MANAGING DEBT STABILITY

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CESIFO WORKING PAPER NO. 1388
CATEGORY 5: FISCAL POLICY, MACROECONOMICS AND GROWTH
JANUARY 2005

CESifo *Working Paper No. 1388*

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Abstract

This paper presents a simple model in which debt management stabilizes the debt-to-GDP ratio in face of shocks to real returns and output growth and thus supports fiscal restraint in ensuring sustainability. The optimal composition of public debt is derived by looking at the relative impact of the risk and cost of alternative debt instruments on the cost of missing the stabilization target. The optimal debt structure is a function of the expected return differentials between debt instruments, of the conditional variance of their returns and of the conditional covariances of their returns with output growth and inflation. We then explore how the relevant covariances and thus the optimal choice of debt instruments depend on the monetary regime and on Central Bank preferences for output stabilization, inflation control and interest-rate smoothing. Finally, we estimate the composition of public debt that would have supported debt stabilization in OECD countries over the last two decades. The empirical evidence suggests that the public debt should have a long maturity and a large share of it should be indexed to the price level.

JEL Code: E63, H63.

Keywords: debt management, debt structure, debt stabilization, inflation indexation, interest rates.

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We thank Adam Posen, Henning Bhon and other seminar participants at the CESifo and LBI Conference on Debt Sustainability for helpful comments and suggestions. The authors are associated with Università di Milano.

1. Introduction

No mention is made of debt management in the debate on debt sustainability, but a careful choice of debt instruments is needed to control interest payments and debt accumulation. Interest-cost minimization is important especially in countries where the level of debt is high and interest payments absorb a large share of the budget. In the same countries avoiding the risk that unfavorable shocks to real returns or output growth lead the debt on an unsustainable path is equally important.

We want to examine whether a concern for debt sustainability justifies the lengthening of debt maturity that has occurred in OECD countries. We are also interested in assessing the scope for inflation-indexed bonds that have been issued only in France, Italy, Sweden and the UK while have been discontinued in the US. To address these issues we rely on a simple model in which debt management stabilizes the debt-to-GDP ratio in face of shocks to real returns and GDP growth and thus supports fiscal restraint in ensuring sustainability.

The optimal debt composition is derived by looking at the relative impact of the risk and cost of alternative debt instruments on the cost of missing the stabilization target. This allows to price risk against the expected cost of debt service and thus to find the optimal combination along the trade off between cost and risk minimization. The optimal debt structure is a function of the expected return differentials between debt instruments, of the conditional variance of their returns, and of the conditional covariances of their nominal returns with output growth and inflation.

We show that debt stabilization is achieved by funding at low cost, and by issuing instruments that provide a hedge against variations in the debt ratio due to lower-than-expected inflation and output growth. For instance, inflation-indexed bonds provide a hedge against variations in the debt ratio due to lower-than-expected inflation. Fixed-rate bonds (as opposed to short-term bills) help to stabilize the debt ratio in cyclical downturns if interest rates and output are negatively correlated.

We find that a stronger fiscal reaction to the debt ratio reduces the importance of expected return differentials for the choice of the debt instruments. In fact, if debt sustainability is on average ensured by a restrictive fiscal stance, minimizing the expected cost of debt service becomes less important than avoiding the risk that unfavorable shocks to real returns or output growth lead the debt on an unsustainable path.

Then, we explore how the relevant covariances between the short-term interest rate, inflation and output growth, and thus the optimal choice of debt instruments, depend on the monetary regime and on Central Bank preferences for output relative to inflation stabilization and interest-rate smoothing. In particular, we compare the implications for debt management of an inflation-targeting regime with those of a fixed-exchange regime.

Finally, we estimate the composition of public debt that would have supported debt stabilization in OECD countries over the past two decades (not taking into account the implications of expected cost differentials). We estimate the conditional covariances between output growth, inflation, and the short-term interest rate using the residuals of forecasting regressions run on yearly data for the period 1960 to 2003. The empirical evidence suggests that, a part from cost considerations, the public debt should have a long maturity and a large share of it should be indexed to the price level.

The paper is organized as follows. Section 2 introduces a simple model of debt stabilization that trades off cost and risk minimization. Then, the optimal debt composition is derived in Section 3 as a function of the risk premia on government bonds and the stochastic relations between output growth, inflation and the interest rate. Section 4 examines the implications for debt management of alternative monetary regimes. Section 5 presents estimates of the stabilizing debt structure for OECD countries. Section 6 concludes.

2. The government problem

In this section we present a simple model where debt management stabilizes the debt ratio and thus helps to ensure debt sustainability. Debt stabilization calls for funding at low cost but also for minimizing the risk of large payments due to unexpected changes in interest rates and inflation. Hence, the choice of debt instruments trades off the risk and the expected cost of debt service.

Risk minimization is accomplished by choosing debt instruments which both ensure a low return variability and provide a hedge against an unexpected economic slowdown (see e.g. Bohn 1990). Reducing the uncertainty of debt returns, for any expected cost of debt service, is valuable in that it lowers the probability that higher than expected real return and/or lower than expected output growth set the debt ratio on an unsustainable path.

To provide insurance against variations in the debt ratio due to lower economic growth, public bonds should be indexed to nominal GDP. However, this would be a costly innovation. Indeed, a high premium would have to be paid: i) for insurance; ii) for the illiquidity of the market and; iii) for the delay in the release of GDP data and their revisions. Therefore, we focus on three main funding instruments currently available to OECD governments: short-term bills (or floating rate notes), fixed-rate long-term bonds and inflation-indexed bonds. We do not consider debt denominated in foreign currency as these instruments are no longer issued by EU governments since the start of the EMU (see Favero et al. 2000).

To examine the role of short- and long-term debt we consider a two period model. Over a two period horizon public debt accumulation is approximately equal to

$$B_{t+1} = (1 + X_{t+1} + X_t)B_{t-1} - S_t \tag{1}$$

where B_{t+1} is the debt-to-GDP ratio, S_t is the primary surplus (relative to GDP) decided at time t for time $t+1$ and X_{t+1} is the real rate of return on public debt minus the rate of output growth:

$$X_{t+1} = I_{t+1} - \pi^d_{t+1} - y_{t+1} \tag{2}$$

where I_{t+1} is the nominal rate of return, π^d_{t+1} is the rate of inflation measured by the GDP deflator and y_{t+1} is the growth rate of GDP.

In order to ensure a sustainable debt, we assume that the government chooses the primary surplus as an increasing function of the debt ratio:

$$S^P_t = \theta B_{t-1} + (X_t - E_{t-1}X_t)B_{t-1} \tag{3}$$

Therefore, the government not only reacts to a higher debt ratio as in Bohn (1998), but it also offsets the increase in the debt ratio due to a higher-than-expected real return net of output growth. The idea is that the government tends to correct a rise in the debt ratio due to past unfavorable shocks such as unexpectedly high returns or low output growth.

Substituting the fiscal rule (3) in equation (1), the change in the debt ratio over the two periods is equal to

$$B_{t+1} - B_{t-1} = (E_{t-1}X_{t+1} + E_{t-1}X_t - \theta)B_{t-1} + (X_{t+1} - E_{t-1}X_{t+1})B_{t-1} \tag{4}$$

Equation (4) shows that, even if the primary surplus is expected to stabilize the debt ratio, so that $(E_{t-1}X_{t+1} + E_{t-1}X_t - \theta)B_{t-1} < 0$, in general, θ might not be high enough to prevent the debt ratio from rising if the real return on debt turns out to be particularly high or the growth rate of GDP particularly low. In particular, if such shocks are permanent the debt ratio may not be stationary. Quoting Bohn (1998): a sufficiently high θ can "keep the debt-to-GDP stationary in the future unless interest rates and growth rates move very unfavorably". In the present model the debt ratio may indeed rise either because of shocks to real interest rates and/or to output growth.

The two terms on the right-hand-side of equation (4) show that the role of debt management in ensuring debt sustainability is twofold. Debt instruments can be chosen either to reduce the expected real return on public debt or to minimize the impact of unfavorable shocks such as unexpectedly high returns or low output growth.

We assume that the government chooses the composition of the debt to stabilize the debt ratio and that deviations above the stabilization target, here set equal to B_{t-1} for simplicity, are increasingly costly. Hence, at time $t-1$ the government minimizes the following quadratic loss[1]

$$L = mE_{t-1}(B_{t+1} - B_{t-1}) + \frac{w}{2}E_{t-1}(B_{t+1} - B_{t-1})^2 \tag{5}$$

with $m + w(B_{t+1} - B_{t-1}) \geq 0$.[2]

There are at least three reasons why the minimization of equation (5) is a sensible objective. First, even a rigorous fiscal rule may not prevent that an unexpected economic slowdown leads the debt ratio onto an unsustainable path. Second, an increase in the debt ratio, according to the fiscal rule (3), requires a revision in the government budget; i.e. a higher primary surplus has to be planned for the following year. Such revisions are costly either because of distortionary taxation or because of political reasons. Third, unlike debt sustainability, debt stabilization is a visible, well defined and politically relevant goal. Therefore, there are economic, political and practical reasons as to why the government efforts are in general directed at preventing a rise in the debt ratio rather than at ensuring that the intertemporal budget constraint holds.

Debt stabilization can be pursued by a combination of policies regarding the choice of primary surpluses and the choice of debt instruments. In what follows we take the choice of θ as given

[1] The analysis can be extended to the case the debt ratio must not exceed a given threshold. In a previous version of the paper the government was assumed to minimize the probability that the debt ratio exceeded B_{t-1}, but the assumption of a constant penalty independent of the deviation from the target can be hardly justified except when debt stabilization is a priority as in Brazil (see Missale and Giavazzi 2004). Moreover, if maximizing the probability of debt stabilization is the only objective of the government this provides an incentive for debt contingent (or derivative) schemes that raise the probability of success in exchange for large payments and thus larger debt deviations in the case of failure. We are indebted to Adam Posen and Henning Bohn for raising these points.

[2] This is the standard assumption with the quadratic utility function that avoids the consideration of losses from large negative deviation from the target.

and focus on the role of debt management. The government can choose between short-term bills (or floating-rate notes), inflation-indexed bonds and fixed-rate long-term bonds. We take the time period as corresponding to one year and assume that short-term bills have a one-year maturity while bonds have a two-year maturity. Focusing on a two-year horizon is obviously a rough approximation given that both inflation-indexed bonds and fixed-rate bonds are issued with much longer maturities. A partial justification for this assumption is provided by the monetary policy model presented in the following section, in which the effects of economic shocks last only two periods.

The composition of the debt chosen at the end of period $t-1$ affects the nominal rate of return between time t and $t+1$, I_{t+1}, as follows

$$I_{t+1} = i_t s + (R^I_{t-1} + \pi_{t+1})h + R_{t-1}(1 - s - h) \tag{6}$$

where s is the share of short-term debt, h is the share of inflation-indexed debt, π_{t+1} is CPI inflation and i_t denotes the short-term interest rate between period t and $t+1$, which determines the nominal rate of return on one-year bills. The interest rate i_t is not known at time $t-1$ when the composition of the debt is chosen. The nominal return on fixed-rate bonds is equal to the long-term interest rate at which fixed-rate bonds are issued, R_{t-1}, and is thus known at time $t-1$. Finally, the nominal rate of return on inflation-indexed bonds is equal to the sum of the real interest rate, R^I_{t-1}, known at the time of issuance, and the rate of CPI inflation, π_{t+1}, to which the bonds are indexed.

3. The choice of debt maturity and indexation

The Treasury chooses the composition of the debt at time $t-1$, and thus s and h, to minimize the expected loss function (5) subject to equations (4), (2) and (6).

The first order conditions are equal to

$$E_{t-1}(i_t + i_{t-1} - 2R_{t-1})[m + w(B_{t+1} - B_{t-1})] = 0 \tag{7}$$

$$E_{t-1}(2R^I_{t-1} + \pi_{t+1} + E_{t-1}\pi_t - 2R_{t-1})[m + w(B_{t+1} - B_{t-1})] = 0 \tag{8}$$

Equations (7) and (8) show that the debt structure is optimal only if the marginal cost of a debt increase, that is associated with the interest cost of additional funding in a particular type of debt, is equalized across debt instruments. If this were not the case, the government could reduce its loss by changing the debt structure; e.g. by substituting fixed-rate bonds for short-term bills or vice versa.[3]

To gain further intuition the difference between the nominal rate of return on short-term bills and fixed-rate bonds can be written as

$$i_t + i_{t-1} - 2R_{t-1} = i_t - E_{t-1}i_t - TP_{t-1} \tag{9}$$

where TP_{t-1} is (two times) the term premium on long-term fixed-rate bonds.

Therefore the expected cost of funding with short-term bills is lower than fixed-rate bonds because of the term premium but, ex-post, the cost may be greater if the short-term rate turns out to be higher than expected.

[3] The argument assumes that there are non-negative constraints to the choice of debt instruments.

The difference between the nominal rate of return on price-indexed bonds and fixed-rate bonds is equal to

$$2R^I_{t-1} + \pi_{t+1} + E_{t-1}\pi_t - 2R_{t-1} = \pi_{t+1} - E_{t-1}\pi_{t+1} - IP_{t-1} \tag{10}$$

where IP_{t-1} is (two times) the inflation risk premium.

Substituting the return differentials (9) and (10) in the first order conditions (7) and (8) yields

$$E_{t-1}(i_t - E_{t-1}i_t)[m + w(B_{t+1} - B_{t-1})] = TP_{t-1}E_{t-1}[m + w(B_{t+1} - B_{t-1})] \tag{11}$$
$$E_{t-1}(\pi_{t+1} - E_{t-1}\pi_{t+1})[m + w(B_{t+1} - B_{t-1})] = IP_{t-1}E_{t-1}[m + w(B_{t+1} - B_{t-1})] \tag{12}$$

Equations (11) and (12) show the trade off between the risk and expected cost of debt service that characterizes the choice of debt instruments. For instance, equation (11) shows that issuing short-term bills is optimal until the uncertainty of their return is expected to raise the marginal cost of deviating from the stabilization target as much as paying the term premium on fixed-rate bonds. The expected marginal gain of reducing the cost of debt servicing by issuing short-term bills must be equal to the marginal cost of deviating from the stabilization target because of the greater risk exposure. Hence, the marginal cost of deviating from the stabilization target can be used to price risk against the expected cost of debt service and thus find the optimal combination along the trade off between cost and risk minimization.

Substituting equations (4), (2) and (6) in the first order conditions (11) and (12) yields the optimal shares of short-term debt, s^*, and inflation-indexed debt, h^*:

$$\begin{aligned} s^* \;=\; & \frac{Cov_{t-1}(\pi^d_{t+1} i_t)}{Var_{t-1}(i_t) + TP^2_{t-1}} + \frac{Cov_{t-1}(y_{t+1} i_t)}{Var_{t-1}(i_t) + TP^2_{t-1}} - h^* \frac{Cov_{t-1}(\pi_{t+1} i_t)}{Var_{t-1}(i_t) + TP^2_{t-1}} + \\ & -h^* \frac{IP_{t-1}TP_{t-1}}{Var_{t-1}(i_t) + TP^2_{t-1}} + \frac{TP_{t-1}[m - w(\theta - X)B_{t-1}]}{(Var_{t-1}(i_t) + TP^2_{t-1})wB_{t-1}} \end{aligned} \tag{13}$$

$$\begin{aligned} h^* \;=\; & \frac{Cov_{t-1}(\pi^d_{t+1} \pi_{t+1})}{Var_{t-1}(\pi_{t+1}) + IP^2_{t-1}} + \frac{Cov_{t-1}(y_{t+1} \pi_{t+1})}{Var_{t-1}(\pi_{t+1}) + IP^2_{t-1}} - s^* \frac{Cov_{t-1}(\pi_{t+1} i_t)}{Var_{t-1}(\pi_{t+1}) + IP^2_{t-1}} + \\ & -s^* \frac{IP_{t-1}TP_{t-1}}{Var_{t-1}(\pi_{t+1}) + IP^2_{t-1}} + \frac{IP_{t-1}[m - w(\theta - X)B_{t-1}]}{(Var_{t-1}(\pi_{t+1}) + IP^2_{t-1})wB_{t-1}} \end{aligned} \tag{14}$$

where $Var_{t-1}(.)$ and $Cov_{t-1}(.)$ denote variances and covariances conditional on the information available at time $t-1$ and $(\theta - X)B_{t-1} = \theta B_{t-1} - [2R_{t-1} - E_{t-1}(y_{t+1} + y_t + \pi^d_{t+1} + \pi^d_t)]B_{t-1} > 0$ is the expected reduction of the debt ratio when all the debt is financed with fixed-rate long-term bonds.

The optimal debt shares, s^* and h^*, depend on both risk and cost considerations. Risk is minimized if a debt instrument provides insurance against variations in the debt ratio due to output and inflation uncertainty, and if the conditional variance of its returns is relatively low. This is captured by the first two terms in equations (13) and (14).

Equation (13) shows that short-term debt is optimal for risk minimization if the short-term interest rate and thus the interest payments are positively correlated with unanticipated output growth and inflation. To pay low interests when output growth and inflation are unexpectedly low is valuable because slow nominal growth increases the debt ratio. Instruments with returns correlated to nominal output growth help to stabilize the debt ratio, thus reducing the risk that it will grow above target. On the other hand, the case for short-term debt weakens as the conditional variance of the short-term interest rate increases, thus producing unnecessary fluctuations in interest payments. The first two terms in equation (13) also decrease with the term premium because a higher premium reduces the insurance motivation for the choice of short-term debt.

Equation (14) shows that the optimal share of inflation-indexed debt increases with the covariance between output growth and inflation. If this covariance is positive, lower interest payments on inflation-indexed debt provide an insurance against unexpected slowdowns in economic activity that raise the debt-to-GDP ratio. However, some inflation-indexed debt would be optimal even if the covariance between output and inflation were zero. The reason is that CPI indexation provides a good hedge against an increase in the debt ratio due to lower than expected GDP inflation. On the other hand a higher inflation volatility or a higher inflation-risk premium reduce the importance of insurance motivations for the choice of inflation-indexed bonds.

Risk minimization also depends on the conditional covariances between the returns on the various debt instruments. For instance, a positive covariance between the returns of two types of debt makes the two instruments substitutes in the government portfolio. This is captured by the third term in equations (13) and (14).

Leaving aside cost considerations, the government should choose the debt composition which offers the best insurance against the risk of deflation and low growth. But insurance is costly; higher expected returns are generally required on hedging instruments, and this leads on average to greater debt accumulation. Debt stabilization thus implies a trade off between cost and risk minimization. The effects of expected return differentials on the optimal debt composition are captured by the last two terms of equations (13) and (14).

The third term shows that the optimal share of short-term debt and inflation-indexed debt decreases with the risk premium of the other type of debt. The last term in equation (13) shows that a higher term premium TP_{t-1}, namely a lower expected return of short-term bills relative to fixed-rate bonds, increases the optimal share of short-term debt. A higher inflation-risk premium, IP_{t-1}, does the same with the share of inflation-indexed debt. Finally, the impact of the expected return differentials on the optimal shares of short-term bills and indexed bonds decreases with the variance of their returns as this makes the cost advantage of such instruments less important.

More important, the impact of the expected cost differentials, TP_{t-1} and IP_{t-1}, depends on the expected reduction of the debt ratio $(\theta - X)B_{t-1}$. A stronger fiscal reaction to the debt ratio, θ, clearly reduces the importance of the expected return differentials for the choice of debt instruments. Intuitively, as debt sustainability is on average ensured by a restrictive fiscal stance, cost considerations become less important than insurance motivations for the choice of debt instruments. In other words, if debt stabilization may fail only for large unfavorable realizations of real returns or output growth, then debt management should be mostly concerned with providing insurance against such events. This result possibly explains why in countries where the dynamics of the debt is out of control interest-cost minimization is the main goal of debt management.

The optimal debt composition depends on both risk and cost considerations. In the following sections, we focus on insurance considerations and, building on the intuition in Bohn (1988), investigate the role of monetary policy in determining the stochastic structure of the economy and thus the optimal debt composition.

4. Monetary policy and debt management

The stochastic relations between output, inflation and the short-term interest rate depend on the reaction of the monetary authority to macroeconomic shocks affecting the economy. Therefore, the monetary regime should play an important role in the choice of debt maturity and indexation.

In this section we rely on a simple aggregate demand and supply model to examine the implications of alternative monetary regimes for the choice of debt instruments.

The supply side of the economy is modeled as a backward looking Phillips curve:

$$\pi_{t+1} = \pi_t + cy_{t+1} + u_{t+1} \tag{15}$$

where c measures the impact of the output gap, y_{t+1}, on inflation π_{t+1}. Inflation is affected by an i.i.d. supply shock, u_t, with mean zero and variance equal to σ_u^2.

Equation (15) implies important nominal rigidities and backward looking behavior in that current inflation entirely depends on lagged inflation as opposed to expected inflation, but its empirical performance is satisfactorily (see Fuhrer 1997).

The aggregate demand is equal to

$$y_{t+1} = \rho y_t - a(i_t - E_t\pi_{t+1} - \bar{r}) + v_{t+1} \tag{16}$$

where $i_t - E_t\pi_{t+1}$ is the real interest rate between period t and $t+1$. The impact of the interest rate depends on the parameter a, while ρ measures output auto-correlation. Finally, v_{t+1} is an i.i.d. demand shock with mean zero and variance equal to σ_v^2.

The Central Bank controls aggregate demand and thus output and inflation with a lag through the choice of the nominal interest rate, i_t set at the beginning of period t.

4.1 Inflation targeting

In an inflation-targeting regime the Central Bank aims at maintaining expected inflation close to the target and, possibly, at stabilizing output. Then, assuming a concern for interest-rate volatility, the loss function of the Central Bank is equal to

$$L^{IT} = E_t(\pi_{t+1} - \pi^T)^2 + \lambda E_t y_{t+1}^2 + \alpha(i_t - i_{t-1})^2 \tag{17}$$

where π^T is the inflation target, and λ and α are the (publicly known) weights given by the Central Bank to output stabilization and interest-rate smoothing relative to the inflation target.

The Central Bank chooses the short-term interest rate, i_t, to minimize the loss function (17) subject to equations (16) and (15). The interest rate rule is, thus, equal to

$$i_t = \mu i_{t-1} + (1-\mu)[\pi^T + \bar{r} + \beta(\pi_t - \pi^T) + \gamma y_t] \tag{18}$$

where

$$\beta = \frac{c + a\lambda}{ac^2 + a\lambda}; \qquad \gamma = \frac{\rho}{a} \qquad \mu = \frac{\alpha(1-ac)^2}{a^2c^2 + a^2\lambda + \alpha(1-ac)^2}$$

and where $ac < 1$ ensures that an increase in the interest rate reduces the inflation rate. Hence, the reaction to current inflation is greater than one, $\beta > 1$, and decreases with the weight, λ, assigned to output stabilization. Finally, the degree of interest-rate smoothing is captured by μ which is increasing in α.

The interest rate rule (17) can be combined with the aggregate demand (16) and the aggregate supply (15) to derive the conditional covariances between output, inflation and the interest rate. In what follows we examine how such covariances are affected by the preferences of the Central Bank regarding output stabilization and interest rate smoothing.

4.1.1 Inflation targeting and debt management

Monetary policy affects the conditional covariances between output, inflation and the interest rate that determine the optimal composition of public debt. The (two-period ahead) unanticipated components of inflation, output and the interest rate are equal to

$$\pi_{t+1} - E_{t-1}\pi_{t+1} = \frac{\lambda}{c^2+\lambda}(\pi_t - E_{t-1}\pi_t) + \mu z c[\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] + u_{t+1} + cv_{t+1} \quad (19)$$

$$y_{t+1} - E_{t-1}y_{t+1} = -\frac{c}{c^2+\lambda}(\pi_t - E_{t-1}\pi_t) + \mu z[\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] + v_{t+1} \quad (20)$$

$$i_t - E_{t-1}i_t = (1-\mu)[\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] \quad (21)$$

where $z = a/(1-ac) > 0$.

Noting that $Var_{t-1}(\pi_t) = c^2\sigma_v^2 + \sigma_u^2$, $Var_{t-1}(y_t) = \sigma_v^2$ and $Cov_{t-1}(y_t\pi_t) = c\sigma_v^2$, we derive the following correlation coefficients of equations (13) and (14):

$$\frac{Cov_{t-1}(y_{t+1}i_t)}{Var_{t-1}(i_t)} = -\frac{c[c(\gamma+c\beta)\sigma_v^2 + \beta\sigma_u^2]}{(c^2+\lambda)(1-\mu)[(\gamma+c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2]} + \frac{\mu z}{1-\mu} \quad (22)$$

$$\frac{Cov_{t-1}(\pi_{t+1}i_t)}{Var_{t-1}(i_t)} = \frac{\lambda[c(\gamma+c\beta)\sigma_v^2 + \beta\sigma_u^2]}{(c^2+\lambda)(1-\mu)[(\gamma+c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2]} + \frac{\mu z c}{1-\mu} \quad (23)$$

$$\frac{Cov_{t-1}(\pi_{t+1}y_{t+1})}{Var_{t-1}(\pi_{t+1})} = \frac{c\sigma_v^2 - \frac{c\lambda}{(c^2+\lambda)^2}(c^2\sigma_v^2 + \sigma_u^2) + \mu z H}{c^2\sigma_v^2 + \sigma_u^2 + \frac{\lambda^2}{(c^2+\lambda)^2}(c^2\sigma_v^2 + \sigma_u^2) + \mu c z Q} \quad (24)$$

where

$$H = \mu c z[(\gamma+c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2] - \frac{c^2-\lambda}{c^2+\lambda}[c(\gamma+c\beta)\sigma_v^2 + \beta\sigma_u^2]$$

$$Q = \mu c z[(\gamma+c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2] + \frac{2\lambda}{c^2+\lambda}[c(\gamma+c\beta)\sigma_v^2 + \beta\sigma_u^2]$$

The correlation coefficients in equations (22), (23) and (24) determine the composition of public debt that offers the best insurance against the risk of deflation and low growth, namely when expected return differentials are not taken into account.

To examine these correlations it is useful to leave aside interest-rate smoothing for a moment; i.e. to assume $\mu = 0$. In this case, the conditional covariance between output and the interest rate in equation (22) is always negative because the interest rate lowers inflation through a contraction of aggregate demand. This negative correlation decreases with λ because a more flexible inflation targeting implies a weaker reaction of the interest rate to inflationary pressure.

The conditional covariance between the interest rate and inflation in equation (23) depends on the weight λ that the monetary authority assigns to output stabilization. In a strict inflation targeting, i.e. with $\lambda = 0$, this covariance is zero since inflation may differ from its expectation two periods earlier only because of contemporaneous shocks (see equation (19)). Intuitively, if the control of inflation is the only objective of monetary policy, then inflation is expected to be uncorrelated with the policy instrument, i_t. On the other hand, if the authorities also care about output stabilization, i.e. if $\lambda > 0$, the interest-rate reaction does not eliminate inflationary pressures and the covariance between inflation and the interest rate is positive. Therefore, in the absence of interest-rate smoothing, a role for short-term debt emerges only if $\lambda > c$, that is, only if the monetary authority cares sufficiently about output stabilization.

Focusing on the choice of inflation-indexed debt, we observe that the correlation coefficient in equation (24) is positive for $\lambda = 0$, it increases with the variance of demand shocks relative to the variance of supply shocks, and decreases with λ. The intuition for these results is as follows. The conditional covariance between output and inflation depends on demand and supply shocks occurring at time $t+1$ and on the correlation induced by monetary policy. Contemporaneous shocks lead to a positive covariance that increases with the variance of demand relative to supply shocks. The effect of monetary policy depends instead on the weight assigned to output stabilization. In a strict inflation targeting, with $\lambda = 0$, unanticipated inflation only depends on contemporaneous shocks and the policy effect vanishes. In a flexible inflation targeting the authorities give up inflation stability and this induces a negative covariance between output and inflation. As a result, the overall covariance decreases and may turn out to be negative for a high weight λ, assigned to output stabilization. A higher inflation volatility also reduces the correlation coefficient in equation (24).

To conclude, in the absence of interest-rate smoothing, there is little or no role for short-term debt. In particular, if the monetary authority only aims at controlling inflation, the debt should either have a long maturity or be indexed to the inflation rate. As implied by tax-smoothing, the optimal share of indexed debt increases with the variance of demand relative to supply shocks (see Missale 1997). On the other hand, if the monetary authority also aims at stabilizing output, then a lower share of inflation-indexed debt is needed to stabilize the debt ratio and a role for short-term debt may emerge.

4.1.2 Debt management and Central Bank preferences

The analysis of the previous section suggests that the preferences of the Central Bank, in particular, the weight, λ, assigned to output stabilization plays an important role for the choice of debt instruments (for given expected return differentials). A concern for output stabilization unambiguously favors short-term debt while the optimal shares of long-term debt and inflation-indexed debt are greater the "more conservative" is the Central Bank in its anti-inflationary policy. It is worth noting that these results are not affected by the consideration of interest-rate smoothing.

The result that a higher share of debt should be indexed to the price level when the monetary authority strictly control inflation is interesting in that it reverses the implications of the time-consistency literature that inflation-indexed debt helps the monetary authority to control inflation by reducing the incentive to inflate.

The analysis also provides insights in how the optimal debt structure may have changed for countries joining the EMU, as policy making moved from national central banks to the European Central Bank (ECB). If the ECB were less concerned with output fluctuations than the national authorities, then, everything else being equal, the optimal policy would call for lengthening the maturity of the debt and issuing inflation indexed debt.

4.1.3 Interest rate smoothing

In this section we relax the assumption that $\mu = 0$ to focus on the implications of interest-rate smoothing. Equations (23) shows that the correlation coefficient between the short-term interest rate and next period inflation increases with the degree of interest-rate smoothing, μ. Intuitively, a concern for interest rate volatility weakens the short-run reaction of the monetary authority to inflationary pressure and leads to a lower variance of the interest rate. More important, as next period inflation is not fully stabilized, current inflationary shocks tend to persist, and this generates a positive covariance between next period inflation and the interest rate.

The correlation coefficient between output growth and the interest rate in equation (22) also increases with the degree of interest-rate smoothing. This is because the impact on output is lower the weaker the interest-rate reaction and this reduces the negative covariance between output and the interest rate.

Hence interest-rate smoothing favors short-term debt besides cost considerations. Short-term debt may even be optimal for insurance if the positive correlation of the interest rate with inflation prevails over the negative correlation with output, which may happen for a sufficiently high λ and μ.

The effect of interest-rate smoothing on the correlation coefficient between output and inflation in equation (24) is ambiguous and so are its implications for inflation-indexed debt. A weaker interest-rate reaction leads to higher inflation volatility, but its effect on the covariance between output and inflation is non-linear and depends on the weight, λ, assigned to output stabilization. In a strict inflation targeting, the covariance between output and inflation decreases because the authorities give up inflation stability in exchange for interest-rate stability and this generates a policy-induced negative covariance between output and inflation. However, this effect is reversed as μ increases and the negative impact on output is reduced. A similar effect arises in a flexible inflation targeting, since the policy-induced covariance between output and inflation is negative. Finally, a greater variance of demand relative to supply shock makes it more likely that interest-rate smoothing reduces the correlation between output and inflation, and thus the need for inflation indexed debt.

To conclude, interest-rate smoothing makes a case for short-term debt. In particular, a positive amount of short-term debt can be optimal for debt stabilization even in a strict inflation-targeting regime, that is, even if the Central Bank does not aim at stabilizing output. In the latter case short-term debt should be issued in exchange for inflation-indexed debt as (some) smoothing reduces the need for indexation.

4.2 Fixed exchange rate

In a fixed exchange-rate regime the Central Bank maintains the parity by pegging the interest rate to the interest rate of the leader country possibly augmented by a currency premium. To the extent that the monetary authority has some flexibility in pursuing domestic stabilization objectives, its loss function is equal to

$$L^{FE} = E_t(\pi_{t+1} - \pi^T)^2 + \lambda E_t y_{t+1}^2 + \varepsilon(i_t - i_t^* - Pe_t)^2 \quad (25)$$

where Pe_t denotes the currency premium, that is, the sum of expected depreciation, the country risk premium and the foreign-exchange premium. Finally, ε is the weight given by the Central Bank to the objective of maintaining the exchange rate fixed.

To simplify the analysis we assume that the exchange rate and foreign output do not affect domestic demand and thus that the structure of the domestic economy is still represented by equations (15) and (16).

The interest rate rule in a fixed exchange regime is derived by minimizing the loss function (25) subject to (15) and (16), and is equal to

$$i_t = \eta(i_t^* + Pe_t) + (1-\eta)[\pi^T + \bar{r} + \beta(\pi_t - \pi^T) + \gamma y_t] \tag{26}$$

where η is an increasing function of ε and captures the extent of interest-rate pegging.

Assuming that the monetary authority of the leader country sets the interest rate to stabilize foreign inflation and output; i.e. $i_t^* = \pi^{T*} + \bar{r}^* + \beta^*(\pi_t^* - \pi^{T*}) + \gamma^* y_t^*$, the domestic interest rate rule is equal to

$$i_t = \eta[Pe_t + \pi^{T*} + \bar{r}^* + \beta^*(\pi_t^* - \pi^{T*}) + \gamma^* y_t^*] + (1-\eta)[\pi^T + \bar{r} + \beta(\pi_t - \pi^T) + \gamma y_t] \tag{27}$$

The interest rate rule (27) can be combined with the aggregate demand (16) and the aggregate supply (15) to derive the conditional covariances between output, inflation and the interest rate.

4.2.1 Fixed exchange rate and debt management

The implications of interest rate pegging for debt management depend on how shocks to the currency premium and foreign monetary policy affect the conditional covariances between output, inflation and the interest rate. To understand the role of foreign policy it is useful to look at the unanticipated component of the domestic interest rate that is equal to

$$\begin{aligned} i_t - E_{t-1}i_t = \eta e_t &+ (1-\eta)[\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] + \\ &+ \eta[\beta^*(\pi_t^* - E_{t-1}\pi_t^*) + \gamma^*(y_t^* - E_{t-1}y_t^*)] \end{aligned} \tag{28}$$

where $e_t = Pe_t - E_{t-1}Pe_t$ defines the unanticipated component of the currency premium due to shocks to the country risk premium and expected depreciation. We assume that e_t, has mean zero, variance equal to σ_e^2, and is uncorrelated to supply and demand shocks u_t and v_t.

Equation (28) shows that the domestic interest rate differs from that prevailing under inflation targeting either because of shocks to the currency premium or because of imported monetary policy. The latter may differ from the optimal policy under inflation targeting for two reasons. First, the timing and magnitude of foreign demand and supply shocks are, in general, different from those of domestic shocks. Second, the economic structure and thus the transmission mechanism of monetary policy may vary across countries, as captured by the reaction parameters, β^* and γ^*. In what follows we focus on each aspect at a time starting from the effects of shocks to the currency premium.

4.2.2 Currency premium shocks and debt management

To focus on the effect of shocks to the country risk premium and expected depreciation, we assume that the domestic and foreign economy are identical and experience the same demand and supply shocks.

The (two-period ahead) unanticipated components of inflation, output and the interest rate are equal to

$$\pi_{t+1} - E_{t-1}\pi_{t+1} = \frac{\lambda}{c^2+\lambda}(\pi_t - E_{t-1}\pi_t) + u_{t+1} + cv_{t+1} - cz\eta e_t \tag{29}$$

$$y_{t+1} - E_{t-1}y_{t+1} = -\frac{c}{c^2+\lambda}(\pi_t - E_{t-1}\pi_t) + v_{t+1} - z\eta e_t \tag{30}$$

$$i_t - E_{t-1}i_t = \eta e_t + [\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] \tag{31}$$

where $z = a/(1-ac) > 0$.

Equations (29), (30) and (31) are the same as in an inflation targeting regime except for the shocks to the currency premium. The latter has the same impact on the economy as an interest rate shock, namely, as a change in interest rate unrelated to supply and demand shocks.

The correlation coefficients of equations (13) and (14) can be derived using (29), (30) and (31) and be expressed in terms of their counterparts (22), (23) and (24) under inflation targeting:

$$\frac{Cov^{FE}_{t-1}(y_{t+1}i_t)}{Var^{FE}_{t-1}(i_t)} = \frac{Cov^{IT}_{t-1}(y_{t+1}i_t) - \eta^2 z\sigma_e^2}{Var^{IT}_{t-1}(i_t) + \eta^2\sigma_e^2} \tag{32}$$

$$\frac{Cov^{FE}_{t-1}(\pi_{t+1}i_t)}{Var^{FE}_{t-1}(i_t)} = \frac{Cov^{IT}_{t-1}(\pi_{t+1}i_t) - \eta^2 cz\sigma_e^2}{Var^{IT}_{t-1}(i_t) + \eta^2\sigma_e^2} \tag{33}$$

$$\frac{Cov^{FE}_{t-1}(\pi_{t+1}y_{t+1})}{Var^{FE}_{t-1}(\pi_{t+1})} = \frac{Cov^{IT}_{t-1}(y_{t+1}\pi_{t+1}) + \eta^2 cz^2\sigma_e^2}{Var^{IT}_{t-1}(\pi_{t+1}) + \eta^2 c^2 z^2\sigma_e^2} \tag{34}$$

Equations (31) and (32) show that the correlation coefficients of output and inflation with the interest rate are lower than the corresponding coefficients under inflation targeting. This is because shocks to the currency premium lead to changes in the interest rate which are unrelated to demand and supply shocks. Since output and inflation fall as the interest rate rises, this generates a negative covariance of the interest rate with both output and inflation.

Therefore shocks to the country risk premium or to expected depreciation make a strong case against short-term debt; its share should be lower than in an inflation-targeting regime.

By contrast inflation-indexed debt can provide a partial insurance against shocks of the currency premium; the correlation coefficient between output and inflation in equation (33) is always higher than the corresponding coefficient under inflation targeting.[4] This is because changes in the interest rate make output and inflation move in the same direction thus reinforcing the positive covariance induced by demand shocks. It is, however, worth noting that this result hinges on the ability of the Central Bank to maintain the exchange rate fixed. If an increase in the country risk premium triggered a depreciation, then the resulting inflation would lead to a negative covariance with output and thus to the opposite result.

To conclude, countries that experience substantial variations in the interest rate due to shocks to the currency premium should abstain from short-term funding of public debt and rely on long-term bonds. If a currency crisis cannot be ruled out, fixed-rate bonds provide the best insurance against such event.

[4]The condition $Cov^{IT}_{t-1}(y_{t+1}\pi_{t+1})/Var^{IT}_{t-1}(\pi_{t+1}) < 1/c$ is verified for $\lambda = 0$ and thus for any λ.

4.2.3 Imported monetary policy and debt management

To understand the implications for debt management of imported monetary policy it is useful to focus on the case where the domestic and foreign economy have the same structure but are hit by different shocks. A simple formalization of this hypothesis is to assume that:

$$u_t^* = ku_t \qquad\qquad v_t^* = kv_t$$

By varying the parameter k we can examine the case of: i) greater foreign shocks, $k > 1$; ii) smaller foreign shocks, $0 < k < 1$, and; iii) asymmetric shocks, $k < 0$.

Noting that $\pi_t^* - E_{t-1}\pi_t^* = k(\pi_t - E_{t-1}\pi_t)$ and $y_t^* - E_{t-1}y_t^* = k(y_t - E_{t-1}y_t)$, the unanticipated components of inflation, output and the interest rate are equal to

$$\pi_{t+1} - E_{t-1}\pi_{t+1} = [1 - cz(\delta\beta - 1)](\pi_t - E_{t-1}\pi_t) + cz\gamma(1-\delta)(y_t - E_{t-1}y_t) + u_{t+1} + cv_{t+1} \quad (35)$$

$$y_{t+1} - E_{t-1}y_{t+1} = -z(\delta\beta - 1)(\pi_t - E_{t-1}\pi_t) + +z\gamma(1-\delta)(y_t - E_{t-1}y_t) + v_{t+1} \quad (36)$$

$$i_t - E_{t-1}i_t = \delta[\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] \quad (37)$$

where $\delta = 1 - \eta + \eta k$.

Equations (35), (36) and (37) allow us to derive the following correlation coefficients:

$$\frac{Cov_{t-1}(y_{t+1}i_t)}{Var_{t-1}(i_t)} = -\frac{z(\delta\beta - 1)[c(\gamma + c\beta)\sigma_v^2 + \beta\sigma_u^2]}{\delta[(\gamma + c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2]} + \frac{z\gamma(1-\delta)(\gamma + c\beta)\sigma_v^2}{\delta[(\gamma + c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2]} \quad (38)$$

$$\frac{Cov_{t-1}(\pi_{t+1}i_t)}{Var_{t-1}(i_t)} = \frac{[1 - cz(\delta\beta - 1)][c(\gamma + c\beta)\sigma_v^2 + \beta\sigma_u^2]}{\delta[(\gamma + c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2]} + \frac{cz\gamma(1-\delta)(\gamma + c\beta)\sigma_v^2}{\delta[(\gamma + c\beta)^2\sigma_v^2 + \beta^2\sigma_u^2]} \quad (39)$$

$$\frac{Cov_{t-1}(\pi_{t+1}y_{t+1})}{Var_{t-1}(\pi_{t+1})} = \frac{c\sigma_v^2 - z(\delta\beta - 1)[1 - cz(\delta\beta - 1)](c^2\sigma_v^2 + \sigma_u^2) + [1 - z\gamma(1-\delta)]z\gamma(1-\delta)c\sigma_v^2}{(c^2\sigma_v^2 + \sigma_u^2)\{1 + [1 - cz(\delta\beta - 1)]^2\} + z\gamma(1-\delta)[z\gamma(1-\delta) - 2 - 2cz(\delta\beta - 1)]c^2\sigma_v^2} \quad (40)$$

Equations (38) and (39) can be compared to equations (22) and (23) to show that whether the correlation coefficients of output and inflation with the interest rate are greater or lower than the corresponding coefficients under inflation targeting depends on $\delta = 1 - \eta + \eta k$. If $0 < \delta < 1$ both coefficients are greater than those under inflation targeting and so is the optimal share of short-term debt. In this case the shocks hitting the foreign economy are smaller than the domestic shocks (and possibly negatively correlated, $-(1-\eta)/\eta < k < 1$), which implies a reaction by the foreign Central Bank that is weaker than what would be optimal for the domestic country. On the contrary, if $\delta > 1$ or $\delta < 0$, both coefficients are smaller than those under inflation targeting, suggesting little or no role for short-term debt in stabilizing the debt ratio. Interestingly, the case against short-term debt arises either because of greater foreign, $k > 1$, or because of negatively correlated shocks, $k < -(1-\eta)/\eta$. In the former case the imported monetary policy leads to a too strong interest-rate reaction, while in the latter it implies a pro-cyclical reaction.

The implications for inflation-indexed debt depend on the correlation between output and inflation in equation (40). Whether this correlation is greater or lower than under inflation targeting, it does not only depend on δ, but also on the values of other parameters and the variance of demand relative to supply shocks. For instance, in the case $\lambda = 0$, the correlation between output and inflation is greater than under inflation targeting if $\delta > 1$ or if $\delta < \bar{\delta} < 1$, where $\bar{\delta}$ can be positive or negative depending on the variance of demand relative to supply shocks. Hence, the optimal share of inflation-indexed debt is greater than under inflation targeting for large shocks to the foreign economy leading to a too strong interest-rate reaction and in the case of small or asymmetric foreign shocks implying a pro-cyclical imported monetary policy.

Therefore, differences in the magnitude and correlation of domestic and foreign shocks in a fixed exchange regime may favor inflation-indexed bonds, and possibly long-term conventional bonds as stabilizing instruments when the foreign policy reaction is too strong or asymmetric for domestic stabilization purposes.

4.2.4 Different transmission mechanisms and debt management

The results of the previous section hinge on the hypothesis that the structures of the domestic and foreign economy are the same. In what follows we show how the implications of different economic structures for the choice of debt instruments can be derived using the framework developed so far.

A first case to consider is when the aggregate demand of the foreign economy is more sensitive to the interest rate than the domestic demand; i.e. when $a^* > a$. If we restrict the analysis to the case where both monetary authorities do not care about output stabilization, then $\beta^* = (a/a^*)\beta$ and $\gamma^* = (a/a^*)\gamma$. Assuming that the two countries experience the same shocks, the interest-rate reaction is equal to

$$i_t - E_{t-1}i_t = \phi[\beta(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] \tag{41}$$

where $\phi = (a/a^*)\eta + (1-\eta) < 1$.

Because $\phi < 1$ the implications for debt management of a greater sensitivity of the aggregate demand of the leader country to the interest rate are the same of those derived earlier under the assumption of a lower variance of foreign shocks. In particular, the optimal share of short-term debt is higher than under inflation targeting because the stronger foreign transmission mechanism implies a weaker reaction by the foreign Central Bank to the same shocks.

A second case to consider is when foreign inflation is less sensitive to the output gap than domestic inflation; i.e. when $c^* < c$. If both monetary authorities do not care about output stabilization, then $\beta^* = (c/c^*)\beta$ and $\gamma^* = \gamma$. Then, assuming that the two countries experience the same shocks, the unanticipated component of the interest rate is equal to

$$i_t - E_{t-1}i_t = [\bar{\beta}(\pi_t - E_{t-1}\pi_t) + \gamma(y_t - E_{t-1}y_t)] \tag{42}$$

where $\bar{\beta} = [(c/c^*)\eta + (1-\eta)]\beta$ is a weighted average of the foreign and domestic interest-rate reactions to inflation shocks, and thus is greater than the domestic reaction under inflation targeting, β. This suggests that a lower sensitivity of foreign inflation to the output gap has the same implications for debt management as those derived earlier for a stronger preference of the Central Bank for inflation control relative to output stabilization. In particular, the stronger reaction to inflationary pressure would reduce the optimal share of short-term debt while it would increase the share of inflation-indexed debt.

5. Estimating the optimal debt structure

In this section we estimate the composition of public debt that would have supported debt stabilization in OECD countries over the past two decades when cost considerations are not taken into account, namely when expected return differentials, TP_{t-1} and IP_{t-1} are set to zero. The debt composition is obtained by estimating the correlation coefficients in equations (13) and (14) using yearly series of output growth, CPI inflation, GDP inflation, and the short-term interest rate for the period 1960-2003.[5]

The conditional correlation coefficients are obtained in two steps. First, the unanticipated components of output growth, CPI inflation, GDP inflation and the short-term interest rate are estimated as the residuals of forecasting equations in their second lags (in the first lag for the interest rate). Second, the correlation coefficients are obtained as the coefficients of the regressions of the residuals of output growth and GDP inflation on the residuals of the interest rate, and as the coefficients of the regression of the residuals of output growth and GDP inflation on the residuals of CPI inflation. The first stage regressions are run recursively using only the information available up to the time when the forecast is made with a constant window of fifteen years. This implies that the series of residuals starts in 1976 or 1980, depending on data availability, despite the fact that the sample period runs from 1960 to 2003.

The estimated coefficients are shown in Table 1; they are remarkably consistent with the stochastic structure derived from the monetary policy model developed in section 4. As expected, the correlation coefficients between output growth and the interest rate are negative and significant for all countries considered except for Spain and Sweden. The coefficients of inflation on the interest rate, though positive, display a different pattern across countries. In Japan, the Netherlands, Spain and the UK, the hypothesis of no conditional correlation between inflation and the interest rate cannot be rejected at the 5% significant level, while in Italy at the 10% level. The absence of a relation is consistent with strict inflation targeting regimes where the authorities place no weight on output stabilization. All the other countries exhibit instead a significant positive correlation between inflation and the interest rate. The coefficient is particularly high for Belgium, Denmark, Germany and the US. This evidence is consistent with monetary regimes where monetary authorities have a concern for output stabilization and/or interest-rate smoothing. This evidence is also consistent with a fixed exchange regime when the interest-rate reaction of the leader countries is weaker than that needed to fully stabilize domestic inflation.

The last column of Table 1 shows that the correlation coefficient between output and inflation is negative but not significant for all countries considered except for Denmark, France, the UK and the US. This is also evidence of a preference for output stabilization and interest-rate smoothing in the conduct of monetary policy.

The estimated coefficients in Table 1 allow us to derive the debt compositions that would have supported debt stabilization over the past two decades when expected cost considerations are not taken into account.

The optimal debt compositions are shown in Table 2. Short-term debt should not have been issued in Denmark, France, Italy, Japan, the Netherlands, Spain, the UK and the US. These countries exhibit a significant negative correlation between output growth and the interest rate that is not offset by the positive correlation between inflation and the interest rate. This evidence suggests that short-term debt in these countries had a role in stabilizing the debt ratio only because of its lower expected return. By contrast, in Ireland, Sweden and, to a lesser extent, in Belgium short-term debt also had a role in stabilizing the debt ratio against variations in output

[5] All data are taken from OECD National Accounts and Main Economic Indicators. In the case of Denmark, Italy, Spain and Sweden we use the long-term interest rate because of the unavailability of sufficiently long series for the short-term interest rate.

growth and inflation, because the correlation between inflation and the interest rate was high and significant.

Indexation to CPI inflation naturally provides insurance against lower-than-expected inflation. The optimal share of inflation-indexed debt is particularly high in Belgium, Germany, Ireland, Italy, Japan, the Netherlands, Spain, Sweden and the UK where the correlation between output growth and inflation is positive or not significant (except for the UK). Thus, insurance considerations suggest a large scope for indexation in these countries in sharp contrast with the little or no reliance on such bonds by actual debt management. The fact that such bonds have been issued only in France, Italy, Sweden, the UK, and the US, and often in limited amounts, can be explained only in part by the cost of introducing such bonds or by their illiquidity.

Finally, a large share of fixed-rate long-term debt should have been issued in Denmark, France, the Netherlands, and the US where short-term debt played no stabilizing role while the insurance provided by inflation indexation was partly offset by the negative correlation between inflation and output growth. In fact, the actual debt maturity in these countries is among the longest in the OECD.

6. Conclusions

In this paper we have presented a model in which debt management stabilizes the debt ratio in face of shocks to real returns and output growth. Although the model is very simple, it helps think about the role of debt management in ensuring debt sustainability, a theme that has, so far, received scant attention in the literature.

The analysis highlights two reasons why the choice of debt instruments is important for debt sustainability. Debt instruments can be chosen to minimize the expected return on public debt and thus the rate at which the debt accumulates. But debt instruments can also be chosen to insulate the debt ratio from unfavorable shocks such as unexpectedly high returns or low output growth. If such shocks are persistent, they may lead the debt on an unsustainable path. Avoiding such risk is a crucial task of debt management.

This finding motivates a further analysis of the stochastic relations between the returns on the main funding instruments with inflation and output growth that affect the dynamics of the debt ratio. We have investigated the role of short-term bills, fixed-rate bonds and inflation-indexed bonds in providing insurance against lower-than-expected inflation and GDP growth in OECD countries over the past two decades.

The empirical evidence suggests that the public debt should have a long maturity and a large share of it should be indexed to the price level. Little or no insurance is instead provided by short-term debt; a role for such instrument in stabilizing the public debt arises only because of its lower expected return.

Although it is tempting to translate these findings into policy indications, caution is needed as the future relevance of the estimated relations is uncertain. Indeed, such relations may reflect the prevalence of particular shocks in the past. Moreover, the estimated optimal compositions do not take into account the implications of expected cost differentials. These may in part explain why OECD governments have so far relied on short-term debt and issued limited amounts of inflation-indexed bonds. However, the estimated optimal share of inflation-indexed bonds is so large that we take this evidence as suggestive of the potential gains that governments could obtain if they issued such bonds.

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Table 1: Estimation results: residuals calculated from recursive regressions.

	$\frac{Cov(y_{t+1},i_t)}{Var(i_t)}$	$\frac{Cov\left(\pi^{def}_{t+1},i_t\right)}{Var(i_t)}$	$\frac{Cov\left(\pi^{def}_{t+1},\pi_{t+1}\right)}{Var(\pi_{t+1})}$	$\frac{Cov(y_{t+1},\pi_{t+1})}{Var(\pi_{t+1})}$
Belgium (1976-2004)	−0.36** (2.28)	0.47** (3.36)	0.77** (11.3)	−0.09 (0.65)
Denmark[a] (1976-2004)	−0.76** (3.44)	0.64** (3.12)	0.60** (5.84)	−0.55** (3.69)
France (1980-2004)	−0.37** (3.19)	0.23** (2.07)	0.50** (5.61)	−0.22* (1.70)
Germany (1976-2004)	−0.42** (2.16)	0.44** (3.91)	0.55** (3.85)	0.22 (0.85)
Ireland (1985-2004)	−0.56** (2.59)	0.82** (3.74)	0.62** (6.71)	−0.17 (1.27)
Italy[a] (1980-2004)	−0.71** (2.73)	0.71* (1.77)	0.97** (19.1)	0.03 (0.27)
Japan (1980-2004)	−0.35* (1.82)	0.15 (0.44)	0.88** (10.4)	−0.11 (0.99)
Netherlands (1976-2004)	−0.31* (1.97)	−0.01 (0.07)	0.49** (3.74)	0.12 (0.72)
Spain[a] (1976-2004)	−0.29 (0.89)	−0.17 (0.45)	0.94** (14.8)	−0.08 (0.57)
Sweden[a] (1976-2004)	−0.31 (0.96)	1.15** (2.67)	0.69** (5.73)	−0.01 (0.61)
UK (1980-2004)	−0.48** (3.15)	0.15 (0.39)	0.98** (18.3)	−0.19** (2.39)
US (1976-2004)	−0.61** (3.58)	0.39** (3.51)	0.53** (8.98)	−0.29* (1.96)

Note: t-statistics in parentheses. * significant at 10% level. ** significant at 5% level.
[a] long-term interest rate have been used in the analysis.
Years in parentheses indicate the recursive estimation sample.

Table 2: Optimal debt composition for debt stabilization.

	Unconstrained			Constrained		
	Short Debt	Inflation Indexation	Fixed-Rate Long Debt	Short Debt	Inflation Indexation	Fixed-Rate Long Debt
Belgium	11	68	21	11	68	21
Denmark[a]	−12	5	107	0	5	95
France	−14	28	86	0	28	72
Germany	2	77	21	2	77	21
Ireland	26	45	29	26	45	29
Italy[a]	0	100	0	0	100	0
Japan	−20	77	43	0	77	23
Netherlands	−32	61	71	0	61	39
Spain[a]	−46	86	60	0	86	14
Sweden[a]	84	68	-52	55	45	0
UK	−33	79	54	0	60	40
US	−22	24	98	0	24	76

Notes: The debt composition is derived from equation (13) and (14) using the coefficients in Table 1. The constrained debt composition is computed assuming that debt shares can not be negative.
[a] long-term interest rate has been used to estimate the coefficients in Table 1.

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