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Material Source and Waste Taxes in Competitive Equilibrium

Abstract

We develop a framework for the representation of material flows in competitive equilibrium. Material balances track material flows, which adjust endogenously to economic transactions. We assume negative environmental effects of resource extraction and waste deposition and show that taxing resource extraction restores efficiency. Taxing waste, where generated, only restores efficiency if producers minimize users' costs of their products, or if there is a dense set of goods with varied material content. We set up the general model structure and use a stylized 3-sector model for illustration. Finally we develop a quantitative stylized assessment of global steel and fossil fuel use.

JEL-Codes: H210, Q290.

Keywords: material balances, material policies, waste policies, upstream versus downstream.

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1 Introduction

Global material consumption is expected to double in the coming 40 years. Indeed, materials are a fundamental component of our economy and its growth. The surge in material use, however, also carries an array of sustainability challenges. Environmental degradation by waste calls for attention, and resource exploitation, including fossil resources and subsequent greenhouse gas emissions, adversely affects climate change (OECD, 2019). Resource scarcity threatens the energy transition, particularly the demand for minerals crucial for renewable energy generation and storage. Material awareness has become integrated into the economic strategies of numerous countries, also resonating in initiatives such as the European Green Deal. Yet there remains a gap between economic models that focus on value of production and consumption, and industrial ecology models that describe the material needs. We contribute by developing a framework for consistent material accounting in a standard economic equilibrium model (Stahel, 2016).

Our analysis elaborates on the recent development of climate-economy models. Since the pioneering work of Nordhaus (1993) and its subsequent inclusion in a standard macro framework by Golosov et al. (2014), the literature in climate economics has understood the need for a good representation of physical material flows, e.g. fossil fuel extraction, processing, and combustion in models that have at their core economic value creation and consumption. For instance, Hassler and Krusell (2012) describe the carbon content of fossil fuels produced, traded, and consumed. Barrage (2020) has more details of upstream material flows. In the climate-economy context, tracking materials is essential for the various policy discussions, such as the debate over consumption versus territorial emissions accounting. We set up a structure for a general tracking of material flows in an economic model. The resulting type of model is key to address sustainability issues going beyond climate change.

Our analysis stands in a long tradition. Incorporating mass balances in macroeconomic models was considered by Ayres and Kneese (1969) and Noll and Trijonis (1971), who deemed it crucial for a comprehensive understanding of resource scarcity and environmental impacts of economic activities. Recent literature focuses on circular economy policies within macroeconomic models (Pittel et al., 2010; Zhou and Smulders, 2021). Yet while macroeconomic models have proven of central significance in assessing environmental policies, to this day they lack physical consistency (Daly, 1997; Couix, 2020).

Ex-ante economic evaluations of environmental policies rely heavily on CGE models (Ibenholt, 2002; McCarthy et al., 2018). They are appreciated for their capacity to describe the intricate interdependencies among sectors, exemplified by the Social Accounting Matrices (SAM) that make up the models' core, reporting the value of deliveries and transfers between all producers, consumers and institutions. Yet CGEs do not represent material flows. Occasionally their outcomes are converted into proxy material flow scenarios, using weight conversion factors and soft-linking economic models with industrial ecology models. We write 'proxy' material flows as these ex-post calculations typically lack material balance consistency. Illustrations include Ibenholt (2003) and Sjöström and Östblom (2010) for waste generation, Masui (2005) who examines the combined impact of reducing greenhouse gas emissions and waste generation, Nechifor et al. (2020) who showcases the potential of available steel scrap as a green input into steel production, and Dellink (2020) who analyzes a global circular economy policy package.

In this study, we set up a Computable General Equilibrium (CGE) model that integrates economic flows with endogenous material flows. We preserve substitution options typical for economic models, fundamental balances for both the economic and material domains, and consistency between the two. Our framework provides a precise description of the relationship between transaction values and material flows, rendering it particularly useful for ex ante analysis of environmental policies.

We complement the SAM by a Physical Input Output Table (PIOT). This table tracks materials throughout the lifecycle, from sourcing to waste accumulation, as emphasized by Ibenholt (2003). We specify the links between both tables, through endogenous variables, and articulate the endogenous adjustments that facilitate internal consistency within each table. We keep the full flexibility of production functions common in CGEs. Our approach thus complements a strand of literature that interprets material balances as constraints on production functions in economic models (Krysiak and Krysiak, 2003; Baumgärtner, 2004). In particular, Krysiak and Krysiak (2003) introduce mass and energy conservation when modeling production and consumption, imposing strict correspondence between material flows and economic goods, including waste. Their approach constrains the set of permissible production functions. However, to the best of our knowledge, their analytical framework has not found practical application in studies. Our approach is more flexible, allowing for the endogenous adjustment of the material content of products while keeping substitution opportunities in terms of value.

Our approach induces an interpretation of the value vis-a-vis the material content as a quality characteristic of a product. A higher-quality product associates more value per unit of material. The method we set up thus naturally connects to the literature on product quality and hedonic pricing of goods Rosen (1974). Our results on downstream taxes is consistent with previous findings that considering quality modifies the competitive equilibrium outcome (Leland, 1977; Drèze and Hagen, 1978).

A key insight that comes out of our framework is the recognition that upstream and downstream material taxes are transmitted differently through the economy. Contrary to conventional wisdom, our findings underscore that up and downstream taxes of materials are not always equivalent. We show that taxing waste where generated does not restore efficiency, in general, while taxing resource extraction does. Equivalence between up and downstream taxes can be restored with hedonic pricing of material contents for goods, or constructing a complete set of all possible goods with varied material content. But even then, efficiency requires full price information to be available to all agents, also for goods not traded in equilibrium.

The analysis of up versus downstream taxes contributes to an extensive micro-economic literature on optimal instruments for controlling waste accumulation and efficient incentives for recycling. The predominant viewpoint advocates for price instruments (Sigman, 1995; Palmer and Walls, 1997; Eichner and Pethig, 2001; Pittel et al., 2010), with a particular emphasis on a deposit refund program targeted at consumers (downstream). Such a program is expected to reduce incentives for illegal dumping (Fullerton and Kinnaman, 1995) and is deemed the least costly policy to enforce (Palmer et al., 1997). However, alternative studies have nuanced this perspective, advocating for the use of multiple instruments (Walls and Palmer, 2001) and highlighting situations where the deposit-refund program may not be optimal (Calcott and Walls, 2000, 2005), especially when considering the recyclability of products. For instance, Calcott and Walls (2000) introduces an upstream instrument. In the context of a climate macroeconomic framework, and focusing on the specifics of fossil fuel combustion, Hassler and Krusell (2012) argue in favor of the implementation of upstream carbon taxation, noting that taxes on consumption have limited effectiveness due to income effects. Our analysis provides a new argument why under certain conditions upstream material regulation can outperform downstream regulation.

The remainder of the paper is organized as follows: Section 2 provides a detailed description of the model, encompassing both its economic and physical components. The theoretical insights from a small general equilibrium model are presented in Section 3. Subsequently, in Section 4, we offer insights derived from calibrated simulations, specifically focusing on the use of iron and fossil fuels in the world economy. Concluding remarks are presented in Section 5. A more detailed description of the model and its algorithm, proofs and the data we use are available in the appendix.

2 Model

2.1 Economic equilibrium

In this section, we introduce a standard model of an economy with firms producing goods $i \in I$, factors of production $f \in F$, and households and institutions $h \in H$ (also referred to as consumers). We use vector notation, e.g. p are prices, and p_I is the sub-vector for good prices, while p_F is the sub vector for factor prices, and p_i, p_f is the price for one good or factor, respectively. We describe taxes on virgin resource use and waste generation. We set up a parsimonious economy, e.g. a more generic version would support more taxes and transfers between institutions and households. We assume that the first consumer receives resource and waste tax revenues, while we abstract from income distributional concerns, need of public revenues for public consumption, and other taxes such as on intermediates, value added, etc. The purpose of the analysis is to assess conditions for material taxes to implement efficient resource use when material use causes a negative externality, e.g. through waste.

We have the following variables: Y_I for output by firms, where we may omit the subscript I for convenience as we do not use the symbol Y for other agents. $X_{J,I}$ for intermediates from firms J to firms I, using the symbol J equivalently (alias) to I. $L_{F,I}$ is factor use by firms, and $\Omega_{F,H}$ is factor supply by consumers. We write $\Omega_{H,F}$ for the inverted matrix of $\Omega_{F,H}$. $C_{I,H}$ is consumption by households. $T_{R/W}$ is taxes on resource use or waste generation with $\tau_{R/W}$ for the per-unit resource/waste tax.

The economy structure is summarized in the following Social Accounting Matrix, detailed below by agents, goods, and factors. The dot-multiplication refers to element-wise multiplication, e.g. $p_J \cdot X_{J,I}$ is the matrix for the value of intermediate deliveries from sectors J (rows) to I (columns).

Table 1: Social Accounting Matrix

	Firms (I)	Factors (F)	Consumers (H)	
J	$p_J \cdot X_{J,I}$		$p_J \cdot C_{J,H}$	Revenues
F	$p_F \cdot L_{F,I}$			Factor costs
H	T_I	$\Omega_{H,F} \cdot p_F$		Income
	Costs	Factor Income	Expenditures	

Section 4 provides a CGE model with specified production and utility functions, deliberately kept simple through Cobb-Douglas production and utility, one region, etc. The aim is to provide the proof of concept such that results remain tractable. It is obvious from the presentation that the integration of material balances into the CGE does not depend on these simplifying assumptions. General CES structures, trade, dynamic integration, etc., are straightforwardly accommodated.

Firms $i \in I$ produce, given a CRS technology, $Y_i = F_i(X_{I,i}, L_{F,i})$, with $X_{J,i} = (X_{j,i})_{j \in I}$, $L_{F,i} = (L_{f,i})_{f \in F}$ intermediate deliveries and factor use, respectively. Production requires virgin resource use of some material m, $R_{m,i} \geq 0$ and produces waste $W_{m,i} \geq 0$. Output Y_i is sold at price p_i, and firms maximize their profit taking into account prices of intermediary deliveries, remuneration of factors, taxes on intermediary inputs and taxes on extraction/waste:

$$\Pi_i = p_i Y_i - p_I X_{I,i} - p_F L_{F,i} - \tau^R_{M,i} R_{M,i} - \tau^W_{M,i} W_{M,i} \tag{1}$$

where we used vector notation for intermediate costs $p_I X_{I,i} = \sum_{j \in J} p_j X_{j,i}$, factor costs $p_F L_{F,i} = \sum_{f \in F} p_f L_{f,i}$, resource taxes $\tau^R_{M,i} R_{M,i} = \sum_m \tau^R_{m,i} R_{m,i}$, and waste taxes $\tau^W_{M,i} W_{M,i} = \sum_m \tau^W_{m,i} W_{m,i}$ from industrial process waste $W_{m,i}$. Note that $\tau^R_{m,i}$ is a per unit tax on resource use for material m in sector i, $\tau^W_{m,i}$ is a per unit waste tax for material m produced in sector i.

We denote by $x_{j,i} = X_{j,i}/Y_i$, $l_{f,i} = L_{f,i}/Y_i$ per output unit inputs of intermediates and factors, while we assume that (virgin) material extraction is proportional to output, and waste production is proportional to all material entering the production process:

$$\rho_{M,i} = \frac{R_{M,i}}{Y_i}, \tag{2}$$

$$\epsilon_{M,i} = \frac{W_{M,i}}{\sum_j X_{M,j,i} + R_{M,i}}, \tag{3}$$

where the subscript M in $X_{M,i,j}$ denotes that we consider *material* flows between sectors measured in tons, rather than in monetary units. We can assume constant virgin resource intensity, $\rho_{M,i}$, or endogenous, and similarly for waste $\epsilon_{M,i}$. The general production function becomes

$$Y_i = F_i(X_{I,i}, L_{F,i}; \rho_{M,i}, \epsilon_{M,i}) \tag{4}$$

with CRS with respect to the vector $X_{I,i}, L_{F,i}$. The structure allows for a flexible description; it encompasses the case when material is embedded in e.g. a production factor. For instance, consider mining which has iron ore as a production factor, then the virgin material inflow of iron corresponds one to one to the factor inflow: $\rho_{m,i} = l_{f,i}$. But the notation also allows to leave out iron ore from the model specifics. The notation can also describe a resource as production factor where the producer can choose how much material to extract from the resource. In terms of notation, it implies that we do not describe material as embedded in production factors, but as an input in production decided as an integral part of the production process. The material accounting also allows for the case when there is no material in factors: $L_{M,F,I} = \Omega_{M,F,H} = 0$.

The material intensity of intermediates is denoted by by $\theta^X_{M,i,j} = X_{M,i,j}/X_{i,j}$ and cannot be controlled by the receiving firm. Thus, one unit of intermediate deliveries from sector j to sector i leads to $\epsilon_{m,i}\theta^X_{m,i,j}$ units of waste of material m, while one unit of output leads to $\epsilon_{m,i}\rho_{m,i}$ units of waste of material m.

Firms maximize profits (1) subject to technology (4), which gives as first order conditions a mapping $p \to (\psi_I, x, l)$ with ψ_I the unit cost price

$$\psi_i(p_I, p_F, \tau) = \min p_J x_{J,i} + p_F l_{F,i} + \tau^R_{M,i}\rho_i + \tau^W_{M,i}\epsilon_{M,i}\left(\rho_{M,i} + \theta^X_{M,J,i} \cdot x_{J,i}\right) \tag{5}$$

where $X_{J,i} = Y_i x_{J,i}$, $L_{F,i} = Y_i l_{F,i}$ and subject to $F(x_{I,i}, l_{F,i}) = 1$. From the FOCs we can derive production intensities and ψ_i unit production costs inclusive of material taxes associated with virgin resource use.

$$x_{j,i} = \frac{\partial \psi_i}{\partial (p_j + \tau^W_{M,i} \epsilon_{M,i} \theta^X_{M,j,i})} \tag{6}$$

$$l_{f,i} = \frac{\partial \psi_i}{\partial p_f} \tag{7}$$

Note that costs for waste associated to intermediate use is included in the first order conditions. In our illustrative calculations, we have raw resource intensity ρ fixed, so that the tax costs associated to resource use is added post-optimization (as in a Leontieff production function). Finite production requires non-positive profits: $\psi_i \geq p_i \perp Y_i \geq 0$. In almost all cases, we have positive output $Y_i > 0$ and $\psi_i = p_i$.

Households $h \in H$ maximize utility through consumption $C_{i,h}$ of good i, subject to their budget constraint. For all consumers, $h \in H$:

$$\max U_h = U(C_{I,h}) \tag{8}$$

$$\text{s.t. } p_I C_{I,h} + \tau^W_{M,h} \theta^C_{M,I,h} C_{I,h} \leq p_F \Omega_{F,h} + T_h \tag{9}$$

with $U(.)$ CRS, $C_{I,h} = (C_{i,h})_i$, where we have added tax on households waste $\tau^W_{M,h}$, factor income $\Omega_{F,h}$; and, T_h are gross tax receipts per consumer.

The material intensity of consumption is denoted by $\theta^C_{M,i,h} = C_{M,i,h}/C_{i,h}$ and is a good characteristic that affects the value of the good, but that is not controlled by the consumer. In this model, we assume that all consumption is disposed of in the same period, so that each unit of material consumption leads to one unit of waste.

First order conditions define a mapping for the cost of one utility unit and relative consumption $(p, \tau) \rightarrow (\psi_H, c_H)$:

$$\psi_h(p_I, \tau^W_{M,h}) = \min p_I c_{I,h} + \tau^W_{M,h} \theta^C_{M,I,h} c_{I,h} \tag{10}$$

subject to $U(c_{I,h}) = 1$, and

$$c_{i,h} = \frac{\partial \psi_h}{\partial (p_i + \theta^C_{M,i,h} \tau^W_{M,h})} \tag{11}$$

where ψ_h are utility unit production costs, and $c_{i,h} = C_{i,h}/U_h$ are per unit utility.

We define the government g as a particular consumer whose budget constraint (9) only has tax revenues T_h as income.

Definition 1. *A* ***competitive equilibrium*** *is conditioned by policies* τ_M^R, τ_M^W*, and supported by price vector* $p = (p_I, p_F)$*. Firms maximize profits with output levels* Y_I*, intermediate deliveries* $X_{j,i} = Y_i x_{j,i}$*, factor use* $L_{f,i} = Y_i l_{f,i}$ *and feasible production* $Y_i = F(X_{J,i}, L_{F,i}; \rho_{M,i}, \epsilon_{M,i})$*. Consumers maximize utility given budget constraints. We assume factor supply* $\Omega_{f,h}$ *inelastic. Constant returns to scale imply zero profits and unit production costs equal prices:* $\psi_I = p_I$*. The good and factor balances require demand equals supply:*

$$\sum_j X_{i,j} + \sum_h C_{i,h} = Y_i \tag{12}$$

$$\sum_i L_{f,i} = \sum_h \Omega_{f,h}. \tag{13}$$

2.2 Material balances

Balance of materials We consider the environment as a source where extracting firms retrieve virgin natural resources R, and, as a sink when due to material loss during production and consumption, all ends up as waste W. In this section, we forego material content of accumulated capital. Thus we consider the environment and economy as a closed system for each material $m \in M$:

$$R_M = W_M \tag{14}$$

The inflow-outflow balance also holds at the level of each good market and household. For each good material embedded in supply must equal material embedded in demand. For each household, material in consumption goods must equal material in waste. The Physical Input Output Table (PIOT, Table 2) tracks these balances.

Table 2: Physical Input Output Table

	Firms (I)	Cons (H)	Environment	Outflow
J H	$X_{M,J,I}$	$C_{M,J,H}$	$W_{M,J}$ $W_{M,H}$	Outflow by J Outflow by H
Env	R_M			Extraction
	Inflow by I	Inflow by H	Waste	

The first row and column represent all material outflows and inflows for firm j. The row reports supply $Y_{M,j}$ which is either used by other firms as intermediates $X_{M,j,I} = \theta^X_{M,j,I} \cdot X_{j,I}$ or consumption $C_{M,i,H} = \theta^C_{M,j,H} \cdot C_{j,H}$. In addition to the material embedded in goods, the firm also produces waste $W_{M,i}$, proportional to virgin resource inputs and material in intermediate inputs, $W_{M,j} = \epsilon_{M,j}(R_{M,j} + \sum_i X_{M,i,j})$. The first column presents the material inflow of firm i. This comes from material embedded in the used intermediates ($X_{M,J,i} = \theta_{X,J,i} \cdot X_{J,i}$) and resource use $R_{M,i} = \rho_{M,i} Y_i$. As the outflow (row-sums) must equal the inflow (column-sums), we have

$$\sum_j X_{M,i,j} + \sum_h C_{M,i,h} + W_{M,i} = \sum_j X_{M,j,i} + R_{M,i} \tag{15}$$

which we can rewrite as

$$\sum_j \theta^X_{M,i,j} X_{i,j} + \sum_h \theta^C_{M,i,h} C_{i,h} = (1 - \epsilon_{M,i}) \left(\sum_j \theta^X_{M,j,i} X_{j,i} + \rho_{M,i} Y_i \right) \tag{16}$$

where we used the identity for waste for firms

$$W_{M,i} = \epsilon_{M,i} \left(\sum_j \theta^X_{M,j,i} X_{j,i} + \rho_{M,i} Y_i \right) \tag{17}$$

The second row and column report the out and inflow for households. Waste as outflow balances the material embedded in consumed goods:

$$W_{M,h} = C_{M,h} = \sum_i \theta_{C,i,h} C_{i,h} \tag{18}$$

The third row and column report the outflow and inflow from and to the environment. Virgin resource use equals waste, as in (14), which in terms of economic flows becomes:

$$\sum_i R_{M,i} = \sum_i W_{M,i} + \sum_h W_{M,h} \Leftrightarrow \tag{19}$$

$$\sum_i \rho_{M,i} Y_i = \sum_i \epsilon_{M,i} (\rho_{M,i} Y_i + \theta^X_{M,J,i} X_{J,i}) + \sum_h \theta^C_{M,I,h} C_{I,h}. \tag{20}$$

Endogenous material intensities When the economy adjusts to changing prices, resource and waste taxes, material intensity θ of intermediate and consumption flows also adjust.

The standard assumption is that goods are homogeneous with unchanging properties between scenarios. In our model, that would imply the material intensity of goods to be independent of use and scenario. Yet such an assumption contrasts the purpose of studying policies on material use, which aim at both the extensive and intensive margin: reducing supply and demand for material-intensive goods and also reducing material intensity of goods.

In the model proposed by Krysiak and Krysiak (2003), material and economic flows are identical, thus matching with the idea of exogenous material intensities that are kept equal to one. By keeping material balance and consistency, they show that defining a general equilibrium is much more constrained and forbids the use of some usual production functions. Practically, fixed material intensities restrain the model to Leontief economies, and/or significantly increases the complexity of the economic model if generality is to be retained. The modeling framework in this paper aims at being as general as possible, as in Krysiak and Krysiak (2003), but also retaining standard economic modelling assumptions.

Thus, here we relax the assumption of constant material intensities (between scenarios) and allow for endogenous intensities. Below, we go one step further and also relax independence of material intensity with respect to users.

As special case consider the assumption that goods are homogeneous, specifically their characteristics such as material intensity does not depend on the user:

Assumption 1.a (Uniform material intensity over all users)**.** *Material intensities are independent of the use of a good:*

$$\theta^X_{M,i,j} = \theta^C_{M,i,h} = \theta_{M,i} \tag{21}$$

In practice, the assumption is empirically falsified as pointed out by McCarthy et al. (2018): when constructing the PIOT, we observe different material intensities for goods produced by the same sector, or firm, dependent on the user, be it consumer or other firm. A car manufacturer may produce light cars and heavy vehicles, sold to different types of customers. Thus we want the flexibility to allow for $\theta_{X,i,j} \neq \theta_{X,i,j'} \neq \theta_{C,i,h}$. We implement this flexibility allowing for a weaker assumption that includes the above case, for which we will show that it still uniquely determines the material intensity in equilibrium:

Assumption 1.b (Heterogeneous material intensities)**.** *When material intensities adjust in equilibrium, they will do so uniformly over all users of* *good i. Let the adjustment be labeled $\lambda_{M,i}$; in formula's the assumption can be written as:*

$$\theta^X_{M,i,j} = \lambda_{M,i}\overline{\theta}^X_{M,i,j} \; ; \theta^C_{M,i,h} = \lambda_{M,i}\overline{\theta}^C_{M,i,h} \tag{22}$$

where the $\overline{\theta}$ is the benchmark material intensity observed in the baseline scenario.

Note that Assumption 1.a is a special case of Assumption 1.b. Our first result is that we show that above assumption uniquely defines the material flows, given a competitive equilibrium.

Lemma 1. *Under Assumption 1.b, given a competitive equilibrium, a unique competitive equilibrium with material balances exists, that is a unique λ so that balances (16) hold.*

Proof. Consider an allocation X, Y, C, ρ, ϵ and reference material intensities $\overline{\theta}$. Define the mapping $\lambda_k \to \lambda_{k+1}$ to generate a sequence for $k = 0, 1, 2, ...$:[1]

$$\lambda_{k+1,i} = \frac{(1-\epsilon_{M,i})\left(\sum_j \lambda_{k,j}\overline{\theta}^X_{M,j,i} \cdot X_{j,i} + \rho_{M,i}Y_i\right)}{\sum_j \overline{\theta}^X_{M,i,j} \cdot X_{i,j} + \sum_h \overline{\theta}^C_{M,i,h}C_{i,h}}. \tag{23}$$

Start the sequence with $\lambda_0 = 0$. Note that if $\lambda_{k+1} > \lambda_k$, then $\lambda_{k+2} > \lambda_{k+1}$, thus by construction the sequence λ_k is strictly increasing and $\lambda_k \geq 0$. Define norms for λ as $|\lambda|_X = \sum_{i,j} \lambda_i \overline{\theta}^X_{M,i,j} \cdot X_{i,j}$ and $|\lambda|_C = \sum_{i,h} \lambda_i \overline{\theta}^C_{M,i,h} \cdot C_{i,h}$. Now consider the increase in norms:

$$|\lambda_{k+2}|_X - |\lambda_{k+1}|_X < |\lambda_{k+2}|_X - |\lambda_{k+1}|_X + |\lambda_{k+2}|_C - |\lambda_{k+1}|_C \tag{24}$$

$$= \sum_i (1-\epsilon_{M,i})\left(\sum_j (\lambda_{k+1,j} - \lambda_{k,j})\overline{\theta}^X_{M,j,i} \cdot X_{j,i}\right) \tag{25}$$

$$\leq |\lambda_{k+1}|_X - |\lambda_k|_X \tag{26}$$

where the first strict inequality follows from C non-zero, and the second (weak) inequality follows from $\epsilon \geq 0$. The norm is additive ($|\lambda + \mu|_X = |\lambda|_X + |\mu|_X$), and as some waste is produced ($\exists i, \epsilon_{M,i} > 0$), there is $0 < \phi < 1$ such that $|\lambda_{k+2} - \lambda_{k+1}|_X < \phi|\lambda_{k+1} - \lambda_k|_X$. It follows that λ is a Cauchy sequence in $\mathbb{R}^I$ with norm $|\cdot|_X$, and thus converges.

Hence, there is a limit $\tilde{\lambda}$ such that we have material balance with equation (23). It is unique, given the definition of the sequence.

□

[1] For the sake of clarity, we drop the subscript M on λ

We now can extend the equilibrium Definition 1 to cover material flows. The standard price-taking behavior of firms assumes the firms minimize costs of producing one unit of output. For reference we state this as explicit assumption:

Assumption 2.a (Minimizing production costs)**.** *Firms minimize costs of production as in:*

$$p_i = \min\{p_J x_{J,i} + p_F l_{F,i} + \tau^R \rho_i + \tau^W \epsilon_i \theta'_i\} \tag{27}$$

$$s.t.\ \theta'_i = \rho_i + \sum_j \theta_{j,i} x_{j,i} \tag{28}$$

where θ'_i is the material inflow intensity into the sector.

The assumption, standard in CGEs, can be challenged when material intensities of goods affect the user value, because users have to pay waste taxes based on material content. If the representative firm represents a set of firms, each of which may have a slightly different material intensity, and buyers do take into account the costs they will have to incur themselves, then only suppliers survive that include those costs in their cost minimization (Pommeret and Pottier, 2024). In an economy with intermediate production, we have to define downstream waste taxes recursively. As alternative for the production cost minimization we list:

Assumption 2.b (Minimizing user costs)**.** *Firms minimize costs for users:*

$$\min\{p_i + \mu_i \theta_i\} \tag{29}$$

$$s.t.\ \theta_i = (1 - \epsilon_i)\theta'_i \tag{30}$$

where p_i is defined by (27), θ'_i by (28), and the second equation states that material content of the produced good equals the material input minus waste within the production process. θ_i newly defined is the material outflow intensity in the sector. The term μ_i measures all downstream waste taxes, caused by material content in good i, as defined in equilibrium through:

$$\mu_i \theta_i Y_i = \sum_h \tau_h^W \theta_{i,h} C_{i,h} + \sum_j (\tau_j^W \epsilon_j + \mu_j (1 - \epsilon_j))\theta_{i,j} X_{i,j}. \tag{31}$$

We remain agnostic whether production or user cost minimization is the most natural assumption. We note that standard CGEs apply production cost minimization. Assuming user cost minimization requires a substantial revision of the framework on top of the material flow tracking specified above. We assume throughout that either one of the two assumptions above holds.

Definition 2. *A* ***competitive equilibrium with material balances*** *is a competitive equilibrium as in Definition 1, with material intensities satisfying Assumption 1.b, so that the global balance* (14) *and the material balances per firm* (16) *hold.*

Then, finally, we can state the equilibrium and policy efficiency properties.

Theorem 1. *Any competitive equilibrium with material balances and non-negative resource taxes uniform over all sectors $\tau^R_{M,I} = \tau^R_M \geq 0$ and absent waste taxes $\tau^W_{M,I} = 0$ implements a cost-efficient allocation, for both production and user cost minimizing firms.*

When waste taxes are strictly positive and uniform over all sectors $\tau^W_{M,I} = \tau^W_M > 0$, the allocation is not cost-efficient if firms are production cost minimizing (Ass 2.a). The allocation is cost-efficient when producers minimize user costs (Ass 2.b).

Given user-cost minimization (Ass 2.b), the equilibrium allocation only depends on total resource plus waste taxes, $\tau^R_M + \tau^W_M$, and is independent of the division.

The proof is in the appendix. The term cost-efficient refers to the exogenously set resource use - waste generation. By increasing the resource or waste tax we can reduce resource use. Any resource tax corresponds to another cost-efficient allocation. We define cost-efficiency formally:

Definition 3. *An allocation Y, X, C, L is cost-efficient if it maximizes $\sum_h \nu_h U_h$ for some welfare weights ν_h, for some exogenously given level of virgin resource use that equals waste ($R = W$).*

The proof of cost efficient resource taxes and waste taxes in case of user cost minimizing producers is provided in the appendix. The proof that waste taxes do not implement a cost-efficient allocation when firms minimize production costs is based on the construction of a counter-example, provided in the following section. That is, we construct a very simple economy where we can derive the analytical solution and thus derive the set of feasible equilibrium allocations. For that economy, we indeed find that resource taxes implement the cost-efficient allocation, while waste taxes result in strictly lower utility for the same resource use and waste generation, given production cost minimization. The illustrative economy will also help understand the mechanism through which Pigouvian waste taxes do not deliver efficiency. It illustrates the price-taking assumption in Definition 2; if agents foresee demand and price changes caused by material content differences, the downstream and upstream tax coincide (Pommeret and Pottier, 2024).

3 A 3-sector economy

In order to highlight some economic mechanisms from our model, we introduce a very simple 3-sectors model without capital, where we have two mechanisms: substitution between sectors and substitution between a resource and factors. We can associate those three sectors to a very stylized economy with mining as the first sector, the manufactured goods as second sector, and services as the third sector.

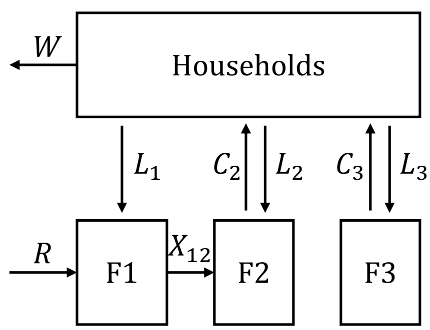


Figure 1: Flows in the 3-sector model

Production and utility is given by:

$$Y_1 = \min\{L_1, R\} \tag{32}$$

$$Y_2 = X_{12}^{\frac{1}{2}} L_2^{\frac{1}{2}} \tag{33}$$

$$Y_3 = L_3 \tag{34}$$

$$U = C_2^{\frac{2}{3}} C_3^{\frac{1}{3}} - \sigma W \tag{35}$$

We introduce σ as the marginal cost of waste flows on welfare. The Leontief production function for mining corresponds to $\rho = 1$ in equation (2), while labor is the only factor of production. The economic and material balances hold:

$$L_1 + L_2 + L_3 = \bar{L} \tag{36}$$
$$X_{12} = Y_1 \tag{37}$$
$$C_2 = Y_2 \tag{38}$$
$$C_3 = Y_3 \tag{39}$$
$$W = R \tag{40}$$

where waste is produced by the consumer use of manufactured goods. Note that material flows down the economy, from mining sector 1, to manufacturing sector 2 to the consumer. This underlying material flow in sector 2 output is equal to $R = W$, given the material balance constraint. For future reference, we define aggregate consumption $C = C_2^{\frac{2}{3}} C_3^{\frac{1}{3}}$.

3.1 Optimal economy

A social planner maximizes welfare under production and labor supply constraints. The Lagrangean becomes (where we substituted $W = R = L_1 = Y_1$, and $C_2 = Y_2, C_3 = Y_3$):

$$\begin{aligned}\mathcal{L} =& Y_2^{\frac{2}{3}} Y_3^{\frac{1}{3}} - \sigma R - p_2(Y_2 - R^{\frac{1}{2}} L_2^{\frac{1}{2}}) - p_3(Y_3 - L_3) \\ & - w(R + L_2 + L_3 - \bar{L}).\end{aligned} \tag{41}$$

For future reference we define $p_1 = w + \tau^R$ as the marginal cost price for producing one unit of Y_1. FOCs for R, L_2, L_3, Y_2, Y_3 give:

$$w + \sigma = \frac{1}{2} p_2 Y_2 / R \tag{42}$$
$$w = \frac{1}{2} p_2 Y_2 / L_2 \tag{43}$$
$$p_3 = w \tag{44}$$
$$p_2 = \frac{2}{3} C / Y_2 \tag{45}$$
$$p_3 = \frac{1}{3} C / Y_3 \tag{46}$$

Combining the second and fourth equation, and third and fifth, gives

$$w L_2 = w L_3 = \frac{1}{3} C \tag{47}$$

Thus $L_2 = L_3$. Note that we have the ratio for labor:

$$\frac{L_1}{L_2} = \frac{w}{w + \sigma} \tag{48}$$

The allocation thus satisfies

$$R = L_1 \leq L_2 = L_3 = \frac{1}{2}(\overline{L} - R) \tag{49}$$

so that the production-resource trade off is given by

$$C = R^{\frac{1}{3}} \left(\frac{1}{2}\overline{L} - \frac{1}{2}R \right)^{\frac{2}{3}} \tag{50}$$

Figure 2 displays the labor shares by sector and aggregate consumption, both as function of resource use R. Absent damages, $\sigma = 0$, we have $R = \bar{L}/3$, and labor shares are all equal. A lower resource inflow R corresponds to a progressive dematerialization of the economy and jointly reduced labor allocated to mining.

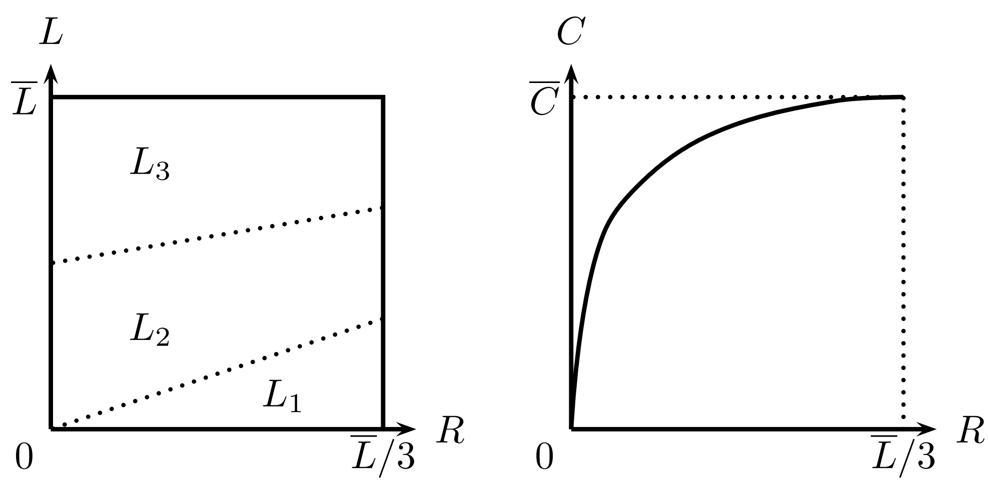


Figure 2: Optimal labour shares and consumption varying with resource use

3.2 Competitive economy

The competitive economy with material balances adds to the above the tracking of material through the (endogenous) material intensity $\theta_2 = R/Y_2$, and specifies virgin resource tax τ^R for the mining sector and a waste tax τ^W for consumers. Below we assume producers minimizing production costs (Assumption 2.a). They do not correct market prices for downstream material (waste) taxes. This definition of a competitive equilibrium differs from Pommeret and Pottier (2024) who assume that upstream producers internalize downstream taxes so that downstream incentives for reducing material intensities are transferred to upstream firms (as in our Assumption 2.b). Profit and utility maximization give (where we reduced the number of variables by substitution $p_3 = w$):

$$(w + \tau^R)R = wL_2 = \frac{1}{2}p_2 Y_2 \tag{51}$$

$$(p_2 + \tau^W \theta_2)Y_2 = \frac{2}{3}C \tag{52}$$

$$wL_3 = \frac{1}{3}C \tag{53}$$

Taxing upstream (τ^R) modifies prices of goods 1 and 2, shifting consumption away from manufacturing, and also modifies the resource intensity. Taxing downstream (τ^W) leaves resource intensity and prices unchanged, however bears on households costs, thus also shifting consumption away from manufacturing to services.

Upstream (resource) and downstream (waste) tax

It is straightforward to see that $\tau^R = \sigma$ and $\tau^W = 0$, reproduces the FOCs of the optimum, confirming the first part of Theorem 1. In the case of a positive waste tax $\tau^W > 0$ and absent a virgin resource tax $\tau^R = 0$, equilibrium gives:

$$R = L_1 = L_2 = Y_2 \Rightarrow L_3 = \overline{L} - 2R \tag{54}$$

$$p_2 = 2w \tag{55}$$

so that the production-resource trade off is given by

$$C = R^{\frac{2}{3}}(\overline{L} - 2R)^{\frac{1}{3}} \tag{56}$$

This proofs the second part of Theorem 1:

Proposition 1. *In a competitive equilibrium of our 3-sector economy with material balances and firms minimizing production costs (Ass 2.a), upstream taxes implement the social optimum while, downstream taxes are suboptimal and always give strictly lower aggregate consumption C for the same resource use R.*

Proof. Reproducing the optimum with $\tau^R = \sigma, \tau^W = 0$ is straightforward. Now compare with the case of $\tau^R = 0, \tau^W > 0$, divide the consumption level equations to get for $0 < R < \frac{1}{3}\overline{L}$:

$$\frac{R^{\frac{1}{3}}(\frac{1}{2}\overline{L} - \frac{1}{2}R)^{\frac{2}{3}}}{R^{\frac{2}{3}}(\overline{L} - 2R)^{\frac{1}{3}}} = \left(\frac{\frac{1}{4}(\overline{L} - R)^2}{R(\overline{L} - 2R)}\right)^{\frac{1}{3}} > 1 \tag{57}$$

The inequality is immediate, as the difference between the numerator and denominator is strictly positive:

$$(\overline{L} - R)^2 - 4R(\overline{L} - 2R) = \overline{L}^2 - 6\overline{L}R + 9R^2 = (\overline{L} - 3R)^2 > 0 \tag{58}$$

The equality occurs when production is maximal and both upstream and downstream taxes absent, i.e. $\tau^R = \tau^W = 0$. □

As represented in figure 3 (RHS), the consumption level with the implementation of a downstream tax is not optimal and always lower than the first best implementation of an upstream tax. In other words, for the same tax level, material use in the economy is more reduced by the upstream tax than the downstream tax.

When taxing resource (upstream), resource use is reduced at both the intensive and extensive margin. First, increasing tax τ^R, thus increasing prices, lead the industry to reduce the material intensity of manufactured goods: θ decreases. Also, the price of the manufactured good increases, leading to a shift for the consumer from manufactured goods 2 to service goods 3. This second effect is visible in the LHS of figure 2.

On the other hand, taxing waste (downstream) focuses on the extensive margin: material intensity and prices remain unchanged but the consumer directly bears the burden of the tax, thus switches from good 2 to good 3, as shown in figure 3 (LHS).

The addition of an endogenous material balance is crucial here to observe an upstream/downstream equivalence that does not hold. The inefficiency when taxing waste arises as information on households preferred material

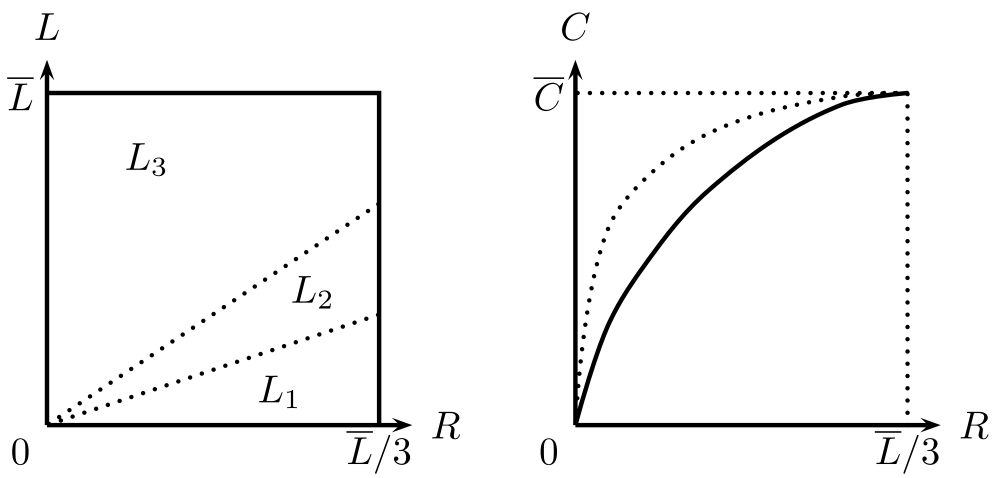


Figure 3: Labour shares for $\tau^R = 0, \tau^W > 0$, and consumption varying with resource use R (straight line in right graph). Right graph: first-best as dotted line

intensities is not transferred to the manufacturing sector 2. This equivalence can be reproduced when pricing also gives full information on the product, including material intensity, as explained in the following subsection.

Leontief economy

When we introduce a fully Leontief economy:

$$Y_2 = \min\{X_{12}, L_2\} = \min\{R, L_2\} \tag{59}$$

it is straightforward that the upstream/downstream equivalence can be reproduced. Indeed, consumption becomes linear in resource use R, thus by adjusting consumption, households directly adjust material content in the economy and thus can transmit the information on preferred material intensity.

This is the case of the General Equilibrium framework proposed by Krysiak and Krysiak (2003). They introduce strict physical/energy balance constraint for all flows, thus not separating material and economic flows as in the model of this paper. As a result, they show that it significantly constrains the shape of production functions, especially not allowing in most cases Cobb-Douglas and CES production functions.

A fine grid of goods

We now define a continuum of good varieties for output of sector 2 and associated production functions for manufacturing sector 2, with all possible material intensities θ. We identify firms with their unique production technology and are double indexed $\{2, \theta\}$, with $\theta = X_{2\theta}/Y_{2\theta}$:[2]

$$U = \left(\int_0^\infty Y_{2,\theta} d\theta\right)^{\frac{2}{3}} Y_3^{\frac{1}{3}} - \sigma W \tag{60}$$

$$Y_{2,\theta} = \min\{\frac{X_{12,\theta}}{\theta}\theta \cdot L_{2,\theta}\} \tag{61}$$

We can then state:

Theorem 2. *For the 3-sector economy, a competitive equilibrium with downstream tax is equivalent to one with an upstream tax if one of the following is true:*

- *the economy is Leontief;*
- *or, the sector-representative production is build through a fine grid of goods varieties each with Leontief production; with complete price information also for goods not produced;*
- *or, if producers minimize user-costs (Ass 2.b).*

Otherwise, under Assumption 2.a, only an upstream resource tax can implement the first-best.

Proof. (i) We have already explained above how a Leontief economy gives equivalence, given the linear relationship between inputs and consumption.
(ii) In case of a fine grid of goods, it is straightforward that upstream taxation restores the optimum. In case of a waste tax, FOC on firms $\{2, \theta\}$ give now the marginal price:

$$p_{2,\theta} = \theta p_1 + \frac{w}{\theta} = w\left(\theta + \frac{1}{\theta}\right) \tag{62}$$

[2]Here, we refer to the work of Jones (2005) who also introduces a continuum of production functions that differ by some characteristics, in his case technology used. In particular, he shows that we can compute a resulting global production function that is Cobb Douglas.

The consumer has access to a continuum of goods, perfectly substitutable, with different weighs on the waste tax. He buys the second-type good that minimizes his user costs ('corner' solution):

$$f(\theta) = p_{2\theta} + \theta\tau^W = \frac{w}{\theta} + (w + \tau^W)\theta \tag{63}$$

The optimal material intensity, i.e. equilibrium, is at:

$$\theta^* = \sqrt{\frac{w}{w + \tau^W}} \tag{64}$$

It follows that $L_1/L_2 = \frac{w}{w+\tau^W}$, same as in optimum as given by equation (48). Thus, we can fully characterize labor shares and we again have:

$$R = L_1 \leq L_2 = L_3 = \frac{1}{2}(\overline{L} - R) \tag{65}$$

That is, the optimum is restored.
(iii) We now show the specific case for Theorem 1, writing the extensive price for consumption good 2 as dependent on its underlying material intensity, $p_2 = p_2(\theta)$ with $\theta = X_{12}/Y_2$. FOC for the producer on Y_2 gives:

$$p_2(\theta) + \tau^W\theta = w\left(\frac{1}{\theta} + \theta\right) + \tau^W\theta \tag{66}$$

with the optimal choice for θ given by (64). □

For equivalence between a source and waste tax, we need to deviate from the standard assumptions in computable general equilibrium, either by introducing an infinite number of agents (in our case for firm 2) most of which do not produce, or, being able to observe all downstream waste taxes. As demonstrated above, the case of a fine grid of goods varieties is equivalent to user-cost minimization.

The natural question thus becomes whether it is natural for producers to focus on minimizing their own production costs, or to also include user costs caused by downstream waste taxes. Full information can be interpreted as free entry/exit on the market; yet as it is somewhat demanding as it relies on precise information on material intensities, as an intrinsic quality characteristic of products. We remain agnostic and consider it a possible but not an obvious property (Rosen, 1974; Leland, 1977; Drèze and Hagen, 1978). We thus consider Theorem 1 and 2 as a suggestion that waste taxes may tend to under perform, compared to source taxes.

4 Simulations: iron and fossil fuels

In this section, we provide a proof of concept through a parametrization of the model, with real data for the world economy, iron usage and carbon emissions. Its purpose is too showcase the possible applications of our framework in environmental policy analysis and provide a calibrated illustration of our theoretical analysis.

4.1 Parametric forms

We assume Cobb-Douglas production and utility functions, with parameters $A_J, \alpha_{J,I}, \beta_{F,I}, \gamma_{I,H}, \eta_{I,G}$. These are calibrated using the FOCs of the model, as described in Section 4.2 below. Production (4) is given by

$$Y_i = A_i \prod_{j \in I} X_{j,i}^{\alpha_{j,i}} \prod_{f \in F} L_{f,i}^{\beta_{f,i}} \tag{67}$$

with CRS: $\sum_j \alpha_{j,i} + \sum_f \beta_{f,i} = 1$.

The first order conditions that define the mapping $p \to (\psi_I, x, l)$, (5) become:

$$\widetilde{\psi}_i = \prod_j \left(\frac{(1+\tau^X_{j,i})p_j + \epsilon_{M,i}\theta^X_{M,j,i}\tau^W_{M,i}}{\alpha_{j,i}} \right)^{\alpha_{j,i}} \prod_f \left(\frac{p_f}{\beta_{f,i}} \right)^{\beta_{f,i}} / A_i \tag{68}$$

$$x_{j,i} = \alpha_{j,i} \frac{\widetilde{\psi}_i}{(1+\tau^X_{j,i})p_j + \epsilon_{M,i}\theta^X_{M,j,i}\tau^W_{M,i}} \tag{69}$$

$$l_{f,i} = \beta_{f,i} \frac{\widetilde{\psi}_i}{p_f} \tag{70}$$

$$\psi_i = \widetilde{\psi}_i + \rho_{M,i}\tau^R_{M,i} + \epsilon_{M,i}\rho_{M,i}\tau^W_{M,i} \tag{71}$$

where ψ_i are unit production costs inclusive of material taxes associated with virgin resource use and taxes $\tau^X_{J,i}$ associated with intermediate input use. Finite production requires non-positive profits: $\psi_i \geq p_i \perp Y_i \geq 0$. In almost all cases, we have positive output $Y_i > 0$ and $\psi_i = p_i$.

Utility (8) is defined as

$$\ln(U_h) = \ln(A_h) + \sum_i \gamma_{i,h} \ln(C_{i,h}) \tag{72}$$

so that utility unit costs (10) and consumption (11) become:

$$\psi_h = \prod_i \left(\frac{(1+\tau^X_{i,h})p_i + \theta^C_{M,i,h}\tau^W_{M,h}}{\gamma_{i,h}} \right)^{\gamma_{i,h}} / A_h \tag{73}$$

$$c_{i,h} = \gamma_{i,h} \frac{\psi_h}{(1+\tau^X_{i,h})p_i + \theta^C_{M,i,h}\tau^W_{M,h}} \tag{74}$$

where ψ_h are utility unit production costs inclusive of material taxes associated with virgin resource use, taxes $\tau^X_{I,h}$ associated with goods consumption. The constant A_h serves for normalization, such that in the baseline U_h equals expenditures in the SAM.

Utility (8) in the particular case where the consumer is the government is defined as

$$\ln(U_g) = \ln(A_g) + \sum_i \eta_{i,g} \ln(C_{i,g}) \tag{75}$$

so that utility unit costs (10) and consumption (11) now become:

$$\psi_g = \prod_i \left(\frac{(1+\tau^X_{i,g})p_i + \theta^C_{M,i,g}\tau^W_{M,g}}{\eta_{i,g}} \right)^{\eta_{i,g}} / A_g \tag{76}$$

$$c_{i,g} = \eta_{i,g} \frac{\psi_g}{(1+\tau^X_{i,g})p_i + \theta^C_{M,i,g}\tau^W_{M,g}} \tag{77}$$

The constant A_g serves for normalization, such that in the baseline U_g equals expenditures in the SAM.

4.2 Baseline calibration

Methodology

The script reads data from the SAM on $X, L, T_I, T_H, \Omega, C_G, C_H$. The parameters α, β are directly derived from (6),(7), given $\psi = p$ in the baseline. Similarly, γ and η come from (11). TFP A_I comes from (68), A_H, A_G from (73), (76) to set $\psi_H = \psi_G = \vec{1}$. Tax rates, are observed from the tax table, as goods consumed by households are taxed at different rates. Note that we also include factor use taxes, factor revenue taxes and direct taxes (However, for the sake of brevity, they are not included in the equations above). We also allow for material embedded in capital accumulation

Table 3: Model's sectors

Sector	Abbreviation
Agriculture, Forestry and Fishing	AGR
Mining	MIN
Basic metal industry	MET
Manufacturing	MAN
Transport	TRA
Services	SER
Fossil fuels	FOS
Electricity generation	ELY

The script then reads the PIOT on $X_M, L_M, \Omega_M C_M, W_M, R_M, \Delta K_M$. It computes embedded material ratios θ and waste/extraction ratios ϵ/ρ from (3) and (2).

Data

The Social Accounting Matrix (SAM) utilized for this calibration is derived from the 2014 GTAP dataset, featuring a singular global region and eight consolidated sectors. Given our emphasis on material utilization within the economy, we maintain a clear distinction between mining and the metal industry. For a comprehensive overview of the sectoral representation, refer to Table 3.

We formulate PIOTs for carbon and iron based on data extracted from the comprehensive global mapping of steel flows provided by Cullen et al. (2012). To ensure a meaningful comparison of upstream and downstream taxation, wherein the entirety of material flows is subject to taxation, we exclude material from capital accumulation in our primary simulations. The PIOT for carbon is derived from the Sankey diagram representing greenhouse gas (GHG) emissions, as computed by the World Resource Institute and aggregated using ClimateWatch data. It is essential to note that carbon is monitored in the model as equivalent CO_2 emissions (in tCO_2e). When necessary, the allocation of carbon usage is distributed proportionally between government and household consumption based on the corresponding values in the SAM data.

Material flow mappings are illustrated through the Sankey diagrams presented in Figures 4 and 5.[3] The primary discrepancy in carbon and iron accounting lies in the inclusion or exclusion of material intermediary inputs during production. Carbon, being associated with fossil fuels, is supplied to sectors and end-consumers solely for combustion (waste). In contrast, iron undergoes incorporation into intermediary products before eventual consumption and subsequent classification as waste. Detailed values are available in the appended PIOTs and SAMs.

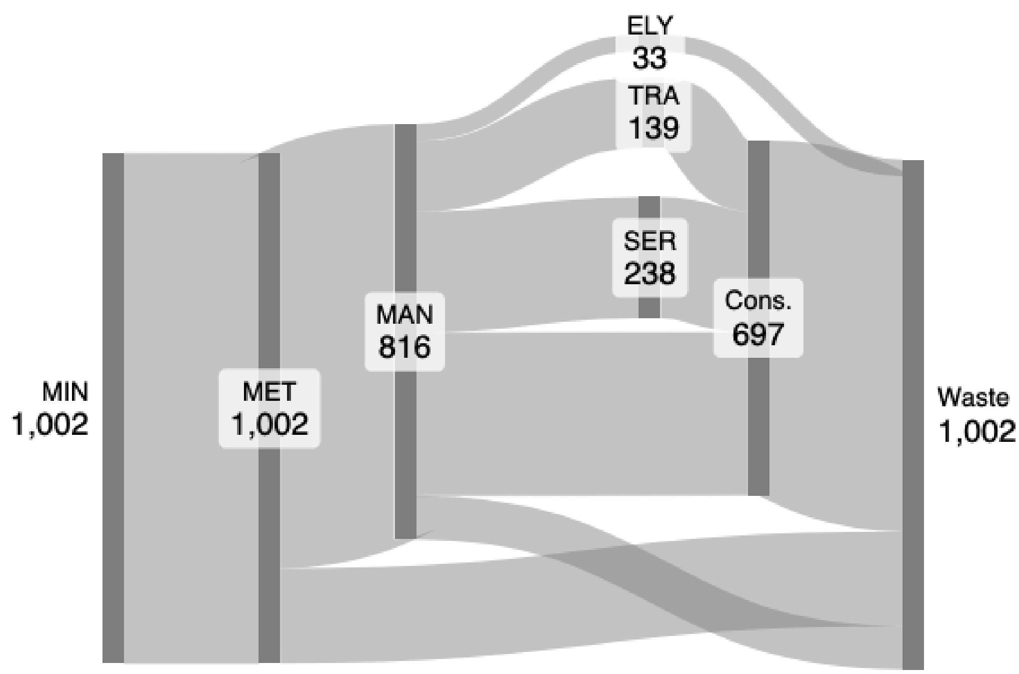


Figure 4: Sankey diagram for iron (Mt)

[3]Diagrams created using SankeyMATIC

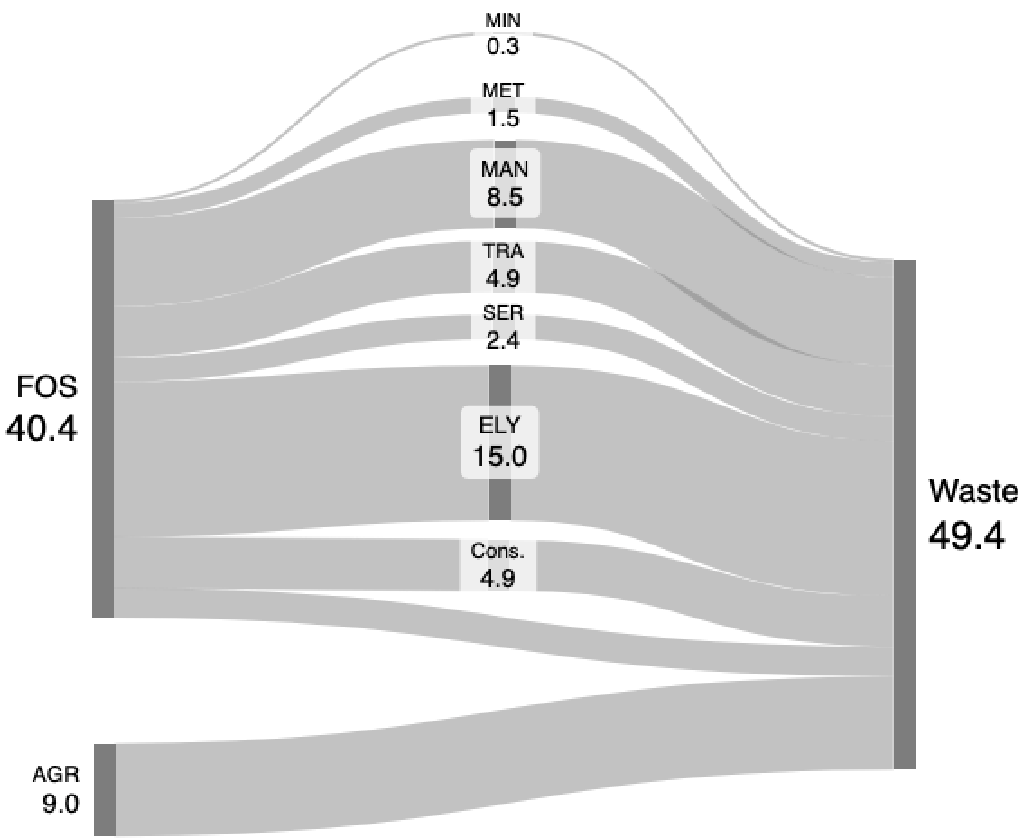


Figure 5: Sankey diagram for carbon ($GtCO_2e$)

4.3 Simulations

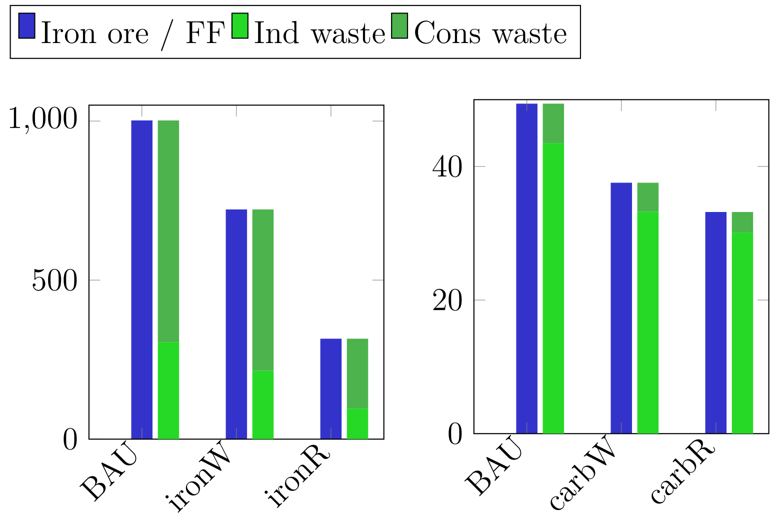
Scenarios. We run five scenarios, starting with a business-as-usual (BAU) condition characterized by the absence of any material tax. Additionally, we explore counterfactual scenarios simulating taxes on carbon and iron flows, both upstream and downstream, as outlined in Table 4. The carbon tax is set at a rate of 50\$/tCO_2e, while the iron tax is set at 2000\$/t, resulting in approximately the same material tax revenues between the carbon and iron tax scenarios. The equilibrium closure rules set a constant share of government consumption in GDP and a closed government budget through adjusting taxes on factors (see the appendix for more details). In a subsequent phase, we conduct a sensitivity analysis and let taxes span the range from 0 to 50\$/tCO_2e for GHG emissions and from 0 to 2000\$/t for iron. Where appropriate, we compare different levels for the extraction and waste taxes that both have a common effect on resource use.

Figure 6 illustrates the impact of material taxes on material use, distinguishing between upstream (iron ore extraction and fossil fuel extraction) and downstream (industry and consumption waste) taxes. The figure provides a succinct depiction of the material balances. The model maintains a balance between material sources (the left bars) and sinks (the right bars). The figure presents the material balance at the macroeconomic level; it also holds for individual sectors.

The figure illustrates that, while upstream and downstream taxes have equal level, the impact is substantially different. Consistent with the theoretical findings outlined in section 2, taxing upstream flows proves to be more effective, resulting in a comparatively higher reduction in material use.

Table 4: Scenarios

Name	Counterfactual tax
BAU	no tax on material flows
ironW	tax on iron waste $\tau^{W}_{\text{iron},I/H/G} = 2000\$/\text{t}$
ironR	tax on iron extraction $\tau^{R}_{\text{iron},I} = 2000\$/\text{t}$
carbW	tax on carbon waste $\tau^{W}_{\text{carbon},I/H/G} = 50\$/\text{tCO}_2\text{e}$
carbR	tax on carbon extraction $\tau^{R}_{\text{carbon},I} = 50\$/\text{tCO}_2\text{e}$



(a) Iron (Mt) (b) Carbon ($GtCO_2e$)

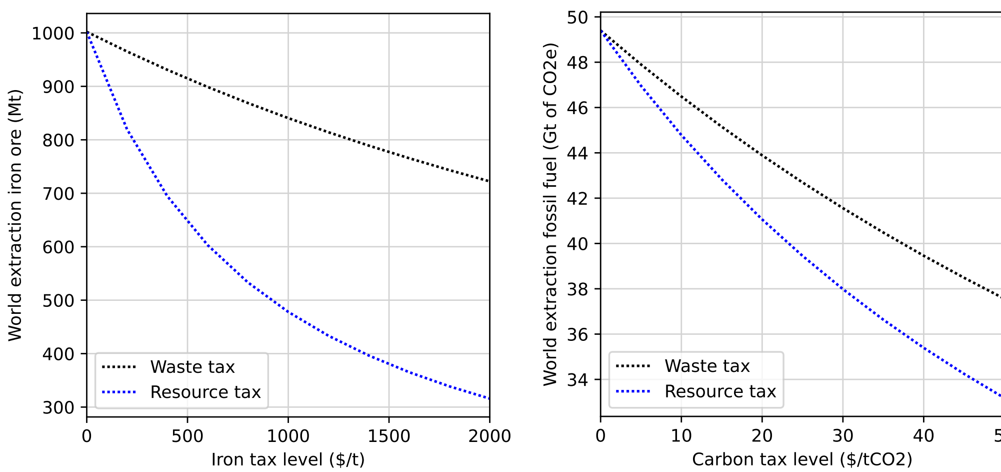
Figure 6: Input/Output material balance

Sensitivity on tax levels. In the sensitivity analysis of tax levels, Figures 7a and 7b depict the impact on total material usage. A 50\$/tCO_2 tax on fossil fuels results in a 28 to 39% reduction in fossil fuel extraction and, consequently, emissions.[4] Despite the highly stylized nature of our model, coupled with strong assumptions regarding production functions, the observed impact of the carbon tax aligns with the literature's findings (Antimiani et al., 2015). However, it's important to acknowledge the stylized nature of our model, particularly in terms of production function assumptions and sectorial representation, e.g. Antimiani et al. (2015) emphasize the sensitivity of outcomes to parameters such as substitution.

It is noteworthy that the efficiency disparity between upstream and downstream appears over the full range of tax levels. As observed in Figures 7a and 7b, the efficiency gap between upstream and downstream taxation is markedly higher for iron, despite comparable tax revenues. Figure 7a indicates a 42% difference for upstream/downstream in the case of iron, while Figure 7b shows an 11% difference for carbon. In line with the analysis of Section 3, the difference can be attributed to the distinct patterns of usage

[4]Note that this graph does not include GHG emissions from the agricultural sector (AGR), contrary to figure 6.

between iron and carbon. There are only few steps between fossil fuel extraction and combustion in the value chain of carbon, limiting the adjustment of carbon embedded in goods. For iron there are more options to reduce material intensity (see Figures 4 and 5). In other words, fossil fuels are procured for the purpose of combustion, with most emissions taking place at the production stage while iron is embedded in many consumption goods. Consequently, the effects of upstream and downstream taxes exhibit a closer alignment for carbon compared to iron.

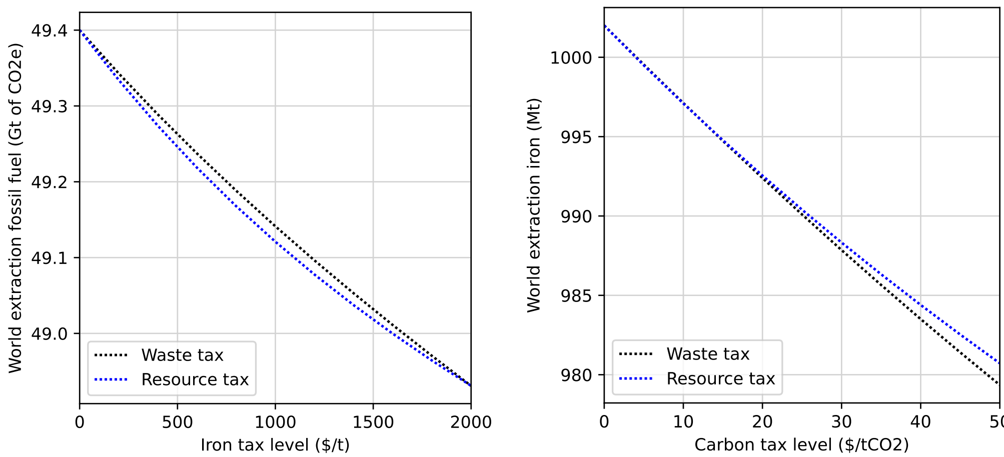


(a) Iron tax

(b) Carbon tax

Figure 7: Effect of material taxation - Upstream VS downstream

Moreover, aside from the direct repercussions of material taxation on material flows, it is essential to consider the interrelated effects on other materials. Given the endogenous nature of material balance within the model, imposing taxes on one material triggers economic consequences that subsequently affect the utilization of other materials. Figure 8 illustrates that both iron and carbon taxes exert an influence respectively on the levels of carbon and iron within the economy. However, these effects are relatively minor compared to the direct impacts of the taxes. Primarily, they stem from reductions in specific sectors; for instance, as the economy decreases its use and production of iron, there is a corresponding decrease in the consumption of fossil fuels needed for production. Nonetheless, this effect is mitigated by the iron reduction process, which also involves improvements in material effi-



(a) Iron tax

(b) Carbon tax

Figure 8: Crossed effects of material taxation - Upstream VS downstream

ciency, leading to substitutions with alternative inputs, including fossil fuels. A similar rationale applies in the case of a carbon tax.

In the simulations, material balances are preserved through the endogenous adjustment of per-sector material intensity per output implemented by Assumption 1.b. Figure 9 illustrates the iron mass per unit of output, denoted as θ^Y, for the manufacturing sector. It shows a decrease in iron intensity when taxing iron both as waste or extraction tax.

Alternatively, one can interpret the inverse $1/\theta^Y$ as product quality, the value per kilogram of iron used in manufactured products. Taxing iron waste/extraction shifts production towards higher value per unit of material. In alignment with the theoretical results presented in Theorem 1, waste taxes are less effective in reducing material intensity. This aligns with the economic intuitions developed in the 3-sectors model outlined in Section 3, where waste taxation does not provide information on preferred material content to upstream sectors, and its impact relies on substitution with other goods at the consumption stage.

Our quantitative model thus replicates the theory of Section 3. If the implicit marginal costs of material intensity cannot be transferred upstream, then a downstream tax cannot provide all incentives for upstream firms to efficiently reduce their material use. If products are sufficiently heterogeneous, up and downstream taxes become equivalent. To capture such mechanisms in the calibrated model with a discrete set of sectors is a topic we leave for future research.

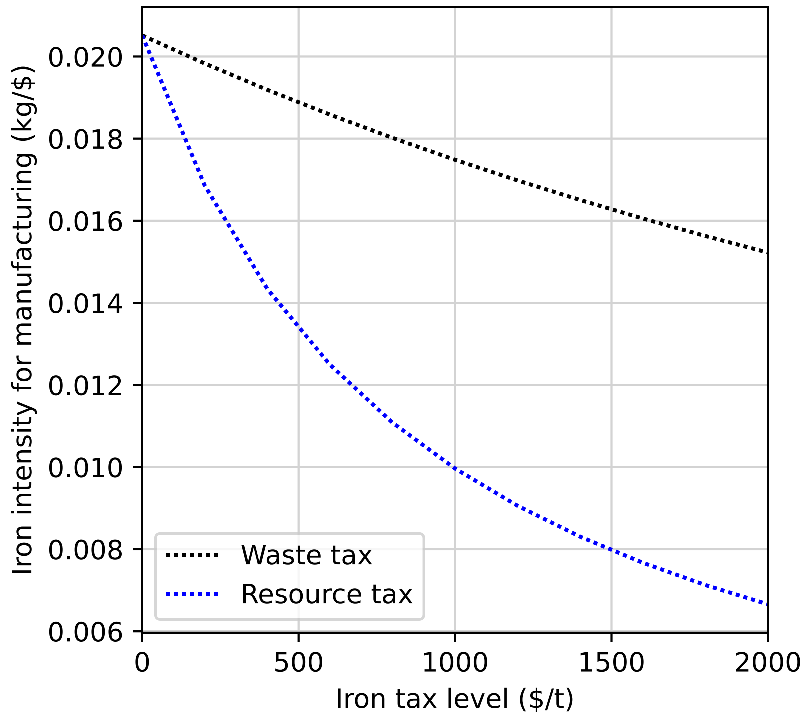


Figure 9: Upstream VS downstream tax sensitivity - Material intensity (iron) - Manufacturing

Labor reallocation To explore paths toward waste reduction, an examination of shifts in labor demand across sectors proves insightful. We now compare two scenarios with an equal 200Mt reduction in iron extraction, and consequently, waste, induced by both upstream and downstream taxes.[5] Figure 10a illustrates absolute changes in employment.

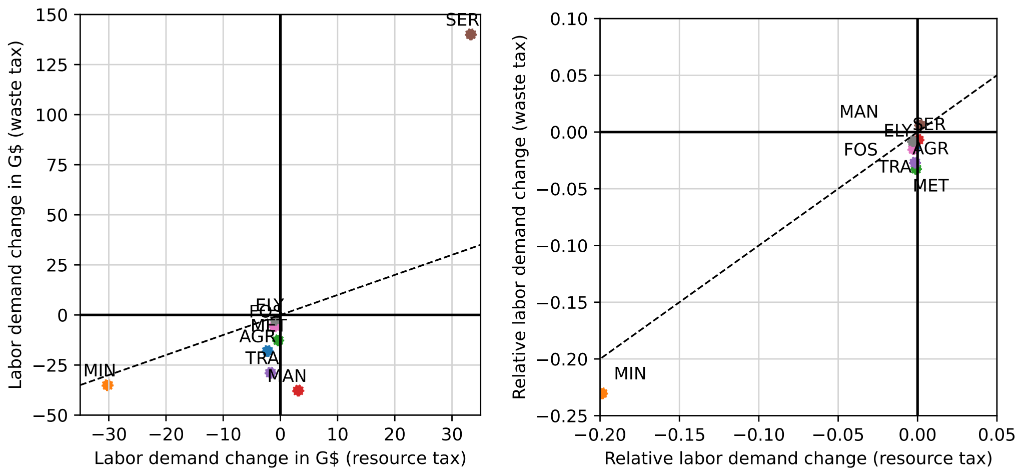
For the mining sector, the reduction in labor is almost symmetrical under both tax systems. Given its status as an extraction sector, diminished iron extraction results from decreased activity, leading to a lower labor input in

[5]This reduction is achieved through either a 250/t or 1500/t tax, respectively, upstream and downstream.

either tax scenario. Examining relative figures (see Figure 10b), this sector stands out as the most significantly impacted, experiencing around a 30% decrease. Conversely, the service sector witnesses an increase in labor under both tax systems. This sector benefits from the reduction in iron extraction as it serves as a substitute good with lower material intensity.

The source and waste tax are strikingly different for the transport and manufacturing sectors, which exhibit contrasting changes in labor demand when using a waste or a resource tax. Transportation is iron intensive and it consistently experiences negative effects. Yet the source tax results in a substantial reduction in material intensity, almost sufficiently to maintain demand and thus employment. In the manufacturing sector, a negative impact is observed under the waste tax, while the resource tax has a small positive impact. The resource tax influences the sector by encouraging the use of high-quality inputs (i.e. with low iron intensity), resulting in higher added value in manufacturing production. In contrast, the waste tax directs consumption to substitute away from iron-intensive goods, leading to an overall reduction in metal production and, consequently, a decrease in labor demand. Note that these quantitative findings are consistent with the left panels of Figs 2 and 3.

In summary, the application of a waste tax reveals a first-order economic effect as production shifts from sectors that heavily rely on iron input to sectors that are much less dependent. This substitution mechanism results in a systematic decrease of labor demand in sectors with high iron intensity (such as mining, metal, manufacturing, and transport). This is in line with the result from the theory model of section 3, where the waste tax leads to labor reduction in material intensive sectors such as manufacturing (see Figure 3). On the contrary, with a resource tax, a trade-off emerges between substitution (leading to decreased labor) and enhanced material efficiency. Labor increases in response to the demand for high-quality goods (characterized by low iron intensity) and/or substitution with inputs that are less iron-intensive. In that case, labor can increase in sectors that become material efficient (e.g. manufacturing), in line with theoretical results from section 3 and exposed in Figure 2.



(a) Absolute change (b) Relative change

Figure 10: Labor reallocation per sector for an iron flow reduction target of 200Mt

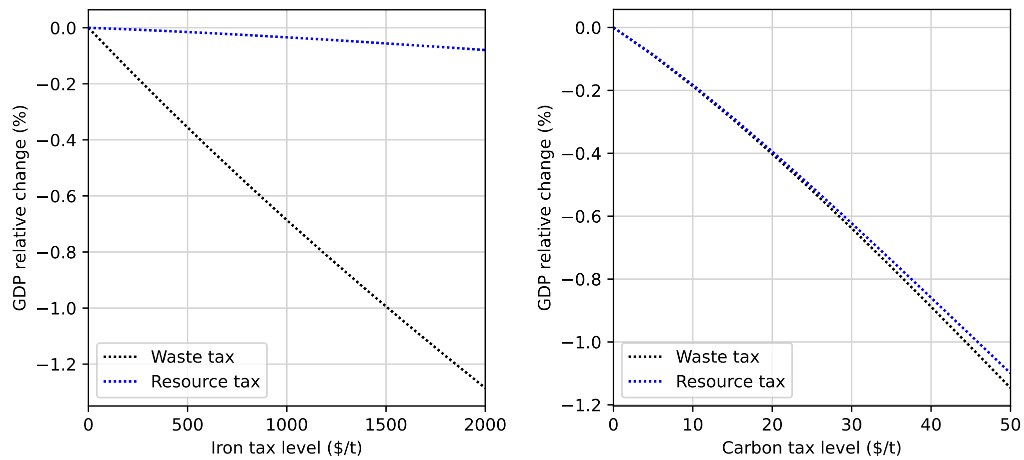
Impacts on GDP Our findings suggest that a the waste tax is more distortive than the resource tax, measured through the GDP decrease (figures 11a and 11b). Note that GDP changes are about 1 to 2%, which is about the same magnitude of the impact of taxes on the other material through activity reduction.

In order to provide further analysis, we analyse the marginal cost of public funds for a tax τ, defined as follows:

$$\mathrm{MCF}_\tau = -\frac{\Delta\mathrm{GDP}}{\Delta\mathrm{TaxRev}} \tag{78}$$

where ΔGDP is the change in GDP and ΔTaxRev is the change in tax revenue for a marginal increase in tax τ. We introduce notations τ_X, $\tau_{X,FOS}$, τ_{FX}, τ_F and τ_D for respectively input (except fossil fuel), fossil fuel input, factor input, factor income and direct taxes.[6] Note that revenue recycling of the waste/resource taxes are recycled through factor tax reduction, almost comparable to lump-sum returns. More details can be found in the appendix.

[6]Note that given the structure of our condensed GTAP data and for the sake of simplification, all factor taxes are on the producer. We separate fossil fuel and other input taxes as they are much higher than the others.



(a) Tax on iron (b) Tax on carbon

Figure 11: GDP changes

Tax	τ_R^{iron}	τ_R^{co2}	τ_W^{iron}	τ_W^{co2}	τ_X	$\tau_{X,FOS}$	τ_{FX}	τ_F	τ_D
MCF	0.276	0.304	0.962	0.353	0.683	0.636	0.003	0.003	0.458
BAU revenue share	0	0	0	0	0.27	0.06	0.51	0	0.16

Figure 12: Marginal cost of public funds

While a carbon tax shows little difference between upstream and dowstream, the iron tax shows an important distortion difference (table 12). Marginal cost of public funds of resource taxes are modest, but quite high for the iron waste tax, even above those for input taxes that are usually viewed as very distortive. This is reflected in the difference in GDP reduction in figure 11a.[7]

5 Concluding remarks

We provided a theoretical framework for the integration of material use in a macro-economic sector model. In particular, we introduce material balances in a CGE tracking material flows through sectors, from extraction to waste management. We complement the standard SAM, used for CGE analysis, with PIOTs for each material tracked in the model. Both accounting matrices are linked through material intensity variables, the inverse of which can be interpreted as quality of the material flow. When we include carbon into the material tracking, we can address climate change jointly with circularity considerations.

The approach allows for policy analysis of both upstream and downstream material unit taxes. Assuming firms minimize their own costs without considering downstream waste taxes, we observe a larger reduction of material extraction with upstream taxation, as industries adjust by changing prices as well as material intensity of the products throughout the life cycle. On the other hand, while waste taxes cause sectoral changes, they have less impact on upstream production processes if the market cannot transfer the downstream preferences for material intensity to the upstream firms.

Both the analytic and quantitative stylized economic models highlight the market failure that leads to waste taxation being a second best policy when producer do not take into account user cost minimization. The result may seem counter-intuitive at first glance. As a waste tax has a Pigouvian flavor, we expect it to fully endogenize the externality when taxing it at its source, the waste stream. The inefficiency stems from the incomplete exchange of information between up and downstream in the economy. The equivalence

[7] The results suggests that revenue recycling for source taxes through input taxes would yield a double dividend (Pearce, 1991). The double dividend hypothesis has been debated extensively, receiving both criticism (Bovenberg and de Mooij, 1994) and support (Goulder, 1995; Chiroleu-Assouline and Fodha, 2006).

between upstream and downstream taxes can be restored when producers are aware of all downstream waste taxes and adjust production accordingly.

The model effectively addresses a common criticism on economics within environmental research, namely the perceived lack of physical consistency in macroeconomics. Through calibrated simulations focusing on the use of iron and fossil fuel in the economy, our approach demonstrates the capability of CGEs to depict stringent material balances and policies geared towards material constraints on inflows/outflows. This includes the taxation of materials (in euros per kg) with exhaustive monitoring across all economic sectors. Notably, the model successfully replicates our theoretical findings regarding the efficiency discrepancy between waste and resource taxes. However, it's essential to acknowledge the limitations of our application, particularly the simplified assumptions made for production functions and the aggregated sectoral description in the calibration. The imposition of assumption 1.b, which enforces a uniform scaling of material intensities for every usage of products, restricts our ability to represent heterogeneous goods produced by one sector. For instance, the assumption does not sit well with a model that has one fossil fuel sector, and specific sectors using either gas or coal as intermediate inputs from the fossil fuel sector. That is, a proper policy simulation requires the granularity suited to the question addressed.

We mention qualifications in several directions. Firstly, introducing more realistic production functions would improve our quantitative illustration. Production functions may need to reflect that material intensity can only be varied within certain limits; unrestricted substitution between material inputs is often not feasible. Secondly, while we considered a fine grid of goods in our theory, we have not developed the general quantitative model to support it. This would make a valuable extension, enabling consumers to have a more active role in adjusting material intensity. Finally, the framework established in this study could be used to explore policy tools in the context of current transition pathways toward a low-carbon economy. Acknowledging potential differences in efficiency between various policy approaches can offer valuable insights for informed decision-making in climate and circular economy policies.

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A Additional material

A.1 Proof of Theorem 1

We drop the subscript M, and throughout the first two sections, and we consider an economy without waste taxes: $\tau_M^W = 0$. We use hats for equilibrium values when producers minimize production costs, and tildes when producers minimize user costs, taking into account downstream waste taxes.

A.1.1 Lagrangean for firms without waste taxes

The Lagrangean for the firm cost-minimization reads:

$$\min \mathcal{L} = p_J x_{J,i} + p_F l_{F,i} + \tau^R \rho_i \tag{5'}$$
$$- \kappa_i \left[F(x_{I,i}, l_{F,i}; \rho_{M,i}, \epsilon_{M,i}) - 1 \right] \tag{79}$$

From this we can derive FOCs for $x_{J,i}, l_{F,i}$[8]

$$\kappa_i F_{i,X} = p_J \tag{80}$$
$$\kappa_i F_{i,L} = p_F \tag{81}$$

Note that because of CRS we have $F_{i,X} x_{J,i} + F_{i,L} l_{F,i} = 1$, so that equilibrium prices satisfy

$$\psi_i = p_i = \kappa_i + \tau^R \rho_i = p_J x_{J,i} + p_F l_{F,i} + \tau^R \rho_i. \tag{82}$$

A.1.2 Lagrangean for consumers without waste taxes

The Lagrangean for cost-minimization per unit of utility for household h reads:

$$\min \mathcal{L} = p_I c_{I,h} \tag{10'}$$
$$+ \nu_h \left(U(c_{I,h}) - 1 \right) \tag{83}$$

which gives as FOCs

$$\nu_h U_{h,C} = p_I \tag{84}$$

where $c_{i,h} = C_{i,h}/U_h$ is consumption per unit utility, and because of CRS of utility we have the cost for a unit of utility

$$\psi_h = p_I c_{I,h}. \tag{85}$$

[8] We can extend the analysis with endogenizing parameters ρ and ϵ.

A.1.3 Lagrangean for firms with waste taxes and user cost minimization

Consider the case with resource and waste taxes τ_i^R, τ_i^W that are not necessarily equal for all sectors and consumers. Under Assumption 2.b, the producers include downstream waste tax costs in their optimization. They understand that the 'true' price paid by the users of their product equals $\widetilde{p}_i = p_i + \mu_i\theta_i$, rather than p_i. The Lagrangean for the firm cost-minimization reads:

$$\min \mathcal{L} = p_J x_{J,i} + p_F l_{F,i} + \tau_i^R \rho_i + (\tau_i^W \epsilon_i + \mu_i(1-\epsilon_i))\theta'_i \tag{5”}$$
$$- \kappa_i \left[F(x_{I,i}, l_{F,i}; \rho_{M,i}, \epsilon_{M,i}) - 1\right] \tag{86}$$
$$- \omega_i \left[\theta'_i - (\rho_i + \sum_j \theta_{j,i} x_{j,i})\right] \tag{87}$$

where $\theta'_i = \theta_i/(1-\epsilon_i)$ is the inflow of material intensity, and ω_i is the social costs of waste downstream plus internal in the sector, caused by embedded material. From this we can derive FOCs for θ'_i, $x_{J,i}$ and $l_{F,i}$

$$\omega_i = \tau^W \epsilon_i + \mu_i(1-\epsilon_i) \tag{88}$$
$$\kappa_i F_{i,X} = p_J + \omega_i \theta_J \tag{89}$$
$$\kappa_i F_{i,L} = p_F \tag{90}$$

Market prices do not include downstream waste taxes, thus if we define inclusive marginal cost of production $\widetilde{\psi}_i$ that embed downstream waste tax costs, we get

$$\begin{aligned}\widetilde{\psi}_i = \widetilde{p}_i &= p_i + \mu_i\theta_i \\ &= p_J x_{J,i} + p_F l_{F,i} + \tau^R \rho_i + \tau^W \epsilon_i \theta'_i\end{aligned} \tag{91}$$

A.1.4 Lagrangean for consumers with waste taxes

The Lagrangean for cost-minimization per unit of utility for household h reads:

$$\min \mathcal{L} = (p_I + \tau_{I,h}^W \theta_{I,h}^C) c_{I,h} \tag{10”}$$
$$+ \nu_h \left(U(c_{I,h}) - 1\right) \tag{92}$$

which gives as FOCs

$$\nu_h U_{h,C} = p_I + \tau_{I,h}^W \theta_{I,h}^C \tag{93}$$

where $c_{i,h} = C_{i,h}/U_h$ is consumption per unit utility, and because of CRS of utility we have the cost for a unit of utility

$$\psi_h = (p_I + \tau^W \theta^C_{I,h}) c_{I,h}. \tag{94}$$

A.1.5 Lagrangean for First Best

We choose Negishi weights ν_h such that the first best maximizes the Langrangean. Note that for the sake of not introducing too many symbols for dual variables, we use notations that are already used in the decentralized program ($\widetilde{p}_i$, p_f, κ_i, τ_R, τ_W, μ_i) and later we show that they actually match the previously introduced variables.

$$\max \mathcal{L} = \sum_h \nu_h U(C_{I,h}) - \sigma^R R - \sigma^W W \tag{95}$$

$$- \sum_i \widetilde{p}_i \left[\sum_j X_{i,j} + \sum_h C_{i,h} - Y_i \right] \tag{12'}$$

$$- \sum_f p_f \left[\sum_i L_{f,i} - \sum_h \Omega_{f,h} \right] \tag{13'}$$

$$- \sum_i \kappa_i \left[Y_i - F_i(X_{J,i}, L_{F,i}; \rho_i, \epsilon_i) \right] \tag{4'}$$

$$- \tau^R \left[\sum_i \rho_i Y_i - R \right] \tag{96}$$

$$- \tau^W \left[\sum_i \epsilon_i (\rho_i Y_i + \lambda_i \overline{\theta}^X_{J,i} X_{J,i}) + \sum_h \lambda_I \overline{\theta}^C_{I,h} C_{I,h} - W \right] \tag{97}$$

$$- \sum_i \mu_i \left[(1 - \epsilon_i) \left(\lambda_J \overline{\theta}^X_{J,i} X_{J,i} + \rho_i Y_i \right) - \sum_j \lambda_i \overline{\theta}^X_{i,j} X_{i,j} - \sum_h \lambda_i \overline{\theta}^C_{i,h} C_{i,h} \right]. \tag{98}$$

We choose to use control variable λ_i instead of θ as lemma 1 states that a unique λ_I exists such that an equilibrium that satisfies material balance given in (97) and (98).

The material balance $R = W$ ensures that the optimal allocation (X, L, Y, C, R, W) is identical for any split of σ that preserves the sum $\sigma = \sigma^R + \sigma^W$. Note that if $\sigma^W = 0$, we can drop the last two equations which become redundant, meaning that $\tau^W = \mu_i = 0$. The inverse does not hold. If

$\sigma^R = 0$, we can drop the equation with R so that τ^R is zero but as the endogenous variable λ_i enters both last two equations, when $\sigma^W = \tau^W > 0$ we have also $\mu_i \neq 0$. As special case, when $\sigma^R = \sigma^W = \tau^R = \tau^W = 0$, we can drop the last 3 equations and we have $\mu_i = 0$.

Note that we put a tilde on the dual variable $\widetilde{p}_i$; it turns out that the dual variable includes the user costs. The dual variable p_f will be seen to be the market price for factors. The dual variable κ_i measures the (marginal) production costs of good i,[9] τ^R, τ^W as the tax on virgin resource use and waste, and μ_i as the implicit price (welfare costs) for material embedded in good i that has not been taxed through upstream virgin taxes. That is, μ_i covers the cumulative downstream waste taxes associated with one unit of output.

The FOCs for $Y_i, X_{J,i}, L_{f,i}, C_{i,h}$ give:

$$\widetilde{p}_i - \mu_i(1-\epsilon_i)\rho_i = \kappa_i + \tau^R\rho_i + \tau^W\epsilon_i\rho_i \tag{99}$$

$$\kappa_i F_{i,X} + \mu_J\lambda_J\overline{\theta}^X_{J,i} = \widetilde{p}_J + \left(\mu_i(1-\epsilon_i) + \tau^W\epsilon_i\right)\lambda_J\overline{\theta}^X_{J,i} \tag{100}$$

$$\kappa_i F_{i,L} = p_F \tag{101}$$

$$\nu_h U_{h,C} + \mu_I\theta^C_{I,h} = \widetilde{p}_I + \tau^W\theta^C_{I,h} \tag{102}$$

A.1.6 Proof of efficient resource and waste taxes (Theorem 1)

We can first establish properties absent waste taxes.

Lemma 2. *A competitive equilibrium with no waste taxes and resource taxes constant over all producers is efficient.*

Proof. Consider a competitive equilibrium with resource taxes $\tau^R \geq 0$ and absent waste taxes $\tau^W = 0$. Prices p_I, p_F support the equilibrium. It follows from (84) that ν_H exists such that $\nu_h U_{h,C} = p_I$. Use those same weights in the welfare program (95), and set $\sigma^R = \tau^R$, $\sigma^W = 0$ (so that $\tau^W = \mu_i = 0$). The FOCs for the First Best from above, for $Y_i, X_{J,i}, L_i$ now become

$$p_i = \kappa_i + \tau^R\rho_i \tag{103}$$

$$\kappa_i F_{i,X} = p_J \tag{104}$$

$$\kappa_i F_{i,L} = p_F \tag{105}$$

[9] Because of CRS marginal equals the average production costs.

which match the FOCs of the producers, which is identical for production and user cost minimization. Thus, the first-best and equilibrium allocations coincide. □

Next we establish equivalence between resource extraction and waste taxes in the first-best allocation.

Lemma 3. *The optimal solution $Y, X, C, L, R, W, \lambda$ only depends on $\sigma^R + \sigma^W$ (the split has no effect). The solution dual variables satisfy $\tau^R = \sigma^R$, $\mu_i = \tau^W = \sigma^W$.*

Proof. Consider an optimal solution for $\widehat{\sigma}^R > 0, \widehat{\sigma}^W = 0$. We use hats for this reference allocation. The FOCs for R give $\widehat{\tau}^R = \widehat{\sigma}^R$, and we have $\widehat{\tau}^W = \widehat{\mu}_i = 0$.

Now consider an alternative case with $\sigma^R + \sigma^W = \widehat{\sigma}^R$ for some $\sigma^W > 0$. We check that the allocation and most dual variables do not change. We keep $\widetilde{p}$ unchanged, but set $\mu_i = \tau^W = \sigma^W$. It is easily verified that the full FOCs (99)-(102) are still satisfied for all dual variables with tildes, that is, the terms with μ, τ^W cancel keeping in mind that $\tau^R + \tau^W = \widehat{\tau}^R$. □

To conclude the proof of Theorem 1, we note that dual variables $\mu_i = \tau^W$ satisfy (31). It is immediate that substitution of $\mu_i = \tau^W$ collapses (100),(101) into (89),(90), where (88) defines $\omega_i = \tau^W$. Furthermore, (102) collapses to (93). Q.E.D.

A.2 Model stocks/flows

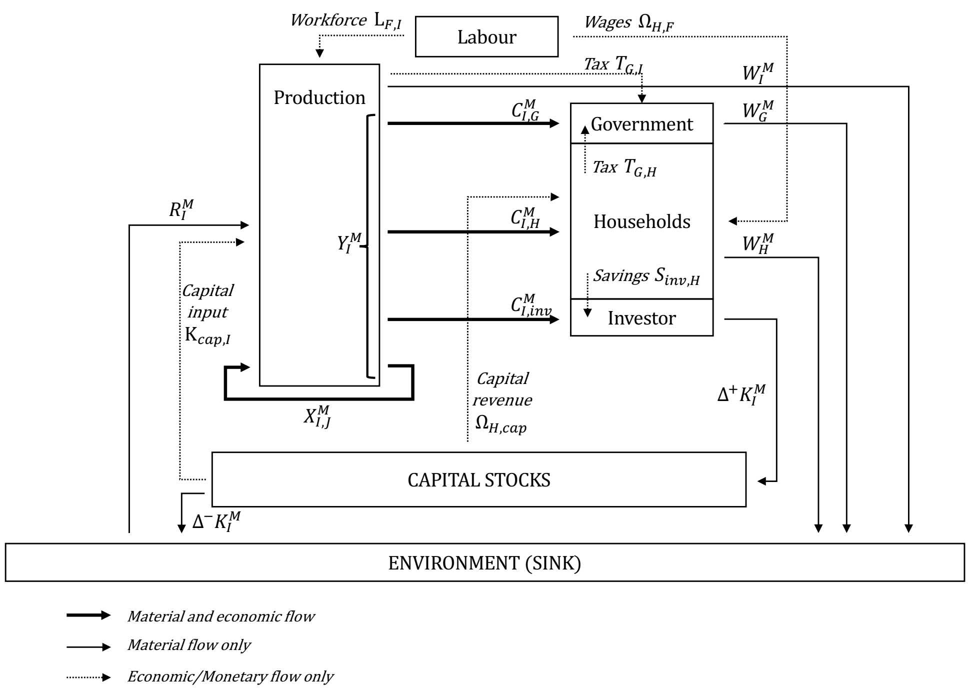


Figure 13: Global economic/material model

A.3 Equilibrium algorithm

For prices and activities, using variables with benchmark normalization to unity has the advantage that the log-control has benchmark zero. Thus, in the script we calculate with log-variables, such as $\ln(p), \ln(y), \ln(z)$ (element-wise) and we store coefficients in standard values.

Mappings Let us collect all parameters in $\chi = (A, \alpha, \beta, \gamma, \eta, \overline{Y}, \overline{C}, \epsilon, \rho, \overline{\theta})$.

By multiplying intensity levels by material activity levels, we construct the mapping MAT: $(\lambda, \zeta) \to \theta$. We then construct mappings FOC_I: $(p, \tau, \theta; \chi) \to (\psi_I, x, l)$ by (68),(69),(70), FOC_H: $(p, \tau, \theta; \chi) \to (\psi_H, c_H)$ by (73),(74), FOC_G: $(p, \tau, \theta; \chi) \to (\psi_G, c_G)$ by (76),(77). Note that θ enters the FOCs as our model includes taxes on material inputs and waste. By multiplying per unit inputs with activity levels, we can then construct the mappings PROD: $(p, \tau, y, \theta; \chi) \to (\psi_I, Y, X, L)$, CONS_H: $(p, \tau, z_H, \theta; \chi) \to C_H$, CONS_G: $(p, z_G, \theta; \chi) \to C_G$. Note that the mapping for public consumption does not depend on tax revenues, as the level of consumption is fully determined by activity z_G.

We define GDP from the demand side, including consumption from households + investor (H) and the government (G):

$$\text{GDP} = \sum_{h,i} p_i C_{i,h} + \sum_{g,i} p_i C_{i,g} \tag{106}$$

We define auxiliary equilibrium functions, such that if the associated control variable increases, the function decreases.

- Zero profit condition by the mapping ZERO_PROFIT: $(\psi_i, p_i) \to E_I = \ln \psi_I - \ln p_I \in \mathbb{R}^I$, element-wise, for excess costs relative to prices (log LHS minus log RHS of (68)) with controls p_I.
- Goods balance mapping COMBAL_I: $(Y, X, C) \to D_{I\setminus i} \in \mathbb{R}^{I-1}$ for goods excess demand (log of LHS minus log of RHS of (12)) with controls y. Here we have one control too many, balanced by the one-dimensional inflation target below which has no control variable. (see discussion below on Walras Law)
- Factor balance mapping COMBAL_F: $(L; \Omega) \to D_F \in \mathbb{R}^F$ for factor excess demand (log of LHS minus log of RHS of (13)) with controls p_F.

- Budget mapping BUD_H: $(p, \tau, \theta, C_H) \to B_H \in \mathbb{R}^H$ and budget mapping BUD_G: $(p, \tau, \theta, C_G) \to B_G \in \mathbb{R}^G$ by (9), for excess budget (log of RHS minus log of LHS of (9)) with controls z_H, z_G.

- Inflation mapping INF: $(p, \tau, \theta) \to \text{INF} \in \mathbb{R}$, which targets a constant CPI (for households, the investor and the government), weighed by BAU consumption shares, for ease of interpretation of results. We note that our economy does not have complete money neutrality, as we allow for material taxes per physical unit.

$$\text{INF} = \sum_{h \in H} \text{share}_h^{BAU} \ln(\psi_h) + \sum_{g \in G} \text{share}_g^{BAU} \ln(\psi_g) \tag{107}$$

  where

$$\text{share}_{h/g}^{BAU} = \sum_i C_{i,h/g}^{BAU} / \text{GDP}^{BAU} \tag{108}$$

- Material balance for the excess inflow and outflow of materials by the mapping FLOWS: $(X_M, C_M, R_M, W_M) \to D_{M,I}^{\text{IO}} \in \mathbb{R}^{MI}$ (preceded by MAT and SIM_MAT: $(Y, X, C, \theta) \to (Y_M, X_M, C_M, R_M, W_M)$), (RHS minus LHS of (16)) with controls $\lambda_{M,I}$.

When material output $Y_{m,i}$ is zero, we can remove control variable $\lambda_{m,i}$ and it's associated balance mapping, as in this case, there can only be intermediate input in and industrial waste out, thus they scale given material balances in other sectors.

Note that Walras' law informs us that, we must take one good i out of the zero excess demand condition; it is always satisfied if the other balances are satisfied. More precisely, without material taxes in physical units, the equilibrium (p^*, y^*, z^*) is over-identified. We can multiply prices p by any factor $a > 0$ into (ap^*, y^*, z^*) and still have an equilibrium. With material taxes, we still have over-identification for $(p^*, y^*, z^*, \lambda^*, \zeta^*)$ but not linearly homogeneous in p^*. We can use the overidentification to normalize prices by imposing an inflationary target; for some household h we impose $\psi_h = 1$. The interpretation is that prices are chosen such that this household observes no inflation. These conditions will uniquely define the equilibrium $(p^*, y^*, z^*, \lambda^*, \zeta^*)$.

Thus, combining all these mappings, we construct the mapping EQUIL: $(p, y, z, \lambda; \tau, \chi) \rightarrow (D_I \backslash i, D_F, E_I, B_H, B_G, \ln(\psi_h), D^{\text{IO}}_{M,I})$. Note that tax ratios τ are exogenous, so that we have a mapping for endogenous variables $(p, y, z, \lambda, \zeta) \in \mathbb{R}^{(2+M)I+F+H+G}$ with the same dimensions for the image (we remove one sector for the goods balance and add the inflation mapping). The equilibrium is the price and activity level $(p, y, z, \lambda, \zeta)$ such that the mapping returns zero. As ex-post check, the Excess Demand D_i should be (close to) zero.

As we allow for material flow in capital accumulation we need to calibrate the initial material stock embedded in capital is not in a SAM/PIOT. One solution to initialize it is to use an initial ratio of embedded material that is equal to the ratio of embedded material in the savings at year 0:

$$K_M = \theta^K_M K = \frac{\sum_i \theta^C_{M,i,inv} C_{i,inv}}{A_{inv} \prod_i C_{i,inv}^{\gamma_{i,inv}}} \sum_h \Omega_{h,cap} \tag{109}$$

This ratio θ^K_M does not have to be constant over time, even with the same depreciation for material and capital value, as savings can change in material content over time. Ideally, we want to observe material intensity of capital stock. The algorithm is further described in appendix.

Baseline After reading the reference (baseline) data, and calibrating the parameters χ, we set the price and activities equal to the unit vector $p = \vec{1}_{I+F}$, $y = \vec{1}_I$, $z = \vec{1}_{H+G}$, $\lambda = \vec{1}_I$. We then check that the mapping EQUIL returns zero.

Counterfactuals For counterfactuals, expressed as policies (taxes), we can calculate those with the 'bruteforce' method by minimizing the squared norm of the equilibrium vector, ensuring a new equilibrium satisfying all equilibrium conditions:

$$\min_{(p,y,z,\lambda)} \|\text{EQUIL}(p, y, z, \lambda; \tau, \chi)\|^2 \tag{110}$$

However, another method is to calculate counterfactuals through an homotopy while maintaining the equilibrium conditions throughout. That is, we construct

$$\tau(\mu) = \mu \tau^{BAU} + (1 - \mu)\tau^* \tag{111}$$

We can then set μ as the first control variable in the minimization procedure, and impose $0 \leq \mu \leq 1$, and minimize μ subject to the equilibrium conditions.

$$\min_{(p,y,z,\lambda,\tau)} \mu \text{ s.t. } \mathrm{EQUIL}(p, y, z, \lambda; \tau(\mu), \chi) = 0 \tag{112}$$

As the calibration starts with an existing solution, the above procedure has a feasible outcome.

When imposing a counterfactual with a budget constraint for the government, we include factor input taxation τ^{FX} as an endogenous variable for revenue recycling.[10] The new constraint is now that the share of government consumption remains the same:

$$\sum_i p_i C_{i,g} = \mathrm{GDP} \times \mathrm{share}_g^{BAU} \tag{113}$$

[10]As demonstrated below, we use factor taxes that are very little distortive in order to remain conservative regarding public spending in the model.

A.4 Results with material in capital

In this subsection, we present the robustness check with input data that includes material (iron-steel) embedded in capital capital accumulation. Practically, we redirect steel used in mechanical equipments from Cullen et al. (2012) data from standard consumption to investor's consumption. Results for iron are presented in the following graphs (Figures 14 and 15).

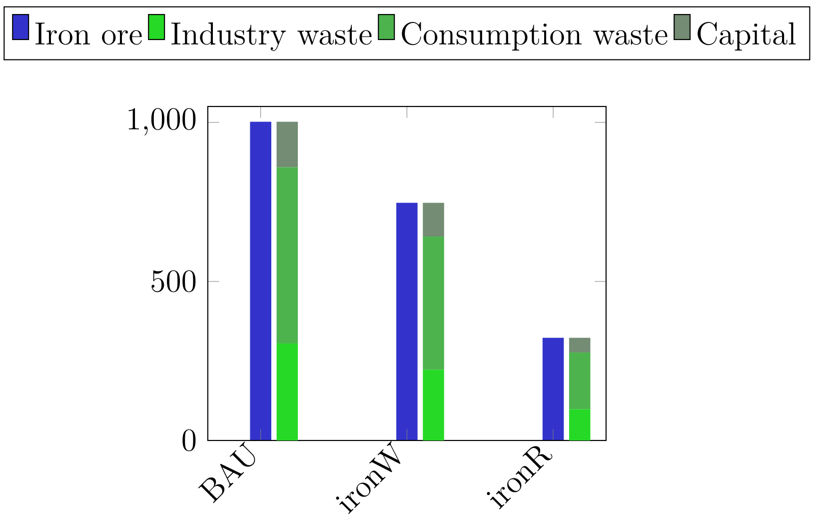
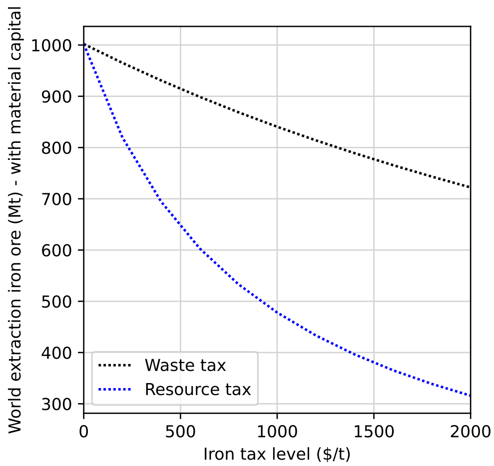
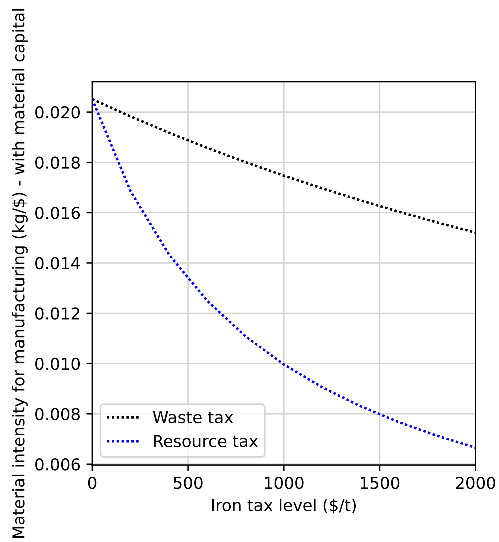


Figure 14: Input/Output iron material balance - with material capital



(a) Iron extraction



(b) Material intensity (MAN)

Figure 15: Upstream VS downstream tax sensitivity

A.5 Simulation input data

Table 5: BAU - SAM

	AGR	MIN	MET	MAN	TRA	SER	FOS	ELY	LAB	FIX	CAP	HOU	INV	GOV	Tot Rev	Prices	Act lvl
AGR		1	1		10		1	1						10		1	1
MIN	4				2		26	3				7	27	0		1	1
MET	2	16			6		43	15				34	48	14	5244	1	1
MAN	686	136	381	17017	362	8858	350	160				10241	6199	259	44653	1	1
TRA	126	79	169	1454	324	2049	261	53				1729	152	126	6527	1	1
SER	446	151	656	6422	1694	20907	609	413				27264	12203	12796	83566	1	1
FOS	102	30	179	807	1391	223	3499	983				921	0	0	8137	1	1
ELY	77	43	271	706	58	715	93	157				778	0	0	2902	1	1
LABOUR	1280	152	388	5544	1072	22085	371	298							31192	1	
FIX	769	71	0	0	0	0	867	0							1709	1	
CAPITAL	679	265	587	5227	1019	20221	1548	644							30194	1	
HOUSEHOLDS									31192	1709	30194				63095		1
INVESTOR									0	0	0	19448			19448		1
GOVERNMENT	7	58	256	2616	585	7225	465	171				1112	710	43	13252		1
Total Expenditures	4663	1124	5244	44653	6527	83566	8137	2902	31192	1709	30194	63095	19448	13252		0	0

Table 6: BAU - PIOT Iron

	AGR	MIN	MET	MAN	TRA	SER	FOS	ELY	LAB	FIX	CAP	HOU	INV	GOV	WASTE	STOCK	OUT	Lambda
AGR	0	0	0	0	0	0	0	0				0	0	0	0		0	1
MIN	0	0	1002	0	0	0	0	0				0	0	0	0		1002	1
MET	0	0	0	816	0	0	0	0				0	0	0	186		1002	1
MAN	0	0	0	186	139	238	0	33				316	0	4	86		1002	1
TRA	0	0	0	0	0	0	0	0				126	0	13	0		139	1
SER	0	0	0	0	0	0	0	0				158	0	80	0		238	1
FOS	0	0	0	0	0	0	0	0				0	0	0	0		0	1
ELY	0	0	0	0	0	0	0	0				0	0	0	33		33	1
LABOUR																	0	
FIX																	0	
CAPITAL																	0	
HOUSEHOLDS															600		600	
INVESTOR																0	0	
GOVERNMENT															97		97	
EXTRACTION	0	1002	0	0	0	0	0	0									1002	
Total input	0	1002	1002	1002	139	238	0	33	0	0	0	600	0	97	1002	0	5115	

Table 7: BAU - PIOT Carbon

	AGR	MIN	MET	MAN	TRA	SER	FOS	ELY	LAB	FIX	CAP	HOU	INV	GOV	WASTE	STOCK	OUT	Lambda
AGR	0	0	0	0	0	0	0	0				0	0	0	5		5	1
MIN	0	0	0	0	0	0	0	0				0	0	0	1		1	1
MET	0	0	0	0	0	0	0	0				0	0	0	2		2	1
MAN	0	0	0	0	0	0	0	0				0	0	0	6		6	1
TRA	0	0	0	0	0	0	0	0				0	0	0	2		2	1
SER	0	0	0	0	0	0	0	0				0	0	0	8		8	1
FOS	5	1	2	6	2	8	0	14				5	0	0	2		49	1
ELY	0	0	0	0	0	0	0	0				0	0	0	14		14	1
LABOUR																	0	
FIX																	0	
CAPITAL																	0	
HOUSEHOLDS															5		5	
INVESTOR																0	0	
GOVERNMENT															0		0	
EXTRACTION	0	0	0	0	0	0	49	0									49	
Total input	5	1	2	6	2	8	49	14	0	0	0	5	0	0	49	0	145	

Table 8: BAU - PIOT Iron (material capital)

	AGR	MIN	MET	MAN	TRA	SER	FOS	ELY	LAB	FIX	CAP	HOU	INV	GOV	WASTE	STOCK	OUT	Lambda
AGR	0	0	0	0	0	0	0	0				0	0	0	0		0	1
MIN	0	0	1002	0	0	0	0	0				0	0	0	0		1002	1
MET	0	0	0	816	0	0	0	0				0	0	0	186		1002	1
MAN	0	0	0	186	139	238	0	33				173	143	4	86		1002	1
TRA	0	0	0	0	0	0	0	0				126	0	13	0		139	1
SER	0	0	0	0	0	0	0	0				158	0	80	0		238	1
FOS	0	0	0	0	0	0	0	0				0	0	0	0		0	1
ELY	0	0	0	0	0	0	0	0				0	0	0	33		33	1
LABOUR																	0	
FIX																	0	
CAPITAL																	0	
HOUSEHOLDS															457		457	
INVESTOR																143	143	
GOVERNMENT															97		97	
EXTRACTION	0	1002	0	0	0	0	0	0									1002	
Total input	0	1002	1002	1002	139	238	0	33	0	0	0	457	143	97	859	143	5115	