

# OPTIMAL INCOME TAXATION WITH A RISKY ASSET – THE TRIPLE INCOME TAX

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# OPTIMAL INCOME TAXATION WITH A RISKY ASSET – THE TRIPLE INCOME TAX

## Abstract

We show in a two-period world with endogenous savings and two assets, one of them exhibiting a stochastic return, that an interest-adjusted income tax is optimal. This tax leaves a riskless component of interest income tax free and taxes the excess return with a special tax rate. There is no trade-off between risk allocation and efficiency in intertemporal consumption. Both goals are reached. As the resulting tax system divides income into three parts, the tax can also be called a *Triple Income Tax*. This distinction and a special tax rate on the excess return are necessary in order to have an optimal risk-shifting effect.

JEL Code: H21.

Keywords: optimal taxation, uncertainty, consumption tax, triple income tax.

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# 1 Introduction

There is an old but still ongoing debate on the best tax system in most OECD countries. Especially in Germany and the U.S., there are proponents of consumption-oriented taxation on the one hand and of comprehensive income taxation on the other. In a major reform, the Nordic countries have implemented in the 1990s a *Dual Income Tax*, which apportions total income into segmented tax bases of labor and capital income (see, e.g., Sørensen, 1994). In all cases, however, this debate fails to consider the effects of risk on personal income and therefore does not deal with risky tax revenue. Consequently, the questions we want to answer in this paper are: First, what is the optimal income taxation in case of aggregate risk? Second, which tax system should therefore be implemented?

In a world without both uncertainty and distributional considerations the optimal tax structure for financing public expenditure is lump sum. In the case of uncertainty about the individual wage rate – so-called private risk, which is unsystematic and can be insured against – this statement does not hold. Eaton and Rosen (1980) showed in their seminal paper that for a one-period world with endogenous labor choice, an income tax with a strictly positive marginal tax rate improves welfare. The government pools the private risk of all individuals and uses a lump-sum transfer in order to return the deterministic tax revenue. The income tax takes on the function of a social insurance scheme against private risk. Varian (1980) showed similar effects for a two-period world where the households work for a known wage rate and have to choose between consumption and savings. In his analysis, individuals face private risk because of uncertainty about the best investment portfolio.

The risk category we are interested in, however, is aggregate risk. This risk is systematic and hits all agents at the same time and in the same manner. Therefore, no insurance is possible, but incurring this risk pays out a risk premium. Amazingly, there are only a few contributions to this topic.

Richter and Wiegard (1991) examine a model with endogenous savings. In their two-period model the households have inelastic labor supply in period zero and divide their exogenous labor income between consumption and savings. Con-

sumption in the following period is financed by savings and a stochastic interest income. Richter and Wiegard show that a tax on this risky interest income improves welfare under certain conditions – even in a world with exogenous labor supply. The optimal tax rate is a trade-off between efficiency and insurance. Therefore, it depends on the elasticity of current consumption with respect to a compensated relative change of the tax rate. Further, they show that a consumption tax cannot achieve this insurance function. Related studies have been done by Richter (1992). He examines the portfolio choice decision in a two-asset world with one asset exhibiting a stochastic return and one safe asset. Richter develops an optimal elasticity rule for the taxation of asset returns and demonstrates that a cash-flow tax is not optimal if tax rates are not differentiated.

Unfortunately, these papers assume risk neutrality in public consumption. Hence, they collapse to the case of private risk and are limited in their statements concerning risky tax bases.

Christiansen (1993), instead, points out some optimal tax rules for portfolio choice in the case of risk aversion in public consumption, but his approach cannot be linked to the tax systems mentioned above.

Nevertheless, all these papers suggest that in case of uncertainty a consumption tax is always inferior to an income tax. This is because an income tax provides superior insurance by taxing capital income, and the sacrifice of a distorted intertemporal consumption decision is more than compensated by the reduction of risk. However, looking at the capital taxation used, in all the papers the return on each single asset is liable to one separate tax rate. Thus, they are not able to deal with risk effects separately, as we point out in the next section.

We show in our paper that the sacrifice of efficiency is not necessary. First, by using an adequately defined tax system, we can achieve both insurance and intertemporal efficiency. Therefore, we use and extend Sandmo's (1977) approach, which interprets assets as goods and assets' returns as their prices, in order to establish an analogy to portfolio choice and consumption theory. Second, we are able to show that the resulting optimal tax scheme is a modern form of consumption taxation. Precisely, we will get a kind of consumption-oriented income tax

with interest adjustment (see, e.g., Rose, 1999, pp. 35), which introduces an additional tax base, being liable to a separated tax rate: the risk premium. We can state that such a consumption tax is able to insure against risk to capital income in an optimal manner.

Amazingly, Croatia implemented a very similar tax system in 1994 for some other reasons (see Rose and Wiswesser, 1998, Rose, 1999). However, it did not deal with the risk issue and set a tax rate on excess capital returns equal to zero. Moreover, Croatia re-reformed its tax system in 2000 towards the traditional comprehensive taxation approach.

Even better fits the shareholder income tax, which has been enacted in Norway in 2006, and which replaces both the Norwegian RISK and its split model. The amendment to the Dual Income Tax identifies the risk premium as additional tax base, and taxes these returns with a combination of the corporate tax and the personal capital income tax.<sup>1</sup> This implies that the effective tax rate roughly equals the top marginal tax rate on labor income, but also that it is not chosen independently. Again, the reasoning behind this tax reform does neglect the risk issue, we are interested in, and solely focus on preventing tax-avoidance.

The remainder of the paper is as follows. In section 2 we present the model and examine the household choice; section 3 discusses the optimal tax structure for a welfare maximum. The paper closes with some conclusions.

## 2 The Model and Household Choice

In order to focus on the effects of aggregate risk, we keep the model as simple, but as general, as possible. Therefore, we use a two-period model without any bequest motive. There is a homogeneous individual, receiving exogenous labor income  $y$  in period 1 and dividing it into first-period consumption  $c_{t-1}$  and savings  $s_{t-1}$ . Assuming that labour supply and wages are perfectly inelastic, is very restrictive,

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<sup>1</sup>See Sørensen (2005), who shows that the shareholder tax is neutral in various aspects, but who does not consider the insurance effect and optimal risk diversification. Moreover, in the Sørensen-setting it is not possible to analyze the Musgrave substitution effect of a net tax.

because it implies that there is a state-independent lump-sum tax available, but it allows to concentrate onto the effects of (risky) capital taxation, in which we are especially interested. We want to focus onto the new view of risk as a ‘bad,’ exhibiting a negative price, and the insurance effect of the risk tax. Extending the analysis for endogenous labor supply is possible, but does not alter the results concerning the risk part. Therefore, it is left for further research, because this implies several new trade-offs and heavily increases complexity.

Savings can be invested in an asset  $A_0$  with a certain return  $r > 0$  and in a risky asset  $A_1$  with a stochastic return  $\tilde{x} \geq -1$ . We assume  $E[\tilde{x}] > r$ . Savings are the only source of consumption in the second period.

The government has to finance a public good  $g$  in the second period and can use both a proportional wage tax  $t^L$  in the first period, and a tax on capital income. For capital-income taxation we use a two-part interest income tax. We tax the riskless return  $r$  on both assets at a rate  $t_0$  and the excess return  $\tilde{x} - r$  at a rate  $t_1$ , and we assume full loss offset. If the realization of the excess return is negative, this loss will lead to a tax refund of  $t_1 \cdot (\tilde{x} - r)$ .

The motivation for the design of the capital tax system is as follows: The investor does not invest in assets in order to own the assets. On the one hand, she makes investments in order to shift resources into the future for financing future consumption. However, postponing consumption creates some kind of disutility as long as her marginal rate of time preference is positive, and she has therefore to be compensated for giving up present consumption. The price of this compensation for resource shifting equals the riskless rate of return and is paid out by both the riskless asset and the risky one. On the other hand, the investor can expand her income for future consumption by buying asset  $A_1$  and incurring risk. Risk creates disutility, and for incurring this risk the investor receives a risk premium. Thus, there are two “goods” traded in our economy, namely, (present versus future) consumption with a net price of  $r$ , and a “bad” called risk with price  $\tilde{x} - r$ .<sup>2</sup> After realization of risk, this price for risk equals the excess return  $\tilde{x} - r$ , which can be negative in bad states of the world. Hence, the investor gets additional (risk)

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<sup>2</sup>This interpretation can be seen as an extension of the Sandmo (1977) approach.

income  $(\tilde{x} - r) \cdot A_1$  according to her risk preferences. This income is positive in expected values.

Taken together, the prices for resource shifting and risk are liable to two separate tax rates under our tax system, whereas in most earlier papers the assets' returns are liable to asset-specific tax rates. This means for the risky asset that the prices mentioned are mixed and taxed at one rate. If so, however, no separate treatment of risk is possible.<sup>3</sup>

Savings can now be written as  $s_{t-1} = A_0 + A_1 = (1 - t^L)y - c_{t-1}$ . Consumption in period 1 is<sup>4</sup>  $\tilde{c}_t = [(1 - t_1)(\tilde{x} - r)]A_1 + [1 + r(1 - t_0)] [(1 - t^L)y - c_{t-1}]$ . We assume that the representative investor is risk-averse in both private and public consumption and her von Neumann–Morgenstern utility function takes the form

$$W = E [U(c_{t-1}, \tilde{c}_t, \tilde{g})] \quad \text{with} \quad U_c > 0, \quad U_{cc} < 0, \quad U_g > 0, \quad U_{gg} < 0.$$

Furthermore, we assume that future consumption and the public good are (weakly) complements:  $U_{c_t g}, U_{g c_t} \geq 0$ . It seems to be reasonable that (aggregate) public consumption is valued more, if there is higher private consumption, when one thinks of  $g$  as cultural supply (theater, opera,...), but also as interior and legal security, or infrastructure. However, this assumption implies that the optimal public spending is pro-cyclical.

In this model, the government chooses the tax rates, and the tax revenue is used entirely to finance the public good in period 1. Therefore, the probability distribution of  $\tilde{g}$  is also an instrument variable of the government.

The household maximizes her expected utility  $W$  for given tax rates by choosing her optimal first-period consumption  $c_{t-1}$  and her optimal savings  $A_0 + A_1 = (1 - t^L)y - c_{t-1}$  with respect to the budget constraint. She does not anticipate the effect of her saving behavior on the level of the public good. Inserting the budget

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<sup>3</sup>Hence, the idea of separated tax rates is to have enough instruments for pursuing two goals, namely, optimal resource allocation as well as efficient risk diversification.

<sup>4</sup>All variables in the paper indicated with a tilde depend on the risky return  $\tilde{x}$ , and are therefore itself stochastic.

constraint for  $\tilde{c}_t$ , the maximization problem can be written as

$$\max_{c_{t-1}, A_1} W = E \left[ U \left( c_{t-1}, (1-t_1)(\tilde{x}-r)A_1 + [1+r(1-t_0)][(1-t^L)y - c_{t-1}], \tilde{g} \right) \right]. \quad (1)$$

The first-order conditions of the household problem are

$$\frac{\partial W}{\partial c_{t-1}} = E [U_{c_{t-1}}] - E [U_{c_t} \cdot [1+r(1-t_0)]] = 0, \quad (2)$$

$$\frac{\partial W}{\partial A_1} = (1-t_1)E [U_{c_t} \cdot (\tilde{x}-r)] = 0. \quad (3)$$

The optimal values of  $c$ ,  $A_1$ , and  $s$  are denoted  $c_{t-1} = c_{t-1}(t_0, t_1, t^L)$ ,  $A_1 = A_1(t_0, t_1, t^L)$ , and  $s_{t-1} = s_{t-1}(t_0, t_1, t^L)$ . For the marginal rate of time preference we obtain

$$\rho = \frac{E [U_{c_{t-1}}]}{E [U_{c_t}]} - 1 = r(1-t_0). \quad (4)$$

Equation (3) indicates that our tax system does not distort portfolio choice, as the FOC is equal to the optimality condition in the case of no taxation.

**Lemma 1** *The tax rate  $t_1$  on the excess return  $\tilde{x} - r$  does not affect overall savings  $s_{t-1}(t_0, t_1, t^L)$ , as  $\partial c_{t-1} / \partial t_1 = 0$ . Furthermore,  $t_1$  has only a Musgrave-substitution effect on  $A_1$ , and  $\partial A_1 / \partial t_1 = A_1 / (1 - t_1)$ .*

**Proof:** See Appendix.

This result corresponds to the result for taxing capital gains in Sandmo (1969, section 8) and is similar to the portfolio choice result for a net tax in the case of several risky assets (Sandmo, 1977). By investing more in the risky asset according to  $\partial A_1 / \partial t_1 = A_1 / (1 - t_1)$  and diminishing the investment in the riskless asset by the same amount, and therefore keeping both first-period and second-period consumption constant, the tax rate change in  $t_1$  does not change the expected utility of the household.

### 3 Optimal Taxes on Interest Income

The government collects tax revenue not only from capital taxation in period  $t$ , but also from the wage tax in that period. However, all spending for the public

good takes place in period  $t$  only. Hence, we assume that the tax revenue of the wage tax in period  $t - 1$  is invested entirely in the safe asset.<sup>5</sup> Thus, in period  $t$ , the government's budget restriction can be written

$$\tilde{g} = (1 + r)t^L \cdot y + t_1(\tilde{x} - r) \cdot A_1(t_0, t_1, t^L) + t_0r \cdot [(1 - t^L)y - c_{t-1}(t_0, t_1, t^L)].$$

The government chooses now the tax rates and the public good  $g$  in order to maximize the social welfare function

$$\Omega = E [U(c_{t-1}(t_0, t_1, t^L), \tilde{c}_t(t_0, t_1, t^L), \tilde{g})]$$

given optimal household choice and subject to its budget restriction. We get the following optimization problem:

$$\max_{t_0, t_1, t^L} E [U(c_{t-1}(\cdot), \tilde{c}_t(\cdot), (1 + r)t^L \cdot y + t_1(\tilde{x} - r) \cdot A_1(\cdot) + t_0r \cdot [(1 - t^L)y - c_{t-1}(\cdot)])]. \quad (5)$$

By using the envelope theorem, we get as first-order conditions

$$E \left[ -U_{c_t} r \cdot s_{t-1} + U_g \cdot \left( t_1(\tilde{x} - r) \frac{\partial A_1}{\partial t_0} + r \cdot s_{t-1} - t_0r \frac{\partial c_{t-1}}{\partial t_0} \right) \right] = 0, \quad (6)$$

$$E \left[ U_g \cdot \left( (\tilde{x} - r)A_1 + t_1(\tilde{x} - r) \frac{\partial A_1}{\partial t_1} - t_0r \frac{\partial c_{t-1}}{\partial t_1} \right) \right] = 0, \quad (7)$$

$$E \left[ -U_{c_t} [1 + r(1 - t_0)]y + U_g \cdot \left( [1 + r(1 - t_0)]y + t_1(\tilde{x} - r) \frac{\partial A_1}{\partial t^L} - t_0r \frac{\partial c_{t-1}}{\partial t^L} \right) \right] = 0. \quad (8)$$

As  $\partial A_1 / \partial t_1 = A_1 / (1 - t_1)$  and  $\partial c_{t-1} / \partial t_1 = 0$  from Lemma 1, (7) can be rewritten as

$$E [U_g \cdot (\tilde{x} - r)] \cdot \frac{A_1}{1 - t_1} = 0. \quad (9)$$

Then, we can conclude from using (9) and simplified versions of (6) and (8) :

**Proposition 1** *An optimal income tax system in the case of exogenous labor income and risky returns to at least one asset does not tax the riskless rate of return ( $t_0 = 0$ ). The optimal tax rate on the excess return  $\tilde{x} - r$  is strictly positive and in*

<sup>5</sup>This may be interpreted as a shortcut to an overlapping-generations model, where the government provides an old-generation-specific public good and where the riskless rate of return equals the population growth in steady state. See, e.g., Sandmo (1985), section 7.

the open interval  $t_1 \in (0, 1)$  if the households are risk-averse in both private and public consumption. The wage tax is used to equate marginal utility of public and private consumption in period 1.

**Proof:** See Appendix.

If  $t_1$  is set optimally, we have  $\text{Cov}(U_{c_t}, \tilde{x}) = \text{Cov}(U_g, \tilde{x})$ . The intuition is straightforward: *ex post*, resources should be efficiently allocated between public and private consumption. Unless one makes special assumptions, this involves adjusting both private and public consumption, given the realization of the shock. Hence, as long as the households are risk-averse in both private and public consumption, the risk must be diversified on both types of consumption. In our model, this can be achieved efficiently by using the risk tax on the excess return.<sup>6</sup>

The diversification depends on the strength of the risk aversion in private consumption relative to that in public consumption. Therefore, the tax rate  $t_1$  depends on this relative strength: The higher the risk aversion in private consumption relative to that in public consumption, the higher the tax rate on the excess return  $\tilde{x} - r$ . The intuition is as follows. The more disutility in private consumption, relative to disutility in public consumption, is caused by risk, the more risk should be transferred to public consumption.

As the government returns the risk to the households by providing a public good, our result is general and independent of any assumption about whether the government can deal better with risk than the capital market. Furthermore, because it provides an additional asset (the public good), which cannot be provided by the capital market, the tax on the risk component is not neutral for the government and achieves positive tax revenue in expected values. Hence, Gordon's (1985) neutrality result does not hold in our model.<sup>7</sup>

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<sup>6</sup>Another possible way to vary public-goods provision is using debt policy. However, in view of the problems of several EU countries in fulfilling the Maastricht criteria and the huge household deficit in the U.S., it seems doubtful that most governments can afford to run large additional deficits in order to secure the optimal provision of the public good – but this might be necessary in case of large shocks.

<sup>7</sup>See Kaplow (1994), p. 795.

Bulow and Summers (1984) state that most risk is embedded in economic depreciations that are not liable to taxation. Thus, the government will not participate in the risk, and there is no insurance effect of taxation. This may be true for corporate taxation, using statutory depreciation rates. However, we tax *ex post* capital income, where the risky rate of return depends on the economic value of the underlying firm.<sup>8</sup> Thus, the government fully participates in all income risk, and the critique of Bulow and Summers (1984) does not apply.

Moreover, we can discuss two limiting cases, if we relax our assumption that the household is neither risk neutral in public consumption nor risk neutral in private consumption. If we assume both risk neutrality in public consumption ( $U_{gg} = 0$ ) and that the marginal utility of future consumption is independent of stochastic fluctuations in the level of the public good ( $U_{c_tg} = 0$ ), we get as special case  $t_1 = 1$ . All risk is concentrated in public consumption, because it does not create any disutility in public consumption.<sup>9</sup> This is in accordance with the Arrow–Lind theorem, where the government can diversify aggregate risk perfectly.

The other (even more) extreme result,  $t_1 = 0$ , is achieved if we assume risk neutrality in private future consumption ( $U_{c_t c_t} = 0$ ) and that the marginal utility in public consumption does not respond to variations in second-period consumption ( $U_{g c_t} = 0$ ). The simplest way to implement such risk preferences is to use a quasi-linear utility function with  $U = v(c_{t-1}) + \tilde{c}_t + h(\tilde{g})$ , where  $h_g > 0$ ,  $h_{gg} < 0$ . If so, the household is willing to bear risk at no cost, and it is optimal to allocate all the risk to private consumption.

The result  $t_1 \in (0, 1)$  can also change drastically, if we relax our assumptions  $U_{c_t g}, U_{g c_t} \geq 0$ , and assume instead that public and private consumption are substitutes. Hence,  $U_{c_t g}, U_{g c_t} \leq 0$ . In this case, public spending has an additional insurance effect, because public spending can compensate for lower private consumption.

If either this effect is strong ( $|U_{c_t g}, U_{g c_t}|$  large) or risk aversion is low ( $|U_{c_t c_t}, U_{g g}|$  small), it can happen that  $\frac{\partial U_{c_t}}{\partial \tilde{x}}, \frac{\partial U_g}{\partial \tilde{x}} > 0$ , or  $\frac{\partial U_{c_t}}{\partial \tilde{x}}, \frac{\partial U_g}{\partial \tilde{x}} < 0$  in case of

<sup>8</sup>Assume asset 1 as a stock. Then its return  $\tilde{x}$  reflects the firm's fluctuating value.

<sup>9</sup>Analytically,  $U_{gg} = 0$  implies  $\text{Cov}(U_g, \tilde{x}) = 0$ , and from (20), this requires  $\text{Cov}(U_{c_t}, \tilde{x}) = 0$ . The latter is solely possible in case of  $t_1 = 1$ , as long as  $U_{c_t c_t} < 0$  and  $U_{c_t g} = 0$ .

$t_1 < 0$ , and the covariances are still equated. The government then subsidizes the investment in the risky asset.

The intuition is as follows: Given the cases above, the government would like to spend more in the bad states of the world and optimally spends less, when the economy is booming. Given the tax instruments in our model, this can be reached efficiently by introducing a subsidy on risk income  $(\tilde{x} - r) \cdot A_1$ . However, the cases of public spending, in which this result applies, are limited. The intuitive idea of unemployment insurance or other low-income related financial transfers does not work, because all these transfers substitute income in private consumption, and, hence, reallocate the aggregate risk directly to households' income. Unfortunately, Gordon's neutrality would then apply. One possible setting, which can instead ensure the result above, is public health care for low-income earners or unemployed, which is provided in the US (e.g., medicaid).

Returning to our original assumptions, we can finally state:

**Proposition 2** *If an interest-adjusted income tax is implemented, taxing the excess return according to Proposition 1 [ $t_1 \in (0;1)$ ] and letting the riskless component of interest be tax-free ( $t_0 = 0$ ), an efficient risk allocation is achieved without disturbing the intertemporal consumption decision. There is no trade-off between risk and efficiency in allocation. The marginal rate of time preference equals the riskless rate of return ( $\rho = r$ ).*

**Proof:** From (4), the marginal rate of time preference is  $\rho = r \cdot (1 - t_0)$ . For  $t_0 = 0$ ,  $\rho = r$ . At the optimum, the marginal rate of time preference is then independent of the tax rates, and the intertemporal consumption decision is not distorted. Additionally,  $\text{Cov}(U_{c_t}, \tilde{x}) = \text{Cov}(U_g, \tilde{x})$  assures efficient risk allocation.

□

As mentioned above, the literature shows that consumption-oriented taxation cannot achieve the insurance function of a traditional income tax in the case of risky capital income. But is it true that an income tax always yields better results? Examining our analysis, this view must be taken with care.

We use a proportional wage tax on exogenous labor income in the first period. The riskless rate of return on savings is tax-free, whereas the excess return (supernormal profits) is taxed at a special rate. This tax scheme equals a modified consumption-oriented income tax with interest adjustment.<sup>10</sup> Therefore, we have a consumption tax, which optimally provides insurance against risky capital income and simultaneously avoids a distortion in the intertemporal consumption decision.

This tax scheme is an extension of the Dual Income Tax approach and can be called a *Triple Income Tax*, as we divide the full income into three different parts. The excess return (or risk premium) is one of them. This distinction is necessary for achieving an optimal risk allocation by taxation:<sup>11</sup>

**Proposition 3** *In the case of aggregate risk the optimal tax system is a Triple Income Tax, which apportions overall income into three separate tax bases: labor income, riskless return to overall savings, and risk income. The last income equals the excess return on risky investment. Such a tax system generalizes the Dual Income Tax approach to include risk effects.*

The intuition behind these results is straightforward. On the one hand, it is optimal to diversify the aggregate risk between private and public consumption. On the other hand, risk shifting has negative welfare effects by disturbing the intertemporal consumption decision, if we tax the risky asset at only one rate. In this case, there is a trade-off and the optimal tax rate depends on the strength of these effects. If we instead tax the excess return at a special rate, then the tax system is well defined and the trade-off can be avoided. Thus, we reach both optimal risk allocation and efficiency in intertemporal consumption simultaneously.

However, we have to note that the proposed tax system cannot implement a First-best optimum. The tax system does better than other income or consumption

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<sup>10</sup>A consumption-oriented income tax with interest adjustment taxes the overall labor income and tax-exempts interest income. For excess returns in capital income a tax with the same tax rate as for labor income is possible. See, e.g., Rose (1999), pp. 35.

<sup>11</sup>Calling a tax system a Triple Income Tax, where one tax rate equals nil, seems not to be very intuitive at first sight. However, the tax system is also suitable in the case of an extended model using endogenous labor choice. Then, all three tax rates are positive.

taxes, and it is superior to a state-independent lump-sum tax. The reason is that it can guarantee an ex-ante optimal risk diversification at no allocation costs, but ex post the risk sharing between different states is linear, due to the fact that the risk tax is proportional to the realization of the risky return  $\tilde{x}$ . Thus, although we can mimic a state-dependent lump sum tax, we cannot ensure an ex post Pareto-optimal risk diversification – at least not without restricting the utility function to special classes.

Last mentioned should be that the results above can also be used to give another justification for the Norwegian shareholder income tax. This tax is (nearby) equivalent to the risk tax in this paper, and thus can be seen as extending the Dual Income Tax towards a Triple Income Tax, because risk (or the return to risky entrepreneur activities) is identified as an additional, independent source of income. However, the effective Norwegian shareholder tax rate is not independent, and determined only by anti-tax avoidance considerations. If the tax policy is extended to incorporate risk diversification and insurance issues, this tax rate should get more ‘slackness.’

## 4 Conclusions

We have shown that an interest-adjusted income tax can guarantee a welfare maximum in a two-period world with two assets, one of them exhibiting a stochastic return. The excess return must be taxed separately, and possible losses in this tax base must be subsidized. In the case of risk aversion in public consumption, we have an inner optimum with  $t_1 \in (0, 1)$ , because the risk must be diversified on both consumption types in order to have an optimal risk allocation.

As such a tax system is a kind of indirect consumption taxation, we showed that a consumption tax is able to insure against risky interest income. Furthermore, the tax system can be called a Triple Income Tax, because it uses three segmented tax bases. This extension of the Dual Income Tax is due to considering risk effects on personal income and tax revenue explicitly. Although not done in the paper, it is straightforward to show that these results also prevail in a multiasset world.

This tax system has an interesting advantage from a political point of view. In public opinion there is usually major support for taxing supernormal profits in capital income, whereas, at least in Germany, such capital gains are mostly tax-free at the moment. Now, this paper shows that taxing excess returns can be done in a welfare-enhancing manner without affecting the utility of private consumption. Therefore, the risk tax creates no incentives for households to engage in tax planning. However, a disadvantage of our tax system is that individuals still have an incentive to declare labor income as preferred taxed riskless capital income in order to avoid taxes. This problem is the same as in case of a (standard) Dual Income Tax with separate tax rates for labor and capital income (see, e.g., Sørensen, 1994, 2005), whereas, as mentioned above, the incentives to tax avoidance in the risk tax (shareholder income tax) may be much less than thought at first glance. The reason is that the risk tax not only has the neutrality properties, shown in Sørensen (2005), but also does, from the perspective of an household, neither affect the budget constraint nor private utility.

Related work can be done in a multiasset world with a fixed amount of savings. In such a world, Richter (1992) and Christiansen (1993) show that there is a trade-off between risk allocation and optimal portfolio choice. If the same tax system is introduced as in this paper, this trade-off should also be overcome, and under special conditions an ex post optimal solution can be achieved.<sup>12</sup> Further work could also examine a multiasset world with endogenous savings and labor supply.

Another interesting topic emerges if we look at further work concerning heterogeneous households. The taxation of the excess return affects utility solely by public consumption. Despite this, the risk tax can be used for redistribution, if low-income households have higher preferences for the public good than high-income households. Here, redistribution is to be thought of as enhancing the utility of low-income households, which is possible by increasing the risk tax and creating higher tax revenue in expected values. Contrary to the traditional literature on redistribution, we get an instrument that avoids any trade-off between efficiency and redistribution, but society has to pay for it with increased risk in consumption.

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<sup>12</sup>See Hilgers and Schindler (2004) for a first analysis concerning first-best optimality.

## 5 Appendix

**Proof of Lemma 1** Let  $p_r = 1 + r(1 - t_0)$  and  $p_A = (1 - t_1)(\tilde{x} - r)$ . Totally differentiating equations (2) and (3) with respect to  $c_{t-1}$ ,  $A_1$ , and  $t_1$  gives

$$\begin{aligned} & \begin{pmatrix} E [U_{c_{t-1}c_{t-1}} - U_{c_{t-1}c_t}p_r - U_{c_t c_{t-1}}p_r + U_{c_t c_t}p_r^2] & E [U_{c_{t-1}c_t}p_A - U_{c_t c_t}p_r p_A] \\ E [U_{c_t c_{t-1}}p_A - U_{c_t c_t}p_A p_r] & E [U_{c_t c_t} \frac{p_A^2}{1-t_1}] \end{pmatrix} \begin{pmatrix} dc_{t-1} \\ dA_1 \end{pmatrix} \\ & = \begin{pmatrix} E [U_{c_{t-1}c_t} \frac{p_A}{1-t_1} - U_{c_t c_t} p_r \frac{p_A}{1-t_1}] \\ E [U_{c_t c_t} \frac{p_A^2}{(1-t_1)^2}] \end{pmatrix} A_1 dt_1. \end{aligned} \quad (10)$$

Using Cramer's rule, we get

$$\frac{\partial c_{t-1}}{\partial t_1} = \frac{\det \alpha_{dt_1 A_1}}{\det \alpha_H} = 0, \quad (11)$$

as the modified determinant

$$\begin{aligned} \det \alpha_{dt_1 A_1} &= \frac{A_1}{(1-t_1)^2} \cdot E [U_{c_{t-1}c_t} \cdot p_A - U_{c_t c_t} \cdot p_r p_A] \cdot E [U_{c_t c_t} \cdot p_A^2] - \\ & \frac{A_1}{(1-t_1)^2} \cdot E [U_{c_t c_t} \cdot p_A^2] \cdot E [U_{c_{t-1}c_t} \cdot p_A - U_{c_t c_t} \cdot p_r p_A] = 0. \end{aligned} \quad (12)$$

Moreover, we have

$$\det \alpha_{c_{t-1} dt_1} = \frac{A_1}{1-t_1} \cdot \det \alpha_H, \quad (13)$$

and, hence,  $\partial A_1 / \partial t_1 = A_1 / (1 - t_1)$  from Cramer's rule.  $\square$

**Proof of Proposition 1** In proving Proposition 1 we use (9) with the first-order conditions (6) and (8) and obtain

$$E [U_g - U_{c_t}] \cdot r \cdot s_{t-1} - E [U_g] \cdot t_0 r \cdot \frac{\partial c_{t-1}}{\partial t_0} = 0, \text{ and} \quad (14)$$

$$E [U_g - U_{c_t}] \cdot [1 + r(1 - t_0)]y - E [U_g] \cdot t_0 r \cdot \frac{\partial c_{t-1}}{\partial t^L} = 0. \quad (15)$$

Combining these expressions results in

$$t_0 \cdot \left( [1 + r(1 - t_0)] \frac{\partial c_{t-1}}{\partial t_0} - r \cdot s_{t-1} \frac{\partial c_{t-1}}{\partial t^L} \right) = 0. \quad (16)$$

A household's budget constraint can be displayed as

$$\tilde{c}_t + p_r \cdot c_{t-1} = (1 - t_1)(\tilde{x} - r)A_1 + p_r \cdot (1 - t^L) \cdot y, \quad (17)$$

where  $p_r = 1 + r(1 - t_0)$  is the price of present consumption,  $p_r \cdot y$  is the value of endowment, and  $I = p_r \cdot (1 - t^L) \cdot y$  is full (lump-sum) income after wage tax.

Thus,  $\partial c_{t-1} / \partial t_0 = (\partial c_{t-1} / \partial p_r)(-r)$ , and  $\partial I / \partial t^L = -p_r \cdot y$ . The wage tax has a pure income effect only, as long as labor supply is exogenous. Hence,  $\partial c_{t-1} / \partial t^L = (\partial c_{t-1} / \partial I) \cdot (\partial I / \partial t^L)$ . Using the Slutsky equation for  $\partial c_{t-1} / \partial p_r$  and endowment effects in (16), and collecting terms, the income effects cancel out, and we are left with the substitution effect:

$$t_0 \cdot [p_r \cdot (-r) \cdot S_{c_{t-1}c_{t-1}}] = 0, \quad (18)$$

and therefore unambiguously  $t_0 = 0$ , because  $S_{c_{t-1}c_{t-1}} < 0$  as long as the tax on riskless return is not another lump-sum tax.

For  $t_0 = 0$ , from (6) and (8) then follows  $E[U_{c_t}] = E[U_g]$ . Using the first-order condition (3) of the household problem and (9), we can write

$$E[U_{c_t} \cdot (\tilde{x} - r)] = 0 = E[U_g \cdot (\tilde{x} - r)]. \quad (19)$$

As  $E[Y \cdot Z] = E[Y] \cdot E[Z] + \text{Cov}(Y, Z)$  and  $E[U_{c_t}] = E[U_g]$ , this expression can be simplified to

$$\text{Cov}(U_{c_t}, \tilde{x}) = \text{Cov}(U_g, \tilde{x}). \quad (20)$$

If  $t_1 \leq 0$ , resources have to be paid out to the households if  $\tilde{x}$  increases, because the risk tax is in fact a subsidy. Thus, we have  $\partial c_t / \partial x > 0$  and  $\partial g / \partial x \leq 0$ . Together with our earlier assumptions,  $U_{c_t g}, U_{g c_t} > 0$ , this implies for  $t_1 \leq 0$

$$\begin{aligned} \frac{\partial U_{c_t}}{\partial x} &= U_{c_t c_t} \cdot \frac{\partial c_t}{\partial x} + U_{c_t g} \cdot \frac{\partial g}{\partial x} < 0, \\ \frac{\partial U_g}{\partial x} &= U_{g c_t} \cdot \frac{\partial c_t}{\partial x} + U_{g g} \cdot \frac{\partial g}{\partial x} > 0. \end{aligned}$$

This implies  $\text{Cov}(U_{c_t}, \tilde{x}) < 0$ , but  $\text{Cov}(U_g, \tilde{x}) > 0$ . By a similar reasoning, in case of  $t_1 \geq 1$ , we get  $\text{Cov}(U_{c_t}, \tilde{x}) > 0$ , but now  $\text{Cov}(U_g, \tilde{x}) < 0$ , as more than the additional income is taxed away.

In both cases, covariances cannot be equated and equation (20) is not fulfilled. Thus, an optimal taxation of the excess return requires  $t_1 \in (0, 1)$ .  $\square$

## References

- Bulow, J. I., and L. H. Summers, 1984. The Taxation of Risky Assets. *Journal of Political Economy* 92, 20–39.
- Christiansen, V., 1993. A Normative Analysis of Capital Income Taxes in the Presence of Aggregate Risk. *The Geneva Papers on Risk and Insurance Theory* 18, 55–75.
- Eaton, J., and Rosen, H.S., 1980. Labor Supply, Uncertainty, and Efficient Taxation. *Journal of Public Economics* 14, 365–374.
- Gordon, R.H., 1985. Taxation of Corporate Capital Income: Tax Revenues Versus Tax Distortions. *Quarterly Journal of Economics* 50, 1–27.
- Hilgers, B., Schindler, D., 2004. First-best Optimality in Capital Income Taxation. NHH Discussion paper SAM 04/26, Norwegian School of Economics and Business Administration, Bergen.
- Kaplow, L., 1994. Taxation and Risk-taking: A General Equilibrium Perspective. *National Tax Journal* 47, 789–798.
- Richter, W.F., 1992. The Optimal Taxation of Risky Capital Income: An Elasticity Rule. *Journal of Economics* 55, 101–111.
- Richter, W.F., and Wiegard, W., 1991. On the Difference between Income and Consumption Taxes when Return to Savings is Uncertain. *Recherches Economiques de Louvain* 77, 379–390.
- Rose, M., 1999. Recommendations on Taxing Income for Countries in Transition to Market Economies. M. Rose (ed.), *Tax Reform for Countries in Transition to Market Economies*. Lucius & Lucius, Stuttgart.

- Rose, M., and R. Wiswesser, 1998. Tax Reform in Transition Economies: Experiences from the Croatian Tax Reform Process of the 1990s. In: Sørensen, P.B. (ed.), *Public Finance in a Changing World*. Macmillan Press.
- Sandmo, A., 1969. Capital Risk, Consumption and Portfolio Choice. *Econometrica* 37, 586–599.
- Sandmo, A., 1977. Portfolio Theory, Asset Demand and Taxation: Comparative Statics with Many Assets. *Review of Economic Studies* 49, 517–528.
- Sandmo, A., 1985. The Effects of Taxation on Savings and Risk Taking. In: Auerbach, A.J., and M. Feldstein (eds.), *Handbook of Public Economics*, Vol. I, Elsevier Science Publishers B.V., 265–311.
- Sørensen, P.B., 1994. From the Global Income Tax to the Dual Income Tax: Recent Reformers in the Nordic Countries. *International Tax and Public Finance* 1, 57–79.
- Sørensen, P.B., 2005. Neutral Taxation of Shareholder Income. *International Tax and Public Finance* 12, 777–801.
- Varian, H.R., 1980. Redistributive Taxation as Social Insurance. *Journal of Public Economics* 14, 49–68.

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