Efficient Tax Policy Ranks Education Higher than Saving

Wolfram F. Richter

CESifo Working Paper No. 2106
Category 1: Public Finance
October 2007

CESifo *Working Paper No. 2106*

EFFICIENT TAX POLICY RANKS EDUCATION HIGHER THAN SAVING

Abstract

Assuming a two-period model with endogenous choices of labour, education, and saving, it is shown to be second-best efficient not to distort the choice of education. In general this implies distorting the saving decision. Hence a strict order of policy priority is derived. Efficient tax policy ranks investment in human capital higher than investment in nonhuman capital. The result assumes an isoelastic earnings function and holds else for arbitrary utility functions. Isoelasticity of earnings is justified with reference to the empirically well-founded power law of learning.

JEL Code: H21, I28, J24.

Keywords: endogenous choice of education, labour, and saving, efficient taxation of human and nonhuman capital investment, power law of learning.

Wolfram F. Richter
University of Dortmund
Department of Economics
44221 Dortmund
Germany
Wolfram.Richter@uni-dortmund.de

August 2007

1. Introduction

One of the most debated issues in public economics is whether capital income should be taxed or not. Although there seems to be broad consensus that taxing savings is presumably "a bad idea" (Atkeson et al., 1999), one has to admit that truly compelling arguments are lacking. While equity arguments are used both to attack and to justify the taxation of capital, efficiency arguments are at best inconclusive. They lend themselves to argument against the taxation of capital in a model of finite horizon only if particular preferences are assumed for which the empirical evidence is weak. If the choice of preferences is to play no critical role, one has to rely on Chamley (1986) and Judd (1985). Their result establishes that the optimal tax rate on capital income tends to zero in a model of infinite horizon. The problem with this result, however, is not only that it does not extend to a finite horizon. More critical is that it is biased in one important respect. It holds for a framework that models the accumulation of nonhuman capital but ignores the accumulation of human capital. If however the two courses of capital accumulation are modelled more symmetrically, the reason for discriminatory taxation disappears. More precisely, Chamley and Judd's result on zero capital-income taxation in the limit is then seen to extend to labour taxes (Jones et al., 1997). This result is not convincing either, however. It relies on blurring differences between human and nonhuman capital, and it raises even more the question as to which ultimate economic feature makes the one differ from the other.

In this paper, the difference between human and nonhuman capital is modelled as a short-run difference in returns. The returns to saving are assumed to be constant, while those of education are assumed to be decreasing. Decreasing returns give rise to pure profit. The profit of education is special for two reasons. First, the immediate outcome of education is ability only. Skills acquired by education do not pay off if not combined with additional own labour effort. It is in this sense that the profit of education is quasi-pure. Secondly, it is not clear what efficient taxation should do with the quasi-pure profit of education. Consumption and labour taxes cut off some profit; they do not however skim it off fully. In this paper it is shown that second-best policy abstains from distorting the decision to invest in one's own human capital. To reach this objective, the supply of labour and even the decision to save have to be distorted in general. Hence there is a strict order of policy priority. Efficient tax policy ranks investment in human capital higher than any other supply-and-demand decision of private households and thus also higher than the decision to invest in nonhuman capital. This result is derived for the standard two-period life-cycle model with endogenous choices of labour, education, and saving. It assumes an isoelastic earnings function and holds else for arbitrary utility functions.

The paper is structured as follows. Section 2 sets up the model of a representative taxpayer. Section 3 introduces policy instruments and the planner's objective function. In Section 4 it is shown to be efficient not to distort human-capital investment. The result generalizes one derived before by Richter (2007) in a less structured model. Section 5 explores the implication for the taxation of nonhuman capital. Section 6 summarizes. Major proofs are relegated to a technical Appendix.

2. A representative-household model

Consider a representative household living for two periods. Lifetime utility is given by $U(C_1, C_2, L_1, L_2)$, where C_i is consumption and L_i is non-leisure time in period i=1,2. Non-leisure time L_2 is identical with second-period labour supply. By contrast, only $L_1 - E$ is time spent in the market; time E is spent on education. First-period labour supply earns a constant wage rate ω_1; the productivity of second-period labour depends on the amount of education. It is paid $\omega_2 H(E)$, where ω_2 is constant while the *earnings function* $H(E)$ displays positive but diminishing returns, $H'>0>H''$. We will also refer to *qualified labour* in the case of L_2 and of *nonqualified labour* in the case of L_1. Education has a cost in *forgone earnings*, which is captured by $\omega_1 E$. *Monetary costs of education* like college fees come on top of these and are modelled by φE. The share of first-period income that is neither spent on education nor on consumption is saved,

$$S = \omega_1(L_1 - E) - \varphi E - qC_1 = \omega_1 L_1 - (\omega_1 + \varphi)E - qC_1. \qquad (1)$$

Second-period consumption is constrained by income earned,

$$qC_2 = \rho S + \omega_2 H(E)L_2 \ . \tag{2}$$

Here q is the consumer price of consumption and ρ is the gross rate of return to saving. All prices are after taxes and subsidies, and the question is which combination of taxes and subsidies is second-best efficient. The representative household is assumed to maximize utility in C_1, C_2, L_1, L_2, E subject to the lifetime budget constraint

$$\rho q C_1 + qC_2 = \rho \omega_1 L_1 + \omega_2 H(E)L_2 - \pi E \tag{3}$$

stated in second-period units. Interpret $\pi \equiv \rho(\omega_1 + \varphi)$ as the *effective* (*unit*) *cost of education*.

The analysis relies on the dual approach to optimal taxation. This means that the focus is shifted from the household's (indirect) utility function to its (net) expenditure function. The task of minimizing (net) expenditures subject to an exogenous utility constraint is best solved in a two-step approach. At the first step income derived from education is maximized while keeping the level of L_2 fixed. Let this income be denoted by $Y(\omega_2, \pi, L_2) \equiv \max_E [\omega_2 H(E)L_2 - \pi E]$, and the optimal amount of education by $E(\omega_2, \pi, L_2)$. The optimal amount is implicitly defined by the first-order condition, $\omega_2 H' L_2 = \pi$. If the second-period labour supply L_2 were exogenous, Y would stand for pure profit. However, the focus is here on an endogenous choice of L_2. Hence Y has to be interpreted as quasi-pure profit, the source of which is education and its diminishing return. Second-period labour supply increases Y only indirectly via increased incentives for education. Let the second-period wage rate before taxes be denoted by w_2, and the effective social cost of education (i.e., the effective cost before taxes and subsidies) by p. *Education is efficient* if the tax wedge between the marginal social return and the effective social cost,

$$\text{tax wedge} \ \equiv \ w_2 H'(E)L_2 - p \ = \ \pi[\frac{w_2 H' L_2}{\pi} - \frac{p}{\pi}] \ = \ \pi[\frac{w_2}{\omega_2} - \frac{p}{\pi}] \ ,$$

vanishes. Obviously, the tax wedge vanishes if, and only if, the rates of return before and after taxes and subsidies are equal,

$$\frac{w_2}{p} = \frac{\omega_2}{\pi} \quad . \tag{4}$$

The main result of this paper states that it is efficient (in the second-best sense) not to compromise on efficiency in education. Note that second-best efficiency refers to total analysis, while efficiency in education is a partial concept.

The expenditure function is defined as

$$e(q,\omega_1,\omega_2,\rho,\varphi;u) \quad \equiv \quad \min[\rho q C_1 + qC_2 - \rho\omega_1 L_1 - Y(\omega_2,\rho(\omega_1+\varphi),L_2)]$$

in C_1, C_2, L_1, L_2 such that $U(C_1,C_2,L_1,L_2) \geq u$.

Hotelling's lemma yields $e_q = \rho C_1 + C_2$, where $C_i = C_i(q,\omega_1,\omega_2,\rho,\varphi;u)$ solves the optimization and where the subscript *q* denotes a partial derivative. By relying on a straightforward generalization of the textbook version of Hotelling's lemma one likewise derives the identities $e_1 \equiv \dfrac{\partial e}{\partial \omega_1} = -\rho(L_1 - E)$, $e_\varphi = \rho E$, and $e_\rho = qC_1 - \omega_1 L_1 + (\omega_1+\varphi)E = -S$. Just like C_i, the functions L_i and *S* are Hicksian ones and have to be evaluated at $q,\omega_1,\omega_2,\rho,\varphi$, and *u*. As a result, the fully specified education function reads $E = E(\omega_2,\rho(\omega_1+\varphi),L_2(q,\omega_1,\omega_2,\rho,\varphi;u))$.

3. Policy instruments

The analysis studies the efficient mix of four policy instruments, each of which is distorting. The first one is a tax t_C>0 on consumption. Treating consumption as a numéraire good with a producer price of one, this implies q=1+t_C.

The second instrument is a tax t_E on the monetary social cost of education *f*, so that $\varphi = (1+t_E)f$ results. Negativity of t_E is not ruled out, so that subsidizing education is a feasible policy.

The third instrument is a tax τ on capital income. Again negative values of τ are not ruled out. As it turns out, it is convenient to define the capital tax in *exclusive* form. Denoting the gross social return to saving by *r*, exclusiveness means that τ satisfies the condition $r=(1+\tau)\rho$. In other words, the base of the capital tax excludes the tax payment.

The fourth policy instrument is a subsidy σ to labour income earned in the first period. This requires $\omega_1=(1+\sigma)w_1$, where w_1 denotes the market wage rate. Second-period labour income is assumed to remain untaxed: $\omega_2=w_2$. Given that consumption is taxable, nothing is gained by introducing a tax on second-period labour. It would only provide a redundant instrument, which could be duplicated by an appropriate choice of the four other policy instruments. As first-period labour is nonqualified while second-period labour is qualified, a positive σ can be interpreted as a policy regime in which labour income is taxed progressively with respect to qualification. All social costs, w_1, w_2, r, and f, are treated as exogenous parameters of the planner's optimization. This holds *a fortiori* for the effective social cost of education, $p=r(w_1+f)$.

There is a need to raise tax revenue in order to finance government expenditures. Government's net tax revenue amounts to

$$\begin{aligned} T &\equiv t_C(rC_1+C_2)+t_E rfE+\tau\rho S-\sigma r w_1(L_1-E) \\ &= (q-1)(rC_1+C_2)+r(\varphi-f)E+(r-\rho)S+r(w_1-\omega_1)(L_1-E). \end{aligned}$$

By invoking Hotelling's lemma this can be written as

$$\begin{aligned} T= {} & (q-1)e_q+\frac{1}{\rho}[\frac{q-1}{q}(\rho-r)\omega_1+r(\omega_1-w_1)]e_1+\frac{\rho-r}{q}e_\rho \\ & +\frac{1}{\rho}[\frac{q-1}{q}(\rho-r)\varphi+r(\varphi-f)]e_\varphi . \end{aligned} \tag{5}$$

The social planner is assumed to maximize tax revenue T subject to the condition that private net expenditure remains constant at zero level, e=0. Let all the conditions of regularity hold that are needed to make the optimization a well-behaved problem. A set of instruments t_C>0, t_E, τ, and σ is said to be second-best efficient if it solves the planner's maximization problem. The assumption that t_C has to be positive rules out the trivial case that all tax rates are zero. The constellation with $t_C>0=t_E=\tau=\sigma$ is feasible. However, it distorts consumption and labour choices, and the key question is whether it is more efficient to reduce t_C and to compensate the reduction by appropriate use of the remaining instruments.

4. Efficient policy

Maximizing government's net revenue *T* in t_C, t_E, τ, and σ subject to a balanced-budget constraint on the taxpayer, *e*=0, yields a problem that can easily be solved by applying the standard Lagrange technique. Maximizing in t_C, t_E, τ, and σ is obviously equivalent to maximizing in q, φ, ρ, and ω_1. Taking partial derivatives with respect to the latter variables, invoking Hotelling's lemma, and eliminating the Lagrange multiplier yields a system of three first-order conditions:

$$\frac{\Delta E}{E} = \frac{\Delta L_1 - \Delta E}{L_1 - E} = \frac{\omega_1 \Delta L_1 - q\Delta C_1}{\omega_1 L_1 - qC_1} = \frac{\rho \Delta C_1 + \Delta C_2}{\rho C_1 + C_2}, \qquad (6)$$

where the differentiation operator Δ applies to arbitrary functions $X = X(q, \omega_1, \omega_2, \rho, \varphi; u)$. The operator is defined as follows:

$$\Delta X \equiv (q-1)X_q + \frac{1}{\rho}\left[\frac{q-1}{q}(\rho - r)\omega_1 + r(\omega_1 - w_1)\right]X_1$$

$$+ \frac{\rho - r}{q}X_\rho + \frac{1}{\rho}\left[\frac{q-1}{q}(\rho - r)\varphi + r(\varphi - f)\right]X_\varphi. \qquad (7)$$

For a proof of (6) see the Appendix. Reshuffling (6) gives us a system of equalities, which is readily interpreted in the spirit of Ramsey:

$$\frac{\Delta E}{E} = \frac{\Delta L_1}{L_1} = \frac{\Delta C_1}{C_1} = \frac{\Delta C_2}{C_2}. \qquad (6')$$

According to (6′), efficiency is achieved if prices are set such that the derivatives of *E*, L_1, and C_i (*i*=1,2) induce equiproportionate changes if taken in the direction of

$$(q-1)dq + \frac{1}{\rho}\left[\frac{q-1}{q}(\rho - r)\omega_1 + r(\omega_1 - w_1)\right]d\omega_1 + \frac{\rho - r}{q}d\rho +$$

$$\frac{1}{\rho}\left[\frac{q-1}{q}(\rho - r)\varphi + r(\varphi - f)\right]d\varphi.$$

More loosely speaking, efficient policy induces equiproportionate reductions in education E, nonqualified labour L_1, and periodic consumption C_i (i=1,2) when all these behavioural functions are interpreted in the Hicksian sense. Note that qualified labour L_2 is missing from this list.

Proposition 1: Efficient policy requires equiproportionate reductions in education, nonqualified labour, and periodic consumption.

Proposition 1 raises the question as to which constellations of q, φ, ρ, and ω_1 induce equiproportionate reductions. Clearly, one should not expect any interesting relationship to hold in full generality. Still, a remarkably strong result is obtained if the individual earnings function is assumed to be isoelastic in education, $H(E) = hE^{\eta}$ with $\eta < 1$.

Proposition 2: If the individual earnings function is isoelastic, it is efficient not to distort the choice of education.

Given that $\omega_2 = w_2$ has been assumed, not to distort education requires leaving the effective cost unchanged:

$$\rho(\omega_1 + \varphi) = \pi \ = \ p = r(w_1 + f) . \qquad (4')$$

The proof is given in the Appendix.

The generality of the proposition is striking. Beyond the standard regularity assumptions of household optimization, there are no additional ones needed to constrain the choice of utility functions. However, isoelasticity of the individual earnings function is indispensable.

From a purely mathematical point of view isoelasticity may look very special. However, its assumption can well be defended by referring to the *power law of* *learning* known from cognitive psychology. The content of the power law is the following. According to common experience, most tasks get faster with practice, and this holds across task size and task type. If the relationship between practice and the completion time of a task is plotted, a power law is generally seen to provide the best fit. Education is undoubtedly broader and more complex than the training for certain tasks. However, "the power law of practice is ubiquitous" (Ritter et al., 2001), and it would not be plausible to doubt its empirical relevance for the formation of abilities, which after all is the economically relevant essence of education.

Combining Propositions 1 and 2 implies that efficient policy well tolerates a reduction in educational effort. This reduction cannot be interpreted, however, as a distortion of education, but only as a distortion in the supply of qualified labour. The intuition underlying Proposition 2 is the following. In deciding on education and labour supply the private household trades off costs against benefits. The benefits are given by labour income and by the private profit from education, which in real terms amounts to *Y/q*. In the general case of $\omega_2 \neq w_2$, the social profit is

$$w_2 H L_2 - pE \;=\; w_2 \frac{Y + \pi E}{\omega_2} - pE \;=\; \frac{w_2}{\omega_2} Y + w_2 [\frac{\pi}{\omega_2} - \frac{p}{w_2}] E \,,$$

which equals $w_2 Y / \omega_2$ whenever (4) holds. The significance of efficiency in education is hence seen to be in the alignment of private and social objectives. By maximizing the private profit from education, *Y/q*, the social profit, $w_2 Y / \omega_2$, is maximized as well. Isoelasticity of *H* serves to ensure that this perfect alignment of private and social objectives need not be compromised. If the earnings function fails to be isoelastic, it may well be optimal to exploit variations in the elasticity of the earnings function and to trade off resulting efficiency gains against distortions in the choice of education.

Proposition 2 has to be contrasted with a result by Bovenberg and Jacobs (2005). These authors likewise identify circumstances under which human-capital investment should remain undistorted. See also Jacobs and Bovenberg (2006). There are notable differences, however. Bovenberg and Jacobs focus on the optimal trade-off between equity and efficiency when skill formation is endogenous, and they enlarge the set of policy instruments by assuming that a poll tax is available. In substituting the equity objective for the objective of generating tax revenue, their analysis goes beyond the present one. On the other hand, these authors are only able to derive efficiency of education for a scenario in which the cost of education is purely monetary. Costs of forgone earnings are ruled out. It is as if nonqualified labour L_1 were exogenously fixed. As a result, education E degenerates to an intermediate good and – in contrast to the authors' own suggestions in Jacobs et al. (2006) – the production efficiency theorem of Diamond and Mirrlees (1971) applies. By contrast, Proposition 2 is unrelated to the production efficiency theorem. It does not assume the availability of a poll tax, and it holds even though education provides disutility and fails to be an intermediate good. Above all, the Diamond–Mirrlees theorem has to assume that no pure profit accrues to the private sector, while Proposition 2 concerns quasi-pure profit, a share of which accrues privately. The suggested policy implications are very different, as will become better noticeable when discussing the efficient taxation of entrepreneurial profits. If the source of such profits is a fixed factor, then Diamond and Mirrlees suggest taxing such profits at one hundred percent. If however profits result from decreasing returns to learning and require personal labour effort, then Proposition 2 suggests letting some positive share accrue to the entrepreneur.

5. Implications for taxing nonhuman capital

On inserting tax variables, the equality $\rho(\omega_1 + \varphi) = \pi = p = r(w_1 + f)$ is seen to imply

$$\tau = \frac{\sigma w_1 + t_E f}{w_1 + f}. \qquad (8)$$

This means that in the second-best optimum the tax on capital income, τ, has to equal the weighted sum of the subsidy to nonqualified labour, σ, and the tax on the monetary cost of education, t_E, with the weights given by the social costs w_1 and f. This tight relationship indicates that one should not expect τ to equal zero except for very special cases. Remember that any efficient set of policy instruments has to solve both (6) and (4′). As a consequence, a pure consumption-tax regime, $t_C > 0 = t_E = \tau = \sigma$, is well feasible and even partially efficient in the sense of not distorting education. As the regime implies $\Delta X = (q-1)X_q$, it is totally efficient, however, only if the elasticities of C_1, C_2, L_1, and E with respect to q happen to be equal at these particular parameter values. In other words, total efficiency requires a well-balanced use of policy instruments, and the efficient τ will only be zero in non-generic cases to be discussed below.

Corollary: It is generically efficient to distort saving.

From combining Proposition 2 and the Corollary an order of policy priority is derived. According to this policy order the decision to invest in human capital ranks higher than the decision to invest in nonhuman capital. The reason is that the former generates quasi-pure profit and the latter does not. Efficiency requires not impairing the incentives to earn ability profit beyond what appears to be unavoidable. Unavoidable are the negative incentives that consumption or wage taxes exert on the supply of qualified labour. By contrast, education should remain undistorted. In general this objective requires distorting the savings decision.

The indicated policy priority is in line with a result of Jacobs and Bovenberg (2005). These authors demonstrate that there are constellations where a positive tax on nonhuman capital serves to alleviate tax distortions in human-capital investment. That result however reverses the intuition conveyed by Nielsen and Sörensen (1997). These authors take the capital income tax as given, and they study its implication for efficient taxation of human-capital investment. Hence they implicitly reverse the suggested policy order. Human-capital policy is used to accommodate distortions in nonhuman-capital policy. More precisely, Nielsen and Sörensen show that a progressive income tax is needed to offset the tendency of a proportional comprehensive income tax to discriminate in favour of human-capital investment. In view of Proposition 2, their result is at best one characterizing third-best policy.

One should mention that Nielsen and Sörensen (1997) ignore monetary costs of education when modelling endogenous labour supply, while Jacobs and Bovenberg

(2005) ignore forgone earnings. Proposition 2 cannot, however, be expected to hold in full generality if just one of these two kinds of education costs is modelled. The set of prices and policy instruments must be sufficiently rich to neutralize the effect that taxation has on human-capital investment. More precisely, it must be possible to tax or to subsidize the monetary cost of education, and it must also be possible to tax or to subsidize nonqualified labour at a rate that may deviate from both the former and the tax applied to qualified labour.

The Corollary does not rule out the possibility that it is efficient to set τ=0 for special utility functions. As a matter of fact, efficiency of τ=0 holds if the utility function takes the form $U(C_1,C_2,L_1,L_2)=U(F(C_1,C_2),L_1,L_2)$ with some homothetic function F. The argument is as follows. If F is homothetic, the optimizing individual allocates lifetime consumption in such a way that the ratio C_1/C_2 only depends on the rate of interest. Hence $\rho C_1 + C_2 = c(\rho)C_2$ holds with some appropriately specified function $c(\rho)$. On normalizing money units (r=1) and setting $\rho \equiv r$, $c(\rho)C_2 \equiv C$, the taxpayer's problem reads

$$\max \tilde{U}(C,L_1,L_2) \qquad \text{in } C,L_1,L_2,E$$

$$\text{subject to} \quad qC = \omega_1 L_1 + \omega_2 H(E)L_2 - (\omega_1 + \varphi)E\,. \tag{3′}$$

Just as before, the social planner maximizes tax revenue. The only change is that the capital tax is no feasible policy instrument. When maximizing revenue with respect to t_C>0, t_E, σ, and $\omega_2 = w_2$, it likewise turns out to be efficient not to distort education, For the details see Richter (2007). In other words, Proposition 2 continues to hold. The intuition is that the policy instrument τ is redundant when the subutility of consumption, $F(C_1,C_2)$, is homothetic. The tax on capital can be set equal to zero without jeopardizing efficiency. In this particular case it is even possible to sign t_E and σ. More precisely, as shown by Richter (2007), it is efficient to levy a *positive* tax on the monetary cost of education and to tax labour *regressively* with respect to qualification. That is, efficiency requires t_E>0=$\tau > \sigma$ when t_C>0, $\omega_2 = w_2$. The only additional assumption needed is that the elasticity of consumption exceeds the elasticity of nonqualified labour when q is varied. In other words, the direct effect that a change in q has on consumption needs to be stronger than the indirect effect that such a change has on the supply of nonqualified labour. The intuition for $t_E > 0 > \sigma$ is that it is efficient to set incentives so that qualified labour is substituted for nonqualified labour. Given that $\omega_2 = w_2$ is assumed to hold, this objective is reached by setting $\omega_1 < w_1 \Leftrightarrow \sigma < 0$. Taxing nonqualified labour, however, means reducing the cost of forgone earnings, the partial effect of which is to encourage human-capital investment. The efficiency condition $\omega_1 + \varphi = w_1 + f$ can then be restored only if the monetary cost of education is positively taxed: $\varphi > f \Leftrightarrow t_E > 0$.

It is an interesting, though still open, question to what extent $t_E > 0 > \sigma$ continues to hold in the non-homothetic case of $U(C_1, C_2, L_1, L_2)$. Equally, it is not clear under which fully general conditions a subsidy to capital, $\tau < 0$, can be shown not to be efficient.

6. Summary

One of the most debated issues in public economics is whether capital income should be taxed or not. This paper suggests that proposed answers may well be useless if human capital, its formation, and its taxation are not appropriately taken into account. If this is done, a strict order of policy priority can be derived. According to this order, human-capital investment ranks higher than nonhuman-capital investment. The former should not be distorted, which in general requires that the latter will be distorted. The reason for this asymmetric treatment is that education – other than saving – generates quasi-pure profit, and taxation should not impair the efficient generation. This result is remarkably robust. In this paper it has been proved for the standard two-period life-cycle model of a representative household with endogenous consumption, labour, and education. The result does not assume particularly selected utility functions, but only an isoelastic earnings function and a sufficiently rich set of policy instruments. Isoelasticity of earnings is justified with reference to the empirically well-founded power law of learning.

The present paper is not the first attempt to study economic implications of decreasing returns to learning. Well known is Arrow’s (1962) attempt to develop a theory of technical change and growth by drawing on the learning curve. In his model, however, the learning curve takes the role of a labour demand curve. Knowledge is completely embodied in capital, and at each moment of time capital goods of different vintages are in use. As Arrow stresses himself in his closing comments, the implicit assumption is that learning takes place only as a by-product of ordinary production. By way of contrast, learning is central in the present model. It is an individual investment in one’s own productivity and the result of endogenous choice.

One cannot summarize without qualifying the obtained results. There are two points of weakness that need to be addressed more than others. One obvious shortcoming of the present analysis is certainly its pure focus on efficiency. Equity considerations have been entirely ignored. This is different from much of the cited literature and from the work of Bovenberg and Jacobs in particular. The conflict between equity and efficiency in human-capital policy is however not a simple one. In any case, it cannot be discussed satisfactorily just in passing. The interested reader is asked instead to refer to the literature as surveyed, e.g., by Carneiro and Heckman (2003).

The other shortcoming can be seen in the lack of balance in the derived efficiency result. The clear policy prescription not to distort the educational choice contrasts with the inability to derive definite policy prescriptions for other areas of taxpayers’ choice. Only if the utility of consumption is assumed to be homothetic and separable from leisure can more be said. In this case, efficiency is enhanced by giving incentives to substitute qualified labour for nonqualified labour while respecting efficiency in education as a constraint. Furthermore, the efficient way of inducing substitution relies on taxing nonqualified labour more heavily than qualified labour and on restoring efficiency in education by taxing its monetary cost. See Richter (2007), where the result is interpreted by referring to the (weak) double-dividend hypothesis known from environmental economics. No doubt, popular conceptions of good education policy look different (Trostel, 2002). The contrast is however nothing but another indication for the need to enrich the present analysis by equity considerations in future research.

7. Appendix

The *proof of* (6) relies on taking partial derivatives of the Lagrange function $T - \lambda e$ with respect to q, φ, ρ, and ω_1:

$$\frac{\partial}{\partial \varphi}[T - \lambda e] = 0 \;\;\Leftrightarrow\;\; (\lambda - 1)e_\varphi + \frac{\rho - r}{q\rho} e_\varphi = \Delta e_\varphi \;. \qquad (9)$$

By Hotelling's lemma and by the definition of the Δ-operator, one obtains

$$e_\varphi = \rho E \;\text{ and }\; \Delta e_\varphi = \Delta(\rho E) = \rho \Delta E + \frac{\rho - r}{q} E \,. \qquad (10)$$

Plugging (10) into (9) yields $\lambda - 1 = \Delta E / E$. Similarly one derives

$$\lambda - 1 \;=\; \frac{\Delta L_1 - \Delta E}{L_1 - E} \;=\; \frac{\omega_1 \Delta L_1 - q \Delta C_1}{\omega_1 L_1 - q C_1} \;=\; \frac{\rho \Delta C_1 + \Delta C_2}{\rho C_1 + C_2} \,. \;\square$$

The *proof of Proposition* 2 requires some preparatory considerations. Let

$$\varepsilon_{X/x} = \frac{x}{X} \frac{\partial X}{\partial x}$$

denote the elasticity of *X* with respect to *x*.

Remark 1: Assuming $H(E) = hE^\eta$, $\eta < 1$, and $\pi \equiv \rho(\omega_1 + \varphi)$, one obtains

$$\varepsilon_{Y/x} \;=\; \varepsilon_{E/x} \;+\; \varepsilon_{\pi/x} \qquad \text{for } x = q, \rho, \omega_1, \varphi \,. \qquad (11)$$

Proof: Relying on the first-order condition $\omega_2 H' L_2 = \pi$ allows one to express the ability rent *Y* as a strictly proportional function of *E*:

$$Y(\omega_2, \pi, L_2) \;=\; \omega_2 H(E) L_2 - \pi E \;=\; H \frac{\pi}{H'} - \pi E \;=\; \pi(\frac{1}{\eta} - 1) E \,.$$

(11) is an obvious implication. □

Remark 2: Assuming $H(E) = hE^{\eta}$, $\eta < 1$, one obtains

$$\frac{\Delta L_2}{L_2} \;=\; \frac{Y}{Y+\pi E}\,\frac{\Delta E}{E} + \frac{\pi - p}{\pi}.$$

Proof: By the definition of the Δ-operator in (7) one obtains

$$[Y+\pi E]\frac{\Delta L_2}{L_2} = \omega_2 H \Delta L_2$$

$$= (q-1)\omega_2 H L_{2q} + \frac{1}{\rho}[\frac{q-1}{q}(\rho - r)\omega_1 + r(\omega_1 - w_1)]\omega_2 H L_{21}$$

$$+ \frac{\rho - r}{q}\omega_2 H L_{2\rho} + \frac{1}{\rho}[\frac{q-1}{q}(\rho - r)\varphi + r(\varphi - f)]\omega_2 H L_{2\varphi}$$

$$= (q-1)Y_q + \frac{1}{\rho}[\frac{q-1}{q}(\rho - r)\omega_1 + r(\omega_1 - w_1)][Y_1 + \rho E]$$

$$+ \frac{\rho - r}{q}[Y_\rho + (\omega_1 + \varphi)E] + \frac{1}{\rho}[\frac{q-1}{q}(\rho - r)\varphi + r(\varphi - f)][Y_\varphi + \rho E],$$

which because of Remark 1 equals

$$(q-1)\frac{Y}{E}E_q + \frac{1}{\rho}[\frac{q-1}{q}(\rho - r)\omega_1 + r(\omega_1 - w_1)]\frac{Y}{E}E_1$$

$$+ \frac{\rho - r}{q}[\frac{Y}{E}E_\rho + \frac{Y}{\rho} + (\omega_1 + \varphi)E] + \frac{1}{\rho}[\frac{q-1}{q}(\rho - r)\varphi + r(\varphi - f)]\frac{Y}{E}E_\varphi$$

$$+ [\frac{q-1}{q}(\rho - r)(\omega_1 + \varphi) + r(\omega_1 + \varphi - w_1 - f)][\frac{Y}{\pi} + E]$$

$$= \frac{Y}{E}\Delta E + \frac{\pi - p}{\pi}[Y + \pi E]. \;\square$$

By relying on the definition of the expenditure function and by invoking Hotelling's lemma one obtains

$$q[\rho C_{1x} + C_{2x}] \;=\; \rho \omega_1 L_{1x} + Y_{L_2} \cdot L_{2x} \quad \text{for} \quad x = q, \omega_1, \rho, \varphi. \tag{12}$$

The relationship (12) extends to the Δ-notation:

$$q[\rho \Delta C_1 + \Delta C_2] = \rho \omega_1 \Delta L_1 + Y_{L_2} \cdot \Delta L_2 \, . \tag{13}$$

Proposition 2 is now easily proved by relying on (6), (13), (6′), and Remark 2:

$$\frac{\Delta E}{E} \underset{(6)}{=} \frac{\rho \Delta C_1 + \Delta C_2}{\rho C_1 + C_2} \underset{(13)}{=} \frac{\rho \omega_1 L_1}{q[\rho C_1 + C_2]} \frac{\Delta L_1}{L_1} + \frac{\omega_2 H L_2}{q[\rho C_1 + C_2]} \frac{\Delta L_2}{L_2}$$

$$= \frac{\rho \omega_1 L_1}{q[\rho C_1 + C_2]} \frac{\Delta E}{E} + \frac{Y}{q[\rho C_1 + C_2]} [\frac{\Delta E}{E} + \frac{\pi - p}{\pi} \frac{Y + \pi E}{Y}]$$

$$= \frac{\Delta E}{E} + \frac{\pi - p}{\pi} \frac{Y + \pi E}{q[\rho C_1 + C_2]} \, .$$

On comparing the first and last members, $\pi = p$ follows. □

References

Arrow, K.J., 1962, The Economic Implications of Learning by Doing, Review of Economic Studies 29, 155−173.

Atkeson, A., V.V. Chari, and P.J. Kehoe, 1999, Taxing capital income: A bad idea, Federal Reserve Bank of Minneapolis Quarterly Review 23, 3−17.

Bovenberg, A.L. and B. Jacobs, 2001, Redistribution and education subsidies are Siamese twins, CEPR Disc. Paper No. 3099.

Bovenberg, A.L. and B. Jacobs, 2005, Redistribution and education subsidies are Siamese twins, Journal of Public Economics 89, 2005–2035.

Carneiro, P. and J.J. Heckman, 2003, Human Capital Policy, in: Inequality in America, J.J. Heckman and A.B. Krueger, eds., MIT Press, 77−239.

Chamley, C., 1986, Optimal taxation of capital income in general equilibrium with infinite lives, Econometrica 54, 607−622.

Diamond, P. and J.A. Mirrlees, 1971, Optimal taxation and public production, I Production efficiency, II Tax rules, American Economic Review 61, 8–27, 261–278.

Jacobs, B. and A.L. Bovenberg, 2005, Human capital and optimal positive taxation of capital income, Tinbergen Institute, Disc. Paper TI 2005-035/3.

Jacobs, B. and A.L. Bovenberg, 2006, Redistribution and education subsidies are Siamese twins (II) – A Correction and Further Generalizations, mimeo.

Jones, L.E., R.E. Manuelli, and P.E. Rossi, 1997, On the optimal taxation of capital income, Journal of Economic Theory 73, 93−117.

Judd, K.L., 1965, Redistributive taxation in a simple perfect foresight model, Journal of Public Economics 28, 59−83.

Nielsen, S.B. and P.B. Sörensen, 1997, On the optimality of the Nordic system of dual income taxation, Journal of Public Economics 63, 311–329.

Richter, W.F., 2007, Taxing human capital efficiently – The double dividend of taxing nonqualified labour more heavily than qualified labour, Ruhr Economic Papers 12.

Ritter, F.E., and Schooler, L.J., 2001, The learning curve, in: International Encyclopedia of the Social and Behavioral Sciences, W. Kintch, N. Smelser, and P. Baltes, eds., Oxford, 8602−8605.

Trostel, P.A., 2002, Should education be publicly provided? Bulletin of Economic Research 54, 373−391.

CESifo Working Paper Series

for full list see www.cesifo-group.org/wp
(address: Poschingerstr. 5, 81679 Munich, Germany, office@cesifo.de)

2044 Annika Alexius and Bertil Holmlund, Monetary Policy and Swedish Unemployment Fluctuations, July 2007

2045 Axel Dreher, Jan-Egbert Sturm and Jakob de Haan, Does High Inflation Cause Central Bankers to Lose their Job? Evidence Based on a New Data Set, July 2007

2046 Guglielmo Maria Caporale and Luis A. Gil-Alana, Long Run and Cyclical Dynamics in the US Stock Market, July 2007

2047 Alessandro Balestrino, It is a Theft but not a Crime, July 2007

2048 Daniel Becker and Michael Rauscher, Fiscal Competition in Space and Time: An Endogenous-Growth Approach, July 2007

2049 Yannis M. Ioannides, Henry G. Overman, Esteban Rossi-Hansberg and Kurt Schmidheiny, The Effect of Information and Communication Technologies on Urban Structure, July 2007

2050 Hans-Werner Sinn, Please Bring me the New York Times – On the European Roots of Richard Abel Musgrave, July 2007

2051 Gunther Schnabl and Christian Danne, A Role Model for China? Exchange Rate Flexibility and Monetary Policy in Japan, July 2007

2052 Joseph Plasmans, Jorge Fornero and Tomasz Michalak, A Microfounded Sectoral Model for Open Economies, July 2007

2053 Vesa Kanniainen and Panu Poutvaara, Imperfect Transmission of Tacit Knowledge and other Barriers to Entrepreneurship, July 2007

2054 Marko Koethenbuerger, Federal Tax-Transfer Policy and Intergovernmental Pre-Commitment, July 2007

2055 Hendrik Jürges and Kerstin Schneider, What Can Go Wrong Will Go Wrong: Birthday Effects and Early Tracking in the German School System, July 2007

2056 Bahram Pesaran and M. Hashem Pesaran, Modelling Volatilities and Conditional Correlations in Futures Markets with a Multivariate t Distribution, July 2007

2057 Walter H. Fisher and Christian Keuschnigg, Pension Reform and Labor Market Incentives, July 2007

2058 Martin Altemeyer-Bartscher, Dirk T. G. Rübbelke and Eytan Sheshinski, Policies to Internalize Reciprocal International Spillovers, July 2007

2059 Kurt R. Brekke, Astrid L. Grasdal and Tor Helge Holmås, Regulation and Pricing of Pharmaceuticals: Reference Pricing or Price Cap Regulation?, July 2007

2060 Tigran Poghosyan and Jakob de Haan, Interest Rate Linkages in EMU Countries: A Rolling Threshold Vector Error-Correction Approach, July 2007

2061 Robert Dur and Klaas Staal, Local Public Good Provision, Municipal Consolidation, and National Transfers, July 2007

2062 Helge Berger and Anika Holler, What Determines Fiscal Policy? Evidence from German States, July 2007

2063 Ernesto Reuben and Arno Riedl, Public Goods Provision and Sanctioning in Privileged Groups, July 2007

2064 Jan Hanousek, Dana Hajkova and Randall K. Filer, A Rise by Any Other Name? Sensitivity of Growth Regressions to Data Source, July 2007

2065 Yin-Wong Cheung and Xing Wang Qian, Hoarding of International Reserves: Mrs Machlup's Wardrobe and the Joneses, July 2007

2066 Sheilagh Ogilvie, 'Whatever Is, Is Right'?, Economic Institutions in Pre-Industrial Europe (Tawney Lecture 2006), August 2007

2067 Floriana Cerniglia and Laura Pagani, The European Union and the Member States: Which Level of Government Should Do what? An Empirical Analysis of Europeans' Preferences, August 2007

2068 Alessandro Balestrino and Cinzia Ciardi, Social Norms, Cognitive Dissonance and the Timing of Marriage, August 2007

2069 Massimo Bordignon, Exit and Voice. Yardstick versus Fiscal Competition across Governments, August 2007

2070 Emily Blanchard and Gerald Willmann, Political Stasis or Protectionist Rut? Policy Mechanisms for Trade Reform in a Democracy, August 2007

2071 Maarten Bosker and Harry Garretsen, Trade Costs, Market Access and Economic Geography: Why the Empirical Specification of Trade Costs Matters, August 2007

2072 Marco Runkel and Guttorm Schjelderup, The Choice of Apportionment Factors under Formula Apportionment, August 2007

2073 Jay Pil Choi, Tying in Two-Sided Markets with Multi-Homing, August 2007

2074 Marcella Nicolini, Institutions and Offshoring Decision, August 2007

2075 Rainer Niemann, The Impact of Tax Uncertainty on Irreversible Investment, August 2007

2076 Nikitas Konstantinidis, Gradualism and Uncertainty in International Union Formation, August 2007

2077 Maria Bas and Ivan Ledezma, Market Access and the Evolution of within Plant Productivity in Chile, August 2007

2078 Friedrich Breyer and Stefan Hupfeld, On the Fairness of Early Retirement Provisions, August 2007

2079 Scott Alan Carson, Black and White Labor Market Outcomes in the 19^{th} Century American South, August 2007

2080 Christian Bauer, Paul De Grauwe and Stefan Reitz, Exchange Rates Dynamics in a Target Zone – A Heterogeneous Expectations Approach, August 2007

2081 Ana Rute Cardoso, Miguel Portela, Carla Sá and Fernando Alexandre, Demand for Higher Education Programs: The Impact of the Bologna Process, August 2007

2082 Christian Hopp and Axel Dreher, Do Differences in Institutional and Legal Environments Explain Cross-Country Variations in IPO Underpricing?, August 2007

2083 Hans-Werner Sinn, Pareto Optimality in the Extraction of Fossil Fuels and the Greenhouse Effect: A Note, August 2007

2084 Robert Fenge, Maximilian von Ehrlich and Matthias Wrede, Fiscal Competition, Convergence and Agglomeration, August 2007

2085 Volker Nitsch, Die Another Day: Duration in German Import Trade, August 2007

2086 Kam Ki Tang and Jie Zhang, Morbidity, Mortality, Health Expenditures and Annuitization, August 2007

2087 Hans-Werner Sinn, Public Policies against Global Warming, August 2007

2088 Arti Grover, International Outsourcing and the Supply Side Productivity Determinants, September 2007

2089 M. Alejandra Cattaneo and Stefan C. Wolter, Are the Elderly a Threat to Educational Expenditures?, September 2007

2090 Ted Bergstrom, Rod Garratt and Damien Sheehan-Connor, One Chance in a Million: Altruism and the Bone Marrow Registry, September 2007

2091 Geraldo Cerqueiro, Hans Degryse and Steven Ongena, Rules versus Discretion in Loan Rate Setting, September 2007

2092 Henrik Jacobsen Kleven, Claus Thustrup Kreiner and Emmanuel Saez, The Optimal Income Taxation of Couples as a Multi-Dimensional Screening Problem, September 2007

2093 Michael Rauber and Heinrich W. Ursprung, Life Cycle and Cohort Productivity in Economic Research: The Case of Germany, September 2007

2094 David B. Audretsch, Oliver Falck and Stephan Heblich, It's All in Marshall: The Impact of External Economies on Regional Dynamics, September 2007

2095 Michael Binder and Christian J. Offermanns, International Investment Positions and Exchange Rate Dynamics: A Dynamic Panel Analysis, September 2007

2096 Louis N. Christofides and Amy Chen Peng, Real Wage Chronologies, September 2007

2097 Martin Kolmar and Andreas Wagener, Tax Competition with Formula Apportionment: The Interaction between Tax Base and Sharing Mechanism, September 2007

2098 Daniela Treutlein, What actually Happens to EU Directives in the Member States? – A Cross-Country Cross-Sector View on National Transposition Instruments, September 2007

2099 Emmanuel C. Mamatzakis, An Analysis of the Impact of Public Infrastructure on Productivity Performance of Mexican Industry, September 2007

2100 Gunther Schnabl and Andreas Hoffmann, Monetary Policy, Vagabonding Liquidity and Bursting Bubbles in New and Emerging Markets – An Overinvestment View, September 2007

2101 Panu Poutvaara, The Expansion of Higher Education and Time-Consistent Taxation, September 2007

2102 Marko Koethenbuerger and Ben Lockwood, Does Tax Competition Really Promote Growth?, September 2007

2103 M. Hashem Pesaran and Elisa Tosetti, Large Panels with Common Factors and Spatial Correlations, September 2007

2104 Laszlo Goerke and Marco Runkel, Tax Evasion and Competition, September 2007

2105 Scott Alan Carson, Slave Prices, Geography and Insolation in 19th Century African-American Stature, September 2007

2106 Wolfram F. Richter, Efficient Tax Policy Ranks Education Higher than Saving, October 2007