Charity Auctions for the Happy Few

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Abstract

Recent literature has shown that all-pay auctions raise more money for charity than winner-pay auctions. We demonstrate that the first and second-price winner-pay auctions generate higher revenue than first-price all-pay auctions when bidders are sufficiently asymmetric. To prove it, we consider a framework with complete information. This analysis is relevant for two main reasons. On the one hand, complete information is more realistic and corresponds to events which occur for instance in a local service club (like in a voluntary organization) or in a show business dinner. Potential bidders are acquaintances or know one another well. On the other hand, our model keeps the qualitative predictions of a private value model under incomplete information in which bidders are ex ante asymmetric, which means that bidders' values are drawn from different distributions. Furthermore, we also analyze second-price all-pay auction. Finally, we show that individual minimum bids could improve the relative revenue performance of first-price all-pay compared to first-price winner-pay auction.

JEL Code: D44, D62, D64.

Keywords: all-pay auctions, charity, complete information, externalities.

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1 Introduction

More and more voluntary organizations wish to raise money for charity purposes through a partnership with firms. Charity auctions have been held in the United States for many years now. However, in China this phenomenon has emerged recently and is in strong progress.[1] In this kind of auction, an object (for example a key case with a zero value or an item given by a luxury brand) is sold. The proceeds then go to charity. Most of these auctions are planned and organized in charity dinners where only wealthy or famous people can participate. Beyond the item value, the valuations of potential bidders depend on their interest for this voluntary organization (their altruism or philanthropy) and also show some kind of conformism "to be seen as the most wealthy and generous". For instance, in China's traditional society, charity auctions were not put forward. The participants preferred to keep a low profile about their bids. However, time has changed: the rich and famous now show their wealth through their involvement in charity auctions. According to the *Beijing Review*:

> *With the development of society, more rich people are emerging. They have their own lifestyle [...] Some day, behind the rich lifestyle, people will find that it is only by offering their love and generosity that they can realize their true class.*

Thus, through charity auctions, potential bidders can build their position in their social class. Everybody wishes, independently of the winner's identity, to raise the highest revenue. Potential bidders make a trade-off between giving money for the fund-raising and keeping it for another personal use. Contrary to non-charity auctions, here the amount paid is "never lost". A wealthy investor, who bought a Dior perfume for 60 000 yuans (about 6 000 euros or 7 700 dollars) – with a reserve price of 20 000 yuans – recently said in the *Beijing Review*:

> *I would never buy perfume for this amount normally, but this time it is for charity. I feel very happy.*

In fact, the money raised will be used to finance a public good. Every participant of the charity auction may take advantage of it, independently of the winner's identity. More precisely, the money raised by each potential bidder impacts the utility of all participants as they take advantage of an externality of the amount of the money raised for the public good or the charity purpose.

Under complete information, these kinds of auctions can be compared to the work of Ettinger (2002) who analyzed a general winner-pay auction framework with two kinds of externalities.[2] One of them does not depend on the winner's identity and can be applied to charity auctions where only the winner pays. Moreover, he shows that there is no "revenue equivalence" with these externalities. Maasland and Onderstal (2006) investigate winner-pay auctions with this kind of linear externalities in an independent private signals model. Their paper can also be applied to charity. They find similar qualitative predictions as Ettinger (2002): the second-price winner-pay auction can outperform[3] the first-price winner-pay auction. In their recent paper, Goeree et al. (2005) analyze charity auctions in the symmetric independent private values model. They

[1] For example, in 2004, at the Formula One Grand Prix opening dinner party in Shanghai (China), *an auction was held of racing suits and crash helmets used by famous racing drivers* (*Beijing Review*, 2005).

[2] To the best of our knowledge, Ettinger (2002) is the only one to consider general externalities which could be non-linear.

[3] In the following, *outperform* means *generate higher revenue*.

show that, given the externality, all-pay auctions raise more money for charity than winner-pay auctions (second-price outperforms first-price) and lotteries. In particular, they determine that the optimal fund-raising mechanism is given by the lowest-price all-pay auction with an entry fee and a reserve price. Engers and McManus (2007) also find closely results to Goeree et al. (2005).[4] Contrary to Goeree et al. (2005), a psychological effect comes into play: the winner benefits from a higher externality with his own bid, the others' bids having a lower effect on him. In their setting, as in Goeree et al. (2005), first-price all-pay auctions and second-price winner-pay auctions are better to raise money than first-price winner-pay auctions. Moreover, first-price all-pay auctions outperform each winner-pay auction only for a sufficiently high number of bidders. Additionally, Engers and McManus (2007) show that there are many optimal charity auctions, among them for example a first-price winner-pay auction with the suitable fees and cancelling threat.

The predictions of Goeree et al. (2005) and Engers and McManus (2007) have been tested experimentally with contradictory results. Onderstal and Schram (2008) have experimented the Goeree et al. (2005)'s result in a laboratory. They are the first to conduct a lab experiment for charity auctions in an independent private value setting. Their results are close to the theoretical predictions: in charity auction, first-price all-pay auction raises higher revenues than other mechanisms (first-price winner-pay auction and lotteries). Carpenter et al. (2008) have tested the predictions of Engers and McManus (2007) and Goeree et al. (2005) in a field experiment. Similar objects are sold in four American pre-schools through three different mechanisms which are first-price all-pay auction, first-price and second-price winner-pay auctions. They study the determinants of the bidders' behavior and the revenue raised. Contrary to the theoretical predictions, first-price all-pay auctions do not produce higher revenues than the winner-pay auctions. Therefore, if auction theory about charity is confirmed in the laboratory, it is not the case in the field. The main explanation for the gap between theory and field experiment can be a non-participation effect, due to the unfamiliarity with these mechanisms and their complexity: the participants didn't know the all-pay design and few took part in second-price auctions on the Internet.

The purpose of this paper is to determine whether or not all-pay auctions can raise higher revenue for charity than winner-pay auctions when the asymmetry between bidders is strong. We consider a complete information framework. As we said before, a lot of charity auctions are conducted among rich people during charity dinners. These events could occur in a local service club (like the Rotary club[5] or another type of voluntary organization) or during a show business dinner. Potential bidders are acquaintances or know one another well. Consequently, a complete information environment is well suited for these kinds of situation. Thus, the paper of Goeree et al. (2005) is revisited with asymmetric bidders in a complete information framework. Our model keeps the qualitative predictions of a private value model under incomplete information in which bidders are *ex ante* asymmetric, which means that bidders' values are drawn from different distributions.

[4]Besides, Engers and McManus (2007) also introduce fees and reserve prices. Then distinguish the issues where the auctioneer can or cannot threaten to cancel the auction, which change their results.

[5]*The Rotary club is a worldwide organization of business and professional leaders that provides humanitarian services, encourages high ethical standards in all vocations, and helps build goodwill and peace in the world.* There are about 32 000 clubs in 200 countries and geographical areas and 1,000 clubs in France like Paris, but also in small town like Niort. http://www.rotary.org/

Following the work of Vartiainen (2007), we analyze all-pay auctions for charity as a mechanism. This approach relies on a general model which can be applied to both first and second-price all-pay auctions. In our setting, every bidder takes as much advantage of his own bid as of her rival's bid thanks to the externalities. Additionally, we defined the bidder i's adjusted-value as the ratio of her value for the item sold and the fraction of her payment which she perceives as a cost given her altruism for the charity purpose. Then, we arrange bidders such as adjusted-values and the valuations are ranked in the same order. We discuss this ranking and its consequences.

First-price all-pay auction equilibrium is characterized and the expected revenue computed; but there is no pure strategy Nash equilibrium. As in a case without externality, only the two bidders with the highest adjusted-values are active. In order to raise money for charity, we set up an optimal lobbying policy based on two steps. We also show the existence of a Nash equilibrium with non-linear externality.

The equilibrium is also characterized and the expected revenue computed for the second-price all-pay auction. In that case, the pure strategy Nash equilibria are degenerated. That is why we determine the mixed strategy Nash equilibrium. We discuss our results by comparing them to Ettinger (2002) who analyzes winner-pay auctions with externalities that do not depend on the identity of the winner and which could be applied to charity auctions.

The second-price all-pay auction can raise more money than other auction designs as long as the bidder with the highest adjusted-value takes part in the auction. Moreover, the revenue of the first-price all-pay auction can be dominated by the winner-pay auctions contrary to the results of Goeree et al. (2005). Indeed, above a certain threshold of asymmetry in the bidders' valuations, winner-pay auctions raise more money for charity than the first-price all-pay auctions. We can also revisit this result by an analysis of the bidders' altruism.

In the last section, we evaluate the impact of individual minimum bids on first-price all-pay and first-price winner-pay auctions. We assume the auctioneer knows the bidders' valuations. This assumption is relevant in a charity dinner which takes place in an isolated environment or in a local service club. The auctioneer gets information through the board of directors of the service club as he does not belong to this environment. Minimal bids could improve the relative revenue performance of the first-price all-pay auction compared to winner-pay auctions. Indeed, minimal bids can offset the effects of asymmetry in the bidders' valuations.

2 The model

Following the work of Vartiainen (2007) with linear cost functions, we analyze all-pay auctions for charity as a mechanism. This approach relies on a general model which can be applied to both first and second-price all-pay auctions. Yet, our approach is different. Moreover, in our case, every bidder takes as much advantage of her own bid as of her rival's bid thanks to introduction of the externalities.

In a charity dinner, an indivisible object (or prize) is sold through an all-pay auction. This prize is allocated to one of the potential bidders $N = \{1, ..., n\}$ contingents upon their bids $\boldsymbol{x} = (x_1, ..., x_n) \in \mathbb{R}^n_+$. As the bidders usually meet each other in these kinds of events, the willingness to pay and the valuation ranking of each bidder, $v_1 > v_2 > ... > v_n$, are common knowledge. An all-pay auction is a pairwise $(\boldsymbol{a}, \boldsymbol{t})$, $\boldsymbol{a}$ being the allocation rule and $\boldsymbol{t}$ the payment rule.

Allocation Rule. The allocation rule $\boldsymbol{a} = (a_1, ..., a_n) : \mathbb{R}_+^n \longrightarrow [0,1]^n$ is such that the winner i gets the object if and only if $a_i(\boldsymbol{x}) = 1$ given the bids and $\sum_{i=1}^n a_i(\boldsymbol{x}) = 1$ for all $\boldsymbol{x}$. The object is allocated to the highest bidder such that

$$\begin{cases} a_i(\boldsymbol{x}) = \frac{1}{\#Q(\boldsymbol{x})} \text{ if } i \in Q(\boldsymbol{x}) \\ a_i(\boldsymbol{x}) = 0 \text{ otherwise} \end{cases}$$

where $Q(\boldsymbol{x}) := \{j | j = \arg\max\{x_k, k \in N\}\}$ is the collection of the highest bids.

Payment Rule. The payment rule $\boldsymbol{t} = (t_1, ..., t_n) : \mathbb{R}_+^n \longrightarrow \mathbb{R}_+^n$ represents for each bidder i her transfer $t_i(\boldsymbol{x})$ to the charity organization for the vector of bids $\boldsymbol{x}$. This payment rule is contingent upon the all-pay design. In fact, in a first-price all-pay auction, each bidder pays her own bid

$$t_i(\boldsymbol{x}) = x_i \ \ \forall i \in N$$

while in the second-price all-pay auction the winner pays the second highest bid and the losers their own bid

$$\begin{aligned} &t_i(\boldsymbol{x}) = x^{(2)} \text{ if } i \in Q(\boldsymbol{x}) \\ &t_i(\boldsymbol{x}) = x_i \text{ otherwise} \end{aligned}$$

with $x^{(2)}$ the second order statistic of the sample $(x_1, ..., x_n)$.

The bidders wish to raise the maximum of money for charity. Every bidder takes advantage of her own participation in the charity auction and of the others' participations as well. In other words, the money raised by each potential bidder impacts the utility of all of the participants including herself. Thus, the bidder's utility function includes an externality which depends on the amount of money raised for the public good or the charity purpose. Denote $h_i(\boldsymbol{t}(\boldsymbol{x}))$ the externality that the bidder i takes advantage of.[6] We could also consider the externality as a function with only one argument $\sum_{j=1}^{n} t_j(\boldsymbol{x})$. Indeed, the externality is independent of the winner's identity and only takes into account the amount raised. Like Goeree et al. (2005) and other papers about charity auctions, we make a linearity assumption on the form of the externality price:

$$h_i(\boldsymbol{t}(\boldsymbol{x})) = h_i\left(\sum_{j=1}^{n} t_j(\boldsymbol{x})\right) = \alpha_i \sum_{j=1}^{n} t_j(\boldsymbol{x})$$

where $\alpha_i \geq 0$ is the coefficient of the bidder i's altruism for the charity purpose. Thus, the bidder i's utility is given by

$$U_i(\boldsymbol{x}) = \tilde{U}_i(a_i, \boldsymbol{t}) = v_i a_i(\boldsymbol{x}) - t_i(\boldsymbol{x}) + \alpha_i \sum_{j=1}^{n} t_j(\boldsymbol{x})$$

Assumption 1 (A1)**.** *$\tilde{U}_i(a_i, \boldsymbol{t})$ is a continuous and differentiable function in the transfer functions t_j for all j.*

Thus, $h_i(\boldsymbol{t}(\boldsymbol{x}))$ is continuous and differentiable in all of its arguments.

Assumption 2 (A2)**.** $\forall x_i \geq 0 \ \dfrac{\partial \tilde{U}_i}{\partial t_i(\boldsymbol{x})}(a_i, \boldsymbol{t}) < 0$ *equivalent to* $\alpha_i \displaystyle\sum_{j=1}^{n} \frac{dt_j(\boldsymbol{x})}{dt_i(\boldsymbol{x})} < 1$.

[6] The vectors $(t_1(y), ..., t_n(y))$ and $(t_1(\boldsymbol{z}), ..., t_n(\boldsymbol{z}))$ are denoted $\boldsymbol{t}(y)$ and $\boldsymbol{t}(\boldsymbol{z})$.

This assumption reminds that the bidder has a strict preference to keep one euro for her own use rather than to give it to the charity auction. This is the limit to the bidders' altruism to give money for charity.[7] The limit of the bidders' altruism is affected by the payment rule. Indeed, the bidder i's transfer can be a function of her opponents' bid. Thus, a change in the payment rule leads to a new limit of the bidders' altruism: in first-price it is $\alpha_i < 1$ while in second-price $\alpha_i < 1/2$.

Denote $F_i(x) \equiv \mathbb{P}(X_i \leq x)$ the cumulative distribution functions such as the bidder i decides to take a bid inferior to x. We denote $F_i(0)$ the probability that bidder i bids 0. When $F_i(0) \neq 0$, bidder i bids zero with a strictly positive probability. When $F_i(0) = 1$, bidder i always bids zero which means that she does not participate to the auction. $F_1, ..., F_n$ can be interpreted as the bidding strategies where the support is $\mathbb{R}_+$. Thus, the expected utility of bidder i is given by:

$$\mathbb{E}U_i(x_i, \boldsymbol{X}_{-i}) = \int_{\mathbb{R}_+^{n-1}} \left(v_i a_i(\boldsymbol{x}) - (1-\alpha_i) t_i(\boldsymbol{x}) + \alpha_i \sum_{\substack{j=1 \\ j \neq i}}^{n} t_j(\boldsymbol{x}) \right) \prod_{j \neq i} dF_j(x_j) \tag{1}$$

$$\begin{aligned} &= v_i \prod_{j \neq i} F_j(x_j) - (1-\alpha_i) \int_{\mathbb{R}_+^{n-1}} t_i(\boldsymbol{x}) \prod_{j \neq i} dF_j(x_j) \\ &\quad + \alpha_i \int_{\mathbb{R}_+^{n-1}} \sum_{\substack{j=1 \\ j \neq i}}^{n} t_j(\boldsymbol{x}) \prod_{j \neq i} dF_j(x_j) \end{aligned} \tag{2}$$

with $\boldsymbol{X}_{-i} = (X_1, ..., X_{i-1}, X_{i+1}, ..., X_n)$. To go from (1) to (2) we can notice that $\{\#Q(\boldsymbol{x}) = 1\}$ and $\{\#Q(\boldsymbol{x}) > 1\}$ are disjoints. Thus, when $\#Q(\boldsymbol{x}) > 1$ the value of the integral is zero. Indeed, a tie is a zero measure event.

Let us denote $\frac{v_i}{1-\alpha_i}$ the bidder i's adjusted-value. The bidders i's adjusted-value is defined as the ratio of her value for the item sold and the fraction of her payment which she perceives as a cost given her altruism for the charity purpose. We can observe this adjusted-value in the expected utility with a normalisation by dividing it by $1 - \alpha_i$. As bidders are *ex ante* asymmetric, we arrange bidders such that $\frac{v_i}{1-\alpha_i}$ decreases with the suffix i and without equality. This is common knowledge. Thus,

$$\frac{v_1}{1-\alpha_1} > \frac{v_2}{1-\alpha_2} > ... > \frac{v_n}{1-\alpha_n}$$

3 First-Price All-Pay Auction

In this section, we study the most popular all-pay auction design, *i.e.* the first-price all-pay auction. Every bidder pays her own bid, but only the one with the highest bid wins the object.

Given assumption $A2$, there is no pure strategy Nash equilibrium. This is a well known result when there is no externality. We only give a sketch proof of this result with two bidders for the first-price all-pay auction with externalities.

[7] If $\alpha_i \sum_{j=1}^{n} \frac{dt_j(\boldsymbol{x})}{dt_i(\boldsymbol{x})} = 1$ then the bidder is indifferent between giving one euro for charity or investing it in an another activity.

Let us assume that $x_i \geq x_j$ and consider some general externality (not necessarily linear) given by $h_i(x_i, x_j)$. In such a framework, two cases can occur. First, if bidder j can overbid, then her best reply is $x_i + \varepsilon$, for $\varepsilon > 0$ such that $v_j - (x_i + \varepsilon) + h_j(x_i, x_i + \varepsilon) \geq -x_j + h_j(x_i, x_j)$. Hence, it is impossible that $x_i \geq x_j$. Second, if j cannot overbid, then his best reply consists in offering zero since, given assumption $A2$, $h_j(x_i, 0) > -x_j + h_j(x_i, x_j)$. Consequently, i's best reply is to offer $\varepsilon > 0$. As a result, the equilibrium is unstable and there is no pure strategy Nash equilibrium.

3.1 Linear Externalities

As we noticed in the last section, assumption $A2$ implies that $\alpha_i < 1$. If bidder i offers x_i, then j will offer less with probability $F_j(x_i)$ and will offer more with probability $1 - F_j(x_i)$. Whatever the outcome, bidder i benefits from the sum of all bids, including her. We call an externality the amount that bidder i benefits from one bid. When computing her expected utility, she takes the amount payed by each opponent into account. Bidder i's expected utility with n potential competitors is given by

$$\mathbb{E}U_i(x_i, \boldsymbol{X}_{-i}) = \prod_{j \neq i} F_j(x_i) v_i - (1 - \alpha_i) x_i + \alpha_i \sum_{j \neq i} \mathbb{E}X_j$$

A potential bidder takes part in the auction if for some bids her expected utility is equal to or higher than the externalities she enjoys when her bid is zero. Formally, a bidder takes part in the auction if

$$\exists \; x \text{ such that } \mathbb{E}U_i(x, \boldsymbol{X}_{-i}) \geq \alpha_i \sum_{j \neq i} \mathbb{E}X_j$$

with $\alpha_i \sum_{j \neq i} \mathbb{E}X_j$ bidder i's expected reservation utility when she takes part in the auction. We call the highest price at which a given bidder is ready to take part in the auction her *indifference pricing*. i's *indifference pricing* is noted $\tilde{x}_i$ and satisfies $\mathbb{E}U_i(\tilde{x}_i) = \alpha_i \sum_{j \neq i} \mathbb{E}X_j$.

Proposition 1. *There is a unique Nash equilibrium and the mixed strategies are given by*

$$F_1(x) = \frac{1 - \alpha_2}{v_2} x \;\; \forall x \in \left[0, \frac{v_2}{1 - \alpha_2}\right] \;\; \textit{and } F_2(x) = 1 - \frac{1 - \alpha_1}{1 - \alpha_2} \frac{v_2}{v_1} + \frac{1 - \alpha_1}{v_1} x \;\; \forall x \in \left(0, \frac{v_2}{1 - \alpha_2}\right]$$

All other bidders use the pure strategy of zero and do not take part in the auction: $F_j(0) = 1$ *for* $j \in \{3, ..., n\}$*. The expected revenue is given by* $\mathbb{E}R = \dfrac{1}{2} \dfrac{v_2}{1 - \alpha_2} \left(\dfrac{1 - \alpha_1}{1 - \alpha_2} \dfrac{v_2}{v_1} + 1 \right)$.

Proof. See in Appendix.

At the Nash equilibrium, only two bidders are active: these bidders have the two highest adjusted-values. i's *indifference pricing* defines her adjusted-value and the second highest adjusted-value specifies the bidders' maximum bid. Hence, the bidders' mixed strategies are represented by uniform distributions and are supported on $[0, \frac{v_2}{1 - \alpha_2}]$ given that bidder 2 (the bidder with the second highest adjusted-value) takes part in the auction with probability

$$1 - F_2(0) = \frac{1 - \alpha_1}{1 - \alpha_2} \frac{v_2}{v_1}$$

Corollary 1. *All bidders obtain a positive payoff. Indeed, the bidders with the two highest adjusted-value obtain a positive payoff* $U_1^\star = v_1 - \frac{1 - \alpha_1}{1 - \alpha_2} v_2 + \frac{\alpha_1}{2} \frac{1 - \alpha_1}{v_1} \left(\frac{v_2}{1 - \alpha_2} \right)^2$ *and* $U_2^\star = \frac{v_2}{2} \frac{\alpha_2}{1 - \alpha_2}$ *and their competitors get* $U_i^\star = \frac{\alpha_i}{2} \frac{v_2}{1 - \alpha_2} \left(\frac{1 - \alpha_1}{v_1} \frac{v_2}{1 - \alpha_2} + 1 \right)$ *for* $i \in \{3, ..., n\}$.

Proof. Computations. ∎

Contrary to the case with no externality, the opponents of the highest bidder get a positive payoff. That is a consequence of externalities: bidders take an advantage of the competitors' behavior.

Remark 1. *Let us assume the difference between α_1 and α_2 is high enough for bidder 1's adjusted-value to be ranked second such that the two highest adjusted-values would be permuted. Then bidder 1 can get a lower payoff than in the case with no externality if and only if her altruism level is lower than $\tilde{\alpha} \equiv 2\frac{v_1 - v_2}{3v_1 - 2v_2}$. We notice that this threshold does not depend on her rival's altruism level, while the changes in the ranking of the adjusted-values is only due to the difference between the players' altruism levels.*

We can notice here that there are two opposite effects. Because of the externalities, the value of one euro that is invested in the auction is less than one euro. Thus, it is possible that the bidders choose more aggressive offers. However, every bidder knows that her competitor is more agressive and that this will affect one's probability of winning. Given an increasing of her competitor's aggressiveness, the bidder's best reply can be increasing or decreasing.

Example 1. *Let us consider two bidders with external effects $\alpha_1 = \alpha_2 = \frac{1}{2\min \tilde{x}_i}$ with $\tilde{x}_i = \frac{x_i}{1-\alpha_i}$. Notice that $A1 - A2$ are satisfied. Furthermore, $\tilde{x}_1 > \tilde{x}_2$, $\tilde{x}_1 = \frac{v_1}{v_2}\left(v_2 + \frac{1}{2}\right)$ et $\tilde{x}_2 = v_2 + \frac{1}{2}$. Thus, we can determine*

$$F_1(x) = \frac{2}{2v_2+1}x, \quad F_2(x) = 1 - \frac{v_2}{v_1} + \frac{2v_2}{(2v_2+1)v_1}x, \mathbb{E}R = \frac{2v_2+1}{4}\left(\frac{v_2}{v_1}+1\right)$$

The bidders' payoffs are $U_1^\star = v_1 - v_2 + \frac{1}{2}\frac{v_2}{v_1}$ and $U_2^\star = \frac{1}{4}$

In order to raise money for charity, we set up an optimal lobbying policy based on two steps. The first step consists in making the low[8] bidder aware of the charity auction and increases her adjusted-value. It is well known reducing the asymmetry that exists between bidders tends to increase competition, and thus leads to a higher rent for the auction. Once the updated-value of the low bidder is equal to the adjusted-value of the high bidder, the second step is to make both agents sensitive to the auction so as to keep their adjusted-values equal. It is important not to work only on the sensitiveness of the bidder with the highest valuation in order to avoid disastrous consequences in terms of revenue.

When the bidders have the same adjusted-value, they get an identical probability to win $F(x) = \frac{x}{v}$ for $x \in [0, v]$. Finally, the optimal level of altruism (α_1, α_2) that gives the maximum revenue for the auction is given by $\alpha_2 = 1 - \frac{v_2}{v_1}(1-\alpha_1)$.

Thus, as opposed to Baye et al. (1993), in charity auctions it is not conceivable to exclude bidders with higher values.

3.2 Non-Linear Externalities

We extend our result to non-linear externalities. We consider two bidders only, such that the expected utility is given by,

$$\mathbb{E}U_1(x_1, X_2) = F_2(x_1)\left(v_1 + \mathbb{E}_{X_2}(h_1(x_1, X_2)\backslash X_2 \leq x_1) - x_1\right) + (1 - F_2(x_1))(\mathbb{E}_{X_2}(h_1(x_1, X_2)\backslash X_2 \geq x_1) - x_1)$$
$$\mathbb{E}U_2(x_2, X_1) = F_1(x_2)\left(v_2 + \mathbb{E}_{X_1}(h_2(X_1, x_2)\backslash X_1 \leq x_2) - x_2\right) + (1 - F_1(x_2))(\mathbb{E}_{X_1}(h_2(X_1, x_2)\backslash X_1 \geq x_2) - x_2)$$

[8] The low and high bidders are respectively the active bidders with the second and the first highest adjusted-values.

with $\mathbb{E}_{X_2}(h_1(x_1, X_2)\backslash X_2 \leq x_1) = \dfrac{1}{F_2(x_1)} \displaystyle\int_0^{x_1} h_1(x_1, x_2) dF_2(x_2)$

It can also be written as

$$\begin{cases} \mathbb{E}U_1(x_1, X_2) = F_2(x_1)v_1 - x_1 + \mathbb{E}_{X_2} h_1(x_1, X_2) \\ \mathbb{E}U_2(x_2, X_1) = F_1(x_2)v_2 - x_2 + \mathbb{E}_{X_1} h_2(X_1, x_2) \end{cases}$$

Bidder i takes part in the auction if her expected utility is higher than her reservation utility:

$$\exists \ x_i \text{ such that } \mathbb{E}U_i(x_i, X_j) \geq \mathbb{E}_{X_j} h_i(0, X_j)$$

Proposition 2. *Given assumptions $A1 - A2$ and that the two bidders have a common support $[0, b]$, the mixed strategy equilibrium exists.*

Proof. See in Appendix. ■

The expected utility's derivative is a Fredholm equation of the second type. The existence of a solution depends on a condition made on the kernel (the kernel being the externality here). Nonetheless, given that the solution is a distribution function defined on a closed and convex set of continuous distribution functions, we are able to show its existence by using the second Schauder's theorem without this standard condition. The solution seems to be unique only in very specific cases, as said in the literature about Fredholm equations.[9]

4 Second-Price All-Pay Auction

In a second-price all-pay auction, the payment rule is the following: the winner pays the second highest bid and others pay their own bid. Our purpose is now to determine bidders' strategies and revenues. In the next section, we will compare the rents obtained in first-price and second-price auctions, as well as winner-pay and all-pay auctions. As a result, we will know which of these designs is the best to raise money for charity.

It is not necessary to find each agent's probability distribution's support in order to determine the mixed strategy Nash equilibrium. Actually, we only need to assume that each bidder i's offer, x_i belongs to a strategy space $X_i \subseteq [0, +\infty)$. For the same reasons as in the first-price auction, the bidders' minimum valuations is zero. As noticed before, assumption $A2$ allows us to write that $\alpha_i < 1/2$.

As for now, we have exclusively studied mixed strategy equilibria. Yet, there are also pure strategy Nash equilibria. Note that these equilibria are degenerated as in the situations without externalities. We give only an intuitive argument for the two bidder case. Bidder i's expected utility is given by

$$U_i(x) = \begin{cases} v_i + (2\alpha_i - 1)x_j & \text{if } x_i > x_j \\ \dfrac{v_i}{2} + (2\alpha_i - 1)x_i & \text{if } x_i = x_j \\ (2\alpha_i - 1)x_i & \text{if } x_i < x_j \end{cases}$$

As before, we note $\tilde{x}_i$ bidder i's *indifference price*, such that $\tilde{x}_1 > \tilde{x}_2$. Let x_i be bidder i's offer. There are two pure strategy Nash equilibria,

$$(0, \beta_1) \text{ with } \beta_1 \in (\tilde{x}_1, +\infty)$$
$$(\beta_2, 0) \text{ with } \beta_2 \in (\tilde{x}_2, +\infty)$$

[9] Kanwal (1971) has written a very complete book about these questions while Ledder (1996) gives a simple method and finds another condition to prove the solution's uniqueness.

The revenue earned for the auction is zero.

4.1 Two Bidders

The strategies' supports are no mass points and are continuous. If two bidders have a mass point, a deviation increases their probability to win. Furthermore, if one bidder has a mass point, his rival will never choose an action below this point. Thus, this bidder's mass point can only be zero. The expected utility given by (2) is

$$\mathbb{E}U_i(x_i, \boldsymbol{X}_{-i}) = \int_0^{x_i} (v_i - (1-2\alpha_i)x)dF_j(x) - (1-2\alpha_i)x_i(1-F_j(x_i))$$

In the second-price all-pay auction with two bidders, the payment rule leads to $t_1(x) = t_2(x)$. Thus, when a bidder wins she pays his rival's bid. Additionally, each bidder benefits from two externalities, one of which is associated to her own bid, and this other one of which is associated to her rival's bid.

Proposition 3. *There is a unique mixed strategy Nash equilibrium. Bidder i's strategy is given by an exponential distribution defined as follows,*

$$F_i \sim \mathcal{E}\left(\frac{1-2\alpha_j}{v_j}\right) \text{ and } \mathbb{E}R = \frac{v_1}{1-2\alpha_1} + \frac{v_2}{1-2\alpha_2}$$

Proof. See in Appendix. ∎

4.2 n Bidders

It is more difficult to find the equilibrium with n bidders. We note $G_i(x) = \prod_{j\neq i} F_j(x)$. It follows that the expected utility (2) can be written

$$\begin{aligned}
\mathbb{E}U_i(x_i, \boldsymbol{X}_{-i}) = & \int_0^{x_i} (v_i - (1-\alpha_i)x)dG_i(x) - (1-\alpha_i)x_i(1-G_i(x_i)) \\
& + \alpha_i \sum_{l\neq i} \int_{\mathbb{R}_+} x_l \Big(1 - \mathbb{1}_{x_i \leq x_l} \prod_{k\neq l,i} F_k(x_l)\Big) dF_l(x_l) \qquad (3) \\
& + \alpha_i \sum_{l\neq i} \Bigg(\int_{\mathbb{R}_+} \int_{x_i}^{x_l} \sum_{k\neq l,i} x_k \prod_{\substack{m\neq i,k,l \\ k\neq l}} F_m(x_k) dF_k(x_k) dF_l(x_l) + x_i \prod_{m\neq i,l} F_m(x_i)(1-F_l(x_i)) \Bigg)
\end{aligned}$$

The transition from (2) to (3) is explained in the proof of the proposition 4 given in appendix. The first line's two terms represent bidder i's payoff condition to her winning or losing the auction, given the externality that arises from her own action. The other lines represent the externalities that come from her competitors' actions (whether they lose or win).
The first of those two lines describes the situation when bidder l ($l \neq i$) loses the auction. In the last line bidder l wins the auction; we distinguish situations where bidder i's bid is the second highest offer from situations in which it is not. Each bidder's offer can be the second highest bid and we hold account of it (sign sum under the integral). The bidder who makes an offer between bidder i and bidder l's offers puts forward the second highest bid. The other part gives the amount of money that bidder l will have to paid when i offers the second highest bid. Indeed, $\prod_{m\neq i,l} F_m(x_i)(1-F_l(x_i))$ is the probability that every bidder except l makes a lower bid than i .
This probability is multiplied by the sum offered by the bidder i.

Note that this expression of expected utility is not valid unless there are at least four bidders. In order to study the three bidders case, it is necessary to (slightly) change the third line. To do this, we must stop computations to the second line of term B_I in the appendix. Thus, this term is writing $\alpha_i \sum_{l \neq i} \left(\int_{\mathbb{R}_+} \int_{x_i}^{x_l} x_k dF_k(x_k) dF_l(x_l) + x_i F_k(x_i)(1 - F_l(x_i)) \right)$, where k is not i, neither l. We do not explain this calcul more.

Proposition 4. *$\forall i > 2$ and suppose assumptions $A1 - A2$ hold, only two bidders among n participate actively to the auction.*

Proof. See in Appendix ■

The bidders' mixed strategies are given by the proposition 3. The weakness of this result is we do not know which bidders are going to participate. Thus, it could happen that the two bidders with the highest values participate or the ones with the lowest values. There are some consequences on the expected revenue.

5 Revenue Comparisons

In this section, we investigate the performance of the revenues and the expected revenues obtained with the different designs.

We consider here that the two bidders have the same altruism level i.e. $\alpha_1 = \alpha_2 = \alpha$. Hence, the bidder with the highest value is also the one with the highest adjusted-value. The expected revenue becomes

$$\mathbb{E}R^{AP1} = \frac{1}{2}\frac{v_2}{1-\alpha}\left(\frac{v_2}{v_1} + 1\right) \text{ et } \mathbb{E}R^{AP2} = \frac{v_i + v_j}{1 - 2\alpha} \quad i, j \in N$$

Indexes AP_i and WP_i correspond to i^{st}-price all-pay and winner-pay auctions. If bidders are complete altruists, i.e. $\alpha^{AP1} \longrightarrow 1$ and $\alpha^{AP2} \longrightarrow 1/2$, the expected revenues diverge as Goeree et al. (2005) predicted. Thus, the altruism level is an essential element to determine the expected revenue. When bidders' altruism levels are the same, the rent for the auction is at least equal to the rent one would obtain with non-altruistic bidders.

In the following, we use Ettinger (2002)'s results about winner-pay auctions with externality to compare the ones with our results about all-pay auctions with externality. These results are sum up in this table:

$v_1 > v_2 > v_3 > v_i\ \forall i > 3$	R^{WP1}	R^{WP2}	$\mathbb{E}R^{AP1}$	$\mathbb{E}R^{AP2}$
$\alpha > 0$	v_2	v_1	$\frac{1}{2}\frac{v_2}{1-\alpha}\left(\frac{v_2}{v_1}+1\right)$	$\frac{v_1+v_i}{1-2\alpha}$, $i \neq 1$
$\alpha = 0$	v_2	v_2	$\frac{v_2}{2}\left(\frac{v_2}{v_1}+1\right)$	$v_1 + v_i$, $i \neq 1$

Table 1: Revenues and expected revenues for every design

Let us consider homogeneous values. Then, we find the same qualitative results as Goeree et al. (2005) does. In particular, the second-price all-pay auctions rent dominates the first-price all-pay auctions rent which dominates the winner-pay auctions rent.

We can notice that the second-price all-pay auction gives a higher rent than other auction designs as long as the bidder with the highest adjusted-value takes part in the auction. On the contrary, when this bidder does not take part in the auction, the ranking of the expected revenue raised in the second-price all-pay auction depends on the asymmetry between bidders' valuations. Moreover, if our setting is suited to charity dinners with complete information (for example dinners organized by a local Rotary Club) first-price all-pay auction contradicts Goeree et al. (2005)'s qualitative results. In order to analyze the impact of asymmetry on rents, we use the following definition.

Definition. *The level of asymmetry between bidders' valuations will be considered "high" if* $v_1 - v_2 > 2\alpha v_1$*, "medium" if* $2\alpha v_1 > v_1 - v_2 > 2\alpha v_1 - v_1 + v_2 \frac{v_2}{v_1}$ *and "low" if* $v_1 - v_2 < 2\alpha v_1 - v_1 + v_2 \frac{v_2}{v_1}$.

Proposition 5. *We assume that* $\alpha_i = \alpha \; \forall i$ *and that the bidder with the highest value takes part in the second-price all-pay auction. Then, this design gives the highest revenues:*

$$\mathbb{E}R^{AP2} > R^{WP2} \geq R^{WP1} \text{ and } \mathbb{E}R^{AP2} > \mathbb{E}R^{AP1}$$

All other things being equal, $\mathbb{E}R^{AP1} > R^{WP2}$ *if and only if the level of asymmetry between valuations is "low",* $R^{WP2} > \mathbb{E}R^{AP1} > R^{WP1}$ *if and only if this level is "medium", and* $R^{WP1} > \mathbb{E}R^{AP1}$ *if and only if it is "high".*

Proof. Computations. ■

The second part of this proposition can be interpreted in two independent ways.

- First of all, given α, the (first-price) all-pay auction is dominated by the first-price winner-pay auction when asymmetry is "high". Furthermore, this all-pay auction raises more money than the second-price winner-pay auction when asymmetry is "low". Thus, in order to determine which design is better to raise money for charity, we need to know the level of asymmetry between bidders.

- Given v_1 and v_2, the (first-price) all-pay auction is dominated by first and second-price winner-pay auctions when the bidders' altruism level is less than $\frac{1}{2}(1 - \frac{v_2}{v_1})$. Moreover, the all-pay auction outperforms the first-price auction and is dominated by the second-price auction if the bidders' altruism level is inferior to $1 - \frac{1}{2}\frac{v_2}{v_1}(\frac{v_2}{v_1} + 1)$ and superior to $\frac{1}{2}(1 - \frac{v_2}{v_1})$. In particular, the threshold above which this all-pay auction raises more money than the first-price winner-pay auction is less than $\frac{1}{2}$. Finally, the first-price all-pay auction outperforms the winner-pay auctions when $\alpha > 1 - \frac{1}{2}\frac{v_2}{v_1}(\frac{v_2}{v_1} + 1)$.

The more asymmetry increases, the more the level of the altruism must also increase for the first-price all-pay auction to give a higher rent than winner-pay auctions. The two graphs below show the limits (in terms of rent domination) for the first-price all-pay auction. We use two parameters: altruism level and the asymmetry among bidders' values (from left to right, $\dfrac{v_2}{v_1}$ varies from 0.9 to its limit in zero with a 0.1 step).

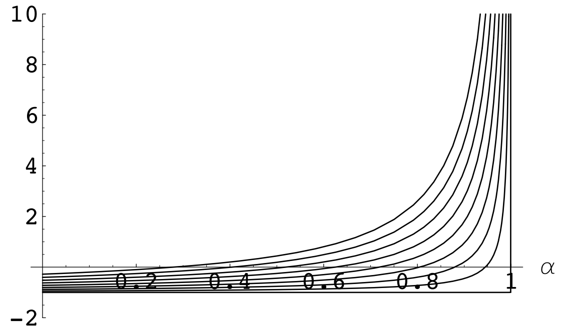


Figure 1: $\mathbb{E}R^{AP1} > R^{WP2}$

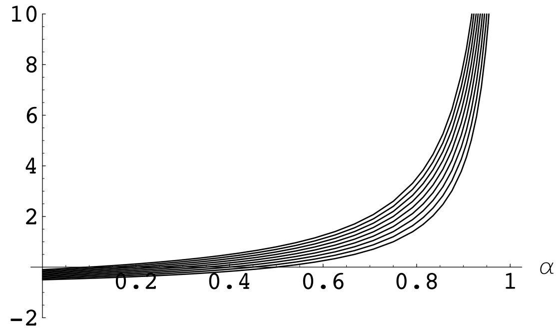


Figure 2: $\mathbb{E}R^{AP1} > R^{WP1}$

As a consequence, in order to determine which design is better to raise money for charity we need to know both the level of asymmetry and altruism. Contrary to the results of Goeree et al. (2005), here the first-price all-pay auction does not outperform the winner-pay auctions every time.

6 Individual Reserve Price

In this section, we determine the impact of minimum bids imposed on rent for two auction designs: first-price all-pay and winner-pay auction.[10] In the rest of the paper, we will note $t_i(x) = x_i$ for all $i \in N$. Moreover, we analyze only the two bidders case (who have the highest valuations). Indeed, only these two bidders participate in all-pay auction as in the third section.

The charity auction organizer imposes an individual bid on everybody: bidder i has to offer a bid at least equal to tv_i so as to take part in the auction. This implies that the auctioneer also knows the bidders' value, so that she can impose a rate t on them. This assumption is not unrealistic. This phenomenon could occur in a local service club (like a local Rotary club) or during a show business dinner. Indeed, the auctioneer could obtain this kind of information through the staff of the local community or because she is herself a member or a friend of the participants.

As expected, there is no pure strategy Nash equilibrium.[11] In order to find the strategies and the probability of entry, we focus on the situation where every bidder wants to participate.

Lemma 1. *At equilibrium, the bidders' minimum bids are asymmetric. They are tv_1 for bidder 1 and tv_2 for bidder 2. In fact, the latter's density is equal to zero on the support $(tv_2, tv_1]$.*

With probability one, bidder i's offer will be at least equal to tv_i. We conclude that $\min x_i \geq tv_i$. Now, let us assume that $\min x_1 = x > tv_1$. Then $\mathbb{P}(X_1 < \{x\}) = 0$, because bidder 1 never makes any offer in the interval (tv_1, x). His competitor offers either tv_2 or $x + \varepsilon$ for $\varepsilon > 0$, a bid between these two values being strictly dominated. Then, if bidder 1 bids $x - \varepsilon$ her probability of winning is not affected. Thus, his minimum bid is tv_1. Moreover, bidding in the interval $(tv_2, tv_1]$ is strictly dominated for bidder 2. Hence, $\mathbb{P}(tv_2 < X_2 \leq tv_1) = 0$ and if she bids $tv_2 < x \leq tv_1$ he loses for sure. When she offers $x = tv_2$ she does not affect his probability of

[10]We would have similar results with the second-price winner-pay auction instead of the first-price. As the latter is more used than the former for charity auctions, we investigate this one.

[11]To see this, let us assume that $x_1 \geq x_2$. As before, we have to consider two situations. First, bidder 2 can overbid. It contradicts the initial assumption. If she cannot overbid, given $A2$, her best reply is to offer tv_2. Hence, bidder 1 bids tv_1. The equilibrium is unstable.

winning but increases her payoff by $A2$. Furthermore, she increases her probability of winning by bidding $x = tv_1 + \varepsilon$ for $\varepsilon > 0$. Bidder 2's density function is zero on the interval $(tv_2; tv_1]$.

Lemma 2. *At equilibrium, bidders offer the same maximum bid $\bar{x} = (1 - \alpha_2 t)\tilde{x}_2$. Every bidder has a mass point for his minimum bid and a mass point can never be on $(tv_1, \bar{x}]$.*

Even if the payoff functions are the same that the ones pointed out in section 3, that is to say

$$\mathbb{E}U_1(x, X_2) = F_2(x)v_1 - (1 - \alpha_1)x + \alpha_1 \mathbb{E}X_2, \quad \mathbb{E}U_2(x, X_1) = F_1(x)v_2 - (1 - \alpha_2)x + \alpha_2 \mathbb{E}X_1$$

the expected level of the bidders' reservation utilities are changed. Indeed, as the minimum bids are positive, bidder i's reservation utility is $\alpha_i \mathbb{E}X_j + \alpha_i t v_i$: he participates to the auction if she gets at least $\alpha_i \mathbb{E}X_j$ (as before) plus the reward of her own minimum bid. Hence, the maximum bid is equal to the lowest of the two bidders' *indifference pricing*. At her *indifference pricing*, bidder i is indifferent between taking part in the auction or not, that is to say to offer tv_i. Thus, the maximum bid is $\bar{x} = (1 - \alpha_2 t)\tilde{x}_2$.
Given the former analysis, bidder 2 has a mass point on tv_2. Bidder 2's strategy space is $\{tv_2\} \cup (tv_1; \bar{x}]$. For similar reasons as in section 3 and for the case without externality, having a mass point on the bidders' common strategy set is dominated for every bidder (since they deviate).[12]

For now, we only consider the bidders' common strategy set, that is to say $(tv_1; \bar{x}]$. A bidder's equilibrium payoff is a constant function on her whole strategy set.[13] Hence,

$$F_2(x)v_1 - (1 - \alpha_1)x + \alpha_1 \mathbb{E}X_2 = v_1 - (1 - \alpha_1)\bar{x} + \alpha_1 \mathbb{E}X_2 \tag{4}$$

for all $x \in (tv_1; \bar{x}]$. The left member of this equation is the bidder 1's expected utility for all bids in $(tv_1; \bar{x}]$, while the right member is bidder 1's payoff when he bids $\bar{x}$. In the same way, bidder 2's bid is such that

$$F_1(x)v_2 - (1 - \alpha_2)x + \alpha_2 \mathbb{E}X_1 = v_2 - (1 - \alpha_2)\bar{x} + \alpha_2 \mathbb{E}X_1 \tag{5}$$

and thus belongs to the interval $\{tv_2\} \cup (tv_1; \bar{x}]$.

In particular, for all bids in the interval $(tv_1, \bar{x}]$ and for $\alpha_1 = \alpha_2$, we find that

$$v_2(1 - F_1(x)) = v_1(1 - F_2(x))$$

As bidder 2 has a mass point on tv_2, the limit in tv_1 gives us the following result[14]

$$F_1(tv_1) = 1 - \frac{v_1}{v_2} + \frac{v_1}{v_2} F_2(tv_1)$$

Using (4) and (5), it is easy to determine the bidders's distribution functions. We specify them in proposition 6 below. As $F_2(tv_1)$ is not equal to $1 - \frac{v_2}{v_1}$ (the value in zero without any externality and minimum bids imposed) bidder 1 has indeed a mass point on tv_1. The bidders' distribution functions are drawn below.

[12] We give here a well-known argument (see for instance Che and Gale (1998)) to support this idea. If only one bidder has a mass point on the support that is common to both bidders, her competitor's density function below this mass point is equal to zero. Hence, she is going to move and her mass point will be the support's lower bound. This action does not affect his probability of winning, but it increases her payoff if she wins. In a similar way, if bidders have a mass point, deviating increases their probability of winning. Consequently, the result follows.

[13] In the following we use similar technical arguments than Che and Gale (1998).

[14] As $\mathbb{P}(tv_2 < X_2 \leq tv_1) = 0$ it follows that $\lim_{x \to tv_1} F_2(x) = F_2(tv_1) = F_2(tv_2)$.

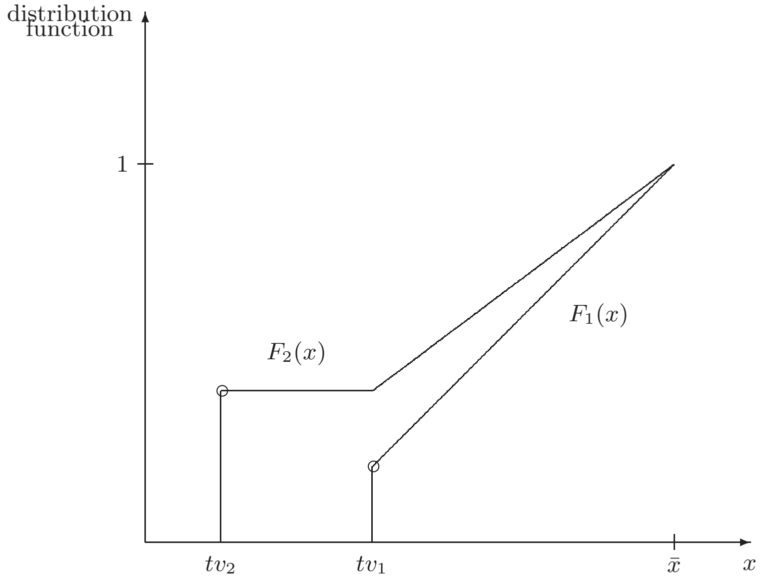


Figure 3: Cumulative distribution functions at the equilibrium

Proposition 6. *Given the bidders' adjusted-values, $(1-\alpha_1 t)\tilde{x}_1$ and $(1-\alpha_2 t)\tilde{x}_2$, there is a unique Nash equilibrium. The bidders' strategies for all $x \in (tv_1; \bar{x}]$ are*

$$F_1(x) = \alpha_2 t + \frac{x}{\tilde{x}_2} \text{ and } F_2(x) = 1 + \frac{x-\bar{x}}{\tilde{x}_1}.$$

Every bidder has one point mass: it is tv_1 for bidder 1 and tv_2 for bidder 2.

A bidder's decision is given by her probability to participate,

$$1 - F_1(tv_1) = 1 - \alpha_2 t - \frac{tv_1}{\tilde{x}_2} \text{ and } 1 - F_2(tv_2) = \frac{\bar{x} - tv_1}{\tilde{x}_1}$$

Additionally, if the maximum bid $\bar{x}$ is inferior to bidder 1's minimum bid $\bar{x} \leq tv_1$, offering a higher bid than their minimum bid is dominated for all bidders. Hence, $\mathbb{E}R = t(v_1+v_2)$ for all $t \geq \bar{t}$ where $\bar{t} \equiv \dfrac{\tilde{x}_2}{v_1 + \alpha_2 \tilde{x}_2}$.
Here, we consider the case where $0 \leq t < 1$ only.[15]

Proposition 7. *Given the distribution functions $F_1(.), F_2(.)$ at equilibrium, the expected revenue raised for charity is*

$$\mathbb{E}R = \begin{cases} \bar{x}^2 \dfrac{\tilde{x}_1 + \tilde{x}_2}{2\tilde{x}_1\tilde{x}_2} + (tv_1)^2 \dfrac{\tilde{x}_1 - \tilde{x}_2}{2\tilde{x}_1\tilde{x}_2} + t^2 v_1 \alpha_2 + tv_2 \left(1 + \dfrac{tv_1 - \bar{x}}{\tilde{x}_1}\right) & \textit{if } t < \bar{t} \\ t(v_1+v_2) & \textit{otherwise} \end{cases}$$

[15] $t > 1$ is not appropriate here. Indeed, the minimum bid of one bidder could be higher than the maximum bid.

Proof. We only have to compute the expected revenue associated to every bidder when $t < \bar{t}$:

$$\begin{aligned}
\mathbb{E}R_i &= \int_{tv_1}^{\bar{x}} x f_i(x)dx + tv_i F_i(tv_1) \\
&= \bar{x}\int_{tv_1}^{\bar{x}} f_i(y)dy - \int_{tv_1}^{\bar{x}}\int_{tv_1}^{x} f_i(y)dydx + tv_i F_i(tv_1) \\
&= \bar{x}(F_i(\bar{x}) - F_i(tv_1)) - \int_{tv_1}^{\bar{x}} F_i(x) - F_i(tv_1)dx + tv_i F_i(tv_1) \\
&= \bar{x} - \int_{tv_1}^{\bar{x}} F_i(x)dx + (tv_i - tv_1)F_i(tv_1)
\end{aligned}$$

Hence, $\mathbb{E}R_1 = \dfrac{\bar{x}^2 + (tv_1)^2}{2\tilde{x}_2} + t^2 v_1 \alpha_2$ and $\mathbb{E}R_2 = \dfrac{\bar{x}^2 - (tv_1)^2}{2\tilde{x}_1} + tv_2\left(1 + \dfrac{tv_1 - \bar{x}}{\tilde{x}_1}\right)$ ■

We must analyze the impact of t on the rent. This will allow us to determine whether imposing a minimal bid to every bidder permits to improve the first-price all-pay auction's efficiency compared to the first-price winner-pay auction or not. In order to do so, we assume that bidders have the same altruism attitude, such that $\alpha = \alpha_1 = \alpha_2$. We analyze only the revenue achievement for $t \leq \bar{t}$. After an increase in t, there are two contradictory effects. The bidders' support's lower bound increases while its upper bound decreases. As a consequence, the expected revenue can increase or decrease. The result depends on which effect dominates the other.

First of all, let us assume that the asymmetry between the bidders' values is considered "high" such that $v_1 - v_2 > 2\alpha v_1$. As a consequence, the all-pay auction expected revenue is increasing in t. The low altruism level of the bidders offsets the impact of t on the bidders' maximum bid, so that the effect on the lower bound dominates.[16] As was pointed out before, when $t = 0$ the all-pay auction expected revenue is strictly dominated by the first-price winner-pay auction revenue.[17] Given this result, the all-pay auction gives a higher revenue than the winner-pay auction for a value of t that offsets the impacts of asymmetry. The graph below illustrates this result for $v_1 = 20$ and $v_2 = 5$. Each curve is the expected revenue when asymmetry is considered "high" and for a specific value of t. The lower envelope curve is given by $t(v_1+v_2)$. The first-price winner-pay auction revenue is given by the dashed curve.

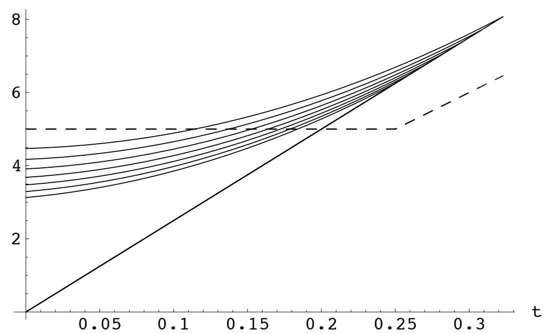


Figure 4: Expected revenue with a "high" asymmetry

[16]We saw in section 5 that $v_1 - v_2 > 2\alpha v_1$ could also be interpreted as a low altruism level.

[17]The first-price winner-pay auction gives a revenue v_2 with a rate t inferior to $\frac{v_2}{v_1} < \bar{t}$. For higher rates, the revenue becomes $tv_1 < t(v_1 + v_2)$.

It is obvious that situations where asymmetry is "medium" or "low" give the same result: all-pay auction raises more money than winner-pay auction. Yet, it is interesting to draw the expected revenues associated to those asymmetry levels. Here, the decreasing effect of the support's upper bound is higher than the increasing effect of the lower bound below a given value of t, where dynamics is reversed.

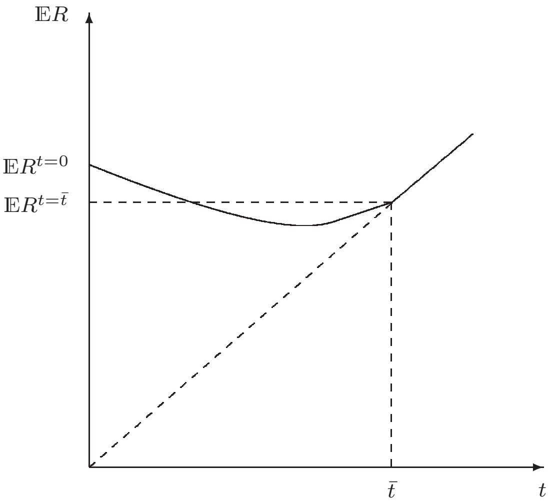


Figure 5: $\mathbb{E}R$ for $\alpha > \max\{\frac{v_1}{v_1+v_2}, \frac{v_1-v_2}{2v_1}\}$

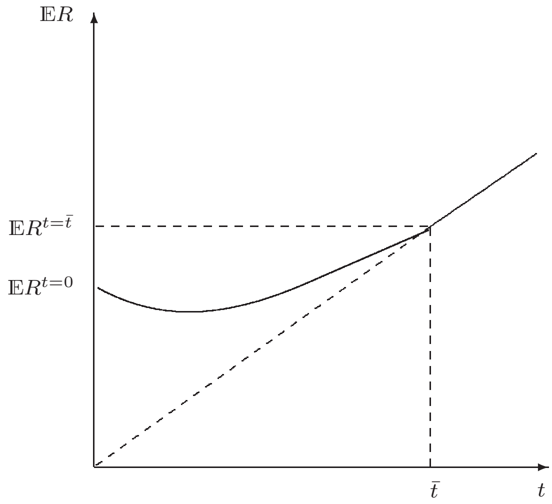


Figure 6: $\mathbb{E}R$ for $\frac{v_1}{v_1+v_2} > \alpha > \frac{v_1-v_2}{2v_1}$

Proposition 8. *Imposing a minimal bid to every bidder permits to improve the first-price all-pay auction's efficiency compared to the first-price winner-pay auction. There is a threshold t above which the all-pay auction dominates the winner-pay auction when the values' asymmetry is considered "high".*

Example 2. *We focus again on the example 1: two bidders benefit the same externality $\alpha_1 = \alpha_2 = \frac{1}{2 \min \tilde{x}_i}$. Hence, the two bidders' maximum bid is $\bar{x} = v_2 + \frac{1-t}{2}$ and the bidders' mixed strategies are*

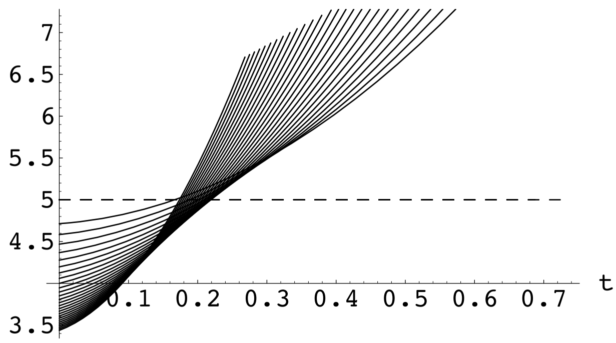
$$F_1(x) = \frac{2x+t}{2v_2+1} \text{ and } F_2(x) = 1 + \frac{v_2(2x - 2v_2 + t - 1)}{v_1(2v_2+1)}$$

Furthermore, the expected revenue when $t < \frac{2v_2+1}{2v_1+1}$ is

$$\mathbb{E}R = \frac{1}{v_1(2v_2+1)}\left[\left(v_2 + \frac{1-t}{2}\right)^2 (v_1+v_2) + (tv_1)^2(v_1-v_2) + t^2v_1^2 + tv_1v_2(2v_2+1) + 2tv_2^2\left(tv_1 - v_2 - \frac{1-t}{2}\right)\right]$$

The graphic below gives all the charts of expected revenue with $t < \bar{t}$, $v_2 = 5$ and v_1 increasing from 7 to 20 with a 0.5 step.[18] *Example 1 (without minimum bids imposed) is equivalent to the situation when $t = 0$. Thus, when the values of t are high enough, we can notice that the first price all-pay auction is better than the first-price winner-pay auction with "high" level of asymmetry.*

[18] $v_1 \geq 7$ ensures that the asymmetry between values is "high".



7 Conclusion

This paper shows that all-pay auctions do not raise higher revenue for charity than winner-pay auctions every time. Indeed, this result depends on the asymmetry between bidders. In particular, winner-pay auctions outperforms first-price all-pay auction when the asymmetry between bidders is strong. That contradicts Goeree et al. (2005)'s results. Our work can be related to the one of Carpenter et al. (2008). Indeed, they have found in a field experiment that first-price winner-pay auction outperforms first-price all-pay auction. One of the explications could be a strong asymmetry between bidders.

This work could be completed by a laboratory experiment. In fact, only one lab experiment (Onderstal and Schram (2008)) has been implemented until now with opposite results to the field experiment of Carpenter et al. (2008). Onderstal and Schram (2008) find similar results to Goeree et al. (2005). However, our results are quite different from Goeree et al. (2005)'s because of the introduction of asymmetric valuations. That is why, it would be interesting to test our prediction with the introduction of asymmetry between the bidders' valuations: all-pay auctions can be dominated by winner-pay auctions. That could also be the occasion to test the impact of altruism on agents' behavior. Finally, theoritical and experimental works should be led about the form of the externalities that we considered here linear.

All-pay auctions with externalities that are independent of the winner's identity but functions of the amount raised have other applications in economy.
Here, we focus on the team theory. This illustration could be connected to other forms of team works (particularly in firms) leading to social promotion. Let's consider, a team sport like basket-ball. Every year during the American championship of basket-ball (the NBA) or the all-stars game finals, the most valuable player (MVP) is elected. During such games, every player makes the highest effort to win the event but also to be elected the MVP of the game. Each player takes advantage of the team's effort to win the game and thus can be elected MVP thanks to the externality of the total amount of the efforts made. v_i represents the player's value for the MVP title. Therefore, her effort x_i has two goals: to win the game and be elected MVP. When a player is not elected MVP, he takes advantage of the externality by winning the game. As a player tries to win the game by making the highest effort, he helps also her team mates to be elected MVP .

In a recent paper, Edlin (2005) displays a tax credit method to incite people to give more for charity purposes. He suggests to deduce the agents' donations to charity organizations from their income tax (limited to a certain percent of their income). The agents are free to choose the organization they want to help. This method should improve the all-pay auctions performance for charity and lets an open question for futur researches.

Appendix

Proof of proposition 1. Let us consider the two bidder case. If we divide the bidders' i expected utility by $1-\alpha_i$, we almost obtain the same bidders' expected utility as in the case without externality given by Hillman and Riley (1989). However, after this operation has been made, there remains an important difference between the bidder's expected utility we find and the one Hillman and Riley (1989) find. Indeed, there is a constant in their function while our function has an externality $\alpha_j \dfrac{\mathbb{E}X_j}{1-\alpha_i}$. Thus, we only have a constant in our function at the equilibrium. That makes an important difference. By this division the result for the two bidder case follows as in Hillman and Riley (1989). Yet, we cannot use the proof of Hillman and Riley (1989) to determine the uniqueness when there are more than two bidders. Indeed, bidders take into account the positive amount payed by each opponent. Thus, even if externalities are constant at the equilibrium, bidders do not take advantage of the same positive externalities. Let us assume that a third bidder takes part in the auction. Her expected utility is equal to or higher than $\alpha_3\mathbb{E}X_1 + \alpha_3\mathbb{E}X_2$. Define $\tilde{x}_i = \frac{x_i}{1-\alpha_i}$. Given her two rivals' mixed strategies, it follows that $F_1(x_3)F_2(x_3)v_3 \geq (1-\alpha_3)x_3$, which is equivalent to $\tilde{x}_1(\tilde{x}_3 - \tilde{x}_2) \geq \tilde{x}_3(\tilde{x}_2 - x_3)$. As $\tilde{x}_2 > \tilde{x}_3$ and $\tilde{x}_3 \geq x_3$, there is a contradiction. This result can be generalized to a game with n bidders. To show that there is a unique solution, here we could apply the lemma 14' of Baye et al. (1990): $\tilde{x}_i = 0\ \forall i > 2$.[19] ■

Proof of proposition 2. The sketch of this proof follows the same logic as the proof of Proposition 2 in Anderson et al. (1998). For similar reasons as the ones pointed out without externalities, the two players make their bids on the common support $[0, b]$ and the density function, $F_i' = f_i$ exists. The set of equilibria in mixed strategies is completely characterized by a Nash equilibria where only pure strategies which are better responses to the others strategies are played with a strictly positive probability. All of these strategies lead to the same expected utility. Next, we denote $\lambda = \frac{1}{v_i}$ and ignore the suffix.

Let T be an operator such as $T : F(x) \longmapsto TF(x)$ and

$$TF(x) \equiv \lambda x - \lambda \int_0^b h(x,y)f(y)dy + \text{ constant} \tag{6}$$

As F is a continuous function, we restrict our study to the set of continuous functions on $[0, b]$ denoted $C[0, b]$. Especially, we consider $D = \{F \in C[0, b] \backslash ||F|| \leq 1\}$ with $||.||$ the supremum norm. The set D, which includes all of the continuous distribution functions, is closed and convex but not compact. Thus, to prove that (6) has a solution, we apply the following Schauder's second theorem:

Theorem (Schauder, 1930)**.** *If D is a closed convex subset of a normed space and E a relatively compact subset of D, then every continuous mapping of D to E has a fixed-point.*

[19] Actually, the proof of this lemma has to be slightly changed and be adapted to our setting. As the modifications are of minor importance, we do not give the details of the proof.

To apply this theorem, we need to prove two parts. First, that $T(D) \equiv E = \{TF \backslash F \in D\}$ is relatively compact.[20] Second, T is a continuous mapping from D to E.

Showing that E is relatively compact is equivalent to showing that E is uniformly bounded and equicontinuous (Ascoli's theorem) on $[0, b]$. Generalization of the assumption $A2$ leads to $\frac{\partial h}{\partial x}(x, y) < 1$ for all $y \in [0, b]$. Then, $TF(x)$ is nondecreasing. Thus $|TF(x)| \leq TF(b) = 1$, for all $x \in [0, b]$. Let us show that E is equicontinuous. We need to show that$\forall \varepsilon, \exists \eta, \forall F \in E$ such that $|TF(x_1) - TF(x_2)| < \varepsilon$ when $|x_1 - x_2| < \eta$.

$$\begin{aligned}
|TF(x_1) - TF(x_2)| &= \left|\lambda(x_1 - x_2) - \lambda \int_0^b [h(x_1, y) - h(x_2, y)]f(y)dy\right| \\
&\leq \lambda\left[|x_1 - x_2| + \left|\int_0^b [h(x_1, y) - h(x_2, y)]f(y)dy\right|\right] \\
&\leq \lambda|x_1 - x_2|\left[1 + \frac{|\sup_{y\in[0,b]}[h(x_1, y) - h(x_2, y)]|}{|x_1 - x_2|}\right] \\
&< \lambda\eta\left[1 + \frac{|\sup_{y\in[0,b]}[h(x_1, y) - h(x_2, y)]|}{|x_1 - x_2|}\right]
\end{aligned}$$

The function h is continuous and bounded on $[0, b]$. $[0, b]$is a compact which explains the result of the last line. Denoted $\kappa \equiv |\sup_{y\in[0,b]}[h(x_1, y) - h(x_2, y)]|$. Thus, $|TF(x_1) - TF(x_2)| < \varepsilon$ for $\eta = \varepsilon \frac{|x_1 - x_2|}{\lambda(|x_1 - x_2| + \kappa)}$.

Now, let us prove the continuity of T. The operator T is continuous if, for all F_1, F_2 and for all $\varepsilon > 0$, there exists a $\eta > 0$ such that $|TF_1(x) - TF_2(x)| < \varepsilon$ when $|F_1 - F_2| < \eta$. Let us write $F_1(x) = F_2(x) + g(x)$ with $-\eta < g(x) < \eta \; \forall x \in [0, b]$. Henceforth

$$\begin{aligned}
|TF_1(x) - TF_2(x)| &= \left| - \lambda \int_0^b h(x, y)(f_1(y) - f_2(y))dy\right| \\
&\leq \lambda \int_0^b |h(x, y)||g'(y)|dy \\
&\leq h(b, b)\lambda \int_0^b |g'(y)|dy \\
&< h(b, b)\lambda\eta
\end{aligned}$$

To go from the first to the second line, notice that $F_1'(x) - F_2'(x) = g'(x)$. We use the fact that h is a continuous function on $[0, b]$ bounded by a maximum $h(b, b)$ to go to the third line. Hence, the difference between TF_1 and TF_2 is inferior to $\varepsilon > 0$ when $\eta = \frac{\varepsilon}{\lambda h(b,b)}$. ■

Proof of proposition 3. All mixed strategies at the equilibrium lead to the same expected utility. Thus, we can completely characterize the set of equilibrium in mixed strategies. In particular, the expected utility is zero for $x_i = 0$:

$$\mathbb{E}U_i(x_i, X_j) = \int_0^{x_i} (v_i - (1 - 2\alpha_i)x)dF_j(x) - (1 - 2\alpha_i)x_i(1 - F_j(x_i)) = 0$$

Hence the Volterra integral equation

$$f_j(x)v_i = (1 - 2\alpha_i)(1 - F_j(x)) \tag{7}$$

[20] A space is relatively compact when his closed span is compact.

The solution is given by

$$F_j(x) = 1 - k_j exp\left(-\frac{(1-2\alpha_i)x}{v_i}\right) \quad x \in X_j \;\; k_j \in \mathbb{R}$$

F_j is a distribution function defined on X_j where the minima is zero and the maxima noted $\tilde{x}$. As the distribution functions must verify $F_j(0) = 0, F_j(\tilde{x}) = 1$ and $\int_0^{\tilde{x}} f_j(x)dx = 1$, we know that X_j and $[0;+\infty)$ are merged but also that $k_j = 1$. Henceforth,

$$F_j(x) = 1 - exp\left(-\frac{(1-2\alpha_i)x}{v_i}\right) \quad x \in [0;+\infty)$$

■

Proof of proposition 4. By (2) we have the expected utility:

$$\mathbb{E}U_i(x_i, \boldsymbol{X}_{-i}) = v_i \prod_{j\neq i} dF_j(x_j) - (1-\alpha_i) \underbrace{\int_{\mathbb{R}_+^{n-1}} t_i(\boldsymbol{x}) \prod_{j\neq i} dF_j(x_j)}_{A} + \alpha_i \underbrace{\int_{\mathbb{R}_+^{n-1}} \sum_{j\neq i} t_j(\boldsymbol{x}) \prod_{j\neq i} dF_j(x_j)}_{B}$$

A represents bidder i's expected payment when we take into account its own external effect. The term B is the expected payment of bidder i's rivals. $\alpha_i B$ is the sum of the externalities of bidder i's rivals that i takes advantage of.

We can write A again as follow

$$\underbrace{\int_{\mathbb{R}_+^{n-1}} x^{(2)} \mathbb{1}_{\substack{x_i \geq x_j \\ \forall j \neq i}} \prod_{j\neq i} dF_j(x_j)}_{A_I} + \underbrace{\int_{\mathbb{R}_+^{n-1}} x_i \mathbb{1}_{\substack{\exists k / x_k > x_i \\ k\neq i}} \prod_{j\neq i} dF_j(x_j)}_{A_{II}}$$

The term A_I is i's expected payment when she wins *i.e.* he pays the second highest bid. A_{II} is i's expected payment when she loses. She could then either be the second highest bidder or a lower bidder.

$$\begin{aligned}
A_I &= \int_{\mathbb{R}_+^{n-1}} \sum_{j\neq i} x_j \mathbb{1}_{\substack{x_k \leq x_j \leq x_i \\ \forall k \neq \{j,i\}, j\neq i}} \prod_{j\neq i} dF_j(x_j) \\
&= \int_{\mathbb{R}_+} \sum_{j\neq i} x_j \mathbb{1}_{x_j \leq x_i} \left\{ \int_{\mathbb{R}_+^{n-2}} \prod_{k\neq i,j} \mathbb{1}_{x_k \leq x_j \leq x_i} \prod_{k\neq i,j} dF_k(x_k) \right\} dF_j(x_j) \\
&= \int_{\mathbb{R}_+} \sum_{j\neq i} x_j \mathbb{1}_{x_j \leq x_i} \left\{ \prod_{k\neq i,j} \int_{\mathbb{R}} \mathbb{1}_{x_k \leq x_j \leq x_i} dF_k(x_k) \right\} dF_j(x_j) \\
&= \int_{\mathbb{R}_+} \sum_{j\neq i} x_j \mathbb{1}_{x_j \leq x_i} \prod_{k\neq i,j} F_k(x_j) dF_j(x_j) \\
&= \int_0^{x_i} x dG_i(x)
\end{aligned}$$

We get the first line from the fact that $x^{(2)} \mathbb{1}_{x_i \geq x_j} = \sum_{j\neq i} x_j \mathbb{1}_{\substack{x_k \leq x_j \leq x_i \\ \forall k \neq \{j,i\}, j\neq i}}$. The independence of the distribution functions explains how we go from the second to the third line. By denoting $dG_i(x) = \sum_{j\neq i} \prod_{k\neq i,j} F_k(x) dF_j(x)$, we obtain the final result.

$$
\begin{aligned}
A_{II} &= \int_{\mathbb{R}_+^{n-1}} x_i(1-\mathbb{1}_{i\in Q(x)})\prod_{j\neq i} dF_j(x_j)\\
&= x_i - x_i\prod_{j\neq i} F_j(x_i)\\
&= x_i(1-G_i(x_i))
\end{aligned}
$$

The independence of the distribution functions, explains how we go from the first line to the second.

B can be written also like

$$
\begin{aligned}
B &= \sum_{l\neq i}\int_{\mathbb{R}_+^{n-1}} t_l(\boldsymbol{x})\prod_{j\neq i} dF_j(x_j)\\
&= \sum_{l\neq i}\Bigg\{\underbrace{\int_{\mathbb{R}_+^{n-1}} x^{(2)}\mathbb{1}_{\substack{x_l\geq x_k\\ \forall k\neq l}}\prod_{j\neq i} dF_j(x_j)}_{B_I} + \underbrace{\int_{\mathbb{R}_+^{n-1}} x_l\mathbb{1}_{\substack{\exists k/x_l<x_k\\ k\neq l}}\prod_{j\neq i} dF_j(x_j)}_{B_{II}}\Bigg\}
\end{aligned}
$$

We add all of the expected external effects. The case where player $l\neq i$ takes the second higher bid is distinguished from the others.

$$
\begin{aligned}
B_I &= \int_{\mathbb{R}_+^{n-1}}\sum_{k\neq l} x_k\mathbb{1}_{\substack{x_m\leq x_k\leq x_l\\ \forall m\neq\{k,l\}}}\prod_{j\neq i} dF_j(x_j)\\
&= \int_{\mathbb{R}_+^{n-1}}\sum_{k\neq l} x_k\prod_{m\neq\{k,l\},k\neq l}\mathbb{1}_{x_m\leq x_k\leq x_l}\prod_{j\neq i} dF_j(x_j)\\
&= \int_{\mathbb{R}_+^{n-1}}\sum_{k\neq i,l} x_k\prod_{\substack{m\neq i,k,l\\ k\neq l}}\mathbb{1}_{x_m\leq x_k\leq x_l} dF_m(x_m)\mathbb{1}_{x_i\leq x_k\leq x_l} dF_k(x_k)dF_l(x_l)\\
&\quad+\int_{\mathbb{R}_+^{n-1}} x_i\prod_{m\neq i,l}\mathbb{1}_{x_m\leq x_i\leq x_l}\prod_{j\neq i} dF_j(x_j)\\
&= \int_{\mathbb{R}_+^2}\sum_{k\neq i,l} x_k\int_{\mathbb{R}_+^{n-3}}\prod_{\substack{m\neq i,k,l\\ k\neq l}}\mathbb{1}_{x_m\leq x_k} dF_m(x_m)\mathbb{1}_{x_i\leq x_k\leq x_l} dF_k(x_k)dF_l(x_l)\\
&\quad+x_i\int_{\mathbb{R}_+}\prod_{m\neq i,l}\left\{\int_0^{x_i} dF_m(x_m)\right\}\mathbb{1}_{x_i\leq x_l} dF_l(x_l)\\
&= \int_{\mathbb{R}_+^2}\sum_{k\neq i,l} x_k\prod_{\substack{m\neq i,k,l\\ k\neq l}} F_m(x_k)\mathbb{1}_{x_i\leq x_k\leq x_l} dF_k(x_k)dF_l(x_l) + x_i\prod_{m\neq i,l} F_m(x_i)(1-F_l(x_i))\\
&= \int_{\mathbb{R}_+}\int_{x_i}^{x_l}\sum_{k\neq i,l} x_k\prod_{\substack{m\neq i,k,l\\ k\neq l}} F_m(x_k)dF_k(x_k)dF_l(x_l) + x_i\left(\prod_{m\neq i,l} F_m(x_i) - G_i(x_i)\right)
\end{aligned}
$$

$$
\begin{aligned}
B_{II} &= \int_{\mathbb{R}_+^{n-1}} x_l(1-\mathbb{1}_{l\in Q(x)})\prod_{j\neq i} dF_j(x_j)\\
&= \int_{\mathbb{R}_+^{n-1}} x_l \prod_{j\neq i} dF_j(x_j) - \int_{\mathbb{R}_+^{n-1}} x_l \prod_{k\neq i,l}\Big(\mathbb{1}_{x_k\leq x_l} dF_k(x_k)\Big)\mathbb{1}_{x_i\leq x_l} dF_l(x_l)\\
&= \int_{\mathbb{R}_+^{n-1}} x_l \prod_{j\neq i} dF_j(x_j) - \int_{\mathbb{R}_+} x_l\mathbb{1}_{x_i\leq x_l}\Big\{\int_{\mathbb{R}_+^{n-2}}\prod_{k\neq i,l}\mathbb{1}_{x_k\leq x_l} dF_k(x_k)\Big\} dF_l(x_l)\\
&= \int_{\mathbb{R}_+} x_l dF_l(x_l) - \int_{\mathbb{R}_+} x_l\mathbb{1}_{x_i\leq x_l}\prod_{k\neq i,l} F_k(x_l) dF_l(x_l)\\
&= \int_{\mathbb{R}_+} x_l(1-\mathbb{1}_{x_i\leq x_l}\prod_{k\neq i,l} F_k(x_l)) dF_l(x_l)
\end{aligned}
$$

Hence

$$
\begin{aligned}
\mathbb{E}U_i(x_i,\boldsymbol{X}_{-i}) = &\int_0^{x_i}(v_i-(1-\alpha_i)x)dG_i(x) - (1-\alpha_i)x_i(1-G_i(x_i))\\
&+\alpha_i\sum_{l\neq i}\int_{\mathbb{R}_+} x_l(1-\mathbb{1}_{x_i\leq x_l}\prod_{k\neq i,l}F_k(x_l))dF_l(x_l)\\
&+\alpha_i\sum_{l\neq i}\Bigg(\int_{\mathbb{R}_+}\int_{x_i}^{x_l}\sum_{k\neq i,l} x_k \prod_{\substack{m\neq i,k,l\\ k\neq l}} F_m(x_k)dF_k(x_k)dF_l(x_l) + x_i\prod_{m\neq i,l}F_m(x_i)(1-F_l(x_i))\Bigg)
\end{aligned}
$$

Next, we will note

$$
G_{il}(x) = \prod_{k\neq i,l} F_k(x) \text{ et } G'_{il}(x) = \sum_{j\neq i,l}\prod_{k\neq i,l,j} F_k(x) dF_j(x)
$$

As the expected utility is constant at the equilibrium, the FOC leads to

$$
v_iG'_i(x) - (1-\alpha_i)(1-G_i(x)) + \alpha_i\sum_{l\neq i}G_{il}(x) - \alpha_i\sum_{l\neq i}G_{il}(x)F_l(x) - \alpha_i x\sum_{l\neq i}G'_{il}(x)F_l(x) = 0
$$

Notice that $(n-1)G_i(x) = \sum_{l\neq i}G_{il}(x)F_l(x)$ and $(n-2)G'_i(x) = \sum_{l\neq i}G'_{il}(x)F_l(x)$ henceforth

$$
(v_i - \alpha_i x(n-2))G'_i(x) + (1-\alpha_i n)G_i(x) = (1-\alpha_i) - \alpha_i\sum_{l\neq i}G_{il}(x) \;\; \forall i\in\{1,...,n\} \tag{A1}
$$

This result is true for all $n > 3$. The closed characterization of the solution is very difficult. Yet, we can deduce the solution by an alternative way. Indeed, let F_i and F_j be the mixed strategies of the two bidders i and j. We can notice that the derivative of the expected utility of a third bidder k $H_k(x) = \frac{\partial \mathbb{E}U_k}{\partial x}(x_i, X_1, X_2)$ is a monotonous increasing function. Furthermore, $H_k(0) = -(1-\alpha_k)$ and $\lim_{x\to+\infty} H_k(x) = 0$. Thus, given the mixed strategies of i and j, k do not participate.
This result can easily be extended to a number n of bidders. For that, we should use recurrence.

■

References

Anderson, S. P., Goeree, J. K. and Holt, C. A. (1998), 'Rent seeking with bounded rationnality : An analysis of the all-pay auction', *Journal of Political Economy* **106**, 828–853.

Baye, M., Kovenock, D. and de Vries, C. (1990), The all-pay auction with complete information. CentER Discussion paper 9051.

Baye, M., Kovenock, D. and de Vries, C. (1993), 'Rigging the lobbying process : An application of the all-pay', *American Economic Review* **83**(1), 289–294.

Carpenter, J., Homes, J. and Matthews, P. H. (2008), 'Charity auctions : A field experiment', *Economic Journal* **118**, 92–113.

Che, Y.-K. and Gale, I. (1998), 'Caps on political lobbying', *American Economic Review* **88**, 643–651.

Edlin, A. S. (2005), 'The choose-your-charity tac : A way to incentivize greater giving', *The Economists' Voice* **2**(3).

Engers, M. and McManus, B. (2007), 'Charity auction', *International Economic Review* **48**(3), 953–994.

Ettinger, D. (2002), 'Bidding among friends and enemies', *Working Paper* .

Goeree, J. K., Maasland, E., Onderstal, S. and Turner, J. L. (2005), 'How (not) to raise money', *Journal of Political Economy* **113**(4), 897–918.

Hillman, A. and Riley, J. (1989), 'Politically contestable rents and transfers', *Economics and Policy* **1**, 17–39.

In a Charitable Mood (2005), Vol. 48, Beijing Review.

Kanwal, R. P. (1971), *Linear Integral Equations : Theory and Technique*, Academic Press, New York.

Ledder, G. (1996), 'A simple introduction to integral equations', *Mathematics Magazine* **69**, 172–181.

Maasland, E. and Onderstal, S. (2006), 'Auctions with financial externalities', *Economic Theory* **32**, 551–574.

Onderstal, S. and Schram, A. (2008), 'Bidding to give : an experimental comparaison of auctions for charity', *International Economic Review* **forthcoming**.

Vartiainen, H. (2007), 'Comparing of all-pay auctions under complete information', *Review of Economic Design* **forthcoming**.

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