IMMEDIATE DEMAND REDUCTION IN SIMULTANEOUS ASCENDING BID AUCTIONS

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Abstract

The present note analyzes the Simultaneous Ascending Bid Auction with arbitrarily many bidders with decreasing marginal valuations under complete information. We show that the game is solvable by iterated elimination of weakly dominated strategies if the efficient allocation assigns at least one unit to every player and if bid increments are sufficiently small. In that unique equilibrium, bidders immediately reduce their demand to the efficient allocation, and the auction ends in the first round of bidding.

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1. INTRODUCTION

Simultaneous Ascending Auctions have become an important device for allocating multiple units of goods[1]. They are also an important theoretical benchmark for models of price formation and resource allocation[2]. Therefore, they deserve a thorough game theoretic analysis.

The present note analyzes the Simultaneous Ascending Auction with arbitrarily many bidders with decreasing marginal valuations under complete information. We show that the game is solvable by iterated elimination of weakly dominated strategies if the efficient allocation assigns at least one unit to every player and if bid increments are sufficiently small. In that unique equilibrium, bidders immediately reduce their demand to the efficient allocation, and the auction ends immediately. We also show by examples that the assumptions we make are necessary.

The practical relevance of demand reduction has been confirmed in a number of Simultaneous Ascending Auctions[3]. The theoretical incentive for strategic demand reduction is also well known[4]. It seems to be less well known that the low price equilibrium is the unique equilibrium that survives iterated elimination of weakly dominated strategies even if there are many bidders.

2. MODEL

There are $M \geq 2$ bidders who bid for $N \geq 2$ objects. Bidders' marginal valuations, $v^i_k, i = 1, \ldots, M, k = 1, \ldots, N$, are strictly decreasing in the number of units k. $w^i_k := \sum_{j=1}^{k} v^i_j$ is the absolute valuation of bidder i for k objects.

The auction is an open, ascending bid clock auction. There, the price clock goes up by the fixed increment $\Delta > 0$, starting at 0, until there is no excess demand. In each round $t = 0, 1, 2, \ldots,$, bidders simultaneously submit a bid $\beta^i(t) \in \{0, 1, 2, \ldots, N\}$, $i = 1, \ldots, M$, which states how many units they demand at

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[1] Starting with Radio Spectrum Auctions for the FCC, variants of the Simultaneous Ascending Auction have been applied in various fields as in electricity, gas, and environmental markets, see, e.g. Cramton (2005), or Milgrom (2004) for recent accounts.

[2] Cf. Milgrom (2000).

[3] For example, Weber (1997) shows that demand reduction was at work in several FCC spectrum auctions. And Grimm, Riedel, and Wolfstetter (2003) report the particularly spectacular case of the second generation GSM spectrum auction in Germany. There, the two dominant market players immediately reduced their demand to one half of the available radio spectrum, and thus immediately ended the auction, after they had succeeded to crowd out the two weaker bidders.

[4] See Cramton (2005), Ausubel and Milgrom (2002), Ausubel and Cramton (2002), Menezes (1996) for dynamic auctions, and Back and Zender (1993), Wilson (1979) for sealed bid auctions, and Engelbrecht-Wiggans and Kahn (1998) and Brusco and Lopomo (2002) for increasing or flat marginal valuations.

the given price $t\Delta$[5]. If the sum of bids does not exceed the supply N, the game ends in round t and bidders pay $t\Delta$ for each of the $\beta^i(t)$ objects they get.

The demand function of bidder i in round t is $d^i(t) = \max\{k : v^i_k \geq \Delta t\}$. Aggregate demand is $D(t) = \sum_i d^i(t)$.

Assumption 1 *All marginal valuations are distinct: for all $i \neq j$,k, l, we have $v^i_k \neq v^j_l$. Marginal valuations do not lie on the price grid: for all $t = 0, 1, 2, \ldots$, $i = 1, \ldots, M$ and $k = 1, \ldots, N$ we have $v^i_k \neq t\Delta$.*

This assumption ensures that there is a competitive solution where demand equals supply in this model because aggregate demands starts at $D(0) = MN$, goes to zero as $t \to \infty$, and has no jumps greater than 1. The smallest competitive price on the price grid determined by Δ is given by $T^*\Delta$ with $T^* \overset{\Delta}{=} \min\{t : D(t) = N\}$. The unique efficient allocation is $a^i \overset{\Delta}{=} d^i(T^*), i = 1, \ldots, M$.

3. UNIQUENESS OF EQUILIBRIUM

Theorem 1 *Assume that the efficient allocations assigns at least one object to each player: $a^i > 0$ for all $i = 1, \ldots, M$. For Δ sufficiently small the game can be solved by iterated removal of weakly dominated strategies. In that solution, the efficient allocation (a^i) obtains and the game ends in round 0 at price 0. The associated equilibrium strategy is: $\beta^i(t) = \min\left\{a^i, d^i(t)\right\}$.*

Proof : Denote by $T^0 = \min\{t : D(t, (N, \ldots, N)) = 0\}$ the first round when aggregate demand is zero. For every $s \geq T^0$, the weakly dominant strategy in the subgame that starts at s is $\beta^i(t) = 0$ for all $t \geq s$. We thus eliminate all strategies which do not to bid zero in rounds $t \geq T^0$.

We now proceed in two steps. In Step 1, we show the only strategy that survives iterated removal of weakly dominated strategies in subgames between T^* and T^0 prescribes to bid one's demand. In Step 2 below, we show that in the early rounds $t = 0, \ldots, T^*$, bidders reduce demand to their efficient quantity a^i.

Step 1. We proceed by backward induction. For $t = T^0$, we know already that the only strategies that survive have $\beta^i(s) = d^i(s) = 0$ for $s \geq T^0$. Assume now that for all $s \geq t + 1$, we have eliminated all strategies that do not have $\beta^i(s) = d^i(s)$. In round t, bidding more than one's demand is weakly dominated by bidding one's demand. Hence, we can eliminate all strategies that have $\beta^i(t) > d^i(t)$. After this elimination, we know that the game ends in round t because $\sum \beta^i(t) \leq D(t) \leq N$. In that case, bidders are just price takers, and take their optimal quantity, $\beta^i(t) = d^i(t)$.

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[5] Bidding strategies may depend on the history of the game, of course. In order to keep the notation simple, we do not make this dependence explicit.

Step 2. We show by backward induction that only $\beta^i(t) = a^i$ survives for $t = 0, \ldots, T^*$. We know by Step 1, that the claim is true for $t = T^*$ since $\beta^i(T^*) = d^i(T^*) = a^i$. Assume now that for all $s \geq t+1, s \leq T^*$, we have eliminated all strategies that do not have $\beta^i(s) = a^i$. Our first claim is that it is weakly dominated to bid $b < a^i$. A bid $b < a^i$ can only lead to a higher payoff, if the game ends in round t with the bid b, and ends in round $t+1$ with the bid a^i. In that case, the bid b yields a payoff $\pi^b \overset{\Delta}{=} w^i(b) - bt\Delta$, whereas the bid a^i leads to a payoff $\pi^* \overset{\Delta}{=} w^i(a^i) - a^i(t+1)\Delta$. Since the demand of player i is at least a^i in round t, we have $\pi^i_b \leq \pi^i_{a^i} = w^i(a^i-1) - (a^i-1)t\Delta$. Hence, it is enough to show that $g := \pi^* - \pi^i(a^i) > 0$. Note that by definition of T^*, there exists a player j with $(T^*-1)\Delta < v^j(a^j+1)$. Since $t < T^*$, we also have $t\Delta < v^j(a^j+1)$. From this, we get $g = v^i(a^i) - t\Delta - a^i\Delta > v^i(a^i) - v^j(a^j+1) - a^i\Delta$. By Assumption 1, we have $v^i(a^i) - v^j(a^j+1) > 0$. Hence, for Δ sufficiently small, $g > 0$ follows. This shows that bidding less than a^i is dominated by bidding a^i.

We thus eliminate all strategies with $\beta^i(t) < a^i$. Given this, every bidder plays at least his efficient quantity $a^j, j = 1, \ldots, M$. After this reduction, it is weakly dominant to play a^i in round t because bidding more just brings the game to the next round where it ends with the allocation (a^j). $\square$

EXAMPLE 1 *The theorem does not hold true if some players do not get objects in the efficient allocation. Consider the following example*[6] *with two players and two objects. Valuations are* $v^1_1 = 101, v^1_2 = 51$, $v^2_1 = 47, v^2_2 = 1$, *and the increment is* $\Delta = 2$. *In this case, the strategy 'truthful bidding'* $\beta^2(t) = d^2(t)$ *cannot be eliminated because player 2 has no incentive to reduce demand to his competitive allocation in early rounds here because he gets nothing in the efficient allocation. Dropping out does not weakly dominate bidding one's demand. On the contrary, for prices* $p = 2, 4, 6, \ldots, 46$ *player 2's weakly dominant strategy is to bid* 1. *Given this strategy, it is optimal for player* 1 *to reduce demand to* 1 *in round* 1 *because he thus obtains a payoff of* 99 *whereas he can get only* 56 *by driving prices up to* 48. *Knowing that outcome, it is weakly dominant to bid* 1 *in round* 0 *for both bidders. Note that the resulting allocation is not efficient but the equilibrium price is still* 0.

On the other hand, if we increase the marginal valuation for the second object of the strong bidder sufficiently, say $v^1_2 = 97$, *a similar argument shows that the unique outcome that survives iterated elimination of weakly dominated strategies implements the efficient allocation at the competitive price* 48. *The resulting allocation is efficient, but the price is not* 0. *However, if one introduces participation fees or bidding costs in this kind of example, the weak bidder drops out immediately, and we get again zero prices.*

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[6] Similar examples can be found in Ausubel and Milgrom (2002) and Milgrom (2000).

EXAMPLE 2 *The assumption that* Δ *be small enough is necessary. Consider a two player auction with* 4 *objects and valuations* $v_1^1 = 101$, $v_1^2 = 91$, $v_3^1 = 61$, $v_4^1 = 11$ *and* $v_1^2 = 103$, $v_2^2 = 63$, $v_3^2 = 13$, *and* $v_4^2 = 1$. *The increment is* $\Delta = 2$. *The efficient allocation assigns two objects to each player. The competitive price is* 62. *At and above this price, every player plays his demand. However, at price* $p = 60$ *in round* $t = 30$, *bidding* 1 *is not weakly dominated by bidding* 2 *for player* 2. *The payoff matrix is (neglecting the bid* 0*)*

	1	*2*	*3*
1	*41,43*	*41,46*	*41, -1*
2	*72,43*	*72,46*	*68,42*
3	*73,43*	*68,42*	*68, 42*

The game has two strict Nash equilbria: $(3, 1)$ *and* $(2, 2)$.

4. CONCLUSIONS

The present paper has not only shown that low price outcomes may be an equilibrium in multi-unit auctions. We showed that the low price equilibria that implement the efficient allocation at the minimum bid is rationalizable as the only equilibrium that survives iterated elimination of dominated strategies.

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