A Comparison of Optimal Tax Policies when Compensation or Responsibility Matter

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Abstract

This paper examines optimal redistribution in a model with high- and low-skilled individuals with heterogeneous tastes for labor. We compare the extent to which optimal policies based on different normative criteria obey the principles of compensation (for differential skills) and responsibility (for preferences for labor) when labor supply is along the extensive margin. With heterogeneity in skills and preferences, traditional Welfarist criteria including Utilitarianism present unappealing policy recommendations in some scenarios as they fail to take compensation and responsibility issues into account. Criteria from the social choice literature perform better in this regard in first-and second-best. More importantly, these equality of opportunity criteria push the second-best policy away from an Earned Income Tax Credit and in the direction of a Negative Income Tax.

JEL-Code: H21, D63.

Keywords: optimal income taxation, equality of opportunity, heterogeneous preferences for labor.

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1 Introduction

Assuming labor supply along the participation (or extensive) margin implies that a larger transfer toward low-paid workers than inactive people, i.e., an Earned Income Tax Credit (EITC), may become part of an optimal tax system (Diamond, 1980; Saez, 2002; Brewer *et al.*, 2008; Choné and Laroque, 2009). This well-known result is obtained under Utilitarian social preferences while agents differ in terms of skills as well as preferences. However, it is commonly admitted that preference heterogeneity poses ethical questions that challenge standard objective functions like Utilitarianism; see, e.g., Rawls (1971), Sen (1980) and Dworkin (1981). Other normative criteria derived from precise axioms of fairness have been proposed in the social choice literature. However, they are scarcely used to derive optimal tax policies.[1] The optimal income tax literature itself considers alternative social preferences but always with labor supply along the intensive margin. For instance, Boadway *et al.* (2002) use a Utilitarian social welfare function where different weights can be assigned to individuals with different preferences for leisure. This amounts to using different cardinalizations of individual utility functions. Paternalistic criteria, in which the planner uses a reference value for the taste for work and maximizes the sum of these adjusted utilities, have also been considered; e.g., by Schokkaert *et al.* (2004). Assuming high- and low-skilled agents with heterogeneous tastes for labor and labor supply along the participation margin, the present paper compares the optimal tax policies under a large set of social preferences from the social choice and the optimal taxation literature.

The first contribution of this paper is to show that the social-choice-inspired criteria provide an additional argument for an optimal tax system away from the EITC. A smaller transfer toward low-paid workers than inactive people, i.e., a Negative Income Tax (NIT), is more likely to become optimal. Moreover, under the assumption that the low-skilled have at least as large a participation elasticity as the high-skilled agents, the labor supply distortion for the high-skilled is tempered.

The second contribution of this paper is to check the optimal tax policies against the equality of opportunity requirements. In the dominant branch of the equality of opportunity literature, liberal egalitarian theories of justice, it is argued that income or welfare inequalities arising from nonresponsibility factors such as innate skills should be eliminated (the compensation principle) and inequalities arising from responsibility factors such as preferences should be respected (the responsibility principle).[2] We follow the applied literature on equality of opportunity assuming that agents are responsible for their preferences but not for their skills.[3] This paper then checks the optimal tax schedules that we obtain using the criteria from social choice and also those from the optimal income tax literature against the compensation and responsibility principles. The criteria that originate from the social choice approach to equality of opportunity perform much better than the traditional criteria, under both full and asymmetric information. Given that social choice criteria were designed to meet one of the principles in the first best, this should not come as a surprise. Under asymmetric information, we also consider an alternative strategy that restricts

[1] Exceptions are Fleurbaey and Maniquet (2006) and (2007) and Luttens and Ooghe (2007). However, these papers model labor supply along the intensive margin, rather than along the extensive margin.

[2] For an overview of this literature, see Fleurbaey (2008) or Fleurbaey and Maniquet (2009). The (in)compatibility of these two principles was first analyzed by Fleurbaey (1995a) and Bossert (1995).

[3] This assumption is useful in order to analyze the basic structure of solutions in a simple way, and many of the qualitative results obtained with it carry over to the more complex settings where agents are held responsible for part of their preferences or of their skills (see Fleurbaey, 2008).

the search for an optimal tax policy satisfying one of the equality of opportunity principles.

The third contribution is to propose five new normative criteria that satisfy priority to the worst-off (and thus weak) versions of the compensation and responsibility principles. They rely on a cardinal or alternatively an ordinal measure of welfare. We show that these criteria, just like the social-choice-inspired criteria, push the optimal tax away from an earned income tax credit and temper the labor supply distortion of the high-skilled.

This paper is organized as follows. In Section 2, we describe the model, provide the characterization of the individuals' behavior, and describe the decision variables of the government under full and asymmetric information. Section 3 states the axioms behind equality of opportunity and presents the distinct objective functions. Section 4 investigates the optimal tax policies under full information, and Section 5 under asymmetric information. Sufficient conditions for an NIT or an EITC are given. Section 6 concludes. The main proofs are presented in the appendix, while some supplementary material is available in Jacquet and Van de gaer (2010).

2 The model

2.1 Individual behavior

Assume that agents decide whether to work or not.[4] They differ along two dimensions: their skill and their disutility of work. Skills take two values, $w_H > w_L > 0$, which correspond to the gross wages in two types of jobs (low- and high-skilled). The disutility of work, α, is distributed according to the cumulative distribution function $F(\alpha) : \mathbb{R}^+ \to [0,1] : \alpha \to F(\alpha)$ and the corresponding density function $f(\alpha)$. The latter is continuous and positive over its domain.[5] These functions are common knowledge. The proportion of low-skilled agents (or w_L-type) in the population is given by γ, while $1-\gamma$ is the proportion of high-skilled people (or w_H-type). We assume that productivity and labor disutility are independently distributed. Utility is quasilinear and represented by:

$$v(x) - \alpha \text{ if they work,}$$

$$v(x) \text{ if they do not work,}$$

where x is consumption, $v(x) : \mathbb{R}^+ \to \mathbb{R} : x \to v(x)$ with $v' > 0 \geq v''$ and $\lim_{x\to\infty} v'(x) = 0$.

2.2 The government's decisions

Under full information (so-called first best), the government implements a tax policy depending on α and $w_Y (Y = L, H)$; hence, it also assigns individuals to low-skilled jobs (where the gross wage is w_L), to high-skilled jobs (where the gross wage is w_H) or to inactivity (activity u). Activity assignment is captured through the functions $\delta_L(\alpha) : \mathbb{R}^+ \to \{0,1\} : \delta_L(\alpha) = 1$ ($\delta_L(\alpha) = 0$) if w_L-agents with this value for α are employed (inactive) and $\delta_H(\alpha) : \mathbb{R}^+ \to \{0,1\} : \delta_H(\alpha) = 1$ ($\delta_H(\alpha) = 0$) if w_H-agents with this value for α are employed (inactive). Low-skilled agents cannot get access to high-skilled jobs, and because efficiency matters, it will never be optimal that high-skilled agents

[4] There is growing evidence that the extensive margin matters a lot; e.g., Meghir and Phillips (2008).

[5] We want to see whether an EITC or an NIT is optimal. This requires us to describe the participation tax rates only. Therefore, it is appropriate to assume a discrete support for skills, such as in Saez (2002). For simplicity, we assume two skill levels, but increasing the number of skills does not modify our main results. Continuity of α is assumed for simplicity.

work in low-skilled jobs. By putting these people in high-skilled jobs instead of low-skilled jobs, they produce more, which can be used to increase someone's consumption. Hence, formally, the government determines four consumption functions: $x_L^w(\alpha)$ for the w_L-workers, $x_H^w(\alpha)$ for the w_H-workers, $x_L^u(\alpha)$ for the w_L-inactive agents and $x_H^u(\alpha)$ for the w_H-inactive agents. All these functions go from $\mathbb{R}^+$ to $\mathbb{R}^+$.

The Government budget constraint can be formulated as follows:

$$\begin{aligned}&\gamma\left[\int_0^\infty \left[\delta_L(\alpha)\left(w_L - x_L^w(\alpha)\right) - \left(1-\delta_L(\alpha)\right)x_L^u(\alpha)\right]dF(\alpha)\right] \\ &+(1-\gamma)\left[\int_0^\infty \left[\delta_H(\alpha)\left(w_H - x_H^w(\alpha)\right) - \left(1-\delta_H(\alpha)\right)\right)x_H^u(\alpha)\right]dF(\alpha)\right] \geq R,\end{aligned} \tag{1}$$

where R is an exogenous revenue requirement, which can be positive or negative. This budget constraint must be binding at the optimum as all government objectives considered in this paper are increasing in individuals' consumption.

The problem for the government in the first best is to determine the functions $x_L^w(\alpha)$, $x_H^w(\alpha)$, $x_H^u(\alpha)$, $x_L^u(\alpha)$ together with $\delta_L(\alpha)$ and $\delta_H(\alpha)$, which are normatively desirable and satisfy the government budget constraint (1).

In the second best, the tax schedule can depend only on income levels $(0, w_L$ or $w_H)$. The government then defines three consumption levels x^u, x_L and x_H, denoting consumption when not participating in the labor force, and when working in low-skilled and in high-skilled jobs, respectively. These consumption levels have to meet the government budget constraint and the set of self-selection or incentive compatibility constraints (which will be stated in Section 5), and have to be normatively desirable. The next section discusses which normative principles or criteria the government can use.

3 Equality of opportunity

The next subsection formally defines equality of opportunity in order to study whether the normative criteria usually assumed in the optimal tax literature succeed in reaching it.

3.1 Two equality of opportunity principles

Define, for the case where $Y = L$ or H, the evaluation of the consumption bundle $(x_Y(\alpha), \delta_Y(\alpha))$ as:

$$u\left(x_Y(\alpha), \delta_Y(\alpha), \alpha^G\right) = \begin{cases} v\left(x_Y^w(\alpha)\right) - \alpha^G & \text{if } \delta_Y(\alpha) = 1, \\ v\left(x_Y^u(\alpha)\right) & \text{if } \delta_Y(\alpha) = 0, \end{cases}$$

where labor disutility is evaluated by parameter α^G. If $\alpha^G = \alpha$, $u\left(x_Y(\alpha), \delta_Y(\alpha), \alpha^G\right)$ coincides with the individual's own utility.

We assume throughout that people are responsible for their tastes for work α but not for their skills[6]. We can then apply Fleurbaey's (1994) approach and capture the intuitions of equality of

[6] We follow here the usual assumption in the applied literature on equality of opportunity. Two further remarks can be made at this point. First, if people are not responsible for anything, from a perspective of equality of opportunity, the only possible objectives are full equality of utility levels or leximin. Second, it is possible to follow the suggestion by Pestieau and Racionero (2009) to decompose the parameter α into two components: $\alpha = \alpha_P + \alpha_D$, where people are responsible for α_P (a preference parameter) but not for α_D (a disability parameter). The present framework can be adjusted to deal with this issue without altering the main results of this paper.

opportunity in two axioms. The first equality of opportunity axiom expresses the idea of compensation.

EWEP (Equal Welfare for Equal Preferences)
$\forall\alpha \in \mathbb{R}^{+} : u\left(x_L\left(\alpha\right), \delta_L\left(\alpha\right), \alpha\right) = u\left(x_H\left(\alpha\right), \delta_H\left(\alpha\right), \alpha\right)$.

An allocation satisfying EWEP is such that differences in skills do not influence a person's welfare. The second axiom of equality of opportunity expresses the idea of responsibility.

ETES (Equal Transfers for Equal Skills)
$\forall\alpha, \alpha' : \delta_L\left(\alpha\right) = \delta_L\left(\alpha'\right) = 1$ and $\forall\alpha'' : \delta_L\left(\alpha''\right) = 0 :$
$x_L^w\left(\alpha\right) - w_L = x_L^w\left(\alpha'\right) - w_L = x_L^u\left(\alpha''\right) = x_L^u,$
$\forall\alpha, \alpha' : \delta_H\left(\alpha\right) = \delta_H\left(\alpha'\right) = 1$ and $\forall\alpha'' : \delta_H\left(\alpha''\right) = 0 :$
$x_H^w\left(\alpha\right) - w_H = x_H^w\left(\alpha'\right) - w_H = x_H^u\left(\alpha''\right) = x_H^u,$

with some abuse of notation for the last term in the expressions for both skill levels. The axiom requires that taxes only depend on skill level. For each skill level, all the inactive get the same benefit, all workers pay the same tax, and the transfer received by the inactive is equal to minus the tax paid by the workers. Therefore, welfare differences that are caused by differential tastes are not compensated and are fully respected.

We formally define full equality of opportunity as follows.

FEO (Full Equality of Opportunity)
An allocation satisfies full equality of opportunity if it satisfies both EWEP and ETES.

In the traditional framework, where the government only (re)distributes consumption, even in the first best, generically, an FEO allocation does not exist; see, e.g., Fleurbaey (1994) and Bossert (1995). For this reason, Fleurbaey (1995b) suggests weakening at least one of the axioms while maintaining the other.[7] This allows him to characterize two allocations. First, the Conditional Equality (CE) allocation keeps ETES but requires EWEP only in the situation where all agents have the reference value (denoted $\widetilde{\alpha}$) for their taste parameter. Second, the Egalitarian Equivalent (EE) allocation keeps EWEP but requires ETES only when all agents have the reference value for the resource bundle, here taken to be the consumption level $\widetilde{x}$ and $\delta_Y = 1$ ($Y = L$ or H).

CE (Conditional Equality)
An allocation is the conditional equality allocation if and only if for all α and all Y it equalizes $u\left(x_Y\left(\alpha\right), \delta_Y\left(\alpha\right), \widetilde{\alpha}\right)$ at the highest feasible level.

EE (Egalitarian Equivalence)
An allocation is egalitarian equivalent if and only if for all α and all $Y : u\left(x_Y\left(\alpha\right), \delta_Y\left(\alpha\right), \alpha\right) = u\left(\widetilde{x}, 1, \alpha\right)$ and $\widetilde{x}$ is at the highest feasible level.

The CE allocation ensures that all individuals are equally well off with their actual bundle of resources when this is evaluated using the reference preference $\widetilde{\alpha}$. The EE allocation makes all individuals indifferent between their actual resource bundle and the reference bundle where they

[7] Of course, it is also possible to weaken both axioms simultaneously; see, e.g., Bossert and Fleurbaey (1996) or Fleurbaey and Maniquet (2009).

work and get a consumption level $\widetilde{x}$.[8] In our definition here, we incorporate that no resources are wasted, in the CE allocation, by equalizing at the highest possible level, and in the EE allocation by pursuing indifference at the highest feasible level of $\widetilde{x}$. The CE allocation can lead to Pareto-dominated allocations because it evaluates bundles with reference preferences.[9] By contrast, the EE allocation always satisfies Pareto efficiency because it uses individual preferences in order to compute equivalent resources $x_Y(\alpha)$ and $\delta_Y(\alpha)$; i.e., the resources that would equalize all utility levels to $u(\widetilde{x}, 1, \alpha)$. A CE or EE allocation need not exist. In particular, in the second best, it will not be possible to equalize the reference utilities as required by CE, and even in the first best, indifference for all individuals with the reference bundle is not feasible in our model. As is standard in the social choice literature, we then formulate maximin social orderings inspired by the CE and EE allocation at the end of the next subsection.

3.2 Different social objective functions

This paper will consider the following social objective functions extensively used in the optimal taxation literature.

The Utilitarian social objective function (used in, e.g., Ebert (1992), Diamond and Sheshinski (1995), Boadway *et al.* (2000), Hellwig (2007)) is the average of all individual utilities; i.e.:

$$\begin{aligned} S^U &= \gamma \int_0^\infty \delta_L(\alpha) \left[v(x_L^w(\alpha)) - \alpha \right] dF(\alpha) + \gamma \int_0^\infty (1 - \delta_L(\alpha))\, v(x_L^u(\alpha)) dF(\alpha) + \\ &(1-\gamma) \int_0^\infty \delta_H(\alpha) \left[v(x_H^w(\alpha)) - \alpha \right] dF(\alpha) + (1-\gamma) \int_0^\infty (1 - \delta_H(\alpha))\, v(x_H^u(\alpha)) dF(\alpha). \quad (2) \end{aligned}$$

Our Welfarist social objective is the average of a concave transformation of individual utilities. The concave transformation allows the expression of inequality aversion with respect to the distribution of utilities. Let the function $\Psi : \mathbb{R} \to \mathbb{R} : a \to \Psi(a)$ be a strictly concave function. Our Welfarist objective function is:

$$\begin{aligned} S^W &= \gamma \int_0^\infty \delta_L(\alpha) \Psi\left(v(x_L^w(\alpha)) - \alpha \right) dF(\alpha) + \gamma \int_0^\infty (1 - \delta_L(\alpha)) \Psi\left(v(x_L^u(\alpha)) \right) dF(\alpha) + \\ &(1-\gamma) \int_0^\infty \delta_H(\alpha) \Psi\left(v(x_H^w(\alpha)) - \alpha \right) dF(\alpha) + (1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) \Psi\left(v(x_H^u(\alpha)) \right) dF(\alpha) \quad (3) \end{aligned}$$

Assumed in the seminal article of Mirrlees (1971), this welfare function has been very popular since then (e.g., Atkinson and Stiglitz (1980), Diamond (1998), Choné and Laroque (2005), Kaplow (2008), Kleven *et al.* (2009)).

The objective function used by Boadway *et al.* (2002) allows us to attach a weight to individuals' utilities that depends on their taste for leisure. Let $W(\alpha) : \mathbb{R}^+ \to \mathbb{R}^+ : \alpha \to W(\alpha)$ be the social welfare weight given to the utility of an individual with disutility of work equal to α. The Boadway

[8] This is similar to the "full-health equivalent income" proposed by Fleurbaey (2005). An alternative egalitarian equivalent allocation could make all individuals indifferent between their actual resource bundle and the reference bundle where they are inactive and get the reference level of consumption.

[9] The idea behind the CE allocation can be preserved if one wants to avoid a clash with Pareto efficiency. A way to do this is to rely on the laissez-faire equivalent budget sets (which reflect the agent's actual preferences) and to apply the reference preference to those; see Fleurbaey and Maniquet (2005, 2007) for more details.

et al. objective function is given by:

$$S^B = \gamma \int_0^\infty \delta_L(\alpha) W(\alpha) [v(x_L^w(\alpha)) - \alpha] dF(\alpha) + \gamma \int_0^\infty (1 - \delta_L(\alpha)) W(\alpha) v(x_L^u(\alpha)) dF(\alpha)$$
$$+(1-\gamma) \int_0^\infty \delta_H(\alpha) W(\alpha) [v(x_H^w(\alpha)) - \alpha] dF(\alpha)$$
$$+(1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) W(\alpha) v(x_H^u(\alpha)) dF(\alpha). \tag{4}$$

This objective function was explicitly introduced to deal with individuals who are heterogeneous in skills and preferences. Also used in Cremer *et al.* (2004 and 2007), for instance, this criterion adopts distinct cardinalizations of individual utilities depending on the individual's taste parameter α. An alternative objective function in the vein of Boadway *et al.* (2002) can be considered by assuming that the weight function $W(.)$ applies only to the disutility of work part of each agent's utility function rather than the part that also involves consumption.[10]

Our next social objective function uses a paternalistic view for the valuation of labor disutility. We define the reference labor disutility as $\overline{\alpha} \geq 0$, which is the weight attached by the government to the α of every individual. The Paternalistic Utilitarian objective is stated as:

$$S^P = \gamma \left[\int_0^\infty \delta_L(\alpha) [v(x_L^w(\alpha)) - \overline{\alpha}] dF(\alpha) \right] + \gamma \int_0^\infty (1 - \delta_L(\alpha)) v(x_L^u(\alpha)) dF(\alpha)$$
$$+(1-\gamma) \int_0^\infty \delta_H(\alpha) [v(x_H^w(\alpha)) - \overline{\alpha}] dF(\alpha) + (1-\gamma) \int_0^\infty (1 - \delta_H(\alpha)) v(x_H^u(\alpha)) dF(\alpha). \tag{5}$$

With this objective function, the social planner has a different idea from the individuals themselves about the 'correct' or reasonable disutility of work. There is then a clear paternalistic motive for taxation that arises from differences between social and private preferences. Schokkaert *et al.* (2004) consider this social objective function, but alternative paternalistic Utilitarian objectives are possible. For instance, it might be argued that the planner only does not respect disutilities of work larger than $\overline{\overline{\alpha}}$, in which case $\overline{\alpha}$ in (5) is replaced by $\min\{\alpha, \overline{\overline{\alpha}}\}$.[11] Marchand *et al.* (2003) and Pestieau and Racionero (2009) consider another alternative paternalistic approach in which the government attaches a larger weight to the labor disutility of disabled individuals. Maximization of Non-Welfarist social objective functions typically selects allocations that are not Pareto efficient.

To state the next two objective functions, which are less standard, we define an operator that takes the first element of a set with two elements if $\delta(\alpha)$ equals one, and the second element otherwise. Formally, we define the operator as:

$$\underset{\delta(\alpha)}{\text{oper}} \{a, b\} = a \text{ if } \delta(\alpha) = 1 \text{ and } \underset{\delta(\alpha)}{\text{oper}} \{a, b\} = b \text{ if } \delta(\alpha) = 0.$$

Roemer (1993 and 1998) proposes that equality of opportunity for welfare holds when the utilities of all those who exercised a comparable degree of responsibility are equal, irrespective of their skills. Assuming that those who have the same preferences have exercised a comparable degree of responsibility, the ideal is to give the same utility to those with the same preferences,

[10] It can then easily be checked that the full information allocation is qualitatively identical to the Utilitarian one. For brevity's sake, we limit our discussion of the second-best solution for this criterion to Appendix C.

[11] We are grateful to one of the referees for pointing out this possibility. For brevity's sake, we limit our discussion of the second-best solution for this criterion to Appendix C.

irrespective of their skills. Because utilities have to be equal for each preference, it will usually (except, as we will see, in the first best) not be possible to achieve this. Roemer therefore suggests to maximize a weighted average of the minimal utilities across individuals having the same tastes. As a result, Fleurbaey (2008) calls this the mean of mins criterion. Roemer's (1998) objective function can be written as:

$$S^R = \int_0^\infty \min \left\{ \underset{\delta_L(\alpha)}{\text{oper}}\{v\left(x_L^w\left(\alpha\right)\right) - \alpha, v\left(x_L^u\left(\alpha\right)\right)\}, \underset{\delta_H(\alpha)}{\text{oper}}\{v\left(x_H^w\left(\alpha\right)\right) - \alpha, v\left(x_H^u\left(\alpha\right)\right)\} \right\} dF\left(\alpha\right). \quad (6)$$

For each α, the government assigns low- and high-skilled individuals to employment or inactivity. The min function in the integral term takes, for each α level, the smallest utility across skill types. The Roemer rule maximizes the sum (over α) of these minimal utility levels. It has been used by Roemer *et al.* (2003) to compare empirically the extent to which fiscal policies manage to equalize opportunities for income acquisition in a set of countries.

While Roemer's proposal is well known, an obvious alternative was proposed by Van de gaer (1993). The starting point is that for each level of skill, utility as a function of the taste parameter can be interpreted as the utilities to which someone with that skill level has access. The proposal is then to maximize the value of the smallest opportunity set, where the opportunity set is the surface under utilities to which she has access, weighted by the frequency with which the corresponding preference parameter occurs. Hence, the proposed social objective function, labeled the min of means criterion by Fleurbaey (2008), is:

$$S^V = \min \left\{ \int_0^\infty \underset{\delta_L(\alpha)}{\text{oper}}\{v\left(x_L^w\left(\alpha\right)\right) - \alpha, v\left(x_L^u\left(\alpha\right)\right)\} dF\left(\alpha\right), \right.$$
$$\left. \int_0^\infty \underset{\delta_H(\alpha)}{\text{oper}}\{v\left(x_H^w\left(\alpha\right)\right) - \alpha, v\left(x_H^u\left(\alpha\right)\right)\} dF\left(\alpha\right) \right\}. \quad (7)$$

This criterion and Roemer's criterion were used to compute optimal linear income taxes in Bossert *et al.* (1999) and Schokkaert *et al.* (2004). Axiomatic characterizations of these criteria can be found in Ooghe *et al.* (2007) and Fleurbaey (2008).

We formulate the maximin objective function inspired by the CE allocation:

$$S^{CE} = \min_{\alpha, w_Y} u\left(x_Y\left(\alpha\right), \delta_Y\left(\alpha\right), \widetilde{\alpha}\right), \quad (8)$$

meaning that the optimal policy is determined such that the lowest level of utility that someone in the population gets with her actual allocation, evaluated at the reference preferences $\widetilde{\alpha}$, is as high as possible. The criterion was explicitly considered by Bossert *et al.* (1999).

Finally, we formulate a maximin objective function inspired by the EE allocation. For each individual, we determine the consumption level that she needs when she has to work and is such that she is indifferent to this bundle and her actual consumption bundle. Evidently, for workers, this is simply their actual consumption level. Inactive people require a consumption level equal to $v^{-1}\left(v\left(x_Y^u\left(\alpha\right)\right) + \alpha\right)$, where $x_Y^u\left(\alpha\right)$ is their actual consumption level. Hence, we can define an EE ordering as maximizing:

$$S^{EE} = \min_{\alpha, w_Y} \left\{ x_L^w\left(\alpha\right), x_H^w\left(\alpha\right), v^{-1}\left(v\left(x_L^u\left(\alpha\right)\right) + \alpha\right), v^{-1}\left(v\left(x_H^u\left(\alpha\right)\right) + \alpha\right) \right\}. \quad (9)$$

In our framework, this social ordering is the natural counterpart of an ordering proposed by Fleurbaey and Maniquet (2005 and 2006). In their papers, the equivalent wage for an individual is defined as the wage rate such that she is indifferent between her actual bundle and the bundle that she could reach if she had her equivalent wage. Their proposed social ordering is then to maximize the minimal equivalent wage. Fleurbaey and Maniquet use an intensive labor supply choice model; the computation of the equivalent wage involves a counterfactual labor supply choice lying between inactivity and full-time employment. In our extensive labor supply model, such a choice is not available. However, we can adjust the concept by comparing the actual consumption bundle with the wage making the individual indifferent between that bundle and full-time employment. Formally, in our extensive margin model, the equivalent wage is defined for the employed as $x_Y^{wE}(\alpha) = x_Y^w(\alpha)$, and for the inactive as $x^{uE}(\alpha) : v\left(x^{uE}(\alpha)\right) - \alpha = v\left(x_Y^u(\alpha)\right)$, which implies that $x^{uE}(\alpha) = v^{-1}\left(v\left(x_Y^u(\alpha)\right) + \alpha\right)$. Maximinning this equivalent wage leads to the social ordering defined in (9).

4 First best

This section studies the first-best optimal policies under the various criteria. The details of the analytical derivations are given in the supplementary material. The optimal activity assignment, denoted A, can vary with the agent's skill level, denoted Y, and with her disutility of work α. Similarly, her optimal consumption can vary with α and Y, but it can also be activity (A) specific. Whether and how these factors influence the optimal policy depends on the government's objective function. The following table summarizes the results.

Table 1: Determinants of optimal policies in the first best.

Social Objective	Determinants of Activity	Determinants of Consumption
Utilitarian	α, Y	–
Welfarist	α, Y	α, A
Boadway *et al.*	α, Y	α
Paternalistic Ut.	Y	–
Roemer	α, Y	α, Y, A
Van de gaer	α, Y	Y
FEO	–	–
Egalitarian Equivalent	α	A
Conditional Egalitarian	Y	A

Note: the entries in the table give the determinants of the optimal policy decision: α indicates the disutility of work, Y the skill level (high or low skill) and A that the optimal consumption varies with activity (employed or inactive). Entry "-" means that the optimal policy is independent of α, Y and A.

Let $X = U$, W, B, P, R, V, CE or EE denote the Utilitarian, Welfarist, Boadway *et al.*, Paternalistic Utilitarian, Roemer, Van de gaer, Conditional Equality and Egalitarian Equivalent objectives, respectively. As far as activity assignment is concerned, there are four families of optimal policies. In the first family, which occurs for $X = U$, W, B, R and V, the individual's activity is determined by her disutility of work α and her skill level Y, as displayed by "α, Y" in Table 1. The planner determines threshold levels of work's disutility α_L^* and α_H^* such that agents with skill level Y and $\alpha \leq \alpha_Y^{X*}$ work while those with $\alpha > \alpha_Y^{X*}$ do not.[12] Because of the higher

[12] Under the Boadway *et al.* criterion with an elasticity smaller than -1 (which requires that $W(\alpha)$ be sufficiently

productivity of the high-skilled, $\alpha_H^{X*} \geq \alpha_L^{X*}$. The second family, which occurs for $X = P$ or CE, determines who has to work according to the skill level Y only (as denoted by "Y" in Table 1). For each skill level, the planner is indifferent whether work is done by those with a high or low disutility of work. This arises because of the nature of these paternalistic criteria, which assign the same disutility of work to everyone. Moreover, because of the higher productivity of the high-skilled, again more high-skilled than low-skilled work. The third family of activity assignment rule occurs for EE. When calculating the equivalent wages of the inactive, individual preferences α matter, as can be seen from the last two terms in (9). The disutility of work increases the equivalent wage of the inactive, and its effect is the same for high- and low-skilled such that activity is α-specific, under EE. This is denoted by the entry "α" in Table 1. Under EE, we then obtain a special case of the first family with $\alpha_L^{EE*} = \alpha_H^{EE*}$. Finally, the fourth family occurs for the FEO objective function. As shown in the supplemental material, either everyone works or everyone is inactive under FEO. Activity assignment is then independent of both skill and disutility of work, which is denoted by the entry "-" in Table 1.

Three of the four families we just described ("Y", "α" and "-") are also valid for consumption assignments, but two additional families can occur ("A" and "α, A"). First, for $X = U$, P and FEO, everybody receives the same consumption level; hence, consumption is independent of individual characteristics. This is denoted by the entry "-" in Table 1. The U and P planners equalize all social marginal utilities of consumption $v'\left(x_Y^X\right)$ ($X = U$ or P) and hence consumption levels. Similarly, the Boadway *et al.* planner aims to equalize the social marginal utilities of consumption. Because his social marginal utilities of consumption x_Y^x equal $W(\alpha)\, v'(x_Y^x)$ (with $x = w$ or u), Boadway *et al.*'s consumption is increasing (decreasing) in α if the weight $W(\alpha)$ is increasing (decreasing) in α. This second family is then denoted by the entry "α" in Table 1. The Welfarist planner also aims to equalize social marginal utilities. These equal $\Psi'(v(x_Y^u(\alpha)))v'(x_Y^u(\alpha))$ for inactive and $\Psi'(v(x_Y^w(\alpha)) - \alpha)v'(x_Y^w(\alpha))$ for workers of skill Y ($Y = L, H$). Therefore, all inactive receive the same consumption level while the consumption of workers is increasing in α. Furthermore, workers with the same α receive the same consumption, whatever their skills. Because we know that agents with $\alpha \in (\alpha_L^{*R}, \alpha_H^{*R}]$ are assigned to work (are inactive) if they have high (low) skill, skills do not (directly) drive consumption differences, but activities do. This explains the entry "α, A" in Table 1 under Welfarism. In the fourth family, which occurs for the Van de gaer criterion, consumption differences only take place when skills differ. This corresponds to the entry "Y" in Table 1. All high-skilled receive the same consumption, and all low-skilled receive the same consumption, the former being at least as big as the latter. Fifth, consumption determination for the Roemer planner can best be explained by the combination of its Utilitarian feature and a concern with compensation. All workers with $\alpha \leq \alpha_L^{R*}$ and all inactive with $\alpha > \alpha_H^{R*}$ get the same consumption level. Moreover, w_H-workers and w_L-inactive having the same $\alpha \in [\alpha_L^{*R}, \alpha_H^{*R})$ reach an identical utility level, which determines their consumption α, skill and activity-specific. Therefore, the entry in Table 1 is "α, Y, A". Finally, for $X = EE$ and CE, all workers get the same consumption bundle (irrespective of their skill level and disutility of work), which differs from the

declining in α), the utility levels of agents with larger a are highly discounted compared with those of agents with lower α. Therefore, the planner assigns those with a high α to work, and those with a low α to inactivity. However, this can be seen as perverse because this activity assignment goes fully against individuals' preferences. To avoid this perverse case, we assume $(\partial W(\alpha)/\partial\alpha)(\alpha/W(\alpha)) > -1$ in our discussion.

consumption bundle offered to the inactive. This is summarized by the entry "A" in Table 1.

We now verify whether the resulting optimal policies satisfy EWEP or ETES. To verify EWEP, consider first the criteria leading to an activity assignment rule of the first (α, Y) or the second (Y) family. For these criteria (U, W, B, R, V, P and CE), typically more high-skilled than low-skilled have to work, such that there exist values of α for which high-skilled, contrary to low-skilled, have to work. By definition, EWEP then requires $v\left(x_L^u\left(\alpha\right)\right) = v\left(x_H^w\left(\alpha\right)\right) - \alpha \Leftrightarrow x_L^u\left(\alpha\right) = v^{-1}\left(v\left(x_H^w(\alpha)\right) - \alpha\right) \; \forall \alpha \in [\alpha_L^{*X}, \alpha_H^{*X})$, which requires consumption to depend on both skills and tastes. Hence none of these criteria, except Roemer's criterion, satisfies EWEP. Finally, both EE and FEO optimal policies satisfy EWEP. Under EE, activity assignment depends only on disutility of work; hence, there are no values for α for which high-skilled work and low-skilled are inactive. Therefore, EWEP is guaranteed under EE because consumption levels vary with activity only. By construction, the FEO allocation satisfies EWEP. This is realized by treating everyone equally in terms of activity assignment and consumption.

ETES requires that for each skill level, the consumption received by the inactive equals the taxes paid by the employed (see Section 3.1). Hence, if there is a skill level with both inactive and employed where the inactive receive the same consumption as the employed, ETES is violated. This occurs for $X = U, P, R$ and V. The Welfarist planner also violates ETES because it gives different consumption bundles to workers depending on their α. The Boadway *et al.* and EE planners give the same consumption to high- and low-skilled workers. Therefore, because of their distinct productivity, the two tax levels that the workers pay cannot be identical, and so these tax levels cannot both be equalized to the inactive's consumption. ETES is again violated. Under CE, the supplemental material emphasizes that whenever for a skill Y there are both inactive and workers, consumptions are determined such that $-x^u = w_Y - x_Y^w$; hence, ETES is satisfied. In the FEO allocation, everybody either works or is inactive, such that the ETES requirement becomes empty and ETES is trivially satisfied. We summarize the performance of all criteria from the equality of opportunity principles in Table 2.

Table 2: Equality of opportunity axioms and social objectives in the first best.

Social Objective	Satisfies EWEP?	Satisfies ETES?
Utilitarian, Welfarist, Boadway *et al.*, Non-Welfarist, Van de gaer	No	No
Roemer	Yes	No
FEO	Yes	Yes
Egalitarian Equivalent	Yes	No
Conditional Egalitarian	No	Yes

Given the origin of these social orderings, it is not surprising to see that the criteria that originate from the social choice approach to equality of opportunity perform much better than the traditional criteria. As emphasized in Sections 3.1 and 3.2, they were designed to do so.

The next section assumes that the tax schedule cannot depend on individual characteristics any more, only on income (e.g., because of asymmetric information).

5 Second-best optima

5.1 Second-best constraints and their implications

In the second best, the government needs to take into account the set of incentive compatibility constraints (hereafter ICC) in order to prevent individuals of a given type from mimicking (i.e., taking the tax treatment designed for) individuals of other types. We first state these IC constraints and then discuss their implications for the social objective functions.

Agents of w_L-type choose between $v(x^u)$ and $v(x_L) - \alpha$. Introducing the threshold value α_L^*, and dropping the superscript X for notational simplicity, the ICC[13] on w_L-agents can be written as:

$$v(x_L) - \alpha_L^* = v(x^u), \tag{10}$$

such that a low-skilled with taste parameter α chooses low-skilled employment instead of inactivity if and only if $\alpha \leq \alpha_L^*$.

Agents of w_H-type choose between $v(x^u)$, $v(x_L) - \alpha$ and $v(x_H) - \alpha$. Because all our objective functions are increasing in individuals' consumption, it will, just like in the first best, never be optimal that high-skilled people work in low-skilled jobs. By putting these people in high-skilled jobs instead of low-skilled jobs, they produce more, which can be used to increase everyone's consumption in a way that respects the ICC and hence increases the social objective's value. Consequently, to induce high-skilled people to work in high-skilled jobs:

$$x_H \geq x_L, \tag{11}$$

and, introducing the threshold value α_H^*, the ICC on agents of w_H-type states:

$$v(x_H) - \alpha_H^* = v(x^u), \tag{12}$$

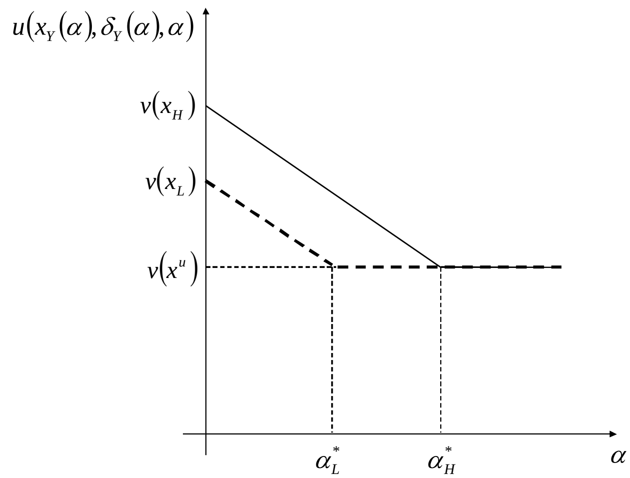
such that a high-skilled agent with taste parameter α prefers high-skilled employment to inactivity if and only if $\alpha \leq \alpha_H^*$. Moreover, from (10), (11) and (12), we have that:

$$\alpha_H^* \geq \alpha_L^*. \tag{13}$$

As a result of the second-best constraints (10), (12) and (13), irrespective of the social objective function, the optimal activity will vary with skill level and disutility of work. Moreover, because of (11), for each value of α, the utility of w_H-workers is at least as high as that of w_L-workers. Hence, the utilities as a function of α, for w_L- and w_H-skilled agents, are presented in Figure 1.

[13] The set of IC constraints for each agent of type (w_Y, α) (with $Y = L, H$ and $\alpha \in \mathbb{R}^+$) can be rewritten as constraints (10)–(12). Moreover, because the labor supply decision is restricted to be binary, the (direct truthful) mechanism that implements the optimal allocations is not fully revealing. Each agent fully reveals her w_Y information but not her α value; she announces only whether α is larger or smaller than α_Y^*.

Figure 1: Utilities in the second best.



The solid line is the utility of a w_H-individual. She works if her disutility of work $\alpha \leq \alpha_H^*$, and she is inactive otherwise. Similarly, the bold dotted line is the utility of a w_L-individual. The latter works for $\alpha \leq \alpha_L^*$ and is inactive otherwise. Different planners choose distinct values for $(x^u, x_L, x_H, \alpha_L^*, \alpha_H^*)$, but the qualitative shape of the utilities as a function of α, for high- and low-skilled individuals, is always as indicated in the graph.

The second-best framework has important implications for the equality of opportunity principles, as stated in the following lemma.

Lemma 1 *Equality of opportunity principles in the second best.*
(a) A necessary and sufficient condition to satisfy EWEP fully is that $\alpha_L^* = \alpha_H^*$*, which requires that* $x_L = x_H$*.*
(b) A necessary and sufficient condition to satisfy ETES fully is that $x_L - w_L = x^u = x_H - w_H$*.*

Part (a) (whose proof is given in Appendix A) says that the threshold values α_L^* and α_H^* have to be the same. To accomplish this, the government has to offer the same consumption level to high- and low-skilled workers. It implies that the same numbers of high- and low-skilled individuals will work. Part (b) of the corollary follows immediately from application of the ETES axiom and has two noteworthy implications. First, because $x_L - w_L = x^u$ and $x_H - w_H = x^u$, the government cannot subsidize or tax the participation decision. Because it cannot do this at the bottom end of the skill distribution, there is neither a negative income tax nor an earned income tax credit. Second, because $x_L - w_L = x_H - w_H$, the government cannot redistribute between low- and high-skilled workers. This is a very severe restriction, which makes the ETES axiom difficult to defend in the second-best context.

As a result of the second-best constraints, the second-best optimal tax problem in its general form reduces to the following maximization problem.

GSBP (General Second-best Problem)

$$\max_{x_L, x_H, x^u, \alpha_L^*, \alpha_H^*} \widetilde{S}^X \left(x_L, x_H, x^u, \alpha_L^*, \alpha_H^*\right),$$

subject to the government budget constraint:

$$\gamma\left[\left(w_L-x_L\right)F\left(\alpha_L^*\right)-x^u\left(1-F\left(\alpha_L^*\right)\right)\right]+(1-\gamma)\left[\left(w_H-x_H\right)F\left(\alpha_H^*\right)-x^u\left(1-F\left(\alpha_H^*\right)\right)\right]-R=0,$$

and constraints (10), (11) and (12).

The second-best framework has important consequences for the specification of the social objective functions. Combining the expressions for the social objective functions (2), (3), (4), (5), (6), (7), (8), (9) with expressions (10), (11), (12) and (13) results in the following objective functions, as shown in Appendix A. Again, we omit the superscripts U, W, B, P, R, V, CE and EE for notational simplicity.

(a) Utilitarian

$$\begin{aligned}\widetilde{S}^U &= \gamma\int_0^{\alpha_L^*}\left[v(x_L)-\alpha\right]dF(\alpha)+\gamma\int_{\alpha_L^*}^{\infty}v(x^u)dF(\alpha)\\ &+(1-\gamma)\int_0^{\alpha_H^*}\left[v(x_H)-\alpha\right]dF(\alpha)+(1-\gamma)\int_{\alpha_H^*}^{\infty}v(x^u)dF(\alpha).\end{aligned}$$

(b) Welfarist

$$\begin{aligned}\widetilde{S}^W &= \gamma\int_0^{\alpha_L^*}\Psi\left(v(x_L)-\alpha\right)dF(\alpha)+\gamma\int_{\alpha_L^*}^{\infty}\Psi\left(v(x^u)\right)dF(\alpha)\\ &+(1-\gamma)\int_0^{\alpha_H^*}\Psi\left(v(x_H)-\alpha\right)dF(\alpha)+(1-\gamma)\int_{\alpha_H^*}^{\infty}\Psi\left(v(x^u)\right)dF(\alpha).\end{aligned}$$

(c) Boadway *et al.*

$$\begin{aligned}\widetilde{S}^B &= \gamma\int_0^{\alpha_L^*}W\left(\alpha\right)\left[v(x_L)-\alpha\right]dF(\alpha)+\gamma\int_{\alpha_L^*}^{\infty}W\left(\alpha\right)v(x^u)dF(\alpha)\\ &+(1-\gamma)\int_0^{\alpha_H^*}W\left(\alpha\right)\left[v(x_H)-\alpha\right]dF(\alpha)+(1-\gamma)\int_{\alpha_H^*}^{\infty}W\left(\alpha\right)v(x^u)dF(\alpha).\end{aligned}$$

(d) Paternalistic Utilitarian

$$\begin{aligned}\widetilde{S}^P &= \gamma\int_0^{\alpha_L^*}\left[v(x_L)-\overline{\alpha}\right]dF(\alpha)+\gamma\int_{\alpha_L^*}^{\infty}v(x^u)dF(\alpha)\\ &+(1-\gamma)\int_0^{\alpha_H^*}\left[v(x_H)-\overline{\alpha}\right]dF(\alpha)+(1-\gamma)\int_{\alpha_H^*}^{\infty}v(x^u)dF(\alpha).\end{aligned}$$

(e) Roemer and (f) Van de gaer

$$\widetilde{S}^R=\int_0^{\alpha_L^*}\left[v\left(x_L\right)-\alpha\right]dF\left(\alpha\right)+\int_{\alpha_L^*}^{\infty}v\left(x^u\right)dF\left(\alpha\right).$$

(g) Conditional Equality

$$\begin{aligned}\text{Low }\widetilde{\alpha} &: \widetilde{S}^{CE_1}=v\left(x^u\right),\\ \text{High }\widetilde{\alpha} &: \widetilde{S}^{CE_2}=v\left(x_L\right).\end{aligned}$$

(h) Egalitarian Equivalent

$$\widetilde{S}^{EE}=x_L.$$

Under asymmetric information, the criteria of Roemer and Van de gaer imply equivalent policies. Because of the second-best constraint, utility as a function of the taste parameter of the low-skilled will never be below utility as a function of the taste parameter of the high-skilled. One implication of this is that the opportunity set for the low-skilled is below that for the high-skilled; hence, in the second best, the mean of mins and min of means criteria will yield the same solutions. Observe further that for low values of $\widetilde{\alpha}$, the Conditional Egalitarian planner identifies the nonemployed as the worst-off and for high values of $\widetilde{\alpha}$, it identifies the working low-skilled as the worst-off.

5.2 Optimal tax formula

Before we can characterize the optimal tax rates, we need to introduce more definitions. Let $T_L = w_L - x_L$, $T_H = w_H - x_H$, and $T_u = -x^u$ be the tax paid by the low-skilled workers, the high-skilled workers and the inactive, respectively. Define the elasticities of participation of the low-skilled with respect to x_L[14] and of the high-skilled with respect to x_H as:

$$\eta\left(x_L, \alpha_L^*\right) \stackrel{\text{def}}{\equiv} \frac{x_L}{F\left(\alpha_L^*\right)} f\left(\alpha_L^*\right) v'\left(x_L\right), \tag{14}$$

$$\eta\left(x_H, \alpha_H^*\right) \stackrel{\text{def}}{\equiv} \frac{x_H}{F\left(\alpha_H^*\right)} f\left(\alpha_H^*\right) v'\left(x_H\right), \tag{15}$$

respectively. Next, observe that the average of the inverse of the private marginal utility of consumption is given by:

$$g_P^X \stackrel{\text{def}}{\equiv} \frac{\gamma F(\alpha_L^{X*})}{v'(x_L^X)} + \frac{\gamma(1-F(\alpha_L^{X*})) + (1-\gamma)(1-F(\alpha_H^{X*}))}{v'(x^{uX})} + \frac{(1-\gamma)F(\alpha_H^{X*})}{v'(x_H^X)}. \tag{16}$$

Let the subscripts on the function $\widetilde{S}$ denote the partial derivative of $\widetilde{S}$ with respect to the argument in the subscript and note that the effect of a uniform increase in private utilities on the social objective function is given by:

$$D^X \stackrel{\text{def}}{\equiv} \frac{\widetilde{S}_{x_L}^X}{v'(x_L)} + \frac{\widetilde{S}_{x_H}^X}{v'(x_H)} + \frac{\widetilde{S}_{x^u}^X}{v'\left(x^u\right)}. \tag{17}$$

Finally, the average social marginal utility of consumption for workers of skill levels L and H are, respectively:

$$g_L^X \stackrel{\text{def}}{\equiv} \frac{\widetilde{S}_{x_L}^X}{\lambda\gamma F\left(\alpha_L^*\right)} \text{ and } g_H^X \stackrel{\text{def}}{\equiv} \frac{\widetilde{S}_{x_H}^X}{\lambda\left(1-\gamma\right) F\left(\alpha_H^*\right)}.$$

The following theorem states the solution for the general second-best problem.

Theorem 1 *Under asymmetric information, the optimal consumption levels have to satisfy the budget constraint, constraints (10), (11) and (12), and the following equations:*

$$\frac{T_L - T_u}{x_L} = \frac{1}{\eta\left(x_L, \alpha_L^*\right)}\left[1 - g_L^X + \frac{\nu}{\lambda\gamma F\left(\alpha_L^*\right)}\right] - \frac{\widetilde{S}_{\alpha_L^*}^X}{\lambda\gamma f\left(\alpha_L^*\right) x_L},$$

$$\frac{T_H - T_u}{x_H} = \frac{1}{\eta\left(x_H, \alpha_H^*\right)}\left[1 - g_H^X - \frac{\nu}{\lambda\left(1-\gamma\right) F\left(\alpha_H^*\right)}\right] - \frac{\widetilde{S}_{\alpha_H^*}^X}{\lambda\left(1-\gamma\right) f\left(\alpha_H^*\right) x_H},$$

$$\left(\lambda^X\right)^{-1} = g_P^X / D^X,$$

[14] $\eta\left(x_L, \alpha_L^*\right) \stackrel{\text{def}}{\equiv} \left(x_L / \gamma F\left(\alpha_L^*\right)\right)\left(\partial\left(\gamma F\left(\alpha_L^*\right)\right)/\partial x_L\right)$. Because $\alpha_L^* = v\left(x_L\right) - v\left(x^u\right)$, we get $\partial \alpha_L^* / \partial x_L = v'\left(x_L\right)$; hence, we obtain (14).

where ν is the Lagrangian multiplier associated with the constraint $x_H \geq x_L$.

A simple heuristic interpretation of the optimal tax formulas, in the spirit of Saez (2002), is given below, while the formal proof is provided in Appendix B.

Consider a small increase in consumption x_L (i.e., a small reduction of the income tax in low-skilled jobs) around the optimal tax schedule. This has a mechanical effect, a behavioral effect (or labor supply response) and an effect on the incentive compatibility constraint.

Mechanical effect

There is an mechanical decrease in tax revenue equal to $-\gamma F(\alpha_L^*)dx_L$ because low-skilled workers have dx_L additional consumption. Each unit of x_L improves the social objective by $\widetilde{S}_{x_L}^X/\lambda$ in terms of government revenue. Hence the total value of the decrease in tax revenue is worth $-\left(\gamma F\left(\alpha_L^*\right) - \widetilde{S}_{x_L}^X/\lambda\right) dx_L$ in terms of government revenue, which can be written as:

$$-\left[1 - \frac{\widetilde{S}_{x_L}^X}{\lambda\gamma F\left(\alpha_L^*\right)}\right]\gamma F\left(\alpha_L^*\right) dx_L.$$

Behavioral effect

The change $dx_L > 0$ induces a change in α_L^* equal to $\left(\partial\alpha_L^*/\partial x_L\right) dx_L$. By (10), $\partial\alpha_L^*/\partial x_L = v'\left(x_L\right)$ and from the definition of the elasticity of participation (14), $v'\left(x_L\right) = \left[F\left(\alpha_L^*\right)\eta\left(x_L, \alpha_L^*\right)\right] / \left[x_L f\left(\alpha_L^*\right)\right]$, such that the induced change in α_L^* is $\left[F\left(\alpha_L^*\right)\eta\left(x_L, \alpha_L^*\right)\right] / \left[x_L f\left(\alpha_L^*\right)\right] dx_L$. A change in the critical value α_L^* has an effect on the social objective, equal to $\widetilde{S}_{\alpha_L^*}^X/\lambda$ in terms of government revenue, and increases government revenue by $\gamma\left[T_L - T_u\right] f\left(\alpha_L^*\right)$. Hence the total behavioral effect in terms of government revenue equals:

$$\left[\frac{\widetilde{S}_{\alpha_L^*}^X}{\lambda} + \gamma\left(T_L - T_u\right) f\left(\alpha_L^*\right)\right]\frac{F\left(\alpha_L^*\right)\eta\left(x_L, \alpha_L^*\right)}{x_L f\left(\alpha_L^*\right)} dx_L.$$

Effect on the Incentive Compatibility Constraint

An increase in x_L of one unit tightens the incentive compatibility constraint, which has an effect on the social objective function equal to $-\nu$, which is worth $-\nu/\lambda$ in terms of government revenue. Hence this ICC effect in terms of government revenue equals:

$$-\left(\nu/\lambda\right) dx_L.$$

At the optimum, the sum of these three effects equals zero. It is easy to verify that this yields the first equation in Theorem 1. The second equation can be given a similar interpretation.

The λ^{-1} equations are similar to Diamond and Sheshinski's (1995) equation (6), p.6, and are associated with an equal marginal change of the consumption of everyone in the economy. Consider a uniform increase in all private utilities of one unit. This does not change the activity decisions. To accomplish this uniform increase, for each w_Y-worker, we need $1/v'\left(x_Y\right)$ extra units of consumption ($Y = L, H$), and for each inactive person, we need $1/v'\left(x^u\right)$ extra units of consumption. Weighting this by the frequencies of these groups in the population, we find that we need an additional $g_P\left(x^u, x_L, x_H, \alpha_L^*, \alpha_H^*\right)$ units of public funds to finance this operation. In terms of social welfare, this is worth $\lambda g_P\left(x^u, x_L, x_H, \alpha_L^*, \alpha_H^*\right)$. This has to be equal to the increase in the social objective function caused by the uniform increase in utilities, which is equal to D. The equation for λ^{-1} thus equates the inverse of the marginal cost of public funds to the ratio between the average of the inverse of the private utilities and the marginal social utility of a uniform increase in all individual utilities.

Observe that the optimal tax formulas in Theorem 1 contain three elements: the deviation of the average social marginal utility of consumption for workers of a particular skill level from unity, $1-g_Y^X$, the Lagrangian multiplier ν and the term $\widetilde{S}^X_{\alpha^*_Y}$ $(Y = L$ or $H)$. The last two terms have not been dealt with in the literature on optimal taxation in the case of extensive labor supply, as they do not appear in the social objective functions U and W that have been considered so far. This is stated in the following lemmata.

Lemma 2 *The value of $\widetilde{S}^X_{\alpha^*_Y}$ $(Y = L, H)$:*
(a) $\widetilde{S}^X_{\alpha^*_Y} = 0$ $(Y = L, H)$ for $X = U$, W, B, R, EE, CE_1 and CE_2.
(b) $\widetilde{S}^P_{\alpha^*_L} = [\alpha^*_L - \overline{\alpha}]\,\gamma f(\alpha^*_L)$ and $S^P_{\alpha^*_H} = [\alpha^*_H - \overline{\alpha}]\,(1-\gamma)\, f(\alpha^*_H)$.

Lemma 3 *The value of the Lagrangian multiplier:*
(a) $\nu = 0$ for $X = U$, W, B, P and CE_1,
(b) $\nu \geq 0$ for $X = R$, EE and CE_2.

Lemma 2 follows from partially differentiating the expressions for the social objective functions with respect to α^*_Y $(Y = L, H)$. These terms represent the direct effects of changes in the critical values on the social objective functions and occur only in the Paternalist case. Lemma 3 is proved in Appendix B. To see how it works, suppose that $\nu > 0$, such that $x_H = x_L$, and take an infinitesimal $\varepsilon > 0$. Next, increase x_H by an amount $dx_H = \varepsilon/(1-\gamma)$ and decrease x_L by an amount $dx_L = \varepsilon/\gamma$. Such an operation has no effect on the social objective functions listed in Lemma 3 (a) but increases government revenue, which can be distributed in an incentive-compatible way to all agents, increasing the value of the social objective function. Observe that under the social objective functions listed in Lemma 3 (b), the effect of this operation depends on whether the negative direct effect on the social objective function because of the decrease in x_L is offset by the increase in the social objective thanks to the increase in x_L because of the incentive-compatibly distributed increase in government revenue of the operation.

Lemma 3 combined with Lemma 1 has implications for the performance of the different social objective functions from the perspective of the equality of opportunity principles. The U, W, B, P and CE_1 criteria have a zero value for ν, and their solutions have $x_H > x_L$ (as shown in the supplemental material). Therefore, $\alpha^*_H > \alpha^*_L$, such that EWEP is violated under those criteria. However, with the R, EE and CE_2 criteria, ν may be strictly positive, in which case $x_H = x_L$ and $\alpha^*_H = \alpha^*_L$ such that EWEP is satisfied.

In order to obtain optimal tax rates with the different social objective functions, we use the relevant properties of these social objective functions and plug them into the equations of Theorem 1. Lemma 2 gives us the values for $\widetilde{S}^X_{\alpha^*_Y}$ $(Y = L, H)$, and Lemma 3 the values for the Lagrangian multipliers ν^X. The average social marginal utility of consumption g_Y^X under objective functions X for agents of skill level Y are given in Table 3. Using these expressions in the equations of Theorem 1, together with $\nu^X = 0$ for $X = U$, W, B, P and CE_1, results in Corollary 1.[15]

[15] The optimal activity assignments are characterized by $\alpha^*_H > \alpha^*_L > 0$ under the U, W, B, P and CE_1 criteria,

Table 3: The average social marginal utility of consumption g_Y^X for social objective X for agents of skill level Y.

X	g_L^X	g_H^X
U,P	$\frac{v'\left(x_L^X\right)}{\lambda^X}$	$\frac{v'\left(x_H^X\right)}{\lambda^X}$
W	$\frac{v'\left(x_L^W\right)}{\lambda^W}\frac{\int_0^{\alpha_L^{W*}}\Psi'\left(v\left(x_L^W\right)-\alpha\right)dF(\alpha)}{F(\alpha_L^{W*})}$	$\frac{v'\left(x_H^W\right)}{\lambda^W}\frac{\int_0^{\alpha_H^{W*}}\Psi'\left(v\left(x_H^W\right)-\alpha\right)dF(\alpha)}{F(\alpha_H^{W*})}$
B	$\frac{v'\left(x_L^B\right)}{\lambda^B}\frac{\int_0^{\alpha_L^{B*}}W(\alpha)dF(\alpha)}{F(\alpha_L^{B*})}$	$\frac{v'\left(x_H^B\right)}{\lambda^B}\frac{\int_0^{\alpha_H^{B*}}W(\alpha)dF(\alpha)}{F(\alpha_H^{B*})}$
$R(=V)$	$\frac{v'\left(x_L^R\right)}{\lambda^R\gamma}$	0
EE	$\frac{1}{\lambda^{EE}\gamma F(\alpha_L^{EE*})}$	0
CE_1	0	0
CE_2	$\frac{v'\left(x_L^{CE}\right)}{\lambda^{CE}\gamma F(\alpha_L^{CE*})}$	0

Corollary 1 *Under asymmetric information, the optimal consumption levels have to satisfy the budget constraint, constraints (10), (11) and (12), and the expressions given in the following table.*

Table 4: Optimal tax formulas under asymmetric information for the social objectives X.

X	$\left(\lambda^X\right)^{-1}$	$\frac{T_H^X-T_u^X}{x_H^X}$	$\frac{T_L^X-T_u^X}{x_L^X}$
U	g_P^U		
W	g_P^W/D^W	$\frac{1}{\eta\left(x_H^X,\alpha_H^{X*}\right)}\left(1-g_H^X\right)$	$\frac{1}{\eta\left(x_L^X,\alpha_L^{X*}\right)}\left(1-g_L^X\right)$
B	g_P^B/D^B		
$R=V$	g_P^X		
EE	g_P^{EE}/D^{EE}	$\frac{1}{\eta\left(x_H^X,\alpha_H^{X*}\right)}\left(1-\frac{\nu^X}{\lambda^X(1-\gamma)F\left(\alpha_H^{X*}\right)}\right)$	$\frac{1}{\eta\left(x_L^X,\alpha_L^{X*}\right)}\left(1-g_L^X+\frac{\nu^X}{\lambda^X\gamma F\left(\alpha_L^{X*}\right)}\right)$
CE_2	$g_P^{CE_2}$		
CE_1	$g_P^{CE_1}$	$\frac{1}{\eta\left(x_H^{CE_1},\alpha_H^{CE_1*}\right)}$	$\frac{1}{\eta\left(x_L^{CE_1},\alpha_L^{CE_1*}\right)}$
P	g_P^P	$\frac{1}{\eta\left(x_H^P,\alpha_H^{P*}\right)}\left(1-g_H^P\right)-\frac{\alpha_H^{P*}-\overline{\alpha}}{\lambda^P x_H^P}$	$\frac{1}{\eta\left(x_L^P,\alpha_L^{P*}\right)}\left(1-g_L^P\right)-\frac{\alpha_L^{P*}-\overline{\alpha}}{\lambda^P x_L^P}$

Note the following definitions: $D^W \overset{\text{def}}{\equiv} \gamma\left[\int_0^{\alpha_L^{W*}}\Psi'\left(v\left(x_L^W\right)-\alpha\right)dF(\alpha)+\int_{\alpha_L^{W*}}^{\infty}\Psi'\left(v\left(x^{uW}\right)\right)dF(\alpha)\right]$ $+\ (1-\gamma)\left[\int_0^{\alpha_H^{W*}}\Psi'\left(v\left(x_H^W\right)-\alpha\right)dF(\alpha)+\int_{\alpha_H^{W*}}^{\infty}\Psi'\left(v\left(x^{uW}\right)\right)dF(\alpha)\right]$, $D^B \overset{\text{def}}{\equiv} \int_0^{\infty}W(\alpha)\,dF(\alpha)$ and $D^{EE}\overset{\text{def}}{\equiv} 1/v'\left(x_L^{EE}\right)$.

Taking as a benchmark case the formula for the popular objective functions U and W, three features of the optimal tax formula are striking. Let us focus on the formula for the low-skilled. First, with the Paternalistic Utilitarian criterion, the extra term $[\alpha_L^*-\overline{\alpha}]/[\lambda x_L]$ appears. It captures the social value of the divergence between private and social preferences. Second, with the social policies inspired by equality of opportunity principles ($X=R$, CE_2 or EE), it is possible that the constraint $x_H\geq x_L$ is binding and $\nu>0$; hence, this term enters the optimal tax formula. If the constraint $x_H\geq x_L$ were absent, the planner would like to increase x_L, which requires a decrease in T_L, such that $\left(T_L^X-T_u^X\right)/x_L^X$ decreases. The presence of the multiplier makes it clear

while $\alpha_H^*\geq\alpha_L^*$ under the R, EE and CE_2 criteria. Moreover, $\alpha_H^*<\infty$ for all criteria. The supplemental material states the proofs.

that when the constraint is binding, $\left(T_L^X - T_u^X\right)/x_L^X$ must be higher. Third, under CE_1, the optimal tax levels have a very simple structure: the (per capita) tax revenue from the low- and high-skilled workers is maximized, as this maximizes the transfer toward the nonemployed.

Since Diamond (1980), it is well known that subsidizing the low-skilled workers more than inactive people (i.e., $T_L < T_u$) can be optimal when the labor supply is modeled along the extensive margin. Using the definition of Saez (2002), an Earned Income Tax Credit (EITC) is then optimal. On the contrary, when $T_L > T_u$, a Negative Income Tax (NIT) is optimal. Alternatively, $T_L < (>) T_u$ can be rewritten as $w_L < (>) x_L - x^u$; i.e., the income gain when a low-skilled agent enters the labor force ($x_L - x^u$) is larger (smaller) than her gross labor income (w_L). In other words, the labor supply of the low-skilled is distorted upwards (downwards), compared with laissez faire. Theorem 1 can be used to study the necessary conditions for an EITC or an NIT under criteria other than the standard Utilitarian and Welfarist ones. Corollary 2 emphasizes that the Roemer, EE, CE and Paternalistic Utilitarian criteria challenge the standard necessary conditions.

Corollary 2 *The following table provides necessary and sufficient conditions for optimality of the EITC or NIT in the second best.*

Table 5: Optimality of EITC or NIT in the second best.

Social Objective	ETES/ NIT/ EITC?
Utilitarian, Welfarist, Boadway *et al.*	NIT (EITC) if $g_L^X < (>)\, 1$
Roemer (=V), Egalitarian Equivalent, Conditional Egalitarian type 2	NIT (EITC) if $g_L^X - \frac{\nu^X}{\lambda\gamma F\left(\alpha_L^{X*}\right)} < (>)\, 1$
Conditional Egalitarian type 1	NIT
Paternalistic Utilitarian	NIT (EITC) if $\frac{1}{\eta\left(x_L^P, \alpha_L^{P*}\right)}\left(1 - g_L^P\right) > (<)\, \frac{\alpha_L^{P*} - \overline{\alpha}}{\lambda^P x_L^P}$
Maximin	NIT

The proof is obvious from the table in Corollary 1. Under the Utilitarian, the Welfarist and the Boadway *et al.* objectives, we retrieve the result that the average social weight of the low-skilled workers larger than one is a necessary condition for the EITC to be optimal. For the Roemer, Egalitarian Equivalent and Conditional Egalitarian type 2 objective functions, this condition has to be adjusted because the constraint that $x_H \geq x_L$ may be binding. If this constraint is binding, an NIT can be optimal even when g_L^X is larger than one. In that sense, these social objective functions that find their inspiration in equality of opportunity theories are more in favor of an NIT. Intuitively, considering any equality of opportunity criterion amounts to modifying the social weight g_L^X to $\breve{g}_L^X \stackrel{\text{def}}{\equiv} g_L^X - \nu^X / \left[\lambda^X \gamma F\left(\alpha_L^{X*}\right)\right]$ with $X = R, V, EE, CE_2$, as can be seen from Table 4. The ICC effect decreases the value of the average social weight on the low-skilled workers to avoid a shift by high-skilled workers to low-skilled jobs. This explains why the government is less willing to transfer income to low-skilled workers under equality of opportunity criteria.

The necessary condition to obtain unambiguous results under the Paternalistic Utilitarian criterion is clearly more complicated: there is no simple relationship between the average social weight of the low-skilled workers being larger than one and the optimality of an EITC. The EITC (NIT) encourages (discourages) participation of the marginal worker, which results in an increased (decreased) utility of consumption equal to α_L^{P*}, which is desirable if this is larger (smaller) than $\overline{\alpha}$, the utility cost of work in the eyes of the Paternalistic Utilitarian planner. The extra term $\left(\alpha_L^{P*} - \overline{\alpha}\right) / \left(\lambda^P x_L^P\right)$ that appears on the right-hand side in Corollary 2 is used as a device to correct undesirable social outcomes. It corrects individual labor supply to correspond to social preferences. Hence, if social preferences are characterized by $\alpha_L^{P*} > (<)\overline{\alpha}$, the government encourages (discourages) participation, and the right-hand side of the inequality in the corollary is positive, such that the EITC (NIT) then becomes more attractive for the Paternalistic Utilitarian planner. This term is sometimes called the paternalistic or the first-best motive for taxation because it arises from differences between social and private preferences (Kanbur *et al.*, 2006). Assuming $\alpha_L^{P*} > \overline{\alpha}$, when the Paternalist government's views on working become more "Calvinistic", i.e., when $\overline{\alpha}$ decreases, the term on the right-hand side becomes larger and hence works in favor of an EITC to promote participation of more people.

As a final point of reference, we compare our policy prescriptions with the policy prescription of the Maximin social objective function. Maximin, which is a subcase of the Welfarist criterion, works in favor of an NIT, as shown in Choné and Laroque (2005). Under Maximin, only the least well-off receive a positive average social marginal utility of consumption. Because of the ICC, the least well-off are the inactive; hence, the Maximin social objective coincides with the Conditional Egalitarian type 1 such that an NIT is always optimal under Maximin.

Empirical studies suggest that participation decisions are more elastic at the bottom of the skill distribution (see the empirical evidence surveyed by Immervoll *et al.*, 2007, and Meghir and Phillips, 2008), which motivates the following assumption.

Assumption 1: $\eta\left(x_L, \alpha_L^*\right) \geq \eta\left(x_H, \alpha_H^*\right)$.

Corollary 3 *Under Assumption 1, for the Utilitarian, Welfarist and Boadway et al. criteria when $W(\alpha)$ is a decreasing function and for the Roemer, EE, CE_1 and CE_2 criteria:*[16]

$$\left(T_L - T_u\right) / x_L < \left(T_H - T_u\right) / x_H .$$

Appendix B states the proof. In our extensive model of labor supply, the degree to which labor supply is distorted downwards depends on the difference between taxes paid when working and taxes paid when inactive (the latter is $-x^u$). The larger this difference, the more labor supply is distorted downwards; if the difference is negative, labor supply is distorted upwards. We now have the following corollary.

Corollary 4 *Under Assumption 1, for the Utilitarian, Welfarist, and Boadway et al. criteria when $W(\alpha)$ is a decreasing function and for the Roemer, EE, CE_1 and CE_2 criteria, the labor supply of the high-skilled is more distorted downwards than the labor supply of the low-skilled.*

Appendix B proves this result. The statement that the labor supply of the high-skilled is more downwardly distorted also allows for the possibility that it is less upwardly distorted than the labor supply of the low-skilled.

[16] For the Roemer, EE and CE_2 criteria when $\nu > 0$, we have $\eta\left(x_L, \alpha_L^*\right) = \eta\left(x_H, \alpha_H^*\right) = \eta\left(x, \alpha^*\right)$.

5.3 Restricted second best

This section searches for the optimal policies that satisfy fully at least one of the equality of opportunity principles, in the second best. The following theorem, proved in Appendix D, shows that there is only one possible allocation that satisfies ETES.

Theorem 2 *Second-best optima satisfying ETES.*

There exists only one second-best allocation satisfying ETES. In this allocation, $x_L = w_L + x^u$ *and* $x_H = w_H + x^u$*. The corresponding values for* x^u, α_L^* *and* α_H^* *are determined by:*

$$x^u \left[1 - 2\left[\gamma F\left(\alpha_L^*\right) + (1-\gamma) F\left(\alpha_H^*\right)\right]\right] = R,$$
$$\alpha_L^* = v\left(w_L + x^u\right) - v\left(x^u\right), \ \alpha_H^* = v\left(w_H + x^u\right) - v\left(x^u\right).$$

In the discussion following Lemma 1, we already noted the restrictive nature of ETES in the context of our model. The severity of the ETES axiom also appears clearly in Theorem 2. Therefore, we think that in the second-best model, priority should be given to the EWEP principle. We now show which allocations are second-best optimal under the different criteria, when the optimum is sought under the allocations satisfying EWEP. Of course, when the optimal policies under the equality-of- opportunity-inspired social objective functions automatically satisfy EWEP (i.e., when $\nu > 0$), the optima derived in this section for $X = R$, EE and CE_2 will be identical to the optima in the previous subsection.

From Lemma 1 (a), we know that the critical values and the consumption levels for both types of workers have to be the same. We denote this critical value by α^* and the workers' consumption by x^w:

$$v\left(x^w\right) - \alpha^* = v\left(x^u\right). \tag{18}$$

The only policy instruments of the planner are now x^w and x^u, which prevents any redistribution between w_L and w_H-workers. Hence, the following programming problem describes the EWEP-restricted general second-best problem.

ERGSBP (EWEP Restricted General Second-best Problem)

$$\max_{x^w, x^u, \alpha^*} \widehat{S}^X\left(x^w, x^u, \alpha^*\right),$$

subject to the government budget constraint:

$$\left[\gamma w_L + (1-\gamma) w_H - x^w\right] F\left(\alpha^*\right) - x^u\left(1 - F\left(\alpha^*\right)\right) - R = 0,$$

and constraint (18).

We define the elasticity of participation (which is any of the previous elasticities where $x^L = x^H = x^w$ is substituted), the average of the inverse of the private marginal utility of consumption, the effect of a uniform increase in private utilities on the social objective function and the average social marginal utility of workers' consumption respectively as:

$$\eta\left(x^w, \alpha^*\right) \stackrel{\text{def}}{\equiv} \frac{x^w}{F\left(\alpha^*\right)} f\left(\alpha^*\right) v'\left(x^w\right), \tag{19}$$

$$g_P^X \stackrel{\text{def}}{\equiv} \frac{F(\alpha^{X*})}{v'(x^{wX})} + \frac{(1 - F(\alpha^{X*}))}{v'(x^{uX})}, \tag{20}$$

$$D^X \stackrel{\text{def}}{\equiv} \frac{\widehat{S}^X_{x^u}}{v'\left(x^u\right)} + \frac{\widehat{S}^X_{x^w}}{v'\left(x^w\right)}. \tag{21}$$

The following theorem states the solution for the EWEP-restricted General Second-best Problem. Its proof is given in the supplementary material.

Theorem 3 *Under asymmetric information, the optimal consumption levels have to satisfy the budget constraint, constraint (18), and the following equations:*

$$\begin{aligned} \frac{\gamma w_L + (1-\gamma) w_H - x^w + x^u}{x^w} &= \frac{1}{\eta(x^w, \alpha^*)} \left[1 - g^X\right] - \frac{\widehat{S}^X_{\alpha^*}}{\lambda f(\alpha^*) x^w}, \\ \lambda^{-1} &= g^X_P / D^X. \end{aligned}$$

The interpretation of the equation for λ^{-1} is similar to the interpretation in the previous section. To obtain more specific expressions for the different social objective functions, observe that $\widehat{S}^X_{\alpha^*} = 0$ for all objective functions, except for the Paternalistic Utilitarian, for which $\widehat{S}^X_{\alpha^*} = (\alpha^* - \overline{\alpha}) f(\alpha^*)$. It is then straightforward to derive the following corollary, which gives the optimal consumption levels in the restricted second best.

Corollary 5 *Under asymmetric information, the second-best optimal consumption levels satisfying EWEP have to satisfy the budget constraint, constraint (18) and the expressions given in the following table.*

Table 6: Optimal tax formula satisfying EWEP under asymmetric information for different social objectives X.

X	$\left(\lambda^X\right)^{-1}$	$\frac{\gamma w_L + (1-\gamma) x_H - x^w + x^u}{x^w}$	g^X
U	g^U_P	$\frac{1}{\eta(x^{wX}, \alpha^{X*})} \left(1 - g^X\right)$	$\frac{v'\left(x^U_L\right)}{\lambda^U}$
W	g^W_P / D^W		$\frac{v'\left(x^{wW}\right)}{\lambda^W} \frac{\int_0^{\alpha^{W*}} \Psi'\left(v\left(x^{wW}\right) - \alpha\right) dF(\alpha)}{F(\alpha^{W*})}$
B	g^B_P / D^B		$\frac{v'\left(x^{wB}\right)}{\lambda^B} \frac{\int_0^{\alpha^{B*}} W(\alpha) dF(\alpha)}{F(\alpha^{B*})}$
$R = V$	g^R_P		$\frac{v'\left(x^{wR}\right)}{\lambda^R}$
EE	g^{EE}_P / D^{EE}		$\frac{1}{\lambda^{EE} F(\alpha^{EE*})}$
CE_2	$g^{CE_2}_P$		$\frac{v'\left(x^{wCE_2}\right)}{\lambda^{CE_2} F(\alpha^{CE_2 *})}$
CE_1	$g^{CE_1}_P$	$\frac{1}{\eta\left(x^{wCE_1}, \alpha^{CE_1 *}\right)}$	0
P	g^P_P	$\frac{1}{\eta(x^{wP}, \alpha^{P*})} \left(1 - g^P\right) - \frac{\alpha^{P*} - \overline{\alpha}}{\lambda^P x^{wP}}$	$\frac{v'\left(x^P_L\right)}{\lambda^P}$

Note the following definitions: $D^W = \left[\int_0^{\alpha^{W*}} \Psi'\left(v\left(x^{wW}\right) - \alpha\right) dF(\alpha) + \int_{\alpha^{W*}}^{\infty} \Psi'\left(v\left(x^{uW}\right)\right) dF(\alpha)\right]$, $D^B = \int_0^{\infty} W(\alpha) dF(\alpha)$ and $D^{EE} = 1/v'\left(x^{wEE}\right)$.

Not surprisingly, the optimal tax formulas have the same shape as in the previous subsection, but now the constraint $x_H = x_L$ is imposed. The major difference is because EWEP impedes the government in distinguishing between low- and high-skilled workers, such that the formula now has to hold for an imaginary worker who has average productivity and thus average wage $\gamma w_L + (1-\gamma) w_H$.

5.4 Priority principles

The social choice literature on equality of opportunity argues that because compensation and responsibility cannot be fully satisfied in general, only a maximin variant makes sense (Fleurbaey, 2008). Therefore, rather than strictly imposing one of the equality of opportunity principles and searching for the optimal allocation satisfying it, this section examines the optimal tax policies when priority to the worst-off is given. The strict equality demanded by each of the principles is weakened and replaced with maximin, and we search for social orderings that embody this weak version of the principle.

EWEP requires that for each value of α, welfares are to be equalized. Rather than insisting on full equality, the priority principle requires that social states be judged, for each α, by the welfare level obtained by the skill level L or H, that has the lowest welfare. It expresses the idea that the allocation of consumption levels and jobs between two individuals with identical tastes should be such that it is impossible to redistribute among them and increase the level of well-being of the least well-off.

The question then becomes how to measure individuals' welfare. A first possibility is to measure welfare by individual utilities. Roemer's criterion applies a Utilitarian aggregation to these minimal levels of welfare, but other aggregation procedures are possible, such as a Welfaristic and a Boadway *et al.* variant, leading to the following Priority Welfare weighted Utility ordering:

$$S^{PWU} = \int_0^\infty \Omega^R \Bigg(\min\{ \underset{\delta_L(\alpha)}{\operatorname{oper}} \{ v\left(x_L^w(\alpha)\right) - \alpha, v\left(x_L^u(\alpha)\right) \},$$

$$\underset{\delta_H(\alpha)}{\operatorname{oper}} \{ v\left(x_H^w(\alpha)\right) - \alpha, v\left(x_H^u(\alpha)\right) \} \} \Bigg) dF(\alpha),$$

where $\Omega^R(\cdot)$ is a welfare function with $\Omega^{R\prime}(\cdot) > 0$ and the Priority Taste weighted Utility ordering:

$$S^{PTU} = \int_0^\infty \Phi^R(\alpha) \Bigg[\min\{ \underset{\delta_L(\alpha)}{\operatorname{oper}} \{ v\left(x_L^w(\alpha)\right) - \alpha, v\left(x_L^u(\alpha)\right) \},$$

$$\underset{\delta_H(\alpha)}{\operatorname{oper}} \{ v\left(x_H^w(\alpha)\right) - \alpha, v\left(x_H^u(\alpha)\right) \} \} \Bigg] dF(\alpha),$$

where $\Phi^R(\alpha) > 0$ weights different tastes. These two objective functions are clearly distinct: S^{PWU} allows the planner to express inequality aversion (preference) with respect to utility differences that arise because of differences in tastes if $\Omega^{R\prime\prime}(\cdot) < (>)\, 0$, while in S^{PTU}, the planner gives different weights to different tastes as such, irrespective of their welfare levels. Both are generalizations of Roemer's criterion, but they do not respect the utilitarian reward principle (see Fleurbaey (2008)), which requires zero aversion to inequalities due to different preferences. However, if the planner wants to express an opinion about welfare inequality that arises because of differences in tastes, these specifications allow the planner to do so.

A second approach consists of taking an ordinal measure of welfare. We can find here inspiration with the reasoning that leads to the Egalitarian Equivalent ordering, and take the consumption level that a person requires when he/she works that makes him/her indifferent to his/her actual consumption bundle. The aggregation of these welfare levels can occur again in a Welfarist or a

Boadway *et al.* way, leading to the Priority Welfare weighted Equivalent ordering:

$$S^{PWE} = \int_0^\infty \Omega^O \left(\min\{ \underset{\delta_L(\alpha)}{\text{oper}} \{ x_L^w(\alpha), v^{-1}(v(x_L^u(\alpha)) + \alpha) \}, \right.$$
$$\left. \underset{\delta_H(\alpha)}{\text{oper}} \{ x_H^w(\alpha), v^{-1}(v(x_H^u(\alpha)) + \alpha) \} \right) dF(\alpha), \tag{22}$$

where $\Omega^O(\cdot)$ is a welfare function with $\Omega^{O\prime}(\cdot) > 0$ and the Priority Taste weighted Equivalent ordering:

$$S^{PTE} = \int_0^\infty \Phi^O(\alpha) \left[\min\{ \underset{\delta_L(\alpha)}{\text{oper}} \{ x_L^w(\alpha), v^{-1}(v(x_L^u(\alpha)) + \alpha) \}, \right.$$
$$\left. \underset{\delta_H(\alpha)}{\text{oper}} \{ x_H^w(\alpha), v^{-1}(v(x_H^u(\alpha)) + \alpha) \} \} \right] dF(\alpha), \tag{23}$$

where $\Phi^O(\alpha) > 0$ weights different tastes. If the welfare function $\Omega^O(\cdot)$ becomes infinitely inequality averse, the social welfare function (22) reduces to the egalitarian equivalent ordering (9).[17]

ETES requires that transfers be the same for all those who have equal skills. To apply the priority principle here, for each level of skill, we have to consider the lowest transfer received by an individual with that skill level. Because we have only two levels of skill, a social ordering embodying the priority principle would be the following Priority Transfer ordering:

$$S^{PT} = \rho \min_{\alpha \in \mathbb{R}^+} \left\{ \underset{\delta_L(\alpha)}{\text{oper}} \{ x_L^w(\alpha) - w_L, x_L^u(\alpha) \} \right\} + (1-\rho) \min_{\alpha \in \mathbb{R}^+} \left\{ \underset{\delta_H(\alpha)}{\text{oper}} \{ x_H^w(\alpha) - w_H, x_H^u(\alpha) \} \right\},$$

where $\rho \in [0,1]$ gives the relative importance attached to the low-skilled agents.

The following lemma gives expressions for these new objective functions in the second-best framework. The proof can be found in the supplementary material.

Lemma 4 *Priority social objective functions in the second best.*

$$\widetilde{S}^{PWU} = \int_0^{\alpha_L^*} \Omega^R(v(x_L) - \alpha)\, dF(\alpha) + \int_{\alpha_L^*}^\infty \Omega^R(v(x^u))\, dF(\alpha).$$
$$\widetilde{S}^{PTU} = \int_0^{\alpha_L^*} \Phi^R(\alpha)[v(x_L) - \alpha]\, dF(\alpha) + \int_{\alpha_L^*}^\infty \Phi^R(\alpha)\, v(x^u)\, dF(\alpha).$$
$$\widetilde{S}^{PWE} = \int_0^{\alpha_L^*} \Omega^O(x_L)\, dF(\alpha) + \int_{\alpha_L^*}^\infty \Omega^O\left(v^{-1}(v(x^u) + \alpha)\right) dF(\alpha).$$
$$\widetilde{S}^{PTE} = x_L \int_0^{\alpha_L^*} \Phi^O(\alpha)\, dF(\alpha) + \int_{\alpha_L^*}^\infty \Phi^O(\alpha)\left(v^{-1}(v(x^u) + \alpha)\right) dF(\alpha).$$
$$\widetilde{S}^{PT} = \rho(x_L - w_L) + (1-\rho)(x_H - w_H).$$

The problem of finding the optimal tax rates with these objective functions has exactly the same structure as the General Second-best Problem formulated in Section 5, whose solution is

[17] In a recent contribution, Hodler (2009) proposes to measure inequality in societies with unequal earning abilities and tastes for work by computing traditional inequality indices (e.g., Gini, Atkinson-Kolm, Theil) for equivalent wages in the *entire* population. When interested in inequality, one can do something similar here, but the priority principle forces us to take, for each value of tastes, only the lowest equivalent wage into account.

given by Theorem 1. Using the same procedure as in Section 5.2, it is easy to derive the terms of the optimal tax formulas; hence, the following lemmata. The detailed proofs are available in the supplementary material.

Lemma 5 *The value of* $\widetilde{S}^X_{\alpha^*_Y}$ $(Y = L, H)$*:*
$\widetilde{S}^X_{\alpha^*_Y} = 0$ *for* $X = PWU$*,* PTU*,* PWE*,* PTE *and* PT*.*

Lemma 6 *The value of the Lagrangian multiplier:*
$\nu \geq 0$ *for* $X = PWU$*,* PTU*,* PWE*,* PTE *and* PT*.*

Combining Lemma 6 with Lemma 1 (a), we see how the different criteria perform from the EWEP perspective: for PWU, PTU, PWE, PTE and PT, the constraint $x_H \geq x_L$ can be binding, in which case $x_H = x_L$, $\alpha^*_H = \alpha^*_L$, and their solution satisfies EWEP.

Continuing further as in Section 5.2, it is straightforward to show that the optimal tax rates have the same structure under the PWU, PTU, PWE, PTE and PT social objective functions in the sense that the multiplier ν pushes the tax system away from the EITC. It is also easy to show under Assumption 1 that for these social objective functions, $(T_L - T_u)/x_L < (T_H - T_u)/x_H$, and the labor supply of the high-skilled is more distorted downwards than the labor supply of the low-skilled.

6 Conclusion

This paper has studied optimal tax policies when agents differ in terms of skills and tastes for labor. We assumed quasilinear utility and that the labor supply decision is at the extensive margin. The optimal tax policies under distinct objective functions were derived, in full and asymmetric information.

The determination of appealing social criteria is important if one looks for social preferences applicable in public economics, in particular when dealing with redistribution. When agents differ in terms of skills and tastes for labor, the equality of opportunity approach is inspiring (Fleurbaey, 1995a) and broadly accepted (Alesina and Angeletos, 2005).

This paper has shown that many criteria in the optimal tax literature (Utilitarianism, Welfarism, Boadway *et al.*, Van de gaer and Paternalistic Utilitarian criteria) fail the requirements of equality of opportunity; i.e., the compensation (EWEP) and responsibility (ETES) principles. It has been shown that in the first best, criteria respecting one of these principles are Roemer's, the Conditional Equality and the Egalitarian Equivalent criterion, the latter two advocated by Fleurbaey (1995b). We also showed that in the second best, these criteria might satisfy EWEP, while the standard criteria from the optimal tax literature never satisfy it.

Our simple optimal taxation exercise illustrates the discrepancy between standard Welfarist approaches and methods proposed in the social choice literature. The standard Welfarist approach equips agents with comparable indices of their well-being and applies an aggregation rule to these indices. The equality of opportunity approach makes the distinction between personal characteristics that are under and beyond individuals' control and constructs (typically, axiomatically) criteria that reach compensation and/or responsibility under full information. Although motivated by a concern for equality of opportunity, Boadway *et al.* and Paternalistic Utilitarian criteria were not axiomatically derived, while the Van de gaer criterion was axiomatically derived from equalization of opportunity sets. Therefore, not surprisingly, they never satisfy EWEP or ETES, even under full information.

In this paper, we have explored two ways to deal with the equality of opportunity principles in the second-best model. One is to search for optimal policies over the allocations that satisfy one of the principles. The other is to weaken the full equality demanded in the equality of opportunity principles and to replace them by priority principles, as advocated in social choice (Fleurbaey, 2008). We therefore build up new criteria, one satisfying an ETES-priority principle and several others satisfying EWEP-priority principles leading to generalizations of Roemer's criterion and the egalitarian equivalent allocation. They have similar properties to the other equality of opportunity principles but allow the researcher to express different kinds and extents of inequality aversion. Throughout, we find that the equality of opportunity approach tends to work against an Earned Income Tax Credit and in favor of a Negative Income Tax.

Appendix A: Proofs of Section 5.1.

PROOF OF LEMMA 1(a).

Suppose the proposition does not hold true. By (13), we then have that $\alpha_H^* > \alpha_L^*$. Hence, there exist α, $\alpha_L^* < \alpha < \alpha_H^*$ for which high-skilled workers get utility $v(x_H) - \alpha$ and low-skilled workers get $v(x^u)$. Because the former depends on α but the latter does not, these two can never be equal for all α, $\alpha_L^* < \alpha < \alpha_H^*$, and so EWEP must be violated.

PROOF OF SOCIAL OBJECTIVE FUNCTIONS IN SECOND BEST.

Parts (a), (b), (c) and (d) are straightforward to prove.

To see part (e), observe that (11) (because of incentive constraints) implies that for all α, $v(x_L) - \alpha \leq v(x_H) - \alpha$. Therefore, Roemer's objective function:

$$\int_0^\infty \min\{\underset{\delta_L(\alpha)}{\text{oper}}\{v(x_L) - \alpha, v(x^u)\}, \underset{\delta_H(\alpha)}{\text{oper}}\{v(x_H) - \alpha, v(x^u)\}\} dF(\alpha),$$

becomes:

$$\int_0^{\alpha_L^*} (v(x_L) - \alpha)\, dF(\alpha) + \int_{\alpha_L^*}^\infty v(x^u)\, dF(\alpha). \tag{24}$$

To see part (f), note that, in the second best, Van de gaer's objective function is:

$$\min\left\{\int_0^\infty \underset{\delta_L(\alpha)}{\text{oper}}\{v(x_L) - \alpha, v(x^u)\} dF(\alpha), \int_0^\infty \underset{\delta_H(\alpha)}{\text{oper}}\{v(x_H) - \alpha, v(x^u)\} dF(\alpha)\right\}.$$

Because of the incentive constraints, this reduces to (24).

To see part (g), observe that, because the policy can no longer depend on α, (8) reduces to:

$$\widetilde{S}^{CE} = \min\{v(x_L) - \widetilde{\alpha}, v(x^u), v(x_H) - \widetilde{\alpha}\}.$$

However, because (11) holds true, $v(x_L) - \widetilde{\alpha}$ is always lower than $v(x_H) - \widetilde{\alpha}$; the low-skilled will always be the worst-off and:

$$\widetilde{S}^{CE} = \min\{v(x_L) - \widetilde{\alpha}, v(x^u)\}. \tag{25}$$

Consider maximization of $v(x_L) - \widetilde{\alpha}$ subject to the relevant constraints. This amounts to maximization of $v(x_L)$, which determines the critical value α_L^*. Two possibilities need to be considered. Either $\widetilde{\alpha} \geq \alpha_L^*$, such that $v(x_L) - \widetilde{\alpha} \leq v(x_L) - \alpha_L^* = v(x^u)$, and the policy just described is the CE policy; or, $\widetilde{\alpha} < \alpha_L^*$, such that $v(x_L) - \widetilde{\alpha} > v(x_L) - \alpha_L^* = v(x^u)$, and the CE policy is found by maximizing $v(x^u)$. The first case occurs for high values of $\widetilde{\alpha}$, the second for low values.

To see part (h), note that the equivalent wages for the employed are equal to x_Y ($Y = H$ or L) and for the inactive $v^{-1}(v(x^u) + \alpha)$. The objective is to maximize the lowest equivalent wage. Consider the inactive. Because $v^{-1}(.)$ is an increasing function, the equivalent wage is lowest for those inactive having the lowest value for α, which are those with $\alpha = \alpha_L^*$. Hence the lowest value for the equivalent wage is $v^{-1}(v(x^u) + \alpha_L^*) = v^{-1}(v(x_L)) = x_L$.

Appendix B: Proofs of Section 5.2.

PROOF OF THEOREM 1.

The Lagrangian function for the general second-best problem is:

$$\begin{aligned}
&\pounds\left(x_L, x_H, x^u, \alpha_L^*, \alpha_H^*, \lambda, \mu_L, \mu_H, \nu, c\right) = \widetilde{S}^X + \nu\left(x_H - x_L - c\right) \\
&+\lambda\left\{\gamma\left(w_L - x_L\right) F\left(\alpha_L^*\right) - \gamma x^u\left(1 - F\left(\alpha_L^*\right)\right)\right. \\
&\left.+\left(1-\gamma\right)\left(w_H - x_H\right) F\left(\alpha_H^*\right) - \left(1-\gamma\right) x^u\left(1 - F\left(\alpha_H^*\right)\right) - R\right\} \\
&+\mu_L\left[v\left(x_L\right) - \alpha_L^* - v\left(x^u\right)\right] + \mu_H\left[v\left(x_H\right) - \alpha_H^* - v\left(x^u\right)\right],
\end{aligned}$$

which has to be maximized with respect to x_L, x_H, x^u, α_L^*, α_H^* and c, taking into account that $c \geq 0$. This leads to the following first-order conditions:

$$\widetilde{S}^X_{x_L} - \lambda\gamma F\left(\alpha_L^*\right) - \nu = -\mu_L v'(x_L), \tag{26}$$

$$\widetilde{S}^X_{x^u} - \lambda\left[\gamma(1 - F(\alpha_L^*)) - (1-\gamma)(1 - F(\alpha_H^*))\right] = (\mu_L + \mu_H)v'(x^u), \tag{27}$$

$$\widetilde{S}^X_{x_H} - \lambda(1-\gamma)F(\alpha_H^*) + \nu = -\mu_H v'(x_H), \tag{28}$$

$$\widetilde{S}^X_{\alpha_L^*} + \lambda\gamma f(\alpha_L^*)(w_L - x_L + x^u) = \mu_L, \tag{29}$$

$$\widetilde{S}^X_{\alpha_H^*} + \lambda\left(1-\gamma\right) f\left(\alpha_H^*\right)\left(w_H - x_H + x^u\right) = \mu_H, \tag{30}$$

$$-\nu \leq 0 \text{ and } \nu c = 0. \tag{31}$$

Solving (26) for μ_L, equating the resulting expression to the left-hand side of (29) and using definition (14), we can write:

$$\frac{w_L - x_L + x^u}{x_L} = \frac{1}{\eta\left(x_L, \alpha_L^*\right)}\left[1 - \frac{\widetilde{S}^X_{x_L} - \nu}{\lambda\gamma F\left(\alpha_L^*\right)}\right] - \frac{\widetilde{S}^X_{\alpha_L^*}}{\lambda\gamma f\left(\alpha_L^*\right) x_L},$$

which is the first equation of Theorem 1.

Similarly, solving (28) for μ_H, equating the resulting expression to the left-hand side of (30) and using definition (15), we get the second equation of Theorem 1:

$$\frac{w_H - x_H + x^u}{x_H} = \frac{1}{\eta\left(x_H, \alpha_H^*\right)}\left[1 - \frac{\widetilde{S}^X_{x_H} + \nu}{\lambda\left(1-\gamma\right) F\left(\alpha_H^*\right)}\right] - \frac{\widetilde{S}^X_{\alpha_H^*}}{\lambda\left(1-\gamma\right) f\left(\alpha_H^*\right) x_H}.$$

Divide Equations (26)–(28) by the marginal utility on their right-hand sides, adding the resulting equation for (26) and (28) and equating the result to (27) yields:

$$\frac{\lambda\gamma F(\alpha_L^*)}{v'(x_L)} - \frac{\widetilde{S}^X_{x_L}}{v'(x_L)} + \frac{\nu}{v'(x_L)} + \frac{\lambda(1-\gamma)F(\alpha_H^*)}{v'(x_H)} - \frac{\widetilde{S}^X_{x_H}}{v'(x_H)} - \frac{\nu}{v'(x_H)}$$
$$= \frac{\widetilde{S}^X_{x^u}}{v'(x^u)} - \frac{\lambda[\gamma(1-F(\alpha_L^*)) + (1-\gamma)(1-F(\alpha_H^*))]}{v'(x^u)}.$$

Collecting the terms in λ gives:

$$\lambda\left[\frac{\gamma F(\alpha_L^*)}{v'(x_L)} + \frac{(1-\gamma)F(\alpha_H^*)}{v'(x_H)} + \frac{[\gamma(1-F(\alpha_L^*)) + (1-\gamma)(1-F(\alpha_H^*))]}{v'(x^u)}\right]$$
$$= \frac{\widetilde{S}^X_{x_L}}{v'(x_L)} + \frac{\widetilde{S}^X_{x_H}}{v'(x_H)} + \frac{\widetilde{S}^X_{x^u}}{v'(x^u)} + \nu\left[\frac{1}{v'(x_H)} - \frac{1}{v'(x_L)}\right].$$

Now, note that from (31), if $\nu > 0$, then $c = 0$, such that $x_H = x_L$ and the last term in the above equation always drops out. Using definitions (16) and (17) gives $\lambda g_P^X = D^X$, and thus $\lambda^{-1} = g_P^X/D^X$, which is the third equation of Theorem 1.

PROOF OF LEMMA 3.

Step 1: we prove the following lemma.

Lemma B: If, evaluated at $x_H = x_L$ and $\alpha_H^* = \alpha_L^*$, $\frac{\widetilde{S}^X_{x_H}+\widetilde{S}^X_{\alpha_H^*}v'(x)}{1-\gamma} = \frac{\widetilde{S}^X_{x_L}+\widetilde{S}^X_{\alpha_L^*}v'(x)}{\gamma}$, then $\nu = 0$.

The proof relies on the necessary conditions that we just derived. Using the necessary condition (30) in (28) and solving for ν, we obtain:

$$\nu = -\widetilde{S}^X_{x_H} - \widetilde{S}^X_{\alpha_H^*}v'(x_H) + \lambda(1-\gamma)[F(\alpha_H^*) - f(\alpha_H^*)(w_H - x_H + x^u)v'(x_H)].$$

Hence, $\nu > 0$ (such that $x_H = x_L = x$ and $\alpha_H^* = \alpha_L^* = \alpha^*$) if and only if:

$$F(\alpha^*) - (w_H - x + x^u)f(\alpha^*)v'(x) > \frac{\widetilde{S}^X_{x_H} + \widetilde{S}^X_{\alpha_H^*}v'(x)}{\lambda(1-\gamma)}. \tag{32}$$

Similarly, using (29) in (26) and solving for ν, we have:

$$\nu = \widetilde{S}^X_{x_L} + \widetilde{S}^X_{\alpha_L^*}v'(x_L) - \lambda\gamma[F(\alpha_L^*) - f(\alpha_L^*)(w_L - x_L + x^u)v'(x_L)],$$

and we find that $\nu > 0$ if and only if:

$$F(\alpha^*) - [w_L - x + x^u]f(\alpha^*)v'(x) < \frac{\widetilde{S}^X_{x_L} + \widetilde{S}^X_{\alpha_L^*}v'(x)}{\lambda\gamma}. \tag{33}$$

If the antecedent of Lemma B holds true, the right-hand sides of (32) and (33) are equal, such that $\nu > 0$ requires:

$$F(\alpha^*) - (w_H - x + x^u)f(\alpha^*)v'(x) > F(\alpha^*) - [w_L - x + x^u]f(\alpha^*)v'(x),$$

but this can only hold true if $w_H < w_L$, which goes against the model's assumptions.

Step 2: we compute the expressions that occur in Lemma B. They are given in the following table.

Table 7: Partial derivative of the social criteria w.r. to x_Y and α_Y^* $(Y = L, H)$

X	$\frac{\widetilde{S}_{x_L}^X}{\gamma}$	$\frac{\widetilde{S}_{x_H}^X}{1-\gamma}$	$\frac{\widetilde{S}_{\alpha_L^*}^X}{\gamma}$	$\frac{\widetilde{S}_{\alpha_H^*}^X}{1-\gamma}$
U	$v'(x) F(\alpha^*)$		0	
W	$v'(x) \int_0^{\alpha^*} \Psi'(v(x) - \alpha) dF(\alpha)$		0	
B	$v'(x) \int_0^{\alpha^*} W(\alpha) dF(\alpha)$		0	
$R = V$	$v'(x) F(\alpha^*)$	0	0	
EE	$1/\gamma$	0	0	
CE_1	0		0	
CE_2	$v'(x)$	0	0	
P	$v'(x) F(\alpha^*)$		$[\alpha^* - \overline{\alpha}] f(\alpha^*)$	

Clearly, for $X = U, W, B, P$ and CE_1, by lemma B, $\nu = 0$.

PROOF OF COROLLARY 3.

Welfarist optimum.

Because $x_H > x_L$, $v(x_L) - \alpha < v(x_H) - \alpha$, and because $\Psi'' < 0$, $\Psi'(v(x_L) - \alpha_1) > \Psi'(v(x_H) - \alpha_1) > \Psi'(v(x_H) - \alpha_2)$ when $\alpha_2 > \alpha_1$, such that $g_H^W < g_L^W$. Combined with $\eta(x_L, \alpha_L^*) \geq \eta(x_H, \alpha_H^*)$, it follows from the expressions in Theorem 1 that $(T_L - T_u)/x_L < (T_H - T_u)/x_H$.

Boadway *et al.* optimum.

Note that:

$$\frac{\int_0^{\alpha_L^*} W(\alpha) dF(\alpha)}{F(\alpha_L^*)} \geq (\leq) \frac{\int_0^{\alpha_H^*} W(\alpha) dF(\alpha)}{F(\alpha_H^*)} \Leftrightarrow$$

$$\frac{\int_0^{\alpha_L^*} W(\alpha) dF(\alpha)}{F(\alpha_L^*)} \geq (\leq) \frac{\int_0^{\alpha_L^*} W(\alpha) dF(\alpha)}{F(\alpha_L^*)} \frac{F(\alpha_L^*)}{F(\alpha_H^*)} + \frac{\int_{\alpha_L^*}^{\alpha_H^*} W(\alpha) dF(\alpha)}{F(\alpha_H^*)} \Leftrightarrow$$

$$\frac{\int_0^{\alpha_L^*} W(\alpha) dF(\alpha)}{F(\alpha_L^*)} \geq (\leq) \frac{\int_{\alpha_L^*}^{\alpha_H^*} W(\alpha) dF(\alpha)}{F(\alpha_H^*) - F(\alpha_L^*)},$$

which holds as $\geq$ automatically if $W(\alpha)$ is a decreasing function, and as $\leq$ if $W(\alpha)$ is an increasing function.

Therefore, assume that $W(\alpha)$ is a decreasing function; hence, $g_L^B > g_H^B$. Because $x_H > x_L$, such that $v'(x_H) < v'(x_L)$, and the assumption that $\eta(x_L, \alpha_L^*) \geq \eta(x_H, \alpha_H^*)$, it follows from the expressions in Theorem 1 that $(T_L - T_u)/x_L < (T_H - T_u)/x_H$.

Roemer, EE and CE$_2$.

There are two cases to consider.

(i) When $\nu = 0$, the proof is straightforward from Table 4 in Corollary 1, $\eta(x_L, \alpha_L^*) \geq \eta(x_H, \alpha_H^*)$ and $g_L^X > 0$.

(ii) When $\nu > 0$, $x_H = x_L = x$; hence, $T_Y = w_Y - x$ $(Y = L, H)$ and $\eta(x_L, \alpha_L^*) = \eta(x_L, \alpha_H^*)$ (using (14)–(15)), which, combined with $w_H > w_L$, yields the inequality $(T_L - T_u)/x_L < (T_H - T_u)/x_H$ (where $x_H = x_L$).

PROOF OF COROLLARY 4.

By definition, $\frac{T_L - T_u}{x_L} < \frac{T_H - T_u}{x_H} \Leftrightarrow \frac{w_L - x_L + x^u}{x_L} < \frac{w_H - x_H + x^u}{x_H}$. Therefore, under Assumption 1, from Corollary 4, we have that for the planners considered in the corollary, $x_H (w_L - x_L + x^u) < x_L (w_H - x_H + x^u)$. Because $x_H \geq x_L$ (from (11)), we have: $w_L - x_L + x^u < w_H - x_H + x^u$.

Appendix C: Analysis of the Modified Boadway *et al.* and Modified Paternalistic criteria under asymmetric information.

Modified Boadway *et al.* criterion

Formally, under asymmetric information, the Modified Boadway *el al.* criterion is defined by:

$$\widetilde{S}^{MB} = \gamma \int_0^{\alpha_L^*} [v(x_L) - W(\alpha)] \, dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} W(\alpha) \, v(x^u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\alpha_H^*} W(\alpha) \, [v(x_H) - W(\alpha)] \, dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} W(\alpha) \, v(x^u) dF(\alpha).$$

Applying Theorem 1, it is easy to verify that the second-best optimal tax formulas are:

$$\frac{T_L^{MB} - T_u^{MB}}{x_L^{MB}} = \frac{1}{\eta\left(x_L^{MB}, \alpha_L^{MB*}\right)} \left[1 - g_L^{MB}\right] - \frac{\left[\alpha_L^{MB*} - W\left(\alpha_L^{MB*}\right)\right]}{\left(\lambda^{MB} x_L^{MB}\right)},$$
$$\frac{T_H^{MB} - T_u^{MB}}{x_H^{MB}} = \frac{1}{\eta\left(x_H^{MB}, \alpha_H^{MB*}\right)} \left[1 - g_H^{MB}\right] - \frac{\left[\alpha_H^{MB*} - W\left(\alpha_H^{MB*}\right)\right]}{\left(\lambda^{MB} x_H^{MB}\right)}.$$

The extra term $-\left[\alpha_Y^* - W(\alpha_Y^*)\right] / \left(\lambda^{MB} x_Y^{MB}\right)$ $(Y = L, H)$ captures the social value of the divergence between private and social preferences, just like under Paternalistic Utilitarianism.

Modified Paternalistic Utilitarian criterion

Formally, under asymmetric information, the Modified Paternalistic criterion equals:

$$\widetilde{S}^{MP} = \gamma \int_0^{\alpha_L^*} \left[v(x_L) - \min\left\{\alpha, \overline{\overline{\alpha}}\right\}\right] dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} v(x^u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\alpha_H^*} \left[v(x_H) - \min\left\{\alpha, \overline{\overline{\alpha}}\right\}\right] dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} v(x^u) dF(\alpha).$$

To find the optimum, consider the solution to $\widetilde{S}^U$. This solution determines values α_L^{U*} and α_H^{U*}. If both of these values are smaller than $\overline{\overline{\alpha}}$, then this solution coincides with the solution to $\widetilde{S}^{MP}$. Remember that $\alpha_H^{U*} > \alpha_L^{U*}$, such that for lower values of $\overline{\overline{\alpha}}$, at some point $\overline{\overline{\alpha}} < \alpha_H^{U*}$. Hence, for intermediate values of $\overline{\overline{\alpha}}$, the solution for the Modified Paternalistic criterion can be found as the maximum to:

$$\gamma \int_0^{\alpha_L^*} [v(x_L) - \alpha] dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} v(x^u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\overline{\overline{\alpha}}} [v(x_H) - \alpha] \, dF(\alpha) + (1-\gamma) \int_{\overline{\overline{\alpha}}}^{\alpha_H^*} \left[v(x_H) - \overline{\overline{\alpha}}\right] dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} v(x^u) dF(\alpha).$$

To find the optimal tax rates, one can use the formula from Theorem 1 to verify that a paternalistic term equal to $-\left[\alpha_H^{MP*} - \overline{\overline{\alpha}}\right] // \left(\lambda^{MP} x_H^{MP}\right)$ enters the optimal tax rate for the high-skilled. The solution to the maximization problem also determines critical values for α_L^* and α_H^*. Decreasing the value for $\overline{\overline{\alpha}}$ further, $\overline{\overline{\alpha}}$ becomes smaller than the critical value for α_L^*, such that for low values of $\overline{\overline{\alpha}}$, the solution for the Modified Paternalistic Utilitarian criterion can be found as the maximum to:

$$\gamma \int_0^{\overline{\overline{\alpha}}} [v(x_L) - \alpha] dF(\alpha) + \gamma \int_{\overline{\overline{\alpha}}}^{\alpha_L^*} [v(x_L) - \overline{\overline{\alpha}}] dF(\alpha) + \gamma \int_{\alpha_L^*}^{\infty} v(x^u) dF(\alpha)$$
$$+(1-\gamma) \int_0^{\overline{\overline{\alpha}}} [v(x_H) - \alpha]\, dF(\alpha) + (1-\gamma) \int_{\overline{\overline{\alpha}}}^{\alpha_H^*} \left[v(x_H) - \overline{\overline{\alpha}}\right] dF(\alpha) + (1-\gamma) \int_{\alpha_H^*}^{\infty} v(x^u) dF(\alpha).$$

To find the optimal tax rates, once more one can use the formula from Theorem 1. It is then easy to verify that a paternalistic term equal to $-\left[\alpha_Y^{MP*} - \overline{\overline{\alpha}}\right] // \left(\lambda^{MP} x_Y^{MP}\right)$ enters the optimal tax rate for skill level Y ($Y = L, H$).

Appendix D: Proof of Theorem 2 (Section 5.3)

Substituting the ETES constraints $w_L - x_L = -x^u$ and $w_H - x_H = -x^u$ into the government budget constraint and rearranging gives the first expression in the lemma. The second and third expressions follow from (10), (12) and the definitions of the critical values a_L^* and α_H^*.

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Supplement to "A comparison of optimal tax policies when compensation or responsibility matter"

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Abstract

In full information, Section 1 derives the optimal allocations under the Welfarist, Utilitarian, Boadway *et al.*, Roemer and Van de gaer criteria as well as the Full Equality of Opportunity (FEO), Conditional Equality (CE) and Egalitarian Equivalent (EE) allocations. Section 2 proves footnote 15. Section 3 derives the optimal EWEP-restricted general second best problem (Theorem 3). Section 4 gives the proofs behind the optimal tax schedules under new normative criteria which satisfy priority versions of the compensation and responsibility principles (Section 5.4).

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1. Welfarist, Utilitarian, Boadway et al., Roemer, Van de gaer, FEO, CE and EE optimal allocations in first best: Proofs

The Lagrangian functions for each of the social objective functions are formed by combining the expressions for the social objective functions given in Section 3.2 and the government budget constraint (1) with its associated Lagrangian multiplier λ. We drop the superscripts W, U, B, P, R, V, FEO, CE and EE corresponding to the respective normative criterion for notational simplicity.

Welfarist and Utilitarian planners

We discuss the Welfarist case first, and show how the properties of the Utilitarian case follow. The first-order conditions of the constrained optimization problem with respect to the four consumption functions are:

$$\begin{aligned}
\delta_L(\alpha)\left[\Psi'(v(x_L^w(\alpha))-\alpha)v'(x_L^w(\alpha))-\lambda\right] &= 0,\\
(1-\delta_L(\alpha))\left[\Psi'(v(x_L^u(\alpha)))v'(x_L^u(\alpha))-\lambda\right] &= 0,\\
\delta_H(\alpha)\left[\Psi'(v(x_H^w(\alpha))-\alpha)v'(x_H^w(\alpha))-\lambda\right] &= 0,\\
(1-\delta_H(\alpha))\left[\Psi'(v(x_H^u(\alpha)))v'(x_H^u(\alpha))-\lambda\right] &= 0.
\end{aligned}$$

Since $\delta_L(\alpha)$ and $\delta_H(\alpha)$ are equal to 1 or 0, for each value of α, only two of these first-order conditions matter; for those that matter the corresponding social marginal utilities of consumption have to be equal, for the other two the consumption function does not matter (as nobody with this value for α is receiving it). So we get for all those that do not work:

$$\Psi'(v(x_L^u(\alpha)))v'(x_L^u(\alpha)) = \lambda = \Psi'(v(x_H^u(\alpha)))v'(x_H^u(\alpha)) . \quad (34)$$

Due to the strict concavity of $\Psi(\cdot)$ and $v(\cdot)$, this can only hold true if

$$\overline{x}^u = x_L^u(\alpha) = x_H^u(\alpha).$$

For those that work, we get

$$\Psi'(v(x_L^w(\alpha)) - \alpha)v'(x_L^w(\alpha)) = \lambda = \Psi'(v(x_H^w(\alpha)) - \alpha)v'(x_H^w(\alpha)). \tag{35}$$

For a given value for α, the requirement is exactly the same for w_L- and w_H-workers. Hence, for a given value of α, both get the same consumption bundle and so, for all α :

$$x_L^w(\alpha) = x_H^w(\alpha). \tag{36}$$

Hence worker's consumption bundles depend on α. Moreover, from the implicit function theorem:

$$\frac{\partial x_L^w(\alpha)}{\partial \alpha} = \frac{\Psi''(v(x_L^w(\alpha)) - \alpha)v'(x_L^w(\alpha))}{\Psi''(v(x_L^w(\alpha)) - \alpha)\left[v'(x_L^w(\alpha))\right]^2 + \Psi'(v(x_L^w(\alpha)) - \alpha)v''(x_L^w(\alpha))} > 0. \tag{37}$$

Therefore, for $\alpha_1 < \alpha_2$, due to the concavity of $v(.)$ we have:

$$v'(x_L^w(\alpha_1)) > v'(x_L^w(\alpha_2)).$$

Combining the last inequality with (35) requires that $\Psi'(v(x_L^w(\alpha_1)) - \alpha_1) < \Psi'(v(x_L^w(\alpha_2)) - \alpha_2)$. Since Ψ is strictly concave, this requires that

$$v(x_L^w(\alpha_1)) - \alpha_1 > v(x_L^w(\alpha_2)) - \alpha_2,$$

and so low-skilled workers with a higher disutility of labor are not fully compensated for this higher disutility. Due to (36), the same holds for high-skilled workers. Note that from (35) with $\alpha = 0$ and (34) we get that

$$x_L^w(0) = x_H^w(0) = \overline{x}_u.$$

The last equation and (36) imply that the optimal consumption varies with α and activity (employed or inactive), as summarized in Table 1.

The government budget constraint depends only on the number of high- and low-skilled that work, not on which high- and low-skilled. From (37), workers' consumption is increasing in their disutility of work, and so it is cheapest and hence optimal for the government to make those work with the lowest α. In view of (36), putting high-skilled and low-skilled at work is equally expensive for the government, but since high-skilled contribute more to the budget than low-skilled, more high-skilled than low-skilled will have to work. Hence, there exist critical values for α_L^* and α_H^* such that

$$\delta_L(\alpha) = 1 \text{ for all } \alpha \leq \alpha_L^*, \delta_H(\alpha) = 1 \text{ for all } \alpha \leq \alpha_H^* \text{ and } \alpha_L^* < \alpha_H^*. \tag{38}$$

Therefore, the optimal activity assignment depends on α and on skill level ($Y = L$ or H), as stated in Table 1.

The Welfarist criterion reduces to the Utilitarian one when $\Psi(y) \overset{def}{\equiv} y$ hence $\Psi'(.) = 1$. Therefore, under the Utilitarian criterion, (34)-(35) yield that the first-order conditions with respect to consumption reduce to $(\forall \alpha)$ (since λ is a constant):

$$\begin{aligned} & v'(x_L^{wU}(\alpha)) = v'(x_L^{uU}(\alpha)) = v'(x_H^{wU}(\alpha)) = v'(x_H^{uU}(\alpha)) = \lambda \\ \Longleftrightarrow\ & \overline{x} = x_L^{wU}(\alpha) = x_L^{uU}(\alpha) = x_H^{wU}(\alpha) = x_H^{uU}(\alpha). \end{aligned} \tag{39}$$

This is summarized in Table 1 when we state that optimal consumption is independent of α, skill level and activity. Since all individuals get the same consumption bundle, it follows from the reasoning leading to (38) that $\alpha_L^{U*} < \alpha_H^{U*}$, hence optimal activity depends on skill and α levels (as stated in Table 1).

Boadway *et al.* planner

The first-order conditions with respect to consumption functions (assuming an interior solution) are:

$$\int_0^\infty \delta_L(\alpha)\left[W(\alpha)\, v'(x_L^w(\alpha)) - \lambda\right] dF(\alpha) = 0,$$
$$\int_0^\infty (1-\delta_L(\alpha))\left[W(\alpha)\, v'(x_L^u(\alpha)) - \lambda\right] dF(\alpha) = 0,$$
$$\int_0^\infty \delta_H(\alpha)\left[W(\alpha)\, v'(x_H^w(\alpha)) - \lambda\right] dF(\alpha) = 0,$$
$$\int_0^\infty (1-\delta_H(\alpha))\left[W(\alpha)\, v'(x_H^u(\alpha)) - \lambda\right] dF(\alpha) = 0.$$

Consequently, we get

$$v'(x_L^w(\alpha)) = v'(x_L^u(\alpha)) = v'(x_H^w(\alpha)) = v'(x_H^u(\alpha)) = \frac{\lambda}{W(\alpha)}$$
$$\Longleftrightarrow x(\alpha) = x_L^w(\alpha) = x_L^u(\alpha) = x_H^w(\alpha) = x_H^u(\alpha)\,. \tag{40}$$

Given α, it is equally costly to have high- and low-skilled at work, but since high-skilled workers contribute more to the government budget, the government always prefers to have more high- than low-skilled at work. From (40), consumption depends on taste for leisure. Application of the implicit function theorem to the equation $v'(x(\alpha)) = \frac{\lambda}{W(\alpha)}$ yields:

$$\frac{\partial x(\alpha)}{\partial \alpha} = -\frac{\lambda}{[W(\alpha)]^2}\frac{W'(\alpha)}{v''(x(\alpha))} \geq (\leq)\, 0 \text{ if } W'(.) \geq (\leq)\, 0.$$

Using (40) in government budget constraint (1) yields that the function $x(\alpha)$ must be such that

$$\int_0^\infty x(\alpha)\, dF(\alpha) = \gamma w_L n_L + (1-\gamma)\, w_H n_H - R.$$

For the government budget constraint it only matters how many high- and low-skilled people work, it does not matter which high- and low-skilled people work. Hence, differential treatment in job assignment between equally skilled people must be based on the objective function. Using (4), the value of the objective function is given by:

$$S^B = \int_0^\infty W(\alpha)\, v(x(\alpha))\, dF(\alpha) - \gamma \int_0^\infty W(\alpha)\, \delta_L(\alpha)\, \alpha dF(\alpha) - (1-\gamma) \int_0^\infty W(\alpha)\, \delta_H(\alpha)\, \alpha dF(\alpha)\,.$$

Whether people with high or low disutility of effort should be working depends on the last two terms of this expression. If $W(\alpha)\, a$ is increasing, having people with a high disutility working is not a good idea. From this it follows that, if the elasticity of the weight function $(\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)})$ is larger than -1, then it is optimal for the government not to employ people that have a high disutility of work. If this elasticity is smaller than -1, it will be optimal to employ people with a high disutility of work. Consequently, the functions $\delta_L(\alpha)$ and $\delta_H(\alpha)$ can have different shapes:

Case 1: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} > -1 : \delta_L(\alpha) = 1$ for all $\alpha \leq \alpha_L^*$, $\delta_H(\alpha) = 1$ for all $\alpha \leq \alpha_H^*$ and $\alpha_L^* < \alpha_H^*$,
Case 2: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} = -1$: see discussion below,
Case 3: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} < -1 : \delta_L(\alpha) = 1$ for all $\alpha \geq \alpha_L^{**}$, $\delta_H(\alpha) = 1$ for all $\alpha \geq \alpha_H^{**}$ and $\alpha_L^* > \alpha_H^*$.

Analyzing Case 2 in more detail, and defining $n_L \overset{\text{def}}{\equiv} \int_0^\infty \delta_L(\alpha)\, dF(\alpha)$ ($n_H \overset{\text{def}}{\equiv} \int_0^\infty \delta_H(\alpha)\, dF(\alpha)$) as the fraction of w_L-agents (w_H-agents) that are employed, the problem facing the planner with $W(\alpha)\alpha$ constant has the following Lagrangian:

$$\pounds(x(\alpha), n_L, n_H, \lambda) = \int_0^\infty W(\alpha)\, v(x(\alpha))\, dF(\alpha) - \gamma W(\alpha)\, \alpha n_L - (1-\gamma) W(\alpha)\, \alpha n_H$$
$$+\lambda\left[\gamma w_L n_L + (1-\gamma) w_H n_H - \int_0^\infty x(\alpha)\, dF(\alpha) - R\right],$$

which leads to the following:

$$\frac{\partial \pounds}{\partial x(\alpha)} = 0 \Leftrightarrow \lambda = \int_0^\infty W(\alpha)\, v'(x(\alpha))\, dF(\alpha),$$
$$\frac{\partial \pounds}{\partial n_L} = -\gamma W(\alpha)\, \alpha + \lambda \gamma w_L,$$
$$\frac{\partial \pounds}{\partial n_H} = -(1-\gamma) W(\alpha)\, \alpha + \lambda (1-\gamma) w_H.$$

Note that the second and third condition cannot hold simultaneously with equality:

$$\frac{\partial \pounds}{\partial n_L} \geq (\leq)\, 0 \Leftrightarrow [\lambda w_L - W(\alpha)\, \alpha] \geq (\leq)\, 0,$$
$$\frac{\partial \pounds}{\partial n_H} \geq (\leq)\, 0 \Leftrightarrow [\lambda w_H - W(\alpha)\, \alpha] \geq (\leq)\, 0.$$

Hence, since $w_H > w_L$, we always have that $\frac{\partial \pounds}{\partial n_L} \geq 0 \Rightarrow \frac{\partial \pounds}{\partial n_H} > 0$ and $\frac{\partial \pounds}{\partial n_H} \leq 0 \Rightarrow \frac{\partial \pounds}{\partial n_L} < 0$. We then get the following possibilities:

Case 2: $\frac{\partial W(\alpha)}{\partial \alpha}\frac{\alpha}{W(\alpha)} = -1$ (i.e. $W(\alpha)\alpha$ is constant):

$$\textit{Activity assignment}\begin{cases} \lambda^B = \int_0^\infty W(\alpha)\, v'\left(x^B(\alpha)\right) dF(\alpha), \\ w_H\lambda^B > w_L\lambda^B > W(\alpha)\,\alpha \Rightarrow n_H^B = n_L^B = 1. \\ w_H\lambda^B > w_L\lambda^B = W(\alpha)\,\alpha \Rightarrow n_H^B = 1, 0 < n_L^B < 1 \\ w_H\lambda^B > W(\alpha)\,\alpha > w_L\lambda^B \Rightarrow n_H^B = 1, n_L^B = 0. \\ w_H\lambda^B = W(\alpha)\,\alpha > w_L\lambda^B \Rightarrow 0 < n_H^B < 1, n_L^B = 0. \\ W(\alpha)\,\alpha > w_H\lambda^B > w_L\lambda^B \Rightarrow n_H^B = n_L^B = 0. \end{cases}$$

From Cases 1-3, we can conclude that the optimal activity depends on skill and α (see Table 1).

Paternalistic Utilitarian social planner

It is easy to see that we obtain the same first-order conditions as with the Utilitarian objective, and so the consumption functions are similar to (39): everybody receives the same level of consumption $\overline{x}$, which, because of the government budget constraint equals $\gamma w_L n_L + (1-\gamma) w_H n_H - R$. Consequently, using (5), the value of our Paternalistic Utilitarian objective function becomes

$$v(\gamma w_L n_L + (1-\gamma) w_H n_H) - \gamma \overline{\alpha} n_L - (1-\gamma)\overline{\alpha} n_H.$$

This expression only depends on the number of low- and high-skilled that are employed; the planner determines n_L and n_H so as to maximize this expression. The derivatives of this expression with respect to n_H and n_L are, respectively

$$(1-\gamma)\left[w_H v'(\overline{x})-\overline{\alpha}\right] \quad \text{and} \quad \gamma\left[w_L v'(\overline{x})-\overline{\alpha}\right].$$

Since $w_H > w_L$, we can distinguish the following cases:

$$\textit{Activity assignment:}\left\{\begin{array}{l} \lambda = v'(\overline{x}) \\ w_H\lambda > w_L\lambda > \overline{\alpha} \Rightarrow n_H = n_L = 1. \\ w_H\lambda > w_L\lambda = \overline{\alpha} \Rightarrow n_H = 1, 0 < n_L < 1. \\ w_H\lambda > \overline{\alpha} > w_L\lambda \Rightarrow n_H = 1, n_L = 0. \\ w_H\lambda = \overline{\alpha} > w_L\lambda \Rightarrow 0 < n_H < 1,\, n_L = 0. \\ \overline{\alpha} > w_H\lambda > w_L\lambda \Rightarrow n_H = n_L = 0. \end{array}\right.$$

Therefore, Table 1 states that the optimal activity assignment of each agent is based only on his skill.

Roemer planner

There is no point in allowing the two elements in the min operator of Roemer's objective function to be different in the first best. Hence there are in principle four possibilities:

(i) $\delta_L(\alpha) = \delta_H(\alpha) = 1 \Rightarrow x_L^w(\alpha) = x_H^w(\alpha)$,

(ii) $\delta_L(\alpha) = 0, \delta_H(\alpha) = 1 \Rightarrow v(x_L^u(\alpha)) = v(x_H^w(\alpha)) - \alpha \Rightarrow x_L^u(\alpha) < x_H^w(\alpha)$,

(iii) $\delta_L(\alpha) = \delta_H(\alpha) = 0 \Rightarrow x_L^u(\alpha) = x_H^u(\alpha)$,

(iv) $\delta_L(\alpha) = 1, \delta_H(\alpha) = 0 \Rightarrow v(x_L^w(\alpha)) - \alpha = v(x_H^u(\alpha)) \Rightarrow x_L^w(\alpha) > x_H^u(\alpha)$.

There is equivalence between the maximin approach and the revenue-maximizing approach. Maximizing tax revenue subject to a minimal utility level is equivalent to maximizing the minimum of utility subject to the revenue constraint. Here, the objective function maximizes the sum of the minimal utility levels but the logic is similar. The government maximizes tax revenue subject to minimal utility levels. Tax revenue will be larger when more people are working, in particular productive people. The minimal utility levels avoid that people with large α work. Therefore, if anyone, we would like the ones with low values for α to work, and since the high-skilled have a higher productivity, we want more highly skilled to work ($\alpha_H^* \geq \alpha_L^*$); for α increasing, we move from (i) over (ii) to (iii). If we plug this in, we get the following objective function:

$$\int_0^{\alpha_L^*} \min\left\{v(x_L^w(\alpha)) - \alpha, v(x_H^w(\alpha)) - \alpha\right\} dF(\alpha)$$
$$+\int_{\alpha_L^*}^{\alpha_H^*} \min\left\{v(x_L^u(\alpha)), v(x_H^w(\alpha)) - \alpha\right\} dF(\alpha) + \int_{\alpha_H^*}^{\infty} \min\left\{v(x_L^u(\alpha)), v(x_H^u(\alpha))\right\} dF(\alpha).$$

Maximizing this objective function implies

$$x_L^w(\alpha) = x_H^w(\alpha) \quad \forall\alpha \in [0, \alpha_L^*), \tag{41}$$
$$x_L^u(\alpha) = v^{-1}\left(v(x_H^w(\alpha)) - \alpha\right) \quad \forall\alpha \in [\alpha_L^*, \alpha_H^*), \tag{42}$$
$$x_L^u(\alpha) = x_H^u(\alpha) \quad \forall\alpha \in [\alpha_H^*, \infty). \tag{43}$$

Therefore, the objective function can be rewritten as

$$\int_0^{\alpha_L^*} \left(v(x_L^w(\alpha)) - \alpha\right) dF(\alpha) + \int_{\alpha_L^*}^{\infty} v(x_L^u(\alpha))\, dF(\alpha). \tag{44}$$

Government budget constraint (1) can be formulated as follows:

$$\gamma \left[\int_0^{\alpha_L^*} \left(w_L - x_L^w(\alpha) \right) dF(\alpha) - \int_{\alpha_L^*}^{\alpha_H^*} x_L^u(\alpha)\, dF(\alpha) \right] - \int_{\alpha_H^*}^{\infty} x_L^u(\alpha)\, dF(\alpha)$$
$$+(1-\gamma) \left[\int_0^{\alpha_L^*} \left(w_H - x_L^w(\alpha) \right) dF(\alpha) + \int_{\alpha_L^*}^{\alpha_H^*} \left(w_H - v^{-1}\left(v\left(x_L^u(\alpha) + \alpha \right) \right) \right) dF(\alpha) \right] \geq R.$$

Forming the Lagrangian with objective function (44), the previous government budget constraint and the Lagrangian multiplier λ, the first-order conditions with respect to $x_L^w(\alpha)$ and $x_L^u(\alpha)$ are:

$$\alpha \leq \alpha_L^* : v'\left(x_L^w(\alpha)\right) = \lambda,$$
$$\alpha_H^* < \alpha : v'(x_L^u(\alpha)) = \lambda,$$
$$\alpha_L^* < \alpha \leq \alpha_H^* : v'(x_L^u(\alpha)) = \lambda \left[\gamma + (1-\gamma) \frac{v'\left(x_L^u(\alpha)\right)}{v'\left(x_H^w(\alpha)\right)} \right].$$

From the first and second first-order conditions and from (41) and (43), we have (since λ is constant):

$$\forall \alpha \in [0, \alpha_L^*) \cup [\alpha_H^*, \infty) : x_L^w(\alpha) = x_H^w(\alpha) = x_L^u(\alpha) = x_H^u(\alpha) = \overline{x}.$$

For $\alpha_L^* < \alpha \leq \alpha_H^*$, from (42), it follows that $x_L^u(\alpha) < x_H^w(\alpha)$ and so $v'\left(x_L^u(\alpha)\right) > v'\left(x_H^w(\alpha)\right)$, such that $v'(x_L^u(\alpha)) > \lambda$ and

$$\forall \alpha \in [\alpha_L^*, \alpha_H^*) : x_L^u(\alpha) = v^{-1}\left(v\left(x_H^w(\alpha)\right) - \alpha \right) < \overline{x}.$$

To summarize, the optimal consumption and activity status depend on skill and α levels.

Van de gaer planner:

In the first best, there is no reason for having different values for opportunity sets of different skill-types. For the same reasons as usual, if anybody works, it will be those with a low disutility of work. Hence the objective function reduces to:

$$\int_0^{\alpha_L^*} \left[v(x_L^w(\alpha)) - \alpha \right] dF(\alpha) + \int_{\alpha_L^*}^{\infty} v\left(x_L^u(\alpha)\right) dF(\alpha). \tag{45}$$

This objective function must be maximized subject to two constraints. The first is that both opportunity sets must have the same value:

$$\int_0^{\alpha_L^*} \left[v(x_L^w(\alpha)) - \alpha \right] dF(\alpha) + \int_{\alpha_L^*}^{\infty} v\left(x_L^u(\alpha)\right) dF(\alpha)$$
$$= \int_0^{\alpha_H^*} \left[v\left(x_H^w(\alpha)\right) - \alpha \right] dF(\alpha) + \int_{\alpha_H^*}^{\infty} v\left(x_H^u(\alpha)\right) dF(\alpha). \tag{46}$$

The second is the budget constraint:

$$\gamma \left[\int_0^{\alpha_L^*} \left(w_L - x_L^w(\alpha) \right) dF(\alpha) - \int_{\alpha_L^*}^{\infty} x_L^u(\alpha)\, dF(\alpha) \right]$$
$$+(1-\gamma) \left[\int_0^{\alpha_H^*} \left(w_H - x_H^w(\alpha) \right) dF(\alpha) - \int_{\alpha_H^*}^{\infty} x_H^u(\alpha)\, dF(\alpha) \right] = R. \tag{47}$$

Forming the Lagrangian with objective function (45), the equality of opportunity set constraint (46) with the associated Lagrangian multiplier μ and government budget constraint (47) with its Lagrangian multiplier λ, the first-order conditions with respect to $x_L^w(\alpha)$, $x_L^u(\alpha)$, $x_H^w(\alpha)$ and $x_H^u(\alpha)$ are:

$$v'(x_L^w(\alpha))(1+\mu) = \lambda\gamma, \tag{48}$$

$$v'(x_L^u(\alpha))(1+\mu) = \lambda\gamma, \tag{49}$$

$$-\mu v'(x_H^w(\alpha)) = \lambda(1-\gamma), \tag{50}$$

$$-\mu v'(x_H^u(\alpha)) = \lambda(1-\gamma). \tag{51}$$

From (48)-(49) and (50)-(51) respectively, we have:

$$x_L^w(\alpha) = x_L^u(\alpha) = \overline{x} \text{ and } x_H^w(\alpha) = x_H^u(\alpha) = \overline{\overline{x}}.$$

Hence, the optimal consumption bundles depend only on skill level. Substituting these two equations into the equality of opportunity sets constraint (46) gives:

$$v(\overline{x}) - \int_0^{\alpha_L^*} \alpha dF(\alpha) = v(\overline{\overline{x}}) - \int_0^{\alpha_H^*} \alpha dF(\alpha).$$

If $\alpha_L^* = \alpha_H^*$, then $\overline{x} = \overline{\overline{x}}$. However, such a situation cannot be optimal, as high-skilled workers contribute more to the government budget than low-skilled workers. Therefore, $\alpha_L^* < \alpha_H^*$ (hence the optimal activity status depends on α and on the level of skill) which yields $\overline{x} < \overline{\overline{x}}$.

Lemma A: for an allocation that satisfies EWEP and ETES, there cannot exist an $\alpha \in \mathbb{R}^+ :$ $\delta_L(\alpha) \neq \delta_H(\alpha)$.

Proof. If such an α existed, we would have by EWEP that for this value either $v(x_L^u(\alpha)) = v(x_H^w(\alpha)) - \alpha$ or $v(x_L^w(\alpha)) - \alpha = v(x_H^u(\alpha))$, both of which are impossible since by ETES the consumption bundles cannot depend on α.

FEO planner

In view of lemma A, we have that for all $\alpha : \delta_L(\alpha) = \delta_H(\alpha)$. Suppose there exists an allocation satisfying EWEP and ETES in which some people work and others do not work. From ETES we know that all low-skilled in work have to get the same consumption bundle, which with some abuse of notation we denote as x_L^w. Similarly, all high-skilled in work get the same consumption bundle, denoted as x_H^w. In addition, by ETES, we need (i) $x_L^w - w_L = x_L^u$ and (ii) $x_H^w - w_H = x_H^u$. EWEP requires that $x_L^u = x_H^u$. Combining this with (i) and (ii) we get that $x_L^w = w_L - w_H + x_H^w$, which because EWEP requires $x_L^w = x_H^w$, reduces to $w_L = w_H$, which was excluded by assumption. Hence an allocation that satisfies EWEP and ETES cannot have some people working and others not working.

It is easy to verify that both axioms are satisfied by the following allocations:

(i) $n_H = n_L = 1$ *and* $x_L^w = x_H^w = \gamma w_L + (1-\gamma) w_H - R$.

(ii) $n_H = n_L = 0$ *and* $x^u = -R$.

The consumption bundles follow from government budget constraint (1). The optimal FEO policy is independent of individual characteristics.

CE planner

A first thing to note is that for the CE allocation to equalize $u\left(x_Y(\alpha), \delta_Y(\alpha), \widetilde{\alpha}\right)$ for all α and $Y = L, H$ requires that $u\left(x_Y(\alpha), \delta_Y(\alpha), \widetilde{\alpha}\right)$ is independent of w_Y. This has the following implications:

i) for all α such that $\delta_L(\alpha) = \delta_H(\alpha) = 1 \Rightarrow x_L^w(\alpha) = x_H^w(\alpha)$. In addition, all those assigned in a job have to get the same level of $u(.,.,\widetilde{\alpha})$, which implies that their consumption bundle cannot depend on α, and thus $x_L^w = x_L^w(\alpha) = x_H^w(\alpha) = x_H^w$;

ii) for all α such that $\delta_L(\alpha) = \delta_H(\alpha) = 0 \Rightarrow x_L^u(\alpha) = x_H^u(\alpha)$. In addition, all those that are inactive have to get the same level of $u(.,.,\widetilde{\alpha})$, implying that their consumption bundle cannot depend on α, such that $x_L^u(\alpha) = x_H^u(\alpha) = x^u$;

iii) for all α such that $\delta_L(\alpha) = 1$ and $\delta_H(\alpha) = 0 \Rightarrow x_L^w(\alpha) = v^{-1}\left(v\left(x_H^u(\alpha)\right) + \widetilde{\alpha}\right)$, which combined with case (i) and (ii) gives $x_L^w = v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)$;

iv) for all α such that $\delta_L(\alpha) = 0$ and $\delta_H(\alpha) = 1 \Rightarrow x_H^w(\alpha) = v^{-1}\left(v\left(x_L^u(\alpha)\right) + \widetilde{\alpha}\right)$, which combined with case 1 and 2 gives $x_H^w = v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)$.

Combining these results, we get

$$x_L^w = x_H^w = v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right). \tag{52}$$

Everybody gets the same level of utility $v\left(x^u\right)$ in the optimum, and so the problem of the first best allocation amounts to maximize the equal utility level $v\left(x^u\right)$ with respect to x^u, n_L and n_H subject to the budget constraint

$$\begin{aligned} R \leq\ & \gamma\left(w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right) n_L - \gamma x^u \left[1 - n_L\right] \\ & + (1-\gamma)\left(w_H - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right) n_H - (1-\gamma) x^u \left[1 - n_H\right]. \end{aligned}$$

The Lagrangian for this problem is

$$\begin{aligned} L = {} & v\left(x^u\right) + \lambda[\gamma\left(w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right) n_L - \gamma x^u \left[1 - n_L\right] \\ & + (1-\gamma)\left(w_H - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right) n_H - (1-\gamma) x^u \left[1 - n_H\right] - R]. \end{aligned}$$

Taking derivatives, we get :

$$\begin{aligned} \frac{\partial L}{\partial x^u} = {} & v'\left(x^u\right) - \lambda\gamma \frac{\partial v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)}{\partial x^u} n_L - \lambda(1-\gamma) \frac{\partial v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)}{\partial x^u} n_H \\ & - \lambda\gamma\left[1 - n_L\right] - \lambda(1-\gamma)\left[1 - n_H\right] = 0, \\ \frac{\partial L}{\partial n_L} = {} & \lambda\gamma\left[w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right] + \lambda\gamma x^u = \lambda\gamma\left[x^u + \left[w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right]\right], \\ \frac{\partial L}{\partial n_H} = {} & \lambda(1-\gamma)\left[w_H - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right] + \lambda(1-\gamma) x^u \\ = {} & \lambda(1-\gamma)\left[x^u + \left[w_H - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right]\right]. \end{aligned}$$

The two last first-order derivatives cannot possibly both be equal to zero at the same time:

$$\begin{aligned} & w_H > w_L \Rightarrow w_H - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right) > w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right) \\ & \Rightarrow x^u + \left[w_H - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right] > x^u + \left[w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right]. \end{aligned}$$

Hence we either have that

(i) $\frac{\partial L}{\partial n_L} > 0 \Rightarrow \frac{\partial L}{\partial n_H} > 0$, implying that $n_H = 1 = n_L$ and from (1): $x_L^w = x_H^w = \gamma w_L + (1-\gamma) w_H - R$,

(ii) $-x^u = \left[w_L - v^{-1}\left(v\left(x^u\right) + \widetilde{\alpha}\right)\right] = w_L - x_L^w$ from (52) and $\frac{\partial L}{\partial n_H} > 0$, implying $n_H = 1$ and n_L $(0 < n_L < 1)$ follows from (1),

(iii) $\frac{\partial L}{\partial n_H} > 0$ and $\frac{\partial L}{\partial n_L} < 0$, implying that $n_H = 1$ and $n_L = 0$ and from (1): $x^u = \left[(1-\gamma)\left(w_H - x_H^w\right) - R\right]/\gamma$,

(iv) $-x^u = \left[w_H - v^{-1}\left(v\left(x^u\right)+\widetilde{\alpha}\right)\right] = w_H - x_H^w$ from (52) and $\frac{\partial L}{\partial n_L} < 0$, implying $n_L = 0$ and n_H $(0 < n_H < 1)$ follows from (1) or

(v) $\frac{\partial L}{\partial n_H} < 0 \Rightarrow \frac{\partial L}{\partial n_L} < 0$, implying that $n_H = 0 = n_L$ and $x^u = -R$ from (1).

Which of these allocations yields the highest value for $v\left(x^u\right)$ depends on the parameters of the model. If $\widetilde{\alpha}$ is sufficiently low, the optimum will be case (i), as $\widetilde{\alpha}$ rises, we move from (i) to (ii), as it increases further we move to (iii) and (iv) and for values of $\widetilde{\alpha}$ sufficiently high, the optimum will be case (v). Table 1 states that the optimal consumption depends on activity and the optimal activity depends on skill, which summarizes these results.

EE planner

We want everybody to be indifferent between his actual resources (consumption and activity) and a reference resource bundle where he works and gets consumption $\widetilde{x}$. The best thing to do is to give all employed exactly this reference consumption bundle: $x_L^w = x_H^w = \widetilde{x}$. Clearly, to bring the equivalent wage of the inactive with a very high α down can lead to negative consumption levels. To prevent this, we impose that $x_Y^u(\alpha) \geq 0$. If this constraint is binding, these individuals get an equivalent wage larger than $\widetilde{x}$; we have to give up the ideal of equalizing equivalent incomes. The logical alternative then becomes Fleurbaey and Maniquet's maximin solution.

To get an equivalent wage of exactly $\widetilde{x}$, a person with taste parameter α needs an inactivity transfer equal to $v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right)$, which is independent of his skill level. Since we maximin the equivalent wages, the transfer for the inactive is $x^u(\alpha) = \min\left\{v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right), 0\right\}$. There exists a value for α, say $\widehat{\alpha}$, such that, if $\alpha \leq \widehat{\alpha}$ we have $x^u(\alpha) = v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right) \geq 0$, and if $\alpha > \widehat{\alpha}$, $x^u(\alpha) = 0$. In both cases, $x^u(\alpha) \leq \widetilde{x}$ such that it is cheaper to have people inactive than to have them working.

However, working people produce w_L or w_H, while inactive people produce nothing. As a consequence, it can never be optimal to have people inactive for which $\alpha \leq \widehat{\alpha}$: they cost $v^{-1}\left(v\left(\widetilde{x}\right)-\alpha\right) \geq 0$, but produce nothing. The best policy that maximizes S^{EE} under budget constraint is therefore $x_L^w = x_H^w = \gamma w_L + (1-\gamma) w_H - R$, $x^u = 0$ (hence the optimal consumption depends only on activity) and $\alpha_L^* = \alpha_H^* = v\left(\gamma w_L + (1-\gamma) w_H\right) - v(0)$ (hence the optimal activity status depends only on α).

2. Footnote 15 and $x_H^X > x_L^X$ for $X = U, W, B, P$ and CE_1: Proofs (Section 5.2)

Step 1: we proof the following lemma:

Lemma C: $x_H^X > x_L^X$ for $X = U, W, B, P$ and CE_1.

Proof. Under $X = U, W, B, P$ and CE_1, $\nu = 0$ from Lemma 3. Assume $x_H = x_L = x$ hence $\alpha_H^* = \alpha_L^* = \alpha^*$. Combining Equations (26) and (29) gives:

$$\frac{\widetilde{S}_{x_L}^X}{\gamma} = \lambda F\left(\alpha^*\right) - \frac{v'(x)}{\gamma}\left[\widetilde{S}_{\alpha_L^*}^X + \lambda\gamma f(\alpha^*)(w_L - x + x^u)\right].$$

Combining Equations (28) and (30) we can write:

$$\frac{\widetilde{S}_{x_H}^X}{1-\gamma} = \lambda F(\alpha^*) - \frac{v'(x)}{1-\gamma}\left[\widetilde{S}_{\alpha_H^*}^X + \lambda(1-\gamma) f\left(\alpha^*\right)\left(w_H - x + x^u\right)\right].$$

Using $\widetilde{S}^X_{x_L}/\gamma = \widetilde{S}^X_{x_H}/(1-\gamma)$, $\widetilde{S}^X_{\alpha^*_L}/\gamma = \widetilde{S}^X_{\alpha^*_H}/(1-\gamma)$ for $X = U, W, B, P$ and CE_1 from Table 7 in Appendix B, the two previous equations yield $\lambda f(\alpha^*)(w_L - x + x^u) = \lambda f(\alpha^*)(w_H - x + x^u)$ but this can only hold true if $w_H = w_L$, which leads to a contradiction. We can conclude that $x_H > x_L$.

From Lemma C, (10) and (12) we have $\alpha^*_H > \alpha^*_L$ under the U, W, B, P and CE_1 criteria.

Step 2: in second best, α^*_H, $\alpha^*_L < \infty$.

Proof. As $\forall \alpha : f(\alpha) > 0$, all low-ability (high-ability) people work means $\alpha^*_L \to \infty$ ($\alpha^*_H \to \infty$) at the optimum. Since consumption levels are finite, from (10) and (resp. (12)), α^*_L and α^*_H cannot tend to ∞.

Step 3: $\alpha^*_L > 0$ when $\nu = 0$.

Proof. Suppose $\alpha^*_L = 0$. From (10), evaluated at $\alpha^*_L = 0$, we have $x_L = x^u$. Since $\nu = 0$ and $F(0) = 0$, from first-order condition (26), $\mu_L = -\widetilde{S}^X_{x_L}/v'(x^u)$. The value $\alpha^*_L = 0$ can only be optimal if $\partial \pounds / \partial \alpha^*_L|_{\alpha^*_L = 0} \leq 0$, which requires, using the previous results

$$\lambda \gamma f(0) w_L \leq -\widetilde{S}^X_{\alpha^*_L} - \widetilde{S}^X_{x_L}/v'(x^u),$$

Going back to Table 7 in Appendix B, it is clear that for all the criteria the right-hand side is not positive, such that $\alpha^*_L = 0$ can only be optimal if $w_L \leq 0$, which, however, was excluded by assumption.

Step 4: to complete the proof, note that we have shown that, for the U, W, B, P and CE_1 criterion, $\nu = 0$, $x_H > x_L$ and thus $\alpha^*_H > \alpha^*_L$. For $X = R$, EE and CE_2, we have shown that $\nu \geq 0$, such that $\alpha^*_H \geq \alpha^*_L$.

3. Proof of Theorem 3 (Section 5.3)

The Lagrangian is

$$\begin{aligned}
&\pounds(x^w, x^u, \alpha^*, \lambda, \mu) = \widehat{S}^X \\
&+\lambda \left\{ \left[\gamma w_L + (1-\gamma) w_H - x^w \right] F(\alpha^*) - x^u (1 - F(\alpha^*)) - R \right\} \\
&+\mu \left[v(x^w) - \alpha^* - v(x^u) \right].
\end{aligned}$$

The first-order conditions are

$$\widehat{S}^X_{x^w} - \lambda F(\alpha^*) = -\mu v'(x^w), \tag{53}$$

$$\widehat{S}^X_{x^u} - \lambda (1 - F(\alpha^*)) = \mu v'(x^u), \tag{54}$$

$$\widehat{S}^X_{\alpha^*} + \lambda \left[\gamma w_L + (1-\gamma) w_H - x^w + x^u \right] f(\alpha^*) = \mu. \tag{55}$$

Combining (53) and (55) and using (19) yields

$$\frac{\gamma w_L + (1-\gamma) w_H - x^w + x^u}{x^w} = \frac{1}{\eta(x^w, \alpha^*)} \left[1 - \frac{\widehat{S}^X_{x^w}}{\lambda F(\alpha^*)} \right] - \frac{\widehat{S}^X_{\alpha^*}}{\lambda f(\alpha^*) x^w}.$$

Dividing Equations (53)-(54) by the marginal utilities on the right-hand side and adding, we obtain

$$\lambda \left[\frac{F(\alpha^*)}{v'(x^w)} + \frac{1 - F(\alpha^*)}{v'(x^u)} \right] = \frac{\widehat{S}^X_{x^u}}{v'(x^u)} + \frac{\widehat{S}^X_{x^w}}{v'(x^w)},$$

from which, using Definitions (20) and (21), we get $\lambda g^X = D^X$, and so $\lambda^{-1} = g^X / D^X$.

4. Proofs of Section 5.4

Proof of lemma 4

(a) Proof for $\widetilde{S}^{PWU}$, $\widetilde{S}^{PTU}$, $\widetilde{S}^{PWE}$ and $\widetilde{S}^{PTU}$.

Observe that in the second best, for $\alpha < \alpha_L^* \leq \alpha_H^*$, $\delta_L(\alpha) = \delta_H(\alpha) = 1$, $x_L^w(\alpha) = x_L$, $x_H^w(\alpha) = x_H$ and that $x_L \leq x_H$. For $\alpha_L^* \leq \alpha \leq \alpha_H^*$, $\delta_L(\alpha) = 0$, and $\delta_H(\alpha) = 1$ and by (12), $v(x^u) = v(x_H) - \alpha_H^*$, which for $\alpha_L^* \leq \alpha \leq \alpha_H^*$ gives $v(x^u) \leq v(x_H) - \alpha$. For $\alpha > \alpha_H^*$, $\delta_L(\alpha) = \delta_H(\alpha) = 0$, and $x_L^u(\alpha) = x_H^u(\alpha) = x^u$.

Substituting these properties into S^{PWU} and S^{PTU} yields $\widetilde{S}^{PWU}$ and $\widetilde{S}^{PTU}$, respectively. Substituting these properties into S^{PWE} and S^{PTE} leads to $\widetilde{S}^{PWE}$ and $\widetilde{S}^{PTE}$. In the procedure, for $\alpha_L^* \leq \alpha \leq \alpha_H^*$ we use $v^{-1}(v(x^u) + \alpha) \leq x_H$ from $v(x^u) \leq v(x_H) - \alpha$.

(b) Proof for $\widetilde{S}^{PT}$.

Since consumption levels do not depend on α in the second best, S^{PT} reduces to

$$\rho \min\{x_L - w_L, x^u\} + (1-\rho)\min\{x_H - w_H, x^u\}.$$

Hence, with the ETES priority principle, the Lagrangian is

$$\begin{aligned}
&\pounds(x_L, x_H, x^u, \alpha_L^*, \alpha_H^*, \lambda, \mu_L, \mu_H, \nu) = \rho\min\{x_L - w_L, x^u\} + (1-\rho)\min\{x_H - w_H, x^u\} \\
&+\lambda\{\gamma F(\alpha_L^*)(w_L - x_L) + (1-\gamma)F(\alpha_H^*)(w_H - x_H) \\
&-[\gamma(1 - F(\alpha_L^*)) + (1-\gamma)(1 - F(\alpha_H^*))]x^u - R\} \\
&+\mu_H[v(x_H) - \alpha_H^* - v(x^u)] + \mu_L[v(x_L) - \alpha_L^* - v(x^u)] + \nu(x_H - x_L - c)
\end{aligned}$$

with $0 \leq \rho \leq 1$.

(i) Suppose $x_L - w_L \geq x^u$. The first-order condition of the Lagrangian with respect to x_L then becomes $-\lambda\gamma F(\alpha_L^*) - \nu = -\mu_L v'(x_L)$, from which $\mu_L > 0$. However, the first-order condition with respect to α_L^* gives $\mu_L = \lambda\gamma f(\alpha_L^*)(w_L - x_L + x^u) \leq 0$ under the assumption made. Hence we obtain a contradiction, such that we know that $x_L - w_L < x^u$.

(ii) Suppose $x_H - w_H \geq x^u$. Then we get $-T_H \geq x^u$; the high-skilled workers receive a larger subsidy than the inactive people which cannot be optimal. Consequently, $x_H - w_H < x^u$.

As a result of (i) and (ii), the ETES priority principle reduces to $\rho(x_L - w_L) + (1-\rho)(x_H - w_H)$.

Proof of Lemma 5

That for all objective functions $\widetilde{S}^X_{\alpha_H^*} = 0$ and that $\widetilde{S}^{PT}_{\alpha_H^*} = \widetilde{S}^{PT}_{\alpha_L^*} = 0$ is evident. Simple differentiation yields $\widetilde{S}^{PWU}_{\alpha_L^*} = \left[\Omega^R(v(x_L) - \alpha_L^*) - \Omega^R(v(x^u))\right]$. Due to (10), $v(x_L) - \alpha_L^* = v(x^u)$, and so $\widetilde{S}^{PWU}_{\alpha_L^*} = 0$. Similarly it can be shown that $\widetilde{S}^X_{\alpha_L^*} = 0$ for $X = PTU$, PWE and PTE.

Proof of Lemma 6

The proof follows the reasoning for Lemma 3 (using Lemma 5) so is skipped here.

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