COMPUTATION OF BUSINESS CYCLE MODELS: A COMPARISON OF NUMERICAL METHODS

BURKHARD HEER
ALFRED MAUSSNER

CESIFO WORKING PAPER NO. 1207
CATEGORY 5: FISCAL POLICY, MACROECONOMICS AND GROWTH
JUNE 2004

CESifo *Working Paper No. 1207*

COMPUTATION OF BUSINESS CYCLE MODELS: A COMPARISON OF NUMERICAL METHODS

Abstract

We compare the numerical methods that are most widely applied in the computation of the standard business cycle model with flexible labor. The numerical techniques imply economically insignificant differences with regard to business cycle summary statistics except for the volatility of investment. Furthermore, these results are robust with regard to the choice of the functional form of the utility function and the model's parameterization. In conclusion, the simplest and fastest method, the log-linearization of the model around the steady state, is found to be most convenient and appropriate for the standard business cycle model.

JEL classification: C68, C63, E32.

Keywords: log-linearization, projection methods, extended path, value function iteration, parameterized expectations, genetic search.

Burkhard Heer
University of Bamberg
Department of Economics
Feldkirchenstrasse 21
96045 Bamberg
Germany

Burkhard.Heer@sowi.uni-bamberg.de

Alfred Maussner
University of Augsburg
Department of Economics
Universitätsstrasse 16
86159 Augsburg
Germany

alfred.maussner@wiwi. uni-augsburg.de

Part of this work has been completed while Burkhard Heer was staying at Georgetown University and Stanford University. Burkhard Heer greatfully acknowledges support from the German Science Foundation (DFG) during this period.

1 Introduction

The dominant paradigma of modern business cycle theory is the stochastic growth model with flexible labor. The stochastic growth model, however, is difficult to compute as multiplicative elements such as the production function interact with additive elements such as depreciation or investment. As a consequence, only special cases (with log utility and full depreciation) can be solved analytically. For this reason, the comparison of different computational techniques that approximate the solution numerically is important. Previous work by Taylor and Uhlig (1980) has focused on the study of the stochastic growth model with inelastic labor supply, while Christiano and Fisher (2000) compare different numerical techniques for the solution of the stochastic growth model with binding constraints on nonnegative investment.

In the present paper, we evaluate the numerical techniques that are most widely applied in recent research on non-linear rational expectations general equilibrium models from a different angle. We analyze the properties of standard methods with regard to their accuracy and appropriateness for business cycle research. Importantly for the study of business cycles, we introduce flexible labor supply in the stochastic growth model. Furthermore, we study the sensitivity of the first two moments of the variables that are important for the business cycle researcher, i.e. output, employment, investment, consumption, and wages, with regard to the computational method varying both the functional form of the utility function and the parameterization of the model. In particular, we apply parameter values in the range that are typically observed across countries.

The paper is organized as follows. In section 2, the model is presented. In section 3, we briefly review the methods most relevant for the computation of modern business cycle models, and section 4 presents summary statistics for the various methods. Section 5 concludes.

2 The standard business cycle model

We consider a decentralized economy with households and firms. Households maximize their expected life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{1}$$

with respect to consumption c_0 and labor supply n_0 in period 0. The time endowment is normalized to one so that $1 - n_t$ denotes leisure in period t. Utility is discounted by $\beta \in (0, 1)$. Instantaneous utility in period t will be chosen among the following functional forms that are commonly applied in business cycle models:

$$u(c_t, n_t) = \begin{cases} \frac{1}{1-\eta}\left[c_t^{1-\eta}(1-n_t)^{\theta(1-\eta)} - 1\right] & \text{I} \\ \frac{1}{1-\eta}\left[\left(c_t - \frac{\theta}{1+\nu}A_t n_t^{1+\nu}\right)^{1-\eta} - 1\right] & \text{II} \\ \ln c_t - \theta n_t & \text{III} \\ \ln c_t + \frac{\theta(1-n_t)^{1-\gamma}}{1-\gamma} & \text{IV} \end{cases} \tag{2}$$

The functions I,III, and IV meet the requirements of King, Plosser, and Rebelo (1988), p.292 that allow for a balanced growth path in the presence of exogenous labor augmenting technical progress A_t.[1] If one uses specification II a balanced growth path exists only if one is willing to assume that the disutility of work is proportional to A_t.[2] We use the common parameter θ to ensure that the fraction of working hours per worker in total hours available equals the respective empirical magnitude.

The households receives income from capital k_t and labor n_t. The budget constraint in period t is given by:

$$k_{t+1} = (1 + r_t)k_t + w_t n_t - c_t, \tag{3}$$

where r_t and w_t denote the real interest rate and the wage rate, respectively.

Firms are of measure one and produce output with labor N_t and capital K_t with constant returns to scale according to:

$$Y_t = Z_t F(K_t, A_t N_t) = Z_t (A_t N_t)^{\alpha} K_t^{1-\alpha}, \tag{4}$$

where A_t denotes the level of labor augmenting technical progress that grows deterministically at the rate $a - 1 \geq 0$:

$$A_{t+1} = a A_t. \tag{5}$$

Total factor productivity Z_t follows the stochastic AR(1)-process:

$$\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t, \tag{6}$$

[1] Function I is the standard functional type. Function III models indivisible labor as in Hansen (1985) and is able to explain the fact that total hours and the number of employed workers is much more variable than individual working hours. Function IV, finally, is used by Castañeda et al. (2004) in their work on heterogeneous agent economies. For this type of the utility function, working hours vary less with individual productivity and are in better accordance with empirical observations.

[2] This functional form is suggested by Greenwood et al. (1998) and has the attractive feature that there is no wealth effect on the labor supply decision. Hence, richer households do not supply less labor ceteris paribus.

where ϵ is a serially uncorrelated, normally distributed random variable with mean 0 and variance σ^2.

In a factor market equilibrium, factors are rewarded with their marginal products:

$$r_t = \frac{\partial Z_t F(K_t, N_t)}{\partial K_t} - \delta, \tag{7}$$

$$w_t = \frac{\partial Z_t F(K_t, N_t)}{\partial N_t}. \tag{8}$$

Capital depreciates at rate δ.

In a competitive equilibrium, aggregate variables equal individual variables, $k_t = K_t$, $n_t = N_t$, $c_t = C_t$, and the economy-wide resource constraint is given by $Y_t = K_{t+1} - (1-\delta)K_t + C_t$. The solution of the model consists of policy functions of the household for consumption $c(k_t, Z_t)$, labor supply $l(k_t, Z_t)$ and next-period capital $k'(k_t, Z_t)$ that cannot be solved analytically in the presence of elastic labor supply.

3 Computation of Solutions

The solution of the model satisfies the following set of conditions, where – different from the previous usage – lower case variables are scaled by A_t, i.e., $x_t \equiv X_t/A_t$, except for the shadow price of wealth Λ_t, where $\lambda_t := \Lambda_t A_t^{\eta}$:

$$\frac{\partial u(c_t, N_t)}{\partial c_t} = \lambda_t, \tag{9a}$$

$$\frac{\partial u(c_t, N_t)}{\partial N_t} = \alpha \lambda_t Z_t N_t^{\alpha-1} k_t^{1-\alpha}, \tag{9b}$$

$$ak_{t+1} = Z_t N_t^{\alpha} k_t^{1-\alpha} + (1-\delta)k_t - c_t, \tag{9c}$$

$$\lambda_t = \beta a^{-\eta} E_t \lambda_{t+1} (1 - \delta + (1-\alpha) Z_{t+1} N_{t+1}^{\alpha} k_{t+1}^{-\alpha}) \tag{9d}$$

We solved this model with five different methods.[3] The *value function iteration method* iterates to convergence over:

$$v^{s+1}(k_t, Z_t) := \max_{k_{t+1}, N_t} u(Z_t N_t^{\alpha} k_t^{1-\alpha} + (1-\delta)k_t - ak_{t+1}, N_t) + \beta a^{1-\eta} E_t v^s(k_{t+1}, Z_{t+1}).$$

Given a pair (k_t, k_{t+1}) we must solve the first order condition for optimal labor supply. Depending on the type of utility function chosen, this requires the solution of the

[3] A detailed description of these methods is provided by Heer/Maußner (2004). The Fortran programs for all five methods can be downloaded from Alfred Maußners homepage 'http://www.wiwi.uni-augsburg.de/vwl/maussner/'.

following (implicit) equation:

$$0 = Z_t N_t^{\alpha} k_t^{1-\alpha} + (1-\delta)k_t - ak_{t+1} - (\alpha/\theta)(1-N_t)Z_t N_t^{\alpha-1} k_t^{1-\alpha}, \tag{I}$$

$$0 = N_t^{1+\nu-\alpha} - (\alpha/\theta)Z_t k_t^{1-\alpha}, \tag{II}$$

$$0 = Z_t N_t^{\alpha} k_t^{1-\alpha} + (1-\delta)k_t - ak_{t+1} - (\alpha/\theta)Z_t N_t^{\alpha-1} k_t^{1-\alpha}, \tag{III}$$

$$0 = Z_t N_t^{\alpha} k_t^{1-\alpha} + (1-\delta)k_t - ak_{t+1} - (\alpha/\theta)(1-N_t)^{\gamma} Z_t N_t^{\alpha-1} k_t^{1-\alpha}. \tag{IV}$$

It can be shown that each of this equations has a unique solution $N \in [0,1]$ given reasonable values of (k_t, k_{t+1}).

To perform the iterations, we use a discrete version of the model: we approximate the AR(1) process for the technological shock Z_t by a finite Markov chain (see Tauchen, 1986) of 9 elements. The difference between the smallest and the largest value of $\ln z$ is three times the size of the unconditional standard deviation of (6). The upper (lower) bound of the capital stock equals the value that the deterministic counterpart of the model would approach if Z_t would equal its maximum (minimum) value all the time. We choose a grid of 5,000 equal spaced points between these bounds.[4]

The *extended deterministic path method* assumes that after a shock in period t no further shock occurs and computes the dynamics for the next T periods. Therefore, it has to solves a set of $2T-1$ equations in the unknowns $N_{t+s}, k_{t+s+1}, s = 0, 2, T-1$ obtained from (9a) to (9d) assuming that k_{t+T} is equal to the respective stationary solution of the deterministic counterpart of the model. From this solution only N_t and k_{t+1} are retained. Then another shock is drawn and the respective systems of equations is solved for N_{t+1} and k_{t+2} and so forth. The accuracy of the solution depends upon T. We found that $T = 150$ gives a very accurate solution, yet at the cost of long computational time.

The log-linear method obtains a linear approximation of the model in the vicinity of the balanced growth path of its deterministic counterpart. This linear rational expectations model can be solved by, e.g., applying the method of Blanchard and Kahn (1980), (see King, Plosser, and Rebelo, 1988) or of King and Watson (2002).

The *parameterized expectations approach* approximates the rhs of (9d) by a polynomial

[4] We also computed the model for a grid of 50,000 equally spaced points and did not find any difference in the solution.

in (Z, k).[5] We use a simple, complete, exponential polynomial of second degree,[6]

$$\psi(\boldsymbol{\gamma}, \ln Z, \ln k) := \exp(\gamma_1 + \gamma_2 \ln Z + \gamma_3 \ln k + \gamma_4 (\ln z)^2 + \gamma_5 (\ln k)^2 + \gamma_6 \ln z \ln k). \quad (11)$$

The vector of parameters $\boldsymbol{\gamma}$ is determined as solution to a non-linear set of equations. This system depends itself on a long series of points obtained from iterations over

$$\lambda_t = \psi(\boldsymbol{\gamma}, \ln Z_t, \ln k_t), \quad (12a)$$

$$\frac{\partial u(c_t, N_t)}{\partial c_t} = \lambda_t, \quad (12b)$$

$$\frac{\partial u(c_t, N_t)}{\partial N_t} = \lambda_t \alpha Z_t N_t^{\alpha-1} k_t^{1-\alpha}, \quad (12c)$$

$$a k_{t+1} = Z_t N_t^{\alpha} k_t^{1-\alpha} + (1-\delta) k_t - c_t, \quad (12d)$$

where Z_t is obtained from (6) using a random number generator that provides pseudo normally distributed innovations ϵ_t. Given λ_t, equations (12b) and (12c) can be reduced to an equation that determines N_t given (Z_t, k_t). Again, this equation depends upon the choice of utility function and is given by

$$0 = (1 - N_t)^{\theta(1-\eta)/\eta - 1} - (\alpha/\theta) \lambda_t^{1/\eta} Z_t N_t^{\alpha-1} k_t^{1-\alpha}, \quad \text{(I')}$$

$$0 = N_t^{1+\nu-\alpha} - (\alpha/\theta) Z_t k_t^{1-\alpha}, \quad \text{(II')}$$

$$0 = N_t^{1-\alpha} - (\alpha/\theta) \lambda_t Z_t k_t^{1-\alpha}, \quad \text{(III')}$$

$$0 = 1 - (\alpha/\theta)(1-N)^{\gamma} \lambda_t Z_t N_t^{\alpha-1} k_t^{1-\alpha} \quad \text{(IV')}$$

Given N_t it is easy to solve for c_t. For initial values of the parameters and the simulated time series, we can compute the least squares of the residuals $R(\ln Z_t, \ln k_t) = C_{t+1}^{-\eta} - \psi(\boldsymbol{\gamma}, \ln Z_t, \ln k_t)$. The parameter vector $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_6)$ is the solution of the following system of non-linear equations:

$$0 = \frac{-2}{T} \sum_{t=0}^{T-1} R(\ln Z_t, \ln k_t) \frac{\partial \psi(\boldsymbol{\gamma})}{\partial \gamma_i}, \quad i = 1, 2 \ldots, 6. \quad (14)$$

The crucial step in applying this algorithm is to find acceptable starting values for the non-linear equations solver. We use a genetic search algorithm to perform this task.[7]

[5] The method of parameterized expectations can be interpreted as a special case of the more general class of projections methods as pointed out by Judd (1996). Furthermore, he emphasizes that the approach was originally developed by Williams and Wright (1982,1984,1991).

[6] See, for example, den Haan and Marcet (1994).

[7] In particular, we applied two different specifications of the genetic search algorithm. In our first specification, we follow Duffy and McNelis (2001). Yet, different from this paper, our fitness criterium

To reduce computational time this algorithm operated over short time series of 4,000 points, say. When a solution was found, we used this as starting values for systems of non-linear equations based on successively longer time series. Usually one additional step with 10,000 observations was necessary to get the final solution based on 50,000 observations.

In the case of our German set of parameter values this strategy always proved successful. Yet, in the case of the US set of parameters the search strategy sometimes failed, especially in the cases of utility function (III) and (IV). Fortunately, an alternative approach worked: we used the policy function of the log-linear solution to compute time series of 500 points, say, for λ_t, k_t, and Z_t. Then, we regressed λ_t on $\psi(\boldsymbol{\gamma}, k_t, Z_t)$ using non-linear least squares.[8] This gives a starting value for solving (14), which – as above – was based on a much longer time series of 50,000 observations. Though this method is much faster than the one based on stochastic search, it did not work in all the cases considered. For instance, we were not able to find acceptable starting values in the case of utility function (II) for the US parameter set.

As in the parameterized expectations approach, the *Galerkin projection method* rests on the approximation of the rhs of (9d). Contrary from the previous method, however, we use a second degree product base, Chebyshev exponential polynomial in $(\ln Z, \ln k)$ as the approximating function. In addition, we do not apply Monte Carlo simulation in order to fit the parameterized function, but compute the weighted residual over the state space with the help of Galerkin methods, i.e. using the weight one and the Chebyshev polynomials as projection functions.[9]

The critical step in the projection method is the choice of a bounded interval for the capital stock so that the algorithm always stays within this interval. We solve this problem in the following way that, to the best of our knowledge, has not been emphasized in the previous literature on projection methods: We simply use a larger interval for the approximation that embeds the latter but integrate the residual over the smaller only. More exactly, the conditional expectation is approxi-

is the minimal absolute value of the rhs of (14). Secondly, we use a different selection scheme and employ a larger set of cross-over operators as Duffy and McNelis (2001) do. In particular, we use stochastic universal sampling as in Mitchell (1996). The genetic search algorithms are described in more detail in Heer and Maußner (2004). None of the two algorithms is found to dominate the other in terms of speed across all calibrations and parameterizations in our model.

[8] Since the error term in $\lambda_t - \psi(\boldsymbol{\gamma}, k_t, Z_t)$ is additive it is inappropriate to regress $\ln(\lambda_t)$ on $\ln(k_t)$ and $\ln(Z_t)$ by means of ordinary least squares.

[9] For a more detailed description of the projection methods, see Judd (1992, 1998) and McGrattan (1999).

mated over the interval $[\ln(k_{min}), \ln(k_{max})]$, while the Galerkin integral is computed over $[\ln(\tilde{k}_{min}), \ln(\tilde{k}_{max})] \subset [\ln(k_{min}), \ln(k_{max})]$. While our program does not find a solution for $\tilde{k}_{min} = k_{min}$ and $\tilde{k}_{max} = k_{max}$, it converges for $\tilde{k}_{min} = 1.1 \cdot k_{min}$ and $\tilde{k}_{max} = k_{max}/1.1$.[10] The basic reason for this behavior of the algorithm is that for initial values of γ in the parameterized function, $\ln k_{t+1}$ might happen to fall outside the interval $[\ln(k_{min}), \ln(k_{max})]$ if we choose $[\ln(\tilde{k}_{min}), \ln(\tilde{k}_{max})]$ too wide. In this case, however, we get highly inaccurate solutions for the policy functions. An additional device helped us to reduce the second interval $[\ln(\tilde{k}_{min}), \ln(\tilde{k}_{max})]$ further. We also implemented a zero constraint on investment from which we know that it never binds in our calibrated model (since the standard deviation of the innovations of (6) is too small). This step requires a third set of equations to be solved for N_t if the constraint binds, i.e., if $c_t = y_t := Z_t N_t^{\alpha} k_t^{1-\alpha}$ holds:

$$0 = N_t - \frac{\alpha/\theta}{1 + (\alpha/\theta)}, \tag{I"}$$

$$0 = N_t^{1+\nu-\alpha} - (\alpha/\theta) Z_t k_t^{1-\alpha}, \tag{II"}$$

$$0 = N_t - (\alpha/\theta), \tag{III"}$$

$$0 = (1 - N_t)^{-\gamma} - (\alpha/\theta)\frac{1}{N_t}. \tag{IV"}$$

It can be shown that in this case $\mu_t := \lambda_t - \psi(\boldsymbol{\gamma}, \ln Z_t, \ln k_t) > 0$ holds, where μ_t denotes the (scaled) Lagrange multiplier of the constraint $ak_{t+1} \geq (1-\delta)k_t$.

Again, starting values for the non-linear equations solver are supplied by a genetic search routine based on a first degree polynomial. In two steps we increase the degree to the final second degree polynomial. Alternatively, we use the solution for λ_t from the log-linearization algorithm and approximate it by a Chebyshev polynomial of second degree.

4 Results

We evaluate the methods with regard to 1) 1st and 2nd moments of employment, output, investment, consumption, and 2) computational time.[11] As we are aiming to assess the suitability of the different methods for business cycle models more generally, we analyze two different sets of calibration parameters. The first set reflects parameters

[10] For the German calibration, we even had to shorten the interval to $[1.2 \cdot \ln(k_{min}), \ln(k_{max})/1.2]$.

[11] In addition, the den Haan/Marcet (1994) statistic and the policy functions are displayed in the appendix.

commonly applied in the business cycle study of the postwar US economy. The second set of parameters is calibrated with the help of German postwar data prior to unification in 1989.[12] Importantly, these two economies are characterized by different institutional settings. In particular, the German capital market is less competitive as many banks are state-owned or subsidized by the state. As a consequence, capital depreciates less rapidly in Germany as capital utilization is lower. Furthermore, the capital coefficient of output, $1-\alpha$, is lower in Germany (0.27) than in the US (0.36). One possible reason may be the presence of unions. Secondly, labor markets are more rigid in Germany and the social security system is more generous. As a consequence, average labor supply is lower in Germany as well.

Table 1
Model Calibration

German Calibration		US Calibration	
Production	Preferences	Production	Preferences
a=1.005	β=0.994	a=1.0055	β=0.99
α=0.73	η=1.0	α=0.64	η=1.00
δ=0.011	ν=5.0	δ=0.025	ν=3.33
ρ=0.90	γ=33.5	ρ=0.95	γ=7.00
σ=0.072	N=0.13	σ=0.0072	N=0.33

For the US economy, we use the set of parameters displayed in Table 1. Except for the rate of per capita output growth they are in accordance with Hansen (1985). The average quarterly growth rate of the US economy exceeds the one of the German economy, $a' = 1.0050$, and amounts to $a = 1.0055$ during 1960-2002 on average. The estimates of the Frisch intertemporal labor supply elasticity $\eta_{n,w}$ implied by microeconometric studies and the implied values of γ and ν vary considerably. MaCurdy (1981) and Altonji (1986) both use PSID data in order to estimate values of 0.23 and 0.28, respectively, while Killingsworth (1983) finds an US labor supply elasticity equal to $\eta_{n,w} = 0.4$.[13] We will use the conservative estimate $\eta_{n,w} = 0.3$ and, accordingly, apply the values $\nu = 3.33$ and $\gamma = 7.0$ in utilities III and IV, respectively.[14] For Germany, we use the

[12] A detailed description of this calibration is provided in Chapter 1 of Heer and Maussner (2004).

[13] Domeij and Floden (2001) argue that these estimates are biased downward due to the omission of borrowing constraints.

[14] Greenwood et al. (1988) even apply a value $\eta_{n,w} = 1.7$ corresponding to $\nu = 0.6$ in their study,

same set of parameters as in Heer and Maußner (2004) (see Table 1).[15] In addition, we use the value $\eta_{n,w} = 0.2$ following Heer and Trede (2003) implying $\nu = 5.0$ and $\gamma = 33.5$.

Our estimation results for the US calibration are displayed in tables 3-6 for the utility functions I-IV, respectively. Consider table 3. The first column presents the variable, the next three columns display the standard deviation of the variable, the variable's correlation with output, and its autocorrelation, respectively. As is obvious from the inspection of the table, the minimum and maximum values obtained in the computation with either methods are close to the mean values. As one exception, the volatility of investment as computed by the different methods varies considerably. The spread between the minimum and maximum value, 3.54 and 4.03, amounts to 13.3%, but is still below the standard deviation of this moment that is displayed in the row below these numbers. These observations also hold for the three other utilities II, III, and IV displayed in tables 4-6, respectively.

Notice further that the moments are very much alike for the four different utility functions. If you consider the mean correlation of investment I, consumption C, employment N, and wages W with output Y, displayed in the sixth columns of tables 3-6, the divergence is small. Similarly, the mean autocorrelations of Y, I, C, N, and W, that are displayed in column 9 in each table are hardly discernable. There are only two marked differences between these four parameterizations: 1) the volatility of output (and, similarly, investment) is more pronounced for utilities I and III, and, 2) employment is much more volatile relative to both output and wages in cases I and III as well. Consequently, utility functions I and III are often applied in business cycle research in order to generate the empirically observed (rather high) volatility of output and employment, while utilities II and IV are often applied in heterogenous-agent models in order to replicate the (rather low) dispersion of hours worked among the workers.

The accuracy of the five different computational methods for the German parameterization does not significantly differ from the one for the US calibration as is obvious from comparison of tables 7-10 with tables 3-6, respectively. Again, the only moment that displays economically significant variation among the different methods applied is the standard deviation of investment. Also in the German case, the standard deviation of this moment is higher than the difference between the lowest and the highest computed value. In summary, all our qualitative results with regard to the computation

while Castañeda et al. (2004) use $\gamma = 5.5$.

[15] In particular, we found no compelling evidence, that the intertemporal elasticity of substitution is different between the US and Germany.

also hold in the model calibrated for the German economy. With regard to its business cycle features, the two economies are also very much alike with one exception. If you compare tables 3 and 7, in particular, you notice that the volatility of output and investment (consumption) is higher (lower) in Germany than the one in the US. As Germans value leisure more than US households (and hence work less in steady state) in this particular case, the relative weight $1/\theta$ of consumption in utility is smaller than in the US for the functional form I of utility. As a consequence, German households have to vary their intertemporal labor supply to a larger extent than US households in order to smooth intertemporal utility and, for this reason, the volatility of hours worked relative to consumption is higher in Germany than in the US. Higher volatility of labor also results in higher volatility of investment and output in Germany.

Table 2
Computational Time

Method	Time
Value function iteration	34 minutes 2.65 seconds
Extended deterministic path	2 hours 34 minutes 29.88 seconds
Parameterized expectations	
- one step with search	36 minutes 29.59 seconds
- three steps with search	3 minutes 31.25 seconds
- given initial value (utility III)	17.17 seconds
Galerkin projection	
- four steps	23 minutes 32.37 seconds
- given initial value (utility IV)	4 minutes 56.08 seconds
Log-linear approximation	0.48 seconds

Notes:
If not otherwise mentioned, the results are based on the solution for utility function (I) and the German parameter set. The program run on a 3.2 Gigahertz Pentium IV personal computer. The program is written in Fortran 95 and compiled with the Compaq Digital Fortran developer environment. Computational time comprises the time needed to compute the solution and to compute the summary statistics from 500 simulations. The stochastic search routine rests on a population of 50 candidate solution vectors and iterates over 100 generations.

With respect to computational time, the algorithms can be ordered as follows: 1) Log-linear approximation is by way the fastest method. Using a 3.2 Gigahertz Pentium IV personal computer it takes less than a second to compute the policy function and to run 500 simulations over 60 periods each. The further ranks depend on our measurement

of computational time and our strategy to find acceptable starting values for the nonlinear equations solver. If we use initial values obtained from the loglinear solution method 2) the parameterized expectations approach (PEA) is much faster than 3) the Galerkin projection method (GPM) and both need less time than 4) the value function iteration method (VIM). The PEA needs less than twenty seconds whereas the GPM consumes almost five minutes of computer time. The reason is that the Galerkin projection must evaluate a sum over 2500 elements (we use 50 nodes for Z and 50 nodes for k to compute the respective double integral), where each summand (namely the approximation of the conditional expectation) is itself a sum over 100 elements. The ranking between the PEA and the GPM does not change, if we use stochastic search. As can be seen from Table 2 it pays in the case of the parameterized expectations approach if we do not operate on the final long time series of 50,000 elements, but start with a much shorter one and use this solution to initialize the computation of the next vector that is based on a longer time series. This approach reduces computation time by a factor of ten. If one uses brute force and operates over 50,000 elements the PEA takes a bit more time than the VIM. One must be either very lucky or very patient if one wants to get the final solution from the GPM in one step. Neither were we lucky nor patient enough. So we started with a 2×2 polynomial and 10 nodes for Z and k. If the search routine was successful, we used this solution and increased the number of nodes and the degree in a few steps. Compared to the stepwise parameterized expectations approach, the Galerkin method is about 5 to six times slower. Since it seems always possible to shorten the search process, the VIM gets rank 4. In any case, 5) the extended deterministic path method is by far the slowest. If you do not own such a fast machine as we do (2.5 hours) you should compute the solution over night, as it may well take six to eight hours.[16] Finally, we like to mention that the solution in the case of utility functions II and III, where an analytical solution for N given λ, k, and Z is available, requires substantially less time. For instance, the extended path method finds the solution in about 50 minutes.

[16] If you do your programming work with GAUSS you must multiply this time by a factor of six.

Table 3

Summary Statistics: Utility I, US Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	1.24	1.28	1.31	1.00	1.00	1.00	0.65	0.65	0.67
Standard Dev.	0.21	0.22	0.23	0.00	0.00	0.00	0.10	0.10	0.11
Investment	3.54	3.74	4.03	0.99	0.99	0.99	0.63	0.64	0.65
Standard Dev.	0.60	0.65	0.73	0.00	0.00	0.00	0.10	0.11	0.11
Consumption	0.38	0.41	0.46	0.86	0.89	0.91	0.75	0.76	0.78
Standard Dev.	0.08	0.09	0.10	0.02	0.03	0.04	0.09	0.09	0.09
Hours	0.58	0.62	0.67	0.98	0.98	0.98	0.63	0.64	0.65
Standard Dev.	0.10	0.11	0.12	0.01	0.01	0.01	0.10	0.11	0.11
Real Wage	0.67	0.68	0.71	0.98	0.98	0.98	0.68	0.69	0.70
Standard Dev.	0.11	0.12	0.13	0.01	0.01	0.01	0.10	0.10	0.10

Notes:

s_x is the standard deviation of variable x listed in the first column. s_{xy} is the correlation of x with output. s_x denotes the first order autocorrelation of x. All moments are averages over 500 simulations. The length of the respective time series was 60 periods in each simulation. All time series are HP-filtered with weight $\lambda = 1600$. The columns labeled 'Min', 'Mean', and 'Max', respectively, display the minimum, the mean, and the maximum from the five different methods. The rows labeled 'Std. Dev.' give the standard deviation of the respective moment, computed from the 500 simulated time series.

Table 4

Summary Statistics: Utility II, US Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	1.03	1.03	1.05	1.00	1.00	1.00	0.66	0.66	0.67
Standard Dev.	0.18	0.18	0.18	0.00	0.00	0.00	0.10	0.10	0.10
Investment	2.52	2.60	2.76	0.99	0.99	0.99	0.64	0.65	0.66
Standard Dev.	0.42	0.44	0.47	0.00	0.00	0.00	0.10	0.11	0.10
Consumption	0.45	0.48	0.50	0.95	0.97	0.98	0.69	0.70	0.71
Standard Dev.	0.08	0.09	0.09	0.01	0.01	0.01	0.10	0.10	0.10
Hours	0.24	0.24	0.24	1.00	1.00	1.00	0.66	0.66	0.67
Standard Dev.	0.04	0.04	0.04	0.00	0.00	0.00	0.10	0.10	0.10
Real Wage	0.79	0.80	0.81	1.00	1.00	1.00	0.66	0.66	0.67
Standard Dev.	0.13	0.14	0.14	0.00	0.00	0.00	0.10	0.10	0.10

Notes: See table 3.

Table 5

Summary Statistics: Utility III, US Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	1.63	1.68	1.77	1.00	1.00	1.00	0.65	0.65	0.66
Standard Dev.	0.26	0.27	0.29	0.00	0.00	0.00	0.10	0.10	0.11
Investment	4.80	5.02	5.56	0.99	0.99	0.99	0.63	0.64	0.65
Standard Dev.	0.78	0.84	0.90	0.00	0.00	0.00	0.10	0.11	0.11
Consumption	0.49	0.51	0.52	0.84	0.87	0.89	0.76	0.77	0.79
Standard Dev.	0.10	0.11	0.11	0.03	0.03	0.04	0.08	0.09	0.09
Hours	1.19	1.26	1.40	0.98	0.98	0.98	0.63	0.64	0.64
Standard Dev.	0.19	0.21	0.22	0.01	0.01	0.01	0.10	0.11	0.11
Real Wage	0.49	0.51	0.52	0.84	0.87	0.89	0.76	0.77	0.79
Standard Dev.	0.10	0.11	0.11	0.03	0.03	0.04	0.08	0.09	0.09

Notes: See table 3.

Table 6

Summary Statistics: Utility IV, US Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	0.96	0.98	1.00	1.00	1.00	1.00	0.66	0.66	0.67
Standard Dev.	0.16	0.17	0.17	0.00	0.00	0.00	0.10	0.10	0.10
Investment	2.67	2.79	3.00	0.99	0.99	0.99	0.64	0.65	0.65
Standard Dev.	0.44	0.46	0.49	0.00	0.00	0.00	0.10	0.10	0.10
Consumption	0.32	0.35	0.37	0.86	0.91	0.93	0.74	0.75	0.75
Standard Dev.	0.07	0.07	0.07	0.02	0.02	0.03	0.09	0.09	0.10
Hours	0.14	0.15	0.17	0.97	0.98	0.98	0.63	0.65	0.65
Standard Dev.	0.02	0.02	0.03	0.01	0.01	0.01	0.10	0.10	0.10
Real Wage	0.82	0.83	0.84	1.00	1.00	1.00	0.66	0.67	0.67
Standard Dev.	0.14	0.14	0.14	0.00	0.00	0.00	0.10	0.10	0.10

Notes: See table 3.

Table 7

Summary Statistics: Utility I, German Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	1.88	1.90	1.92	1.00	1.00	1.00	0.61	0.62	0.63
Standard Dev.	0.28	0.31	0.32	0.00	0.00	0.00	0.11	0.11	0.11
Investment	8.55	8.72	9.04	0.99	0.99	1.00	0.60	0.62	0.63
Standard Dev.	1.35	1.40	1.45	0.00	0.00	0.00	0.11	0.11	0.11
Consumption	0.29	0.32	0.34	0.75	0.81	0.83	0.76	0.77	0.79
Standard Dev.	0.06	0.07	0.08	0.04	0.04	0.05	0.08	0.09	0.09
Hours	1.41	1.44	1.48	0.99	0.99	0.99	0.60	0.62	0.63
Standard Dev.	0.22	0.23	0.24	0.00	0.00	0.00	0.11	0.11	0.11
Real Wage	0.47	0.50	0.52	0.93	0.94	0.95	0.68	0.68	0.69
Standard Dev.	0.08	0.09	0.10	0.02	0.02	0.02	0.10	0.10	0.10

Notes: See table 3.

Table 8

Summary Statistics: Utility II, German Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	0.95	0.98	1.00	1.00	1.00	1.00	0.62	0.63	0.64
Standard Dev.	0.14	0.16	0.17	0.00	0.00	0.00	0.10	0.11	0.11
Investment	3.83	3.87	3.90	0.99	0.99	1.00	0.60	0.62	0.64
Standard Dev.	0.57	0.62	0.65	0.00	0.00	0.00	0.11	0.11	0.11
Consumption	0.28	0.30	0.31	0.91	0.95	0.96	0.62	0.67	0.69
Standard Dev.	0.04	0.05	0.06	0.01	0.02	0.02	0.10	0.10	0.11
Hours	0.16	0.16	0.17	1.00	1.00	1.00	0.62	0.63	0.64
Standard Dev.	0.02	0.03	0.03	0.00	0.00	0.00	0.10	0.11	0.11
Real Wage	0.79	0.82	0.83	1.00	1.00	1.00	0.62	0.63	0.64
Standard Dev.	0.12	0.13	0.14	0.00	0.00	0.00	0.10	0.11	0.11

Notes: See table 3.

Table 9

Summary Statistics: Utility III, German Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	2.34	2.39	2.41	1.00	1.00	1.00	0.61	0.62	0.63
Standard Dev.	0.35	0.38	0.41	0.00	0.00	0.00	0.10	0.11	0.11
Investment	10.62	11.06	11.57	0.99	0.99	1.00	0.60	0.62	0.63
Standard Dev.	1.74	1.78	1.84	0.00	0.00	0.00	0.10	0.11	0.11
Consumption	0.34	0.38	0.39	0.74	0.79	0.83	0.76	0.78	0.79
Standard Dev.	0.07	0.08	0.09	0.05	0.05	0.05	0.08	0.09	0.09
Hours	2.03	2.10	2.17	0.99	0.99	0.99	0.60	0.62	0.63
Standard Dev.	0.31	0.33	0.35	0.00	0.00	0.00	0.10	0.11	0.11
Real Wage	0.34	0.38	0.39	0.74	0.79	0.83	0.76	0.78	0.79
Standard Dev.	0.07	0.08	0.09	0.05	0.05	0.05	0.08	0.09	0.09

Notes: See table 3.

Table 10

Summary Statistics: Utility IV, German Calibration

Variable	s_x			s_{xy}			r_x		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Output	0.93	0.96	0.97	1.00	1.00	1.00	0.61	0.63	0.64
Standard Dev.	0.13	0.15	0.16	0.00	0.00	0.00	0.10	0.10	0.11
Investment	4.22	4.24	4.25	0.99	0.99	1.00	0.61	0.62	0.63
Standard Dev.	0.61	0.68	0.71	0.00	0.00	0.00	0.10	0.10	0.11
Consumption	0.18	0.20	0.20	0.75	0.84	0.87	0.67	0.73	0.75
Standard Dev.	0.03	0.04	0.04	0.03	0.04	0.05	0.09	0.09	0.10
Hours	0.13	0.13	0.13	0.99	0.99	0.99	0.60	0.62	0.63
Standard Dev.	0.02	0.02	0.02	0.00	0.00	0.00	0.10	0.11	0.11
Real Wage	0.80	0.83	0.84	1.00	1.00	1.00	0.61	0.63	0.64
Standard Dev.	0.12	0.13	0.14	0.00	0.00	0.00	0.10	0.10	0.11

Notes: See table 3.

5 Conclusion

This paper has shown that several numerical methods can be applied in order to study the standard business cycle model. Using either value function iteration, log-linearization, deterministic extended path, parameterized expectations, or projection methods basically results in the same values for the first and second moments of the variables that the business cycle researcher is most interested in, ie output, employment, investment, consumption, and wages. Log-linearization, of course, is very easy to implement and by far the fastest method. Furthermore, the solution from this method can often be successfully applied as an initial value for more sophisticated non-linear methods like parameterized expectations or projection methods where the computation of a good initial value with genetic search algorithm or homotopy methods may become very time-consuming, as may be the case in more complex multi-dimensional state-space applications. Our results, therefore, suggest that the researcher may benefit from using log-linearization methods in the first place and, possibly, also use non-linear methods such as parameterized expectations or projection methods to improve the accuracy of the computation in more non-linear problem that may arise, for example, in the presence of binding constraints, e.g. a non-negativity constraint on investment or a constraint on the maximum number of working hours.

Our work also emphasized an important detail in the application of projection methods for the approximation of polynomial solution function in more complex dynamic models. In our example of the stochastic growth model with flexible labor supply, standard projection methods failed to converge even for good initial values that were computed with the help of the log-linear solution. We found that the basic reason for this observation is the poor approximation of functions with Chebyshev polynomials outside the approximation interval. As a solution to this problem, we suggest to use a wider interval for the appoximation of the function than for the integration over the residual function.

References

Altonij, J.G., 1986, Intertemporal Substitution in Labor Supply: Evidence from Micro Data, *Journal of Political Economics*, vol. 94, S17-S215.

Blanchard, O. J. and Ch. M. Kahn, 1980, The Solution of Linear Difference Models Under Rational Expectations, *Econometrica*, vol. 48, 1305-1311.

Castañeda, A., J. Díaz-Giminénez, J.-V. Ríos-Rull, 2004, Accounting for the US Earnings and Wealth Inequality, *Journal of Political Economy*, forthcoming.

Christiano, L.J., and J.D.M. Fisher, 2000, Algorithms for solving dynamic models with occasionally binding constraints, *Journal of Economic Dynamics and Control*, vol. 24, 1179-1232.

Den Haan, W. J. and A. Marcet, 1994, Accuracy in Simulations, *Review of Economic Studies*, vol. 61, 3-17

Domeij, D., and M. Floden, 2001, The labor supply elasticity and borrowing constraints: Why estimates are biased, SSE/FEI working paper series in economics and finance, no. 480.

Duffy, J., and P.D. McNelis, 2002, Approximating and Simulating the Stochastic Growth Model: Parameterized Expectations, Neural Networks, and the Genetic Algorithm, *Journal of Economic Dynamics and Control*, vol. 25, 1273-1303.

Greenwood, J., Z. Hercowitz, and G.W. Huffman, 1998, Investment, capacity utilization, and the real business cycle, *American Economic Review*, vol. 78, 402-17.

Hansen, G., 1985, Indivisible Labor and the Business Cycle, *Journal of Monetary Economics*, vol. 16, 309-327.

Heer, B., and A. Maußner, 2004, *Dynamic General Equilibrium Modelling: Computational Methods and Applications*, Springer, Berlin, forthcoming.

Heer, B., and M. Trede, 2003, 'Efficiency and Distribution Effects of a Revenue-neutral Income Tax Reform', *Journal of Macroeconomics*, vol. 25, 87-107.

Judd, K.L., 1992, Projection methods for aggregate growth models, *Journal of Economic Theory*, vol. 58, 410-52.

Judd, K.L., 1996, Approximation, Perturbation, and Projection Methods in Economic Analysis, in: H.M. Amman, D.A. Kendrick, and J. Rust, eds., *Handbook of Computational Economics*, vol. I, Elsevier Science B.V., Amsterdam.

Judd, K.L., 1998, *Numerical Methods in Economics*, MIT Press: Cambridge, Ma.

Killingsworth, M.R., 1983, *Labor Supply*, Cambridge University Press, Cambridge, MA.

King, R. G., Ch. I. Plosser, and S. Rebelo, 1988, Production, Growth and Business Cycles I, The Basic Neoclassical Model, *Journal of Monetary Economics*, vol. 21, 195-232

King, R.G. and M.W. Watson, 2002, System Reduction and Solution Algorithms for Singular Linear Difference Systems under Rational Expectations, *Computational Economics*, vol. 20, 57-86

MaCurdy, T.E., 1981, An Empirical Model of Labor Supply in a Life-Cycle Setting, *Journal of Political Economy*, vol. 89, 1059-85.

McGrattan, E.R., 1999, Application of Weighted Residual Methods to Dynamic Economic Models, in: R. Marimon and A. Scott (Eds.), *Computational Methods for the Study of Dynamic Economies*, Oxford University Press: Oxford and New York, 114-142

Mitchell, M., 1996, *An introduction to genetic algorithms*, Harvard University Press, Cambridge, MA.

Tauchen, G., 1986, Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions, *Economics Letters*, vol. 20, 177-181.

Taylor, J.B., and H. Uhlig, 1990, Solving nonlinear stochastic growth models: A comparison of alternative solution methods, *Journal of Business and Economic Statistics*, vol. 8, 1-17.

Wright, B.D., and J.C. Williams, 1982, The Economic Role of Commodity Storage, *Economic Journal*, vol. 92, 596-614.

Wright, B.D., and J.C. Williams, 1984, The Welfare Effects of the Introduction of Storage, *Quarterly Journal of Economics*, vol. 99, 169-82.

Wright, B.D., and J.C. Williams, 1991, *Storage and Commodity Markets*, Cambridge University Press, Cambridge, UK.

Appendix

DM-Statistic One of the measures of the accuracy of the approximate solution is the statistic proposed by den Haan and Marcet (1994). It is a Wald test of the null that the residual

$$e_t := \beta a^{-\eta} \lambda_{t+1} (1 - \delta + (1 - \alpha) * Z_{t+1} N_{t+1}^{\alpha} k_{t+1}^{-\alpha}) - \lambda_t$$

cannot be predicted by past information. We use five lags of consumption, of working hours, and of the productivity shock to test this hypotheses. Towards this end we compute the residual from a time series of 3,000 points, regress it on a constant, c_{t-j}, N_{t-j}, Z_{t-j}, $j = 1, 2, \ldots, 5$ and compute the Wald statistic of the hypothesis that the 16-vector of coefficients $\underline{a}$ is not different from the zero vector. We repeat this experiment 500 times. Table 11 reports the fraction of simulations, in which the respective Wald statistic is below (above) the respective 2.5 (97.5) percentile of the $\chi^2(16)$-distribution. In a large sample, this fraction should be not very different from 0.025.

Table 11
DM Statistics for Selected Solutions, German Calibration

Method	Utility I		Utility II	
	Below	Above	Below	Above
Value function iteration	0.038	0.030	0.020	0.016
Parameterized expectations	0.018	0.024	0.026	0.036
Galerkin projection	0.030	0.024	0.014	0.016
Log-linear approximation	0.024	0.030	0.032	0.026

Notes:
The columns labeled Above (Below) display the fraction out of 500 simulations in which the DM-Statistic was below (above) the 2.5 (97.5) percentile of the $\chi^2(16)$ distribution.

Consumption Function Another measure of the accuracy of the solution is the policy function for consumption, which is a non-linear function of the stock of capital k and the level of the productivity shock Z. We use the consumption function from value function iteration as a benchmark and plot its graph against those found by the respective other methods. Since the extended path algorithm does not provide the consumption function directly, we approximate this function in the following way: we use the time paths computed from 500 simulations and project the solution for λ on the same polynomial used to compute the parameterized expectations function. Given this, we compute the policy function in the same way as the policy function from the PEA.

In each case the policy function is plotted over the range of k used in the value function iteration method.

Figure 1
Policy Functions in the Case of Utility Function I

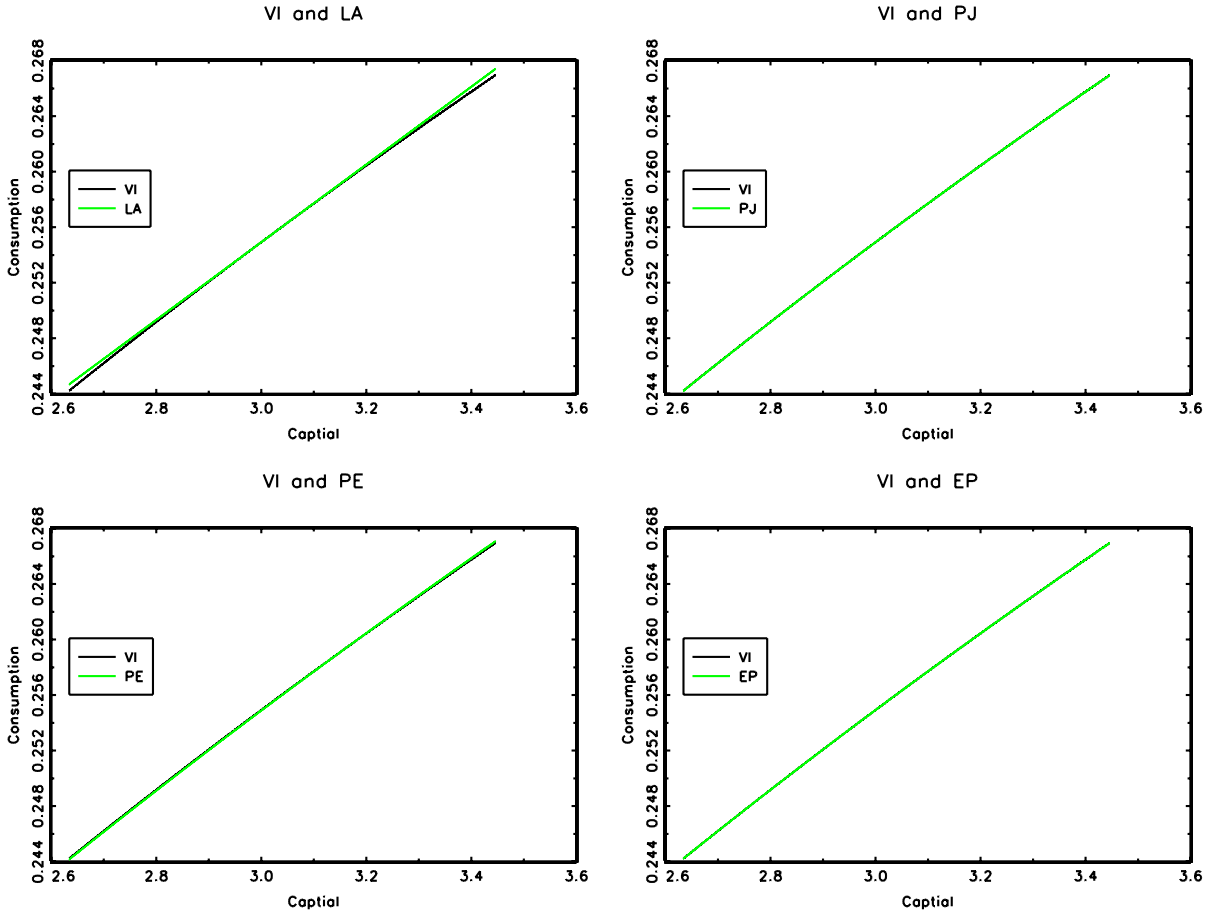
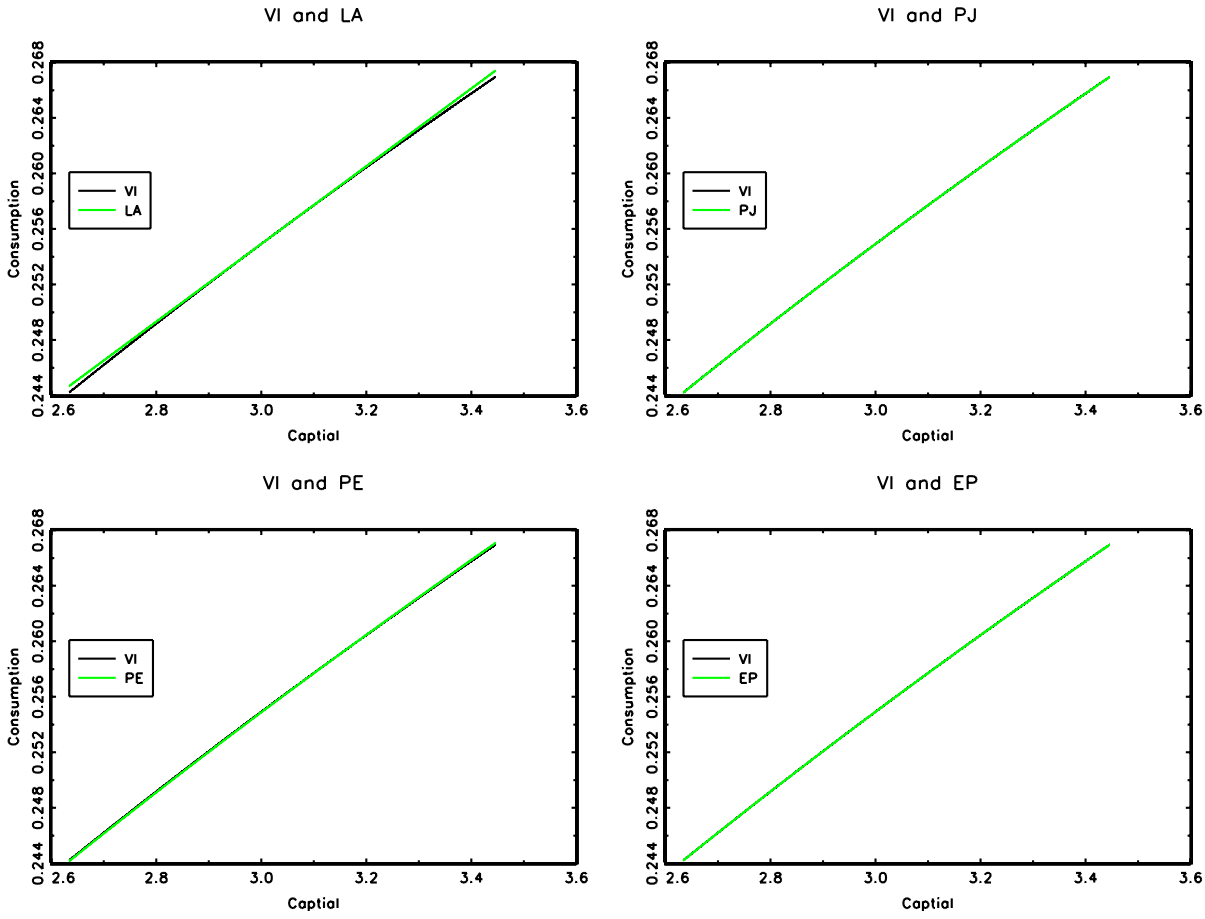


Figure 1 compares the policy functions obtained from the German calibration in the case of utility function I and Figure 2 does the same in the case of utility function II. In both figures Z is kept at the stationary value of $Z = 1$.

As can be seen from both figures the log-linear consumption function (LA) is flatter than the solution obtained via value function iteration (VI). Thus, for low as well as for large values of the capital stock, it does not provide a good approximation. Yet, since in the simulations the capital stock remains close to the stationary capital stock, this is of no consequence for the second moments. For large values of the capital stock the consumption function found from the parameterized expectations approach (PE) lies a bit above the VI function. Very good approximations are provided by both, the Galerkin projection (PJ) and the extended deterministic path (EP) method.

Figure 2
Policy Functions in the Case of Utility Function II



CESifo Working Paper Series

(for full list see www.cesifo.de)

1144 Sascha O. Becker, Samuel Bentolila, Ana Fernandes, and Andrea Ichino, Job Insecurity and Children's Emancipation, March 2004

1145 Pascalis Raimondos-Møller and Alan D. Woodland, Non-Preferential Trading Clubs, March 2004

1146 Robert Fenge and Matthias Wrede, EU Regional Policy: Vertical Fiscal Externalities and Matching Grants, March 2004

1147 Chi-Yung Ng and John Whalley, Geographical Extension of Free Trade Zones as Trade Liberalization: A Numerical Simulation Approach, March 2004

1148 Marc-Andreas Muendler, Trade, Technology, and Productivity: A Study of Brazilian Manufacturers, 1986-1998, March 2004

1149 Eugene Beaulieu, Vivek H. Dehejia, and Hazrat-Omar Zakhilwal, International Trade, Labour Turnover, and the Wage Premium: Testing the Bhagwati-Dehejia Hypothesis for Canada, March 2004

1150 Giorgio Brunello and Francesca Gambarotto, Agglomeration Effects on Employer-Provided Training: Evidence from the UK, March 2004

1151 S. Brock Blomberg, Gregory D. Hess, and Athanasios Orphanides, The Macroeconomic Consequences of Terrorism, March 2004

1152 Bodo Sturm and Joachim Weimann, Unilateral Emissions Abatement: An Experiment, March 2004

1153 Wolfgang Ochel, Welfare-to-Work Experiences with Specific Work-First Programmes in Selected Countries, March 2004

1154 Jan K. Brueckner and Eric Pels, European Airline Mergers, Alliance Consolidation, and Consumer Welfare, March 2004

1155 Aaron Tornell, Frank Westermann, and Lorenza Martínez, NAFTA and Mexico's Economic Performance, March 2004

1156 George Economides, Sarantis Kalyvitis, and Apostolis Philippopoulos, Do Foreign Aid Transfers Distort Incentives and Hurt Growth? Theory and Evidence from 75 Aid-recipient Countries, March 2004

1157 Robert Fenge and Volker Meier, Are Family Allowances and Fertility-related pensions Siamese Twins?, March 2004

1158 Bruno S. Frey, Simon Luechinger, and Alois Stutzer, Valuing Public Goods: The Life Satisfation Approach, March 2004

1159 Jerome L. Stein and Guay C. Lim, Asian Crises: Theory, Evidence, Warning-Signals, March 2004

1160 Romain Ranciere, Aaron Tornell, and Frank Westermann, Crises and Growth: A Re-Evaluation, March 2004

1161 Assaf Razin and Efraim Sadka, Transparency, Specialization and FDI, March 2004

1162 Ludger Woessmann, How Equal Are Educational Opportunities? Family Background and Student Achievement in Europe and the United States, March 2004

1163 B.M.S. van Praag and Barbara E. Baarsma, Using Happiness Surveys to Value Intangibles: The Case of Airport Noise, March 2004

1164 Aaron Tornell, Frank Westermann, and Lorenza Martínez, The Positive Link Between Financial Liberalization, Growth, and Crises, March 2004

1165 Helge Berger and Carsten Hefeker, One Country, One Vote? Labor Market Structure and Voting Rights in the ECB, March 2004

1166 Clemens Fuest and Martin Kolmar, A Theory of User-Fee Competition, March 2004

1167 Friedrich Schneider and Robert Klinglmair, Shadow Economies around the World: What Do We Know?, April 2004

1168 Horst Raff and Nicolas Schmitt, Exclusive Dealing and Common Agency in International Markets, April 2004

1169 M. Hashem Pesaran and Allan Timmermann, Real Time Econometrics, April 2004

1170 Sean D. Barrett, Privatisation in Ireland, April 2004

1171 V. Anton Muscatelli, Patrizio Tirelli and Carmine Trecroci, Can Fiscal Policy Help Macroeconomic Stabilisation? Evidence from a New Keynesian Model with Liquidity Constraints, April 2004

1172 Bernd Huber and Marco Runkel, Tax Competition, Excludable Public Goods and User Charges, April 2004

1173 John McMillan and Pablo Zoido, How to Subvert Democracy: Montesinos in Peru, April 2004

1174 Theo Eicher and Jong Woo Kang, Trade, Foreign Direct Investment or Acquisition: Optimal Entry Modes for Multinationals, April 2004

1175 Chang Woon Nam and Doina Maria Radulescu, Types of Tax Concessions for Attracting Foreign Direct Investment in Free Economic Zones, April 2004

1176 M. Hashem Pesaran and Andreas Pick, Econometric Issues in the Analysis of Contagion, April 2004

1177 Steinar Holden and Fredrik Wulfsberg, Downward Nominal Wage Rigidity in Europe, April 2004

1178 Stefan Lachenmaier and Ludger Woessmann, Does Innovation Cause Exports? Evidence from Exogenous Innovation Impulses and Obstacles, April 2004

1179 Thiess Buettner and Johannes Rincke, Labor Market Effects of Economic Integration – The Impact of Re-Unification in German Border Regions, April 2004

1180 Marko Koethenbuerger, Leviathans, Federal Transfers, and the Cartelization Hypothesis, April 2004

1181 Michael Hoel, Tor Iversen, Tore Nilssen, and Jon Vislie, Genetic Testing and Repulsion from Chance, April 2004

1182 Paul De Grauwe and Gunther Schnabl, Exchange Rate Regimes and Macroeconomic Stability in Central and Eastern Europe, April 2004

1183 Arjan M. Lejour and Ruud A. de Mooij, Turkish Delight – Does Turkey's accession to the EU bring economic benefits?, May 2004

1184 Anzelika Zaiceva, Implications of EU Accession for International Migration: An Assessment of Potential Migration Pressure, May 2004

1185 Udo Kreickemeier, Fair Wages and Human Capital Accumulation in a Global Economy, May 2004

1186 Jean-Pierre Ponssard, Rent Dissipation in Repeated Entry Games: Some New Results, May 2004

1187 Pablo Arocena, Privatisation Policy in Spain: Stuck Between Liberalisation and the Protection of Nationals' Interests, May 2004

1188 Günter Knieps, Privatisation of Network Industries in Germany: A Disaggregated Approach, May 2004

1189 Robert J. Gary-Bobo and Alain Trannoy, Efficient Tuition Fees, Examinations, and Subsidies, May 2004

1190 Saku Aura and Gregory D. Hess, What's in a Name?, May 2004

1191 Sjur Didrik Flåm and Yuri Ermoliev, Investment Uncertainty, and Production Games, May 2004

1192 Yin-Wong Cheung and Jude Yuen, The Suitability of a Greater China Currency Union, May 2004

1193 Inés Macho-Stadler and David Pérez-Castrillo, Optimal Enforcement Policy and Firms' Emissions and Compliance with Environmental Taxes, May 2004

1194 Paul De Grauwe and Marianna Grimaldi, Bubbles and Crashes in a Behavioural Finance Model, May 2004

1195 Michel Berne and Gérard Pogorel, Privatization Experiences in France, May 2004

1196 Andrea Galeotti and José Luis Moraga-González, A Model of Strategic Targeted Advertising, May 2004

1197 Hans Gersbach and Hans Haller, When Inefficiency Begets Efficiency, May 2004

1198 Saku Aura, Estate and Capital Gains Taxation: Efficiency and Political Economy Consideration, May 2004

1199 Sandra Waller and Jakob de Haan, Credibility and Transparency of Central Banks: New Results Based on Ifo's *World Economicy Survey*, May 2004

1200 Henk C. Kranendonk, Jan Bonenkamp, and Johan P. Verbruggen, A Leading Indicator for the Dutch Economy – Methodological and Empirical Revision of the CPB System, May 2004

1201 Michael Ehrmann, Firm Size and Monetary Policy Transmission – Evidence from German Business Survey Data, May 2004

1202 Thomas A. Knetsch, Evaluating the German Inventory Cycle – Using Data from the Ifo Business Survey, May 2004

1203 Stefan Mittnik and Peter Zadrozny, Forecasting Quarterly German GDP at Monthly Intervals Using Monthly IFO Business Conditions Data, May 2004

1204 Elmer Sterken, The Role of the IFO Business Climate Indicator and Asset Prices in German Monetary Policy, May 2004

1205 Jan Jacobs and Jan-Egbert Sturm, Do Ifo Indicators Help Explain Revisions in German Industrial Production?, May 2004

1206 Ulrich Woitek, Real Wages and Business Cycle Asymmetries, May 2004

1207 Burkhard Heer and Alfred Maußner, Computation of Business Cycle Models. A Comparison of Numerical Methods, June 2004