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Abstract

Integrated assessment models (IAMs) provide key inputs to decision-makers on economically efficient climate policies, and technical change is one of the key assumptions in any IAM that estimates mitigation costs. We conduct a systematic survey of how technical change is currently represented in the main IAMs and find that a diversity of approaches continues to exist. This makes it important to conduct an up-to-date assessment of what difference technical change makes to IAM results. Here we attempt such an assessment, using an analytical IAM with a reduced-form representation of technical change, which we can calibrate on the relationship between abatement costs and the timing of abatement in 109 IAM scenarios from two major databases. We first show in theory how a range of technical-change mechanisms can be adequately captured in a reduced-form model, in which the key difference is whether technical change is a function of time, i.e., exogenous, or cumulative past emissions abatement, i.e., endogenous. We then derive analytical and quantitative results on the effect of technical change on optimal climate policy, for both cost-benefit and cost-effectiveness policy problems. Under cost-benefit analysis, technical change has a quantitatively large, negative effect on long-run emissions and temperatures. The effect on carbon prices differs markedly depending on whether technical change is exogenous or endogenous, and whether clean technology deployment is incentivised by carbon prices or a dedicated deployment subsidy. Under cost-effectiveness analysis, technical change has a small effect on transient emissions and temperatures, but it has a large, negative effect on carbon prices almost irrespective of the policy instruments available. We make several practical recommendations for how IAMs can better incorporate TC, particularly when facing computational constraints.

JEL-Codes: C610, O300, Q540, Q550, Q580.

Keywords: climate change, cost-benefit analysis, induced innovation, integrated assessment models, technical change.

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1 Introduction

Integrated assessment models (IAMs) provide key inputs to decision-makers on economically efficient climate policies (e.g. Stern, 2007; IPCC, 2018, 2022a), and technical change (hereafter TC) is one of the key assumptions in any IAM that estimates emissions abatement costs, because it can reduce future costs.

TC is a nebulous concept covering many mechanisms. Some are exogenous to abatement policy decisions. For example, there may be technological spillovers from non-climate R&D, such as general-purpose membrane technologies developed in the chemicals industry, which can also reduce the costs of clean hydrogen production. Some mechanisms are endogenous because they depend on climate R&D specifically. For example, in the current early stage of development of nuclear fusion, cost reductions depend on R&D investment rather than deployment. Still, investment in nuclear fusion is more attractive in a low-emissions scenario because the market for the technology is bigger. Some mechanisms are endogenous to abatement policy because technology costs depend directly on deployment. For example, photovoltaic cells would still be expensive today if they had not been deployed at large scale. The sources of these cost reductions are various and include learning by doing and economies of scale in manufacturing (Elia et al., 2021). Even for R&D, deployment will often be helpful – it allows companies to get feedback on their technologies, scaling up may require R&D in itself (producing a few kilos in a lab can require different methods to producing Megatonnes), etc. Often, perhaps usually, exogenous and endogenous TC will co-exist. For example, the development of Lithium-ion batteries for smartphones can reduce the future cost of electric-vehicle batteries (exogenous TC). However, the specific requirements of car batteries (e.g., large capacity and peak power) will be met more quickly if electric vehicles are produced at scale (endogenous TC).

IAMs need to make sense of the complex and diverse mechanisms of TC, simplify them and build appropriate model abstractions. Previous reviews have shown that modellers have taken different approaches to TC (Löschel, 2002; Grubb et al., 2002; Sue Wing, 2006; Gillingham et al., 2008). At the same time, it is well known from other reviews that there is wide variation between IAMs in their estimated abatement costs of meeting pre-determined climate goals (Clarke et al., 2014; van Vuuren et al., 2020; Riahi et al., 2022), or alternatively in their prescriptions of optimal warming (Gillingham et al., 2018). The missing piece of the puzzle is knowing what role TC plays in this variation. It is difficult to know because IAMs are rich and complicated, with many relevant differences. Therefore, it is not obvious how to construct a controlled comparison that leads to an understanding of the effect of TC on optimal climate policy. This is our aim in this paper.

We start with a systematic survey of how TC is represented in the current crop of major IAMs, based on 22 families of models. We establish that the diversity of modelling approaches to TC identified in older literature still exists. We then turn to making sense of the diversity.

Our approach is to construct a reduced-form model of TC, which is capable of nesting the various TC mechanisms described above and distilling them into their most salient differences for climate policy, defined as optimal emissions, temperatures and abatement costs/carbon prices. The critical difference is whether TC is exogenous or endogenous to decisions on emissions abatement/pricing. We show that a model of exogenous, time-dependent TC captures not only pure time-dependent TC mechanisms such as spillovers from other sectors, it is also an adequate representation of TC driven by early-stage R&D, because early-stage R&D has dynamic properties that will be approximately the same as exogenous TC under reasonable assumptions (it boils down to assuming R&D investment costs for emissions abatement have a negligible effect on overall economic growth). We show that a model of endogenous TC captures not only the mechanical dependence of future abatement costs on current abatement via learning by doing, economies of scale, etc., but also that the same endogenous TC model can be obtained from rewriting a model where TC is a function of past carbon/energy prices instead. Thus, most varieties of TC represented by the IAM literature can be adequately characterised by two reduced-form models, one of exogenous TC and one of endogenous TC.

We then take these models of TC and place them within a broader IAM of the 'analytical' type, with a view to understanding how they affect optimal climate policy. TC makes future abatement cheaper than today; a *cost-reduction effect*. This is true of both exogenous and endogenous TC. We show theoretically that it results in a lower carbon price than without TC, which creates an incentive to abate later. In the case of endogenous TC, there is an additional and opposite *endogenous future gain effect*: abatement today induces cheaper abatement in the future, which creates an incentive for early abatement. If a carbon price is the only policy instrument, the endogenous future gain effect merits a higher initial carbon price, and slower carbon price growth. If a clean-technology deployment subsidy is also available as a second policy instrument, then the carbon price is just set at the social cost of carbon. We also show how the effect of TC on optimal climate policy depends on how the policy problem is set up. In a cost-benefit analysis, TC reduces steady-state/peak warming. In a cost-effectiveness analysis, where costs are minimised to stay below a given temperature target, this effect is absent.

The last step in our analysis is to use the model for numerical simulations. We develop a method of structural estimation, which enables us to calibrate the abatement cost and TC parameters of our model on the current crop of IAMs. We use 'observed' variation in the timing of abatement and associated abatement costs across 109 IAM scenarios collected in two major databases (the Intergovernmental Panel on Climate Change or IPCC and the Network of Central Banks and Supervisors for Greening the Financial System or NGFS). This gives us an estimate of how much TC drives down abatement costs in current IAMs, supposing the process is either exogenous or endogenous. We then take this calibration of abatement costs/TC, combine it with a parameterisation of the remainder of the IAM, and numerically solve for optimal carbon prices, emissions and temperatures.

We find that under cost-benefit analysis, TC has a quantitatively large, negative effect on long-run emissions and temperatures. Temperatures are 0.2-0.3°C lower in 2100. The effect on carbon prices differs markedly depending on whether TC is exogenous or endogenous, and whether a dedicated deployment subsidy is available to complement carbon prices. Under exogenous TC, the cost-reduction effect depresses the carbon price. Under endogenous TC, the cost-reduction effect is almost exactly cancelled out by the endogenous future gain effect initially. Thus, if a carbon price is the only policy instrument, the net effect of TC on the carbon price is minimal, but over time the cost-reduction effect comes to dominate and the carbon price grows more slowly than without TC. If a deployment subsidy is also available, the carbon price is the same as under exogenous TC. Under cost-effectiveness analysis, TC has only a small effect on emissions and temperatures, but it has a large, negative effect on carbon prices, which are 12-14% lower initially than without TC, with the gap widening over time. We further analyse the sensitivity of the results to the discount rate and speed of TC – they are qualitatively the same but TC has larger quantitative effects under a low discount rate and (mechanically) with faster TC.

In the discussion section we make several practical recommendations for how IAMs can better incorporate TC, particularly when facing computational constraints.

Related literature

We connect to three main strands of literature. The first is the literature investigating the effect of TC on optimal carbon prices and emissions. Fundamental contributions include: Goulder and Mathai (2000); Van der Zwaan et al. (2002); Popp (2004); Manne and Richels (2004); Popp et al. (2010). Therefore, this paper is not the first to consider how TC affects optimal climate policy. In particular, our framework generalises the theory of Goulder and Mathai (2000). However, it is time to bring this literature, much of which dates back 15 years or more, up to date and apply it to the contemporary question of how much current IAM scenarios and results depend on TC and differences in how it is modelled.[1] In addition to the aforementioned papers, there is a large literature conceptualising TC in specific ways, such as patented R&D into abatement or clean technologies (Gerlagh et al., 2009, 2014; Greaker and Pade, 2009; Acemoglu et al., 2012), and learning by doing (Bramoullé and Olson, 2005). Our approach differs from this type of contribution by taking a reduced-form approach. While our approach overlooks micro-foundations, it enables us to be much more agnostic on the mechanisms of TC, which is critical for our purpose.

Second, we contribute to the recent literature developing and applying analytical IAMs (Golosov et al., 2014; Rezai and Van der Ploeg, 2016; Van den Bijgaart et al., 2016; Dietz and Venmans, 2019a; Traeger, forthcoming). As the moniker suggests, these models are intended to give analytical insights into the role of different parameters and assumptions, shining a light into the black box of richer numerical IAMs.

Third, we contribute to the literature reviewing and synthesising IAMs. Many of the scenario runs of these models are summarized in the IPCC reports (Shukla et al., 2022). Weyant (2017) and Nikas et al. (2019) provide recent, general overviews of IAMs, Löschel (2002), Grubb et al. (2002), Sue Wing (2006) and Gillingham et al. (2008) are examples of earlier overviews of TC in these models, and van Vuuren et al. (2020) analyse the large differences in abatement costs

[1]With the benefit of hindsight, some results in Goulder and Mathai (2000), such as optimal carbon prices of roughly \$35/tCO2 in 2100, and optimal CO_2 concentrations above 800ppm in 2200 (causing 5°C warming), reflect understandings of the costs and benefits of abatement that have become out of date, with the utmost respect.

between models.

The rest of the paper is structured as follows. Section 2 describes our systematic review of TC in IAMs and the results of that review. Section 3 develops analytical models of exogenous and endogenous TC and shows how a range of model representations can be grouped into these two reduced-form classes. Section 4 takes this model and places it within a broader, analytical IAM. This model yields some theoretical results on optimal climate policy. Section 5 describes the calibration/estimation of this model and our quantitative results. Section 6 provides a concluding discussion.

2 Model survey

This section describes our systematic review of how TC is represented in IAMs currently. We first compiled a list of candidate IAMs, populating the list using a set of international databases/web resources and previous reviews on the topic.[2] The resulting long list comprised 87 models. We then screened this long list of IAMs based on the following criteria for inclusion. First, the model must be global. Second, the model must be in current/recent use, which we defined as having yielded a publication within the three years prior to undertaking our review. Third, the model must have been designed to estimate mitigation costs from the energy system (this excluded models primarily intended to estimate damages/the social cost of carbon, and it also excluded specialist land-use models). Fourth, the model must have been used in multiple papers or projects (we excluded 'one-off' models). Lastly, we consolidated versions of the same model into a single 'family'. After screening and combining, we were left with 22 model families for analysis of their representation of TC. These are listed in Table 1.

The table describes the type of model and then classifies the models' representation of TC into three categories: exogenous, endogenous and what we call 'semi-endogenous'. We use semi-endogenous to describe models in which current deployment of an abatement technology makes the technology cheaper in the future, but where this learning mechanism does not affect the optimal carbon price or marginal abatement cost trajectory. In other words, models with semi-endogenous TC include larger future deployment as technologies become cheaper, but omit the incentive for early abatement anticipating the endogenous future gain effect.[3]

The results of our systematic review are as follows. First, the diversity of modelling approaches to TC identified in earlier reviews endures today. Second, we find that TC is exogenous in the majority of models. Four models include endogenous TC, and five other models have semi-endogenous TC. TC is exogenous in the remaining 13 models. A pre-requisite for

[2] In particular, we used the IPCC AR6 Scenario Explorer and Database hosted by IIASA (`https://data.ece.iiasa.ac.at/ar6/#/workspaces`); the web resources of the Integrated Assessment Modeling Consortium or IAMC (`https://www.iamconsortium.org/resources/models-documentation/`), the United Nations Framework Convention on Climate Change or UNFCCC response measures modelling tools (`https://unfccc.int/topics/mitigation/workstreams/response-measures/modelling-tools-to-assess-the-impact-of-the-implementation-of-response-measures`), the Stanford University Energy Modeling Forum (Böhringer et al., 2021), and previous review articles by Gillingham et al. (2008) and Nikas et al. (2019).

[3] To ensure our classification of models as semi-endogenous was reasonable, we contacted the relevant modelling teams to explain our concept of semi-endogenous TC and check our characterisation of their model. We contacted eight modelling teams (IMAGE, GTEM, POLES, E3ME, GEM-E3, EPPA, IMACLIM-R, IGEM) and received answers from seven of them.

fully endogenous TC is the ability to optimise the model intertemporally; only eight models do this, as column 3 shows. Lastly, of the models with semi-endogenous or endogenous TC, the majority represent learning by doing, with R&D investments explicitly represented in only two models. In their sample, Gillingham et al. (2008) identified a larger share of models with R&D, but most of these models were one-off developments: R&D is less prevalent as a TC mechanism in the most commonly used core versions of IAMs that feed into inter-comparison exercises like the IPCC scenario database.

3 A general model of TC

TC encompasses many phenomena, each with their own dynamics, for example, R&D investments in green technology, learning by doing, and spillovers from innovation of general purpose technologies. In this section, we develop an analytical model that is general enough to capture most if not all of these phenomena. It builds on Goulder and Mathai (2000), who develop two special cases of the general model. The general model allows us to show how different types of TC affect the social planner's optimal climate policy. We show that the key difference lies in whether current technology costs depend on past abatement.

To introduce path dependence of TC, a technological state variable affecting abatement costs is required. The most intuitive interpretation of such a state variable is a knowledge stock, but it can be any technological parameter that is path-dependent. The knowledge stock H accumulates according to the following general equation of motion, which depends on time t, the existing knowledge stock, investment I, and abatement $a = E_{BAU} - E$,

$$\dot{H} = \psi(t, H, I, a). \tag{1}$$

The presence of H in the function may represent the phenomenon of 'standing on shoulders', where existing knowledge makes new knowledge easier to develop ($\psi_H > 0$), or 'fishing out', where it becomes harder to find new ideas, the larger is the existing knowledge stock ($\psi_H < 0$). The knowledge stock may increase over time ($\frac{\partial \psi}{\partial t} \geq 0$), e.g., via technological spillovers from non-green sectors, with R&D investments ($\psi_I \geq 0$), or with green technology deployment ($\psi_a \geq 0$), e.g., via learning by doing. This dependence of H on abatement is the most fundamental feature, as it leads to deviations from the standard optimality rules in either cost-benefit or cost-effectiveness settings – see below.

We will now define the other, more standard elements of the model. Building on recent developments to reflect contemporary climate science in economic models (Dietz and Venmans, 2019b; Dietz et al., 2021), temperature T is proportional to cumulative emissions S, with ζ the Transient Climate Response to cumulative carbon Emissions or TCRE,

$$T = \zeta S. \tag{2}$$

Consider a consumption function $c\,(a, H, I, T, t)$, twice differentiable in all its arguments, where positive abatement is costly $-c_a \geq 0$, the marginal abatement cost function is increasing ($-c_{aa} > 0$), and emissions beyond BAU are useless $-c_a|_{a \leq 0} = 0$. Knowledge decreases total and

Model	Type	Intertemporal optimisation	Endogenous or exogenous TC	LbD/R&D	References
AIM/CGE	CGE	No	exogenous		Fujimori et al. (2017a,b)
DICE	Optimal growth	Yes	exogenous		Nordhaus (2007, 2017)
DNE21+	Energy System	Yes	exogenous*		Sano et al. (2006); Wada et al. (2012)
E3ME	Macroeconometric	No	semi-endogenous	LbD/R&D	Mercure et al. (2018) Cambridge Econometrics (2019)
ENV-Linkages	CGE	No	exogenous		Château et al. (2014)
EPPA	CGE	No	exogenous		Chen et al. (2015); Jacoby et al. (2006) Octaviano et al. (2016)
GCAM	Other IAMs	No	exogenous		Bond-Lamberty et al. (2022); Calvin et al. (2017)
GEM-E3	CGE	No	semi-endogenous	LbD	van Regemorter et al. (2013)
GTAP-E	CGE	No	exogenous		Burniaux and Truong (2002); Corong et al. (2017)
GTEM	CGE	No	semi-endogenous	LbD	Jakeman et al. (2004); Pant (2007)
ICES	CGE	No	exogenous*		Eboli et al. (2010); Parrado and De Cian (2014)
IGEM	CGE	Yes	exogenous		Goettle et al. (2007)
IMACLIM-R	CGE	No	semi-endogenous	LbD	Bibas et al. (2022)
IMAGE	Other IAMs	Yes	endogenous	LbD	Stehfest et al. (2014)
MARKAL/ TIMES	Energy System	Yes	endogenous**	LbD	Loulou et al. (2016)
MESSAGEix-GLOBIOM	Other IAMs	Yes	exogenous*		Fricko et al. (2017); Krey et al. (2020) Messner (1997)
PACE	CGE	Yes***	exogenous		Böhringer et al. (2009); Gavard et al. (2022)
Phoenix	CGE	No	exogenous		Wing et al. (2011); Lucena et al. (2018)
POLES	Energy System	No	semi-endogenous	LbD	Keramidas et al. (2017)
REMIND	Optimal growth	Yes	endogenous	LbD	Luderer et al. (2015)
WEM	Energy System	No	exogenous		IEA (2021)
WITCH	Optimal growth	Yes	endogenous	LbD/R&D	Emmerling et al. (2016)

* A study adding endogenous change exists, but this is not incorporated in the main model (e.g., Messner (1997) for MESSAGE; Parrado and De Cian (2014) for ICES; Sano et al. (2006) for DNE21)
** Most applications do not use the endogenous TC feature of the model.
*** There is an extension allowing intertemporal optimisation of the model. (Böhringer and Löschel (2004)).

Table 1: Representation of technical change in 22 IAMs. In models with learning by doing (LbD), the cost of a technology depends on past (cumulative) deployment. In models with R&D, the cost of a technology depends on past investments in R&D. The "Other IAMs" category contains Energy System models coupled with other modules such as land use.

marginal abatement costs ($c_H > 0; -c_{aH} > 0$), and investment in R&D reduces consumption ($c_I = 1$). Climate warming causes convex damages ($c_T < 0, c_{TT} < 0$).

Population at time zero is normalised to one and grows at rate n.[4] The utility function has the standard properties $u_c > 0, u_{cc} < 0$. The social planner maximizes welfare as discounted utility,

$$\max_{\{a,I\}} \int_0^\infty e^{-(\delta-n)t} u\left(c\left(a, H, I, T, t\right)\right) dt, \tag{3}$$

subject to

$$\dot{S} = E_{BAU} - a; \dot{H} = \psi(t, H, I, a); S_0, H_0 \text{ given.} \tag{4}$$

The FOCs include

$$c_a = \frac{\lambda}{u_c} - \frac{\mu}{u_c}\psi_a, \tag{5}$$

where λ is the shadow price of cumulative emissions (the marginal damage cost or 'social cost' of carbon [SCC] in utils) and μ is the shadow price of the knowledge stock.

Eq. (5) shows that whenever the knowledge stock depends on past abatement ($\psi_a \neq 0$), a wedge is created between marginal abatement costs and the SCC. On the optimal path, the MAC equals the SCC, plus the future marginal gains of endogenous TC (see Appendix A for the derivation),

$$\underbrace{c_{a_t}}_{MAC} = \underbrace{\int_t^\infty e^{-r(\tau-t)} \left(-\zeta c_{T_\tau}\right) d\tau}_{SCC} + \underbrace{\overbrace{\psi_{a_t}}^{knowledge\ increment} \overbrace{\int_t^\infty e^{-\int_t^\tau (r-\psi_H)ds} c_{H_\tau} d\tau}^{...and\ its\ effect\ on\ abatement\ costs}}_{Endog.\ Future\ Gain}. \tag{6}$$

The *endogenous future gain effect* takes into account that green technology deployment – abatement – increases the knowledge stock ψ_a, and that this knowledge stock will reduce future abatement costs c_H. This future cost reduction is discounted at rate $r - \psi_H$. In the case of $\psi_H > 0$, standing on shoulders, a larger knowledge stock leads to faster accumulation, which amplifies future effects and reduces the discount rate. By contrast, in the case of $\psi_H < 0$, fishing out of ideas, a current invention makes it harder to find future inventions and the discount rate increases. Appendix A shows that in models with many technologies with different TC dynamics, Eq. (6) applies to each technology. If after reaching peak warming ($E = 0$) the MAC and the SCC are constant, i.e., peak warming is the model's steady state, it is also possible to prove that TC decreases optimal peak warming (see Appendix C). This is intuitively clear from Eq. (6): TC decreases the future MAC, so at the time of peak warming the left-hand side is lower and the optimal SCC must be lower accordingly, implying a lower temperature.

In a cost-effectiveness setting, the SCC is replaced by a Hotelling path, since the problem

[4]In our numerical model, we allow population growth to decrease over time.

becomes one of optimal intertemporal use of a fixed carbon budget (Appendix A),

$$\underbrace{c_{a_t}}_{MAC} = \underbrace{\lambda_0 e^{rt}}_{Hotelling} + \underbrace{\overbrace{\psi_{a_t}}^{knowledge\ increment} \overbrace{\int_t^{\infty} e^{-\int_t^{\tau}(r+\psi_H)ds} c_{H_\tau} d\tau}^{...and\ its\ effect\ on\ abatement\ costs}}_{Endog.\ Future\ Gain} . \tag{7}$$

In a decentralised competitive market economy where deployment spillovers cannot be appropriated, optimal climate policy can be brought about by a carbon tax/price equal to the SCC or Hotelling price, and a deployment subsidy equal to the endogenous future gain effect (see Rezai and van der Ploeg, 2017, for a derivation). If only a carbon price is available, this is optimally set equal to the SCC/Hotelling price plus the endogenous future gain effect. If companies can partially appropriate the spillovers from their deployment activity, this lowers the optimal deployment subsidy or the optimal supplement to the carbon price.[5]

Special case 1: exogenous TC

In the case of exogenous TC, knowledge accumulation depends on time and possibly also on the existing knowledge stock $\dot{H} = \psi(t, H)$. Since the time path of the knowledge stock does not depend on the decision variables, it can be solved independently as $H^* = f(t)$ and the model is isomorphic to a model with a time-dependent abatement cost function $c(a, f(t), T, t)$. The standard cost-benefit and cost-effectiveness decision rules – MAC = SCC and MAC = Hotelling price respectively – are maintained.

Special case 2: endogenous TC via early-stage R&D

Assume the knowledge stock is independent of abatement, but responds to investment I: early-stage R&D (Goulder and Mathai, 2000), $\dot{H} = \psi(t, H, I)$. The technology function ψ is independent of abatement, but the dependence on investment makes TC endogenous. Investment in green technology becomes more attractive in a lower emissions scenario, even if current abatement still does not affect future technology costs. Nonetheless, a model with early-stage R&D has optimal emissions, temperature and MAC dynamics almost identical to a model with purely exogenous TC. To see this, ignore investment costs c_I for a moment. Call $H^*(t)$ and $I^*(t)$ the optimal knowledge stock and investment respectively of the model with early-stage R&D. Compare this to a model with exogenous TC, where the paths of the knowledge stock and investment respectively are replaced by an exogenous function of time $f(t)$, such that $\forall t : c\,(a, H^*(t), I^*(t), T, t) = c\,(a, f(t), T, t)$. The consumption function will be identical over the path, resulting in the same MAC and marginal damage functions. Moreover, since $\psi_a = 0$, the optimal abatement path a^* of the early-stage R&D model also satisfies Eq. (6) for cost-benefit analysis and Eq. (7) for cost-effectiveness analysis with the exogenous abatement function. Hence the exogenous TC model and the early-stage R&D model have the same optimal abatement path. Now bring back the R&D investment costs – these reduce consumption and therefore alter marginal damages and the discount rate. However, the critical observation

[5]In our quantitative modelling below, we add a penalty on the speed of abatement to reflect adjustment costs and capital inertia. Appendix B.2 shows in theory that this adds abatement speed costs to the optimal MAC.

is that this effect on consumption growth will generally be negligible, given the small size of the investment costs when converted into a growth impact (IPCC, 2022b). Therefore, we will treat early R&D as exogenous TC in the remainder of the paper.[6]

Special case 3: price-induced TC

In some models, relative price trajectories affect TC. Consider a model where the carbon price p affects the knowledge stock, $\dot{H} = \psi(t, H, I, p)$. Using an inverse MAC function, which maps the carbon price into abatement $a = \alpha(p, H, t)$, we can rewrite the consumption function as $c\left(\underbrace{\alpha(p, H, t)}_{a}, H, I, T, t\right)$ and the equation of motion for the knowledge stock as $\dot{H} = \psi\left(t, H, I, \underbrace{\alpha(p, H, t)}_{a}\right)$. The FOC equivalent to Eq. (5) becomes $\underbrace{-c_a\alpha_p}_{-c_p} + \mu\underbrace{\psi_a\alpha_p}_{\psi_p} = \lambda\alpha_p$, which is identical after dividing by α_p. That is, price-induced TC is isomorphic to endogenous TC defined as TC that depends on past abatement.

In some models, technology parameters (or knowledge) may be a function of the current price rather than past prices, so technology is not modeled as a stock variable but rather as an instantaneous variable: $H = f(p)$. As a result, the model has no path-dependent TC (no knowledge stock) and it makes more sense the consider it as a model without TC (Gillingham et al., 2008). For models in which the function ψ depends on other prices, it is not obvious how to develop analytical results, but Eq. (5) can give the following insight. If abatement affects the other prices and these other prices affect the knowledge stock dynamics, an extra endogenous future gain term will drive a wedge between the SCC and the MAC. If abatement affects the other prices, but the other prices only affect total abatement costs, not technological progress ψ, the dynamics are those of exogenous TC at most.

4 A more specialised model

The previous section showed how the essence of a variety of TC processes can be distilled into reduced-form models of either exogenous or endogenous TC, for the purposes of optimal carbon pricing, emissions and warming.

We now specialise the model further in order to obtain additional theoretical insights, as well as to ready it for quantification. A key element of our approach to model calibration is fitting the model's abatement cost/TC parameters to data from the IPCC and NGFS databases of IAM model runs. This anchors our results in rates of TC estimated by the IAMs. It also guides how we specialise the model, because the model parameters must be identifiable from the data, meaning the IAM outputs available (or not) from the databases constrain the model structure.

In particular, from IPCC and NGFS we can obtain consistent data on total abatement costs, MACs (carbon prices), and emissions. We do not have data on green investments, so we need to

[6] Note that solving the model for the optimal R&D investment rule is not straightforward, because investment depends on the abatement scenario and *vice versa*. Yet, provided that the modeller has estimated MAC curves in line with the R&D induced by the scenario, modeling the MAC as a function of time (i.e., an exogenous process) will give the same solution.

omit investment from the knowledge accumulation function ψ. Instead, we assume knowledge accumulation is proportional to abatement (as in Rezai and van der Ploeg, 2017),[7]

$$\dot{H} = \varrho a. \tag{8}$$

We normalise the unit of H such that $\varrho = 1$. Call A cumulative abatement. All else equal, an extra unit of cumulative emissions implies a unit less of cumulative abatement, $A_S = -1$. Integrating Eq. (8) allows us to write the knowledge stock as a function of time and cumulative emissions,

$$H(t, S) = A_t = A_0 + \int_0^t E_{BAU} d\tau - S_t + S_0. \tag{9}$$

This reduces the number of state variables to one and the shadow price of carbon now includes both the effect of damages and TC.

We model an endowment economy where exogenous, labour-augmenting TC improves labour productivity, leading to BAU consumption growth of rate g. The MAC function is linear in abatement for fixed technology and consumption. TC shifts the slope downwards, and MACs scale with consumption[8]:

$$MAC \overset{def}{=} -c_a = \varphi_t a \left(A/A_0\right)^{-\chi} c. \tag{10}$$

We also tried fitting a quadratic MAC function to the data but the quadratic term was both economically and statistically insignificant. *Exogenous TC* is captured by the parameter φ_t, which is the slope of the MAC curve and gradually decreases over time according to $\varphi_t = \varphi_\infty + (\varphi_0 - \varphi_\infty) e^{-g_\varphi t}$.[9] *Endogenous TC* is captured by the factor $(A/A_0)^{-\chi}$. For every percent increase in cumulative abatement, the MAC decreases by χ percent.

Finally, we assume that climate damages are quadratic and proportional to consumption. Marginal damages are therefore $\gamma T c$. All the above leads to the following expression for consumption per capita,

$$c = c_0 \exp\left(gt - \frac{\varphi_t}{2} a^2 \left(A/A_0\right)^{-\chi} - \frac{\gamma}{2} T^2\right), \tag{11}$$

where c_0 is a constant, representing initial consumption in the absence of climate damages and abatement costs.

Appendix B.1 derives the optimal solution of the specialised model. From this we can obtain some further theoretical results before quantification. The intuition behind these results is contained in Figures 1 and 2. Formal proofs are contained in Appendix C.

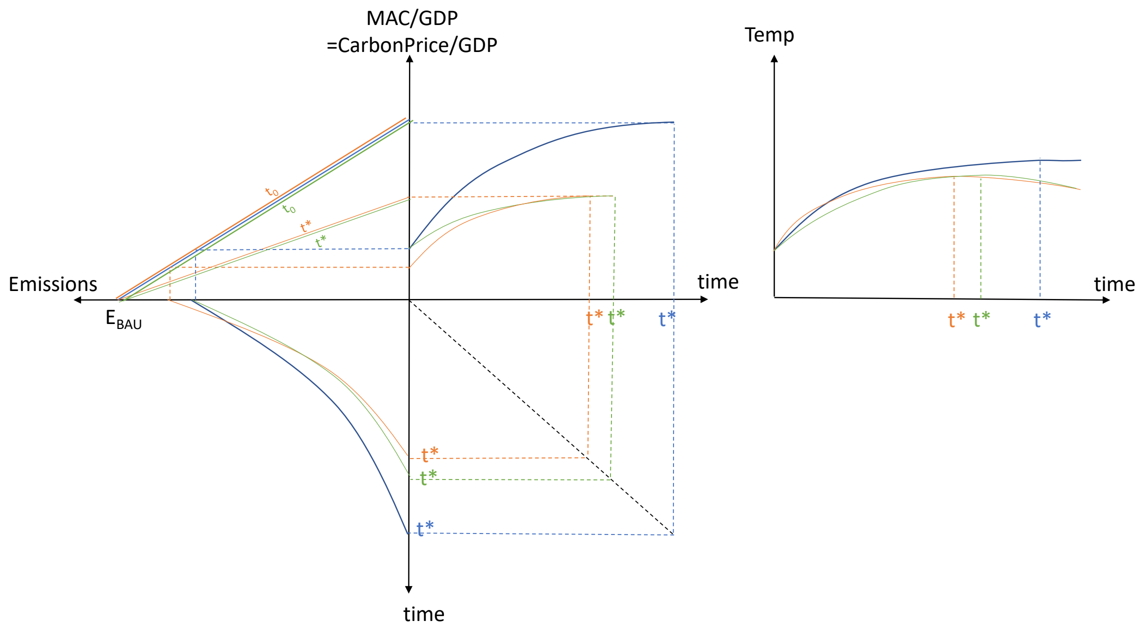

Figure 1: Graphical representation of analytical results for cost-benefit analysis. Blue lines represent the case without TC, orange lines represent exogenous TC and green lines represent endogenous TC. The NW quadrant is the MAC curve, expressed as a proportion of GDP, which is static for the case without TC and decreasing over time in the case of TC. The NE quadrant is the time path of the carbon price (again as a % of GDP). The SW quadrant shows the time path of emissions. Warming, in the separate graph, is proportional to cumulative emissions, i.e., the area under the emissions path. t* is the time when zero emissions is reached and coincides with the time of peak warming.

Cost-benefit analysis

1. Compared to a model without TC, *peak warming is lower with TC (exogenous or endogenous)* except in the unrealistic case where the model starts in the neighbourhood of peak warming (see Appendix C).

2. Compared to a model without TC, *a model with TC has a steeper abatement path initially.* This is visually apparent in the SW quadrant of Figure 1, where a steeper abatement path gives rise to a steeper emissions path. This result is clear from the expression for the optimal growth rate of abatement,

[7]This would be equivalent to a model where ϱ is a function of R&D investments and time $\dot{H} = \varrho(I,t)a$, but where optimal investments are compensated by time trends such that ϱ is constant.

[8]We assume that, all else equal, abatement costs increase with the size of the economy, because the natural resources used for abatement are finite. In a larger economy, land for biofuels, advantageous locations for wind farms, carbon sinks in soils and forests, olivines for mineral weathering and geological space for carbon storage will become scarcer. Also, higher consumption in hard-to-abate sectors such as meat and aviation will increase the need for negative emissions technologies. Note that these increasing scarcity effects can be offset by TC.

[9]As mentioned before, this can include R&D investments which reduce abatement costs over time, but do not require deployment. By contrast, R&D which is facilitated by deployment is included in our endogenous TC.

$$\frac{\dot{a}}{a} = \underbrace{\delta - n + (\eta - 1)\frac{\dot{c}}{c}}_{r-g} - \frac{\dot{\varphi}}{\varphi} + \frac{\chi a}{A} - \frac{1}{2}\frac{\chi a}{A} - \frac{\gamma \zeta^2 S}{\varphi a \left(A/A_0\right)^{-\chi}}. \tag{12}$$

The first three terms correspond to the growth-adjusted discount rate.[10] Next, the positive terms $-\dot{\varphi}/\varphi + \chi a/A$ represent the cost-reduction effects of exogenous and endogenous TC, respectively. For a given carbon price, more abatement is obtained in the future than today. This leads to a steeper abatement trajectory. The next term $-\frac{1}{2}\frac{\chi a}{A}$ corresponds to the endogenous future gain effect and flattens the abatement path. Since this term is dominated by the preceding term, endogenous TC also steepens the abatement path. The last term, $-\frac{\gamma \zeta^2 S}{\varphi a (A/A_0)^{-\chi}}$, is the effect of marginal damages, which creates an incentive to abate earlier (i.e., a flatter abatement path). TC has an ambiguous but small effect on this term, because it decreases both the denominator (lower MAC) and the numerator (lower peak warming) in an approximately proportional way (see Appendix C Corollary 4 and 5).[11]

3. Compared to a model without TC, *a model of exogenous TC will have lower initial abatement, a lower initial carbon price, and, assuming abatement costs have a negligible effect on consumption growth, the same carbon price growth rate.* Conjecture that TC would lead to the same MAC path. Due to TC, the same MAC will lead to more abatement and lower temperatures over the entire path. This violates Eq. (6), since the left-hand side is identical, while the SCC is lower and there are no endogenous future gains. Therefore, our conjecture is wrong: decreasing marginal damages must lead to a lower carbon price over the entire path. As a result of lower long-term temperatures, the initial carbon price is lower.

4. By contrast, *a model of endogenous TC has an ambiguous effect on initial abatement because of the endogenous future gain effect. The effect on the initial carbon price is also ambiguous if the carbon price is the only policy instrument.* The green trajectory in Figure 1 is a possible solution.

Cost-effectiveness analysis

1. In a cost-effectiveness analysis, abatement costs are minimised subject to a temperature constraint, so *peak warming is unaffected by TC.* The temperature constraint requires cumulative emissions to be identical under a linear temperature response (Eq. 2).

2. *TC implies that emissions start higher, decrease at a faster rate and the temperature constraint will be hit earlier, compared to no TC. The carbon price will be lower over the entire trajectory.* Equation (12) shows that the abatement path is steeper under TC.[12] This implies that initial abatement is lower, which is illustrated in the SE quadrant of Figure 2.

[10] The term g stems from the hypothesis that abatement costs and damages are proportional to production.

[11] Both effects are approximately proportional because at the optimum, the MAC equals the integral of marginal damages. Moreover, the endogenous future gains will reduce the magnitude of this ratio.

[12] The last term in Equation (12) will be zero, because damages are replaced by the constraint.

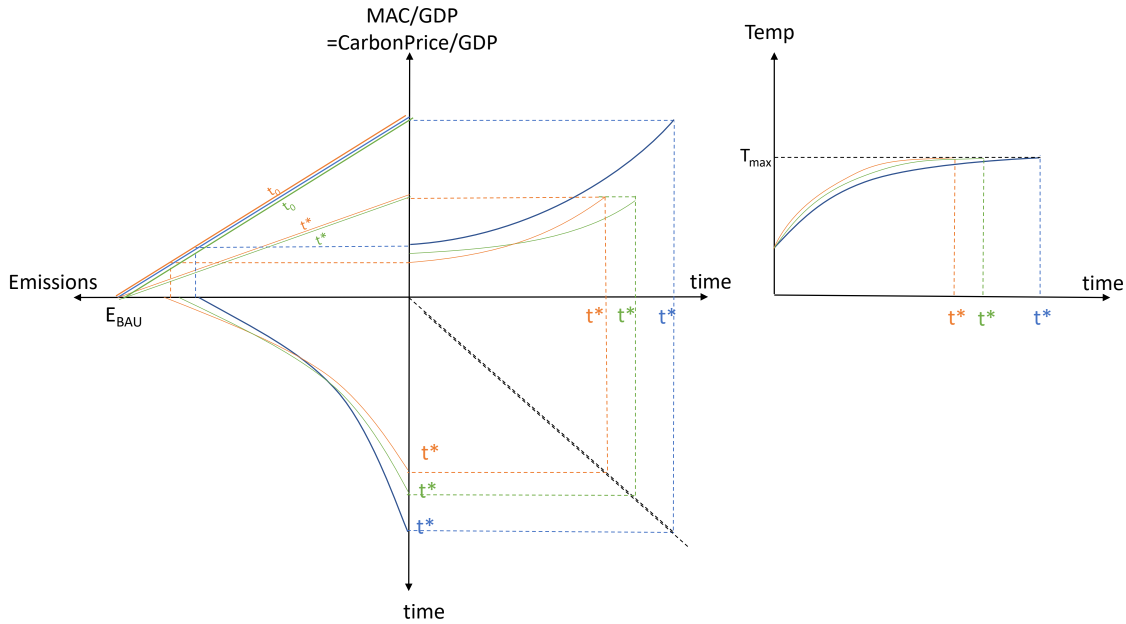

Figure 2: Graphical representation of analytical results for cost-effectiveness analysis. Blue lines represent the case without TC, orange lines represent exogenous TC and green lines represent endogenous TC. The NW quadrant is the MAC curve, expressed as a proportion of GDP, which is static for the case without TC and decreasing over time in the case of TC. The NE quadrant is the time path of the carbon price (again as a % of GDP). The SW quadrant shows the time path of emissions. Warming, in the separate graph, is proportional to cumulative emissions, i.e., the area under the emissions path. t* is the time when zero emissions is reached and coincides with the time of peak warming.

The intuition for this result is that TC leads to lower abatement costs in the long run and this is anticipated at time zero.

3. If a carbon price is the only policy instrument, *endogenous TC will reduce the initial carbon price and initial abatement by less than exogenous TC*. Compared with no TC, exogenous TC does not affect the growth rate of the carbon price, which still follows the Hotelling rule à la Eq. (7). But endogenous TC will flatten the carbon price path due to the endogenous future gain effect.

5 Quantitative modelling

In this section, we calibrate the specialised model and use it to quantify the effects of exogenous and endogenous TC on optimal climate policies. To obtain more realistic estimates, we make two further extensions to the model outlined in the previous section. First, we add a penalty on the speed of abatement. This is a simple way to factor in the effect of adjustment costs and capital inertia, without explicitly modelling them. Rapid abatement may require costly repurposing/stranding of fossil-fuel-based capital, and green capital accumulation may also face bottlenecks. We define abatement speed as $v = \dot{a}$ and assume a quadratic total speed penalty, i.e., a linear marginal speed penalty, $\partial c/\partial v = \theta vc$.[13] Second, we allow for decreasing population growth, $n = n_0 e^{-g_n t}$.

The abatement cost/TC parameters are calibrated on 109 scenarios from eight leading IAM families,[14] obtained by pooling results from the IPCC and NGFS databases. We exploit variation between IAM scenarios in total abatement costs, MACs, and emissions. If the underlying IAMs were static, a given quantity of abatement would cost the same whenever it happens. But with TC, a given quantity of abatement is more costly the earlier it happens. This is the variation we use. A model of exogenous TC can be estimated by assuming that the observed cost reduction in the dataset is driven solely by time. Alternatively, a model of endogenous TC can be estimated by assuming that the same observed cost reduction is a function solely of cumulative abatement. We cannot estimate a mixed exogenous/endogenous model, as there are insufficient data to separately identify the effects of time and cumulative abatement. However, the parameter estimates recovered from the pure exogenous and endogenous TC models could still be used in a mixed TC model, where the MAC function is a weighted average of the two.

Parameter estimation is made using the Generalised Method of Moments, estimating total and marginal abatement cost functions simultaneously. This allows us to obtain more robust results: although the MAC function is more economically meaningful as it determines the FOCs of the optimum, the MAC functions of the underlying IAMs could be non-linear. We give equal weight to the errors of both the total and marginal abatement cost functions and assume they are normally distributed. Table 2 provides the resulting parameter estimates.

We then plug the estimated abatement cost/TC parameters into our model. The model consists of a system of four differential equations in four variables (S, a, v, λ^S) and is solved as

[13] The consumption function now becomes $c = c_0 \exp\left(gt - \frac{\varphi_t}{2} a^2 \left(A/A_0\right)^{-\chi} - \frac{\theta_2}{2} v^2 - \frac{\gamma}{2} T^2\right)$.

[14] Alternatively, 18 different IAMs counting multiple members of the same family, e.g., different model versions, or energy models with and without coupling to land-use models.

Variable	No TC		Exogenous TC		Endogenous TC	
	No inertia	Inertia	No Inertia	Inertia	No Inertia	Inertia
φ_0	3.53e-05	3.46e-05	6.17e-05	5.11e-05	6.15e-05	4.74e-05
	(1.49e-06)	(1.48e-06)	(4.87e-06)	(4.77e-06)	(4.60e-06)	(3.64e-06)
θ		set at .00176		.00175		.00178
				(.000279)		(.00030)
g_φ			.0579	.0481		
			(.0104)	(.0143)		
φ_∞			3.37e-05	3.35e-05		
			(1.68e-06)	(1.73e-06)		
χ					.147	.109
					(.0304)	(.0452)
A_0					37.3	100.7
					(12.37)	(134.7)
N	1850	1850	1850	1848	1850	1848
Log-likelihood	7094.400	7120.121	7636.438	7639.879	7113.744	7121.040
BIC	-14181.278	-14232.719	-15250.307	-15249.67	-14204.919	-14211.992
AIC	-14186.8	-14238.242	-15266.876	-15271.757	-14221.488	-14234.08

Table 2: Parameter estimates for fitting both total and marginal abatement costs to the climate scenarios database of IPCC and NGFS. The model with exogenous TC has the following formula for the slope of the MAC function $\varphi_t = \varphi_\infty + (\varphi_0 - \varphi_\infty)\, e^{-g_\varphi t}$. The endogenous TC model with inertia has the parameter A_0, which is estimated endogenously. Standard errors in parenthesis.

a boundary value problem with MATLAB's bvp5c function.[15] The boundary conditions are presented in Appendix B.2. Table 3 reports the additional, exogenous parameter values. All models are constrained to have the same initial MAC function, which is achieved by setting identical values for θ_2 and φ_0.[16] The model is run out to 2500 in order to avoid terminal values affecting the optimal paths during our period of interest.

[15]For the cost-effectiveness analysis, we minimise the total cost choosing a every two years until 2200 using MATLAB's fmincon function

[16]We take the average from the exogenous and endogenous TC models ($\theta = 0.00176; \varphi_0 = 0.0000492$).

Parameter	Value	Source
δ	0.008	Drupp et al. (2018), 20% trimmed mean of experts
η	1.3	As above
n, $-g_n$	0.0105, 0.013	United Nations (2022)
g	0.02	By assumption
ζ	0.0006	IPCC AR6 WGI (IPCC, 2021)
γ	0.0154	Howard and Sterner (2017)
GDP_{2020}	US$84.537 trn	International Monetary Fund (2021)
E_{BAU}	60 GtCO2eq	IPCC AR6 WGIII (IPCC, 2022b)

Table 3: Exogenous parameter values for the numerical simulations.

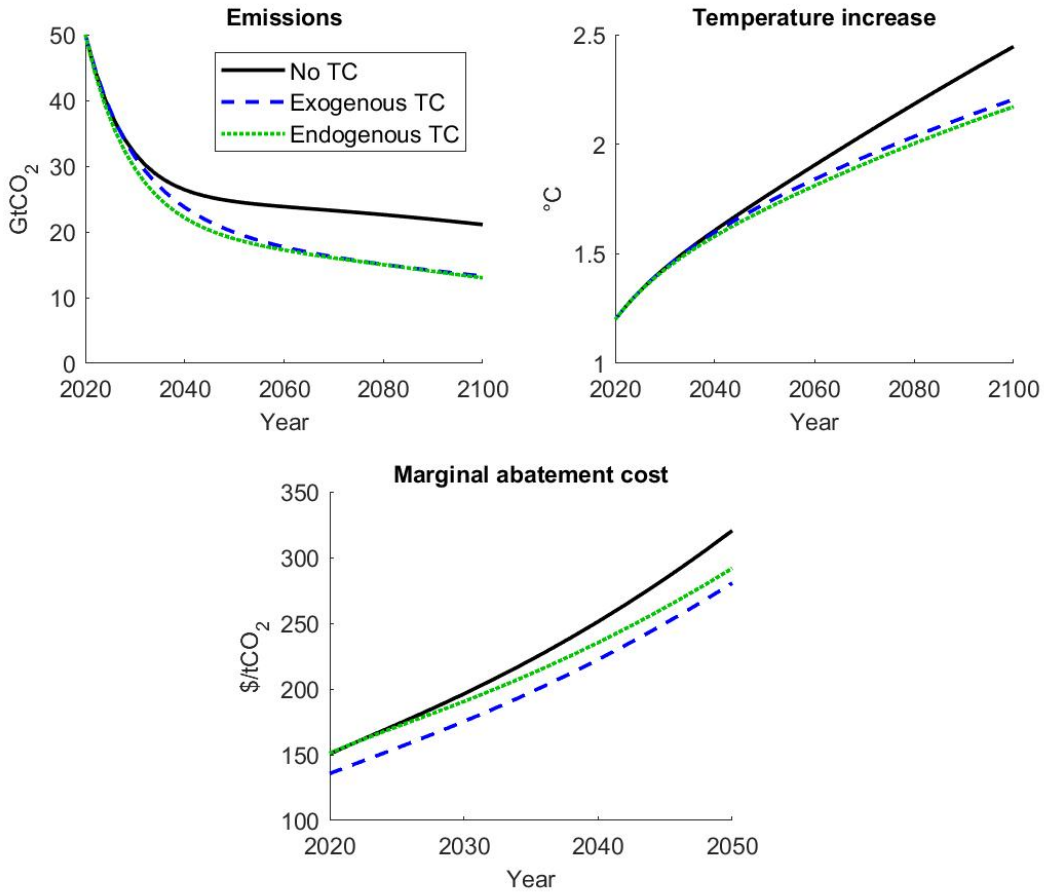

Figure 3: Optimal, cost-benefit climate policies without TC, with exogenous TC and with endogenous TC. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.

Cost-benefit analysis

Figure 3 plots optimal, cost-benefit climate policies for no TC, exogenous TC and endogenous TC. The presence of capital inertia in the form of an abatement speed penalty constrains emissions to be similar in the three models in early periods. This contrasts with the results of the previous section and with numerical simulations of the model without capital inertia (see Appendix E). However, without an abatement speed penalty, initial variations in emissions are unrealistic. Once the constraint imposed by capital inertia starts to wear off, TC of either sort has a quantitatively large effect on optimal emissions and temperatures. By 2050, emissions are 19% lower under exogenous TC and 23% lower under endogenous TC. Optimal emissions are about 40% lower in 2100 under exogenous or endogenous TC, and optimal warming in 2100 is about 2.2°C with TC, compared to over 2.4°C without. In the case of exogenous TC, the initial MAC/carbon price is 10% lower than in the model without TC and this difference widens over time. By contrast, with endogenous TC the MAC/carbon price starts at approximately the same level as without TC. But then it grows more slowly than without TC, such that it is 9% lower than without TC in 2050.

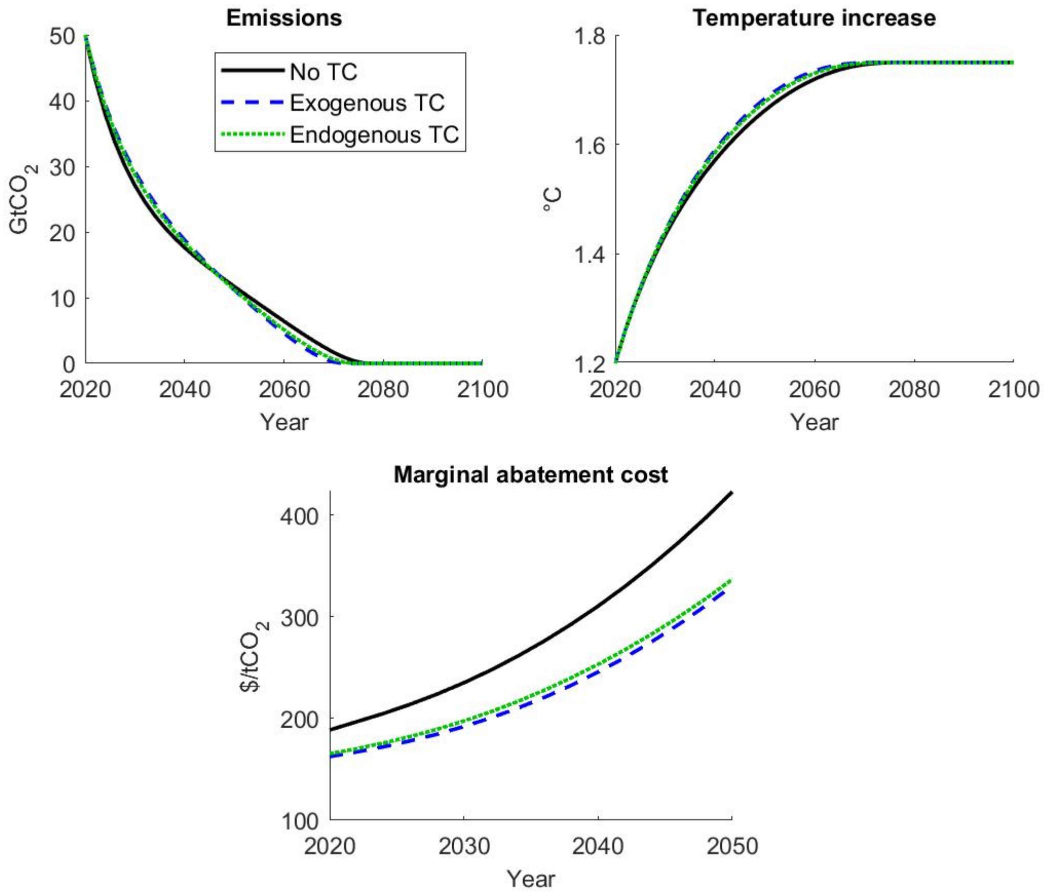

Figure 4: Optimal, cost-effective climate policies without TC, with exogenous TC and with endogenous TC. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.

Cost-effectiveness analysis

Figure 4 plots optimal, cost-effective climate policies, imposing a temperature constraint of 1.75°C as representative of the UN Paris Agreement goal of "well below 2°C". Unlike the cost-benefit case, TC has a small effect on emissions and temperatures not only in the short run but also in the long run. Emissions are constrained to be identical in 2020 but then are slightly higher under TC of either kind, before crossing the no-TC emissions pathway in the 2040s and thereafter being slightly lower. Consequently temperature hits the 1.75°C constraint slightly earlier. TC has a larger effect on the MAC/carbon price, however, and the effect is quantitatively similar whether the TC is exogenous or endogenous. In the case of exogenous TC, the initial MAC/carbon price is 14% lower than in the model without TC. In 2030, it is 18% lower and in 2050 it is 22% lower. In the case of endogenous TC, it is 12% lower initially, a difference that increases to 16% in 2030 and 20% in 2050. It can be inferred from the similarity of the carbon price trajectories that the endogenous future gain effect is relatively less important in the cost-effectiveness case.

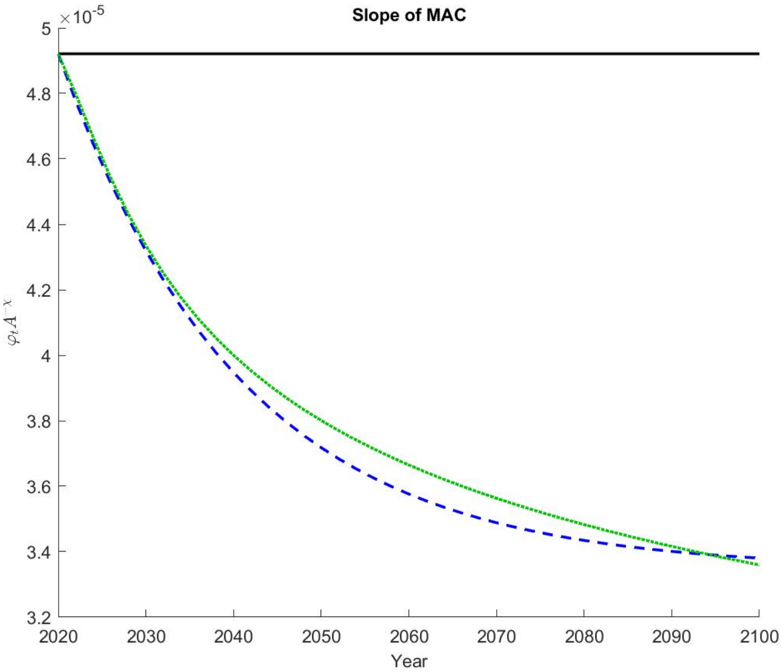

Figure 5: Slope of the MAC curve with exogenous TC (blue dashes) and endogenous TC (green dots), under optimal, cost-benefit climate policies. Constant MAC curve without TC is included for comparison (solid black).

Estimated rate of technical change

Using the optimal solutions to the cost-benefit problem, Figure 5 plots the slopes of the MAC curves. This provides a measure of the estimated rate of TC. Our fit of the IAM results indicates that the slope of the MAC curve is reduced by roughly 1.4% per year initially, 0.5% per year in 2050, converging to about 70% of today's cost in 2100. In the case of endogenous TC, abatement costs are reduced by 7.4% for each doubling of cumulative abatement. This is know as the learning rate, 2^{χ}. Larger historical learning rates have been recorded for some technologies, such as photovoltaics (32%), wind power (19%) and battery technologies (42%) (Way et al., 2022). Lower learning rates have been recorded for other technologies, such as hydroelectricity (0%) and nuclear (0%) (Way et al., 2022). Earlier studies tend to find lower learning rates (Neij, 2008).

Estimating the endogenous future gain effect

The above figures show different effects of exogenous and endogenous TC on optimal climate policies. The main conceptual difference between the two TC processes is the endogenous future gain effect, as explained in Sections 3 and 4. However, the above results do not isolate the endogenous future gain effect. So, Figure 6 decomposes the optimal MAC/carbon price into its two components, the SCC/Hotelling price and the endogenous future gain effect.

Looking first at the cost-benefit problem, the SCC is \$132/tCO_2 in 2020, \$172/tCO_2 in 2030 and \$275/tCO_2 in 2050. The endogenous future gain effect is worth an additional \$19/tCO_2 in 2020, \$21/tCO_2 in 2030 and \$20/tCO_2 in 2050. If climate policy uses both a Pigouvian carbon

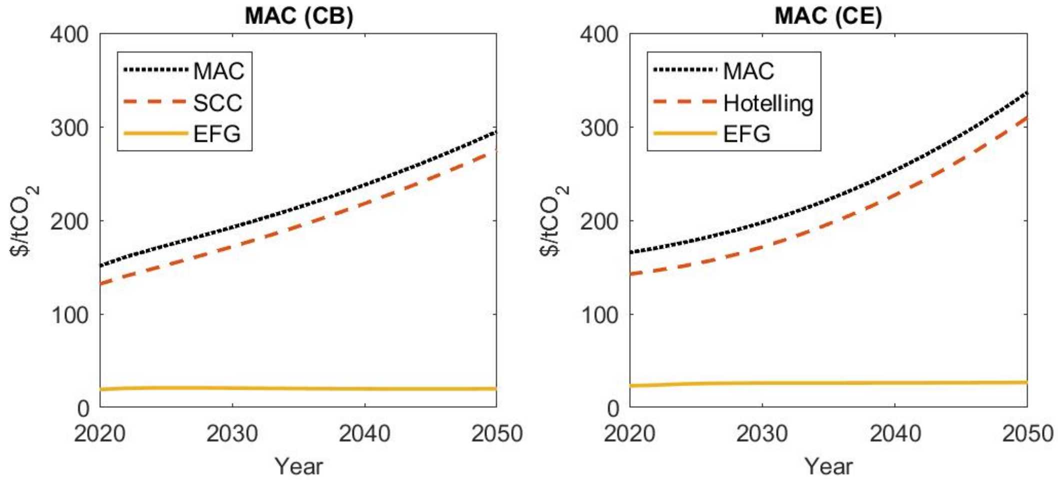

Figure 6: Decomposition of the optimal MAC into the SCC/Hotelling price and the endogenous future gain effect (EFG), for the cost-benefit (left) and cost-effectiveness (right) problems.

price/tax and a technology deployment subsidy, the former is equal to the SCC and the latter to the endogenous future gain effect. If only a carbon price is used, then it should be equal to the SCC plus the endogenous future gain effect, which is \$151/tCO_2 in 2020, \$192/tCO_2 in 2030 and \$295/tCO_2 in 2050. Thus, the carbon price/tax adjusted for endogenous TC (the optimal MAC) is higher and grows at a slightly slower rate. In the cost-effectiveness case, the endogenous future gain effect is larger in absolute terms at \$23/tCO_2 in 2020, \$26/tCO_2 in 2030 and \$27/tCO_2 in 2050. This is consistent with the fact that cumulative abatement is higher in the cost-effectiveness case. However, as indicated above, the relative contribution of the endogenous future gain effect to the optimal MAC/carbon price is smaller in most periods, because the Hotelling price is higher than the SCC in most periods, at \$142/tCO_2 in 2020, \$171/tCO_2 in 2030 and \$310/tCO_2 in 2050.

But how does the dynamic incentive created by endogenous TC impact on optimal emissions, temperatures and MACs/carbon prices? This requires a different kind of analysis, because this dynamic incentive also affects the SCC. That is, take the endogenous future gain effect away and the SCC also changes. Here we develop a method of isolating the dynamic incentive of endogenous TC. The method proceeds in two steps.

Figure 5 clearly shows that the MAC functions, separately estimated for pure exogenous and pure endogenous TC, are only approximately the same due to differences in statistical fit. Therefore, the first step is to estimate a time-dependent MAC function (i.e., exogenous TC), with a MAC that is identical to the endogenous TC model at each point in time. This can be achieved using a sufficiently high-order polynomial. The second step is to take the exogenous TC MAC curve so estimated, and use it to recalculate optimal model trajectories. Any difference between the optimal paths of the endogenous TC model and its exogenous replica must then be down to the dynamic incentive of endogenous TC (see Appendix D).

Figure 7 plots the results. Emissions are 9% lower in 2050 and 4% lower in 2100 due to the endogenous future gain effect, leading to 0.06°C less warming at the end of the century. The MAC/carbon price starts 12% higher and is 4% higher in 2050. These results indicate that, insofar as TC is endogenous, models that do not include the incentive structure of endogenous TC

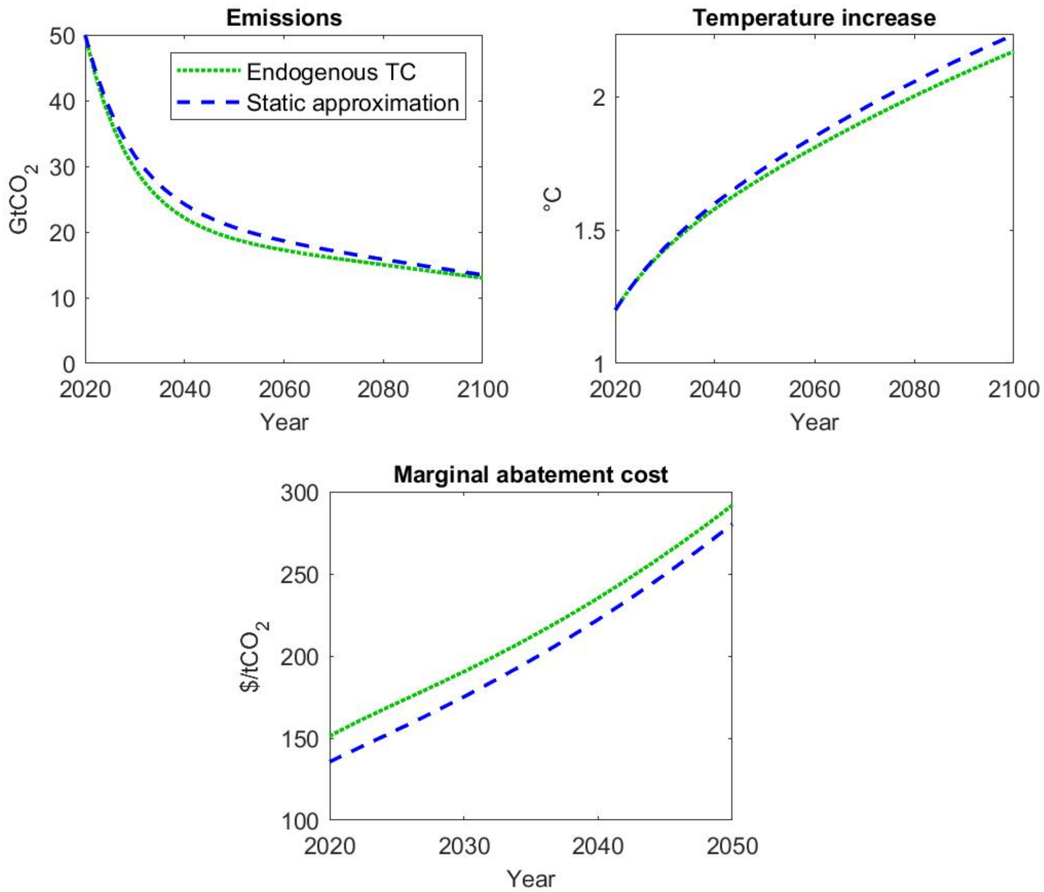

Figure 7: Comparison of a model of endogenous TC with a model without TC, but where the static MAC function is calibrated to give the same marginal cost and quantity of abatement at each point in time as the endogenous TC model. The difference between the paths is exactly the endogenous future gain effect. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.

produce too little abatement, optimal MACs/carbon prices that are too low, and temperatures that are too high. In our review of models, we showed that TC is exogenous in most IAMs, but this observation also applies to models of semi-endogenous TC, as these also omit the dynamic incentive. The estimates here provide an upper bound on the bias though, because they are derived from comparing a model of pure exogenous TC and a model of pure endogenous TC. Reality is likely a mixture.

Appendix D further compares a model of exogenous TC with a static model with identical MACs at each point in time. For example, if, say, the exogenous TC model projects zero emissions in 2050 at a marginal cost of $\$250/tCO_2$, the static model would have the same MAC at zero emissions. The static MAC function will be concave, since TC makes the linear instantaneous MAC function fall over time. We show theoretically that the static approximation of the exogenous TC model is exact under certain assumptions and quantitatively that it is almost exact under more general assumptions. By contrast, a static model cannot imitate the dynamics of a model with endogenous TC.

	Emissions ($GtCO_2$)			Temperatures (°C)			Carbon prices ($\$/tCO_2$)	
	2030	2050	2100	2030	2050	2100	2020	2050
	Main specification							
No TC	31.89	24.62	21.11	1.44	1.76	2.45	150.66	320.46
Exogenous TC	31.38	19.93	13.26	1.44	1.73	2.20	135.68	280.52
Endogenous TC	29.61	18.94	13.00	1.43	1.70	2.17	151.26	291.79
	Low discount rate							
No TC	14.92	5.22	7.14	1.37	1.46	1.66	252.19	478.37
Exogenous TC	17.14	2.19	0.87	1.38	1.47	1.49	214.82	393.69
Endogenous TC	14.31	0.40	0.80	1.37	1.43	1.44	234.13	397.37
	High discount rate							
No TC	43.30	39.06	33.83	1.48	1.97	3.06	78.08	192.55
Exogenous TC	42.11	34.82	26.34	1.47	1.93	2.83	74.39	179.13
Endogenous TC	41.54	35.01	26.79	1.47	1.92	2.85	82.41	188.50
	Slow technical change							
No TC	31.89	24.62	21.11	1.44	1.76	2.45	150.66	320.46
Exogenous TC	31.81	22.62	17.06	1.44	1.75	2.33	143.19	300.36
Endogenous TC	30.75	21.80	16.95	1.43	1.73	2.31	151.56	307.21
	Fast technical change							
No TC	31.89	24.62	21.11	1.44	1.76	2.45	150.66	320.46
Exogenous TC	30.59	15.43	7.37	1.43	1.69	1.99	124.02	249.95
Endogenous TC	27.84	14.43	7.34	1.42	1.66	1.96	148.55	265.41

Table 4: Sensitivity analysis on the discount rate and the rate of technical change. The low discount rate corresponds to $\delta = 0$ and the high discount rate to $\delta = 0.02$. Further sensitivity analysis on η can be found in Appendix X. Fast (slow) technical change corresponds to plus (minus) one standard deviation for χ and +/-50% for A_0 in the model of endogenous TC. Comparable variation in exogenous TC is then created by varying g_{φ} so as to fit the endogenous TC MAC in 2100.

Sensitivity analysis

Here we analyse the sensitivity of the optimal, cost-benefit policy to variation in the abatement cost/TC parameters, and the discount rate. For the former variation we use the standard errors from the GMM estimates in Table 2. For the latter we use the range of responses from the expert survey on discounting by Drupp et al. (2018). The results are summarised in Table 4, with detailed results contained in Appendix E. There are four scenarios: low discount rate; high discount rate; slow TC; fast TC.

Reducing the discount rate leads to higher carbon prices, lower emissions and temperatures, as expected. Increasing the discount rate leads to the opposite effects. The effects of TC are qualitatively the same under different discount rates[17] but they are quantitatively larger under a low discount rate, both in absolute and relative terms. If TC is at the low end of the range, carbon prices are higher, but so are emissions and temperatures. The opposite is true for TC at the high end of the range. The comparative effects of exogenous and endogenous TC are qualitatively the same whether TC is slow or fast. However, assuming TC is fast, including TC has a large effect on optimal pathways. For example, optimal warming in 2100 is now around 0.5°C lower under either form of TC.

[17]very small differences between pathways can be due to model error and should not be over-interpreted.

6 Discussion

In this paper, our aim was to assess, both qualitatively and quantitatively, how the representation of TC in the current crop of IAMs affects their results. Although the literature has explored IAMs' representation of TC before, most of it was published more than 15 years ago. Therefore, we think it is important to bring the literature up-to-date and our approach of constructing a reduced-form model of TC, nested within an IAM and calibrated on the breadth of current IAM results, introduces some novel methodological elements to the existing suite of tools. Among these existing tools are standard meta-analysis, i.e., in this context, regression of IAM outputs on model features (e.g. Kuik et al., 2009), sensitivity analysis using a single model (e.g. Manne and Richels, 2004), and harmonised runs of multiple IAMs (e.g. Gillingham et al., 2018). Each has its pros and cons. Standard meta-analysis provides a convenient way to explore a wide range of model features but typically yields qualitative results on the effects of how TC is represented (since model features are represented as dummy variables), and given the nature of the data faces identification problems such as low statistical power and multi-collinearity. Sensitivity analysis with individual IAMs permits a tightly controlled experiment into the effect of TC, all else equal, but is limited to an individual model structure. By contrast, harmonised runs of multiple IAMs allow model uncertainty to be explored but it is practically difficult to evaluate many models this way, let alone multiple parameterisations of each model. Our approach strictly speaking uses a single IAM, but the IAM is set up to perform more of a meta-analytical role. In that regard, it has some similarities with the meta-model idea of van Vuuren et al. (2020), but has considerably more structure, as well as yielding analytical insights.

Summary of results

We first conducted a systematic survey of how TC is represented in the main extant IAMs, using international databases/web resources and previous reviews to populate our list. We found that IAMs continue to represent TC in diverse ways, although TC is exogenous in the majority of models, and even where there is a mechanism linking current abatement costs with past cumulative deployment/abatement, this is not always fully endogenised by figuring in the planner's incentives. R&D is less prevalent as a TC mechanism in the IAMs that commonly feed into inter-comparison exercises like IPCC and NGFS, compared with learning by doing.

We then set about building a simple, structural model of TC, embedded in a wider IAM capable of describing the causal chain from economic and population growth to rising temperatures and climate damages. The resulting IAM is of the 'analytical' type. Rather than positing micro-foundations, we took a reduced-form approach capable of embedding the most important TC mechanisms in the literature and distilling them into their most salient differences for climate policy.

We showed analytically that in terms of optimal emissions, temperatures and MACs/carbon prices, the critical difference is whether TC is exogenous in the sense of being time-dependent, or endogenous in the sense that current abatement costs depend on past abatement. Although other TC models exist, notably TC driven by early-stage R&D and price-induced TC, we showed that optimal trajectories under early-stage R&D should be almost identical to those under exogenous

TC (this rests on the defensible assumption that R&D investment costs negligibly reduce overall economic growth), and that TC dependent on past carbon prices is isomorphic to endogenous TC defined as TC that depends on past abatement.

We further showed that TC reduces optimal steady-state/peak warming, when the policy problem is discounted net benefit maximisation (cost-benefit analysis). Under exogenous TC, the prospect of lower future abatement costs – the cost-reduction effect – reduces initial abatement and it reduces carbon prices in all periods. Under endogenous TC, however, the effects are ambiguous because the cost-reduction effect is offset by the endogenous future gain effect. When the policy problem is instead to minimise the costs of meeting a temperature constraint (cost-effectiveness analysis), peak warming is naturally unaffected by TC. However, carbon prices are affected – they are unambiguously lower under TC, but exogenous TC reduces them by more than endogenous TC.

Lastly, we used the model for quantification of these effects. A key element of this was to calibrate the TC parameters on the relationship between abatement costs and the timing of abatement in more than 100 IAM scenarios from the IPCC and NGFS databases. In this way, we estimate the effects of statistically representative TC rates in the IAM literature on optimal policies. Solving the model numerically also enabled us to introduce some more realism, notably capital inertia and falling population growth rates. This analysis confirmed that TC lowers optimal warming in the long run and further showed that the effect is quantitatively relatively large, especially under high-end TC rates, where the difference is around 0.5°C in 2100. Exogenous TC depresses the initial optimal carbon price by 10%. Under endogenous TC, the endogenous future gain effect almost exactly offsets the cost-reduction effect initially, although over time the latter effect comes to dominate, and the carbon price grows more slowly than without TC as a consequence. The implications of this for optimal policy depend also on whether the planner can use a technology deployment subsidy alongside a carbon price/tax. Under cost-effectiveness analysis with a temperature constraint of well below 2°C, as expected the analysis showed little impact on optimal emissions/temperatures, but optimal carbon prices are 12-14% lower initially and 16-20% lower in 2050. Thus, the endogenous future gain effect is relatively less important in the cost-effectiveness case. We further explored how the dynamic incentive created by endogenous TC impacts on optimal trajectories, by constraining MACs to be identical under both types of TC. We provided an upper-bound estimate of how much abatement and carbon prices are underestimated, when an IAM does not reflect endogenous TC in the planner's incentives. The dynamic incentive reduces emissions by 9% in 2050 and 4% in 2100, while the carbon price is 12% higher in 2020 and 4% higher in 2050.

Uncertainty about TC

Sensitivity analysis reveals that our results are qualitatively robust to variation in the rate of TC (and the discount rate). The sensitivity analysis on the rate of TC, carried out using the standard errors of our statistical estimates, also provides an insight into the likely impacts of TC on optimal trajectories, in case the IAM literature as a whole underestimates future technology cost reductions. This is an implication of recent work by Way et al. (2022), who argue IAMs underestimate deployment rates for renewable energy technologies and overestimate their costs.

Supposing this critique has some validity, we might then be more guided by our results for high-end TC as calibrated on the IAM databases.[18] Mechanically, fast TC, so defined and measured, results in a larger quantitative effect on optimal emissions, temperatures and carbon prices.

Uncertainty about TC is an important feature of the problem. Although there exist many papers with sensitivity analysis on technological parameters, like ours, to the best of our knowledge the literature has not yet investigated optimal emissions under uncertainty in a dynamic model where information on TC is gradually discovered. A fully dynamic stochastic model goes beyond the scope of this paper, but from our analytical solutions we can speculate about the effect of uncertainty.

Since total abatement costs are convex in abatement, Jensen's inequality indicates that the expected value of future abatement costs increases under uncertainty. This increases future abatement costs relative to current, certain abatement costs, and is an argument for earlier abatement with a higher initial MAC. The higher MAC can be understood as a risk premium society is willing to pay to insure against the possible outcome of slower-than-expected TC.

But how much this matters depends as usual on whether TC is exogenous or endogenous, and whether the objective is cost-benefit or cost-effectiveness. When TC is exogenous and the objective is cost-effectiveness, the above effect is the only one at play and in principle the risk premium on TC uncertainty could be quantitatively large. In the case of endogenous TC, we would expect the risk premium to be lower, because we know from our analysis that the endogenous future gain effect attenuates the effect of TC uncertainty on the initial MAC. In other words, since endogenous TC has a smaller impact on the optimal MAC (compared to exogenous TC), the societal cost of wrongly anticipating TC is lower. Similarly, in a cost-benefit analysis, lower-than-expected TC leads to a higher optimal peak temperature, attenuating the initial price/MAC adjustment. Again, since the effect of TC on the initial MAC is lower than under cost-effectiveness, the social cost of wrongly anticipating TC and the corresponding risk premium are lower. Combining both effects, i.e. in a cost-benefit analysis with endogenous TC, the effect of TC on the initial MAC should be negligible.

Note that these are mere first-order effects of adding uncertainty on parameters χ and g_φ. There are other effects that go beyond this intuition, for example when the uncertainty regarding the growth rate of the economy is correlated with uncertainty regarding TC. Also, uncertainty affects investment incentives. In the case of long-term, irreversible investments with large uncertainty over the profitability of the technology, uncertainty leads to an incentive to postpone the investment and wait for new information, a.k.a. option value, the value of keeping options open. For example, uncertainty about the availability of nuclear fusion in the future can create an incentive to postpone the investment in a new nuclear fission plant today. Investigating optimal emissions under technological uncertainty is therefore a fruitful avenue for future research.

[18] In our fast TC scenario, abatement costs are reduced by one third between 2020 and 2040. This corresponds to the cost reduction for wind technologies in the fast transition scenario of Way et al. (2022). Their cost reductions for batteries and electrolysers are even larger because these technologies start with a very low cumulative installed capacity. By contrast, they argue that other technologies such as carbon capture and storage and nuclear have seen almost no cost reductions over the last decades.

Implications for IAMs

Lastly, our results have several practical implications for integrated assessment modellers.

1. If due to computational constraints it is impossible to add TC to the model, exogenous TC can be approximated by adjusting the shape of the MAC, such that it represents the MAC at the time a given level of abatement is reached. For example, if a model reaches zero emissions in 2060 and the marginal abatement technology is assumed to be direct air capture (DAC), the MAC at zero should be the cost of DAC in 2060, not the current cost of DAC. In our data, the best fit for such a model is a linear MAC, not the widely used convex MAC implied by nested CES functions, say.

2. If due to computational constraints it is impossible to fully endogenise TC, an adjustment can be applied to the marginal abatement cost path. In our model, this correction factor is $-\chi a/2A$, leading to a model with a larger initial marginal abatement cost and a flatter trajectory.

3. Models with detailed information about endogenous TC for different technologies with different learning rates should allow for a higher MAC for the technologies with a larger potential for cost reductions. Equation (6) can be applied to each technology separately. Technologies should start to be deployed when their MAC is lower than the SCC plus the endogenous future gain effect.

References

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., 2012. The environment and directed technical change. American economic review 102, 131–66.

Bibas, R., Cassen, C., Crassous, R., Guivarch, C., Hamdi-Cherif, M., Hourcade, J.C., Leblanc, F., Méjean, A., Broin, E.Ó., Rozenberg, J., et al., 2022. Impact assessment of climate policies with imaclim-r 1.1. model documentation version 1.1. .

Van den Bijgaart, I., Gerlagh, R., Liski, M., 2016. A simple formula for the social cost of carbon. Journal of Environmental Economics and Management 77, 75–94.

Böhringer, C., Löschel, A., 2004. Policy analysis based on computable equilibrium. TranSust Model Comparisons .

Böhringer, C., Löschel, A., Moslener, U., Rutherford, T.F., 2009. Eu climate policy up to 2020: An economic impact assessment. Energy economics 31, S295–S305.

Böhringer, C., Peterson, S., Rutherford, T.F., Schneider, J., Winkler, M., 2021. Climate policies after paris: Pledge, trade and recycle: Insights from the 36th energy modeling forum study (emf36). Energy Economics 103, 105471. URL: https://www.sciencedirect.com/science/article/pii/S0140988321003571, doi:https://doi.org/10.1016/j.eneco.2021.105471.

Bond-Lamberty, B., Patel, P., Lurz, J., pkyle, kvcalvin, Smith, S., abigailsnyder, Dorheim, K.R., russellhz, mbins, Link, R., skim301, nealtg, Narayan, K., Turner, S.W.D., S., A., Feng, L., enlochner, cwroney, Lynch, C., jhoring, Khan, Z., siddarthd96, orourkepr, JonathanHuster, Haewon, Ou, Y., Iyer, G., mwisepnnl, marideeweber, 2022. Jgcri/gcam-core: Gcam 6.0. URL: https://doi.org/10.5281/zenodo.6619287, doi:10.5281/zenodo.6619287.

Bramoullé, Y., Olson, L.J., 2005. Allocation of pollution abatement under learning by doing. Journal of Public Economics 89, 1935–1960.

Burniaux, J.M., Truong, T.P., 2002. Gtap-e: an energy-environmental version of the gtap model. GTAP Technical Papers , 18.

Calvin, K., Bond-Lamberty, B., Clarke, L., Edmonds, J., Eom, J., Hartin, C., Kim, S., Kyle, P., Link, R., Moss, R., McJeon, H., Patel, P., Smith, S., Waldhoff, S., Wise, M., 2017. The ssp4: A world of deepening inequality. Global Environmental Change 42, 284–296. URL: https://www.sciencedirect.com/science/article/pii/S095937801630084X, doi:https://doi.org/10.1016/j.gloenvcha.2016.06.010.

Cambridge Econometrics, 2019. E3me technical manual v6. 1. Version March .

Château, J., Dellink, R., Lanzi, E., 2014. An overview of the oecd env-linkages model .

Chen, Y.H., Paltsev, S., Reilly, J.M., Morris, J.F., Babiker, M.H., 2015. The MIT EPPA6 model: Economic growth, energy use, and food consumption. Technical Report. MIT Joint Program on the Science and Policy of Global Change.

Clarke, L., Jiang, K., Akimoto, K., et al., 2014. Asessing transformation pathways, in: Edenhofer, O., Pichs-Madruga, R., Sokona, Y., et al. (Eds.), Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, NY, USA.

Corong, E.L., Hertel, T.W., McDougall, R., Tsigas, M.E., Van Der Mensbrugghe, D., 2017. The standard gtap model, version 7. Journal of Global Economic Analysis 2, 1–119.

Dietz, S., van der Ploeg, F., Rezai, A., Venmans, F., 2021. Are economists getting climate dynamics right and does it matter? Journal of the Association of Environmental and Resource Economists 8, 895–921.

Dietz, S., Venmans, F., 2019a. Cumulative carbon emissions and economic policy: In search of general principles. Journal of Environmental Economics and Management 96, 108–129. URL: https://www.sciencedirect.com/science/article/pii/S0095069618302122, doi:https://doi.org/10.1016/j.jeem.2019.04.003.

Dietz, S., Venmans, F., 2019b. Cumulative carbon emissions and economic policy: in search of general principles. Journal of Environmental Economics and Management 96, 108–129.

Drupp, M.A., Freeman, M.C., Groom, B., Nesje, F., 2018. Discounting disentangled. American Economic Journal: Economic Policy 10, 109–34. URL: https://www.aeaweb.org/articles?id=10.1257/pol.20160240, doi:10.1257/pol.20160240.

Eboli, F., Parrado, R., Roson, R., 2010. Climate-change feedback on economic growth: explorations with a dynamic general equilibrium model. Environment and Development Economics 15, 515–533.

Elia, A., Kamidelivand, M., Rogan, F., Gallachóir, B.Ó., 2021. Impacts of innovation on renewable energy technology cost reductions. Renewable and Sustainable Energy Reviews 138, 110488.

Emmerling, J., Drouet, L., Reis, L.A., Bevione, M., Berger, L., Bosetti, V., Carrara, S., De Cian, E., D'Aertrycke, G.D.M., Longden, T., et al., 2016. The WITCH 2016 model-documentation and implementation of the shared socioeconomic pathways. Technical Report. JSTOR.

Fricko, O., Havlik, P., Rogelj, J., Klimont, Z., Gusti, M., Johnson, N., Kolp, P., Strubegger, M., Valin, H., Amann, M., Ermolieva, T., Forsell, N., Herrero, M., Heyes, C., Kindermann, G., Krey, V., McCollum, D.L., Obersteiner, M., Pachauri, S., Rao, S., Schmid, E., Schoepp, W., Riahi, K., 2017. The marker quantification of the shared socioeconomic pathway 2: A middle-of-the-road scenario for the 21st century. Global Environmental Change 42, 251–267. URL: https://www.sciencedirect.com/science/article/pii/S0959378016300784, doi:https://doi.org/10.1016/j.gloenvcha.2016.06.004.

Fujimori, S., Hasegawa, T., Masui, T., 2017a. Aim/cge v2. 0: basic feature of the model, in: Post-2020 climate action. Springer, pp. 305–328.

Fujimori, S., Hasegawa, T., Masui, T., Takahashi, K., Herran, D.S., Dai, H., Hijioka, Y., Kainuma, M., 2017b. Ssp3: Aim implementation of shared socioeconomic pathways. Global Environmental Change 42, 268–283. URL: https://www.sciencedirect.com/science/article/pii/S0959378016300838, doi:https://doi.org/10.1016/j.gloenvcha.2016.06.009.

Gavard, C., Voigt, S., Genty, A., 2022. Using emissions trading schemes to reduce heterogeneous distortionary taxes: The case of recycling carbon auction revenues to support renewable energy. Energy Policy 168, 113133.

Gerlagh, R., Kverndokk, S., Rosendahl, K.E., 2009. Optimal timing of climate change policy: Interaction between carbon taxes and innovation externalities. Environmental and resource Economics 43, 369–390.

Gerlagh, R., Kverndokk, S., Rosendahl, K.E., 2014. The optimal time path of clean energy r&d policy when patents have finite lifetime. Journal of Environmental Economics and Management 67, 2–19.

Gillingham, K., Newell, R.G., Pizer, W.A., 2008. Modeling endogenous technological change for climate policy analysis. Energy Economics 30, 2734–2753. URL: https://www.sciencedirect.com/science/article/pii/S0140988308000443, doi:https://doi.org/10.1016/j.eneco.2008.03.001. technological Change and the Environment.

Gillingham, K., Nordhaus, W., Anthoff, D., Blanford, G., Bosetti, V., Christensen, P., McJeon, H., Reilly, J., 2018. Modeling uncertainty in integrated assessment of climate change: A multimodel comparison. Journal of the Association of Environmental and Resource Economists 5, 791–826.

Goettle, R.J., Ho, M.S., Jorgenson, D.W., Slesnick, D.T., Wilcoxen, P.J., EP-W, E.C., 2007. Igem, an inter-temporal general equilibrium model of the us economy with emphasis on growth, energy and the environment. Model documentation in support of ongoing policy analysis for the US Environmental Protection Agency, EPA Contract No. EP-W-05-035 .

Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. Econometrica 82, 41–88.

Goulder, L.H., Mathai, K., 2000. Optimal co2 abatement in the presence of induced technological change. Journal of Environmental Economics and Management 39, 1–38. URL: https://www.sciencedirect.com/science/article/pii/S0095069699910896, doi:https://doi.org/10.1006/jeem.1999.1089.

Greaker, M., Pade, L.L., 2009. Optimal carbon dioxide abatement and technological change: should emission taxes start high in order to spur r&d? Climatic Change 96, 335–355.

Grubb, M., Köhler, J., Anderson, D., 2002. Induced technical change in energy and environmental modeling: Analytic approaches and policy implications. Annual Review of Energy and the Environment 27, 271–308.

Howard, P.H., Sterner, T., 2017. Few and not so far between: a meta-analysis of climate damage estimates. Environmental and Resource Economics 68, 197–225.

IEA, 2021. World energy model documentation.

International Monetary Fund, 2021. World economic outlook database .

IPCC, 2018. Global Warming of 1.5 C: An IPCC Special Report on the Impacts of Global Warming of 1.5C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty. Cambridge University Press, Cambridge, UK and New York, NY, USA.

IPCC, 2021. Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, NY, USA.

IPCC, 2022a. Climate Change 2022: Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, NY, USA. URL: https://www.ipcc.ch/report/ar6/wg3/downloads/report/IPCC_AR6_WGIII_FullReport.pdf, doi:10.1017/9781009157926.

IPCC, 2022b. Climate change 2022: Mitigation of climate change. contribution of working group iii to the sixth assessment report of the intergovernmental panel on climate change. Cambridge University Press doi:https://doi.org/10.1017/9781009157926.

Jacoby, H.D., Reilly, J.M., McFarland, J.R., Paltsev, S., 2006. Technology and technical change in the mit eppa model. Energy Economics 28, 610–631. URL: https://www.sciencedirect.com/science/article/pii/S0140988306000624, doi:https://doi.org/10.1016/j.eneco.2006.05.014. modeling Technological Change in Climate Policy Analyses.

Jakeman, G., Hanslow, K., Hinchy, M., Fisher, B.S., Woffenden, K., 2004. Induced innovations and climate change policy. Energy Economics 26, 937–960. URL: https://www.sciencedirect.com/science/article/pii/S0140988304000799, doi:https://doi.org/10.1016/j.eneco.2004.09.002.

Keramidas, K., Kitous, A., Després, J., Schmitz, A., Vazquez, A.D., Mima, S., Russ, P., Wiesenthal, T., et al., 2017. Poles-jrc model documentation. Publications Office of the European Union 10, 225347.

Krey, V., Havlik, P., Kishimoto, P., Fricko, O., Zilliacus, J., Gidden, M., Strubegger, M., Kartasasmita, G., Ermolieva, T., Forsell, N., et al., 2020. Messageix-globiom documentation-2020 release .

Kuik, O., Brander, L., Tol, R.S., 2009. Marginal abatement costs of greenhouse gas emissions: A meta-analysis. Energy policy 37, 1395–1403.

Löschel, A., 2002. Technological change in economic models of environmental policy: a survey. Ecological economics 43, 105–126.

Loulou, R., Goldstein, G., Kanudia, A., Lehtila, A., Remne, U., 2016. Documentation for the times model-part 1 2016. URL https://iea-etsap. org/docs/Documentation for the TIMES Model-Part-I July-2016. pdf .

Lucena, A.F., Hejazi, M., Vasquez-Arroyo, E., Turner, S., Köberle, A.C., Daenzer, K., Rochedo, P.R., Kober, T., Cai, Y., Beach, R.H., et al., 2018. Interactions between climate change mitigation and adaptation: The case of hydropower in brazil. Energy 164, 1161–1177.

Luderer, G., Leimbach, M., Bauer, N., Kriegler, E., Baumstark, L., Bertram, C., Giannousakis, A., Hilaire, J., Klein, D., Levesque, A., et al., 2015. Description of the remind model (version 1.6) .

Manne, A., Richels, R., 2004. The impact of learning-by-doing on the timing and costs of co2 abatement. Energy Economics 26, 603–619. URL: https://www.sciencedirect.com/science/article/pii/S0140988304000441, doi:https://doi.org/10.1016/j.eneco.2004.04.033. eMF 19 Alternative technology strategies for climate change policy.

Mercure, J.F., Pollitt, H., Edwards, N.R., Holden, P.B., Chewpreecha, U., Salas, P., Lam, A., Knobloch, F., Vinuales, J.E., 2018. Environmental impact assessment for climate change policy with the simulation-based integrated assessment model e3me-ftt-genie. Energy Strategy Reviews 20, 195–208. URL: https://www.sciencedirect.com/science/article/pii/S2211467X18300129, doi:https://doi.org/10.1016/j.esr.2018.03.003.

Messner, S., 1997. Endogenized technological learning in an energy systems model. Journal of Evolutionary Economics 7, 291–313.

Neij, L., 2008. Cost development of future technologies for power generationâa study based on experience curves and complementary bottom-up assessments. Energy Policy 36, 2200–2211. URL: http://linkinghub.elsevier.com/retrieve/pii/S0301421508001237, doi:10.1016/j.enpol.2008.02.029.

Nikas, A., Doukas, H., Papandreou, A., 2019. A detailed overview and consistent classification of climate-economy models. Understanding risks and uncertainties in energy and climate policy , 1–54.

Nordhaus, W., 2007. Accompanying notes and documentation on development of dice-2007 model: Notes on dice-2007. delta. v8 as of september 21, 2007. Miscellaneous publication, Yale University, New Haven, NE .

Nordhaus, W.D., 2017. Revisiting the social cost of carbon. Proceedings of the National Academy of Sciences 114, 1518–1523.

Octaviano, C., Paltsev, S., Gurgel, A.C., 2016. Climate change policy in brazil and mexico: Results from the mit eppa model. Energy Economics 56, 600–614. URL: https://www.

sciencedirect.com/science/article/pii/S0140988315001292, doi:https://doi.org/10.1016/j.eneco.2015.04.007.

Pant, H., 2007. Global trade and environment model (gtem). Australian Bureau of Agricultural and Resource Economics and Sciences Canberra, Australia .

Parrado, R., De Cian, E., 2014. Technology spillovers embodied in international trade: Intertemporal, regional and sectoral effects in a global cge framework. Energy Economics 41, 76–89. URL: https://www.sciencedirect.com/science/article/pii/S0140988313002417, doi:https://doi.org/10.1016/j.eneco.2013.10.016.

Popp, D., 2004. Entice: endogenous technological change in the dice model of global warming. Journal of Environmental Economics and Management 48, 742–768. URL: https://www.sciencedirect.com/science/article/pii/S0095069603001220, doi:https://doi.org/10.1016/j.jeem.2003.09.002.

Popp, D., Newell, R.G., Jaffe, A.B., 2010. Energy, the environment, and technological change. Handbook of the Economics of Innovation 2, 873–937.

van Regemorter, D., Capros, P., Paroussos, L., Karkatsoulis, P., 2013. GEM-E3 model documentation. Publications Office. doi:doi/10.2788/47872.

Rezai, A., Van der Ploeg, F., 2016. Intergenerational inequality aversion, growth, and the role of damages: Occamâs rule for the global carbon tax. Journal of the Association of Environmental and Resource Economists 3, 493–522.

Rezai, A., van der Ploeg, F., 2017. Abandoning fossil fuel: How fast and how much. The Manchester School 85, e16–e44. doi:10.1111/manc.12189.

Riahi, K., Schaeffer, R., Arango, J., Calvin, K., Guivarch, C., Hasegawa, T., Jiang, K., Kriegler, E., Matthews, R., Peters, G., Rao, A., Robertson, S., Sebbit, A., Steinberger, J., Tavoni, M., Van Vuuren, D., 2022. Mitigation pathways compatible with long-term goals, in: Shukla, P., Skea, J., Slade, R., Khourdajie, A.A., van Diemen, R., McCollum, D., Pathak, M., Some, S., Vyas, P., Fradera, R., Belkacemi, M., Hasija, A., Lisboa, G., Luz, S., Malley, J. (Eds.), Climate Change 2022: Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, NY, USA. book section 3. URL: https://www.ipcc.ch/report/ar6/wg3/downloads/report/IPCC_AR6_WGIII_Chapter03.pdf, doi:10.1017/9781009157926.005.

Sano, F., Akimoto, K., Homma, T., Tomoda, T., 2006. Analysis of technological portfolios for co2 stabilizations and effects of technological changes. The Energy Journal .

Shukla, P., Skea, J., Slade, R., Khourdajie, A.A., van Diemen, R., McCollum, D., Pathak, M., Some, S., Vyas, P., Fradera, R., Belkacemi, M., Hasija, A., Lisboa, G., Luz, S., Malley, J., (eds.), 2022. Climate change 2022: Mitigation of climate change. contribution of working group iii to the sixth assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge, UK and New York, NY, USA URL: https://www.sciencedirect.com/science/article/pii/S0959378016300681, doi:10.1017/9781009157926.

Stehfest, E., van Vuuren, D., Bouwman, L., Kram, T., et al., 2014. Integrated assessment of global environmental change with IMAGE 3.0: Model description and policy applications. Netherlands Environmental Assessment Agency (PBL).

Stern, N.H., 2007. The economics of climate change: the Stern review. cambridge University press.

Sue Wing, I., 2006. Representing induced technological change in models for climate policy analysis. Energy Economics 28, 539–562.

Traeger, C., forthcoming. Ace–analytic climate economy. AEJ: Economic Policy .

United Nations, 2022. World Population Prospects 2022, Online Edition. United Nations, Department of Economic and Social Affairs, Population Division.

van Vuuren, D.P., van der Wijst, K.I., Marsman, S., van den Berg, M., Hof, A.F., Jones, C.D., 2020. The costs of achieving climate targets and the sources of uncertainty. Nature Climate Change 10, 329–334.

Wada, K., Sano, F., Akimoto, K., Homma, T., 2012. Assessment of copenhagen pledges with long-term implications. Energy Economics 34, S481–S486. URL: https://www.sciencedirect.com/science/article/pii/S0140988312000023, doi:https://doi.org/10.1016/j.eneco.2012.01.001. the Asia Modeling Exercise: Exploring the Role of Asia in Mitigating Climate Change.

Way, R., Ives, M.C., Mealy, P., Farmer, J.D., 2022. Empirically grounded technology forecasts and the energy transition. Joule 6, 2057–2082. URL: https://doi.org/10.1016/j.joule.2022.08.009, doi:10.1016/j.joule.2022.08.009.

Weyant, J., 2017. Some contributions of integrated assessment models of global climate change. Review of Environmental Economics and Policy .

Wing, I.S., Daenzer, K., Fisher-Vanden, K., Calvin, K., 2011. Phoenix model documentation. Maryland: Joint Global Change Research Institute, Pacific Northwest National Laboratory .

Van der Zwaan, B.C., Gerlagh, R., Schrattenholzer, L., et al., 2002. Endogenous technological change in climate change modelling. Energy economics 24, 1–19.

Appendices for Online Publication

Appendix A Solving the general model of TC

The knowledge stock evolves according to $\dot{H} = \psi(t, H, I, a)$. Assume the function ψ is twice differentiable in all its arguments. Abatement leads to accumulation of knowledge and has diminishing marginal returns ($\psi_a \geq 0; \psi_{aa} < 0$). The knowledge stock reduces total and marginal abatement costs ($\forall a > 0 : c_H > 0$ & $c_{aH} > 0$). One dollar of investment reduces consumption by one dollar $c_I = -1$. The marginal effectiveness of investments is subject to diminishing returns to scale $\psi_I > 0$; $\psi_{II} < 0$. We assume $\psi_H < r$ to avoid a bang-bang solution. The other assumptions are as stated in Section 3.

The social planner chooses optimal abatement and investment in knowledge to maximize utility U, which is a function of consumption,

$$\max_{a,I} \int_0^\infty e^{-(\delta-n)t} u\left(c\left(a, H, I, T(S), t\right)\right) dt, \tag{13}$$

s.t. $\dot{S} = E_{BAU} - a; \dot{H} = \psi(t, H, I, a)$.

The current value Hamiltonian of the problem is

$$\mathcal{H} = u\left(c\left(a, H, I, T(S), t\right)\right) - \lambda\left(E_{BAU} - a\right) + \mu\psi\left(t, H, I, a\right). \tag{14}$$

The FOCs include

$$u_c c_a + \mu\psi_a = \lambda, \tag{15}$$

$$\dot{\lambda} = (\delta - n)\lambda - u_c \zeta c_T, \tag{16}$$

$$u_c = \mu\psi_I, \tag{17}$$

$$\dot{\mu} = (\delta - n)\mu - u_c c_H - \mu\psi_H. \tag{18}$$

The transversality conditions are $\lim_{t\to\infty}^{-(\delta-n)t} = 0$ and $\lim_{t\to\infty} \mu e^{-(\delta-n)t} = 0$.

Integrating Equations 16 and 18, and plugging the integrals into Equation 15, we obtain

$$c_{a_t} = \int_t^\infty e^{-(\delta-n)(\tau-t)} \frac{u_{c_\tau}}{u_{c_t}} \zeta\left(-c_{T_\tau}\right) d\tau + \psi_{a_t} \int_t^\infty e^{-\int_t^\tau ((\delta-n)+\psi_H) ds} \frac{u_{c_\tau}}{u_{c_t}} c_{H_\tau} d\tau. \tag{19}$$

Acknowledging that $\ln \frac{u_{c_\tau}}{u_{c_t}} = \int_t^\tau \frac{\dot{u_c}}{u_c} ds$ and defining the discount rate $r = \delta - n - \frac{\dot{u_c}}{u_c}$, we can show that the MAC equals the present value of future marginal damages (the SCC) and the future gains of TC as a result of an extra tonne of abatement today, i.e., Equation (6) in the main text:

$$\underbrace{c_{a_t}}_{MAC} = \underbrace{\int_t^\infty e^{-\int_t^\tau r ds} \zeta\left(-c_{T_\tau}\right) d\tau}_{SCC} + \underbrace{\overbrace{\psi_{a_t}}^{knowledge\ increment} \overbrace{\int_t^\infty e^{-\int_t^\tau (r+\psi_H) ds} c_{H_\tau} d\tau}^{...and\ its\ effect\ on\ abatement\ costs}}_{Endog.\ Future\ Gain}.$$

Differentiating Equation 15 and combining it with the other FOC gives

$$\dot{c_a} = rc_a + \frac{\psi_H \psi_a - \dot{\psi_a}}{\psi_I} + c_H \psi_a - \zeta c_T, \tag{20}$$

which after dividing through by c_a gives the growth rate of the MAC,

$$\frac{\dot{c_a}}{c_a} = r + \underbrace{\frac{c_H \psi_a + \frac{\psi_H}{\psi_I}\psi_a - \frac{\dot{\psi_a}}{\psi_I}}{c_a}}_{\textit{Endog. Future Gain}} - \underbrace{\frac{\zeta c_T}{c_a}}_{\textit{Damages}} \tag{21}$$

Model with many technological processes

We will show the derivation for two families of abatement technology, each with a different process of TC. The extension to more technologies is straightforward. Assume the same conditions as above apply to each technology, $c_a < 0; c_{aa} < 0; c_a|_{a<0} = 0$, meaning that within each abatement family there are diminishing returns to scale (see Bramoullé and Olson, 2005, for a model with constant MACs and a technology accumulation function $\dot{H} = a$). The assumption of differentiability and zero MACs for negative emissions implies zero MAC at zero abatement. This will ensure an interior solution for each technology family.[19] The planner's objective is

$$\max_{a,I} \int_0^\infty e^{-(\delta - n_t)t} u\left(c\left(a_1, a_2, H_1, H_2, I_1, I_2, T(S), t\right)\right) dt, \tag{22}$$

s.t. $\dot{S} = E_{BAU} - a_1 - a_2; \dot{H}_i = \psi\left(t, H_i, I_i, a_i\right)$.

The current value Hamiltonian of the problem is

$$\mathcal{H} = u\left(c\left(a_1, a_2, H_1, H_2, I_1, I_2, T(S), t\right)\right) - \lambda\left(E_{BAU} - a_i\right) + \sum_{i=1,2} \mu_i \psi^i\left(t, H_i, I_i, a_i\right). \tag{23}$$

The seven FOCs for $i = 1, 2$ include

$$u_c c_{a_i} + \mu_i \psi^i_{a_i} = \lambda, \tag{24}$$

$$u_c = \mu_i \psi^i_{I_i}, \tag{25}$$

$$\dot{\lambda} = \left(\delta - n_t\right) \lambda - u_c \zeta u_T, \tag{26}$$

$$\dot{\mu_i} = \left(\delta - n_t\right) \mu_i - u_c c_{H_i} - \mu \psi_{H_i}. \tag{27}$$

Integrating the costate Equations (27) for each technology family and substituting in Equation (24) allows us to write Equation (6) for each technology family separately.

[19] In case abatement costs are non-continuous at zero, a non-negativity condition $a_i \geq 0$ should be added and will lead to the obvious conclusion that families of technologies that are more expensive than alternative families of technologies, even at very low levels of deployment, should not be deployed yet.

Cost-effectiveness analysis

In the case of cost-effectiveness analysis, damages are replaced by a maximum temperature $T \leq \bar{T}$, which corresponds to a constraint on cumulative emissions $\bar{T} = \zeta \bar{S}$. The Hamiltonian in Equation (14) is replaced by the following Lagrangian,

$$\mathcal{L} = u\left(c\left(a, H, I, t\right)\right) - \lambda\left(E_{BAU} - a\right) + \mu\psi\left(t, H, I, a\right) - \theta\left(E_{BAU} - a\right), \tag{28}$$

where the Lagrange multiplier θ indicates that whenever $S = \bar{S}$, emissions cannot be positive. The FOCs are

$$u_c c_a + \mu\psi_a = \lambda + \theta, \tag{29}$$

$$\dot{\lambda} = \left(\delta - n\right)\lambda, \tag{30}$$

$$\begin{aligned} &\text{If} \quad S = \bar{S}: \quad \theta > 0;\ \dot{\theta} \leq 0;\ E \leq 0. \\ &\text{If} \quad S < \bar{S}: \quad \theta = 0. \end{aligned} \tag{31}$$

Equations (25) and (27) remain the same. Call time $\bar{t}$ the time when the constraint hits. Before time $\bar{t}$, the Lagrange multiplier is zero and the integral expression for the MAC is Equation (7), i.e.,

$$\underbrace{c_{a_t}}_{MAC} = \underbrace{\lambda_0 e^{rt}}_{Hotelling} + \underbrace{\overbrace{\psi_{a_t}}^{knowledge\ increment} \overbrace{\int_t^{\infty} e^{-\int_t^{\tau}(r+\psi_H)ds} c_{H_\tau} d\tau}^{...and\ its\ effect\ on\ abatement\ costs}}_{Endog.\ Future\ Gain}.$$

At time $\bar{t}$, the continuity of the costate variable implies continuous MACs and therefore continuous emissions at zero. Using boundary conditions at time $\bar{t}$, $a_{\bar{t}} = E_{BAU}$, $S_{\bar{t}} = \bar{S}$, $\int_0^{\bar{t}} E dt = \bar{S} - S_0$, and assuming that endogenous future gains are negligible after time $\bar{t}$, we establish that λ_0 is the present value of the MAC at zero emissions at time $\bar{t}$, $\lambda_0 = e^{-r\bar{t}}\ c_{a_{\bar{t}}}|_{a=E_{BAU}}$ (this requires the knowledge stock at time $\bar{t}$, so there will be no closed form solution in the general case).

For early R&D or exogenous TC, we have $\psi_a = 0$ and the Hotelling rule is preserved. For general cases of endogenous TC, the Hotelling rule is no longer valid.

Appendix B Solving the specialised model

B.1 Without inertia

Assume the knowledge stock builds up proportional to abatement $\dot{H} = \varrho\left(I, t\right) a$. Since we have no data on R&D investments, we assume the investment decisions affecting the function $\varrho(I, t)$ are such that optimal investments make the function $\varrho\left(I^*, t\right)$ constant over time (the full model would have a second optimality condition $u_c = \mu\rho_I a$). We normalise the unit of H such that $\varrho = 1$.

Integrating the differential equation for $\dot{H}$ allows us to write the variable H as a function of time and the state variable S as follows:

$$H(t,S) = A = \int_0^t E_{BAU} dt - S + S_0. \tag{32}$$

Now the model can be solved with only one state variable and its shadow price λ now incorporates both damages and endogenous TC.

We assume exogenous TC reduces abatement costs over time, such that the slope of the MAC curve φ is a decreasing function of time. The slope of the MAC curve is also affect by a separate, early-stage R&D knowledge stock, which is accumulated by R&D investments (but not by abatement), $\dot{\tilde{H}} = \tilde{\psi}\left(\tilde{I}, t\right)$. This stock is optimized according to a third FOC, $u_c = \tilde{\mu}\tilde{\psi}_{\tilde{I}}$, but since we have no data on R&D investments, we do not solve the model for optimal R&D, and calibrate the slope of the MAC curve as a function of time, encompassing both exogenous and early R&D. The consumption function is now $c\left(a, A, S, \tilde{H}^*(t), t\right)$.

We assume quadratic total abatement costs and quadratic damages, which lead to the following expression for consumption per capita:

$$c = c_0 e^{gt - \frac{\varphi_t}{2} a^2 (A/A_0)^{-\chi} - \frac{\gamma}{2}\zeta^2 S^2}. \tag{33}$$

We use a CES utility function $u = \frac{c^{1-\eta}}{1-\eta}$, with η the elasticity of marginal utility. We assume decreasing population growth $n = n_0 e^{-g_n t}$ and standardise initial population to one. The welfare functional, discounted at the utility discount rate δ, is

$$\max \int_0^\infty e^{-(\delta - n_t)t} u \; dt. \tag{34}$$

The present Value Hamiltonian is

$$H^{PV} = e^{-(\delta - n_t)t} u\left(c\left(t, S, a\right)\right) - \lambda^S \left(E_{BAU} - a\right). \tag{35}$$

The FOCs are

$$\lambda^S = -e^{-(\delta - n_t)t} u_c c_a, \tag{36}$$

$$\dot{\lambda^S} = e^{-(\delta - n_t)t} u_c c_S, \tag{37}$$

where $c_S = c\left(-\gamma\zeta^2 S - \frac{\chi\varphi_t}{2A_0} a^2 \left(A/A_0\right)^{-\chi-1}\right)$ includes both marginal damages and endogenous gains from TC.

Integrate Equation (37) between time t and infinity with terminal condition $\lim_{t\to\infty}\lambda^S = 0$:

$$\lambda_t^S = \int_t^\infty e^{-(\delta - n_\tau)\tau} c_\tau^{1-\eta} \left(\gamma\zeta^2 S + \frac{\chi\varphi_t}{2A_0} a^2 \left(A/A_0\right)^{-\chi-1}\right) d\tau. \tag{38}$$

Combining this result with Equation (36) and dividing by $-e^{-(\delta - n_t)t} u_{c_t} c_t$ gives

$$c_t \varphi_t a \left(A/A_0\right)^{-\chi} = \int_t^\infty e^{-\delta(\tau - t) + (n_\tau \tau - n_t t)} \left(\frac{c_\tau}{c_t}\right)^{-\eta} c_\tau \left(-\gamma\zeta^2 S - \frac{\chi\varphi_t}{2A_0} a^2 \left(A/A_0\right)^{-\chi-1}\right) d\tau. \tag{39}$$

Acknowledging $\ln\frac{c_\tau}{c_t} = \int_t^\tau d\ln c_s \Leftrightarrow \frac{c_\tau}{c_t} = e^{\int_t^\tau \frac{\dot{c}}{c} ds} \Leftrightarrow \left(\frac{c_\tau}{c_t}\right)^{-\eta} = e^{-\eta\int_t^\tau \frac{\dot{c}}{c} ds}$ shows that the first factor in the integral is the discount factor with the Ramsey discount rate $\delta - n_t + \eta\frac{\dot{c}}{c}$. Note that since both marginal abatement costs and damages are proportional to consumption, we can write this equation relative to consumption while reducing the discount rate by the growth rate of consumption:

$$\varphi_t a\left(A/A_0\right)^{-\chi} = \int_t^\infty e^{-\delta(\tau-t)+(n_\tau\tau-n_t t)-(\eta-1)\int_t^\tau \frac{\dot{c}}{c}ds}\left(\gamma\zeta^2 S_\tau + \frac{\chi\varphi_\tau}{2A_\tau}a_\tau^2\left(A_\tau/A_0\right)^{-\chi}\right)d\tau. \quad (40)$$

To find a differential equation for the carbon price (MAC), first take the time derivative of Equation (36):

$$\dot{\lambda^S} = -e^{-(\delta-n_t)t}(-\delta+n_t+\dot{n}_t t)u_c c_a - e^{-(\delta-n_t)t}\dot{u_c}c_a - e^{-(\delta-n_t)t}u_c\dot{c_a}. \quad (41)$$

Substitute out $\dot{\lambda^S}$ from Equations (37) and (41), divide by $e^{-(\delta-n_t)t}u_c$, and use $-\frac{\dot{u_c}}{u_c} = \eta\frac{\dot{c}}{c}$ to give

$$-\dot{c_a} = \left(\delta - n_t - \dot{n}_t t + \eta\frac{\dot{c}}{c}\right)(-c_a) + c_S, \quad (42)$$

which corresponds to

$$\underbrace{\frac{\dot{c_a}}{c_a}}_{MAC\ growth} = r - \underbrace{\frac{\zeta c_T}{c_a}}_{Marginal\ Damages} + \underbrace{\frac{c_A}{c_a}}_{Endogenous\ Future\ Gains}. \quad (43)$$

To find a differential equation for abatement, we plug in the expression for the MAC function $-c_a = c\varphi_t a\left(A/A_0\right)^{-\chi}$ in Equation (42):

$$c\varphi_t a\left(A/A_0\right)^{-\chi}\left(\tfrac{\dot{c}}{c} + \tfrac{\dot{\varphi}}{\varphi} + \tfrac{\dot{a}}{a} - \chi\tfrac{a}{A}\right)$$
$$= \left(\delta - n_t - \dot{n}_t t + \eta\tfrac{\dot{c}}{c}\right)c\varphi_t a\left(A/A_0\right)^{-\chi} + c\left(-\gamma\zeta^2 S - \tfrac{\chi\varphi_t}{2A}a^2\left(A/A_0\right)^{-\chi}\right)$$

Dividing by the MAC results in Equation (12), (extended to decreasing population growth), i.e.,

$$\frac{\dot{a}}{a} = \underbrace{\delta - n_t - \dot{n}_t t + (\eta-1)\frac{\dot{c}}{c}}_{r-g} - \frac{\dot{\varphi}}{\varphi} + \frac{\chi a}{A} - \frac{1}{2}\frac{\chi a}{A} - \frac{\gamma\zeta^2 S}{\varphi a\left(A/A_0\right)^{-\chi}}.$$

A more detailed model may have several groups of abatement technologies, each with a MAC function $\varphi_{i,t}a_i\left(\frac{A_i}{A_{0i}}\right)^{\chi_i}$. Cumulative emissions are now $S_t = S_0 + E_{BAU}t - \sum A_i$. For N groups of technologies, the model now has N decision variables and N stock variables (cumulative abatement for each group of technologies). Assuming a constant discount rate, the integral form of the Euler equations for each technology is

$$\underbrace{\varphi_{i,t}a_i\left(A_i/A_{0,i-}\right)^{-\chi_i}}_{MAC_i^{\%}} = \int_t^\infty \underbrace{e^{-(r-g)t}}_{Discount\ factor}\left(\underbrace{\gamma\zeta^2 S_\tau}_{Marg\ damages\%} + \underbrace{\frac{\chi_i\varphi_{i,\tau}}{2A_{i,\tau}}a_\tau^2\left(A_{i,\tau}/A_{i,0}\right)^{-\chi_i}}_{Endogenous\ future\ gains_i^{\%}}\right)d\tau. \quad (44)$$

B.2 With inertia

Call $v = \dot{a}$ the abatement speed. Assume a quadratic penalty on abatement speed (as a proportion of GDP) of $\frac{\theta}{2}v^2$. Consumption per capita now becomes

$$c = c_0 e^{\left(gt - \frac{\varphi_t}{2}a^2(A/A_0)^{-\chi} - \frac{\theta}{2}v^2 - \frac{\gamma}{2}\zeta^2 S^2\right)}. \tag{45}$$

The present value Hamiltonian is

$$H^{PV} = e^{(-\delta+n_t)t}u(c) - \lambda^S(E_{BAU} - a) + \lambda^a v, \tag{46}$$

with FOCs

$$\lambda^a = e^{(-\delta+n_t)t}u_c c\theta v, \tag{47}$$
$$\dot{\lambda^a} = e^{(-\delta+n_t)t}u_c c\varphi_t a\,(A/A_0)^{-\chi} - \lambda^S, \tag{48}$$
$$\dot{\lambda^S} = e^{(-\delta+n_t)t}u_c c_S. \tag{49}$$

Differentiate the FOC of the maximisation:

$$\dot{\lambda^a} = e^{(-\delta+n_t)t}u_c c\theta v\left[-\delta + n_t + \dot{n}t - \eta\frac{\dot{c}}{c} + \frac{\dot{c}}{c} + \frac{\dot{v}}{v}\right], \tag{50}$$

substitute this result in Equation (48) and divide by $e^{(-\delta+n_t)t}u_c c\theta$:

$$v\left[-\delta + n_t + \dot{n}_t t - (\eta - 1)\frac{\dot{c}}{c}\right] + \dot{v} = \frac{\varphi_t a\,(A/A_0)^{-\chi}}{\theta} - \frac{\lambda^S}{e^{(-\delta+n_t)t}u_c c\theta}, \tag{51}$$

with

$$n_t + \dot{n}t = n_0 e^{-g_n t}(1 - g_n t), \tag{52}$$

and

$$\frac{\dot{c}}{c} = g + \frac{(\varphi_0 - \varphi_\infty)\,g_\varphi}{2}a^2(A/A_0)^{-\chi} - \varphi_t a v\,(A/A_0)^{-\chi} + \frac{\varphi_t}{2}a^2\chi\,(A/A_0)^{-\chi}\,\frac{a}{A} - \theta v\dot{v} - \gamma\zeta^2 S\,(E_{BAU} - a). \tag{53}$$

The growth rate of consumption is very close to g, but the component $-\theta v\dot{v}$ cannot be neglected. Reorganise to obtain a differential equation in $\dot{v}$,

$$\dot{v} = \frac{1}{1 + (\eta - 1)\theta v^2}\left[\left(\delta - n_0 e^{-g_n t}(1 - g_n t) + (\eta - 1)\overbrace{\left(\frac{\dot{c}}{c} + \theta v\dot{v}\right)}^{Doesn't\ contain\ \dot{v}}\right)v + \frac{\varphi_t a\,(A/A_0)^{-\chi}}{\theta} - \frac{\lambda^S}{e^{(-\delta+n_t)t}u_c c\theta}\right]. \tag{54}$$

We now have a system of four differential equations in four variables S, a, v, λ^S (Equations 4, 12, 54 and 49). The boundary conditions are

$$\begin{aligned} S(0) &= S_0, \\ a(0) &= a_0 = E_{BAU} - E_0, \\ a(\infty) &= E_{BAU}, \\ v(\infty) &= 0. \end{aligned} \tag{55}$$

We define the carbon price as the current value shadow price of carbon, expressed in consumption units, i.e.,

$$p = \frac{\lambda^S e^{\delta t}}{u_c}. \tag{56}$$

From Equation (51) we can write the price in the form of a MAC augmented by extra inertia costs: [20]

$$p = e^{n_t t} \left\{ \underbrace{c\varphi_t a \left(A/A_0\right)^{-\chi}}_{\partial c/\partial a \ standard \ MAC} \underbrace{+ c\theta v \left[\delta - n_t - \dot{n}_t t + (\eta - 1)\frac{\dot{c}}{c}\right] - c\theta \dot{v}}_{Abatement \ speed \ costs \ (pos)} \right\}. \tag{57}$$

Alternatively, the integration of Equation (49) gives the carbon price as the sum of both the SCC and the endogenous future gains:

$$p = \frac{\lambda^S e^{\delta t}}{u_c} = \int_t^\infty e^{n_\tau \tau} \underbrace{e^{-\delta(\tau - t) - \eta \int_t^\tau \frac{\dot{c}}{c} ds}}_{Discount \ factor} c_\tau \left(\gamma \zeta^2 S + \frac{\chi \varphi_\tau}{2A} a^2 \left(A/A_0\right)^{-\chi} \right) d\tau. \tag{58}$$

The discount factor is the standard Ramsey discount factor as can be seen from $\left(\frac{c_\tau}{c_t}\right)^{-\eta} = e^{-\eta \int_t^\tau \frac{\dot{c}}{c} ds}$.

Appendix C Proofs of analytical results

Proposition 1 *Assume that from the point at which peak warming is reached the marginal abatement cost and damage functions are static. In the general model of Appendix A, TC decreases peak warming.*

Proof: If the marginal abatement cost and damage functions are static from the point at which peak warming is reached, peak warming is also the steady state – if we conjecture that $\dot{E} = 0$, the MAC, temperature and hence marginal damages will be constant, Equation (6) is satisfied and has the solution

$$MAC^* = \frac{-\zeta c_T^*}{r}. \tag{59}$$

[20]For analytical simplicity in the theory part, we focus on the marginal effect of abatement on consumption per capita, i.e. c_a. However, since a is expressed as worldwide abatement, the marginal effect of abatement on total consumption is more relevant, because it corresponds to costs in production. So we report the latter and we multiply costs by population size $e^{n_t t}$.

Since a model with TC has a lower MAC in the steady state, from Eq. (59) $-c_T^*$ is lower. And since marginal damages are increasing in temperature, the steady state temperature is lower. ■

Proposition 1 depends on static MACs and marginal damages after peak warming, which ensure peak warming is the model's steady state. However, peak warming may not be a steady state. One special case is where both the marginal abatement cost and damage functions are proportional to consumption. In this case, r in Eq. (59) is replaced by $r-g$ and the proposition still holds, in fact. More generally, in the specialised model of Section 4/Appendix B.1, the assumption of static marginal abatement costs and damages after peak warming is not satisfied, and the model has no steady state at peak warming. Yet, we can still show that TC decreases peak warming provided a highly plausible condition is met:

Proposition 2 *In the specialised model of Appendix B.1, TC decreases peak warming if at the time of peak warming optimal abatement satisfies*

$$\frac{\left(r-g\overbrace{-\frac{\dot{a}}{a}}^{-ve}\overbrace{-\frac{\dot{\varphi}}{\varphi}+\frac{\chi a}{2A}}^{+ve}\right)}{(r-g)}\,\frac{\varphi_{t^*}\left(\frac{A}{A_0}\right)^{-\chi}}{\varphi_0}<1. \tag{60}$$

Proof: The proof follows from rewriting Equation (12) at the time of peak warming as

$$\gamma\zeta T^* = \left(r-g\overbrace{-\frac{\dot{a}}{a}-\frac{\dot{\varphi}}{\varphi}+\frac{\chi a}{2A}}^{-\frac{d}{dt}\left(\frac{MAC}{c}\right)-\frac{c_A}{c}}\right)\varphi a\left(\frac{A}{A_0}\right)^{-\chi}, \tag{61}$$

and dividing this by the equivalent expression for the model without TC. ■

This condition is satisfied unless peak warming is reached implausibly quickly. The first factor on the left-hand side of the inequality converges to one in the long term and tends to be very close to one at peak warming. The numerator comprises the discount rate, plus the degrowth rate of the MAC (the cost-reduction effect, $-d/dt(MAC/c)$), minus the endogenous future gains of TC. In the static model, this is just $r-g$, but in the presence of TC we have additional terms. The second factor is much smaller than one. It is the relative reduction of the slope of the MAC curve due to TC at the time of peak warming.

Proposition 3 *In a cost-benefit setting, exogenous TC ($\psi_a = 0$) results in a lower initial carbon price, lower initial abatement, and a lower initial carbon price growth rate than a model without TC.*

Proof: This can be proved for the general model. Conjecture that the model with exogenous TC has the same initial carbon price as the model without TC. From Eq. (21), this implies that initial price growth is also the same (the initial conditions ensure marginal damages are the same). The same carbon price growth rate combined with a decreasing MAC curve in the model with exogenous TC will lead to faster abatement and lower temperatures after the start. This will lead in turn to faster growth of the carbon price in later periods (from Eq. 21). From

Eq. (6), lower emissions and temperatures lead to a lower carbon price, which contradicts our conjecture. As a result, the initial carbon price needs to be lower. If the carbon price is lower and the MAC function is identical at time zero, initial abatement will be lower. Regarding the initial carbon price growth rate, Eq. (21) shows that since c_T is identical from the initial condition on temperature and c_a is lower, the change in the carbon price will initially be lower. ■

In later periods, the lower carbon price is compensated by lower marginal damages and the effect of exogenous TC on the growth rate is ambiguous. Moreover:

Corollary 1 *In a cost-benefit setting, endogenous TC ($\psi_a > 0$) has ambiguous effects on the initial carbon price, initial abatement and the initial carbon price growth rate compared to a model without TC.*

In a model with endogenous TC, there is the additional endogenous future gains component (Equation 6), which increases the carbon price, all else being equal. Hence the effect of endogenous TC on the initial carbon price, and in turn initial abatement and the initial carbon price growth rate, is in general ambiguous.

Proposition 4 *Consider the specialized model of Appendix B.1, denote the (negative) growth rate of exogenous TC as $g_{TC} = \dot{\varphi}/\varphi$ and define $\frac{\partial a}{\partial g_{TC}}$ as the difference in abatement between two identical models with marginally different TC growth rates. In a cost-benefit setting, exogenous TC increases the initial abatement speed iff*

$$\frac{\partial a}{\partial g_{TC}}\frac{1}{a} < \frac{\varphi a(A/A_0)^{-\chi}}{\gamma\zeta^2 S}. \tag{62}$$

Proof: Consider Equation 12 and acknowledge that a change in the exogenous TC rate will affect initial abatement, but not A, S and φ, because they are defined by their initial conditions. Define $g_a = \dot{a}/a$, and denote $\frac{\partial a}{\partial g_{TC}}$ as the difference in abatement between two models, where the TC has been marginally altered.[21] Taking the derivative of Equation (12) wrt g_{TC} at time zero gives,

$$-\frac{\partial g_a}{\partial g_{TC}} = 1 - \frac{\gamma\zeta^2 S}{(\varphi a(A/A_0)^{-\chi})^2}\varphi\frac{\partial a}{\partial g_{TC}}(A/A_0)^{-\chi} \tag{63}$$

Imposing that this equation is positive gives our result. ■

This should always be the case for real-world parameters. The LHS is typically between one and five (in our model initial abatement is 5% lower for an initial growth rate of ϕ of -1.6%, so the LHS equals three), whereas the RHS is typically larger than 100 (it is the inverse of the growth-adjusted discount rate $r - g$ at the steady state and considerably larger at the start of the transition).

Proposition 5 *Consider the specialised model of Appendix B.1, denote the (negative) growth rate of endogenous TC as $g_{TC} = \frac{\frac{d}{dt}(A/A_0)^{-\chi}}{(A/A_0)^{-\chi}} = -\frac{\chi a}{A}$. In a cost-benefit setting, endogenous TC increases the initial abatement speed iff*

[21] In dynamic programming jargon, this is called the derivative of the policy function wrt g_{TC} as a parametric state variable.

$$\frac{\partial a}{\partial g_{TC}}\frac{1}{a} < \frac{1}{2}\frac{\varphi a(A/A_0)^{-\chi}}{\gamma\zeta^2 S}. \tag{64}$$

Proof: Consider Equation 12 and acknowledge that an change in the endogenous TC rate will affect initial abatement, but not A, S and φ, because they are defined by there initial conditions. Call $g_a = \dot{a}/a$. Taking the derivative of equation 12 wrt g_{TC}, at time zero gives,

$$-\frac{\partial g_a}{\partial g_{TC}} = 1/2 - \frac{\gamma\zeta^2 S}{(\varphi a(A/A_0)^{-\chi})^2}\varphi\frac{\partial a}{\partial g_{TC}}\ (A/A_0)^{-\chi} \tag{65}$$

Imposing that this equation is positive gives our result. ■

Again, this should always be the case for real-world parameters, because, compared to exogenous TC, $\frac{\partial a/a}{\partial g_{TC}}$ is not only smaller, it is often negative (initial abatement increases with endogenous TC), as it is for our parameters. The factor 1/2 comes from the fact that the endogenous future gain incentive creates a flattening effect on the abatement path that is half the size of the opposite decreasing cost effect.

Proposition 6 *In a cost-effectiveness setting, exogenous TC has no effect on the growth rate of the carbon price (the Hotelling rule is unaffected), but results in a lower carbon price over the entire path, less initial abatement, higher later abatement, and peak warming being reached earlier.*

Proof: From Equation 21 and the assumptions of cost-effectiveness ($c_T = 0$) and no endogenous TC ($\psi_a = 0$), we see that the carbon price increases according to the Hotelling rule at rate r in both models. Conjecture that the initial carbon price would start at the same price. The model with TC has lower abatement costs after time zero and would have weakly higher abatement over the whole path and reach zero emissions earlier. This would violate the condition that cumulative emissions before reaching zero emissions must be equal in both models ($\zeta\int_0^\infty E = \bar{T}$ in both models). Hence the carbon price must start lower. Since the MAC function is identical at the start, this must result in lower abatement at the start. Since cumulative emissions must be identical, the abatement path must cross the no-TC abatement path and lead to zero emissions earlier. ■

Proposition 7 *In a cost-effectiveness setting, compared to a model without TC, the specialised model of Appendix B.1 with endogenous TC will have lower carbon price growth, a lower carbon price over the entire path, less initial abatement, higher later abatement, and reach peak warming earlier.*

Proof: The emissions path will still be steeper, because the cost-reduction effect dominates the endogenous future gain effect (Eq. 12). Since cumulative emissions are identical with and without TC in the cost-effectiveness case, the emissions paths must cross each other, with emissions higher at the start under endogenous TC, lower at the end, and zero emissions reached earlier. Higher initial emissions implies a lower initial carbon price. Eq. (43) shows that endogenous future gains will reduce the growth rate of the optimal carbon price (marginal damages are zero in the cost-effectiveness analysis). A lower initial carbon price combined with lower growth implies a lower carbon price over the entire path. ■

Note that the shadow price of carbon, λ in Equation 30, always follows a Hotelling path irrespective of TC. This result is also reported in Goulder and Mathai (2000).

Appendix D Method of isolating endogenous future gain effect

Our method of isolating the endogenous future gain effect proceeds in two steps. The first step is to estimate a time-dependent MAC function (i.e., exogenous TC), with a MAC that is identical to the endogenous TC model at each point in time. This can be achieved using a sufficiently high-order polynomial. The second step is to take the exogenous TC MAC curve and use it to recalculate optimal model trajectories. Any difference between the optimal paths of the endogenous TC model and its exogenous replica must then be down to the endogenous future gain effect.

We can show this in theory. Call a^* the optimal abatement path of the model with endogenous TC. Conjecture that this is also the optimal path of the model with exogenous TC. By the assumption of identical MACs, the abatement path a^* will result in the same left-hand side of Equation (6) at each point in time. Given that we use the same climate model (Equation 2), the model without TC results in the same temperature path. Figure A1 shows that since MACs are identical, total abatement costs are also identical. Hence, consumption and the discount rate are the same. However, in the exogenous TC model, the absence of the endogenous future gain effect decreases the right-hand side of Equation (6). Hence, our conjectured abatement path a^* does not solve Equation (6), and the initial carbon price and abatement are lower in the model with exogenous TC. Lower abatement results in higher temperatures, higher marginal damages, and a higher right-hand side of Equation (6). Therefore, both models converge in the long run. If TC is zero after peak warming ($\forall t > t^{peak} : \dot{H} = 0$), Equation (6) shows that the optimal abatement path of the endogenous TC model also solves the exogenous TC model, i.e., peak warming and long-run temperatures are identical (although peak warming comes earlier in the endogenous TC model).

Static approximation of MAC function under exogenous TC

We can also test the ability of a suitably calibrated static model to approximate the optimal solution of a model of exogenous TC. Figure A2 shows our approach visually. Exogenous TC results in an optimal pair $\{MAC^{\%*}, a^*\}$ at each moment in time, where again $MAC^{\%} = \varphi_t a$, i.e., the MAC as a proportion of consumption. Next, we construct a static MAC function that is fitted to be identical at each point in time (i.e., at each level of abatement on the optimal path of the exogenous TC model). This static MAC function will be concave, since TC makes the MAC function, which is assumed to be linear, fall over time. We fit based on a polynomial such that

$$MAC^{\%*} = \left(\sum_{n=0}^{N} \varphi_n^{nolearn} a_n^* \right), \tag{66}$$

where the coefficients $\varphi_n^{nolearn}$ are independent of time, unlike in the model with exogenous TC.

We optimise the model with the static MAC function and check the solution is close to the optimum of the model with exogenous TC. Figure A3 shows that the correspondence is almost

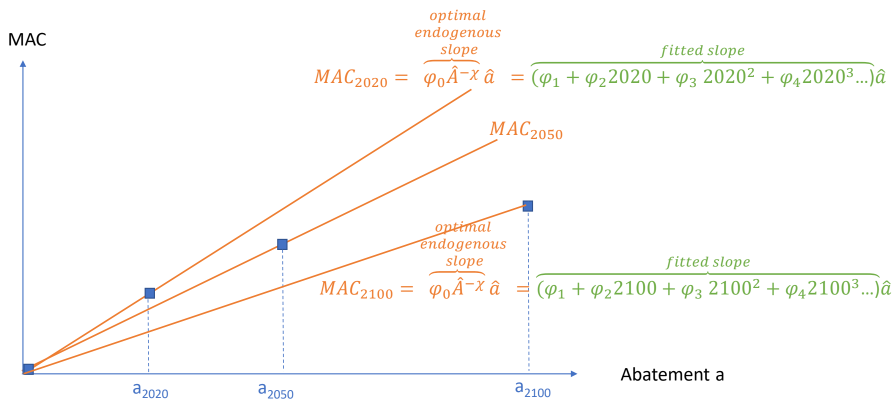

Figure A1: Graphical representation of the comparison between endogenous TC and exogenous TC with identical MAC function. The endogenous TC model has a linear marginal abatement cost function $MAC^{\%} = \varphi A^{-\chi} a$. The optimal path (which depends on cumulative abatement) results in optimal slopes at each point in time. Next, a 25th-degree polynomial in time is fitted to obtain the same MAC but without the dependence on cumulative abatement. $MAC = \left(\varphi_1 + \varphi_2 t + \varphi_3 t^2 + \varphi_4 t^3 ...\right) a$.

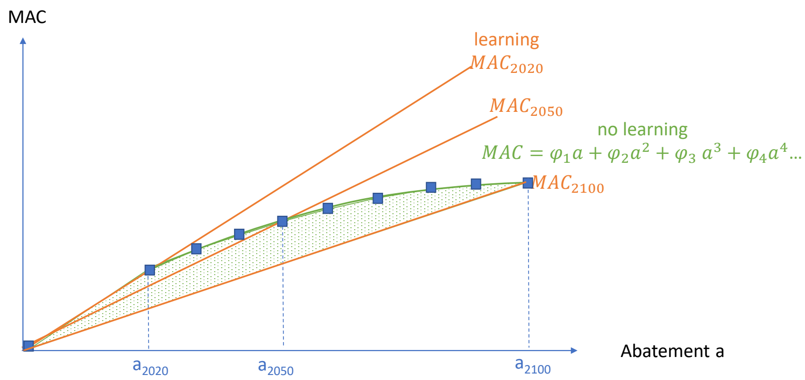

Figure A2: Graphical representation of the comparison between TC and no TC with identical but static MAC. The model with TC has a linear abatement cost function but with a time-dependent slope $\varphi_t a = \left(\varphi_\infty + (\varphi_0 - \varphi_\infty) e^{-g\varphi t}\right) a$ in the case of exogenous TC. This model results in an optimal set of $\{MAC^{\%*}, a^*\}$ depicted by the blue squares. Next, a polynomial in a (of order 15) is fitted to obtain a time-independent MAC that is identical on the optimal path. The polynomial also goes through the origin, in order to ensure similar total abatement costs, represented by the area under the curve. The dotted area is the difference between the total abatement costs of both models in 2100.

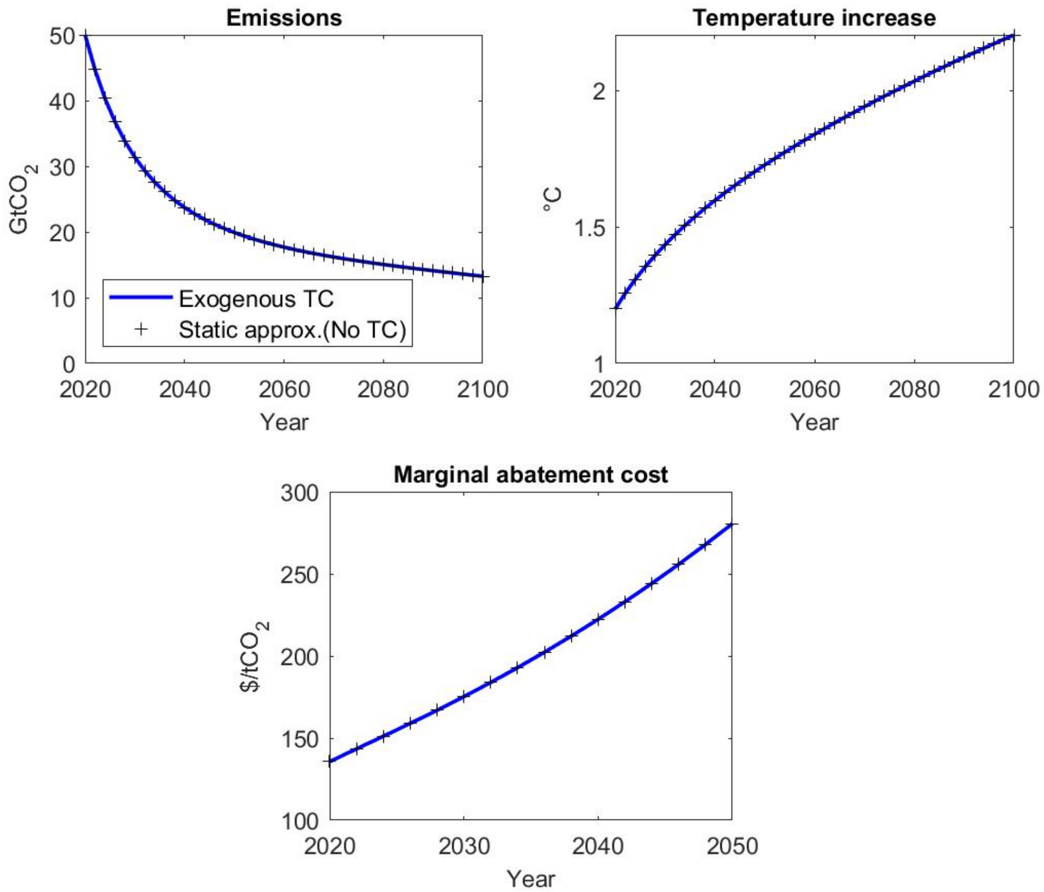

Figure A3: Exogenous TC versus no TC: fitting a polynomial in abatement to the MAC of the exogenous TC curve. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.

exact (less than 0.1% difference on emissions or carbon prices).

In fact, under certain specific functional forms the correspondence is exact in theory. In particular, if marginal abatement costs and marginal damages are proportional to consumption to the power ν and marginal utility is CES, the static MAC function is a *perfect* substitute for exogenous TC. The assumption of constant elasticities implies that we can factorise the marginal cost functions into a factor that does not depend on consumption and a power function of consumption, respectively: $c_a = -MAC^{\%}(a,t)c^{\nu}$; $c_T = -MD^{\%}(T)c^{\nu}$. Integrating Equation (16) yields

$$\lambda_t = \int_t^{\infty} e^{-(\delta-n)(\tau-t)} u_{c_\tau} \zeta c_\tau^{\nu} MD^{\%}(T) d\tau. \tag{67}$$

Substituting Equation (15) gives

$$u_{c_t} c_t^{\nu} MAC^{\%}(a,t) = \int_t^{\infty} e^{-(\delta-n)(\tau-t)} u_{c_\tau} \zeta c_\tau^{\nu} MD^{\%}(T) d\tau. \tag{68}$$

$u_{c_t} c_t^{\nu}$ is a constant (independent of time τ) and can therefore be included in the integral,

$$MAC^{\%}(a,t) = \int_t^{\infty} e^{-(\delta-n)(\tau-t)} \left(\frac{c_\tau}{c_t}\right)^{\nu-\eta} \zeta MD^{\%}(T) d\tau. \tag{69}$$

Call a^* the optimal abatement path of the model with exogenous TC. We try this path as a candidate solution to the model without TC. By assumption of identical MACs, the abatement path a^* will result in the same left-hand side of Equation (69) at each point in time. Given that we use the same climate model (Equation 2), the model without TC will result in the same temperature path. Figure A2 shows that although MACs are identical, total abatement costs will be higher in the model without TC (dotted area). Hence, consumption will be slightly lower in the model without TC. But since we assume $\nu = \eta$, the right-hand side of Equation 69 will also be identical at each point in time. Hence our candidate solution solves the Euler equation and is the optimal solution of the model without TC.

What if $\eta > \nu$? Figure A2 shows that although MACs are identical, total abatement costs will be lower in the model with TC (dotted area). Hence, consumption will be slightly higher in the model with TC. As a result, the discount factor $\left(\frac{c_\tau}{c_t}\right)^{\varsigma-\eta}$ will be slightly lower, and MACs and abatement will be lower too. The effect is too small to be visible on a graph though.

Appendix E Additional quantitative results

Figure A4 presents optimal, cost-benefit climate policies in the absence of capital inertia (i.e., no abatement speed penalty). Without inertia, the social planner is free to choose large variations in initial emissions. Accordingly, initial emissions are much lower than in the presence of capital inertia, regardless of the existence and type of TC. This results in more slowly increasing temperatures, but since capital inertia is relevant in the short to medium run but less so in the long run, temperatures in 2100 are similar.

Consistent with the theoretical results in Section 4, emissions under exogenous TC are initially higher (abatement is lower) than without TC, with a lower carbon price. Conversely, under endogenous TC emissions are initially lower than without TC, with a higher carbon price. The abatement path is steeper under both forms of TC, which is also consistent with the theory.

Figures A5 to A8 present optimal, cost-benefit climate policies with low/high discount rates and slow/fast TC. These plots provide further details on the results summarised in Table 4.

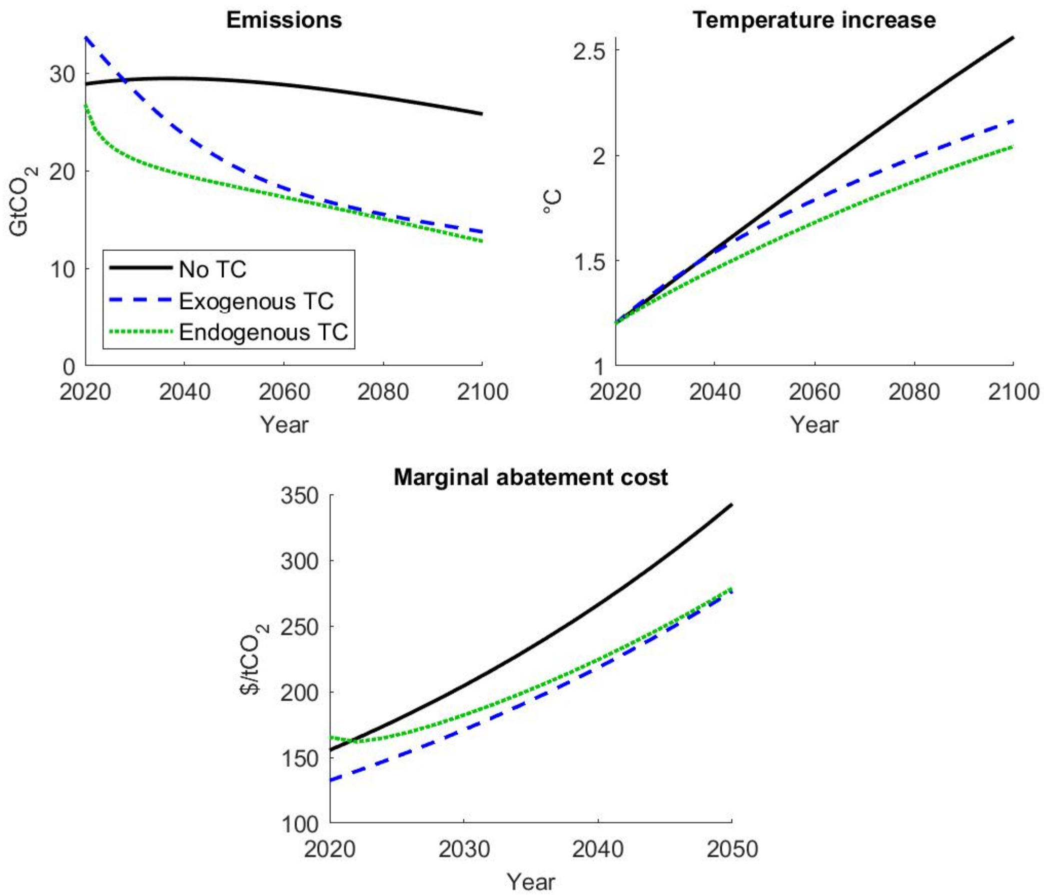

Figure A4: Optimal, cost-benefit climate policies without TC, with exogenous TC and with endogenous TC, for the case of **no capital inertia**. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom. Note that the modest increase in emissions in scenarios without TC is the result of population growth and marginal damages, which both create an incentive for early abatement (both factors exceed the effect of the discount rate $r - g$ in Equation (12)).

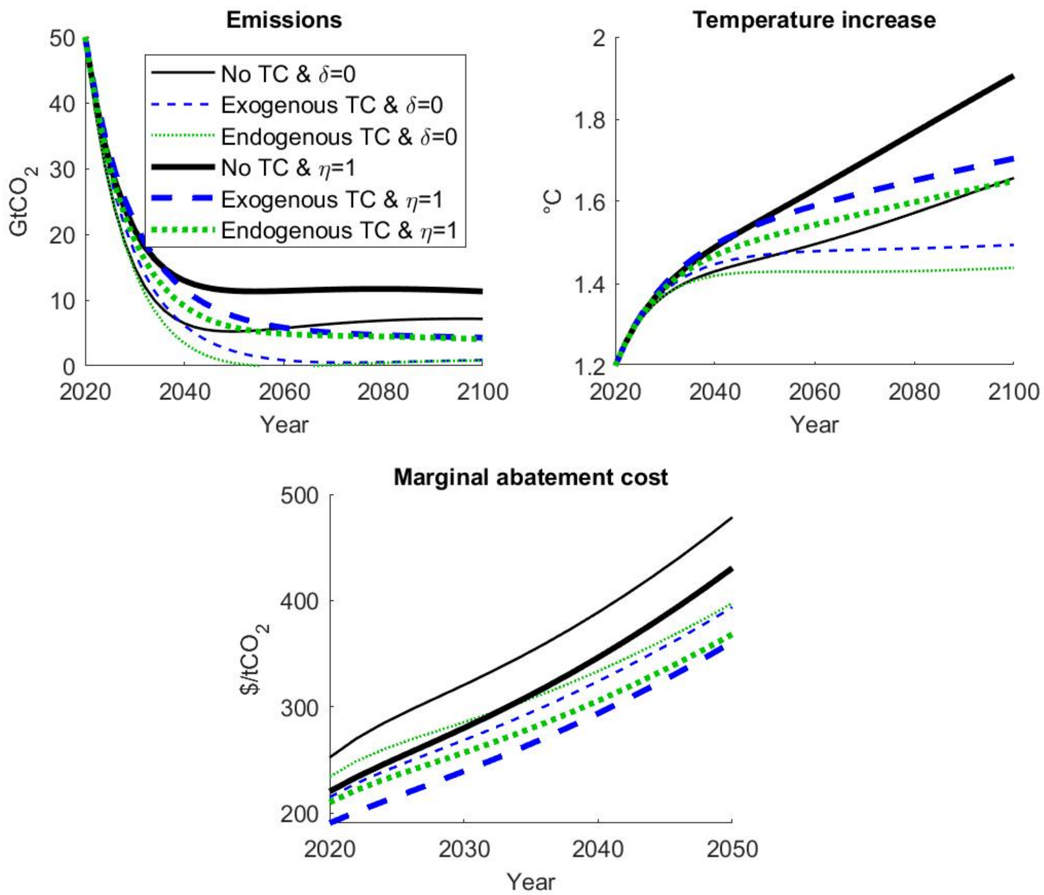

Figure A5: Optimal, cost-benefit climate policies without TC, with exogenous TC and with endogenous TC, under **low discount rates** ($\delta = 0$ or $\eta = 1$). Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom. Note that the modest increase in emissions in scenarios without TC is the result of population growth and marginal damages, which both create an incentive for early abatement (both factors exceed the effect of the discount rate $r - g$ in Equation (12)).

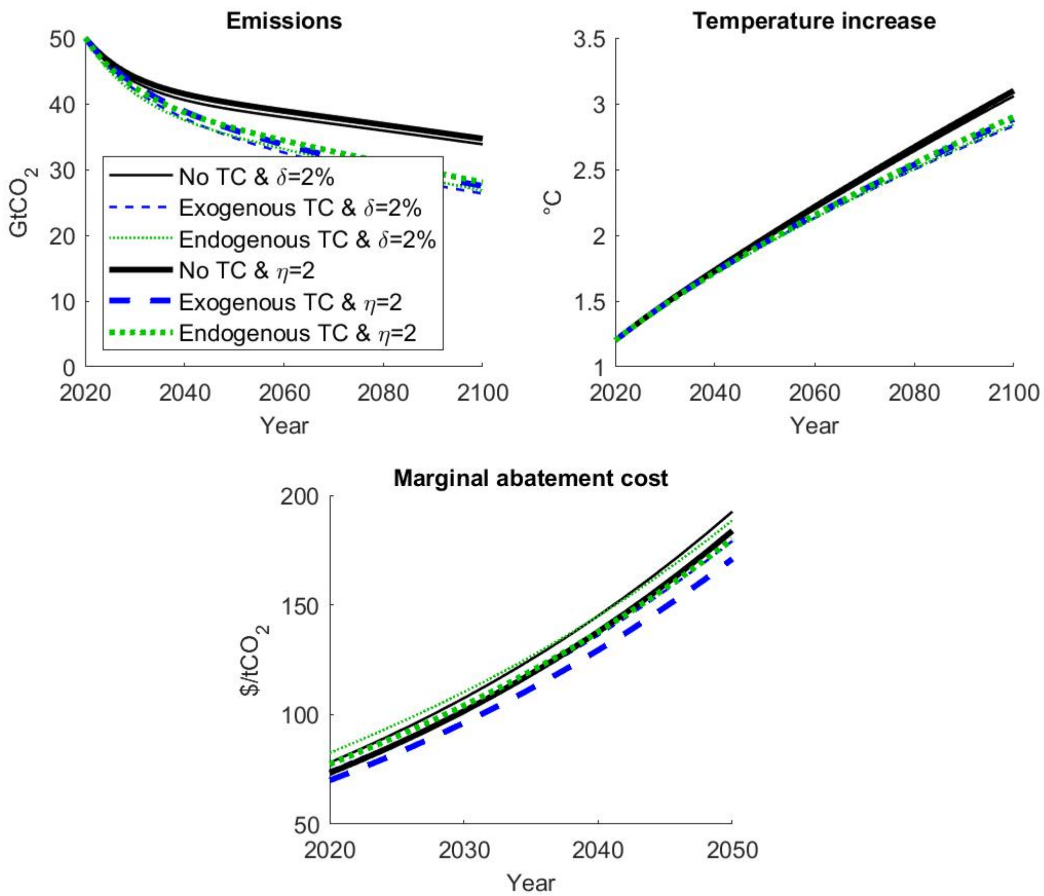

Figure A6: Optimal, cost-benefit climate policies without TC, with exogenous TC and with endogenous TC, under **high discount rates** ($\delta = 0.02$ or $\eta = 2$). Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.

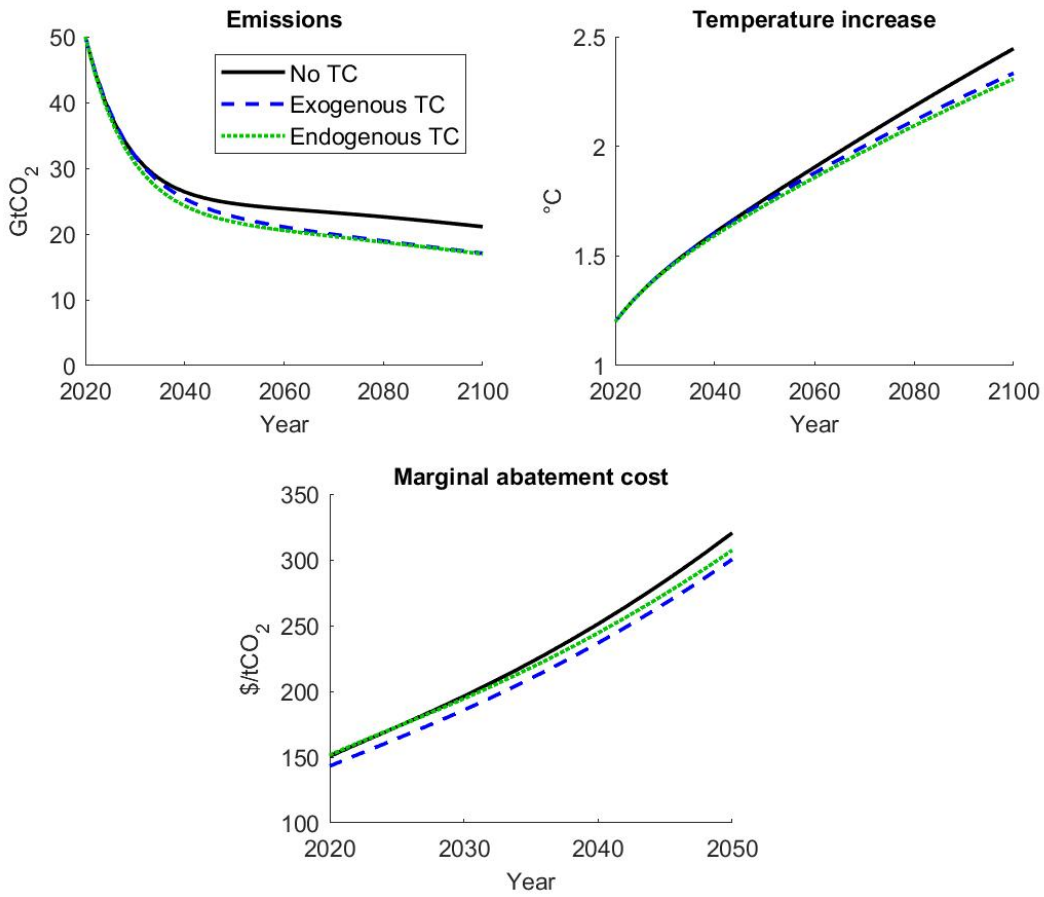

Figure A7: Optimal, cost-benefit climate policies without TC, with exogenous TC and with endogenous TC, under **slow TC**. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.

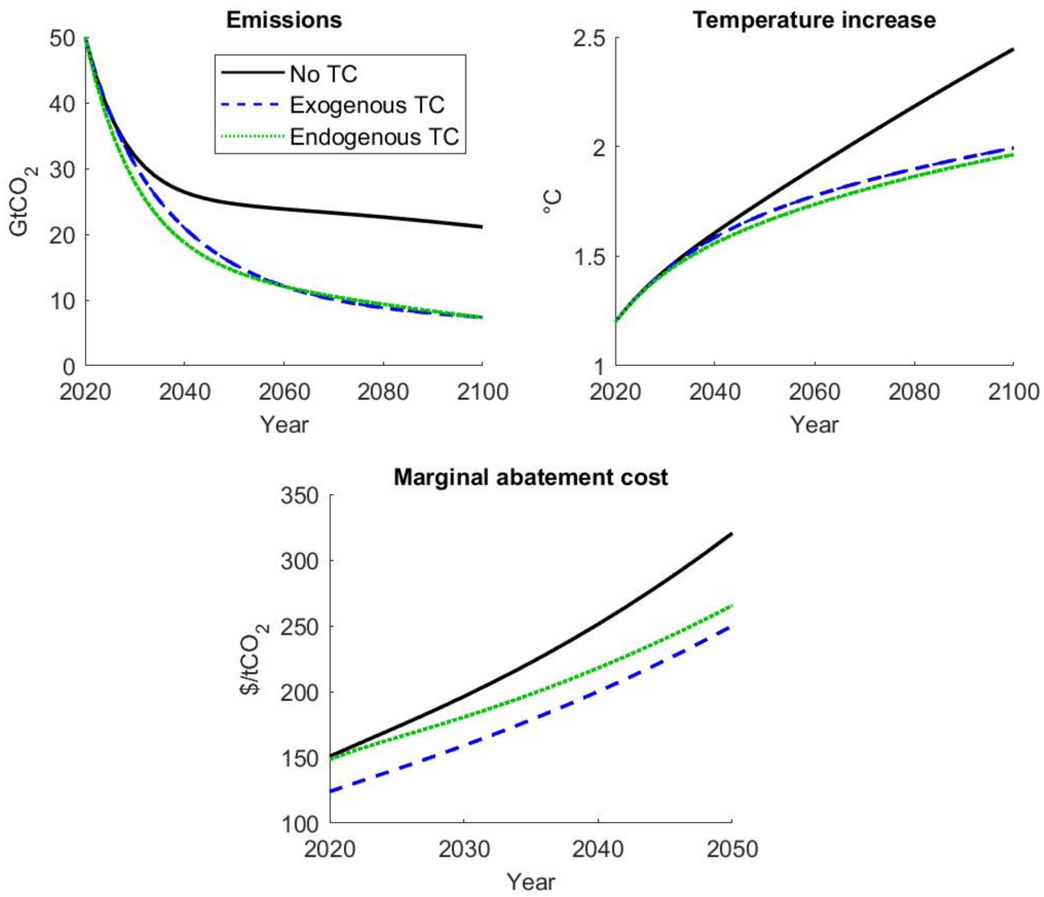

Figure A8: Optimal, cost-benefit climate policies without TC, with exogenous TC and with endogenous TC, under **fast TC**. Emissions in the top-left panel, temperature above pre-industrial in the top right, and the MAC/carbon price on the bottom.