Efficient Education Policy – A Second-Order Elasticity Rule

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Abstract

Assuming a two-period model with endogenous choices of labor, education, and saving, efficient education policy is characterized for a Ramsey-like scenario in which the government is constrained to use linear instruments. It is shown that education should be effectively subsidized if, and only if, the elasticity of the earnings function is increasing in education. The strength of second-best subsidization increases in the elasticity of the elasticity of the earnings function. This second-order elasticity rule extends the well-known Ramsey-Boiteux inverse elasticity rule.

JEL-Code: H21, I28, J24.

Keywords: endogenous choice of education, second-best efficient taxation, linear instruments, finite periods, Ramsey's Rule, inverse elasticity rule.

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1. Introduction

The inverse elasticity rule states that optimally chosen commodity tax rates should be inversely related to the price elasticities of demand. The rule is usually attributed to Ramsey (1927), but a structurally equivalent formula was derived independently by Boiteux (1956) for the problem of optimal monopoly pricing. The assumptions underlying optimal taxation in Ramsey's tradition have been criticized from a conceptual point of view. Still, there will be no introductory course in optimal taxation that does not mention the rule. This paper demonstrates how the rule has to be extended if it is to guide efficient education policy in Ramsey's tradition. It is shown that the standard role taken by the elasticity of demand or supply has to be replaced with the elasticity of elasticity – also called second-order elasticity in what follows – if the rule is to apply to education. The extended rule calls for subsidizing education effectively if, and only if, the elasticity of the earnings function is increasing in education. In particular, the strength of subsidization should increase in the function's second-order elasticity.

This note is structured as follows. Section 2 sets up the model of a representative taxpayer. Section 3 derives the extended elasticity rule for education. Section 4 provides the proof.

2. A Representative-Household Model

The model is taken from Richter (2009). It assumes a representative taxpayer living for two periods and deriving strictly increasing utility U from consumption C_i and strictly decreasing disutility from nonleisure time L_i in periods i=1,2. The function U=$U(C_1, C_2, L_1, L_2)$ is strictly quasi-concave. L_2 is identical with the second-period labor supply. By contrast, only $L_1 - E$ is time spent in the market, while E is time spent on education. The first-period labor supply earns a constant wage rate ω_1; the return to second-period labor depends on the amount of education. It is paid $\omega_2 H(E)$, where ω_2 is constant while the earnings function $H(E)$ displays positive but diminishing returns, H'>0>H''. The quantity L_2 is interpreted as qualified labor. Likewise, the quantities $L_1 - E$ and L_1 are interpreted as nonqualified labor and nonqualified nonleisure, respectively. Education causes an opportunity cost in forgone earnings and a monetary cost of tuition. Both costs are assumed to be linear in time. The cost of foregone earnings is denoted by $\omega_1 E$, and the cost of tuition is denoted by φE. The share of first-period income that is spent neither on education nor on consumption is saved:

$$S = \omega_1(L_1 - E) - \varphi E - C_1 = \omega_1 L_1 - (\omega_1 + \varphi)E - C_1 \tag{1}$$

By way of normalization, the price of consumption is set equal to one. The gross rate of return to saving is denoted by ρ. Second-period consumption is constrained by income earned:

$$C_2 = \rho S + \omega_2 H(E) L_2 \tag{2}$$

Substituting for S in (1) and (2) yields the lifetime budget constraint:

$$C_1 + C_2/\rho = \omega_1 L_1 + \omega_2 H(E) L_2/\rho - (\varphi + \omega_1)E \tag{3}$$

The sole objective of this note is to extend the inverse elasticity rule to education. For this purpose consideration is restricted to utility functions that are quasilinear in first-period consumption and additive in periodic sub-utilities:

$$U(C_1, C_2, L_1, L_2) = C_1 - V_1(L_1) + U(C_2, L_2) \tag{4}$$

The function V_1 is strictly increasing and strictly convex. The representative taxpayer maximizes (4) in $C_1, C_2, L_1, L_2, E \geq 0$ subject to $L_1 \geq E$ and (3). In what follows it is assumed that this maximization is well behaved. This means that there exists an interior unique solution that is differentiable in $\omega_1, \omega_2, \rho, \varphi$ and that the second-order conditions are fulfilled. The first-order conditions are

$$\omega_1 = V_1^{'}(L_1), \quad \rho = 1/U_C, \tag{5}$$

$$U_L + \omega_2 H(E)/\rho = 0, \quad \omega_2 H'(E) L_2/\rho = \omega_1 + \varphi. \tag{6}$$

The following example satisfies all the assumptions needed in the present study:

- $H(E) \equiv hE^{\overline{\eta}} + H_0$ with $h > 0$, $1 > \overline{\eta} > 0$, $H_0 \geq 0$, (7)
- $U(C_2, L_2) \equiv u_C(C_2) - V_2(L_2)$ with $V_2^{'} > 0$,

and some elasticity of the marginal disutility of labor which is sufficiently large at the second-best level of L_2:

- $\nu_2 \equiv L_2 V_2^{''}/V_2^{'} > \overline{\eta}/(1-\overline{\eta})$ (8)

The case of an increasing elasticity of the earnings function, $\eta \equiv EH'/H$, will play a prominent role in the following discussion of second-best policy. Because $\eta = \overline{\eta}(1 - H_0/H(E))$, the specification (7) provides an example of an earnings function featuring an increasing elasticity, $\eta' > 0$, if and only if $H_0 > 0$. The inequality (8) is needed to ensure that the second-order conditions of the taxpayer's maximization are fulfilled. This can be verified as follows. Let

$$Y(L_2) \equiv \max_E [\omega_2 H L_2 / \rho - (\omega_1 + \varphi) E]$$

be the ability-rent income. The second-order condition with respect to L_2 requires

$$0 > Y'' - V_2'' \underset{(6)}{=} -\frac{\omega_2}{\rho}\frac{H'^2}{H'' L_2} - V_2'' \underset{(6)}{=} -\frac{V_2'}{H}\frac{H'^2}{H'' L_2} - V_2'' \Leftrightarrow$$

$$\nu_2 = L_2 V_2'' / V_2' > \frac{H' E / H}{-H'' E / H'} = \frac{\bar{\eta}}{1-\bar{\eta}}\frac{H - H_0}{H}.$$

This follows from (8) and (7).

3. Second-Best Policy

The government faces the need to raise an exogenous amount of revenue *G*. Four linear tax instruments are available, each of which is distorting. The taxes are levied on period *i*'s labor income, on the return to saving, and on the cost of tuition. They are modeled implicitly as the difference between prices before and after taxes. The prices after taxes and subsidies are endogenous and denoted by $\omega_1, \omega_2, \rho, \varphi$. The prices before taxes and subsidies are exogenous and denoted by w_1, w_2, r, f. Little would change if the wage rate of qualified labor, w_2, were normalized to one. By way of contrast, the wage rate after tax and subsidy, ω_2, cannot be normalized, as it is determined endogenously. The tax on period *i*'s labor income is modeled by $w_i - \omega_i$, the tax on capital income by $r - \rho$, and the tax on the cost of tuition by $\varphi - f$. It goes without saying that each tax can well take on a negative value so that it is effectively a subsidy. Government's net revenue has to balance the budget:

$$(w_1 - \omega_1)(L_1 - E) + (\varphi - f)E + [(w_2 - \omega_2)H(E)L_2 + (r - \rho)S]/r$$

$$\underset{(2)}{=} (w_1 - \omega_1)L_1 + [(\varphi + \omega_1) - (f + w_1)]E + [\frac{w_2}{r} - \frac{\omega_2}{\rho}]HL_2 + [\frac{1}{\rho} - \frac{1}{r}]C_2 = G. \qquad (9)$$

The planner maximizes the representative taxpayer's utility (4) in the quantities C_1, C_2, L_1, L_2, E and prices $\omega_1, \omega_2, \rho, \varphi$ subject to the behavioral constraints (3), (5), (6), and the budget constraint (9). Assume that the planner's maximization is well behaved. The sole objective of this note is to characterize efficient policy for education in relation to the taxation of nonqualified labor.

Efficiency is characterized in terms of wedges. Denote by

$$\Delta_{L_1} \equiv \frac{w_1 - \omega_1}{\omega_1} \qquad \text{the wedge on nonqualified labor, and by} \tag{10}$$

$$\Delta_E \equiv \frac{w_2 H' L_2 / r - f - w_1}{\varphi + \omega_1} \underset{(6)}{=} \frac{w_2 / r}{\omega_2 / \rho} - \frac{f + w_1}{\varphi + \omega_1} \qquad \text{the wedge on education.} \tag{11}$$

According to (11), the wedge on education equals the difference between two ratios. The first ratio relates present returns before and after taxes and subsidies, and the second ratio relates costs before and after taxes and subsidies. Hence the wedge vanishes if the ratio of returns equals the ratio in costs. Let us speak of *effective* subsidization if Δ_E is negative. According to (11), a negative value of Δ_E is the combined result of all four policy instruments. Effective subsidization is clearly reached by the statutory subsidization of the cost of tuition. This is however not the only way of reducing Δ_E. Other effective means are (i) increasing the tax on nonqualified labor and thus reducing the opportunity cost of education, (ii) reducing the tax on qualified labor and thus increasing the return to education, and finally (iii) taxing saving and thus increasing the return to education.

Denote by

$$\nu_1 \equiv L_1 V_1'' / V_1' > 0$$

the elasticity of marginal disutility of nonqualified labor, i.e., the reciprocal of the wage elasticity, and by

$$\eta_\eta \equiv E\eta' / \eta$$

the second-order elasticity of the earnings function.

Elasticity Rule for Education: If ω_1, ω_2, and φ are optimally chosen, then

$$\frac{\Delta_E}{\Delta_{L_1}} = -\frac{\eta_\eta}{\nu_1} . \tag{12}$$

Note that the rule holds even if the planner does not optimize with respect to ρ. Saving does not need to be taxed efficiently, and yet education policy should respect (12). A straightforward implication is that education should not be distorted (Δ_E =0) if the elasticity of the earnings function, η, is constant. This is a well-known result, also named the education efficiency proposition (Richter, 2009). It had been derived before in more elaborate models with heterogeneous taxpayers by Bovenberg and Jacobs (2005) and Jacobs and Bovenberg (2008). Another implication of (12) is that education should be subsidized if the elasticity of the earnings function is increasing.

Related results have been derived by Braun (2009) and Richter (2009). What has been lacking so far is the simple elasticity formula (12), which allows one to characterize efficient subsidization by means of the second-order elasticity of the earnings function. For the purpose of interpreting (12), assume that nonqualified labor income is taxed (Δ_{L_1} >0) and that the elasticity of the earnings function is increasing. Then (12) suggests that the second-best tax on (nonqualified) labor should vary inversely with the wage elasticity. This is the known part of the elasticity rule. The innovative part is the suggestion that the effective subsidization of education should increase monotonically in the second-order elasticity of education.

The recommendation to effectively subsidize education raises the question of how to translate it into specific tax and subsidy rates. Unfortunately, no simple formula exists for the Ramsey framework. As one may well suppose, the efficient set of tax and subsidy rates strongly depends on the specification of the taxpayer’s utility function. There is only one known robust result. This states that qualified labor should be less distorted than nonqualified labor (Richter, 2009). It holds for arbitrary utility and earnings functions.

4. Proof

The proof is fairly straightforward. Start by simplifying the planner’s problem. Replace ω_1 by $V_1^{'}$, and drop the constraint $\rho = 1/U_C$ because the proof does not rely on any derivatives with respect to ρ, C_2, L_2. The restated planner’s problem is:

$$\max\ [\, L_1 V_1^{'}(L_1) + \omega_2 H(E) L_2 / \rho - (\varphi + V_1^{'}(L_1))E - V_1(L_1) + U(C_2, L_2) - C_2/\rho \,] \qquad (13)$$

in $\varphi, L_1, \omega_2, E$ subject to

$$U_L(C_2, L_2) + \omega_2 H(E)/\rho = 0\,, \qquad (\lambda) \qquad (14)$$

$$\omega_2 H'(E)L_2/\rho = \varphi + V_1'(L_1)\ , \qquad (\mu) \tag{15}$$

$$(w_1 - V_1'(L_1))L_1 + [(\varphi + V_1'(L_1)) - (f + w_1)]E$$

$$+ \left(\frac{w_2}{r} - \frac{\omega_2}{\rho}\right) H(E)L_2 + \left[\frac{1}{\rho} - \frac{1}{r}\right] C_2 = G. \quad (\gamma) \tag{16}$$

The first-order conditions with respect to $\varphi, L_1, \omega_2, E$ are as follows:

$$\frac{\partial}{\partial\varphi}: \quad (\gamma - 1)E = \mu; \tag{17}$$

$$\frac{\partial}{\partial L_1}: \quad \gamma(w_1 - V_1') \underset{(17)}{=} (\gamma - 1)L_1 V_1''; \tag{18}$$

$$\frac{\partial}{\partial \omega_2}: \quad \lambda \underset{(17)}{=} (\gamma - 1)(1-\eta)L_2; \tag{19}$$

$$\frac{\partial}{\partial E}: \quad \gamma[f + w_1 - w_2 H' L_2 / r] \underset{(17),(19)}{=} (\gamma - 1)[\varphi + V_1' - \eta\omega_2 H' L_2/\rho + \omega_2 H'' E L_2/\rho]$$

$$\underset{(6)}{=} (\gamma - 1)\left[1 - \eta + \frac{H''E}{H'}\right](\varphi + V_1') = (\gamma - 1)\frac{E\eta'}{\eta}(\varphi + \omega_1). \tag{20}$$

The Lagrange multiplier γ is clearly positive. The multiplier γ has to be greater than one if w_1 is to exceed $V_1' = \omega_1$. Dividing (18) by V_1' and (20) by $\varphi + \omega_1$ and dividing the resulting equation (20) by the resulting equation (18) yields (12).

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