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Abstract

Countries with ambitious climate targets are concerned about carbon leakage to countries with more lenient or no carbon pricing. A common policy measure against leakage is output-based allocation of emissions allowances, whose effectiveness could be further enhanced by consumption taxes levied on the carbon intensity of goods. We combine theoretical and numerical analysis to derive optimal combinations of output-based allocation and consumption taxes for different assumptions on the stringency of emissions reduction targets, the coverage of emissions in regulated sectors, and their trade exposure. A key analytical finding is that output-based allocation and consumption taxes are complements rather than substitutes, i.e., the extent of output-based allocation should be higher if combined with a consumption tax. A key numerical finding is that the optimal output-based allocation and consumption tax rates should be set at almost the same rate and increase substantially with the stringency of the emissions reduction targets.

JEL-Codes: D610, F180, H230, Q540.

Keywords: carbon leakage, output-based allocation, consumption taxes.

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1. Introduction

To keep global warming well below 2°C, anthropogenic greenhouse gas emissions must be drastically reduced worldwide. However, there are huge differences in emissions pricing between countries,[1] which raise concerns on carbon leakage, i.e., the counterproductive relocation of emissions from regions with strict regulations to other regions with no or more lenient restrictions. In the climate policy debate, such concerns are particularly relevant for emissions-intensive and trade-exposed (EITE) industries such as steel, cement and aluminum.

A common unilateral measure to curb carbon leakage via international markets for EITE goods has been to combine an emissions trading system (ETS) with generous allocation of free emissions allowances to EITE firms. This was implemented, for example, with the introduction of the EU Emissions Trading Scheme in 2005. In order to make free allowances an effective (incentive-compatible) instrument for curbing leakage, the allocation of allowances must be linked to the economic activities of individual firms. This is the case for output-based allocation (OBA) (Böhringer and Lange 2005ab), which comes very close to what the EU has implemented since 2013.

However, the allocation of free allowances via OBA has negative side effects, as has been shown in previous studies (e.g., Böhringer et al., 2014a, 2017b; Martin et al., 2014). OBA to EITE firms acts as an implicit subsidy to EITE production, and hence tends to encourage excessive use of the EITE goods and insufficient substitution with other goods. This is particularly unfavorable for those EITE goods which are not so much exposed to leakage. Alternative instruments to combat carbon leakage have therefore been discussed. In particular, border carbon adjustments (BCAs), i.e., import tariff on embodied carbon and possibly an export rebate of domestic carbon payments, have been appraised in the academic literature as more efficient instruments (Markusen, 1975; Hoel, 1996; Fischer and Fox, 2012; Böhringer et al., 2014a). BCAs are meanwhile considered for implementation in several countries. The EU has recently decided to introduce a variant of BCA – the so-called Carbon Border Adjustment Mechanism (CBAM) – which will gradually replace the allocation of free allowances from 2026 onwards.[2] However, there are several potential drawbacks with BCAs in general and the EU's CBAM in particular (see e.g. Mehling et al., 2019; Böhringer et al.,

[1] https://www.i4ce.org/en/publication/global-carbon-accounts-2023-climate/
[2] https://taxation-customs.ec.europa.eu/carbon-border-adjustment-mechanism_en

2022; Clausing and Wolfram, 2023), which are related to WTO compatibility and the fact that EU's CBAM is only targeting imports and not exports of EITE goods.

An alternative strategy proposed and analyzed in some recent studies (Böhringer et al., 2017b, 2021; Kaushal and Rosendahl, 2020) is to supplement OBA with a tax on the domestic use of EITE goods based on their carbon content. Such a "consumption tax" will mitigate the negative side effects of OBA by curbing the use of EITE goods and incentivizing substitution with other goods. Moreover, Böhringer et al. (2017b) show that the combination of output-based allocation and a consumption tax for EITE goods can be equivalent to BCA on global efficiency grounds if the import tariff is set equal across importers of the same good. The import tariff in the EU's CBAM, however, applies to the individual importer, potentially giving importers incentives to reduce their emissions intensity. Alternatively, it may simply lead to reshuffling of trade (Böhringer et al., 2022). Whether the combination of OBA with a consumption tax outperforms the EU's CBAM or not is thus an open question. For EITE sectors with a large share of exports outside the EU, however, the former policy is likely better from a global efficiency perspective (since CBAM does not protect exporters, while OBA acts partially as an export rebate of domestic carbon payments). Moreover, from a WTO perspective, such a policy has furthermore the appeal to be less legally controversial.

A challenge for designing efficient anti-leakage policies is the fact that a substantial share of the total emissions associated with the production of EITE goods do not come directly from the emissions taking place at the EITE plant itself (Scope 1), but is indirectly embodied in electricity use (Scope 2) and other intermediate inputs (Scope 3).[3] As a consequence, EITE industries that are very electricity-intensive will suffer from higher electricity prices (along with higher emissions prices) to the extent that electricity generation is based on fossil fuels such as gas and coal. In the EU, this is partly accounted for by allowing Member States to compensate its industries for higher electricity prices,[4] but this support scheme will be gradually replaced by an extended CBAM that takes into account Scope 2 emissions.

The objective of this paper is to derive, both analytically and numerically, optimal combinations of output-based allocation and consumption taxes. We investigate the extent to

[3] As shown in Böhringer et al. (2017a), a BCA that extends the scope of coverage from Scope1 to include also Scope 2 emissions can increase its cost-effectiveness as it extends the outreach of unilateral emissions regulations.
[4] Member States are allowed to support electricity-intensive industries exposed to carbon leakage for higher electricity prices caused by the EU ETS. The maximum aid may be either proportional to output or use of electricity (https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52012XC0605%2801%29, see paragraph 27). If proportional to output, this works very similar to OBA. If proportional to electricity use, the similarity with OBA depends e.g. on how easy it is for the firms to substitute electricity with other input factors.

which the two policy instruments complement or substitute each other, and explore how the optimal OBA rate depends on the consumption tax rate (and vice versa). In our numerical analysis, we examine how the optimal policy is affected by different assumptions about the stringency of emissions reduction targets, the coverage of emissions in regulated sectors, and their trade exposure. In terms of practical policy design, we are particularly interested in how the optimal mix compares with an OBA rate of 100%, that is, when aggregate allocation of allowances for the EITE firms equals the aggregate EITE (Scope 1) emissions, as is the case in current practice.

To date, there are only a few studies that address the combination of OBA and a consumption tax. The study which comes closest to the present analysis is Böhringer et al. (2017b). Compared to the latter, our analysis makes the following contributions: *i)* we derive optimal consumption tax rates at different OBA rates, and vice versa, thereby identifying the optimal combinations of the two instruments (Böhringer et al. (2017b) only consider an OBA rate of 100%), *ii)* in addition to Scope 1 emissions we include Scope 2 emissions, which are gaining more and more policy relevance, *iii)* instead of a partial equilibrium model our theoretical analysis relies on a general equilibrium model that also incorporates intermediate use of EITE goods, and *iv)* we investigate how more stringent emissions reductions targets, emissions coverage and trade exposure affect the optimal policy mix.

Other related studies are Böhringer et al. (2021) and Kaushal and Rosendahl (2020), who consider combinations of OBA and consumption tax in the context of the EU ETS. Böhringer et al. (2021) also derive analytically the optimal consumption tax given constant returns to scale technology and an OBA rate of 100%. Other studies analyzing consumption taxes from a climate perspective are Holland (2012), Eichner and Pethig (2015a,b), Pollitt et al. (2020) and Grubb et al. (2022).[5] For our research, we combine theoretical general equilibrium analysis with computable general equilibrium (CGE) analysis to complement qualitative findings with quantitative estimates on the magnitude of policy impacts. We set up a canonical CGE model which is calibrated to the latest economic accounts for the world economy provided by the Global Trade Analysis Project (GTAP version 11, Aguiar et al., 2022).

[5] More generally, there is a large literature on carbon leakage and measures to mitigate leakage, e.g., Baccianti and Schenker (2022), Dechezleprêtre et al. (2022), Fowlie and Reguant (2022), McAusland (2021). Fewer studies have analyzed the effects of the compensation scheme for higher electricity prices, but two recent empirical studies are Ohlendorf (2022) and Ferrara and Giua (2022).

In our theoretical analysis, we show that the optimal OBA rate depends positively on the consumption tax rate and vice versa. Hence, the two policy instruments are complements, not substitutes. The intuition is that both instruments dampen the negative side effects of the other instrument. We derive reduced-form expressions for the optimal combination of OBA rates and consumption tax rates. A key result is that the optimal policy depends on the extent to which the instruments are able to reduce emissions abroad (Scope 1 and 2), relative to their distortionary impacts on the EITE markets (measured as changes in net imports).

The numerical results confirm our theoretical finding on the complementarity of the two policy instruments: When we increase the consumption tax rate, the optimal OBA rate increases (and vice versa). The simulation results indicate that the optimal OBA rate stand-alone is strictly positive and ranges below 100% (unless emissions reduction targets are very stringent). Another policy-relevant finding from the simulation analysis is that consumption taxes can *increase* leakage unless intermediate use of EITE goods in EITE industries is exempted. Hence, the optimal consumption tax rate stand-alone is strictly positive only if the intermediate use in EITE industries is exempted. Once consumption taxes are applied economy-wide, the optimal OBA and consumption tax rates should be set at almost the same levels.

The remainder of this paper is organized as follows. Section 2 presents our theoretical analysis, while Section 3 contains the numerical analysis. Section 4 concludes.

2. Theoretical analysis

We consider a stylized general equilibrium model with two regions, $j = \{1, 2\}$, and three goods $i = \{x, y, z\}$. Good x is emissions-free and tradable, good y is emissions-intensive and tradable (EITE), while good z is emissions-intensive and non-tradable and only used as input in production of y; z can be interpreted as electricity, in which case we disregard that electricity may also be used in consumption and production of emissions-free goods (as we want to focus on the importance of indirect emissions in EITE production). We also disregard other emissions-intensive and non-tradable goods such as transportation. Goods of the same variety which are produced in different regions are assumed to be homogenous and not subject to trade cost.[6] The market price (excluding taxes and subsidies) of good i in region j is denoted p^{ij}. For the tradeable goods, prices are equal and hence we have p^x and p^y.

Production of the three goods requires use of an input factor denoted l^{ij}, which is given in fixed supply L^j in each region. Hence, we have $L^j = l^{xj} + l^{yj} + l^{zj}$. The input factor L^j can be interpreted as labor or capital (or a combination), but we will refer to it as labor. Emissions e^{ij} are a by-product of production of y and z, and can be decreased either by reducing production or switching to other inputs.

Production of the good x is given by $x^j = f^j(x^{xj}, y^{xj}, l^{xj})$, where x^x and y^x are intermediates used in production of the good x. Production of the goods y and z are given by respectively $y^j = g^j(x^{yj}, y^{yj}, z^j, l^{yj}, e^{yj})$ and $z^j = h^j(l^{zj}, e^{zj})$. The representative consumer has utility $u^j = u^j(\bar{x}^j, \bar{y}^j)$ from consuming goods x and y, where $\bar{x}^j$ and $\bar{y}^j$ denote final consumption of the two goods.

Available climate policies are a tax t^j on emissions in region j, a subsidy s^j to production of the y good in region j, and a consumption tax v^j on purchases of the y good in region j. For the consumption tax, we will distinguish between an economy-wide tax and a tax that may differ between different segments of the economy, i.e., production sectors and the representative consumer. We are especially interested in the case where the tax is exempted for own (intermediate) use in the y sector. Note that a subsidy s^j mimics a situation where an emissions trading system (ETS) is combined with output-based allocation (OBA) to EITE industries.[7]

The maximization problems for producers of good x, y and z, with profits π_x, π_y and π_z, are respectively:

$$
\begin{aligned}
&\max_{x^{xj}, y^{xj}, l^{xj}} \left[p^x f^j(x^{xj}, y^{xj}, l^{xj}) - p^x x^{xj} - (p^y + v^j) y^{xj} - p^{lj} l^{xj} \right] \\
&\max_{x^{yj}, y^{yj}, z^j, l^{yj}, e^{yj}} \left[(p^y + s^j) g^j(x^{yj}, y^{yj}, z^j, l^{yj}, e^{yj}) - p^x x^{yj} - (p^y + v^j) y^{yj} - p^{zj} z^j - p^{lj} l^{yj} - te^{yj} \right] \quad (1) \\
&\max_{l^{zj}, e^{zj}} \left[p^{zj} h^j(l^{zj}, e^{zj}) - p^{lj} l^{zj} - t^j e^{zj} \right]
\end{aligned}
$$

[6] In the numerical simulations, we consider the case where goods of the same variety produced in different regions are heterogenous, and we add another emission-intensive and non-tradable sector (which includes transportation) as well as an explicit sector for fossil fuels.

[7] As laid out in several previous studies (e.g., Böhringer and Lange, 2005a,b), OBA can be regarded as an implicit production subsidy.

The representative consumer maximizes its utility, given its income which it takes as exogenous. Assuming the consumer receives all labor income, profits and net government revenues, its budget constraint becomes equivalent to the balance of trade constraint below. First-order conditions for producers and the consumer are provided in Appendix A.

Net imports of the goods *x* and *y* from region 2 to region 1 are given by:

$$\begin{aligned} x^T &= \overline{x}^1 + x^{x1} + x^{y1} - x^1 = x^2 - \overline{x}^2 - x^{x2} - x^{y2} \\ y^T &= \overline{y}^1 + y^{x1} + y^{y1} - y^1 = y^2 - \overline{y}^2 - y^{x2} - y^{y2} \end{aligned} \tag{2}$$

We assume balance of trade, that is:

$$p^x x^T + p^y y^T = 0 \tag{3}$$

Country *j*'s welfare is given by:

$$W^j = u^j(\overline{x}^j, \overline{y}^j) - \tau_j^j e^j - \tau_{-j}^j e^{-j} \tag{4}$$

where $e^j = e^{yj} + e^{zj}$. τ_j^j reflects the valuation of domestic emissions while τ_{-j}^j reflects the valuation of foreign emissions.

We will throughout assume that $\tau_2^2 = \tau_1^2 = 0$ (i.e., region 2 does not care about any emissions) and $\tau_1^1 \geq \tau_2^1 \geq 0$. Since our main interest is in policies to mitigate carbon leakage, we will focus on the case where region 1 considers emissions as a global externality causing damage regardless of the region of origin: $\tau^1 \equiv \tau_1^1 = \tau_2^1 > 0$.[8] We assume that the Pigouvian tax $t^1 = \tau^1$ is implemented to regulate emissions in region 1. There is no price on emissions (or any other climate policy) in region 2 ($t^2 = \tau^2 = 0$). Hence, we will refer to region 1 as the policy/domestic region, and region 2 as the foreign region.

Before we go into the welfare analysis of the production subsidy s^1 and the consumption tax v^1 in the policy region 1, we set out two assumptions underlying our analytical results below:

[8] In Appendix B, we consider the case $\tau_1^1 > 0$ and $\tau_2^1 = 0$, which is more appropriate for local pollutants, and derive the optimal consumption tax v^1 if a subsidy s^1 is already implemented for some reason.

Assumption A1. Introducing and increasing a subsidy to the y good in region 1 (s^1) increases (decreases) production of the y good in region 1 (2) and decreases emissions in region 2.

Assumption A2. Introducing and increasing a consumption tax on the y good in region 1 (v^1) reduces production and imports of the y good in region 1 as well as emissions in region 2 if own use in the y sector is exempt from the tax.

A subsidy to production of the emissions-intensive and trade-exposed good y will likely increase domestic production of this good at the expense of foreign production (and corresponding emissions), which explains A1. Furthermore, a tax on the use of the y good, with exemption for own use in the y sector, will likely reduce production in both regions, and hence also emissions abroad, explaining A2. See Appendix A for a more formal discussion of A1 and A2. Importantly, it is ambiguous whether emissions in region 2 will decrease or increase if an *economy-wide* consumption tax is levied in region 1, that is, if the y sector's own use of y as an input factor in production is also subject to the consumption tax. The reason is that the tax increases production costs for domestic y producers in region 1, which increases output of the y good and hence emissions in region 2. Whether or not this competitiveness effect dominates the effect of lower overall demand for the y good is ambiguous.

We first examine the optimal s^1 for any exogenous v^1 (and t^1). Define $\tilde{y}^1 = \bar{y}^1 + y^{x1} + y^{y1}$, i.e., total use of good y in region 1 (consumption plus use of y as a factor of production). Obviously, this equals domestic production plus net import of y ($y^1 + y^T$, cf. equation (2)).

Maximizing welfare (4) with respect to s^1 gives (see Appendix A for full derivation):

$$\frac{dW^1}{ds^1} = -s^1 \frac{dy^1}{ds^1} + v^1 \frac{d\tilde{y}^1}{ds^1} - t^1 \frac{de^2}{ds^1} - \left(\frac{dp^x}{ds^1} x^T + \frac{dp^y}{ds^1} y^T \right) \tag{5}$$

The last term in equation (5) denotes a terms-of-trade effect, which for the sake of tractability we disregard throughout our theoretical analysis. Our omission of terms-of-trade effects is akin to the assumption that countries do not misuse climate policies as a means to exploit terms of trade. Furthermore, we show in Appendix A that maximizing global welfare with respect to s^1 also yields equation (5), but without the terms-of-trade effects. Hence, the results obtained below are also valid from a global welfare perspective. We then derive the optimal rates of s^1 as follows:[9]

$$s^{1*} = \frac{-\frac{de^2}{ds^1}}{\frac{dy^1}{ds^1}} t^1 + \frac{\frac{d\tilde{y}^1}{ds^1}}{\frac{dy^1}{ds^1}} v^1 \tag{6}$$

Both fractions are positive, indicating that the optimal rate of s^1 increases with both the Pigouvian tax and the consumption tax. The optimal subsidy to *y* production is higher the more emissions abroad decrease *relative to* the increased domestic production of the *y* good. This reflects that the purpose of the subsidy is *not* to increase domestic output as such (quite the opposite), but to indirectly reduce emissions abroad. A negative side effect of s^1 is that it causes too high a production of *y* by distorting relative prices. This is reflected in the denominator in equation (6).

It should also be noticed that both direct (e^{y2}) and indirect (e^{z2}) emissions abroad should be taken into account (recall that $e^2 = e^{y2} + e^{z2}$), while domestic emissions intensities are not relevant in first place (we return to this below). Further, if a consumption tax is in place ($v^1 > 0$), the optimal subsidy is higher the more it increases the domestic use of the product relative to domestic output. That is, the subsidy should correct for the negative side effect of a potential consumption tax v^1, which distorts relative prices causing too little use of the *y* good: One purpose of each instrument (here: s^1) is to correct for the negative side effect of the other instrument (here: v^1). Note that the fraction in front of v^1 is less than one (cf. Assumption A1), which means that in the absence of t^1 the subsidy rate should be lower than v^1.

In the special (but policy-relevant) case without an initial consumption tax, the optimal *s* becomes:

$$s^{1*} = \frac{-\frac{de^2}{ds^1}}{\frac{dy^1}{ds^1}} t^1 \tag{7}$$

[9] Throughout we use * to denote the optimal rate of a policy instrument. Note that Assumption A1 and equation (5) imply an interior solution for s^{1*}, because $s^1\left(dy^1 / ds^1\right) = 0$ when $s^1 = 0$ (such that $dW^1 / ds^1 > 0$ at $s^1 = 0$).

In this case, we see that the optimal *s* increases proportionally with the Pigouvian tax as long as the fraction is not changed, and we refer to this as a first-order effect.

We summarize our results in the following proposition:

Proposition 1. *Suppose region 1 cares equally about domestic and foreign emissions. Then it is optimal to implement a subsidy* s^{1*} *to the production of* y^1*. The first-order effect of increasing either the Pigouvian tax* t^1 *or the consumption tax* v^1 *is to increase the optimal subsidy, too.*

Proof. The proposition follows from the evaluation of equation (6), which again follows from the first-order condition $dW^1 / ds^1 = 0$ associated with equation (5) and Assumption A1.

Note that the proposition holds both from a regional welfare perspective, if the region does not strategically misuse climate policy to improve its terms of trade, and from a global welfare perspective (where terms-of-trade effects balance out by definition).

Next, we consider the optimal consumption tax, assuming that the Pigouvian tax and the subsidy rate are exogenous. We first derive optimal consumption tax rates $v^{1\alpha}$ differentiated across sectors *x* and *y* as well as the representative consumer (*c*), indicated by the superscript $\alpha = \{c, x, y\}$. One motivation for deriving the differentiated tax rates is to show why it might make sense to exempt own use of sector *y*. We then discuss the optimal economy-wide consumption tax v^1.

Maximizing welfare (4) with respect to $v^{1\alpha}$ and solving for the optimal differentiated consumption taxes yields (see Appendix A for derivation):

$$\begin{aligned}
v^{1c*} &= \left[s^1 \frac{dy^1}{dv^{1c}} - v^{1x} \frac{dy^{x1}}{dv^{1c}} - v^{1y} \frac{dy^{y1}}{dv^{1c}} + t^1 \frac{de^2}{dv^{1c}} \right] / \frac{d\bar{y}^1}{dv^{1c}} \\
v^{1x*} &= \left[s^1 \frac{dy^1}{dv^{1x}} - v^{1y} \frac{dy^{y1}}{dv^{1x}} - v^{1c} \frac{d\bar{y}^1}{dv^{1x}} + t^1 \frac{de^2}{dv^{1x}} \right] / \frac{dy^{x1}}{dv^{1x}} \\
v^{1y*} &= \left[s^1 \frac{dy^1}{dv^{1y}} - v^{1x} \frac{dy^{x1}}{dv^{1y}} - v^{1c} \frac{d\bar{y}^1}{dv^{1y}} + t^1 \frac{de^2}{dv^{1y}} \right] / \frac{dy^{y1}}{dv^{1y}}
\end{aligned} \qquad (8)$$

where terms-of-trade effects are again omitted.

Consider first the consumption tax imposed on the consumer, v^{1c}. This tax reduces both consumption and domestic production of the *y* good, i.e., $d\bar{y}^1 / dv^{1c} < 0$ and $dy^1 / dv^{1c} < 0$ (cf. Assumption A2). Hence, the denominator is negative, and so is also the first term inside the square bracket (if $s^1 > 0$). The second term is also likely negative (if $v^{1x} > 0$) as the consumption tax *ceteris paribus* reduces the equilibrium price for *y* and thereby stimulates the use of *y* as a factor of production in *x* ($dy^{x1} / dv^{1c} > 0$).[10] The third term is less clear. On the one hand, the reduced equilibrium price for *y* stimulates own use of *y* in producing *y*. On the other hand, as mentioned above the tax reduces domestic production of *y*. These two mechanisms have opposite effects on the sign of dy^{y1} / dv^{1c}, which is then a priori ambiguous. Last but not least, the fourth term is negative, reflecting that the consumption tax dampens production of *y* and hence emissions abroad (cf. Assumption A2). Note that this mechanism is present even without existing subsidies or consumption taxes. It follows that the optimal consumption tax imposed on the consumer is positive ($v^{1c*} > 0$) when evaluated at $v^{1x} = v^{1y} = 0$ (and most likely also if $v^{1x} > 0$). By similar reasoning, we find that the optimal v^{1x*} is positive.

For the optimal consumption tax on own use of *y* in the *y* sector, v^{1y}, the sign is no longer clear as the term involving foreign emissions in equation (8) is ambiguous. There are two opposing effects on foreign emissions: First, the consumption tax reduces domestic consumption of *y*. Some of this decrease falls on imports, which reduces foreign production and hence emissions. This effect is the same as for v^{1c} and v^{1x}. However, v^{1y} also has a second effect: It increases the *y* sector's costs and, hence, reduces its competitiveness relative to the foreign *y* sector. This effect tends to increase foreign production and emissions. It follows that the sign on the optimal consumption tax v^{1y*} is ambiguous. Because we restrict the analysis to $v^{1\alpha} \geq 0$, this implies that the corner solution $v^{1y*} = 0$ is possible.

To summarize, we have the following result for optimal differentiated consumption taxes:

Lemma 1. *Suppose region 1 cares equally about domestic and foreign emissions. Then it is optimal to implement consumption taxes* $v^{1\alpha*}$ *on the y good used in the x sector and in consumption. It is ambiguous whether it is optimal to implement the tax on own use of y in the y sector. In general, we have* $v^{1c*} \neq v^{1x*} \neq v^{1y*}$.

[10] It will also increase the demand for *x* if *x* and *y* are substitutes in consumption. On the other hand, if they are complements, the consumption tax on *y* can reduce the demand for *x*, which again may reduce factor demand for *y* in the production of *x*.

Proof: The lemma follows from equation (8) and Assumption A2.

Optimal differentiated consumption taxes as given by equation (8) may be difficult to implement in practice. A reasonable alternative is an economy-wide consumption tax, with a possible exemption for own use in the *y* sector.[11] As discussed above, the impacts on emissions abroad from decreased domestic demand of the good *y* depend crucially on whether the demand reduction is due to reduced own use in sector *y* or due to reductions in other demands of the good. In the latter case, domestic and foreign producers of the *y* good are affected symmetrically. In the former case, the incidence is predominantly on domestic supply as it has to bear an increase in input cost which further reduces domestic supply. This might suggest that the *y* sector, or more generally leakage-exposed sectors, should be exempted from a consumption tax. As we see from equation (8), this is more likely to be the case if the consumption tax is implemented alone (with $s = 0$), and less likely if combined with the subsidy. We return to this below.

With an economy-wide tax ($v^{1x} = v^{1y} = v^{1c} = v^1$), equation (8) simplifies to (see Appendix A):

$$v^{1*} = \frac{\frac{de^2}{dv^1}}{\frac{d\tilde{y}^1}{dv^1}} t^1 + \frac{\frac{dy^1}{dv^1}}{\frac{d\tilde{y}^1}{dv^1}} s^1 \tag{9}$$

The same expression applies if the *y* sector is exempted ($v^{1y} = 0$). The optimal v^1 increases in the subsidy rate s^1. Moreover, if a subsidy is already implemented, the consumption tax is higher the more domestic production decreases relative to domestic use (consumption and input in production) of the *y* product. The optimal consumption tax v^{1*} also increases in the Pigouvian tax *if* the tax decreases foreign emissions (this is always the case if $v^{1y} = 0$). Finally, v^{1*} is higher the more emissions abroad decrease relative to the domestic use of the *y* product. Equation (9) reflects that the purpose of the consumption tax is *not* to reduce consumption as such, but to *i)* indirectly reduce emissions abroad (which are not regulated by any emissions price), and *ii)* correct for the negative side effect of the production subsidy *s*. Finally, we note that the economy-wide tax is given by $v^{1*} = s^1$ when there is no trade, i.e., the tax exactly offsets the distortions caused by the subsidy for the case of a closed economy.

[11] The economy-wide tax is then either $v^1 = v^{1c} = v^{1x} = v^{1y}$ or, in the case of excluding the *y* sector's own use, $v^1 = v^{1c} = v^{1x}$ and $v^{1y} = 0$.

The fraction dy^1 / dv^1 in equation (9) will be negative for the economy-wide consumption tax, because the demand for *y* falls when domestic consumption of *y* is taxed. However, this need not be the case if own use in the *y* sector is exempt from the consumption tax. The reason is that the price of *y* as a factor of production in the *y* sector falls when the other sectors have to pay the consumption tax. The consumption tax then has two counteracting effects on the *y* sector in region 1: *(i)* a lower producer price for output, and *(ii)* a lower price for *y* in intermediate own use. Therefore, the sign of dy^1 / dv^1 in equation (9) depends on the relative strength of these two opposite effects. Note that the relative strength of these effects can change for different levels of s^1 and v^1.[12]

Summarizing, we have the following result for optimal consumption taxes:

Proposition 2. *Suppose region 1 cares equally about domestic and foreign emissions. Then it is optimal to implement a consumption tax* v^{1*} *on the domestic use of the* y *good if own use in the* y *sector is exempted. The first order effect of increasing either the Pigouvian tax* t^1 *or the subsidy* s^1 *is to increase the optimal consumption tax, too. Whether it is optimal to implement an economy-wide consumption tax is ambiguous.*

Proof. The proposition follows from Lemma 1 and the evaluation of equation (9) above.

Again, the proposition holds both from a regional welfare perspective when disregarding terms-of-trade, and from a global welfare perspective.

In the formulas for optimal production subsidies and consumption taxes emissions intensities in the policy region 1 play no direct role. It is only emissions abroad that matter explicitly. The reason for this is that domestic emissions are controlled by the Pigouvian tax $t^1 = \tau$. Still, a stricter climate policy (higher t^1) caused by a higher shadow price on emissions τ^1 tends to reduce domestic emissions intensities. At the same time, it tends to increase the optimal subsidy and consumption tax (cf. (6) and (9)). Thus, there will be a tendency of both the optimal subsidy and the optimal consumption tax to increase as domestic emissions intensities decline.

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[12] The fractions in front of s^1 and v^1 in equations (6) and (9) are the inverses of each other if $dy^1 / dv^1 = -dy^1 / ds^1$ and $d\tilde{y}^1 / dv^1 = -d\tilde{y}^1 / ds^1$. This is true in a closed economy, since an increase in *v* is then equivalent to a decrease in *s*, and it can occur approximately for an open economy if *y* is a homogeneous good. In this case, the two curves in Figure 1 in the numerical section will lie on top of each other.

It is insightful to relate the optimal subsidy rate s^* to the output-based allocation rate a^e, that is, the ratio between allocated emissions allowances and actual (direct) emissions $a^e = a^y y^1 / e^{y1}$, where a^y denotes the allowances allocated per unit of production. The ratio can also be written as $a^e = (a^y t y^1)/(t e^{y1})$, where t is the emissions price and $s^{OBA} \equiv a^y t$ is the implicit subsidy rate per unit of production. A common benchmark for OBA is 100% allocation ($a^e = 1$), that is, $s^{OBA} = t e^{y1} / y^1$. How does this s^{OBA} compare with the optimal s^{1*} in (6)? If we disregard the influence of v^1 ($v^1 = 0$), we are left with comparing e^{y1} / y^1, that is, the (direct) domestic emissions intensity in sector *y*, with $(-de^2 / ds^1)/(dy^1 / ds^1)$. This last expression can be either lower or higher than the former. This depends on to what degree an increase in domestic production affects production and emissions abroad. On the one hand, increased domestic supply will typically not completely replace displace foreign supply, so that a 100% free allocation tends to be too generous. The extent to which foreign supply is displaced depends on the extent to which the sector is exposed to leakage. On the other hand, emissions intensities abroad may differ from those at home, and when accounting for indirect emissions abroad as well, it can be the case that the sum of direct and indirect emissions intensities abroad are (possibly much) higher than the direct emissions intensities at home. This argument is strengthened as the domestic price of emissions increases, incentivizing reduced domestic emissions intensities. Which of the two counteracting factors predominates will vary from (sub)sector to (sub)sector. Allowing for $v^1 > 0$ (and considering the optimal combination of s^1 and v^1) increases the optimal OBA rate (cf. the second term in (6)) – still it is ambiguous how the optimal rate compares with 100% free allocation.

We now derive the optimal combination of a subsidy s^1 and an economy-wide consumption tax v^1 in region 1 for a given Pigouvian tax t^1. Combining the expressions for s^{1*} (6) and v^{1*} in equations (6) and (9), we can derive the following optimal combination (see Appendix A):

$$\begin{aligned} s^{1*} &= \left[\frac{d\tilde{y}}{ds^1} \frac{de^2}{dv^1} - \frac{d\tilde{y}}{dv^1} \frac{de^2}{ds^1} \right] \frac{t^1}{\Gamma} \\ v^{1*} &= \left[\frac{dy^1}{ds^1} \frac{de^2}{dv^1} - \frac{dy^1}{dv^1} \frac{de^2}{ds^1} \right] \frac{t^1}{\Gamma} \end{aligned} \tag{10}$$

where $\Gamma = \frac{dy^1}{ds^1} \frac{dy^T}{dv^1} - \frac{dy^1}{dv^1} \frac{dy^T}{ds^1}$ is negative if the *y* sector's own use of *y* is exempt from the consumption tax, but otherwise ambiguous (cf. Assumptions A1 and A2). Similarly, both brackets (for s^{1*} and v^{1*}) are negative if the *y* sector's own use is exempt, but otherwise ambiguous. Hence, if the *y* sector is exempt from the consumption tax, both expressions are positive. Moreover, the first-order effect of raising the Pigouvian tax is to increase both s^{1*} and v^{1*} proportionally.

We see that *both* instruments should be higher the more emissions abroad are reduced when increasing the same *or the other* instrument. Further, we notice that the optimal subsidy is higher the more responsive domestic *supply* (domestic production plus import) is to each of the two instruments. On the other hand, the optimal consumption tax is higher the more responsive domestic *production* is to each of the two instruments. If the subsidy is increased, domestic production responds more strongly than domestic supply (since net imports drop), while the opposite is likely to be the case if the consumption tax is increased.

Equation (10) implies the following result (see Appendix A):

$$s^{1*} \geq (\leq) v^{1*} \Leftrightarrow \frac{\frac{de^2}{ds^1}}{\frac{dy^T}{ds^1}} \geq (\leq) \frac{\frac{de^2}{dv^1}}{\frac{dy^T}{dv^1}} \qquad (11)$$

where both fractions are positive if the *y* sector is exempt from the consumption tax (otherwise the right-hand side is ambiguous).[13]

The numerators in the two fractions capture the key positive effects from the instruments, which are lower emissions abroad. The denominators capture the key negative effects of the instruments, i.e., the distortions to respectively domestic production and domestic use of *y* (as captured by dy^T / ds^1 and dy^T / dv^1). Hence, both fractions increase in the positive effects from the instrument and decrease in the negative effects. We also observe the similarity between the two fractions and the optimal subsidy and consumption tax in equations (7) and (9).

We summarize these findings in the following proposition:

[13] Most likely the right-hand side is also positive for the case of an economy-wide consumption tax, as the numerator and denominator will typically have the same sign. The only exception is if the reduction in p^y (due to v^1) stimulates demand in region 2 to such an extent that y^2 increases while y^T (i.e., net export from region 2) decreases.

Proposition 3. *Suppose region 1 cares equally about domestic and foreign emissions. Then it is optimal to implement a strictly positive subsidy to production of the y good and a strictly positive consumption tax on the y good (possibly exempting own use in sector* y*).*

Proof. The proposition follows directly from equations (10) and (11) and Assumptions A1 and A2.

This proposition also holds both from a regional welfare perspective, if one disregards terms-of-trade effects, and from a global welfare perspective.

3. Numerical Analysis

We develop a canonical computable general equilibrium (CGE) model which is used for quantitative simulation analysis of production subsidies and consumption taxes in unilateral climate policy. The numerical model mirrors the basic features of the analytical model but adopts explicit functional forms to capture technologies and preferences based on empirical data. We begin with a non-technical summary of the model structure (an algebraic model summary can be found in Appendix C), and briefly lay out the data used for model parameterization. We then describe our policy scenarios before presenting and discussing simulation results.

3.1 Non-technical model summary

The model consists of two composite regions (policy region 1 and no-policy region 2) with five production sectors and one final demand segment. Three of the sectors correspond to the sectors *x*, *y* and *z* in the analytical model: carbon-free and tradable production (*NC_T*), carbon-intensive and tradable production (*C_T*), and electricity generation (*ELE*). In addition, we include other carbon-intensive and non-tradable production (*C_NT*) and fossil energy production (*FE*). CO_2 emissions are proportional to fossil energy use. All five production goods can be used both for final consumption and as intermediate inputs in production, with the only exception that *NC_T* does not use fossil energy.

Primary factors of production include capital and labor, which are mobile within a region but not between regions; in addition, fossil energy production uses sector-specific fossil resources. Primary factors and intermediate inputs are combined to produce a good at minimum cost subject to technological constraints. For non-energy goods, three-level constant-elasticity-of-substitution (CES) cost functions capture the production possibilities.

At the top level, non-energy intermediates trade off with a composite of energy, capital and labor. At the next level, energy trades off with a value-added composite of capital and labor, while at the third level, capital and labor trade off. In fossil energy production, the fossil resource trades off with a Leontief composite of all other inputs.

In each region, a representative consumer maximizes utility subject to a budget constraint, where income comes from factor earnings and net revenues from emissions pricing. Utility is given as a two-level CES expenditure function. At the first level, energy trades off with a composite of non-energy goods. At the second level, the different non-energy goods trade off.

Emissions can be reduced by decreasing the use of fossil energy - either by substituting fossil energy with other inputs in production or by reducing the production output.

In our standard model specification, regions can trade the two goods *C_T* and *NC_T*. Trade in fossil fuels is not considered initially, as our analysis focuses on leakage via spillover effects on international markets for carbon-intensive and trade-exposed goods – the so-called competitiveness channel as opposed to the so-called fossil fuel market channel which captures leakage through price responses on international fuel markets (see e.g. Böhringer et al., 2012).[14] For the same reason – i.e., in order to suppress leakage through energy price responses in region 2 – we keep the fossil energy price in the no-policy region 2 fixed at the benchmark level.[15]

A balance of payment constraint is imposed for both regions. In the theoretical analysis, goods produced in different regions are treated as homogenous. In the numerical simulations, we consider the more realistic case of product heterogeneity, following the seminal proposition by Armington (Armington, 1969) which differentiates goods by region of origin. Goods of the same variety that are produced in different regions trade off at a constant elasticity of substitution to form the composite Armington good. The Armington good then serves both intermediate input demand as well as final consumption demand.

3.2 Model parametrization

The free parameters of functional forms that characterize production technologies (cost functions) and consumer preferences (expenditure functions) are calibrated based on cost and

[14] The focus on the competitiveness channel is motivated by the fact that production subsidies and consumption taxes are demand-side instruments that can only target leakage through the competitiveness channel. We consider the implications of trade in fossil energy as part of our sensitivity analysis.
[15] The fuel price targeting in region 2 is implemented as an endogenous rationing of fossil fuel supply in that region.

expenditure shares from empirical observations. We use the latest version of the GTAP database which provides input-output and final demand transactions in a globally consistent set of social accounts for the base-year 2017. The 65 GTAP sectors are first mapped to our five sectors (see Table D.1 in Appendix D for the sector mapping). We then construct an input-output table for global production and consumption where we aggregate the 160 GTAP regions.[16]

To follow the symmetry assumption in the theoretical analysis, the global economy is divided into two identical regions. For each of the two regions, each entry in the region-specific input-output table is set equal to half of the corresponding entry in the input-output table for the global economy. When it comes to initial trade between the two regions, we assume that half of the gross GTAP trade flows for commodities *C_T* and *NC_T* takes place between the two regions. The resulting input-output table for each of the two symmetric regions which captures the benchmark situation without any climate policy is shown in Table D.2 in Appendix D.

The GTAP database also includes estimates for (Armington) trade elasticities, which determine the degree of substitutability in (intermediate and final) consumption between domestically and foreign produced traded goods. This elasticity is a key determinant of leakage through the competitiveness channel and hence also of the impacts of production subsidies and consumption taxes as policy instruments to curb leakage through the competitiveness channel.

3.3 Policy scenarios

Our policy simulations start from a reference scenario (*ref*) where a single region – in our case region 1 – levies an economy-wide price on its CO_2 emissions. The unilateral CO_2 price is set to achieve a specific domestic emissions reduction target and can be implemented either as an explicit tax or through an emissions trading system (ETS). By default, the emissions reduction target is set to 20% below benchmark emissions. Note that the refence scenario describes a situation in which no production subsidies in the form of output-based allocation (referred thereafter as *oba*) or consumption taxes (referred thereafter as *ctax*) are used to

[16] As mentioned above, fossil energy use in the *NC_T* sector is set equal to zero. In the original GTAP dataset, this sector only accounts for a few percent of total fossil energy use.

combat leakage and increase global cost-effectiveness of unilateral emissions pricing. In region 2, there is no CO_2 price (or other climate policy) in any of the scenarios.

We then consider combinations of a production subsidy (*oba*) and a consumption tax (*ctax*) for the carbon-intensive and tradable good *C_T* in region 1. Consistent with our theoretical analysis, the consumption tax on the use of *C_T* can either apply to the entire economy or exempt intermediates use in the *C_T* sector. The rates of the production subsidy and the consumption tax are varied and are reported as *oba* and *ctax* rates. Both rates are defined as percentages of the direct (Scope 1) emissions intensity in *C_T* production prevailing in the *ref* scenario times the emissions price. So, if the *oba* rate is 100%, *C_T* producers receive 100% of the emissions allowances they would need if they did not change their emissions intensity.[17] A 100% *ctax* rate implies a consumption tax which taxes the use of *C_T* at the full direct emissions intensity times the actual emissions price.

To ensure a consistent global cost-effectiveness analysis across different climate policy designs, we keep global emissions constant at the level of the *ref* scenario.[18] This is done by endogenously scaling the domestic emissions cap in region 1 (or likewise the domestic emissions price) so that global emissions are always the same as in the *ref* scenario. Thus, if the additional policies (*oba* and *ctax*) reduce leakage, the domestic emissions target is relaxed. With a constant global emissions level, we can investigate policy designs which are either maximizing global welfare or the welfare of the unilaterally acting region. For the former – which will be the focus of our numerical analysis – we adopt a utilitarian perspective which maximizes the sum of money-metric utility across regions.[19] In line with our theoretical analysis, we will search for optimal *ctax* rates for given *oba* rates (cf. equation (9) in Section 2), optimal *oba* rates for given *ctax* rates (cf. equation (6)), and optimal combinations of the two (cf. equations (10)). Since we keep global emissions constant across policy scenarios, "optimal" here means from a global cost-effectiveness perspective.

To check the robustness of our results, we conduct sensitivity analyses along four dimensions: i) the stringency of unilateral climate policy, i.e. the level of the domestic

[17] Both *oba* and *ctax* hardly change the *ref* emissions intensity.
[18] Otherwise, cost-benefit analysis would require an explicit valuation of climate damages from emissions which is subject to considerable uncertainty as estimates on climate damages from the integrated assessment literature vary widely depending on critical assumptions such as discount rates or the choice of damage functions (Wang et al., 2019).
[19] Note that region-specific welfare maximization includes a strategic dimension as unilateral policy design is not only driven by the motive to reduce leakage but also by the incentive to exploit terms of trade at the expense of trading partners (Böhringer et al., 2014b). If terms of trade effects are disregarded, the optimal policy is the same when maximizing regional or global welfare (cf. the theorical analysis in Section 2).

emissions reduction target, ii) the degree of leakage exposure via the competitiveness channel, i.e. the ease of substitution between the domestic and foreign variety of a traded good, iii) the importance of intermediate use of *C_T* goods in *C_T* production, i.e., the share of own use, and iv) the coverage of indirect emissions from electricity use in *oba* and *ctax* policy design, i.e., the role of Scope 2 emissions.

3.4 Simulation results

The main objective of our analysis is to investigate how the additional use of production subsidies (*oba*) and consumption taxes (*ctax*) can improve (global) cost-effectiveness of unilateral emissions pricing (*ref*). We therefore start with the economic and environmental impacts of the reference scenario *ref*, where we impose a 20% emissions reduction in region 1 (which in our case of two symmetric regions and abstracting from carbon leakage would imply a global emissions reduction of 10%).

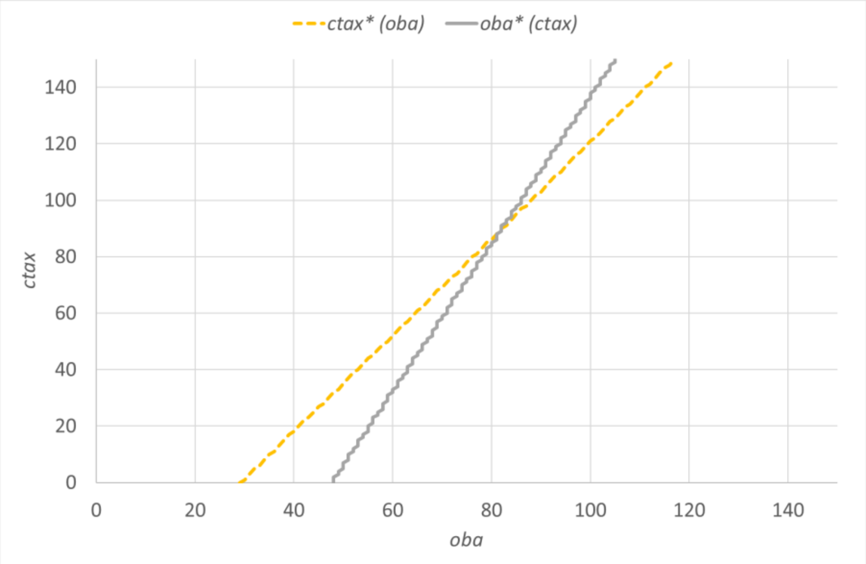
To achieve the domestic emissions reduction target, region 1 needs to raise an economy-wide emissions price of \$78/tCO_2, leading to a regional economic adjustment cost of 0.23% of regional welfare.[20] The production of *C_T* in region 1 declines by 3.8%, while it increases by 2.8% in region 2. This means that around three quarter of the domestic *C_T* production is offset by increased production abroad. The economy-wide carbon leakage rate is quite small (3.2%),[21] whereas leakage in the *C_T* sector alone (Scope 1) is 21%. Welfare in region 2 also drops via negative trade spillover effects, but to a much lesser extent, so that global welfare only decreases by 0.13% (when disregarding the benefits from lower global emissions).

Next, we identify optimal combinations of *oba* and *ctax* in region 1 from the perspective of global cost-effectiveness, which disregards terms-of-trade effects (as in our theoretical analysis). Figure 1 shows the optimal *oba* rates as a function of the *ctax* rate and vice versa.

[20] Recall that we do not value benefits from emissions reduction such that binding emissions constraints will inevitably lead to losses in overall allocative efficiency. Economic adjustment cost – of likewise welfare losses – are measured in terms of Hicksian equivalent variation which denotes the amount of money one has to give to the region in order to make this region in the policy counterfactual as well off as in the benchmark situation.

[21] Recall that we do not consider trade in fossil energy in our central case scenarios, and therefore suppress leakage via the fossil fuel market channel (cf. discussion above). Thus, leakage only takes place via the competitiveness channel for the *C_T* sector, which accounts for merely 18% of total emissions in the benchmark data. Moreover, leakage in our simulations is limited by the fact that we consider climate policy in a large region covering 50% of the global economy. As shown by e.g. Böhringer et al. (2014a), carbon leakage declines significantly with the size of the climate coalition.

Figure 1. Optimal *oba* rates (as a function of the *ctax* rate) and *ctax* rates (as a function of the *oba* rate) from a global cost-effectiveness perspective.



Three main results can be observed. First, the optimal *oba* rate stand-alone is 47%, meaning that *C_T* producers receive 47% of the allowances they would need to cover their emissions.[22] This is well below the allocation rate commonly used in policy practice for the sectors most exposed to leakage (cf. EU ETS). Second, the optimal *ctax* rate stand-alone would be negative (or zero as we restrict *oba* and *ctax* rates to be non-negative). The reason for this is that the consumption tax has a negative impact on the competitiveness of domestic *C_T* producers, as it increases the cost of intermediate inputs (especially the use of *C_T* goods in *C_T* production). Therefore, foreign production of *C_T* goods and thus leakage increases if the *ctax* is introduced stand-alone.

Third, the optimal rates of each instrument increase with the rate of the other instrument. As suggested by our theoretical analysis, the two instruments are complements, which is an important insight for unilateral climate policy design: if policy makers consider consumption taxes additional to unilateral emissions pricing, they also must consider higher production subsidies in an optimal policy mix – the reason is that the consumption tax counteracts the negative side effects of production taxes. While consumption taxes on their own are not desirable for leakage reduction, in combination with production subsidies they improve global cost-effectiveness of unilateral carbon pricing. The optimal combination for the case

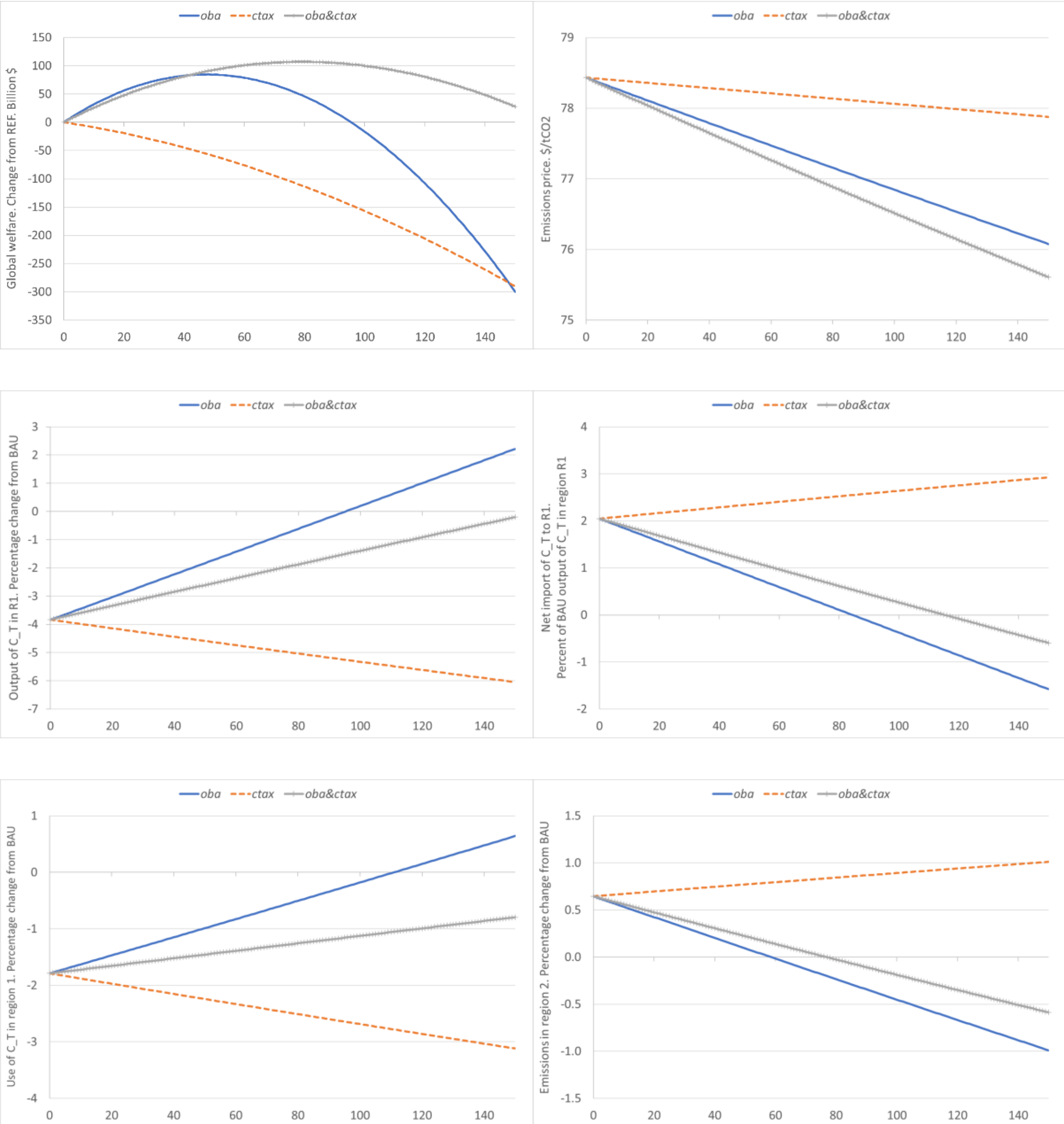
[22] Strictly speaking, the *oba* rate is related to the emissions intensity in the *ref* scenario, but the intensity hardly changes with the *oba* rate (by only 0.1% when the *oba* rate increases from 0 to 100%).

of the 20% unilateral emissions reduction target consists of *oba* and *ctax* rates of 81% and 88% respectively. Since the optimal combination of *oba* and *ctax* consists of quite similar rates (also in most of the other scenarios considered, except when the *C_T* sector itself is exempted from the *ctax*, see below), in the following we will mostly examine either *oba* or *ctax* stand-alone, or a combination with equal *oba* and *ctax* rates (referred to as *oba*&*ctax*). Recall that equal *oba* and *ctax* rates imply that the additional cost per input unit of *C_T* is equal to the subsidy received per output unit of *C_T*.

To better understand the numerical results, it is useful to revisit the analytical expressions derived in Section 2, for the optimal *oba* and *ctax* rates (s^{1*} and v^{1*} in (6) and (9), respectively). The optimal rates depend on how the instruments affect foreign emissions (de^2), domestic output (dy^{y1}) and domestic use ($d\tilde{y}^1$), where the latter is equal to output plus net import (i.e., domestic supply). As shown in Figure 2.d, an increase in *ctax* leads to an increase in imports of *C_T* goods, which explains the increase in emissions abroad (see Figure 2.f). In fact, domestic use of this good decreases significantly less than domestic output – as captured by the slope of the *ctax* curve in Figure 1, which reflects how domestic output changes relative to domestic use (cf. equation (9)).

The opposite is the case when we consider *oba*. A small increase in the *oba* rate leads to a 2.5 times larger increase in domestic output than in domestic supply. Since the slope of the *oba* curve reflects how domestic use changes relative to domestic output (i.e., the inverse of the slope of the *ctax* curve, see equation (6)), the optimal *oba* rate is much less responsive to the *ctax* rate than vice versa. On the other hand, foreign emissions are declining in the *oba* rate (see Figure 2.f), which has a positive effect on the optimal *oba* rate. Around the optimal combination of *oba* and *ctax* in Figure 1, the emissions effects (first part of equation (6)) account for about 60% of the optimal *oba* rate (i.e., value of s^{1*}), while the *ctax* rate (second part of the equation) accounts for the rest.

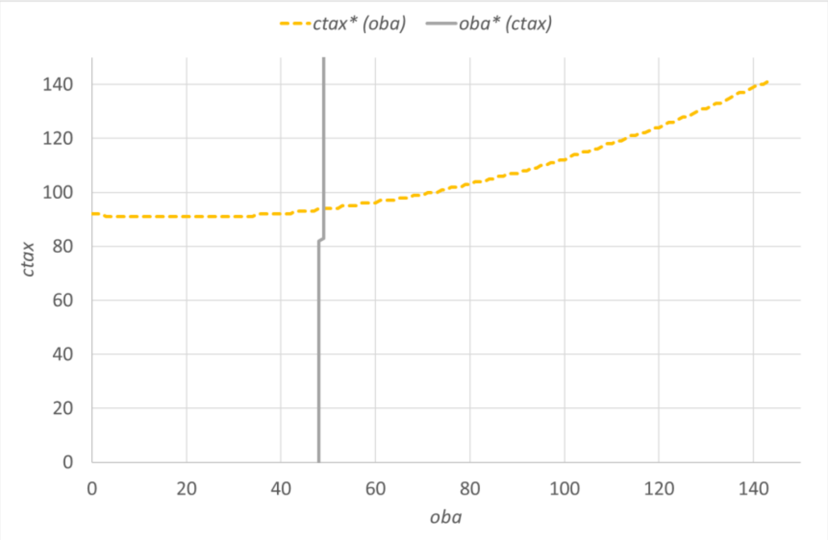
Figure 2. Effects of increasing only *oba*, only *ctax* or both (*oba*&*ctax*) on a) global welfare, b) emissions price in region 1, c) output of *C_T* in region 1, d) net import of *C_T* in region 1, e) domestic use of *C_T* in region 1, and f) emissions in region 2.



We have so far considered an economy-wide *ctax*, but pointed out that there may be a rationale for exempting *C_T* producers from this tax, as the tax reduces their competitiveness and actually *increases* emissions abroad (given the empirical data for our numerical simulations). Figure 3 shows the results of introducing *ctax* for all sectors except for *C_T*. The optimal *ctax* stand-alone now changes substantially, from zero to 92%. That is, even if there were no compensation to *C_T* producers for their emissions costs via *oba*, a high consumption tax on the use of *C_T* goods is warranted. That tax dampens the demand for

C_T goods and thus also the *C_T* production and associated emissions in region 2. As a result, leakage declines with the partial *ctax*, whereas it increases with an economy-wide *ctax*. Figure 2 shows that the optimal *oba* rate is almost independent of the *ctax* rate when intermediate use in the *C_T* sector is exempt from the tax. The optimal combination involves an *oba* rate of 49% and a *ctax* rate of 94%.[23]

Figure 3. Optimal *oba* and *ctax* rates from a global cost-effectiveness perspective when *C_T* producers are exempt from the consumption tax.



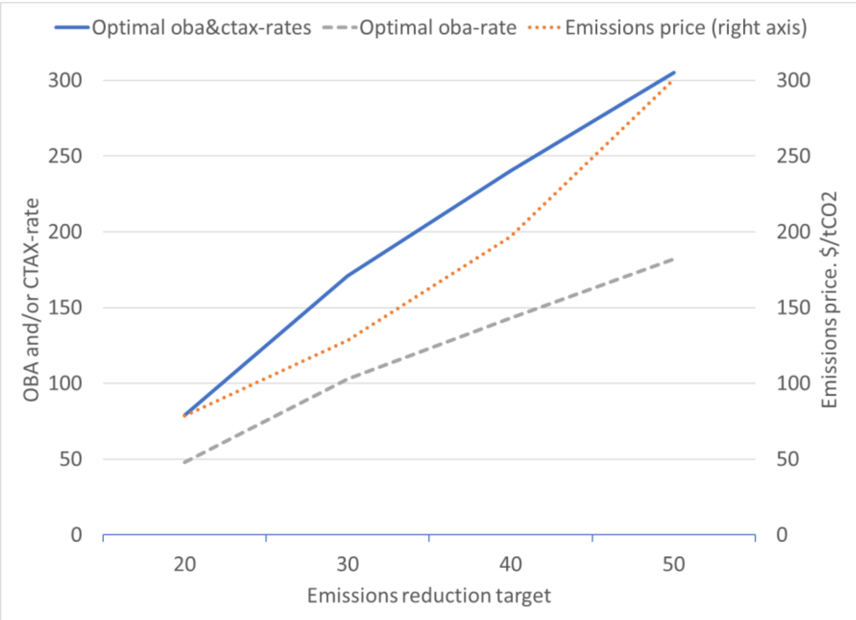
Comparing the optimal combination of policies with or without *ctax* exemptions for *C_T* producers, it turns out that non-exemption is desirable from a global cost-effectiveness perspective. While it is efficient to exempt *C_T* producers when there is no free allocation of allowances, this is not the case when *oba* can be optimally set. When *C_T* producers are not exempt, the optimal *oba* rate increases to counteract the negative effects on the competitiveness for domestic *C_T* producers. The subsequent discussion of simulation results therefore focuses on the setting with an economy-wide consumption tax.

An obvious driver for leakage and hence the potential need for anti-leakage measures is the ambition level of the unilateral climate policy. Many countries have pledged to reduce emissions to net zero within the next few decades. Figure 4 shows that the optimal *oba* and

[23] At low *oba* rates, we notice that the optimal *ctax* rate actually has a slight U-shape, that is, the optimal *ctax* rate *decreases* (marginally) in the *oba* rate. The explanation is that *ctax* leads to marginally *higher* domestic *C_T* production in region 1 (when the *oba* rate is low). As seen from equation (9), the optimal *ctax* rate is then decreasing in the *oba* rate. Increased domestic *C_T* production from a higher *ctax* rate is due to general equilibrium effects in region 1, following from the fact that other sectors (and final demand) have to pay for the *C_T* goods, while the *C_T* sector itself does not. This domestic competitiveness gain then dominates the effects from reduced demand for *C_T* goods in region 1, which affects *C_T* producers in both regions.

oba&ctax rates both increase substantially as we increase the unilateral emissions reduction target – from 48% (*oba*) and 79% (*oba&ctax*) in our central case setting with a 20% unilateral emissions reduction target to 182% and 305% with a 50% emissions reduction target. The implicit production subsidy and consumption tax rates are increasing even more since the emissions price required to meet the emissions reduction targets rises steeply (see Figure 4). A relevant concern with the results in Figure 4 is that even though an *oba* rate (or *oba&ctax* rates) above 100% may be optimal for more ambitious emissions reductions targets, it could be politically difficult to implement, particularly because of WTO regulations.

Figure 4. Optimal *oba* and *oba&ctax* rates from global cost-effectiveness perspective, as functions of the unilateral emissions reduction target.

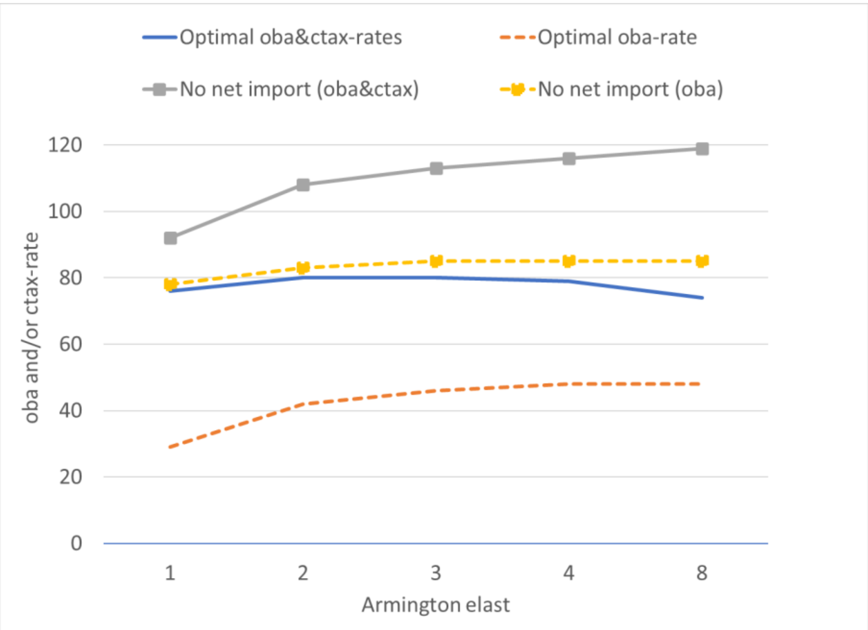


It is clear that the case for additional instruments to prevent leakage such as *oba* and *ctax* depends on the risk of carbon leakage through the competitiveness channel. The key driver in this regard is the Armington elasticity which determines the level of substitutability between domestic and imported varieties of traded goods. For our central case simulations, we adopt a value of 4 as an average estimate provided by the GTAP database.

We examine how sensitive our results are to alternative choices of the Armington elasticities for the *C_T* good. As expected, net imports of the *C_T* good into region 1 in the *ref* scenario (as a share of domestic output) increase with the Armington elasticity. More specifically, net imports are seven times higher with an elasticity of 8 than with an elasticity of 1, indicating substantial differences in leakage exposure.

Figure 5 shows how the optimal policy is affected by the degree of trade responsiveness. Intuitively, the optimal *oba* rate increases with the Armington elasticity – yet, mainly at low elasticity levels. The optimal *oba&ctax* rate is less sensitive to the elasticity. As before, optimal rates are consistently higher when both *oba* and *ctax* instruments are used. Figure 5 furthermore shows that high *oba* and especially *oba&ctax* rates are needed to bring the net import of the *C_T* good back to zero. This illustrates that a cost-effective anti-leakage policy is not only about creating a "leveling playing field" between domestic and foreign producers, but also about taking into account the emissions changes abroad (these two effects are of course interconnected).

Figure 5. Optimal *oba* and *oba&ctax* rates from a global cost-effectiveness perspective, and *oba* and *oba&ctax* rates implying no net import of *C_T* goods to region 1. Different Armington elasticities for the *C_T* good.



The potency of instruments to reduce leakage will hinge on the scope to which they cover emissions embodied in traded goods. In our reference scenario, Scope 2 emissions from electricity use amount to 50% of Scope 1 emissions in the *C_T* sector of region 1. Hence, coverage of both Scope 1 and 2 emissions via output-based allocation would entail an *oba* rate of 150%, which is much higher than what is needed to avoid net import from region 2 (cf. Figure 2.d).[24]

To investigate the importance of Scope 2 emissions for the optimal anti-leakage policy, we set the input of electricity in *C_T* production to zero in both regions (and rearranged the input-output table accordingly). The optimal *oba* and *oba&ctax* rates then drop markedly, from 48% to 13% (*oba*) and from 79% to 23% (*oba&ctax*), respectively. There are two main drivers for this drop. First, there is a lower increase in emissions abroad from *C_T* production since there are no Scope 2 emissions (cf. equations (6) and (9)). Second, the cost increase for domestic *C_T* producers due to the carbon price is reduced as they are no longer affected in addition by higher electricity prices (caused by the carbon price). Hence, domestic *C_T* production declines less than in our central case, and thus there is less need for counter measures. To conclude, indirect emissions from the use of electricity appear to be quite important for the optimal design of unilateral anti-leakage measures.

In our central case simulations, we omit energy trading for two reasons: first, to keep the numerical setting closely aligned with our theoretical analysis, and second to focus on leakage through the competitiveness channel. When opening up for trade in (fossil) energy, economy-wide leakage in the *ref* scenario increases dramatically – from 3% to 42%. The difference is in part due to the fact that the *C_T* sector accounts for only 18% of total emissions in the benchmark data. More importantly, however, is that without trade in energy, we suppress the fossil fuel market channel for leakage. With international trade in energy, emissions reductions in region 1 lead to a downward pressure on the international price of fossil fuel which incentives fuel use and hence higher emissions in region 2 – leakage in the *C_T* sector alone is likewise much higher with trade in fossil energy (74% compared to 21%).

The much higher leakage rates trigger much higher optimal *oba* and/or *ctax* rates from a global cost-effectiveness perspective. Without any *ctax*, the optimal *oba* rate increases from 48% to 166%, while with *oba&ctax* (equal rates), the optimal rates increase from 79% to 284%. Implicitly, the higher optimal rates for *oba* and *ctax* reflect a second-best outcome to combat leakage through the fossil fuel markets given the lack of other more targeted policies such as taxing export of fossil fuels (Hoel, 1994). Obviously, results are not only sensitive to

[24] In the EU, output-based allocation is combined with a compensation scheme for high electricity prices due to the ETS, a scheme which is delegated to (and differs between) Member States. The compensation is not directly linked to the Scope 2 emissions, but rather to the ETS price and either intermediate electricity use or the production level of the firms, cf. footnote 4.

the magnitude of energy trade in the benchmark, but also to the size of the supply elasticity of energy (which is set to one in our simulations). Without energy trade, the optimal rates increase with the supply elasticity, while *with* energy trade the optimal rates decrease with the elasticity.

4. Conclusions

Combating climate change is a challenge, not least because of the global nature of the problem and the lack of harmonized global climate policies. Instead of coordinated uniform carbon pricing across countries, which would be desirable from a global cost-effectiveness perspective, there are considerable asymmetries in emissions pricing across national and subnational jurisdictions (World Bank, 2023). The landscape of divergent carbon prices with limited geographical scope offers a conduit for international trade to undermine their effectiveness through carbon leakage. Climate-ambitious countries are in particular concerned about leakage via the so-called competitiveness channel: more stringent unilateral regulation imposes additional costs on domestic emissions-intensive and trade-exposed (EITE) industries as compared to international rivals, leading to the relocation of EITE production and emissions to abroad.

A widespread strategy to combat leakage via the competitiveness channel has been to allocate free allowances to domestic EITE firms, typically conditioned on output or some other economic activity. However, the use of output-based allocation has adverse side effects in stimulating excessive domestic consumption of EITE goods and insufficient substitution by other goods. As a “corrective” measure, additional consumption taxes on the domestic consumption of EITE goods based on their carbon content have been suggested.

In this paper, we have deepened previous economic research on the combination of output-based allocation and consumption taxes to improve the cost-effectiveness of unilateral emissions pricing. Based on theoretical and numerical general equilibrium analyses, we show that in optimal unilateral policy design the two instruments are complements – the higher the output-based allocation rate, the higher the consumption tax rate should be (and vice versa). We furthermore caution against an economy-wide consumption tax without combining it with a sufficiently high output-based allocation rate, as otherwise domestic EITE producers may suffer excessively with adverse effects on global emissions. In fact, the consumption tax may increase carbon leakage if own use in EITE productions is not exempt from the tax. This is because the consumption tax increases the cost of EITE own use, which makes it more difficult for domestic EITE producers to compete with rivals abroad.

Another policy-relevant insight that emerges from our analysis is that the optimal output-based allocation and consumption tax rates increase significantly in the stringency of domestic climate policies, with the rates exceeding 100% at more ambitious emissions reduction targets – suggesting that optimal unilateral policies might be difficult to implement without violating WTO rules. Our results also highlight the pitfall of allocating larger amounts of free allowances to EITE sectors that are only slightly exposed to leakage (although the optimal output-based allocation rate increases when supplemented with a consumption tax). With regard to the ongoing policy debate on the coverage of emissions, our analysis shows that the inclusion of Scope 2 emissions, i.e., emissions embodied in electricity use, plays an important role for the optimal output-based allocation and consumption tax rates.

The combination of output-based allocation and consumption taxes has similarities with border carbon adjustments, i.e., import tariffs on embodied carbon and (possibly) export rebates of carbon payments. One prominent example of border carbon adjustments is the EU Carbon Border Adjustment Mechanism (CBAM). CBAM can incentivize importers in the EU to reduce their emission intensities to the extent that this lowers import tariffs. This is not the case for the combined policy of output-based allocation and consumption taxes, where all products of the same variety are treated equally. On the other hand, CBAM only targets imports and only a subset of EITE products is included in the initial phase. Furthermore, Scope 2 emissions, which according to our results need to be taken into account, are (at least initially) only included for cement and fertilizers. As discussed in Böhringer et al. (2022), there are also other challenges with border carbon adjustments, such as the risk of reshuffling, relocation down the value chain, legal issues (with respect to GATT rules), and the risk of trade retaliation. Therefore, a combination of output-based allocation and consumption taxes could be a better and less controversial policy choice than border carbon adjustments. From a practical point of view, the administrative costs of consumption taxes are likely to be moderate, as the benchmarks and scope of output-based allocation are already set.

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Appendix A: Derivations and proofs

Derivation of production, emissions and use of input factors under competitive equilibrium and social planner (welfare maximum):

Competitive equilibrium first order conditions for consumers: The representative consumer maximizes utility w.r.t $\bar{x}^j$ and $\bar{y}^j$ subject to the binding budget constraint (3). The Lagrangian is:

$$
\begin{aligned}
L^u &= u\left(\bar{x}^j, \bar{y}^j\right) - \lambda_T\left(p^x x^T - p^y y^T\right) \\
&= u\left(\bar{x}^j, \bar{y}^j\right) - \lambda_T\left(p^x\left(\bar{x}^j + x^{xj} + x^{yj} - x^j\right) - \left(p^y + v^j\right)\left(\bar{y}^j + y^{xj} + y^{yj} - y^j\right)\right),
\end{aligned}
$$

with first order conditions:

$$\bar{x}^j: \;\; u_x^j / \lambda_T^j - p^x = 0$$

$$\bar{y}^j: \;\; u_y^j / \lambda_T^j - \left(p^y + v^j\right) = 0$$

Together the first-order conditions imply the familiar result that the marginal rate of substitution equals the ratio of consumer prices (producer prices plus consumption tax), $u_x^j / u_y^j = p^x / \left(p^y + v^j\right)$ in the interior solution. Note that the Lagrange multiplier λ_T can be interpreted as the marginal utility of income.

Competitive equilibrium first order conditions for producers: The firms solve the maximization problems (1) given in the text. The first order conditions are:

$$x^{xj}: \;\; f_x^j - 1 = 0$$

$$y^{xj}: \;\; p^x f_y^j - \left(p^y + v^j\right) = 0$$

$$l^{xj}: \;\; p^x f_l^j - p^{lj} = 0$$

$$x^{yj}: \;\; (p^y + s^j) g_x^j - p^x = 0$$

$$y^{yj}: \;\; (p^y + s^j) g_y^j - \left(p^y + v^j\right) = 0$$

$$z^{yj}: \;\; (p^y + s^j) g_z^j - p^{zj} = 0$$

$$l^{yj}: \ (p^y + s^j)g_l^j - p^{lj} = 0$$

$$e^{yj}: \ (p^y + s^j)g_e^j - t^j = 0$$

$$l^{zj}: \ p^{zj}h_l^j - p^{lj} = 0$$

$$e^{zj}: \ p^{zj}h_e^j - t^j = 0$$

First order conditions for welfare maximizing solution: Suppose a social planner maximizes welfare subject to two constraints: (*i*) the budget/trade balance constraint $p^x x^T - p^y y^T$ and, (*ii*), the labor market constraint $L = l^x + l^y + l^z$. The Lagrangian is:

$$\begin{aligned}
&L^{sp} = u^j(\overline{x}^j, \overline{y}^j) - \tau_j^j e^j - \tau_{-j}^j e^{-j} \\
&+\lambda_T^j \left(p^x \left(f\left(x^{xj}, y^{xj}, l^{xj}\right) - \overline{x}^j - x^{xj} - x^{yj} \right) + p^y \left(g\left(x^{yj}, y^{yj}, l^{yj}, e^{yj}\right) - \overline{y}^j - y^{xj} - y^{yj} \right) + p^{zj} \left(h^j\left(l^{zj}, e^{zj}\right) - z^{yj} \right) \right) \\
&+\lambda_L^j \left(l^{xj} + l^{yj} + l^{zj} - L^j \right)
\end{aligned}$$

The first order conditions are:

$$\overline{x}^j: \ \frac{u_x^j}{\lambda_T^j} - p^x = 0$$

$$\overline{y}^j: \ \frac{u_y^j}{\lambda_T^j} - p^y = 0$$

$$x^{xj}: \ f_x^j - 1 = 0$$

$$y^{xj}: \ p^x f_y^j - p^y = 0$$

$$l^{xj}: \ p^x f_l^j - \frac{\lambda_L^j}{\lambda_T^j} = 0$$

$$x^{yj}: \ p^y g_x^j - p^x = 0$$

$$y^{yj}: \ g_y^j - 1 = 0$$

$$z^{yj}: \ p^y g_z^j - p^{zj} = 0$$

$l^{yj}:\quad p^y g_l^j - \frac{\lambda_L^j}{\lambda_T^j} = 0$

$e^{yj}:\quad p^y g_e^j - \frac{\tau^j}{\lambda_T^j} = 0$

$l^{zj}:\quad p^{zj} h_l^j - \frac{\lambda_L^j}{\lambda_T^j} = 0$

$e^{zj}:\quad p^{zj} h_z^j - \frac{\tau^j}{\lambda_T^j} = 0$

Competitive equilibrium equals social planner's solution if $v^j = s^j = 0$ and $t^j = \tau^j / \lambda_T^j$ (note that this implies $p^{lj} = \lambda_L^j / \lambda_T^j$, because the systems of equations are identical).

Substantiation of Assumptions 1 and 2:

The producer first order conditions implicitly define supply functions for the *y*-good, denoted $G^j(p^y + s^j; p^y + v^j, p^x, p^{zj}, p^{lj}, t^j)$. Here the first argument ($p^y + s^j$) refers to the price on output, whereas the second to sixth arguments refer to prices on factors of production. Supply is increasing and concave in the producer price, which includes the subsidy *s*, and decreasing in the price of input factors used in production (incl. the consumption tax on input of *y*). Let G_v denote the derivative of the function G with respect to its *v*'th argument. Then the firms' first order conditions imply the following: $G_1^j > 0, G_2^j < 0, G_3^j < 0, G_4^j < 0, G_5^j < 0$ and $G_6^j < 0$.

The firms' first order conditions also implicitly define factor demand functions for the goods used as inputs in production. The factor demand functions for good $g \in \{x, y, l\}$ are decreasing in the price of input factor g, and increases in the price of the good produced. How the demand for the other input factors react to a change in the price on g depends on whether the goods are substitutes or complements in production. Let $FD^{yj}(p^y + s^j; p^y + v^j, p^x, p^{zj}, p^{lj}, t^j)$ denote the factor demand function for *y* in region *j*. Factor demand for in the *y* sector increases in production of y, and hence in the producer price of y, $p^y + s^j$. Factor demand for good g also decreases in the price on good g as an input factor. Hence, the derivatives satisfy $FD_1^{yj} > 0, FD_2^{yj} < 0, FD_3^{yj} < 0, FD_4^{yj} < 0, FD_5^{yj} < 0$ and $FD_6^{yj} < 0$. The cross derivatives of input factor prices are positive (negative) if the factors are substitutes (complements) in production.

The equation $u_x^j / u_y^j = p^x / \left(p^y + v^j \right)$ and the budget constraint (3) implicitly defines consumer demand functions for the two goods, which are convex and decreasing in the consumer price. Let $CD^{yj}(p^y + v^j, p^x, m^j)$ denote the consumer demand function for *y* in region *j*, where m^j is disposable income as determined by the budget constraint (3), and the derivatives satisfy $dCD^{yj} / d(p^y + v^j) = CD_1^{yj} < 0$, $CD_2^{yj} \geq 0$ and $CD_3^{yj} \geq 0$. Note that m^j will depend on e.g. government transfers and firm profits.

Market equilibrium for the *y*-good requires:

$$\begin{aligned} & CD^{y1}(p^y + v^1, p^x, p^{l1}, m^1) + FD^{y1}(p^y + s^1; p^y + v^1, p^x, p^{z1}, p^{l1}, t^1) + CD^{y2}(p^y, p^x, p^{l2}, m^2) \\ & + FD^{y2}(p^y; p^y, p^x, p^{z2}, p^{l2}) = G^1(p^y + s^1; p^y + v^1, p^x, p^{z1}, p^{l1}, t^1) + G^2(p^y; p^y, p^x, p^{z2}, p^{l2}). \end{aligned} \quad (12)$$

We first consider *Assumption 2* on the effects a change in the consumption tax has on production and imports of the y good in region 1, as well as emissions in region 2. Differentiating (12)w.r.t the consumption tax v^1, we get:

$$\begin{aligned} & CD_1^{y1}\left(\frac{dp^y}{dv^1} + 1\right) + CD_2^{y1}\frac{dp^x}{dv^1} + CD_3^{y1}\frac{dp^{l1}}{dv^1} + CD_4^{y1}\frac{dm^1}{dv^1} \\ & + FD_1^{y1}\frac{dp^y}{dv^1} + FD_2^{y1}\left(\frac{dp^y}{dv^1} + 1\right) + FD_3^{y1}\frac{dp^x}{dv^1} + FD_4^{y1}\frac{dp^z}{dv^1} + FD_5^{y1}\frac{dp^{l1}}{dv^1} \\ & + CD_1^{y2}\frac{dp^y}{dv^1} + CD_2^{y2}\frac{dp^x}{dv^2} + CD_3^{y2}\frac{dp^{l2}}{dv^1} + CD_4^{y2}\frac{dm^2}{dv^1} \\ & + FD_1^{y2}\frac{dp^y}{dv^1} + FD_2^{y2}\frac{dp^y}{dv^1} + FD_3^{y2}\frac{dp^x}{dv^1} + FD_4^{y2}\frac{dp^z}{dv^1} + FD_5^{y2}\frac{dp^{l2}}{dv^1} \\ & = G_1^1\frac{dp^y}{dv^1} + G_2^1\left(\frac{dp^y}{dv^1} + 1\right) + G_3^1\frac{dp^x}{dv^1} + G_4^1\frac{dp^{z1}}{dv^1} + G_5^1\frac{dp^{l1}}{dv^1} \\ & + G_1^2\frac{dp^y}{dv^1} + G_2^2\frac{dp^y}{dp^y} + G_3^2\frac{dp^x}{dv^1} + G_4^2\frac{dp^{z2}}{dv^1} + G_5^2\frac{dp^{l2}}{dv^1}. \end{aligned} \quad (13)$$

Let us consider the direct effects in the market for *y* and assume that the general equilibrium (GE) effects are approximately zero, i.e. we have $dm^j / dv^1 = dp^x / dv^1 = dp^{zj} / dv^1 = dp^{lj} / dv^1 \approx 0$. Then equation (13) can be simplified to:

$$CD_1^{y1}\left(\frac{dp^y}{dv^1}+1\right)+FD_1^{y1}\frac{dp^y}{dv^1}+FD_2^{y1}\left(\frac{dp^y}{dv^1}+1\right)+CD_1^{y2}\frac{dp^y}{dv^1}+FD_1^{y2}\frac{dp^y}{dv^1}+FD_2^{y2}\frac{dp^y}{dv^1}$$
$$\approx G_1^1\frac{dp^y}{dv^1}+G_2^1\left(\frac{dp^y}{dv^1}+1\right)+G_1^2\frac{dp^y}{dv^1}+G_2^2\frac{dp^y}{dp^y}$$

Which can be rewritten as:

$$CD_1^{y1}+FD_2^{y1}-G_2^1\approx\left(G_1^1+G_2^1+G_1^2+G_2^2-CD_1^{y1}-FD_1^{y1}-FD_2^{y1}-CD_1^{y2}-FD_1^{y2}-FD_2^{y2}\right)\frac{dp^y}{dv^1}\quad (14)$$

Case 1: Assume (*i*) that the firms in the *y* sector do not pay a consumption tax on the y-good (i.e., the firms do not pay a consumption tax on y if y is used as a factor in production of y), and (*ii*) that the firms' net supply of good *y* does not decline in the price of *y*. Assumption (*i*) implies that the terms FD_2^{y1} and G_2^1 cancel from equation (14) (unless they are multiplied with the term dp^y/dv^1), because they are the direct effect of the tax payments. Equation (14) then simplifies to:

$$CD_1^{y1}\approx\left(\underbrace{G_1^1+G_2^1-FD_1^{y1}-FD_2^{y1}}_{A^1}+\underbrace{G_1^2+G_2^2-FD_1^{y2}-FD_2^{y2}}_{A^2}-CD_1^{y1}-CD_1^{y2}\right)\frac{dp^y}{dv^1},\quad (15)$$

where $A^1\geq 0$ and $A^2\geq 0$ cf. assumption (*ii*) above. Because $CD_1^{yj}<0$ (i.e., consumer demand decreases when taxed), the above (approximate) equality can only be sustained if $dp^y/dv^1<0$. It follows that production in both regions decreases, whereas consumption in region 1 decreases (given $d(p^y+v^1)/dv^1>0$) and consumption in region 2 increases. Hence net imports of the y-good in region 1 declines. Assumption 2 follows. Note, however, that some of the reduced consumption in region 1 is offset by increased foreign consumption of the y-good.

The GE effects are somewhat harder to pin down. For example, the consumption tax may increase the disposable income in region 1, because imports from region 2 is taxed and transferred to the consumer in region 1. Note that the results in the preceding paragraph, and thereby Assumption 2, apply in Case 1 under the reasonable assumption that the first order effects dominate the second order GE effects in the case of opposite signs.

Case 2: Assume that the firms in the *y* sector pay a consumption tax on the y-good (i.e., the firms pay a consumption tax on y also if y is used as a factor in production of y). A similar calculation as performed above yields:

$$CD_1^{y1} + FD_2^{y1} - G_2^1 \approx \left(\underbrace{G_1^1 + G_2^1 - FD_1^{y1} - FD_2^{y1}}_{A^1} + \underbrace{G_1^2 + G_2^2 - FD_1^{y2} - FD_2^{y2}}_{A^2} - CD_1^{y1} - CD_1^{y2} \right) \frac{dp^y}{dv^1} \quad (16)$$

This is equal to equation (15) above, except for the terms $FD_2^{y1} - G_2^1$ on the left-hand side. Here $FD_2^{y1} < 0$ is the decline factor demand for *y* in the *y* sector caused by the consumption tax (not controlled for the change in p^y), whereas $G_2^1 > 0$ is the decline in supply of y following the consumption tax (again not controlled for the change in p^y). We observe that the left-hand side of equation (16) can be positive under reasonable assumptions in case 2. For example, if the absolute value on G_2^1 is large (i.e., the derivative of supply of y in region 1 with respect to the consumption tax v). This may, e.g., be the case if it is hard for the *y* sector to substitute away from the use of *y* as an input factor, and the supply of the *y*-good in region 2 is very price elastic. If this is the case, and the left-hand side of equation (16) is negative, we have $dp^y / dv^1 > 0$ (given $A^1 \geq 0$ and $A^2 \geq 0$), which implies that the consumption tax increases foreign emissions. We also observe that the left-hand side of equation (16) is more likely to be negative if consumer demand in Region 1 is not very sensitive to the consumer price ($p^y + v^1$). This is why Assumption 2 requires own use in the *y* sector to be exempted from the tax.

We now consider *Assumption 1* and the effects of a subsidy s^1 on production of the y good and emissions in region 2. Differentiating equation (12) w.r.t s^1 we get:

$$\begin{aligned} &CD^{y1}(p^y + v^1, p^x, p^{l1}, m^1) + FD^{y1}(p^y + s^1; p^y + v^1, p^x, p^{z1}, p^{l1}, t^1) + CD^{y2}(p^y, p^x, p^{l2}, m^2) \\ &+FD^{y2}(p^y; p^y, p^x, p^{z2}, p^{l2}) = G^1(p^y + s^1; p^y + v^1, p^x, p^{z1}, p^{l1}, t^1) + G^2(p^y; p^y, p^x, p^{z2}, p^{l2}) \end{aligned}$$

$$
\begin{aligned}
& CD_1^{y1}\frac{dp^y}{ds^1}+CD_2^{y1}\frac{dp^x}{ds^1}+CD_3^{y1}\frac{dp^{l1}}{ds^1}+CD_4^{y1}\frac{dm^1}{ds^1} \\
& +FD_1^{y1}\left(\frac{dp^y}{ds^1}+1\right)+FD_2^{y1}\frac{dp^y}{ds^1}+FD_3^{y1}\frac{dp^x}{ds^1}+FD_4^{y1}\frac{dp^z}{ds^1}+FD_5^{y1}\frac{dp^{l1}}{ds^1} \\
& +CD_1^{y2}\frac{dp^y}{ds^1}+CD_2^{y2}\frac{dp^x}{ds^2}+CD_3^{y2}\frac{dp^{l2}}{ds^1}+CD_4^{y2}\frac{dm^2}{ds^1} \\
& +FD_1^{y2}\frac{dp^y}{ds^1}+FD_2^{y2}\frac{dp^y}{ds^1}+FD_3^{y2}\frac{dp^x}{ds^1}+FD_4^{y2}\frac{dp^z}{ds^1}+FD_5^{y2}\frac{dp^{l2}}{ds^1} \\
& =G_1^1\left(\frac{dp^y}{ds^1}+1\right)+G_2^1\frac{dp^y}{ds^1}+G_3^1\frac{dp^x}{ds^1}+G_4^1\frac{dp^{z1}}{ds^1}+G_5^1\frac{dp^{l1}}{ds^1} \\
& +G_1^2\frac{dp^y}{ds^1}+G_2^2\frac{dp^y}{ds^y}+G_3^2\frac{dp^x}{ds^1}+G_4^2\frac{dp^{z2}}{ds^1}+G_5^2\frac{dp^{l2}}{ds^1}.
\end{aligned}
$$

Let us again consider the direct effects in the market for *y* and assume that the general equilibrium (GE) effects are approximately zero, i.e. we have $dm^j / ds^1 = dp^x / ds^1 = dp^{zj} / ds^1 = dp^{lj} / ds^1 \approx 0$. Then the above equation can be simplified to:

$$
\begin{aligned}
& CD_1^{y1}\frac{dp^y}{ds^1}+FD_1^{y1}\left(\frac{dp^y}{ds^1}+1\right)+FD_2^{y1}\frac{dp^y}{ds^1}+CD_1^{y2}\frac{dp^y}{ds^1}+FD_1^{y2}\frac{dp^y}{ds^1}+FD_2^{y2}\frac{dp^y}{ds^1} \\
& =G_1^1\left(\frac{dp^y}{ds^1}+1\right)+G_2^1\frac{dp^y}{ds^1}+G_1^2\frac{dp^y}{ds^1}+G_2^2\frac{dp^y}{ds^y}.
\end{aligned}
$$

Which can be rearranged to:

$$
G_1^1-FD_1^{y1}=\left(\underbrace{G_1^1+G_2^1-FD_1^{y1}-FD_2^{y1}}_{A^1}+\underbrace{G_1^2+G_2^2-FD_1^{y2}-FD_2^{y2}}_{A^2}-CD_1^{y1}-CD_1^{y2}\right)\frac{-dp^y}{ds^1} \qquad (17)
$$

where again $A^1 \geq 0$ and $A^2 \geq 0$ (cf. assumption (*ii*) above). Hence, the parenthesis in equation (17) is positive (because $CD_1^{yj} < 0$). The left hand side of equation (17) is positive, given that the net supply from the *y* sector in region 1 increase in the subsidy to production of y (i.e., the *y* sector's output of y increases more than its use of y as a factor of production, which is assumption (*ii*)). We see that equation (17) then requires that $dp^y / ds^1 < 0$. Hence, consumption of *y* increases in both regions, whereas production of *y* in region 1 (2) increases (decreases). This implies that net imports of *y* to region 1 decreases in s^1, given that the first order effects dominate the second order GE effects (if of opposite signs). Assumption 1 follows.

Derivation of Equation (5) (the optimal s^1 for any given v^1):

The social planner in region 1 maximizes welfare w.r.t s^1, subject to the budget/trade constraint (3) and the labor market constraint $L^1 = l^{x1} + l^{y1} + l^{z1}$. The Lagrangian is:

$$L^{sp} = u^1(\bar{x}^1, \bar{y}^1) - \tau^1\left(e^1 + e^2\right) - \lambda_T^1\left(p^x x^T + p^y y^T\right) - \lambda_L^1\left(l^{x1} + l^{y1} + l^{z1} - L^1\right).$$

Remember that the Lagrange multiplier λ_T can be interpreted as the marginal utility of income. It is positive and finite given (local) non-satiation and a bounded utility function. The first order condition is:

$$\frac{dL^{sp}}{ds^1} = u_x^1 \frac{d\bar{x}^1}{ds^1} + u_y^1 \frac{d\bar{y}^1}{ds^1} - \tau^1 \frac{de^1}{ds^1} - \tau^1 \frac{de^2}{ds^1}$$

$$-\frac{d\lambda_T^1}{ds^1}\left(p^x x^T + p^y y^T\right) - \lambda_T^1\left(\frac{dp^x}{ds^1} x^T + p^x \frac{dx^T}{ds^1} + \frac{dp^y}{ds^1} y^T + p^y \frac{dy^T}{ds^1}\right) - \frac{d\lambda_L^1}{ds^1}\left(l^{x1} + l^{y1} + l^{z1} - L^1\right) - \lambda_L^1\left(\frac{l^{x1}}{ds^1} + \frac{l^{y1}}{ds^1} + \frac{l^{z1}}{ds^1} - \frac{L^1}{ds^1}\right) = 0$$

$$\Leftrightarrow u_x^1 \frac{d\bar{x}^1}{ds^1} + u_y^1 \frac{d\bar{y}^1}{ds^1} - \tau^1 \frac{de^1}{ds^1} - \tau^1 \frac{de^2}{ds^1} = 0$$

$$\Leftrightarrow \frac{u_x^1}{\lambda_T^1}\frac{d\bar{x}^1}{ds^1} + \frac{u_y^1}{\lambda_T^1}\frac{d\bar{y}^1}{ds^1} - \frac{\tau^1}{\lambda_T^1}\left(\frac{de^1}{ds^1} + \frac{de^2}{ds^1}\right) = 0$$

$$\Leftrightarrow p^x \frac{d\bar{x}^1}{ds^1} + \left(p^y + v^1\right)\frac{d\bar{y}^1}{ds^1} - t^1\left(\frac{de^1}{ds^1} + \frac{de^2}{ds^1}\right) = 0$$

$$\Leftrightarrow p^x\left(\frac{dx^T}{ds^1} + \frac{dx^1}{ds^1} - \frac{dx^{x1}}{ds^1} - \frac{dx^{y1}}{ds^1}\right) + p^y\left(\frac{dy^T}{ds^1} + \frac{dy^1}{ds^1} - \frac{dy^{x1}}{ds^1} - \frac{dy^{y1}}{ds^1}\right) + v^1\frac{d\bar{y}^1}{ds^1} - t^1\left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^{y2}}{ds^1} + \frac{de^{z2}}{ds^1}\right) = 0$$

$$\Leftrightarrow p^x\left(\frac{dx^T}{ds^1} + f_x^1\frac{dx^{x1}}{ds^1} + f_y^1\frac{dy^{x1}}{ds^1} + f_l^1\frac{dl^{x1}}{ds^1} - \frac{dx^{x1}}{ds^1} - \frac{dx^{y1}}{ds^1}\right)$$

$$+p^y\left(\frac{dy^T}{ds^1} + g_x^1\frac{dx^{y1}}{ds^1} + g_y^1\frac{dy^{y1}}{ds^1} + g_z^1\frac{dz^1}{ds^1} + g_l^1\frac{dl^{y1}}{ds^1} + g_e^1\frac{de^{y1}}{ds^1} - \frac{dy^{x1}}{ds^1} - \frac{dy^{y1}}{ds^1}\right) + v^1\frac{d\bar{y}^1}{ds^1} - t^1\left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^{y2}}{ds^1} + \frac{de^{z2}}{ds^1}\right) = 0$$

$$\Leftrightarrow p^x\left(f_x^1 - 1\right)\frac{dx^{x1}}{ds^1} + \left(p^x f_y^1 - p^y\right)\frac{dy^{x1}}{ds^1} + \left(-p^x + p^y g_x^1\right)\frac{dx^{y1}}{ds^1} + p^y\left(g_y^1 - 1\right)\frac{dy^{y1}}{ds^1} + \left(p^x f_l^1\frac{dl^{x1}}{ds^1} + p^y g_l^1\frac{dl^{y1}}{ds^1}\right)$$

$$+p^y g_z^1\left(h_l^1\frac{dl^{z1}}{ds^1} + h_e^1\frac{de^{z1}}{ds^1}\right) + p^y g_e^1\frac{de^{y1}}{ds^1} + \left(p^x\frac{dx^T}{ds^1} + p^y\frac{dy^T}{ds^1}\right) + v^1\frac{d\bar{y}^1}{ds^1} - t^1\left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^{y2}}{ds^1} + \frac{de^{z2}}{ds^1}\right) = 0$$

$$\Leftrightarrow v^1\frac{dy^{x1}}{ds^1} - s^1 g_x^1\frac{dx^{y1}}{ds^1} + \left(v^1 - s^1 g_y^1\right)\frac{dy^{y1}}{ds^1} + \left(p^{l1}\frac{dl^{x1}}{ds^1} + \left(p^{l1} - s^1 g_l^1\right)\frac{dl^{y1}}{ds^1} + \left(p^{l1} - s^1 g_z^1 h_l^1\right)\frac{dl^{z1}}{ds^1}\right)$$

$$+\left(p^{z1} - s^1 g_z^1\right)h_e^1\frac{de^{z1}}{ds^1} + \left(t^1 - s^1 g_e^1\right)\frac{de^{y1}}{ds^1} + v^1\frac{d\bar{y}^1}{ds^1} + \left(p^x\frac{dx^T}{ds^1} + p^y\frac{dy^T}{ds^1}\right) - t^1\left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^{y2}}{ds^1} + \frac{de^{z2}}{ds^1}\right) = 0$$

$$\Leftrightarrow s^1\left(-g_x^1\frac{dx^{y1}}{ds^1} - g_y^1\frac{dy^{y1}}{ds^1} - g_l^1\frac{dl^{y1}}{ds^1} - g_z^1 h_l^1\frac{dl^{z1}}{ds^1} - g_z^1 h_e^1\frac{de^{z1}}{ds^1} - g_e^1\frac{de^{y1}}{ds^1}\right) + v^1\left(\frac{dy^{x1}}{ds^1} + \frac{dy^{y1}}{ds^1} + \frac{d\bar{y}^1}{ds^1}\right)$$

$$+\left(p^x\frac{dx^T}{ds^1} + p^y\frac{dy^T}{ds^1}\right) - t^1\left(\frac{de^{y2}}{ds^1} + \frac{de^{z2}}{ds^1}\right) = 0$$

$$\Leftrightarrow -s^1\frac{dy^1}{ds^1} + v^1\left(\frac{dy^1}{ds^1} + \frac{dy^T}{ds^1}\right) - t^1\left(\frac{de^{y2}}{ds^1} + \frac{de^{z2}}{ds^1}\right) - \left(\frac{dp^x}{ds^1}x^T + \frac{dp^y}{ds^1}y^T\right) = 0$$

Derivation of Equations (8) and (9) (optimal v^1 given exogenous s^1):

The social planner in region 1 maximizes welfare w.r.t v^1, subject to (3) and $L^1 = l^{x1} + l^{y1} + l^{z1}$. The Lagrangian is $L^{sp} = u^1(\bar{x}^1, \bar{y}^1) - \tau^1\left(e^1 + e^2\right) - \lambda_T^1\left(p^x x^T + p^y y^T\right) - \lambda_L^1\left(l^{x1} + l^{y1} + l^{z1} - L^1\right)$ with associated first order condition:

$$\frac{dL^{sp}}{dv^{1\alpha}} = u_x^1 \frac{d\bar{x}^1}{dv^{1\alpha}} + u_y^1 \frac{d\bar{y}^1}{dv^{1\alpha}} - \tau^1 \frac{de^1}{dv^{1\alpha}} - \tau^1 \frac{de^2}{dv^{1\alpha}}$$

$$-\frac{d\lambda_T^j}{dv^{1\alpha}}\left(p^x x^T + p^y y^T\right) - \lambda_T^j\left(\frac{dp^x}{dv^{1\alpha}} x^T + p^x \frac{dx^T}{dv^{1\alpha}} + \frac{dp^y}{dv^{1\alpha}} y^T + p^y \frac{dy^T}{dv^{1\alpha}}\right)$$

$$-\frac{d\lambda_L^1}{dv^{1\alpha}}\left(l^{x1} + l^{y1} + l^{z1} - L^1\right) - \lambda_L^j\left(\frac{l^{x1}}{dv^{1\alpha}} + \frac{l^{y1}}{dv^{1\alpha}} + \frac{l^{z1}}{dv^{1\alpha}} - \frac{L^1}{dv^{1\alpha}}\right) = 0$$

$$\Leftrightarrow u_x^1 \frac{d\bar{x}^1}{dv^{1\alpha}} + u_y^1 \frac{d\bar{y}^1}{dv^{1\alpha}} - \tau^1 \frac{de^1}{dv^{1\alpha}} - \tau^1 \frac{de^2}{dv^{1\alpha}} = 0$$

$$\Leftrightarrow \frac{u_x^1}{\lambda^T} \frac{d\bar{x}^1}{dv^{1\alpha}} + \frac{u_y^1}{\lambda^T} \frac{d\bar{y}^1}{dv^{1\alpha}} - \frac{\tau^1}{\lambda^T}\left(\frac{de^1}{dv^{1\alpha}} + \frac{de^2}{dv^{1\alpha}}\right) = 0$$

$$\Leftrightarrow p^x \frac{d\bar{x}^1}{dv^{1\alpha}} + \left(p^y + v^1\right)\frac{d\bar{y}^1}{dv^{1\alpha}} - t^1\left(\frac{de^1}{dv^{1\alpha}} + \frac{de^2}{dv^{1\alpha}}\right) = 0$$

$$\Leftrightarrow p^x\left(\frac{dx^T}{dv^{1\alpha}} + \frac{dx^1}{dv^{1\alpha}} - \frac{dx^{x1}}{dv^{1\alpha}} - \frac{dx^{y1}}{dv^{1\alpha}}\right) + p^y\left(\frac{dy^T}{dv^{1\alpha}} + \frac{dy^1}{dv^{1\alpha}} - \frac{dy^{x1}}{dv^{1\alpha}} - \frac{dy^{y1}}{dv^{1\alpha}}\right)$$

$$+v^{1c} \frac{d\bar{y}^1}{dv^{1\alpha}} - t^1\left(\frac{de^{y1}}{dv^{1\alpha}} + \frac{de^{z1}}{dv^{1\alpha}} + \frac{de^{y2}}{dv^{1\alpha}} + \frac{de^{z2}}{dv^{1\alpha}}\right) = p^x\left(\frac{dx^T}{dv^{1\alpha}} + f_x^1 \frac{dx^{x1}}{dv^{1\alpha}} + f_y^1 \frac{dy^{x1}}{dv^{1\alpha}} + f_l^1 \frac{dl^{x1}}{dv^{1\alpha}} - \frac{dx^{x1}}{dv^{1\alpha}} - \frac{dx^{y1}}{dv^{1\alpha}}\right)$$

$$+p^y\left(\frac{dy^T}{dv^{1\alpha}} + g_x^1 \frac{dx^{y1}}{dv^{1\alpha}} + g_y^1 \frac{dy^{y1}}{dv^{1\alpha}} + g_z^1 \frac{dz^1}{dv^{1\alpha}} + g_l^1 \frac{dl^{y1}}{dv^{1\alpha}} + g_e^1 \frac{de^{y1}}{dv^{1\alpha}} - \frac{dy^{x1}}{dv^{1\alpha}} - \frac{dy^{y1}}{dv^{1\alpha}}\right)$$

$$+v^{1c} \frac{d\bar{y}^1}{dv^{1\alpha}} - t^1\left(\frac{de^{y1}}{dv^{1\alpha}} + \frac{de^{z1}}{dv^{1\alpha}} + \frac{de^{y2}}{dv^{1\alpha}} + \frac{de^{z2}}{dv^{1\alpha}}\right) = 0$$

$$\Leftrightarrow p^x\left(f_x^1 - 1\right)\frac{dx^{x1}}{dv^{1\alpha}} + \left(p^x f_y^1 - p^y\right)\frac{dy^{x1}}{dv^{1\alpha}} + \left(-p^x + p^y g_x^1\right)\frac{dx^{y1}}{dv^{1\alpha}} + p^y\left(g_y^1 - 1\right)\frac{dy^{y1}}{dv^{1\alpha}} + \left(p^x f_l^1 \frac{dl^{x1}}{dv^{1\alpha}} + p^y g_l^1 \frac{dl^{y1}}{dv^{1\alpha}}\right)$$

$$+p^y g_z^1\left(h_l^1 \frac{dl^{z1}}{dv^{1\alpha}} + h_e^1 \frac{de^{z1}}{dv^{1\alpha}}\right) + p^y g_e^1 \frac{de^{y1}}{dv^{1\alpha}} + v^{1c} \frac{d\bar{y}^1}{dv^{1\alpha}} + \left(p^x \frac{dx^T}{dv^{1\alpha}} + p^y \frac{dy^T}{dv^{1\alpha}}\right) - t^1\left(\frac{de^{y1}}{dv^{1\alpha}} + \frac{de^{z1}}{dv^{1\alpha}} + \frac{de^{y2}}{dv^{1\alpha}} + \frac{de^{z2}}{dv^{1\alpha}}\right) = 0$$

$$\Leftrightarrow v^{1x} \frac{dy^{x1}}{dv^{1\alpha}} - s^1 g_x^1 \frac{dx^{y1}}{dv^{1\alpha}} + \left(v^{1y} - s^1 g_y^1\right)\frac{dy^{y1}}{dv^{1\alpha}} + \left(p^{l1} \frac{dl^{x1}}{dv^{1\alpha}} + \left(p^{l1} - s^1 g_l^1\right)\frac{dl^{y1}}{dv^{1\alpha}} + \left(p^{l1} - s^1 g_z^1 h_l^1\right)\frac{dl^{z1}}{dv^{1\alpha}}\right)$$

$$+\left(p^{z1} - s^1 g_z^1\right)h_e^1 \frac{de^{z1}}{dv^{1\alpha}} + \left(t^1 - s^1 g_e^1\right)\frac{de^{y1}}{dv^{1\alpha}} + v^{1c} \frac{d\bar{y}^1}{dv^{1\alpha}} + \left(p^x \frac{dx^T}{dv^{1\alpha}} + p^y \frac{dy^T}{dv^{1\alpha}}\right) - t^1\left(\frac{de^{y1}}{dv^{1\alpha}} + \frac{de^{z1}}{dv^{1\alpha}} + \frac{de^{y2}}{dv^{1\alpha}} + \frac{de^{z2}}{dv^{1\alpha}}\right) = 0$$

$$\Leftrightarrow s^1\left(-g_x^1 \frac{dx^{y1}}{dv^{1\alpha}} - g_y^1 \frac{dy^{y1}}{dv^{1\alpha}} - g_l^1 \frac{dl^{y1}}{dv^{1\alpha}} - g_z^1 h_l^1 \frac{dl^{z1}}{dv^{1\alpha}} - g_z^1 h_e^1 \frac{de^{z1}}{dv^{1\alpha}} - g_e^1 \frac{de^{y1}}{dv^{1\alpha}}\right)$$

$$+v^{1x} \frac{dy^{x1}}{dv^{1\alpha}} + v^{1y} \frac{dy^{y1}}{dv^{1\alpha}} + v^{1c} \frac{d\bar{y}^1}{dv^{1\alpha}} - \tau^1\left(\frac{de^{y2}}{dv^{1\alpha}} + \frac{de^{z2}}{dv^{1\alpha}}\right) - \left(\frac{dp^x}{dv^{1\alpha}} x^T + \frac{dp^y}{dv^{1\alpha}} y^T\right) = 0$$

$$\Leftrightarrow -s^1 \frac{dy^1}{dv^{1\alpha}} + v^{1x} \frac{dy^{x1}}{dv^{1\alpha}} + v^{1y} \frac{dy^{y1}}{dv^{1\alpha}} + v^{1c} \frac{d\bar{y}^1}{dv^{1\alpha}} - t^1\left(\frac{de^{y2}}{dv^{1\alpha}} + \frac{de^{z2}}{dv^{1\alpha}}\right) - \left(\frac{dp^x}{dv^{1\alpha}} x^T + \frac{dp^y}{dv^{1\alpha}} y^T\right) = 0$$

The first-order conditions yield the following interior solutions the optimal differentiated consumption taxes:

$$v^{1c}=\left[s^1\frac{dy^1}{dv^{1c}}-v^{1x}\frac{dy^{x1}}{dv^{1c}}-v^{1y}\frac{dy^{y1}}{dv^{1c}}+t^1\left(\frac{de^{y2}}{dv^{1c}}+\frac{de^{z2}}{dv^{1c}}\right)+\left(\frac{dp^x}{dv^{1c}}x^T+\frac{dp^y}{dv^{1c}}y^T\right)\right]/\frac{d\overline{y}^1}{dv^{1c}}$$

$$v^{1x}=\left[s^1\frac{dy^1}{dv^{1x}}-v^{1y}\frac{dy^{y1}}{dv^{1x}}-v^{1c}\frac{d\overline{y}^1}{dv^{1x}}+t^1\left(\frac{de^{y2}}{dv^{1x}}+\frac{de^{z2}}{dv^{1x}}\right)+\left(\frac{dp^x}{dv^{1x}}x^T+\frac{dp^y}{dv^{1x}}y^T\right)\right]/\frac{dy^{x1}}{dv^{1x}}$$

$$v^{1y}=\left[s^1\frac{dy^1}{dv^{1y}}-v^{1x}\frac{dy^{x1}}{dv^{1y}}-v^{1c}\frac{d\overline{y}^1}{dv^{1y}}+t^1\left(\frac{de^{y2}}{dv^{1y}}+\frac{de^{z2}}{dv^{1y}}\right)+\left(\frac{dp^x}{dv^{1y}}x^T+\frac{dp^y}{dv^{1y}}y^T\right)\right]/\frac{dy^{y1}}{dv^{1y}}$$

Inserting $v^{1c}=v^{1x}=v^{1y}=v^1$ in the expression for $dW^1/dv^{1\alpha}$ above yields:

$$\frac{dW^1}{dv^1}=-s^1\frac{dy^1}{dv^1}+v^1\left(\frac{dy^{x1}}{dv^1}+\frac{dy^{y1}}{dv^1}+\frac{d\overline{y}^1}{dv^1}\right)-t^1\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\left(\frac{dp^x}{dv^1}x^T+\frac{dp^y}{dv^1}y^T\right)$$

The associated first-order condition gives equation (9) for the economy-wide tax. Last, inserting $v^{1c}=v^{1x}=v^1$ and $v^{1y}=0$ in the expression for $dW^1/dv^{1\alpha}$ above yields:

$$\frac{dW^1}{dv^1}=-s^1\frac{dy^1}{dv^1}+v^1\left(\frac{dy^{x1}}{dv^1}+\frac{d\overline{y}^1}{dv^1}\right)-t^1\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\left(\frac{dp^x}{dv^1}x^T+\frac{dp^y}{dv^1}y^T\right)$$

of which associated first-order condition also gives the expression in equation (9).

Derivations of Equations (5) and (8) given that the regulator maximizes global welfare:

We first observe that equation (2) implies $\overline{x}^1+\overline{x}^2=x^1+x^2-x^{x1}-x^{x2}-x^{y1}-x^{y2}$ and, hence,

$$\frac{d\overline{x}^1}{ds^1}+\frac{d\overline{x}^2}{ds^1}=\frac{dx^1}{ds^1}-\frac{dx^{x1}}{ds^1}-\frac{dx^{y1}}{ds^1}+\frac{dx^2}{ds^1}-\frac{dx^{x2}}{ds^1}-\frac{dx^{y2}}{ds^1}=\sum_{j=1}^{2}\left(\frac{dx^j}{ds^1}-\frac{dx^{xj}}{ds^1}-\frac{dx^{yj}}{ds^1}\right).$$

Similarly, we have the following for *y*:

$$\frac{d\overline{y}^1}{ds^1}+\frac{d\overline{y}^2}{ds^1}=\frac{dy^1}{ds^1}-\frac{dy^{x1}}{ds^1}-\frac{dy^{y1}}{ds^1}+\frac{dy^2}{ds^1}-\frac{dy^{x2}}{ds^1}-\frac{dy^{y2}}{ds^1}=\sum_{j=1}^{2}\left(\frac{dy^j}{ds^1}-\frac{dy^{xj}}{ds^1}-\frac{dy^{yj}}{ds^1}\right).$$

Let global welfare be given by $W^g=u^1(\overline{x}^1,\overline{y}^1)-\tau_1^1e^1-\tau_2^1e^2+u^2(\overline{x}^2,\overline{y}^2)$. Note that region 2 does not value emissions in this formulation of global welfare. The Lagrangian is

$$L^{spg}=u^1(\overline{x}^1,\overline{y}^1)-\tau^1\left(e^1+e^2\right)-\lambda_T^1\left(p^xx^T+p^yy^T\right)-\lambda_L^1\left(l^{x1}+l^{y1}+l^{z1}-L^1\right)+u^2(\overline{x}^2,\overline{y}^2)-\lambda_L^2\left(l^{x2}+l^{y2}+l^{z2}-L^2\right),$$

with associated first order condition:

$$\frac{dL^{spg}}{ds^1} = u_x^1 \frac{d\overline{x}^1}{ds^1} + u_y^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \frac{de^1}{ds^1} - \tau^1 \frac{de^2}{ds^1}$$

$$-\frac{d\lambda_T^1}{ds^1}\left(p^x x^T + p^y y^T \right) - \lambda_T^1 \left(\frac{dp^x}{ds^1} x^T + p^x \frac{dx^T}{ds^1} + \frac{dp^y}{ds^1} y^T + p^y \frac{dy^T}{ds^1} \right) - \frac{d\lambda_L^1}{ds^1}\left(l^{x1} + l^{y1} + l^{z1} - L^1 \right) - \lambda_L^1 \left(\frac{l^{x1}}{ds^1} + \frac{l^{y1}}{ds^1} + \frac{l^{z1}}{ds^1} - \frac{L^1}{ds^1} \right)$$

$$+u_x^2 \frac{d\overline{x}^2}{ds^1} + u_y^2 \frac{d\overline{y}^2}{ds^1} - \frac{d\lambda_L^2}{ds^1}\left(l^{x2} + l^{y2} + l^{z2} - L^2 \right) - \lambda_L^2 \left(\frac{l^{x2}}{ds^1} + \frac{l^{y2}}{ds^1} + \frac{l^{z2}}{ds^1} - \frac{L^2}{ds^1} \right)$$

$$= u_x^1 \frac{d\overline{x}^1}{ds^1} + u_y^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \frac{de^1}{ds^1} - \tau^1 \frac{de^2}{ds^1} + u_x^2 \frac{d\overline{x}^2}{ds^1} + u_y^2 \frac{d\overline{y}^2}{ds^1}$$

$$= p^x \frac{d\overline{x}^1}{ds^1} + \left(p^y + v^1 \right) \frac{d\overline{y}^1}{ds^1} - \tau^1 \frac{de^1}{ds^1} - \tau^1 \frac{de^2}{ds^1} + p^x \frac{d\overline{x}^2}{ds^1} + p^y \frac{d\overline{y}^2}{ds^1}$$

$$= p^x \left(\frac{d\overline{x}^1}{ds^1} + \frac{d\overline{x}^2}{ds^1} \right) + p^y \left(\frac{d\overline{y}^1}{ds^1} + \frac{d\overline{y}^2}{ds^1} \right) + v^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \left(\frac{de^1}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= p^x \sum_{j=1}^{2} \left(\frac{dx^j}{ds^1} - \frac{dx^{xj}}{ds^1} - \frac{dx^{yj}}{ds^1} \right) + p^y \sum_{j=1}^{2} \left(\frac{dy^j}{ds^1} - \frac{dy^{xj}}{ds^1} - \frac{dy^{yj}}{ds^1} \right) + v^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \left(\frac{de^1}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= p^x \sum_{j=1}^{2} \left(\frac{dx^j}{ds^1} - \frac{dx^{xj}}{ds^1} - \frac{dx^{yj}}{ds^1} \right) + p^y \sum_{j=1}^{2} \left(\frac{dy^j}{ds^1} - \frac{dy^{xj}}{ds^1} - \frac{dy^{yj}}{ds^1} \right) + v^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= p^x \sum_{j=1}^{2} \left(f_x^j \frac{dx^{xj}}{ds^1} + f_y^j \frac{dy^{xj}}{ds^1} + f_l^j \frac{dl^{xj}}{ds^1} - \frac{dx^{xj}}{ds^1} - \frac{dx^{yj}}{ds^1} \right)$$

$$+ p^y \sum_{j=1}^{2} \left(g_x^j \frac{dx^{yj}}{ds^1} + g_y^j \frac{dy^{yj}}{ds^1} + g_z^j \frac{dz^{yj}}{ds^1} + g_l^j \frac{dl^{yj}}{ds^1} + g_e^j \frac{de^{yj}}{ds^1} - \frac{dy^{xj}}{ds^1} - \frac{dy^{yj}}{ds^1} \right) + v^1 \frac{d\overline{y}^1}{ds^1}$$

$$-\tau^1 \left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= \sum_{j=1}^{2} \left(p^x \left(f_x^j - 1 \right) \frac{dx^{xj}}{ds^1} + \left(p^x f_y^j - p^y \right) \frac{dy^{xj}}{ds^1} + \left(p^y g_x^j - p^x \right) \frac{dx^{yj}}{ds^1} + p^y \left(g_y^j - 1 \right) \frac{dy^{yj}}{ds^1} + p^x f_l^j \frac{dl^{xj}}{ds^1} + p^y g_l^j \frac{dl^{yj}}{ds^1} \right)$$

$$+ \sum_{j=1}^{2} \left(p^y g_z^j \left(h_l^j \frac{dl^{zj}}{ds^1} + h_e^j \frac{de^{zj}}{ds^1} \right) + p^y g_e^j \frac{de^{yj}}{ds^1} \right) + v^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= \sum_{j=1}^{2} \left(v^j \frac{dy^{xj}}{ds^1} - s^j g_x^j \frac{dx^{yj}}{ds^1} + \left(v^j - s^j g_y^j \right) \frac{dy^{yj}}{ds^1} + p^{lj} \frac{dl^{xj}}{ds^1} + \left(p^{lj} - s^j g_l^j \right) \frac{dl^{yj}}{ds^1} + \left(p^{lj} - s^j g_z^j h_l^j \right) \frac{dl^{zj}}{ds^1} \right)$$

$$+ \sum_{j=1}^{2} \left(\left(p^{zj} - s^j g_z^j \right) h_e^j \frac{de^{zj}}{ds^1} + \left(t^j - s^j g_e^j \right) \frac{de^{yj}}{ds^1} \right) + v^1 \frac{d\overline{y}^1}{ds^1} - \tau^1 \left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= s^1 \left(-g_x^1 \frac{dx^{y1}}{ds^1} - g_y^1 \frac{dy^{y1}}{ds^1} - g_l^1 \frac{dl^{y1}}{ds^1} - g_z^1 h_l^1 \frac{dl^{z1}}{ds^1} - g_z^j h_e^j \frac{de^{zj}}{ds^1} - g_e^j \frac{de^{yj}}{ds^1} \right) + v^1 \left(\frac{dy^{xj}}{ds^1} + \frac{dy^{yj}}{ds^1} + \frac{d\overline{y}^1}{ds^1} \right)$$

$$+ \sum_{j=1}^{2} \left(p^{lj} \left(\frac{dl^{xj}}{ds^1} + \frac{dl^{yj}}{ds^1} + \frac{dl^{zj}}{ds^1} \right) + p^{zj} h_e^j \frac{de^{zj}}{ds^1} \right) + t^1 \frac{de^{y1}}{ds^1} - \tau^1 \left(\frac{de^{y1}}{ds^1} + \frac{de^{z1}}{ds^1} + \frac{de^2}{ds^1} \right)$$

$$= -s^1 \frac{dy^1}{ds^1} + v^1 \left(\frac{dy^{xj}}{ds^1} + \frac{dy^{yj}}{ds^1} + \frac{d\overline{y}^1}{ds^1} \right) + p^{z2} h_e^2 \frac{de^{z2}}{ds^1} - \tau^1 \frac{de^2}{ds^1}$$

$$= -s^1 \frac{dy^1}{ds^1} + v^1 \left(\frac{dy^{xj}}{ds^1} + \frac{dy^{yj}}{ds^1} + \frac{d\overline{y}^1}{ds^1} \right) - \tau^1 \frac{de^2}{ds^1}$$

This is equal to equation (5) when removing the terms of trade. Similarly, differentiating global welfare with respect to the consumption tax in region 1, the first order condition yields:

$$\frac{dW^g}{dv^1} = u_x^1 \frac{d\overline{x}^1}{dv^1} + u_y^1 \frac{d\overline{y}^1}{dv^1} - \tau^1 \frac{de^1}{dv^1} - \tau^1 \frac{de^2}{dv^1} + u_x^2 \frac{d\overline{x}^2}{dv^1} + u_y^2 \frac{d\overline{y}^2}{dv^1}$$

$$= p^x \left(\frac{d\overline{x}^1}{dv^1} + \frac{d\overline{x}^2}{dv^1} \right) + p^y \left(\frac{d\overline{y}^1}{dv^1} + \frac{d\overline{y}^2}{dv^1} \right) + v^1 \frac{d\overline{y}^1}{dv^1} - \tau^1 \left(\frac{de^1}{dv^1} + \frac{de^2}{dv^1} \right)$$

$$= p^x \sum_{j=1}^{2} \left(\frac{dx^j}{dv^1} - \frac{dx^{xj}}{dv^1} - \frac{dx^{yj}}{dv^1} \right) + p^y \sum_{j=1}^{2} \left(\frac{dy^j}{dv^1} - \frac{dy^{xj}}{dv^1} - \frac{dy^{yj}}{dv^1} \right) + v^1 \frac{d\overline{y}^1}{dv^1} - \tau^1 \left(\frac{de^{y1}}{dv^1} + \frac{de^{z1}}{dv^1} + \frac{de^2}{dv^1} \right)$$

$$= p^x \sum_{j=1}^{2} \left(f_x^j \frac{dx^{xj}}{dv^1} + f_y^j \frac{dy^{xj}}{dv^1} + f_l^j \frac{dl^{xj}}{dv^1} - \frac{dx^{xj}}{dv^1} - \frac{dx^{yj}}{dv^1} \right)$$

$$+ p^y \sum_{j=1}^{2} \left(g_x^j \frac{dx^{yj}}{dv^1} + g_y^j \frac{dy^{yj}}{dv^1} + g_z^j \frac{dz^{yj}}{dv^1} + g_l^j \frac{dl^{yj}}{dv^1} + g_e^j \frac{de^{yj}}{dv^1} - \frac{dy^{xj}}{dv^1} - \frac{dy^{yj}}{dv^1} \right) + v^1 \frac{d\overline{y}^1}{dv^1}$$

$$- \tau^1 \left(\frac{de^{y1}}{dv^1} + \frac{de^{z1}}{dv^1} + \frac{de^2}{dv^1} \right)$$

$$= \sum_{j=1}^{2} \left(p^x \left(f_x^j - 1 \right) \frac{dx^{xj}}{dv^1} + \left(p^x f_y^j - p^y \right) \frac{dy^{xj}}{dv^1} + \left(p^y g_x^j - p^x \right) \frac{dx^{yj}}{dv^1} + p^y \left(g_y^j - 1 \right) \frac{dy^{yj}}{dv^1} + p^x f_l^j \frac{dl^{xj}}{dv^1} + p^y g_l^j \frac{dl^{yj}}{dv^1} \right)$$

$$+ \sum_{j=1}^{2} \left(p^y g_z^j \left(h_l^j \frac{dl^{zj}}{dv^1} + h_e^j \frac{de^{zj}}{dv^1} \right) + p^y g_e^j \frac{de^{yj}}{dv^1} \right) + v^1 \frac{d\overline{y}^1}{dv^1} - \tau^1 \left(\frac{de^{y1}}{dv^1} + \frac{de^{z1}}{dv^1} + \frac{de^2}{dv^1} \right)$$

$$= \sum_{j=1}^{2} \left(v^j \frac{dy^{xj}}{dv^1} - s^j g_x^j \frac{dx^{yj}}{dv^1} + \left(v^j - s^j g_y^j \right) \frac{dy^{yj}}{dv^1} + p^{l^j} \frac{dl^{xj}}{dv^1} + \left(p^{lj} - s^j g_l^j \right) \frac{dl^{yj}}{dv^1} + \left(p^{lj} - s^j g_z^j h_l^j \right) \frac{dl^{zj}}{dv^1} \right)$$

$$+ \sum_{j=1}^{2} \left(\left(p^{zj} - s^j g_z^j \right) h_e^j \frac{de^{zj}}{dv^1} + \left(t^j - s^j g_e^j \right) \frac{de^{yj}}{dv^1} \right) + v^1 \frac{d\overline{y}^1}{dv^1} - \tau^1 \left(\frac{de^{y1}}{dv^1} + \frac{de^{z1}}{dv^1} + \frac{de^2}{dv^1} \right)$$

$$= s^1 \left(-g_x^1 \frac{dx^{y1}}{dv^1} - g_y^1 \frac{dy^{y1}}{dv^1} - g_l^1 \frac{dl^{y1}}{dv^1} - g_z^1 h_l^1 \frac{dl^{z1}}{dv^1} - g_z^j h_e^j \frac{de^{zj}}{dv^1} - g_e^j \frac{de^{yj}}{dv^1} \right) + v^1 \left(\frac{dy^{xj}}{dv^1} + \frac{dy^{yj}}{dv^1} + \frac{d\overline{y}^1}{dv^1} \right)$$

$$+ \sum_{j=1}^{2} \left(p^{l^j} \left(\frac{dl^{xj}}{dv^1} + \frac{dl^{yj}}{dv^1} + \frac{dl^{zj}}{dv^1} \right) + p^{zj} h_e^j \frac{de^{zj}}{dv^1} \right) + t^1 \frac{de^{y1}}{dv^1} - \tau^1 \left(\frac{de^{y1}}{dv^1} + \frac{de^{z1}}{dv^1} + \frac{de^2}{dv^1} \right)$$

$$= -s^1 \frac{dy^1}{dv^1} + v^1 \left(\frac{dy^{xj}}{dv^1} + \frac{dy^{yj}}{dv^1} + \frac{d\overline{y}^1}{dv^1} \right) + p^{z2} h_e^2 \frac{de^{z2}}{dv^1} - \tau^1 \frac{de^2}{dv^1}$$

$$= -s^1 \frac{dy^1}{dv^1} + v^1 \left(\frac{dy^{xj}}{ds^1} + \frac{dy^{yj}}{ds^1} + \frac{d\overline{y}^1}{ds^1} \right) - \tau^1 \frac{de^2}{ds^1}$$

(which can also be written $\frac{dW^g}{dv^1} = -s^1 \frac{dy^1}{dv^1} + v^1 \left(\frac{dy^1}{dv^1} + \frac{dy^T}{dv^1} \right) - \tau^1 \frac{de^2}{ds^1}$). This is equation (8) after removing the terms of trade terms.

Derivation of equation (10) (the optimal combination of v[1] and s[1]):

Solving the system of equations constituted by equations (6) and (9) yields the following reduced form solutions:

$$s^1 = \frac{\left(\frac{dy^1}{ds^1}+\frac{dy^T}{ds^1}\right)\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\left(\frac{dy^1}{dv^1}+\frac{dy^T}{dv^1}\right)\left(\frac{de^{y2}}{ds^1}+\frac{de^{z2}}{ds^1}\right)}{\frac{dy^1}{ds^1}\frac{dy^T}{dv^1}-\frac{dy^1}{dv^1}\frac{dy^T}{ds^1}}\tau^1$$

$$v^1 = \frac{\frac{dy^1}{ds^1}\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\frac{dy^1}{dv^1}\left(\frac{de^{y2}}{ds^1}+\frac{de^{z2}}{ds^1}\right)}{\frac{dy^1}{ds^1}\frac{dy^T}{dv^1}-\frac{dy^1}{dv^1}\frac{dy^T}{ds^1}}\tau^1 \qquad (18)$$

We further have $\frac{dy^1}{ds^1}+\frac{dy^T}{ds^1}=\frac{d\tilde{y}}{ds^1}$ and $\frac{dy^1}{dv^1}+\frac{dy^T}{dv^1}=\frac{d\tilde{y}}{dv^1}$ using equation (2) and the definition $\tilde{y}=\bar{y}^1+y^{x1}+y^{y1}$. Insertion yields equation (10).

Derivation of equation (11):

Using equation (10) can be shown that:

$$s^1-v^1=\frac{\tau^1}{\frac{dy^1}{ds^1}\frac{dy^T}{dv^1}-\frac{dy^1}{dv^1}\frac{dy^T}{ds^1}}\left(\frac{dy^T}{ds^1}\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\frac{dy^T}{dv^1}\left(\frac{de^{y2}}{ds^1}+\frac{de^{z2}}{ds^1}\right)\right)$$

Given that the denominator is negative (see the main text), and that τ^1 is positive, we then have:

$$\begin{aligned}
& s^1-v^1\geq 0 \\
\Leftrightarrow\ & \frac{\tau^1}{\frac{dy^1}{ds^1}\frac{dy^T}{dv^1}-\frac{dy^1}{dv^1}\frac{dy^T}{ds^1}}\left(\frac{dy^T}{ds^1}\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\frac{dy^T}{dv^1}\left(\frac{de^{y2}}{ds^1}+\frac{de^{z2}}{ds^1}\right)\right)\geq 0 \\
\Leftrightarrow\ & \frac{dy^T}{ds^1}\left(\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}\right)-\frac{dy^T}{dv^1}\left(\frac{de^{y2}}{ds^1}+\frac{de^{z2}}{ds^1}\right)\leq 0 \\
\Leftrightarrow\ & \frac{\frac{de^{y2}}{dv^1}+\frac{de^{z2}}{dv^1}}{\frac{dy^T}{dv^1}}-\frac{\frac{de^{y2}}{ds^1}+\frac{de^{z2}}{ds^1}}{\frac{dy^T}{ds^1}}\leq 0
\end{aligned}$$

Equation (11) follows.

Appendix B: Optimal consumption tax with local pollutant

Here we consider the case $\tau_1^1 > 0$ and $\tau_2^1 = 0$, which is relevant for local pollutants. We derive the optimal consumption tax v^1 in a situation where an emissions tax $t^1 = \tau_1^1$ is already imposed, but also a subsidy to domestic production of the *y* good, $s^1 > 0$. The reason for introducing s^1 (e.g., via output-based allocation) in this case may be lobbying from the EITE industries or some other motivation that we do not consider here.

To derive the optimal v^1, we differentiate the welfare function (4) with respect to v^1, and find:[25]

$$\frac{dW^1}{dv^1} = -s^1 \frac{dy^1}{dv^1} + v^1\left(\frac{dy^1}{dv^1} + \frac{dy^T}{dv^1}\right) - \left(\frac{dp^x}{dv^1}x^T + \frac{dp^y}{dv^1}y^T\right) \quad (19)$$

The last term is terms-of-trade effects, which we disregard. Increasing the consumption tax in region 1 will depress consumption and, as a consequence, domestic production and net import of the *y* good (cf. Assumption 2). Thus, the first term is positive if $s^1 > 0$. Hence, increasing the consumption tax from zero is welfare-improving – the optimal consumption tax is strictly positive. The explanation is (as before) that the consumption tax (partly) corrects for the deadweight loss created by the subsidy.

The second term then turns negative as the consumption tax is increased from zero. We further see that if $v^1 = s^1$, then $\frac{dW^1}{dv^1} = v^1 \frac{dy^T}{dv^1} < 0$. That is, the consumption tax should be set below the subsidy rate. The more trade exposed country 1 is, that is, the stronger import responds relative to domestic production (of good *y*) to a consumption tax, the lower should the optimal consumption tax be. Even though the subsidy creates a deadweight loss, the consumption tax is not able to fully correct for this if there is trade. In a closed economy, however, the optimal consumption tax is equal to the subsidy, as production equals consumption plus use of intermediates.

The optimal v^1, for a given s^1, is:

[25] The derivation is very similar to the ones shown in Appendix A, and thus not included here.

$$v^{1*} = \frac{\frac{dy^1}{dv^1}}{\left(\frac{dy^1}{dv^1} + \frac{dy^T}{dv^1}\right)} s^1 \tag{20}$$

We see that v^{1*} increases proportionally with s^1 as long as the fraction is not changed. Most likely, the fraction will change somewhat though, but it is difficult to say in which direction.

We summarize these results as follows:

If region 1 only cares about domestic emissions, and has already implemented a subsidy s^1 to the production of y^1, then it is optimal to implement a consumption tax v^{1*} on domestic use of y ($\bar{y}^1, y^{x1}, y^{y1}$), with the optimal tax given by (20). The optimal tax is strictly positive but smaller than the subsidy: $0 < v^{1*} < s^1$. Furthermore, the first order effect of increasing the subsidy is to increase the optimal consumption tax proportionally.

Appendix C: Algebraic summary of the numerical CGE model

The canonical multisectoral multiregional computable general equilibrium model is formulated as a system of nonlinear inequalities. The inequalities correspond to the two classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero profit) for producers with constant returns to scale; and (ii) market clearance for all goods and factors. The first class determines activity levels, the second price levels. In equilibrium, each variable is associated with an inequality condition: an activity level to an exhaustion of product condition and a commodity price to a market clearance condition. In our algebraic representation, the notation is Π_{gr}^{z} used to denote the unit profit function (calculated as the difference between unit revenue and unit cost) for production with constant returns to scale of sector g in region r, where z is the name assigned to the associated production activity. Differentiating the unit profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotelling's lemma), which appear subsequently in the market clearance conditions. We use g as an index for all sectors/commodities except primary fossil energy and index r (aliased with s) to denote region. Furthermore, we indicate complementarity between equilibrium conditions and variables with the operator $\perp$.

Tables C1–C6 explain the notations for variables and parameters employed within our algebraic exposition. Figures C1-C3 sketch the nesting of functional forms in production and consumption together with the default elasticities underlying our central case simulations. Numerically, the model is implemented in GAMS (Rosenthal, 2007)[26] and solved using PATH (Ferris and Munson, 1999).[27]

[26] Rosenthal, R.E. (2007): *GAMS: A User's Guide*. GAMS Development Corporation: Washington DC, USA.
[27] Ferris, M. and T.S. Munson (1999): Interfaces to PATH 3.0: Design, Implementation and Usage, *Computational Optimization and Applications* 12: 1-3, 207-227.
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Table C.1. Indices and sets

G	Set of all commodities {*NC_T, C_T, C_NT, ELE, FE*}
EG	Subset of primary energy goods {*FE*}
R	Set of regions {1, 2}
g (*alias i*)	Index for sectors and commodities
r (*alias s*)	Index for regions

Table C.2. Activity variables

Y_{gr}	Production of commodity g in region r
M_{gr}	Material composite for commodity g in region r
KL_{gr}	Value-added composite for commodity g in region r
A_{gr}	Armington aggregate of commodity g in region r
IM_{gr}	Import aggregate of commodity g in region r
C_r	Consumption composite in region r

Table C.3. Price variables

p_{gr}	Price of commodity g in region r
p_{gr}^{M}	Price of material composite for commodity g in region r
p_{gr}^{KL}	Price of value-added composite for commodity g in region r
p_{gr}^{A}	Price of Armington aggregate of commodity g in region r
p_{gr}^{IM}	Price of aggregate imports of commodity g in region r
p_r^{C}	Price of consumption composite in region r
w_r	Price of labor (wage rate) in region r
v_r	Price of capital services (rental rate) in region r
q_r	Rent for primary energy resource in region r
p_r^{CO2}	Price of carbon emissions in region r

Table C.4. Cost shares

θ_{gr}^{M}	Cost share of material composite in production of commodity g in region r
θ_{gr}^{FE}	Cost share of primary energy in capital-labor-energy composite input to production of commodity g in region r
θ_{igr}^{MN}	Cost share of input i in material composite of commodity g in region r
θ_{gr}^{K}	Cost share of capital within the value-added of commodity g in region r
θ_{r}^{Q}	Cost share of primary energy resource in primary energy production in region r
$\theta_{FE,r}^{LN}$	Cost share of labor in non-resource composite of primary energy production in region r
$\theta_{FE,r}^{KN}$	Cost share of capital in non-resource input to primary energy production in region r
$\theta_{g,FE,r}^{N}$	Cost share of good g in non-resource input to primary energy production in region r
θ_{gr}^{A}	Cost share of domestic input g in the Armington composite of commodity g in region r
θ_{gsr}^{IM}	Cost share of commodity g from region s in import composite of region r
θ_{gr}^{C}	Cost share of commodity g in consumption composite of region r

Table C.5. Elasticities of substitution

σ_{gr}^{KLEM}	Substitution between the material composite and the energy-value-added aggregate in production of commodity g in region r
σ_{gr}^{KLE}	Substitution between primary fossil energy and the value-added nest in production of commodity g in region r
σ_{gr}^{M}	Substitution between material inputs within the material composite in production of commodity g in region r
σ_{gr}^{KL}	Substitution between the capital and labor within the value-added composite in production of commodity g in region r
σ_{gr}^{Q}	Substitution between natural resource input and the composite of other inputs in primary energy production in region r
σ_{gr}^{A}	Substitution between import composite and domestic input to Armington production of commodity g in region r
σ_{gr}^{IM}	Substitution between imports from different regions within the import composite of commodity g in region r
σ_{r}^{C}	Substitution between commodity inputs to composite consumption in region r

Table C.6. Endowments

$\overline{L}_r$	Aggregate labor endowment in region r
$\overline{K}_r$	Capital endowment in region r
$\overline{Q}_r$	Resource endowment of primary fossil energy in region r
$\overline{CO2}_r$	Endowment with CO_2 emissions allowances in region r
$a_{FE,r}^{CO_2}$	CO_2 emissions coefficient for primary fossil energy in region r

Zero profit conditions

- Production of goods except fossil primary energy ($g \notin EG$):

$$\Pi_{gr}^{y} = p_{gr} - \left[\theta_{gr}^{M} p_{gr}^{M\left(1-\sigma_{gr}^{KLEM}\right)} + \left(1-\theta_{gr}^{M}\right) \left[\theta_{gr}^{FE} \left(p_{FE,r} + a_{FE,r}^{CO_2} p_{r}^{CO_2} \right)^{\left(1-\sigma_{gr}^{KLE}\right)} + \left(1-\theta_{gr}^{FE}\right) p_{gr}^{KL\left(1-\sigma_{gr}^{KLE}\right)} \right]^{\frac{\left(1-\sigma_{gr}^{KLEM}\right)}{\left(1-\sigma_{gr}^{KLE}\right)}} \right]^{\frac{1}{\left(1-\sigma_{gr}^{KLEM}\right)}} \leq 0 \qquad \perp \; Y_{gr}$$

- Sector-specific material composite ($g \notin EG$):

$$\Pi_{gr}^{M} = p_{gr}^{M} - \left[\sum_{i \notin EG} \theta_{igr}^{MN} p_{ir}^{A\left(1-\sigma_{gr}^{M}\right)} \right]^{\frac{1}{\left(1-\sigma_{gr}^{M}\right)}} \leq 0 \qquad \perp \; M_{gr}$$

- Sector-specific value-added aggregate ($g \notin EG$):

$$\Pi_{gr}^{KL} = p_{gr}^{KL} - \left[\theta_{gr}^{K} v_{r}^{\left(1-\sigma_{gr}^{KL}\right)} + \left(1-\theta_{gr}^{K}\right) w_{r}^{\left(1-\sigma_{gr}^{KL}\right)} \right]^{\frac{1}{\left(1-\sigma_{gr}^{KL}\right)}} \leq 0 \qquad \perp \; KL_{gr}$$

- Production of primary fossil fuel:

$$\Pi_{FE,r}^{Y} = p_{FE,r} - \left[\theta_{r}^{Q} q_{r}^{\left(1-\sigma_{r}^{Q}\right)} + \left(1-\theta_{r}^{Q}\right) \left[\theta_{FE,r}^{LN} w_{r} + \theta_{FE,r}^{KN} v_{r} + \sum_{g \notin EG} \theta_{g,FE,r}^{N} p_{gr}^{A} \right]^{\left(1-\sigma_{r}^{Q}\right)} \right]^{\frac{1}{\left(1-\sigma_{r}^{Q}\right)}} \leq 0 \qquad \perp \; Y_{FE,r}$$

- Armington aggregate ($g \notin EG$):

$$\Pi_{gr}^{A} = p_{gr}^{A} - \left[\theta_{gr}^{A} p_{gr}^{\left(1-\sigma_{gr}^{A}\right)} + \left(1-\theta_{gr}^{A}\right) p_{gr}^{IM\left(1-\sigma_{gr}^{A}\right)} \right]^{\frac{1}{\left(1-\sigma_{gr}^{A}\right)}} \leq 0 \qquad \perp \; A_{gr}$$

- Import composite ($g \notin EG$):

$$\Pi_{gr}^{IM} = p_{gr}^{IM} - \left[\sum_{s \neq r} \theta_{gsr}^{IM} p_{gs}^{\left(1-\sigma_{gr}^{IM}\right)} \right]^{\frac{1}{\left(1-\sigma_{gr}^{IM}\right)}} \leq 0 \qquad \perp \; IM_{gr}$$

- Consumption composite:

$$\Pi_{r}^{C} = p_{r}^{C} - \left[\sum_{g \notin EG} \theta_{gr}^{C} p_{gr}^{A\left(1-\sigma_{gr}^{C}\right)} \right]^{\frac{1}{\left(1-\sigma_{gr}^{C}\right)}} \leq 0 \qquad \perp \; C_{r}$$

Market clearance conditions

- Labor:

$$\bar{L}_r \geq Y_{FE,r} \frac{\partial \Pi^Y_{FE,r}}{\partial w_r} + \sum_{g \notin EG} KL_{gr} \frac{\partial \Pi^{KL}_{gr}}{\partial w_r} \quad \perp \quad w_r$$

- Capital:

$$\bar{K}_r \geq Y_{FE,r} \frac{\partial \Pi^Y_{FE,r}}{\partial v_r} + \sum_{g \notin EG} KL_{gr} \frac{\partial \Pi^{KL}_{gr}}{\partial v_r} \quad \perp \quad v_r$$

- Primary fossil energy resource:

$$\bar{Q}_r \geq Y^Y_{FE,r} \frac{\partial \Pi^Y_{FE,r}}{\partial q_r} \quad \perp \quad q_r$$

- Material composite ($g \notin EG$):

$$M_{gr} \geq Y_{gr} \frac{\partial \Pi^Y_{gr}}{\partial p^M_{gr}} \quad \perp \quad p^M_{gr}$$

- Value-added ($g \notin EG$):

$$KL_{gr} \geq Y_{gr} \frac{\partial \Pi^Y_{gr}}{\partial p^{KL}_{gr}} \quad \perp \quad p^{KL}_{gr}$$

- Armington aggregate ($g \notin EG$):

$$A_{gr} \geq C_r \frac{\partial \Pi^C_r}{\partial p^A_{gr}} + Y_{FE,r} \frac{\partial \Pi^Y_{FE,r}}{\partial p^A_{gr}} + \sum_{i \notin EG} M_{ir} \frac{\partial \Pi^M_{ir}}{\partial p^A_{gr}} \quad \perp \quad p^A_{gr}$$

- Import composite ($g \notin EG$):

$$IM_{gr} \geq A_{gr} \frac{\partial \Pi^A_{gr}}{\partial p^{IM}_{gr}} \quad \perp \quad p^{IM}_{gr}$$

- Goods except primary energy ($g \notin EG$):

$$Y_{gr} \geq A_{gr} \frac{\partial \Pi^A_{gr}}{\partial p_{gr}} + \sum_{s \neq r} IM_{gs} \frac{\partial \Pi^{IM}_{gs}}{\partial p_{gs}} \quad \perp \quad p_{gr}$$

- Primary energy:

$$Y_{FE,r} \geq \sum_{g \notin EG} Y_{gr} \frac{\partial \Pi_{gr}^{Y}}{\partial \left(p_{FE,r} + a_{FE,r}^{CO_2} p_r^{CO_2} \right)} \quad \perp \quad p_{FE,r}$$

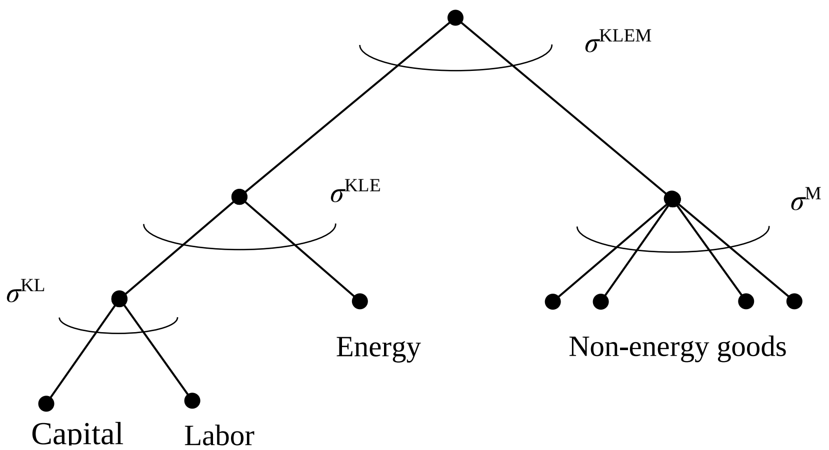
- Private consumption ($g = C$):

$$p_r^C C_r \geq w_r \bar{L}_r + v_r \bar{K}_r + q_r \bar{Q}_r + p_r^{CO_2} \overline{CO2}_r \quad \perp \quad p_r^C$$

- Carbon emissions:

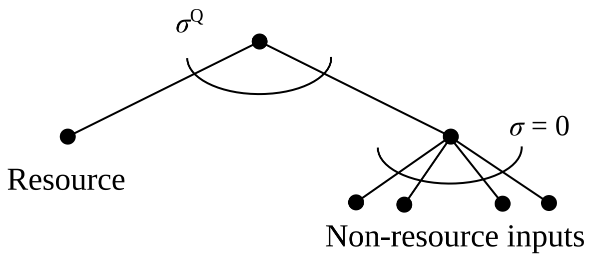
$$\overline{CO2}_r \geq a_{FE,r}^{CO_2} Y_{FE,r} \quad \perp \quad p_r^{CO_2}$$

Figure C.1. Nesting in non-energy production



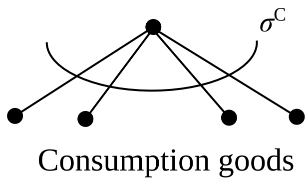
Elasticities: $\sigma^{KLEM} = 0.25$; $\sigma^{KLE} = 0.5$; $\sigma^{M} = 0.1$; $\sigma^{KL} = 1$

Figure C.2. Nesting in fossil energy production



Elasticities: σ^{Q} =0.5 (calibrated to an initial supply elasticity of 1)

Figure C.3. Nesting in final consumption



Elasticities: σ^{C} =0.5

Appendix D: Mapping of GTAP sectors and base-year data

Table D.1 shows the mapping of the 65 GTAP sectors to the five composite sectors in our model.

Table D.1. Mapping of GTAP sectors to composite model sectors

Model sectors	GTAP sectors
FE: fossil energy composite	Coal; Crude oil; Gas (extraction and distribution)
ELE: electricity	Electricity
C_T: carbon-intensive and tradable goods	Refined oil; Ferrous metals; Non-ferrous metals; Non-metallic minerals; Chemical rubber products; Other machinery and equipment; Paper and paper products
C_NT: carbon-intensive and non-tradable goods	All transportation sectors (air, water, rail, road)
NC_T: carbon-free and tradable goods	All remaining goods and services

Table D.2 shows the input-output table for the two symmetric regions. The entries indicate value flows with negative values constituting inputs (demands) and positive values constituting outputs or endowments (supplies). Since the base-year data are monetary values, we have to choose units for goods and factors to separate price and quantity observations. Typically, the units for goods and factors are chosen to have a market price of 1 such that the input-output values can be easily converted into quantities.[28]

Table D2. Input-output table (in bn USD) for each region based on GTAP11 data*

	FE	*ELE*	*C_NT*	*C_T*	*NC_T*	*FD*	*Export*	**Demand**
FE	-1039.5	-376.0	-441.5	-318.0		-237.5		-2412.5
ELE	-49.5	-73.5	-30.5	-346.5	-580.0	-391.5		-1471.5
C_NT	-47.5	-36.5	-201.5	-237.0	-1337.0	-1366.5		-3226.0
C_T	-71.5	-37.5	-45.5	-3062.0	-4707.0	-788.0	-2156.0	-10867.5
NC_T	-330.5	-301.0	-1017.5	-2062.5	-23649.0	-36016.5	-6866.0	-70243.0
Labor	-107.2	-236.5	-748.0	-1190.0	-17568.8			
Capital	-506.1	-410.5	-741.5	-1495.5	-15535.2			
Resource	-260.7							
Output	2412.5	1471.5	3226.0	8711.5	63377.0			
Import				2156.0	6866.0			
Supply	2412.5	1471.5	3226.0	10867.5	70243.0			

* *FE*: fossil energy; *ELE*: electricity; *C_NT*: carbon-intensive and non-tradable goods, *C_T*: carbon-intensive and tradable goods; *NC_T*: carbon-free and tradable goods; *FD*: final demand

[28] We abstract from explicit tax wedges and use gross-of-tax values throughout to suppress initial tax distortions which are also absent in our theoretical analysis.