LONGEVITY AND AGGREGATE SAVINGS

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Abstract

For the last fifty years, countries in Asia and elsewhere have witnessed a surge in aggregate savings per capita. Some empirical studies attribute this trend to the increases in life longevity of the populations of these countries. It has been argued that the rise in savings is short-run, eventually to be dissipated by the dissaving of the elderly, whose proportion in the population rises along with longevity. This paper examines whether these conclusions are supported by economic theory. A model of life-cycle decisions with uncertain survival is used to derive individuals' consumption and chosen retirement age response to changes in longevity from which changes in individual savings are derived. Conditions on the age-profile of improvements in survival probabilities are shown to be necessary in order to predict the direction of this response. Population theory (e.g. Coale, 1952) is used to derive the steady-state population age density function, enabling the aggregation of individual response functions and a comparative steady-state analysis. Under certain conditions, increased longevity is shown to increase aggregate savings per capita. These conclusions pertain to an economy with a competitive annuity market. The absence of such market compels individuals to leave unintended bequests, whose size depends on the (random) age of death. While an increase in longevity raises individual savings for given endowments, it is shown that the effect on expected steady-state aggregate savings, taking into account the endogenous ergodic distribution of endowments, cannot be determined a-priori.

JEL Code: D1, D6, E2, H0.

Keywords: longevity, annuities, life cycle savings, retirement age, steady-state, aggregate savings.

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1 Introduction

Mortality has fallen substantially in the past hundred years: in 1900 about 2.5 people per 100 died in a typical year. Today, mortality is two-thirds lower (Cutler (2004)). This translates to substantially greater life expectancy both at birth and, particularly in recent history, at later ages. In many countries, the rise in longevity was accompanied by an increase in aggregate (real) savings per capita[1]. Investigators who studied these trends regard the former as a major cause of the latter (for example, Deaton and Paxson (1997)). In particular, the surge in savings in East Asia between 1950 and 1990 is attributed largely to the rapidly improving life expectancy in the region (see, for example, Lee, Mason and Miller (1998, 2000) and Lorentzen, McMillan and Wacziarg (2005)).

Does economic theory provide the underpinning for this conclusion? The answer rests on two levels of analysis. First, the foundation is an analysis of individual life-cycle decisions when facing survival probabilities. This will produce endogenous consumption functions and optimum retirement ages. A crucial assumption for modeling individual decisions is the availability of insurance. Specifically, with respect to longevity risks, the possibility of annuitizing savings.

Close examination of an individual multi period model reveals that to predict individual's response it is important to specify the *age profile* of improvements in survival probabilities. Savings and retirement age response can be expected to be quite different when the decrease in mortality rates is concentrated at younger ages compared to the case when this decrease is concentrated at older ages. The data supports the significance of this observation.

As Cutler (2004) points-out, the trend of declining mortality had three distinct phases. Early in the twentieth century there has been a significant drop in infant mortality due to improvements in nutritional and general health conditions. This was followed by a major reduction in mortality rates of adults due to infectious diseases. "Until the 1950's there was no evidence in any society of people reducing mortality from chronic diseases of old age... and then

[1] Aggregate savings in absolute terms are naturally expected to increase with the growth of population (due, say, to higher longevity, birth rates or other reasons).

cardiovascular disease mortality started declining extremely rapidly" (Cutler, (2004, p.8)). The trend of life lengthening due to medical advances aimed at older people is still present. This history of uneven age specific declines in mortality rates underlines the importance of the theoretical question addressed in this paper: how do alternative patterns of decline in mortality rates affect individual and aggregate behavior?

The second level, building on the first, is an aggregation analysis of individuals' behavior. Changes in survival probabilities affect the medium and long-run *population age density function*. The dynamics of the demographic process generated by such changes is quite complex. There exists, however, a well developed theory of the dependence of *steady-state* population age density distributions on the underlying parameters (e.g. Coale (1952, 1957)). To study the effects of changes in longevity on aggregate savings these endogenous changes in the age density function have to be taken into account.

This paper performs these two tasks: first, it sets up a model of individual decisions under longevity risks and, second, it aggregates individual decisions, linking survival functions with the population age density function.

Our objective is to formulate precise conditions which enable one to determine the micro and long-run macro effects on savings of changes in longevity.

2 A Simple Life-Cycle Model With Uncertain Survival

Consider a simple individual life-cycle model with uncertain survival. At age 0, the probability of surviving to age z is $F(z) : F(0) = 1$, and $F'(z) \leq 0$, $z \geq 0$. There may be a finite age $T > 0$ for which $F(T) = 0$, but this is immaterial.[2]

Individuals derive instantaneous utility $u(c)$ ($u' > 0$, $u'' < 0$), independent of age, from consumption, c, and can decide to work or retire (disregarding the choice of labor intensity). Work is normalized to a level of unity.

[2] Philipson and Becker (1998) allow individuals to affect their survival functions (say, by improved health care) which may create a 'moral hazard' problem. We disregard this effect.

Disutility from work at age z, $e(z) > 0$, is assumed to be independent of consumption and, in order to ensure that work precedes retirement, non-decreases with age, $(e'(z) \geq 0)$. In the absence of time-preference, expected lifetime utility, V, is therefore

$$V = \int_0^\infty u(c(z))F(z)dz - \int_0^R e(z)F(z)dz \tag{1}$$

where $c(z)$ is consumption at age z and R is the age of retirement.

Let $a(z)$ be the amount of annuities held by an age z individual[3]. Then the dynamic budget constraint is

$$\dot{a}(z) = r(z)a(z) + w(z) - c(z) \tag{2}$$

where $\dot{a}(z)$ is the amount of annuities purchased (> 0) or sold (< 0), $r(z)$ is the instantaneous rate of return on annuities and $w(z)$ is the wage rate ($w(z) = 0$ for $z > R$) for an age z individual. Solving (2) (with $a(0) = 0$), the demand for annuities at age z is

$$a(z) = e^{\int_0^z r(x)dx} \int_0^z e^{-\int_0^x r(h)dh}(w(x) - c(x))dx \tag{3}$$

In a competitive annuity market equilibrium, the rate of return on annuities is equal to the *Hazard-Rate*, the conditional probability of dying at age z

$$r(z) = -\frac{d \ln F(z)}{dz} = \frac{f(z)}{F(z)} \tag{4}$$

where $f(z) = -\dfrac{dF(z)}{dz}$ is the probability of dying at age z[4].

From (3), (4) and the transversality condition $\lim\limits_{z \to \infty} a(z)\ e^{-\int_0^z r(x)dx} = 0$, we obtain the lifetime budget constraint

$$\int_0^\infty c(z)F(z)dz - \int_0^R w(z)F(z)dz = 0, \tag{5}$$

---

[3] We know from Yaari (1965) that when longevity is the only uncertainty then individuals annuitize all their assets. The modifications required when individuals have a positive time preference and/or there is a positive interest rate on non-annuitized assets are well-known.

[4] See Sheshinski (2006).

Thus, equilibrium condition (4) implies that expected consumption is equal to expected wages, that is, zero expected profits. Maximization of (1) s.t.(5) yields constant optimum consumption, c^*, and a retirement age, R^*, which satisfy

$$c^* = \frac{\int_0^{R^*} w(z)F(z)dz}{\overline{z}} \tag{6}$$

$$u'(c^*)w(R^*) - e(R^*) = 0 \tag{7}$$

where $\overline{z} = \int_0^\infty F(z)dz$ is life expectancy[5].

Condition (7) equates the marginal benefits and costs of a small postponement of retirement.

For simplicity, assume that the wage rate is constant: $w(z) = w$. This ensures that the solution $(c^*,\ R^*)$ to (6) - (7) is unique[6].

Individual savings at age z, $s^*(z)$, are positive during the working phase and negative during retirement:

$$s^*(z) = \begin{cases} w - c^* = \dfrac{w\int_{R^*}^\infty F(z)dz}{\overline{z}}, & 0 \leq z \leq R^* \\ -c^* = -\dfrac{w\int_0^{R^*} F(z)dz}{\overline{z}}, & R^* < z < \infty \end{cases} \tag{8}$$

In the absence of a bequest motive, expected savings over the whole lifetime are, of course, zero: $\int_0^\infty s^*(z)F(z)dz = 0$.

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[5] Integrating by parts, $\overline{z} = \int_0^\infty zf(z)dz$.

[6] A sufficient condition for the existence of an interior solution is that $e(z)$ strictly increases from zero to ∞ as z rizes from zero to T.

3 Effects of Longevity Changes on Individual Decisions

Suppose that the survival function depends on a parameter denoted α, $F(z,\alpha)$, representing longevity. We take a decrease in α to cause an upward shift in survival probabilities, $\dfrac{\partial F(z,\alpha)}{\partial \alpha} < 0$, at all ages, $z > 0$[7]. This implies, of course, that expected lifetime, $\bar{z}(\alpha) = \int_0^\infty F(z,\alpha)dz$, decreases with α.

Differentiating (3) partially w.r.t. α, holding R^* constant, yields

$$\frac{1}{c^*}\frac{\partial c^*}{\partial \alpha} = \varphi(R^*,\alpha) \tag{9}$$

where

$$\begin{aligned}\varphi(R^*,\alpha) &= \int_0^{R^*} \frac{\partial F(z,\alpha)}{\partial \alpha} dz \Big/ \int_0^{R^*} F(z,\alpha)dz - \\ &- \int_0^\infty \frac{\partial F}{\partial \alpha}(z,\alpha)dz \Big/ \int_0^\infty F(z,\alpha)dz\end{aligned} \tag{10}$$

Clearly, $\lim_{R^* \to \infty} \varphi(R^*,\ \alpha) = 0$. Hence, if $\dfrac{\partial \varphi}{\partial R^*}(R^*,\ \alpha) \leq 0$ $\left(\dfrac{\partial \varphi}{\partial R^*}(R^*,\ \alpha) \geq 0\right)$ for all $R^* > 0$ (with strict inequality for some R^*), then $\varphi(R^*,\ \alpha) > 0$ $(\varphi(R^*,\ \alpha) < 0)$.

We have

$$\begin{aligned}\frac{\partial \varphi(R^*,\ \alpha)}{\partial R^*} &= \frac{F(R^*)}{\int_0^{R^*} F(z,\alpha)dz} \int_0^{R^*} \left[\frac{\partial F(R^*,\alpha)}{\partial \alpha} / F(R^*,\alpha) - \right. \\ &\left. - \frac{\partial F(z,\alpha)}{\partial \alpha} / F(z,\alpha)\right] F(z,\alpha) \Big/ \int_0^{R^*} F(z,\alpha)dz\end{aligned} \tag{11}$$

[7] Though $F(0,\alpha) = 1$ for any α. If the effect of a change in α on $F(z,\alpha)$ is continuous, the implication is that this effect around $z = 0$ is small. See Assumption 1 below. When there is a finite T for which $F(T,\alpha) = 0$, this makes T depend on α. In view of the rise in survival probabilities at very old ages, this is to be expected.

The more reasonable case is when increases in survival probabilities, holding retirement age unchanged, lead to a decrease in consumption. Hence the following assumption which ensures that (11) is negative:

Assumption 1. $\dfrac{\partial F(z,\alpha)}{\partial \alpha}/F(z,\alpha)$ *is monotone non-increasing in* z *for all* $z > 0$.

This assumption has a straightforward interpretation: *improvements in survival rates are proportionately larger at later ages.* It is equivalent to assuming that an increase in α raises the *Hazard-Rate*[8].

It follows from (10) and (11) that under Assumption 1, $\dfrac{\partial c^*}{\partial \alpha} > 0$, that is, an increase in longevity decreases consumption. Note that when $\dfrac{\partial F(z,\alpha)}{\partial \alpha}/F(z,\alpha)$ is monotone non-decreasing in z, then $\dfrac{\partial c^*}{\partial \alpha} < 0$. When increases in survival probabilities are proportionately larger at early ages compared to later ages then individuals, naturally, increase consumption (and decrease savings).

The effect of a change in survival probabilities on optimum retirement is obtained by totally differentiating (6) – (7) w.r.t. α. In elasticity form:

$$\frac{\alpha}{R^*}\frac{dR^*}{d\alpha} = -\frac{\sigma \dfrac{\alpha}{c^*}\dfrac{\partial c^*}{\partial \alpha}}{\sigma \dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} + \dfrac{R^* e'(R^*)}{e(R^*)}} \tag{12}$$

where $\sigma = -\dfrac{u''(c^*)c^*}{u'(c^*)} > 0$ is the *coefficient of relative risk aversion.*

From (6), $\dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} = \dfrac{F(R^*,\alpha)R^*}{\int_0^{R^*} F(z,\alpha)dz}$. Since F non-increases in z, it is seen that

$0 < \dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} < 1$. Hence, $\dfrac{dR^*}{d\alpha} \lesseqgtr 0 \Leftrightarrow \dfrac{\partial c^*}{\partial \alpha} \gtreqless 0$.

[8] According to a standard definition of *Stochastic Dominance* (see Sheshinski (2003)), when this assumption is satisfied then a survival function with a lower α stochastically dominates any survival function with a higher α.

The total change in consumption is, using (12),

$$\frac{dc^*}{d\alpha} = \frac{\partial c^*}{\partial R}(\frac{\alpha dR^*}{R^* d\alpha}) + \frac{\partial c^*}{\partial \alpha} = \left(\frac{\frac{R^* e'(R^*)}{e(R^*)}}{\sigma \frac{R^*}{c^*}\frac{\partial c^*}{\partial R} + \frac{R^* e'(R^*)}{e(R^*)}} \right) \frac{\partial c^*}{\partial \alpha}. \tag{13}$$

By Assumption 1, an increase in longevity increases the optimum retirement age, but this only partially compensates for the decrease in optimum consumption (increase in optimum savings) and hence $\frac{dc^*}{d\alpha} > 0$.

We summarize the analysis so far:

Proposition 1 *Under Assumption 1, an increase in longevity increases optimum retirement,* $\frac{dR^*}{d\alpha} < 0$*, and decreases optimum consumption,* $\frac{dc^*}{d\alpha} > 0$*.*

It is of interest to find the effect of a change in α on expected optimum lifetime utility, $V^* = u(c^*)\overline{z} - \int_0^{R^*} e(z)F(z,\alpha)dz$.

By the envelope theorem, (3) – (4), (6) and (7),

$$\begin{aligned} \frac{dV^*}{d\alpha} &= \frac{\partial V^*}{\partial \alpha} = [u(c^*) - u'(c^*)c^*]\int_0^{\infty} \frac{\partial F(z,\alpha)}{\partial \alpha} dz + \\ &+ \int_0^{R^*} [e(R^*) - e(z)] \frac{\partial F(z,\alpha)}{\partial \alpha} dz \end{aligned} \tag{14}$$

Strict concavity of $u(c)$ and the assumption that $e'(z) \geq 0$ ensure that $\frac{dV^*}{d\alpha} < 0$. As expected, an increase in longevity always increases welfare[9].

[9] This result depends on our assumption that $u(c) > 0$ independent of age, compared to zero utility at death. In discussions of life extending treatments this assumption has at times been questioned.

4 Longevity Changes and Aggregate Savings

Suppose that the population grows at a constant rate, g. The *steady-state* age density function of the population, denoted $h(z,\alpha,g)$, is given by[10]

$$h(z,\alpha,g) = me^{-gz}F(z,\alpha) \tag{15}$$

where $m = \dfrac{1}{\int_0^\infty e^{-gz}F(z,\alpha)dz}$ is the birth rate.

The growth rate g, in turn, is determined by the second fundamental equation of stable population theory:

$$\int_0^\infty e^{-gz}F(z,\alpha)b(z)dz = 1 \tag{16}$$

where $b(z)$ is the age specific fertility function.

The effect on g of a change in α, can be determined by totally differentiating (16):

$$\frac{dg}{d\alpha} = \frac{\int_0^\infty e^{-gz}\dfrac{\partial F(z,\alpha)}{\partial\alpha}b(z)dz}{\int_0^\infty e^{-gz}zF(z,\alpha)b(z)dz} < 0. \tag{17}$$

---

[10]Equations (15) and (16) are derived as follows (see, for example, Coale (1952) and (1957)): let the current number of age z females be $n(z)$, while the total number is N. When population grows at a rate g, the number of females z periods ago was Ne^{-gz}. If m is the birth rate, then z periods ago mNe^{-gz} females were born. Given the survival function $F(z,\alpha)$,

$$h(z,\alpha,g) = \frac{n(z)}{N} = \frac{Ne^{-gz}mF(z,\alpha)}{N} = me^{-gz}F(z,\alpha).$$

Since $\int_0^\infty h(z,\alpha,g)dz = 1$ if follows that the birth rate m is equal to $m = 1 \int_0^\infty e^{-gz}F(z,\alpha)dz$. This yields equation (15). By definition $m = \int_0^\infty h(z,\alpha,g)b(z)dz$, where $b(z)$ is the age z *fertility* rate. Substituting the above definition of $h(z,\alpha,g)$ we obtain (16).

An increase in longevity raises the steady-state growth rate of the population. The magnitude of g depends implicitly on the form of the survival, $F(z,\ \alpha)$, and fertility, $b(z)$, functions. It can be solved explicitly in some special cases. For example, for $F(z,\alpha) = e^{-\alpha z}$ and $b(z) = b > 0$, constant, for all $z \geq 0$, (16) yields $g = b - \alpha$. Indeed, substituting $\frac{1}{F}\frac{\partial F}{\partial \alpha} = -z$ into (17), we obtain that $\frac{dg}{d\alpha} = -1$.

Aggregate steady-state savings per capita, S, are

$$
\begin{aligned}
S &= \int_0^\infty s^*(z,\alpha) h(z,\alpha,g)dz = \\
&\quad \text{from (6), (8) and (17)} \\
&= w\int_0^{R^*} h(z,\alpha,g)dz - c^* = \\
&= w\int_0^{R^*} \left[e^{-gz} \Big/ \int_0^\infty e^{-gz} F(z,\alpha)dz - 1 \Big/ \int_0^\infty F(z,\alpha)dz \right] F(z,\alpha)dz.
\end{aligned}
\tag{18}
$$

It is seen that $S = 0$ when $g = 0$. A stationary economy without population growth has no aggregate savings per capita, corresponding to zero personal lifetime savings.

We shall now show that $S > 0$ when $g > 0$. Denote average life expectancy of the population below a certain age, R, by $\tilde{z}(R)$. From (15),

$$
\tilde{z}(R) = \int_0^R e^{-gz} z F(z,\alpha)dz \Big/ \int_0^R e^{-gz} F(z,\alpha)dz \tag{19}
$$

Accordingly, the average population age, $\tilde{z}$, is

$$
\tilde{z} = \tilde{z}(\infty) = \int_0^\infty e^{-gz} z F(z,\alpha)dz \Big/ \int_0^\infty e^{-gz} F(z,\alpha)dz. \tag{20}
$$

Clearly, $\tilde{z}(R) < \tilde{z}$ for any R.

Differentiating (18) partially w.r.t. g

$$\frac{\partial S}{\partial g} = \left(\int_0^R e^{-gz} F(z,\alpha) dz \Big/ \int_0^\infty e^{-gz} F(z,\alpha) dz \right) (\widetilde{z} - \widetilde{z}(R)) > 0 \tag{21}$$

Hence, a positive population growth rate, $g > 0$, implies that aggregate steady-state savings per capita are positive.

In order to isolate the effect of the change in longevity from other factors, such as a change in fertility, we shall assume that $b(z) = b > 0$, is unchanged throughout.

A sufficient condition for an increase in longevity to raise aggregate savings, as we show below, is the following:

Assumption 2: $\frac{1}{z}\frac{\partial F(z,\alpha)}{\partial \alpha} / F(z,\alpha)$ *non-decreases in* z *for all* $z \geq 0$.

The interpretation of Assumption 2 seems clear. Proportional improvements in survival rates *relative to age* are non-increasing. For example, a 5 percent improvement in survival probability at age 20 and a 6 percent improvement at age 30, satisfy this assumption. Increases in survival rates generate an increase in the population's steady-state growth rate. The implied change in the age density function should be weighed towards younger ages, when individuals have positive personal savings.

In an example analyzed below, $F(z,\alpha) = e^{-\alpha z}$, the term in Assumption 2 is equal to minus one for all z and α. Hence, the assumption is satisfied. Note, importantly, that this example also satisfies Assumption 1.

We now state:

Proposition 2 *Under Assumptions 1 and 2,* $\frac{dS}{d\alpha} < 0$.

Proof. Differentiating (18) totally w.r.t. α, taking into account the dependence of g on α via (16),

$$\frac{dS}{d\alpha} = wme^{-gR^*}F(R^*,\alpha)\frac{dR^*}{d\alpha}+$$

$$+ wm\left[\int_0^{R^*} e^{-gz}\frac{\partial F(z,\alpha)}{\partial\alpha}dz - m\left(\int_0^{R} e^{-gz}F(z,\alpha)dz\right)\left(\int_0^{\infty} e^{-gz}\frac{\partial F(z,\alpha)}{\partial\alpha}dz\right)\right]+$$

$$+ wm\left[\left(\int_0^{R^*} e^{-gz}F(z,\alpha)dz\right)\left(\int_0^{\infty} e^{-gz}zF(z,\alpha)dz\right) - \int_0^{R^*} e^{-gz}zF(z,\alpha)dz\right]\frac{dg}{d\alpha} - \frac{dc^*}{d\alpha} =$$

substituting from (17),

$$\begin{aligned} &= wme^{-gR^*}F(R^*,\alpha)\frac{dR^*}{d\alpha} + \\ &+m\left[\frac{\int_0^{R^*} e^{-gz}\frac{\partial F(z,\alpha)}{\partial\alpha}dz}{\int_0^{R^*} e^{-gz}zF(z,\alpha)dz} - \frac{\int_0^{\infty} e^{-gz}\frac{\partial F(z,\alpha)}{\partial\alpha}dz}{\int_0^{\infty} e^{-gz}zF(z,\alpha)dz}\right] - \frac{dc^*}{d\alpha}. \qquad (22)\end{aligned}$$

We have shown, (11) and (12), that under Assumption 1, $\frac{dR^*}{d\alpha} < 0$ and $\frac{dc^*}{d\alpha} > 0$. The term in square brackets in (19) approaches 0 as $R^* \to \infty$. The derivative of the first term w.r.t. R^* is

$$\frac{d}{dR^*}\left(\frac{\int_0^{R^*} e^{-gz}\frac{\partial F(z,\alpha)}{\partial\alpha}dz}{\int_0^{R^*} e^{-gz}zF(z,\alpha)dz}\right) = \frac{e^{-gR^*}F(R^*,\alpha)R^*}{\int_0^{R^*} e^{-gz}zF(z,\alpha)dz}\int_0^{R^*}\left[\frac{1}{R^*}\frac{\partial F(R^*,\alpha)}{\partial\alpha}/F(R^*,\alpha) - \frac{1}{z}\frac{\partial F(z,\alpha)}{\partial\alpha}/F(z,\alpha)\right]\frac{e^{-gz}zF(z,\alpha)}{\int_0^{R^*} e^{-gz}zF(z,\alpha)dz}dz \qquad (23)$$

Assumption 2 ensures that (23) is non-negative and hence the term in square brackets in (22) is non-positive for all R^*. We conclude that $\frac{dS}{d\alpha} < 0$ ||.

It is worth explaining further the seeming tension between Assumptions 1 and 2 above. Assumption 1, postulating that longevity improvements are relatively larger at later ages, ensures that individuals (though postponing retirement) increase personal savings. Aggregate savings depend on personal savings times the number of individuals in each age. Assumption 1 has two opposite effects. For a given population growth rate, it tilts the age density towards older ages. Since individuals dissave at older ages, this has a negative effect on aggregate savings. However, higher longevity also increases the steady-state population growth rate. This tilts the age density towards younger ages and has, therefore, a positive effect on aggregate savings. Assumption 2 ensures (it is a *sufficient* condition) that the latter effect is dominant.

Formally, let $\Delta_F(R)$ denote the population weighted average improvement in survival probabilities at ages below R :

$$\Delta_F(R) = \frac{\int_0^R e^{-gz} F(z,\alpha) \left(\frac{1}{F(z,\alpha)} \frac{\partial F(z,\alpha)}{\partial \alpha} \right) dz}{\int_0^R e^{-gz} F(z,\alpha) dz} \; (< 0) \tag{24}$$

The population weighted average age below R, (19), is

$$\tilde{z}(R) = \int_0^R e^{-gz} F(z,\alpha) z dz \Big/ \int_0^R e^{-gz} F(z,\alpha) dz \, .$$

As seen from (22), Assumption 2 states that $\frac{\Delta_F(R)}{\tilde{z}(R)}$ non-decreases in R. That is, *weighted by the population density*, longevity improvements are concentrated at the younger ages.

5 Example: Exponential Survival Function

Some of the above expressions can be solved explicitly for the particular survival function $F(z,\alpha) = e^{-\alpha z}$, $z \geq 0$.

Equation (6) becomes

$$c^* = w(1 - e^{-\alpha R^*}) \tag{25}$$

and (11) and (12) are (in elasticity form):

$$\frac{\alpha}{R^*}\frac{dR^*}{d\alpha} = -\frac{\sigma}{\sigma + \dfrac{R^* a'(R^*)}{a(R^*)}\left(\dfrac{e^{\alpha R^*}-1}{\alpha R^*}\right)} \tag{26}$$

$$\frac{\alpha}{c^*}\frac{dc^*}{d\alpha} = \frac{\alpha R^*}{e^{\alpha R^*}-1}\left(1+\frac{\alpha}{R^*}\frac{dR^*}{d\alpha}\right) \tag{27}$$

Clearly, $-1 \leq \dfrac{\alpha}{R^*}\dfrac{dR^*}{d\alpha} \leq 0$ and $0 \leq \dfrac{\alpha}{c^*}\dfrac{dc^*}{d\alpha} \leq 1$.

The steady-state age density function, (15), is

$$h(z,\alpha,g) = (g+\alpha)e^{-(g+\alpha)z} \tag{28}$$

while the population growth rate, g, with constant birth rate, b, is solved from (16), $g = b - \alpha$.

Hence, holding b constant, $\dfrac{dg}{d\alpha} = -1$.

Aggregate steady-state savings, (18), are given by

$$S = e^{-\alpha R^*}(1 - e^{-gR^*}) \tag{29}$$

Totally differentiating (30),

$$\frac{dS}{d\alpha} = -we^{-\alpha R^*}\left\{1 + \frac{\alpha}{R^*}\frac{dR^*}{d\alpha}\left[1 - \frac{be^{-gR^*}}{\alpha}\right]\right\} < 0 \tag{30}$$

6 No Annuitization

We have assumed throughout that annuitization is available, which means that individuals can take advantage of risk pooling. To demonstrate that this is a critical assumption, consider the case of no insurance[11]. The budget constraint (5) now becomes:

$$\int_0^{\infty} c(z)dz - \int_0^{R} w(z)dz = 0 \tag{31}$$

In the absence of insurance, there is also a constraint that assets must be non-negative at all ages (individuals cannot die with debt). Equating expected marginal utility across ages yields decreasing optimum consumption, whose shape reflects the individual's degree of risk aversion. To demonstrate that the effects of a change in longevity on savings and retirement are, in general, indeterminate, it suffices to take particular utility and survival functions. Thus, assume that $u(c) = \ln c$ and $F(z, \alpha) = e^{-\alpha z}$. For a constant wage $w(z) = w$, optimum consumption, $\hat{c}(z)$, now becomes (instead of (6)):

$$\hat{c}(z) = w\alpha\hat{R}e^{-\alpha z} \tag{32}$$

Accordingly, individual savings, (8), are now:

$$\hat{s}(z) = \begin{cases} w(1 - \alpha\hat{R}\, e^{-\alpha z}) & 0 \leq z \leq \hat{R} \\ \\ -w\alpha\hat{R}\, e^{-\alpha z} & \hat{R} \leq z \leq \infty \end{cases} \tag{33}$$

where optimum retirement is obtained from condition (7):

$$\frac{1}{\alpha\hat{R}} e^{\alpha\hat{R}} = e(\hat{R}). \tag{34}$$

For this condition to have a unique solution it is assumed that the L.H.S. strictly decreases with $\hat{R}$. This holds iff $\hat{R} < \dfrac{1}{\alpha}$, i.e. optimum retirement age is lower than expected lifetime. This is reasonable but not necessary. When this condition holds then $\dfrac{d\hat{R}}{d\alpha} \leq 0$, that is, as before, an increase in longevity leads to an increase in retirement age.[12]

[11] Social Security systems provide such annuitization which may, however, be inadequate for some individuals. See Sheshinski (in Aliprantis *et-al*, (2003), 27-54).

[12] The same condition ensures the non-negativity of assets at all ages ($S^*(0) = w(1 - \alpha R^*) > 0$).

Aggregate steady-state savings, (14), now become:

$$S = w\left[1 - e^{-(g+\alpha)\hat{R}} - \frac{\alpha\hat{R}(g+\alpha)}{g+2\alpha}\right] \tag{35}$$

Taking into account that $\dfrac{dg}{d\alpha} = -1$, it is seen that, holding $\hat{R}$ constant, a decrease in α affects S positively. However, when the change in $\hat{R}$ is also taken into account, the direction of the change in S is indeterminate, depending on parameter configuration.

7 Unintended Bequests

The analysis in the previous section disregards the fact that in the absence of full annuitization there are *unintended bequests* which affect individual behavior, in particular individual savings[13]. A general equilibrium analysis of longevity effects on aggregate savings has to take these intergenerational transfers into account.

In the absence of full annuitization, uncertain lifetime generates a distribution of bequests which depends on survival probabilities. A proper comparison of steady-states with and without annuitization requires derivation of the *ergodic*, *long-term*, *distribution of bequests* which, in turn, generates a distribution of individual and aggregate savings. A general analysis of this process is beyond the scope of this paper. The issue can, however, be clarified by means of a simple example.

Suppose that individuals live one period and with probability p, $0 \leq p \leq 1$, two periods. With no time preference, expected lifetime utility, V, is

$$V = u(c) + pu(c_1) \tag{36}$$

[13] The empirical importance of bequests and intergenerational transfers is debated extensively, among the inconclusive issues is the separation of planned bequests from those due to lack of annuity markets.

See, for example, Kotlikoff and Summers (1981) and most recently Kopczuk and Lupton (2005).

where c is first period consumption and c_1 is second period consumption. Without annuities and a zero interest rate, the budget constraint is

$$c + c_1 = w + b \tag{37}$$

where $w > 0$ is income and $b \geq 0$ is initial endowment. Let $u(c) = \ln c$. Then optimum consumption, $\hat{c}$ and $\hat{c}_1$, is

$$\hat{c}(b) = \frac{w+b}{1+p}, \qquad \hat{c}_1(b) = \frac{p(w+b)}{1+p} \tag{38}$$

Having no bequest motive, individuals who live two periods leave no bequest. Consequently, some individuals will have no initial endowments. Others will have positive endowments which depend on the history of parental survivals. In fact, the steady-state distribution of initial endowments is a *Renewal Process.*

Denote by $\hat{b}_k$ the initial endowment of an individual whose k previous generations of parents lived one period only. If p_0 is the probability of a zero endowment, then the probability of $\hat{b}_k$ is $(1-p)^k\ p_0$. Since $p_0 \sum\limits_{k=0}^{\infty}(1-p)^k = 1$, it follows that $p_0 = p$. We can calculate $\hat{b}_k$ from (38):

$$\begin{aligned}\hat{b}_k &= w + \hat{b}_{k-1} - \hat{c}(\hat{b}_{k-1}) = \left[\frac{p}{1+p} + (\frac{p}{1+p})^2 + ... + (\frac{p}{1+p})^k\right] w = \\ &= p\left(1 - (\frac{p}{1+p})^{k-1}\right) w \qquad\qquad k = 1, 2, ...\end{aligned} \tag{39}$$

Thus, savings of an individual with endowment $\hat{b}_k$, $s(\hat{b}_k)$, is

$$s(\hat{b}_k) = w - \hat{c}(\hat{b}_k) = (\frac{p}{1+p})^{k+1}\ w \tag{40}$$

and expected total savings, S, is

$$S = p\sum_{k=1}^{\infty} s(\hat{b}_k)(1-p)^k = \frac{p^2}{1+p}\sum_{k=1}^{\infty}\left(\frac{p(1-p)}{1+p}\right)^k \tag{41}$$

While $S > 0$ for any $0 < p < 1$, the sign of the effect on S of an increase in the survival probability p is indeterminate.

Incorporating a positive birth rate would not change this conclusion: *in the absence of a competitive annuity market, the effect of increased longevity on steady-state aggregate savings is indeterminate.*

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