Optimum Taxation of Inheritances

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Abstract

We incorporate the fact that inheritances create a second distinguishing characteristic of individuals, in addition to earning abilities, into an optimum income taxation model with bequests motivated by joy of giving. We show that a tax on inheritances and a uniform tax on all expenditures including bequests are equivalent and that either is desirable, according to an intertemporal social objective, if on average high-able individuals have larger inherited endowments than low-able. We demonstrate that such a situation results as the outcome of a process with stochastic transition of abilities over generations, if all descendants are more probable to have their parent's ability rank than any other.

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1. Introduction

The tax on estates or inheritances has been a highly controversial issue for long. On the political level, opponents consider it morally inappropriate to use the moment of death as a cause for imposing a tax, and stress its negative consequences on economic variables, in particular on capital accumulation and on family business. Supporters find these consequences exaggerated and claim that a tax on bequests is desirable for redistributive reasons, contributing to "equality of opportunity".[1]

In the academic literature, no widely accepted view on this tax seems to have evolved either. One reason may be that there is too little empirical knowledge of the magnitude of its effect on the economy. Another reason may be that also on the theoretical level the consequences of inheritance taxation on efficiency and equity have not been worked out clearly. Indeed, we argue that studies in optimum tax theory, which provides the appropriate framework for such an analysis, have not yet succeeded in clarifying the role of this tax within the entire tax system.

The intention of this paper is to propose an optimum-taxation model, which allows a discussion of the central question: is a shift from labour income taxation to a tax on intergenerational wealth transfers a desirable means of redistribution? To answer this question, we extend the standard optimum income taxation approach in the tradition of Mirrlees (1971) to a sequence of generations and introduce a bequest motive. As the adequate version of the bequest motive we consider bequests as consumption (or joy-of giving, see, e.g., Cremer and Pestieau 2006): the amount left to the descendants increases welfare of the parents similar to the consumption of a good.[2] Individuals live for one period and differ in their earning abilities, inherited wealth increases their

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[1] Specifically in the USA, there has been a heated debate on the proposal to repeal the federal estate tax permanently. In 2006 it failed the needed majority narrowly in the Senate, after the House of Representatives had voted overwhelmingly for the permanent repeal. Some countries like Sweden and Singapore have just recently abolished taxation of inherited wealth, or, like Austria, phase out this tax. However, many other countries, in particular in Europe, still stick to their taxes on inheritance.

[2] Though, of course, bequests require savings of the parents. Another motive would be pure altruism, where the parents' utility function has utility of the descendants as an argument. This motive leads to dynastic preferences. We do not intend to model redistribution between dynasties, but between individuals in each generation. We also leave out the strategic bequest motive as well as unintended bequests (for the latter, see Blumkin and Sadka 2003; they also study estate taxation in case of dynastic preferences).

budget on top of their net income; the budget can be used for consumption and bequests left to the next generation.

The essential point of our analysis comes from the observation that inherited wealth creates a second distinguishing characteristic of individuals, in addition to earning abilities, and it is this fact which motivates the above-mentioned view that a tax on estates or inheritances enhances equality of opportunity. Straightforward as this observation is, it seems to have not always been incorporated appropriately by former contributions, which discuss bequest taxation in an optimum taxation framework. Instead of taking into account the differences caused by bequests within the generation of heirs, authors discuss the specific nature of bequests as one form of consumption, for which members of the bequeathing generation spend their budget. Such an analysis, referring to a standard result in optimum-taxation theory (Atkinson and Stiglitz 1972, among others), leads to the question of whether preferences are separable between leisure and consumption (then the latter should not be taxed at all) or whether leaving bequests represents a complement or a substitute to enjoying leisure.[3] We argue in the present paper that this is the inappropriate question, because what matters is not that bequests represent a particular use of the budget, and because the Atkinson-Stiglitz result is derived for a model where individuals only differ in earning abilities.

On the other hand, there is already a series of papers which pay attention to the fact that inheritances create a second distinguishing characteristic, in addition to earning abilities. However, to our knowledge none of these provides a unified framework for an analysis of the role of bequest taxation within an optimum tax system. Cremer et al. (2001) resume the discussion of indirect taxes, given that individuals differ in endowments (inheritances) as well as abilities and that an optimum nonlinear tax on labour income is imposed. They assume, however, that inheritances are unobservable and concentrate on the structure of indirect tax rates. Similarly, Cremer et al. (2003) and Boadway et al. (2000) study the desirability of a tax on capital income, because it can serve as a surrogate for the taxation of inheritances, which are considered unobservable.

3 See Gale and Slemrod (2001, p.33) and Kaplow (2001), as well as Blumkin and Sadka (2003) in the context of a dynastic model.

In contrast to these contributions, we study a comprehensive tax system where a nonlinear tax on labour income can be combined with taxes on endowments and on expenditures. Therefore, we take all these variables as being observable (only abilities are unobservable). In our view, this is indeed the assumption on the basis of which real-world tax systems, including the tax on bequests, operate, and it is important to investigate its consequences. Our intention is to clarify precisely how wealth transfers can (or should) be taxed and how this affects the welfare of different generations.

As a starting point we consider a static model with two types of individuals, who hold exogenously given initial endowments. We discuss the role of two taxes, both in combination with an optimum tax on labour income: (i) a (direct) tax on the initial endowments, and (ii) an (indirect) tax on expenditures, that is, when spending for all purposes (which might include leaving bequests) is taxed at a uniform rate. We show that these two taxes are equivalent (though, at first glance, only the first is lump-sum) and that either is desirable according to a utilitarian objective, if endowments and earning abilities are positively correlated. The underlying reason is that introducing these taxes allows further redistribution on top of what can be achieved through labour income taxation alone.

Then we turn to an analysis of the dynamic model. That is, when discussing the two equivalent ways of imposing a tax (either directly on – inherited – initial endowments or indirectly on expenditures), we now take into account that bequests left by some generation influence the welfare of future generations. It turns out, contrary to what one expects, that introducing dynamic effects does not change anything compared to the result of the static model: the positive relation between endowments and earning abilities remains the only decisive issue for both ways of taxation. All other welfare effects – including those falling on later generations – associated with the introduction of the tax on endowments (or on expenditures including bequests), are neutralised by the simultaneous adaptation of the optimum tax on labour income. Thus, we also find that the "double-counting" problem, which typically arises in models where bequests for joy-of-giving (or generally: altruistic) motives enter a social objective twice through welfare of the donor as well as the recipient[4], does not occur in our framework.

[4] This is so, because spending causes two positive effects on the involved individuals (the donor enjoys giving, the beneficiary likes receiving), and the welfare of both appears in the social welfare function. In

This result has to be modified somewhat if the first instrument (a tax directly imposed on initial endowments) is applied and if one assumes that the bequeathing individuals care for bequests net of the inheritance tax falling on the heirs. Then collecting the tax in some period will have repercussions on the previous generation, which affects their bequest decision. This problem does not arise with an expenditure tax (which includes bequests).

In a next step, we generalise the model to one with arbitrarily many types of individuals and with a stochastic relation between endowments and earning abilities. Restricting the analysis to quasilinear preferences, we show that the results remain essentially unchanged, the crucial point for the desirability of a tax on inheritances (or expenditures) being that expected endowments increase with abilities. Furthermore, we provide a theoretical argument that this is a plausible situation, by constructing a stochastic process of abilities which is built on the key assumption that all descendants are more probable to have their parent's ability rank than any other.[5] We show that if each parent has a descendant, to whom she leaves her bequests, this process indeed generates a positive relation between endowments and abilities in any generation.[6]

Our work is related to contributions which study a stochastic process describing the transition of wealth over generations, and analyse the evolution of inequality. They show that, depending on the assumptions of the model, a tax on bequests may increase inequality (by reducing the role of inheritances as compensating for income shocks of the descendants, see, e.g., Becker and Tomes 1979) or decrease inequality (by redistributing wealth, see, e.g., Bossmann, Kleiber and Wälde, 2007, Davies and Kuhn 1991). In contrast to these contributions, which concentrate on inequality measures, but do not discuss welfare effects and typically assume fixed labour supply, we follow the optimum-taxation approach, which allows a combined consideration of

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the case of bequests, this calls for a subsidy instead of a tax. Some authors discuss "laundering out" this double counting from the social welfare function, see, e. g., Cremer and Pestieau (2006).

5 This assumption is justified by various empirical studies which find that the children's incomes are positively correlated with those of their parents. For instance, Solon 1992 and Zimmerman 1992 both find an intergenerational correlation in income of 0.4 for the US economy.

6 To our knowledge, there is no direct empirical evidence on this issue. However, it has been found that earnings are positively correlated with wealth (see, e.g., Díaz-Giménez et al. 2002 for the US economy, who find a positive correlation between earnings and wealth of 0.47). This can be seen as a partial support for our result, as wealth consists of inheritances to a substantial extent (for an overview see Kessler and Masson 1989).

efficiency and redistributive effects of the taxation of bequests, and we analyse its role within the tax system.

In the following Section 2 the model with two types of individuals is introduced and the results for the static as well as for the dynamic formulation are derived in turn. In Section 3 the model is generalised to more types and a stochastic relation between ability levels and inheritances. Moreover, a transition process which generates such a stochastic relation is studied. Section 4 provides concluding remarks.

2. Two ways of taxing inherited endowments

We begin this Section with an analysis of a static model, which will be extended to a dynamic framework with many generations in Subsection 2.2. The economy consists of two individuals i = L, H, characterised by differing earning abilities $\omega_L < \omega_H$, and by exogenous initial endowments of (inherited) "capital" e_i, i = L,H. The individuals live for one period. By supplying labour time l_i, each individual earns pre-tax income $z_i = \omega_i l_i$, i = L,H. After-tax income is denoted by x_i, which, together with initial endowment, is spent on general consumption c_i and some specific good b_i. We call the latter good bequests to be consistent with the terminology later on, though – taken literally – it makes no sense to have bequests in a static model. The individuals have common preferences, described by the concave utility function u(c,b,l).

2.1 A basic equivalence

In our model, the tax system consists of a tax on labour income, described implicitly by the function σ: $\mathbb{R} \rightarrow \mathbb{R}$, which relates gross and net income: $x = \sigma(z)$ (thus the tax is $z - \sigma(z)$), of a proportional tax τ_e on initial endowments, and of proportional taxes τ_c and τ_b on consumption and bequests, resp. Assuming that the prices of consumption and bequests are one, the budget equation of an individual reads:

$$(1+\tau_c)c_i + (1+\tau_b)b_i = \sigma(z_i) + (1-\tau_e)e_i \,. \tag{1}$$

Obviously, τ_e is a lump-sum tax in this case.

The budget set $B(\sigma(z_i), e_i, \tau_c, \tau_b, \tau_e)$ contains all nonnegative pairs (c_i,b_i) which fulfil the budget equation (1) with "≦". If, for given e_i, two tax systems lead to identical budget sets, for any z_i, then the two tax systems induce the same decision of the individuals with respect to the choice of l_i, c_i, b_i. Therefore we call the two tax systems equivalent in this case.

It is well known that in the absence of endowments a tax system consisting of an income tax plus a uniform expenditure tax is equivalent to an income tax alone. This is no longer true, if there exist endowments: then there is a case for a second tax instrument, in addition to the tax on labour income. We find immediately:

Lemma 1:

(a) A tax system $(\sigma, \tau_e, \tau_c, \tau_b)$ is equivalent to a tax system $(\hat{\sigma}, \hat{\tau}_e, \hat{\tau}_c, \hat{\tau}_b)$, where one of $(\hat{\tau}_e, \hat{\tau}_c, \hat{\tau}_b)$ is zero. Moreover,

if $\hat{\tau}_e = 0$, then $\hat{\sigma} = \sigma/(1-\tau_e)$ and $\hat{\tau}_c = \dfrac{\tau_e + \tau_c}{1-\tau_e}, \hat{\tau}_b = \dfrac{\tau_e + \tau_b}{1-\tau_e}$,

if $\hat{\tau}_c = 0$, then $\hat{\sigma} = \sigma/(1+\tau_c)$ and $\hat{\tau}_b = \dfrac{\tau_b - \tau_c}{1+\tau_c}, \hat{\tau}_e = \dfrac{\tau_e + \tau_c}{1+\tau_c}$,

if $\hat{\tau}_b = 0$, then $\hat{\sigma} = \sigma/(1+\tau_b)$ and $\hat{\tau}_c = \dfrac{\tau_c - \tau_b}{1+\tau_b}, \hat{\tau}_e = \dfrac{\tau_e + \tau_b}{1+\tau_b}$.

(b) A tax system $(\sigma, \tau_e, \tau_c, \tau_b)$ with $\tau_c = \tau_b$ is equivalent to a tax system $(\hat{\sigma}, \hat{\tau}_e, \hat{\tau}_c, \hat{\tau}_b)$, where $\hat{\tau}_c = \hat{\tau}_b$ and either $\hat{\tau}_e = 0$ or $\hat{\tau}_c = \hat{\tau}_b = 0$. The formulas in (a) apply.

Proof: Follows immediately from appropriate manipulations of the budget equation (1).

In the following we make use of the observation, expressed in Lemma 1(b) that a tax on initial endowments is essentially the same as a uniform tax on expenditures for consumption and bequests (which in fact are a form of consumption), because the income tax can be adjusted accordingly. In particular, the uniform expenditure tax represents a kind of lump-sum tax in this framework, as does the tax on endowments, though consumption is variable, while endowments are fixed.

Note that the switch to a tax system without a tax on endowments means that the income tax has to be reduced (net income $\sigma(z)$ is increased), while the taxes on c_i and b_i have to be increased. Similarly, a switch such that expenditures are untaxed (consider case (b)) means an increase of the income tax and of the tax on endowments (if $\tau_e < 1$).

The equivalence extends to the welfare effect of a marginal change of the tax system, which we discuss in an optimum income taxation framework.

We introduce the indirect utility function

$$v^i(x_i, z_i, e_i, \tau_e, \tau) \equiv \max\left\{u(c_i, b_i, z_i / \omega_i) \mid (1+\tau)(c_i + b_i) \leq x_i + (1-\tau_e)e_i\right\},$$

where we consider only a tax system with a uniform tax rate τ on expenditures, equivalent to the tax rate τ_e on initial endowments.

As usual, we assume that the tax authority cannot tie a tax directly with individual abilities, because they are not observable, therefore it imposes an income tax. For the determination of the latter, we take some tax rate τ and/or τ_e as fixed for the moment. In case that there are no restrictions on the functional form of the income tax, the appropriate way to determine the optimum nonlinear schedule is to maximise a social welfare function with respect to the individuals' income bundles (x,z), subject to the self-selection constraints and the resource constraints.

As is standard in optimum income taxation models, we assume that the condition of "agent monotonicity" (Seade 1982) holds, which guarantees single crossing of indifference curves of the two individuals in x-z-space. Define $MRS_{zx}^i \equiv -(\partial v^i / \partial z_i) / (\partial v^i / \partial x_i)$, then for any given e_i, τ_e, τ:

$$\text{AM: } MRS_{zx}^L > MRS_{zx}^H \text{ at any vector } (x, z).$$

As is well-known, this condition essentially guarantees that for any income tax function the high-able individual does not choose to earn less income than the low-able. It is fulfilled, if the high-able individual requires less additional net income to be compensated for earning one additional unit of gross income than what the low-able individual requires. This is plausible, because the former needs less additional working time. It should be noted, however, that in the presence of initial (non-human) endowments this assumption is more critical than in the standard model à la Mirrlees: if the initial endowment of the high-able individual is much larger than that of the low-able, the former (because of a quite low marginal utility of net income) might require a larger amount of net income as a compensation for her effort to earn one more unit of gross income, than what the latter requires.[7]

We assume a utilitarian social welfare function with weights $f_L, f_H, f_L \geq f_H$, of the two individuals, then the objective is

$$\max_{x_i, z_i} f_L v^L(x_L, z_L, e_L, \tau_e, \tau) + f_H v^H(x_H, z_H, e_H, \tau_e, \tau). \tag{2}$$

There is a resource constraint

$$x_L + x_H \leq z_L + z_H + \tau_e \left(e_L + e_H\right) + \tau\left(c_L(\cdot) + b_L(\cdot) + c_H(\cdot) + b_H(\cdot)\right) - g, \tag{3}$$

where g denotes the requirement of the state. $c_i(\cdot)$, $b_i(\cdot)$ are demand functions with the same arguments as $v^i(\cdot)$, i = L, H.

Finally we have to introduce the self-selection constraints: the government must determine the two bundles of gross and net income in such a way that no individual prefers the bundle assigned to the other. We follow the frequently made assumption of a sufficient importance of the low-able individual in the objective function (2). Then the social objective favours redistribution from the high- to the low-able individual, and one can show that only the self-selection constraint of the high-able individual is binding in the optimum and needs to be considered:

$$v^H(x_H, z_H, e_H, \tau_e, \tau) \geq v^H(x_L, z_L, e_H, \tau_e, \tau). \tag{4}$$

Let, for given τ_e, τ, the optimum value of the social welfare function (2) subject to the constraints (3) and (4) be denoted by S(τ_e, τ), and let the Lagrange multiplier of the

[7] Such a potential problem does not occur, if we work with quasilinear preferences, as we do in Section 3.

self-selection constraint (4) be denoted by μ. μ is positive as a consequence of the above assumption that (4) is binding in the optimum. $\partial v^H[L]/\partial x_L > 0$ describes marginal utility of income of the high-able individual in case of mimicking.[8] We find

Theorem 1: The welfare effect of a marginal increase of τ_e and τ, resp., reads:

(a) $$\frac{\partial S}{\partial \tau_e} = \mu \frac{\partial v^H[L]}{\partial x_L}(e_H - e_L),$$

(b) $$\frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial \tau_e}\frac{1-\tau_e}{1+\tau}.$$

Hence, both taxes increase welfare, if the initial endowment of the high-able individual is larger than that of the low-able.

Proof: see Appendix A.

Given a larger endowment of the high-able individuals, the social objective calls for further redistribution than what is possible through an income tax alone. Such an additional redistribution can equivalently be achieved by a tax on initial endowments or on expenditures. In particular, it turns out that, in a sense, the justification for indirect taxation is uniquely linked to the existence of differing endowments: given these, the expenditure tax combined with an optimum income tax is indeed a lump-sum tax, being equivalent to the tax on endowments.

The positive effect on welfare comes from a relaxation of the self-selection constraint induced by an increase of τ_e (or τ). The intuition can be explained as follows: Assume, as a first step, that after an increase of τ_e by $\Delta\tau_e$, each individual i is just compensated through an increase of net labour income x_i by $\Delta\tau_e e_i$. If $e_H > e_L$, the high-able individual experiences a larger increase of the net labour income than the less able which makes mimicking less attractive and gives slack to the self-selection constraint. As a consequence, in a second step additional redistribution from the high- to the low-able individuals becomes possible, which increases social welfare. In our model, this

[8] Mimicking refers to a situation where the high-able individual opts for the (x,z)-bundle designed for the low-able.

mechanism works as long as the social objective favours further redistribution; if the desired extent of redistribution via τ_e (or τ) is attained, μ becomes zero.

One may object to our model that assuming a fixed relation between (unobservable) abilities and (observable) initial endowments (or expenditures) makes the imposition of an income tax not reasonable from the beginning. Namely, the tax authority can use information on initial endowments (or on expenditures) to identify individuals, and then impose a tax on abilities directly, which is first-best. In reality, however, such a method of identification is not employed, and the main reason seems to be that endowments (or expenditures) are not precise indicators for earning abilities. By incorporating this idea in our model we will show in Section 3 that an accordingly modified version of Theorem 1 also holds when endowments are stochastic.

2.2 Taxation of inheritances and the welfare of future generations

As a next step we formulate a simple intertemporal model within which we discuss the optimum taxation of inheritance. We assume that the (static) economy described above represents the situation in some single period t and we take into account that bequests (and taxes on them) affect the welfare of future generations.

In view of the results of the foregoing subsection, the ultimate reason, why the intergenerational transfer of wealth may represent an object for taxation is that receiving inheritances creates a second distinguishing characteristic of the individuals, in addition to their earning abilities. In order to account for this, two possible instruments can be applied (in some period t):

- taxing inherited wealth e_{it} as a direct source of inequality within the receiving generation. That is, a tax τ_{et} is employed for generation t in our model.
- using a general expenditure tax (that is, in our terminology, a uniform tax τ_t on consumption c_{it} and bequests b_{it}) as a surrogate taxation of unequal inherited wealth e_{it} of the bequeathing generation t.

In a static framework, these two instruments proved equivalent (and lump-sum). We now ask what can be said in an intertemporal setting, that is, when effects on future generations are taken into account. Let arbitrary τ_{et}, τ_t be given (possibly zero). The government imposes an optimum income tax and considers a change of τ_{et}, τ_t. The revenues from τ_{et}, τ_t, run into the budget of this generation t and are redistributed through a reduced need for labour-income tax revenues.

We work with the indirect utility functions as before, now being defined as

$$v_t^i(x_{it}, z_{it}, e_{it}, \tau_{et}, \tau_t) \equiv \max\left\{u(c_{it}, b_{it}, z_{it} / \omega_i) \mid (1+\tau_t)(c_{it} + b_{it}) \le x_{it} + (1-\tau_{et})e_{it}\right\}.$$

The (inherited) endowments e_{it} of an individual i of generation t are exogenous. They arise as a result of some allocation of the sum of the bequests b_{it-1} left by the previous generation to the individuals of generation t. For the analysis of this Section, the rules guiding this allocation need not be specified.

On the other hand, the bequests $b_{it}(\cdot)$ left by generation t represent endowments for the individuals of the next generation t + 1 and enter their utility. Moreover, they also influence bequests left by generation t + 1 and, by this, utility of generation t + 2, and so on. We take account of all these effects through a very general formulation: we simply assume that (discounted) welfare of all future generations from t + 1 onwards can be described by some general (intertemporal) social welfare function $W(b_{Lt}, b_{Ht})$, which depends on the bequests left to generation t + 1.[9] In order to determine the tax rates in period t, the planner must incorporate how the tax rates influence future welfare, and this happens only via bequests of generation t in our model. Thus, W must be known to the planner, but it can be any suitable function.

Then the objective function of the planner to determine the optimum nonlinear income tax in period t reads

$$\max_{x_{it}, z_{it}} \sum_{i=L,H} f_i v_t^i(x_{it}, z_{it}, e_{it}, \tau_{et}, \tau_t) + (1+\gamma)^{-1} W(b_{Lt}(\cdot), b_{Ht}(\cdot)), \tag{5}$$

9 Obviously, if there is a positive rate of return on (bequeathed) capital, its welfare effect is also included in W.

where $\gamma > 0$ represents the planner's one-period discount rate. (5) is to be maximised subject to the resource constraint

$$x_{Lt} + x_{Ht} \leq z_{Lt} + z_{Ht} + \tau_{et}\left(e_{Lt} + e_{Ht}\right) + \tau_t\left(c_{Lt}(\cdot) + b_{Lt}(\cdot) + c_{Ht}(\cdot) + b_{Ht}(\cdot)\right) - g_t \tag{6}$$

and to the self-selection constraint

$$v_t^H(x_{Ht}, z_{Ht}, e_{Ht}, \tau_{et}, \tau_t) \geq v_t^H(x_{Lt}, z_{Lt}, e_{Ht}, \tau_{et}, \tau_t)\,. \tag{7}$$

Note again, that b_{Lt}, b_{Ht}, influenced by the income tax and by τ_{et}, τ_t, enter welfare W of future generations.[10] We find the surprising result that this effect plays no role for the desirability of τ_{et}, τ_t. Let $S^d(\tau_{et}, \tau_t)$ denote the optimum value of the maximisation of (5), subject to (6) and (7), and μ^d the Lagrange multiplier corresponding to the self-selection constraint (7):

Theorem 2: In a dynamic model, the welfare effect of a marginal increase of τ_e and τ, resp., reads:

(a) $\dfrac{\partial S^d}{\partial \tau_{et}} = \mu^d \dfrac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht} - e_{Lt})$,

(b) $\dfrac{\partial S^d}{\partial \tau_t} = \dfrac{\partial S^d}{\partial \tau_{et}} \dfrac{1 - \tau_{et}}{1 + \tau_t}$.

Hence, as in the static model, both taxes increase welfare, if the inheritance received by the high-able individual is larger than that received by the low-able.

Proof: see Appendix B.

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[10] To give a natural example for W: assume that all later generations consist of the same two types of individuals and in each period all bequests left by type i go to the same type i of the next generation ($e_{is} = b_{is-1}(1 + r_s)$ with r as the interest rate). We define $W(b_{Lt}, b_{Ht})$ as the maximum (discounted) future welfare, from t+1 onwards, for given b_{Lt}, b_{Ht}, if an optimum nonlinear income tax is imposed in each period, i.e.,

$$W(b_{Lt}, b_{Ht}) \equiv \max_{x_{is}, z_{is}} \sum_{s=t+1}^{\infty} (1+\gamma)^{t+1-s} \sum_{i=L,H} f_i v_s^i(\cdot)\,,$$

subject to the resource and the self-selection constraints (6) and (7), for every period s = t+1,…,∞. Then by differentiating the solution of (5)-(7) with respect to τ_{et}, τ_t we find the effect of a change of these tax rates on present and future welfare, given an optimum income tax in each period. Note that bequests $(1 + r_t)b_{it} = e_{it+1}$ of generation t enter $v_{t+1}^i(\cdot)$.

Thus, the dynamic character of the present problem does not change anything regarding the desirability of a tax on inherited endowments or on expenditures. Contrary to the intuition, the same condition as in the static case applies, though the tax on endowments (or expenditures) will affect (negatively) the amount of bequests left to the next generation. The reason is the simultaneous adaptation of the optimum non-linear income tax, as can be seen from an inspection of the proof of Theorem 2. Indeed, an increase in τ_{et} or τ_t allows an increase in net income from labour which can, for each individual, be designed in such a way that all other welfare consequences of the increase of τ_{et} (or τ_t), in particular, the consequences for the subsequent generations via bequests, cancel out, except the one appearing in Theorem 2(a). The latter effect, which operates via a change of the self-selection constraint, is positive, if the high-able individual also has a higher endowment, as discussed earlier.

This result may be interpreted as a rationale for the common idea that inheritance taxation serves the target of equality of opportunity. Its proponents implicitly assume that the group with the higher earning abilities also has higher inherited wealth. In the political decision it is also frequently taken as granted that taxation of bequests via an estate tax is an appropriate instrument for redistribution. However, when considered alone, an estate tax leads to a distortion of the bequest decision, which is avoided if all expenditures are taxed at a uniform rate.

A specifically interesting aspect of this "cancelling out" of all other welfare effects is that obviously the value of the social discount rate γ - the weight of future generations – plays no direct role for the desirability of τ_{et}, τ_t (it influences the magnitude of the Lagrange multiplier μ^d). Moreover, as mentioned in the Introduction, our result shows that the well-known "double-counting" of bequests, which usually in joy-of-giving models causes a counter effect against the introduction of an estate or inheritance tax (and in fact calls for a subsidy), can be ignored as well. The point is again that in an appropriate formulation it is not the specific use of the budget for leaving a bequest which is taxed, but the initial endowment.

However, it must be added that up to now we have considered the introduction or the increase of taxes on endowments and/or expenditures in some period t alone. What we had in mind was that a-priori there exists a sequence of generations (periods) $t' < t$, with fixed values of $\tau_{et'}$ and $\tau_{t'}$ (both possibly zero) and an optimum non-linear income tax in each period t'. In some period t, unexpected by the prior generations up to t-1 (i.e., after they have already made their decisions), the tax τ_{et} or τ_t is increased (or introduced), but only for that period t, and Theorem 2 describes the consequences. Obviously, the same logic applies, if – one period later – the taxes τ_{et+1} and/or τ_{t+1} are introduced, unexpected by the prior generations up to t.

As a final step of our analysis in this section, we now ask whether something changes, if taxes on endowments or expenditures are at the same time not only increased or introduced in some period t but also for the subsequent period t+1, and this is anticipated by the individuals of generation t.

The answer to this question depends on the interpretation of the bequest motive in our model: bequests are considered as some form of consumption which per-se provides utility to the bequeathing individual. Thus, concerning the expenditure tax, a simultaneous introduction of τ_t and τ_{t+1} will not change anything with the above analysis, because the expenditure tax in period t + 1 does, by definition, not change the value of the bequest b_{it} for the bequeathing individual i of generation t.

But the situation may be different when it comes to the tax on inherited endowments. Taking the bequest-as-consumption model literally, one might still argue that the introduction of τ_{et+1}, simultaneously to that of τ_{et}, again does not change anything with the above result, because individuals simply care for what they leave as (gross) bequests to their descendants. On the other hand, however, the bequeathing generation may be modelled as caring for net bequests, i.e. for $b_{it}^{net} \equiv b_{it}(1-\tau_{et+1})$, instead of gross bequests b_{it}[11]. Such a formulation means that bequeathing individuals only pay attention to the amount going directly to the descendants. They do not care for the revenues raised by τ_{et+1} (notice that these run into the public budget of the descendants' generation and may reduce their income tax burden).[12]

[11] Note that we use the expression "gross bequests" for b_{it} from the viewpoint of the receiving generation t+1, i.e. only in reference to the inheritance tax τ_{et+1}. For the bequeathing generation, however, b_{it} is pre-tax concerning the expenditure tax τ_t.

[12] Brunner (1997) analyses the case that tax revenues run into the budget of the parent generation in a mode, where the focus is on a specific tax on bequests.

With this formulation, the introduction of a tax τ_{et+1} on inheritances causes a negative effect on the bequest decision of the previous generation t, which has not been considered so far. To analyse this effect, we extend the problem (5) – (7) by adding τ_{et+1} as an argument of v_t^i, c_{it} and b_{it}. Let $\tilde{S}^d(\tau_{et}, \tau_t, \tau_{et+1})$ denote the optimum value function of the extended problem and $\tilde{\mu}_t^d$ the Lagrange multiplier corresponding to the self-selection constraint (in period t). The total welfare effect of an increase of τ_{et+1}, which is found by differentiation of the Lagrangian function (see Appendix B) reads :

$$\begin{aligned}\frac{\partial \tilde{S}^d}{\partial \tau_{et+1}} = \frac{1+\tau_t}{(1-\tau_{et+1})^2}[\sum_{i=L,H} -f_i b_{it}^{net}\frac{\partial v_t^i}{\partial x_{it}} - \tilde{\mu}_t^d(b_{Ht}^{net}\frac{\partial v_t^H}{\partial x_{Ht}} - b_{Ht}^{net}[L]\frac{\partial v_t^H[L]}{\partial x_{Lt}})] + \\ + (1+\gamma)^{-1}[\frac{\partial W}{\partial \tau_{et+1}} + \sum_{i=L,H}\frac{\partial W}{\partial b_{it}}\frac{\partial b_{it}}{\partial \tau_{et+1}}].\end{aligned} \tag{8}$$

The expression in the first square brackets of (8) shows us how generation t is affected. As can be seen from the first term, the increase (introduction, resp.) of a tax τ_{et+1} on inherited wealth in period t+1 has a direct negative effect on welfare of the previous, bequeathing generation t. (This is a result of double-counting in the social welfare function: in the present model the inheritance tax diminishes welfare of two generations (t and t+1), while the revenues from the tax and their redistribution to the individuals have a positive impact only on generation t+1.) The second term shows that the increase of τ_{et+1} also affects the self-selection constraint of generation t; its sign is undetermined for arbitrary preferences[13]. (Clearly, τ_{et+1} does not change the available resources in period t, therefore the resource constraint is unaffected.)

The expression in the second square brackets of (8) describes the welfare effect of the increase (introduction, resp.) of τ_{et+1} for generation t + 1 (and all future generations). It can be decomposed into two parts: The first, $\partial W/\partial \tau_{et+1}$, is the (positive) direct effect, analysed more precisely in Theorem 2(a) with respect to τ_{et}, now appearing one period later. The second part arises because individuals of generation t will adapt the amount of gross bequests left to their descendants. Taking $\partial W / \partial b_{it}$ as positive (bequests raise welfare of the subsequent generations), the sign of

[13] However, for quasilinear preferences (which will be introduced in Section 3) the sign is negative as well, because the marginal utility of net income is constant and individual H leaves less net bequests in case of mimicking. Then the increase of a tax τ_{et+1} makes mimicking more attractive for the high-able individual and, hence, reduces the scope for redistribution via the labour income tax.

$\sum(\partial W/\partial b_{it})(\partial b_{it}/\partial\tau_{et+1})$ depends on the reaction of gross bequests. They may increase or decrease, depending on the elasticity of net bequests $b_{it}^{net} = b_{it}(1-\tau_{et+1})$. In case of an elasticity of 1, as with Cobb-Douglas preferences over c_{it} and b_{it}^{net} (and separability with respect to labour time), gross bequests remain unchanged and, hence, the indirect effect on welfare of future generations is zero.

Altogether, we find that the positive welfare effect of an increase of the inheritance tax τ_{et+1} in period t+1 is reduced, if this increase is anticipated by the previous generation t and individuals care for net instead of gross bequests. The main reason for this is the twofold appearance of bequests in the social welfare function, besides, a distortion of the bequest decision occurs. As discussed above, these opposite effects do not arise in our model, if instead of the inheritance tax τ_{et+1} the expenditure tax τ_{t+1} is increased in period t+1.

3. Taxation of inheritances in a stochastic framework

As already mentioned, an objection against the models of Section 2 could be that with a fixed one-to-one relation between abilities and endowments it is possible to identify individuals by their endowments or by their expenditures (which we consider observable) and to impose a first-best tax. In reality, no tax authority follows this strategy, because the relation between endowments (or expenditures) and skills is not fixed, but stochastic. In order to capture this issue, we now assume that inherited endowments are random and prove a stochastic version of Theorem 2, where still a positive relation between endowments and abilities is decisive. Furthermore, we offer a theoretical argument for the plausibility of such a relation: It results as the outcome of a stochastic process of abilities, if a mild condition on the probabilities relating the possible realisations of the child's ability to the parent's ability holds.

In order to make the model tractable, we assume in this Section that the utility function (identical for all individuals) is quasilinear, i.e., $u(c,b,l) = \varphi(c,b) + \psi(l)$, where $\varphi: \mathbb{R}^2 \to \mathbb{R}$ is linear-homogeneous with $\partial\varphi/\partial c > 0$, $\partial\varphi/\partial b > 0$, and $\psi: \mathbb{R} \to \mathbb{R}$ is strictly concave with $\psi' < 0$. One observes immediately that for quasilinear utility the following statements hold for indirect utility and demand:[14]

(a) $\partial v / \partial x = \rho/(1+\tau)$. ρ is a constant for any ability ω and any x, z. $\partial v / \partial e = \rho(1-\tau_e)/(1+\tau)$.

(b) $\partial b / \partial z = \partial c / \partial z = 0$. Demand is independent of gross income and labour time.

(c) $c = \alpha_c(x+(1-\tau_e)e)/(1+\tau)$ and $b = \alpha_b(x+(1-\tau_e)e)/(1+\tau)$. α_c, α_b are the constant shares of consumption and bequests in the available budget, after correcting for τ, with $\alpha_c+\alpha_b = 1$. For later use, we define $\tilde{\alpha} \equiv \alpha_b/(1+\tau)$, $\hat{\alpha} \equiv \alpha_b(1-\tau_e)/(1+\tau)$.

The most important consequence of (a) is that the self-selection constraint is independent of income effects, that is, of inheritances (see (11) later on).

We generalise the model by introducing n (not just two) different types of individuals, characterised by their earning abilities ω_{it}, i = 1,…,n, with $\omega_{it} < \omega_{i+1t}$ in period t.

3.1. A stochastic relation between ability levels and inheritances

Let some tax rates τ_{et}, τ_t (possibly zero) be given in period t. At the beginning of this period the planning tax authority determines the optimum tax on labour income (that is, the optimum bundles x_{it}, z_{it}, i = 1, ..., n) and, possibly, a change of the tax rates τ_{et}, τ_t (or the introduction of these taxes).

When making the decision, the planner knows the ability levels ω_{1t}, ..., ω_{nt} of the individuals of generation t period, but cannot identify individuals. Moreover, we assume that the planner knows the aggregate amount of bequests, e_t^{agg}, left to the generation t in total (no uncertainty on aggregate resources in period t exists). There is, however, only a stochastic relation between the ability level and the amount of inheritances an individual receives. Thus, the planner cannot, even when the realisation of inheritances

[14] For simplicity we drop the indices referring to the types and periods, because the statements hold for individuals of any ability level ω in any period.

is known, infer the ability type of the receiving individual. (Nor is identification possible from the expenditures of an individual.)

More formally, we assume that there exist a (finite) number k of ways of how the aggregate amount e_t^{agg} may be distributed to the individuals of generation t, where each specific allocation j, j = 1,…k, occurs with probability κ_{jt} (with $\kappa_{1t} + ... + \kappa_{kt} = 1$) and transfers e_{it}^j to individual i, with $e_{1t}^j + ... + e_{nt}^j = e_t^{agg}$. The possible realisations and their probabilities are known.

Facing uncertainty, the planner wants to maximise expected social welfare in period t. With $f_1 > f_2 > ... > f_n$ being the weights of the different types in the social objective, the optimisation problem which determines the optimum income tax (that is, the bundles x_{it}, z_{it}) reads, for given τ_{et}, τ_t:

$$\max_{x_{it}, z_{it}} \sum_{j=1}^{k} \Big(\sum_{i=1}^{n} f_i v_t^i(x_{it}, z_{it}, e_{it}^j, \tau_{et}, \tau_t) \Big) \kappa_{jt} + (1+\gamma)^{-1} \sum_{j=1}^{k} W(b_{1t}^j, ..., b_{nt}^j) \kappa_{jt}, \tag{9}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{it} \le \sum_{i=1}^{n} z_{it} + \tau_{et} \sum_{j=1}^{k} \Big(\sum_{i=1}^{n} e_{it}^j \Big) \kappa_{jt} + \tau_t \sum_{j=1}^{k} \Big(\sum_{i=1}^{n} (c_{it}^j + b_{it}^j) \Big) \kappa_{jt} - g_t, \tag{10}$$

$$\frac{\rho}{1+\tau_t}(x_{it} - x_{i-1t}) \ge \psi\Big(\frac{z_{i-1t}}{\omega_i}\Big) - \psi\Big(\frac{z_{it}}{\omega_i}\Big), \quad i = 2, ..., n. \tag{11}$$

Here c_{it}^j, b_{it}^j denote consumption of individual i and bequests left by her in case that allocation j is realised. Moreover, similar to the formulation in Section 2, W describes how future social welfare is influenced by the bequests of generation t. We have assumed that only the self-selection constraints (11) for the respective higher-able individuals are relevant in the optimum.[15] This is justified, if the social objective implies downward redistribution, which follows from our assumption $f_i > f_{i+1}$.[16]

We have to check, whether this problem is well defined, that is, whether it can be solved by the planner without knowing the actual realisation of the endowments. For this, the constraints (10) and (11) must be independent of the actual realisation. As the

[15] It is well-known that only the self-selection constraints of pairs of individuals with adjacent ability levels need to be considered.

[16] Note that with quasilinear preferences the marginal utility of income is identical for all individuals, therefore a utilitarian objective with equal weights would not imply downward redistribution.

b_{it}^{j} do not appear in the self-selection constraints (11) (due to the consequence (a) of quasilinear utility, as already mentioned), the required property is clearly fulfilled for these constraints. Moreover, exchanging the order of summation in the resource constraint (10) and using the property (c) of quasilinear utility, it can be written as

$$\sum_{i=1}^{n} x_{it} \le \sum_{i=1}^{n} z_{it} + \tau_{et} e_t^{agg} + \frac{\tau_t}{1+\tau_t}[\sum_{i=1}^{n} x_{it} + (1-\tau_{et}) e_t^{agg}] - g_t. \tag{10'}$$

Thus, the resource constraint is independent of the particular realisation of the inheritances as well. Only the aggregate amount of inheritances matters, which we assume to be known. This proves

Lemma 2: The optimum bundles (x_{it}, z_{it}), i = 1, ..., n of problem (9) - (11) are independent of the particular realisation of individual endowments b_{it-1}.

To derive the following theorem, we need the assumption that W has some "quasilinear property", namely that, given any i, the derivatives $\partial W / \partial b_{it}^{j}$ are independent of j. In other words, the marginal welfare effect of an increase of an individual's bequests on the welfare of future generations is constant and is, in particular independent of the specific realisation of endowments. This is obviously fulfilled, if W is a discounted sum of future expected social welfare (see the remarks in footnote 10), with quasilinear individual utility in each period.

Let now $S^r(\tau_{et},\tau_t)$ be the optimum value of (9) subject to (10) and (11), for given τ_{et}, τ_t, and let $\bar{e}_{it}$ denote the expected value of the inheritances e_{it}^{j}, that individual i of generation t receives. As criteria for a change (or the introduction) of taxes on endowments and/or expenditures we find:

Theorem 3: With stochastic inheritances, the welfare effect of a marginal increase of τ_{et} and τ_t, resp., reads:

(a) $$\frac{\partial S^r}{\partial \tau_{et}} = \frac{\rho}{1+\tau_t} \sum_{i=2}^{n} \mu_i^r (\bar{e}_{it} - \bar{e}_{i-1t}),$$

(b) $$\frac{\partial S^r}{\partial \tau_t} = \frac{\partial S^r}{\partial \tau_{et}} \frac{1-\tau_{et}}{1+\tau_t}.$$

Proof: See Appendix C.

Thus, we arrive at a direct stochastic analogon of Theorem 2, referring to expected values instead of deterministic endowments. A sufficient (but not necessary) condition for the desirability of a tax on inheritances (or on expenditures) is that the order of expected inheritances is the same as the order of earning abilities, then the right-hand side of (a) is positive.

3.2. An intertemporal model with stochastic transition of abilities

In this subsection we want to provide a theoretical argument for the plausibility of the sufficient condition of Theorem 3. We do so by studying a stochastic process which determines how the relation between abilities and endowments evolves over time. The essential elements of the process we consider are the following:

(A) In each period t there exists the same number n of individuals with identical quasilinear utility, as introduced at the beginning of Section 3. They differ in their earning abilities, with order $\omega_{1t} < \omega_{2t} \ldots < \omega_{nt}$.[17]

(B) Each individual has a single descendant to which she leaves all her bequests.

(C) The order of ability levels of the descendants can be any permutation of the order of the parent individuals' abilities.

(D) In each period t the identical permutation, where each individual's ability is ranked just as her parent's ability (in period t – 1), has a higher probability p_{Et} than any other permutation. All other permutations occur with the same probability p_t, with $(n!-1)p_t = 1 - p_{Et}$.

(A) – (D) seem to be reasonable properties. In particular – as mentioned in footnote 5 – there is much empirical evidence indicating a positive correlation between children's and parents' earning abilities, which we capture by property (D).[18] Note that the process has the well-known property of “regression to the mean” in the following sense:

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[17] Thus, we allow any change of the ability levels, e.g., they could grow by some common growth rate.

[18] An alternative way would be to assume that the probability of a descendant's ability level having the same rank as the parent's is higher than the probability of having any other rank. This would imply our assumption of a higher probability of the identical permutation.

if we consider a parent with ability rank i in the upper half (i > (n + 1)/2), then the descendant's ability has a higher probability to be ranked below i, and vice versa.[19]

In addition, we assume that in each period t a tax system exists, consisting of a tax on labour income, inheritances and expenditures (all possibly zero). Individuals earn gross income z_{it} and net income x_{it} and choose c_{it}, b_{it}.

Generally, the transfer of wealth over generations and the stochastic nature of how abilities are linked to inheritances in each generation generate a very complex process, whose properties are difficult to analyse. The reason is that in each period the amount which an individual receives as inheritance depends on the combination of ability level and inheritance that characterised her parent, and the inheritance of the latter in turn depended on the combination characterising the grandparent and so on. Thus, the number of possible combinations grows rapidly over time.

The key observation, which allows us to derive a clear-cut result on the long-run stochastic properties of the distribution of inherited wealth and earning abilities, as introduced above, is the following: Assume that in some starting period 0 there are no initial endowments. With quasilinear preferences, each individual with ability level ω_{i0} leaves bequests $b_{i0} = \tilde{\alpha}_0 x_{i0}$ (remember the definition at the beginning of Section 3; we add a period index to indicate that $\tilde{\alpha}_t$ depends on the tax rate of the respective period) to her descendant with ability level ω_{j1}. The latter in turn, for whom $b_{i0}(1-\tau_{e1}) = e_{j1}(1-\tau_{e1})$ is part of the budget, bequeaths an amount $\hat{\alpha}_1 b_{i0} = \hat{\alpha}_1\tilde{\alpha}_0 x_{i0}$ out of b_{i0} to her descendant[20] (with some ability level ω_{m2}), who again leaves $\hat{\alpha}_2\hat{\alpha}_1\tilde{\alpha}_0 x_{i0}$ out of it, and so on.[21]

That is, each net income x_{i0} initiates an own series of bequests, which can, given quasilinear utility, be described by a simple formula. Obviously, this observation can be

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[19] As for the descendant any rank j ≠ i has the same probability $(n-1)!p_t$, the probability that her rank is lower than i is $(i-1)(n-1)!p_t$ for her, while that of a higher rank is $(n-i)(n-1)!p_t$. $i > (n+1)/2$ implies $i-1 > n-i$. See also the proof of Lemma 3 below.

[20] In addition, of course, the individual of type ω_{j1} also bequeaths $\tilde{\alpha}_1 x_{j1}$ out of her own net income.

[21] Here we have assumed that bequeathing individuals care for gross bequests b_{it}. If they anticipate the next period's inheritance tax and care for net bequests $b_{it}(1-\tau_{et+1})$, the respective definitions of $\tilde{\alpha}_t$ and $\hat{\alpha}_t$ continue to hold, but with a different value of the parameter α_b, which now depends on the inheritance tax τ_{et+1} of the next period.

generalised to later periods: Out of the net incomes x_{it} of that period, each generation t initiates a new series, which we call a *bequest series*, denoted by β_t. β_t consists of the elements β_{it}^{s}, where i indicates the ability level of the first bequeathing individual and s denotes the receiving generation, thus $\beta_{it}^{t+1} = \tilde{\alpha}_t x_{it}$ and $\beta_{it}^{s+1} = \hat{\alpha}_s \beta_{it}^{s}$ for s > t + 1. One observes immediately that each bequest series vanishes in the course of time, as all $\tilde{\alpha}_t, \hat{\alpha}_t < 1$. Note also that the ability levels of the receiving individuals of any generation t' > t do not influence the value of subsequent β_{it}^{s}, s > t'.

From the perspective of a receiving individual in some period s, her inheritance is the sum of what she receives through all bequest series β_t initiated by earlier generations. We study the joint evolution of a single bequest series and of the earning abilities. Let $P_{js}(\beta_{it}^{s})$ denote the probability that individual j in period s receives the bequest initiated by individual i in period t. An immediate consequence of the properties (A) - (D) is

Lemma 3: For any bequest series β_t there exist probabilities π_E^{t+1}, π^{t+1} for any i = 1,...,n, with the following properties:

(i) $P_{it+1}(\beta_{it}^{t+1}) = \pi_E^{t+1}$, $P_{it+1}(\beta_{jt}^{t+1}) = \pi^{t+1}$ for any $j \neq i$, $\pi_E^{t+1} + (n-1)\pi^{t+1} = 1$,

(ii) $\pi_E^{t+1} > \pi^{t+1}$.

Proof: See Appendix D.

When a bequest series begins, an individual of the first generation of heirs has a higher probability π_E^{t+1} of receiving the bequests left by a parent with identical ability rank than receiving the bequests of any other parent (with probability π^{t+1}).

The next Lemma shows that this property remains to hold over further generations, but becomes less pronounced:

Lemma 4: Assume that for some bequest series β_t and for some s > t probabilities $\pi_E^{s} > \pi^{s}$ exist, such that $\pi_E^{s} + (n-1)\pi^{s} = 1$ and $P_{is}(\beta_{it}^{s}) = \pi_E^{s}$, $P_{is}(\beta_{jt}^{s}) = \pi^{s}$, for any i = 1,…,n and $j \neq i$. Then there exist probabilities π_E^{s+1}, π^{s+1} with the properties:

(i) $P_{is+1}(\beta_{it}^{s+1}) = \pi_E^{s+1}$, $P_{is+1}(\beta_{jt}^{s+1}) = \pi^{s+1}$ for any i = 1,…,n and $j \neq i$,
$\pi_E^{s+1} + (n-1)\pi^{s+1} = 1$,

(ii) $\pi_E^{s+1} > \pi^{s+1}$,

(iii) $\pi_E^{s} > \pi_E^{s+1}$, $\pi^{s} < \pi^{s+1}$.

Proof: See Appendix D.

We know from Lemma 3 that, for any bequest series, an individual i of the first generation of heirs is more likely to receive the bequests left by an equally ranked parent than to receive any other bequests. Lemma 4 then tells us that this property is transferred to the next generation and from this, obviously, to the third generation and so on. However, the difference in probabilities becomes smaller with any additional transition and disappears eventually, as s goes to infinity. On the other hand, this equalisation occurs for lower and lower values of the transfers in a bequest series, as this series diminishes with $\hat{\alpha}_s < 1$. What dominates the inheritances received by some generation are the bequest series initiated by rather recent generations, which are more unequally distributed.

A consequence of the properties of the wealth transfer as described above is that for any bequest series the order of expected values of inheritances coincides with the order of ability levels, if in the initial period net incomes rise with abilities. Let $E_{is}[\beta_t]$ denote the expected value of the inheritance received by an individual with ability ω_{is} in period s from the bequest series β_t.

Lemma 5: Assume that $x_{it} < x_{i+1t}$. Then for any $s > t$, $E_{is}[\beta_t] < E_{i+1s}[\beta_t]$ for all $i = 1,...,n-1$.

Proof: See Appendix D.

Note that the condition $x_{it} < x_{i+1t}$ is indeed fulfilled, if preferences have the property AM (see Section 2.1) and marginal income tax rates are lower than 1, as we will assume in the following.

Observing finally that the inheritances received by the individuals of some generation s are the sum of what they receive out of the bequest series β_t initiated by all earlier generations, we arrive at the desired characterisation of the relation between expected inheritances $\bar{e}_{is}$ and ability levels ω_{is}:

Theorem 4: Assume that in period 0 there are no initial endowments. Then $\bar{e}_{is} < \bar{e}_{i+1s}$ for all periods s > 0 and all $i=1,...,n-1$.

Proof: See Appendix D.

Theorem 4 allows us to formulate a definite result on the desirability of a tax on endowments and on expenditures in our model. We consider an economy developing according to the stochastic process described by (A) – (D), where in each period a tax system may exist. Then, in some period s, the planner chooses an optimum nonlinear income tax and thinks of a change of the tax rates τ_{es}, τ_s. She aims at maximising present and (discounted) future welfare and knows the aggregate amount of inheritances received by generation s, and its possible distributions. Thus, (9) – (11) is the relevant optimisation problem and we find:

Theorem 5: Assume that in period 0 there are no initial endowments. Then in period s an increase of the taxes on inheritances and/or on expenditures, combined with an optimum nonlinear income, is desirable.

Proof: Combine Theorems 3 and 4.

Note that Theorem 5, as far as the inheritance tax is concerned, rests on the assumption that decisions of prior generations are already made, when the increase of τ_{et} is announced (see the discussion following Theorem 2).

4. Conclusion

In this paper we have clarified the role of inheritance taxation in an optimum-taxation framework with a bequest-as-consumption motive. In particular, we have worked out how different generations are affected by this tax. More generally, our results shed new light on the role of indirect taxes as well as of a tax on endowments in combination with an optimum nonlinear income tax. In our view, there are two main messages:

First, it is desirable according to a utilitarian social objective to shift some tax burden from labour income to endowments or expenditures, if there is a positive relation between earning abilities and endowments. From a theoretical point of view, this result is a consequence of the information constraint which motivates income taxation in the Mirrlees-model: if the tax authority could observe individual earning abilities, it would impose the tax directly on these, as a (differentiated) first-best instrument. It seems obvious, then, that the authority can improve the tax system by use of information on endowments (that is, taxing them), given that they are observable and correlated with abilities. (In fact, if the relation were negative, endowments should be subsidised.) Equivalently, a tax on all expenditures is also appropriate for this purpose.

The second message is that this result remains unchanged, even if the social welfare function accounts for effects on future generations: these effects cancel out when the optimum labour income tax is adapted accordingly. This is at least true, if a uniform tax on all expenditures including bequests is imposed, as a surrogate for a tax on inherited endowments. If inheritances are taxed directly and the parent individuals care for net instead of gross bequests, there occurs a negative effect on the previous generation because of "double-counting" of bequests.

Obviously, for the second message the assumption of the joy-of-giving motive for leaving bequests is important. With this motive, individuals care for the amount they leave to their descendants (and possibly for its reduction through an inheritance tax). However, they do not care for which purpose the descendants use their inheritance, nor, in particular, to which extent the descendants are subjected to a tax, when they use the inherited amount for own consumption as well as for bequests in favour of a further generation. This is a frequently made assumption and we indeed consider it realistic, given the extreme difficulty to forecast tax rates that will be imposed on later generations. It implies that a uniform tax on expenditures, including bequests, produces no negative effects for the parent generation.

Clearly, this assumption is opposed to the view that parents, when making their bequest decision, have a purely altruistic motive, that is, care for overall welfare of their descendants. As mentioned in the Introduction, perpetuating this view leads to dynastic preferences (Barro 1974) with an infinite time horizon of the planning individuals, that is, to a rather demanding model concerning the capacity of human decision making.

Finally, we have shown that the results on the taxation of inheritances remain essentially valid, if there is a stochastic instead of a deterministic connection between abilities and inheritances: taxation is desirable, if expected inheritances of higher-able individuals are larger. As a theoretical argument that this is indeed a plausible situation, we have shown that it arises as the outcome of a stochastic process, when the descendants' ability ranks are more likely to be the same as their parents' ranks than any other.

Throughout this paper we have assumed that earning abilities are exogenous. In reality, of course, they depend on human capital investments, which are financed out of the parents' budget, as are inheritances of non-human capital. Given that both increase with the budget, this provides an additional argument for the positive relation between abilities and inherited endowments within the generation of heirs.

When investigating the welfare consequences of the taxation of inheritances, we confined our analysis to a uniform expenditure tax and to a proportional tax on endowments, and proved that, in principle, they are equivalent. We did not consider the possibility that a differentiation of tax rates according to the type of expenditures might increase welfare further, as it does in the Atkinson-Stiglitz model. Moreover, also the welfare consequences of other tax schedules, for instance a linear (instead of a nonlinear) income tax or a nonlinear tax on inheritances, deserve further analysis.

Appendix A: Proof of Theorem 1

(a) The Lagrangian to the maximisation problem (2) – (4) reads

$$L = f_L v^L(x_L, z_L, e_L, \tau_e, \tau) + f_H v^H(x_H, z_H, e_H, \tau_e, \tau) -$$
$$- \lambda\left(x_L + x_H - z_L - z_H - \tau_e\left(e_L + e_H\right) - \tau\left(c_L(\cdot) + b_L(\cdot) + c_H(\cdot) + b_H(\cdot)\right) + g\right) +$$
$$+ \mu\left(v^H(x_H, z_H, e_H, \tau_e, \tau) - v^H(x_L, z_L, e_H, \tau_e, \tau)\right)$$

which gives us the first-order condition with respect to x_L, x_H, i = L,H (we use the abbreviation $v^H[L] \equiv v^H(x_L, z_L, e_H, \tau_e, \tau)$):

$$f_L \frac{\partial v^L}{\partial x_L} - \lambda + \lambda\tau(\frac{\partial c_L}{\partial x_L} + \frac{\partial b_L}{\partial x_L}) - \mu \frac{\partial v^H[L]}{\partial x_L} = 0\,, \tag{A1}$$

$$f_H \frac{\partial v^H}{\partial x_H} - \lambda + \lambda\tau(\frac{\partial c_H}{\partial x_H} + \frac{\partial b_H}{\partial x_H}) + \mu \frac{\partial v^H}{\partial x_H} = 0\,. \tag{A2}$$

Using the Envelope Theorem we get for the optimal value function $S(\tau_e, \tau)$

$$\begin{aligned} \frac{\partial S}{\partial \tau_e} = f_L \frac{\partial v^L}{\partial \tau_e} + f_H \frac{\partial v^H}{\partial \tau_e} + \lambda(e_L + e_H) + \lambda\tau(\frac{\partial c_L}{\partial \tau_e} + \frac{\partial b_L}{\partial \tau_e} + \frac{\partial c_H}{\partial \tau_e} + \frac{\partial b_H}{\partial \tau_e}) + \\ + \mu(\frac{\partial v^H}{\partial \tau_e} - \frac{\partial v^H[L]}{\partial \tau_e}). \end{aligned} \tag{A3}$$

We use $\partial v^i / \partial \tau_e = -e_i\, \partial v^i / \partial x_i$, $\partial v^H[L] / \partial \tau_e = -e_H\, \partial v^H[L] / \partial x_L$, $\partial c_i / \partial \tau_e = -e_i\, \partial c_i / \partial x_i$, $\partial b_i / \partial \tau_e = -e_i \partial b_i / \partial x_i$, compute $f_i\, \partial v^i / \partial x_i$, i = L,H, from (A1) and (A2) and transform, thus, (A3) to

$$\frac{\partial S}{\partial \tau_e} = \mu \frac{\partial v^H[L]}{\partial x_L})(e_H - e_L)\,. \tag{A4}$$

(b) We determine

$$\begin{aligned} \frac{\partial S}{\partial \tau} = f_L \frac{\partial v^L}{\partial \tau} + f_H \frac{\partial v^H}{\partial \tau} + \lambda(c_L + b_L + c_H + b_H) + \lambda\tau(\frac{\partial c_L}{\partial \tau} + \frac{\partial b_L}{\partial \tau} + \frac{\partial c_H}{\partial \tau} + \frac{\partial b_H}{\partial \tau}) + \\ + \mu \frac{\partial v^H}{\partial \tau} - \mu \frac{\partial v^H[L]}{\partial \tau}. \end{aligned} \tag{A5}$$

The individual i's budget equation can be written as $c_i + b_i = B_i$, where $B_i \equiv (x_i + (1 - \tau_e)e_i)/(1 + \tau)$. Thus, $\partial c_i / \partial \tau = (\partial c_i / \partial B_i)(\partial B_i / \partial \tau) = -(c_i + b_i)\partial c_i / \partial x_i$ (use

$\partial B_i / \partial \tau = -(x_i + (1-\tau_e)e_i)/(1+\tau)^2 = -(c_i + b_i)/(1+\tau))$ and $\partial c_i / \partial x_i = \partial c_i / \partial B_i /(1+\tau))$; equivalently $\partial b_i / \partial \tau = -(c_i + b_i)\partial b_i / \partial x_i$. Substituting these terms, together with $\partial v^i / \partial \tau = -(c_i + b_i)\partial v^i / \partial x_i$, $\partial v^H[L] / \partial \tau = -(c_H[L] + b_H[L])\partial v^H[L] / \partial x_L$ (where $c_H[L]$, $b_H[L]$, resp., denotes consumption and bequests of individual H, having L's gross and net income), and with (A1),(A2) into (A5) yields

$$\frac{\partial S}{\partial \tau} = \mu \frac{\partial v^H[L]}{\partial x_L}\left((c_H[L] + b_H[L]) - (c_L + b_L)\right). \tag{A6}$$

Inserting the (transformed) budget equations of individual H when mimicking and of individual L, i.e., $c_H[L] + b_H[L] = (x_L + (1-\tau_e)e_H)/(1+\tau)$ and $c_L + b_L = (x_L + (1-\tau_e)e_L)/(1+\tau)$, together with (A4), into (A6), we obtain

$$\frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial \tau_e}\frac{1-\tau_e}{1+\tau}. \qquad \text{QED}$$

Appendix B

Proof of Theorem 2

(a) From the Lagrangean to the optimisation problem (5) – (7) we derive the first-order conditions with respect to x_{Lt}, x_{Ht}, where λ^d, μ^d are the multipliers corresponding to the resource constraint and to the self-selection constraint, resp.:

$$f_L \frac{\partial v_t^L}{\partial x_{Lt}} + (1+\gamma)^{-1}\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial x_{Lt}} - \lambda^d + \lambda^d \tau_t\left(\frac{\partial c_{Lt}}{\partial x_{Lt}} + \frac{\partial b_{Lt}}{\partial x_{Lt}}\right) - \mu^d \frac{\partial v_t^H[L]}{\partial x_{Lt}} = 0, \tag{B1}$$

$$f_H \frac{\partial v_t^H}{\partial x_{Ht}} + (1+\gamma)^{-1}\frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial x_{Ht}} - \lambda^d + \lambda^d \tau_t\left(\frac{\partial c_{Ht}}{\partial x_{Ht}} + \frac{\partial b_{Ht}}{\partial x_{Ht}}\right) + \mu^d \frac{\partial v_t^H}{\partial x_{Ht}} = 0. \tag{B2}$$

The derivative of the optimum-value function S^d with respect τ_{et} is found by differentiating the Lagrangean:

$$\begin{aligned}\frac{\partial S^d}{\partial \tau_{et}} = {} & f_L \frac{\partial v_t^L}{\partial \tau_{et}} + f_H \frac{\partial v_t^H}{\partial \tau_{et}} + (1+\gamma)^{-1}\left(\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial \tau_{et}} + \frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial \tau_{et}}\right) + \lambda^d(e_{Lt} + e_{Ht}) + \\ & + \lambda^d \tau_t\left(\frac{\partial c_{Lt}}{\partial \tau_{et}} + \frac{\partial b_{Lt}}{\partial \tau_{et}} + \frac{\partial c_{Ht}}{\partial \tau_{et}} + \frac{\partial b_{Ht}}{\partial \tau_{et}}\right) + \mu^d\left(\frac{\partial v_t^H}{\partial \tau_{et}} - \frac{\partial v_t^H[L]}{\partial \tau_{et}}\right).\end{aligned} \tag{B3}$$

By use of the formulas below (A3), (B3) can be transformed to

$$\frac{\partial S^d}{\partial \tau_{et}} = -f_L e_{Lt}\frac{\partial v_t^L}{\partial x_{Lt}} - f_H e_{Ht}\frac{\partial v_t^H}{\partial x_{Ht}} + (1+\gamma)^{-1}(-e_{Lt}\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial x_{Lt}} - e_{Ht}\frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial x_{Ht}}) +$$
$$+\lambda^d(e_{Lt}+e_{Ht}) + \lambda^d\tau_t[-e_{Lt}(\frac{\partial c_{Lt}}{\partial x_{Lt}} + \frac{\partial b_{Lt}}{\partial x_{Lt}}) - e_{Ht}(\frac{\partial c_{Ht}}{\partial x_{Ht}} + \frac{\partial b_{Ht}}{\partial x_{Ht}})] - \tag{B4}$$
$$-\mu^d e_{Ht}(\frac{\partial v_t^H}{\partial x_{Ht}} - \frac{\partial v_t^H[L]}{\partial x_{Lt}}).$$

Multiplying (B1), (B2) by e_{Lt}, e_{Ht}, resp., and substituting into (B4) gives us

$$\frac{\partial S^d}{\partial \tau_{et}} = \mu^d \frac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht} - e_{Lt}).$$

(b) Differentiating the Lagrangean of problem (5) - (7) with respect to τ_t gives:

$$\frac{\partial S^d}{\partial \tau_t} = f_L\frac{\partial v_t^L}{\partial \tau_t} + f_H\frac{\partial v_t^H}{\partial \tau_t} + (1+\gamma)^{-1}(\frac{\partial W}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial \tau_t} + \frac{\partial W}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial \tau_t}) +$$
$$+\lambda^d[c_{Lt} + b_{Lt} + c_{Ht} + b_{Ht} + \tau_t(\frac{\partial c_{Lt}}{\partial \tau_t} + \frac{\partial b_{Lt}}{\partial \tau_t} + \frac{\partial c_{Ht}}{\partial \tau_t} + \frac{\partial b_{Ht}}{\partial \tau_t})] + \tag{B5}$$
$$+\mu^d(\frac{\partial v_t^H}{\partial \tau_t} - \frac{\partial v_t^H[L]}{\partial \tau_t}).$$

By use the formulas below (A5), (B5) can be transformed to

$$\frac{\partial S^d}{\partial \tau_t} = \sum_{i=L,H}\{-f_i(c_{it} + b_{it})\frac{\partial v_t^i}{\partial x_{it}} - (1+\gamma)^{-1}((c_{it} + b_{it})\frac{\partial W}{\partial b_{it}}\frac{\partial b_{it}}{\partial x_{it}} +$$
$$+\lambda^d[c_{it} + b_{it} - \tau_t(c_{it} + b_{it})(\frac{\partial c_{it}}{\partial x_{it}} + \frac{\partial b_{it}}{\partial x_{it}}]\} - \tag{B6}$$
$$-\mu^d(c_{Ht} + b_{Ht})\frac{\partial v_t^H}{\partial x_{Ht}} + \mu^d(c_{Ht}[L] + b_{Ht}[L])\frac{\partial v_t^H[L]}{\partial x_{Lt}}).$$

Multiplying (B1), (B2) by $(c_{Lt} + b_{Lt})$, $(c_{Ht} + b_{Ht})$, resp., and substituting into (B6) gives us

$$\frac{\partial S^d}{\partial \tau_t} = \mu^d \frac{\partial v_t^H[L]}{\partial x_{Lt}}(c_{Ht}[L] + b_{Ht}[L] - c_{Lt} - b_{Lt}),$$

or, as shown in the proof of Theorem 1(b),

$$\frac{\partial S^d}{\partial \tau_t} = \frac{\partial S}{\partial \tau_{et}} \frac{1-\tau_{et}}{1+\tau_t}.$$ QED

Derivation of Formula (8)

If individuals of generation t care for net bequests, indirect utility depends also on τ_{et+1}:

$$\begin{aligned} &v_t^i(x_{it}, z_{it}, e_{it}, \tau_{et}, \tau_t, \tau_{et+1}) \equiv \\ &\qquad \max\left\{u(c_{it}, b_{it}^{net}, z_{it}/\omega_i) \mid (1+\tau_t)(c_{it} + b_{it}^{net}/(1-\tau_{et+1})) \leq x_{it} + (1-\tau_{et})e_{it}\right\} \end{aligned}$$

Obviously, $c_{it}(\cdot)$ and net bequests $b_{it}^{net}(\cdot)$ depend on the same arguments as $v_t^i(\cdot)$. For generation $t+1$ as a whole, gross bequests $b_{it}(\cdot) = b_{it}^{net}(\cdot)/(1-\tau_{et+1})$ are relevant, because tax revenues are redistributed to the members of generation $t+1$. The maximisation problem (5) – (7) of the planner in period t changes to

$$\max_{x_{it}, z_{it}} \sum_{i=L,H} f_i v_t^i(\cdot) + (1+\gamma)^{-1} W(\tau_{et+1}, b_{Lt}(\cdot), b_{Ht}(\cdot)) \tag{B7}$$

$$\text{s.t.} \quad \sum_{i=L,H} x_{it} \leq \sum_{i=L,H} [z_{it} + \tau_{et} e_{it} + \tau_t (c_{it}(\cdot) + b_t^{net}(\cdot)/(1-\tau_{et+1}))] - g_t \tag{B8}$$

$$v_t^H(x_{Ht}, z_{Ht}, e_{Ht}, \tau_{et}, \tau_t, \tau_{et+1}) \geq v_t^H(x_{Lt}, z_{Lt}, e_{Ht}, \tau_{et}, \tau_t, \tau_{et+1}) \tag{B9}$$

Note that τ_{et+1} has a direct effect on the welfare W of generation t + 1 and all future generations, as well as an indirect effect via the gross bequests, left by generation t.

Using the Envelope Theorem we get for the optimum value function $\tilde{S}^d(\tau_{et}, \tau_t, \tau_{et+1})$ of the maximisation problem (B7) – (B9) ($\tilde{\lambda}_t^d, \tilde{\mu}_t^d$ are the multipliers corresponding to the (B8) and (B9), resp.)

$$\begin{aligned} \frac{\partial \tilde{S}^d}{\partial \tau_{et+1}} = &\sum_{i=L,H} f_i \frac{\partial v_t^i}{\partial \tau_{et+1}} + (1+\gamma)^{-1}\left(\frac{\partial W}{\partial \tau_{et+1}} + \sum_i \frac{\partial W}{\partial b_{it}} \frac{\partial b_{it}}{\partial \tau_{et+1}}\right) + \\ &+ \tilde{\lambda}_t^d \tau_t \sum_i \left(\frac{\partial c_{it}}{\partial \tau_{et+1}} + \frac{b_{it}^{net}}{(1-\tau_{et+1})^2} + \frac{1}{(1-\tau_{et+1})} \frac{\partial b_{it}^{net}}{\partial \tau_{et+1}}\right) + \\ &+ \tilde{\mu}_t^d \left(\frac{\partial v_t^H}{\partial \tau_{et+1}} - \frac{\partial v_t^H[L]}{\partial \tau_{et+1}}\right). \end{aligned} \tag{B10}$$

Substituting the demand functions $c_{it}(\cdot)$, $b_{it}^{net}(\cdot)$ into the (transformed) budget equation of an individual i and differentiating $c_{it}(\cdot)+b_{it}^{net}(\cdot)/(1-\tau_{et+1})=x_{it}(1-\tau_{et})e_{it}/(1+\tau_t)$ with respect to we τ_{et+1} obtain

$$\frac{\partial c_{it}}{\partial \tau_{et+1}}+\frac{b_{it}^{net}}{(1-\tau_{et+1})^2}+\frac{1}{(1-\tau_{et+1})}\frac{\partial b_{it}^{net}}{\partial \tau_{et+1}}=0\,. \tag{B11}$$

Using (B11), together with Roy's Lemma, viz. $\partial v_t^i/\partial \tau_{et+1}=-\left((1+\tau_t)b_{it}^{net}/(1+\tau_{et+1})^2\right)\partial v_t^i/\partial x_{it}$, gives us formula (8). QED

Appendix C: Proof of Theorem 3

(a) From the Lagrangean to the problem (9), (10'), (11), we derive the first-order conditions for the optimum x_{it}, i = 1,...,n, where λ^r, μ_i^r, i = 2,...,n are the multipliers corresponding to the resource constraint and the self-selection constraints, respectively (remember that $\partial v/\partial x=\rho/(1+\tau_t)$, $\partial c/\partial x=\alpha_c/(1+\tau)$, $\partial b/\partial x=\alpha_b/(1+\tau)$ and $\alpha_c+\alpha_b = 1$):

$$\frac{f_1\rho}{1+\tau_t}+(1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{j=1}^{k}\frac{\partial W}{\partial b_{1t}^j}\kappa_{jt}-\lambda^r+\lambda^r\frac{\tau_t}{1+\tau_t}-\frac{\mu_2^r\rho}{1+\tau_t}=0, \tag{C1}$$

$$\begin{aligned}&\frac{f_i\rho}{1+\tau_t}+(1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{j=1}^{k}\frac{\partial W}{\partial b_{it}^j}\kappa_{jt}-\lambda^r+\lambda^r\frac{\tau_t}{1+\tau_t}+\frac{\mu_i^r\rho}{1+\tau_t}-\\ &\quad-\frac{\mu_{i+1}^r\rho}{1+\tau_t}=0, \qquad\qquad i=2,\ldots,n-1,\end{aligned} \tag{C2}$$

$$\frac{f_n\rho}{1+\tau_t}+(1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{j=1}^{k}\frac{\partial W}{\partial b_{nt}^j}\kappa_{jt}-\lambda^r+\lambda^r\frac{\tau_t}{1+\tau_t}+\frac{\mu_n^r\rho}{1+\tau_t}=0. \tag{C3}$$

Next we consider the derivative of the Lagrangean with respect to τ_{et}:

$$\frac{\partial S^r}{\partial \tau_{et}}=\sum_{j=1}^{k}\sum_{i=1}^{n}f_i\frac{\partial v_t^i}{\partial \tau_{et}}\kappa_{jt}+(1+\gamma)^{-1}\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{\partial W}{\partial b_{it}^j}\frac{\partial b_{it}^j}{\partial \tau_{et}}\kappa_{jt}+\lambda^r e_t^{agg}-\lambda^r\frac{\tau_t}{1+\tau_t}e_t^{agg} \tag{C4}$$

Using $\partial v_t^i(\cdot,e_{it}^j,\cdot)/\partial\tau_{et}=-e_{it}^j\partial v_t^i/\partial x_{it}=-e_{it}^j\rho/(1+\tau_t)$ and $\partial b_{it}^j/\partial\tau_{et}=-e_{it}^j\alpha_b/(1+\tau_t)$, (C4) reads

$$\frac{\partial S^r}{\partial \tau_{et}} = -\frac{\rho}{1+\tau_t}\sum_{i=1}^{n} f_i \bar{e}_{it} - (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\sum_{i=1}^{n}\bar{e}_{it}\frac{\partial W}{\partial b_{it}} + \lambda^r e_t^{agg} - \lambda^r \frac{\tau_t}{1+\tau_t} e_t^{agg}. \qquad \text{(C5)}$$

Here we have used the property that $\partial W / \partial b_{it}^j$ is assumed independent of j, as mentioned in the text (we write $\partial W / \partial b_{it}$). Using this property again in (C1) – (C3) and multiplying each equation by the appropriate $\bar{e}_{it}$ gives

$$-\frac{f_1 \rho}{1+\tau_t}\bar{e}_{1t} = (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\bar{e}_{1t}\frac{\partial W}{\partial b_{1t}} - \lambda^r \bar{e}_{1t} + \lambda^r \frac{\tau_t}{1+\tau_t}\bar{e}_{1t} - \frac{\mu_2^r \rho}{1+\tau_t}\bar{e}_{1t}, \qquad \text{(C6)}$$

$$\begin{aligned} -\frac{f_i \rho}{1+\tau_t}\bar{e}_{it} = (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\bar{e}_{it}\frac{\partial W}{\partial b_{it}} - \lambda^r \bar{e}_{it} + \lambda^r \frac{\tau_t}{1+\tau_t}\bar{e}_{it} + \frac{\mu_i^r \rho}{1+\tau_t}\bar{e}_{it} - \\ -\frac{\mu_{i+1}^r \rho}{1+\tau_t}\bar{e}_{it}, \qquad\qquad i = 2,\ldots,n-1, \end{aligned} \qquad \text{(C7)}$$

$$-\frac{f_n \rho}{1+\tau_t}\bar{e}_{nt} = (1+\gamma)^{-1}\frac{\alpha_b}{1+\tau_t}\bar{e}_{nt}\frac{\partial W}{\partial b_{nt}} - \lambda^r \bar{e}_{nt} + \lambda^r \frac{\tau_t}{1+\tau_t}\bar{e}_{nt} + \frac{\mu_n^r \rho}{1+\tau_t}\bar{e}_{nt}. \qquad \text{(C8)}$$

Substituting (C6) – (C8) into (C5) and observing that, by assumption

$$\sum_{i=1}^{n}\bar{e}_{it} = \sum_{i=1}^{n}\sum_{j=1}^{k} e_{it}^j \kappa_{jt} = \sum_{j=1}^{k}\kappa_j \sum_{i=1}^{n} e_{it}^j = e_t^{agg},$$

gives

$$\frac{\partial S^r}{\partial \tau_{et}} = \frac{\rho}{1+\tau_t}\sum_{i=2}^{n}\mu_i^r(\bar{e}_{it} - \bar{e}_{i-1t}).$$

(b) The proof of Theorem 3(b) is analogous. QED

Appendix D

Proof of Lemma 3

(i) There are (n - 1)! permutations that have the property that the descendant of an individual with ability rank i has the same rank. One of these permutations is the identical, which has probability p_{Et+1}, while the others have probability p_{t+1}, therefore

$$P_{it+1}(\beta_{it}^{t+1}) = p_{Et+1} + [(n-1)! - 1]p_{t+1} \equiv \pi_E^{t+1} \tag{D1}$$

Analogously, there are (n - 1)! permutations with the property that a descendant with rank i has a parent of some rank $j \neq i$. All these permutations have probability p_{t+1}, thus

$$P_{it+1}(\beta_{jt}^{t+1}) = (n-1)!\,p_{t+1} \equiv \pi^{t+1}. \tag{D2}$$

Using the definitions (D1) and (D2), one checks immediately that

$$\begin{aligned}\pi_E^{t+1} + (n-1)\pi^{t+1} &= p_{Et+1} + [(n-1)! - 1]p_{t+1} + (n-1)(n-1)!\,p_{t+1} \\ &= p_{Et+1} + n!\,p_{t+1} - p_{t+1} = 1,\end{aligned}$$

where the latter follows from assumption (D).

(ii) $\pi_{Et+1}^{t+1} > \pi^{t+1}$ is equivalent to $p_{Et+1} - p_{t+1} > 0$, which holds by assumption. QED

Proof of Lemma 4

(i) Considering only the last transition, there are two ways for a type-i individual to receive, in period $s+1$, the bequest left by an identically ranked individual in some period t < s: Either from the type-i individual in period s (who has received the i-bequest with probability π_E^s) or from some other (type-j) individual in period s (who has received the i-bequest with probability π^s). Therefore (remember the first paragraph of the proof of Lemma 3)

$$P_{is+1}(\beta_{it}^{s+1}) = \pi_E^s[p_{Es+1} + ((n-1)! - 1)p_{s+1}] + (n-1)\pi^s(n-1)!\,p_{s+1} \equiv \pi_E^{s+1}. \tag{D3}$$

Analogously, the three ways for a type-i individual in period $s+1$ to receive the bequest left by some type-j individual in period t < s are: Either from the type-i individual in period s or from the type-j individual in period s or from any other individual ($\neq$ i,j) in period s. Therefore

$$\begin{aligned}P_{is+1}(\beta_{jt}^{s+1}) = \pi^s[p_{Es+1} + ((n-1)! - 1)p_{s+1}] + \pi_E^s(n-1)!\,p_{s+1} + \\ + (n-2)\pi^s(n-1)!\,p_{s+1} \equiv \pi^{s+1}.\end{aligned} \tag{D4}$$

Using the definitions (D3) and (D4), we obtain, by appropriate grouping,

$$\pi_E^{s+1} + (n-1)\pi^{s+1} = \pi_E^s p_{Es+1} + \pi_E^s p_{s+1}[(n-1)! - 1 + (n-1)(n-1)!] + \pi^s p_{Es+1}(n-1) + \\ + \pi^s p_{s+1}(n-1)[(n-1)! + (n-1)! - 1 + (n-2)(n-1)!] \\ = \pi_E^s [p_{Es+1} + p_{s+1}(n! - 1)] + \pi^s (n-1)[p_{Es+1} + p_{s+1}(n! - 1)]$$

which is equal to 1, as $p_{Es+1} + (n! - 1)p_{s+1} = 1$ and $\pi_E^s + (n-1)\pi^s = 1$.

(ii) Straightforward transformations show that $\pi_E^{s+1} > \pi^{s+1}$ is equivalent to $\pi_E^s(p_{Es+1} - p_{s+1}) > \pi^s(p_{Es+1} - p_{s+1})$, which holds, because $\pi_E^s > \pi^s$ and $p_{Es+1} > p_{s+1}$.

(iii) Finally, π_E^{s+1} is a convex combination of π_E^s and π^s, because by definition $\pi_E^{s+1} = \pi_E^s[p_{Es+1} + ((n-1)! - 1)p_{s+1}] + \pi^s(n-1)(n-1)!\,p_{s+1}$ (see (D3)) and the sum of the coefficients of π_E^s and π^s is

$$p_{Es+1} - p_{s+1} + (n-1+1)(n-1)!\,p_{s+1} = p_{Es+1} + (n! - 1)p_{s+1} = 1.$$

Thus, π_E^{s+1} lies between π_E^s and π^s. As $\pi_E^s > \pi^s$, we conclude that $\pi_E^{s+1} < \pi_E^s$. From $\pi_E^s + (n-1)\pi^s = 1$ as well as $\pi_E^{s+1} + (n-1)\pi^{s+1} = 1$, it follows that $\pi^{s+1} > \pi^s$.

QED

Proof of Lemma 5

We have seen in the main text that a bequest series β_t, initiated in t as $b_{it} = \tilde{\alpha}_t x_{it}$, leads to net inheritances Γx_{it} in period s with $\Gamma \equiv \tilde{\alpha}_t \prod_{s'=t+1}^{s-1} \hat{\alpha}_{s'}$.

Therefore $E_{is}[\beta_t] < E_{i+1s}[\beta_t]$ is equivalent to

$$\pi_E^s \Gamma x_{it} + \sum_{j \neq i} \pi^s \Gamma x_{jt} < \pi_E^s \Gamma x_{i+1t} + \sum_{m \neq i+1} \pi^s \Gamma x_{mt}$$

and further to

$$\pi_E^s x_{it} + \pi^s x_{i+1t} < \pi_E^s x_{i+1t} + \pi^s x_{it}.$$

The validity of the latter relation follows from $\pi_E^s > \pi^s$ and $x_{it} < x_{i+1t}$. QED

Proof of Theorem 4

The inheritances of an individual i in period s can be written as being the sum of all bequest series initiated in periods t < s. That is,

$$\overline{e}_{is} = E_{is}[\sum_{t=0}^{s-1} \beta_t] = \sum_{t=0}^{s-1} E_{is}[\beta_t].$$

Therefore, $\overline{e}_{is} < \overline{e}_{i+1s}$ follows immediately from Lemma 5. QED

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