

Monetary Policy and Radical Uncertainty

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Monetary Policy and Radical Uncertainty

Abstract

In a world of radical uncertainty the frequency distributions of economic variables deviate from the normal distribution and typically exhibit fat tails. We show that this feature is obtained in simple models where agents have cognitive limitations and fail to understand the underlying model. Although the model is simple, we obtain great complexity. We analyse the implications for monetary policy. We show that in such models the central bank bears a much greater responsibility to stabilize an otherwise unstable system than in mainstream models that assume Rational Expectations. We also question the use of impulse responses to exogenous shocks when the distribution of these impulse responses is not normal.

JEL-Codes: E520, E580, E700.

Keywords: radical uncertainty, monetary policy.

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1. Introduction

The nature of uncertainty matters a great deal for the conduct of macroeconomic policies in general and monetary policies in particular. This has always been the case. It will probably be even more so in the future when climate change is likely to create existential crises and thus new dimensions of uncertainty for everybody, including policymakers. In this connection the notion of “radical uncertainty” has become popular (see Kay and King(2020)); a kind of uncertainty that will radically affect the nature of policymaking.

In order to understand how radical uncertainty may affect policies it is necessary to make clear what is meant by “radical uncertainty”. Radical uncertainty can be defined in different ways. Here we will consider two definitions. We distinguish between a strong and a soft definition.

The strong definition interprets radical uncertainty in the sense of Frank Knight (Knight(1921)). Uncertainty is radical when we cannot quantify it. In particular, it is impossible to know the frequency distribution of macroeconomic shocks and macroeconomic variables in general. We are in the realm of the “unknown unknowns”. Anything can happen. Large shocks can occur, but there is no way of knowing the nature and the timing of these shocks.

There is a softer definition of radical uncertainty. This is a situation where the frequency distributions deviate from Gaussian (normal) distribution. In particular, they have fat tails (“Black swans”, Talib(2007)) and they may not be one-modal. We will mostly discuss radical uncertainty in this second sense.

Purists may counter that the second definition is not really radical, and this may be true. The advantage of using this soft definition of radical uncertainty, however, is that we can model it, and we can come to some conclusions that deviate from mainstream macroeconomics. But, we will occasionally refer to radical uncertainty in the first sense to find out how it interacts with the second one.

How can one model radical uncertainty (in its soft definition)? We will show that radical uncertainty arises because agents fail to understand the underlying model. Thus, one can have an underlying model that is relatively simple, but when we assume that agents do not understand its structure we obtain complexity and radical uncertainty. There is no need to model complexity to obtain this result.

As will be shown, the intriguing thing is that in a world where agents do not understand the model one creates complexity that is very difficult to understand, thereby validating why agents do not understand the model.

This contrasts with mainstream macroeconomic models that assume Rational Expectations (RE), i.e. that assume that agents understand the underlying model. Such a RE-model does not generate radical uncertainty (as defined here in its soft sense). Radical uncertainty can only come from outside the model. Mainstream macroeconomics only recognizes the exogenous shocks as sources of radical uncertainty. There is no place for endogenous sources of radical uncertainty because agents with rational expectations understand the workings of the underlying model of capitalism. Once the shock has occurred they can compute with great confidence how it will be transmitted to the economy (for a profound analysis of how modern macroeconomics went wrong, see Stiglitz(2018)).

2. A simple behavioural macroeconomic model

We use a simple behavioural macroeconomic. It consists of a standard aggregate demand equation, a New-Keynesian supply equation and a Taylor rule equation. Aggregate demand is a function of expected future demand and the real interest rate. The New-Keynesian supply curve explains the rate of inflation by the expected inflation and the output gap. The Taylor rule describes the behaviour of the central bank that manipulates the nominal interest rate so as to keep inflation close to its target and so as to stabilize the output gap.

It is assumed that agents do not know the structure of the model in which they operate. The model is too complex to be understood by humans. Therefore, they use simple rules (heuristics) to guide their behaviour. These agents are rational, however, in that they are willing to learn from their mistakes. Thus when they find out that the rule they are using performs less well than alternative rules, they are willing to switch rules. This switching rule is a way for agents to learn about the economy. We show this model in the appendix (see also De Grauwe (2012) and De Grauwe and Ji(2019)).

We will illustrate how in this model radical uncertainty emerges in different forms in sections 2.1 and 2.2, respectively:

- movements in macroeconomic variables that are not normally distributed and that exhibit fat tails even if the exogenous shocks are normally distributed.

- impulse responses to shocks that are not normally distributed leading to the problem that conditional forecasting (“what if”) cannot be answered properly; it also leads to the problem that policy analyses based on representing the effect of policy actions (e.g. interest rate hikes) by impulse responses cannot be analyzed properly either.

2.1 Deviations from normal distributions

We start by presenting basic results of the model. Given the non-linear nature of the model we have recourse to numerical methods. We simulated the model using numerical values of the coefficients obtained from the literature and imposing i.i.d. shocks in the demand and supply equations and the Taylor rule equation, with zero mean and standard deviations of 0.5.

Figure 1 presents the movements of the output gap in the time domain (left panel) and in the frequency domain (right panel). Figure 2 shows the movements of “animal spirits” which express market sentiments of optimism and pessimism generated endogenously in the model (see appendix). It is an index varying between -1 (extreme pessimism) and + 1 (extreme optimism). We observe that the model produces waves of optimism and pessimism (animal spirits) that can lead to a situation where everybody becomes optimist ($S_t = 1$) or pessimist ($S_t = -1$).

As can be seen from the left hand side panels, the correlation of these animal spirits and the output gap is high. In the simulations reported in Figure 1 this correlation reaches 0.94. Underlying this correlation is the self-fulfilling nature of expectations. When a wave of optimism is set in motion, this leads to an increase in aggregate demand. This increase in aggregate demand leads to a situation in which those who have made optimistic forecasts are vindicated. This attracts more agents using optimistic forecasts. This leads to a self-fulfilling dynamics in which most agents become optimists. It is a dynamics that leads to a correlation of the same beliefs. The reverse is also true. A wave of pessimistic forecasts can set in motion a self-fulfilling dynamics leading to a downturn in economic activity (output gap). At some point, most of the agents have become pessimists.

The right hand side panels show the frequency distribution of output gap and animal spirits. We find that the output gap is not normally distributed, with excess kurtosis and fat tails. A Jarque-Bera test rejects normality of the distribution of the output gap. The origin of the non-

normality of the distribution of the output gap can be found in the distribution of the animal spirits. We find that there is a concentration of observations of animal spirits around 0. This means that much of the time there is no clear-cut optimism or pessimism. We can call these periods of “Great Moderation”. The excess kurtosis tells us that there is a high concentration of such periods. There is also, however, a concentration of extreme values at either -1 (extreme pessimism) or +1 (extreme optimism). These extreme values of animal spirits explain the fat tails observed in the distribution of the output gap. The interpretation of this result is as follows. When the market is gripped by a self-fulfilling movement of optimism (or pessimism) this can lead to a situation where everybody becomes optimist (pessimist). This then also leads to an intense boom (bust) in economic activity.

Figure 1: Output gap

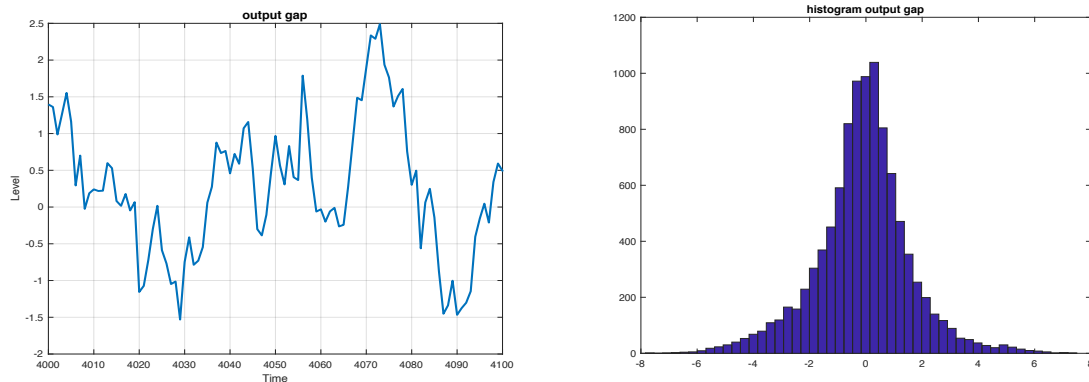
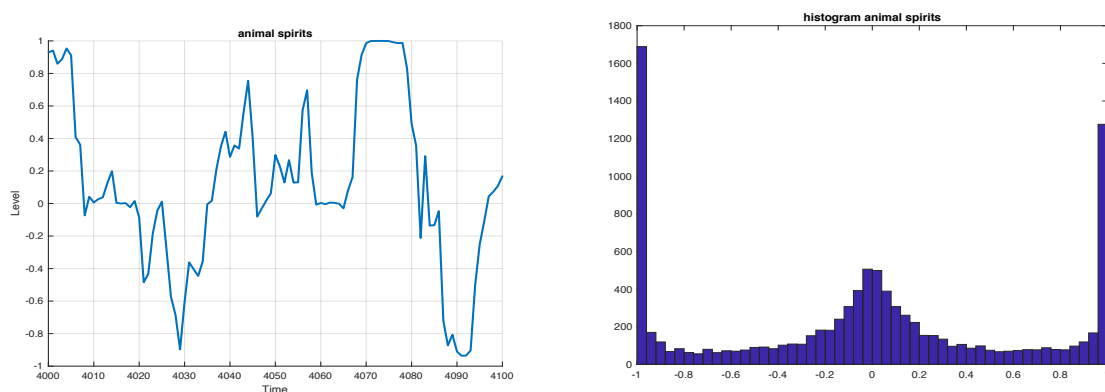


Figure 2: Animal Spirits



Source: De Grauwe and Ji(2019)

Note: the model was simulated over 10,000 periods. The representations in the time domain show a representative sample of 100 periods. The frequency domain shows all periods.

The economy switches from normal periods to extreme movements of booms and busts in an unpredictable way. This is what produces complexity and radical uncertainty, which we

defined earlier as deviations from Gaussian distributions. (Note that the exogenous shocks are normally distributed). Two factors explain this complexity. First, there is the ignorance of agents about the underlying model, and second, their attempt to understand by a “trial and error” learning mechanism. The complexity that is created in this way justifies the assumption that agents have insufficient cognitive abilities to understand the underlying model.

Note also that the switching from Great Moderation to booms-and-bust regimes gives the impression of changes in the structure of the model, although no such structural changes occur. This also adds to the complexity of the dynamics of the model.

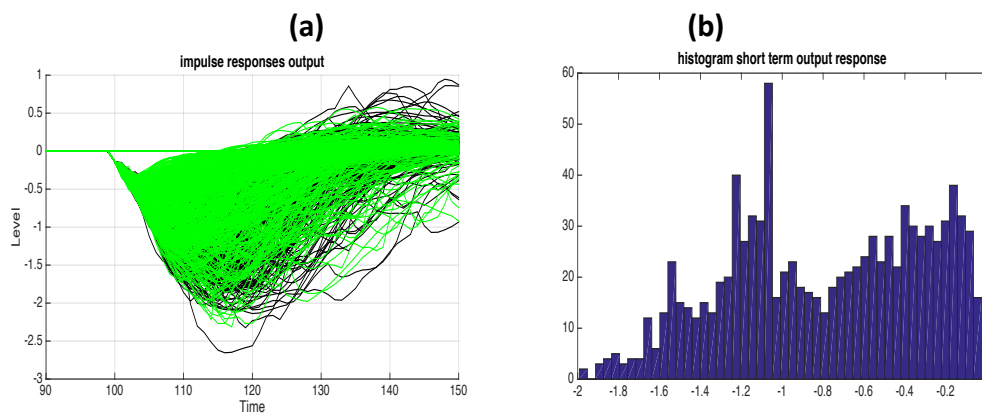
2.2 Impulse responses

In contrast to linear rational expectations models the impulse responses depend on the timing of the shock. Put differently, an impulse response computed with one realization of the stochastic shocks in the equations of the model will be different from an impulse response to exactly the same shock but performed using another realization of these stochastic shocks. This is the case even when all parameters of the model are identical.

In order to illustrate this we simulated 1000 impulse responses of the output gap to the same (one standard deviation) negative supply shock occurring at a particular point in time, assuming each time a different realization of stochastic shocks of the model. We show these impulse responses in Figure 3, in the time domain and the frequency domain. We obtain a collection of 1000 impulse responses. Note that the responses in the frequency domain are obtained by collecting these responses 12 periods (3 years) after the supply shock. So, the frequency domain figure is just the intersection of the observations of the time series 12 periods after the supply shock. Several features of these impulse responses stand out.

First, there is sensitivity to initial conditions. We obtain very different impulse responses to the same shock, depending on the initial conditions. The representation in the frequency domain shows that the distribution is not at all Gaussian. It is difficult to infer any structure in this distribution. As a result, it is very difficult to make conditional forecasts about how a negative supply shock will affect the output gap, except that the effect is negative, and that after a sufficiently long period this negative effect will tend to disappear.

Figure 3: Impulse response output gap to a (1 standard deviation) negative supply shock



Source: De Grauwe and Ji(2024)

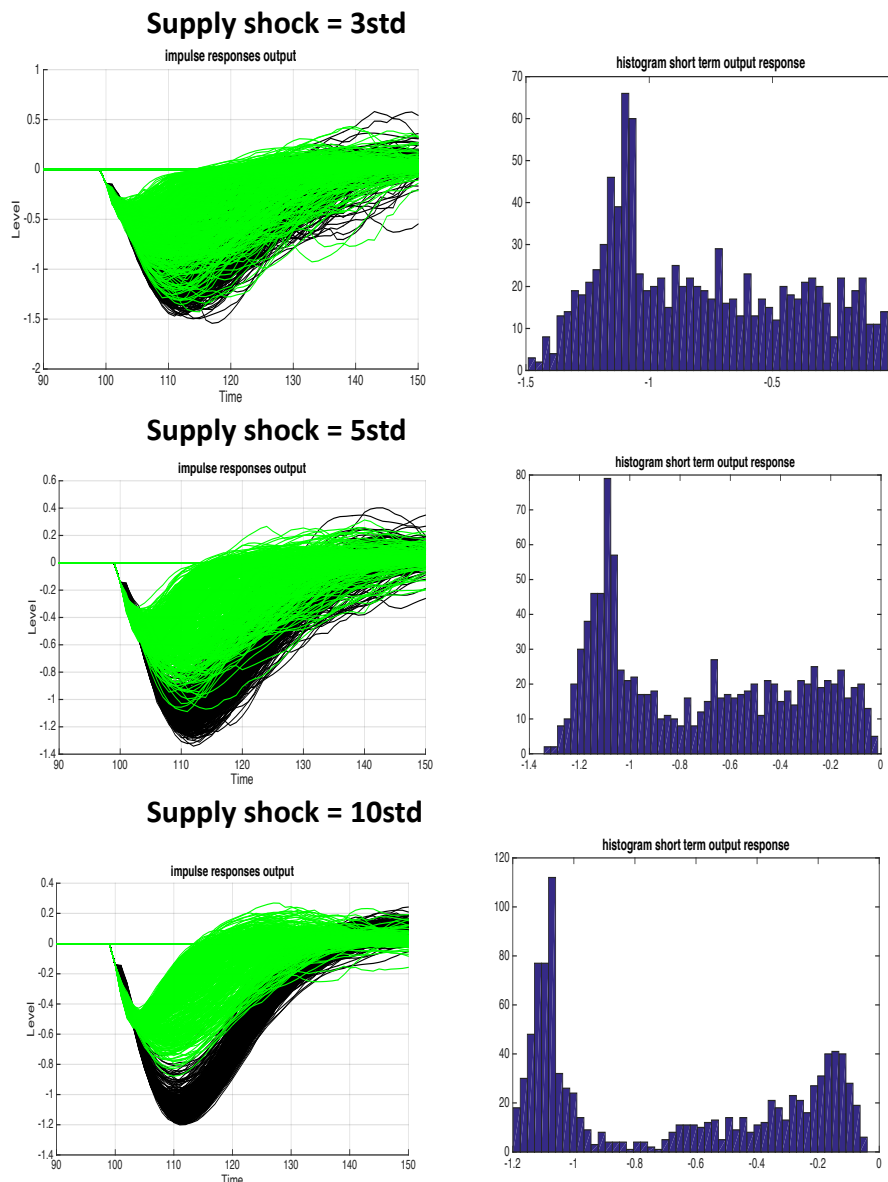
Second, the impulse responses are sensitive to the size of the shock. We show this in Figure 4 by increasing the size for the negative supply shock to respectively, 3 std, 5 std and 10 std. A striking result is that by increasing the size of the shock, there appears to be more structure in the distribution of the impulse responses. However, this structure does not show any relation to the normal distribution. What appears is a movement to a bi-modal distribution. This is especially evident when the impulse responses following a very large negative supply shock of 10std. This is the kind of shock experienced during the pandemic of 2020-21, when many countries saw their GDP decline by 10 percent or more. Clearly, this was a shock of the “unknown unknown” type and arises from radical uncertainty in its strong definition.

Figure 4 makes clear that such a large shock changes the transmission mechanism of the shock in a fundamental way. This transmission mechanism now shows a strong bi-modal structure. It appears that there are two types of trajectories taken by the impulse responses when the supply shock is very large. There is a set of “good” trajectories (coloured green) which shows the collection of output responses to the negative supply shock that are relatively mild, and a set of “bad” trajectories (coloured black) showing a collection of responses that lead to significantly stronger declines in the output gap. The existence of these two trajectories is clearly shown in the frequency distribution of the output gap responses 12 periods after the shock. We observe a concentration of responses around -0.15 and a concentration around -1.1. (These numbers are the multipliers of the supply shocks on output).

One interesting aspect of these results is that these two trajectories depend on the initial conditions. When the latter are favourable, (i.e. initial inflation and inflation expectations are

low) we end up in a good trajectory. When in contrast the initial conditions are unfavourable (high initial inflation and inflation expectations) we end up in the bad trajectory characterized by a deep recession that lasts much longer than in the good trajectory. Note also that within these two trajectories there is a lot of variation of the particular path the impulse responses will take. (For more detail on the role of the initial conditions and for an interpretation of these results, see De Grauwe and Ji(2024)).

Figure 4: Impulse responses to negative supply shocks of varying sizes

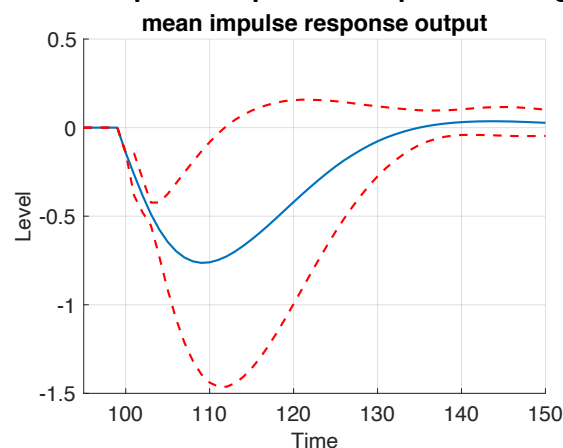


Source: De Grauwe and Ji(2024)

The frequency distributions of impulse responses that we analyzed in the previous sections show strong departures from the Gaussian distribution. As a result, the mean response and

the standard deviations of these responses are not informative about the true underlying distribution. We illustrate this problem as follows. We use the impulse responses of output to a 10 std deviation negative supply shock from Figure 4 and compute the mean and the two standard deviations below and above the mean. We show the results in Figure 5. Comparing these with Figure 4 it is clear that the mean and the standard deviations are not only uninformative, but even misleading about the true underlying distribution because Figure 5 gives the impression of the existence of a central tendency, the mean, that is representative of the impulse responses. In fact, there are almost no observations close to the mean as the impulse responses are clustered away from the mean. In addition, the representation in Figure 5 gives the wrong impression that, as one moves away from the mean, observations become less likely. In fact, the opposite is true.

Figure 5: Mean impulse responses output after large supply shock



This leads to the following problem. A standard assumption made in mainstream RE models is that agents know the distribution of the shocks, typically assumed to be Gaussian. The impulse responses derived from such an analysis typically have a representation as in Figure 5. This only makes sense if the distribution of these responses is Gaussian. If they are not, as is the case in our model, these representations are generally misleading.

The main business of macroeconomists is to produce conditional forecasts i.e. producing mean effects of some shock and a band of uncertainty around the mean. This could be a policy shock, a demand and supply shock, and many others. In a non-Gaussian world these conditional forecasts cannot be trusted. This leads to the idea that when making conditional forecasts one has to think in terms of *scenarios*. There are good scenarios and bad scenarios. In our model the probability of each of these scenarios is 50%. We can, however, make more

precise forecasts if we know the initial conditions when the shock occurred (See De Grauwe and Ji (2024)).

In this connection, it is useful to introduce the notion of *ambiguity*. There is strong ambiguity about the effects of shocks because the same shock can lead us into different universes of adjustment. In other words, without the knowledge of initial conditions, the distribution of the impulse responses is ambiguous.

3. Forecasting and radical uncertainty

Modern macroeconomics has given central stage to forward looking agents. This means that agents are assumed to make decisions based on forecasts of the variables that matter. Consumers, for example, are supposed to base their decision to consume on what they expect their future income to be. Similarly, policymakers in a rational expectations setting are assumed to make decisions based on forecasts of the variables they wish to influence. In this logic central bankers should set the interest rate based on their expectations (forecasts) of future inflation and output gap (or growth rate). See Clarida, Gali, Gertler(2000), Batini, and Haldane (1999), Svensson(1997).

The question that arises here is whether this is a sensible decision rule when the future is radically uncertain. When central banks rely on forecasts to make their decisions, they are likely to often make significant policy mistakes in a world of radical uncertainty. The question then is whether they can improve the quality of their policy decisions by not relying on forecasts of inflation and of the output gap, but rather by relying on currently observed values of these variables.

We analyzed this question in the context of our behavioral macroeconomic model (De Grauwe and Ji (2019)). We used two versions of the Taylor rule equations. The first one uses currently observed values of inflation and output gap. We called this the “current Taylor rule”. The second one uses the market forecasts of inflation and output gap. We called this the “forward Taylor rule”.

We then simulated the model using i.i.d. shocks in the demand and supply equations and calculated the forecast errors made by agents and by central bank under the current and forward Taylor rules. We plot the squared forecast errors of output gap (Figure 6) and inflation

(Figure 7) against the animal spirits. We find that when animal spirits are close to zero (tranquil times) the forecast errors tend to be the same in the two Taylor rule regimes. As animal spirits increase (in absolute values) the forecast errors increase and more so under the forward-looking Taylor rule.

This leads to the following insight. When extreme optimism or pessimism prevails (animal spirits are then close to +1 or -1) the economy is in a boom-bust regime with extreme volatility of output and inflation. Given the extreme volatility of these variables when animal spirits are intense, the central bank that uses market expectations will make many policy errors that have to be corrected afterwards. It is then better for the central bank to use currently observed output and inflation to set the interest rate. This leads to lower forecasting errors so that the central bank is likely to make fewer policy errors.

Figure 6: Squared forecast errors output gap and animal spirits

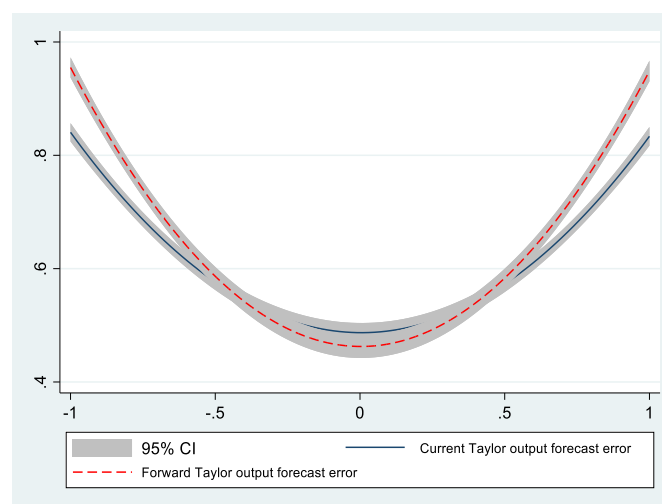
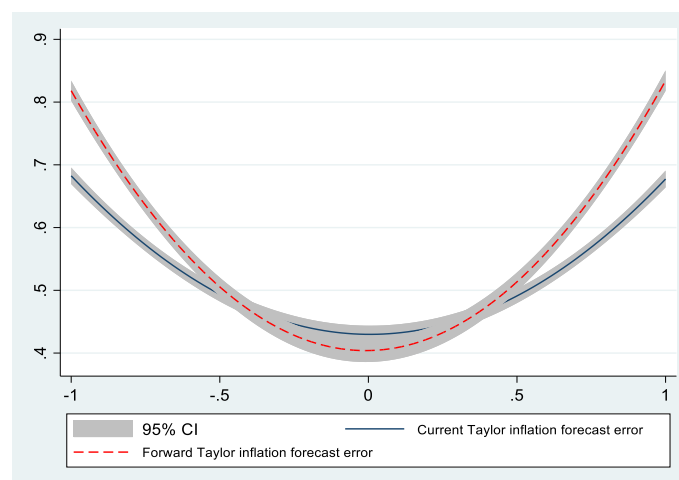


Figure 7: Squared forecast errors inflation and animal spirits



Source: De Grauwe and Ji(2019)

4. Boom-bust and stabilization

The previous discussion leads to the question of stabilization. We have found out that a world in which agents have cognitive limitations preventing them from having RE is one characterized by frequent boom-bust scenarios that destabilize the economy and that also have the potential of undermining the fabric of society. Can monetary policy do something about this and “stabilize an otherwise unstable” economy? The answer is positive. We simulated our behavioural model assuming normally distributed (and small) shocks. We have seen earlier that under those conditions the model produces regimes of relative tranquillity alternating with occasional bursts of boom and busts. We asked the question of whether the central bank can reduce the occurrence of booms and busts by attaching an increasing weight to output stabilization in the Taylor rule equation. We find that in general this reduces the deviation of the distribution of the variables from normality.

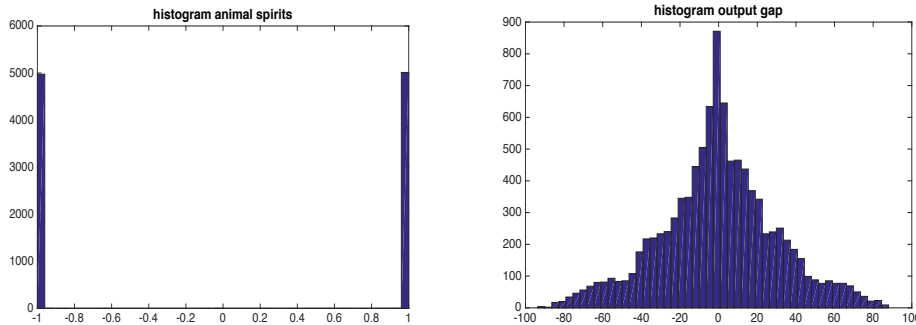
Figure 8 shows how an increase in the stabilization effort reduces the intensity of booms and busts. We show the frequency distribution of animal spirits and the corresponding distribution of the output gap for increasing values of the output parameter (c_2) in the Taylor rule equation. We observe several features. First, when $c_2=0$ we have a qualitatively very different result compared with the results obtained when $c_2>0$. This has to do with the fact that when $c_2=0$ we have a chaotic dynamics (see De Grauwe and Ji(2019)). There are then only extreme values of animal spirits and extreme fluctuations of the output. Chaotic dynamics disappears when $c_2>0$.

Second, as c_2 increases the frequency with which extreme values of optimism and pessimism occur declines and the concentration around the mean increases. Third, the variability of the output gap declines significantly. This can be seen on the horizontal axis of the distribution of the output gap. With low c_2 the output gaps varies between much larger values than when c_2 is high. Thus, the intensity of output stabilization has a double effect: it reduces the variability of the output and it reduces the frequency with which extreme booms and busts occur as a result of extreme variation of animal spirits.

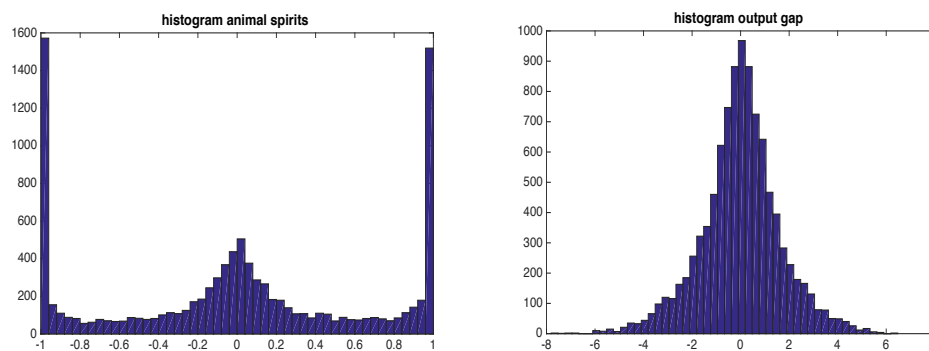
The result of this stabilization effort by the central bank is that fat tails become less fat leading to less intense booms and busts. Thus, there is a role for the central banks to stabilize the business cycle. Just keeping inflation low will not be sufficient. Central banks that pursue strict inflation targeting, without concern for output stabilization, maximize the probability of

boom-bust scenarios and the occurrence of fat tails. Paradoxically, as in boom-bust scenarios output becomes very volatile, inflation will also be volatile. It is therefore in the interest of a central bank concerned about price stability to actively stabilize output.

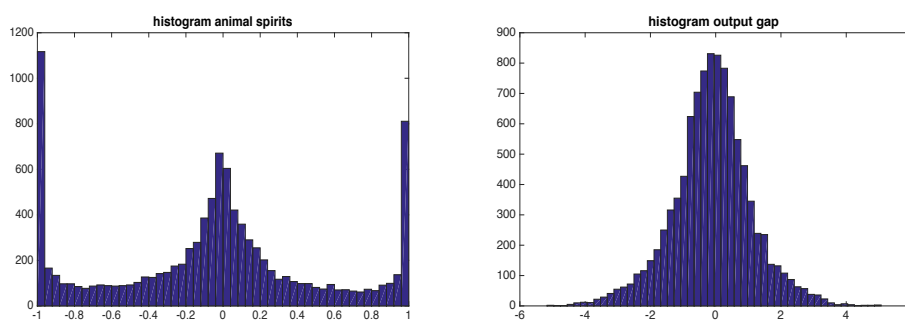
Figure 8: Frequency distribution animal spirits and output gap ($c_1=1.5$)
 $c_2=0$



$c_2=0.5$



$c_2=1$

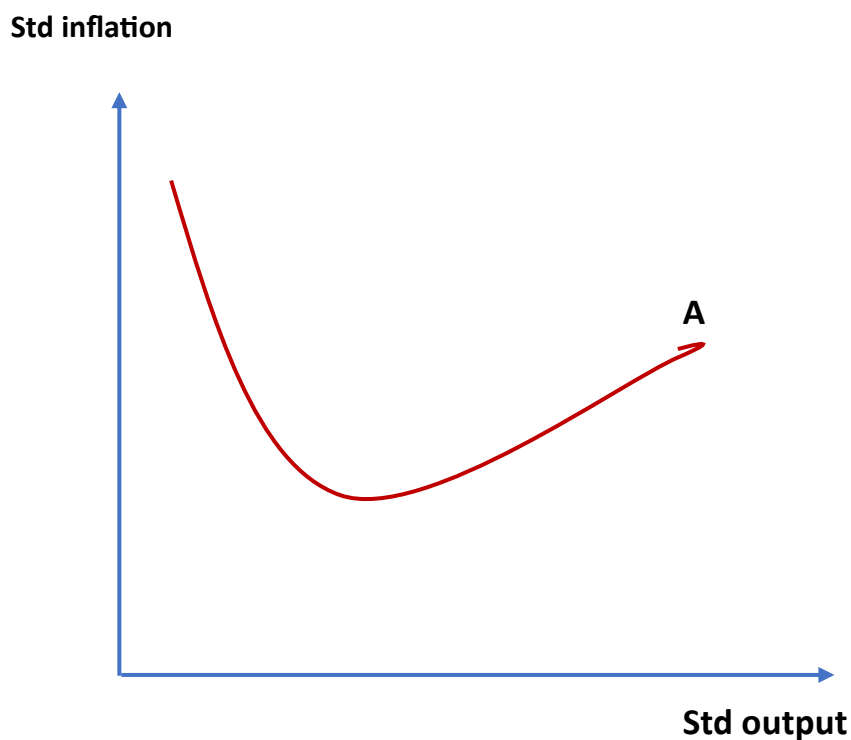


Mainstream DSGE macroeconomics has taken the view that apart from maintaining price stability, the task of the central bank is to reduce the inefficiencies arising from wage and price rigidities. Stabilizing output is motivated by the need to reduce inefficiencies. The idea that the central bank may be called upon to stabilize an otherwise unstable system is completely absent (Woodford(2003), Smets and Wouters(2003), Gali(2008)).

Here we have shown that there is a need to stabilize a system that is regularly gripped by waves of optimism and pessimism. These waves can lead to violent movements of output and employment. As a result, the need to stabilize runs much deeper and creates greater responsibilities of the monetary authorities than one obtains from mainstream macroeconomics.

Clearly, that does not eliminate trade-offs. In De Grauwe and Ji(2019) we derive the trade-offs between inflation and output stabilization. In contrast to RE models where these trade-offs are negative, (i.e. the pursuit of more output stability always leads to less inflation stability), we find in our behavioural model that this trade-off is non-linear. We show an example in Figure 9. This shows the standard deviation of inflation on the vertical axis and the standard deviation of output on the horizontal axis.

Figure 9: Trade-off volatility of inflation and of output



To understand this trade-off start from point A. This is the point where the central bank only pursues price stability, with no effort to stabilize output. When the central bank increases its intensity to stabilize output (by increasing the output coefficient in the Taylor rule) we move downwards on the trade-off curve. This means that by doing more output stabilization, the central bank reduces *both* output and inflation volatility. At some point, however, one hits

the minimum point on the trade-off curve. Further attempts to stabilize output will then lead to more inflation volatility. We then reach the standard negative segment on the trade-off. This leads to the conclusion that there is some optimal degree of output stabilization. It also leads to the conclusion that a no-output stabilization strategy is sub-optimal.

5. Conclusion

In this paper we have analyzed how radical uncertainty in its various appearances affects the movements of macroeconomic variables. We have argued that in a world of radical uncertainty there will be deviations from normality in the frequency distributions of macroeconomic variables. This then becomes a world of frequency distributions with fat tails. It is also a world in which the transmission of large shocks cannot be forecasted.

Climate scientists have made it clear that climate change will have dramatic effects on living conditions on our planet. Yet, there is considerable uncertainty about how and when these effects will hit us. There is, in other words, radical uncertainty about how and when the planet will be affected.

In a recent article Annicchiarico, et al. (2024) analyze the macroeconomic implications of climate change using a behavioural macroeconomic model similar to the one used here, i.e. it is a model where agents face cognitive limitations to understand the complexity of the world, and as a result use simple heuristics to make forecasts. This model produces similar business cycle behaviour, and departures from Gaussian distributions as those discussed earlier. These authors find that in such a model it will be more difficult to stabilize the economy and to keep inflation low when climate change occurs, compared to a model where agents are assumed to have Rational Expectations (RE). This is not really surprising. Agents with RE understand the nature of the climate change hitting them and take the necessary precautions in terms of saving and consumption, helping to keep the economy on a steadier path. When agents have cognitive limitations this becomes more difficult to achieve as boom-bust scenarios (fat tails) will undermine the stability of the economy.

Appendix: A simple behavioural macroeconomic model

1 Basic equations

We decided to use the simplest possible behavioural model, (for more complex models see e.g. Delli Gatti, et al. (2005)). The basic model consists of an aggregate demand equation, an aggregate supply equation and a Taylor rule as described by De Grauwe (2012, 2019 and 2020). The aggregate demand and supply equations can be derived from expected utility maximization of consumers and expected profit maximization of firms (Hommes and Lustenhouwer(2019) and De Grauwe and Ji(2019)). In De Grauwe and Ji(2019) we provide a microfoundation.

The aggregate demand equation obtained from this microfoundation can be expressed in the following way:

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + v_t \quad (1)$$

where y_t is the output gap in period t , r_t is the nominal interest rate, π_t is the rate of inflation and two forward looking components, $\tilde{E}_t \pi_{t+1}$ and $\tilde{E}_t y_{t+1}$. The tilde above E refers to the fact that expectations are not formed rationally. How exactly these expectations are formed will be specified in section 2.2.

The aggregate supply equation is represented in (2). This New Keynesian Philips curve includes a forward looking component, $\tilde{E}_t \pi_{t+1}$, and a lagged inflation variable. Inflation π_t is sensitive to the output gap y_t . The parameter b_2 measures the extent to which inflation adjusts to changes in the output gap.

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \quad (2)$$

The Taylor rule describes the central bank's behaviour in setting the interest rate. This behaviour can be described as follows:

$$r_t = (1 - c_3) [c_1 (\pi_t - \pi^*) + c_2 y_t] + c_3 r_{t-1} + u_t \quad (3)$$

where r_t is the interest rate in period t , π_t is the inflation rate, π^* is the target rate of inflation and y_t is the output gap.

This Taylor rule tells us that the central bank increases (reduces) the interest rate when currently observed inflation exceeds (falls short of) the target and when the currently

observed output gap is positive (negative). We assume that the central bank wants to smoothen interest rate changes (see Woodford (2003)).

There are error terms in each of the equations (1) to (3) which describe the nature of the different shocks that can hit the economy. They include demand shocks, v_t , supply shocks, η_t and interest rate shocks, u_t . These shocks are assumed to be normally distributed with mean zero and a constant standard deviation.

2 Expectations formation

We analyze how the forecast of output gap $\tilde{E}_t y_{t+1}$ and inflation $\tilde{E}_t \pi_{t+1}$ are formed in the model. The rational expectations hypothesis requires agents to understand the complexities of the underlying model and to know the frequency distributions of the shocks that will hit the economy. We take it that agents have cognitive limitations that prevent them from understanding and processing this kind of information. These cognitive limitations have been confirmed by laboratory experiments and survey data (see Branch, 2004; Hommes (2011, 2021)).

Forecasting the output gap

We assume two types of rules agents follow to forecast the output gap. A first rule is called a “fundamentalist” one. Agents use the steady state value of the output gap (which is normalized at 0) to forecast the future output gap. A second forecasting rule is a “naïve” extrapolative one. Following this rule, agents extrapolate the previous observed output gap into forecasting the future. The fundamentalist and extrapolator rules for output gap are specified as follows:

$$\tilde{E}_t^f y_{t+1} = 0 \quad (4)$$

$$\tilde{E}_t^e y_{t+1} = y_{t-1} \quad (5)$$

This kind of simple heuristic has often been used in the behavioral macroeconomics and finance literature where agents are assumed to use fundamentalist and chartist rules (see Brock and Hommes(1997), Branch and Evans(2006), Brazier et al. (2008)).

The market forecast can be obtained as a weighted average of these two forecasts, i.e.

$$\tilde{E}_t y_{t+1} = \alpha_{f,t} \tilde{E}_t^f y_{t+1} + \alpha_{e,t} \tilde{E}_t^e y_{t+1} \quad (6)$$

$$\alpha_{f,t} + \alpha_{e,t} = 1 \quad (7)$$

where $\alpha_{f,t}$ and $\alpha_{e,t}$ are the probabilities that agents use the fundamentalist and the naïve rule respectively.

We specify a switching mechanism of how agents adopt specific rule. Using discrete choice theory (see Anderson, de Palma, and Thisse, (1992) and Brock & Hommes (1997)) to work out the probability of choosing a particular rule (see De Grauwe and Ji(2019) for more detail. We obtain:

$$\alpha_{f,t} = \frac{\exp(\gamma U_{f,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})} \quad (8)$$

$$\alpha_{e,t} = \frac{\exp(\gamma U_{e,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})} \quad (9)$$

where $U_{f,t}$ and $U_{e,t}$ ¹ are the past forecast performance (utility) of using the fundamentalist and the naïve rules. The parameter γ measures the “intensity of choice”. It can also be interpreted as expressing a willingness to learn from past performance. When $\gamma = 0$ this willingness is zero; it increases with the size of γ .

Forecasting inflation

Agents also forecast inflation using a similar heuristic, with one rule that could be called a fundamentalist rule and the other a naïve extrapolative rule (see Brazier et al.(2008) for a similar setup). In an institutional set-up, the central bank announces an explicit inflation target. The fundamentalist rule will be called an “inflation targeting” rule described in (10), i.e. agents who have confidence in the credibility of the central bank use the announced inflation target to forecast inflation.

$$\tilde{E}_t^f \pi_{t+1} = \pi^* \quad (10)$$

where the inflation target is π^* . Agents who do not trust the announced inflation target use the naïve rule, which consists in extrapolating inflation from the past into the future. The “naïve” rule is defined by

$$\tilde{E}_t^e \pi_{t+1} = \pi_{t-1} \quad (11)$$

¹Note $U_{f,t} = -\sum_{k=0}^{\infty} \omega_k [y_{t-k-1} - \tilde{E}_{f,t-k-2} y_{t-k-1}]^2$ and $U_{e,t} = -\sum_{k=0}^{\infty} \omega_k [y_{t-k-1} - \tilde{E}_{e,t-k-2} y_{t-k-1}]^2$

The market forecast is a weighted average of these two forecasts, i.e.

$$\tilde{E}_t \pi_{t+1} = \beta_{f,t} \tilde{E}_t^f \pi_{t+1} + \beta_{e,t} \tilde{E}_t^e \pi_{t+1} \quad (12)$$

$$\beta_{f,t} + \beta_{e,t} = 1 \quad (13)$$

Where $\beta_{f,t}$ and $\beta_{e,t}$ are the probabilities that agents use the fundamentalist and the extrapolative rules respectively. The same selection mechanism is used as in the case of output forecasting to determine the probabilities of agents trusting the inflation target and those who do not trust it and revert to extrapolation of past inflation. This inflation forecasting heuristics can be interpreted as a procedure of agents to find out how credible the central bank's inflation targeting is. If this is credible, using the announced inflation target will produce good forecasts and as a result, the probability, $\beta_{f,t}$, that agents will rely on the inflation target will be high. If on the other hand the inflation target does not produce good forecasts (compared to a simple extrapolation rule) the probability that agents will use it will be small. Using the switching mechanism similar to the one specified in equations (8) and (9), we can compute the probability of choosing a particular rule.

$$\beta_{f,t} = \frac{\exp(\gamma U'_{f,t})}{\exp(\gamma U'_{f,t}) + \exp(\gamma U'_{e,t})} \quad (14)^2$$

$$\beta_{e,t} = \frac{\exp(\gamma U'_{e,t})}{\exp(\gamma U'_{f,t}) + \exp(\gamma U'_{e,t})} \quad (15)$$

The probability, $\beta_{f,t}$, that agents will rely on the inflation target to make inflation forecasts can also be interpreted as the fraction of agents who trust the central bank's inflation target.

The forecasts made by extrapolators and fundamentalists play an important role in the model. In order to highlight this role we define an index of market sentiments, which we call "animal spirits", and which reflects how optimistic or pessimistic these forecasts are (see also Franke and Westerhoff(2017)).

The definition of animal spirits is as follows:

$$S_t = \begin{cases} \alpha_{e,t} - \alpha_{f,t} & \text{if } y_{t-1} > 0 \\ -\alpha_{e,t} + \alpha_{f,t} & \text{if } y_{t-1} < 0 \end{cases} \quad (23)$$

² Note $U'_{f,t} = -\sum_{k=0}^{\infty} \omega_k [\pi_{t-k-1} - \tilde{E}_{f,t-k-2} \pi_{t-k-1}]^2$ and $U'_{e,t} = -\sum_{k=0}^{\infty} \omega_k [\pi_{t-k-1} - \tilde{E}_{e,t-k-2} \pi_{t-k-1}]^2$

where S_t is the index of animal spirits. This can change between -1 and +1. There are two possibilities:

- When $y_{t-1} > 0$, extrapolators forecast a positive output gap. The fraction of agents who make such a positive forecasts is $\alpha_{e,t}$. Fundamentalists, however, then make a pessimistic forecast since they expect the positive output gap to decline towards the equilibrium value of 0. The fraction of agents who make such a forecast is $\alpha_{f,t}$. We subtract this fraction of pessimistic forecasts from the fraction $\alpha_{e,t}$ who make a positive forecast. When these two fractions are equal to each other (both are then 0.5) market sentiments (animal spirits) are neutral, i.e. optimists and pessimists cancel out and $S_t = 0$. When the fraction of optimists $\alpha_{e,t}$ exceeds the fraction of pessimists $\alpha_{f,t}$, S_t becomes positive. As we will see, the model allows for the possibility that $\alpha_{e,t}$ moves to 1. In that case there are only optimists and $S_t = 1$.
- When $y_{t-1} < 0$, extrapolators forecast a negative output gap. The fraction of agents who make such a negative forecasts is $\alpha_{e,t}$. We give this fraction a negative sign. Fundamentalists, however, then make an optimistic forecast since they expect the negative output gap to increase towards the equilibrium value of 0. The fraction of agents who make such a forecast is $\alpha_{f,t}$. We give this fraction of optimistic forecasts a positive sign. When these two fractions are equal to each other (both are then 0.5) market sentiments (animal spirits) are neutral, i.e. optimists and pessimists cancel out and $S_t = 0$. When the fraction of pessimists $\alpha_{e,t}$ exceeds the fraction of optimists $\alpha_{f,t}$ S_t becomes negative. The fraction of pessimists, $\alpha_{e,t}$, can move to 1. In that case there are only pessimists and $S_t = -1$.

We can rewrite (23) as follows:

$$S_t = \begin{cases} \alpha_{e,t} - (1 - \alpha_{e,t}) = 2\alpha_{e,t} - 1 & \text{if } y_{t-1} > 0 \\ -\alpha_{e,t} + (1 - \alpha_{e,t}) = -2\alpha_{e,t} + 1 & \text{if } y_{t-1} < 0 \end{cases} \quad (24)$$

3. Solving the model

The solution of the model is found by first substituting (3a) into (1a) and rewriting in matrix notation. This yields:

$$\begin{aligned}
& \begin{bmatrix} 1 & -b_2 \\ -a_2c_1 & 1 - a_2c_2 \end{bmatrix} \begin{bmatrix} \pi'_t \\ y_t \end{bmatrix} \\
&= \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \widetilde{E}_t \pi'_{t+1} \\ \widetilde{E}_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 1 - b_1 & 0 \\ 0 & 1 - a_1 \end{bmatrix} \begin{bmatrix} \pi'_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ a_2c_3 \end{bmatrix} r'_{t-1} \\
&+ \begin{bmatrix} \eta_t \\ a_2u_t + \varepsilon_t \end{bmatrix}
\end{aligned}$$

i.e.

$$\mathbf{AZ}_t = \mathbf{B}\widetilde{\mathbf{E}}_t \mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{b}r'_{t-1} + \mathbf{v}_t \quad (25)$$

where bold characters refer to matrices and vectors. The solution for \mathbf{Z}_t is given by

$$\mathbf{Z}_t = \mathbf{A}^{-1}[\mathbf{B}\widetilde{\mathbf{E}}_t \mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{b}r'_{t-1} + \mathbf{v}_t] \quad (26)$$

The solution exists if the matrix \mathbf{A} is non-singular, i.e. $(1-a_2c_2)-a_2b_2c_1 \neq 0$. The system (26) describes the solutions for y_t and π'_t given the forecasts of y_t and π'_t . The latter have been specified in equations (7) and (17) and therefore can be substituted into (26). Finally, the solution for r'_{t-1} is found by substituting y_t and π_t obtained from (26) into (3a).

The model has non-linear features making it difficult to arrive at analytical solutions. That is why we will use numerical methods to analyze its dynamics. In order to do so, we have to calibrate the model, i.e. to select numerical values for the parameters of the model. In Table 1 the parameters used in the calibration exercise are presented. The values of the parameters are based on what we found in the literature (see Gali(2008) and Blattner and Margaritov(2010)). The model was calibrated in such a way that the time units can be considered to be quarters. The three shocks (demand shocks, supply shocks and interest rate shocks) are independently and identically distributed (i.i.d.) with standard deviations of 0.5%. These shocks produce first moments of the output gap and inflation that mimic the first moments found in the empirical data (see Reifschneider and Williams(1999) and Chung, et al. (2012)).

Table 1: Parameter values of the calibrated model

$a_1 = 0.5$	coefficient of expected output in output equation
$a_2 = -0.2$	interest elasticity of output demand
$b_1 = 0.5$	coefficient of expected inflation in inflation equation
$b_2 = 0.05$	coefficient of output in inflation equation
$c_1 = 1.5$	coefficient of inflation in Taylor equation
$c_2 = 0.5$	coefficient of output in Taylor equation
$c_3 = 0.8$	interest smoothing parameter in Taylor equation
$\gamma = 2$	intensity of choice parameter
$\sigma_\varepsilon = 0.5$	standard deviation shocks output
$\sigma_\eta = 0.5$	standard deviation shocks inflation
$\sigma_u = 0.5$	standard deviation shocks Taylor
$\rho = 0.5$	measures the speed of declining weights in mean squares errors (memory parameter)

Note: the parameter values used here are similar to the ones obtained in Kukacka and Sacht (2022).

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