

# HOUSEHOLD MODELS: AN HISTORICAL PERSPECTIVE

PATRICIA APPS  
RAY REES

CESIFO WORKING PAPER NO. 2172  
CATEGORY 4: LABOUR MARKETS  
DECEMBER 2007

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the RePEc website:* [www.RePEc.org](http://www.RePEc.org)
- *from the CESifo website:* [www.CESifo-group.org/wp](http://www.CESifo-group.org/wp)

# HOUSEHOLD MODELS: AN HISTORICAL PERSPECTIVE

## Abstract

This paper is a survey of the literature on theoretical models of the household, paying particular attention to some of the earlier contributions, and using them to place the current state of the theory in perspective. One of its aims is to suggest that the literature's neglect of Samuelson's proposal, that households can be modelled as if they maximised a form of social welfare function, was a mistake. However, the idea following directly from the Nash bargaining models, that the household's preference ordering over the utility profiles of its members depends on exogenous variables, in particular wage rates and non-wage incomes, is an important one. Combined with Samuelson's proposal, it can be made the basis for a general approach to modelling household decision taking, flexible enough to encompass non-cooperative behaviour and Pareto inefficiencies arising out of the inevitable incompleteness and unenforceability of domestic agreements. We also point out the importance of household production and some of the implications of its neglect in modelling households. Above all, the aim is to provide a deeper understanding of the current theoretical literature on household economics by means of a survey of its history.

JEL Code: D10, D13, B21, J22, J16, J13, D60, D01.

Keywords: household behavior, family economics, household welfare, time allocation, labor supply, household production, child care, gender, discrimination, cooperative models, non-cooperative models, trade models, microeconomic history.

*Patricia Apps*  
*Public Economics in Law*  
*University of Sydney*  
*Room 1119, Sydney Law School*  
*Sydney 2000, NSW*  
*Australia*  
*p.apps@usys.edu.au*

*Ray Rees*  
*University of Munich*  
*Department of Economics*  
*Ludwigstr. 28*  
*80539 Munich*  
*Germany*  
*ray.rees@lrz.uni-muenchen.de*

# 1 Introduction

This paper is a survey of the literature on theoretical models<sup>1</sup> of the household, paying particular attention to some of the earlier contributions, and using them to place the current state of the theory in perspective. The first and most basic distinction we make is that between models which analyse the decisions of a single consumer/worker, and those that begin with the view of a household as consisting of more than one individual. We reserve the term “household models” for the latter, and call the former “individual models”. We define a household as a set of two or more individuals who live together and are involved in joint (pairwise or group) decision taking in respect of their allocations of time and money. Models of the household in this sense, also commonly called “family models”,<sup>2</sup> are the subject of this paper.

A second distinction concerns the purpose the models are meant to serve. Here we can contrast the approaches of the two early pioneers in this field, Paul A. Samuelson and Gary S. Becker. We would phrase the question motivating Samuelson’s approach in general terms as:

What becomes of the standard propositions of economics, whether theoretical or empirical, positive or normative, when we realise that the basic decision taking unit is a household and not an individual?

In his classic paper on social indifference curves,<sup>3</sup> Samuelson noted that his proof that these indifference curves could not exist, in the sense that they constitute a preference map with the same properties as that in the individual model, presented major problems for the theory of consumer demand, since observed household demands must in general be aggregates of demands of the individuals in the household. Then, how can a household be modelled in such a way that its preference ordering possesses the properties of those in the individual model? The answers given by Samuelson and others to the general questions raised by the nature of household decision taking is the main theme of this paper.

Becker’s approach, on the other hand, was concerned not with reformulating the existing body of economic propositions, but with extending the domain of economic analysis to areas that before him had not generally been considered amenable to or appropriate for it. He was concerned to show that economic methodology could give important new insights into the workings of the household as an economic institution, in matters such as marriage, divorce, fertility, bequests, human capital formation, role allocation, and so on. Here, we will not review this important body of literature,<sup>4</sup> but simply note that his results were

---

<sup>1</sup>A companion paper to this, Apps and Rees (2007a), surveys the empirical literature.

<sup>2</sup>A problem with this terminology is that without further qualification, for example by terms such as “nuclear” or “extended”, it is hard to decide if a “family” meets this definition of a household or not, while a household in the sense defined here may not be a family. See also Pollak (1985).

<sup>3</sup>Samuelson (1956).

<sup>4</sup>For an extensive survey and discussion of Becker’s work in general see Pollak (2003). Bergstrom (1997) also gives a wide-ranging survey of the broad field of household economics.

mainly achieved in the context of a model that adopted the simplest possible solution to Samuelson's problem, by assuming that the household's preferences are those of an individual, the "head of the household". This is a much more drastic and simplistic step than that proposed by Samuelson himself, even though in the later literature the two are often, quite incorrectly, lumped together.

A third distinction we make in this introduction is that between models that do and those that do not incorporate household production, the use of market goods and household members' time to produce goods and services for consumption within the household. It is perfectly possible to introduce household production into the individual consumer model, as was done by Becker (1965), Gorman (1956), Lancaster (1966), Muth (1966) and others, and a lot can be learned from this.<sup>5</sup> It is also possible to formulate household models without household production, where, as in the individual model, each worker/consumer divides her time between just two uses, market labour supply and "leisure", the direct consumption of one's own time. Throughout his discussion Samuelson, in order "to eliminate nonessential complications", assumes a two-person pure exchange economy, so that his household consists of individuals with given endowments of consumption goods.<sup>6</sup> Whatever the formal difficulties, it seems to us to be essential to either of the basic purposes of household models - the reformulation of the standard results of economics, and the extension of its domain of application - to incorporate household production. We will make this argument in detail at various points in the rest of this paper.

We organise this overview of the theoretical literature on household models by classifying them into three types, according to the principle by which the resource allocation decisions of the household are assumed to be solved. We distinguish between *cooperative* models, *non-cooperative* models, and *equilibrium* models.

Under the first heading we group Samuelson (1956), Becker (1981), the bargaining models of Chen and Woolley (2001), Lundberg and Pollak (1993), McElroy and Horney (1981), Manser and Brown (1980), and Ott (1992), as well as the non-bargaining models of Apps and Rees (1988), (1997), (2001), Basu (2006), Browning and Chiappori (1998) and Chiappori (1988), (1992)<sup>7</sup>, and regard them

---

Jacob Mincer was of course also an influential early contributor to the field, see for example Mincer (1963).

<sup>5</sup>These models should be distinguished from the models of household farm production that are found in the development economics literature, see for example Singh et. al. (1986). The key difference is that in this literature the goods produced in the household have perfect substitutes in the market. The basic model is that of a small open economy determining its production equilibrium by equating domestic marginal costs with world prices, and exporting or importing the difference between its production and consumption. We would argue that in this case household production, in the sense defined in this paper, becomes uninteresting. We are interested only in household goods that are consumed within the household and that have imperfect, even if close, substitutes on the market.

<sup>6</sup>He obviously saw it as unproblematic to extend the model to include production. If there are joint costs and increasing returns to scale, the analysis does become more complicated, and some authors have seen these as central aspects of household production. See in particular the critique of Becker's household production model by Pollak and Wachter (1975).

<sup>7</sup>Vermeulen (2002) surveys the theoretical and empirical work on the model presented in

as particular formulations of Samuelson's consensual approach. We will try to distinguish the contribution each type of model makes to the development of this approach (regardless of whether or not the authors concerned saw themselves as in fact working in the Samuelson tradition).

Non-cooperative models, such as those of Leuthold (1968), Ashworth and Ulph (1981), and Konrad and Lommerud (1995), (2000), characterise the household allocation as the Nash equilibrium of a non-cooperative game. In the literature we often find the assertion that a key difference between cooperative and non-cooperative models is that the equilibrium allocations in non-cooperative models are not assumed to be Pareto efficient, while those in cooperative models are assumed to be. This could however be misleading.

To see this, it is useful to recall the distinction between the *Pareto property* of social welfare functions and the *first or second best Pareto efficiency* of equilibrium resource allocations. The cooperative models all have, explicitly or implicitly, maximands that we will call household welfare functions, and these all have the Pareto property - an increase in the utility of any one individual, *ceteris paribus*, is always a good thing. Non-cooperative models do not possess maximands in this sense, *i.e.* they are not based on the idea that the household as a group solves a well-defined maximisation problem, even though the individuals within it maximise their own utilities. Nevertheless, models adopting the cooperative approach need not result in first best Pareto efficient equilibria, while non-cooperative models can result in first best Pareto efficient outcomes. The efficiency of the realised resource allocation depends on the constraints that are imposed on the allocations that are feasible, *i.e.* on the entire structure of the model.

For example, in a two-period Nash bargaining model the inability to make binding commitments between periods may lead to second best outcomes,<sup>8</sup> even though the Nash bargaining model assumes the Pareto property in the function to be maximised. On the other hand, in an infinitely repeated non-cooperative game, first best Pareto efficient allocations are often in the set of subgame perfect Nash equilibria. In other words, it is neither necessary nor sufficient that a model be cooperative for it to produce first best Pareto efficient allocations, and so it is a source of confusion to define the differences between these classes of model in terms of the Pareto efficiency of their outcomes. It should also be noted that some of the cooperative models use aspects of the results of the non-cooperative models, for example in defining threat points in the cooperative Nash bargaining game, or in drawing on the results for private-contribution public goods games.<sup>9</sup> We discuss these issues further in sections 2, 3 and especially 4 below.

Finally, we have models such as the Walrasian exchange model of Apps (1981), (1982), the marriage market models of Becker (1973) (1974), and the marriage labour market model of Grossbard (1976), (1984), (2003). Here, unlike both cooperative and non-cooperative models, the household members do not act as if they were in a situation of strategic interdependence, but simply

---

these papers by Chiappori and Browning and Chiappori, referred to as the collective model.

<sup>8</sup>See for example Ott (1992) and Lundberg and Pollak (2003).

<sup>9</sup>See Warr (1983) and Bergstrom, Blume and Varian (1987) for the theory of these games.

take individually optimal decisions subject to the constraints presented by competitive markets. Then, while household members do not behave cooperatively in the decision making sense, competitive market forces determine equilibrium solutions to their resource allocation problems that are typically Pareto efficient (though possibly very unequal). An important insight that these models provide, in common with the bargaining models, is that conditions on markets external to the household have an important influence on the equilibrium allocation of resources within it. In this paper we briefly discuss the Apps model, but, for reasons of space constraints, leave it to the reader to pursue the important contributions of Becker and Grossbard.

## 2 Cooperative Models

The common element of the models reviewed in this section is that the objective functions they postulate for the household, used to solve the household's resource allocation problem, are based on Samuelson's idea of a consensual or cooperative household agreement. The models differ in the attention they pay to the question of *how* the household reaches this agreement, and the effect this has on the form of its objective function. Furthermore, they assume that the household members pool their incomes in deriving the household budget constraint, in the sense that it is feasible to make any lump sum income transfers between individuals that may be required to reach the household optimum. This does not mean that all the models imply the so-called "Pooling Hypothesis", which suggests the inappropriateness of this terminology and the need for an alternative.

### 2.1 Samuelson's theorem

To solve the problem of how to derive transitive household preference orderings that allow derivation of well-behaved household demand functions for goods, Samuelson proposed the application of the Bergson-Samuelson social welfare function (SWF), which in this context we will call the household welfare function (HWF). If we denote the utility functions of the  $h$  household members by  $u^i(x^i)$ ,  $i = 1, \dots, h$ , with  $x^i \in \mathbf{R}_+^n$  their consumption vectors, we can define the HWF as  $H(u^1(x^1), \dots, u^h(x^h))$ . Note that the individual utility functions are "selfish", in the sense that they represent the individual preference orderings over consumption bundles.<sup>10</sup> Aspects of household relationships such as love and caring can best be thought of as entering into the determination of the HWF.

To understand what is implied by the translation of this concept from general equilibrium welfare economics to the analysis of the household, it is useful to spend a little more time on the meaning of the SWF than Samuelson did in his paper.<sup>11</sup> The central idea is to abstract from the issue of exactly how the social

<sup>10</sup>Though of course they maybe extended to incorporate whatever externalities and public goods may be present within the household.

<sup>11</sup>For fuller discussion of the concept, in the context that preoccupied theoretical and applied

group, in this case the household, arrives at its complete, transitive preference ordering over levels of well-being of its members, as well as leaving open the choice of a specific function to represent this preference ordering. Whatever this process may be, it is assumed that the outcome can be represented as a function possessing a minimum of three basic properties:

- *the Pareto property*:  $\partial H/\partial u^i \equiv H_i > 0$ , it is strictly increasing in the well-being of each individual;
- *quasi-concavity*: given two utility profiles  $[u^i(\cdot)]_{i=1}^h$  and  $[\hat{u}^i(\cdot)]_{i=1}^h$  such that  $H(u^1(\cdot), \dots, u^h(\cdot)) \leq H(\hat{u}^1(\cdot), \dots, \hat{u}^h(\cdot))$ , and a third utility profile  $[\bar{u}^i(\cdot)]_{i=1}^h = \lambda[u^i(\cdot)]_{i=1}^h + (1 - \lambda)[\hat{u}^i(\cdot)]_{i=1}^h$ , for  $\lambda \in [0, 1]$ , we have that  $H(\bar{u}^1(\cdot), \dots, \bar{u}^h(\cdot)) \geq H(u^1(\cdot), \dots, u^h(\cdot))$ . Loosely, household indifference surfaces in the space of individual utilities are linear or strictly convex to the origin;
- *differentiability* to any required order at all points in the domain of the function.

The first of these properties seems reasonable for any household. As far as quasi-concavity is concerned, the welfare economics literature takes as limiting cases the weighted utilitarian case,  $H = \sum \alpha_i u^i(x^i)$ ,  $\alpha_i \geq 0$ ,  $\sum \alpha_i = 1$ , and the Rawlsian  $H = \min_{u^i} [u^i(x^i)]_{i=1}^h$ . The first displays no inequality aversion, in the sense that all profiles that give the same (weighted) sum of utilities are equally good, whatever they may imply for inequality of utility among the household members. The Rawlsian possesses infinite inequality aversion, in that the household would be seeking to maximise the utility of its worst-off member. All strictly quasi-concave functions between these extremes are possible household welfare functions, with varying degrees of inequality aversion. The third property is a useful technical assumption.<sup>12</sup>

The key idea is that the function provides an analytical device that allows the economist to derive the implications for resource allocation of any specific set of ethical or value judgements that would determine its precise form, or indeed of all sets of value judgements that possess these three properties. Its widespread use in public economics, international economics and applied welfare economics generally over the past half-century confirms its usefulness.

As we argue below, the assumption that the household chooses its resource allocation by a process of Nash bargaining is equivalent to choosing a specific form for the HWF. The assumption about the process by which the household consensus is reached is sufficient to place a specific structure on the HWF. The

---

welfare economists at the time, see Bergson (1938) and Samuelson (1947). Needless to say there has been considerable further development since then. For good surveys see Boadway and Bruce (1984) Jehle and Reny (2001) and Mas-Colell et al (1995).

<sup>12</sup>Note that the limiting case of the Rawlsian HWF possesses only the weak Pareto property  $H_i \geq 0$  and is not everywhere differentiable, and so strictly speaking is excluded by these assumptions. It can however be approximated arbitrarily closely by a function that does possess these properties.

key innovation that resulted from this, yielding a significant extension of Samuelson's approach, was, in effect, to introduce exogenous variables such as prices, wages and non-wage income, as well as other "extrahousehold environmental parameters"<sup>13</sup> (EEP's) into the HWF as conditioning variables that determine the household's ordering over the utility profiles of its members.

Suppose now that the household faces a price vector  $p \in \mathbf{R}_{++}^n$  and the household members have exogenously given individual non-wage incomes  $\mu_i$ , with  $\mu = \sum_i \mu_i$ . If the individuals pool their incomes, we can write the household's budget constraint as  $px \leq \mu$ , and Samuelson formulated the household's resource allocation problem as that of maximising  $H(\cdot)$  subject to this constraint. We summarise his discussion of this problem by the following<sup>14</sup>

**Theorem** (Samuelson, 1956): If there exists a HWF  $H(\cdot)$  with the three properties given above, the following statements hold:

(i) *Decentralisation*: if  $\hat{x}^i$  are solutions to the problem  $\max_{x^i} H(\cdot)$  s.t.  $px \leq \mu$ , there exist functions  $s_i(p, \mu)$ , with  $\sum_i s_i(p, \mu) = \mu$ , such that  $\hat{x}^i$  are also solutions to the problems  $\max_{x^i} u^i(x^i)$  s.t.  $px^i \leq s_i(p, \mu)$ ,  $i = 1, \dots, h$ . Samuelson termed the functions  $s_i(p, \mu)$  the *sharing rule*. Conversely, existence of this sharing rule implies existence of a HWF.

(ii) *Aggregation*: Let  $v^i(p, s_i)$  be the indirect utility functions derived as the value functions of the problems  $\max_{x^i} u^i(x^i)$  s.t.  $px^i \leq s_i$  for any  $s_i$ . Then the  $s_i(p, \mu)$  are the solutions to the problem<sup>15</sup>  $\max_{s_i} H(v^1(p, s_1), \dots, v^h(p, s_h))$  s.t.  $\sum_i s_i \leq \mu$ . Furthermore let  $x = \sum_i x^i$  be the household's aggregate demand vector. Then the value function in this latter problem, say  $v(p, \mu)$ , possesses all the properties of an indirect utility function, with in particular  $\nabla_p v(p, \mu) = -v_\mu x$  (Roy's Identity). We can call  $v_\mu$  the household's marginal utility of income.

Intuitively, we can interpret the HWF as formally identical to an individual utility function with weak separability in consumptions, with the quantity of each physical good consumed by each household member viewed as a separate good. Then the standard results on two-stage budgeting<sup>16</sup> apply: the household can be modelled as first allocating its total expenditure among its members, and then within each expenditure category, *i.e.* for each individual, allocating the given expenditure optimally over goods.

Note that the sharing rule functions necessarily contain as arguments prices and total non-wage income, because these are the exogenous variables in the problem that generates them. Intuitively, shifts in the household's budget constraint and corresponding utility possibility set can be expected to change the

<sup>13</sup>The terminology introduced in McElroy (1990).

<sup>14</sup>We state this theorem here in its modern form. This is meant to summarise Samuelson's lengthy discussion of sharing rules as well as his formal theorem. For proofs, see Mas-Colell et al, (1995), ch. 4. The usual context for this theorem is the economy as a whole, but of course it immediately applies to the household as a small economy. The key idea is "[The] problems of home economics are, abstractly conceived, exactly of the same logical character as the general problem of government and social welfare." (Samuelson, *Ibid.*, p 10).

<sup>15</sup>We are free to choose utility functions which make  $H(\cdot)$  here a quasiconcave function of the  $s_i$ , so that the second order conditions will be satisfied.

<sup>16</sup>See Gorman (1961) and Deaton and Muellbauer (1980).



distribution of income among household members *given* its unchanged preferences over their relative well being. Note, however, that they do not depend on the *individual* non-wage incomes: any reallocation of these that leaves the total unchanged always also leaves the share functions unchanged. This, as we shall see below, is generally in contrast to the results of bargaining models.

The aggregation aspect of the theorem is also important. This implies that the aggregate household demands for goods, the vector  $x(p, \mu)$ , can be modelled *as if* they were the demands of a single fictitious consumer with the indirect utility function  $v(p, \mu)$ . This is a complete description of preferences, and an expenditure function and a direct utility function  $u(x)$  (which we refer to as the *household utility function*) for this fictitious individual can be derived from it in the usual way. The assumptions on the HWF, combined with the form of its maximisation problem (including the pooled budget constraint) give a problem in the space of aggregate household consumption vectors,  $\max_x u(x)$  *s.t.*  $px \leq \mu$ , that is identical in form to the problem of an individual consumer, and so the resulting aggregate demands will have all the regularity properties of the demand functions derived from the model of the individual consumer. We can derive aggregate demands by maximising a household utility function subject to the aggregate budget constraint. We should however not lose sight of the fact that underlying these demands is an allocation of consumptions among household members that reflects the household's sharing rule, or equivalently, maximises the HWF.

Becker's approach is in one sense a special case of Samuelson's. On the other hand, the models of the household Becker formulates are much richer than Samuelson's general equilibrium pure exchange model and are concerned with the analysis of a far wider range of issues. Opinions can differ on the extent to which the significance of the insights Becker derives outweighs any reservations one may have about the ethical presuppositions represented by the form of his HWF, as well as about its descriptive realism.<sup>17</sup>

Samuelson also considered the approach to modelling household demand that has since become known as Gorman Aggregation.<sup>18</sup> This can be thought of as providing a solution to the problem of deriving well-behaved aggregate demand functions for a group of consumers, for example a household, without having to specify explicitly a resource allocation process within the group, other than that individuals within it receive income shares, somehow determined. Suppose each household member  $i$  has a strictly positive demand for each good  $j = 1, \dots, n$  that takes the functional form

$$x_{ij} = a_{ij}(p) + b_j(p)s_i \tag{1}$$

where  $s_i$  is  $i$ 's total expenditure on goods and  $a_{ij}(p)$  and  $b_j(p)$  are given functions. Then  $\sum_i x_{ij}$ , the aggregate household demand for each good  $j$ , is a function only of the price vector and aggregate household income  $\mu = \sum_i s_i$ . This is

---

<sup>17</sup>We have had students who maintain that Becker's HWF is fully descriptive of the households they have grown up in. For a good overall assessment see Pollak (2003).

<sup>18</sup>See Samuelson (1956), Gorman (1959) as well as Deaton and Muellbauer (1980).

because, when all demands are strictly positive, a reallocation of this aggregate income among household members leaves aggregate demand unchanged, since the coefficients  $b_j(p)$  are the same for all  $i$ .

Note however that if a good, for example one person's leisure, is always consumed in a positive amount by only that person, and in zero quantity by all others, then its total household demand is a function only of that individual's income, and not of aggregate household income. A pure redistribution of total income that changed one individual's income share would change the demand for her leisure.

The demands in (1) are generated by a utility maximisation process if and only if each household member has the Gorman polar form of indirect utility function

$$v^i(p, s_i) = \alpha_i(p) + \beta(p)s_i \quad (2)$$

Thus one could argue that placing these very strong restrictions on preferences would allow an econometrician to work with aggregate household demand functions without having to construct a household model. Samuelson dismissed this approach as much too restrictive. Moreover, if demands are generated by individual utility maximisation given some distribution of total expenditures, then the resulting household resource allocation must be Pareto efficient, as a specific implication of the fact that all consumers face the same price ratios of all pairs of goods. So even assuming the Gorman polar form implies some restriction on the household decision process.

Samuelson clearly felt confident that he had given a satisfactory solution to the problem he had posed. Why was this solution apparently rejected by most of the subsequent contributors to the literature? One reason put forward<sup>19</sup> was that the process of household formation cannot be analysed by a household utility function approach. This is not correct. Prior to formation of a household, the potential members must be able to predict the allocations they will obtain within the household and compare them to the next best alternative. They must do this by solving the HWF-maximisation problem. This is qualitatively no different to what is assumed in models that use Nash bargaining, *i.e.* a specific HWF, to analyse household formation or the marriage decision, and much more general. Another<sup>20</sup> was that Samuelson's approach misses the elements of conflict as well as cooperation that characterise household decision taking. This is also not correct (though it is true of Becker's special case of the HWF). Samuelson's formulation is quite general as to the processes that generate the HWF, and these may well involve conflict. The HWF simply represents the outcome of its resolution.

McElroy and Horney, in motivating their Nash bargaining approach, argue that Samuelson's solution implies that the outcomes of family decisions are "empirically indistinguishable from those of constrained utility maximisation" in the individual consumer model. This is the key point, and we now turn to it.

<sup>19</sup>See for example Nerlove (1974).

<sup>20</sup>See Manser and Brown (1980).

## 2.2 Symmetry, anonymity and pooling

### 2.2.1 Symmetry

The aggregation part of Samuelson's Theorem says that the household can be analysed *as if* it generates its aggregate demands  $x(p, \mu)$  by solving the problem  $\max_x u(x)$  s.t.  $px \leq \mu$ , where  $u(\cdot)$  has all the properties of an individual utility function. This implies that the aggregate household demands will have the same properties as those of individual demands in the standard consumer model, and in particular that the Slutsky matrix of compensated demand derivatives, where "compensation" in this case must be defined as holding the value of the HWF constant, will be symmetric and negative semidefinite.

For later reference, it is useful to illustrate Samuelson's Theorem in terms of a simple two-person labour supply model. Define the first two goods as the respective leisure consumptions of the two household members, with therefore the first two prices as their wage rates  $w_i$ ,  $i = 1, 2$ . The remaining  $n-2$  goods are consumption goods, and let us assume that their prices remain constant relative to each other, so that we can apply Hicks's Composite Commodity Theorem and write the utility functions as  $u^i(x_i, l_i)$ , where  $x$  is the composite consumption commodity and  $l$  is leisure. The pooled household budget constraint is  $\sum_i (w_i l_i + x_i) \leq \sum_i (w_i + \mu_i)$ , where we have normalised wages and prices so that the composite consumption commodity has a price of 1, and total time endowment of each household member is normalised at 1, so  $(1 - l_i)$  is that individual's labour supply.<sup>21</sup>

Given the sharing rule interpretation of the household equilibrium, each individual solves  $\max_{x_i, l_i} u^i(x_i, l_i)$  s.t.  $w_i l_i + x_i \leq s_i$  for given  $s_i$  to yield the indirect utility function  $v^i(w_i, s_i)$  with, from Roy's Identity,

$$\frac{\partial v^i}{\partial w_i} = -\frac{\partial v^i}{\partial s_i} l_i \quad (3)$$

The solution to the household's distribution problem

$$\max_{s_i} H(v^1(w_1, s_1), v^2(w_2, s_2)) \text{ s.t. } \sum_i s_i \leq \sum_i (w_i + \mu_i)$$

has first order conditions

$$H_i \frac{\partial v^i}{\partial s_i} = \lambda \quad i = 1, 2 \quad (4)$$

The value function of this problem  $v(w_1, w_2, \mu_1, \mu_2)$  has, by the Envelope Theorem, the derivatives

$$\frac{\partial v}{\partial w_i} = H_i \frac{\partial v^i}{\partial w_i} + \lambda \quad i = 1, 2 \quad (5)$$

$$\frac{\partial v}{\partial \mu_i} = \lambda \quad (6)$$

---

<sup>21</sup>Given the assumption that each  $i$  has a positive market labour supply. Where one of the household members, for example a child, *necessarily* has zero market labour supplies, we can simply set the corresponding  $l_i = 1$ . To deal with cases in which an individual may choose between zero and positive labour supply, we simply have to extend the formulation of the problem to include the possibility of corner solutions.

Then substitution gives

$$\frac{\partial v}{\partial w_i} = \lambda(1 - l_i) \quad i = 1, 2 \quad (7)$$

as should be the case if  $v(\cdot)$  is an indirect utility function for the fictitious aggregate household member. The direct utility function corresponding to this indirect utility function can be written as  $u(x, l_1, l_2)$  with  $x = \sum_i x_i$ .

Now, since  $u(x, l_1, l_2)$  has the properties of an individual utility function, maximising it subject to the budget constraint  $\sum_i w_i l_i + x \leq \sum_i (w_i + \mu_i)$  yields leisure demand functions  $l_i(w_1, w_2, \mu)$  to which the standard results of the individual labour supply model apply. In particular, we have symmetry of the compensated leisure demands for some given constant utility level<sup>22</sup>  $u^0$

$$\left[ \frac{\partial l_1}{\partial w_2} \right]_{u=u^0} = \left[ \frac{\partial l_2}{\partial w_1} \right]_{u=u^0} \quad (8)$$

These are in this model the negatives of the compensated labour supply derivatives.

In the literature it is claimed that the symmetry of compensated labour supply derivatives of husbands and wives is rejected by the data, which in turn implies rejection of this implication of Samuelson's Theorem. As we argue at some length in Apps and Rees (2007a), because they are based on the simple division of time between market work and leisure and do not take account of household production, these studies do not in fact provide a satisfactory empirical test of Samuelson's model. Nevertheless, we doubt that symmetry would hold in an empirical analysis that did correctly specify the model.

### 2.2.2 Anonymity and pooling

The formulation of the budget constraint, with aggregate household expenditure constrained by aggregate household income, rests, as we pointed out earlier, on the assumption that members of the household in effect pool their incomes, in the sense that they are prepared implicitly or explicitly to make whatever transfers of income between themselves are necessary to achieve the household optimum. The individual shares in the total value of consumption are derived from the household optimisation, rather than being fixed *a priori* by some sharing rule *not* derived in this way.<sup>23</sup> In the interests of clarity, we believe that the term "income pooling" should be reserved for this kind of formulation of the budget constraint in a household model.

As we just saw, in Samuelson's model consumption and leisure demands, labour supplies and the household sharing rule are functions only of aggregate household income. This implies in turn that the effect on these demands, supplies and income shares of an increase in an individual's non-wage income does not depend on who that individual is - an additional \$1 of income has the same

<sup>22</sup>That is, some given value of the HWF.

<sup>23</sup>What Samuelson called a "shibboleth sharing rule".

effect regardless of to whom it accrues. This has come to be known in the literature as the "Pooling Hypothesis", since it appears to be the result of the assumption that income is pooled to obtain the budget constraint.

This terminology is unfortunate, because there are several models, including the bargaining models that we look at in the next section, which have household budget constraints that assume income pooling, but in which the effects of a change in individual income do depend on the identity of that individual, at least for some types of income change. Moreover, in non-cooperative models of household public good provision, where income pooling is not assumed, if both individuals supply positive amounts of the public good at the Nash equilibrium, then a redistribution of income between the two individuals has no effect, and only total income matters. In other words, income pooling as such is neither necessary nor sufficient for the "Pooling Hypothesis" to be a result of a model, and whether it is or is not depends crucially on the rest of the structure of the model, as well as on what type of income change is being considered. It seems to us therefore that it would be much better to call this effect the "Anonymity Hypothesis", since it is really saying that the identity of the income recipient does not matter. It is this result of a model, and not income pooling *per se*, that would be rejected when the data seem to show that the effects of an income change depend on precisely whose income has in fact changed. Samuelson's model does imply the Anonymity Hypothesis. There is now quite a substantial body of empirical work that concludes that this hypothesis is rejected by the data.<sup>24</sup>

This discussion suggests that Samuelson's formulation was *too* successful in solving the problem of household decision taking - it produced a model with identical results to the individual model, as McElroy and Horney pointed out. For them and other proponents of bargaining models, the answer was to base the model on the *process* by which a household consensus is achieved. Before considering the Nash bargaining approach in some depth, we look at three generalisations of Samuelson's model: the incorporation of household production; the inclusion of children; and the generalisation of the household welfare function.

## 2.3 Generalising Samuelson

### 2.3.1 Household production

The papers by Apps (1981), (1982), and Apps and Rees (1988), (1997), (1999), (2001), fall within the Samuelson tradition, in that they were concerned with extending existing results, primarily in labour and public economics rather than demand theory in general, to household models. They generalise Samuelson in that they include household production<sup>25</sup> as an essential component of the models. Indeed, for the issues with which these papers were concerned, it was seen to be essential not only to model the multi-person household, but also to

---

<sup>24</sup>For the early contributions see Schultz (1990) and Thomas (1990).

<sup>25</sup>Drawing on Gronau (1986).

incorporate household production. There was no point in doing one without the other.

Apps (1981) used a two-sector general equilibrium model to analyse the incidence of income taxation in an economy in which differences in market wage rates associated with non-economic characteristics, in particular gender, are due not to innate differences in productivity,<sup>26</sup> as in the standard optimal tax models, but rather to labour market discrimination, which "crowds" women into low-wage occupations. In this model, men supply labour in a high-wage sector to produce a market good, and women work in a low-wage sector to produce a second market good, as well as working within the household to produce a domestic good. The implicit price a man pays for this domestic good cannot be less than the opportunity cost of his wife's time, valued at the wage women receive in the low-wage market sector, since women are assumed free to move between market and household work. Women are not however allowed access to the high-wage sector. There is a perfectly competitive marriage market so that, in equilibrium, no woman could demand more than the going market price for the household good. There are no transfers or income pooling within the household: each spouse consumes goods to a value given by the income from his or her own labour supply plus individual non-wage income. Goods prices and men's and women's wage rates are determined in a competitive general equilibrium by equalities of demands and supplies of all goods and labour.

Anonymity does not hold in this model – an increment of income to a man has in general different effects to those of an income increment to a woman. Moreover, symmetry of compensated labour supply derivatives with respect to each other's wage rates also does not hold. For example, an increase in the male net wage (caused, say, by a reduction in the tax rate) affects the labour supply of a woman not through a pooled budget constraint, but rather through a set of comparative statics effects on the price of the market good produced by men and the demands for the household and market goods produced by women.

Although it does not involve bargaining or threat points, the model highlights the important idea that the utility level partners can achieve within the household depends crucially on their outside opportunities, in this case conditions on their respective labour markets. Labour market discrimination that reduces the market wage of women will also reduce what they can obtain within the household. It embodies a form of the "separate spheres" idea of Lundberg and Pollak (1993), but applied to household production rather than purchases of household public goods.

The later papers cited above generalise this early model by basing the solution for the household resource allocation on Samuelson's household consensus, thus allowing for the possibility of intrahousehold transfers and imperfect markets. They note, for example in Apps and Rees (1988), that in characterising a household equilibrium, it is enough simply to assume that the household seeks to achieve Pareto efficiency, since this is a common element in all HWF's. However,

---

<sup>26</sup>At least, not initially, though in the long run the effect of labour market discrimination is to create differences in the accumulation of human capital and therefore in individual productivities. These effects are modelled endogenously.

they also go on to point out (p. 365)

The general characterisation of the household equilibrium by the [Pareto] efficiency conditions [...] allows us to bring out in a general way the significance of household distribution effects, but does not in itself allow a precise analysis of these. For this it is necessary to specify how the household distributes income among its members, and this requires a more substantive hypothesis than that the final allocation is Pareto efficient.

Thus, in carrying out the comparative statics of the models, they assume the existence of a HWF or, equivalently, a sharing rule.

The papers cited above use a variety of models, each designed to fit the problem at hand. Apps and Rees (1988) is concerned with the implications for tax policy of the relationship between the distributional preferences of the policy-maker or "planner" and those of the household. It shows how intra-household distributional terms will enter expressions for optimal taxation and tax reform, and also derives conditions under which they can be ignored - the "non-dissonance" case. Of course, the standard optimal taxation and tax reform literature does indeed ignore these terms, because it is based on an individual model.<sup>27</sup> But in addition to this, one may want to carry out tax analysis that requires a household model but focusses on inter- rather than intra-household distributional effects,<sup>28</sup> and for this it is useful to know what is being assumed in suppressing the latter. The later papers deal with issues such as the problem of "retrievability" or identifiability of the sharing rule in the presence of household production, the choice of the individual *vs.* the couple as the tax unit in an income tax system, the measurement of child costs, and the choice of tax/subsidy policies to solve the problem of fertility decline. Rather than going through the variety of models, we present here a general model,<sup>29</sup> which suggests in particular how the model with household production can be extended to include children.

### 2.3.2 Children and child care

We would argue that in formulating models that explicitly include children, we should treat them as individuals with their own utility functions, rather than as "public goods" consumed by their parents. This approach seems to be far more consistent with the spirit of household models. The "children as household public goods" approach seems to have originated with the Nash bargaining models, which are constrained to two-person bargaining and therefore cannot handle children as individuals. In the spirit of Samuelson, we leave open the question of the extent to which children participate in the decision process of the household.

---

<sup>27</sup>See for example the seminal papers in this literature, Mirrlees (1971) and Sheshinski (1972).

<sup>28</sup>See for example Apps and Rees (2007c).

<sup>29</sup>This closely follows that in Apps and Rees (2001).

It can be argued that in the absence of children, household production takes on far less significance as an economic activity than when they are present. The arrival of children generates a very large demand for child care and it is this that makes inclusion of household production in the model absolutely essential. This in turn has important implications for female labour supply and household income, and indeed the entire economics of the household.<sup>30</sup>

As in the two-adult household considered in the previous sections, the household with children can be modelled as if it were a small economy, and the standard results of general equilibrium theory can be applied in a straightforward way. The household buys a vector  $x$  of market goods which it consumes directly, a vector  $b$  of market goods which it uses as intermediate goods in domestic production, including possibly child care, a vector  $g$  of household public goods, and it produces a vector  $y$  of household private goods. We identify domestic child care outputs explicitly as  $c_k$ ,  $k = 1, \dots, K$  where  $K \geq 1$  is the number of children in the household and  $k$  denotes an individual child. The household technology is assumed subject to non-increasing returns and no joint production, and is given by the twice-differentiable, concave production functions

$$y_j = h^j(t_f^j, t_m^j, b^j) \quad j = 1, \dots, n \quad (9)$$

$$c_k = c^k(t_f^k, t_m^k, b^k) \quad k = 1, \dots, K \quad (10)$$

where  $t_i^j$  is  $i$ 's time input into production of household good  $j$  and  $t_i^k$  is the time  $i$  spends on care of child  $k$ .  $i = f, m$  are the two adults and only they supply time to household production.<sup>31</sup> We have of course  $b = \sum_j b^j + \sum_k b^k$ .

The utility functions of adults are defined on their consumptions of the vector of market goods,  $x^i$ , the vector of household private goods,  $y^i$ , pure leisure (a scalar),  $l_i$ , and the vector of household public goods,  $g$ :

$$u_i = u^i(x^i, y^i, l_i, g) \quad i = f, m \quad (11)$$

The utility functions of children are defined on market goods,  $x^k$ , household private goods,  $y^k$ , child care outputs,  $c_k$ , and household public goods,  $g$ :

$$u_k = u^k(x^k, y^k, c_k, g) \quad k = 1, \dots, K \quad (12)$$

All utility functions are strictly quasi-concave and increasing, and at least twice continuously differentiable.

Each adult divides his or her time between general household production, child care, market labour supply  $z_i$  and pure leisure. Thus we have the time constraints:

$$t_i + l_i + z_i = 1 \quad i = f, m \quad (13)$$

---

<sup>30</sup>See Apps and Rees (2005), where this view is supported by data from a number of countries.

<sup>31</sup>If domestic help, including child care, is bought on the market then this is one of the market goods with the corresponding wage rate as its price. The essential assumption is that children do not contribute to domestic production.



where  $t_i = \sum_j t_i^j + \sum_k t_i^k$  is the total time  $i$  spends in household goods production and child care. It is assumed that all of a child's time is pure leisure<sup>32</sup>.

To complete the model we specify the household budget constraint. Corresponding to the vector  $x$  is the given price vector  $p$ , to the vector  $b$  the given price vector  $q$ , and to the vector  $g$  the given price vector  $r$ . Some market goods may of course be both final consumption and intermediate goods, in which case the corresponding elements of  $p$  and  $q$  will be identical. Given the market wage rates  $w_i$  and non-wage incomes  $\mu_i$ , the budget constraint is

$$px + qb + rg \leq \sum_{i=f,m} (w_i z_i + \mu_i) \quad (14)$$

which, eliminating the time constraints, can be equivalently written as

$$px + qb + rg + \sum_{i=f,m} w_i(t_i + l_i) \leq \sum_{i=f,m} (w_i + \mu_i) \quad (15)$$

The household acts as if it maximises  $H(\cdot)$  subject to the constraints on the domestic technology and expenditure given in (9), (10) and (15). Given our assumptions, the first-order conditions for this problem are both necessary and sufficient for an optimum and are thoroughly familiar from standard general equilibrium theory.<sup>33</sup> Assuming all choice variables are strictly positive at the optimum we have:

- marginal rates of substitution between any pair of goods are equalised across household members;
- inputs are allocated so as to equalise their marginal value products in all uses;
- where both adults supply market labour,  $i$ 's market wage measures the marginal value product of time  $i$  spends in any household production or child care activity and also the marginal value of leisure;<sup>34</sup>
- the Samuelson conditions characterise the consumption of household public goods, and in particular the optimal output of such a good equates its price to a weighted sum of its marginal utilities to the household members, where the weights are the values of the partial derivatives  $H_i^*$ ,  $H_k^*$  at the optimal solution.<sup>35</sup>

---

<sup>32</sup>In both developing and developed countries children as usually defined, *i.e.* persons under the age of 16, may in fact supply both market and household labour. This can be handled formally by increasing the number of "adults" in the household.

<sup>33</sup>As long as we are only concerned with characterising the household equilibrium, we could just as well have assumed the household simply finds a Pareto efficient equilibrium, rather than maximising a HWF. Comparative statics analysis however requires a HWF.

<sup>34</sup>Where one adult supplies no market labour, the marginal value product of his/her time in household production is equal to the marginal value of leisure and is equal to or greater than the market wage.

<sup>35</sup>That is, an individual's weight in the household resource allocation, as measured by this derivative, determines the extent to which his or her preferences for the household public good influence its total output.

- the amount of care given to child  $k$  equates the marginal cost of this care to its weighted marginal value product, where the weight is again the equilibrium value  $H_k^*$ .

We can solve for the vectors of demands, outputs of household goods and child care, and labour supplies as functions of prices, wage rates and non-wage incomes. All these functions are in principle observable, though existing datasets provide only very incomplete information that does not allow them to be estimated at the level of generality of this model. Apps and Rees (2001) provide an example of the sorts of assumptions that have to be made to estimate this model with existing datasets.

An objection to this approach might be: What is the point of having a model that cannot be estimated? Our reply would be that model-building should not be constrained by what happens to be the currently available datasets.<sup>36</sup> Model-building unconstrained by data availability should lead to an awareness of the kinds of data that need to be collected. It also helps us to see the way in which empirical models make assumptions that amount to making up the missing data.

This model helps to clarify exactly what we might mean by "the costs of children". The cost of a child is simply the value of the bundle of goods - market, household and public - plus the cost of child care, that the child receives at the equilibrium household allocation, *i.e.* as the value of the resources the household chooses to allocate to her.

On the given assumptions, there can be associated with the equilibrium allocation a set of implicit prices for the domestically produced goods, including child care, given by their marginal costs at the equilibrium. Let the vector of implicit prices of household private goods be denoted by  $\pi^*$  and let  $\tau_k^*$  denote the equilibrium household price of child care for the  $k$ 'th child. Then the *full consumption cost of child  $k$*  is given by

$$C_k^* = px^{k*} + \pi^* y^{k*} + \tau_k^* c_k^* + r[g^* - g_{-k}^*] \quad (16)$$

that is, by the cost of the bundle of market, public and domestic private goods and child care that the child consumes at the equilibrium household allocation (denoted by asterisks). Here  $r[g^* - g_{-k}^*]$  is the cost associated with the increased demand for household public goods that child  $k$  imposes, with  $g_{-k}^*$  denoting the vector of household public goods that would optimally be provided in the absence of child  $k$ .<sup>37</sup> Our contention then is that it is this measure of the cost of a child that should ideally be estimated in child cost studies. More usually, such studies construct "adult equivalence scales" for children based either upon minimal physiological needs, or estimates of the increase in income required to hold parental utility constant, given their expenditure on consumption goods for the children, *i.e.*  $\sum_k px^{k*}$ .

<sup>36</sup>For example, macroeconomics would not exist if that principle had been followed. National income accounting came after Keynes.

<sup>37</sup>For example, if a shared good such as housing space is a household public good, then its consumption is likely to increase with the number of children. If not, this term is just zero.

### 2.3.3 Generalising the Household Welfare Function

In Section 2.4.1 we discuss the Nash bargaining approach in some detail and argue there that the key general idea it contains is that the household's preference ordering over the utility profiles of its members depends on their wage rates and non-wage incomes. It is the absence of these in Samuelson's HWF that leads it to give the anonymity and symmetry results. Returning, for simplicity of notation, to the two-person household model, and writing the HWF now<sup>38</sup> as

$$H = H(u^1(x_1, l_1), u^2(x_2, l_2); w_1, \mu_1, w_2, \mu_2) \quad (17)$$

allows us to take over the idea that the household preference ordering depends on these exogenous variables, without constraining us to accept the specific rationale - their embodiment in formal threat points - that makes their presence natural in the Nash bargaining model. We call this the *generalised household welfare function* (GHWF).

We then define the *generalised Samuelson model* by the problem<sup>39</sup>

$$\max H(u^1(x_1, l_1), u^2(x_2, l_2); w_1, \mu_1, w_2, \mu_2) \quad (18)$$

$$s.t. \quad \sum_i (w_i l_i + x_i) \leq \sum_i (w_i + \mu_i) + t \quad (19)$$

where  $t$  is thought of as a transfer to the household that is not assigned to any one individual, and does not affect the household's preferences over utility profiles.

It is perhaps pointless to speculate on whether Samuelson would have approved of this step at the time he wrote his paper on social indifference curves. On the one hand, he clearly conceived of social welfare functions in very general terms indeed. Thus in his discussion of social welfare functions in the context of the economy as a whole<sup>40</sup> he writes

Without inquiring into its origins, we take as a starting point for our discussion a function of all the economic magnitudes of a system which is supposed to characterize some ethical belief.....Any possible opinion is permissible... We only require that the belief is such as to admit of an unequivocal answer as to whether one configuration of the economic system is "better" or "worse" than any other, or

---

<sup>38</sup>We could also adopt McElroy's suggestion of including a vector of EEP's that influence this preference ordering though not the household budget constraint. Examples of such variables typically cited are: indicators of gender discrimination in labour markets; the ratio of marriagable men to women, the so-called sex ratio; taxes and transfers that change with the individual's marital status; and the nature of divorce laws. These variables will often be specific to the precise context of the analysis. For example, analysis of household decisions in a developing country may include dowries and bride prices among these variables.

<sup>39</sup>In the general notation of the consumption good model introduced initially, the problem is  $\max_{x^i} H(u^1(x^1), \dots, u^h(x^h); p, \mu_1, \dots, \mu_h)$   
 $s.t. \quad p \sum_i x^i \leq \sum_i \mu_i + t$

<sup>40</sup>Samuelson (1947) pp. 221, 222.

"indifferent", and that these relationships are transitive....numerous individuals find it of interest to specialize the form of [the SWF]...For one thing, prices are not usually included in the welfare function itself, except very indirectly through the effects of different prices and wages upon the quantities of consumption, work etc.

Applied to the household, we could certainly accept that the opinion that the household's preferences over utility profiles of its members should depend on their wages and non-wage incomes is a permissible ethical belief, in the sense that we may well expect household members to agree that it should be the case.<sup>41</sup> Then we can model their objective function in a way that corresponds to this belief. Samuelson's exclusion of prices from the *social* welfare function is no doubt due to the fact that in the case of the economy as a whole, these are endogenous variables. Indeed, in applications of the social welfare function, for example in optimal tax theory, it is usual to write it as a function of individual utilities alone. However, in the case of the household, wages and non-wage incomes are exogenous, and so an objection on grounds of endogeneity would not apply. We regard the GHWF as the appropriate extension of the idea of a social welfare function to the case of the household.

In Apps and Rees (2007b) we suggest reasonable general restrictions on the way in which the variables  $w_i, \mu_i$   $i = 1, 2$  should affect the preference ordering over utility profiles. Increases in  $w_1$  and  $\mu_1$  should increase the relative weight the household gives to 1's utility at any point, and increases in  $w_2$  and  $\mu_2$  should reduce it. It is shown that anonymity will not in general hold in this model, and, as we should expect from Pollak's (1977) analysis of price-dependent preferences, the Slutsky matrix is not in general symmetric and negative semidefinite. Furthermore, Slutsky equations for the sharing rule can be derived, which provide testable implications of the model. These results generalise those obtained from specific cooperative models, such as the bargaining models and the collective model, which can be regarded as being based on specific forms of the GHWF. We now turn to these.

## 2.4 Nash bargaining and the collective model

### 2.4.1 Nash bargaining

The two papers that introduced Nash bargaining models into the household economics literature illustrate nicely the two main concerns of this literature pointed out earlier in the Introduction. The paper by Manser and Brown (1980) is concerned with analysing the marriage decision, and takes as its point of departure the work by Becker on the theory of marriage.<sup>42</sup> The paper stresses the importance of modelling the household as consisting of individuals with their

---

<sup>41</sup>Possibly because they define the values of outside options, but other reasons are also possible. For example, they may be taken as an indicator of the contribution an individual makes to the household budget, and therefore of his or her entitlement to a share in household full income.

<sup>42</sup>See Becker (1973), (1974).

own utility functions, and of retaining the elements of conflict as well as cooperation that are involved in the household resource allocation decision. Though the paper is correct in arguing that Becker's formulation of the household objective function, as the utility function of the "head of the household", suppresses these aspects of household decision taking, they are wrong in dismissing Samuelson's approach as if it also did this. The HWF retains the separate utility functions of individuals, is quite general about the types of considerations that determine the trade-offs between their utilities - by no means ruling out conflict - and, in the equivalent concept of the sharing rule, provides a flexible and convenient way of analysing the intra-household income distribution.

The second paper, by McElroy and Horney (1981), is on the other hand concerned with the issue Samuelson saw as central: what can be said about the properties of household demands when we recognise that the household consists of more than one individual? The paper shows that by modelling the household resource allocation process as a cooperative Nash bargaining game, well-defined demand functions can be derived on which testable restrictions can be placed, differing in important ways from those derived from the standard demand theory (and therefore from Samuelson's model as set out in Section 2.1). The paper provides a detailed analysis of the structure of the household demand functions implied by the Nash bargaining hypothesis.

The two papers differ in the details of the formulation of their models. Manser and Brown emphasise the existence of household public goods and love and caring in explaining why marriage generates a surplus of utility over that which is obtained when the two individuals live separately. The existence of this surplus explains why marriage (or more generally formation of a household) takes place, while the analytical problem, solved by Nash bargaining, is to determine how this surplus is shared. In McElroy and Horney's paper, goods may also be private (*i.e.* consumption by one individual reduces the amount available for the other), love plays no explicit role (though the utility of each individual depends also on the consumptions of the other), and the existence of a surplus from marriage is simply assumed.

The existence of a surplus to be bargained over is one necessary component of a Nash bargaining model, the specification of the "threat points", the utilities that will be achieved by the respective parties if no agreement is reached, is the other. Both papers take the maximum (expected) utility each individual could achieve outside the household in question as the threat point. Since McElroy and Horney are concerned with an ongoing household, this has come to be called a "divorce threat point", though in the case of Manser and Brown, since they are analysing conditions under which a not-yet-existing household will be formed, this is a misnomer. Their threat point is the next best expected utility an individual can obtain on the marriage market.

We formalise the bargaining approach in a way that emphasises its application to labour supply decisions, although this was not in fact the approach of either of the two original papers. Let  $v^i(w_i, \mu_i)$  denote the highest expected utility that individual  $i = 1, 2$  could obtain outside the household in question. These are exogenously given reservation constraints. They are obtained

by solving the problem  $\max u^i(x_i, l_i) \quad s.t. \quad w_i l_i + x_i \leq w_i + \mu_i$ , where  $l_i$  is again  $i$ 's leisure consumption and  $x_i$  consumption of a household public good<sup>43</sup> with price normalised at 1. The two-person household is then assumed to solve the problem

$$\max H^N = [u^1(x, l_1) - v^1(w_1, \mu_1)][u^2(x, l_2) - v^2(w_2, \mu_2)] \quad (20)$$

$$s.t. \quad \sum_i w_i l_i + x \leq \sum_i (w_i + \mu_i) \quad (21)$$

Note that in this model, as in Samuelson's, the budget constraint implies income pooling, the possibility of unrestricted lump sum transfers between the individuals. Leisure and consumption demands of the household are now  $l_i(w_1, \mu_1, w_2, \mu_2)$ ,  $x(w_1, \mu_1, w_2, \mu_2)$ . The household surplus is generated because with the same leisure consumptions each member can have more of the household public good than when living separately. The demand functions have the "adding-up" and zero degree homogeneity<sup>44</sup> properties of standard Marshallian demand functions, but the properties of anonymity and symmetry of compensated leisure demands discussed in the previous section no longer hold in general, as McElroy and Horney show.

Note that there is a specific form of GHWF in this problem, for, after all, what is the Nash product  $H^N$ , the maximand in the Nash bargaining problem, but a representation of the household's preference ordering over the individual utilities? As with the Stone-Geary individual utility function, it is a type of Cobb-Douglas function (defined on utility pairs rather than goods) on a restricted coordinate space<sup>45</sup> with origin at  $(v^1, v^2)$  rather than  $(0, 0)$ . It is easy to show<sup>46</sup> that it possesses the general properties of a GHWF mentioned in the previous section, though not for all types of redistribution of non-wage incomes.

Lundberg and Pollak (1993) pointed out that the model has a problem, in that it does predict anonymity with respect to individual non-wage income changes that leave threat points unchanged. Thus suppose, in the event of divorce, individual 1 would receive a state transfer  $t$ , whereas, while married, individual 2 receives this transfer. Now consider a policy change that switches payment of  $t$  from 2 to 1 also while they are married. This leads to no change in the threat points, and also no change in total income, the right hand side of the budget constraint, so there would be no effect on the Nash bargaining equilibrium. Against this, Lundberg and Pollak (1993) argued that it might be expected that such a change would increase 1's influence over the household

---

<sup>43</sup>Taking consumption  $x$  as a household public rather than private good is the simplest way, in the absence of household production, to ensure a utility surplus from formation of the household. If  $x$  here were a private good, it would not be possible for both individuals to be better off in the joint household. Our own preference however is to follow Becker and show how such a surplus can be generated by improvements in the production set arising from joint rather than individual household production.

<sup>44</sup>The indirect utility functions  $v^i(\cdot)$  are invariant to equiproportionate changes in all prices and incomes, and these also leave the budget constraint unaffected.

<sup>45</sup>See also Lundberg and Pollak (1993), p.995.

<sup>46</sup>See Apps and Rees (2007b).

resource allocation, while Lundberg, Pollak and Wales (1997) later presented evidence to show that a similar policy change in the UK did in fact cause a change in household consumption patterns in the expected direction. This argument motivated their formulation of the "separate spheres bargaining model", to which we now turn.

#### 2.4.2 Nash bargaining with non-cooperative threat points

The idea underlying the separate spheres bargaining model is to base the threat points in the Nash bargaining game on separate budget constraints for the two partners in the household, in a way that ensures that anonymity will not hold, even under the kind of transfer policy change just discussed.

The first step is to introduce the idea, originally proposed by Ulph (1988), and Woolley (1988), of basing the threat points not on the utilities the partners would receive if they divorced, but rather on the utilities they achieve in a non-cooperative Nash equilibrium within the existing household. There are in any case good arguments against post-divorce utilities as threat points - divorce may simply be too drastic and costly a fall-back point in the event of failure to agree on division of the utility surplus generated by the two-person household, and so may not be a credible threat. But a Nash bargaining model must have threat points, and so an alternative is to base them on a non-cooperative equilibrium within an ongoing household.

The next step is to characterise these threat points. How would people behave if, while remaining within the household, they do not cooperate to achieve the full benefits that are feasible, because they cannot agree on how to share them? Suppose now that there are two household public goods,  $x_1$  and  $x_2$ , so that the individual utility functions are  $u^i(x_1, x_2, l_i)$ ,  $i = 1, 2$ . In a cooperative equilibrium, the individuals will jointly agree on how much of these goods to buy and consume.<sup>47</sup> In a non-cooperative game with Nash equilibrium as the solution concept (sometimes called Cournot-Nash equilibrium to distinguish it more sharply from the Nash *bargaining* equilibrium), the partners will independently choose how much of these to buy to maximise their own individual utilities, taking as given the amount bought by the other. As in standard oligopoly models, this defines a set of reaction functions, the solution of which determines the Nash equilibrium.

The papers on voluntary-contribution public goods games by Warr (1983) and Bergstrom *et. al.* (1987) show that two types of Nash equilibrium are possible in such games. Both players may choose to contribute positive amounts of each public good, in which case a key result is that a small transfer of income between them will leave the supplies of the public goods and the individual utilities of the players unaffected. In that case therefore a Nash bargaining model with its threat points defined by this non-cooperative equilibrium would, as pointed out earlier, actually predict anonymity with respect to the kinds of

---

<sup>47</sup>Note that there is no household production in this model, as is the case with all the bargaining models considered so far. Ott (1992) appears to have been the first to introduce household production into the Nash bargaining model.

policy change Lundberg and Pollak are concerned with, for exactly the same reason as before: such a transfer changes neither the threat points nor the budget constraint.

Alternatively, there may be a non-cooperative equilibrium at a corner solution, where each of the players supplies nothing of one of the public goods, free-riding on the expenditure made by the other. In this type of equilibrium, a small transfer between the players does affect the equilibrium, increasing or decreasing the supply of a public good according to whether the transfer is away from or towards the free rider for that good.

The final step therefore is to argue that households are characterized by a division of responsibilities based on "socially recognised and sanctioned gender roles", under which each partner specialises in buying just one of the public goods, so we *necessarily* have the corner solution case in the non-cooperative equilibrium. In this equilibrium, assuming partner  $i = 1, 2$  buys only the one public good  $x_i$ , each finds his or her equilibrium consumptions  $x_i^*, l_i^*$  by solving the problem

$$\max_{x_i, l_i} u^i(x_i, x_j^*, l_i) \quad s.t. \quad w_i l_i + p_i x_i \leq w_i + \mu_i \quad i, j = 1, 2, \quad i \neq j \quad (22)$$

where  $x_j^*$  is the equilibrium supply of the other's public good, and  $p_i$  is the price of  $x_i$ . Note that in this non-cooperative game there is no income pooling: the partners are constrained in their choices by their own incomes. Then the threat points in the associated Nash bargaining game are given by the indirect utilities

$$v_i^* = u^i(x_i^*, x_j^*, l_i^*) = v^i(w_1, w_2, p_1, p_2, \mu_1, \mu_2) \quad (23)$$

In general, each threat point is a function of both partners' wage rates and non-wage incomes, since the non-cooperative Nash equilibrium values  $x_i^*, l_i^*$  are determined by all these variables. The cooperative Nash bargaining solution is then found by solving

$$\max_{x_i, l_i} N^S = [u^1(x_1, x_2, l_1) - v_1^*][u^2(x_1, x_2, l_2) - v_2^*] \quad (24)$$

$$s.t. \quad \sum_i (w_i l_i + p_i x_i) \leq \sum_i (w_i + \mu_i) \quad (25)$$

Thus changes in either one of the  $\mu_i$  affect both the threat points by changing the Nash equilibrium of the non-cooperative game, and anonymity will not hold, even though the budget constraint in the cooperative game assumes pooling. Note that the solution of the Nash bargaining game is Pareto efficient, since the GHWF  $N^S$  possesses the Pareto property, and unrestricted transfers are permitted by the pooled budget constraint. On the other hand, the threat points are not Pareto efficient, because in that equilibrium each partner takes no account of the utility the other derives from his or her supply of the public good when choosing how much of that good to buy.

The exclusion of household production in this model makes it necessary to insist on a corner solution for the non-cooperative equilibrium, and this implies



assigning a very powerful role to "socially recognised and sanctioned gender roles". Can these however really be so powerful that an individual cannot *buy* both of the public goods on the market, for example increasing his or her consumption of the other's public good if not enough of it is being provided in the non-cooperative equilibrium? This of course undoes the model, since in equilibrium both contributions to each public good will be positive. Examples of gender specialisation in household provision that come to mind, for example with females supplying child care and males house maintenance, relate to specialisation in household production activities rather than expenditures. Even here it is hard to believe that only complete specialisation is feasible or permissible.

Konrad and Lommerud (1995) show in fact that the required non-neutrality of pure transfers between the individuals in a non-cooperative Nash equilibrium can be far more plausibly rationalised in a model where individuals differ in their productivities in household production, so that specialisation is based on comparative advantage rather than socially sanctioned gender roles.<sup>48</sup> Thus they show that it is not necessary to have a corner solution in the non-cooperative game in order to obtain non-anonymity with respect to transfers.

Furthermore, it is of course possible that one of the spouses supplies no time to the labour market, which after all is the case in a large proportion of households. In that case, the non-cooperative equilibrium in the separate spheres model involves a zero supply of her public good (absent non-wage income) which strengthens the argument against a corner solution in this equilibrium. Moreover, in the final Nash bargaining equilibrium she can only buy this public good with a transfer from her spouse. As with all household models that ignore household production, the role of a spouse who supplies no market labour is essentially parasitic. Thus, although we agree that non-cooperation rather than divorce on the whole provides a more plausible rationalisation of threat points, and succeeds in extending the scope of the non-anonymity result, the model certainly gains in plausibility from being based on separate spheres in household production rather than simply in expenditures.

The model of Nash bargaining with non-cooperative Nash equilibrium threat points in Chen and Woolley (2001) does not make the separate spheres assumption, and allows the possibility that both partners buy positive amounts of the public good in the threat point equilibrium. However, in this model the overall Nash equilibrium solution is still always sensitive to the non-wage incomes<sup>49</sup> of the partners, essentially because the bargaining is assumed to be over transfers of income, rather than over the allocations of expenditures to private and public consumption goods. The partners take their Nash bargained equilibrium income shares and then individually maximise their own utility in choosing their own consumption and the amount of the household public good they buy. In terms of Samuelson's Theorem, bargaining is over the sharing rule. The result of this is that, although their Nash bargaining GHWF possesses the Pareto property,

<sup>48</sup>Though of course the reasons for these differences in productivities might be related to conventional modes of upbringing.

<sup>49</sup>There is no labour supply in their model, all income is in any case exogenous.

the final household consumption allocation is *not* Pareto efficient, and this is perhaps the most striking result of the model. The reason is that in spending their incomes individually rationally, they choose their expenditures on the household public good non-cooperatively.

The question then of course arises: why do the partners not perceive that they could both be better off if they bargained directly over expenditure allocations, rather than subjecting themselves to the two-stage process of a cooperative income allocation and then a non-cooperative expenditure allocation? Second best allocations always arise out of some additional constraint on the first best allocation process. In this case, this is the constraint that bargaining must be over income shares and not consumption choices. It is hard to see however why rational households would subject themselves to such a constraint, when it leads to Pareto-inferior outcomes. They are after all free to choose what they bargain about.

In conclusion: Bargaining models impose a very specific and not especially convenient structure on the GHWF. Amongst the proponents of the approach, there seems to be no consensus on a satisfactory specification of the threat points, while the results of the models are sensitive to these. Moreover, the "ethical basis" for Nash bargaining, defined by its four axioms - the Pareto property, invariance to positive linear transformations of the utility functions, independence of irrelevant alternatives, and symmetry<sup>50</sup> - are not as a group particularly compelling as a representation of family decision taking, however intuitively appealing the idea is that some kind of explicit or implicit negotiation takes place within households. For example, there is no place for interpersonal utility comparisons in Nash bargaining, yet casual observation suggests that these are extremely prevalent in families. The requirement that just two individuals determine the household allocation is restrictive, and no justification is given for assuming that the bargaining game is cooperative, *i.e.* that binding commitments are possible. We believe that these qualifications leave a lot of room for alternative approaches to the formal modelling of household decision taking. In the rest of this paper we discuss some of these alternatives.

### 2.4.3 The collective model

The history of the collective model begins with something of a puzzle. In introducing the model, Chiappori (1988) makes the apparently unequivocal claim:

[I]t tries to derive falsifiable conditions upon household behavior from a "collective rationality" concept; however, instead of referring to some definite bargaining concept, it only makes a very weak and general assumption - namely that the household always reaches Pareto-efficient agreements. The question which is investigated through the paper is thus the following: does Pareto efficiency

---

<sup>50</sup>This says that in a symmetric game both players should receive the same utilities in the bargaining solution. A game is symmetric if in the  $(u_1, u_2)$ -coordinate plane the feasible set of utility pairs is symmetric about the 45° line and the threat point utilities lie on this line.

alone imply restrictions upon observable household behavior?

and similarly, Chiappori (1992) states

Indeed, it is only assumed that agents are either egoistic or "caring" in the Beckerian sense and that internal decision processes are cooperative, in the sense that they systematically lead to Pareto-efficient outcomes.

Now it is straightforward to characterize a given household equilibrium allocation by Pareto efficiency conditions alone, and, by the Second Theorem of Welfare Economics, to interpret this as being generated as if the household first shared its aggregate income among its members, who then maximise their individual utilities. However, the first point made by Samuelson (1956) was precisely to insist that Pareto efficiency alone is not enough to enable restrictions to be placed on the results of a comparative statics analysis of this equilibrium. Simply knowing that, following some change in prices or income, the new equilibrium is Pareto efficient, does not allow us to say where this will be in the Pareto efficient set, in relation to the previous equilibrium. Restrictions on demand functions expressing the results of this comparative statics analysis therefore cannot be derived from Pareto efficiency alone.

The puzzle is resolved later on in Chiappori (1988) and (1992), where it becomes clear that he is actually assuming the existence of a household sharing rule or, equivalently, of *some* GHWF. This becomes most explicit in Browning and Chiappori (1998), where the "household utility function" or, in the present terminology, the GHWF, is explicitly written<sup>51</sup> as

$$H^C = \alpha(w_1, w_2, \mu)u^1(x_1, l_1) + [1 - \alpha(w_1, w_2, \mu)]u^2(x_2, l_2) \quad (26)$$

Thus the collective model is characterised by a weighted utilitarian household welfare function, the weights varying with the wage rates and total household income. The phrase "assuming Pareto efficiency alone" can be interpreted as being intended to emphasise that the model does not adopt a specific Nash bargaining formulation, but abstracts, in the spirit of Samuelson, from explicit consideration of the process by which the household agreement is reached. It cannot mean however that no HWF is being assumed.

The well-known property of a weighted utilitarian SWF is that it possesses no inequality aversion: all utility pairs that yield the same total are equally good. The fact that the model developed as a reaction against Nash bargaining may explain why the choice of a GHWF with zero inequality aversion is not discussed in the literature on the collective model, even though it implies a substantive

---

<sup>51</sup>In the notation of the model we are presently using. In Chiappori (1988) and (1992) the subject of the analysis was labour supplies, and so the GHWF would be written as here. In Browning and Chiappori (1998) the analysis was of consumption expenditures and the function is written as:  $\alpha(p, \mu)u^1(x^1, x^2, X) + [1 - \alpha(p, \mu)]u^2(x^1, x^2, X)$  where  $p$  is a price vector,  $x^i$  are consumption vectors and  $X$  is a household public good.

restriction on the distributional preferences of the household.<sup>52</sup> It follows the Nash bargaining models in generalising the HWF to contain as arguments prices and income, and possibly also EEP's, though it does not distinguish between the individual non-wage incomes.<sup>53</sup>

A central issue raised in Chiappori (1988) and (1992) is that of whether the household's sharing rule  $s_i(w_1, w_2, \mu)$  can be identified by using empirical labour supply functions of individual household members, estimated on typically available datasets. The general answer is that it cannot, but there are special cases in which its *partial derivatives* can be derived from estimated labour supply functions. Thus in Chiappori (1988) and (1992) the household consists only of two individuals, each of whom supplies labour to the market, and there is no household production. Chiappori shows that in this model it is possible to identify the partial derivatives of the sharing rule, essentially because the only way in which the household non-wage income and the other partner's wage rate enter the labour supply function of each individual is through this sharing rule. Estimates of the coefficients of the partner's wage and of household non-wage income from empirical labour supply functions then provide the basis for identification of the partial derivatives of the sharing rule. Necessary for this identification however is that we have as many labour supply functions as individuals with shares in the household income. If there are non-working children, or if one of the adults supplies no labour to the market, there are not enough parameter estimates and the model is underidentified.

Chiappori (1992) places some emphasis on the importance of identification of the partial derivatives of the sharing rule for the analysis of public policy, for example income taxation. In any optimal tax or tax reform analysis based on a household model, the derivatives of individual income shares with respect to net of tax wage rates appear prominently in the conditions determining tax rates.<sup>54</sup> However, knowledge of these derivatives alone is not in general sufficient to allow solution of the conditions for the tax rates. Knowing by how much an individual's income share would change does not help us to decide on the desirability of such a change in the absence of information about the *levels* of the income shares. Marginal social utilities of income depend on levels of income as well as the income share derivatives. For example, the fact that 55 cents of a one dollar increase in income would flow to 1 and 45 cents to 2 cannot be judged to be a good or bad thing unless we know how well off relative to each other 1 and 2 are in the first place. Moreover, if, because of data limitations, it really is the case that only aggregate household non-wage income is observable, while income shares are functions of individual non-wage incomes, then we have no way of retrieving the derivatives of the sharing rule with respect to individual

---

<sup>52</sup>In the social choice literature of course, there is extensive discussion of the arguments for and against utilitarianism as a basis for the social welfare function.

<sup>53</sup>Strictly speaking therefore, the model does not make any predictions about anonymity with respect to individual non-wage income changes, since it does not identify these. Chiappori (1988) explains the choice of total non-wage income on the grounds that typical datasets do not provide information on individual non-wage incomes.

<sup>54</sup>See for example Apps and Rees (1988).

non-wage incomes.

However, as Apps and Rees (1997) show, and Chiappori (1997) acknowledges, the major limitation to the applicability of the result arises out of the fact that the model excludes household production. In the presence of household production, it is no longer sufficient to have empirical estimates of the labour supply functions, since these do not yield the leisure demand functions. Non-market time is divided between household production and leisure. It is therefore also necessary to have estimates of the individual demand functions for time spent in household production, together with data on domestic output. Even when these are available, further restrictions are required in order to identify the derivatives of the sharing rule along the lines suggested by Chiappori. One possibility is to assume constant returns to scale in household production. Apps and Rees show that this assumption is necessary to reduce the number of unknown partial derivatives to equal the number of equations assumed to be empirically estimated.<sup>55</sup>

When data on domestic output are missing, an alternative assumption that has the effect of constructing the missing data is necessary. One possibility is the assumption that household goods have perfect market substitutes, as in the farm household production model mentioned earlier (though note that in the empirical work on this model farm output data are also collected). In this case the price of the good produced by the household is exogenously given and observable. Chiappori shows that in this case his earlier results on retrievability continue to hold.

In the more general case where the household goods do not have perfect market substitutes and output data are unavailable, even with the constant returns to scale assumption the sharing rule can be identified only up to an additive function of wages, *i.e.* not even its partial derivatives can be identified, as confirmed by Chiappori. The problem is that in this case the implicit (endogenous) price of the domestic good is identified only up to a multiplicative constant. In other words it could be anything (positive). This supports our basic contention: Empirical applications of the sharing rule approach simply have to have more comprehensive datasets.

### 3 Non-cooperative Models

The distinction between cooperative and non-cooperative games hinges on the ability to make binding commitments to implement an agreed set of actions. Thus in Nash bargaining, the parties can somehow commit themselves to implement whatever actions produce the utility pair given by the Nash bargaining solution. This possibility of making binding commitments, for example in the

---

<sup>55</sup>This is shown in Proposition 4 of Apps and Rees (1997) and Proposition 2 of Chiappori (1997). Somewhat puzzlingly, Chiappori claims that the latter “generalises proposition 4 in Apps and Rees since we do not need to assume that the production function is linear.” (p.199). This must be based on a misreading, because Proposition 4 in Apps and Rees clearly states that the condition is linear *homogeneity* of the production function.

form of legally enforceable contracts, is exogenously given. In non-cooperative games no such exogenously given commitment possibilities exist, and although equilibria that are "cooperative", in the sense of being Pareto efficient, may be possible, they must be supported by the self interest of the players, and so their existence is endogenous to the game being played.

It seems clear that, descriptively speaking, household decision taking is better characterised as non-cooperative in this sense. There may be formal laws, social norms and customs that constrain individual actions within the household, but it seems fanciful to suggest that these amount to a mechanism for making complete, binding commitments. There are three possible responses to this observation.

The first is to try to rescue the Nash bargaining approach by appealing to some results in the theory of non-cooperative bargaining games.<sup>56</sup> Thus as Binmore et al (1986) show, in an infinitely repeated non-cooperative bargaining game where two players alternately make offers at fixed intervals of time, as the length of this time interval goes to zero, the solution of the game converges to that of the corresponding cooperative Nash bargaining game. Though this is a beautiful result, it does not in our view provide a realistic basis for adopting the cooperative bargaining approach to household decision taking.

The second response is to take the non-cooperative approach seriously and to model household decision taking as a game in which the solution concept is Nash equilibrium or some refinement of it. Here there are two approaches in the literature. One, already discussed in the previous section, is to draw on the theory of voluntary-contribution public goods games, modelling household consumption as a public good. The main general limitation of this is that it gives too prominent a place to household public goods in the household's consumption decisions. The second approach, which is designed for the analysis of labour supply decisions, is to derive best-response or reaction functions in these variables and solve for the Nash equilibrium. This approach was introduced by Leuthold (1968), and further developed by Ashworth and Ulph (1981), though neither paper points out explicitly that it is finding a non-cooperative Nash equilibrium of the household. A comprehensive and insightful discussion of both the theory and econometrics of these kinds of models is provided by Kapteyn and Kooreman (1990).

Leuthold takes a household consisting of two individuals with utilities defined only on total household consumption of a market good and own leisure consumption, and with a pooled budget constraint. Each independently maximises his/her own utility function, assumed to have the Stone-Geary form, subject to this budget constraint, but with the labour supply of the other, which enters into this constraint *via* the wage income variable, taken as given. This is how the strategic interdependence between the players in this game comes about. The relationships that emerge from this maximisation are the individ-

---

<sup>56</sup>The proponents of the Nash bargaining models discussed above saw no reason for such a justification. They took the ability to commit simply as given, and saw the bargaining approach as appropriately capturing the elements of conflict as well as cooperation in household decision taking.

ual reaction functions. The household equilibrium is then found by solving the simultaneous equations defined by these functions, giving the Nash equilibrium. This solution is then the basis for the comparative statics analysis of the model and its empirical estimation.

Ashworth and Ulph allow utilities to depend also on the partner's leisure consumption, the case of "caring preferences", and adopt a flexible functional form for the utility function, but given the data that are available in the typical datasets, including theirs, the cases of "selfish" and "caring" preferences are observationally equivalent. They explicitly test and reject the hypothesis that the parameters of the two utility functions are identical, which they interpret as being equivalent to rejection of the model that treats the household as if it were a single consumer. Neither paper explores the question of the allocation of total consumption between the two individuals.

Kapteyn and Kooreman argue that an important and questionable aspect of this Nash equilibrium solution is that it is not Pareto efficient. Each individual could be made strictly better off by moving from this equilibrium to a point on the Pareto frontier, but such a move cannot be supported as an equilibrium in this one-shot non-cooperative game. Thus Kapteyn and Kooreman follow Ulph (1988) and Woolley (1988) in suggesting the equilibrium as the basis for threat points or reservation values, though whereas the latter two would embed them in a Nash bargaining game, Kapteyn and Kooreman impose them as constraints on the problem of maximising a Samuelsonian HWF. The difference between the two approaches is that whereas proposing Nash bargaining implies assuming a specific cooperative game without a justification for the possibility of binding agreements, Samuelson's approach leaves the process by which the HWF is derived entirely open. Kapteyn and Kooreman go on to show that, given the datasets typically available for estimation of these types of models, they will in general be under-identified, and illustrate with a specific example how more data can improve identification.

The third and most satisfactory response to the observation that household members are engaged in a non-cooperative game is to recognise that this is in fact a repeated game, rather than the one-shot game that has been modelled in the literature discussed so far. This opens up the possibility of application of a rich body of game-theoretic literature, which so far seems only to have been exploited by Lundberg and Pollak (1994), and Basu(2006).

The intuitive idea that rational households ought to be able to do better than the one-shot non-cooperative Nash equilibrium receives support in the theory of repeated non-cooperative games, where it is shown how "cooperative", *i.e.* Pareto efficient, equilibria can be supported by threats of punishment for deviation from them by any individual player. In a sense, this could be thought of as the generalisation of the idea of a threat point. Since a potential deviant will weigh up the cost of future punishment against the immediate gain from deviation, the future must not be too heavily discounted for such threats to work, which we will assume to be the case in the following discussion. The simplest form of punishment following deviation from the agreed equilibrium would be reversion to the one-shot non-cooperative equilibrium for the rest of

the game,<sup>57</sup> but more sophisticated "carrot and stick" strategies<sup>58</sup>, in which punishments harsher than Nash reversion are imposed for a limited period, followed by return to the cooperative equilibrium, can support Pareto efficient outcomes as subgame perfect equilibria.

There are three important conceptual issues in the analysis of such games, each of which is relevant to the application of these ideas to the economics of the household. First is the issue of the finiteness of the repeated game. Since the lives of household members are finite, attempting to apply the theory developed for infinitely repeated games to the household runs into the paradox of backward induction. In the last period of the game there is no possibility of supporting cooperative behaviour by the threat of future punishment for a deviation, and so the only equilibrium is the one-shot Nash equilibrium. In the second-to-last period, everyone knows the equilibrium that will be played in the last period and so no threat can support cooperation in this period - again only the one-shot Nash equilibrium can be sustained. This argument can then be applied period-by-period right back to the first. Thus "cooperative" equilibria are not possible.

There are however a number of cases in which this problem can be circumvented. One important case is that in which there is uncertainty about the terminal period of the game: for almost all periods, there is some probability that there will be a future period. Then, cooperation may still be sustained by the threat of punishment in a possible future. Also important is the case in which there is some probability that any given household member is simply of the cooperative type,<sup>59</sup> because, say, she has internalised social and cultural norms that would make one play the game cooperatively rather than selfishly (individually rationally). If all players are of this type then there is no problem, but even if one is not, she may find it individually rational to behave for at least part of the game *as if* she were, in order to maintain a reputation for being a cooperative type. In both these types of games, we would expect to see the breakdown of cooperative behaviour in the later stages of the game. A third example is the case in which there are multiple Nash equilibria in the one-shot game, which seems perfectly possible for household models in general.<sup>60</sup> Then the backward induction chain could be broken, for example by supporting cooperation in the next-to-last period by the threat of playing the worst possible Nash equilibrium instead of the best possible Nash equilibrium in the last period. This would work if the discounted value of the difference in payoffs between the two equilibria exceeds the immediate gain from deviation.

The second issue concerns renegotiation proofness.<sup>61</sup> Punishment for devia-

---

<sup>57</sup>See Friedman (1977) who has extensively analysed such strategies. Note that these games, known as supergames, take the form of infinitely repeated plays of the same one-shot game.

<sup>58</sup>See Abreu (1986).

<sup>59</sup>For analysis of such games see Kreps et al (1982).

<sup>60</sup>Though models using non-cooperative equilibria, such as Chen and Woolley (2001), Leuthold (1968), and Lundberg and Pollak (1993) usually exclude this possibility by using specific functional forms for the utility functions. For the game theory in this case see Benoit and Krishna (1985).

<sup>61</sup>See for example Farrell and Maskin (1989).



tion could be costly to the punisher, and, given that a deviation has occurred, it might appear rational to "kiss and make up", "forgive and forget", and not carry out the self-lacerating punishment. But anticipation of this *ex ante* would then make the cooperative solution unsustainable. Thus attention would have to be restricted to cooperative equilibria that can be supported by punishments that would credibly be carried out, for example because they give the punisher, for the duration of the punishment, a higher payoff than she obtains at the cooperative equilibrium that is to be supported.

The third issue concerns completeness of information. Information in a game is complete if each player knows at each point in time the actions chosen by all players at all previous points in time. Suppose however that the previous choice of action by the other players is not observable by any one player, and, because of some kind of underlying uncertainty, cannot be inferred from the outcomes of the game.<sup>62</sup> When a particular set of payoffs is realised, there is a positive probability that it was generated by actions that deviated from the cooperative agreement. The optimal punishment strategies in this case take the form of setting a critical level of payoff and carrying out the punishment if the actual payoff deviates from this, where this critical level reflects a choice of probability that it was not due to chance.

As a simple example: suppose that if he studies appropriately hard for a math exam, your son should obtain an A with probability 0.5, a B with probability 0.4, and a C with probability 0.1. You cannot observe how hard he studies, so promise him a ticket to the next game of his favourite football team if he gets an A, nothing if he gets a B, and that he's grounded for a week and has to work on his math if he gets a C. You know that if you do this, he will certainly study hard. The slightly paradoxical thing about this is that if he does get a C, you have to carry out the punishment, even though you know that he was simply unlucky and did work hard. If you take him to the football game anyway, since you yourself get pleasure from doing that, he has no incentive to work hard in future, and if he anticipates your doing this, he has no incentive to study hard now.

The theory of finitely-repeated non-cooperative games offers a rich set of ideas for application to the economics of the household. It suggests that under some circumstances a household may achieve Pareto efficient outcomes, though this cannot be taken for granted, and in some contexts it may be interesting and important to analyse this explicitly within the framework of a properly formulated non-cooperative game. We pursue this point in the next section.

Finally, it should be noted that typically, the set of equilibria that may be supported by threat strategies in a repeated game can be quite large, so that there is still a problem of equilibrium selection. One proposal might be that the equilibrium could be selected by Nash bargaining over the set of utility pairs sustainable as equilibria by threats of punishment for deviation, but for reasons given in the previous section we regard this as too special and constricting.

---

<sup>62</sup>For analysis of models of this type in the context of oligopoly see Abreu, Pearce and Stacchetti (1986), Green and Porter (1984), Rees (1985) and Rotemberg and Saloner (1986).

Lundberg and Pollak (1994) see this multiplicity as allowing a role to be played by social and cultural norms, custom and tradition. An obvious way to formalise this is by the GHWF. We can explore the implications of any specific set of assumptions about these norms and customs, as well as those holding for all sets of assumptions that result in a GHWF with the properties set out earlier, by maximising this over a set of utility possibilities that can be regarded as supportable as equilibria in an appropriately-formulated finitely-repeated non-cooperative game. We now turn to an example of the application of this idea.

## 4 The Pareto property and Pareto (in)efficiency

Cooperative household models see the household as acting as if it maximises a GHWF. The Pareto property, that the GHWF is strictly increasing in the utilities of its members, is one of the general properties of this function. Whether the household actually achieves a first best Pareto efficient allocation is a separate issue, and in this section we illustrate this point with a cooperative model which has a Pareto inefficient equilibrium.

This inefficiency arises because we assume that the household members are unable to make binding commitments over time, although within any given time period they are able to do so. The model tries to capture the following situation. A newly-formed household sees its future in two phases. In the first, it will have a high demand for household production. If one of the partners specialises in this, as is usually the case, she will reduce her market labour supply and correspondingly accumulate less work-related human capital. In the second phase, there is a much lower demand for household production, but the partner who previously specialised in it will face a lower market wage rate. If the couple can commit in the first period to consumption levels in the second, they can achieve a Pareto efficient allocation which takes into account that their joint income will be lower in the second period the more household production they have in the first. The weight each receives in the GHWF is determined by their wage rates in the *first* period, because that is when they negotiate the allocation. However, if they cannot make a binding commitment to consumption levels in the second phase, they must recognise that any first period agreement would be renegotiated at that time in the light of the then-prevailing wage rates. Thus, we have to impose as an *additional* constraint on their choice of allocations in the first period, the restriction on the possible allocations they will be able to negotiate in the second. This creates the second best Pareto inefficiency.

We take a two-period model in which only one partner,  $f$ , carries out household production, in the first period only. The key assumption is that, because human capital acquisition is work-related, her market wage in the second period is a decreasing function of the amount of household production she carries out in the first period, since this displaces time spent in market work in that period. We model the household's choices first on the assumption that it is able to commit in the first period to individual consumption levels in the second, and show that we obtain a Pareto efficient equilibrium. We then show the inefficiency

that results when commitment is not possible.<sup>63</sup>

### The Commitment Case

Assume that  $m$  supplies 1 unit of market labour inelastically and that his wage is constant over time at  $w_m$ , while  $f$ 's wage is  $w_{f1}$  in period 1 and  $w_{f2} = \omega(y)$ , in period 2, with  $\omega'(y) < 0$ . Production of the household good  $y$  is carried out in the first period only. We assume that one unit of  $f$ 's time produces one unit of  $y$ , and so the marginal opportunity cost or implicit price of  $y$  is  $w_{f1}$ . When the household can commit to future consumption values, it solves the problem

$$\max H(u_{f1}(x_{f1}, y_f) + u_{f2}(x_{f2}), u_{m1}(x_{m1}, y_m) + u_{m2}(x_{m2}), w_{f1}, w_m) \quad (27)$$

$$s.t. \sum_i (x_{i1} + w_{f1}y_i) \leq w_m + w_{f1} \quad (28)$$

$$\sum_i x_{i2} \leq w_m + \omega(y) \quad (29)$$

where  $x$  is a market consumption good with price normalised at 1 and  $\sum_i y_i = y$ . We assume time-separable utility with no utility discounting. To concentrate on essentials, we also assume no capital market. Since the household chooses its allocation at time 1, the relevant argument in the GHWF, which determines the marginal weight  $f$ 's utility receives, is her first period wage  $w_{f1}$ . Then the first order conditions with respect to consumptions can be written as:

$$\frac{\partial u_{f1}/\partial y_f}{\partial u_{f1}/\partial x_{f1}} = w_{f1} - \delta\omega' = \frac{\partial u_{m1}/\partial y_m}{\partial u_{m1}/\partial x_{m1}} \quad (30)$$

$$\frac{\partial u_{f1}/\partial y_f}{u'_{f2}} = \frac{w_{f1}}{\delta} - \omega' = \frac{\partial u_{m1}/\partial y_m}{u'_{m2}} \quad (31)$$

implying of course Pareto efficiency. Here  $\delta = \lambda_2/\lambda_1$  is a discount factor, where  $\lambda_1$  and  $\lambda_2$  are the marginal utilities of income in periods 1 and 2 respectively.

The household takes full account of the fact that part of the cost of household production in the first period is a lower wage for  $f$  in the second, arising from the loss of her human capital, and so expresses the implicit relative price of the domestic good in period 1 as  $w_{f1} - \delta\omega'$  (recall  $\omega'(y) < 0$ ). The term  $-\delta\omega'$  acts as a tax on current consumption of the domestic good. It arises because the household's income in period 2 will be lower, the higher is  $f$ 's domestic output in period 1.

---

<sup>63</sup>This type of problem has been thoroughly analysed in general terms in the "transactions cost" literature, associated primarily with Coase, Grossman, Hart and Williamson. See Hart (1995) and Williamson (1989) for comprehensive accounts of this literature. The problem arises because complete contracts cannot be written or enforced. Marriage seems a particularly striking case of an incomplete contract. Pollak (1985) appears to have been the first to introduce the ideas from this literature into household economics. The model in Apps (1981), (1982) has the idea of market productivity decreasing with specialisation in household production. The present model is a simpler version of a Nash bargaining formulation of the non-commitment problem developed by Ott (1992) ch. 6. See also Lundberg and Pollak (2003) for a similar type of Pareto inefficiency. The simplicity of the present model is perhaps an argument for the GHWF approach.

### The Non-Commitment Case

The model is as before, but now both partners realise that any prior agreement on consumptions in period 2 will be renegotiated in the light of wage rates prevailing at that time. Then the time 2 allocation will be chosen as the solution to the following problem

$$\max H(u_{f2}(x_{f2}), u_{m2}(x_{m2}), \omega(y), w_m) \quad (32)$$

$$s.t. \sum_i x_{i2} \leq w_m + \omega(y) \quad (33)$$

That is, the weight given to  $f$  in the second period will reflect her wage in that period. It follows that the optimal solutions to this problem are functions  $x_{i2}^*[\omega(y)]$ . Moreover, given the assumptions on the GHWF set out in Section 2.3.2, straightforward comparative statics yields

$$\frac{dx_{f2}^*}{dy} \neq \frac{dx_{m2}^*}{dy} \quad (34)$$

as we would expect. A change in  $y$  changes each period 2 consumption not only because of an income effect, but also because the relative weights on the individuals' utilities change in  $m$ 's favour.

At time period 1 the household solves the problem

$$\max H(u_{f1}(x_{f1}, y_f) + u_{f2}\{x_{f2}^*[\omega(y)]\}, u_{m1}(x_{m1}, y_m) + u_{m2}\{x_{m2}^*[\omega(y)]\}, w_{f1}, w_m) \quad (35)$$

$$s.t. \sum_i (x_{i1} + w_{f1}y_i) \leq w_m + w_{f1} \quad (36)$$

From the first order conditions for this problem we obtain

$$\frac{\partial u_{i1}/\partial y_i}{\partial u_{i1}/\partial x_{i1}} = w_{f1} - \frac{\lambda_2}{\lambda_1} \frac{dx_{i2}^*}{dy} \quad i = f, m \quad (37)$$

Then, since the second terms on the right hand side are unequal for each  $i$ , the first period allocation will not be Pareto efficient. In choosing their consumptions, they take into account the effects of  $f$ 's loss of human capital on their individual consumptions and utilities in the household equilibrium in period 2 and, since these are different, their marginal rates of substitution between the two goods in period 1 will differ. They cannot correct this because of the inability to make binding commitments to the allocation in period 2.

## 5 Conclusions

One aim of this paper has been to suggest that the literature's neglect of Samuelson's proposal that cooperative households can be modelled as if they maximised a form of social welfare function, the household welfare function, was a mistake. McElroy and Horney's criticism of the implications of the standard kind of

HWF, defined only on utilities of household members, was however well-taken, and the idea which follows directly from their Nash bargaining model, that the household's preference ordering over the utility profiles of its members depends on exogenous variables, in particular wage rates and non-wage incomes, is an important and fruitful one. It has however much wider application than to bargaining models alone, and can be made the basis for a general approach to modelling household decision taking, flexible enough to encompass non-cooperative behaviour and Pareto inefficiencies arising out of the inevitable incompleteness and unenforceability of domestic agreements. We have also tried to point out some of the implications for existing models of the neglect of household production. Above all, the aim has been to provide a deeper understanding of the current theoretical literature on household economics by means of a survey of its history.

## References

- [1] Abreu, D. (1986). Extremal equilibria of oligopolistic supergames. *Journal of Economic Theory*, 39, 191-235.
- [2] Abreu, D., Pearce, D. & Stacchetti, E., (1986). Optimal cartel equilibria with incomplete monitoring. *Journal of Economic Theory*, 39, 251-269.
- [3] Apps, P. F. (1981), *A Theory of Inequality and Taxation*. Cambridge: Cambridge University Press.
- [4] Apps, P.F. (1982). Institutional inequality and tax incidence. *Journal of Public Economics* 18, 217-242.
- [5] Apps, P.F., & Rees, R. (1988), Taxation and the household, *Journal of Public Economics*, 35, 355-369.
- [6] Apps, P.F., & Rees, R. (1997). Collective labor supply and household production. *Journal of Political Economy* 105, 178-190.
- [7] Apps, P. F. & Rees, R. (1999), Joint vs individual taxation in models of household production", *Journal of Political Economy*, 107, 178-190.
- [8] Apps, P.F., & Rees, R. (2001). Household production, full consumption and the costs of children, *Labour Economics*, 8, 621-648.
- [9] Apps, P.F., & Rees, R. (2005). Gender, time use and public policy over the life cycle. *Oxford Review of Economic Policy*. 21(3), 439-461.
- [10] Apps, P.F., & Rees, R. (2007a). Empirical models of the household: A survey. mimeo.
- [11] Apps, P.F., & Rees, R. (2007b). Cooperative household models. IZA Discussion Paper 3127.

- [12] Apps, P.F., & Rees, R. (2007c). The taxation of couples. IZA Discussion Paper 2910.
- [13] Ashworth, J. S., & Ulph, D. T., (1981), Household models, In Brown C. V. (ed.), *Taxation and Labour Supply*, London: Allen & Unwin.
- [14] Basu, K., (2006), Gender and say: A model of household behavior with endogenously-determined balance of power. *Economic Journal*, 116, 558-580.
- [15] Becker, G. S. (1965). A theory of the allocation of time. *Economic Journal*, 75, 493-517
- [16] Becker, G. S. (1973). A theory of marriage (Part I). *Journal of Political Economy*, 81, 813-846.
- [17] Becker, G. S. (1974). A theory of marriage (Part II). *Journal of Political Economy*, 82, 511-526.
- [18] Becker, G. S. (1981). *A Treatise on the Family*. (1st ed.) Cambridge: Harvard University Press.
- [19] Benoit, J.-P. & Krishna, V., (1985). Finitely repeated games. *Econometrica* 53, 905-922.
- [20] Bergson, A. (1938). A reformulation of certain aspects of welfare economics. *Quarterly Journal of Economics*, 52, 30-34.
- [21] Bergstrom, T. C. (1997), A survey of theories of the family. In Rosenzweig, M., & Stark, O., (eds), *Handbook of Population and Family Economics*, North-Holland, Elsevier.
- [22] Bergstrom, T., Blume, L., & Varian, H., (1987), On the private provision of public goods. *Journal of Public Economics*, 29, 25-49.
- [23] Binmore, K., Rubinstein, A., & Wolinsky, A. (1986), The Nash bargaining solution in economic modelling. *Rand Journal of Economics*, 17, 176-188.
- [24] Boadway, R. & Bruce, N. (1984). *Welfare Economics*. Oxford: Basil Blackwell.
- [25] Browning, M., & Chiappori, P.-A. (1998). Efficient intra-household allocation: a characterisation and tests. *Econometrica*, 66(6), 1241-1278.
- [26] Chen, Z., & Woolley, F. (2001). A Cournot-Nash model of family decision making. *Economic Journal*, 111(474), 722-748.
- [27] Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica*, 56(1), 63-90.
- [28] Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy*, 100, 437-467.

- [29] Chiappori, P.-A. (1997). Introducing household production in collective models of labor supply". *Journal of Political Economy*, 105, 191-209.
- [30] Deaton, A. & Muellbauer, J. (1980). *Economics and Consumer Behaviour*. Cambridge: Cambridge University Press.
- [31] Farrell, J. & Maskin, E. (1989). Renegotiation in repeated games. *Games and Economic Behavior*, 1, 327-360.
- [32] Friedman, J. (1977). *Oligopoly and the Theory of Games*. Amsterdam. North-Holland.
- [33] Gorman, W. M., (1956), A possible procedure for analysing quality differentials in the egg market. Ames: Iowa State College, mimeo. (Reissued as Discussion Paper No. B4, London School of Economics Econometrics Program, London 1976).
- [34] Gorman, W. M., (1959), Separable utility and aggregation, *Econometrica*, 27, 469-481.
- [35] Gorman, W. M., (1961), On a class of preference fields. *Metroeconomica* 13, 53-56.
- [36] Green, E., & Porter, R., (1984), Noncooperative collusion under imperfect price information. *Econometrica*, 52, 87-100.
- [37] Gronau, R. (1986), Home production - a survey. In Ashenfelter, O., & Layard, R. (eds), *Handbook of Labour Economics*, Vol I, 273-304. North-Holland: Elsevier.
- [38] Grossbard, A., (1976). An economic analysis of polygamy: the case of Maidiguru. *Current Anthropology* 17, 701-707.
- [39] Grossbard-Schechtman, A., (1984). A theory of the allocation of time in markets for labor and marriage. *Economic Journal* 94, 863-882.
- [40] Grossbard-Schechtman, S., (2003). A consumer theory with competitive markets for work in marriage. *The Journal of Socio-Economics*, 31, 609-645.
- [41] Hart, O., (1995). *Firms, Contracts and Financial Structure*. Oxford: Oxford University Press.
- [42] Jehle, G. A. & Reny, P. J. (2001). *Advanced Microeconomic Theory*. (2nd ed.) Boston: Addison-Wesley.
- [43] Konrad, K. A. & Lommerud, K. E., (1995), Family policy with non-cooperative families. *Scandinavian Journal of Economics* 97, 581-601.
- [44] Konrad, K. A. & Lommerud, K. E., (2000), The bargaining family revisited. *Canadian Journal of Economics*, 33(2), 471-487.

- [45] Kooreman, P., and Kapteyn, A., (1990). On the empirical implementation of some game-theoretic models of household labor supply. *Journal of Human Resources*, 25, 25, 584-598.
- [46] Kreps, D. M., Milgrom, P., Roberts, J. & Wilson, R. (1982). Rational cooperation in the finitely repeated prisoners' dilemma, *Journal of Economic Theory*, 27, 245-252.
- [47] Lancaster, K. J., (1966), A new approach to consumer theory. *Journal of Political Economy*, 74, 132-157.
- [48] Leuthold, J. H., (1968), An empirical study of formula income transfers and the work decision of the poor. *Journal of Human Resources*, 3, 312-323.
- [49] Lundberg, S., & Pollak, R. A. (1993). Separate spheres bargaining and the marriage market. *Journal of Political Economy*, 101, 988-1010.
- [50] Lundberg, S., & Pollak, R. A. (1994). Noncooperative bargaining models of marriage. *American Economic Review*,
- [51] Lundberg, S., & Pollak, R. A. (1996). Bargaining and distribution in marriage. *Journal of Economic Perspectives*, 10(4), 139-158.
- [52] Lundberg, S., & Pollak, R. A. (2003). Efficiency in marriage. *Review of Economics of the Household*, 153-167.
- [53] Lundberg, S., Pollak, R. A. & Wales, T. J. (1997). Do husbands and wives pool their resources? Evidence from the UK child benefit. *Journal of Human Resources*, 32(3), 463-480.
- [54] McElroy, M. B. (1990). The empirical content of Nash bargained household behaviour. *Journal of Human Resources*, (25)4, 559-583.
- [55] McElroy, M. B. & and Horney, M. (1981). Nash-bargained decisions: Towards a generalization of the theory of demand. *International Economic Review*, 22, 333-349.
- [56] Manser, M., & Brown, M. (1980). Marriage and household decision making: A bargaining analysis. *International Economic Review*, 21(1), 31-34.
- [57] Mas-Colell, A., Whinston, M. D. & Green J. R., (1995), *Microeconomic Theory*, Oxford: Oxford University Press.
- [58] Mirrlees, J., (1971). An exploration in the theory of income taxation", *Review of Economic Studies*, 38, 175-208.
- [59] Mincer, J., (1963). Market Prices, opportunity costs and income effects, in *Measurement in Economics*, C. Christ (ed.), Stanford: Stanford University Press.



- [60] Muth, R. F., (1966). Household production and consumer demand. *Econometrica*, 34, 699-708.
- [61] Nerlove, M., (1974), Toward a new theory of population and economic growth. In Schultz, T. W.. (ed), *Economics of the Family*, Chicago: University of Chicago Press.
- [62] Ott, N., (1992). *Intrafamily Bargaining and Household Decisions*. Berlin: Springer-Verlag, .
- [63] Pollak, R. A. (1977). Price dependent preferences. *American Economic Review*, 67, 64-75.
- [64] Pollak, R. A. (1985). A transaction cost approach to families and households. *Journal of Economic Literature*, 23(2), 581-608.
- [65] Pollak, R. A. (2003). Gary Becker's contribution to family and household economics. *Review of Economics and the Household*.
- [66] Pollak, R. A., & Wachter, M. L., (1975). The relevance of the household production function and its implications for the allocation of time. *Journal of Political Economy*, 68, 349-359.
- [67] Rees, R., (1985), Cheating in a duopoly supergame. *Journal of Industrial Economics*, 23, 387-400.
- [68] Rotemberg, J., & Saloner, G., ((1986), A supergame-theoretic model of business cycles and price wars during booms. *American Economic Review*, 76, 390-407.
- [69] Samuelson, P. A. (1947). *Foundations of Economic Analysis*. Cambridge, Mass: Harvard University Press.
- [70] Samuelson, P. A. (1956). Social indifference curves. *Quarterly Journal of Economics*, 70, 1-22.
- [71] Schultz, T. P. (1990). Testing the neoclassical model of family labour supply and fertility. *Journal of Human Resources*, 25, 599-634.
- [72] Sheshinski, E. (1972). The optimal linear income tax". *Review of Economic Studies*, 39, 297-302.
- [73] Singh, I., Squire, L., and Strauss, J. (eds), *Agricultural Household Models: Extensions, Applications, and Policy*. Baltimore,: Johns Hopkins Press (for World Bank), 1986.
- [74] Thomas, D. (1990). Intra-household resource allocation: An inferential approach. *Journal of Human Resources*, 25, 635-664.
- [75] Ulph, D. (1988). A general non-cooperative Nash model of household consumption behaviour. Working paper 88-205, Department of Economics, University of Bristol.

- [76] Vermeulen, F. (2002). Collective household models: Principles and main results. *Journal of Economic Surveys*, 16, 533-564.
- [77] Warr, P., (1983). The private provision of a public good is independent of the distribution of income. *Economics Letters*, 13, 207-211.
- [78] Williamson, O. E. (1989). Transactions cost economics, in Schmalensee R. & Willig, R. D. (eds), *The Handbook of Industrial Organisation*, Vol. 1. Amsterdam: North-Holland.
- [79] Woolley, F. (1988). A non-cooperative model of family decision making. Discussion Paper TIDI 125, London School of Economics.

# CESifo Working Paper Series

for full list see [www.cesifo-group.org/wp](http://www.cesifo-group.org/wp)

(address: Poschingerstr. 5, 81679 Munich, Germany, [office@cesifo.de](mailto:office@cesifo.de))

---

- 2108 Torben M. Andersen and Michael Svarer, Flexicurity – Labour Market Performance in Denmark, October 2007
- 2109 Jonathan P. Thomas and Tim Worrall, Limited Commitment Models of the Labor Market, October 2007
- 2110 Carlos Pestana Barros, Guglielmo Maria Caporale and Luis A. Gil-Alana, Identification of Segments of European Banks with a Latent Class Frontier Model, October 2007
- 2111 Felicitas Nowak-Lehmann D., Sebastian Vollmer and Immaculada Martínez-Zarzoso, Competitiveness – A Comparison of China and Mexico, October 2007
- 2112 Mark Mink, Jan P.A.M. Jacobs and Jakob de Haan, Measuring Synchronicity and Co-movement of Business Cycles with an Application to the Euro Area, October 2007
- 2113 Ossip Hühnerbein and Tobias Seidel, Intra-regional Tax Competition and Economic Geography, October 2007
- 2114 Christian Keuschnigg, Exports, Foreign Direct Investment and the Costs of Corporate Taxation, October 2007
- 2115 Werner Bönnte, Oliver Falck and Stephan Heblich, Demography and Innovative Entrepreneurship, October 2007
- 2116 Katrin Assenmacher-Wesche and M. Hashem Pesaran, Assessing Forecast Uncertainties in a VECX Model for Switzerland: An Exercise in Forecast Combination across Models and Observation Windows, October 2007
- 2117 Ben Lockwood, Voting, Lobbying, and the Decentralization Theorem, October 2007
- 2118 Andrea Ichino, Guido Schwerdt, Rudolf Winter-Ebmer and Josef Zweimüller, Too Old to Work, too Young to Retire?, October 2007
- 2119 Wolfgang Eggert, Tim Krieger and Volker Meier, Education, Unemployment and Migration, October 2007
- 2120 Stefan Napel and Mika Widgrén, The European Commission – Appointment, Preferences, and Institutional Relations, October 2007
- 2121 Bertil Holmlund and Martin Söderström, Estimating Income Responses to Tax Changes: A Dynamic Panel Data Approach, October 2007
- 2122 Doina Maria Radulescu, From Separate Accounting to Formula Apportionment: Analysis in a Dynamic Framework, October 2007

- 2123 Jelle Brouwer, Richard Paap and Jean-Marie Viaene, The Trade and FDI Effects of EMU Enlargement, October 2007
- 2124 Kurt R. Brekke, Luigi Siciliani and Odd Rune Straume, Competition and Waiting Times in Hospital Markets, October 2007
- 2125 Alexis Direr, Flexible Life Annuities, October 2007
- 2126 Johannes Becker and Clemens Fuest, Quality versus Quantity – The Composition Effect of Corporate Taxation on Foreign Direct Investment, October 2007
- 2127 Balázs Égert, Real Convergence, Price Level Convergence and Inflation Differentials in Europe, October 2007
- 2128 Marko Koethenbueger, Revisiting the “Decentralization Theorem” – On the Role of Externalities, October 2007
- 2129 Axel Dreher, Silvia Marchesi and James Raymond Vreeland, The Politics of IMF Forecasts, October 2007
- 2130 Andreas Knabe and Ronnie Schöb, Subsidizing Extra Jobs: Promoting Employment by Taming the Unions, October 2007
- 2131 Michel Beine and Bertrand Candelon, Liberalization and Stock Market Co-Movement between Emerging Economies, October 2007
- 2132 Dieter M. Urban, FDI Technology Spillovers and Wages, October 2007
- 2133 Valentina Bosetti, Carlo Carraro, Emanuele Massetti and Massimo Tavoni, Optimal Energy Investment and R&D Strategies to Stabilise Greenhouse Gas Atmospheric Concentrations, October 2007
- 2134 David-Jan Jansen and Jakob de Haan, The Importance of Being Vigilant: Has ECB Communication Influenced Euro Area Inflation Expectations?, October 2007
- 2135 Oliver Falck, Heavyweights – The Impact of Large Businesses on Productivity Growth, October 2007
- 2136 Xavier Freixas and Bruno M. Parigi, Banking Regulation and Prompt Corrective Action, November 2007
- 2137 Jan K. Brueckner, Partial Fiscal Decentralization, November 2007
- 2138 Silvia Console Battilana, Uncovered Power: External Agenda Setting, Sophisticated Voting, and Transnational Lobbying, November 2007
- 2139 Alan J. Auerbach, Michael P. Devereux and Helen Simpson, Taxing Corporate Income, November 2007

- 2140 Lorenzo Cappellari, Paolo Ghinetti and Gilberto Turati, On Time and Money Donations, November 2007
- 2141 Roel Beetsma and Heikki Oksanen, Pension Systems, Ageing and the Stability and Growth Pact, November 2007
- 2142 Hikaru Ogawa and David E. Wildasin, Think Locally, Act Locally: Spillovers, Spillbacks, and Efficient Decentralized Policymaking, November 2007
- 2143 Alessandro Cigno, A Theoretical Analysis of the Effects of Legislation on Marriage, Fertility, Domestic Division of Labour, and the Education of Children, November 2007
- 2144 Kai A. Konrad, Mobile Tax Base as a Global Common, November 2007
- 2145 Ola Kvaløy and Trond E. Olsen, The Rise of Individual Performance Pay, November 2007
- 2146 Guglielmo Maria Caporale, Yannis Georgellis, Nicholas Tsitsianis and Ya Ping Yin, Income and Happiness across Europe: Do Reference Values Matter?, November 2007
- 2147 Dan Anderberg, Tax Credits, Income Support and Partnership Decisions, November 2007
- 2148 Andreas Irmen and Rainer Klump, Factor Substitution, Income Distribution, and Growth in a Generalized Neoclassical Model, November 2007
- 2149 Lorenz Blume, Jens Müller and Stefan Voigt, The Economic Effects of Direct Democracy – A First Global Assessment, November 2007
- 2150 Axel Dreher, Pierre-Guillaume Méon and Friedrich Schneider, The Devil is in the Shadow – Do Institutions Affect Income and Productivity or only Official Income and Official Productivity?, November 2007
- 2151 Valentina Bosetti, Carlo Carraro, Emanuele Massetti and Massimo Tavoni, International Energy R&D Spillovers and the Economics of Greenhouse Gas Atmospheric Stabilization, November 2007
- 2152 Balázs Égert and Dubravko Mihaljek, Determinants of House Prices in Central and Eastern Europe, November 2007
- 2153 Christa Hainz and Hendrik Hakenes, The Politician and his Banker, November 2007
- 2154 Josef Falkinger, Distribution and Use of Knowledge under the “Laws of the Web”, December 2007
- 2155 Thorvaldur Gylfason and Eduard Hochreiter, Growing Apart? A Tale of Two Republics: Estonia and Georgia, December 2007
- 2156 Morris A. Davis and François Ortalo-Magné, Household Expenditures, Wages, Rents, December 2007

- 2157 Andreas Haufler and Christian Schulte, Merger Policy and Tax Competition, December 2007
- 2158 Marko Köthenbürger and Panu Poutvaara, Rent Taxation and its Intertemporal Welfare Effects in a Small Open Economy, December 2007
- 2159 Betsey Stevenson, Title IX and the Evolution of High School Sports, December 2007
- 2160 Stergios Skaperdas and Samarth Vaidya, Persuasion as a Contest, December 2007
- 2161 Morten Bennedsen and Christian Schultz, Arm's Length Provision of Public Services, December 2007
- 2162 Bas Jacobs, Optimal Redistributive Tax and Education Policies in General Equilibrium, December 2007
- 2163 Christian Jaag, Christian Keuschnigg and Mirela Keuschnigg, Pension Reform, Retirement and Life-Cycle Unemployment, December 2007
- 2164 Dieter M. Urban, Terms of Trade, Catch-up, and Home Market Effect: The Example of Japan, December 2007
- 2165 Marcelo Resende and Rodrigo M. Zeidan, Lionel Robbins: A Methodological Reappraisal, December 2007
- 2166 Samuel Bentolila, Juan J. Dolado and Juan F. Jimeno, Does Immigration Affect the Phillips Curve? Some Evidence for Spain, December 2007
- 2167 Rainald Borck, Federalism, Fertility and Growth, December 2007
- 2168 Erkki Koskela and Jan König, Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment, December 2007
- 2169 Egil Matsen and Øystein Thøgersen, Habit Formation, Strategic Extremism and Debt Policy, December 2007
- 2170 Torben M. Andersen and Allan Sørensen, Product Market Integration and Income Taxation: Distortions and Gains from Trade, December 2007
- 2171 J. Atsu Amegashie, American Idol: Should it be a Singing Contest or a Popularity Contest?, December 2007
- 2172 Patricia Apps and Ray Rees, Household Models: An Historical Perspective, December 2007