HETEROGENEITY WITHIN COMMUNITIES: A STOCHASTIC MODEL WITH TENURE CHOICE

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Abstract

Standard explanations for the observed income heterogeneity within communities rely on differences of preferences across households and heterogeneity of the housing stock. We propose a dynamic stochastic model of location choice where households differ according to income only, and homes are identical within locations. Households choose whether to own or rent their home motivated by concerns over housing expenditure risk. The model highlights how differences in the timing of moves generate income heterogeneity across homeowners within neighborhoods, in particular in cities that experience strong positive demand shocks. US Census data provides evidence in favor of the income mixing mechanism we identify. In communities that have experienced strong price growth, the heterogeneity of homeowners' incomes is positively correlated with the heterogeneity of the times since they bought their homes. Homeowners who moved in more recently earn higher incomes than homeowners who bought earlier, more so in cities with strong housing price growth. These relationships do not hold for renters.

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1 Introduction

There is considerable income heterogeneity within neighborhoods. Epple and Sieg (1999) estimate that 89 percent of the income variance in the Boston metropolitan area in 1980 could be explained by within-community variance. Davidoff (2005) finds that only 6 percent of the variation of household incomes within US metropolitan areas could be explained by differences across jurisdictions in 1990. Hardman and Ioannides (2004) report that in 1993 more than two-thirds of U.S. metropolitan neighborhoods in the American Housing Survey included at least one household with income in the bottom quintile of the metropolitan income distribution; more than half the neighborhoods had at least one household with income in the top quintile.

Standard explanations for income mixing rely on dimensions of heterogeneity beyond that of household income, e.g., heterogeneity of households' place of work (Brueckner, 1994), heterogeneity of preferences for local amenities (Epple and Platt, 1998), or heterogeneity of the housing stock (Nechyba, 2000).

We propose an alternative explanation that is motivated by the following observation. There are neighborhoods in central London where taxicab drivers live next to investment bankers. The taxicab drivers' income has not kept up with housing rents. It now takes a banker's income to afford a home in these neighborhoods. What distinguishes the taxicab drivers who live next to the bankers is that they bought their homes several years prior, at a time when they were much more affordable. Had these taxicab drivers chosen to rent, they would have moved out due to the rise in rents.

This observation leads us to study a dynamic model of the housing market where households appear at different times and choose not only where to locate but also whether to own or rent their home. The model has two locations and two periods. The locations differ in the amenities they provide to their residents and in their housing supply elasticities. Native households populate the model from the start. With some probability, newcomers appear in the second period. In equilibrium, second-period housing rents in the more desirable location depend on whether or not the city population grows.

The model features a single dimension of household heterogeneity: the size of a lump-sum endowment that we interpret as human capital. In the second period, once the uncertainty about the arrival of newcomers is resolved, the equilibrium problem reduces to a standard static sorting problem à la Tiebout (1956) with total wealth as the one relevant dimension of household heterogeneity. The key modeling innovation is that the mapping from human capital to wealth for native households depends on their location and tenure choices the period before. For newcomers, whom we interpret to be young households who have not accumulated any assets yet, total wealth simply coincides with human capital.

We assume that native households choose whether to own or rent a home in the first period motivated by concerns over housing expenditure risk. The only natives who buy a home in the more desirable location in the first period are those who plan to remain there independently of the population shock. This makes their second-period wealth risky, but perfectly insures them against the shock: wealth is higher precisely when newcomers move in and rents rise.

For both natives and newcomers, there is a common critical level of second-period wealth such that households with wealth above that level choose the more desirable location, while all others choose the other location. When the arrival of newcomers generates a sufficiently large rent rise in the more desirable location, some native homeowners realize capital gains that lift their wealth above the critical level even though their human capital is below it. By contrast, the newcomers who become their neighbors all have human capital above this threshold, and so their human capital exceeds that of the poorest native homeowners.

Thus the model delivers a very simple explanation for our observation about central London. While the cab drivers have lower income than their banker neighbors, they enjoyed substantial capital gains on their homes. This wealth increase let them stay in the neighborhood despite the strong increase in the user cost of housing relative to their incomes.

To ascertain the broader relevance of the income mixing mechanism we identify, we turn to the 5 percent sample of the 2000 US Census. The predictions of the model are contingent on housing price growth over the period when households move to the city. We focus on the 1351 urban Public Use Micro Areas (PUMAs) that we can match to metropolitan areas covered by the Freddie Mac housing price index. The housing price data starts in 1975; as an indicator of housing price growth, we take the growth over the period 1975-1999. We split the metropolitan areas into four quartiles according to the price growth they experienced. This gives us four sub-samples of PUMAs differentiated along the dimension relevant to the model.

We first study the correlation between income heterogeneity and heterogeneity of time since moved among homeowners. We regress income heterogeneity on heterogeneity of time since moved and control variables that capture the heterogeneity of the age of the heads of households and the heterogeneity of property values. We find a significant positive correlation between heterogeneity of income and time since moved only in cities that have experienced greater than median price growth over the period 1975-1999. We find a larger coefficient on heterogeneity of time since moved in the sub-sample of PUMAs located in cities that experienced price growth in the top quartile.

The relationship between heterogeneity of income and of time since moved in the model is due to the fact that the most recent movers have higher income than their neighbors, in cities with strong price growth. In the data, we find that households who bought a home more recently than their neighbors have a higher income relative to their neighbors, holding constant the age of the head of the household and the property value. Furthermore, the regression coefficient on differences in time since moved is larger the greater the local housing price growth over the past 25 years.

The predictions of the model concern homeowners and the capital gains they realize on their home. Renters do not experience capital gains when housing prices grow. As expected, we do not find the same empirical patterns when we run the above regressions on renters. To complement our empirical analysis, we also report evidence from the American Housing Survey which provides data on small neighborhoods of about 10 households on average.

We abstract from transaction costs in the model. Transactions costs would also generate hysteresis in the allocation of properties across households. Absent any wealth effect, if transaction costs were the drivers of the income heterogeneity we observe, we would expect a positive relationship between income heterogeneity and heterogeneity of time since moved in places that experienced weak housing price growth. In places that experienced strong housing price growth, we would expect transaction costs to be irrelevant, and hence no relationship between income heterogeneity and heterogeneity of time since moved. This goes counter to our empirical finding that the relationship between the heterogeneity of homeowners' incomes and the heterogeneity of the times since they moved is strongest in the locations that experienced the largest price growth.

We are not the first to study a dynamic sorting model. Bénabou (1996a, 1996b), Durlauf (1996) and Fernandez and Rogerson (1996, 1998) propose dynamic sorting models to analyze macroeconomic and policy issues. They assume that the benefits of living in a community depend on the make-up of the community and are therefore determined endogenously. The same is true in static models that determine the benefits of each community by a political equilibrium; see, for example, Epple, Filimon and Romer (1984, 1993). Common to all these models is that households make only one location decision in equilibrium, either by assumption or because of a focus on stationary environments. We instead take the amenities of a community as given, but we allow households to relocate and to choose whether to own or rent their property in the face of endogenous fluctuations in housing costs.

What distinguishes the more expensive community in our model is a combination of greater desirability and a more inelastic housing supply. Gyourko, Mayer and Sinai (2004) find that the households that move to desirable cities with inelastic housing supplies tend to be richer than the households already living in these cities. Although our discussion is cast in terms of communities within the same urban area, our arguments seem to apply equally to cities within a country.

As in Ortalo-Magné and Rady (2002), we focus on tenure choice driven by concerns over future housing expenditure risk. Davidoff (2003), Diaz-Serrano (2005), Han (2004) and Hilber (2005) provide evidence of the relevance of this driver of tenure choice. Our two-period model captures the idea that, at short horizons, household concerns over period-to-period rent risk are dominated by concerns over end-of-holding-period price risk, and vice versa at long horizons (Sinai and Souleles, 2005). From a modeling standpoint, the innovation in the present paper relative to this literature is that we cast such tenure concerns within an equilibrium model of the housing market.

2 The Model

There are two periods, 1 and 2, and two communities, 0 and 1. In community 0, the supply of homes is perfectly elastic at a constant rent normalized to zero. In community 1, there is a measure S of identical homes owned initially by absentee landlords. For simplicity, the landlords are assumed to be risk-neutral. They discount rents at the same exogenous interest rate at which households can borrow and save. Without loss of generality, we assume that this interest rate is zero.

Initially, the area is populated by a measure one of households that we call the natives. Natives derive additively separable utility from the consumption of housing and a numeraire good. There is no discounting of utility across periods. Community 1 is more desirable than community 0: housing utility derived from a home in community 0 is normalized to zero, whereas a home in community 1 yields an additive utility premium of $\mu > 0$ per period, whether the home is owned or rented.

The numeraire good is enjoyed at the end of period 2 only. The utility derived from consumption of c units of the numeraire good is described by the constant absolute risk aversion function $U(c) = -e^{-c}$ where the coefficient of absolute risk aversion is assumed to be 1 to economize on notation. Within each period, trading takes place before consumption.

There is uncertainty in period 2. With probability $\pi \in (0, 1)$, state H occurs: A measure ν of newcomer households moves to the area at the start of period 2. With probability $1 - \pi$, state L occurs: Nobody moves in. Although the shock is asymmetric by design, we will see later that from the point of view of the natives, it amounts to either a rent increase (state H), or a rent decline (state L). Our specific modelling choice for the shock is motivated by our interest in the allocation of homes between households that had the opportunity to buy their homes early and those who move in later.

Each native household is characterized by an endowment of $W \geq 0$ units of the numeraire good that we interpret as its human capital. The distribution of native households' endowments has a strictly positive density on $(0, \infty)$. The corresponding cumulative distribution function is $F\colon [0, \infty] \to [0, 1]$. We assume perfect capital markets, so the household faces a single budget constraint: life-time expenditures on housing and numeraire consumption cannot exceed W.

The distribution of the endowments of newcomer households also has support $[0, \infty)$ and a strictly positive density on $(0, \infty)$. The corresponding cumulative distribution function is $\tilde{F}\colon [0, \infty] \to [0, 1]$.

For ease of exposition, we assume $S < \frac{1}{2}$ throughout. This limits the number of cases we will have to consider without taking anything away from the results.

2.1 Tenure choice

Whether a household owns or rents a home in community 0, the cost is nil by assumption. Since we also assume that housing utility does not depend on tenure, all households are indifferent between renting and owning a home in community 0.

Tenure matters for homes in community 1. We denote R_1 their rent in period 1; R_H in period 2, state H; and R_L in period 2, state L. We assume $R_L < R_H$ throughout this section. We will see later that this inequality holds in equilibrium.

Arbitrage on the part of the landlords ensures that the price of a home in period 1, p_1, equals the first-period rent plus discounted expected second-period rent:

$$p_1 = R_1 + \bar{R}_2 \tag{1}$$

where R_1 denotes the first-period rent and $\bar{R}_2 = \pi\, R_H + (1-\pi)\, R_L$ the expected second-period rent. Since period 2 is the last period of the economy, renting a home in period 2 is equivalent to buying it, so the price of a home in period 2 coincides with the rental cost of that home in period 2.

Further notation describes location and tenure choices. A native household's *location plan* is denoted by (h_1, h_H, h_L), where h_1, h_H and h_L take the value of 1 for community 1, and 0 for community 0. To indicate the tenure choice when $h_1 = 1$, we denote the combined *location-tenure plan* by $(1_B, h_H, h_L)$ if the household buys a home, and $(1_R, h_H, h_L)$ if it rents one. Figure 1 summarizes the location-tenure choices available to a native household.

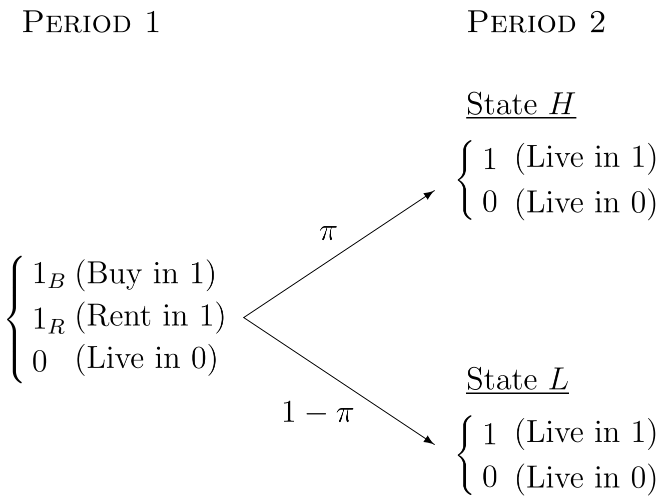

Figure 1: Native households' housing choices

Natives choose among twelve location-tenure plans. There are eight location plans. There are two alternatives for each of period 1, period 2, state H, and period 2, state L. For the four location plans that involve living in location 1 in period 1, native households must decide whether to buy or rent.

The tenure choice affects how shocks to the housing markets translate into shocks to the household's cost of housing and then through the budget constraint into shocks to non-housing consumption. The stochastic properties of numeraire consumption are therefore what is at issue with regard to the choice of tenure.

For example, consider the expected numeraire consumption of a household that chooses to live in location 1 in period 1 and in period 2, whatever the shock. If the household rents in period 1, it pays first-period rent and then realized rent in period 2. Its expected numeraire consumption is $W - R_1 - \bar{R}_2$. If the household buys in period 1, its numeraire consumption is $W - p_1$. By equation (1), expected numeraire consumption is independent of tenure choice. The same holds for every other plan that involves a tenure choice.

Because households are risk-averse, this property of expected numeraire consumption implies that the tenure decision reduces to choosing the option that produces the smallest absolute difference between the numeraire consumption levels in the two states of the economy.

For the location plans with a deterministic horizon in the type 1 home, $(1, 0, 0)$ and $(1, 1, 1)$, the tenure choice is obvious as one of the tenure modes provides full insurance and the other does not. A household that rents in period 1 and moves to location 0 in period 2 does not suffer any shock to its consumption of numeraire. A household that buys in period 1 and remains in location 1 in period 2 does not face any numeraire consumption risk either. The plans $(1_R, 0, 0)$ and $(1_B, 1, 1)$ therefore dominate the plans $(1_B, 0, 0)$ and $(1_R, 1, 1)$, respectively.

Under the location plans $(1, 1, 0)$ and $(1, 0, 1)$, however, either tenure mode imposes some risk on the household. Under $(1, 1, 0)$, if the household rents, it pays the rent R_H in state H and no rent in state L; its numeraire consumption is lower by R_H in state H than in state L. If the household buys in the first period, it sells the home if the state L occurs. The price of a location 1 home in state L is R_L. The household's numeraire consumption is therefore lower by R_L in state H than in state L if it buys in period 1. Buying is thus less risky, given our working assumption that R_L is lower than R_H. The location-tenure plan $(1_B, 1, 0)$ therefore dominates the plan $(1_R, 1, 0)$. Under $(1, 0, 1)$, the logic is reversed: the location-tenure plan $(1_R, 0, 1)$ dominates the plan $(1_B, 0, 1)$.

We summarize these findings in

Lemma 1 *If $R_L < R_H$, a native household wanting to live in location 1 in the first period prefers to own its home if and only if it plans to stay in location 1 should state H occur in the second period.*

2.2 Location choice

We are left with eight plans to consider: $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,0,1)$, $(1_B,1,0)$, and $(1_B,1,1)$. Each of these plans determines a curve in the plane with coordinates W (the household's endowment) and EU (the expected overall utility level). Determining the optimal plan for every W amounts to characterizing the upper envelope of the expected utility curves. In the discussion, we maintain our working assumption that $R_L < R_H$.

First, the CARA specification of non-housing utility implies that the expected utility of any location-tenure plan can be written as $EU = -A\,e^{-W} + B$ with plan-specific constants $A > 0$ and $B \geq 0$, where $B \in \{0,\ \pi\mu,\ (1-\pi)\mu,\ \mu,\ (1+\pi)\mu,\ (2-\pi)\mu,\ 2\mu\}$ is the expected utility of housing. For example, for the plan $(1_B,1,1)$, the expected utility takes the form

$$EU_{(1_B,1,1)} = -\,e^{p_1}\,e^{-W} + 2\mu.$$

It is easy to check that if the expected utility curves of two plans cross, the curve associated with the plan that promises a longer expected time in community 1 (and so has the higher B) is steeper at all endowment levels (has the greater A). Note also that the higher B, the greater the expected utility as W increases (the limit of EU as W tends to infinity is B). This immediately yields

Lemma 2 *The amount of housing a native household expects to consume in community 1 increases weakly with the household's endowment.*

Second, using CARA utility, it is easy to verify that the preference ranking of the plans $(1_R,0,0)$ and $(0,1,1)$ does not depend on the household's endowment. In other words, the expected utility curves associated with these two plans are either identical or do not intersect. Both plans generate the same utility of housing, μ; their ranking is determined by the cost difference alone. We thus have

Lemma 3 *The plan* $(1_R,0,0)$ *weakly dominates* $(0,1,1)$ *if and only if*

$$e^{R_1} \leq \pi e^{R_H} + (1-\pi)e^{R_L}, \tag{2}$$

with a strict preference if the inequality is strict.

Third, we find that the plans $(0,1,0)$ and $(1_B,1,0)$ are not chosen by any native household. This yields

Lemma 4 *If $R_L < R_H$, each native household chooses one of the location-tenure plans* $(0,0,0)$, $(0,0,1)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,0,1)$ *and* $(1_B,1,1)$.

To see this, note that the location choice in period 2 obeys a simple cutoff rule in terms of period 2 wealth, W', which is the endowment minus the cost of housing consumed in period 1. In state $s \in \{H, L\}$, a household with wealth W' strictly prefers location 1 if and only if $W' > W'_s$ where W'_s solves

$$-e^{-(W'_s - R_s)} + \mu = -e^{-W'_s} \tag{3}$$

or

$$\mu e^{W'_s} = e^{R_s} - 1. \tag{4}$$

If $R_L < R_H$, then $W'_L < W'_H$.

Households that spend period 1 in location 0 have the same wealth at the start of period 2 in either state. If this wealth is such that they choose location 1 in state H, they obviously also choose location 1 in state L. This rules out the plan $(0,1,0)$.

Households that buy a location 1 home in period 1 enjoy gains or losses depending on the state in period 2. The difference between the corresponding second-period wealth levels is $R_H - R_L$. As

$$W'_H - W'_L = \ln\left(e^{R_H} - 1\right) - \ln\left(e^{R_L} - 1\right) > \ln\left(e^{R_H}\right) - \ln\left(e^{R_L}\right) = R_H - R_L \tag{5}$$

by the concavity of the logarithm, the inequality $W' > W'_H$ implies $W' > W'_L$. This rules out the plan $(1_B, 1, 0)$.

The newcomers appear in the second period only if state H occurs. Any newcomer with endowment above W'_H chooses location 1, all others choose location 0.

2.3 Equilibrium

An equilibrium is a triple of rents, (R_1, R_H, R_L), and a period 1 price, p_1, for homes in community 1, together with a location-tenure plan for each native household and a location choice for each newcomer. The equilibrium price of homes in community 1 must be such that landlords are indifferent between selling a home in period 1 and renting it in both periods at the equilibrium rents. The equilibrium allocation must be such that housing markets clear and each household's utility is maximized, given its budget constraint and the prices and rents of homes in community 1.

Proposition 1 *There is a unique equilibrium. The equilibrium rents satisfy* $R_L < R_1 < R_H$*. There is a unique size* $\nu^* > 0$ *of the newcomer cohort such that condition* (2) *holds as an equality for all* $\nu \leq \nu^*$*, and as a strict inequality for all* $\nu > \nu^*$*. Native households' equilibrium choices are characterized by critical endowment levels* $0 < W_1 < W_2 < W_3 < W_4$ *such that*

- *all native households with endowment smaller than* W_1 *choose* $(0, 0, 0)$*;*
- *all native households with endowment between* W_1 *and* W_2 *choose* $(0, 0, 1)$*;*
- *if* (2) *holds as a strict inequality, all native households with endowment between* W_2 *and* W_3 *choose* $(1_R, 0, 0)$*;*
- *if* (2) *holds as an equality, more than half of all native households with endowment between* W_2 *and* W_3 *choose* $(1_R, 0, 0)$*, and the rest* $(0, 1, 1)$*;*
- *all native households with endowment between* W_3 *and* W_4 *choose* $(1_R, 0, 1)$*;*
- *all native households with endowment greater than* W_4 *choose* $(1_B, 1, 1)$.

All newcomers with endowment greater than W'_H *choose community 1 in state* H*, all others community 0.*

Proof: See Appendix A.1. The formulas for the endowment cutoffs are given in Appendix A.4. ■

The inequality $R_L < R_H$ reflects the price pressure newcomers exert when they appear in state H. That the opportunity cost of choosing community 1 in the first period, R_1, lies strictly in between R_L and R_H is then dictated by market clearing. Intuitively, the cost of living in community 1 in period 1 cannot be too different from the cost of living in community 1 in period 2 for sure, a cost that lies in between R_L and R_H.

How much price pressure newcomers exert depends on the size of their cohort, ν. If it is large enough, location 1 is sufficiently expensive in state H for the plan $(1_R, 0, 0)$ to strictly dominate $(0, 1, 1)$.

Figure 2 summarizes native households' choices in equilibrium for this case and graphs the mapping from endowments to second-period wealth. A household's endowment, W, is on the horizontal axis. Wealth at the time when the housing market opens in period 2,

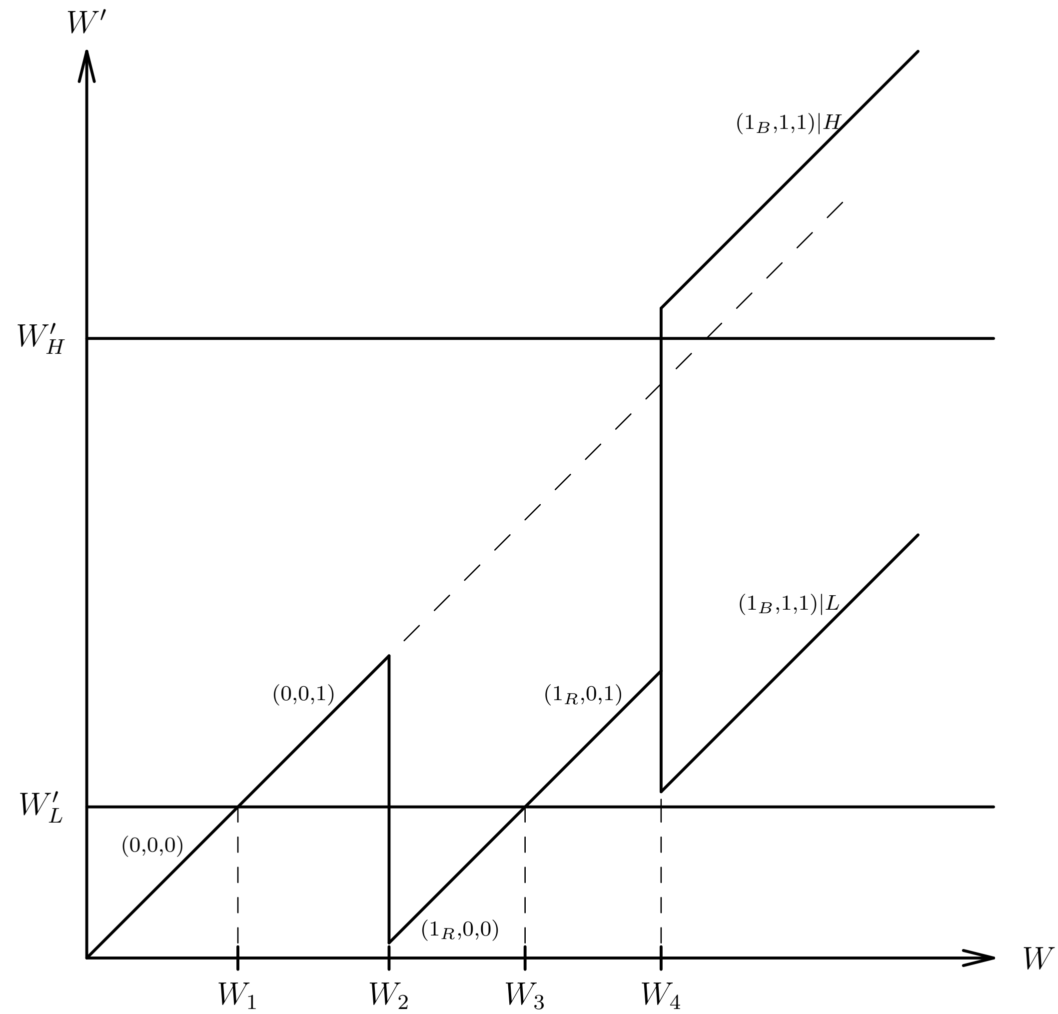

Figure 2: Equilibrium location choices and second-period wealth of native households

W', is on the vertical axis; it equals W minus the cost of housing consumed in period 1. Up to $W = W_2$, W' equals W. From $W = W_2$ to W_3, W' equals W minus first-period rent R_1 for those households that choose $(1_R, 0, 0)$, and W for those that choose $(0, 1, 1)$. From $W = W_3$ to W_4, W' equals W minus first-period rent. For W above W_4, W' depends on the realization of the shock. In state H, $W' = W - p_1 + R_H$; in state L, $W' = W - p_1 + R_L$.

The figure also shows the critical wealth levels W'_H and W'_L that determine second-period location choice. By time consistency, the second-period wealth of the poorest native households that follow plan $(1_B, 1, 1)$ must be at least W'_H in state H. In fact, we have the strict inequality $W_4 - p_1 + R_H > W'_H$. To prove it, suppose that $W_4 - p_1 + R_H = W'_H$. The households with endowment W_4 are then indifferent between the plans $(1_B, 0, 1)$ and $(1_B, 1, 1)$. However, we know from Lemma 1 that the plan $(1_R, 0, 1)$ dominates the plan $(1_B, 0, 1)$. This is inconsistent with the definition of W_4.

Similarly, by time consistency, the second-period wealth of the poorest native households that follow plan $(1_B, 1, 1)$ must be at least W'_L in state L. Again, we have the strict inequality $W_4 - p_1 + R_L > W'_L$. To prove it, suppose that $W_4 - p_1 + R_L = W'_L$. The households with endowment W_4 are then indifferent between the plans $(1_B, 1, 0)$ and $(1_B, 1, 1)$. However, Lemma 4 implies that households that are indifferent between the plans $(1_B, 1, 0)$ and $(1_B, 1, 1)$ strictly prefer the plan $(1_R, 0, 1)$ to the plan $(1_B, 1, 1)$. This is inconsistent with the definition of W_4.

The second-period wealth of the poorest native households that buy a location 1 home in period 1 is thus strictly larger than the endowment of the poorest newcomers who move to location 1 in both states H.

When the wealth of the poorest natives who buy in location 1 is boosted by sufficiently large capital gains $R_H - p_1$ on location 1 homes, their *endowment* can still be strictly smaller than the endowment of these newcomers. This is the situation depicted in Figure 2. We find that it arises if the price pressure from newcomers is sufficiently large.

Proposition 2 *There is a $\nu^{**} > 0$ such that for all $\nu \geq \nu^{**}$, the endowment of the poorest native households that buy a location 1 home in period 1 is smaller than the endowment of the poorest newcomers that move to location 1 in state H; i.e., $W_4 < W'_H$.*

PROOF: See Appendix A.5. ■

Proposition 2 allow us to rationalize our observation about central London. The taxicab drivers moved in when housing prices were low relative to their income. By buying their home, they insured themselves against their income not growing as fast as rents. Subsequently, they enjoyed capital gains large enough to more than compensate their income disadvantage relative to their new neighbors. The wealth of the newcomers, who are young, is primarily their human capital. Moving next to the old taxicab drivers requires substantial earning power. This is why the young people who move next to the old taxicab drivers are investment bankers and not young university professors.[1]

3 Empirical Evidence

The model yields predictions with regards to households' location and tenure choices as a function of their human capital. In communities that experienced strong price growth,

[1]Both authors struggled to find housing in central London at the start of their academic career.

the model predicts a positive correlation between the heterogeneity of homeowners' human capital and the heterogeneity of the time since they moved in.

To what extent is this relevant for income mixing? To answer this question, we turn to the household data from the 2000 5% Census sample (Ruggles et al., 2004). To identify locations, we use the smallest geographic unit that is identifiable in this data set, the Public Use Microdata Area (PUMA).

The predictions of the model depend on the extent to which housing prices have grown over the period during which homeowners moved into their community. Freddie Mac has been publishing a quality adjusted housing price index for US metropolitan areas since 1975. For the purpose of our study, we compute real housing price growth between the first quarter of 1975 and the fourth quarter of 1999. As a price deflator, we use the CPI-US index for all urban consumers from the Bureau of Labor Statistics.

We are able to match 1351 PUMAs to the metropolitan housing price data. There are between 82 and 4467 homeowner households within each PUMA with a median of 1060 households per PUMA. We group the PUMAs into four quartiles according to their price growth experience. The price growth cutoff points are 4.2%, 19.5% and 58.2%. The highest price growth in the sample was 188% for a PUMA located in San Jose, CA. Appendix A.6 reports definitions of the variables we use and summary statistics.

If our theoretical findings are relevant for income mixing within community, we should find a positive correlation between income heterogeneity and heterogeneity of time since moved in places that have experienced strong price growth. We measure income heterogeneity with the coefficient of variation of income in order to take out mean effects. From a theoretical point of view, it makes no difference whether households moved in 1 and 2 years ago or 11 and 12 years ago. We therefore use the standard deviation to measure the heterogeneity of the time since households moved to the neighborhood.

We control for two additional factors. First, income heterogeneity may arise as a function of time since moved simply because people who moved in at different times do not have the same age. This would arise for example in a world where everyone has same human capital, buys a home at 25, and income increases with age. We therefore control for the standard deviation of the age of the head of household. Second, communities with more heterogeneous properties are likely to house more heterogeneous households. We therefore control for the coefficient of variation of property values. We choose the coefficient of variation in order to be consistent with our measure of income heterogeneity.

Table 1: Coefficient of variation of homeowners' income within PUMAs

PUMAs	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
S.D. time since moved	−0.0046	−0.0116	0.0095	0.0184
	(0.0026)	(0.0043)*	(0.0040)*	(0.0045)*
S.D. age head	0.0237	0.0295	0.0184	0.0150
	(0.0033)*	(0.0049)*	(0.0041)*	(0.0044)*
C.V. home value	0.4549	0.4044	0.4127	0.4466
	(0.0277)*	(0.0271)*	(0.0267)*	(0.0354)*
Intercept	0.2097	0.2429	0.1824	0.1510
	(0.0466)*	(0.0529)*	(0.0453)*	(0.0599)*
R^2	0.56	0.54	0.54	0.45

*Indicates statistical significance at the 0.05 level.

Table 1 presents the results.[2] The relationship between income heterogeneity and heterogeneity of time since moved is negative for PUMAs located in cities with below median price growth. It is positive and significant for PUMAs located in cities with above median price growth. The coefficient estimated for PUMAs in the top price growth quartile is almost twice that for PUMAs in the second quartile. The greater the heterogeneity of the age of heads of households, the greater the income heterogeneity. The greater the heterogeneity of property values, the greater the income heterogeneity.

The relationship between income heterogeneity and time-since-moved heterogeneity in the model is due to the fact that in markets with strong housing price growth, homeowners who moved in more recently have higher human capital than homeowners who moved in earlier. To check whether the same relationship holds in the data, we regress relative household income on the relative time since the household moved to the PUMA. We compute a household's relative income as the ratio of its income to the median household income for all homeowners who live in the same community. We compute a household's relative time since moved as the difference between the time since the household bought its current home and the median time since homeowner households bought their current home in the community. Again, we control for age differences by including as covariates the difference between the age of the head of the household and the median age of household

[2]Standard errors are in parentheses in this and all subsequent tables where relevant.

heads for the community. We also control for differences in property value by including the ratio of the value of the household's property to the median property value in the community. We compute robust standard errors, accounting for the clustering at the PUMA level.

Table 2: Homeowners' income relative to other homeowners in the same PUMA

PUMAs	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Time since moved	−.0011	−.0021	−.0023	−.0056
	(.0002)*	(.0002)*	(.0002)*	(.0003)*
Age household head	−.0074	−.0079	−.0082	−.0084
	(.0002)*	(.0001)*	(.0002)*	(.0002)*
Home value	.4924	.5112	.5349	.6367
	(.0077)*	(.0107)*	(.0105)*	(.0134)*
Intercept	.6750	.6524	.6328	.5667
	(.0080)*	(.0109)*	(.0106)*	(.0118)*
R^2	0.20	0.19	0.18	0.18

*Indicates statistical significance at the 0.05 level.

The regression results reported in Table 2 indicate that households who moved in more recently tend to have a higher income than households who moved in earlier. The coefficient on relative time since moved is larger in communities located in metropolitan areas that have experienced the strongest price growth. Households younger than their neighbors tend to have greater income. Households who own a more valuable property also tend to have greater income.

The Census data indicates a positive relationship between the heterogeneity of homeowners' income and the heterogeneity of the time since they bought their home when the community is located in a city that has experienced strong price growth. The rationalization for this fact that is offered by the model is not rejected by the data: we find evidence that homeowners who moved in more recently have greater income than their neighbors, again more so in communities that have experienced strong price growth.

The predictions of the model concern homeowners, not renters. As a further check on the mechanism that generates income heterogeneity in the model, we replicate the above regressions for renters. We recompute the same income, time since moved and age measures for renters within each PUMA. We replace the coefficient of variation of property values and the relative property value with the coefficient of variation of gross monthly rents and relative gross monthly rent.

Table 3: Coefficient of variation of renters' income within PUMAs

PUMAs	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
S.D. time since moved	0.0328	0.0213	0.0366	0.0337
	(0.0067)*	(0.0080)*	(0.0040)*	(0.0075)*
S.D. age head	−0.0095	−0.0162	−0.0032	−0.0130
	(0.0041)*	(0.0050)*	(0.0044)	(0.0059)*
C.V. gross rent	0.3551	0.7928	0.3941	0.3249
	(0.1030)*	(0.1181)*	(0.0838)*	(0.1277)*
Intercept	0.7654	0.7094	0.5711	0.7932
	(0.0664)*	(0.0815)*	(0.0741)*	(0.0816)*
R^2	0.20	0.20	0.34	0.20

*Indicates statistical significance at the 0.05 level.

The results we obtain for the heterogeneity of income are reported in Table 3. We find a positive relationship between income heterogeneity and heterogeneity in the time since moved. However, the coefficients that we estimate are similar across all sub-samples. Differences in local housing price growth do not seem to affect the correlation between the heterogeneity of renters' incomes and that of time since moved.

Contrary to what we obtained for owners, Table 4 shows a positive relationship between differences in time since moved and relative income for renters. The renters who moved in more recently tend to have *lower* income than the renters who moved in earlier, ceteris paribus. Again, estimates for renters do not differ significantly across the four sub-samples.

PUMAs are large communities with the advantage of containing sufficiently many households for us to study the relationship between relative income and time since moved. Moreover, they are sufficiently large for metropolitan area housing prices to provide a good indicator of price growth over the long period we focused on.

The neighborhood cluster samples of the American Housing Survey (AHS) offer an opportunity to examine the relationship between the heterogeneity of incomes and of time since moved at a much lower level of aggregation. For a description and detailed analysis

Table 4: Renters' income relative to other renters in the same PUMA

PUMAs	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Time since moved	.0104	.0114	.0113	.0099
	(.0006)*	(.0005)*	(.0006)*	(.0006)*
Age household head	−.0033	−.0051	−.0062	−.0055
	(.0003)*	(.0003)*	(.0003)	(.0003)*
Gross rent	.7139	.7060	.7840	.8361
	(.0155)*	(.0257)*	(.0186)*	(.0207)*
Intercept	.5462	.5622	.4951	.4133
	(.0154)*	(.0232)*	(.0173)*	(.0186)*
R^2	0.08	0.08	0.09	0.11

*Indicates statistical significance at the 0.05 level.

of this survey data, see Ioannides (2004), who finds in particular that the coefficient of variation in neighborhood incomes increases with the *mean* time since moved. When we regress the coefficient of variation of incomes in AHS neighborhoods on the *standard deviation* of time since moved, the standard deviation of the age of heads of households and the coefficient of variation of property values, we find a significant positive coefficient on the standard deviation of time since moved for homeowners. For renters, this coefficient is not significant.[3]

4 Concluding Remarks

The empirical literature concerned with housing and location choices has flourished recently thanks to econometric advances that enable researchers to estimate households' preferences and willingness to pay for various amenities, the returns to educational expenditures, the benefits of social interactions and peer effects; e.g., Bajari and Kahn (2005), Bayer, Ferreira and McMillan (2003), Bayer, Ross and Topa (2005), Calabrese et al. (forthcoming), Sieg et al. (2004).

[3]The results are available from the authors on request. Owing to lack of data, we cannot control for the heterogeneity of rents in the regression for renters. The AHS data samples are too small to replicate the other regressions we carry out on the Census data, especially once we exclude neighborhoods for which we do not have metropolitan area price data.

Data requirements with regards to housing consumption restrict these studies to cross-sectional data sets with no ability to track households over time. These data sets provide household income but not household wealth. This is the case with the widely used Census data, for example.

As a result, it is common for researchers to approach the data through the lens of a static model of housing choice constrained by income. But a household's housing choice is the outcome of a dynamic optimization constrained by wealth, not income. The typical empirical approach therefore suffers from the fact that income is a poor predictor of a household's wealth; e.g., Kennickell (1999).

This paper offers a partial remedy. We show that people who moved in at different times are likely to have different wealth even if they have the same income, in particular if they own their home and their location has a history of strong housing price growth.

Some researchers restrict their samples to recent movers, usually motivated by concerns on housing consumption hysteresis because of moving costs. At the very least, our findings provide an additional justification for such a sample restriction.

More generally, our findings should encourage researchers to study the predictive power of differences in tenure choice and in time since moved. This information may help researchers to disentangle the contribution of wealth heterogeneity to observed housing and location choices from the contribution of preference heterogeneity. This point is particularly relevant for the numerous studies that focus on coastal metropolitan areas (e.g., Boston, Los Angeles, San Francisco). Our empirical findings indicate that time since moved is particularly informative in cities with a history of strong housing price growth.

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Appendix

A.1 Proof of Proposition 1

The proof draws on auxiliary results that are established in Sections A.2–A.4 of this appendix. Lemma A.4 shows that in equilibrium, second period rents satisfy $R_L < R_H$. Lemma A.5 shows that (2) holds. Lemma A.6 shows that the equilibrium configuration must be as stated in the proposition. This implies that the relevant market clearing conditions are (A.10)–(A.13). Lemma A.7 shows that these conditions are equivalent to the system of equations (A.14)–(A.17). Lemmas A.8 and A.9 show that this system admits a unique solution. Lemma A.10 shows that this solution yields an equilibrium. Lemma A.11 shows the existence of ν^*. ■

A.2 Auxiliary results on household behavior

To ease the notational burden, we define

$$e_1 = e^{(1+r)R_1}, \quad e_H = e^{R_H}, \quad e_L = e^{R_L}, \quad e_2 = e^{\bar{R}_2}. \tag{A.1}$$

Lemma A.1 *Let $R_L < R_H$. Then:*

(i) *the plan $(1_R, 0, 1)$ is preferred over both $(1_R, 0, 0)$ and $(1_B, 1, 1)$ at all endowments in some set of positive measure;*

(ii) *at least one of the plans $(0, 0, 1)$ and $(1_R, 0, 0)$ is preferred over both $(0, 0, 0)$ and $(1_R, 0, 1)$ at all endowments in some set of positive measure;*

(iii) *the plan $(0, 0, 1)$ is preferred over both $(0, 0, 0)$ and $(0, 1, 1)$ at all endowments in some set of positive measure.*

Proof: Part (i): Let W_* be the endowment at which a native household would be indifferent between the plans $(1_R, 0, 0)$ and $(1_B, 1, 1)$, and $W_\dagger$ the endowment at which it would be indifferent between the plans $(1_R, 0, 1)$ and $(1_B, 1, 1)$. To show that the plan $(1_R, 0, 1)$ is preferred to both $(1_R, 0, 0)$ and $(1_B, 1, 1)$ on a set of endowment levels of positive measure, it is enough to show that $W_* < W_\dagger$. To see this, recall from Section 2.1 that if the expected utility curves of two plans cross, the curve associated with the plan that promises a larger amount of housing consumption in location 1 ex ante is steeper at all endowment levels. The curve associated with $(1_R, 0, 1)$ is above the curve associated with $(1_B, 1, 1)$ to the left of $W_\dagger$, and the latter is above the curve associated with $(1_R, 0, 0)$ to the right of W_*. If $W_* < W_\dagger$, therefore, $(1_R, 0, 1)$ is preferred to both $(1_R, 0, 0)$ and $(1_B, 1, 1)$ at all endowments strictly between W_* and $W_\dagger$.

It is straightforward to verify that the endowments W_* and $W_\dagger$ are defined by

$$\mu e^{W_*} = e_1\,(e_2 - 1), \tag{A.2}$$

$$\mu e^{W_\dagger} = e_1 \left[e_2 - 1 + \frac{1-\pi}{\pi}\,(e_2 - e_1) \right]. \tag{A.3}$$

The inequality $W_* < W_\dagger$ is easily seen to be equivalent to $e_L < e_2$, which in turn is the same as $e_L < e_H$.

Part (ii): An argument similar to the one used for part (i) shows first that for $e_1 \leq e_L$, $(1_R, 0, 0)$ is preferred to $(0, 0, 0)$ and $(1_R, 0, 1)$ on some open interval of endowments; and second, that for $e_1 \geq e_L$, $(0, 0, 1)$ is preferred to $(0, 0, 0)$ and $(1_R, 0, 1)$ on some open interval of endowments.

Part (iii): Let $W_\#$ be the endowment level at which a native household would be indifferent between the plans $(0, 0, 0)$ and $(0, 1, 1)$, and $W_\flat$ the endowment level at which it would be indifferent between the plans $(0, 0, 1)$ and $(0, 1, 1)$:

$$\mu e^{W_\#} = \pi e_H + (1-\pi) e_L - 1, \tag{A.4}$$

$$\mu e^{W_\flat} = e_H - 1. \tag{A.5}$$

It suffices to show that $W_\# < W_\flat$. This is easily seen to be equivalent to $e_L < e_H$. ■

If $R_L > R_H$, the roles of the two states in period 2 are reversed. The following result is therefore just a mirror image of Lemma 4 and does not require proof.

Lemma A.2 *If $R_L > R_H$, each native household chooses one of the plans $(0,0,0)$, $(0,1,0)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,1,0)$ and $(1_B,1,1)$.*

If $R_H = R_L$, the tenure mode is irrelevant, so native households' decisions concern location only.

Lemma A.3 *If $R_L = R_H$, only the location plans $(0,0,0)$, $(0,1,1)$, $(1,0,0)$ and $(1,1,1)$ may be chosen by a positive measure of native households.*

Proof: If $R_L = R_H$, the wealth cutoffs that determine second-period location choice satisfy $W'_L = W'_H$. The result thus follows by backward induction. ■

A.3 Auxiliary results on equilibrium prices and configurations

In the following, we shall write D_1, D_H, and D_L for native households' aggregate demand for location 1 housing in period 1, period 2 state H and period 2 state L, respectively.

Lemma A.4 *In equilibrium, second-period rents satisfy $R_L < R_H$.*

Proof: Suppose that $R_L \geq R_H$. Then, Lemmas A.2 and A.3 imply that $D_L \leq D_H$. In state H, a positive measure of newcomers demand housing in location 1. So total demand for housing in location 1 is strictly higher in state H than in state L. Given that the supply of housing in location 1 is the same in both states, this is incompatible with market clearing. ■

Lemma A.5 *In equilibrium, the measure of native households that choose the plan $(1_R,0,0)$ is larger than the measure of native households that choose the plan $(0,1,1)$. As a consequence, $(0,1,1)$ cannot dominate $(1_R,0,0)$, so (2) holds.*

Proof: If it were otherwise, Lemmas A.4 and 4 would imply $D_1 < D_L$, which is incompatible with market clearing. ■

Lemma A.6 *In equilibrium, the location-tenure plans chosen by positive measures of native households are $(0,0,0)$, $(0,0,1)$, $(1_R,0,0)$, $(1_R,0,1)$ and $(1_B,1,1)$ plus possibly $(0,1,1)$.*

Proof: From Lemma A.4, we know that $R_L < R_H$. From Lemma 4, we know that the only plans that may be chosen by a positive measure of native households are $(0,0,0)$, $(0,0,1)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,0,1)$ and $(1_B,1,1)$. As the endowment distribution for native households has support $[0,\infty)$, we know that $(1_B,1,1)$ is chosen. (Here and in what follows, we interpret the word "chosen" to mean "chosen by a positive measure of native households.")

First, suppose $(0,0,0)$ is not chosen. Then, market clearing in period 1 implies $m_{001} + m_{011} = 1 - S$ where m_{001} denotes the measure of native households choosing $(0,0,1)$, and m_{011} the measure of native households choosing $(0,1,1)$. Market clearing in period 2 state L implies that the measure of native households choosing $(1_R,0,0)$ is $m_{100} = 1 - S$. Adding these two equations yields $m_{001} + m_{011} + m_{100} = 2(1-S) > 1$, which contradicts the fact that the total native population has size 1.

Second, suppose that $(1_R,0,1)$ is not chosen. By part (i) of Lemma A.1, $(1_R,0,0)$ is then not chosen either. By Lemma A.5, the same is true for $(0,1,1)$. Once these plans are eliminated, however, one either has $D_1 = D_H < D_L$ or $D_1 = D_H = D_L$ depending on whether $(0,0,1)$ is chosen or not. Both cases are incompatible with market clearing, which requires $D_1 = D_L > D_H$. So $(1_R,0,1)$ must be chosen.

Third, suppose that $(1_R,0,0)$, and hence $(0,1,1)$, is not chosen. By part (ii) of Lemma A.1, $(0,0,1)$ is then chosen. However, this implies $D_1 < D_L$, which is again incompatible with market clearing. So $(1_R,0,0)$ must be chosen.

Finally, suppose that $(0,0,1)$ is not chosen. By part (iii) of Lemma A.1, $(0,1,1)$ is then not chosen either. But then $D_1 > D_L$, again in contradiction to market clearing. So $(0,0,1)$ must be chosen. ■

A.4 Auxiliary results on equilibrium existence and uniqueness

Four critical endowment levels fully characterize native households' equilibrium choices. For indifference between $(0,0,0)$ and $(0,0,1)$, the critical endowment level is W_1 with

$$\mu e^{W_1} = e_L - 1. \tag{A.6}$$

For indifference between $(0,0,1)$ and $(1_R,0,0)$, the critical endowment level is W_2 with

$$\mu e^{W_2} = \left[e_1 - \pi - (1-\pi)e_L\right]/\pi. \tag{A.7}$$

For indifference between $(1_R,0,0)$ and $(1_R,0,1)$, the critical endowment level is W_3 with

$$\mu e^{W_3} = e_1\left(e_L - 1\right). \tag{A.8}$$

For indifference between $(1_R,0,1)$ and $(1_B,1,1)$, the critical endowment level is W_4 with

$$\mu e^{W_4} = \left[e_1\left(e_2 - \pi - (1-\pi)e_L\right)\right]/\pi. \tag{A.9}$$

Clearly, $W_1 = W_L'$ and $W_3 = W_L' + R_1$. Given our results on the set of possible equilibrium configurations, these critical endowment levels satisfy $0 < W_1 < W_2 < W_3 < W_4$.

For $k = 1,\ldots,4$, we let $i_k = F(W_k)$ denote the measure of native households with endowments lower than W_k. Thus, $0 < i_1 < i_2 < i_3 < i_4 < 1$. For the newcomers, $n_1 = \tilde{F}(W_H')$ denotes the measure of newcomer households with wealth lower than W_H'; it satisfies $0 < n_1 < 1$. With this notation, the market clearing conditions for location 1 housing in period 1, period 2 state H and period 2 state L take the form

$$\begin{aligned} S &= 1 - i_3 + \rho(i_3 - i_2), && \text{(A.10)}\\ S &= 1 - i_4 + (1-\rho)(i_3 - i_2) + (1-n_1)\nu, && \text{(A.11)}\\ S &= 1 - i_3 + (1-\rho)(i_3 - i_2) + i_2 - i_1, && \text{(A.12)} \end{aligned}$$

where ρ is the fraction of native households with endowments between W_2 and W_3 that choose $(1_R,0,0)$. By Lemma A.5, we have $\frac{1}{2} < \rho \leq 1$ and (2). Moreover, Lemma 3 implies that

$$(1-\rho)\left[e_1 - \pi e_H - (1-\pi)e_L\right] = 0. \tag{A.13}$$

We write W_{1-S} for the $(1-S)$-quantile of the endowment distribution of native households; that is, $F(W_{1-S}) = 1 - S$. We set $\psi = \mu e^{W_{1-S}} + 1$.

Lemma A.7 *The system of equations* (A.10)–(A.13) *is equivalent to the system of equations*

$$\begin{aligned} 2(1-S) &= i_1 + i_3, && \text{(A.14)}\\ 2(1-S) + \nu &= i_2 + i_4 + \nu n_1, && \text{(A.15)}\\ e_1 &= \pi \min\{e_H, \psi\} + (1-\pi)e_L, && \text{(A.16)} \end{aligned}$$

and

$$\rho = \frac{i_3 - (1-S)}{i_3 - i_2}. \tag{A.17}$$

Proof: Adding equations (A.10) and (A.11), we obtain (A.15). Adding equations (A.10) and (A.12), we obtain (A.14). Now, if $e_1 < \pi e_H + (1-\pi)e_L$ then $\rho = 1$ by (A.13). Equation (A.10) then implies $i_2 = 1 - S$ and $W_2 = W_{1-S}$, which by the definition of W_2 yields

$$e_1 = \pi\psi + (1-\pi)e_L < \pi e_H + (1-\pi)e_L. \tag{A.18}$$

If $e_1 = \pi e_H + (1-\pi)e_L$ then $\rho \leq 1$ and the definition of W_2 becomes $\mu e^{W_2} = e_H - 1$. Moreover, (A.10) implies $i_2 \leq 1 - S$, hence $W_2 \leq W_{1-S}$ and $e_H \leq \psi$. Therefore:

$$e_1 = \pi e_H + (1-\pi)e_L \leq \pi\psi + (1-\pi)e_L. \tag{A.19}$$

So equation (A.16) holds. Finally, rearranging (A.10) yields (A.17).

Conversely, equation (A.16) gives us two possible cases. First, if $\psi < e_H$, then (A.16) plus the definitions of W_2 and i_2 imply $i_2 = 1 - S$, which yields $\rho = 1$ by equation (A.17) and implies that equations (A.10) and (A.13) hold. Then, replacing one term $1-S$ by i_2 in equations (A.15) and (A.14) yields equations (A.11) and (A.12) for the case $\rho = 1$. Second, if $\psi \geq e_H$, then (A.16) implies that (A.13) holds. Using (A.17) to replace one term $1-S$ in equations (A.15) and (A.14) yields equations (A.11) and (A.12). Rearranging (A.17) yields (A.10). ■

For our next result, define $\underline{e} > 1$ as the unique real number satisfying the equality

$$2(1-S) = F\left(\ln\frac{\underline{e}-1}{\mu}\right) + F\left(\ln\frac{\underline{e}\,(\underline{e}-1)}{\mu}\right). \tag{A.20}$$

It is straightforward to see that $1 < \underline{e} < \psi$. We write W_{1-2S} for the $(1-2S)$-quantile of the endowment distribution of native households and set $\phi = \mu e^{W_{1-2S}} + 1$.

Lemma A.8 *Equations* (A.14) *and* (A.16) *yield* e_1 *and* e_L *as continuous functions of* e_H *on* $[1,\infty[$. *For* $e_H < \psi$, e_1 *is strictly increasing and* e_L *strictly decreasing in* e_H, *with* $e_L = e_1 = e_H$ *if and only if* $e_H = \underline{e}$, *and* $\phi < e_L < \underline{e} < e_1 < e_H$ *if* $\underline{e} < e_H < \psi$. *For* $e_H \geq \psi$, e_1 *and* e_L *do not vary with* e_H, *and* $\phi < e_L < e_1 < \psi$.

Proof: By the definitions of i_1 and i_3, the right-hand side of (A.14) is strictly increasing in e_L and e_1. Equation (A.14) thus defines e_L as a strictly decreasing function of e_1 which assumes the value ψ at $e_1 = 1$ and tends to ϕ as e_1 goes to infinity. Rearranging equation (A.16) into

$$(1-\pi)e_L = e_1 - \pi\min\{e_H,\ \psi\} \tag{A.21}$$

defines e_L as a strictly increasing function of e_1, given e_H. This function assumes a value strictly below 1 at $e_1 = 1$ and tends to infinity as e_1 does. This implies that for any given e_H, (A.14) and (A.16) determine unique values of e_1 and e_L with $\phi < e_L < \psi$. When $e_H < \psi$, an increase in e_H shifts the second function down and leaves the first unchanged; when $e_H \geq \psi$, an increase in e_H leaves both functions unchanged. Continuity is obvious.

Next, note that in the (e_1, e_L)-plane, the graph of the function defined by (A.21) cuts the 45 degree line from below at $e_1 = \min\{e_H,\ \psi\}$, while the graph of the function defined by (A.14) cuts the 45 degree line from above at $e_1 = \underline{e}$. Using these facts, it is easy to verify the statements about the ranking of e_1, e_H and e_L. ■

Lemma A.9 *The system of equations* (A.14)–(A.16) *has a unique solution* (e_1, e_H, e_L) *in* $[1,\infty[^3$. *This solution satisfies* $e_H > \underline{e}$ *and* $e_L < e_1 < e_H$. *Moreover,* e_H *is strictly increasing in* ν *with* $e_H \to \underline{e}$ *as* $\nu \to 0$, *and* $e_H \to \infty$ *as* $\nu \to \infty$.

Proof: We want to to establish that equation (A.15) admits a unique solution e_H once e_1 and e_L are solved for as functions of e_H according to Lemma A.8. First, we note that i_2 is strictly increasing in e_1 and strictly decreasing in e_L. This implies that i_2 is weakly increasing in e_H. Second, n_1 is strictly increasing in e_H. Third, the definition of W_4 can be rearranged into

$$\mu e^{W_4} = e_1 e_L - e_1 + e_1 e_L z, \tag{A.22}$$

where $z = [(e_H/e_L)^\pi - 1]/\pi$ is strictly increasing in e_H and non-negative when $e_H \geq \underline{e}$. Note that $e_1 e_L - e_1 = \mu e^{W_3}$, which is weakly increasing in e_H by equation (A.14) and the fact that i_1 is weakly decreasing in e_H. As e_1 is weakly increasing in e_H, the product $e_1 e_L$ on its own is weakly increasing. So

i_4 is strictly increasing in e_H. This establishes that the right-hand side of (A.15) is strictly increasing in e_H.

At $e_H = \underline{e}$, we have $i_2 = i_1$ and $i_4 = i_3$, so (A.14) implies that the right-hand side of (A.15) is smaller than the left-hand side. For $e_H \geq \psi$, equation (A.16) and the definition of i_2 imply that $i_2 = 1 - S$. As e_H tends to ∞, the right-hand side of (A.15) therefore converges to $2 - S + \nu$ which is greater than the left-hand side. This establishes existence and uniqueness of a solution to the system of equations (A.14)–(A.16) with the stated properties.

As $n_1 < 1$, raising ν makes the left-hand side of (A.15) exceed the right-hand side. As the latter is strictly increasing in e_H once e_1 and e_L are solved for as functions of this variable, we have the claimed comparative statics and asymptotics for e_H. ■

Lemma A.10 *The solution to the system of equations* (A.14)–(A.16) *identified in Lemma A.9 constitutes an equilibrium.*

PROOF: Lemma A.8 implies that $0 < i_1 < i_2 < i_3 < i_4$. Thus, the ranking of the measures i_1 through i_4 is the one that we assumed when formulating the market clearing conditions (A.10)–(A.13). The solution we have identified thus constitutes an equilibrium. ■

Lemma A.11 *There is a unique* $\nu^* > 0$ *such that condition* (2) *holds as an equality for all* $\nu \leq \nu^*$*, and as a strict inequality for all* $\nu > \nu^*$*.*

PROOF: By the last part of Lemma A.9, there is a unique ν such that $e_H = \psi$; call this ν^*. The result then follows from equation (A.16). ■

A.5 Proof of Proposition 2

The definitions of W_4 and W'_H yield

$$\mu e^{W_4 - W'_H} = \frac{e_1 \left[e_2 - \pi - (1-\pi)e_L\right]}{\pi(e_H - 1)} \,. \tag{A.23}$$

For $\nu \geq \nu^*$, e_1 and e_L are independent of e_H by Lemma A.8. Using the fact that $e_2 = e_H^{\pi} e_L^{1-\pi}$, we find that the derivative of the right-hand side of (A.23) with respect to e_H is

$$-\frac{e_1 \left[\pi(e_2 - 1) + (1-\pi)(e_2 - e_L)\right]}{\pi(e_H - 1)^2} \,, \tag{A.24}$$

hence strictly negative and bounded away from zero. By the last part of Lemma A.9, there are thus two cases. Either $W_4 - W'_H < 0$ at $\nu = \nu^*$ and we can take $\nu^{**} = \nu^*$; or $W_4 - W'_H \geq 0$ at $\nu = \nu^*$ and there is a $\nu^{**} > \nu^*$ with the stated property. ■

A.6 Data sources and summary statistics

Real housing prices are built from the MSA Conventional Mortgage Home Price Index produced by the Office of the Chief Economist at Freddie Mac. To obtain real housing prices, we use the CPI-US index (series cuur0000sa0) from the US Bureau of Labor Statistics.

We build all other variables from the Census data provided at www.ipums.org. The website provides detailed definitions for each variable. For each household in the sample, we download household income (HHINCOME), tenure (OWNERSHP), home value (VALUEH), gross monthly rent (RENTGRS) and location indicators (PUMA, STATEFIP, METAREA). The census questionnaire in 2000 did not ask households to explicitly identify the head of household. To compute the age of the head of household, we download the age of each person in the household (AGE) and its wage income (INCWAGE). We define the age of the head of household as the age of the person with the highest wage income in the household. If no person receives a wage in the household, we take the age of the oldest person in the household. To determine the number of years since the household moved into its current home, we use the number of years since our defined head of household moved into residence (MOVEDIN). The variables VALUEH and INCWAGE are coded in intervals. We replace each interval code with the median value of the interval.

We restrict the sample to households that live in the 1351 PUMAs located in one of the 164 MSAs for which we have real housing prices. We end up with 2,035,611 households that own their home and 1,084,878 households that rent their home.

We group PUMAs according to the housing price growth in the MSA where they are located. The groups vary in size because we have more than one PUMA for most MSAs (between 1 and 67 PUMAs, with a median of 4). Each group is computed including PUMAs with growth strictly greater than the low cutoff value and less than or equal to the high cutoff value. Note that the results we report are not sensitive to changes in the grouping rule.

Table 5 reports summary statistics at the PUMA level computed over owners and renters separately.

Table 6 reports summary statistics at the household level where again households are grouped according to the same PUMA groups as above. Recall that relative income, relative home value and relative gross rents are computed as ratios to the median of the PUMA. For time since moved and age, we use the difference between the value of the head of the household and the median of the PUMA.

Table 5: PUMAs: Summary statistics

PUMAs	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Number of PUMAs	349	327	398	277
	——— Homeowners ———			
Mean C.V. household income	0.8948	0.8670	0.8394	0.8524
Standard deviation	0.1162	0.1134	0.1112	0.0963
Mean S.D. time since moved	10.6521	11.4233	11.2733	11.5503
Standard deviation	1.7649	1.3880	1.3549	1.1108
Mean S.D. age head	16.0495	15.9543	15.7162	15.8541
Standard deviation	1.4446	1.2912	1.3724	1.1346
Mean C.V. home value	0.7785	0.7084	0.6328	0.5604
Standard deviation	0.1591	0.1693	0.1542	0.1220
	——— Renters ———			
Mean C.V. household income	0.9693	0.9628	0.9644	0.9480
Standard deviation	0.1526	0.1774	0.1661	0.1416
Mean S.D. time since moved	5.6055	6.4128	6.9176	6.2490
Standard deviation	1.3377	1.3223	1.9325	1.2618
Mean S.D. age head	16.9028	17.6863	16.8909	16.3005
Standard deviation	2.0836	2.0382	1.7669	1.6309
Mean C.V. gross rent	0.5095	0.5080	0.4909	0.4784
Standard deviation	0.0860	0.0870	0.0894	0.0757

Table 6: Households: Summary statistics

PUMAs	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Number of Households	349	327	398	277
	——— Homeowners ———			
Median income	51,000	53,000	62,200	68,950
Median time since moved	8	8	8	8
Median age head	49	48	48	48
Median home value	95,000	112,500	162,500	225,000
S.D. relative income	1.1660	1.1034	1.0504	1.0760
S.D. diff. time since moved	10.8661	11.7818	11.5142	11.7488
S.D. diff. age head	16.1431	16.0068	11.5142	15.7554
S.D. relative home value	1.0287	.9023	.7856	.6534
	——— Renters ———			
Median income	27,000	26,100	30,500	34,000
Median time since moved	4	4	4	4
Median age head	37	38	39	38
Median gross rent	573	560	667	767
S.D. relative income	1.3253	1.3337	1.3788	1.3033
S.D. diff. time since moved	5.8399	6.6317	7.9955	6.5087
S.D. diff. age head	16.8338	17.7254	17.0313	16.2931
S.D. relative gross rent	.5272	.5347	.5243	.5087

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