

# Optimal Investment and Financial Strategies under Tax Rate Uncertainty

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# Optimal Investment and Financial Strategies under Tax Rate Uncertainty

## Abstract

In this paper we apply a real-option model to study the effects of tax rate uncertainty on a firm's decisions. In doing so, we depart from the relevant literature, which focuses on fully equity-financed investment project. By letting a representative firm borrow optimally, we show that debt finance not only encourages investment activities but can also substantially mitigate the effect of tax rate uncertainty on investment timing.

JEL-Code: H25.

Keywords: capital levy, corporate taxation, default risk, real options.

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# 1 Introduction

Over the last decades, increase in capital mobility has lead to a sharp rise in FDIs and multinational activity, thereby creating the conditions for international tax competition.<sup>1</sup> As shown in Figure 1, among the 106 countries surveyed by KPMG (2009), the average of corporate statutory tax rates (All) has fallen from 31.4 to 25.9% over the 1999-2008 period. While tax cuts were less pronounced in Latin American (LAT) and Asian Pacific (ASPAC) countries, much more dramatic tax cuts occurred in industrialized countries: in the EU, for example, the decrease was sharper (i.e., from 34.2 to 23.2%). We can therefore say that, due to tax competition, firms have been operating in a tax-rate-cut scenario where further reductions might occur in the future.<sup>2</sup>

Despite this generalized tax-cut policies, the recent world crisis has lead politicians to discuss a possible tax rate increase aimed at financing the policies implemented to offset the dramatic effects of the 2008-2009 recession.<sup>3</sup> As pointed out by Mintz (1995, p. 61): "When capital is sunk, governments may have the irresistible urge to tax such a capital at a high rate in the future. This endogeneity of government decisions results in a problem of *time consistency* in tax policy, whereby governments may wish to take actions in the future that would be different from what would be originally planned". In this case, the commitment failure leads to the well-known "capital levy problem", which is related to a firm's fear that a government can decide to raise taxes on capital already invested.<sup>4</sup> Firms are aware that the government can take a different action from that initially planned and try to anticipate its tax choices.

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<sup>1</sup>See, e.g., Devereux et al. (2008) and Ghinamo et al. (2009).

<sup>2</sup>KPMG experts add that "we have found no country anywhere that has raised its rate since last year" (KPMG 2009, p. 6).

<sup>3</sup>The sharp increase in public deficits throughout the world will probably be tackled not only by cutting public spending but also by increasing tax rates. Some US States (such as Oregon and Illinois) have already planned or are planning to raise statutory tax rates. Similarly, public budget concerns in the Eurozone make tax rate increases more likely in some countries.

<sup>4</sup>For further details on the capital-levy problem, see Eichengreen (1990).

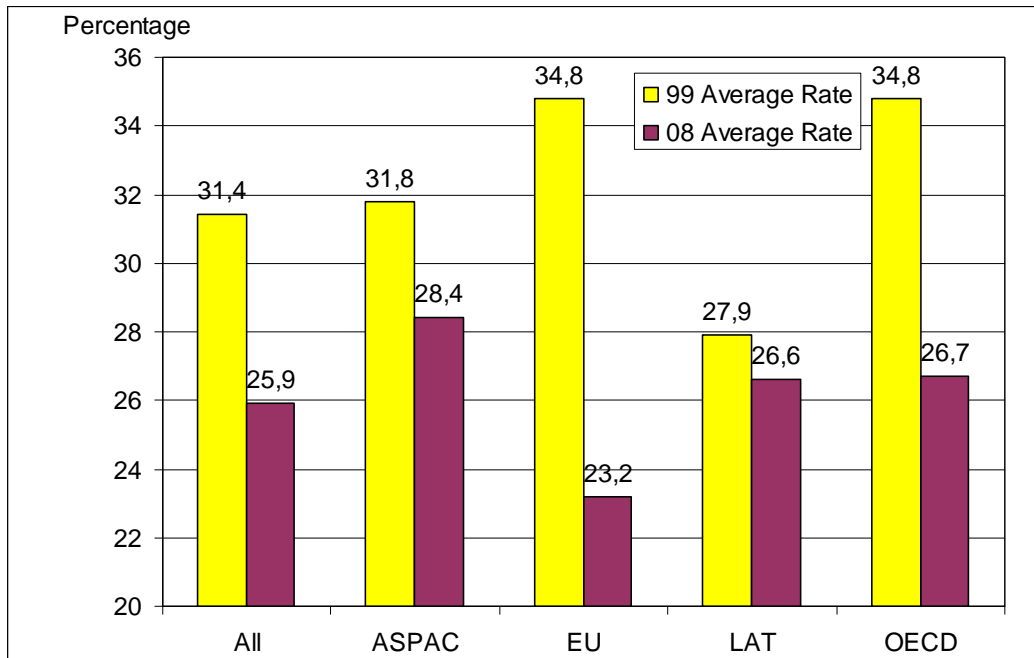


FIGURE 1: A TAX RATE COMPARISON BY GLOBAL REGION, 1999-2008.  
KPMG (2009).

Whatever the sign of the tax rate change is, the empirical evidence shows that tax rate changes are not only frequent but also are difficult to foresee by taxpayers. This means that tax changes are a source of uncertainty.

Tax uncertainty is a fairly important problem and must be analyzed with appropriate techniques. Over the last decade, scholars have used the real-option approach to deal with tax uncertainty.<sup>5</sup> Hassett and Metcalf (1999) used a model with an output price following a geometric Brownian motion and an uncertain investment tax credit to explain the effects of tax policy uncertainty on aggregate investment. They concluded that tax policy uncertainty tends to delay investment under a continuous-time random walk, but increases the capital stock under a Poisson jump process.

Böhm and Funke (2000) showed that investment is not very sensitive to the degree of tax policy uncertainty, when capital is gradually accumulated. Agliardi (2001) analyzed investment effects of uncertain investment tax credits following a jump-diffusion process. She found that tax policy uncertainty delays investment. Böckem (2001) studied whether the threat of imposing a sales tax can lead to a systematic delay of investment. In a dynamic investment model with demand and tax uncertainty, she showed that no systematic delay of investment is expected

<sup>5</sup> As argued by Pindyck (2007): "sunk costs *do matter* in decision-making when those costs *have yet to be sunk*". This implies that the effects of tax uncertainty must be analyzed from an ex-ante perspective, i.e., when firms are still free to choose not only whether but also when to invest.

to occur.

Panteghini (2001a, 2001b) used a Poisson process for the tax rate and proved that investment may be unaffected by tax policy uncertainty, if an ACE-type system is applied. Niemann (2004) defined two neutrality conditions: first-order neutrality, which requires the complete ineffectiveness of taxation on investment decisions; second-order neutrality, which means that the stochastic nature of taxation does not alter investment decisions. In a subsequent article (Niemann, 2006), he analyzed combined tax rate and tax base uncertainty by assuming a stochastic tax payment. He showed that the uncertainty of tax payments has an ambiguous impact on investment timing.<sup>6</sup> Recently, Chen and Funke (2008) have found that political uncertainty (including discontinuous changes in taxation) discourages FDI decisions.

It is worth noting that all these articles deal with tax uncertainty by assuming fully equity-financed investment decisions. However, the evidence shows that investment and financial decisions are related and, hence, should be jointly analyzed. To provide a more realistic analysis, therefore, we depart from the relevant literature and let a firm borrow optimally. Given the importance that the statutory tax rate has on financial choices (see, e.g., Leland, 1994), we will focus on tax rate uncertainty. Moreover, since the evidence shows that tax rate changes are discrete, we will describe them with a Poisson process. In doing so, we will be able to study two possible scenarios: a standard capital-levy one, where the tax rate is expected to rise, and a tax-competition one, where there is a downward trend in tax rates.

Given this model, we will prove that debt finance not only encourages investment activities but also can substantially mitigate the effect of tax rate uncertainty on investment timing. In particular, using a numerical simulation, based on realistic parameter values, we will show that a highly volatile tax system may have a negligible impact on investment choices, if firms can choose their capital structure. If however, they are credit-constrained the impact of tax-rate uncertainty is much more significant. Our results have implications in terms of both empirical analysis and policy decision-making. Firstly, we can say that econometric investigation should control for the existence (absence) of financial flexibility. Otherwise, estimates would be misleading. Secondly, the effects of a hot policy debate on future (and uncertain) tax-rate changes crucially depend on the efficiency of financial market and, in particular, on the existence/absence of credit constraints.

The structure of the paper is as follows. Section 2 describes the model. Section 3 shows our main findings and discusses how tax rate uncertainty affects a firm's choices. Section 4 summarizes our main findings and discusses its implications.

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<sup>6</sup>On this point see also Sureth (2002).

## 2 The model

In this section we introduce an EBIT-based model in the spirit of Goldstein et al. (2001). By focusing on cash flows rather than stocks, we can better describe the investment and financial strategies of an infinitely-lived risk-neutral firm.<sup>7</sup>

Let us denote  $\Pi_t$  as the firm's Earning Before Interest and Taxes (EBIT) at time  $t$  and assume that it evolves as follows:

$$\frac{d\Pi_t}{\Pi_t} = \alpha dt + \sigma dz_t, \text{ with } \Pi_0 \geq 0, \quad (1)$$

where  $\alpha$  is the expected rate of growth,  $\sigma$  is the instantaneous standard deviation of  $\frac{d\Pi_t}{\Pi_t}$  and  $dz_t$  is the increment of a Brownian motion. Moreover, let us introduce the following hypotheses.

**Assumption 1** *The firm must pay a sunk start-up cost, denoted by  $I$ , to undertake a risky project.*

**Assumption 2** *The firm can borrow from a perfectly competitive risk-neutral credit sector, characterized by a given risk-free interest rate  $r$ .*

**Assumption 3** *The firm can decide how much to borrow by choosing a non-renegotiable coupon  $C$ .*

**Assumption 4** *Default takes place when  $\Pi_t$  goes to  $C$ .*

**Assumption 5** *The cost of default is equal to  $vC$  with  $v > 0$ .*

According to Assumption 1, the firm must pay a sunk cost. This means that investment projects are irreversible. Assumption 2 entails a simple framework where lenders are price-takers and become shareholders in the event of default. In line with Leland (1994), the firm chooses an optimal coupon (Assumption 3).<sup>8</sup> For simplicity, the capital structure is assumed to be static, i.e., financial policy cannot be reviewed later.<sup>9</sup> Moreover, according to Assumption 4, default occurs

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<sup>7</sup>For a study on risk-averse firms' investment choices, see Niemann and Sureth (2004, 2005).

<sup>8</sup>Given  $C$  and the risk-free interest rate  $r$ , the market value of debt can be calculated. It is worth noting that, in the absence of arbitrage, setting the coupon first, and then calculating the market value of debt is equivalent to first choosing the value of debt and then calculating the effective interest rate. The ratio between  $C$  and the market value of debt is equal to the effective interest rate (which is given by the sum between  $r$  and the default risk premium).

<sup>9</sup>Ruling out the option to renegotiate debt does not affect the qualitative properties of the model. For a detailed analysis of *dynamic* trade-off strategies, with costly debt renegotiation, see, e.g., Goldstein et al. (2001), and Hennessy and Whited (2005).

when the firm's profit, net of its debt obligations, is nil.<sup>10</sup> When default takes place, a sunk default cost, equal to  $vC$ , is faced (Assumption 5).<sup>11</sup>

Let us next introduce taxation. We define  $\tau$  as the tax rate and assume that interest payments are fully deductible. As to the treatment of the lender's receipts, the evidence shows that effective tax rates on capital income are fairly low. With no loss of generality, therefore, we assume that the lender's pre-default tax burden is nil, so that her after-tax profit function at any instant  $t$  is simply  $C$ . When, however, default takes place, the lender becomes shareholder and is then subject to corporate taxation.

Given these assumptions a firm's after-tax profit function, at time  $t$ , is equal to

$$\Pi^N = (1 - \tau_t)(\Pi_t - C). \quad (2)$$

Given Assumption 4 therefore, default takes place when  $\Pi^N = 0$ . This means that the default threshold point is  $C$ .

Let us finally model tax rate uncertainty. We assume that the tax rate follows a Poisson process. Given an initial tax rate  $\tau_0$ , at any short time interval  $dt$  there is a probability  $\lambda dt$  that the tax rate changes to  $\tau_1$  ( $\leq \tau_0$ ). Hence we can write:

$$d\tau = \begin{cases} 0 & \text{w.p. } 1 - \lambda dt, \\ \Delta\tau & \text{w.p. } \lambda dt, \end{cases} \quad (3)$$

where  $\Delta\tau = \tau_1 - \tau_0$ . Given (3), we can therefore focus on:

1. a capital-levy scenario, where the tax rate is expected to rise (i.e.,  $\Delta\tau > 0$ );
2. a tax-competition scenario, where the tax rate is expected to decrease (i.e.,  $\Delta\tau < 0$ ).

For simplicity, hereafter we will omit the time variable  $t$ .

## 2.1 The value function

Let us next calculate the firm's value function. Using a backward approach, we will first focus on the value function after the tax rate change: in this case the relevant rate is  $\tau_1$  and the value function will be denoted by  $V_1(\Pi; C)$ . Subsequently, we will focus on the before-tax-change scenario, where the current statutory rate is  $\tau_0$ .

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<sup>10</sup> Assumption 4 implies that debt is protected. As pointed out by Leland (1994) minimum net-worth requirements, implied by protected debt, are common in short-term debt financing. For further details on default conditions see Brennan and Schwartz (1978), and Smith and Warner (1979). For a comparison between protected and unprotected debt financing, see also Panteghini (2007a).

<sup>11</sup> The quality of results does not change if, like Leland (1994), we assume that default costs are proportional to a firm's value.

### 2.1.1 After the tax-rate change

The value function  $V_1(\Pi; C)$  is given by the sum between the equity value  $E_1(\Pi; C)$  and the debt value  $D_1(\Pi; C)$ , net of the investment cost  $I$ . As shown in Appendix A.1, it amounts to:

$$V_1(\Pi; C) = \begin{cases} \frac{(1-\tau_1)\Pi}{\delta} - I, & \text{after default,} \\ \frac{(1-\tau_1)\Pi}{\delta} + \frac{\tau_1 C}{r} - (\frac{\tau_1}{r} + v) C \left(\frac{\Pi}{C}\right)^{\beta_2} - I, & \text{before default,} \end{cases} \quad (4)$$

where  $\delta \equiv r - \alpha > 0$  is the so-called "dividend yield". The term  $\frac{(1-\tau_1)\Pi}{\delta}$  measures the gross value of an unlevered firm;<sup>12</sup>  $\frac{\tau_1 C}{r}$  is the tax benefit due to deductibility of the interest expenses. The third term,  $\left[-(\frac{\tau_1}{r} + v) C \left(\frac{\Pi}{C}\right)^{\beta_2}\right]$ , measures the contingent cost of default. In other words, in the event of default, a firm not only faces a sunk cost  $vC$  but also loses the tax benefit of interest deductibility  $(\frac{\tau_1 C}{r})$ . The present value of the default cost is multiplied by  $\left(\frac{\Pi}{C}\right)^{\beta_2}$ , with  $\beta_2 < 0$  (see Appendix A), which measures the contingent value of 1€ in the event of default.

### 2.1.2 Before the tax-rate change

Let us now calculate the value function before the tax rate change,  $V_0(\Pi; C)$ . As shown in Appendix A.2,  $V_0(\Pi; C)$  is given by the sum of the equity value  $E_0(\Pi; C)$  and the debt value  $D_0(\Pi; C)$  before the tax rate change, net of the investment cost  $I$ , i.e.,

$$V_0(\Pi; C) = E_0(\Pi; C) + D_0(\Pi; C) - I = V_1(\Pi; C) + X(\Pi; C) + Y(\Pi; C) - I, \quad (5)$$

where

$$\begin{aligned} X(\Pi; C) &\equiv E_0(\Pi; C) - E_1(\Pi; C) \\ &= (\tau_1 - \tau_0) \left[ \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) - \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \right] \end{aligned}$$

and

$$Y(\Pi; C) \equiv D_0(\Pi; C) - D_1(\Pi; C) = (\tau_1 - \tau_0) \frac{C}{\delta + \lambda} \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)}$$

are the expected changes in the equity and debt value, respectively. The term  $\left(\frac{\Pi}{C}\right)^{\beta_2(\lambda)}$ , with  $\beta_2(\lambda) < 0$  (see again Appendix A), is the contingent value of 1€, under tax rate uncertainty. As can be seen, the relevant discount rates are  $\delta + \lambda$  and  $r + \lambda$  (instead of  $\delta$  and  $r$ ), respectively. This is due to the fact that, before the tax-rate jump, present value calculations must account for the probability  $\lambda$  of this tax change.

<sup>12</sup>The relevant discount rate of the first term is  $\delta \leq r$ . This means that the present value of future cash flow accounts for the expected growth rate  $\alpha \geq 0$  of  $\Pi$ .



It is worth noting that  $\beta_2(\lambda)$  depends on the probability of the tax-rate change. This implies that the contingent value of default is affected by tax uncertainty. To understand this important effect, let us compare the tax uncertainty case with the tax certainty one. Since  $\beta_2(\lambda) < \beta_2 < 0$ , the inequality  $\left(\frac{\Pi}{C}\right)^{\beta_2} > \left(\frac{\Pi}{C}\right)^{\beta_2(\lambda)}$  holds. This means that the contingent value of 1€ under tax rate uncertainty is less than that under tax rate certainty.

Both the expected changes in the equity and debt value account for the tax rate change and are proportional to the differential  $(\tau_1 - \tau_0)$ . In particular,  $X(\Pi; C)$  is given by the product between the tax rate differential  $(\tau_1 - \tau_0)$  and the term in square brackets, which measures the contingent value of equity: this term is given by the difference between a firm's equity value, with zero default risk, i.e.,  $\left(\frac{\Pi}{\delta+\lambda} - \frac{C}{r+\lambda}\right)$ , and the contingent value of equity in the event of default, i.e.,  $\left(\frac{C}{\delta+\lambda} - \frac{C}{r+\lambda}\right) \left(\frac{\Pi}{C}\right)^{\beta_2(\lambda)}$ . Function  $Y(\Pi; C)$  measures the impact of the tax-rate change on the benefit of interest deductibility. It is thus equal to the product between the tax rate differential  $(\tau_1 - \tau_0)$  and the contingent value of  $C$  Euros (i.e.,  $C \left(\frac{\Pi}{C}\right)^{\beta_2(\lambda)}$ ), divided by the relevant discount rate  $(\delta + \lambda)$ .

## 2.2 The option value

Let us next deal with the option to invest. Accordingly, we denote the option value  $O_1(\Pi; C)$  and  $O_0(\Pi; C)$ , under tax rate certainty and uncertainty, respectively. Again, we will follow a backward approach.

### 2.2.1 After the tax-rate change

Let us start with the after-change scenario. Since the tax-rate change has already occurred, policy uncertainty has vanished. As shown in Appendix B.1, the option value is therefore equal to:

$$O_1(\Pi; C; \bar{\Pi}) = \left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_1} \left[ \frac{(1 - \tau_1)\bar{\Pi}}{\delta} + \frac{\tau_1 C}{r} - (\tau_1 + rv) \frac{C}{r} \left(\frac{\bar{\Pi}}{C}\right)^{\beta_2} - I \right], \quad (6)$$

where  $\bar{\Pi}$  is the threshold EBIT level, above which investment is undertaken. As can be seen, the option value is given by the product between  $\left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_1}$ , i.e., the present value of 1€ contingent on the entry decision, and

$$\left[ \frac{(1 - \tau_1)\bar{\Pi}}{\delta} + \frac{\tau_1 C}{r} - (\tau_1 + rv) \frac{C}{r} \left(\frac{\bar{\Pi}}{C}\right)^{\beta_2} - I \right],$$

that is, the Net Present Value (NPV) at point  $\Pi = \bar{\Pi}$  (i.e., when the investment project is optimally undertaken).

## 2.2.2 Before the tax-rate change

The option value under tax-rate uncertainty can be written as follows (see Appendix B.2):

$$O_0(\Pi; C; \hat{\Pi}; \bar{\Pi}) = O_1(\Pi; C; \bar{\Pi}) + Z(\Pi; C; \bar{\Pi}; \hat{\Pi}), \quad (7)$$

where  $\hat{\Pi}$  is the threshold point, above which investment is profitable in a tax-uncertainty context. Function

$$Z(\Pi; C; \bar{\Pi}; \hat{\Pi}) = \left[ V_0(\hat{\Pi}; C) - V_1(\bar{\Pi}; C) \left( \frac{\hat{\Pi}}{\bar{\Pi}} \right)^{\beta_1} \right] \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1(\lambda)} \quad (8)$$

measures the contingent effect of tax-rate uncertainty on the option value (see Appendix B). Terms  $V_0(\hat{\Pi}; C)$  and  $V_1(\bar{\Pi}; C)$  are the value functions at the relevant threshold levels  $\hat{\Pi}$  and  $\bar{\Pi}$ , respectively, and  $\left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1(\lambda)}$  is the contingent value of 1€ invested when  $\Pi$  reaches  $\hat{\Pi}$ . Finally,  $\left( \frac{\hat{\Pi}}{\bar{\Pi}} \right)^{\beta_1}$  measures the impact of tax rate uncertainty on the contingent evaluation of assets.<sup>13</sup>

## 2.3 The firm's problem

Given the NPV and the option value, let us next analyze a firm's decision on both the coupon  $C$  and the investment timing. Again, we will start with the after-tax reform, and then focus on the before-tax rate change case.

### 2.3.1 After the tax-rate change

If the tax rate change has already occurred, a firm's problem is:

$$\max_{\bar{\Pi} > 0, C > 0} \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \left[ \frac{(1 - \tau_1)\bar{\Pi}}{\delta} + \frac{\tau_1 C}{r} - (\tau_1 + rv) \frac{C}{r} \left( \frac{\bar{\Pi}}{C} \right)^{\beta_2} - I \right], \quad (9)$$

Solving problem (9) gives (Appendix C.1):

$$\begin{aligned} \bar{\Pi}_1^* &= \frac{1}{1+m_1} \frac{\beta_1}{\beta_1-1} \frac{rI}{1-\tau_1}, \\ \text{with } m_1 &\equiv \frac{\tau_1}{1-\tau_1} \frac{\beta_2}{\beta_2-1} \left( \frac{1}{1-\beta_2} \frac{\tau_1}{\tau_1+rv} \right)^{-\frac{1}{\beta_2}} > 0, \\ \bar{C}_1 &= \left( \frac{1}{1-\beta_2} \frac{\tau_1}{\tau_1+rv} \right)^{-\frac{1}{\beta_2}} \bar{\Pi}_1^*. \end{aligned} \quad (10)$$

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<sup>13</sup>If tax-rate uncertainty vanishes, the equality  $\hat{\Pi} = \bar{\Pi}$  holds and  $\left( \frac{\hat{\Pi}}{\bar{\Pi}} \right)^{\beta_1}$  goes to unity.

The threshold point  $\bar{\Pi}_1^*$  is given by the product between the term  $\frac{1}{1+m_1}$ , which measures the effect of debt financing on the firm's trigger point, and  $\frac{\beta_1}{\beta_1-1} \frac{rI}{1-\tau_1}$ , which is the value of the threshold point under full equity finance (see, Panteghini, 2007a and 2007b). Since  $\frac{1}{1+m_1} < 1$ , debt financing encourages investment (namely, induces a firm to invest earlier). The reasoning behind this result is straightforward: if a firm can borrow, it will invest earlier in order to benefit from interest deductibility (see Panteghini 2007b).

The optimal coupon  $\bar{C}_1$  is proportional to the threshold point  $\bar{\Pi}_1^*$  and is negatively affected by the default cost parameter  $v$ . As shown in Appendix C.2, both  $\bar{\Pi}_1^*$  and  $\bar{C}_1$  are increasing in  $\tau_1$ . The positive effect of  $\tau_1$  on the threshold point  $\bar{\Pi}_1^*$  is due to the fact that the higher the tax rate, the greater the option value is (i.e., the higher the opportunity cost of immediate investment is), and the lower the after-tax value of the project is. Both effects cause a delay in investment. The positive sign of  $\frac{\partial \bar{C}_1}{\partial \tau_1}$  means that the higher the rate  $\tau_1$ , the greater the benefit of interest deductibility, the higher the optimal coupon is. Moreover, we can show (see again Appendix C.2) that the ratio  $\left(\frac{\bar{C}_1}{\bar{\Pi}_1^*}\right)$  is increasing in  $\tau_1$ . This means that a higher tax rate stimulates borrowing for any given level of EBIT.

### 2.3.2 Before the tax-rate change

Under tax-rate uncertainty, a firm's problem is as follows:

$$\max_{\hat{\Pi} > 0, C > 0} O_0(\Pi; C; \hat{\Pi}; \bar{\Pi}), \quad (11)$$

where  $O_0(\Pi; C; \hat{\Pi}; \bar{\Pi})$  is defined by (6), (7) and (8). Solving (11) gives the following first order conditions:

$$\begin{aligned} \frac{\partial O_0(\Pi; C; \hat{\Pi}; \bar{\Pi})}{\partial \hat{\Pi}} &= V_1(\bar{\Pi}; C) \left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_1} \frac{\beta_1(\lambda) - \beta_1}{\hat{\Pi}} \left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1(\lambda) - \beta_1} + \\ &+ \left[ \frac{\partial V_0(\hat{\Pi}; C)}{\partial \hat{\Pi}} \left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1(\lambda)} - \frac{\beta_1(\lambda)}{\hat{\Pi}} V_0(\hat{\Pi}; C) \left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1(\lambda)} \right] = 0. \end{aligned} \quad (12)$$

and

$$\begin{aligned} \frac{\partial O_0(\Pi; C; \hat{\Pi}; \bar{\Pi})}{\partial C} &= \frac{\partial V_1(\bar{\Pi}; C)}{\partial C} \left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_1} \left[ 1 - \left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1(\lambda) - \beta_1} \right] \\ &+ \left[ \frac{\partial V_0(\hat{\Pi}; C)}{\partial C} \left(\frac{\Pi}{\hat{\Pi}}\right)^{\beta_1(\lambda)} \right] = 0. \end{aligned} \quad (13)$$

$\widehat{\Pi}^*$  and  $C^*$  are the solutions of system (12)-(13). Although this it has no closed-form solution, the analysis of equations (12)-(13) gives us some hint about the effects of tax rate uncertainty. To do so, let us start with the tax-certainty scenario. Setting  $\lambda = 0$  gives  $\beta_1(\lambda)|_{\lambda=0} = \beta_1$ , and therefore, the terms

$$V_1(\bar{\Pi}; C) \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \frac{\beta_1(\lambda) - \beta_1}{\widehat{\Pi}} \left( \frac{\Pi}{\widehat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1} \quad (14)$$

and

$$\frac{\partial V_1(\bar{\Pi}; C)}{\partial C} \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1} \left[ 1 - \left( \frac{\Pi}{\widehat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1} \right] \quad (15)$$

go to zero. In this case the two-equation system (12)-(13) collapses to the tax-rate certainty system (48)-(49) (in Appendix C.1), with the relevant tax rate  $\tau_0$ .

This first step allows us to show that terms (14) and (15) measure the distortions caused by tax-rate uncertainty. In particular, term (14) measures the marginal distortion on the threshold EBIT level. As can be seen, (14) is proportional to the firm's value after the tax-rate change  $V_1(\bar{\Pi}; C)$ . Moreover, it depends on both  $\left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1}$  and  $\left( \frac{\Pi}{\widehat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1}$ : while former measures the contingent value of 1€ invested when  $\Pi$  reaches  $\bar{\Pi}$ , i.e., when it is optimal to invest in the absence of tax-rate uncertainty, the latter measures the wedge on contingent evaluation due to tax-rate uncertainty. As we can see, the wedge  $\left( \frac{\Pi}{\widehat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1}$  depends on the tax-rate-uncertainty trigger point  $\widehat{\Pi}$  and on the difference  $[\beta_1(\lambda) - \beta_1]$ . This means that the higher the parameter  $\lambda$ , the larger the tax-rate uncertainty wedge is, or equivalently, the greater the distortion caused by tax-rate uncertainty.

A similar reasoning holds for term (15), which enters Eq. (13). As can be seen, the marginal condition on the coupon depends on the marginal benefit of debt financing, after the tax-rate change (i.e.,  $\frac{\partial V_1(\bar{\Pi}; C)}{\partial C}$ ). Moreover, it is proportional to the contingent value of 1€ invested when  $\Pi$  reaches  $\bar{\Pi}$  (i.e.,  $\left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_1}$ ). Finally, it depends on the tax-rate uncertainty wedge  $\left( \frac{\Pi}{\widehat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1}$ .

Notice that  $\left( \frac{\Pi}{\widehat{\Pi}} \right)^{\beta_1(\lambda) - \beta_1}$  has two opposing effects on Equations (12)-(13): while it raises the LHS of (12), it reduces the LHS of (13). As will be shown in the next section, this opposing effect will lead to an increase in the marginal cost of immediate investment and, at the same time, a drop in the marginal cost of debt finance.

### 3 A numerical analysis

In order to analyze how a firm's ability to borrow affects investment decisions, let us compare the tax-uncertainty scenario with the tax-certainty one. In both

cases, we assume that the starting tax rate is  $\tau_0$ . While in the tax-certainty case it will be unchanged, under tax-rate uncertainty this rate is expected to jump (either up or down).

Under tax-rate certainty, the firm's problem is equivalent to (9), with the assumption that here, the relevant tax rate is  $\tau_0$  (instead of  $\tau_1$ ). Hence, its solutions will have the same form as solution (10):

$$\begin{aligned}\bar{\Pi}_0^* &= \frac{1}{1+m_0} \frac{\beta_1}{\beta_1-1} \frac{rI}{1-\tau_0}, \\ \bar{C}_0 &= \left( \frac{1}{1-\beta_2} \frac{\tau_0}{\tau_0+rv} \right)^{-\frac{1}{\beta_2}} \bar{\Pi}_0^*, \\ m_0 &\equiv \frac{\tau_0}{1-\tau_0} \frac{\beta_2}{\beta_2-1} \left( \frac{1}{1-\beta_2} \frac{\tau_0}{\tau_0+rv} \right)^{-\frac{1}{\beta_2}} > 0.\end{aligned}\tag{16}$$

Since solutions (16) have the same form as solutions (10), the comparative statics results of Appendix C.2 (on solutions solutions (10)) still hold. In other terms, the threshold point and the optimal coupon of (16), as well as the ratio  $\left(\frac{\bar{C}_0}{\bar{\Pi}_0^*}\right)$  are increasing in the tax rate ( $\tau_0$ ).

As pointed out, the tax-rate uncertainty problem (11) has no closed-form solution. For this reason, we need a numerical analysis to compare the tax-certainty with the tax-uncertainty case. In doing so, we will use the benchmark parameter values of Table 1. In line with Dixit and Pindyck (p. 157 and p. 193, 1994; 1999) we set  $r = \delta = 0.05$  and  $\sigma = 0.4$ .<sup>14</sup> Furthermore, we assume that  $v = 3$ . This means that, given  $r = 0.05$ , the default cost is about 10% of the debt value.<sup>15</sup>

As we have seen in Figure 1, over the last decade, the average statutory tax rate has been about 30%. Accordingly, we set  $\tau_0$  equal to 0.3. Given the high heterogeneity of the tax-rate jumps occurred over the past decade, we will assume that  $\tau_1$  ranges from 0.1 to 0.55.<sup>16</sup> We will therefore be able to study both the capital-levy and the tax-competition case.

Let us first look at the effects of the tax rate differential  $\Delta\tau$  on both the threshold point and the optimal coupon. Figure 2 shows that both  $\hat{\Pi}^*$  and  $C^*$  are positively affected by the tax rate differential  $\Delta\tau$ . The reasoning behind this result is as follows. An increase in  $\Delta\tau$  means that, given  $\tau_0$  and  $\lambda$ , a higher average tax rate (which must account for both the current rate  $\tau_0$  and the expected future one  $\tau_1$ ) will be levied. The heavier the expected tax burden, the higher the investment trigger point is (i.e., the later the investment will be undertaken). Therefore, if  $\Delta\tau > 0$ , an increase in  $\Delta\tau$  raises the expected tax burden, and therefore, causes a delay in investment timing. The opposite is true if  $\Delta\tau < 0$ .

<sup>14</sup>The quality of results does not change if we use different values of  $\sigma$ . For further details see, e.g., Leland (1994).

<sup>15</sup>This percentage is in line with Branch's (2002) estimates.

<sup>16</sup>Of course, the quality of results does not change if a different value of  $\tau_0$  is assumed.

Table 1: The parameter values.

Parameter	Value
$\sigma$	.4
$I$	100
$\tau_0$	.3
$\tau_1$	[.1, .55]
$r$	.05
$\delta$	.05
$v$	3
$\lambda$	[.1, 1]

Similarly, an increase in  $\Delta\tau$  raises the optimal coupon  $C^*$ . This is due to the fact that a higher average tax rate leads to a higher tax benefit of interest deductibility, thereby encouraging borrowing.

Notice that point  $\Delta\tau = 0$  describes the tax-rate certainty case. Therefore, Figure 2 allows us to compare the tax-rate certainty case with the tax-rate uncertainty one. If a firm foresees a tax cut ( $\Delta\tau < 0$ ), both the threshold point and the optimal coupon are less than under tax-rate certainty. The opposite is true if  $\Delta\tau > 0$ .

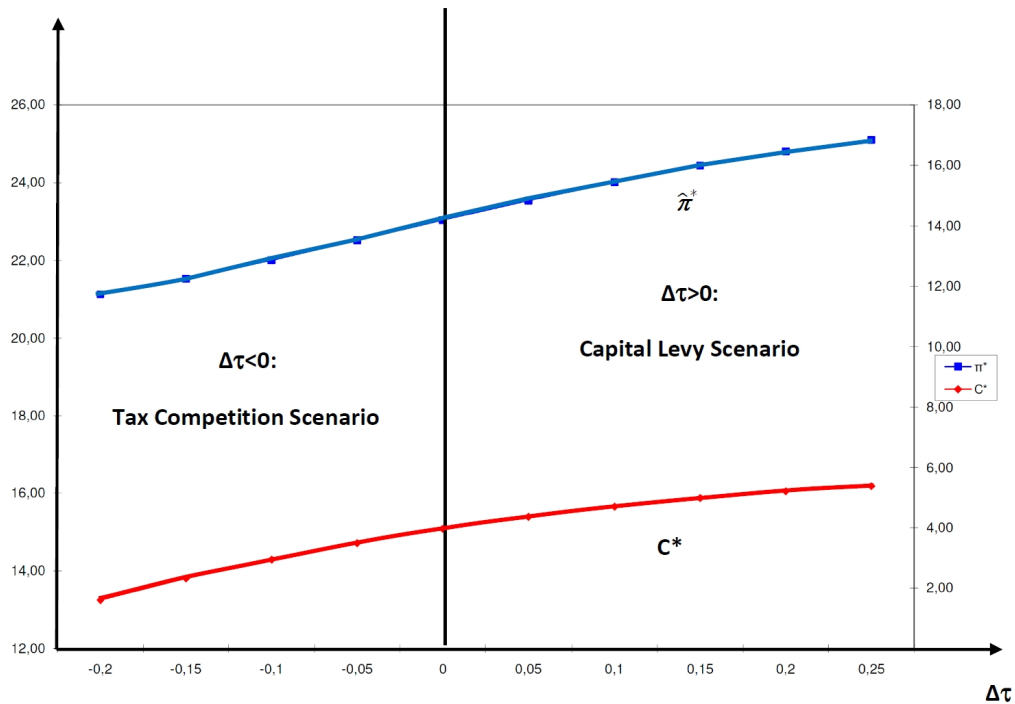


FIGURE 2: THE OPTIMAL VALUES  $C^*$  AND  $\hat{\Pi}^*$  AS FUNCTION OF  $\Delta\tau$ .

So far we have focused on the absolute values of  $\Pi$  and  $C$ . Let us next

focus on the coupon/EBIT ratio. We will then compare the tax-rate-certainty ratio,  $\left(\frac{\bar{C}_0}{\bar{\Pi}_0^*}\right)$ , with the tax-rate uncertainty one,  $\left(\frac{C^*}{\hat{\Pi}^*}\right)$ . As shown in Figure 3, ratio  $\left(\frac{C^*}{\hat{\Pi}^*}\right)$  is increasing in  $\Delta\tau$  (blue line); in other words, a firm's propensity to borrow increases with  $\Delta\tau$ . Of course, the ratio  $\left(\frac{\bar{C}_0}{\bar{\Pi}_0^*}\right)$ , depicted by the purple line, is constant. It is worth noting that the blue line (tax-uncertainty case) is below (above) the purple line (tax-certainty case) if  $\Delta\tau < 0$  ( $\Delta\tau > 0$ ). This means that when a tax rate is expected to decrease, the expected tax benefit of interest deductibility will be lower and therefore, debt will be less profitable than under tax-rate certainty. The converse is true if  $\Delta\tau > 0$ : in this case, a firm operating in a uncertain tax environment is stimulated to raise leverage.

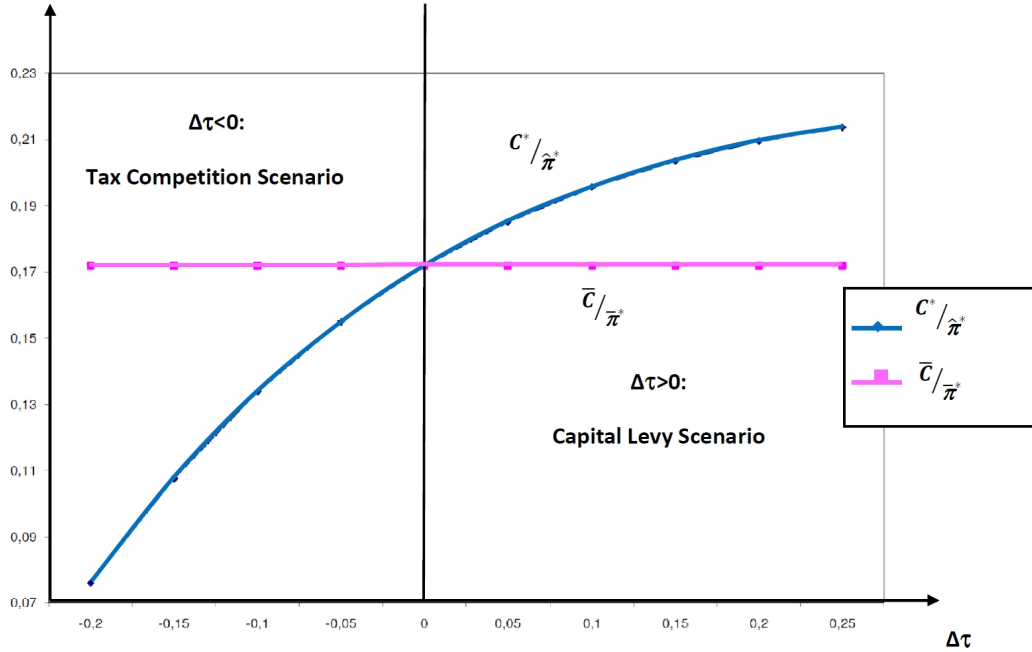


FIGURE 3: THE OPTIMAL RATIOS  $C^*/\hat{\Pi}^*$  AND  $\bar{C}_0/\bar{\Pi}_0^*$  AS FUNCTION OF  $\Delta\tau$ .

In Tables 2, 3 and 4 we provide a sensitivity analysis for different values of  $\Delta\tau$ ,  $\lambda$ ,  $\sigma$  and  $v$ . In order to analyze how a firm's ability to borrow affects its investment decisions, we will compare the well-known full-equity finance case ( $C = 0$ ), with the optimal-leverage case (i.e.,  $C = C^*$ ). Accordingly, we will calculate the threshold values under both full-equity financing (i.e.,  $\hat{\Pi}^*(C = 0)$ ) and optimal-leverage financing (i.e.,  $\hat{\Pi}^*(C = C^*)$ ). Moreover, we will calculate the differential  $\Delta\hat{\Pi}^* \equiv \hat{\Pi}^*(C = 0) - \hat{\Pi}^*(C = C^*)$ .

$\lambda$	0.10			0.25			0.50			0.75			1.00		
	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$
<b>0.25</b>	21.86	20.82	1.04	20.87	19.47	1.40	20.12	18.24	1.88	19.72	17.54	2.18	19.46	17.07	2.39
<b>-0.20</b>	22.12	21.12	1.00	21.28	20.10	1.18	20.64	19.18	1.46	20.30	18.64	1.67	20.08	18.26	1.82
<b>-0.15</b>	22.41	21.53	0.88	21.74	20.79	0.95	21.24	20.17	1.07	20.97	19.79	1.18	20.79	19.52	1.28
<b>-0.10</b>	22.75	22.00	0.74	22.27	21.54	0.73	21.91	21.17	0.74	21.72	20.95	0.77	21.60	20.79	0.81
<b>-0.05</b>	23.12	22.51	0.61	22.87	22.30	0.57	22.68	22.14	0.53	22.57	22.05	0.52	22.51	21.99	0.52
<b>0.00</b>	23.54	23.03	0.51	23.54	23.03	0.51	23.54	23.03	0.50	23.53	23.03	0.50	23.53	23.03	0.50
<b>0.05</b>	24.02	23.54	0.48	24.30	23.71	0.59	24.51	23.80	0.70	24.62	23.85	0.77	24.69	23.87	0.82
<b>0.10</b>	24.55	24.01	0.54	25.15	24.30	0.85	25.61	24.43	1.18	25.86	24.47	1.39	26.02	24.48	1.54
<b>0.15</b>	25.14	24.44	0.70	26.11	24.80	1.31	26.87	24.91	1.96	27.27	24.90	2.37	27.54	24.86	2.68
<b>0.20</b>	25.80	24.80	0.99	27.21	25.18	2.02	28.31	25.22	3.08	28.91	25.14	3.77	29.30	25.03	4.26
<b>0.25</b>	26.54	25.09	1.44	28.46	25.44	3.01	29.98	25.37	4.61	30.81	25.19	5.62	31.35	25.02	6.33

TABLE 2: THE OPTIMAL VALUES  $\hat{\Pi}^*$  WITH  $C=0$  AND  $C=C^*$  AS FUNCTION OF  $\Delta T$  AND  $\lambda$ .



Let us first focus on the effects of tax rate uncertainty ( $\lambda$ ) on a firm's choices. As shown in Table 2, we can see that:

1. for any given  $\lambda$ , the inequality  $\widehat{\Pi}^*(C = C^*) < \widehat{\Pi}^*(C = 0)$  holds. As expected, financial flexibility allows firms to invest earlier, in order to enjoy the tax benefit of interest deductibility.
2. A similar reasoning holds when  $\Delta\tau = 0$ . However, in the certainty-case, the differential  $\Delta\widehat{\Pi}^*$  is quite small. This means that a firm's ability to borrow allows a firm to invest earlier, although this effect is almost negligible.
3. When  $\Delta\tau \neq 0$ , the distortive effect of tax-rate uncertainty is always larger in the standard full-equity financing case. Indeed, the ability to borrow can substantially smooth the effects of tax rate uncertainty.
4. Under the capital-levy scenario (i.e., with  $\Delta\tau > 0$ ), the threshold points  $\widehat{\Pi}^*(C = 0)$  and  $\widehat{\Pi}^*(C = C^*)$  are positively affected by  $\lambda$ . The reasoning behind this result is simple: the higher the probability of a given tax rate increase, the higher the expected tax burden and the higher a firm's trigger point is (and hence, the later an investment is made). The opposite is true in a tax-competition environment (i.e., with  $\Delta\tau < 0$ ): an increase in  $\lambda$  reduces  $\widehat{\Pi}^*$ .

To sum up, we can say that, when debt finance is allowed, tax-rate uncertainty (in terms of both a higher differential  $\Delta\tau$  and a higher probability) has a smaller impact on investment timing. For instance, our numerical analysis shows that if a dramatic tax rate increase ( $\tau_1$  from 0.3 to 0.55) was expected, its effect would be almost negligible for an optimally-leveraged firm. When, instead, a borrowing constraint is binding and a firm can finance its investment only through equity issues, the effect of tax-rate uncertainty is much more significant.

$\sigma$	0.10			0.20			0.30			0.40			0.50			0.60		
	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$	$\#(C=0)$	$\#(C=C^*)$	$\Delta\#$
-0.25	10.50	7.31	3.19	12.29	10.74	1.56	16.47	15.22	1.25	21.86	20.82	1.04	28.51	27.62	0.88	36.47	35.68	0.79
-0.20	9.53	7.60	1.94	12.39	11.01	1.37	16.67	15.49	1.18	22.12	21.12	1.00	28.82	27.99	0.83	36.82	36.14	0.69
-0.15	9.20	7.87	1.33	12.54	11.36	1.18	16.90	15.87	1.03	22.41	21.53	0.88	29.16	28.43	0.73	37.22	36.63	0.59
-0.10	9.21	8.10	1.12	12.74	11.71	1.03	17.17	16.29	0.88	22.75	22.00	0.74	29.55	28.95	0.60	37.66	37.19	0.47
-0.05	9.32	8.28	1.04	12.98	12.02	0.95	17.49	16.71	0.77	23.12	22.51	0.61	29.99	29.52	0.46	38.14	37.81	0.34
0.00	9.47	8.39	1.08	13.26	12.28	0.97	17.85	17.11	0.73	23.54	23.03	0.51	30.47	30.13	0.34	38.68	38.47	0.21
0.05	9.66	8.44	1.23	13.58	12.49	1.10	18.25	17.47	0.78	24.02	23.54	0.48	31.00	30.75	0.25	39.27	39.18	0.09
0.10	9.89	8.38	1.51	13.95	12.62	1.33	18.71	17.78	0.93	24.55	24.01	0.54	31.60	31.37	0.22	39.93	39.91	0.02
0.15	10.15	8.22	1.93	14.37	12.67	1.70	19.22	18.02	1.20	25.14	24.44	0.70	32.26	31.97	0.29	40.65	40.64	0.01
0.20	10.45	7.97	2.48	14.84	12.63	2.22	19.79	18.18	1.61	25.80	24.80	0.99	32.99	32.53	0.46	41.45	41.36	0.09
0.25	10.77	7.61	3.16	15.38	12.48	2.89	20.45	18.25	2.19	26.54	25.09	1.44	33.81	33.04	0.78	42.34	42.06	0.29

TABLE 3:  $\Delta\hat{\Pi}^* = \hat{\Pi}^*(C=0) - \hat{\Pi}^*(C=C^*)$  WITH  $C=0$  AND  $C=C^*$  AS FUNCTION OF  $\Delta T$  AND  $\sigma$ .

Let us next look at the effects of both EBIT volatility ( $\sigma$ ) and default cost ( $v$ ) on a firm's decisions. Results can be summarized as follows.

1. For any given couple  $(\sigma, \Delta\tau)$  and  $(v, \Delta\tau)$ , the differential  $\Delta\hat{\Pi}^*$  is positive. Again, we can see that financial flexibility allows firms to invest earlier in order to enjoy the tax benefit of interest deductibility.
2. For any given value of  $\sigma$  and  $v$ , the value  $\Delta\hat{\Pi}^*$  is U-shaped. This means that: (i) the differential  $\Delta\hat{\Pi}^*$  is large when  $|\Delta\tau|$  is large, i.e., a firm's ability to borrow allows to invest much earlier when a large variation of the tax rate is expected; (ii) the differential  $\Delta\hat{\Pi}^*$  is minimum for  $\Delta\tau \rightarrow 0$ . In this case, the effect of a firm's ability to borrow on investment timing is still positive, though it is relatively small.
3. Table 3 shows that the positive effect of  $\sigma$  on both thresholds points  $\hat{\Pi}^*$  ( $C = 0$ ) and  $\hat{\Pi}^*$  ( $C = C^*$ ) is substantial. This is due to the fact that volatility raises a firm's option value, with the effect that the investment is delayed. On the contrary, Table 4 shows that the effect of parameter  $v$ , representing the unitary cost of default, on  $\hat{\Pi}^*$  ( $C = C^*$ ) is almost negligible.

To sum up, we can say that the results of Tables 3 and 4 show that our results are robust: when debt finance is allowed, tax rate uncertainty has a smaller effect on investment timing decision for a wide range of parameter values.

v	1,000			3,000			6,000			9,000			12,000			15,000		
	$\bar{\pi}(C=0)$	$\bar{\pi}(C=C^*)$	$\Delta \bar{\pi}$	$\bar{\pi}(C=0)$	$\bar{\pi}(C=C^*)$	$\Delta \bar{\pi}$	$\bar{\pi}(C=0)$	$\bar{\pi}(C=C^*)$	$\Delta \bar{\pi}$	$\bar{\pi}(C=0)$	$\bar{\pi}(C=C^*)$	$\Delta \bar{\pi}$	$\bar{\pi}(C=0)$	$\bar{\pi}(C=C^*)$	$\Delta \bar{\pi}$	$\bar{\pi}(C=0)$	$\bar{\pi}(C=C^*)$	$\Delta \bar{\pi}$
-0.25	21.86	20.05	1.81	21.86	20.82	1.04	21.86	21.08	0.78	21.86	21.14	0.72	21.86	21.16	0.70	21.86	21.17	0.69
-0.20	22.12	20.48	1.64	22.12	21.12	1.00	21.12	21.46	-0.34	22.11	21.57	0.54	22.12	21.61	0.51	22.12	21.63	0.49
-0.15	22.41	21.02	1.39	22.41	21.53	0.88	22.41	21.87	0.54	22.41	22.00	0.41	22.41	22.06	0.35	22.41	22.09	0.32
-0.10	22.74	21.39	1.15	22.75	22.00	0.74	22.75	22.32	0.43	22.75	22.45	0.30	22.75	22.51	0.23	22.75	22.55	0.20
-0.05	23.12	22.13	0.99	23.12	22.51	0.61	23.12	22.79	0.33	23.12	22.91	0.21	23.12	22.97	0.15	23.12	23.01	0.11
0.00	23.54	22.65	0.89	23.54	23.03	0.51	23.54	23.28	0.27	23.55	23.39	0.16	23.55	23.44	0.11	23.55	23.47	0.07
0.05	24.00	23.12	0.88	24.02	23.54	0.48	24.02	23.77	0.24	24.02	23.86	0.15	24.02	23.91	0.11	24.02	23.93	0.09
0.10	24.53	23.53	1.00	24.55	24.01	0.54	24.55	24.26	0.29	24.55	24.34	0.21	24.55	24.37	0.18	24.55	24.39	0.16
0.15	25.12	23.89	1.23	25.14	24.44	0.70	25.14	24.71	0.43	25.15	24.80	0.35	25.15	24.83	0.32	25.15	24.84	0.31
0.20	25.77	24.18	1.59	25.80	24.80	0.99	25.80	25.13	0.67	25.81	25.23	0.58	25.82	25.26	0.55	25.82	25.27	0.54
0.25	26.51	24.39	2.12	26.54	25.09	1.44	26.56	25.30	1.06	26.55	25.63	0.93	26.57	25.67	0.90	26.57	25.68	0.89

TABLE 4:  $\Delta \bar{\pi}^* = \bar{\pi}^*(C=0) - \bar{\pi}^*(C=C^*)$  WITH  $C=0$  AND  $C=C^*$  AS FUNCTION OF  $\Delta t$  AND  $v$ .

## 4 Conclusion

In this article, we have applied a real-option model to study the effects of tax rate uncertainty on both investment timing and the optimal capital structure of a representative firm.

By departing from the relevant literature, which has extensively analyzed fully equity-financed investment decisions, we have shown that the ability to borrow allows firms to invest earlier in order to enjoy the tax benefit of interest deductibility. More importantly, we have shown that a highly volatile tax system may have a negligible impact on investment choices, when firms can choose their capital structure. This leads us to conclude that debt finance allows a firm to substantially smooth the distortive effects of tax-rate uncertainty.

Our results have interesting implications in terms of both empirical analysis and policy decisions. Given our findings, we can say that when investment activities are studied, empirical investigation should control for the existence (absence) of financial flexibility. Indeed, disregarding the characteristics of financial markets would be misleading.

Similarly, the effect of a hot policy debate on future (and uncertain) tax-rate changes may have a significantly negative impact on investment, if firms are credit-constrained. If however, financial markets are efficient and hence provide a sufficient amount of resources, the same debate may lead to a negligible impact on investment, since firms can smooth the effects of tax-rate uncertainty by optimally adjusting their capital structure. It is worth noting that we assumed the absence of any renegotiation of debt. Of course we expect that, whenever renegotiation is allowed, a firm enjoys a higher degree of flexibility and therefore, the effect of tax-rate uncertainty is further mitigated.

In this article we have used other simplifying assumptions, such as the symmetric treatment of profits and losses, as well as the absence of personal taxation, agency costs and any bargaining process between stakeholders (including renegotiation and partial conversion of debt into equity). We believe that the elimination of any of these simplifying assumptions is an interesting topic that we leave for future research. Finally, the evidence shows that tax uncertainty is caused by both tax-rate and tax-base changes (e.g., via changes in investment tax credits and fiscal depreciation allowances). We therefore believe that a promising extension of our model would entail the joint analysis of tax-rate and tax-base uncertainty.

## A The value functions

In order to calculate a firm's value function with tax rate uncertainty we must first focus on the value function after the tax-rate change, i.e., when tax rate is

$\tau_1$ . Subsequently, we will deal with the value function under tax rate uncertainty, i.e., when the current tax rate is  $\tau_0$ .

## A.1 The value function (4)

Using dynamic programming, let us calculate the equity value  $E_1(\Pi; C)$  as a summation between the net cash flow  $(1 - \tau_1)(\Pi - C)$ , in the short interval  $dt$ , and its future value after the instant  $dt$  has passed:

$$E_1(\Pi; C) = (1 - \tau_1)(\Pi - C) dt + e^{-rdt} \xi [E_1(\Pi + d\Pi; C)], \quad (17)$$

where  $\xi [E(\Pi + d\Pi; C)]$  is the expected value of equity at time  $t + dt$ . Expanding the RHS of (17), applying Itô's Lemma and rearranging gives the following non-arbitrage condition:

$$rE_1(\Pi; C) = (1 - \tau_1)(\Pi - C) + (r - \delta)\Pi E_{1\Pi}(\Pi; C) + \frac{\sigma^2}{2}\Pi^2 E_{1\Pi\Pi}(\Pi; C), \quad (18)$$

where  $\delta \equiv r - \alpha$ ,  $E_{1\Pi} \equiv \frac{\partial E}{\partial \Pi}$  and  $E_{1\Pi\Pi} \equiv \frac{\partial^2 E}{\partial \Pi^2}$ . The general-form solution of (18) is

$$E_1(\Pi; C) = \begin{cases} 0 & \text{after default,} \\ (1 - \tau_1) \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) + \sum_{i=1}^2 A_i \Pi^{\beta_i} & \text{before default,} \end{cases} \quad (19)$$

where  $\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$  and  $\beta_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$  are the roots of the characteristic equation  $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + (r-\delta)\beta - r = 0$ .

Let us next calculate  $A_1$  and  $A_2$ . In the absence of any financial bubbles,  $A_1$  is nil. To calculate  $A_2$ , we must consider that default occurs when  $\Pi$  drops to  $C$ , namely the condition

$$E_1(C; C) = (1 - \tau_1) \left( \frac{C}{\delta} - \frac{C}{r} \right) + A_2 C^{\beta_2} = 0$$

holds. Rearranging this equation gives:

$$A_2 = - \left[ (1 - \tau_1) \left( \frac{C}{\delta} - \frac{C}{r} \right) \right] C^{-\beta_2}.$$

Using these results we can therefore rewrite (19) as

$$E_1(\Pi; C) = \begin{cases} 0 & \text{after default,} \\ (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] & \text{before default.} \end{cases} \quad (20)$$

Following the same procedure we can write  $D_1(\Pi; C)$  as:

$$D_1(\Pi; C) = C + e^{-rdt} \xi [D_1(\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô's Lemma and rearranging gives the following non-arbitrage condition:

$$rD_1(\Pi; C) = \begin{cases} (1 - \tau_1)\Pi + (r - \delta)\Pi D_{1\Pi}(\Pi; C) + \frac{\sigma^2}{2}\Pi^2 D_{1\Pi\Pi}(\Pi; C) & \text{after default,} \\ C + (r - \delta)\Pi D_{1\Pi}(\Pi; C) + \frac{\sigma^2}{2}\Pi^2 D_{1\Pi\Pi}(\Pi; C) & \text{before default.} \end{cases} \quad (21)$$

The closed-form solution of (21) is:

$$D_1(\Pi; C) = \begin{cases} \frac{(1-\tau_1)\Pi}{\delta} + \sum_{i=1}^2 B_i \Pi^{\beta_i} & \text{after default,} \\ \frac{C}{r} + \sum_{i=1}^2 D_i \Pi^{\beta_i} & \text{before default.} \end{cases} \quad (22)$$

To calculate  $B_2$  we use the boundary condition  $D(0; C) = 0$ , which means that when  $\Pi$  falls to zero the lender's post-default claim is nil. This implies that  $B_2 = 0$ . In the absence of any financial bubble, we have  $B_1 = D_1 = 0$ . Finally, to calculate  $D_2$  we let the pre-default branch of (22) equate its after-default one, net of the default cost  $vC$ , at point  $\Pi = C$ . We thus obtain:

$$\frac{C}{r} + D_2 C^{\beta_2} = \frac{(1 - \tau)C}{\delta} - vC. \quad (23)$$

Solving (23) for  $D_2$  gives  $D_2 = \left[ \frac{(1-\tau_1)C}{\delta} - \frac{C}{r} - vC \right] C^{-\beta_2}$ , and hence, (22) reduces to:

$$D_1(\Pi; C) = \begin{cases} \frac{(1-\tau_1)\Pi}{\delta} & \text{after default,} \\ \left[ \frac{1}{r} + \left( \frac{1-\tau_1}{\delta} - \frac{1}{r} - v \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] C & \text{before default.} \end{cases} \quad (24)$$

The summation of (20) and (24), net of the investment cost  $I$ , gives:

$$\begin{aligned} V_1(\Pi; C) &= E_1(\Pi; C) + D_1(\Pi; C) - I \\ &= \frac{(1-\tau_1)\Pi}{\delta} + \frac{\tau_1 C}{r} - \left( \frac{\tau_1}{r} + v \right) C \left( \frac{\Pi}{C} \right)^{\beta_2} - I. \end{aligned} \quad (25)$$

## A.2 The value function (5)

Following the same procedure we can calculate the value function before the tax rate change. Again, we write the value of equity as:

$$E_0(\Pi; C) = (1 - \tau_0)(\Pi - C) + (1 - \lambda dt) e^{-rdt} \xi [E_0(\Pi + d\Pi; C)] + \lambda dt \xi [E_1(\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô's Lemma and rearranging gives:

$$(r + \lambda) E_0(\Pi; C) = (1 - \tau_0)(\Pi - C) + (r - \delta)\Pi E_{0\Pi}(\Pi; C) + \frac{\sigma^2}{2}\Pi^2 E_{0\Pi\Pi}(\Pi; C) + \lambda E_1(\Pi; C). \quad (26)$$

Let us next subtract (18) from (26) so that:

$$(r + \lambda) X(\Pi; C) = (\tau_1 - \tau_0)(\Pi - C) + (r - \delta)\Pi X_\Pi(\Pi; C) + \frac{\sigma^2}{2}\Pi^2 X_\Pi(\Pi; C), \quad (27)$$

where

$$X(\Pi; C) \equiv E_0(\Pi; C) - E_1(\Pi; C). \quad (28)$$

Solving (27) gives

$$X(\Pi; C) = (\tau_1 - \tau_0) \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) + \sum_{i=1}^2 F_i \Pi^{\beta_i(\lambda)}, \quad (29)$$

where

$$\beta_1(\lambda) = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(r + \lambda)}{\sigma^2}} > 1,$$

$$\beta_2(\lambda) = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(r + \lambda)}{\sigma^2}} < 0,$$

are the roots of the characteristic equation

$$\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + (r - \delta)\beta - (r + \lambda) = 0.$$

Notice that, in the absence of bubbles, we have  $F_1 = 0$ . Using (28) and (29), and rearranging, we obtain:

$$\begin{aligned} E_0(\Pi; C) &= E_1(\Pi; C) + X(\Pi; C) \\ &= (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] \\ &\quad + (\tau_1 - \tau_0) \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) + F_2 \Pi^{\beta_2(\lambda)}. \end{aligned}$$

Using condition  $E_0(C; C) = 0$  and solving for  $F_2$  we can find

$$F_2 = -(\tau_1 - \tau_0) \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) C^{-\beta_2(\lambda)}.$$

Hence, the value of equity is equal to:

$$\begin{aligned} E_0(\Pi; C) &= E_1(\Pi; C) + X(\Pi; C) \quad (30) \\ &= (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] \\ &\quad + (\tau_1 - \tau_0) \left[ \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) - \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \right]. \end{aligned}$$



Let us now calculate the value of debt before the tax rate change. As usual, we can write it as:

$$D_0(\Pi; C) = C + (1 - \lambda dt) e^{-rdt} \xi [D_0(\Pi + d\Pi; C)] + \lambda dt \xi [D_1(\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô's Lemma and rearranging gives the following non-arbitrage condition:

$$(r + \lambda) D_0(\Pi; C) = C + (r - \delta) \Pi D_{0\Pi}(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 D_{0\Pi\Pi}(\Pi; C) + \lambda D_1(\Pi; C). \quad (31)$$

Subtracting (21) from (31) gives:

$$(r + \lambda) Y(\Pi; C) = (r - \delta) \Pi Y_{\Pi}(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 Y_{\Pi\Pi}(\Pi; C), \quad (32)$$

with

$$Y(\Pi; C) \equiv D_0(\Pi; C) - D_1(\Pi; C). \quad (33)$$

The solution of (32) has the following form:

$$Y(\Pi; C) = \begin{cases} \frac{(\tau_1 - \tau_0)\Pi}{\delta + \lambda} + \sum_{i=1}^2 L_i \Pi^{\beta_i(\lambda)} & \text{after default,} \\ \sum_{i=1}^2 G_i \Pi^{\beta_i(\lambda)} & \text{before default.} \end{cases}$$

Notice that, after default (but before the tax-rate change), the boundary condition  $Y(0; C) = 0$  holds. This implies that  $L_2 = 0$ . Moreover, in the absence of bubbles, we have  $L_1 = G_1 = 0$ .

Remember that, after default, the lender becomes shareholders. Therefore, using (32) and rearranging, we can write the firm's value after default:

$$D_0(\Pi; C) = \left[ \frac{(1 - \tau_1)}{\delta} + \frac{(\tau_1 - \tau_0)}{\delta + \lambda} \right] \Pi.$$

Following the same procedure we can write the before-default value of debt:

$$D_0(\Pi; C) = \left\{ \frac{1}{r} + \left[ \frac{(1 - \tau_1)}{\delta} - \frac{1}{r} - v \right] \left( \frac{\Pi}{C} \right)^{\beta_2} \right\} C + G_2 \Pi^{\beta_2(\lambda)}. \quad (34)$$

To find  $G_2$ , we let the two branches of the debt function meet at point  $\Pi = C$ , and account for the default cost. This means that the equality

$$\frac{1 - \tau_1}{\delta} C + G_2 C^{\beta_2(\lambda)} = \left( \frac{1 - \tau_1}{\delta} + \frac{\tau_1 - \tau_0}{\delta + \lambda} \right) C - vC \quad (35)$$

holds. Solving (35) for  $G_2$  and substituting the result into (34) gives:

$$D_0(\Pi; C) = \left[ \frac{1}{r} + \left( \frac{1 - \tau_1}{\delta} - \frac{1}{r} - v \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] C + \frac{\tau_1 - \tau_0}{\delta + \lambda} C \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)}. \quad (36)$$

Using (30) and (36) we can finally calculate the firm's NPV:

$$\begin{aligned} V_0(\Pi; C) = & (1 - \tau_1) \left[ \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) - \left( \frac{C}{\delta} - \frac{C}{r} \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] + \\ & + (\tau_1 - \tau_0) \left[ \left( \frac{\Pi}{\delta + \lambda} - \frac{C}{r + \lambda} \right) - \left( \frac{C}{\delta + \lambda} - \frac{C}{r + \lambda} \right) \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} \right] \\ & + \left[ \frac{1}{r} + \left( \frac{1 - \tau_1}{\delta} - \frac{1}{r} - v \right) \left( \frac{\Pi}{C} \right)^{\beta_2} \right] C + \frac{\tau_1 - \tau_0}{\delta + \lambda} C \left( \frac{\Pi}{C} \right)^{\beta_2(\lambda)} - I. \end{aligned} \quad (37)$$

## B The option functions

### B.1 The option function (6)

Using dynamic programming we can write a firm's option to invest under tax rate certainty as:

$$O_1(\Pi; C) = e^{-rdt} \xi [O_1(\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô's Lemma and rearranging gives the following non-arbitrage condition:

$$rO_1(\Pi; C) = (r - \delta)\Pi O_{1\Pi}(\Pi; C) + \frac{\sigma^2}{2}\Pi^2 O_{1\Pi\Pi}(\Pi; C). \quad (38)$$

As shown in Dixit and Pindyck (1994), the general-form solution of (38) is:

$$O_1(\Pi; C) = H_1 \Pi^{\beta_1}, \quad (39)$$

where  $H_1$  is an unknown. To calculate  $H_1$ , we apply the VMC at the entry threshold level ( $\Pi = \bar{\Pi}$ ), i.e.,

$$V_1(\Pi; C)|_{\Pi=\bar{\Pi}} = O_1(\Pi; C)|_{\Pi=\bar{\Pi}}, \quad (40)$$

and therefore, obtain

$$\begin{aligned} H_1 = & V_1(\bar{\Pi}; C) \bar{\Pi}^{-\beta_1} = \\ = & \left[ \frac{(1 - \tau_1)\bar{\Pi}}{\delta} + \frac{\tau_1 C}{r} - (\tau_1 + rv) \frac{C}{r} \left( \frac{\bar{\Pi}}{C} \right)^{\beta_2} - I \right] \bar{\Pi}^{-\beta_1} \end{aligned} \quad (41)$$

Substituting (41) into (39), using (40) and rearranging gives the option function (6).

## B.2 The option function (7)

Following the same procedure, let write a firm's option to invest as:

$$O_0(\Pi; C) = (1 - \lambda dt) e^{-rdt} \xi [O_0(\Pi + d\Pi; C)] + \lambda dt \xi [O_1(\Pi + d\Pi; C)].$$

Expanding its RHS, applying Itô's Lemma and rearranging gives the following non-arbitrage condition:

$$(r + \lambda) O_0(\Pi; C) = (r - \delta) \Pi O_{0_{\text{III}}}(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 O_{0_{\text{III}}}(\Pi; C) + \lambda O_1(\Pi; C). \quad (42)$$

Subtracting (38) from (42) gives

$$(r + \lambda) Z(\Pi; C) = (r - \delta) \Pi Z_{\text{II}}(\Pi; C) + \frac{\sigma^2}{2} \Pi^2 Z_{\text{III}}(\Pi; C), \quad (43)$$

where  $Z(\Pi; C) \equiv O_0(\Pi; C) - O_1(\Pi; C)$ . Solving (43) we have:

$$Z(\Pi; C) = \sum_{i=1}^2 Z_i \Pi^{\beta_i(\lambda)}.$$

Since  $Z(0; C) = 0$ , we obtain

$$Z(\Pi; C) = Z_1 \Pi^{\beta_1(\lambda)},$$

and therefore, the option value can be rewritten as follows:

$$\begin{aligned} O_0(\Pi; C) &= O_1(\Pi; C) + Z(\Pi; C) \\ &= H_1 \Pi^{\beta_1} + Z_1 \Pi^{\beta_1(\lambda)}. \end{aligned} \quad (44)$$

To calculate  $Z_1$  we apply the VMC at the threshold point  $\Pi = \hat{\Pi}$ :

$$V_0(\Pi; C)|_{\Pi=\hat{\Pi}} = O_0(\Pi; C)|_{\Pi=\hat{\Pi}}. \quad (45)$$

Using (44) and (45) we obtain:

$$H_1 \hat{\Pi}^{\beta_1} + Z_1 \hat{\Pi}^{\beta_1(\lambda)} = V_0(\hat{\Pi}; C), \quad (46)$$

which gives

$$Z_1 = \left[ V_0(\hat{\Pi}; C) - H_1 \hat{\Pi}^{\beta_1} \right] \hat{\Pi}^{-\beta_1(\lambda)}.$$

Substituting (41) into  $Z(\Pi; C)$  gives

$$Z_1 \Pi^{\beta_1(\lambda)} = \left[ V_0(\hat{\Pi}; C) - H_1 \hat{\Pi}^{\beta_1} \right] \left( \frac{\Pi}{\hat{\Pi}} \right)^{\beta_1(\lambda)}. \quad (47)$$

Substituting (47) into (46), using (45) and rearranging gives (7).

## C The firm's choice under tax-rate certainty

Under tax-rate certainty, the firm's problem is (9).

### C.1 The solutions

The first order conditions of (9) w.r.t.  $C$  and  $\bar{\Pi}$  are

$$\left(\frac{\bar{\Pi}}{\bar{\Pi}}\right)^{\beta_1} \frac{1}{r} \left[ \tau_1 - (1 - \beta_2)(\tau_1 + vr) \left(\frac{\bar{\Pi}}{C}\right)^{\beta_2} \right] = 0, \quad (48)$$

$$\left(\frac{\bar{\Pi}}{\bar{\Pi}}\right)^{\beta_1} \frac{1}{\bar{\Pi}} \left\{ \frac{(1-\tau_1)\bar{\Pi}}{r} - \beta_2 \frac{C}{r} (\tau_1 + vr) \left(\frac{\bar{\Pi}}{C}\right)^{\beta_2} + \right. \\ \left. - \beta_1 \left[ \frac{(1-\tau_1)\bar{\Pi}}{r} + \frac{C}{r} \left( \tau_1 - (\tau_1 + vr) \left(\frac{\bar{\Pi}}{C}\right)^{\beta_2} \right) - I \right] \right\} = 0, \quad (49)$$

respectively. Rearranging (48) gives

$$C = \left( \frac{1}{1 - \beta_2} \frac{\tau_1}{\tau_1 + vr} \right)^{-\frac{1}{\beta_2}} \bar{\Pi}. \quad (50)$$

Moreover, from (48) we obtain

$$\frac{\tau_1}{1 - \beta_2} = (\tau_1 + vr) \left(\frac{\bar{\Pi}}{C}\right)^{\beta_2},$$

and hence, we can rewrite (49) as

$$\left[ \frac{(1 - \tau_1)\bar{\Pi}}{r} - \beta_2 \frac{C}{r} \frac{\tau_1}{1 - \beta_2} \right] - \beta_1 \left[ \frac{(1 - \tau_1)\bar{\Pi}}{r} + \frac{C}{r} \left( \tau_1 - \frac{\tau_1}{1 - \beta_2} \right) - I \right] = 0. \quad (51)$$

Rearranging and dividing (51) by  $\frac{(1-\beta_1)(1-\tau_1)}{r}$  one obtains:

$$\bar{\Pi} + \frac{\tau_1}{1 - \tau_1} \frac{\beta_2}{\beta_2 - 1} C - \frac{\beta_1}{\beta_1 - 1} \frac{r}{1 - \tau_1} I = 0. \quad (52)$$

Substituting (50) into (52) gives

$$\bar{\Pi}_1^* = \frac{1}{1 + m_1} \frac{\beta_1}{\beta_1 - 1} \frac{r}{1 - \tau_1} I,$$

where  $m_1 \equiv \frac{\tau_1}{1 - \tau_1} \frac{\beta_2}{\beta_2 - 1} \left( \frac{1}{1 - \beta_2} \frac{\tau_1}{\tau_1 + vr} \right)^{-\frac{1}{\beta_2}} > 0$ . Solution (10) is thus obtained.

## C.2 Comparative statics

Let us next provide some comparative statics for  $\bar{\Pi}_1^*$ ,  $\bar{C}_1$  and  $m_1$ . Their derivatives w.r.t.  $\tau_1$  are

$$\begin{aligned}\frac{\partial \bar{\Pi}_1^*}{\partial \tau_1} &= \frac{\beta_1}{\beta_1 - 1} \frac{rI}{(1 - \tau_1)^2} \frac{1 + m_1 - \frac{\partial m_1}{\partial \tau_1} (1 - \tau_1)}{(1 + m_1)^2}, \\ \frac{\partial \bar{C}_1}{\partial \tau_1} &= \left( \frac{(1 - \beta_2)(\tau_1 + vr)}{\tau_1} \right)^{\frac{1}{\beta_2}} \left( \frac{\partial \bar{\Pi}_1^*}{\partial \tau_1} - \frac{1}{\beta_2 \tau_1} \frac{rv}{\tau_1 + vr} \bar{\Pi}_1^* \right), \\ \frac{\partial m_1}{\partial \tau_1} &= \frac{\beta_2}{\beta_2 - 1} \left( \frac{1}{1 - \beta_2} \right)^{-\frac{1}{\beta_2}} \left( 1 + \frac{vr}{\tau_1} \right)^{\frac{1}{\beta_2}} \frac{(\tau_1 + vr)\beta_2 - vr(1 - \tau_1)}{(1 - \tau_1)^2 \beta_2 (\tau_1 + vr)} > 0.\end{aligned}\tag{53}$$

It is easy to show that

$$\frac{\partial \bar{\Pi}_1^*}{\partial \tau_1} > 0 \text{ if } \frac{\partial m_1}{\partial \tau_1} < \frac{1 + m_1}{1 - \tau_1},$$

where

$$\frac{\partial m_1}{\partial \tau_1} < \frac{1 + m_1}{1 - \tau_1} \iff \frac{\beta_2(\tau_1 + vr) - vr}{(\beta_2 - 1)(\tau_1 + vr)} \left( \frac{1}{1 - \beta_2} \frac{\tau_1}{\tau_1 + vr} \right)^{-\frac{1}{\beta_2}} < 1.$$

Since  $\frac{\beta_2(\tau_1 + vr) - vr}{(\beta_2 - 1)(\tau_1 + vr)} < 1$  and  $\left( \frac{\tau_1}{(1 - \beta_2)(\tau_1 + vr)} \right)^{-\frac{1}{\beta_2}} < 1$ , inequalities  $0 < \frac{\partial m_1}{\partial \tau_1} < \frac{1 + m_1}{1 - \tau_1}$  hold. This implies that  $\frac{\partial \bar{\Pi}_1^*}{\partial \tau_1} > 0$ .

Let us next study the derivative  $\frac{\partial \bar{C}_1}{\partial \tau_1}$ . It is worth noting that, given  $\frac{\partial \bar{\Pi}_1^*}{\partial \tau_1} > 0$ , it is always positive.

Given these results one can easily obtain:

$$\frac{\partial \left( \frac{\bar{C}_1}{\bar{\Pi}_1^*} \right)}{\partial \tau_1} = - \left( \frac{1}{1 - \beta_2} \right)^{-\frac{1}{\beta_2}} \frac{1}{\beta_2} \left( \frac{\tau_1}{\tau_1 + vr} \right)^{-\frac{1 - \beta_2}{\beta_2}} \frac{rv}{(\tau_1 + vr)^2} > 0.$$

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