_{j,t}^*}, \quad (\text{A.2})$$

for  $k \in \{L, E\}$ . Savers are indifferent about their portfolio composition in the disaster state. From the expressions above, we obtain eqn. (2) and (3). Differentiating the HJB

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<sup>1</sup>Formally, the solution is also subject to a state-constraint boundary condition. See [Achdou, Han, Lasry, Lions, and Moll \(2017\)](#) for a discussion of such constraints in continuous-time savings problems.



equation in the no-disaster state with respect to  $B_{j,t}$ , we obtain the envelope condition:<sup>2</sup>

$$\rho_{j,t} \frac{\partial V_{j,t}}{\partial B_{j,t}} = \frac{\partial V_{j,t}}{\partial B_{j,t}} (i_t - \pi_t) + \frac{\mathbb{E}_{j,t} [d \left( \frac{\partial V_{j,t}}{\partial B_{j,t}} \right)]}{dt}. \quad (\text{A.3})$$

Using the optimality condition for consumption and the condition above, we obtain:

$$i_t - \pi_t - \rho_{j,t} = - \frac{\mathbb{E}_t [dC_{j,t}^{-\sigma}]}{C_{j,t}^{-\sigma} dt} = \frac{\sigma C_{j,t}^{-\sigma-1} \dot{C}_{j,t} - \lambda_j [(C_{j,t}^*)^{-\sigma} - C_{j,t}^{-\sigma}]}{C_{j,t}^{-\sigma}}, \quad (\text{A.4})$$

using the fact that  $dC_{j,t} = \dot{C}_{j,t} dt + [C_{j,t}^* - C_{j,t}] d\mathcal{N}_t$  and Ito's lemma. Rearranging the expression above, we obtain eqn. (1). A similar envelope condition holds in the disaster state, which gives the Euler equation for the disaster state

$$\frac{\dot{C}_{j,t}^*}{C_{j,t}^*} = \sigma^{-1} (i_t - \pi_t - \rho_{j,t}^*). \quad (\text{A.5})$$

The relative net worth of optimistic and pessimistic savers evolves according to

$$\frac{\dot{B}_{o,t}}{B_{o,t}} - \frac{\dot{B}_{p,t}}{B_{p,t}} = \sum_{k \in \{L,E\}} r_{k,t} \left( \frac{B_{o,t}^L}{B_{o,t}} - \frac{B_{p,t}^k}{B_{p,t}} \right) - \left( \frac{C_{o,t} - T_{s,t}}{B_{o,t}} - \frac{C_{p,t} - T_{s,t}}{B_{p,t}} \right). \quad (\text{A.6})$$

**Workers' problem.** The HJB for the workers' problem is given by

$$\begin{aligned} \rho_w V_{w,t} = & \max_{\tilde{C}_{w,t}, N_{w,t}, B_{w,t}^L} \frac{\tilde{C}_{w,t}^{1-\sigma}}{1-\sigma} + \frac{\partial V_{w,t}}{\partial B_{w,t}} \left[ (i_t - \pi_t) B_{w,t} + r_{L,t} B_{w,t}^L + \frac{W_t}{P_t} N_{w,t} + T_{w,t} - \tilde{C}_{w,t} - \frac{N_{w,t}^{1+\phi}}{1+\phi} \right] \\ & + \frac{\partial V_{w,t}}{\partial t} + \lambda_w [V_{w,t}^* - V_{w,t}] \end{aligned} \quad (\text{A.7})$$

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<sup>2</sup>Here we used the fact that  $\mathbb{E}_{j,t} [dF(B_{j,t}, t)] = [F_t + \lambda_j [F^* - F] + F_B ((i - \pi) B_j + r_L B_j^L + r_E B_j^E - C_j)] dt$  for any function  $F(B_{j,t}, t)$ , according to Ito's lemma.

subject to the state-constraint boundary condition

$$\frac{\partial V_{w,t}(0)}{\partial B_{w,t}} \geq \left( \frac{W_t}{P_t} N_{w,t} - \frac{N_{w,t}^{1+\phi}}{1+\phi} + T_{w,t} \right)^{-\sigma}, \quad (\text{A.8})$$

where we adopted the change of variables  $\tilde{C}_{w,t} \equiv C_{w,t} - \frac{N_{w,t}^{1+\phi}}{1+\phi}$ .

For simplicity, we have already imposed that  $B_{w,t}^E = 0$ . We show below that  $B_{w,t}^L = 0$  and a similar argument shows that workers would be against the short-selling constraint for equities when  $B_{w,t}^E$  is a choice variable.

The optimality condition for labor supply is given by

$$N_{w,t}^\phi = \frac{W_t}{P_t}. \quad (\text{A.9})$$

We focus on an equilibrium where workers are always constrained. To derive the conditions that ensure this is indeed the case, we start by considering a stationary equilibrium where all variables are constant conditional on the state. If workers are constrained in the stationary equilibrium, then they will also be constrained if fluctuations are small enough.

In a stationary equilibrium, net consumption  $\tilde{C}_w$  in the no-disaster state is given by

$$\tilde{C}_w = \frac{W}{P} N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w, \quad (\text{A.10})$$

and an analogous expression holds in the disaster state. Notice that the expression above does not depend on  $\rho_w$  or  $\lambda_w$ .

For workers to be unconstrained, the following condition would have to hold:

$$\frac{\dot{\tilde{C}}_{w,t}}{\tilde{C}_{w,t}} = \sigma^{-1}(r_n - \rho_w) + \frac{\lambda_w}{\sigma} \left[ \left( \frac{\tilde{C}_{w,t}}{\tilde{C}_{w,t}^*} \right)^\sigma - 1 \right]. \quad (\text{A.11})$$

For  $\rho_w$  sufficiently large, workers would want a declining path of consumption, so cur-

rent consumption would be above  $\frac{W}{P}N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w$ , which would violate the state-constraint. Hence, the constraint must be binding for  $\rho_w$  sufficiently large.

If the workers hold a positive amount of the long-term bonds, then the following condition must hold

$$r_L = \lambda_w \left( \frac{\tilde{C}_w}{\tilde{C}_w^*} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L}. \quad (\text{A.12})$$

As  $C_w$  and  $C_w^*$  are independent of  $\lambda_w$ , the equation above would hold only if  $\lambda_w$  equals the value  $\bar{\lambda}_w \equiv \frac{r_L}{\left(\frac{C_w}{C_w^*}\right)^\sigma \frac{Q_L - Q_L^*}{Q_L}}$ . For  $\lambda_w > \bar{\lambda}_w$ , borrowers would want a smaller dispersion between  $C_w$  and  $C_w^*$ , which requires holding less risky bonds, violating the non-negativity constraint on long-term bonds. Therefore, borrowers will hold zero long-term bonds for  $\lambda_w$  sufficiently large.

**Firms' problem.** The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s \geq t}} \mathbb{E}_t \left[ \int_t^{t^*} \frac{\eta_s}{\eta_t} \left( \frac{P_{i,s}}{P_s} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A_s} - \frac{\varphi}{2} \pi_s^2(j) \right) ds + \frac{\eta_{t^*}}{\eta_t} Q_{i,t^*}^*(P_{i,t^*}) \right],$$

subject to  $Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$  and  $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$ , given  $P_{i,t} = P_i$ .

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left( \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dt + \mathbb{E}_t [d(\eta_t Q_{i,t})], \quad (\text{A.13})$$

where  $\frac{\mathbb{E}_t [d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda_t \frac{\eta_t^*}{\eta_t} [Q_{i,t}^* - Q_{i,t}]$ .

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in  $\pi_t$  conditional on no disaster is then given by

$$\left( \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} \right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} = \varphi \dot{\pi}_{i,t}. \quad (\text{A.14})$$

The envelope condition with respect to  $P_{i,t}$  is given by

$$0 = \left( (1 - \epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} - (i_t - \pi_t) \frac{\partial Q_{i,t}}{\partial P_i} + \lambda_t \frac{\eta_t^*}{\eta_t} \left( \frac{\partial Q_{i,t}^*}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i} \right). \quad (\text{A.15})$$

Multiplying the expression above by  $P_{i,t}$  and using eqn. (A.14), we obtain

$$0 = \left( (1 - \epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t + \varphi \dot{\pi}_t - (i_t - \pi_t) \varphi \pi_{i,t} + \lambda_t \varphi \frac{\eta_t^*}{\eta_t} (\pi_{i,t}^* - \pi_{i,t}).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = \left( i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t} \right) \pi_t - \frac{\epsilon \varphi^{-1}}{A} \left( \frac{W_t}{P_t} - (1 - \epsilon^{-1}) A \right) Y_t,$$

where we have assumed that  $P_{i,t} = P_t$  for all  $i \in [0, 1]$  and that  $\pi_i^* = 0$ .

## A.2 The stationary equilibrium

**Aggregate output.** Consider a stationary equilibrium with zero inflation. From the New Keynesian Phillips curve, we obtain

$$\frac{W}{P} = (1 - \epsilon^{-1}) A, \quad \frac{W^*}{P} = (1 - \epsilon^{-1}) A^*. \quad (\text{A.16})$$

Combining the expressions above with the labor supply condition, we obtain

$$Y = \mu_w(1 - \epsilon^{-1})^{\frac{1}{\phi}} A^{\frac{1+\phi}{\phi}}, \quad Y^* = \mu_w(1 - \epsilon^{-1})^{\frac{1}{\phi}} (A^*)^{\frac{1+\phi}{\phi}}. \quad (\text{A.17})$$

**Disaster state.** From the Euler equation for short-term bonds, an allocation with constant consumption must satisfy  $r_n^* = \rho_j^*$ . Uzawa preferences implies that this condition is eventually satisfied. For simplicity, we assume that  $\rho_j^*(\cdot)$  is constant and  $\rho_o^* = \rho_p^*$ . This assumption is not necessary for our results, but it simplifies presentation, as it ensures that allocations are constant as the economy switches to the disaster state. We set  $\rho_j^* = \rho_s$ , so there is no jump in the discount rate of the representative saver. In this case, the real interest rate in the disaster state is given by  $i_t^* - \pi_t^* = r_n^* = \rho_s$ .

The excess return on long-term bonds and equity are equal to zero,  $r_L^* = r_E^* = 0$ , so the price of the long-term bond is given by

$$Q_L^* = \frac{1}{r_n^* + \psi_L}, \quad (\text{A.18})$$

and the equity price is given by  $Q_E^* = \frac{\Pi^*}{r_n^*}$ .

The consumption of borrowers is given by

$$C_w^* = (1 - \epsilon^{-1}) \frac{Y^*}{\mu_w} + T_w^*. \quad (\text{A.19})$$

We assume that the government chooses fiscal transfers so workers have a given share  $0 < \mu_{Y,w} < 1$  of output, so  $C_w^* = \mu_{Y,w} \frac{Y^*}{\mu_w}$ . The parameter  $\mu_{Y,w}$  captures the government's preference for redistribution. This requires that the government sets  $T_w^* = \left[ \frac{\mu_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w} \right] Y^*$ . In the main text, we focus on the case  $\mu_{Y,w} = \mu_w$ .

Savers' consumption is given by

$$C_j^* = r_n^* B_j^* + T_j^*, \quad (\text{A.20})$$

where  $B_j^* = B_j + B_j^L \frac{Q_L^* - Q_L}{Q_L} + B_j^E \frac{Q_E^* - Q_E}{Q_E}$ .

Aggregate consumption of savers is given by

$$C_s^* = r_n^* \frac{\bar{D}_G^*}{\mu_s} + \frac{\Pi^*}{\mu_s} + T_s. \quad (\text{A.21})$$

Transfers to savers must satisfy  $T_s = (1 - \mu_{Y,w} - \epsilon^{-1}) \frac{Y^*}{\mu_s} - r_n^* \frac{\bar{D}_G^*}{\mu_s}$  such that the government's budget constraint is satisfied. This implies that the aggregate consumption of savers is given by  $C_s^* = (1 - \mu_{Y,w}) \frac{Y^*}{\mu_s}$ .

We focus on a symmetric allocation in the disaster state, so we assume that  $T_{o,t}^* - T_{p,t}^* = -r_n^*(B_o^* - B_p^*)$ , for  $t \geq t^*$ . This implies that  $C_j^* = C_s^*$ .

**No-disaster state.** The consumption of workers is given by

$$C_w = \left[ (1 - \epsilon^{-1}) A \right]^{\frac{1+\phi}{\phi}} + T_w. \quad (\text{A.22})$$

As in the disaster state, the government chooses fiscal transfers so workers have a given share  $0 < \mu_{Y,w} < 1$  of output, so  $C_w = \mu_{Y,w} \frac{Y}{\mu_w}$  and  $C_s = (1 - \mu_{Y,w}) \frac{Y}{\mu_s}$ . This requires that the government sets  $T_w = \left[ \frac{\mu_{Y,w}}{\mu_w} - \frac{1 - \epsilon^{-1}}{\mu_w} \right] Y$ .

From the market clearing condition for assets, we obtain

$$B_s = \frac{\bar{D}_G + Q_E}{1 - \mu_w}, \quad B_s^L = \frac{\bar{D}_G}{1 - \mu_w}, \quad B_s^E = \frac{Q_E}{1 - \mu_w}. \quad (\text{A.23})$$

The consumption of individual savers is given by

$$C_j = r_n B_j + r_L B_j^L + r_E B_j^E - T_j \quad (\text{A.24})$$

From the Euler equation for short-term bonds to be satisfied for both types of savers, the following condition must be satisfied:  $\rho_o - \rho_p = \lambda_p - \lambda_o$ , where  $\rho_j$  is an increasing

function of  $\frac{C_j}{C_s}$ . As the consumption of type- $j$  savers is increasing in  $B_j$ ,  $\rho_o - \rho_p$  is increasing in  $B_o$ . Hence, there is a unique value of  $B_o$  such that  $\rho_o - \rho_p = \lambda_p - \lambda_o$ . We assume the function  $\rho_j(\cdot)$  is such that this equality is achieved when  $B_o = B_p$ .

Using the fact that  $B_o = B_p$  and  $T_o = T_p$  in a stationary equilibrium, we can write the consumption of optimistic and pessimistic savers as follows:

$$C_o = C_s + r_L \frac{\mu_p}{\mu_o + \mu_p} (B_o^L - B_p^L) + r_E \frac{\mu_p}{\mu_o + \mu_p} (B_o^E - B_p^E) \quad (\text{A.25})$$

$$C_p = C_s - r_L \frac{\mu_o}{\mu_o + \mu_p} (B_o^L - B_p^L) - r_E \frac{\mu_o}{\mu_o + \mu_p} (B_o^E - B_p^E). \quad (\text{A.26})$$

We can use the Euler equations for risky assets to eliminate  $r_L$  and  $r_E$  from the expressions above, which gives us

$$C_o = C_s \left[ 1 + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o \right], \quad C_o^* = C_s^*, \quad (\text{A.27})$$

$$C_p = C_s \left[ 1 - \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{\mu_o}{\mu_o + \mu_p} \mathcal{R}_o \right], \quad C_p^* = C_s^*, \quad (\text{A.28})$$

where  $\mathcal{R}_o \equiv \frac{Q_L - Q_L^*}{Q_L} \frac{B_o^L - B_p^L}{C_s} + \frac{Q_L - Q_L^*}{Q_E} \frac{B_o^E - B_p^E}{C_s}$  represents optimistic relative risk exposure.

From the optimality condition for risky assets, we obtain

$$\left( 1 + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o \right)^\sigma = \frac{\lambda_p}{\lambda_o} \left( 1 - \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{\mu_o}{\mu_o + \mu_p} \mathcal{R}_o \right)^\sigma. \quad (\text{A.29})$$

Rearranging the expression above, we obtain

$$\lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \mathcal{R}_o = \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\frac{\mu_o}{\mu_o + \mu_p} \lambda_p^{\frac{1}{\sigma}} + \frac{\mu_p}{\mu_o + \mu_p} \lambda_o^{\frac{1}{\sigma}}}, \quad (\text{A.30})$$

which is positive if  $\lambda_p > \lambda_o$ . The value of  $\mathcal{R}_o$  pins down only a linear combination of  $B_o^L - B_p^L$  and  $B_o^E - B_p^E$ . For concreteness, we assume that  $B_o^E = B_p^E$ , so savers have different

exposure to bonds in equilibrium.

Given  $\mathcal{R}_o$ , we can solve for the share of consumption of optimistic savers:

$$\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = \frac{\mu_o}{\mu_o + \mu_p} \left[ 1 + \frac{\mu_p (\lambda_o^{-\frac{1}{\sigma}} - \lambda_p^{-\frac{1}{\sigma}})}{\mu_o \lambda_o^{-\frac{1}{\sigma}} + \mu_p \lambda_p^{-\frac{1}{\sigma}}} \right]. \quad (\text{A.31})$$

Given the expression above, we obtain the market-implied disaster probability:

$$\lambda = \left[ \frac{\mu_o C_o}{\mu_p C_p + \mu_p C_p} \lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_p}{\mu_p C_p + \mu_p C_p} \lambda_p^{\frac{1}{\sigma}} \right]^{\sigma}. \quad (\text{A.32})$$

From the Euler equations for short-term and long-term bonds, we obtain

$$r_n = \rho_j - \lambda_j \left[ \left( \frac{C_j}{C_j^*} \right)^{\sigma} - 1 \right], \quad r_k = \lambda_j \left( \frac{C_j}{C_j^*} \right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k}, \quad (\text{A.33})$$

for  $k \in \{L, E\}$ , where  $r_L = \frac{1}{Q_L} - \psi_L - r_n$ ,  $r_E = \frac{\Pi}{Q_E} - r_n$ , and  $\Pi = \epsilon^{-1} Y$ .

Using the fact that  $\lambda \left( \frac{C_s}{C_s^*} \right)^{\sigma} = \lambda_j \left( \frac{C_j}{C_j^*} \right)^{\sigma}$ , we can write the Euler equations in terms of aggregate savers' consumption:

$$r_n = \rho_s - \lambda \left[ \left( \frac{C_s}{C_s^*} \right)^{\sigma} - 1 \right], \quad r_k = \lambda \left( \frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k}, \quad (\text{A.34})$$

for  $k \in \{L, E\}$ , where  $\rho_s$  satisfy the condition  $\rho_s + \lambda = \rho_j + \lambda_j$  for  $j \in \{o, p\}$ .

We solve next for the price of risky assets. Given  $r_L$ , we can solve for  $Q_L$ :

$$\frac{1}{Q_L} - \psi_L - r_n = \lambda \left( \frac{C_s}{C_s^*} \right)^{\sigma} \left( 1 - \frac{Q_L^*}{Q_L} \right) \Rightarrow Q_L = Q_L^* \frac{r_n^* + \psi_L + \lambda \left( \frac{C_s}{C_s^*} \right)^{\sigma}}{r_n + \psi_L + \lambda \left( \frac{C_s}{C_s^*} \right)^{\sigma}}, \quad (\text{A.35})$$

where  $Q_L > Q_L^*$ , as  $r_n < r_n^*$  due to the precautionary motive in the no-disaster state.



The loss in long-term bonds in the disaster state is given by

$$\frac{Q_L - Q_L^*}{Q_L} = \frac{r_n^* - r_n}{r_n^* + \psi_L + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma}, \quad (\text{A.36})$$

which is positive as  $r_n^* > r_n$ . Long-term interest rates are higher than short-term interest rates in the stationary equilibrium, i.e., the yield curve is upward sloping in this economy.

The equity price is given by

$$\frac{\Pi}{Q_E} - r_n = \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \left( 1 - \frac{Q_E^*}{Q_E} \right) \Rightarrow Q_E = \frac{\Pi + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma Q_E^*}{r_n + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma}, \quad (\text{A.37})$$

so the loss on equity in the disaster state is given by

$$\frac{Q_E - Q_E^*}{Q_E} = \frac{\Pi - r_n Q_E^*}{\Pi + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma Q_E^*} = \frac{\rho_s \zeta_\Pi + \lambda \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right] (1 - \zeta_\Pi)}{\rho_s + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma (1 - \zeta_\Pi)}, \quad (\text{A.38})$$

where  $\zeta_\Pi \equiv 1 - \frac{\Pi^*}{\Pi}$  is the size of the drop in profits. As the expression above is positive, the equity premium is positive in the stationary equilibrium.

### A.3 Log-linear approximation

We consider next the effects of an unexpected monetary shock for an economy starting at the stationary equilibrium described above.

**Disaster state.** As there is no monetary shock in the disaster state, inflation is equal to zero,  $\pi_t^* = 0$ , and output is kept at the stationary-equilibrium level,  $y_t^* = 0$ . Wages and hours are unchanged, so  $c_{w,t}^* = 0$ . Savers' aggregate consumption is also the same as in the stationary equilibrium,  $c_{s,t}^* = 0$ . Savers' flow budget constraint is given by

$\mu_s C_{s,t}^* = r_{n,t}^* (D_{G,t} \frac{Q_{L,t}^*}{Q_{L,t}} + Q_{E,t}^*) + T_{s,t}^*$ . Notice that  $r_{n,t}^* = r_n^*$ ,  $Q_{L,t}^* = Q_{L'}^*$ , and  $Q_{E,t}^* = Q_E^*$ . For simplicity, we further assume that the government chooses transfers in the no-disaster state such that  $D_{G,t} = D_G q_{L,t}$ , so transfers must satisfy  $T_{s,t}^* = T_s^*$ . Consumption of type- $j$  saver is then given by  $\frac{C_{j,t}^*}{B_j^*} c_{j,t}^* = r_n^* b_{j,t}^*$ .

**Market-based disaster probability.** Linearizing eqn. (4) around the stationary equilibrium, we obtain

$$\frac{\lambda^{\frac{1}{\sigma}}}{\sigma} \hat{\lambda}_t = \mu_{c,o} \mu_{c,p} \left( \lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}} \right) [c_{p,t} - c_{o,t}], \quad (\text{A.39})$$

where  $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p}$  and  $c_{j,t} \equiv \log C_{j,t} / C_j$ , for  $j \in \{o, p\}$ .

**Euler equation for short-term bonds.** Using the fact that  $\lambda_j \left( \frac{C_{j,t}}{C_{j,t}^*} \right)^\sigma = \lambda_t \left( \frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma$ , we can write the Euler equation for short-term bonds as follows

$$\dot{c}_{j,t} = \sigma^{-1} (i_t - \pi_t - (\rho_{j,t} + \lambda_j)) + \frac{\lambda_t}{\sigma} \left( \frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma. \quad (\text{A.40})$$

Linearizing the discount-rate function, we obtain  $\rho_{j,t} = \rho_j + \sigma \xi (c_{j,t} - c_{s,t})$ , where we assumed a common slope for both types  $\sigma \xi$ , so we obtain the linearized Euler equation

$$\dot{c}_{j,t} = \sigma^{-1} (i_t - \pi_t - r_n) + \frac{\lambda}{\sigma} \left( \frac{C_s}{C_s^*} \right)^\sigma (\hat{\lambda}_t + \sigma c_{s,t}) - \xi (c_{j,t} - c_{s,t}). \quad (\text{A.41})$$

Aggregating the expression above, and using  $c_{s,t} = \sum_{j \in \{o,p\}} \mu_{c,j} c_{j,t}$ , we obtain

$$\dot{c}_{s,t} = \sigma^{-1} (i_t - \pi_t - r_n) + \frac{\lambda}{\sigma} \left( \frac{C_s}{C_s^*} \right)^\sigma (\hat{\lambda}_t + \sigma c_{s,t}). \quad (\text{A.42})$$

**Relative consumption.** From the optimality condition for risky assets, we obtain

$$\lambda_0^{\frac{1}{\sigma}} \frac{C_{o,t}}{C_{o,t}^*} = \lambda_p^{\frac{1}{\sigma}} \frac{C_{p,t}}{C_{p,t}^*} \Rightarrow c_{p,t} - c_{o,t} = c_{p,t}^* - c_{o,t}^* \quad (\text{A.43})$$

Relative consumption in the no-disaster evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\bar{\zeta}(c_{p,t} - c_{o,t}). \quad (\text{A.44})$$

**Relative net worth.** Linearizing eqn. (A.6), we obtain

$$\begin{aligned} \dot{b}_{p,t} - \dot{b}_{o,t} = & \sum_{k \in \{L,E\}} r_k \left[ \hat{r}_{k,t} \left( \frac{b_p^k}{b_p} - \frac{b_o^k}{b_o} \right) + \frac{b_p^k}{b_p} (b_{p,t}^k - b_{p,t}) - \frac{B_o^k}{B_o} (b_{o,t}^k - b_{o,t}) \right] \\ & - \left( \frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t} \right) + \frac{C_p - T_p}{B_p} b_{p,t} - \frac{C_o - T_o}{B_o} b_{o,t}, \end{aligned} \quad (\text{A.45})$$

where  $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_k^*}{Q_k - Q_k^*} q_{k,t}$ . Using the fact that  $\frac{C_j - T_j}{B_j} = r_n + \sum_{k \in \{L,E\}} r_k \frac{B_j^k}{B_j}$ , we can write the expression above as follows

$$\begin{aligned} \dot{b}_{p,t} - \dot{b}_{o,t} = & \sum_{k \in \{L,E\}} r_k \left[ \hat{r}_{k,t} \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) + \frac{B_p^k}{B_p} b_{p,t}^k - \frac{B_o^k}{B_o} b_{o,t}^k \right] - \left( \frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t} \right) \\ & + r_n (b_{p,t} - b_{o,t}). \end{aligned} \quad (\text{A.46})$$

The relative net worth in the disaster state at  $t = t^*$  is given by

$$\frac{B_p^*}{B_p} b_{p,t^*}^* - \frac{B_o^*}{B_o} b_{o,t^*}^* = b_{p,t^*} - b_{o,t^*} - \sum_{k \in \{L,E\}} \left[ \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) \frac{Q_k^*}{Q_k} q_{k,t^*} + \frac{Q_k - Q_k^*}{Q_k} \left( \frac{B_p^k}{B_p} b_{p,t^*}^k - \frac{B_o^k}{B_o} b_{o,t^*}^k \right) \right]. \quad (\text{A.47})$$

**Relative risk exposure.** Consumption of savers in the disaster state is given by  $c_{j,t}^* = \frac{r_n^* B_j^*}{C_s^*} b_{j,t}^*$ , so we obtain that  $c_{p,t}^* - c_{o,t}^* = \frac{r_n^*}{C_s^*} (B_p^* b_{p,t}^* - B_o^* b_{o,t}^*)$ . Using this expression and the

fact that  $c_{p,t}^* - c_{o,t}^* = c_{p,t} - c_{o,t}$ , we can solve for the relative risk exposure:

$$\sum_{k \in \{L,E\}} \frac{Q_k - Q_k^*}{Q_k} \left( \frac{B_p^k}{B_p} b_{p,t}^k - \frac{B_o^k}{B_o} b_{o,t}^k \right) = b_{p,t} - b_{o,t} - \frac{C_s^*}{r_n^* B_s} (c_{p,t} - c_{o,t}) - \sum_{k \in \{L,E\}} \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) \frac{Q_k^*}{Q_k} q_{k,t}. \quad (\text{A.48})$$

**The dynamic system.** Using the expression above to eliminate the relative risk exposure, the relative net worth at the no-disaster state is given by

$$\begin{aligned} \dot{b}_{p,t} - \dot{b}_{o,t} &= (\hat{\lambda}_t + (\sigma - 1)c_{s,t}) \sum_{k \in \{L,E\}} r_k \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) + \rho(b_{p,t} - b_{o,t}) \\ &\quad - \left( r_n + \frac{T_s}{B_s} + \frac{C_s^*(\rho - r_n)}{r_n^* B_s} \right) (c_{p,t} - c_{o,t}) - \sum_{k \in \{L,E\}} r_k \left( \frac{B_p^k}{B_p} (c_{p,t} - c_{s,t}) - \frac{B_o^k}{B_o} (c_{o,t} - c_{s,t}) \right), \end{aligned} \quad (\text{A.49})$$

using  $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_k^*}{Q_k - Q_k^*} q_{k,t}$ ,  $\frac{C_j}{B_j} = r_n + \frac{T_j}{B_j} + \sum_{k \in \{L,E\}} r_k \frac{B_j^k}{B_j}$ , and  $\lambda \left( \frac{C_s}{C_s^*} \right)^\sigma = \rho - r_n$ .

The deviation of consumption from average can be written as

$$c_{p,t} - c_{s,t} = \mu_{c,o}(c_{p,t} - c_{o,t}), \quad c_{o,t} - c_{s,t} = -\mu_{c,p}(c_{p,t} - c_{o,t}). \quad (\text{A.50})$$

Combining the expressions above, we can write  $\dot{b}_{p,t} - \dot{b}_{o,t}$  as follows

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \rho(b_{p,t} - b_{o,t}) - \chi_{b,c}(c_{p,t} - c_{o,t}) + \chi_{b,c_s} c_{s,t}, \quad (\text{A.51})$$

where  $\chi_{b,c_s} \equiv (\sigma - 1) \sum_{k \in \{L,E\}} r_k \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right)$ , and

$$\begin{aligned} \chi_{b,c} &\equiv \sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \sum_{k \in \{L,E\}} r_k \left( \frac{B_o^k}{B_o} - \frac{B_p^k}{B_p} \right) + \left( r_n + \frac{T_s}{B_s} + \frac{C_s^*(\rho - r_n)}{r_n^* B_s} \right) \\ &\quad + \sum_{k \in \{L,E\}} r_k \left( \mu_{c,o} \frac{B_p^k}{B_p} + \mu_{c,p} \frac{B_o^k}{B_o} \right). \end{aligned} \quad (\text{A.52})$$

Note that  $r_n + \frac{T_s}{B_s} = \frac{C_j}{B_j} - \sum_{k \in \{L, E\}} r_k \frac{B_j^k}{B_j}$ , so  $r_n + \frac{T_s}{B_s} = \mu_{c,p} \frac{C_o}{B_o} + \mu_{c,o} \frac{C_p}{B_p} - \sum_{k \in \{L, E\}} r_k \left( \mu_{c,p} \frac{B_o^k}{B_o} + \mu_{c,o} \frac{B_p^k}{B_p} \right)$ .

We can then write  $\chi_{b,c}$  as follows:

$$\chi_{b,c} = \sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \sum_{k \in \{L, E\}} r_k \left( \frac{B_o^k}{B_o} - \frac{B_p^k}{B_p} \right) + \mu_{c,p} \frac{C_o}{B_o} + \mu_{c,o} \frac{C_p}{B_p} + \frac{C_s^* (\rho - r_n)}{r_n^* B_s}, \quad (\text{A.53})$$

so  $\chi_{b,c} > 0$ , as  $r_n < \rho$ .

In general, we would have to simultaneously solve for the aggregate variables and the relative net worth and relative consumption of pessimistic savers, which would increase the dimensionality of the problem relative to the standard New Keynesian. We assume that  $r_k c_{s,t} = \mathcal{O}(\|i_t - r_n\|^2)$ , so this term is small and can be ignored in our approximate solution. This implies that the system is now *block recursive*, where we can solve for the dynamics of relative consumption and relative net worth before fully characterizing the behavior of other aggregate variables. Under this assumption, we can write the joint dynamics of  $b_{p,t} - b_{o,t}$  and  $c_{p,t} - c_{o,t}$  as follows:

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} -\xi & 0 \\ -\chi_{b,c} & \rho \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix}. \quad (\text{A.54})$$

**Persistence of  $\hat{\lambda}_t$ .** The system above has a positive and a negative eigenvalue, so there is a unique bounded solution given by

$$\begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} = \begin{bmatrix} \frac{\rho + \xi}{\chi_{b,c}} \\ 1 \end{bmatrix} e^{-\psi_\lambda t} (b_{p,0} - b_{o,0}) \quad (\text{A.55})$$

where  $\psi_\lambda = \xi$ .

We can then write the market-implied disaster probability as follows:

$$\hat{\lambda}_t = e^{-\psi_\lambda t} \hat{\lambda}_0, \quad (\text{A.56})$$

where

$$\hat{\lambda}_0 \equiv \sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \frac{\rho + \xi}{\chi_{b,c}} (b_{p,0} - b_{o,0}). \quad (\text{A.57})$$

Hence,  $\psi_\lambda$  captures the persistence of  $\hat{\lambda}_t$ . If  $\xi = 0$ , then  $\psi_\lambda = 0$  and changes in  $\lambda_t$  are permanent. For high values of  $\psi_\lambda$ , the effects on  $\lambda_t$  are transitory and  $\psi_\lambda$  controls the speed of the convergence.

**Wealth revaluation and  $\hat{\lambda}_0$ .** The revaluation of the relative net worth is given by

$$b_{p,0} - b_{o,0} = \sum_{k \in \{L,E\}} \left( \frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) q_{k,0}. \quad (\text{A.58})$$

The price of the long-term bond satisfies the condition

$$-\frac{1}{Q_L} q_{L,t} + \dot{q}_{L,t} - (i_t - r_n) = r_L \left[ \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_L^*}{Q_L - Q_L^*} q_{L,t} \right] \quad (\text{A.59})$$

Rearranging the expression above, we obtain

$$\dot{q}_{L,t} - (\rho + \psi_L) q_{L,t} = (i_t - r_n) + r_L (\hat{\lambda}_t + \sigma c_{s,t}). \quad (\text{A.60})$$

Solving the differential equation above, we obtain

$$q_{L,0} = - \int_0^\infty e^{-(\rho + \psi_L)t} (i_t - r_n) dt - \int_0^\infty e^{-(\rho + \psi_L)t} r_L (\hat{\lambda}_t + \sigma c_{s,t}) dt. \quad (\text{A.61})$$

Suppose  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$  and  $r_L \sigma_{c_s, t} = \mathcal{O}(\|i_t - r_n\|^2)$ , then

$$q_{L,0} = -\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} - \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}. \quad (\text{A.62})$$

We focus on the case  $\frac{B_p^E}{B_p} = \frac{B_o^E}{B_o}$ , so the initial relative wealth revaluation is given by

$$b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right) \left[ \frac{i_0 - r_n}{\rho + \psi_L + \psi_m} + \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda} \right]. \quad (\text{A.63})$$

Plugging the expression above into the expression for  $\hat{\lambda}_0$

$$\hat{\lambda}_0 \equiv \frac{\sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \frac{\rho + \xi}{\chi_{b,c}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{i_0 - r_n}{\rho + \psi_L + \psi_m}}{1 - \sigma \mu_{c,o} \mu_{c,p} \left( \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \frac{\rho + \xi}{\chi_{b,c}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{r_L}{\rho + \psi_L + \psi_\lambda}}. \quad (\text{A.64})$$

Notice that there is an amplification mechanism between the price of the long-term bond and the change in disaster probability. A wealth redistribution towards pessimistic investors tends to increase  $\hat{\lambda}_0$ . An increase in  $\hat{\lambda}_0$  depresses the value of long-term bonds, redistributing towards pessimistic investors, further increasing  $\hat{\lambda}_t$ .

**Workers' consumption.** Log-linearizing workers' budget constraint, we obtain

$$c_{w,t} = \frac{WN_w}{PC_w} (w_t - p_t + n_{w,t}) + \frac{Y}{C_w} T'_w(Y) y_t. \quad (\text{A.65})$$

Using the fact that  $w_t - p_t + n_{w,t} = (1 + \phi) y_t$ , we can write the expression above as follows

$$c_{w,t} = \chi_y y_t. \quad (\text{A.66})$$

where  $\chi_y \equiv \frac{WN_w}{PC_w} (1 + \phi) + \frac{Y}{C_w} T'_w(Y)$ .

**Phillips curve.** Linearizing the Phillips curve, we obtain

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \quad (\text{A.67})$$

where  $\kappa \equiv \frac{\phi\epsilon}{\varphi} \frac{WN}{P}$ .

**Stock prices.** Linearizing the expression for  $r_{E,t}$ , we obtain

$$\frac{\Pi}{Q_E}(\hat{\Pi}_t - q_{E,t}) + \dot{q}_{E,t} - (i_t - \pi_t - r_n) = r_E \left[ \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_E^*}{Q_E - Q_E^*} q_{E,t} \right]. \quad (\text{A.68})$$

Rearranging the expression above, we obtain

$$\dot{q}_{E,t} - \rho q_{E,t} = -\frac{1}{Q_E} \hat{\Pi}_t + (i_t - \pi_t - r_n) + r_E [\hat{\lambda}_t + \sigma c_{s,t}], \quad (\text{A.69})$$

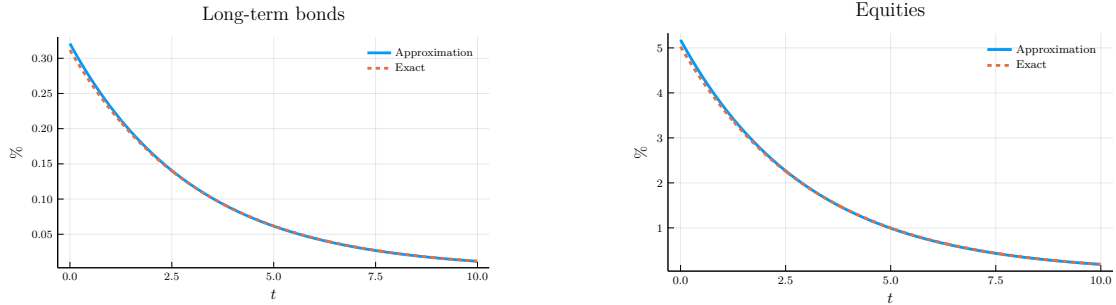
Solving the differential equation above, we obtain

$$q_{E,t} = \frac{1}{Q_E} \int_t^\infty e^{-\rho(s-t)} \hat{\Pi}_s ds - \int_t^\infty e^{-\rho(s-t)} [(i_s + \pi_s - r_n) + r_E(\hat{\lambda}_t + \sigma c_{s,t})] ds. \quad (\text{A.70})$$

#### A.4 The approximation in the price of risk

Proposition 3 shows that an approximate block recursivity property holds when  $r_k \sigma c_{s,t} = \mathcal{O}(\|i_t - r_n\|^2)$ , for  $k \in \{L, E\}$ . The term premium at  $t = 0$  is given by  $\int_0^\infty e^{-(\rho+\psi_L)t} r_L (\sigma c_{s,t} + \hat{\lambda}_t) dt$ , so this assumption implies that we can approximate the term premium, up to first order, by the expression  $\int_0^\infty e^{-(\rho+\psi_L)t} r_L \hat{\lambda}_t dt$ . Similarly, the drop in the stock price caused by changes in risk premia is given by  $\int_0^\infty e^{-\rho t} r_E (\sigma c_{s,t} + \hat{\lambda}_t) dt \approx \int_0^\infty e^{-\rho t} r_E \hat{\lambda}_t dt$  under our assumption about  $r_k \sigma c_{s,t}$ . To assess the quantitative importance of this assumption, we compare the discount rate effect on long-term bonds and equities when we include the term  $r_k \sigma c_{s,t}$  to the corresponding solution when this term is ignored.





**Figure A.1:** Risk premium effect on long-term bonds and stocks.

Note: The left panel shows the term premium when the term  $r_L\sigma c_{s,t}$  is included in the calculation (exact) and when this term is omitted (approximation). The right panel shows the drop on the stock price due to changes in the price of risk when the term  $r_L\sigma c_{s,t}$  is included in the calculation (exact) and when this term is omitted (approximation).

Figure A.1 shows the effect of this approximation on the pricing of stocks and bonds.<sup>3</sup> The left panel shows that the response of the yield on the long-term bond when we omit the term  $r_L\sigma c_{s,t}$  is nearly identical to the one when this term is included. A similar pattern emerges for stocks. The right panel shows the magnitude of the drop in the stock price caused by movements in the price of risk. The solid line represent the calculation when the term  $r_E\sigma c_{s,t}$  is included, and the dashed line shows the calculation when this term is omitted. Once again the approximate solution is nearly identical to the exact one.

## B Derivations for Section 3

### B.1 Trading in stocks

We consider next an extension where investors trade in stocks in the stationary equilibrium. In this case, the wealth effect of individual investors depends on the amount they trade on short-term bonds, long-term bonds, and stocks. However, as in the baseline model, the aggregate wealth effect depends only on the amount of government bonds

<sup>3</sup>Notice that we are only assessing the role of the assumption  $\mathcal{O}(\|i_t - r_n\|^2)$ . The lines we refer as “Exact” in Figure A.1 still corresponds to a linearized solution.

traded, as the household sector as a whole act as buy-and-hold investors on stocks.

**The replicating portfolio.** Let  $i \in \mathcal{I}_j$  denote saver  $i$  of type  $j$  and assume that saver  $i$  receives real income  $I_{j,t}(i) = a_j(i)e^{-\psi_E t} \Pi_t$ . We assume that  $\int_{i \in \mathcal{I}_j} a_j(i) di = 0$  and that the following condition is satisfied in a stationary equilibrium:

$$B_{j,0}(i) + \mathbb{E} \left[ \int_0^\infty \frac{\eta_t}{\eta_0} I_{j,t}(i) dt \right] = B_{j,0}, \quad (\text{B.1})$$

where  $B_{j,0}(i)$  is the initial wealth of saver  $i$  and  $B_{j,0}$  is the average wealth of type- $j$  savers. This implies that the consumption of all savers is the same in the stationary equilibrium. Let  $B_{j,t}^S(i) = B_j^S + \tilde{B}_{j,t}^S(i)$  and  $B_{j,t}^E(i) = B_j^E + \tilde{B}_{j,t}^E(i)$ , then

$$\tilde{B}_{j,t}^S + \tilde{B}_{j,t}^E + Q_{I_j(i),t} = 0, \quad \tilde{B}_{j,t}^S + \tilde{B}_{j,t}^E \frac{Q_E^*}{Q_E} + Q_{I_j(i),t}^* = 0. \quad (\text{B.2})$$

We can then solve for the portfolio of individual  $i$ :

$$\tilde{B}_{j,t}^S(i) = Q_{I_j(i),t} \frac{Q_E^*}{Q_E - Q_E^*} - Q_{I_j(i),t}^* \frac{Q_E}{Q_E - Q_E^*}, \quad (\text{B.3})$$

$$\tilde{B}_{j,t}^E(i) = Q_{I_j(i),t}^* \frac{Q_E}{Q_E - Q_E^*} - Q_{I_j(i),t} \frac{Q_E}{Q_E - Q_E^*}. \quad (\text{B.4})$$

**Pricing.** Notice that we can write the expression for  $\tilde{B}_{j,t}^E(i)$  as follows:

$$\frac{Q_E - Q_E^*}{Q_E} \tilde{B}_{j,t}^E(i) = - \frac{Q_{I_j(i),t} - Q_{I_j(i),t}^*}{Q_{I_j(i),t}} Q_{I_j(i),t}^*, \quad (\text{B.5})$$

so  $r_E \tilde{B}_{j,t}^E(i) = -r_{I_j(i)} Q_{I_j(i),t}^*$ . Assuming the economy is in the stationary equilibrium, the value of the income claim in the disaster state is given by

$$Q_{I_j(i),t}^* = a_j(i) \frac{e^{-\psi_E t} \Pi^*}{r_n^* + \psi_E}, \quad (\text{B.6})$$

and the value of the income claim in the no-disaster state is given by

$$Q_{I_j(i),t} = \frac{a_j(i)\Pi e^{-\psi_E t} + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma Q_{I_j(i),t}^*}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma + \psi_E}. \quad (\text{B.7})$$

We can then write the portfolio holdings of investor  $i$  as follows:

$$\tilde{B}_{j,t}^E(i) = -a_j(i)e^{-\psi_E t} \frac{Q_E}{Q_E - Q_E^*} \frac{\Pi - \frac{r_n + \psi_E}{r_n^* + \psi_E} \Pi^*}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma + \psi_E} \quad (\text{B.8})$$

$$\tilde{B}_{j,t}^S(i) = a_j(i)e^{-\psi_E t} \frac{Q_E}{Q_E - Q_E^*} \left[ \frac{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\Pi^*}{r_n^* + \psi_E} \frac{Q_E^*}{Q_E}}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma + \psi_E} - \frac{\Pi^*}{r_n^* + \psi_E} \right]. \quad (\text{B.9})$$

Notice that  $r_{I_j(i)}$  is given by

$$r_{I_j(i)} = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\Pi - \frac{r_n + \psi_E}{r_n^* + \psi_E} \Pi^*}{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\Pi^*}{r_n^* + \psi_E}}. \quad (\text{B.10})$$

Linearizing the pricing condition for the income claim, we obtain

$$q_{I_j,0} = \frac{a_j(i)Y}{Q_{I_j,0}} \int_0^\infty e^{-(\rho + \psi_E)t} \hat{\Pi}_t dt - \int_0^\infty e^{-(\rho + \psi_E)t} \left( i_t - \pi_t - r_n + r_{I_j(i)} p_{d,t} \right) dt. \quad (\text{B.11})$$

**Wealth effects.** The intertemporal budget constraint for household  $i$  is given by

$$\mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} C_{j,t}(i) dt \right] = B_{j,0}(i) + \mathbb{E} \left[ \int_0^\infty \frac{\eta_t}{\eta_0} (I_{j,t}(i) + T_{j,t}) dt \right]. \quad (\text{B.12})$$

Linearizing the equation above, we obtain

$$\Omega_{j,0}(i) = \frac{1}{C_j} \left[ B_j^L q_{L,0} + B_{j,0}^E(i) q_{E,0} + Q_{T_j} q_{T_j,0} + Q_{I_j(i),0} q_{I_j(i),0} \right] + \frac{Q_{C_j}}{C_j} \int_0^\infty e^{-\rho t} \left( i_t - \pi_t - r_n + r_{C_j} p_{d,t} \right) dt, \quad (\text{B.13})$$

where  $Q_{I_j(i),0}$  is the value at 0 of a claim on  $I_{j,t}(i)$  for all  $t \geq 0$ .

Using the fact that  $Q_{C_j} = B_{j,0}^S(i) + B_j^L + B_{j,0}^E(i) + Q_{I_j(i),0} + Q_{T_j}$  and  $Q_{C_j} r_{C_j} = B_j^L r_L + B_{j,0}^E(i) r_E + Q_{I_j(i),0} r_{I_j(i)} + Q_{T_j} r_{T_j}$ , we can write the wealth effect as follows:

$$\begin{aligned} \Omega_{j,0}(i) &= \Omega_{j,0} + \frac{Y}{C_j} \int_0^\infty e^{-\rho t} \left( \frac{B_{j,0}^E(i)}{Q_E} + e^{-\psi_E t} a_j(i) \right) \hat{\Pi}_t dt \\ &\quad + \frac{\tilde{B}_{j,0}^S(i)}{C_j} \int_0^\infty e^{-\rho t} (i_t - \pi_t - r_n) dt \\ &\quad + \frac{Q_{I_j(i),0}}{C_j} \int_0^\infty e^{-\rho t} (1 - e^{-\psi_E t}) (i_t - \pi_t - r_n + r_{I_j(i)} p_{d,t}) dt \end{aligned} \quad (\text{B.14})$$

Notice that  $(1 - e^{-\psi_E t}) Q_{I_j(i),0} = Q_{I_j(i),0} - Q_{I_j(i),t}$ ,  $Q_{I_j(i),t} = -\tilde{B}_{j,t}^S(i) - \tilde{B}_{j,t}^E(i)$ , and  $r_{I_j} Q_{I_j(i),t} = r_E \tilde{B}_{j,t}^E(i)$ . We can then write the expression above as follows:

$$\begin{aligned} \Omega_{j,0}(i) &= \Omega_{j,0} + \frac{Y}{C_j} \int_0^\infty e^{-\rho t} \left( \frac{\tilde{B}_{j,0}^E(i)}{Q_E} + e^{-\psi_E t} a_j(i) \right) \hat{\Pi}_t dt \\ &\quad + \frac{1}{C_j} \int_0^\infty e^{-\rho t} \Delta B_{j,t}^S (i_t - \pi_t - r_n) dt \\ &\quad + \frac{1}{C_j} \int_0^\infty e^{-\rho t} \Delta B_{j,t}^E (i_t - \pi_t - r_n + r_E p_{d,t}) dt, \end{aligned} \quad (\text{B.15})$$

where  $\Delta B_{j,t}^E = \tilde{B}_{j,t}^E(i) - \tilde{B}_{j,0}^E(i)$  and  $\Delta B_{j,t}^S = \tilde{B}_{j,t}^S(i)$ . Notice that as  $\int_{i \in I_j} a_j(i) di = 0$ , then  $\frac{1}{\mu_j} \int_{i \in I_j} \Omega_{j,0}(i) di = \Omega_{j,0}$ .

The equation above express the wealth effect in terms of cumulative purchases of assets. We can equivalently write the expression above in terms of instantaneous net purchases of assets, as in [Fagereng et al. \(2022\)](#). For simplicity, assume there is no cash-flow

effect. We can then write the integral above involving equities as follows:

$$\begin{aligned} \int_0^\infty e^{-\rho t} \Delta B_{j,t}^E (i_t - \pi_t - r_n + r_E p_{d,t}) dt &= \int_0^\infty e^{-\rho t} (1 - e^{-\psi_E t}) B_{j,0}^E (\dot{q}_{E,t} - \rho q_{E,t}) dt \\ &= B_{j,0}^E \left[ \int_0^\infty d(e^{-\rho t} q_{E,t}) - \int_0^\infty d(e^{-(\rho+\psi_E)t} q_{E,t}) \right] + \int_0^\infty e^{-\rho t} N_{j,t}^E q_{E,t} dt \end{aligned} \quad (\text{B.16})$$

$$= - \int_0^\infty e^{-\rho t} N_{j,t}^E q_{E,t} dt. \quad (\text{B.17})$$

where  $N_{j,t}^E = -\psi_E B_{j,t}^E$  denotes the net purchases at period  $t$ , using the fact that  $i_t - \pi_t - r_n + r_E p_{d,t} = \dot{q}_{E,t} - \rho q_{E,t}$

## B.2 Wealth effects and Hicksian compensation

**Hicksian compensation.** We show next that  $\Omega_{j,0}$  corresponds to (minus) the *Hicksian wealth compensation* for each household. Let  $e_j(\eta, U)$  define the expenditure function

$$e_j(\eta, U) = \min_{\{C_j\}} \mathbb{E}_{j,0} \left[ \int_0^{t^*} \frac{\eta_{j,t}}{\eta_{j,0}} C_{j,t} dt + \int_{t^*}^\infty \frac{\eta_{j,t}^*}{\eta_{j,0}} C_{j,t}^* dt \right], \quad (\text{B.18})$$

subject to  $\mathbb{E}_{j,0} \left[ \int_0^{t^*} e^{-\int_0^t \rho_{j,s} ds} \frac{C_{j,t}^{1-\sigma} - C_j^{1-\sigma}}{1-\sigma} dt + \int_{t^*}^\infty e^{-\int_0^t \rho_{j,s} ds} \frac{(C_{j,t}^*)^{1-\sigma} - (C_j^*)^{1-\sigma}}{1-\sigma} dt \right] = U$ . We subtracted the utility of the stationary-equilibrium consumption bundle, so  $U = 0$  corresponds to the utility obtained in the stationary equilibrium. The solution to this problem is the Hicksian demand  $C_{j,t}^h(\eta_j, U)$  and  $C_{j,t}^{h,*}(\eta_j, U)$  in the no-disaster and disaster states.

Let  $\eta'$  denote an alternative price process and  $U'$  the corresponding utility under the new equilibrium. [Mas-Colell et al. \(1995\)](#) (see page 62) defines the Hicksian wealth compensation as  $e_j(\eta'_t, U) - e_j(\eta'_t, U')$ . We focus on a first-order approximation, that is,  $\eta'_t/\eta'_0 = \eta_t/\eta_0 + \tilde{\eta}_t$ , where  $\tilde{\eta}_t$  is small. Let  $\tilde{c}_{j,t} \equiv \log C_{j,t}^h(\eta', U)/C_{j,t}^h(\eta, U)$ . Plugging the

expression for  $C_{j,t}^h(\eta', U)$  into the constraint and linearizing, we obtain

$$\mathbb{E}_{j,0} \left[ \int_0^{t^*} e^{-\rho_j t} C_{j,t}^h(\eta, U)^{1-\sigma} \tilde{c}_{j,t} dt + e^{-\rho_j t^*} \int_{t^*}^{\infty} e^{-\rho_j^*(t-t^*)} C_{j,t}^{h,*}(\eta, U)^{1-\sigma} \tilde{c}_{j,t}^* dt \right] = 0. \quad (\text{B.19})$$

Notice this implies that  $\mathbb{E}_{j,0} \left[ \int_0^{t^*} \frac{\eta_{j,t}}{\eta_{j,0}} C_{j,t}^h(\eta, U) \tilde{c}_{j,t} dt + \int_{t^*}^{\infty} \frac{\eta_{j,t}^*}{\eta_{j,0}} C_{j,t}^{h,*}(\eta, U) \tilde{c}_{j,t}^* dt \right] = 0$ . As workers do not engage in intertemporal substitution, we set  $\tilde{c}_{w,t} = \tilde{c}_{w,t}^* = 0$ , so this equation would hold for them as well. We can then write  $e_j(\eta', U)$  up to first order as follows

$$\begin{aligned} e_j(\eta', U) &= \mathbb{E}_0 \left[ \int_0^{t^*} \frac{\eta'_t}{\eta'_0} C_{j,t}^h(\eta, U) dt + \int_{t^*}^{\infty} \frac{\eta'_t}{\eta'_0} C_{j,t}^{*,h}(\eta, U) dt + \int_0^{t^*} \frac{\eta_t}{\eta_0} C_{j,t}^h(\eta, U) \tilde{c}_{j,t} dt + \int_{t^*}^{\infty} \frac{\eta_t^*}{\eta_0} C_{j,t}^{h,*}(\eta, U) \tilde{c}_{j,t}^* dt \right], \\ &= \mathbb{E}_0 \left[ \int_0^{t^*} \frac{\eta'_t}{\eta'_0} C_{j,t}^h(\eta, U) dt + \int_{t^*}^{\infty} \frac{\eta'_t}{\eta'_0} C_{j,t}^{*,h}(\eta, U) dt \right]. \end{aligned} \quad (\text{B.20})$$

We assume that the initial equilibrium corresponds to the stationary equilibrium, so  $C_{j,t}^h(\eta, U) = C_j$  and  $C_{j,t}^{h,*}(\eta, U) = C_j^*$ . Let  $\eta'_j$  denote the SDF after the monetary shock and  $U'$  the corresponding utility level. Therefore, the Hicksian wealth compensation is given by

$$e_j(\eta'_j, U) - e_j(\eta'_j, U') = \mathbb{E}_{j,0} \left[ \int_0^{t^*} \frac{\eta'_{j,t}}{\eta'_{j,0}} C_j dt + \int_{t^*}^{\infty} \frac{\eta'_{j,t}}{\eta'_{j,0}} C_j^* dt \right] - \mathbb{E}_{j,0} \left[ \int_0^{t^*} \frac{\eta'_{j,0}}{\eta'_{j,0}} C_{j,t} dt + \int_{t^*}^{\infty} \frac{\eta'_{j,t}}{\eta'_{j,0}} C_{j,t}^* dt \right], \quad (\text{B.21})$$

which corresponds to  $-\Omega_{j,0} C_j$  as defined in the text.

**Compensating and equivalent variation.** From the derivation above, we obtain that  $\Omega_{j,0} C_j = e_j(\eta'_j, U') - e_j(\eta'_j, U)$ , which corresponds to the compensating variation. We show next that  $\Omega_{j,0} C_j$  also coincides with the equivalent variation up to first order. The EV is given by  $e_j(\eta_j, U') - e_j(\eta_j, U)$ . Up to first order,  $C_{j,t}^h(\eta'_j, U') - C_{j,t}^h(\eta_j, U) = C_{j,t}^h(\eta_j, U') - C_{j,t}^h(\eta_j, U) + C_{j,t}^h(\eta'_j, U) - C_{j,t}^h(\eta_j, U)$ . As the present discounted value of  $C_{j,t}^h(\eta'_j, U) - C_{j,t}^h(\eta_j, U)$  is equal to zero, evaluated at the initial SDF, then the present discounted value of the left-

hand side,  $C_{j,t}^h(\eta'_j, U') - C_{j,t}^h(\eta_j, U)$ , equals the present discounted value of  $C_{j,t}^h(\eta_j, U') - C_{j,t}^h(\eta_j, U)$ . The present discounted value of  $C_{j,t}^h(\eta'_j, U') - C_{j,t}^h(\eta_j, U)$  evaluated at  $\eta$  (or  $\eta'$ ) corresponds to  $C_j\Omega_{j,0}$ . The present discounted value of  $C_{j,t}^h(\eta_j, U') - C_{j,t}^h(\eta_j, U)$  evaluated at  $\eta$  equals the equivalent variation, so  $e_j(\eta_j, U') - e_j(\eta_j, U) = C_j\Omega_{j,0}$ .

### B.3 Welfare

The indirect utility function is given by

$$V_j(\eta, \omega) = \mathbb{E}_{j,0} \left[ \int_0^{t^*} e^{-\int_0^t \rho_{j,s} ds} \frac{C_{j,t}^{1-\sigma}}{1-\sigma} dt + \int_{t^*}^{\infty} e^{-\int_0^t \rho_{j,s} ds} \frac{(C_{j,t}^*)^{1-\sigma}}{1-\sigma} dt \right], \quad (\text{B.22})$$

subject to  $\mathbb{E}_{j,0} \left[ \int_0^{t^*} \frac{\eta_{j,t}}{\eta_{j,0}} C_{j,t} dt + \int_{t^*}^{\infty} \frac{\eta_{j,t}^*}{\eta_{j,0}} C_{j,t}^* dt \right] = \omega$ .

Let  $\bar{V}_j$  denote welfare associated with the stationary-equilibrium consumption bundle. The change in welfare of deviating from the stationary equilibrium can be written as

$$\begin{aligned} V_j - \bar{V}_j &= C_j^{-\sigma} \int_0^{\infty} \lambda_j e^{-\lambda_j t^*} \left[ \int_0^{t^*} e^{-\rho_j c_{j,t}} dt + e^{-\rho_j t^*} \left( \frac{C_j^*}{C_j} \right)^{-\sigma} \frac{C_j^*}{C_j} \int_{t^*}^{\infty} e^{-\rho_j^*(t-t^*)} c_{j,t}^* dt \right] C_j \\ &= C_j^{-\sigma} \int_0^{\infty} e^{-\rho t} [c_{j,t} + \chi_{c_j^*} c_{j,t}^*] C_j \\ &= C_j^{1-\sigma} \Omega_{j,0}. \end{aligned} \quad (\text{B.23})$$

Hence, the wealth effect captures the impact on welfare for household  $j$ . We showed in the previous subsection that  $\Omega_{j,0}C_j$ , so the wealth effect corresponds to the compensating variation and equivalent variation. We show next that the wealth effect also corresponds to the welfare measure proposed by [Fagereng et al. \(2022\)](#).

Following [Fagereng et al. \(2022\)](#), let's assume there is no cash-flow effect, so we can

write

$$\Omega_{j,0} = \int_0^\infty e^{-\rho t} \frac{\Delta B_j^S}{C_j} (i_t - \pi_t - r_n) dt + \int_0^\infty e^{-\rho t} \frac{\Delta B_{j,t}^L}{C_j} (i_t - \pi_t - r_n + r_L p_{d,t}) dt. \quad (\text{B.24})$$

Notice that  $i_t - \pi_t - r_n + r_L p_{d,t} = \dot{q}_{L,t} - (\rho + \psi_L) q_{L,t}$ . We can then write the second integral above as follows:

$$\begin{aligned} \int_0^\infty e^{-\rho t} \frac{\Delta B_{j,t}^L}{C_j} (i_t - \pi_t - r_n + r_L p_{d,t}) dt &= \frac{B_j^L}{C_j} \int_0^\infty \left( e^{-\rho t} - e^{-(\rho + \psi_L)t} \right) (\dot{q}_{L,t} - (\rho + \psi_L) q_{L,t}) dt \\ &= \frac{B_j^L}{C_j} \left[ \int_0^\infty d(e^{-\rho t} q_{L,t}) dt - \int_0^\infty d(e^{-(\rho + \psi_L)t} q_{L,t}) dt - \int_0^\infty e^{-\rho t} \psi_L q_{L,t} dt \right] \\ &= -\frac{1}{C_j} \int_0^\infty N_{j,t}^L q_{L,t} dt, \end{aligned} \quad (\text{B.25})$$

where  $N_{j,t}^L = \psi_L B_j^L$  denotes the net purchases of long-term bonds in period  $t$ .

## B.4 iMPCs

The problem of saver  $j$  can be written as

$$V(\eta, \omega) = \mathbb{E}_{j,0} \left[ \int_0^\infty e^{-\int_0^t \rho_{j,s} ds} \frac{\tilde{C}_{j,t}^{1-\sigma}}{1-\sigma} dt \right], \quad (\text{B.26})$$

subject to

$$\mathbb{E}_{j,0} \left[ \int_0^\infty \frac{\tilde{\eta}_{j,t}}{\eta_{j,0}} \tilde{C}_{j,t} dt \right] = \omega, \quad (\text{B.27})$$

where  $\eta_{j,t}$  denotes the SDF under saver  $j$ 's beliefs, which evolves according to  $\frac{d\eta_{j,t}}{\eta_{j,t}} = - \left[ i_t - \pi_t + \lambda_j \frac{\eta_{j,t}^* - \eta_{j,t}}{\eta_{j,t}} \right] dt + \frac{\eta_{j,t}^* - \eta_{j,t}}{\eta_{j,t}} d\mathcal{N}_t$ ,  $\tilde{C}_{j,t} = C_{j,t}$  if the economy is in the no-disaster state, and  $\tilde{C}_{j,t} = C_{j,t}^*$  if the economy is in the disaster state. The SDF satisfies the change of measure conditions:  $\lambda_j \frac{\eta_{j,t}^*}{\eta_{j,t}} = \lambda_t \frac{\eta_t^*}{\eta_t}$  and  $\frac{\eta_{j,t}}{\eta_{0,t}} = e^{-\int_0^t (\lambda_s - \lambda_j) ds} \frac{\eta_t}{\eta_0}$ .



The first-order conditions for this problem are given by

$$e^{-\int_0^t \rho_{j,s} ds} \tilde{C}_{j,t}^{-\sigma} = \Lambda \frac{\tilde{\eta}_{j,t}}{\eta_{j,0}}, \quad (\text{B.28})$$

where  $\Lambda$  is the Lagrange multiplier on the intertemporal budget constraint.

Applying a change of measure, we can write the expression above as follows:

$$e^{-\int_0^t (\rho_{j,s} + \lambda_j) ds} \tilde{C}_{j,t}^{-\sigma} = \Lambda e^{-\int_0^t \lambda_s ds} \frac{\tilde{\eta}_{j,t}}{\eta_{j,0}}, \quad (\text{B.29})$$

The intertemporal budget constraint can be written as

$$\int_0^\infty e^{-\lambda_j t} \frac{\eta_{j,t}}{\eta_{j,0}} \left[ C_{j,t} + \lambda_j \frac{\eta_{j,t}^*}{\eta_{j,t}} Q_{C_{j,t}}^* \right] dt = \omega, \quad (\text{B.30})$$

where  $Q_{C_{j,t}}^* = \int_t^\infty \frac{\eta_{j,s}^*}{\eta_{j,t}^*} C_{j,s}^* ds$  is the value of a consumption claim for an economy that switches to the disaster state at time  $t$ . Applying a change of measure, we can write the equation above as follows:

$$\int_0^\infty e^{-\int_0^t \lambda_s ds} \frac{\eta_t}{\eta_0} \left[ C_{j,t} + \lambda_t \frac{\eta_t^*}{\eta_t} Q_{C_{j,t}}^* \right] dt = \omega, \quad (\text{B.31})$$

**The stationary equilibrium.** In a stationary equilibrium, we have that  $\eta_{j,t}^* = e^{-r_n^*(t-t^*)} \eta_{j,t^*}^*$ . Given our assumption that  $\rho_j^* = r_n^*$ , then  $C_{j,t}^* = C_{j,t^*}^*$ , so  $Q_{C_{j,t^*}}^* = \frac{C_{j,t^*}^*}{r_n^*}$ . From the optimality condition for risky assets,  $\lambda \left( \frac{C_s}{C_s^*} \right)^\sigma = \lambda \left( \frac{C_j}{C_j^*} \right)^\sigma$ , we obtain

$$C_j^* = \frac{\lambda_j^{\frac{1}{\sigma}} C_s^*}{\lambda^{\frac{1}{\sigma}} C_s} C_j. \quad (\text{B.32})$$

Plugging the condition above, and using the fact that consumption in the no-disaster

state is constant, we obtain

$$\int_0^\infty e^{-\rho t} \left[ C_j + \lambda \left( \frac{C_s}{C_s^*} \right)^{\sigma-1} \frac{\lambda_j^{\frac{1}{\sigma}} C_j}{\lambda^{\frac{1}{\sigma}} r_n^*} \right] dt = \omega. \quad (\text{B.33})$$

Rearranging the expression above, we obtain

$$C_j = \underbrace{\frac{\rho}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}}}_{\text{MPC}_j} \omega, \quad C_j^* = \underbrace{\frac{\rho \bar{\chi}^* \lambda_j^{\frac{1}{\sigma}}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}}}_{\text{MPC}_j^*} \omega \quad (\text{B.34})$$

where  $\bar{\chi} \equiv \frac{\lambda^{\frac{\sigma-1}{\sigma}}}{r_n^*} \left( \frac{C_s}{C_s^*} \right)^{\sigma-1}$  and  $\bar{\chi}^* \equiv \lambda^{-\frac{1}{\sigma}} \frac{C_s^*}{C_s}$ . The expressions above show that savers have heterogeneous MPCs. Optimistic investors have higher MPCs in the no-disaster state, while they have lower MPCs (out of initial wealth) in the disaster state.

**Perturbation.** Consider a perturbation of the environment above, where wealth and the SDF are subject to small shocks. From the Euler equation for riskless and risky assets, we obtain

$$\dot{c}_{j,t} = \sigma^{-1} (i_t - \pi_t - r_n) + \frac{\lambda}{\sigma} \left( \frac{C_s}{C_s^*} \right)^\sigma (\hat{\lambda}_t + \sigma c_{s,t}) - \zeta (c_{j,t} - c_{s,t}), \quad (\text{B.35})$$

and

$$\hat{\lambda}_t + \sigma (c_{s,t} - c_{s,t}^*) = \sigma (c_{j,t} - c_{j,t}^*). \quad (\text{B.36})$$

We can write the equations above as follows:

$$c_{j,t} = c_{s,t} + e^{-\zeta t} (c_{j,0} - c_{s,0}), \quad c_{j,t}^* = c_{j,t} - c_{s,t} - \frac{1}{\sigma} \hat{\lambda}_t. \quad (\text{B.37})$$

Linearizing the intertemporal budget constraint, we obtain

$$\int_0^{\infty} e^{-\rho t} [c_{j,t} + \chi_{c_j}^* c_{j,t}^*] dt = \Omega_{j,0}, \quad (\text{B.38})$$

where  $\chi_{c_j}^* = \bar{\chi} \lambda_j^{\frac{1}{\sigma}} = \frac{\lambda}{\rho_s} \left( \frac{C_s}{C_s^*} \right)^{\sigma} \frac{C_j^*}{C_j}$ .

Combining the expressions for consumption with the intertemporal budget constraint, we obtain

$$\frac{1 + \chi_{c_j}^*}{\rho + \xi} (c_{j,0} - c_{s,0}) = \Omega_{j,0} - \Omega_{s,0} + \frac{\chi_{c_j}^*}{\sigma} \frac{\hat{\lambda}_0}{\rho + \xi}. \quad (\text{B.39})$$

Rearranging the expression above, we obtain

$$c_{j,0} = c_{s,0} + \frac{\rho + \xi}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} (\Omega_{j,0} - \Omega_{s,0}) + \frac{\bar{\chi} \lambda_j^{\frac{1}{\sigma}}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} \frac{\hat{\lambda}_0}{\sigma}. \quad (\text{B.40})$$

Consumption at date  $t$  in the no-disaster state is given by

$$c_{j,t} = c_{s,t} + \frac{(\rho + \xi) e^{-\xi t}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} (\Omega_{j,0} - \Omega_{s,0}) + \frac{\bar{\chi} \lambda_j^{\frac{1}{\sigma}}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} \frac{\hat{\lambda}_t}{\sigma}. \quad (\text{B.41})$$

Consumption at date  $t$  in the disaster state is given by

$$c_{j,t}^* = \frac{(\rho + \xi) e^{-\xi t}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} (\Omega_{j,0} - \Omega_{s,0}) - \frac{1}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} \frac{\hat{\lambda}_t}{\sigma}. \quad (\text{B.42})$$

An increase in  $\Omega_{j,0}$  raises consumption in both states, while an increase in  $\hat{\lambda}_t$  raises consumption in the no-disaster state and reduces consumption in the disaster state.

**MPCs and iMPCs.** Define the *intertemporal MPCs*, or *iMPCs*, for saver  $j$  in the no-disaster and disaster states as follows

$$\mathcal{M}_{j,t} \equiv \frac{\partial c_{j,t}}{\partial \Omega_{j,0}} = \frac{(\rho + \xi)}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} e^{-\xi t}, \quad \mathcal{M}_{j,t}^* \equiv \frac{C_j^*}{C_j} \frac{\partial c_{j,t}^*}{\partial \Omega_{j,0}} = \frac{(\rho + \xi) \bar{\chi}^* \lambda_j^{\frac{1}{\sigma}}}{1 + \bar{\chi} \lambda_j^{\frac{1}{\sigma}}} e^{-\xi t}. \quad (\text{B.43})$$

Optimistic investors have higher iMPCs than pessimistic investors in the no-disaster state, while pessimistic investors have higher iMPCs than optimistic investors in the disaster state. However, the average iMPC is the same for both types of savers:

$$\int_0^\infty e^{-\rho t} \left[ \mathcal{M}_{j,t} + \frac{\lambda}{\rho_s} \left( \frac{C_s}{C_s^*} \right)^\sigma \mathcal{M}_{j,t}^* \right] dt = 1. \quad (\text{B.44})$$

The difference in consumption at date  $t$  is given by

$$c_{p,t} - c_{o,t} = \mathcal{M}_{p,t}(\Omega_{p,0} - \Omega_{s,0}) - \mathcal{M}_{o,t}(\Omega_{o,0} - \Omega_{s,0}) + \left( \mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^* \right) \frac{\bar{\chi}^* \hat{\lambda}_t}{\bar{\chi}} \frac{1}{\sigma} \frac{1}{\rho + \xi}. \quad (\text{B.45})$$

We can write the expression above as follows:

$$c_{p,t} - c_{o,t} = [\mathcal{M}_{p,t} \mu_{c,o} + \mathcal{M}_{o,t} \mu_{c,p}] (\Omega_{p,0} - \Omega_{o,0}) + \left( \mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^* \right) \frac{\bar{\chi}^* \hat{\lambda}_t}{\bar{\chi}} \frac{1}{\sigma} \frac{1}{\rho + \xi}. \quad (\text{B.46})$$

As  $\hat{\lambda}_t = \chi_{\lambda,c}(c_{p,t} - c_{o,t})$ , then

$$c_{p,t} - c_{o,t} = \frac{\mathcal{M}_{p,t} \mu_{c,o} + \mathcal{M}_{o,t} \mu_{c,p}}{1 - \left( \mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^* \right) \frac{\bar{\chi}^* \chi_{\lambda,c}}{\bar{\chi}} \frac{1}{\sigma} \frac{1}{\rho + \xi}} [\Omega_{p,0} - \Omega_{o,0}]. \quad (\text{B.47})$$

Therefore,  $\hat{\lambda}_t$  is given by

$$\hat{\lambda}_t = \frac{\chi_{\lambda,c} (\mathcal{M}_{p,t} \mu_{c,o} + \mathcal{M}_{o,t} \mu_{c,p})}{1 - \left( \mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^* \right) \frac{\bar{\chi}^* \chi_{\lambda,c}}{\bar{\chi}} \frac{1}{\sigma} \frac{1}{\rho + \xi}} [\Omega_{p,0} - \Omega_{o,0}].$$

## B.5 Minimum State Variable Solution

**General formulation.** Consider a general dynamic system involving the vector of endogenous variables  $Z_t = [K_t', Y_t']'$ , where  $Y_t$  is a vector of non-predetermined variables and  $K_t$  a vector of predetermined variables. The dynamics of  $Z_t$  is given by

$$\dot{Z}_t = AZ_t + BV_t, \quad (\text{B.48})$$

given  $K_0$ , where  $V_t$  is a vector of disturbances following the dynamics  $\dot{V}_t = \Psi_v V_t$ .

The minimum state-variable (MSV) solution takes the form:

$$Y_t = \Phi_{YK}K_t + \Phi_{YV}V_t, \quad \dot{K}_t = \Phi_{KK}K_t + \Phi_{KV}V_t. \quad (\text{B.49})$$

We can obtain the MSV solution using the method of undetermined coefficients. Importantly, the method produces a unique solution even when the number of negative eigenvalues exceed the number of predetermined variables.

**MSV solution of baseline model.** Consider the dynamic system given by (10) and (11), given a process for  $i_t$  and  $\hat{\lambda}_t$ . In particular, we assume that  $i_t$  follows the continuous-time analog of an AR(K) process:  $i_t - r_n = \Gamma_i' V_t$ , where  $\dot{V}_t = \Psi_V V_t$ , for  $\Psi_V$  diagonal.<sup>4</sup> We know that  $\hat{\lambda}_t = e^{-\psi_\lambda t} \hat{\lambda}_0$ , where  $\hat{\lambda}_0$  is a function of the path for  $i_t - r_n$ . We assume that one of the variables in  $V_t$  decay at rate  $\psi_\lambda$ , so we can write  $\hat{\lambda}_t = \Gamma_\lambda' V_t$ . After replacing  $i_t - r_n$  and  $\hat{\lambda}_t$  for the appropriate linear functions of  $V_t$ , we obtain a dynamic system in  $Z_t = [y_t, \pi_t]'$ . The MSV solution is given by

$$y_t = \Phi_y' V_t, \quad \pi_t = \Phi_\pi' V_t. \quad (\text{B.50})$$

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<sup>4</sup>In discrete time, we can write an AR(K) as  $(1 - a_1 L - \dots - a_K L^K)y_t = v_t$ , so  $y_t = \frac{v_t}{(1-\lambda_1 L) \dots (1-\lambda_K L)} = \sum_{k=1}^K \Gamma_{ik} V_{k,t}$ , assuming  $\lambda_i$  are distinct, where  $V_{k,t} \equiv \frac{v_t}{1-\lambda_i L}$ . Hence,  $y_t$  is a sum of K AR(1) variables.

Using the method of undetermined coefficients, we obtain

$$\Phi'_y \Psi_V = \tilde{\sigma}^{-1}(\Gamma'_i - \Phi'_\pi) + \delta \Phi'_y + \chi_{pd} \Gamma'_\lambda, \quad \Phi'_\pi \Psi_v = \rho \Phi'_\pi - \kappa \Phi'_y. \quad (\text{B.51})$$

Rearranging the expression above, we obtain the linear system

$$\begin{bmatrix} -\psi_k - \delta & \tilde{\sigma}^{-1} \\ \kappa & -\psi_k - \rho \end{bmatrix} \begin{bmatrix} \Phi_{yk} \\ \Phi_{\pi k} \end{bmatrix} = \begin{bmatrix} \tilde{\sigma}^{-1} \Gamma_{ik} + \chi_{pd} \Gamma_{\lambda k} \\ 0 \end{bmatrix}, \quad (\text{B.52})$$

where  $-\psi_k$  is the  $k$ -th element of the diagonal of  $\Psi_V$ . Solving the system above, we obtain

$$\begin{bmatrix} \Phi_{yk} \\ \Phi_{\pi k} \end{bmatrix} = -\frac{1}{(\bar{\omega} + \psi_k)(\underline{\omega} + \psi_k)} \begin{bmatrix} \rho + \psi_k \\ \kappa \end{bmatrix} \left( \tilde{\sigma}^{-1} \Gamma_{ik} + \chi_{pd} \Gamma_{\lambda k} \right), \quad (\text{B.53})$$

assuming  $\psi_k \neq -\underline{\omega}$ .

We show next how to implement the MSV solution using a Taylor rule. Suppose  $u_t = \sum_{k=1}^K \varphi_k u_{k,t}$ , where  $u_{k,t} = V_{k,t}$ . We adopt the normalization  $V_{k,0} = i_0 - r_n$ . The nominal rate under the Taylor rule is given by

$$i_t - r_n = \sum_{k=1}^K \varphi_k \frac{(\bar{\omega} + \psi_k)(\underline{\omega} + \psi_k)}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} u_{k,t} - \frac{\phi_\pi \kappa \chi_\lambda}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} e^{-\psi_\lambda t} (i_0 - r_n).$$

In the case  $\psi_k \neq \psi_\lambda$ , the coefficient  $\varphi_k = \Gamma_{ik} \frac{(\omega_1 + \psi_k)(\omega_2 + \psi_k)}{(\bar{\omega} + \psi_k)(\underline{\omega} + \psi_k)}$ . In the case  $\psi_k = \psi_\lambda$ , the coefficient is given by  $\varphi_k = \frac{\Gamma_{ik}(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda) + \phi_\pi \kappa \chi_\lambda}{(\bar{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)}$ .

Output is then given by

$$y_t = -\sum_{k=1}^K \Gamma_{ik} \frac{\rho + \psi_k}{(\bar{\omega} + \psi_k)(\underline{\omega} + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{(\rho + \psi_\lambda) \tilde{\chi}_\lambda}{(\bar{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)} e^{-\psi_\lambda t} (i_0 - r_n)$$

$$\pi_t = -\sum_{k=1}^K \varphi_k \frac{\kappa}{(\bar{\omega} + \psi_k)(\underline{\omega} + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{\kappa \tilde{\chi}_\lambda}{(\bar{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)} e^{-\psi_\lambda t} (i_0 - r_n),$$

where

$$\tilde{\chi}_\lambda = \chi_\lambda \left[ \frac{\phi_{\pi\kappa}\tilde{\sigma}^{-1}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} + \frac{(\bar{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \right] = \chi_\lambda. \quad (\text{B.54})$$

Finally, the coefficient  $\epsilon_\lambda$  is given by

$$\epsilon_\lambda = \frac{\chi_{\lambda,c} \frac{\rho + \xi}{\chi_{b,c}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right)}{1 - \chi_{\lambda,c} \frac{\rho + \xi}{\chi_{b,c}} \left( \frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{r_L}{\rho + \psi_L + \psi_\lambda}} \sum_{k=1}^K \Gamma_{ik} \frac{i_0 - r_n}{\rho + \psi_L + \psi_k} = \sum_{k=1}^K \Gamma_{ik} \epsilon_{\lambda,k}. \quad (\text{B.55})$$

Hence, given an interest rate  $i_t - r_n = \sum_{k=1}^K \Gamma_{ik} e^{-\psi_k t} (i_0 - r_n)$ , we can write the solution for output and inflation as  $y_t = \sum_{k=1}^K \Gamma_{ik} y_{k,t}$  and  $\pi_t = \sum_{k=1}^K \Gamma_{ik} \pi_{k,t}$ , where  $y_{k,t}$  and  $\pi_{k,t}$  correspond to the solution when the interest rate follows the process  $e^{-\psi_k t} (i_0 - r_n)$ .

The case where  $u_t = \varphi_1 e^{-\psi_m t} (i_0 - r_n)$ ,  $\psi_m \neq \psi_\lambda$ , corresponds to the coefficients:

$$\Gamma_{i1} = 1 + \frac{\phi_{\pi\kappa}\chi_\lambda}{(\bar{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda) + \tilde{\sigma}^{-1}\phi_{\pi\kappa}'}, \quad \Gamma_{i2} = -\frac{\phi_{\pi\kappa}\chi_\lambda}{(\bar{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda) + \tilde{\sigma}^{-1}\phi_{\pi\kappa}'}, \quad (\text{B.56})$$

where  $\psi_1 = \psi_m$  and  $\psi_2 = \psi_\lambda$ .

In the case  $\psi_m = \psi_\lambda$ , we have  $\Gamma_{i1} = 1$ , which requires

$$\varphi_1 = 1 + \frac{(\tilde{\sigma}^{-1} + \chi_\lambda)\phi_{\pi\kappa}}{(\bar{\omega} + \psi_m)(\underline{\omega} + \psi_m)}. \quad (\text{B.57})$$

## C Derivations for Section 4

### C.1 Bond pricing and forward curve

In this section, we solve for prices, yields, and forward rates of zero-coupon bonds of different maturity. While in the main text we focused on the price of a single bond with exponentially decaying coupons, we solve here for the entire yield and forward curves.

Let  $Q_{B,t}(h)$  denote the period  $t$  price of a nominal zero-coupon bond maturing at pe-

riod  $t + h$ ,  $y_{B,t}(h)$  denotes the corresponding yield on the bond, and  $f_{B,t}(h)$  denotes the instantaneous forward rate. The bond price satisfy the standard pricing condition

$$Q_{B,t}(h) = \mathbb{E}_t \left[ \frac{\eta_{t+h} P_t}{\eta_t P_{t+h}} \right], \quad (\text{C.1})$$

using the fact that  $\eta_t/P_t$  is the nominal SDF in this economy.

**Stationary equilibrium.** The price of the bond in the no-disaster state of the stationary equilibrium is given by

$$Q_B(h) = \int_h^\infty \lambda e^{-\lambda t^*} e^{-\rho_s h} dt^* + \int_0^h \lambda e^{-\lambda t^*} e^{-\rho_s t^*} \left( \frac{C_s}{C_s^*} \right)^\sigma e^{-r^*(h-t^*)} dt^* \quad (\text{C.2})$$

$$= e^{-\rho h} + (1 - e^{-\lambda h}) e^{-\rho_s h} \left( \frac{C_s}{C_s^*} \right)^\sigma. \quad (\text{C.3})$$

while the price of the bond in the disaster state is simply  $Q_B^*(h) = e^{-\rho_s h}$ . Notice that  $\int_0^\infty e^{-\psi_L h} P(h) dh = \frac{1 + Q_L^* \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma}{\rho + \psi_L} = Q_L$ , so this is consistent with our derivation for  $Q_L$ .

The yield on the bond is given by

$$y_B(h) = \rho_s + \lambda - \frac{1}{h} \log \left[ 1 + (e^{\lambda h} - 1) \left( \frac{C_s}{C_s^*} \right)^\sigma \right]. \quad (\text{C.4})$$

Notice that  $\lim_{h \rightarrow 0} y_B(h) = r_n^*$  and  $\lim_{h \rightarrow \infty} y_B(h) = \rho > r_n^*$ , capturing the fact that the yield curve is upward-sloping.

The forward rate is given by

$$f_B(h) = -\frac{\partial \log Q_B(h)}{\partial h} = \rho_s - \frac{\lambda \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right]}{(e^{\lambda h} - 1) \left( \frac{C_s}{C_s^*} \right)^\sigma + 1}. \quad (\text{C.5})$$



**The linearized PDE.** Let  $r_{B,t}(h)$  denote the excess holding-period return on a bond maturing  $h$  periods ahead conditional on no disaster:

$$r_{B,t}(h) \equiv \frac{1}{Q_{B,t}(h)} \left[ -\frac{\partial Q_{B,t}(h)}{\partial h} + \frac{\partial Q_{B,t}(h)}{\partial t} \right] - i_t. \quad (\text{C.6})$$

The Euler equation for the bond is given by

$$r_{B,t}(h) = \lambda_t \left( \frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma \frac{Q_{B,t}(h) - Q_{B,t}^*(h)}{Q_{B,t}(h)}. \quad (\text{C.7})$$

Let  $q_{b,t}(h) \equiv \log Q_{B,t}(h) - \log Q_B(h)$ , then linearizing the equation above we obtain

$$-\frac{\partial q_{B,t}(h)}{\partial h} + \frac{\partial q_{B,t}(h)}{\partial t} = i_t - r_n + r_B(h) \left[ \hat{\lambda}_t + \frac{Q_B^*(h)}{Q_B(h) - Q_B(h)^*} q_{b,t}(h) \right], \quad (\text{C.8})$$

where we used the assumption that  $r_B(h)\sigma_{C_{s,t}}$  is second-order.

**From PDE to system of ODEs.** Assuming that the nominal interest rate is exponentially decaying,  $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ , we will guess-and-verify that the solution takes the form:

$$q_{B,t}(h) = \chi_{B,i}(h)(i_t - r_n) + \chi_{B,\lambda}(h)\hat{\lambda}_t, \quad (\text{C.9})$$

where  $\chi_{B,i}(0) = \chi_{B,\lambda}(0) = 0$ . Plugging the expression above into the PDE, we obtain

$$-\chi'_{B,i}(h)(i_t - r_n) - \chi'_{B,\lambda}(h)\hat{\lambda}_t - \psi_m \chi_{B,i}(h)(i_t - r_n) - \psi_\lambda \chi_{B,\lambda}(h)\hat{\lambda}_t = \quad (\text{C.10})$$

$$i_t - r_n + r_B(h)\hat{\lambda}_t + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} [\chi_{B,i}(h)(i_t - r_n) + \chi_{B,\lambda}(h)\hat{\lambda}_t]. \quad (\text{C.11})$$

The equation above has to hold for any values of  $i_0 - r_n$  and  $\hat{\lambda}_0$ , then we obtain a

system decoupled ODEs

$$-\chi'_{B,i}(h) - \psi_m \chi_{B,i}(h) = 1 + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \chi_{B,i}(h) \quad (\text{C.12})$$

$$-\chi'_{B,\lambda}(h) - \psi_\lambda \chi_{B,\lambda}(h) = r_B(h) + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \chi_{B,i}(h), \quad (\text{C.13})$$

given the initial conditions  $\chi_{B,i}(0) = \chi_{B,\lambda}(0) = 0$ , where

$$r_B(h) = \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{(1 - e^{-\lambda h}) \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right]}{e^{-\lambda h} + (1 - e^{-\lambda h}) \left( \frac{C_s}{C_s^*} \right)^\sigma}, \quad \frac{Q_B^*(h)}{Q_B(h)} = \frac{1}{e^{-\lambda h} + (1 - e^{-\lambda h}) \left( \frac{C_s}{C_s^*} \right)^\sigma}. \quad (\text{C.14})$$

We can write the ODEs above as follows:

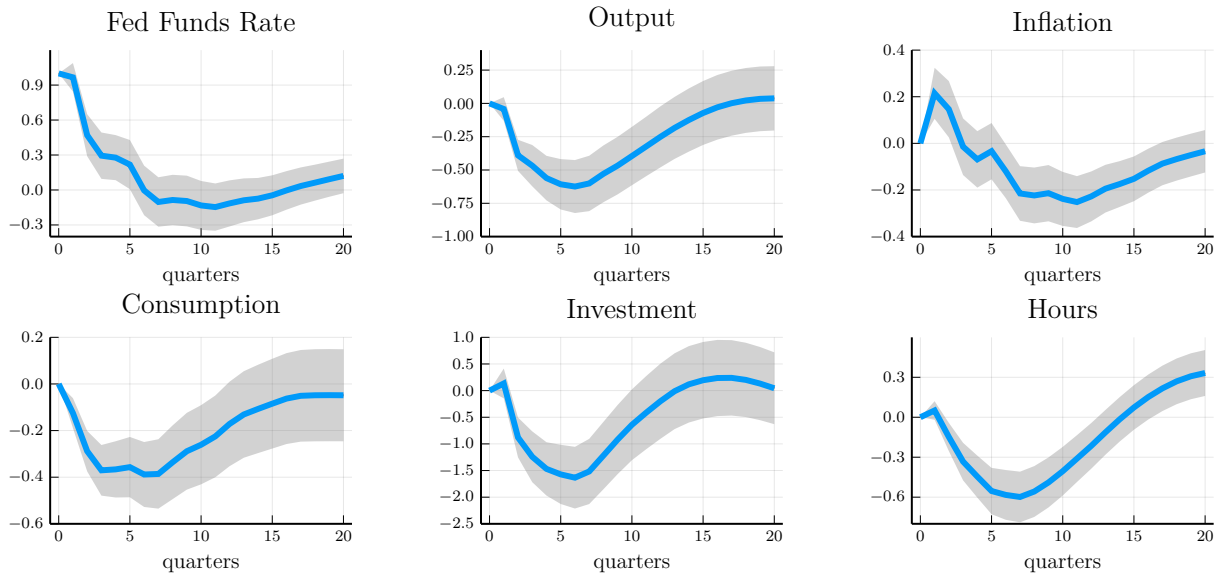
$$\chi'_{B,i}(h) = -1 - \left[ \psi_m + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \right] \chi_{B,i}(h) \quad (\text{C.15})$$

$$\chi'_{B,\lambda}(h) = -r_B(h) - \left[ \psi_\lambda + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \right] \chi_{B,i}(h). \quad (\text{C.16})$$

The system above can easily solve numerically using a finite-differences scheme. Given the bond prices, we can find the yield  $y_{B,t}(h) = -\frac{1}{h} \log Q_{B,t}(h) = -\frac{1}{h} \log Q_B(h) - \frac{1}{h} q_{B,t}(h)$ . Let  $\hat{y}_{B,t}(h)$  denote the deviation of the yield on the bond from its value in the stationary equilibrium. The forward rate is given by  $f_{B,t}(h) = -\frac{\partial \log Q_{B,t}(h)}{\partial h} = -\frac{\log Q_B(h)}{\partial h} - \frac{\partial q_{B,t}(h)}{\partial h}$ , so  $\hat{f}_{B,t}(h) \equiv -\frac{\partial q_{B,t}(h)}{\partial h}$  denotes the deviation of the forward rate from its value in the stationary equilibrium.

## D Estimation of Fiscal Response to a Monetary Shock

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in [Christiano, Eichenbaum, and Evans \(1999\)](#), extended to include fiscal variables. The variables included are: real GDP per capita, CPI inflation, real consumption per capita,



**Figure D.1:** Estimated IRFs.

real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that react contemporaneously to the monetary shock are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including tax revenues and expenditures, react with a lag of one quarter.

**Data sources.** The data sources are: **Nominal GDP:** BEA Table 1.1.5 Line 1; **Real GDP:** BEA Table 1.1.3 Line 1, **Consumption Durable:** BEA Table 1.1.3 Line 4; **Consumption Non Durable:** BEA Table 1.1.3 Line 5; **Consumption Services:** BEA Table 1.1.3 Line 6; **Private Investment:** BEA Table 1.1.3 Line 7; **GDP Deflator:** BEA Table 1.1.9 Line 1; **Capacity Utilization:** FRED CUMFNS; **Hours Worked:** FRED HOANBS; **Nominal Hourly Com-**

	(1) Revenues	(2) Interest Payments	(3) Transfers & Expenditures	(4) Debt in $T$	(5) Initial Debt	(1) - (2) - (3) + (4) - (5) Residual
Data	10.54 [-14.11,35.18]	36.2 [20.07,52.33]	2.68 [-16.99,22.34]	1.42 [-14.77,17.61]	-17.62 [-21.62,-13.63]	9.3 [-16.69,35.29]

**Table D.1:** The impact on fiscal variables of a monetary policy shock

Note: Calculations correspond to a a 100 bps unanticipated interest rate increase. Confidence interval at 68% level.

**pensation:** FRED COMPNFB; **Civilian Labor Force:** FRED CNP16OV; **Nominal Revenues:** BEA Table 3.1 Line 1; **Nominal Expenditures:** BEA Table 3.1 Line 21; **Nominal Transfers:** BEA Table 3.1 Line 22; **Nominal Gov't Investment:** BEA Table 3.1 Line 39; **Nominal Consumption of Net Capital:** BEA Table 3.1 Line 42; **Effective Federal Funds Rate (FF):** FRED FEDFUNDS; **5-Year Treasury Constant Maturity Rate:** FRED DGS5; **Market Value of Government Debt:** [Hall, Payne and Sargent \(2018\)](#).

All the variables are obtained from standard sources, except for the real value of debt, which we construct from the series provided by [Hall et al. \(2018\)](#). We transform the series into quarterly frequency by keeping the market value of debt in the first month of the quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the *quantity* of debt after a monetary shock instead of changes in *prices*.

**VAR estimation.** Figure [D.1](#) shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

**The Government's Intertemporal Budget Constraint.** The fiscal response in the model corresponds to the present discounted value of transfers over an infinite horizon, that is,

$\sum_{t=0}^{\infty} \tilde{\beta}^t T_t$ , where  $\tilde{\beta} = \frac{1-\lambda}{1+\rho_s}$ . We next consider its empirical counterpart. First, we calculate a truncated intertemporal budget constraint from period zero to  $\mathcal{T}$ :

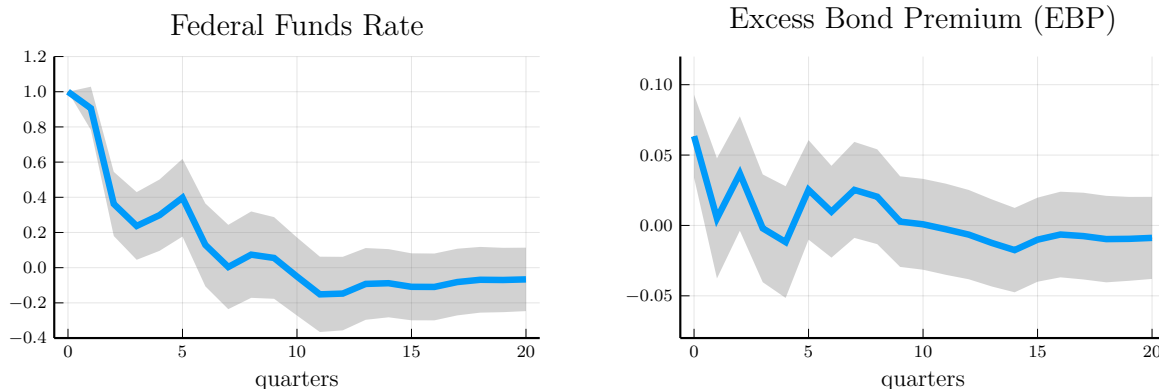
$$\underbrace{b_y b_0}_{\text{debt revaluation}} = \sum_{t=0}^{\mathcal{T}} \tilde{\beta}^t \left[ \underbrace{\tau y_t + \tau_t}_{\text{tax revenue}} - \underbrace{\tilde{\beta}^{-1} b_y (i_{t-1}^m - \pi_t - r^n)}_{\text{interest payments}} \right] - \underbrace{T_{0,\mathcal{T}} + \tilde{\beta}^{\mathcal{T}} b_y b_{\mathcal{T}}}_{\text{other transfers/expenditures \& final debt}} \quad (\text{D.1})$$

The right-hand side of (D.1) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. The second term represents the change in interest payments on government debt that results from change in nominal rates. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period  $\mathcal{T}$ , respectively. In particular,  $T_{0,\mathcal{T}}$  represents the present discounted value of transfers from period 0 through  $\mathcal{T}$ . Provided that  $\mathcal{T}$  is large enough, such that  $(y_t, \tau_t, i_t)$  have essentially converged to the steady state, then the value of debt at the terminal date,  $b_{\mathcal{T}}$ , equals (minus) the present discounted value of transfers and other expenditures from period  $\mathcal{T}$  onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity. Finally, the left-hand side represents the revaluation effect of the *initial* stock of government debt.

Table D.1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We first apply equation (D.1) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. The residual is calculated as

$$\text{Residual} = \text{Revenues} - \text{Interest Payments} - \text{Transfers} + \text{Debt in } \mathcal{T} - \text{Initial Debt}$$

We truncate the calculations to quarter 60, that is,  $\mathcal{T} = 60$  (15 years) in equation (D.1).



**Figure D.2:** IRFs for the federal funds rate and excess bond premium.

The results reported in Table D.1 imply that we cannot reject the possibility that the residual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 2. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

**EBP.** To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates in the literature.

## References

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