Competition and Quality in Regulated Markets with Sluggish Demand

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Abstract

We investigate the effect of competition on quality in regulated markets (e.g., health care, higher education, public utilities), using a Hotelling framework, in the presence of sluggish demand. We take a differential-game approach, and derive the open-loop solution (providers choose the optimal quality investment plan based on demand at the initial period) and the feedback closed-loop solution (providers observe demand in each period and choose quality in response to current demand). If production costs are strictly convex, the steady state quality is higher under the open-loop solution than under the feedback solution. In both solutions, quality and demand move in opposite directions over time on the equilibrium path to the steady state. While fiercer competition (lower transportation costs or less sluggish demand) leads to higher quality in both solutions, the quality response to increased competition is weaker when players use feedback strategies.

JEL-Code: H42, I18, I21, L13, L51.

Keywords: regulated markets, competition, quality, differential games.

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1 Introduction

Competition leads to better quality when prices are regulated. This is a fairly robust prediction from economic theory.[1,2] Given that the regulated price is above the marginal costs, firms have an incentive to invest in quality in order to attract (or avoid losing) consumers. Tougher competition – measured for instance by the number of competing firms or by the degree of substitutability among products – amplifies the incentives to invest in quality. Prime examples of the types of regulated industries we have in mind are health care and education, where consumer choices – at least in most European countries – are mainly driven by considerations of quality rather than price.

In theoretical models, the positive relationship between competition and quality in regulated markets is generally derived within a static framework, neglecting potentially important dynamic issues related to quality. In particular, static models assume that demand responds immediately to quality changes. This assumption is unrealistic. Demand tends to be sluggish. If a provider increases quality, sluggish demand implies that it will take some time before the potential demand increase is fully realised. Such demand sluggishness can typically arise for two different reasons. First, imperfect information on the demand side, which is particularly relevant in markets where quality is the main competition variable: while prices usually are easily and immediately observable, quality is often less readily observable and much more difficult to measure. Second, sticky behaviour of consumers, motivated by (personal or familiar) habits, or by trust or confidence in one specific provider. Let us think, for example, of the cases of people who choose a dentist simply because relatives went to her/him; or a child going to a specific school (or college) simply because brothers went there. Moreover, the relational content in the service exchange between a provider and a consumer in fields like education or health play an important role in making demand sluggish.[3]

Our basic framework is the widely-used Hotelling model for quality competition with regulated prices.[4] In this model there are two firms offering one product each.

[1] See, for instance, the survey by Gaynor (2006) and references therein. See also the papers by Ma and Burgess (1993), Wolinsky (1997), and Brekke, Nuscheler and Straume (2006). See also Lambertini (2006) for theoretical analyses of vertical differentiation in industrial organization.

[2] If firms can set prices as well, the effect of competition on quality is in general ambiguous. In this case, competition depresses prices and, thus, the marginal revenues from quality investment (see e.g., Economides, 1993).

[3] Brekke, Cellini, Siciliani and Straume (2008) investigate a different dynamic issue. They assume that demand responds instantaneously to quality but quality is a stock variable, which increases over time only if investment in quality is higher than its deterioration rate.

[4] See, for instance, Ma and Burgess (1993), Calem and Rizzo (1995), Wolinsky (1997), Lyon (1999), Del Rey (2001), Beitia (2003), Brekke, Nuscheler and Straume (2006, 2007), and De Fraja and Landeraas (2006).

We consider the case in which the spatial locations are given, while the products are horizontally and vertically differentiated, and the firms choose quality to maximise profits. These quality choices are modelled in a dynamic framework under the key assumption that demand adjusts sluggishly. We model sluggish demand such that, at each point in time, only a fraction of consumers respond to quality changes. Thus, it will take some time before potential demand is fully realised. The time it takes for potential and actual demand to align is determined by the degree of demand sluggishness.

We use a differential-game approach to analyse dynamic competition between the two firms.[5] There are two main solution concepts. First, we derive the open-loop solution, where each firm commits to an optimal (quality) investment plan at the initial period. This solution is reasonable if it is very difficult or costly to obtain information about competitors and/or the quality variable is subject to some rigidity (e.g., investment regulations). Second, we derive the feedback (closed-loop) solution, where each firm knows the quality of the competitor at each point in time, not just the initial state. Here, firms choose an optimal rule connecting the current value of their choice variable to the current value of states. The fundamental difference between the two solution concepts is the degree of commitment, but the feedback solution is also sometimes interpreted as a more competitive solution in the sense that firms can at each point in time change their investments in response to the dynamics of the states.[6]

Our main result is that if production costs are strictly convex in output, quality is lower in the feedback than in the open-loop solution. The reason is that quality choices are *strategic complements* in this case. In a dynamic game, this provides an incentive to compete less aggressively. Otherwise, if production costs are linear in output, quality choices become strategically independent, and the two solutions – open-loop and feedback – coincide. In both solutions, the steady state level of quality is increasing in price level and decreasing in level of transportation costs and the degree of demand sluggishness, as expected. Thus, the positive relationship between competition and quality is confirmed also in a dynamic setting. More interesting, perhaps, is that the quality response to increased competition is weaker

[5] See Dockner, Jørgensen, Van Long and Sorger (2000) for an introduction. The Hotelling model in a differential game framework is used, *inter alia*, by Laussel, de Montmarin and Van Long (2004) and by Piga (1998). Differently from our present paper, however, the former focuses on network effects and competition is on prices (rather than quality), while the latter studies the role of advertising and price competition.

[6] In dynamic capital accumulation games the closed-loop solution is typically more competitive (see Dockner, 1992; Dockner, Jørgensen, Van Long and Sorger, 2000). In these models, providers compete a la Cournot but face capacity constraints that can be relaxed by capital accumulation through investments. It turns out that investments under the closed-loop solution is *higher* than under open-loop.

in the feedback solution, compared with the open-loop equilibrium.

A second main finding is that demand and quality move in opposite directions over time on the equilibrium path to the steady state, given that production costs are strictly convex. This result contradicts the static relationship between quality and demand. Consider a situation where actual demand is below the steady-state level. In this situation, the provider will raise quality above the steady-state level. However, as demand increases, the marginal profit gain becomes lower due to increasing marginal costs, and the provider will gradually reduce quality until the steady state is reached. As a result, we obtain a negative relationship between (actual) demand and quality. This result might have implications for empirical studies. Unless sufficient care is taken to account for dynamic adjustment over time, this kind of equilibrium dynamics could potentially lead to spurious relationships between quality and demand.

We also consider welfare and policy implications. Deriving the first-best quality path over time, we show that there is a trade-off between improving allocative (cost) efficiency and increasing consumer benefit from quality investments. If the former incentive dominates, the first-best solution prescribes a lower quality level for the provider with the higher demand, implying that first-best quality and demand always move in the same direction over time. The strengths of these two incentives depend on production cost convexity and demand sluggishness. If demand is sufficiently sluggish, the latter incentive dominates, implying that first-best quality and demand move in opposite direction over time. We show how the first-best quality path can be implemented by dynamic price regulation, where the regulator sets time-dependent and provider-specific prices. Finally, we point out that demand sluggishness might be affected by regulatory policy as well. The regulator might spend resources on publishing quality indicators of the providers. If this is the case, then reducing demand sluggishness is a policy substitute to providing high-powered incentives

We believe our analysis is relevant for several regulated industries. A prime example is health care. In this industry prices are either set by the insurer (government or private insurer) or settled in negotiations with the providers (hospitals and physicians). Consumers (patients) are insured against medical expenditures, so non-price measures like quality and distance are more relevant for provider choice than price.[7] Since health care providers typically receive payments per patient (or per treatment), they might find it profitable to improve quality to attract (or avoid losing) patients and, in turn, increase revenues.[8]

[7] The empirical studies by Kessler and McClellan (2000) and Tay (2003) show that distance and quality are the main predictors of hospital choice. These papers also assess the relationship between competition and quality in the US Medicare hospital market.

[8] Related theoretical studies on competition in health care are, for instance, Calem and Rizzo

Another example is the market for (especially higher) education. In most European countries, tuition fees play a negligible role, and funding of educational institutions is to a large extent based on student attendances.[9] A student's choice of school or university is typically based on the quality of the institution, as well as the institution's location (geographically and/or in product space). As for hospitals, universities might find it profitable to invest in quality (new facilities, better laboratories, hiring of top researchers, etc.) in order to attract more students, thereby increasing revenues.[10]

In both health care and education, quality is a major concern. In recent years, many European countries have implemented marked-based reforms exposing providers to competition. In particular, the introduction of provider choice and activity-based payments are aimed at stimulating competition and in turn quality. For example, in the UK, hospitals are paid a tariff for every patients treated (Payment by Results): providers who attract more patients receive more resources (money follows the patient). Similar initiatives have been introduced in Norway, Denmark, Italy and several other European countries. Our study is also relevant for the US Medicare system, where hospitals are paid a fixed price per treatment within a specific diagnosis related group (DRG), a system that has been adopted by many European countries.

In both sectors, governments spend resources on collecting information on quality indicators and publishing scores and rankings of institutions (e.g., league tables of hospitals, universities, schools, etc.). Obviously, the main purpose of this activity is to make demand more responsive to quality differences. Our purpose is to contribute to the understanding of the impact of competition on quality in regulated markets characterised by demand sluggishness.

The rest of the paper is organised as follows. In Section 2, we present the model framework. In Section 3 we derive and characterise the equilibrium quality under the open-loop solution, while a corresponding analysis for the feedback solution is provided in Section 4. Welfare and policy implications are presented in Section 5. In Section 6 we extend the model to the case of elastic market demand. Section 7 concludes the paper.

(1995); Lyon (1999); Gravelle (2000); Gravelle and Masiero (2000); Beitia (2003); Nuscheler (2003); Brekke, Nuscheler and Straume (2006, 2007); and Karlsson (2007).

[9] See Kaiser, Raymond, Koelman and van Vught (1992) for an overview.

[10] For related theoretical studies in education, see papers by Del Rey (2001), De Fraja and Ioassa (2001), and De Fraja and Landeras (2006). For empirical studies on competition and quality in education, see e.g., Dee (1998), Epple and Romano (1998) and Hoxby (2000).

2 Model

In line with previous literature on quality competition in regulated markets, we conduct the analysis within a Hotelling framework (Hotelling, 1929). Consider a market with two providers located (exogenously) at either end of the unit line $S = [0,1]$.[11,12] On this line segment there is a uniform distribution of individuals, with total mass normalised to 1. We assume unit demand, where each individual demands one unit of output. The utility of an individual who is located at $x \in S$ and chooses provider i, located at z_i, is given by

$$U(x, z_i) = v + kq_i - \tau |x - z_i|, \tag{1}$$

where v is the gross valuation from consumption, $q_i \geq \underline{q}$ is the quality at provider i, k is a parameter measuring the (marginal) utility of quality, and τ is a transportation cost parameter.[13] The lower bound $\underline{q}$ on quality represents the minimum quality providers are allowed to offer.[14] For simplicity, we set $\underline{q} = 0$. Moreover, we normalise the marginal utility of quality to one, i.e., $k = 1$, without loss of generality. This implies that τ can be interpreted as the marginal disutility of travelling *relative* to quality. Thus, a low (high) τ means that quality is of relatively more (less) importance to the patient than travelling distance.

Since the distance between providers is equal to one (exogenously fixed), the individual who is indifferent between provider i and provider j is located at D^*, given by

$$v - \tau D^* + q_i = v - \tau (1 - D^*) + q_j, \tag{2}$$

[11] S is typically interpreted either as a geographical space or a product (taste) space.

[12] A limiting assumption in our analysis is that locations are exogenous. In our dynamic setting, allowing for endogenous locations will severely complicate the analysis (see Brekke, Nuscheler and Straume, 2006, for a static analysis with endogenous locations and qualities). However, the assumption of exogenous locations is arguably closer to reality in the particular sectors that we consider to be our main applications, such as health care and education. For example, locations of hospitals or universities are typically fixed in all but the very long run. Even in the long run, location choices might also be restricted due to regulation.

[13] The assumption that k is the same for all consumers implies (along with the other symmetry assumptions) that there will be no vertical differentiation in the steady state equilibrium (though there will be vertical differentiation on the equilibrium dynamic path to the steady state). We stick to this assumption for two reasons. Most importantly, introducing consumer heterogeneity along the vertical dimension (as, for example, in Neven and Thisse, 1990) will severely complicate the analysis. Furthermore, by keeping k constant across consumers, our model is more in line with the previously mentioned (static) analyses of quality competition with fixed prices.

[14] We can think of $\underline{q}$ as the minimum quality level set by a regulator and/or defined through legislation. If $q < \underline{q}$, the provider might be sued or lose his licence. In health care, we can think of $q < \underline{q}$ as malpractice or failure to meet licence standards.

yielding the *notional* (or *potential) demand* for provider i,

$$D^* = \frac{1}{2} + \frac{q_i - q_j}{2\tau}, \tag{3}$$

implying that the provider with a higher quality has a potential demand in excess of 1/2. Notice how lower transportation costs make it less costly for consumers to switch between providers, increasing the demand responsiveness to changes in quality.[15]

In the existing literature, it is typically assumed that demand responds instantaneously to quality changes. This is obviously a simplifying assumption. Demand is generally sluggish. If a provider increases quality, sluggish demand responses imply that it will take some time before the potential demand increase is fully realised. Such demand sluggishness can typically arise from imperfect information on the demand side, which is particularly relevant in markets where quality is the main competition variable. While prices usually are easily and immediately observable, quality is often much more difficult to measure and thus less readily observable. For example, assume that, at each point in time, only a proportion $\gamma \in [0, 1]$ of consumers become aware of quality changes in the market. This would imply that, at each point in time, only a fraction γ of any potential change in demand is realised. A different set of reasons why demand is sluggish has to do with personal or familiar habits in fields like education or health: people trust in one specific provider, for personal or familiar considerations, apart from the objective quality of the service; sticky behaviour, and in some cases even addiction to a specific provider, lead to sluggish demand.

Define $D(t)$ as the *actual demand* of provider i at time t (as opposed to *potential demand* $D^*(t)$). Analytically, the law of motion of actual demand is given by

$$\frac{dD(t)}{dt} \equiv \dot{D}(t) = \gamma(D^*(t) - D(t)), \tag{4}$$

where $\gamma \in [0, 1]$ is an inverse measure of demand sluggishness. The higher is γ, the less sluggish is demand. If $\gamma = 0$, the demand facing each provider is completely inelastic, as actual demand does not respond to quality changes, while, if $\gamma = 1$, potential demand changes are immediately and fully realised. Such a specification is widely used in theoretical IO models to describe market price stickiness (see, e.g.,

[15] Notice that our assumption of maximal differentiation (i.e., locations at the endpoints of the Hotelling line) is not crucial for our results. In fact, any pair of *symmetric* locations would give exactly the same potential demand function as (3). Furthermore, even for asymmetric locations, the marginal effect of quality on potential demand ($\partial D^*/\partial q_i$) is identical as long as the transportation cost function is linear in distance.

Simaan and Takayama, 1978; Fershtman and Kamien, 1987; Dockner et al., 2000 Sect. 10.1; Cellini and Lambertini, 2007).

Since market demand is inelastic, notice that both providers face the same dynamic constraint, given by (4). To see this, notice that *actual demand* for provider j at time t is given by $1 - D(t)$ (as opposed to *potential demand* $1 - D^*(t)$). Analytically, the law of motion of actual demand for provider j is then given by

$$\frac{d[1 - D(t)]}{dt} = \gamma \left[(1 - D^*(t)) - (1 - D(t)) \right], \tag{5}$$

which can easily be rewritten as (4). Thus, the dynamics of the demand for provider i automatically determines the demand for provider j.[16]

We assume that providers maximise profits. The instantaneous objective function of provider i is assumed to be given by

$$\pi_i(t) = T + pD(t) - C(D(t), q_i(t)) - F, \tag{6}$$

where p is a regulated price per unit of output provided (for example, a treatment or a patient in the context of health care markets; a student in the context of education markets).[17] T is a potential lump-sum transfer (or a fixed grant/budget) received from a third-party payer.

On the cost-side, each provider i faces a fixed cost F and variable cost $C(\cdot)$ that depends on the quality q_i and the actual demand D. For simplicity, we assume that $C(\cdot)$ takes the following quadratic form:

$$C(D, q_i) = \frac{\theta}{2} q_i^2 + \frac{\beta}{2} D^2, \tag{7}$$

where $\theta > 0$ and $\beta \geq 0$. Thus, production costs are increasing and strictly convex in quality, while increasing and weakly convex in output.[18] Notice that the case where production costs are linear in output is captured by setting $\beta = 0$.[19]

[16] In Section 6 we extend the model to allow for elastic market demand.

[17] As long as prices are fixed, whether payments are collected directly from the consumers (as for public utilities) or from a third-party payer (which is more relevant for health care and, to a certain extent, education markets) is immaterial for our results.

[18] We make the simplifying assumption that the cost function is separable in quality and output: $C_{Dq_i} = 0$. The assumption of cost separability between quality and quantity is widely used in the related literature (see, e.g., Economides, 1989, 1993; Calem and Rizzo, 1995; Lyon, 1999; Gravelle and Masiero, 2000; Nuscheler, 2003; Brekke, Nuscheler and Straume, 2006, 2007). Relaxing the cost separability assumption should not qualitatively affect our results as long as $C_{DD} > |C_{Dq_i}|$.

[19] To see this, notice that the cost specification $C(D, q_i) = \frac{\theta}{2} q_i^2 + \sigma D + \frac{\beta}{2} D^2$ (i.e., adding the term σD to (7)) is captured by re-defining the parameter p in (6) as $p := \widetilde{p} - \sigma$, where $\widetilde{p}$ is the regulated per-unit price received by the provider. The case of a constant marginal production cost $\sigma > 0$ is then obtained by setting $\beta = 0$.

Defining ρ as the (constant) preference discount rate, the provider's objective function over the infinite time horizon is

$$\int_{0}^{+\infty} \pi_i(t) e^{-\rho t} dt.$$

In reality, providers may not have an infinite-time horizon, but may have reasonably long finite horizons. If the optimal path does not differ significantly from the solution with a very large but finite horizon, the convenience of working with an infinite-horizon model may be worth the loss of realism (see Léonard and van Long, 1992, p. 285). Also, when decision-makers retire, they may well be replaced by other decision-makers with similar utility functions, thus generating an infinite-time horizon.

In this type of dynamic models with strategic interactions – i.e., differential games – there are two main solution concepts for the Nash equilibrium: a) *open-loop solution*, where each provider knows the initial state of the system and then nothing else, i.e., each provider knows the initial quality (and thus potential demand) of the other provider, but not in the following periods; b) *closed-loop solution*, where each provider knows the initial state of the system, but also later knows the state variable values, i.e., each provider knows the quality of the other provider, not only in the initial state, but also in all of the subsequent periods. Within the closed-loop solutions, further distinctions can be made: if one assumes that players take into account only the initial state and the current state, the 'memoryless' closed-loop solution is obtained; if players take into account the whole history of states, the 'perfect state' closed-loop is obtained; finally, if players in each instant take into account the current value of states (i.e., the whole past history is summarised by the current value of states), the feedback rule is obtained, and this is a case of stationary Markovian strategy. A strategy is said to be Markovian if the rule (i.e., the function) connecting the choice variable $x(t)$ to the states $y(t)$ is of type $x(t) = f(y(t), t)$, i.e., the choice variables depend on the current value of states but not on the path followed by the states until t. If $x(t) = f(y(t))$ the Markovian strategy is stationary (see, Dockner et al. 2000, Sect. 4.3). Typically, the feedback closed-loop Markovian solution, which is the one we apply in the present analysis, is obtained based on the Bellman equation.

In order to establish which is the most appropriate solution concept, it is essential to evaluate the relevant information set used by players when they take their decisions. In cases where collection of information over time is difficult, it is reasonable to model the choice according to the open-loop rule;[20] on the contrary, when players

[20] One example from the education sector could be the Research Assessment Exercise in the UK,

can observe the current state of the world and they behave accordingly, the closed-loop rules are more appropriate. Arguably, closed-loop solutions are more appealing, but solving for closed-loop is more difficult. However, in some cases – and health care markets can be a good example – players might have to commit to investment plans and stick to them for long periods of time. In this case, the open-loop solution might be the relevant one. Nevertheless, there is a wide range of problems where the two solutions coincide.[21] Below, we compare the Nash equilibria under the open-loop and feedback (closed-loop) solution concepts. The next section provides the open-loop Nash equilibrium, while Section 4 provides the feedback solution.

3 Open-loop solution

Consider first the case where the providers use open-loop decision rules. Provider i's maximisation problem is given by

$$\underset{q_i}{\text{Maximise}} \int_0^{+\infty} \pi_i(t) e^{-\rho t} dt,$$

$$\text{subject to } \dot{D}(t) = \gamma(D^*(t) - D(t)), \quad (8)$$

$$D(0) = D_0 > 0, \quad (9)$$

where q_i is the control variable. Let $\mu_i(t)$ be the current value co-state variable associated with the state equation. The current-value Hamiltonian is:[22]

$$H_i = T + pD - C(D, q_i) - F + \mu_i \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D \right), \quad (10)$$

where $C(D, q_i)$ is given by (7). The solution is given by (a) $\partial H_i / \partial q_i = 0$, (b) $\dot{\mu}_i = \rho \mu_i - \partial H_i / \partial D$, (c) $\dot{D} = \partial H_i / \partial \mu_i$, or more extensively:

$$\frac{\gamma}{2\tau} \mu_i = \theta q_i, \quad (11)$$

$$\dot{\mu}_i = \mu_i (\rho + \gamma) - (p - \beta D), \quad (12)$$

$$\dot{D} = \gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right), \quad (13)$$

that produces quality profiles of higher education institutions every 8 years. Arguably, with a time span of this length, quality becomes observable only quite rarely.

[21] Games where this coincidence arises are presented in Clemhout and Wan (1974); Reinganum (1982); Mehlmann and Willing (1983); Dockner, Feichtinger and Jørgensen (1985). See also Mehlmann (1988), Fershtman, Kamien and Muller (1992), Dockner, Jørgensen, Van Long, Sorger (2000, ch. 7) for reviews.

[22] The indication of time (t) is omitted to ease notation.

to be considered along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu_i(t)D(t) = 0$. The second order conditions are satisfied if the Hamiltonian is concave in the control and state variables (Léonard and Van Long, 1992).[23]

Totally differentiating (11) with respect to time we obtain $\frac{\gamma}{2\tau}\dot{\mu}_i = \theta\dot{q}_i$, or, after substitution, $\frac{\gamma}{2\tau}\left(\mu_i\left(\rho+\gamma\right) - \left(p - \beta D\right)\right) = \theta\dot{q}_i$. Using $\mu_i = \theta q_i \frac{2\tau}{\gamma}$, we obtain

$$\dot{q}_i = q_i\left(\rho+\gamma\right) - \frac{\gamma}{2\tau\theta}\left(p - \beta D\right), \tag{14}$$

which, together with (13), describe the dynamics of the equilibrium.

As to the possible steady state, setting $\dot{q}_i = 0$ and totally differentiating yields

$$\frac{\partial D}{\partial q_i}|_{\dot{q}_i=0} = -\frac{\theta\left(\rho+\gamma\right)2\tau}{\gamma\beta} < 0. \tag{15}$$

The locus of quality, $\dot{q}_i = 0$, is negatively sloped. The second locus around the steady state (i.e. when qualities are symmetric), is $\dot{D} = 0$, or $D = 1/2$: each provider has half of the market. The dynamics of quality and demand around the steady state can be represented in matrix form as follows:

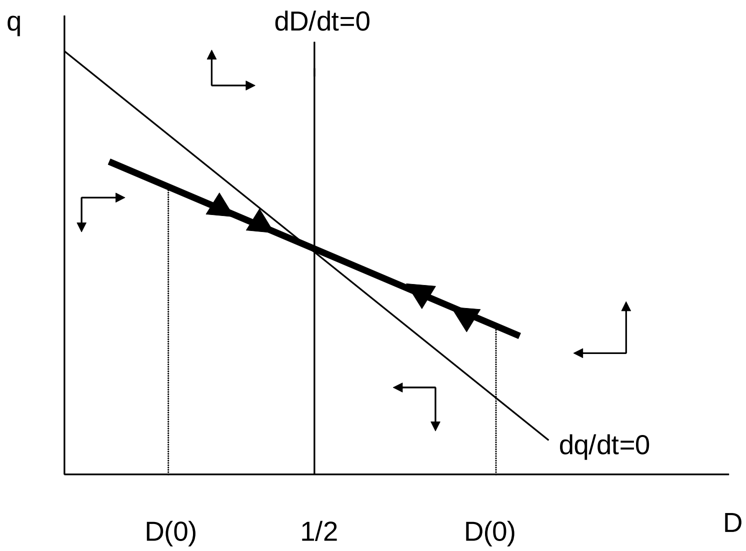
$$\begin{bmatrix} \dot{q}(t) \\ \dot{D}(t) \end{bmatrix} = \begin{bmatrix} (\rho+\gamma) & \frac{\gamma\beta}{2\tau\theta} \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} q(t) \\ D(t) \end{bmatrix} + \begin{bmatrix} -\frac{\gamma p}{2\tau\theta} \\ \frac{\gamma}{2} \end{bmatrix}, \tag{16}$$

where the 2-by-2 matrix is the Jacobian J of the dynamic system. As for the dynamic properties of the system, it is immediate to check that the Jacobian matrix J in (16) is such that $tr(J) = \rho > 0$, and $\det(J) = -\gamma\left(\rho+\gamma\right) < 0$, implying that

[23] This is the case since (a) $H_{q_iq_i} = -\theta < 0$; (b) $H_{DD} = -\beta < 0$; (c) $H_{DD}H_{q_iq_i} > \left(H_{Dq_i}\right)^2$ or $\theta\beta > 0$.

the equilibrium is stable in the saddle sense. The solution is described in Figure 1.[24]

Figure 1. Equilibrium is a saddle point



Let $D^s = 1/2$ be the steady state level of demand. Suppose we start off steady state at a level where the initial demand is low: $D(0) < D^s$. One possible interpretation is the case of a provider who at time 0 enters a previously monopolistic market. The solution is then characterised by a period of increasing demand and decreasing quality. Notice that the optimal solution for the 'incumbent' is precisely the opposite and it is equivalent to the case where the demand is high ($D(0) < 1/2 \iff 1 - D(0) > 1/2$). For this provider, we should observe a period of decreasing demand and increasing quality. These dynamic patterns establish our first main result:

Proposition 1 *If production costs are strictly convex in output, demand and quality move in opposite directions over time on the equilibrium path to the steady state.*

In the next section, we will show that the above result holds also when the players use feedback decision rules. In an extension to the main model, we will also show (Section 6) that this result is robust to the case of elastic market demand. Notice, however, that the result in Proposition 1 holds only if the cost function is strictly convex in output.

To grasp the intuition behind this result, it is useful to consider, as a benchmark for comparison, the special case of linear production costs, $\beta = 0$, implying that

[24] As shown in Appendix 1, the system is not only locally stable (around the steady state), but also globally stable.

the quality locus is horizontal at the steady state level of quality, q^s. In this case, if $D(0) \neq 1/2$, the two providers will immediately set their qualities at the steady state level, q^s, and maintain this quality level at all times. The demand dynamics, (13), will then eventually bring demand to the steady state level, $D^s = 1/2$, with the speed of adjustment depending of the degree of demand sluggishness. The reason is that, with constant marginal cost of output (and fixed prices), marginal profits $(\partial \pi_i / \partial q_i)$ are independent of output. Thus, the profit-maximising choice of quality is q^s irrespective of demand, and each provider will therefore keep quality at this level at each point in time.

On the other hand, when the cost function is strictly convex in output, $\beta > 0$, marginal profits depend on actual demand. More specifically, for a given level of quality, the marginal profit gain of higher quality is monotonically decreasing in the actual demand facing the provider, since new consumers are increasingly costly to serve. Thus, if a provider faces actual demand $D < D^s$, he will set quality $q > q^s$. As demand increases along the equilibrium dynamic path, the marginal profit gain of quality decreases; consequently, the provider will gradually reduce quality until the steady state level is reached. Obviously, the inverse logic applies for $D > D^s$.

We believe that the result in Proposition 1 has potential implications for empirical analyses of the effect of quality on demand and, in turn, of the relationship between competition and quality. In addition to the opposite movement of quality and demand over time, notice that, at a given point in time, a comparison of the two providers – off the steady state – unambiguously predicts a negative relationship between quality and demand. Thus, it is tempting to speculate that this type of equilibrium dynamics could potentially lead to spurious relationships between quality and demand in empirical studies, unless sufficient care is taken to account for endogeneity. Folland (1983), Luft et al. (1990), Burns and Wholey (1992), Hodgkin (1996), Tay (2003) and Howard (2005) for example find that higher quality, as measured by outcome measures (like standardised mortality ratios and complication rates) or process measures (like the ratio of staff per bed and the availability of specialised services), increases the demand for hospital care. Our model suggests that these estimates might be biased downwards as they may not take into account the fact that higher demand may simultaneously reduce quality.

Finally, to obtain the steady state level of quality, we set $\dot{q}_i = 0$, which, combined with $D^s = 1/2$, yields

$$q^{OL} = \left(\frac{1}{1 + \frac{\rho}{\gamma}} \right) \left(\frac{p - \frac{\beta}{2}}{2\tau\theta} \right). \tag{17}$$

If we consider the comparative statics properties of (17), the results are reasonable and intuitive. If the price is above the marginal cost, then lower transportation costs

(τ) or a higher price (p) increase quality. Similarly, a higher marginal cost of quality (β), a higher marginal cost of provision (θ) or a higher time preference discount rate (ρ) reduce quality. Steady state quality is also decreasing in the degree of demand sluggishness (measured by γ^{-1}). Notice that the steady state quality converges to the equilibrium quality in a corresponding static model for $\rho \to 0$ and $\gamma \to 1$. Thus, the dynamic open-loop solution yields a lower level of steady state quality than a corresponding static Nash equilibrium, and this difference is increasing with the degree of demand sluggishness.

4 Feedback solution

In solving for the feedback solution, we restrict attention to stationary Markovian strategies. More specifically, we obtain a stationary Markovian Nash equilibrium in linear strategies.[25] The full derivation of the feedback solution is given in Appendix 2. The equilibrium dynamic decision rules are found to be given by:[26]

$$q_i = \phi_i(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2 D\right) \tag{18}$$

and

$$q_j = \phi_j(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2(1-D)\right), \tag{19}$$

where[27]

$$\alpha_1 = \frac{p + \frac{\gamma\alpha_2}{2}\left(1 - \frac{\gamma\alpha_2}{2\theta\tau^2}\right)}{\gamma + \rho - \frac{\gamma^2}{4\theta\tau^2}\alpha_2} > 0, \tag{20}$$

$$\alpha_2 = -\frac{2\theta\tau^2\left(\psi - 2\gamma - \rho\right)}{3\gamma^2} < 0, \tag{21}$$

and

$$\psi := \sqrt{\left(2\gamma + \rho\right)^2 + \frac{3\beta\gamma^2}{\theta\tau^2}}. \tag{22}$$

The quality difference at each point in time is thus given by

$$q_i - q_j = \frac{\gamma\alpha_2}{\theta\tau}\left(D - \frac{1}{2}\right). \tag{23}$$

The first observation we want to highlight is the negative sign of α_2, implying a negative relationship between demand and quality over time along the equilibrium

[25] The strategy is said to be *Markovian* since the control at time t does not depend on the path followed by the state until t, but only on the current value of the state; it is *stationary* since it depends only on the current vaule of the state, and it is autonomous from the time t.

[26] To ease notation, we continue the practice of dropping time indications.

[27] The positive sign of α_1 is explicitly confirmed in the Appendix.

dynamic path. Thus, as previously mentioned, the result reported in Proposition 1 carries over to the feedback case, and the intuition is equivalent to the one given in the previous section, for the open-loop case. Once more, notice that this result holds only when the cost function is strictly convex in output, as $\alpha_2 = 0$ if $\beta = 0$.

Applying the steady state condition $D^s = 1/2$ to (18)-(19), steady state quality in the feedback solution is equal to

$$q^F = \left(\frac{1}{1 + \frac{\rho}{\gamma} - \frac{\gamma\alpha_2}{4\theta\tau^2}} \right) \left(\frac{p - \frac{\beta}{2}}{2\tau\theta} \right). \tag{24}$$

Comparing the steady state equilibria of the open-loop and feedback solutions, we see that, if production costs are linear in output ($\beta = 0$), implying $\alpha_2 = 0$, the two solutions coincide: $q^F = q^{OL}$. Otherwise, if the cost function is strictly convex in output ($\beta > 0$), implying $\alpha_2 < 0$, steady state quality is lower in the feedback solution. The reason for the coincidence result is related to the previously discussed implication of linear production costs, namely that the profit margin becomes independent of output. This implies an absence of strategic interaction between the two players that causes the two solution concepts to coincide. From (18)-(19), it is straightforward to verify that the optimal dynamic decision rules imply that each player sets quality at the steady state level at every point in time, irrespective of the quality chosen by the other player.[28]

However, a cost function that is strictly convex in output ($\beta > 0$) introduces a strategic interaction between the providers that implies that the optimal quality choice of provider j depends, at each point in time, on the actual demand of provider j. More specifically, qualities are *strategic complements* when $\beta > 0$. The intuition is the following: a quality increase by provider i will shift demand from provider j to provider i. This causes a reduction in marginal production costs for provider j. Since the price is constant, this leads to an increase in the profit margin of provider j, which, in turn, makes quality investments more profitable for this provider. Thus, a quality increase from provider i will be strategically met by a quality increase from provider j. This strategic complementarity establishes a positive relationship between q_i and q_j at each point in time in the optimal dynamic decision rules.[29] Moreover, notice that there exists an *intertemporal strategic complementarity* between the variables, according to the definition provided by Jun and Vives (2004), as long as the control of a player responds positively to a change in

[28] Notice also that the game becomes 'linear state' under $\beta = 0$, and it is well known that open-loop and closed-loop solutions coincide in such games (see, e.g., Dockner et al, 2000, Section 7.2).

[29] We can easily see this from (18)-(19) by noticing that D is increasing (decreasing) in q_i (q_j) and that $\alpha_2 < 0$ when $\beta > 0$.

the state pertaining to the opponent ($\partial q_j/\partial D > 0$, $\partial q_i/\partial(1-D) > 0$).

The above explained strategic complementarity introduces an intertemporal trade-off for the providers. When revising their quality choices according to the evolution of actual demand, each provider knows that an increase in the quality level will provoke a quality-increasing response by the competitor in the future. Thus, when contemplating an increase in the quality level, each provider must weigh the instantaneous gain in market share against the future loss due to the strategic response by the competitor. As long as the providers value future profits, this dynamic strategic interaction leads to a lower steady state level of quality in the feedback equilibrium (compared with the open-loop solution).[30]

The comparative statics properties of the feedback solution is qualitatively similar to the open-loop case. It is relatively straightforward to show[31] that more competition – measured either by less demand sluggishness or lower transportation costs – will increase steady state quality. However, the strength of the quality responses to increased competition differ. We can measure the relative quality response to an increase in competition by the following elasticities:

$$\eta_\gamma := \left| \frac{\partial q}{\partial \gamma} \frac{\gamma}{q} \right|, \tag{25}$$

$$\eta_\tau := \left| \frac{\partial q}{\partial \tau} \frac{\tau}{q} \right|. \tag{26}$$

Inserting the steady state levels of quality from the two solutions yields

$$\eta_\gamma^{OL} = \frac{\rho}{\rho+\gamma}; \quad \eta_\gamma^{F} = \frac{\rho\,(2\gamma+\rho+5\psi)}{\psi\,(4\gamma+5\rho+\psi)}; \tag{27}$$

$$\eta_\tau^{OL} = 1; \quad \eta_\tau^{F} = \frac{(2\gamma+\rho)^2 + (4\gamma+5\rho)\,\psi}{\psi\,(4\gamma+5\rho+\psi)}. \tag{28}$$

It is fairly straightforward to verify that $\eta_\gamma^{OL} > \eta_\gamma^{F}$ and $\eta_\tau^{OL} > \eta_\tau^{F}$ for all parameter configurations. Thus, an increase in competition – either through less sluggish demand or lower transportation costs – will have a stronger (weaker) impact on quality if the players use open-loop (feedback) decision rules.

The following Proposition summarises the most important steady state characteristics of the open-loop and feedback solutions:

[30] Notice that the strategic complementarity property is robust to relaxing the assumption of symmetric locations. As long as quality has a business-stealing effect, and as long as production costs are strictly convex in output, a quality increase by one provider will always increase the incentive for the competitor to increase quality as well. This is also the case if market demand is elastic, as we show in Section 6.

[31] See Appendix 2 for the details of the calculations.

Proposition 2 *(i) If production costs are linear in output, then* $q^{OL} = q^F$*.*

(ii) If production costs are strictly convex in output, then $q^{OL} > q^F$*.*

(iii) Less sluggish demand and/or lower transportation costs will increase the steady state level of quality under both solution concepts.

(iv) The positive impact of increased competition on quality is weaker in the feedback equilibrium.

5 Welfare and policy implications

In this section we derive the welfare properties of our model and analyse optimal price regulation. We start out by deriving the optimal (first-best) quality in the steady state solution and the corresponding optimal price for the two solution concepts. Subsequently, we derive the optimal quality paths off the steady state and show how these can be implemented by dynamic price regulation. Finally, we briefly discuss other policy measures that could be undertaken to affect the provision of quality in the market.

5.1 Optimal quality in the steady state

First-best quality in the steady state is derived by maximising (instantaneous) aggregate consumer utility net of quality and provision costs. The maximisation problem is given by

$$\underset{q_i,q_j}{\text{Maximise}} \int_0^{\frac{1}{2}+\frac{q_i-q_j}{2\tau}} (v + q_i - \tau x)\, dx + \int_{\frac{1}{2}+\frac{q_i-q_j}{2\tau}}^{1} (v + q_j - \tau (1 - x))\, dx$$
$$-\frac{\theta}{2}\left(q_i^2 + q_j^2\right) + \frac{\beta}{2}\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right)^2 + \frac{\beta}{2}\left(\frac{1}{2} + \frac{q_j - q_i}{2\tau}\right)^2.$$

Maximising this expression with respect to qualities yields

$$q_i = q_j = q^* = \frac{1}{2\theta}, \tag{29}$$

which implies $D = 1/2$.

With competition along only one dimension, namely quality, the first-best steady state level of quality can always be implemented by appropriate choice of the regulated price, p. Since equilibrium quality is monotonically increasing in the price, under both solution concepts, the optimal price in the steady state is such that

$$\frac{\beta}{2} + \tau\left(1 + \frac{\rho}{\gamma}\right) = p^{OL} < p^F = \frac{\beta}{2} + \tau\left(1 + \frac{\rho}{\gamma}\right) - \frac{\alpha_2\gamma}{4\theta\tau}. \tag{30}$$

Thus, if players use dynamic decision rules of the feedback type, more high-powered incentives, in the form of higher regulated prices, are necessary to induce first-best quality in the steady state.

5.2 Optimal dynamic price regulation

When demand is sluggish, the quality level given by (29) will typically not be optimal off the steady state, and neither will the steady state price rule given by (30). Instead there will generally be a time dependent welfare maximising quality path that can be implemented by an optimally chosen dynamic pricing rule. As before, we consider the cases of open-loop and feedback behaviour.

5.2.1 Open-loop behaviour

Suppose that the regulator can directly set the providers quality levels at each point in time. The first-best dynamic quality paths are then given by the solution to the following problem:

$$\underset{q_i, q_j}{\text{Maximise}} \int_0^{+\infty} W(t)\, e^{-\rho t} dt,$$

subject to

$$\frac{dD(t)}{dt} \equiv \dot{D}(t) = \gamma(D^*(t) - D(t)), \tag{31}$$

$$D(0) = D_0 > 0, \tag{32}$$

where

$$\begin{aligned} W(t) &= \int_0^{D(t)} (v + q_i(t) - \tau x)\, dx + \int_{D(t)}^1 (v + q_j(t) - \tau(1-x))\, dx \\ &\quad - \frac{\theta}{2}\left(q_i(t)^2 + q_j(t)^2\right) - \frac{\beta}{2}\left(D(t)^2 + (1 - D(t))^2\right). \end{aligned} \tag{33}$$

Let $\lambda(t)$ be the current value co-state variable associated with the state equation. The current-value Hamiltonian is

$$\begin{aligned} H &= \int_0^{D} (v + q_i - \tau x)\, dx + \int_{D}^1 (v + q_j - \tau(1-x))\, dx - \frac{\theta}{2}\left(q_i^2 + q_j^2\right) \\ &\quad - \frac{\beta}{2}\left(D^2 + (1 - D)^2\right) + \lambda\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right), \end{aligned} \tag{34}$$

The solution is given by (a) $\partial H/\partial q_i = 0$, $\partial H/\partial q_j = 0$, (b) $\dot{\lambda} = \rho\lambda - \partial H/\partial D$, (c) $\dot{D} = \partial H/\partial \lambda$, or more extensively:

$$D + \frac{\gamma}{2\tau}\lambda = \theta q_i, \tag{35}$$

$$(1 - D) - \frac{\gamma}{2\tau}\lambda = \theta q_j, \tag{36}$$

$$\dot{\lambda} = \lambda(\rho + \gamma) - [q_i - q_j + (\beta + \tau)(1 - 2D)], \tag{37}$$

$$\dot{D} = \gamma(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D), \tag{38}$$

to be considered along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\lambda(t)D(t) = 0$. The above conditions are also sufficient if the H is concave in (q_i, q_j, D), which requires that $\theta > \frac{1}{\beta+\tau}$.[32]

From (35) and (36), we find that $\lambda = (\theta q_i - D)\frac{2\tau}{\gamma} = (1 - D - \theta q_j)\frac{2\tau}{\gamma}$, which implies $q_j = \frac{1}{\theta} - q_i$. By totally differentiating (35), and using (37)-(38) and $q_j = \frac{1}{\theta} - q_i$, we derive

$$\dot{q_i}^* = q_i^*(\rho + \gamma) - \frac{\beta\gamma}{2\tau\theta} + \frac{\beta\gamma - \tau(\gamma + \rho)}{\theta\tau}D. \tag{39}$$

Setting $\dot{q_i}^* = 0$ and differentiating yields

$$\frac{\partial D}{\partial q_i^*}|_{\dot{q}_i^*=0} = \frac{\theta\tau(\rho + \gamma)}{\tau(\rho + \gamma) - \gamma\beta} < (>)\, 0 \quad if \quad \beta > (<)\, \tau\left(1 + \frac{\rho}{\gamma}\right). \tag{40}$$

Using again $q_j = \frac{1}{\theta} - q_i$, we can write

$$\dot{D} = \gamma(\frac{1}{2} + \frac{2\theta q_i^* - 1}{2\tau\theta} - D),$$

Setting $\dot{D} = 0$ and differentiating yields $\frac{\partial D}{\partial q_i^*}|_{\dot{D}=0} = \frac{1}{\tau} > 0$. The first-best solution is described in Figures 2 and 3. Notice that, in contrast with the equilibrium path (under open-loop or feedback behaviour), it is possible that quality and demand move together on the socially optimal path. This happens if the degree of production cost convexity is sufficiently low or if demand is sufficiently sluggish ($\beta < \tau\left(1 + \frac{\rho}{\gamma}\right)$),

[32] H is concave in (q_i, q_j, D) if the Hessian matrix

$$\begin{bmatrix} H_{q_iq_i} & H_{q_iq_j} & H_{q_iD} \\ H_{q_jq_i} & H_{q_jq_j} & H_{q_jD} \\ H_{Dq_i} & H_{Dq_j} & H_{DD} \end{bmatrix} = \begin{bmatrix} -\theta & 0 & 1 \\ 0 & -\theta & -1 \\ 1 & -1 & -2(\beta + \tau) \end{bmatrix}$$

is negative semidefinite. This is true if $\theta > \frac{1}{\beta+\tau}$.

which produces an *upward sloping* locus of quality.

Figure 2. First best. Case 1. quality increasing over time

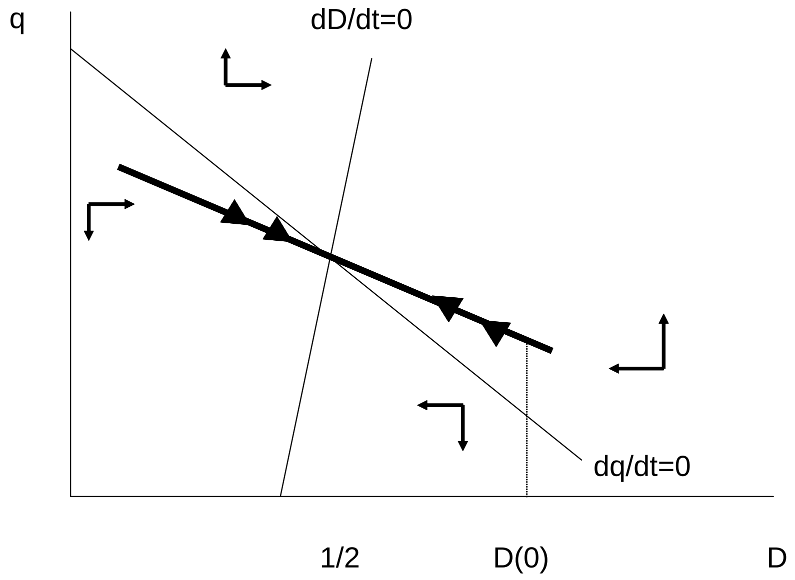
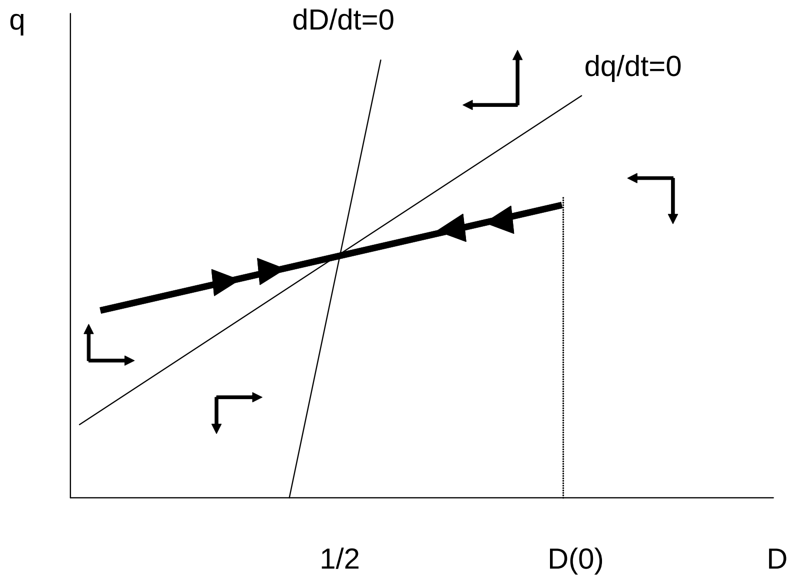


Figure 3. First best. Case 2. quality decreasing over time



In Figure 2 the provider with high initial demand starts from a low quality level which *increases* over time towards the steady state. Contrastingly, in Figure 3 the provider with high initial demand starts from a high quality level which *decreases* towards the steady state. The intuition for these results relates to the following regulatory trade-off: The provider with the higher initial demand has the higher marginal production costs (if $\beta > 0$). For this reason, the regulator has an incentive to choose a low quality level for the high-demand provider in order to shift demand towards the other provider, thereby improving allocative (cost) efficiency. On the other hand, consumers benefit more (in sum) from quality improvements by the provider with the higher demand. Thus, considerations for aggregate consumer utility indicate that the regulator should choose a high quality level for the high-demand provider. Notice how the relative strengths of these two incentives depend on the convexity of production costs (β) and the degree of demand sluggishness. In the extreme case of linear production costs ($\beta = 0$) the former incentive is eliminated, implying that quality and demand always move together on the optimal dynamic path (i.e., the provider with higher demand has always higher quality). Similarly, if demand is extremely sluggish ($\gamma \to 0$), reducing quality for the provider with high initial demand has little or no effect on actual demand, effectively eliminating the former incentive. Figure 2 illustrates the case where the former incentive dominates, while Figure 3 illustrates the opposite case. In the knife-edge case of $\beta = \tau\left(1 + \frac{\rho}{\gamma}\right)$ the two incentives exactly cancel each other and the optimal quality is constant over time. In the steady state, where $\dot{q_i}^* = \dot{D} = 0$, we obviously recover the solution derived in the previous sub-section, where $q^* = \frac{1}{2\theta}$ and $D = \frac{1}{2}$.

Let us now see how the first-best quality paths can be implemented by dynamic price regulation. This requires that the regulator can set different time-dependent prices for different providers. Comparing the first-best solution (39) with the equilibrium open-loop solution (14) for provider i, the first-best solution can be implemented by setting the price

$$p_i^{OL}(t) = \beta + \left(\frac{2\tau(\gamma+\rho)}{\gamma} - \beta\right) D(t). \tag{41}$$

In the steady state, where $D = 1/2$, we recover the optimal price p^{OL} given by (30), which is obviously equal for both providers. An instructive way to express the optimal time-dependent price is in the form of deviations from the steady-state first-best price. Using (30) and (41), the optimal dynamic price rule for provider i is given by

$$p_i^{OL}(t) = p^{OL} + \left[\tau\left(1 + \frac{\rho}{\gamma}\right) - \frac{\beta}{2}\right][2D(t) - 1]. \tag{42}$$

Notice that if provider i has a high initial market share, i.e. $D_0 > \frac{1}{2}$, then the price at time zero is higher than the steady state price only if $\beta < 2\tau\left(1 + \frac{\rho}{\gamma}\right)$.

In order to provide an intuitive characterisation of the optimal dynamic pricing rule, consider first the knife-edge case of $\beta = \tau\left(1 + \frac{\rho}{\gamma}\right)$, which implies a first-best quality level that is constant over time. In this case, the optimal price is given by

$$p_i^{OL}(t) = p^{OL} + \frac{\beta}{2}\left[2D(t) - 1\right] \tag{43}$$

If the price is fixed, we know from the analysis in Section 3 that the provider with high initial demand will provide low quality at the beginning and then increase quality over time. Thus, to induce the first-best quality, which in this example is constant over time, the regulator has to set a price above p^{OL} at time zero and then decrease the price over time, so that quality remains constant.

Assume that provider i has the higher initial demand. In general, there are then three different regimes to consider:

1. $\beta < \tau\left(1 + \frac{\rho}{\gamma}\right)$: First-best quality for provider i increases over time. The optimal price for this provider starts out higher than the steady-state price and decreases over time while quality increases.
2. $\beta \in \left(\tau\left(1 + \frac{\rho}{\gamma}\right), 2\tau\left(1 + \frac{\rho}{\gamma}\right)\right)$: First-best quality for provider i decreases (slowly) over time. The optimal price for this provider still starts out higher than the steady-state price and decreases over time along with the quality level.
3. $\beta > 2\tau\left(1 + \frac{\rho}{\gamma}\right)$: First-best quality for provider i decreases over time. The optimal price for this provider now starts out lower than the steady-state price and increases over time, while quality decreases.

Cases 1 and 3 are perhaps counter-intuitive. Case 1 (case 3) implies that reductions (increases) in prices over time are followed by increases (reductions) in quality. This is in contrast to static models (or to steady-state comparative statics) where an increase in price generates higher quality.

5.2.2 Feedback behaviour

In order to facilitate comparison with the feedback solution (Section 4) we can solve for the first-best quality path using the Bellman equation rather than the Hamiltonian. Using this approach (see Appendix 3 for details), the first-best quality is given, at each point in time, by

$$q_i^* = \alpha_1' \frac{\gamma}{2\tau\theta} + \left(\alpha_2' \frac{\gamma}{2\tau\theta} + \frac{1}{\theta}\right) D, \tag{44}$$

where

$$\alpha_1' = \frac{\left(\beta + \tau - \frac{1}{\theta}\right) + \frac{\gamma}{2}\left(1 - \frac{1}{\theta\tau}\right)\alpha_2'}{\gamma + \rho - \frac{\gamma}{\theta\tau}\left(1 + \frac{\gamma}{2\tau}\alpha_2'\right)} \tag{45}$$

and

$$\alpha_2' = -\frac{2\theta\tau^2}{\gamma^2}\left(\sqrt{\left(\frac{\gamma}{\theta\tau}-\gamma-\frac{\rho}{2}\right)^2+\frac{\gamma^2}{\theta\tau^2}\left(\beta+\tau-\frac{1}{\theta}\right)}+\left(\frac{\gamma}{\theta\tau}-\gamma-\frac{\rho}{2}\right)\right) < 0. \quad (46)$$

Consistent with the first-best solution derived from the Hamiltonian, it is easily shown, from (44) and (46), that $\frac{\partial q_i^*}{\partial D} > (<) \, 0$ if $\beta < (>) \, \tau \left(1+\frac{\rho}{\gamma}\right)$. Also, setting $D = \frac{1}{2}$ in (44) yields the first-best steady-state level $q^* = \frac{1}{2\theta}$. Since (44)-(46) describe the same dynamic path as (38) and (39), no further comments on this solution are necessary.

Once more, the first-best solution can be implemented also under feedback behaviour, using time-dependent and firm-specific prices. Comparing the first-best solution (44) with the equilibrium feedback solution (18) for provider i, the first-best solution can be implemented by setting the price

$$\begin{aligned} p_i^F(t) &= \frac{1}{\theta}\left((\gamma+\rho)\frac{2\tau\theta}{\gamma}-\frac{\gamma}{2\tau}\alpha_2\right)\left(1+\frac{\gamma}{2\tau}\left(\alpha_2'-\alpha_2\right)\right)D(t) \\ &\quad +\alpha_1'\left(\gamma+\rho-\frac{\gamma^2\alpha_2}{4\theta\tau^2}\right)-\frac{\gamma\alpha_2}{2}\left(1-\frac{\gamma\alpha_2}{2\theta\tau^2}\right), \end{aligned} \quad (47)$$

where α_1 and α_2 are given by (20) and (21), respectively. It is relatively straightforward to show that there exists a positive threshold value of β, given by

$$\widehat{\beta} := \frac{\tau}{8\gamma^2}\left(12\gamma(\gamma+\rho)+\theta\tau(2\gamma+\rho)\left(\sqrt{\frac{8\gamma}{\theta\tau}(\gamma+\rho)+(2\gamma+\rho)^2}-(2\gamma+\rho)\right)\right), \quad (48)$$

such that

$$\frac{\partial p_i^F(t)}{\partial D(t)} > (<) \, 0 \quad if \quad \beta < (>) \, \widehat{\beta}.$$

In qualitative terms, this corresponds exactly with the optimal dynamic price rule under open-loop behaviour, where the threshold value of β was given by $2\tau\left(1+\frac{\rho}{\gamma}\right)$. If production costs are sufficiently convex (high β), the optimal price for the high-demand provider starts out a level that is lower than the steady-state price, and then increases over time while demand decreases. Otherwise (low β), the high-demand provider faces a starting price that is higher then the steady-state level. On the equilibrium path to the steady state, the price is then gradually reduced while demand increases.

5.3 Other policy measures

Besides price regulation, we can also think of other policy measures that could be used in order to affect the providers' supply of quality. For example, a policy maker could take measures to reduce demand sluggishness in the market. One possible way to achieve this would be to develop and publish frequently updated quality indicators that increase consumers' awareness of quality differences in the market.[33] Since the providers' incentives to provide quality increases with the regulated price and decreases with the degree of demand sluggishness, notice that measures taken to increase the amount of information available to consumers would be a policy substitute to exposing the providers to high-powered incentive schemes (i.e., high prices). The less sluggish demand is (i.e., the higher γ is), the lower is the optimal price in the steady state.[34]

We have seen that, for given prices and demand sluggishness, equilibrium quality depends on whether the providers use open-loop or feedback strategies, where quality incentives are lower in the latter case. In principle, it could also be possible for a policy maker to influence the strategic context that the providers face; i.e., whether they find themselves in an open-loop or a feedback setting. Suppose that the policy maker would like to increase the degree of competition between the providers. From our previous analysis we know that this is equivalent to saying that the policy maker would prefer the providers to play open-loop (rather than feedback) strategies. The fundamental difference between the two decision rules is the degree of commitment. Thus, any institutional arrangement that increases the degree of commitment could make the providers' strategic context more similar to the open-loop setting and thereby induce quality decisions that are closer to the open-loop solution. One way to do this could be to require that the providers (e.g., hospitals or universities) make long-term investment plans with respect to quality that they have to commit to. Presumably, the commitment effect will be stronger the longer the time horizon of the required plans and the more detailed the plans are required to be. Obviously, any potential gains of requiring such plans must be weighed against the costs, which include a loss of flexibility in the case of changing market conditions.

[33] The publication of hospital and school 'League Tables' in the UK are examples of such policy measures.

[34] From (30) we immediately see that p^{OL} is decreasing in γ. It is relatively straightforward to show that the same is true for p^{F}.

6 Extension: Elastic market demand

Suppose that consumers are distributed on the entire real line with a constant density of 1. The two providers (i and j) are still located at 0 and 1, respectively. By this formulation, three indifferent consumers can be identified. The consumer who is indifferent between the two providers is located at $\widehat{x}$, implicitly given by $v+q_i-\tau\widehat{x} = v + q_j - \tau(1-\widehat{x})$. In addition, there are consumers who make the choice between acquiring the good from the nearest provider and staying out of the market. In the 'hinterland' of provider i, the consumer who is indifferent between these two choices is located at $\widehat{x}_i$, implicitly given by $v+q_i+\tau\widehat{x}_i = 0$, while the corresponding indifferent consumer in the hinterland of provider j is located at $\widehat{x}_j$, implicitly given by $v + q_j - \tau(\widehat{x}_j - 1) = 0$. *Potential demand* for provider i is then given by

$$D_i^* = \widehat{x} - \widehat{x}_i = \frac{1}{2} + \frac{q_i - q_j}{2\tau} + \frac{v + q_i}{\tau}, \tag{49}$$

while the provider's *actual demand* at time t is given by

$$\frac{dD_i(t)}{dt} = \gamma(D_i^* - D_i) = \gamma(\frac{1}{2} + \frac{q_i - q_j}{2\tau} + \frac{v + q_i}{\tau} - D_i). \tag{50}$$

6.1 Open-loop solution

When the total demand is not fixed, the dynamic optimisation problem has two state variables, D_i and D_j. Thus, with open-loop behaviour, each provider maximises discounted profits over the infinite time horizon, subject to two state equations:

$$\dot{D}_i(t) = \gamma(D_i^*(t) - D_i(t)), \tag{51}$$

$$\dot{D}_j(t) = \gamma(D_j^*(t) - D_j(t)), \tag{52}$$

along with some initial conditions. Letting $\mu_i(t)$ and $\mu_j(t)$ be the current value co-state variables associated with the two state equations, the current-value Hamiltonian is

$$\begin{aligned} H_i &= T + pD_i - C(D_i, q_i) - F + \mu_i\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} + \frac{v + q_i}{\tau} - D_i\right) \\ &\quad + \mu_j\gamma\left(\frac{1}{2} + \frac{q_j - q_i}{2\tau} + \frac{v + q_j}{\tau} - D_j\right). \end{aligned} \tag{53}$$

The solution is given by (a) $\partial H_i/\partial q_i = 0$, (b) $\dot{\mu_i} = \rho\mu_i - \partial H_i/\partial D_i$, (c) $\dot{\mu_j} = \rho\mu_j - \partial H_i/\partial D_j$, (d) $\dot{D} = \partial H_i/\partial \mu_i$, or more extensively:

$$\frac{3\gamma}{2\tau}\mu_i = \theta q_i, \tag{54}$$

$$\dot{\mu_i} = \mu_i(\rho+\gamma) - (p - \beta D_i), \tag{55}$$

$$\dot{\mu_j} = \mu_j(\rho+\gamma), \tag{56}$$

$$\dot{D_i} = \gamma(\frac{1}{2} + \frac{q_i - q_j}{2\tau} + \frac{v + q_i}{\tau} - D_i), \tag{57}$$

to be considered along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu_i(t)D_i(t) = 0$.

Totally differentiating (54) with respect to time we obtain $\frac{3\gamma}{2\tau}\dot{\mu_i} = \theta\dot{q_i}$, or, after substitution, $\frac{\gamma}{2\tau}(\mu_i(\rho+\gamma) - (p-\beta D_i)) = \theta\dot{q_i}$. Using $\mu_i = \theta q_i\frac{2\tau}{3\gamma}$, we obtain

$$\dot{q_i} = q_i(\rho+\gamma) - \frac{3\gamma(p-\beta D_i)}{2\tau\theta}, \tag{58}$$

which, together with (57), describe the dynamics of the equilibrium.

Setting $\dot{q_i} = 0$ and totally differentiating yields

$$\frac{\partial D_i}{\partial q_i}|_{\dot{q_i}=0} = -\frac{\theta(\rho+\gamma)2\tau}{3\gamma\beta} < 0. \tag{59}$$

As before, the locus of quality, $\dot{q_i} = 0$, is negatively sloped. The second locus around the steady state is $\dot{D_i} = 0$, or $D_i = \frac{1}{2} + \frac{v+q_i}{\tau}$. The locus of actual demand, $\dot{D_i} = 0$, is positively sloped:

$$\frac{\partial D_i}{\partial q_i}|_{\dot{D_i}=0} = \frac{1}{\tau} > 0. \tag{60}$$

This is quite intuitive, since we now allow quality to have a market expanding effect. Nevertheless, it is easily confirmed that the result in Proposition 1 still holds: quality and demand move in opposite direction over time on the equilibrium dynamic path.

Setting $\dot{q_i} = \dot{D_i} = 0$ in (57) and (58), the steady state solution is given by

$$q^{OL} = \frac{3}{2}\frac{(\tau(2p-\beta) - 2v\beta)\gamma}{3\beta\gamma + 2\theta\tau^2(\gamma+\rho)} \tag{61}$$

and

$$D^{OL} = \frac{3p\gamma + \theta\tau(\gamma+\rho)(2v+\tau)}{3\beta\gamma + 2\theta\tau^2(\gamma+\rho)}. \tag{62}$$

An interior solution ($q_i > 0$) requires that p is sufficiently high relative to β and v. If marginal production costs are constant ($\beta = 0$) we always have an interior solution as long as the price-cost margin is positive ($p > 0$).

It is also worth noticing that the relationship between transportation costs and steady state quality is now ambiguous if $\beta > 0$, as is easily confirmed from (61). The reason is that, when total demand is elastic, τ is no longer a 'pure' competition measure but also affects total demand. A reduction of τ will increase demand from the monopolistic segments (consumers located in the 'hinterlands'), which increases the marginal cost of treatment (if $\beta > 0$). All else equal, the optimal response for each provider is to dampen this demand increase by reducing quality. With constant marginal cost of treatment this effect vanishes and lower transportation costs always lead to higher quality.

6.2 Feedback solution

When market demand is elastic, a full analytical derivation of the feedback solution is no longer feasible. We will therefore derive the solution as function of the price only, where we choose the following numerical values for the remaining parameters: $\tau = \theta = v = \beta = 1$, $\gamma = \frac{1}{2}$ and $\rho = 0.95$.

In Appendix 4, we show that, with the above parameterisation, the equilibrium dynamic decision rule is given by

$$q_i = \frac{1}{4}\left(3\alpha_1 - \alpha_3 + (3\alpha_2 - \alpha_5)\, D_i + (3\alpha_5 - \alpha_4)\, D_j\right), \tag{63}$$

where

$$\alpha_1 := 0.6167p - 0.2182; \quad \alpha_2 := -0.4555, \tag{64}$$

$$\alpha_3 := 0.0283p - 0.0174; \quad \alpha_4 := -0.0010; \quad \alpha_5 := -0.0157. \tag{65}$$

As in the open-loop solution, the negative dynamic relationship between quality and demand is preserved, as $3\alpha_2 - \alpha_5 < 0$.

In the steady state, quality is given by

$$q^F = 0.3376p - 0.5063, \tag{66}$$

while demand is $D = 0.9937 + 0.337\,6p$. Using the same parameterisation in (61), the corresponding steady state quality level in the open-loop solution is given by

$$q^{OL} = 0.3409p - 0.5114. \tag{67}$$

Comparing the two solutions, we find that $q^{OL} > q^F$ if $p > 1.4970$. Since an interior solution ($q > 0$) requires $p > 1.5$, the result from this numeric example is in line with our previous conclusion, that steady state quality is lower in the feedback solution than in the open-loop solution. With elastic market demand, it follows that steady state output is also lower in the feedback solution.

Although the inability to derive a full analytical solution restricts the generality of the analysis, we have no reason to believe that qualitatively different results could be obtained with other parameter configurations. The reason is that the key feature of the model – qualities being strategic complements – is unaffected by whether market demand is elastic or not.

7 Concluding remarks

In this paper, we have analysed the impact of competition on quality in a market with regulated prices and sluggish demand. The basic model is the widely used Hotelling model where products are horizontally and (potentially) vertically differentiated. We have considered the case in which the spatial locations are exogenously fixed, while firms choose quality. These choices are studied within a dynamic framework, where demand responds to quality changes with some degree of sluggishness, implying a divergence between actual and potential demand (out of steady state). We would like to stress that our assumptions fit quite well with the features of markets with regulated price – let us think of education or health: the spatial locations of providers are given; competition among providers is based mainly on the product quality; prices play a limited role in the competitive process; the consumer behavior is characterised by a certain degree of stickiness.

Using a differential-game approach, we have derived the open-loop and the feedback (closed-loop) solutions. In the open-loop solution, each provider knows the quality of the competitor in the initial state, and chooses the time path of quality efforts at the beginning, and then stick to this plan for the whole length of the game. In the feedback solution, each provider knows the quality of the competitor, not only in the initial state, but also in all subsequent periods, and thus can choose the quality effort at each point in time, possibly responding to quality changes by the competitor. Specifically, we have found the feedback closed-loop Markovian solution, in which the current choice of each player depends on the current value of the state variables.

The analysis has provided two main findings, both of which relate to the case of strictly convex production costs. First, we found (under both solution concepts) a negative relationship between quality and demand off the steady state, which is contrary to the static relationship. The reason is that the marginal profit gain is decreasing in quality. Second, we showed that the feedback solution results in lower quality than the open-loop solution. On the other hand, if production costs are linear in output, the two solutions coincide. Once again, the reason for such results is that with strictly convex production costs, quality choices are strategic complements, while with linear production costs they are strategically independent. Thus, when firms can observe (and respond to) the competitors' quality at any time period, and quality choices are strategic complements, quality will be lower.

Regarding social welfare, we have shown how a regulator in principle can implement the first-best quality paths by dynamic price regulation, choosing time-dependent and firm-specific prices. The regulator might also use non-price instruments to induce more socially preferable outcomes. For example, our analysis of the open-loop versus feedback solutions shows that by forcing firms to stick to long-term investment plans (i.e., to adopt open-loop rules in terms of differential game theory), lower monetary incentives (i.e., prices) are needed to reach the first-best quality level, compared with the situation in which firms can make quality choices at each point in time. Moreover, if demand sluggishness could be affected by the regulator, for instance, by public disclosure of quality indicators, then this would also be a policy substitute to high-powered incentives.

We find this analysis relevant for several regulated industries, especially health care and education. In these markets, quality is a major concern, and prices are less crucial when consumers choose a provider. Many European governments have introduced (elements of) competition in health care and education in order to stimulate quality. In the US competition has been in place for many years. Recently, we have seen a trend in both the US and in Europe towards publishing quality rankings (league tables) of hospitals, universities, schools, etc. Obviously, this is done to stimulate demand responses to quality differences. The purpose of our paper has been to analyse the impact of competition on quality in regulated markets when demand is not responding instantaneously to quality differences. Hopefully, our analysis can shed some light on the recent reforms in health care and education.

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Appendix

1. Global stability

The dynamics of qualities and demand derived in Section 2 are: $\dot{q_i} = q_i(\rho+\gamma) - \frac{\gamma(p-\beta D)}{2\tau\theta}$, $\dot{q_j} = q_j(\rho+\gamma) - \frac{\gamma(p-\beta(1-D))}{2\tau\theta}$, $\dot{D} = \gamma(\frac{1}{2} + \frac{q_i-q_j}{2\tau} - D)$, or in matrix form:

$$\begin{bmatrix} \dot{q_i}(t) \\ \dot{q_j}(t) \\ \dot{D}(t) \end{bmatrix} = \begin{bmatrix} (\rho+\gamma) & 0 & \frac{\gamma\beta}{2\tau\theta} \\ 0 & (\rho+\gamma) & -\frac{\gamma\beta}{2\tau} \\ \frac{\gamma}{2\tau} & -\frac{\gamma}{2\tau} & -\gamma \end{bmatrix} \begin{bmatrix} q_i(t) \\ q_j(t) \\ D(t) \end{bmatrix} + \begin{bmatrix} -\frac{\gamma p}{2\tau\theta} \\ -\frac{\gamma p}{2\tau\theta} \\ \frac{\gamma}{2} \end{bmatrix},$$

There are two positive eigenvalues ($\gamma+\rho>0$, $\frac{\Lambda+\tau\rho}{2\tau}>0$) and a negative one ($-\frac{\Lambda-\tau\rho}{2\tau}<0$), where $\Lambda := \sqrt{\tau^2\rho^2 + \gamma^2\beta\left(1+\frac{1}{\theta}\right) + 4\tau^2\gamma(\gamma+\rho)}$, which implies full stability, i.e. there is only one admissible path which leads to the steady state. Moreover, define $Q := q_i - q_j$. Then, we can re-write the dynamics of the system as: $\dot{Q} = (\rho+\gamma)Q + \frac{\gamma\beta}{\tau\theta}\left(\frac{1}{2} - D\right)$, $\dot{D} = \gamma(\frac{1}{2} - D + \frac{Q}{2\tau})$. A phase diagram analysis in the (Q, D) space reveals that if $D(0) > D^s$, then $Q < 0$, and $q_i < q_j$: the provider with higher initial demand provides lower quality. The dynamics described in Figure 1 is therefore also global.

2. The feedback solution in the basic framework

Provider i's instantaneous objective function is

$$T + pD - \frac{\theta}{2}q_i^2 - \frac{\beta}{2}D^2 \tag{A1}$$

in which time index is suppressed to ease notation. Eq.(A1), together with the linear dynamic constraint, (4), gives rise to a linear-quadratic problem. Hence, we define the value function of provider i as

$$V^i(D) = \alpha_0 + \alpha_1 D + (\alpha_2/2)D^2, \tag{A2}$$

implying $V_D^i(D) = \alpha_1 + \alpha_2 D$. Notice that $\alpha_2 < 0$ is required to ensure concavity of the value function, and hence stability of the strategies.

The optimal investment strategies are functions of actual demand at each point in time. Thus, we define $q_i = \phi_i(D)$ and $q_j = \phi_j(D)$. We are focusing on stationary Markovian linear strategies. The value function has to satisfy the Hamilton-Jacobi-Bellman (HJB) equation, which, for provider i, is given by

$$\rho V^i(D) = \max\left\{T + pD - \frac{\theta}{2}q_i^2 - \frac{\beta}{2}D^2 + V_D^i\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right)\right\}. \tag{A3}$$

Maximisation of the right-hand-side yields $-\theta q_i + V^i_D \frac{\gamma}{2\tau} = 0$, which, after substitution of V^i_D, yields

$$q_i = \phi_i(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2 D\right). \tag{A4}$$

By symmetry, the optimal investment strategy for provider j is given by

$$q_j = \phi_j(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2(1-D)\right), \tag{A5}$$

implying that the quality difference at time t is given by $q_i - q_j = \frac{\gamma\alpha_2}{\theta\tau}\left(D - \frac{1}{2}\right)$. Notice that quality of provider i is higher than quality of provider j if demand is lower than half of the market (assuming $\alpha_2 < 0$).

Substituting $q_i = \phi_i(D)$, $q_j = \phi_j(D)$ and $V^i_D(D) = \alpha_1 + \alpha_2 D$ into (A3), we obtain

$$\rho V^i(D) = \left\{ \begin{array}{c} T + pD - \frac{\theta}{2}\frac{\gamma^2}{(2\tau\theta)^2}\left(\alpha_1 + \alpha_2 D\right)^2 - \frac{\beta}{2}D^2 \\ +\left(\alpha_1 + \alpha_2 D\right)\gamma\left(\frac{1}{2} + \frac{\gamma\alpha_2}{2\tau^2\theta}\left(D - \frac{1}{2}\right) - D\right) \end{array} \right\}. \tag{A6}$$

For the above equality to hold, the parameters must satisfy the following equations:

$$\rho\alpha_0 - \frac{\gamma\alpha_1}{2} - T + \frac{\gamma^2\alpha_1^2}{8\theta\tau^2} + \frac{\gamma^2\alpha_1\alpha_2}{4\theta\tau^2} = 0, \tag{A7}$$

$$\left(\gamma\alpha_1 - p - \frac{\gamma\alpha_2}{2} + \rho\alpha_1 + \frac{\gamma^2\alpha_2^2}{4\theta\tau^2} - \frac{\gamma^2\alpha_1\alpha_2}{4\theta\tau^2}\right) D = 0, \tag{A8}$$

$$\left(\frac{\beta}{2} + \gamma\alpha_2 + \frac{\rho\alpha_2}{2} - \frac{3\gamma^2\alpha_2^2}{8\theta\tau^2}\right) D^2 = 0. \tag{A9}$$

From (A9), solving for α_2, we obtain two candidate solutions:

$$\alpha_2 = \frac{2\tau\theta\tau^2}{3\gamma^2}\left(2\gamma + \rho \pm \sqrt{\left(2\gamma + \rho\right)^2 + \frac{3\beta\gamma^2}{\theta\tau^2}}\right). \tag{A10}$$

The condition that the value function be concave leads us to select the negative root; this condition on α_2 ensures the global stability of the steady state. From (A8) we have

$$\alpha_1 = \frac{p + \frac{\gamma\alpha_2}{2}\left(1 - \frac{\gamma\alpha_2}{2\theta\tau^2}\right)}{\gamma + \rho - \frac{\gamma^2}{4\theta\tau^2}\alpha_2}. \tag{A11}$$

In order to establish the sign of α_1, notice that the numerator in (A11) is monotonically increasing in α_2.[35] Furthermore, it is straightforward to verify that $\frac{\partial\alpha_2}{\partial\rho} > 0$ and $\frac{\partial\alpha_2}{\partial\beta} < 0$. An interior solution requires that price is higher than marginal production costs. Thus, α_2 approaches its lowest permissible value if $\rho \to 0$ and $\beta \to 2p$. In this case, $\alpha_2 = 2\theta\tau^2\frac{2-\sqrt{4+\frac{6p}{\theta\tau^2}}}{3\gamma}$, and the numerator of (A11) is given

[35] $\frac{\partial\left(p + \frac{\gamma\alpha_2}{2}\left(1 - \frac{\gamma\alpha_2}{2\theta\tau^2}\right)\right)}{\partial\alpha_2} = \frac{1}{2}\gamma\frac{\theta\tau^2 - \gamma\alpha_2}{\theta\tau^2} > 0$.

by $\frac{1}{9}\left(3p+\theta\tau^2\left(\sqrt{2}\sqrt{2+\frac{3p}{\theta\tau^2}}-2\right)\right)$, which is unambiguously positive. Thus, we conclude that α_1 is positive for all permissible parameter configurations.

In the steady state $D^s = 1/2$, so that $q_i = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \frac{\alpha_2}{2}\right)$, which, after substitution of α_1, leads to

$$q^F = \left(\frac{1}{1+\frac{\rho}{\gamma}-\frac{\gamma\alpha_2}{4\theta\tau^2}}\right)\left(\frac{p-\frac{\beta}{2}}{2\tau\theta}\right), \tag{A12}$$

where $\alpha_2 < 0$ is given by the negative root in (A10). The comparative statics properties of (A12) are given by

$$\frac{\partial q^F}{\partial\gamma} = \frac{3\rho\left(2\gamma+\rho+5\psi\right)\left(p-\frac{\beta}{2}\right)}{\left(4\gamma+5\rho+\psi\right)^2\tau\theta\psi} > 0 \tag{A13}$$

and

$$\frac{\partial q^F}{\partial\tau} = -\frac{\left(p-\frac{\beta}{2}\right)\left[\left(2\gamma+\rho\right)^2+\psi\left(4\gamma+5\rho\right)\right]3\gamma}{\left[\begin{array}{c}\theta\tau^2\psi\left[\left(4\gamma+5\rho\right)^2+\psi^2\right]+24\beta\gamma^3\\ +2\theta\tau^2\left(16\gamma^3+5\rho^3\right)+24\theta\tau^2\gamma\rho\left(3\gamma+2\rho\right)\end{array}\right]} < 0, \tag{A14}$$

where $\psi := \sqrt{\left(2\gamma+\rho\right)^2+\frac{3\beta\gamma^2}{\theta\tau^2}}$.

3. First-best quality using the Bellman equation

Instantaneous social welfare is given by

$$\int_0^D\left(v+q_i-\tau x\right)dx+\int_D^1\left(v+q_j-\tau\left(1-x\right)\right)dx-\frac{\theta}{2}\left(q_i^2+q_j^2\right)-\frac{\beta}{2}\left(D^2+\left(1-D\right)^2\right), \tag{A15}$$

We define the value function of provider i as

$$V^i(D) = \alpha_0' + \alpha_1' D + (\alpha_2'/2)D^2, \tag{A16}$$

implying $V_D^i(D) = \alpha_1' + \alpha_2' D$. Notice that $\alpha_2' < 0$ is required to ensure concavity of the value function. The optimal investment strategies are functions of actual demand at each point in time. Thus, we define $q_i = \phi_i(D)$ and $q_j = \phi_j(D)$. The value function has to satisfy the Hamilton-Jacobi-Bellman (HJB) equation, which, for the regulator, is given by

$$\rho V(D) = \max_{q_i,q_j}\left\{\begin{array}{c} v+q_iD+q_j(1-D)-\frac{\tau+\beta}{2}\left(D^2+(1-D)^2\right)\\ -\frac{\theta}{2}\left(q_i^2+q_j^2\right)+V_D\gamma\left(\frac{1}{2}+\frac{q_i-q_j}{2\tau}-D\right)\end{array}\right\}. \tag{A17}$$

Maximisation of the right-hand-side yields $D - \theta q_i + V_D \frac{\gamma}{2\tau} = 0$ and $(1-D) - \theta q_j - V_D \frac{\gamma}{2\tau} = 0$, which, after substitution of $V_D = \alpha_1' + \alpha_2' D$, yields

$$q_i = \alpha_1' \frac{\gamma}{2\tau\theta} + \left(\alpha_2' \frac{\gamma}{2\tau\theta} + \frac{1}{\theta} \right) D \tag{A18}$$

and

$$q_j = \frac{(1-D)}{\theta} - \left(\alpha_1' + \alpha_2' D \right) \frac{\gamma}{2\tau\theta}. \tag{A19}$$

Substituting $q_i = \phi_i(D)$, $q_j = \phi_j(D)$, $V_D^i(D) = \alpha_1' + \alpha_2' D$ and $V^i(D)$ into (A17), we obtain

$$\begin{aligned} 0 &= v - \frac{\beta}{2} - \frac{\tau}{2} + \frac{1}{2\theta} + \frac{\gamma\alpha_1'}{2} - \rho\alpha_0 + \frac{\gamma^2 (\alpha_1')^2}{4\theta\tau^2} - \frac{\gamma\alpha_1'}{2\theta\tau} \\ &\quad + D \left(- \left(\frac{1}{\theta} - \beta - \tau \right) + \alpha_2' \frac{\gamma}{2} \left(1 - \frac{1}{\theta\tau} \right) + \alpha_1' \left(\frac{\gamma}{\theta\tau} - \gamma - \rho + \frac{\gamma^2 \alpha_2'}{2\theta\tau^2} \right) \right) \\ &\quad + D^2 \left(\frac{1}{\theta} - \beta - \tau - \gamma\alpha_2' - \frac{\rho\alpha_2'}{2} + \frac{\gamma^2 (\alpha_2')^2}{4\theta\tau^2} + \frac{\gamma\alpha_2'}{\theta\tau} \right). \end{aligned} \tag{A20}$$

The solution is found to be

$$\alpha_1' = \frac{\left(\beta + \tau - \frac{1}{\theta} \right) + \left(\frac{\gamma}{2} - \frac{\gamma}{2\theta\tau} \right) \alpha_2}{\gamma + \rho - \frac{\gamma}{\theta\tau} - \frac{\gamma^2}{2\theta\tau^2} \alpha_2} \tag{A21}$$

and

$$\alpha_2' = \frac{\pm\sqrt{\left(\frac{\gamma}{\theta\tau} - \gamma - \frac{\rho}{2} \right)^2 + \frac{\gamma^2}{\theta\tau^2} \left(\beta + \tau - \frac{1}{\theta} \right)} - \left(\frac{\gamma}{\theta\tau} - \gamma - \frac{\rho}{2} \right)}{\frac{\gamma^2}{2\theta\tau^2}}. \tag{A22}$$

The condition that the value function be concave leads us to select the negative root.

4. Elastic market demand (feedback solution)

We define the value function of provider i as

$$V^i(D_i, D_j) = \alpha_0 + \alpha_1 D_i + (\alpha_2/2) D_i^2 + \alpha_3 D_j + (\alpha_4/2) D_j^2 + \alpha_5 D_i D_j, \tag{A23}$$

implying $V_{D_i}^i = \alpha_1 + \alpha_2 D_i + \alpha_5 D_j$, $V_{D_j}^j = \alpha_1 + \alpha_2 D_j + \alpha_5 D_i$ and $V_{D_j}^i = \alpha_3 + \alpha_4 D_j + \alpha_5 D_i$. Notice that $\alpha_2 < 0$ is required to ensure concavity of the value functions.

The optimal investment strategies are functions of actual demand at each point in time: $q_i = \phi_i(D_i, D_j)$ and $q_j = \phi_j(D_j, D_i)$. The value function has to satisfy the

Hamilton-Jacobi-Bellman (HJB) equation, which, for provider i, is given by

$$\rho V^i(D_i, D_j) = \max \left\{ \begin{array}{c} T + pD_i - \frac{\theta}{2}q_i^2 - \frac{\beta}{2}D_i^2 \\ +V^i_{D_i}\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} + \frac{v+q_i}{\tau} - D_i\right) \\ +V^i_{D_j}\gamma\left(\frac{1}{2} - \frac{q_i - q_j}{2\tau} + \frac{v+q_j}{\tau} - D_j\right) \end{array} \right\}. \tag{A24}$$

Maximisation of the right-hand-side yields $\theta q_i = V^i_{D_i}\frac{3\gamma}{2\tau} - V^i_{D_j}\frac{\gamma}{2\tau}$, which, after substitution of $V^i_{D_i}$ and $V^i_{D_j}$, yields the optimal dynamic decision rule for provider i:

$$q_i = \phi_i(D_i, D_j) = \frac{\gamma}{2\tau\theta}\left(3\alpha_1 - \alpha_3 + (3\alpha_2 - \alpha_5) D_i + (3\alpha_5 - \alpha_4) D_j\right), \tag{A25}$$

with a symmetric expression for the optimal decision rule of provider j. Substituting $q_i = \phi_i(D_i, D_j)$, $q_j = \phi_j(D_j, D_i)$, $V^i_{D_i}$ and $V^i_{D_j}$ into (A24), we obtain

$$\rho V^i(D_i, D_j) = \left\{ \begin{array}{c} T + pD_i - \frac{\theta}{2}\left(\frac{\gamma}{2\tau\theta}\left(3\alpha_1 - \alpha_3 + (3\alpha_2 - \alpha_5) D_i + (3\alpha_5 - \alpha_4) D_j\right)\right)^2 - \frac{\beta}{2}D_i^2 \\ +(\alpha_1 + \alpha_2 D_i + \alpha_5 D_j)\gamma \left(\begin{array}{c} \frac{1}{2} + \frac{(D_i - D_j)\left(\frac{3\gamma}{2\tau\theta}(\alpha_2 - \alpha_5) - \frac{\gamma}{2\tau\theta}(\alpha_5 - \alpha_4)\right)}{2\tau} \\ + \frac{v + \left(\frac{\gamma}{2\tau\theta}(3\alpha_1 - \alpha_3 + (3\alpha_2 - \alpha_5)D_i + (3\alpha_5 - \alpha_4)D_j)\right)}{\tau} - D_i \end{array} \right) \\ +(\alpha_3 + \alpha_4 D_j + \alpha_5 D_i)\gamma \left(\begin{array}{c} \frac{1}{2} - \frac{(D_i - D_j)\left(\frac{3\gamma}{2\tau\theta}(\alpha_2 - \alpha_5) - \frac{\gamma}{2\tau\theta}(\alpha_5 - \alpha_4)\right)}{2\tau} \\ + \frac{v + \left(\frac{\gamma}{2\tau\theta}(3\alpha_1 - \alpha_3 + (3\alpha_2 - \alpha_5)D_j + (3\alpha_5 - \alpha_4)D_i)\right)}{\tau} - D_j \end{array} \right) \end{array} \right\} \tag{A26}$$

At this point, we need to attach some numerical values to the parameters of the model in order to obtain a solution. Setting $\tau = \theta = v = \beta = 1$, $\gamma = \frac{1}{2}$ and $\rho = 0.95$, and substituting the expression for $V^i(D_i, D_j)$ from (A23) into (A26), the solution is given by

$$\begin{aligned} & \left(\frac{5}{32}\alpha_3^2 - \frac{3}{32}\alpha_1^2 - \frac{7}{16}\alpha_1\alpha_3 - \frac{3}{4}\alpha_1 - \frac{3}{4}\alpha_3 + 0.95\alpha_0\right) \\ & +D_i\left(\begin{array}{c} 1.45\alpha_1 - \frac{3}{8}\alpha_1\alpha_2 - \frac{1}{16}\alpha_1\alpha_4 - \frac{3}{16}\alpha_1\alpha_5 + \frac{1}{8}\alpha_2\alpha_3 \\ -\frac{3}{4}\alpha_2 + \frac{3}{16}\alpha_3\alpha_4 - \frac{7}{16}\alpha_3\alpha_5 - \frac{3}{4}\alpha_5 - p \end{array} \right) \\ & +D_j\left(\begin{array}{c} \frac{3}{16}\alpha_1\alpha_2 - \frac{3}{8}\alpha_1\alpha_4 - \frac{7}{16}\alpha_1\alpha_5 - \frac{9}{16}\alpha_2\alpha_3 + \frac{1}{8}\alpha_3\alpha_4 \\ +\frac{5}{16}\alpha_3\alpha_5 + 1.45\alpha_3 - \frac{3}{4}\alpha_4 - \frac{3}{4}\alpha_5 \end{array} \right) \\ & +D_i^2\left(\frac{1}{2} - \frac{9}{32}\alpha_2^2 - \frac{1}{16}\alpha_2\alpha_4 + \frac{3}{8}\alpha_2\alpha_5 + 0.975\,\alpha_2 + \frac{3}{16}\alpha_4\alpha_5 - \frac{19}{32}\alpha_5^2\right) \\ & +D_j^2\left(\frac{3}{16}\alpha_2\alpha_5 - \frac{9}{16}\alpha_2\alpha_4 - \frac{1}{32}\alpha_4^2 + \frac{3}{8}\alpha_4\alpha_5 + 0.975\,\alpha_4 - \frac{11}{32}\alpha_5^2\right) \\ & +D_iD_j\left(\frac{3}{16}\alpha_2^2 + \frac{3}{16}\alpha_2\alpha_4 - \frac{19}{16}\alpha_2\alpha_5 + \frac{3}{16}\alpha_4^2 - \frac{11}{16}\alpha_4\alpha_5 + \frac{9}{16}\alpha_5^2 + 1.95\alpha_5\right) \\ = & \ 0 \end{aligned} \tag{A27}$$

For the equality to hold, the terms in brackets in the above equation must be equal to zero. Since the last three terms only depend on α_2, α_4 and α_5, we start by solving the following system of equations:

$$\begin{aligned}
\left(\frac{1}{2}-\frac{9}{32}\alpha_2^2-\frac{1}{16}\alpha_2\alpha_4+\frac{3}{8}\alpha_2\alpha_5+0.975\,\alpha_2+\frac{3}{16}\alpha_4\alpha_5-\frac{19}{32}\alpha_5^2\right) &= 0,\\
\left(\frac{3}{16}\alpha_2\alpha_5-\frac{9}{16}\alpha_2\alpha_4-\frac{1}{32}\alpha_4^2+\frac{3}{8}\alpha_4\alpha_5+0.975\,\alpha_4-\frac{11}{32}\alpha_5^2\right) &= 0,\\
\left(\frac{3}{16}\alpha_2^2+\frac{3}{16}\alpha_2\alpha_4-\frac{19}{16}\alpha_2\alpha_5+\frac{3}{16}\alpha_4^2-\frac{11}{16}\alpha_4\alpha_5+\frac{9}{16}\alpha_5^2+1.95\alpha_5\right) &= 0.
\end{aligned}$$

There are six possible solutions:

$$\left\{\begin{array}{c}
[\alpha_2=3.9951,\quad \alpha_4=-0.4800,\quad \alpha_5=2.3900]\,,\\
[\alpha_2=0.3924,\quad \alpha_4=0.9675,\quad \alpha_5=-0.9275]\,,\\
[\alpha_2=-0.4555,\quad \alpha_4=-0.0010,\quad \alpha_5=-0.0157]\,,\\
[\alpha_2=2.5681,\quad \alpha_4=2.8403,\quad \alpha_5=2.9188]\,,\\
[\alpha_2=1.8194,\quad \alpha_4=-2.3528,\quad \alpha_5=-1.4563]\,,\\
[\alpha_2=4.8430,\quad \alpha_4=0.4885,\quad \alpha_5=1.4782]\,.
\end{array}\right\} \tag{A28}$$

We choose the solution with negative α_2, as this is required for the value function to be concave. We still need to compute α_1 and α_3. Using the second and third term in (A27), we can find α_1 and α_3 by simultaneously solving the following two equations:

$$\begin{aligned}
\left(\begin{array}{c} 1.45\alpha_1-\frac{3}{8}\alpha_1\alpha_2-\frac{1}{16}\alpha_1\alpha_4-\frac{3}{16}\alpha_1\alpha_5+\frac{1}{8}\alpha_2\alpha_3\\ -\frac{3}{4}\alpha_2+\frac{3}{16}\alpha_3\alpha_4-\frac{7}{16}\alpha_3\alpha_5-\frac{3}{4}\alpha_5-p\end{array}\right) &= 0,\\
\left(\begin{array}{c} \frac{3}{16}\alpha_1\alpha_2-\frac{3}{8}\alpha_1\alpha_4-\frac{7}{16}\alpha_1\alpha_5-\frac{9}{16}\alpha_2\alpha_3+\frac{1}{8}\alpha_3\alpha_4\\ +\frac{5}{16}\alpha_3\alpha_5+1.45\alpha_3-\frac{3}{4}\alpha_4-\frac{3}{4}\alpha_5\end{array}\right) &= 0.
\end{aligned}$$

Inserting the values of α_2, α_4 and α_5 found above, the solution is

$$\alpha_1=0.6167p-0.2182,\quad \alpha_3=0.0283p-0.0174. \tag{A29}$$

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