MODEL AVERAGING AND VALUE-AT-RISK BASED EVALUATION OF LARGE MULTI ASSET VOLATILITY MODELS FOR RISK MANAGEMENT

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Abstract

This paper considers the problem of model uncertainty in the case of multi-asset volatility models and discusses the use of model averaging techniques as a way of dealing with the risk of inadvertently using false models in portfolio management. In particular, it is shown that under certain conditions portfolio returns based on an average model will be more fat-tailed than if based on an individual underlying model with the same average volatility. Evaluation of volatility models is also considered and a simple Value-at-Risk (VaR) diagnostic test is proposed for individual as well as 'average' models and its exact and asymptotic properties are established. The model averaging idea and the VaR diagnostic tests are illustrated by an application to portfolios of daily returns based on twenty two of Standard & Poor's 500 industry group indices over the period January 2, 1995 to October 13, 2003, inclusive.

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1 Introduction

Multivariate models of conditional volatility are of crucial importance for optimal asset allocation, risk management, derivative pricing and dynamic hedging. Yet there are few published empirical studies of the performance of multivariate volatility models as applied to portfolios with a relatively large number of assets. Although many alternative parametric and semi-parametric multivariate volatility models have been advanced in academic literature, until recently most have been limited in the number of assets that they can handle. In an attempt to provide operationally feasible volatility models for the analysis of portfolios with a large number of assets, many investigators (in financial markets as well as in academia) have focused on highly restricted versions of multivariate generalized autoregressive conditional heteroscedastic (GARCH) specification. These include the conditionally constant correlation (CCC) model of Bollerslev (1990), the Riskmetrics specifications popularized by J.P.Morgan/Reuters (1996), and used predominantly by practitioners, the orthogonal GARCH model of Alexander (2001), and more recently the dynamic conditional correlation (DCC) model advanced by Engle (2002). Bauwens, Laurent, and Rombouts (2003) provides a survey of this literature. Multivariate stochastic volatility (SV) models have also been considered in literature, with excellent reviews by Ghysels, Harvey, and Renault (1995) and Shephard (2004). So far the focus of the SV literature has been on univariate and multivariate models with a small number of assets, with the notable exceptions of Diebold and Nerlove (1989), Engle, Ng, and Rothschild (1990), King, Sentana, and Wadhwani (1994) and Harvey, Ruiz, and Shephard (1994), that are similar in structure to the class of factor GARCH models that we do consider below.

The highly restricted nature of the multivariate volatility models could present a high degree of model uncertainty which ought to be recognized at the outset. This is particularly important since due to data limitations and operational considerations it is not possible to subject these models to rigorous statistical testing either. Application of model selection procedures also involves additional risks when the number of assets is moderately large, and might very well be that no single model choice would be satisfactory in practice and could carry risks that are difficult to asses *a priori*. This paper considers model averaging as a risk diversification strategy in dealing with model uncertainty, and provides a detailed application of recent developments in model evaluation and model averaging techniques to multi-asset volatility models, in the case where the number of assets under consideration is relatively large. As a part of this research strategy we also develop a simple criterion for evaluation of alternative volatility forecasts by examining the Value-at-Risk (VaR) performance of their associated portfolios. The VaR based diagnostic tests developed in this paper can be used both for risk monitoring of given portfolios as well as for construction of new portfolios.

From a methodological perspective, following Granger and Pesaran (2000b), our approach aims to represent a more unified treatment of the empirical portfolio analysis from a decision-theoretic perspective rather than from a merely statistical one. Granger and Pesaran (2000a) clarify the importance of concentrating on probability forecasts rather than on just event forecasts. Within a risk-management perspective the motivations for doing so appear even stronger since the ultimate goal is not simply finding the best approximating volatility model but how to best approximate the entire predictive density of asset returns, or at least its tail behavior. The standard forecast evaluation techniques that focus on standard metrics such as root mean square forecast errors (RMSFE), also run into difficulties when considering volatility models. Since volatility is not directly observable, it is often proxied by square of daily returns or more recently by the standard error of intra-daily returns, known as realized volatility (see, for example, Andersen, Bollerslev, Diebold, and Labys (2003)). In multi-asset contexts the use of standard metrics such as RMSFE is further complicated by the need to select weights to be attached to different types of errors in forecasts of individual asset volatilities and their cross-volatility correlations and choice of such weights is not innocuous in a multivariate framework (see Pesaran and Skouras (2002)).

The probability forecast combination approach also attempts to avoid the pre-testing problem associated with the standard two-stage procedure where the decision problem is based on a probability model selected as the 'best' from a given set of candidate models according to a suitable criteria. Frequently used selection criteria are Akaike Information Criterion (AIC) and the Schwartz Bayesian Information Criterion (SBC). However, such a two-step procedure is subject to the pre-test (selection) bias problem and tends to under-estimate the uncertainty that surrounds the forecasts. Of course, the use of model averaging techniques in econometrics is not new and dates back to the seminal work of Granger and Newbold (1977) on forecast combination.[1]

[1] For reviews of the forecast combination literature see Clemen (1989), Granger (1989), Diebold and Lopez (1996) and Hendry and Clements (2002).

However, this literature focusses on combining point forecasts and does not address the problem of combining forecast probability distribution functions which is relevant in risk management.

The remainder of the paper is organized as follows: the decision problem that underlies the VaR analysis and the associated diagnostic tests is set out in Section 2. Section 3 provides a brief outline of the different types of multivariate volatility models considered in the paper. Bayesian and non-Bayesian approaches to model averaging are reviewed and discussed in Section 4. Section 5 introduces a simple Value-at-Risk (VaR) diagnostic test and establishes its distribution. Section 6 provides a detailed empirical analysis using daily returns on twenty two of Standard and Poor's 500 industry indices over the period January 2 1995 to October 13 2003. Section 7 concludes with a summary of the main results and provides suggestions for future research. The mathematical proofs are collected in the Appendix.

2 The Decision Problem

At the heart of all econometric analysis lies a decision problem. In this paper we are concerned with the decision of an individual fund manager who is interested in controlling the risk of his/her portfolio over a given trading day. Denote the fund manager's asset positions at the close of business on day $t-1$ by the $N \times 1$ vector, $\mathbf{a}_{t-1} = (a_{1,t-1}, a_{2,t-1}, ..., a_{N,t-1})'$. The change in the value of this portfolio is given by

$$\Delta V_t = \sum_{j=1}^{N} (P_{jt} - P_{j,t-1})\, a_{j,t-1} = \sum_{j=1}^{N} a_{j,t-1} P_{j,t-1} r_{jt}, \tag{1}$$

where P_{jt} is the price of the j^{th} asset at time t and $r_{jt} = (P_{jt} - P_{j,t-1})\,/P_{j,t-1}$ is the associated daily rate of return.[2] The rate of return of the portfolio can now be written as

$$\rho_t = \frac{\Delta V_t}{V_{t-1}} = \boldsymbol{\omega}'_{t-1}\mathbf{r}_t, \tag{2}$$

where $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{Nt})'$, and $\boldsymbol{\omega}_t = (\omega_{1t}, \omega_{2t}, ..., \omega_{Nt})'$, with

$$\omega_{it} = a_{it} P_{it} / \sum_{j=1}^{N} a_{jt} P_{jt}.$$

[2] We are assuming dividend payments are negligible and can be ignored, although the analysis can be readily extended to allow for dividends.

By construction $\boldsymbol{\tau}'\boldsymbol{\omega}_{t-1} = 1$ where $\boldsymbol{\tau}$ is an $N \times 1$ vector of unity. In the case of a fund manager who has been given the task of allocating a given sum, V_{t-1} on the N assets without the possibility of shorting, we have the additional non-negativity restrictions, $\omega_{it} \geq 0$, for all i.

The fund manager faces two different but closely related tasks, which we refer to as 'passive' and 'active' risk management problems. Under the latter the portfolio weights are treated as unknown and are determined by maximizing the expected utility of the portfolio, derived with respect to the conditional multivariate distribution of $\mathbf{r}_t$, subject to the non-negativity constraints (if applicable) and the following VaR constraint

$$\Pr\left(\rho_t < -\bar{\rho}_{t-1} \left| \mathcal{F}_{t-1}\right.\right) \leq \alpha, \tag{3}$$

where $\mathcal{F}_{t-1}$ is the available information, $\bar{\rho}_{t-1} > 0$ is a pre-specified rate of return and α is a probability value (typically taken to be 1%) which captures the trader's attitude to risk in the case of large losses. Under passive risk management, which might also be viewed as a risk monitoring activity, $\boldsymbol{\omega}_{t-1}$ and α are assumed as given and the aim would to solve for $\bar{\rho}_{t-1}(\boldsymbol{\omega}_{t-1},\alpha)$ using

$$\Pr\left(\rho_t < -\bar{\rho}_{t-1}(\boldsymbol{\omega}_{t-1},\alpha) \left| \mathcal{F}_{t-1}\right.\right) \leq \alpha.$$

The capital at risk of the portfolio would then be given by $L_{t-1}(\boldsymbol{\omega}_{t-1},\alpha) = V_{t-1}\bar{\rho}_{t-1}(\boldsymbol{\omega}_{t-1},\alpha)$, namely the maximum loss tolerated over day $t-1$ to t with probability α.

3 Multivariate Models of Asset Returns

It is clear that for active risk management a complete knowledge of the joint probability distribution of the vector of returns, $\mathbf{r}_t$ conditional on available information, denoted by $\Pr(\mathbf{r}_t \left|\mathcal{F}_{t-1}\right.)$, would be needed.[3] But for passive risk management it is clearly possible to work *directly* with the conditional distribution of ρ_t, the portfolio return, with no apparent need for multivariate volatility modeling. Such a startegy is relatively simple to implement, but will be portfolio specific and could lead to contradictory outcomes if different portfolios are considered. In comparing the risk of different portfolios

[3] Here we are ruling out the possibility of feedbacks from portfolio decisions to the 'true' probability distribution of the returns, by assuming that the value of the portfolio under management is small relative to the market (in the case of all assets).

it is important that the distribution of all portfolio returns are based on the same underlying multivariate model of $\mathbf{r}_t$. Also our primary concern in this paper is on modeling and evaluation of alternative multivariate volatility models in a wider context that nests both passive and active risk management problems. Therefore, in what follows we shall focus on alternative specifications of $\Pr(\mathbf{r}_t |\mathcal{F}_{t-1})$. For this purpose it is convenient to work with the de-volatized returns, $\mathbf{z}_t$, defined by

$$\mathbf{z}_t = \mathbf{\Sigma}_t^{-\frac{1}{2}}\mathbf{r}_t.$$

where $\mathbf{\Sigma}_t = Var(\mathbf{r}_t |\mathcal{F}_{t-1})$ is the conditional variance-covariance matrix of the returns assumed to be positive definite. Typically one would also need to model the conditional mean, $E(\mathbf{r}_t |\mathcal{F}_{t-1}) = \boldsymbol{\mu}_t$, although given the focus of the present paper on multivariate volatility models and the daily nature of the returns data that we shall be using to illustrate our approach we shall maintain that $\boldsymbol{\mu}_t = \mathbf{0}$, throughout. This assumption can be readily relaxed in our mathematical exposition, but its relaxation in the empirical application would involve considerable additional computations and data requirements. Past returns are unlikely to be sufficient for modelling of $\boldsymbol{\mu}_t$, and data sets on other industry-specific and macro indicators such as size, book value, interest rates and exchange rates would be needed.

Therefore, a complete specification of $\Pr(\mathbf{r}_t |\mathcal{F}_{t-1})$ can be achieved by a non-singular choice of $\mathbf{\Sigma}_t$, and by specification of the distribution of devolatized values, $\mathbf{z}_t$. As far as the latter is concerned, we focus on distributions that are closed under linear transformations. This includes the case of standard multivariate Gaussian, and the multivariate Student t with v degrees of freedom. These are the two specifications that are most commonly encountered in practice. With respect to the specification of $\mathbf{\Sigma}_t$, we focus on parametric volatility models, the classical example of which is represented by the multivariate generalized autoregressive heteroskedasticity model of order $1,1$ (multivariate GARCH$(1,1)$), which in its most general specification (see Bollerslev, Engle, and Wooldridge (1988, eq. 4)) is

$$vech(\mathbf{\Sigma}_{MGARCH,t}) = \mathbf{\Omega}_0 + \mathbf{A}_0 vech(\mathbf{\Sigma}_{MGARCH,t-1}) + \mathbf{B}_0 vech\left(\mathbf{r}_{t-1}\mathbf{r}'_{t-1}\right) \quad (4)$$

where $vech(.)$ denotes the column stacking operator of the lower portion of a symmetric matrix. Concerning model parameters, $\mathbf{\Omega}_0$ is an $N(N+1)/2 \times 1$ vector and $\mathbf{A}_0$, $\mathbf{B}_0$ are $N(N+1)/2 \times N(N+1)/2$ matrices of coefficients. It is evident that even such a low-order model already contains a large number of parameters even for moderate values of N which makes model (4) effectively unfeasible for practical applications.

Many alternative multivariate models have been proposed, all of which entails some simplification of the multivariate GARCH specification given by (4). Denote by $\mathbf{\Sigma}_{it}$ the $\mathbf{\Sigma}_t$ implied by model M_i with $i = 1, 2, .., m$. Let $\boldsymbol{\theta}_{i0}$ be the $k_i \times 1$ vector of true parameters characterizing model M_i yielding $\mathbf{\Sigma}_{it}= \mathbf{\Sigma}_{it}(\boldsymbol{\theta}_{i0})$. For estimation of the models we shall be using the Gaussian pseudo maximum likelihood estimator (PMLE), defined by

$$\hat{\boldsymbol{\theta}}_{iT_0} = \arg\max_{\boldsymbol{\theta}_i \in \Theta_i} \left\{ -\frac{1}{2} \sum_{t=\tau-T_0+1}^{\tau} \left(\log \mid \mathbf{\Sigma}_{it}(\boldsymbol{\theta}_i) \mid + \mathbf{r}_t' \mathbf{\Sigma}_{it}^{-1}(\boldsymbol{\theta}_i) \mathbf{r}_t \right) \right\}, \tag{5}$$

where Θ_i defines a suitable parameter space, τ is the end of the estimation period, T_0 is the size of the estimation period. Correspondingly, let $\hat{\mathbf{\Sigma}}_{it} = \mathbf{\Sigma}_{it}(\hat{\boldsymbol{\theta}}_{iT_0})$. We view Gaussian PMLE as a robust method, delivering consistent and asymptotically normal estimates of $\boldsymbol{\theta}_i$ under M_i even for non Gaussian $\mathbf{z}_t$. In particular we shall assume that as $T_0 \to \infty$,

$$\hat{\boldsymbol{\theta}}_{iT_0} \xrightarrow{p} \boldsymbol{\theta}_{i0} \tag{6}$$

and

$$\sqrt{T_0} \left(\hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \right) \mid M_i \xrightarrow{d} N \left[\mathbf{0}, \mathbf{\Omega}_i \left(\boldsymbol{\theta}_{i0} \right) \right], \tag{7}$$

where $\mathbf{\Omega}_i \left(\boldsymbol{\theta}_{i0} \right)$ is a positive definite matrix, $\xrightarrow{p}$ denotes convergence in probability and $\xrightarrow{d}$ convergence in distribution. The asymptotic properties of the Gaussian PMLE have been established for certain classes of multivariate GARCH-type volatility models (see Ling and McAleer (2003)) and it is reasonable to expect that results such as (6) and (7) would hold for the more general class of models considered in this paper, under suitable regularity conditions.

In what follows we shall assume that under model M_i

$$M_i: \qquad \mathbf{r}_t = \mathbf{\Sigma}_{it}^{\frac{1}{2}} \mathbf{z}_t, \;\; \mathbf{z}_t \mid \; \mathcal{F}_{t-1} \sim (F_{it}, \; \mathbf{0}, \; \mathbf{I}_N), \tag{8}$$

meaning that $E \left(\mathbf{z}_t \mid \mathcal{F}_{t-1}, M_i \right) = \mathbf{0}$, $E \left(\mathbf{z}_t \mathbf{z}_t' \mid \mathcal{F}_{t-1}, M_i \right) = \mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $F_{it}(.)$ is the the conditional joint probability distribution function of the $\mathbf{z}_t$. Note that the above formulation allows the higher order moments of $\mathbf{z}_t$ to be time varying. This would be the case, for example, when $\mathbf{z}_t$ is distributed as the multivariate Student t with time varying degrees of freedom, v_t.

Almost all the multivariate volatility models considered in the literature can be cast in terms of the following decomposition of $\mathbf{\Sigma}_t$, originally used in the literature by Bollerslev (1990):

$$\mathbf{\Sigma}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \tag{9}$$

where $\mathbf{R}_t$ is the one-step-ahead conditional correlation matrix with its (h, j)th element given by $\rho_{hj,t}$, and $\mathbf{D}_t$ is a diagonal matrix with $\sqrt{\sigma_{hh,t}}$ on its (h, h)th element. This is a convenient decomposition and allows separate specification of the conditional volatilities and conditional cross-asset returns correlations. The models used in our empirical applications also belong to the class of models spanned by different specifications of $\sqrt{\sigma_{hh,t}}$ and $\rho_{hj,t}$, which are computationally feasible for estimation and forecasting in the case of portfolios with a large number of assets ($N = 22$ in our applications).

As previously indicated, throughout the paper by 'model' we mean a given specification of the multivariate conditional covariance matrix, $\mathbf{\Sigma}_{it}$, *together* with a particular distribution of scaled innovations, $\mathbf{z}_t \mid \mathcal{F}_{t-1}$. This also applies to the equal and exponential weighted specifications set out in Sections 3.1-3.5 where they are typically known as *filters*. However, in the context of the present paper it seems reasonable to refer to them as models since they will be considered in conjunction with particular distributions of the scaled innovations.

3.1 Equal-Weighted Moving Average (EQMA(n_0))

In the absence of reliable intra-daily observations on returns, a simple estimate of $\mathbf{\Sigma}_t$ can be obtained using the following rolling moment estimates based on the last n_0 observations:

$$\mathbf{\Sigma}_{1t} = \frac{1}{n_0} \sum_{s=1}^{n_0} \mathbf{r}_{t-s} \mathbf{r}'_{t-s}.$$

For $\mathbf{\Sigma}_{1t}$ to be positive definite we must have $n_0 > N$. In the empirical applications we shall consider four variants of $\mathbf{\Sigma}_{1t}$, using $n_0 = 50, 75, 125$, and 250. Subject to $n_0 > N$, care should be taken so that n_0 is not set too high; otherwise $\mathbf{\Sigma}_{1t}$ could behave like the unconditional variance matrix of the returns.

3.2 Exponential-Weighted Moving Average (EWMA(n_0, λ_0))

This is the popular *Riskmetrics* estimate of $\mathbf{\Sigma}_t$ (see J.P.Morgan/Reuters (1996)) which is defined by the following recursion

$$\mathbf{\Sigma}_{2t} = \lambda_0 \mathbf{\Sigma}_{2,t-1} + \frac{(1-\lambda_0)}{(1-\lambda_0^n)} \mathbf{r}_{t-1}\mathbf{r}'_{t-1} - \frac{(1-\lambda_0)}{(1-\lambda_0^{n_0})} \lambda_0^{n_0-1} \mathbf{r}_{t-n_0-1}\mathbf{r}'_{t-n_0-1}, \quad (10)$$

for a constant parameter $0 < \lambda_0 < 1$, and a window of size n_0. Typically, the initialization of the recursion in (10) is based on estimates of the unconditional variances using a pre-sample of data. For the (i, j)th entry of $\mathbf{\Sigma}_{2t}$ we have

$$\sigma_{2,ijt} = \frac{(1-\lambda_0)}{(1-\lambda_0^{n_0})} \sum_{s=1}^{n_0} \lambda_0^{s-1} r_{i,t-s} r_{j,t-s}.$$

The Riskmetrics model is characterized by the fact that n_0 and λ_0 is fixed *a priori*.[4] The value of $\lambda_0 = 0.94$ is suggested in J.P.Morgan/Reuters (1996). In our analysis we shall consider the values $\lambda_0 = 0.94, 0.95$, and 0.96, and set $n_0 = 250$. We only consider one value for the window size since there is an obvious trade-off between λ_0 and n_0, with a small λ_0 yielding similar results to a small n_0. Note that for $\mathbf{\Sigma}_{2t}$ to be non singular one requires $n_0 \geq N$. Nevertheless, the model does admit a well-defined forecasting function and indeed $\mathbf{\Sigma}_{2,t+1}$ represents the one-step ahead forecast of the conditional variance for period $t+1$, based on the information available up to time t.

3.3 Two-parameter Exponential-Weighted Moving Average (EWMA (n_0, λ_0, ν_0))

Practitioners and academics have often pointed out that the effects of shocks on conditional variances and conditional correlations could decay at different rates, with correlations typically responding at a slower pace than volatilities (see De Santis and Gerard (1997)). This suggests using two different parameter values for the decay coefficients, one for volatilities and the other for correlations (see De Santis, Litterman, Vesval, and Winkelmann (2003, p.14)). Therefore, the diagonal elements of (10) define conditional variances $\sigma_{3,hht}$, $h = 1, ..., N$ the square-root of which form the diagonal matrix $\mathbf{D}_{3t}$.

[4] Moreover, it has been recently pointed out that it is not possible to formally estimate the model statistically, due to its asymptotic degenerateness. See Zaffaroni (2003).

The covariances are based on the same recursion as (10) but using a smoothing parameter ν_0, generally different from λ_0 ($\nu_0 \leq \lambda_0$) yielding

$$\sigma_{3,hjt} = \frac{(1-\nu_0)}{(1-\nu_0^{n_0})} \sum_{s=1}^{n_0} \nu_0^{s-1} r_{h,t-s} r_{j,t-s}, \text{ for } h \neq j.$$

We assume that the same window size, n_0, applies to variance and covariance recursions. The ratio

$$\sigma_{3,hjt} / \sqrt{\sigma_{3hh,t}\, \sigma_{3jj,t}} \tag{11}$$

represents the (h, j)th entry of the matrix $\mathbf{R}_{3t}$. Combining terms according to (9) one gets $\mathbf{\Sigma}_{3t}$. The parameters ν_0 and λ_0 are not estimated but calibrated *a priori*, as for the one-parameter EWMA model.

3.4 Mixed Moving Average (MMA(n_0, ν_0))

This is a generalization of the equal-weighted MA model discussed above. Under this specification, the conditional variances are computed as in the equal-weighted MA model, the square root of which yields the diagonal matrix $\mathbf{D}_{4t}$. Then we estimate the conditional covariances using a Riskmetrics type filter: $\sigma_{4,hjt} = \frac{(1-\nu_0)}{(1-\nu_0^{n_0})} \sum_{s=1}^{n_0} \nu_0^{s-1} r_{h,t-s} r_{j,t-s}$, which after normalization according to (11) yields $\mathbf{R}_{4t}$. Re-combining the results according to (9) we then obtain $\mathbf{\Sigma}_{4t}$.

3.5 Generalized Exponential-Weighted Moving Average (EWMA(n_0, p, q, ν_0))

This is a generalization of the two-parameter EWMA. In the first stage N different univariate GARCH(p, q) models are estimated for each r_{ht} by PMLE. The conditional covariances are then obtained using the Riskmetrics filter (10), with the parameters n_0 and ν_0 fixed *a priori*. The results are then normalized using (11), with the resultant variances and correlations re-combined according to (9), thus yielding $\mathbf{\Sigma}_{5t}$. The estimated number of parameters of this model is $k_5 = N(1 + p + q)$, which will be used in the computation of AIC and SBC.

3.6 Constant Conditional Correlation (CCC(p, q))

Bollerslev (1990) introduced a multivariate GARCH model with the simplifying assumption that the one-step ahead conditional correlations are constant. Under this model, (9) takes the form $\mathbf{\Sigma}_{6t} = \mathbf{D}_{6t}\mathbf{R}_6\mathbf{D}_{6t}$, where $\mathbf{D}_{6t}$ is a diagonal matrix containing the square-root of the $\sigma_{6,hht}$, each of which follow the GARCH(p, q) model of Bollerslev (1986)

$$\sigma_{6,hht} = c_{0h} + \sum_{k=1}^{q} \alpha_{0hk} r_{h,t-k}^2 + \sum_{j=1}^{p} \beta_{0hj}\sigma_{6,hht-j},$$

for constant positive parameters c_{0h}, $\alpha_{0h1}, ..., \alpha_{0hq}, \beta_{0h1}, ..., \beta_{0hp}$. Positivity of these parameters is sufficient but not necessary to ensure $\sigma_{6,hht} > 0$ $a.s.$ (see Nelson and Cao (1992)). The positive definite matrix $\mathbf{R}_6$, made by $N(N-1)/2$ constant parameters, contains the (constant) conditional correlations of the $r_{ht}, h = 1, 2, ..., N$.

Bollerslev (1990) proposed to estimate the model by the PMLE and noting that (5) simplifies due to the constant correlation assumption. The estimated number of parameters of this model is given by $k_6 = N(p + q + 1) + N(N-1)/2$.

3.7 Orthogonal GARCH (O-GARCH(p, q))

This model is proposed by Alexander (2001) and uses a static principle component decomposition of standardized returns defined by

$$\tilde{r}_{it} = \frac{r_{it} - \bar{r}_{iT}}{s_{iT}},\ t = 1, 2, ..., T,$$

where $\bar{r}_{iT}$ and s_{iT} are the sample mean and standard deviations of the returns. Denote the sample covariance matrix of the standardized returns by

$$\tilde{\mathbf{S}}_T = \frac{\sum_{t=1}^{T} \tilde{\mathbf{r}}_t \tilde{\mathbf{r}}_t'}{T},\ \ \tilde{\mathbf{r}}_t = (\tilde{r}_{1t}, , ..., \tilde{r}_{Nt})'.$$

Then

$$\tilde{\mathbf{S}}_T \mathbf{W}_T = \mathbf{W}_T \mathbf{\Lambda}_T, \tag{12}$$

where and $\mathbf{W}_T$ and $\mathbf{\Lambda}_T$ are the corresponding $N \times N$ matrices of eigenvectors and eigenvalues, respectively. Then setting (see Alexander (2001))

$$\mathbf{\Sigma}_{7t}(u) = \mathbf{V}\,\mathbf{W}(u)\mathbf{\Gamma}_t(u)\mathbf{W}(u)'\mathbf{V},$$

where $\mathbf{W}(u) = (\mathbf{w}_1, ..., \mathbf{w}_u)$ denotes the $N \times u$ matrix of eigenvectors corresponding to the first largest u eigenvalues, $\mathbf{V}$ is a diagonal matrix with the sample standard deviation of r_{ht} on the (h, h)th entry and $\mathbf{\Gamma}_t(u)$ is a $u \times u$ diagonal matrix whose (j, j)th entry, γ_{jt}, $j = 1, ..., u$, is assumed to satisfy the following univariate GARCH(p, q) specification

$$\gamma_{jt} = c_{0j} + \alpha_{0j1} s_{jt-1}^2 + ... + \alpha_{0jp} s_{jt-p}^2 + \beta_{0j1} \gamma_{jt-1} + ... + \beta_{0jq} \gamma_{jt-q}, \quad j = 1, ..., u,$$

where $\mathbf{s}_j = (\mathbf{r}_1, ..., \mathbf{r}_T)' \mathbf{w}_j$, $j = 1, ..., N$. Note that this method makes use of the fact that the factors are unconditionally orthogonal, but there is no guarantee that they will also be conditionally orthogonal. Also to ensure that $\mathbf{\Sigma}_{7t}(u)$ is non-singular we must have $u = N$, which is the value considered here, yielding $\mathbf{\Sigma}_{7t} = \mathbf{\Sigma}_{7t}(N)$. Hence for the O-GARCH$(p, q)$ specification we have $k_7 = N(p + q + 1)$.

3.8 Factor GARCH of Harvey, Ruiz and Sentana (Factor HRS $(p, q, 1, 1)$)

We consider the one factor model

$$\mathbf{r}_t = \mathbf{b}_0 f_t + \mathbf{v}_t, \tag{13}$$

where $\mathbf{b}_0$ is a $N \times 1$ vector and

$$\begin{pmatrix} f_t \\ \mathbf{v}_t \end{pmatrix} \mid \mathcal{F}_{t-1} \sim \begin{pmatrix} 0 \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \lambda_t & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_t \end{pmatrix}$$

where $\mathbf{\Gamma}_t$ is a diagonal matrix, and f_t and $\mathbf{v}_t$ are mutually independent. It easily follows that

$$\mathbf{\Sigma}_{8t} = (\mathbf{b}_0 \mathbf{b}_0') \lambda_t + \mathbf{\Gamma}_t. \tag{14}$$

With respect to the standard formulation of factor models, Harvey, Ruiz, and Sentana (1992) assume that the conditional variance λ_t is a function of past observations, not of past f_t, yielding

$$\lambda_t = c_0 + \alpha_{01}(\hat{f}_{t-1|t-1}^2 + g_{t-1|t-1}) + ... + \alpha_{0q}(\hat{f}_{t-q|t-q}^2 + g_{t-q|t-q}) + \beta_{01} \lambda_{t-1} + ... + \beta_{0p} \lambda_{t-p}, \tag{15}$$

for

$$\hat{f}_{t|t} = \lambda_t \mathbf{b}_0' \mathbf{\Sigma}_{8t}^{-1} \mathbf{r}_t, \quad g_{t|t} = \lambda_t \left(1/\lambda_t + \mathbf{b}_0' \mathbf{\Sigma}_{8t}^{-1} \mathbf{b}_0\right) \lambda_t.$$

Similarly, the (i, i)th entry of $\mathbf{\Gamma}_t$ is specified as a GARCH$(1, 1)$ with γ_{it} being a function of $\gamma_{i,t-1}$ and $\hat{v}_{i,t-1}^2$ where $\hat{\mathbf{v}}_t = (\hat{v}_{1,t}, ..., \hat{v}_{N,t})' = \mathbf{r}_t - \mathbf{b}_0 \hat{f}_{t|t}$. The model, estimated by PMLE, has $k_8 = N + (p+q+1) + N(1+1+1)$ parameters.

3.9 Dynamic Conditional Correlation (DCC$(p, q, 1, 1)$)

Engle (2002) relaxed the assumption of constant conditional correlation of the CCC model of Bollerslev (1990). The conditional variances of individual returns are estimated as univariate GARCH(p, q) specifications, and the diagonal matrix, $\mathbf{D}_{9t}$, is formed with their square roots. Unlike the CCC, the conditional correlations are now allowed to be time-varying and are obtained as follows. Starting with the standardized residuals, $\tilde{\mathbf{r}}_{9t} = (\mathbf{D}_{9t})^{-1}\,\mathbf{r}_t$, the DCC model assumes that the (h, j)th entry of the conditional covariance matrix of $\tilde{\mathbf{r}}_{9t}$, namely $\mathbf{R}_{9t}$, is given by $q_{hjt}/\sqrt{q_{hht}\,q_{jjt}}$, where q_{hjt} is the (h, j)th element of matrix $\mathbf{Q}_t$ defined by

$$\mathbf{Q}_t = \overline{\mathbf{Q}}\,(1 - \gamma_{01} - \delta_{01}) + \gamma_{01}\tilde{\mathbf{r}}_{9,t-1}\tilde{\mathbf{r}}'_{9,t-1} + \delta_{01}\mathbf{Q}_{t-1}.$$

for a fixed positive definite matrix $\overline{\mathbf{Q}}$, and positive parameters satisfying $\gamma_{01} + \delta_{01} < 1$. Finally, $\mathbf{\Sigma}_{9t}$ is obtained re-combining $\mathbf{D}_{9t}$ and $\mathbf{R}_{9t}$ based on (9). The estimation of the parameters of the DCC model is carried out using a two-stage Gaussian PMLE procedure. The log-likelihood function (5) is first optimized with respect to the parameters driving the individual conditional variances. Conditional on these parameter estimates, in the second step the log-likelihood function is maximized with respect to the parameters driving conditional correlations. See Engle (2002, Section 4) for details. For this model we have $k_9 = N(p + q + 1) + N(N + 1)/2 + 2$.

3.10 Asymmetric Dynamic Conditional Correlation (ADCC$(p, q, 1, 1)$)

Cappiello, Engle, and Sheppard (2002) generalized the DCC allowing for the possibility of asymmetric effects on conditional variances and correlations. The conditional variances of the individual returns are specified using the specification advanced by Glosten, Jagannathan, and Runkle (1993) given by:

$$\sigma_{10,hht} = c_{0h} + \sum_{k=1}^{q} \alpha_{0hk} r_{h,t-k}^2 + \sum_{k=1}^{q} \vartheta_{0hk} I(r_{h,t-k} < 0) r_{h,t-k}^2 + \sum_{j=1}^{p} \beta_{0hj}\sigma_{10,hh,t-j},$$

where $I(\mathcal{A})$ denotes the indicator function which takes the value of unity if $\mathcal{A} > 0$, and zero otherwise. Let $\tilde{\mathbf{r}}_{10t} = (\mathbf{D}_{10t})^{-1}\,\mathbf{r}_t$, where $\mathbf{D}_{10t}$ is the diagonal matrix formed with the square roots of $\sigma_{10,hht}$. The ADCC model assumes that the (h,j)th entry of the conditional covariance matrix of $\tilde{\mathbf{r}}_{10t}$, namely $\mathbf{R}_{10t}$, is given by $q_{hjt}/\sqrt{q_{hht}\,q_{jjt}}$ where q_{hjt} is the (h,j)th element of matrix $\mathbf{Q}_t$ defined by

$$\mathbf{Q}_t = \overline{\mathbf{Q}}\,(1-\gamma_{01}-\delta_{01}-\vartheta_{01}) + \gamma_{01}\tilde{\mathbf{r}}_{10,t-1}\tilde{\mathbf{r}}'_{10,t-1} + \delta_{01}\mathbf{Q}_{t-1} + \vartheta_{01}\underline{\mathbf{r}}_{10,t-1}\underline{\mathbf{r}}'_{10,t-1}$$

where $\underline{\mathbf{r}}_{10t} = \tilde{\mathbf{r}}_{10t} \odot I(\mathbf{r}_{10,t-1} < 0)$ (here $\odot$ denotes the Hadamard product), $\overline{\mathbf{Q}}$ is a fixed positive definite matrix, and γ_{01}, δ_{01}, and ϑ_{01} are positive parameters satisfying $\gamma_{01}+\delta_{01}+\vartheta_{01} < 1$. Finally, $\mathbf{\Sigma}_{10t}$ is constructed using $\mathbf{D}_{10t}$ and $\mathbf{R}_{10t}$ as in (9). The estimation of the parameters of the ADCC model is carried out as for the DCC, where now we have $k_{10} = N(p+2q+1) + N(N+1)/2 + 3$.

3.11 Factor GARCH of Diebold and Pesaran (Factor DP $(p,q,1,1)$)

This procedure starts with the one factor model, (13), but relaxes the condition that λ_t does not directly depend on past values of the factors, replacing (15) with

$$\lambda_t = c_0 + \alpha_{01}f_{t-1}^2 + ... + \alpha_{0q}f_{t-q}^2 + \beta_{01}\lambda_{t-1} + ... + \beta_{0p}\,\lambda_{t-p}. \qquad (16)$$

The estimation of this more general factor model by the maximum likelihood method poses significant computational problems and so far recent advances on efficient estimation of such models have been simulation-based; see Fiorentini, Sentana, and Shephard (2004). We adopt here the estimation approach proposed by Diebold and Pesaran (1999) which is relatively simple to compute and can be implemented in the following manner. For N sufficiently large, f_t is estimated (up to a linear transformation) consistently by $\hat{f}_t = N^{-1}\sum_{i=1}^{N} r_{it}$. Then each r_{it} is regressed on $\hat{f}_t$ yielding the factor loadings, $\hat{\mathbf{b}}_0 = (\hat{b}_{01}, \hat{b}_{02}, ..., \hat{b}_{0N})'$ as in (13).[5] Finally, a univariate GARCH(p,q) models is fitted by PMLE to $\hat{f}_t$, and GARCH$(1,1)$ models are estimated for the individual residuals $\hat{v}_{it} = r_{it} - \hat{b}_{0i}\hat{f}_t$. The model has $k_{11} = N-1+(N+1)(p+q+1)$ unknown parameters.

[5] Note that by construction $\sum_{i=1}^{N}\hat{b}_{0i} = 1$, and in effect only $N-1$ factor loadings are estimated.

4 Average Volatility Models

Considering the restrictive nature of the multivariate volatility models discussed in the previous section, model averaging techniques that explicitly allow for parameter and model uncertainty could be particularly important in risk management. An important example is the Bayesian Model Averaging (BMA) that combines the models under consideration using their respective posterior probabilities.[6] Let $\Pr(M_i \,|\mathcal{F}_{t-1})$ be the posterior probability of model M_i, and let $\Pr(\mathbf{r}_t \,|\mathcal{F}_{t-1}\,, M_i)$, be the predictive density of $\mathbf{r}_t$ conditional on model M_i and the in-sample information available, namely $\mathcal{F}_{t-1}$. Finally, let the space of the models under consideration be $\mathcal{M} = \bigcup_{i=1}^{m} \{M_i\}$.. Then the predictive density of $\mathbf{r}_t$ conditional on $\mathcal{F}_{t-1}$ is given by the so-called BMA formula

$$\Pr(\mathbf{r}_t \,|\mathcal{F}_{t-1},\, \mathcal{M}) = \sum_{i=1}^{m} \Pr(M_i \,|\mathcal{F}_{t-1}) \Pr(\mathbf{r}_t \,|\mathcal{F}_{t-1}\,, M_i).$$

where $\Pr(M_i \,|\mathcal{F}_{t-1})$ is the posterior probability of model M_i, given by

$$\Pr(M_i \,|\mathcal{F}_{t-1}) = \frac{\Pr(M_i) \int_{\mathbf{\Theta}_i} \Pr(\boldsymbol{\theta}_i \,|M_i)\, L_i(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_{t-1} \,|\mathbf{r}_0, ..., \mathbf{r}_{-s_i+1}, \boldsymbol{\theta}_i) d\boldsymbol{\theta}_i}{\sum_{j=1}^{m} \Pr(M_j) \int_{\mathbf{\Theta}_j} \Pr(\boldsymbol{\theta}_j \,|M_i)\, L_j(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_{t-1} \,|\mathbf{r}_0, ..., \mathbf{r}_{-s_j+1}, \boldsymbol{\theta}_j) d\boldsymbol{\theta}_j}, \quad (17)$$

$\Pr(M_i)$ is the prior probability of model M_i, $L_i(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_{t-1} \,|\mathbf{r}_0, ..., \mathbf{r}_{-s_i+1}, \boldsymbol{\theta}_i)$ is the joint probability distribution of the observations $\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_{t-1}$ conditional on the given initial values $\mathbf{r}_0, ..., \mathbf{r}_{-s_i+1}$, and $\Pr(\boldsymbol{\theta}_i \,|M_i)$ is the prior on $\boldsymbol{\theta}_i$ conditional on M_i. The Bayesian approach requires *a priori* specifications of $\Pr(M_i)$ and $\Pr(\boldsymbol{\theta}_i \,|M_i)$ for $i = 1, 2, ..., m$, and can be quite demanding computationally, particularly in the case of multi-variate volatility models with many unknown parameters. As a result other approaches to the determination of the model weights, $\lambda_{i,t-1} = \Pr(M_i \,|\mathcal{F}_{t-1})$, have been proposed in the literature. For example, in the case of the univariate models it is suggested that the model weights are based on the relative past forecast accuracy of the models under consideration. Alternatively, $\lambda_{i,t-1}$ are approximated by

[6] A formal Bayesian solution to the problem of model uncertainty is reviewed, for example, in Draper (1995) and Hoeting, Madigan, Raftery, and Volinsky (1999). Recent applications to time series econometrics are provided in Fernandez et al. (2001a,b), Garratt, Lee, Pesaran, and Shin (2003) and Godsill, Stone, and Weeks (2004).

the Akiake weights (discussed in Chapter 4 of Burnham and Anderson (1998)) or the Schwartz weights - the latter gives a Bayesian approximation with t the estimator sample sufficiently large. In the present applications, due to large time series data being available, parameter uncertainty is likely to be of second order importance in the case of volatility models where the parameters can be estimated consistently under each model. See, for example, Burnham and Anderson (1998). In particular, we have

$$\Pr(\mathbf{r}_t \,|\mathcal{F}_{t-1},\, \mathcal{M}) \approx \sum_{i=1}^{m} \lambda_{i,t-1} \Pr(\mathbf{r}_t \,|\mathcal{F}_{t-1} \,, M_i), \tag{18}$$

with the pre-determined weights

$$\lambda_{i,t-1} = \frac{\exp\left(\Delta_{i,t-1}\right)}{\sum_{j=1}^{m} \exp(\Delta_{j,t-1})}. \tag{19}$$

Note that the model weights $\lambda_{i,t-1}, i = 1, 2, ..., m$ are pre-determined when the decision over the the portfolio weights, $\omega_{j,t-1},\ j = 1, 2, ..., N$, is taken. This is possible since it is assumed that there are no feedbacks from trade decisions to the probability models being considered. In the case of AIC and SBC we have, respectively,

$$\Delta_{i,t-1}^{AIC} = AIC_{i,t-1} - Max_j \left(AIC_{j,t-1}\right),\ \ AIC_{i,t-1} = LL_{i,t-1} - k_i, \tag{20}$$

$$\Delta_{i,t-1}^{SBC} = SBC_{i,t-1} - Max_j \left(SBC_{j,t-1}\right),\ \ SBC_{i,t-1} = LL_{i,t-1} - \left(\frac{k_i}{2}\right) \ln(t-1). \tag{21}$$

We do, however, recognize that for the small to moderate sample sizes used in macro-economic applications the choice of priors could be important, particularly if the object of exercise is the estimation of the marginal probability densities.

The Bayesian model averaging *formula* also provides a simple 'optimal' solution to the problem of pooling of the point forecasts, namely

$$E\left(\mathbf{r}_t \,|\mathcal{F}_{t-1},\, \mathcal{M}\right) = \sum_{i=1}^{m} \lambda_{i,t-1} E(\mathbf{r}_t \,|\mathcal{F}_{t-1}, M_i),$$

with the pooled variance matrix given by

$$V\left(\mathbf{r}_t\,|\mathcal{F}_{t-1},\,\mathcal{M}\right) = \sum_{i=1}^{m}\lambda_{i,t-1}V(\mathbf{r}_t\,|\mathcal{F}_{T-1},M_i) + \sum_{i=1}^{m}\lambda_{i,t-1}\left[E(\mathbf{r}_t\,|\mathcal{F}_{t-1},M_i) - E\left(\mathbf{r}_t\,|\mathcal{F}_{t-1},\,\mathcal{M}\right)\right]^2 \tag{22}$$

It is also worth noting that the combined model, $\Pr(\mathbf{r}_t\,|\mathcal{F}_{t-1},\,\mathcal{M})$, will not belong to the class of multivariate normal distribution even if all the underlying models, M_i, $i = 1, 2, ..., m$, belong to that class, so long as the models being considered are not the same and none of the underlying models are assigned a weight of unity. This means that under model averaging it is not sufficient to combine point and volatility forecasts. Combining of probability models also affect the shape of the resultant probability distribution, which can not be easily inferred from those of the constituent models averaging.

Finally, a number of non-Bayesian model averaging procedures have also been considered in the literature, the most recent example being the so-called 'thick' modeling discussed in Granger and Jeon (2004) and applied to point forecasts of excess returns across different models by Aiolfi, Favero, and Primiceri (2001) and Aiolfi and Favero (2002). This procedure typically involves averaging the top 10%, 20% or 50% of point forecasts from models ranked by model selection criteria such as AIC or SBC. In our empirical analysis we shall extend the application of these techniques to multivariate volatility models.

5 Value-at-Risk Based Diagnostic Tests

This section considers how the different modeling strategies set out above can be evaluated from the perspective of their use in risk management. At first we describe approaches suitable for assessing the validity of any given individual model M_i. Next, we describe how to extend the analysis to models obtained by application of Bayesian-type model averaging techniques.

5.1 VaR Diagnostics for Individual Models

In the econometric literature models are often evaluated by their out-of-sample forecast performance using standard metrics such as the RMSFE but, as discussed in Section 2, the application of this approach to volatility models is subject to a number of difficulties. An alternative approach would be to employ decision-based evaluation techniques and compare different volatility models in terms of their performance in trading and risk management.[7] In this section we propose simple examples of such a procedure based on the VaR problem set out in Section 2.[8]

Consider first the VaR constraint (3) associated with the passive version of the risk management problem where the portfolio weights, $\boldsymbol{\omega}_{t-1}$, are given, and suppose that the analysis is carried out conditional on model M_i. In this setting the VaR constraint will be given by

$$\Pr\left(\rho_t < -\bar{\rho}_{i,t-1}\,|\mathcal{F}_{t-1}, M_i\right) \leq \alpha, \tag{23}$$

and $\bar{\rho}_{i,t-1}$ will be a function of α and the assumed volatility model, M_i. To fully specify the model, assume that the de-volatized returns, $\mathbf{z}_t$, have a joint cumulative distribution function $F_{it}(\cdot)$ which is closed under linear combinations so that $\mathbf{c}'\mathbf{z}_t$ also has (univariate) distribution $F_{it}(\cdot)$ for any N-dimensional vector $\mathbf{c}$. A special case of our results is obtained if $\mathbf{z}_t$ is assumed to follow the multivariate normal or the Student t distribution, since both distributions are closed under linear transformations. Conditional on $\mathcal{F}_{t-1}$ and model M_i being true, ρ_t will have mean zero and variance

$$\sigma^2_{\rho t}(M_i) = \boldsymbol{\omega}'_{t-1}\boldsymbol{\Sigma}_{it}\,\boldsymbol{\omega}_{t-1}.$$

Therefore, under (8) we have

$$z_{\rho t}(M_i) = \frac{\boldsymbol{\omega}'_{t-1}\mathbf{r}_t}{\sigma_{\rho t}(M_i)}\,|\mathcal{F}_{t-1}, M_i \sim (F_{it}, 0, 1), \tag{24}$$

which implies that under M_i, $z_{\rho t}(M_i)$ is a martingale difference sequence with a unit variance. Note, however, that $z_{\rho\, t}(M_i)$ need not be independent across time. Temporal dependence in $z_{\rho\, t}(M_i)$ could arise not only due to possible higher-order moment dependence of the underlying innovations $\mathbf{z}_t$, but also because of possible serial dependence of portfolio weights and the temporal dependence of $\boldsymbol{\Sigma}_{it}$.

[7] For a general discussion of decision-based evaluation techniques see Granger and Pesaran (2000a,b) and Pesaran and Skouras (2002).

[8] For a review of existing approaches to the evaluation of the VaR estimates see Lopez (1999). These procedures tend to be purely statistical in nature and are not explicitly related to particular multivariate volatility models.

Denoting the value of , $\bar{\rho}_{i,t-1}$that satisfies (23) by $\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1},\alpha)$ and assuming that (24) holds, then

$$F_{it}\left(-\frac{\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1},\alpha)}{\sigma_{\rho t}(M_i)}\right) \leq \alpha.$$

But since $F_{it}(\cdot)$ is a continuous and monotonically non-decreasing function we have

$$-\frac{\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1},\alpha)}{\sigma_{\rho t}(M_i)} = F_{it}^{-1}(\alpha) = -c_{it}(\alpha),$$

or

$$\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1},\alpha) = c_{it}(\alpha)\ \sigma_{\rho t}(M_i), \tag{25}$$

where $-c_{it}(\alpha)$ is the $\alpha\%$ critical value of the distribution of $z_{\rho t}(M_i)$ conditional on $\mathcal{F}_{t-1}$, and model M_i. Note that $c_{it}(\alpha)$ and $\sigma_{\rho t}(M_i)$ are based on observations available at time $t-1$, and this is highlighted in the notation used for $\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1},\alpha)$.. For certain choice of the joint distribution of the $F_{it}(\cdot)$ of the devolatized returns $\mathbf{z}_t$, however, $F_{it}(\cdot) = F_i(\cdot)$, meaning that the conditional distribution is independent of t. This is the case when F_{it} is the multivariate normal or the multivariate Student t distribution with a time invariant degrees of freedom. The above derivations hold even if the portfolio weights, $\boldsymbol{\omega}_{t-1}$, are derived conditional on model M_i. In that case the portfolio weights should be denoted by $\boldsymbol{\omega}_{i,t-1}$ to highlight their dependence on the choice of the volatility model. But to simplify the notations we continue to represent the portfolio weights without the subscript i.

The evaluation of model M_i can now proceed in the following manner. Suppose that the evaluation exercise starts on day $t = \tau + 1$ with the available sample of T observations split at this date into $T = T_0 + (T - T_0)$ for some $0 < T_0 < T$. Further suppose that the first T_0 observations before day $\tau + 1$ are used for estimation whereas the last $T_1 = T - T_0$ observations are used for evaluation purposes. Accordingly, we define the sets of estimation and evaluation dates by $\mathcal{T}_0 = \{\tau - T_0 + 1, \tau - T_0 + 2, ..., \tau\}$, and $\mathcal{T}_1 = \{\tau + 1, \tau + 2, ..., \tau + T_1\}$, respectively.

A simple test of the validity of model M_i from the perspective of the VaR can then be based on the proportion of days in the evaluation sample where the VaR is violated:

$$\hat{\pi}_i = \frac{1}{T_1}\sum_{t\in\mathcal{T}_1} d_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}\right),$$

where

$$d_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}\right) = I[-\rho_t - c_{it}(\alpha)\ \hat{\sigma}_{\rho t}\,(M_i)],$$

and

$$\hat{\sigma}_{\rho t}(M_i) = \left(\boldsymbol{\omega}'_{t-1}\hat{\boldsymbol{\Sigma}}_{it}\,\boldsymbol{\omega}_{t-1}\right)^{\frac{1}{2}},\ \hat{\boldsymbol{\Sigma}}_{it} = \boldsymbol{\Sigma}_{it}(\hat{\boldsymbol{\theta}}_{iT_0}),$$

Recall that $\hat{\boldsymbol{\theta}}_{iT_0}$ is the PMLE of the unknown parameters (if any) of $\boldsymbol{\Sigma}_{it}$ under model M_i (see (5)), and $I(\cdot)$ as an indicator function.

We now present two Theorems. The first establishes the distribution of $T_1\hat{\pi}_i$ under the null hypothesis defined by

$$H_{i0}:\ \boldsymbol{\Sigma}_t =\ \boldsymbol{\Sigma}_{it} \text{ and } \mathbf{z}_t \mid \mathcal{F}_{t-1}, M_i \sim (F_{it},\ \mathbf{0}, \mathbf{I}_N). \tag{26}$$

for $T_1 < \infty$ and as $T_0 \to \infty$. The second Theorem establishes the asymptotic distribution of the following standardized test statistic based on $\hat{\pi}_i$

$$z_{\hat{\pi}_i} = \frac{\sqrt{T_1}(\hat{\pi}_i - \alpha)}{\sqrt{\alpha(1-\alpha)}} \tag{27}$$

under H_{i0}, and as $T_1/T_0 + 1/T_1 \to 0$. The proofs of both theorems are provided in the Appendix.

Theorem 1 *(finite-T_1 distribution) Assume that $\boldsymbol{\Sigma}_{it}(\theta_i)$ is continuous in θ_i and that (7) holds. Let $Bi(T_1, \alpha)$ define a Binomial distribution with parameters T_1 and α. Then under H_{i0},*

$$T_1\hat{\pi}_i \xrightarrow{d} Bi(T_1, \alpha) \qquad as\ T_0 \to \infty, \tag{28}$$

for any finite T_1, $0 < \alpha < 1$, and any sequence of portfolio weights, $\boldsymbol{\omega}_{t-1}$, $t = 0, \pm 1...$, satisfying $\parallel \boldsymbol{\omega}_{t-1} \parallel > 0$.

Remark. This result is important for cases when T_1 is small or, alternatively, when one is interested in testing VaR performance of a given set of portfolios for small values of α. In such cases the asymptotic normal distribution presented below might not provide a sufficiently accurate approximation.

Theorem 2 *(asymptotic distribution) Assume that (i) $f_{it}(\cdot) = F'_{it}(\cdot)$ exists and $\bar{f}_{it} = \sup_x f_{it}(x) < \infty$ for any t; (ii) condition (7) holds and $\boldsymbol{\theta}_{i0}$ belongs to the interior of the compact set Θ_i; (iii) $\boldsymbol{\Sigma}_{it}(\theta_i)$ is twice continuously* *differentiable in* $\boldsymbol{\theta}_i$ *such that for some* $\delta > 1$

$$\inf_{\boldsymbol{\theta}_i \in \Theta_i} \underline{\lambda}_{it}(\boldsymbol{\theta}_i) > 0 \ a.s.,$$

$$E\left\{\sup_{\boldsymbol{\theta}\in\Theta_i} \frac{\| \partial\bar{\lambda}_{it}(\boldsymbol{\theta})/\partial\boldsymbol{\theta} \|}{\underline{\lambda}_{it}^{\frac{1}{2}}(\boldsymbol{\theta})\underline{\lambda}_{it}^{\frac{1}{2}}(\boldsymbol{\theta}_{i0})}\right\}^{\delta} = \mu_{it}, \ \frac{1}{T_1}\sum_{t\in\mathcal{T}_1}^{T} \bar{f}_{it}\mu_{it}^{1/\delta} = O(1), \qquad (29)$$

where $\bar{\lambda}_{it}(\boldsymbol{\theta}_i)$ *and* $\underline{\lambda}_{it}(\boldsymbol{\theta}_i)$ *define, respectively, the maximum and the minimum eigenvalues of* $\boldsymbol{\Sigma}_{it}(\boldsymbol{\theta}_i)$, *with* $\| \cdot \|$ *being the Euclidean norm; (iv) for* T_0 *sufficiently large*

$$E \| \hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \|^{\frac{\delta}{\delta-1}} = O(T_0^{-\delta/(2(\delta-1))}). \qquad (30)$$

Then under H_{i0}, *any* $0 < \alpha < 1$,

$$z_{\hat{\pi}_i} \xrightarrow{d} N(0,1) \quad \text{as } \frac{T_1}{T_0} + \frac{1}{T_1} \to 0,$$

for any sequence of portfolios $\boldsymbol{\omega}_{t-1}$, $t = 0, \pm 1...$, *satisfying* $\| \boldsymbol{\omega}_{t-1} \| > 0$.

Remarks.
(i) It is important to note that the null distribution of $z_{\hat{\pi}_i}$ does not depend on the portfolio weights, $\boldsymbol{\omega}_{t-1}$, although the power of the test typically does depend on $\boldsymbol{\omega}_{t-1}$.
(ii) The mild condition for consistency of the test is that $\hat{\pi}_i$ does not converge in probability to α as $T_1/T_0 + 1/T_1 \to 0$. This can happen if either we use the wrong conditional covariance matrix or the wrong innovation distribution, or both. For example, in the first case, under $M_j : \boldsymbol{\Sigma}_{jt} \neq \boldsymbol{\Sigma}_{it}$ we have

$$E(\hat{\pi}_i | M_j) = \frac{1}{T_1}\sum_{t\in\mathcal{T}_1} E\left[F_{it}\left(-c_{it\,\alpha} q_{ij,t}\right)\right],$$

where

$$q_{ij,t} = \left(\frac{\boldsymbol{\omega}'_{t-1}\hat{\boldsymbol{\Sigma}}_{it}\,\boldsymbol{\omega}_{t-1}}{\boldsymbol{\omega}'_{t-1}\boldsymbol{\Sigma}_{jt}\,\boldsymbol{\omega}_{t-1}}\right)^{1/2}, \text{ for } t \in \mathcal{T}_1.$$

It is clear that under M_j, $q_{ij,t}$ does not tend to unity and in general $E(\hat{\pi}_i | M_j)$ will diverge from its hypothesized value of α, and the power of the test tends to unity with T_1.

(iii) Most likely, the assumptions required for (6) and (7) will imply (29) but we felt it is necessary to make the additional explicit assumptions since the former have been formally established only for a sub-class of multivariate volatility models considered in this paper.
(iv) When model M_i is not subjected to estimation, such as for some of the models we consider, then the theorem applies by setting $\hat{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_{i0}$ and the conditions (29) and (30) are no longer needed. In particular, the non-singularity condition of the model conditional covariance matrix is not required.
(v) Under the null hypothesis $H_{i0} : E(z_{\rho t}(M_i) \mid \mathcal{F}_{t-1}) = 0$. This is a key property since it implies that $I(-z_{\rho t}(M_i) - c_{it\,\alpha}) - \alpha$ is also a martingale difference process. Strict stationarity of the asset returns is not required.
(vi) It is likely that (29) holds for a relatively large δ implying a weaker moment condition in (30). Obviously the limit of $\sqrt{T_0} \parallel \hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \parallel$ has all the moments by asymptotic normality but we require a stronger moment condition.
(vii) The importance of the condition $T_1/T_0 \to 0$ in cross validation of forecasts was put forward by West (1996). McCracken (2000) extends West's framework to allow for non-differentiable loss functions in a regression set-up.

5.2 VaR-Based Diagnostics for Average Models

Suppose that set of m models is described by

$$\mathbf{r}_t \,|\mathcal{F}_{t-1}, M_i \sim (F_{it}, \mathbf{0}, \boldsymbol{\Sigma}_{it}),\ \ i = 1, 2, ..., m.$$

Therefore, $F_{it}(\cdot)$ defines the conditional distribution of the observed return $\mathbf{r}_t$, given $\mathcal{F}_{t-1}$ and model M_i.

The probability distribution function of portfolio return, ρ_t, based on the average model obtained with respect to these models using the weights, $\lambda_{i,t-1}$, is then given by

$$\Pr(\rho_t < a \ \ |\mathcal{F}_{t-1}, M) = \sum_{i=1}^{m} \lambda_{i,t-1} F_{it} \left(\frac{a}{\sigma_{\rho t}(M_i)} \right).$$

In cases where $\Pr(\rho_t < a \,|\mathcal{F}_{t-1}\,, M_i)$ does not have a closed form it needs to be computed by stochastic simulations, noting that conditional on model M_i

we have, as $J \to \infty$,

$$\frac{1}{J}\sum_{j=1}^{J} I(-\boldsymbol{\omega}'_{t-1}\mathbf{r}_{jt}^{(i)} + a) \to \Pr(\rho_t < a\,|\mathcal{F}_{t-1}\,, M_i) \quad \text{almost surely,}$$

where J is the number of replications and $\mathbf{r}_{jt}^{(i)}$ is the j^{th} draw from the assumed distribution of $\mathbf{r}_t$ under M_i. On the other hand, when the probability distribution of $\mathbf{r}_t$ under M_i are closed under linear transformations, as with Gaussian or multivariate t distribution, the computations can be simplified considerably by drawing from the distribution of $\rho_t = \boldsymbol{\omega}'_{t-1}\mathbf{r}_t$ under M_i directly or using the closed-form expression when the latter exists.

It is now easy to generalize the diagnostic test statistics given by (27) for an individual model M_i, to the case of an average model. For a given α we need to find the value for $\bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1}, \alpha)$, the VaR associated with the BMA forecast probabilities, for which

$$\sum_{i=1}^{m} \lambda_{i,t-1} F_{it}\left(-\frac{\bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1},\alpha)}{\sigma_{\rho t}(M_i)}\right) \leq \alpha.$$

To solve for $\bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1},\alpha)$, let

$$g(\kappa) = \sum_{i=1}^{m} \lambda_{i,t-1} F_{it}\left(-\frac{\kappa}{\sigma_{\rho t}(M_i)}\right) - \alpha = 0, \tag{31}$$

and note that $g(\kappa) = 0$ has a unique positive solution under the additional assumption that all the model densities $f_{it}(\cdot) = F'_{it}(\cdot)$ are differentiable and have a unique maximum at zero. In the case of such distributions $\bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1},\alpha)$ can be easily computed using numerical techniques such as the Newton-Raphson iterative procedure. The VaR diagnostic statistic, given by (27), can then be computed for the average model using

$$\hat{d}_{bt} = I\left(-\rho_t - \bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1},\alpha)\right),$$

in place of $d_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}\right)$.

5.3 Tail Behavior of Average Volatility Models

It is well known that linear combinations of normal distributions is not normal, although the moments of the mixture distribution are effectively linear combinations of the corresponding moments of the individual normal distributions, with the same weights. For instance we have seen that the pooled volatility forecast of portfolio return, with zero conditional mean, is given by (see (22))

$$V\left(\rho_t \left|\mathcal{F}_{t-1}, M\right.\right) = \sum_{i=1}^{m} \lambda_{it-1}\sigma^2_{\rho t}(M_i).$$

However

$$\sum_{i=1}^{m} \lambda_{it-1}\Phi\left(\frac{a}{\sigma_{\rho t}(M_i)}\right) \neq \Phi\left(\frac{a}{\sqrt{\sum_{i=1}^{m} \lambda_{it-1}\sigma^2_{\rho t}(M_i)}}\right), \tag{32}$$

unless $\mathbf{\Sigma}_{it} = \mathbf{\Sigma}_t$ for all i, where $\Phi(\cdot)$ defines the normal cumulative distribution function. The following Theorem, whose proof is reported in the Appendix, characterizes the direction of the bias. In risk management applications where $a < 0$ and one is interested in tail probabilities, it is easily seen that the correctly combined model, on the left hand side of (32), will be more fat-tailed than the associated Gaussian model with the same average volatility measure, on the right hand side of (32), whenever $a < -\sqrt{3}\sigma_{\rho t}(M_i)$, $i = 1, ..., m$. As we shall see this result has direct bearing on some of the empirical results that we shall be reporting.

Theorem 3 *Let $f(x)$ be a differentiable real function, with f' denoting its first-derivative, with $\int_{-\infty}^{\infty} \mid f(u) \mid du < \infty$. Let $F(z) = \int_{-\infty}^{z} f(u)du$. Then, for any constant a and any finite sequence $b_1, ..., b_N$ of strictly positive constants satisfying*

$$a\left[\left(\frac{a}{\sqrt{b_i}}\right) f'(\frac{a}{\sqrt{b_i}}) + 3f(\frac{a}{\sqrt{b_i}})\right] > 0, \;\; i = 1, ..., N, \tag{33}$$

it follows that

$$\sum_{i=1}^{N} \lambda_i F(\frac{a}{\sqrt{b_i}}) > F\left(\frac{a}{\sqrt{\sum_{i=1}^{N} \lambda_i b_i}}\right), \tag{34}$$

for any finite sequence $\lambda_1, ..., \lambda_N$ of non-negative constants such that $\lambda_1 + \lambda_2 + ... + \lambda_N = 1$, $\lambda_i < 1$, $i = 1, 2, ..., N$.

Remarks.

(i) When $f(u)$ is the standard normal density, for $a<0$ condition (33) is

$$\frac{a}{\sqrt{b_i}} < -\sqrt{3}, \quad i = 1, ..., n. \tag{35}$$

When $a>0$ condition (33) is instead

$$0 < \frac{a}{\sqrt{b_i}} < \sqrt{3}, \quad i = 1, ..., n.$$

although note that when $a>0$ (34) expresses the case where the tail probability of the average model is smaller than for the model with the average parameter $\sum_{i=1}^{n}\lambda_i b_i$.
(ii) When $f(u)$ is the standardized Student t with $\nu>2$ degrees of freedom, for $a<0$ the same condition (35) applies, independently from ν.

6 An Empirical Application

6.1 Data and Some Preliminary Analysis

The approach developed in this paper (the model averaging and the associated VaR evaluation tests) can be applied to a variety of problems in finance. Here we shall consider the daily VaR of portfolios constructed from 22 main industry indices of the Standard & Poor's 500. The source of our data is Datastream, which provides twenty four S&P 500 industry price indices according to the Global Industry Classification Standard. To ensure a sufficiently long span of daily prices we have excluded the 'Semiconductors & Semiconductor Equipment' and 'Real Estates' from our analysis. The list of the $N = 22$ industries included in our analysis is given in Table 1. Our data set covers the industry indices from 2nd January 1995 to 13th October 2003 ($T = 2291$ observation). Daily returns are computed as $r_{jt} = 100 * \ln\left(P_{jt}/P_{jt-1}\right)$, $j = 1, ..., 22$, where P_{jt} is the j^{th} price index. The realized returns $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{22,t})'$ exhibit all the familiar stylized features over our sample period. They are highly cross-correlated, with an average pair-wise cross-correlation coefficient of 0.5. A standard factor analysis yields that the two largest estimated eigenvalues are equal to 11.5 and 1.7, with the remaining being all smaller than unity. The unconditional daily volatility differs significantly across industries and lie in range of 1.13% (Food, Beverage & Tobacco) to 2.39% (Technology Hardware & Equipment). See Table

2. The first-order autocorrelation coefficients of the individual returns are quantitatively very small (ranging from -0.049 to 0.054) and are statistically significant only in the case of four out of the twenty two industries (Automobiles & Components, Health Care Equipment & Services, Diversified Financial, and Utilities). We decided not to filter out any serial correlation in the data since this would have probably induced a sizeable amount of noise, probably more harmful than the small amount of serial correlation present in the case of four of the assets. We derived non-parametric estimates of the density functions for the standardized returns, confirming that the marginal distributions tend to be symmetric and slightly fat-tailed. These results provide some support for our working assumption of zero mean returns, and highlight the non-Gaussian and the highly cross correlated nature of the asset returns by industries.

Estimates of univariate GARCH $(1,1)$ models for the returns summarized in Table 3 also provide some support in favour of a Student t distribution with a low degree of freedom for the conditional distribution of the asset returns. The degrees of freedom estimated for the different assets lie in the relatively narrow range of $(5.2-11.7)$ with an average estimate of 7.3, and a mid-point value of 8.5. Estimation of multivariate volatility models with non-Gaussian distributions present considerable technical difficulties and are unlikely to significantly affect the QMLE estimates. As an operational compromise it seems justified to use the QMLE estimates with multivariate Student t distributions with low degrees of freedom. Based on the univariate estimates, 6 and 8 seem sensible choices and will be considered below.

6.2 Recursive Estimation of Multivariate Volatility Models

The types of volatility models included in the empirical analysis are set out in sub-sections (3.1) to (3.11). For each of these types a number of variations were considered, depending on the choice of the window size (n_0) when applicable, the pre-specified parameters of the Riskmetrics specifications (λ_0, ν_0) and the orders of the multivariate GARCH models (p, q, r, s). In particular, we considered the following parameter values $n_0 = 50, 75, 125, 250$, $\lambda_0 = 0.94, 0.95, 0.96$, $\nu_0 = 0.6, 0.8, 0.94$, $p, q \in \{1, 2\}$ and $r = s = 1$. In the case of the factor models of section 3.8 and 3.11, we considered only one factor.

All models were estimated recursively using an expanding window starting with 1784 observations as the first estimation sample, with the parameter values (when applicable) updated at monthly intervals. Clearly, the parameters of the volatility models could also have been updated daily. But it is not clear if the extra computations that such daily updates entail would have been worthwhile. Given the relatively large initial estimation sample of 1784, the addition of one extra data point seems unlikely to have made that much of a difference. On the other hand not updating the parameter estimates over the whole of the evaluation sample of 507 observations would also have been rather extreme and could have been favouring the models with pre-specified parameter values such as the Riskmetrics models. The monthly updates of the parameters can be viewed as a plausible and practical solution to a highly computer intensive problem.[9] Therefore, the models were estimated twenty-four times over the evaluation sample.

Since for certain values of p, q the estimation algorithm did not converge for all models and all data periods, we ended up with $m = 63$ different (nested and non-nested) models with convergent estimates. However, for some models the algorithm converged except for a few isolated time periods . In such cases the estimation results for the model in question was ignored by assigning a zero weight to it in the model averaging procedure in the non-convergent periods.[10]

All the models (estimated or with pre-specified parameter values) were then evaluated over the last two years of data (from November 2, 2001 to October 13, 2003, inclusive), with $T_1 = 507$, using one-day ahead forecasts of $\mathbf{\Sigma}_t$ under M_i, denoted by $\hat{\mathbf{\Sigma}}_{it}$, $i = 1, ..., 63$. The evaluations were carried out using two different types of portfolio weights: an equal-weighted portfolio, namely $\boldsymbol{\omega}_{t-1} = \boldsymbol{\omega} = N^{-1}(1, ..., 1)'$, yielding the portfolio return $\rho_t = \boldsymbol{\omega}'_{t-1}\mathbf{r}_t = \bar{r}_t$, and a sequence of artificially generated random portfolios given by

$$\boldsymbol{\omega}_t = U_t^{-1}(u_{1t}, ..., u_{Nt})', \;\; U_t = u_{1t} + ... + u_{Nt},$$

[9] We also carried out a straightforward cross-validation test where all models (when relevant) were estimated once using the first $T_0 = 1784$ observations and then evaluated using the last $T_1 = 507$ observations. Perhaps not surprisingly, the results were generally less satisfactory than those based on the recursively computed parameter updates. These pure cross-validation results are available from the authors on request.

[10] All the computations have been carried out in MatLab and the codes are available upon request. For estimation of CCC, DCC and O-GARCH we used the UCSD_GARCH Toolbox developed by Sheppard (2002). All other codes are our own.

where each N-dimensional vector $(u_{1t}, ..., u_{Nt})$ is a random draw from a multivariate uniform distribution in $(0, 1)$, with independent components. The portfolio weights, $\boldsymbol{\omega}_{t-1}$, are drawn independently across time.

It is also possible to use model-specific portfolio weights, $\boldsymbol{\omega}_{i,t-1}$, where the weights are determined recursively by a suitable expected utility maximization subject to the VaR constraint. Such an exercise would also involve modelling of the conditional mean returns, which has not been addressed in this paper.[11] Choice of the multivariate volatility model is clearly essential for this purpose, and the associated model-specific weights are likely to play an important role in any evaluation exercise involving alternative multivariate volatility models. However, such an exercise is well beyond the scope of the present study and deserves serious consideration elsewhere.

6.3 Modelling Strategies

A number of different modelling strategies may now be considered. One possibility would be to follow the classical approach and select the 'best' model from the set of models under consideration using model selection criteria such as AIC or SBC. Alternatively, as argued in this paper, the model uncertainty can be explicitly taken into account using Bayesian type model averaging procedures. A third approach would be to follow the so-called 'thick' modelling and average across a pre-specified fraction of top models ranked, for example, by AIC or SBC. We refer to these as 'best', 'Bayesian average' and 'thick average' modelling strategies. See Section 4 for further details and references to the literature.

When considering normal innovations, AIC selects the ADCC$(1, 2, 1, 1)$ throughout the evaluation period whereas SBC first selects the ADCC$(1, 1, 1, 1)$, then switches to ADCC$(1, 2, 1, 1)$ from the middle of the sample onwards. In the case of models under multivariate Student t with 6 and 8 degrees of freedom, AIC selected the O-GARCH$(2, 2)$ in the first three weeks of the evaluation sample, switching to DCC$(1, 2, 1, 1)$ up to the middle of the sample, with ADCC$(1, 2, 1, 1)$ being selected thereafter. Similar results were also obtained with SBC. With few exceptions, the DCC type models tended to dominate the remaining specifications. This outcome is particularly interesting since the evaluation sample includes the recent periods of large stock

[11] See Pesaran and Timmermann (2004) for a discussion of such an estimation strategy in real time.

market falls and contrast the outcome of recursive modelling applied to S&P mean returns reported in Pesaran and Timmermann (1995) where the best model selected for the monthly excess returns tend to change quite frequently over time. This could be due to the relative stability of volatility models as compared to models of mean returns that are known to be subject to structural breaks.

To provide some idea of the extent to which the DCC type models dominate other specifications, in Table 4 we summarize selected values of the AIC-penalized log-likelihood values, $AIC_{i,t-1}$, defined by (20), for all the 63 models computed using a multivariate Student t distribution with 8 degrees of freedom. Table 4 reports $AIC_{i,t-1}$ at the beginning of the evaluation sample (2 November 2001), at the end of the evaluation sample (13 October 2003) and its average value over the evaluation sample.[12] As can be seen the DCC (and CCC) type models systematically fit the data better than the other models, and the differences in the AIC-penalized log-likelihood values for the DCC and other models are sufficiently large for the model weights, $\lambda_{i,t-1}$, of the DCC models defined by (19), to take the extreme value of unity for most periods.[13] It is also interesting to note that on average the simplest of the data filters, namely the equal weighted moving average specification, EQMA, with $n_0 = 125$, or 250, do considerably better than the other filters and perform well even when compared to estimated models such as O-GARCH or Factor GARCH models. Similar conclusions are also reached if one uses the Gaussian innovations or the SBC criteria.

In implementing the thick modelling we used the top 25% and 50% of the models selected according to AIC or SBC and constructed equal-weighted average models. As an extreme benchmark we also considered an equal-weighted average model using all the 63 specifications.

6.4 VaR Diagnostic Test Results

For the individual and average models we recursively computed the VaR thresholds, $\bar{\rho}_{t-1}(\boldsymbol{\omega}_{t-1},\alpha)$, using (25) and (31), respectively, for equal-weighted and random portfolio weights, $\boldsymbol{\omega}_{t-1}$, assuming Gaussian and Student t dis-

[12] Recall that all estimations are carried out recursively using an expanding window starting in 2, January 1995.

[13] Notice that the model weights are obtained by exponentiation of the AIC-penalized log-likelihood values and even seemingly small differences in the average fit of the models can translate into major differences in model weights for sufficiently large sample sizes.

tributed devolatized returns with 6 and 8 degrees of freedom and two different values of α, namely $\alpha = 1\%$ and $\alpha = 5\%$. Using these threshold estimates we then computed, $\hat{\pi}$, the percentage of times (out of the 507 portfolio returns observed over the evaluation sample) that the VaR constraint were violated, and hence the VaR diagnostic statistic, $z_{\hat{\pi}}$, defined by (27). The results are summarized in Tables 5a and 5b for $\alpha = 1\%$ and $\alpha = 5\%$, respectively[14]. In view of the model selection results discussed above the test outcomes are very similar, and in many instances are identical for the AIC and SBC selection criteria. Furthermore, as predicted by the theory, the VaR diagnostic test results are insensitive to the choice of the portfolio weights. In contrast, choice of the distribution of the devolatized returns appears important. For example, the 'best' modelling strategy is rejected by the VaR test when the underlying distribution is assumed to be Gaussian but not if the Student t is used.

Also in the present application there are no differences in the test results for the average 'Bayesian' and the 'best' modelling strategies. As noted earlier, this is due to the fact that for most periods in the evaluation sample the 'best' model happen to totally dominate all other models, and as a result the average 'Bayesian' and the best models end up being the same for all practical purposes.

Comparing across strategies, the best outcome is found with respect to the thick modeling strategy when averaging across the best 15 models (best 25 % percentile). The test results are quite robust with respect to the choice of the conditional distribution of the innovations, although they deteriorate as we move from the normal distribution towards Student t with 6 degrees of freedom. This is in line with the theoretical result discussed in Section 5.3, where it was shown that the average model will be more fat-tailed than the underlying Gaussian or Student t models with the same average volatility. In cases where the underlying models are already fat tailed, the model averaging (without any single model dominating) can induce an excessive degree of fat-tailness. As can be seen from the results in Tables 5a and 5b, this tendency is accentuated as the coverage of thick modeling is increased, and is most acute when all the 63 models are included.

[14] We also computed the exact version of the VaR test defined by (28) and, not surprisingly given the size of the evaluation sample, we obtained very similar results.

6.5 Statistical Diagnostic Test Results

The different modelling strategies can also be evaluated using purely statistical techniques, in contrast to the decision-theoretic approach of the previous section. A statistical procedure, which is closer to ours, focuses on the probability density forecasts of a given portfolio return, $\rho_t = \boldsymbol{\omega}'_{t-1}\mathbf{r}_t$, and considers the following probability integral transforms

$$\hat{v}_{it} = \int_{-\infty}^{\rho_t} \hat{f}(x\,|\mathcal{F}_{t-1}, M_i)dx, \text{ for } t = \tau+1, ..., \tau+T_1,$$

where $\hat{f}(x\,|\mathcal{F}_{t-1}, M_i)$ is the estimated probability density of ρ_t under model M_i and conditional on the information available at time $t-1$. Making use of a well-known result due to Rosenblatt (1952) it is now easily seen that the sequence $\{\hat{v}_{it},\, t \in \mathcal{T}_1\}$ will be *i.i.d.* uniformly distributed on the interval $[0,1]$ if $\hat{f}(x\,|\mathcal{F}_{t-1}, M_i)$ coincides with the 'true' but unknown conditional predictive density of ρ_t. See Diebold, Gunther, and Tay (1998) and Diebold, Hahn, and Tay (1999) for further development of this idea in econometrics.

To test the hypothesis that $\hat{v}_{it}$ are random draws from the uniform $[0,1]$ distribution, we consider the standard Kolmogorov-Smirnov (KS) test

$$KS = \max_{1\leq j\leq T_1} \left| \frac{j}{T_1} - \hat{v}_j^* \right|, \tag{36}$$

as well as the Kuiper test

$$Ku = \max_{1\leq j\leq T_1} (\frac{j}{T_1} - \hat{v}_j^*) + \max_{1\leq j\leq T_1} (\hat{v}_j^* - \frac{j}{T_1}), \tag{37}$$

where $\hat{v}_1^* \leq \hat{v}_2^* \leq ... \leq \hat{v}_{T_1}^*$ represent the ordered values of the $\hat{v}_{i\tau+1}, ..., \hat{v}_{i\tau+T_1}$. The Kuiper test has the added advantage of placing greater emphasis on the tail behavior of the distribution.

Table 6 reports the p-values of these tests, computed using the analytic approximations provided in Stephens (1970), for the three modelling strategies and the two portfolios. The test results for the 'best' and the 'average' modelling strategies are identical, for the same reasons as noted above, and indicate a mild rejection (between 7 and 9 per cent level) of the models with Gaussian de-volatilized returns if the Kuiper test is used, but strongly in favor of the Student t distribution especially for 8 degrees of freedom. None of the specifications are rejected by the KS test. The Student t distribution is favored when considering the 'thick' modelling approach which includes the best 15 models but tend to be rejected when the average include a larger number of models. The opposite is observed with respect to the normal distribution. These conclusions tend to be robust to the choice of the portfolio weights.

Overall, the above statistical tests support the main conclusions reached using the VaR based diagnostics, although they appear to be less informative and less clear cut as far as the tail properties of the portfolio return distributions are concerned.

7 Summary and Conclusions

This paper focusses on the problem of model uncertainty in the case of large multivariate volatility models for use in asset management and risk monitoring. The problem is particularly important considering the highly restrictive nature of the multivariate volatility models that are used in practice. To deal with model uncertainty we advocate the use of model averaging techniques where an 'average' model is constructed by combining the predictive densities of the models under consideration, using a set of weights that reflect the models' relative in-sample performance. In a formal Bayesian framework the model weights would be set at their posterior probabilities, but as argued by Burnham and Anderson (1998), other weights based on model selection criteria could also be considered.

The paper also proposes a simple decision-based model evaluation technique that compares the volatility models in terms of their Value-at-Risk performance. The proposed test is applicable to individual as well as to average models, and can be used in a variety of contexts. Under certain regularity conditions, the test is shown to have a Binomial distribution when evaluation sample (T_1) is finite and T_0 (the estimation sample) is sufficiently large. The proposed test converges to a standard Normal variate provided $T_1/T_0 + 1/T_1 \to 0$, a condition encountered in forecast evaluation literature, as discussed in West (1996). The proposed VaR test is also invariant to the portfolio weights and is shown to be consistent under departures from the null hypothesis. The Binomial version of the VaR test could have important applications in credit risk literature where the evaluation samples are typically short.

In the empirical application we experimented with AIC and SBC weights and found that, due to the large sample sizes available, they led to very similar results with the selected models often totally dominating the rest. The model most often selected by both criteria turned out to be the Asymmetric Dynamic Conditional Correlation (ADCC) model of Cappiello, Engle and Sheppard (2002). In the out of sample evaluation tests, only the multivariate Student t version of the ADCC model with 8 degrees of freedom managed to pass the VaR diagnostic tests. Interesting enough, the simplest of all data filters used in this paper, namely the Equal Weighted Moving Average filter also performed well; doing better than other data filters as well as the remaining estimated models, namely O-GARCH and Factor GARCH specifications.

We also considered other model averaging techniques, such as the so-called 'thick' modelling, and found strong evidence in their favor in our empirical applications, especially when averaging across a small number of models. Our empirical analysis clearly shows the relevance of the proposed VaR test for the evaluation of the multivariate volatility models from a decision making perspective. It also shows the potential importance of model averaging for risk management.

Finally, it is important to note that while model averaging provides a useful alternative to the two-step model selection strategy, it is nevertheless subject to its own form of uncertainty, namely the choice of the space of models to be considered and their respective weights. It is therefore important that applications of model averaging techniques are investigated for their robustness to such choices. In the case of our application it is clearly desirable that other forms of multivariate volatility models are also considered, which could be the subject of future research.

Mathematical Appendix

Proof of Theorem 1. Clearly as $T_0 \to \infty$

$$\hat{\pi}_i \to_p \pi_i = \frac{1}{T_1} \sum_{t \in \mathcal{T}_1} d_{it}, \;\; d_{it} = I\left(-\rho_t - c_{it\,\alpha}\sigma_{\rho\,t}(M_i)\right).$$

Consider now the moments of $T_1\pi_i$ and note that for any integer $n \geq 1$,

$$E\left(T_1\pi_i\right)^n = \sum_{t_1, t_2, \ldots, t_n \in \mathcal{T}_1} \left\{E\left(d_{it_1} d_{it_2} \ldots d_{it_n}\right)\right\}. \tag{38}$$

However, for any $\delta > 0$ we have

$$E(d_{it}^{\delta} \mid \mathcal{F}_{t-1}, M_i) = \alpha,$$

or unconditionally

$$E(d_{it}^{\delta} \mid M_i) = \alpha.$$

Hence, all the terms $E\left(d_{it_1} d_{it_2} \ldots d_{it_n}\right)$ in (38) coincide with the case when the d_{it_j}, $j = 1, .., n$, are $i.i.d$ Bernoulli distributed random variables with parameter α, for any choice of $t_1, \ldots, t_n$. Also, since $T_1 < \infty$, the support of the distribution of $T_1\pi_i$ is bounded and as a consequence its moment generating function exists and is the same as that of a Binomial distribution with parameters T_1 and α. Therefore, by method of moments (see Billingsley (1986, Theorem 30.1)), $T_1\pi_i$ will also have a Binomial distribution. ■

Proof of Theorem 2. Assume H_{i0} defined by (26) holds. Set

$$q_{it} = q_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}, \boldsymbol{\theta}_{i0}\right) = \left(\frac{\hat{\sigma}_{\rho\,t}(M_i)}{\sigma_{\rho\,t}(M_i)}\right) = \left(\frac{\boldsymbol{\omega}'_{t-1}\hat{\boldsymbol{\Sigma}}_{it}\,\boldsymbol{\omega}_{t-1}}{\boldsymbol{\omega}'_{t-1}\boldsymbol{\Sigma}_{it}\,\boldsymbol{\omega}_{t-1}}\right)^{1/2}.$$

Then

$$E\left[d_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}\right)|\mathcal{F}_{t-1}, M_i\right] = F_{it}(-c_{it\,\alpha}q_{it})$$

and

$$E\left[\hat{\pi}_i\,|M_i\right] = \frac{1}{T_1}\sum_{t \in \mathcal{T}_1} E\left\{F_{it}\left(-c_{it\,\alpha}q_{it}\right)\right\}.$$

As $T_0 \to \infty$, $\hat{\boldsymbol{\theta}}_{iT_0} \xrightarrow{p} \boldsymbol{\theta}_{i0}$ and since $\boldsymbol{\Sigma}_{it}(\boldsymbol{\theta}_i)$ is a continuous function of $\boldsymbol{\theta}_i$ it also follows that $q_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}, \boldsymbol{\theta}_{i0}\right) \xrightarrow{p} 1$, for all values of $t \in \mathcal{T}_1$. Hence, for any given *finite* evaluation sample size, T_1, and as $T_0 \to \infty$,

$$E\left[\hat{\pi}_i\,|M_i\right] = \frac{1}{T_1}\sum_{t \in \mathcal{T}_1} E\left\{F_{it}\left(-c_{it\,\alpha}q_{it}\right)\right\} \xrightarrow{p} F_{it}\left(-c_{it\,\alpha}\right) = \alpha..$$

Consider now the statistic $\sqrt{T_1}(\hat{\pi}_i - \alpha)$ and write it as

$$\sqrt{T_1}(\hat{\pi}_i - \alpha) = \sqrt{T_1}(\pi_i - \alpha) + \sqrt{T_1}(\hat{\pi}_i - \pi_i), \tag{39}$$

where

$$\pi_i = \frac{1}{T_1} \sum_{t \in \mathcal{T}_1} d_{it}\left(\boldsymbol{\theta}_{i0}\right).$$

Also note that

$$\sqrt{T_1}(\hat{\pi}_i - \pi_i) = \sqrt{\frac{T_1}{T_0}} \left(\frac{\sum_{t \in \mathcal{T}_1} X_{it,T_0}}{T_1} \right),$$

where

$$X_{it,T_0} = \sqrt{T_0} \left[d_{it}(\hat{\boldsymbol{\theta}}_{iT_0}) - d_{it}\left(\boldsymbol{\theta}_{i0}\right) \right].$$

But it is easily seen that,

$$|X_{it,T_0}| = \begin{cases} \sqrt{T_0} & \text{if } (\rho_t + c_{i\,\alpha} \hat{\sigma}_{\rho\,t}(M_i))(\rho_t + c_{i\,\alpha}\sigma_{\rho\,t}(M_i)) < 0, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, for all $t \in \mathcal{T}_1$

$$\Pr\left(|X_{it,T_0}| = \sqrt{T_0}\, |\mathcal{F}_{t-1}, M_i \right) \leq \left| F_{it}\left(-c_{i\,\alpha} q_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}, \boldsymbol{\theta}_{i0} \right) \right) - F_{it}\left(-c_{it\,\alpha}\right) \right|,$$

and consequently

$$E\left(|X_{it,T_0}| \quad |\mathcal{F}_{t-1}, M_i\right) \leq \sqrt{T_0} \left| F_{it}\left(-c_{it\,\alpha} q_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}, \boldsymbol{\theta}_{i0} \right) \right) - F_{it}\left(-c_{it\,\alpha}\right) \right|.$$

Using the mean-value expansion of $F_{it}(-c_{it\,\alpha} q_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}, \boldsymbol{\theta}_{i0} \right))$ around $\hat{\boldsymbol{\theta}}_{iT_0}$

$$\begin{aligned} F_{it}\left(-c_{it\,\alpha} q_{it}\left(\hat{\boldsymbol{\theta}}_{iT_0}, \boldsymbol{\theta}_{i0} \right) \right) &= F_{it}\left(-c_{it\,\alpha}\right) - c_{it\,\alpha} f_{it}\left(-c_{it\,\alpha} q_{it}\left(\bar{\boldsymbol{\theta}}_i, \boldsymbol{\theta}_{i0} \right) \right) \times \\ & \frac{\partial q_{it}\left(\bar{\boldsymbol{\theta}}_i, \boldsymbol{\theta}_{i0} \right)}{\partial \hat{\boldsymbol{\theta}}_{iT_0}'} \left(\hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \right), \end{aligned}$$

where the elements of $\bar{\boldsymbol{\theta}}_i$ are convex combinations of the corresponding elements of $\hat{\boldsymbol{\theta}}_{iT_0}$ and $\boldsymbol{\theta}_{i0}$. By the Holder's inequality for norm of matrices, since $\| \boldsymbol{\omega}_t \| > 0$, we have

$$\begin{aligned} E\left(|X_{it,T_0}| \quad |\mathcal{F}_{t-1}, M_i\right) &\leq c_{it\,\alpha} f_{it}\left(-c_{it\,\alpha} q_{it}\left(\bar{\boldsymbol{\theta}}_i, \boldsymbol{\theta}_{i0} \right) \right) \times \\ & \left\| \frac{\partial q_{it}\left(\bar{\boldsymbol{\theta}}_i,\ \boldsymbol{\theta}_{i0} \right)}{\partial \hat{\boldsymbol{\theta}}_{iT_0}} \right\| \sqrt{T_0} \left\| \hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \right\| \\ &\leq c_{it\,\alpha} f_{it}\left(-c_{it\,\alpha} q_{it}\left(\bar{\boldsymbol{\theta}}_i, \boldsymbol{\theta}_{i0} \right) \right) \left\{ \sup_{\theta \in \Theta_i} \frac{\| \partial \bar{\lambda}_{it}(\boldsymbol{\theta}) / \partial \boldsymbol{\theta} \|}{\underline{\lambda}_{it}^{\frac{1}{2}}(\theta) \underline{\lambda}_{it}^{\frac{1}{2}}(\theta_0)} \right\} \times \\ \sqrt{T_0} \quad & \left\| \hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \right\|. \end{aligned}$$

Taking the unconditional mean and using the Holder inequality again yields

$$
\begin{aligned}
& E\left(|X_{it,T_0}| \quad |M_i\right) \\
\leq \; & c_{it\,\alpha} \sup_x f_{it}\left(x\right) \left(E \left| \sup_{\theta \in \Theta_i} \frac{\parallel \partial \bar{\lambda}_{it}(\boldsymbol{\theta})/\partial \boldsymbol{\theta} \parallel}{\underline{\lambda}_{it}^{\frac{1}{2}}(\theta)\underline{\lambda}_{it}^{\frac{1}{2}}(\theta_0)} \right|^{\delta} \right)^{\frac{1}{\delta}} \sqrt{T_0}\left(E \parallel \hat{\boldsymbol{\theta}}_{iT_0} - \boldsymbol{\theta}_{i0} \parallel^{\frac{\delta}{\delta-1}} \right)^{1-1/\delta}.
\end{aligned}
$$

Therefore, $T_1^{-1} \sum_{t \in \mathcal{T}_1} X_{it,T_0} = O_p(1)$ and the second term in (39) vanishes as $T_1/T_0 + 1/T_1 \to 0$. Hence

$$
\sqrt{T_1}(\hat{\pi}_i - \alpha) - \sqrt{T_1}(\pi_i - \alpha) = o_p(1),
$$

where

$$
\sqrt{T_1}(\pi_i - \alpha) = \frac{1}{\sqrt{T_1}} \sum_{t \in \mathcal{T}_1} g_{it}, \;\; g_{it} = I\left(-\rho_t - c_{it\,\alpha}\sigma_{\rho\,t}(M_i)\right) - \alpha.
$$

Therefore, it remains to establish the asymptotic distribution of $\sqrt{T_1}(\pi_i - \alpha)$. This easily follows by the martingale central limit theorem of Brown (1971, Theorem 2) since the g_{it} are a bounded, martingale difference sequence with the constant variance $\alpha(1-\alpha)$. ■

Proof of Theorem 3. Inequality (34) can be expressed as

$$
\sum_{i=1}^{N} \lambda_i g(b_i) \; > g\left(\sum_{i=1}^{N} \lambda_i b_i\right),
$$

for the function $g(x) \equiv F(\frac{a}{\sqrt{x}})$. Jensens's inequality ensures that the latter inequality is satisfied whenever $g(\cdot)$ is strictly convex. Since $g(\cdot)$ is twice differentiable by construction, we just need to check the conditions such that the second derivative of $g(x)$ satisfies $g''(x) > 0$. Straightforward calculations yield the required condition (33).■

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Table 1: Standard & Poor 500 Industry Groups

	Codes	*Industries*
1	EN	Energy
2	MA	Materials
3	IC	Capital Goods
4	CS	Commercial Services & Supplies
5	TRN	Transportation
6	AU	Automobiles & Components
7	LP	Consumer Durables & Apparel
8	HR	Hotels, Restaurants & Leisure
9	ME	Media
10	MS	Retailing
11	FD	Food & Staples Retailing
12	FBT	Food, Beverage & Tobacco
13	HHPE	Household & Personal Products
14	HC	Health Care Equipment & Services
15	PHB	Pharmaceuticals & Biotechnology
16	BK	Banks
17	DF	Diversified Financials
18	INSC	Insurance
19	IS	Software & Services
20	TEHW	Technology Hardware & Equipment
21	TS	Telecommunication Services
22	UL	Utilities

Note: The codes in the second column are taken from REUTERS for the S & P 500 industry groups according to the Global Industry Classification Standard. 'Real States' and 'Semiconductors & Semiconductor Equipment' industries are excluded.
Source: *Datastream.*

Table 2: Summary statistics

Sector	Mean	St.Dev.	Skewness	Kurtosis	Ljung-Box(20)
EN	0.031	1.386	0.049	5.435	40.3
MA	0.015	1.367	0.141	6.347	22.1
IC	0.040	1.395	-0.156	6.784	33.1
CS	0.022	1.318	-0.466	8.777	18.7
TRN	0.027	1.407	-0.501	10.644	28.4
AU	0.011	1.628	-0.172	7.017	36.3
LP	0.015	1.194	-0.099	6.750	25.1
HR	0.034	1.422	-0.393	9.241	16.4
ME	0.030	1.660	-0.056	8.168	37.2
MS	0.057	1.739	0.017	6.120	48.5
FD	0.028	1.328	-0.217	6.597	30.8
FBT	0.032	1.132	0.008	6.312	32.4
HHPE	0.042	1.445	-1.581	30.256	55.1
HC	0.039	1.274	-0.295	7.008	57.4
PHB	0.054	1.472	-0.172	5.821	53.1
BK	0.051	1.590	0.045	5.324	37.0
DF	0.075	1.840	0.036	5.013	47.4
INSC	0.044	1.549	0.415	11.045	38.8
IS	0.062	2.246	0.060	5.019	32.6
TEHW	0.043	2.393	0.165	5.719	30.6
TS	0.000	1.605	-0.072	5.969	22.7
UL	0.005	1.197	-0.363	9.881	25.8

Note: Columns 2 to 4 report the sample mean, standard deviation, skewness and kurtosis. Column 5 reports the Ljung-Box statistic of order 20 for testing autocorrelations in individual asset returns. The critical value of χ^2_{20} at the 1% significance level is 37.56. The sample period is 2^{nd} January 1995 - 13^{th} October 2003.

Table 3: Estimation Results for Univariate GARCH$(1,1)$ Models

	Normal innovations			Student t innovations			
Sector	$\hat{c}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\hat{c}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\hat{\nu}_i$
EN	0.0241	0.0620	0.9276	0.0287	0.0665	0.9210	10.25
MA	0.0246	0.1016	0.8932	0.0101	0.0625	0.9355	6.71
IC	0.0222	0.0710	0.9216	0.0201	0.0590	0.9329	6.96
CS	0.1265	0.0759	0.8566	0.0559	0.0755	0.8967	5.20
TRN	0.0163	0.0563	0.9399	0.0282	0.0756	0.9135	6.99
AU	0.0352	0.0604	0.9289	0.0371	0.0635	0.9251	6.51
LP	0.0517	0.0697	0.8975	0.0316	0.0586	0.9212	6.51
HR	0.0841	0.0696	0.8943	0.0327	0.0443	0.9402	6.03
ME	0.0328	0.0854	0.9090	0.0199	0.0617	0.9335	6.17
MS	0.0337	0.0560	0.9346	0.0250	0.0470	0.9459	7.29
FD	0.0474	0.0768	0.8996	0.0300	0.0681	0.9179	7.41
FBT	0.0232	0.0693	0.9150	0.0178	0.0568	0.9311	7.06
HHPE	0.0133	0.0741	0.9259	0.0360	0.0678	0.9147	6.70
HC	0.0950	0.1431	0.8086	0.0911	0.1035	0.8443	6.45
PHB	0.0640	0.0701	0.9032	0.0587	0.0736	0.9030	6.95
BK	0.0557	0.0801	0.8998	0.0457	0.0843	0.9008	8.66
DF	0.1033	0.0686	0.9030	0.0890	0.0732	0.9034	8.53
INSC	0.0375	0.0852	0.9044	0.0344	0.0750	0.9149	5.49
IS	0.0915	0.0647	0.9187	0.0639	0.0587	0.9303	10.14
TEHW	0.0730	0.0730	0.9159	0.0532	0.0608	0.9309	11.73
TS	0.0255	0.0478	0.9437	0.0231	0.0438	0.9484	7.09
UL	0.0213	0.1162	0.8735	0.0183	0.1105	0.8838	6.42
average	0.0501	0.0762	0.9052	0.0387	0.0677	0.9177	7.34

Note: Columns 2-4 report the PMLE estimates of the univariate GARCH$(1,1)$ model for each sector $i = 1, ..., 22$ assuming *Gaussian innovations*:

$$\sigma_{ii,t} = c_{0i} + \alpha_{0i} r_{i,t-1}^2 + \beta_{0i}\sigma_{ii,t-1}.$$

Columns 5-8 report the PMLE estimates of the univariate GARCH$(1,1)$ model for each sector $i = 1, ..., 22$ assuming *Student t innovations* with ν_i degrees of freedom. All the estimates reported for c_i, α_i, and β_i are statistically significant at 5% or less. The estimation period is 2^{nd} January 1995 - 13^{th} October 2003.

Table 4: AIC-penalized likelihood values with Student (8) distribution

Model		2 Nov'01	13 Oct'03	Average	Model		2 Nov'01	13 Oct'03	Average
EWMA	(n_0)								
	(50)	-52921	-69477	-61666	DCC	(p,q,r,s)			
	(75)	-50167	-65931	-58500		(1,1,1,1)	-47851	-61964	-55310
	(125)	-48815	-64185	-56943		(2,1,1,1)	-47849	-61998	-55326
	(250)	-48096	-63437	-56182		(1,2,1,1)	-47843	-61895	-55272
						(2,2,1,1)	-47876	-62044	-55371
EWMA	(λ_0,ν_0)								
	(0.96,0.94)	-53681	-70350	-62493	ADCC	(p,q,r,s)			
	(0.96,0.80)	-87212	-114517	-101336		(1,1,1,1)	-48126	-61856	-55356
	(0.96,0.60)	-167254	-220654	-194364		(2,1,1,1)	-48199	-62141	-55521
	(0.95,0.94)	-53696	-70367	-62505		(1,2,1,1)	-48099	-61737	-55282
	(0.95,0.80)	-87180	-114515	-101309		(2,2,1,1)	-48194	-62018	-55462
	(0.95,0.60)	-167207	-220659	-194334					
	(0.94,0.94)	-53726	-70406	-62537	CCC	(p,q)			
	(0.94,0.80)	-87142	-114495	-101274		(1,1)	-48299	-62847	-55980
	(0.94,0.60)	-167143	-220626	-194279		(2,1)	-48298	-62881	-55997
	(0.95,0.95)	-52279	-68510	-60868		(1,2)	-48293	-62780	-55944
	(0.96,0.96)	-50970	-66802	-59361		(2,2)	-48069	-62698	-55785
MMA	(n_0,ν_0)				O-GARCH	(p,q,r,s)			
	(50,0.60)	-169511	-221486	-195659		(1,1,1,1)	-47899	-66423	-57518
	(75,0.60)	-169503	-221370	-195615		(2,1,1,1)	-47926	-66529	-57670
	(125,0.60)	-169533	-221210	-195607		(1,2,1,1)	-47909	-66818	-57588
	(250,0.60)	-169754	-221114	-195832		(2,2,1,1)	-47703	-66180	-57271
	(50,0.80)	-90156	-116706	-103702					
	(75,0.80)	-90089	-116523	-103586	Factor HRS	(p,q,r,s)			
	(125,0.80)	-90130	-116411	-103591		(1,1,1,1)	-50625	-66506	-59004
	(250,0.60)	-90317	-116311	-103764		(2,1,1,1)	-50119	-65747	-58411
	(50,0.94)	-58962	-76663	-68266		(1,2,1,1)	-50102	-65698	-58382
	(75,0.94)	-57016	-74134	-66038		(2,2,1,1)	-50116	-65733	-58403
	(125,0.94)	-56667	-73624	-65604					
	(250,0.94)	-58962	-76663	-68266	Factor DP	(p,q,r,s)			
						(1,1,1,1)	-50180	-65722	-58445
Gen.EWMA	(n_0,ν_0)					(2,1,1,1)	-50207	-65759	-58479
	(2,2,0.94)	-53821	-63768	-59212		(1,2,1,1)	-50183	-65726	-58449
	(2,2,0.80)	-86819	-94612	-91299		(2,2,1,1)	-50190	-65735	-58457
	(2,2,0.60)	-167006	-202297	-185255					
	(1,2,0.94)	-53767	-63550	-59072					
	(1,2,0.80)	-86705	-93863	-90911					
	(1,2,0.60)	-166798	-201250	-184666					
	(2,1,0.94)	-53751	-63638	-59108					
	(2,1,0.80)	-86708	-94096	-91028					
	(2,1,0.60)	-166863	-201696	-184932					
	(1,1,0.94)	-53773	-63607	-59102					
	(1,1,0.80)	-86782	-94010	-91036					
	(1,1,0.60)	-166953	-201546	-184936					

Note: The figures report the maximized values of the Student t (8) log likelihoods, penalized by the AIC criterion: $AIC_{i,t-1}= LL_{i,t-1}-k_i$, as defined by (cf. (20)) where $LL_{t-1,i}$ is the maximized log likelihood at time t for model i and k_i is the number of parameters of model i. Columns 2 and 6 report the AIC-penalized log likelihood at the initial date of the evaluation period (2 Nov '01), columns 3 and 7 report the AIC-penalized log likelihood at the final date of the evaluation period (13 Oct '03), and columns 4 and 8 report the average AIC-penalized log likelihood values over the days between these two dates.

Table 5a: VaR Diagnostic Tests for $\alpha = 1\%$
(Recursive Estimation With Expanding Window)

Panel A: Portfolio weights: $\boldsymbol{\omega}_t = (1/22, 1/22, ..., 1/22)'$

	Normal		Student (8)		Student (6)	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
Modelling Strategies						
Best						
AIC	1.972	2.200	0.592	-0.924	0.394	-1.370
SBC	1.972	2.200	0.592	-0.924	0.197	-1.817
'Bayesian' Average						
AIC	1.972	2.200	0.592	-0.924	0.394	-1.370
SBC	1.972	2.200	0.592	-0.924	0.197	-1.817
Thick Average						
AIC best 15 (25%)	0.394	-1.370	0.197	-1.817	0	-2.263
SBC best 15 (25%)	0.394	-1.370	0.197	-1.817	0	-2.263
AIC best 32 (50%)	0.592	-0.924	0.197	-1.817	0	-2.263
SBC best 32 (50%)	0.592	-0.924	0.197	-1.817	0	-2.263
All (100%)	0.197	-1.817	0	-2.263	0	-2.263

Panel B: Portfolio weights: $\boldsymbol{\omega}_t = \left(\sum_{j=1}^{22} u_{jt}\right)^{-1} (u_{1t}, u_{2t}, ..., u_{22t})'$, $u_{it} \sim Uniform(0, 1)$

	Normal		Student (8)		Student (6)	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
Modelling Strategies						
Best						
AIC	2.167	2.647	0.789	-0.477	0	-2.263
SBC	2.167	2.647	0.789	-0.477	0	-2.263
'Bayesian' Average						
AIC	2.167	2.647	0.789	-0.477	0	-2.263
SBC	2.167	2.647	0.789	-0.477	0	-2.263
Thick Average						
AIC best 15 (25%)	0.394	-1.370	0.197	-1.817	0	-2.263
SBC best 15 (25%)	0.394	-1.370	0.197	-1.817	0	-2.263
AIC best 32 (50%)	0.592	-0.924	0.197	-1.817	0	-2.263
SBC best 32 (50%)	0.592	-0.924	0.197	-1.817	0	-2.263
All (100%)	0	-2.263	0	-2.263	0	-2.263

Table 5b: VaR Diagnostic Tests for $\alpha = 5\%$
(Recursive Estimation With Expanding Window)

Panel A: Portfolio weights: $\boldsymbol{\omega}_t = (1/22, 1/22, ..., 1/22)'$

	Normal		Student (8)		Student (6)	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
Modelling Strategies						
Best Models						
AIC	7.100	2.170	4.733	-0.275	3.550	-1.497
SBC	6.903	1.966	4.733	-0.275	3.353	-1.701
'Bayesian' Average Models						
AIC	7.100	2.170	4.733	-0.275	3.550	-1.497
SBC	6.903	1.966	4.733	-0.275	3.353	-1.701
Thick Average Models						
AIC best 15 (25%)	4.536	-0.478	4.142	-0.886	3.155	-1.905
SBC best 15 (25%)	4.536	-0.478	4.142	-0.886	2.958	-2.109
AIC best 32 (50%)	4.733	-0.275	2.761	-2.312	2.169	-2.924
SBC best 32 (50%)	4.733	-0.275	2.761	-2.312	2.169	-2.924
All (100%)	3.155	-1.905	1.577	-3.535	1.577	-3.535

Panel B: Portfolio weights: $\boldsymbol{\omega}_t = \left(\sum_{j=1}^{22} u_{jt}\right)^{-1} (u_{1t}, u_{2t}, ..., u_{22t})'$, $u_{it} \sim Uniform(0,1)$

	Normal		Student (8)		Student (6)	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
Modelling Strategies						
Best						
AIC	7.100	2.170	4.536	-0.478	3.944	-1.090
SBC	7.100	2.170	4.339	-0.682	3.747	-1.294
'Bayesian' Average						
AIC	7.100	2.170	4.536	-0.478	3.944	-1.090
SBC	7.100	2.170	4.339	-0.682	3.747	-1.294
Thick Average						
AIC best 15 (25%)	4.142	-0.886	4.142	-0.886	3.944	-1.090
SBC best 15 (25%)	4.142	-0.886	4.142	-0.886	3.550	-1.497
AIC best 32 (50%)	4.536	-0.478	2.761	-2.312	2.169	-2.924
SBC best 32 (50%)	4.536	-0.478	2.761	-2.312	2.169	-2.924
All (100%)	2.958	-2.109	1.972	-3.127	1.577	-3.535

Table 6: Probability Values for Kupier and Kolmogorov-Smirnov Tests
(Recursive Estimation With Expanding Window)

Panel A: Portfolio weights $\boldsymbol{\omega}_t = (1/22, 1/22, ..., 1/22)'$

	Normal		Student (8)		Student (6)	
Modelling Strategy	Ku	KS	Ku	KS	Ku	KS
Best						
AIC	0.091	0.158	0.485	0.303	0.179	0.174
SBC	0.091	0.181	0.362	0.247	0.292	0.279
'Bayesian' Average						
AIC	0.091	0.158	0.485	0.303	0.179	0.174
SBC	0.091	0.181	0.362	0.247	0.292	0.279
Thick Average						
AIC (25%)	0.003	0.004	0.154	0.187	0.052	0.107
SBC (25%)	0.003	0.004	0.148	0.163	0.025	0.080
AIC (50%)	0.143	0.066	0.005	0.003	0.000	0.016
SBC (50%)	0.143	0.066	0.006	0.003	0.000	0.016
All (100%)	0.016	0.049	0.000	0.002	0.000	0.014

Panel B: Portfolio weights: $\boldsymbol{\omega}_t = \left(\sum_{j=1}^{22} u_{jt}\right)^{-1} (u_{1t}, u_{2t}, ..., u_{22t})'$, $u_{it} \sim Uniform(0, 1)$

	Normal		Student (8)		Student (6)	
Modelling Strategy	Ku	KS	Ku	KS	Ku	KS
Best						
AIC	0.079	0.148	0.767	0.607	0.410	0.421
SBC	0.075	0.141	0.754	0.679	0.567	0.610
'Bayesian' Average						
AIC	0.079	0.148	0.767	0.607	0.410	0.421
SBC	0.075	0.141	0.754	0.679	0.567	0.610
Thick Average						
AIC (25%)	0.006	0.012	0.284	0.355	0.114	0.249
SBC (25%)	0.006	0.012	0.163	0.212	0.026	0.093
AIC (50%)	0.224	0.144	0.004	0.035	0.000	0.017
SBC (50%)	0.224	0.144	0.006	0.044	0.000	0.017
All (100%)	0.052	0.115	0.000	0.031	0.000	0.016

Note: The columns indicated by KS report the p-values of the Kolmogorov-Smirnov test (36) and the columns indicated by Ku reports the p-values of the Kupier test (37).

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