Risk Premia in General Equilibrium

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Abstract

This paper shows that non-linearities imposed by a neoclassical production function alone can generate time-varying and asymmetric risk premia over the business cycle. These (empirical) key features become relevant, and asset market implications improve substantially when we allow for non-normalities in the form of rare disasters. We employ analytical solutions of dynamic stochastic general equilibrium models, including a novel solution with endogenous labor supply, to obtain closed-form expressions for the risk premium in production economies. In contrast to endowment economies, the curvature of the policy functions affects the risk premium through controlling the individual's effective risk aversion.

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Keywords: risk premium, continuous-time DSGE.

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1 Introduction

"... the challenge now is to understand the economic forces that determine the stochastic discount factor, or put another way, the rewards that investors demand for bearing particular risks." (Campbell, 2000, p.1516)

In general equilibrium models, the stochastic discount factor is not only determined by the consumption-based first-order condition, but also linked to business cycle characteristics. In macroeconomics, dynamic stochastic general equilibrium models (DSGE) have been very successful in explaining co-movements in aggregate data, but relatively less progress has been made to reconcile their asset market implications with financial data (cf. Grinols and Turnovsky, 1993; Jermann, 1998, 2010; Tallarini, 2000; Lettau and Uhlig, 2000; Boldrin, Christiano and Fisher, 2001; Lettau, 2003; Campanale, Castro and Clementi, 2010).[1] One main advantage of using general equilibrium models to explain asset market phenomena is that the asset-pricing kernel is consistent with the macro dynamics, which offers excellent guidance to the future development of models in both macroeconomics and finance.

However, surprisingly little is known about the risk premium in non-linear DSGE models.[2] At least two primary questions present themselves. Which economic forces determine the market risk premium in general equilibrium? What are the implications of using production based models compared to the endowment economy? This paper fills the gap by studying asset pricing implications of prototype DSGE models analytically.[3] Why is this important? We argue that a clear understanding of the risk premium can best be achieved by working out analytical solutions. These solutions are shown to be important knife-edge cases which can therefore be used to shed light on our numerical results.

In a nutshell, this paper shows that a neoclassical production function alone generates key features of the risk premium. The economic intuition is that individual's effective risk aversion, excluding singular cases, is not constant in a neoclassical production economy.

We use analytical solutions of DSGE models. For this purpose we readopt formulating models in continuous-time (as in Merton, 1975; Eaton, 1981; Cox, Ingersoll and Ross, 1985) which gives closed-form solutions for a broad class of models and parameter sets.[4] Recent research emphasizes the importance of non-linearities and non-normalities in explaining busi-

[1]There is an increasing interest in DSGE models in finance (cf. Kaltenbrunner and Lochstoer, 2006). A survey of the literature on the intersection between macro and finance is Cochrane (2008, chap. 7).

[2]Grinols and Turnovsky (1993) and Turnovsky and Bianconi (2005) study asset pricing implications of aggregate risk and/or idiosyncratic shocks in stochastic endogenous growth models with a quasi-linear production technology. Our formulation focuses on non-linear DSGE models with transitional dynamics.

[3]Our approach differs from the 'analytical' approach of Campbell (1994), as we obtain exact solutions.

[4]Analytical solutions to continuous-time DSGE models can be found in Turnovsky (1993, 2000), Corsetti (1997), Wälde (2005), Turnovsky and Smith (2006), and Posch (2009).

ness cycle dynamics for the US economy (Fernández-Villaverde and Rubio-Ramírez, 2007; Justiniano and Primiceri, 2008; Posch, 2009). To illustrate our general equilibrium pricing approach, the starting point is Lucas' fruit-tree endowment economy with rare disasters. We obtain closed-form expressions for the 'implicit risk premium' from the Euler equation and relate it to the 'market risk premium'. Subsequently the framework is extended to a neoclassical production economy and (non-tradable) human wealth with endogenous labor supply. Our approach still gives closed-form expressions under parametric restrictions.

The major findings can be summarized as follows. Non-linearities in DSGE models can generate time-varying and asymmetric risk premia over the business cycle.[5] Although these key features of the risk premium are negligible in the standard real business cycle model, we show that they become relevant, and asset market implications improve substantially, when we allow for non-normalities in the form of rare disasters (Rietz, 1988; Barro, 2006, 2009). Our result is based on the finding that the individual's effective risk aversion is not constant for non-homogeneous consumption functions (cf. Carroll and Kimball, 1996).[6]

One caveat of discrete-time models is the lack of analytical solutions. To some extent, the gap between the literature of asset pricing models in finance using endowment models and typically non-linear production economies in dynamic macroeconomics is due to the difficulty of solving these models. In particular by focusing on the effects of uncertainty, the traditional approach of linearization about the non-stochastic steady state does not provide an adequate framework. Alternatively, the literature suggests the use of risk-sensitive objectives (Hansen, Sargent and Tallarini, 1999; Tallarini, 2000) or log-linearization methods (Campbell, 1994; Lettau, 2003). Similarly, numerical strategies employ perturbation and higher-order approximation schemes (cf. Taylor and Uhlig, 1990; Schmitt-Grohé and Uribe, 2004; van Binsbergen, Fernández-Villaverde, Koijen and Rubio-Ramírez, 2010). Although these methods usually are locally highly accurate, the effects of large economic shocks, such as rare disasters on approximation errors, are largely unexplored.

Our continuous-time formulation does not suffer from such limitations. First, we exploit closed-form solutions, which are available for reasonable parametric restrictions, to study the determinants of the risk premium analytically. Second, we use powerful numerical methods to examine the properties of the risk premium for a broader parameter range without relying on local approximations (Posch and Trimborn, 2010). We obtain the optimal policy functions and risk premia in the neoclassical production economy, while our closed-form solutions can

[5]While the time-varying feature is well documented empirically (Welch and Goyal, 2008), there is some evidence that the risk premium increases more in bad times than it decreases in good times (Mele, 2008).

[6]Other contributions to Mehra and Presott's (1985) equity premium puzzle for endowment economies, e.g. Epstein and Zin (1989); Abel (1990, 1999); Constantinides (1990); Campbell and Cochrane (1999); Veronesi (2004); Bansal and Yaron (2004), are generating time-varying risk aversion through different channels.

be used to gauge the accuracy of the numerical method for large economic shocks. Thus we propose this formulation as a workable paradigm in the macro-finance literature.

Our paper is closely related to Lettau (2003), who derives asset pricing implications in a real business cycle model using log-linear approximations. Using this approach gives the endogenous variables (in logs) as a linear function of the state variables (in logs). What are we missing by using log-linear approximations? As a result, the risk premium is a function of the (constant) elasticities of consumption with respect to the state variables, comparable to our knife-edge solutions. But in fact the risk premium exhibits time-varying and asymmetric behavior due to changes in effective risk aversion. Thus, we overlook potentially important properties of the risk premium implied by the neoclassical production economy. We show that closed-form results can be obtained in production economies for parametric restrictions, which in turn shed light on the risk premium in the general case.

Our finding relates to Jermann (2010), who studies the determinants of the risk premium as implied by producers' first-order conditions. The author identifies the adjustment cost curvature and the investment volatility as key determinants of the risk premium, similar to our ingredients such as the policy function curvature and the consumption volatility.

There is a literature documenting that the Barro-Rietz rare disaster hypothesis generates a sizable risk premium.[7] The most fundamental critique, however, is on the calibration of rare disasters. Although there is empirical evidence that economic disasters have been sufficiently frequent and large enough to make the hypothesis viable (cf. Barro, 2006), we emphasize that our results do not crucially depend on the rare disaster hypothesis. Two reasons make the hypothesis an excellent candidate for making the implications of the neoclassical production function visible (which for small risk premia would be negligible). First, it substantially increases the level of the risk premium without loosing analytical tractability. Second, it does not require other forms of non-linearities such as habit formation or recursive preferences which allows us to obtain very sharp results. Thus we do not contribute to the debate of why the historic equity premium seems too high given the low aggregate consumption volatility and our priors about risk aversion. In contrast, we confirm that the ability to buffer risk makes it even more challenging to generate sizable risk premia in production economies.

The remainder of the paper is organized as follows. Section 2 solves in closed form a continuous-time version of Lucas' fruit-tree model with exogenous, stochastic production and obtains the risk premium. Section 3 studies the effects of non-linearities on the risk premium in Merton's neoclassical growth model. Section 4 concludes.

[7] Gabaix (2008) and Wachter (2009) suggest variable intensity versions together with recursive preferences, which not only generates a time-varying risk premium but also increases the level of the premium, as a viable explanation for several macro-finance puzzles. A more critical view is in Julliard and Gosh (2008).

2 An endowment economy

This section illustrates our general equilibrium approach to compute the risk premium in an endowment economy, i.e., the minimum difference an individual requires in normal times to accept an uncertain rate of return between the expected value and the (shadow) risk-free rate the individual is indifferent to. It also shows how rare disasters can account for the equity premium puzzle which became known as the Barro-Rietz 'rare disaster hypothesis'.

2.1 Lucas' fruit-tree model with rare disasters

Consider a fruit-tree economy (one risky asset or equity), and a riskless asset in normal times with default risk (government bond) similar to Barro (2006).

2.1.1 Description of the economy

Technology. Consider an endowment economy (Lucas, 1978). Suppose production is entirely exogenous: no resources are utilized, and there is no possibility of affecting the output of any unit at any time, $Y_t = A_t$ where A_t is the stochastic technology. Output is perishable. The law motion of A_t will be taken to follow a Markov process,

$$dA_t = \bar{\mu} A_t dt + \bar{\sigma} A_t dB_t + (e^{\bar{\nu}} - 1) A_{t-} dN_t, \tag{1}$$

where B_t is a standard Brownian motion, N_t is a standard Poisson process at arrival rate λ, whereas $\bar{\mu}$ and $\bar{\sigma}$ determine the conditional instantaneous mean and variance of percentage changes in output. The jump size is assumed to be a constant fraction of output, $e^{\bar{\nu}} - 1$, an instant before the jump, A_{t-}, ensuring that A_t does not jump negative.

In this economy the bonds with default risk are issued exogenously by the government. Suppose that the price of the government bill follows

$$dp_0(t) = p_0(t) r dt + p_0(t_-) D_t dN_t, \tag{2}$$

where D_t is a random variable denoting a random default risk in case of a disaster, and q is the probability of default (cf. Barro, 2006). For illustration, we assume

$$D_t = \left\{ \begin{array}{rcl} 0 & \text{with} & 1-q \\ e^{\kappa} - 1 & \text{with} & q \end{array} \right. .$$

Ownership of fruit-trees is determined at each instant in a competitive stock market, and the production unit has one outstanding perfectly divisible equity share. A share entitles its owner to all of the unit's instantaneous output in t. Shares are traded at a competitively determined price, p_t. Suppose that for the risky asset,

$$dp_t = \mu p_t dt + \sigma p_t dB_t + p_{t-} J_t dN_t, \tag{3}$$

where J_t is a random variable denoting the jump risk.

Because prices fully reflect all available information, the parameters r, μ, σ and J_t will be determined in general equilibrium. The objective is to relate exogenous productivity changes to the market determined movements in asset prices. In fact, the evolution of prices ensures that assets are priced such that individuals are indifferent between holding more assets and consuming. Given initial wealth, we are looking for the optimal consumption path.

Preferences. Consider an economy with a single consumer, interpreted as a representative "stand in" for a large number of identical consumers. The consumer maximizes discounted expected life-time utility discounted at the subjective rate of time preference $\rho > 0$,

$$E \int_0^\infty e^{-\rho t} u(C_t) dt, \quad u' > 0, \; u'' < 0.$$

Assuming no dividend payments, the budget constraint reads

$$dW_t = ((\mu - r) w_t W_t + r W_t - C_t) \, dt + w_t \sigma W_t dB_t + ((J_t - D_t) w_{t-} + D_t) W_{t-} dN_t, \tag{4}$$

where W_t is real financial wealth, and w_t denotes a consumer's share holdings.

Equilibrium properties. In this economy, it is easy to determine equilibrium quantities of consumption and asset holdings. The economy is closed and all output will be consumed, $C_t = Y_t$, and all shares will be held by capital owners.

2.1.2 The short-cut approach

In a companion paper we solve the more comprehensive approach considering both portfolio selection and consumption. It is straightforward to show that the portfolio selection problem can be separated from the consumption problem - a result we use throughout the paper.

Suppose that the only asset is the *market portfolio*,

$$dp_M(t) = \mu_M p_M(t) dt + \sigma_M p_M(t) dB_t - \zeta_M(t_-) p_M(t_-) dN_t, \tag{5}$$

where $\zeta_M(t)$ is considered as an exogenous stochastic jump-size. With probability q it takes the value ζ_M, and with probability $1 - q$ the jump size is ζ_M^0. Thus, the consumer obtains capital income and has to finance his or her consumption stream from wealth,

$$dW_t = (\mu_M W_t - C_t) \, dt + \sigma_M W_t dB_t - \zeta_M(t_-) W_{t-} dN_t. \tag{6}$$

One can think of the original problem with budget constraint (4) as having been reduced to a simple Ramsey problem in which we seek an optimal consumption rule given that income is generated by the uncertain yield of a (composite) asset (cf. Merton, 1973).

Define the *value function* as

$$V(W_0) \equiv \max_{\{C_t\}_{t=0}^{\infty}} E_0 \int_0^{\infty} e^{-\rho t} u(C_t) dt, \quad s.t. \quad (6), \quad W_0 > 0. \tag{7}$$

Choosing the control $C_s \in \mathbb{R}_+$ at time s, the Bellman equation reads

$$\begin{aligned} \rho V(W_s) \;\; = \;\; & \max_{C_s} \left\{ u(C_s) + (\mu_M W_s - C_s) V_W + \tfrac{1}{2}\sigma_M^2 W_s^2 V_{WW} \right. \\ & \left. + (V((1-\zeta_M)W_s)q + V((1-\zeta_M^0)W_s)(1-q) - V(W_s))\lambda \right\}. \end{aligned} \tag{8}$$

Because it is a necessary condition, the *first-order conditions* reads

$$u'(C_s) - V_W(W_s) \;\; = \;\; 0 \quad \Rightarrow \quad V_W(W_s) = u'(C_s) \tag{9}$$

for any interior solution at any time $s = t \in [0, \infty)$.

It can be shown that the *Euler equation* is (cf. appendix)

$$\begin{aligned} du'(C_t) \;\; = \;\; & \big((\rho - \mu_M + \lambda)u'(C_t) - \sigma_M^2 W_t u''(C_t) C_W - u'(C((1-\zeta_M)W_t))(1-\zeta_M)q\lambda \\ & - u'(C((1-\zeta_M^0)W_t))(1-\zeta_M^0)(1-q)\lambda\big) dt \\ & + \pi u'(C_t) dB_t + (u'(C((1-\zeta_M(t_-))W_{t-})) - u'(C(W_{t-}))) dN_t, \end{aligned} \tag{10}$$

which implicitly determines the optimal consumption path, where we define the market price of diffusion risk as $\pi \equiv \sigma_M W_t u''(C_t) C_W / u'(C_t)$. Moreover, we define C_W as the marginal propensity to consume out of wealth, i.e., the slope of the *consumption function*. Using the inverse function, we are able to determine the path for consumption ($u'' \neq 0$).

To shed light on the effects of uncertainty we follow an approach similar to Steger (2005), rewriting the Euler equation (10) and obtaining

$$\begin{aligned} \rho - \frac{1}{dt} E\left[\frac{du'(C_t)}{u'(C_t)}\right] \;\; = \;\; & \mu_M - E\left[-\frac{u''(C_t)}{u'(C_t)} C_W W_t \sigma_M^2 + \frac{u'(C((1-\zeta_M)W_t))}{u'(C(W_t))} \zeta_M q \lambda \right. \\ & \left. + \frac{u'(C((1-\zeta_M^0)W_s))}{u'(C(W_t))} \zeta_M^0 (1-q)\lambda \right]. \end{aligned} \tag{11}$$

In equilibrium, the *certainty equivalent rate of return*, i.e., the expected rate of return on saving (conditioned on no disasters) less the expected *implicit risk premium*,

$$RP_t \;\equiv\; -\frac{u''(C_t)}{u'(C_t)} C_W W_t \sigma_M^2 + E^{\zeta}\left[\frac{u'(C((1-\zeta_M(t))W_t))}{u'(C(W_t))} \zeta_M(t)\lambda\right], \tag{12}$$

equals expected cost of forgone consumption, i.e., the subjective rate of time preference, and the expected rate of change of marginal utility on the left-hand side in (11). It denotes the percentage spread between the certainty equivalent rate of return (shadow risk-free rate) and the average rate of return of risky asset in normal times. For samples which include sufficiently many disasters such that the observed frequency is equal to the true probability, the (unconditional) expected rate of return on the market portfolio is $\mu_M - E(\zeta_M(t))\lambda$.

The implicit risk premium as from (12) extends the 'proportional probability premium' in static utility-of-wealth models (Pratt, 1964) to dynamic consumption-portfolio models. It is related to the effective relative risk aversion of the indirect utility function,

$$RRA_W = -\frac{V_{WW}W_t}{V_W} = -\frac{u''(C_t)C_W W_t}{u'(C_t)}. \tag{13}$$

Hence, the indirect utility function, i.e., the value function, and the utility function are intimately linked by the optimality condition (9). This condition demands that the marginal utility of consumption equals the marginal utility of wealth (cf. Breeden, 1979, p.274).

2.1.3 General equilibrium prices

This section shows that general equilibrium conditions pin down the prices in the economy. From the Euler equation (10), we obtain

$$\begin{aligned} dC_t \;=\; & \big((\rho - \mu_M + \lambda)u'(C_t)/u''(C_t) - \sigma_M^2 W_t C_W - \tfrac{1}{2}\sigma_M^2 W_t^2 C_W^2 u'''(C_t)/u''(C_t) \\ & - E^{\zeta}\left[u'(C((1-\zeta_M(t))W_t))(1-\zeta_M(t))\right]\lambda/u''(C_t)\big)dt \\ & + \sigma_M W_t C_W dB_t + (C((1-\zeta_M(t))W_{t-}) - C(W_{t-}))dN_t, \end{aligned} \tag{14}$$

where we employed the inverse function $c = g(u'(c))$ which has

$$g'(u'(c)) = 1/u''(c), \quad g''(u'(c)) = -u'''(c)/(u''(c))^3.$$

Economically, concave utility ($u'(c) > 0$, $u''(c) < 0$) implies risk aversion, whereas convex marginal utility, $u'''(c) > 0$, implies a positive precautionary saving motive. Accordingly, $-u''(c)/u'(c)$ measures absolute risk aversion, whereas $-u'''(c)/u''(c)$ measures the degree of absolute prudence, i.e., the intensity of the precautionary saving motive (Kimball, 1990b).

Because output is perishable, using the market clearing condition $Y_t = C_t = A_t$,

$$dC_t = \bar{\mu}C_t dt + \bar{\sigma}C_t dB_t + (e^{\bar{\nu}} - 1)C_{t-}dN_t. \tag{15}$$

Thus, the general equilibrium approach pins down asset prices as follows. Defining optimal jump in consumption as $\tilde{C}(W_t) \equiv C((1-\zeta_M(t))W_t)/C(W_t)$, market clearing requires the percentage jump in aggregate consumption to match the disaster size, $e^{\bar{\nu}} - 1 = \tilde{C}(W_t) - 1$, which implies a constant jump term. For example, if consumption is linearly homogeneous in wealth (as shown for CRRA preferences below), the jump of the asset price satisfies[8]

$$C((1-\zeta_M(t))W_{t-})/C(W_{t-}) = 1 - \zeta_M(t) \quad \Rightarrow \quad \zeta_M = \zeta_M^{\text{o}} = 1 - e^{\bar{\nu}}. \tag{16}$$

[8]Conditioning on no default, $(\zeta_M(t)|D_t = 0) = \zeta_M^{\text{o}}$, gives $e^{\bar{\nu}} - 1 = -\zeta_M^{\text{o}}$, whereas conditioning on default, $(\zeta_M(t)|D_t = e^{\kappa} - 1) = \zeta_M$, demands $e^{\bar{\nu}} - 1 = -\zeta_M$.

Similarly, the market clearing condition pins down $\sigma_M W_t C_W = \bar{\sigma} C_t$, and thus

$$\mu_M - r = -\frac{u''(C_t)C_t^2}{u'(C_t)C_W W_t}\bar{\sigma}^2 - \frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}\left((1-e^{\kappa})q + e^{\bar{\nu}} - 1\right)\lambda,$$

where

$$r = \rho - \frac{u''(C_t)C_t}{u'(C_t)}\bar{\mu} - \tfrac{1}{2}\frac{u'''(C_t)C_t^2}{u'(C_t)}\bar{\sigma}^2 + \lambda - (1-(1-e^{\kappa})q)\frac{u'(e^{\bar{\nu}}C_t)}{u'(C_t)}\lambda. \tag{17}$$

As a result, the higher the subjective rate of time preference, ρ, the higher is the general equilibrium interest rate to induce individuals to defer consumption (cf. Breeden, 1986). For convex marginal utility (decreasing absolute risk aversion), $u'''(c) > 0$, a lower conditional variance of dividend growth, $\bar{\sigma}^2$, a higher conditional mean of dividend growth, $\bar{\mu}$, and a higher default probability, q, decrease the bond price and increase the interest rate.

2.1.4 Components of the risk premium

Observe that the implicit risk premium (12) in general equilibrium simplifies to

$$RP_t = \underbrace{-\frac{u''(C_t)}{u'(C_t)}C_W W_t \sigma_M^2}_{\text{diffusion risk}} + \underbrace{\frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}\zeta_M \lambda}_{\text{total jump risk}}, \tag{18}$$

whereas the conditional market premium reads

$$\begin{aligned}
\mu_M - r &= \underbrace{-\frac{u''(C_t)C_W W_t}{u'(C(W_t))}\sigma_M^2}_{\text{diffusion risk}} + \underbrace{(\zeta_M - (1-e^{\kappa})q)\frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}\lambda}_{\text{disaster risk}} \\
&= \underbrace{-\frac{u''(C_t)C_W W_t}{u'(C(W_t))}\sigma_M^2}_{\text{diffusion risk}} + \underbrace{\frac{u'(e^{\bar{\nu}}C(W_s))}{u'(C(W_t))}\zeta_M \lambda}_{\text{total jump risk}} - \underbrace{(1-e^{\kappa})\, q \frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}\lambda}_{\text{default risk}}.
\end{aligned} \tag{19}$$

Note that one would expect $\bar{\nu} < 0$ and $\kappa < 0$ for a 'disaster' hypothesis.

In the presence of default risk, the conditional market premium differs from the implicit risk premium. The reason is that we obtain the implicit risk premium from the certainty equivalent rate of return (shadow risk-free rate), but the government bill has a risk of default. This default risk is not rewarded in the market as there is no truly riskless asset, but it is reflected in the implicit risk premium. If there was no default risk, the implicit risk premium would have the usual interpretation of the conditional market premium.

2.1.5 Explicit solutions

As shown in Merton (1971), the standard dynamic consumption and portfolio selection problem has explicit solutions where consumption is a linear function of wealth. For later reference, we provide the solution for constant relative risk aversion (CRRA).

Proposition 2.1 (CRRA preferences) *If utility exhibits constant relative risk aversion, i.e.,* $-u''(C_t)C_t/u'(C_t) = \theta$*, optimal consumption is linear in wealth,* $C_t = C(W_t) = bW_t$*, where the marginal propensity to consume out of wealth is*

$$b \equiv \left(\rho + \lambda - (1-\theta)\mu_M - (1-\zeta_M)^{1-\theta}\lambda + (1-\theta)\theta\tfrac{1}{2}\sigma_M^2\right)/\theta.$$

The effective relative risk aversion of the indirect utility in (13) is constant, $RRA_W = \theta$*.*

Proof. see Appendix A.1.3 ■

Corollary 2.2 *Use the optimal policy function,* $C_t = C(W_t) = bW_t$*, and the conditional market premium in general equilibrium (19) to obtain*

$$\mu_M - r = \theta\sigma_M^2 + e^{-\theta\bar{\nu}}\zeta_M\lambda - e^{-\theta\bar{\nu}}\left(1 - e^{\kappa}\right)q\lambda, \tag{20}$$

with the conditional variance of the market portfolio $\sigma_M = \bar{\sigma}$*, the jump size of the market portfolio* $\zeta_M = 1 - e^{\bar{\nu}}$*, and the riskless rate in (17) as*

$$r = \rho + \theta\bar{\mu} - \tfrac{1}{2}\theta(1+\theta)\bar{\sigma}^2 + \lambda - \left(1 - (1-e^{\kappa})q\right)e^{-\theta\bar{\nu}}\lambda.$$

The unconditional market premium, i.e., for long samples, is $\mu_M - \zeta_M\lambda - (r - (1-e^{\kappa})q\lambda)$*.*

Corollary 2.3 *Use the optimal policy function,* $C_t = C(W_t) = bW_t$*, and the implicit risk premium in general equilibrium (18), to obtain*

$$RP = \theta\bar{\sigma}^2 + e^{-\theta\bar{\nu}}(1-e^{\bar{\nu}})\lambda = \mu_M - r + e^{-\theta\bar{\nu}}(1-e^{\kappa})q\lambda. \tag{21}$$

Similar to Barro (2006), the traditional risk premium in (21) increases by $e^{-\theta\bar{\nu}}(1-e^{\bar{\nu}})\lambda$, which can be sizable. Thus, the intuition why rare disaster may solve the equity premium puzzle is straightforward. Even for logarithmic utility, $\theta = 1$, and for low-probability events, $\lambda = 0.01$, the premium for the jump risk in percentage points, $e^{-\bar{\nu}} - 1$, can be very large. For the case of 'disasters' one would expect $\bar{\nu}$ to be negative. The more negative is the parameter, the more severe is the disaster and $\bar{\nu} \to -\infty$ denotes complete destruction.

As we show below, the reason that the risk premium is constant is that the consumption function is homogeneous (of degree $k = 1$), which implies that effective risk aversion is constant. A time-varying disaster size and/or arrival rate (i.e., stochastic volatility) can even lead to a level increase of the risk premium (cf. Gabaix, 2008; Wachter, 2009).

2.1.6 Stochastic discount factor

This section illustrates the link between the implicit risk premium and the stochastic discount factor (SDF). Similar to the implicit risk premium, the SDF follows from the Euler equation (10), which in general equilibrium is

$$\begin{aligned} du'(C_t) &= (\rho - r)u'(C_t)dt + (1-e^{\kappa})u'(C(e^{\bar{\nu}}W_t))q\lambda dt - (u'(C(e^{\bar{\nu}}W_t)) - u'(C_t))\lambda dt \\ &\quad +\pi u'(C_t)dB_t + (u'(C(e^{\bar{\nu}}W_{t-})) - u'(C(W_{t-})))dN_t, \end{aligned}$$

where the deterministic term consists of, firstly, the difference between the subjective rate of time preference and the riskless rate, secondly, a term which transforms this rate into the certainty equivalent rate of return (shadow risk-free rate) and, thirdly, the compensation which transforms the Poisson process into a martingale. For $s \geq t$, we may write

$$\begin{aligned} d\ln u'(C_t) &= \left(\frac{u''(C_t)C_t}{u'(C_t)}\bar{\mu} + \tfrac{1}{2}\frac{u'''(C_t)C_t^2}{u'(C_t)}\bar{\sigma}^2 - \tfrac{1}{2}\pi^2 \right) dt \\ &\quad +\pi dB_t + \left(\ln u'(C(e^{\bar{\nu}}W_{t-})) - \ln u'(C(W_{t-}))\right) dN_t \\ \Leftrightarrow \quad \frac{e^{-(s-t)\rho}u'(C_s)}{u'(C_t)} &= \exp\left(-\int_t^s \left(\rho - \frac{u''(C_v)C_v}{u'(C_v)}\bar{\mu} - \tfrac{1}{2}\frac{u'''(C_v)C_v^2}{u'(C_v)}\bar{\sigma}^2 + \tfrac{1}{2}\pi^2 \right) dv \right) \\ &\quad \times \exp\left(\int_t^s \pi dB_v + \int_t^s \left(\ln u'(C(e^{\bar{\nu}}W_{t-})) - \ln u'(C(W_{t-}))\right) dN_v \right), \end{aligned}$$

i.e., equating discounted marginal utility in s and t. Therefore,

$$m_s/m_t \equiv \frac{e^{-(s-t)\rho}u'(C_s)}{u'(C_t)} \tag{22}$$

defines the *stochastic discount factor* (also known as the pricing kernel or state-price density) which can be used to price any asset in this economy. For CRRA preferences, it reads

$$\begin{aligned} m_s/m_t &= \exp\left(-(r - e^{-\theta\bar{\nu}}(1-e^{\kappa})q\lambda + \tfrac{1}{2}(\theta\bar{\sigma})^2 + (e^{-\bar{\nu}\theta} - 1)\lambda)(s-t)\right) \\ &\quad \times \exp\left(\theta\bar{\sigma}(B_s - B_t) - \theta\bar{\nu}(N_s - N_t)\right) \\ &= \exp\left(-(\rho + \theta\bar{\mu} - \tfrac{1}{2}\theta\bar{\sigma}^2)(s-t) + \theta\bar{\sigma}(B_s - B_t) - \theta\bar{\nu}(N_s - N_t)\right) \end{aligned}$$

which has the standard properties (cf. Campbell, 2000). This result illustrates that the Euler equation (10) can be used to compute both the implicit risk premium and the SDF in any continuous-time DSGE model without explicitly studying asset pricing implications.

3 A neoclassical production economy

This section illustrates that non-linearities in a neoclassical DSGE model imply interesting asset market implications, in particular these can generate a time-varying risk premium. We use a version of Merton's (1975) asymptotic theory of growth under uncertainty.

3.1 A model of growth under uncertainty

This section assumes that there is no truly riskless asset. We employ the certainty equivalent rate of return - or the shadow risk-free rate - to obtain the implicit risk premium.

3.1.1 Description of the economy

Technology. At any time, the economy has some amounts of capital, labor, and knowledge, and these are combined to produce output. The production function is a constant return to scale technology $Y_t = A_t F(K_t, L)$, where K_t is the aggregate capital stock, L is the constant population size, and A_t is the stock of knowledge or total factor productivity (TFP), which in turn is driven by a standard Brownian motion B_t,

$$dA_t = \bar{\mu} A_t dt + \bar{\sigma} A_t dB_t. \tag{23}$$

A_t has a log-normal distribution with $E_0(\ln A_t) = \ln A_0 + (\bar{\mu} - \frac{1}{2}\bar{\sigma}^2)t$, and $Var_0(\ln A_t) = \bar{\sigma}^2 t$.

The capital stock increases if gross investment exceeds stochastic capital depreciation,

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t + (e^{\nu} - 1)K_{t-} dN_t, \tag{24}$$

where Z_t is a standard Brownian motion (uncorrelated with B_t), and N_t is a standard Poisson process with arrival rate λ. Unlike in Merton's (1975) model, the assumption of stochastic depreciation introduces instantaneous riskiness making physical capital indeed a risky asset (for similar examples see Turnovsky, 2000). The fundamental difference to Lucas' endowment economy is that the outstanding equity shares follow a stochastic process as well, i.e., not only the production technology but also the number of trees is stochastic.

Preferences. Consider an economy with a single consumer interpreted as a representative "stand in" for a large number of identical consumers. The consumer maximizes expected life-time utility

$$E_0 \int_0^{\infty} e^{-\rho t} u(C_t) dt, \quad u' > 0, \ u'' < 0 \tag{25}$$

subject to

$$dW_t = ((r_t - \delta)W_t + w_t^L - C_t)dt + \sigma W_t dZ_t + (e^{\nu} - 1)W_{t-} dN_t. \tag{26}$$

$W_t \equiv K_t/L$ denotes individual wealth, r_t is the rental rate of capital, and w_t^L is labor income. The paths of factor rewards are taken as given by the representative consumer.

Equilibrium properties. In equilibrium, factors of production are rewarded with value marginal products, $r_t = Y_K$ and $w_t^L = Y_L$. The goods market clearing condition demands

$$Y_t = C_t + I_t. \tag{27}$$

Solving the model requires the aggregate accumulation constraints (23) and (24), the goods market equilibrium (27), equilibrium factor rewards of competitive firms, and the first-order condition for consumption. It gives a system of stochastic differential equations which, given initial conditions, determines the paths of K_t, Y_t, r_t, w_t^L and C_t, respectively.

3.1.2 The short-cut approach

Define the value of the optimal program as

$$V(W_0, A_0) = \max_{\{C_t\}_{t=0}^{\infty}} E_0 \int_0^{\infty} e^{-\rho t} u(C_t) dt \quad s.t. \quad (26) \quad \text{and} \quad (23), \quad W_0, A_0 > 0, \tag{28}$$

which denotes the present value of expected utility along the optimal program. Similar to the endowment economy, we obtain the first-order condition for the problem as

$$u'(C_t) = V_W(W_t, A_t) \tag{29}$$

for any $t \in [0, \infty)$, making consumption a function of the state variables $C_t = C(W_t, A_t)$.

It can be shown that the *Euler equation* is (cf. appendix)

$$\begin{aligned} du'(C_t) \;=\; & (\rho - (r_t - \delta) + \lambda)u'(C_t)dt - u'(C(e^{\nu}W_t, A_t))e^{\nu}\lambda dt - \sigma^2 u''(C_t)C_W W_t dt \\ & + u''(C_t)(C_A A_t \bar{\sigma} dB_t + C_W W_t \sigma dZ_t) \\ & + [u'(C(e^{\nu}W_{t-}, A_{t-})) - u'(C(W_{t-}, A_{t-}))]dN_t, \end{aligned} \tag{30}$$

which implicitly determines the optimal consumption path. To shed some light on the effects of uncertainty in the production economy, we rewrite the Euler equation and obtain

$$\rho - \frac{1}{dt}E\left[\frac{du'(C_t)}{u'(C_t)}\right] \;=\; E(r_t) - \delta - E\left[-\frac{u''(C_t)}{u'(C_t)}C_W W_t \sigma^2 + \frac{u'(C(e^{\nu}W_t, A_t))}{u'(C(W_t, A_t))}(1 - e^{\nu})\lambda\right].$$

In equilibrium, the *certainty equivalent rate of return*, i.e., the expected return on the risky asset net of depreciation, $E(r_t - \delta)$, less the expected *implicit risk premium*,

$$RP_t \equiv -\frac{u''(C_t)}{u'(C_t)}C_W W_t \sigma^2 + \frac{u'(C(e^{\nu}W_t, A_t))}{u'(C(W_t, A_t))}(1 - e^{\nu})\lambda, \tag{31}$$

equals the cost of forgone consumption. It is remarkable that the structure is equivalent to the endowment economy (18), but the premium in (31) has quite interesting properties. The most obvious result is that the implicit risk premium indeed refers to the rewards that investors demand for bearing the systematic risk, while it does not directly account for the risk of a stochastically changing total factor productivity (23).

3.1.3 Explicit solutions

A convenient way to describe the behavior of the economy is in terms of the evolution of C_t, A_t and W_t. Similar to the endowment economy we obtain explicit solutions. Due to the non-linearities they are available only for specific parametric restrictions. Below we use two known restrictions where the *policy function* $C_t = C(A_t, W_t)$ (or consumption function) is available, and all economic variables can be solved in closed form.

Proposition 3.1 (linear-policy-function) *If the production function is Cobb-Douglas, $Y_t = A_t K_t^\alpha L^{1-\alpha}$, utility exhibits constant relative risk aversion, i.e., $-u''(C_t)C_t/u'(C_t) = \theta$, and $\alpha = \theta$, then optimal consumption is linear in wealth.*

$$\alpha = \theta \quad \Rightarrow \quad C_t = C(W_t) = \phi W_t$$
$$\textit{where} \quad \phi \equiv (\rho - (e^{(1-\theta)\nu} - 1)\lambda + (1-\theta)\delta)/\theta + \tfrac{1}{2}(1-\theta)\sigma^2 \tag{32}$$

Proof. see Appendix A.2.2 ■

Corollary 3.2 *Using the policy function $C_t = C(W_t) = \phi W_t$ and (31),*

$$RP = \theta\sigma^2 + e^{-\theta\nu}(1 - e^\nu)\lambda. \tag{33}$$

Proposition 3.3 (constant-saving-function) *If the production function is Cobb-Douglas, $Y_t = A_t K_t^\alpha L^{1-\alpha}$, utility exhibits constant relative risk aversion, i.e., $-u''(C_t)C_t/u'(C_t) = \theta$, and the subjective rate of time preference is*

$$\bar{\rho} \equiv (e^{(1-\alpha\theta)\nu} - 1)\lambda - \theta\bar{\mu} + \tfrac{1}{2}\left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\theta)\sigma^2\right) - (1-\alpha\theta)\delta, \tag{34}$$

then optimal consumption is proportional to current income (non-linear in wealth).

$$\rho = \bar{\rho} \quad \Rightarrow \quad C_t = C(W_t, A_t) = (1-s)A_t W_t^\alpha, \quad \theta > 1, \quad \textit{where} \quad s \equiv 1/\theta \tag{35}$$

Proof. see Appendix A.3.2 ■

Corollary 3.4 *Using the policy function $C_t = C(W_t, A_t) = (1-s)A_t W_t^\alpha$ and (31),*

$$RP = \alpha\theta\sigma^2 + e^{-\alpha\theta\nu}(1 - e^\nu)\lambda. \tag{36}$$

We are now in a position to understand why the (implicit) risk premium depends on the curvature of the policy function (or consumption function): Any homogenous consumption function, where $C_W(W_t, A_t)W_t = kC(W_t, A_t)$ or equivalently $C(cW_t, A_t) = c^k C(W_t, A_t)$ for $c, k \in \mathbb{R}_+$, implies a constant risk premium. Technically, the policy function is homogenous of degree k in wealth. Because these functions are obtained only for knife-edge restrictions, we conclude that the (implicit) risk premium generally will be dependent on wealth which in turn implies a time-varying behavior since wealth is changing stochastically.

Economically, the reason why the risk premium depends on the curvature of the policy function (and can vary over time) is that the optimal response to disasters or shocks depends on the level of wealth. An individual with high levels of financial wealth will adjust his or her optimal expenditures for consumption differently from an individual that has no financial wealth. Though the utility function has CRRA with respect to consumption, the indirect utility function (the value function) does *not* exhibit CRRA with respect to wealth except for the knife-edge cases above. This finding is closely related to the link Kimball (1990a) shows between the marginal propensity to consume and the effective risk aversion of the value function. Accordingly, a higher marginal propensity to consume out of gross wealth (inclusive of labor income) raises the effective risk aversion. A concave consumption function implies that effective risk aversion falls more quickly with wealth than it does for the case of a constant marginal propensity to consume (Carroll and Kimball, 1996, p.982).

There are two important differences to the earlier work. First, our consumption function is defined by the optimal policy rule which gives consumption as a function of financial wealth (exclusive of labor income), i.e., the only observable and tradable asset. Hence, it cannot easily be interpreted as a function of gross wealth (inclusive of labor income) or total wealth (i.e., financial and human wealth). Thus, the marginal propensity to consume out of wealth is defined by the slope of the consumption function with respect to financial wealth. In contrast, the consumption and saving rates - in the tradition of the Solow model - refer to current income (i.e., labor and capital income). Second, the effects of uncertainty are studied in a general equilibrium environment which allows us to obtain analytical solutions for linear and strictly concave consumption functions in a DSGE model for certain parametric restrictions, while Carroll and Kimball restrict their focus on partial equilibrium models leaving the processes for labor income and capital returns exogenous.

Our finding sheds light on how total factor productivity (TFP) generally affects the risk premium in (31) though the consumer is not interested in hedging TFP risk directly. For any consumption function non-homogeneous in wealth, effective relative risk aversion depends on TFP through the consumption function, e.g., for CRRA preferences,

$$RP_t = \theta\sigma^2 C_W(W_t, A_t)W_t/C(W_t, A_t) + (1 - e^\nu)\lambda(C(e^\nu W_t, A_t)/C(W_t, A_t))^{-\theta}. \qquad (37)$$

Unfortunately, an analytical study of the structural parameters in the general case is not possible. Though clearly being knife-edge cases, our explicit solutions are important to understand the mechanisms that affect the risk premium in DSGE models. Both solutions imply that the consumption function is homogenous and thus a constant risk premium.[9] Below we study the implications when allowing the parameters to take different values.

3.1.4 Numerical solutions

This section implements the algorithm as in Posch and Trimborn (2010) to obtain a numerical solution for the case where $\sigma = \bar{\sigma} = \bar{\mu} = 0$, and with $A = 1$. As it is seen below, this assumption does not affect our conclusions, but reduces the computational burden as the reduced form representing the dynamics of the DSGE model can be summarized as

$$\begin{aligned} dW_t &= ((r_t - \delta)W_t + w_t^L - C_t)dt - (1 - e^\nu)W_{t-}dN_t, \\ dC_t &= -\frac{u'(C_t)}{u''(C_t)}(r_t - \delta - \rho - \lambda)dt - \frac{u'(C(e^\nu W_t))}{u''(C(W_t))}e^\nu \lambda dt + [C(e^\nu W_{t-}) - C(W_{t-})]dN_t, \end{aligned}$$

where $r_t = Y_K$ and $w_t^L = Y_L$. For the case of Cobb-Douglas production, $Y_t = AK_t^\alpha L^{1-\alpha}$, and CRRA preferences with relative risk aversion θ, we obtain from (31) or (37)

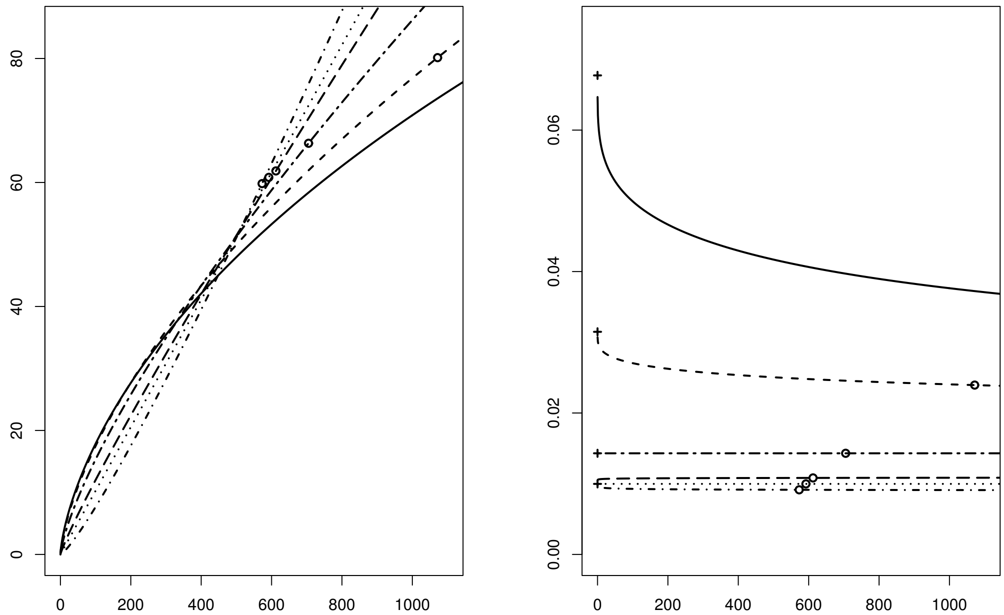
$$RP_t \equiv \frac{C(e^\nu W_t)^{-\theta}}{C(W_t)^{-\theta}}(1 - e^\nu)\lambda. \tag{38}$$

The numerical solution to the non-linear system of stochastic differential equations is the policy function, $C_t = C(W_t)$, which is obtained from the optimal paths of control and state variables computed for the complete state space $W_t \in \mathbb{R}_+$. In particular our procedure does not rely on local approximation methods, but directly solves the system using the Waveform relaxation algorithm (cf. Posch and Trimborn, 2010). According to (38), we obtain the risk premium by evaluating the optimal policy function at two points in the state space.

Figure 1 illustrates the optimal policy function and the resulting implicit risk premium (38) for different values for the parameter of relative risk aversion. For $\theta = \alpha$ the policy function is a linear-homogenous function with slope ϕ which corresponds to the analytical solution in (32). In this singular case the risk premium is $e^{-\theta\nu}(1 - e^\nu)\lambda$ (equivalent to the risk premium in the endowment economy). At each point, the change of the expected proportional decline in marginal utility equals the change in capital rewards, implying a constant risk premium in (38). For $\theta < \alpha$ the policy function is convex, and the marginal propensity to consume increases with wealth, $C(e^\nu W_t) < e^\nu C(W_t)$. This increase, however, is less rapid than the increase of the consumption-wealth ratio which lowers the effective level of risk aversion. Hence, the risk premium is convex and has the upper bound $e^{-\alpha\nu}(1 - e^\nu)\lambda$ for wealth approaching zero. For $\theta > \alpha$, which is the empirically most plausible scenario,

[9] For $\alpha = \theta$, the consumption function becomes a linear function in wealth, i.e., it is linearly homogeneous or homogeneous of degree one. In the case of $\rho = \bar{\rho}$, which is only possible for values $\theta > 1$, the consumption function becomes homogeneous of degree α.

Figure 1: Risk premia in a production economy



Notes: These figures illustrate the optimal policy functions (left panel) and the risk premium (right panel) as functions of financial wealth for different levels of relative risk aversion for the case of $\sigma = \bar{\sigma} = \bar{\mu} = 0$. The calibrations of other parameters is $(\rho, \alpha, \theta, \delta, \lambda, 1 - e^{\nu}) = (.05, .75, \cdot, .1, .017, .4)$ where $\theta = .5$ (dotdash), $\theta = .75$ (dotted), $\theta = 1$ (longdash), $\theta = 1.9406$ (twodash) which refers to the knife-edge case $\rho = \bar{\rho}$ in (35) with a constant saving rate, $\theta = 4$ (dashed), and $\theta = 6$ (solid).

the consumption function has the standard form, i.e., strictly concave and the marginal propensity to consume is decreasing with wealth, $C(e^{\nu}W_t) > e^{\nu}C(W_t)$. In this case, the properties of the risk premium (38) depend on whether the subjective rate of time preference ρ exceeds or falls short of the knife-edge value $\bar{\rho}$ in (34).

At the knife-edge value of $\rho = \bar{\rho}$ the policy function is homogeneous of degree α which refers to the analytical solution in (35) where the savings rate is constant, $s = 1/\theta$, and the risk premium is $e^{-\alpha\theta\nu}(1 - e^{\nu})\lambda$. For $\rho < \bar{\rho}$ the individual prefers a higher savings rate, $s(W_t) > s$, and the marginal propensity to consume out of wealth decreases more rapidly than it would if the saving rate (or consumption-income ratio) were constant which lowers the effective risk aversion. Because the saving rate is increasing in wealth and bounded by unity, $s < s(W_t) < 1$, the risk premium is convex and has the upper bound $e^{-\alpha\theta\nu}(1 - e^{\nu})\lambda$ for wealth approaching zero. For $\rho > \bar{\rho}$ the saving rate is smaller, $s(W_t) < s$, and the marginal propensity to consume out of wealth decreases less rapidly than it would if the saving rate were constant which raises the effective risk aversion. Since the saving rate is decreasing in wealth, the risk premium in (38) is concave with lower bound $e^{-\theta\alpha\nu}(1-e^{\nu})\lambda$ for sufficiently risk averse individuals, $\theta \geq 1$. Otherwise the substitution effect dominates the precautionary savings effect which depresses savings and increases the marginal propensity to consume (Weil, 1990). Since the consumption function is concave for $\theta > \alpha$ due to the non-linear production function, effective risk aversion remains higher than for $\theta = \alpha$, such that the lower bound is $e^{-\max(\theta,1)\alpha\nu}(1-e^{\nu})\lambda$ for wealth approaching zero.

In our numerical study $\bar{\rho}$ depends on the arrival rate, λ, the disaster size, $e^{\nu}-1$, the output elasticity of capital, α, and the risk aversion, θ, which coincides with the inverse of the intertemporal elasticity of substitution (IES), and the rate of depreciation, δ. For the case $\alpha\theta > 1$, that is when the output elasticity of capital exceeds the IES, this critical value is positive, $\bar{\rho} > 0$, and vice versa. Thus, for the empirically most plausible calibrations, e.g., for $\alpha \approx 0.5$ and $\theta \approx 4$, we have $\alpha\theta > 1$ and obtain a positive knife-edge value, $\bar{\rho} > 0$.

Our numerical study of the risk premium thus gives the following results. Considering the empirically most plausible scenarios, we find that the risk premium is time-varying and asymmetric (concave) over the 'business cycle'. In prosperous states of the economy with higher transitional growth rates (capital scarcity), the risk premium is higher than in periods with lower - or even negative - growth rates (capital abundance). In other words, immediately after a disaster has occurred, the risk premium would jump to a higher level and then subsequently return to lower values as more capital is accumulated.

Allowing for (Gaussian) stochastic depreciation, $\sigma > 0$, and/or a second state variable in the form of time-varying TFP, $\bar{\mu} \neq 0$, $\bar{\sigma} > 0$, the implicit risk premium is (37), and the same analysis could be conducted. The consumption function is concave in wealth for $\theta \geq \alpha$ and the risk premium has the same properties as in Figure 1, conditioned on the state A_t. However, there are three main differences. First, since the individual is willing to hedge against the diffusion risk (stochastic investment opportunities), the risk premium will be slightly higher.[10] Second, the risk premium in general also depends indirectly on TFP trough the optimal consumption function, as described above. Finally, the knife-edge value $\bar{\rho}$ as from (35) decreases in the mean, $\bar{\mu}$, but increases in the variance $\bar{\sigma}^2$ of TFP growth. For the case $\alpha\theta > 1$ it increases in the variance of stochastic depreciation, σ^2.

3.1.5 Human wealth and financial wealth

Economically, the individual implicitly solves an intertemporal consumption problem in a stochastically changing investment opportunity set. In this view, the state variables which determine investment opportunities are the *aggregate* capital stock, K_t, and total factor productivity A_t, whereas the asset returns $r_t = r(A_t, K_t)$ and the wage rate $w_t = w(A_t, K_t)$ depend on the state variables. The DSGE model at hand is a specific case where general equilibrium conditions pin down asset prices as well as cost of capital and leisure (hours) in

[10]Since the diffusion risk is of less importance, this effect is negligible (cf. Tables A.1.3 and A.1.3).

the economy (cf. Campbell and Viceira, 2002, chap. 6).

In particular, the optimal decisions of households can be thought of in terms of financial wealth (physical assets) and human wealth, i.e., the present value of future labor income (Bodie, Merton and Samuelson, 1992, p.431).[11] It thus seems important to allow for flexible labor supply when studying the risk premium. Labor flexibility introduces an additional margin along which an individual can buffer risk (Turnovsky and Bianconi, 2005, p.325).

3.2 An extension: endogenous labor supply

This section allows for elastic labor supply in the neoclassical DSGE model. Our objective is to study how the time-varying property of the risk premium is affected by the ability of individuals to buffer risk through their labor-leisure choice. For reading convenience, this section replicates some of the equations from the previous section.

3.2.1 Description of the economy

Technology. The production function exhibits constant return to scale, $Y_t = A_t F(K_t, H_t)$, where K_t is the aggregate capital stock, H_t is total hours worked and A_t is total factor productivity, which in turn is driven by a standard Brownian motion B_t

$$dA_t = \bar{\mu} A_t dt + \bar{\sigma} A_t dB_t. \tag{39}$$

The capital stock increases if gross investment exceeds stochastic capital depreciation,

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t + (e^{\nu} - 1)K_{t-} dN_t, \tag{40}$$

where Z_t is a standard Brownian motion (uncorrelated with B_t), and N_t is a standard Poisson process with arrival rate λ, describing a counting process for the number of disasters.

Preferences. Consider an economy with a single consumer, interpreted as a representative "stand in" for a large number of identical consumers, such that $C_t = Lc_t = c_t$, where L is normalized to one, and $H_t = 1 - l_t$ with l_t denoting the amount of leisure the individual 'purchases' (cf. Bodie et al., 1992). The consumer seeks to maximize

$$E_0 \int_0^{\infty} e^{-\rho t} u(C_t, H_t) dt, \quad u_C > 0,\ u_H < 0,\ u_{CC} \leq 0,\ u_{CC} u_{HH} - (u_{CH})^2 \geq 0, \tag{41}$$

subject to

$$dW_t = ((r_t - \delta)W_t + H_t w_t^H - C_t)dt + \sigma W_t dZ_t + J_t W_{t-} dN_t. \tag{42}$$

[11] Bodie et al. (1992) show that the individual's human capital, which essentially is the same as a financial asset except that it is not traded, is valued by the individual as if it were a traded asset.

$W_t \equiv K_t/L$ denotes individual wealth, r_t is the rental rate of capital, and $H_t w_t^H$ is labor income. The paths of factor rewards are taken as given by the representative consumer.

Equilibrium properties. In equilibrium, factors of production are rewarded with value marginal products, $r_t = Y_K$ and $w_t^H = Y_H$. The goods market clearing condition demands

$$Y_t = C_t + I_t. \tag{43}$$

Solving the model requires the aggregate accumulation constraints (39) and (40), the goods market equilibrium (43), equilibrium factor rewards of competitive firms, and the first-order condition for consumption and hours. It gives a system of equations which, given initial conditions, determines the paths of K_t, Y_t, r_t, w_t^H, C_t and H_t, respectively.

3.2.2 The short-cut approach

Define the value function as

$$V(W_0, A_0) = \max_{\{C_t, H_t\}_{t=0}^{\infty}} E_0 \int_0^{\infty} e^{-\rho t} u(C_t, H_t) dt \quad s.t. \quad (42) \quad \text{and} \quad (39), \quad W_0, A_0 > 0, \tag{44}$$

denoting the present value of expected utility along the optimal program. It can be shown that the first-order conditions for any interior solution are (cf. appendix)

$$u_C(C_t, H_t) = V_W(W_t, A_t), \tag{45}$$

$$-u_H(C_t, H_t) = w_t^H V_W(W_t, A_t), \tag{46}$$

for any $t \in [0, \infty)$, making optimal consumption and hours functions of the state variables $C_t = C(W_t, A_t)$ and $H_t = H(W_t, A_t)$, respectively. Specifically it pins down the opportunity cost (or price) of leisure,

$$w_t^H = -\frac{u_H(C_t, H_t)}{u_C(C_t, H_t)}. \tag{47}$$

It can be shown that the *Euler equation* for consumption is (cf. appendix)

$$\begin{aligned} du_C = {} & (\rho - (r_t - \delta) + \lambda) u_C dt - u_C(C(e^{\nu} W_t, A_t), H(e^{\nu} W_t, A_t)) e^{\nu} \lambda dt \\ & -\sigma^2 \left(u_{CC}(C_t, H_t) C_W + u_{CH}(C_t, H_t) H_W \right) W_t dt \\ & +(C_A A_t \bar{\sigma} dB_t + C_W W_t \sigma dZ_t) u_{CC} + (H_A A_t \bar{\sigma} dB_t + H_W W_t \sigma dZ_t) u_{CH} \\ & + \left[\frac{u_C(C(e^{\nu} W_{t-}, A_{t-}), H(e^{\nu} W_{t-}, A_{t-}))}{u_C(C(W_{t-}, A_{t-}), H(W_{t-}, A_{t-}))} - 1 \right] u_C(C_{t-}, H_{t-}) dN_t, \end{aligned} \tag{48}$$

which implicitly determines the optimal consumption path. To shed some light on the effects of uncertainty in this economy, we rewrite the Euler equation and obtain

$$\begin{aligned} \rho - \frac{1}{dt} E \left[\frac{du_C(C_t, H_t)}{u_C(C_t, H_t)} \right] = {} & E(r_t - \delta) - E \left[-\frac{u_{CC}(C_t, H_t) C_W + u_{CH}(C_t, H_t) H_W}{u_C(C_t, H_t)} W_t \sigma^2 \right] \\ & -E \left[\frac{u_C(C(e^{\nu} W_t, A_t), H(e^{\nu} W_t, A_t))}{u_C(C(W_t, A_t), H(W_t, A_t))} (1 - e^{\nu}) \lambda \right]. \end{aligned}$$

Similar to the previous approaches the *implicit risk premium* is defined as

$$RP_t \equiv -\frac{u_{CC}C_W + u_{CH}H_W}{u_C(C_t, H_t)} W_t \sigma^2 + \frac{u_C(C(e^\nu W_t, A_t), H(e^\nu W_t, A_t))}{u_C(C(W_t, A_t), H(W_t, A_t))}(1 - e^\nu)\lambda. \quad (49)$$

Observe that the structure is equivalent to (31), with the notable difference that the curvature of both the policy function for consumption and hours matters for effective risk aversion, and thus the risk premium (for consumption and leisure not being separable).

Technically, to examine whether the time-varying property survives in this economy for utility exhibiting constant relative risk aversion, it is sufficient to examine whether consumers have the labor-leisure choice such that $u_{CC}C_W + u_{CH}H_W = u_{CC}C_t$ which again would yield the constant effective risk aversion similar to the endowment economy.

3.2.3 Explicit solutions

As before, a convenient way to describe the behavior of the economy is in terms of the evolution of C_t, H_t, A_t and W_t. Similar to the endowment economy there exist explicit solutions, however, due to the non-linearities these are only available for specific parameter restrictions. Below we use one restriction where the *policy functions* $C_t = C(A_t, W_t)$ and $H_t = H(A_t, W_t)$, and most economic variables of interest can be solved in closed form.

In what follows, we restrict our attention to the class of utility functions which exhibits constant relative risk aversion with respect to both consumption and leisure,

$$u(C_t, H_t) = \frac{(C_t(1 - H_t)^\psi)^{1-\theta}}{1 - \theta}, \quad \theta > 0, \ \psi \geq 0. \quad (50)$$

Similar to Turnovsky and Smith (2006), the parameter ψ measures the preference for leisure. To ensure concavity, we restrict $\theta - (1 - \theta)\psi \geq 0$. For the case where $\psi = 0$, the solutions in Proposition 3.1 and Proposition 3.5 apply accordingly. For the broader case where $\psi > 0$, a closed-form solution is obtained below. Both the following solution and the numerical solution techniques used for the general case are novel in the macro literature.

Proposition 3.5 (constant-saving-function) *If the production function is Cobb-Douglas, $Y_t = A_t K_t^\alpha H_t^{1-\alpha}$, utility exhibits constant relative risk aversion with respect to both leisure and consumption, i.e., $u_{HH}H_t/u_H = 1 - (1 - \theta)\psi$ and $-u_{CC}C_t/u_C = \theta$, respectively, and the subjective rate of time preference is*

$$\bar{\rho} \equiv (e^{\nu(1-\alpha\theta)} - 1)\lambda - (1 - \alpha\theta)\delta - \theta\bar{\mu} + \tfrac{1}{2}\left(\theta(1 + \theta)\bar{\sigma}^2 - \alpha\theta(1 - \alpha\sigma)\sigma^2\right),$$

then optimal consumption is proportional to income, and optimal hours are constant.

$$\rho = \bar{\rho} \quad \Rightarrow \quad C_t = (1 - s)A_t W_t^\alpha H^{1-\alpha}, \quad H = \frac{\theta(1 - \alpha)}{\theta(1 - \alpha) - \psi(1 - \theta)} \quad \theta > 1, \quad \psi \neq 0,$$

$$\textit{where} \quad s \equiv 1/\theta$$

Proof. see Appendix A.3.2 ■

Corollary 3.6 *Using the policy function* $C_t = C(W_t, A_t) = (1-s)A_t W_t^\alpha H^{1-\alpha}$ *and (49),*

$$RP = e^{-\alpha\theta\nu}(1-e^\nu)\lambda + \alpha\theta\sigma^2. \tag{51}$$

This particular rate of time preference, $\bar{\rho}$, clearly is a knife-edge condition which ensures that the optimal leisure, the saving rate and the implicit risk premium are constant. In this singular case, the parameter measuring the preference for leisure, ψ, does not affect the risk premium or the saving rate, though it affects hours. To study the dynamic effects of labor supply flexibility for a broader parameter set, we employ numerical solutions.

3.2.4 Numerical solutions

This section again uses the algorithm as in Posch and Trimborn (2010) to obtain a numerical solution for the case $\sigma = \bar{\sigma} = \bar{\mu} = 0$ and $A = 1$. In particular, the reduced-form dynamics then can be summarized by the budget constraint (42) and two Euler equations for both consumption and hours. Moreover, the condition (47) implies that optimal consumption can be expressed as a function of hours and wealth which again reduces the dimensionality.

For illustration, consider the case of Cobb-Douglas production, $Y_t = AK_t^\alpha H_t^{1-\alpha}$, with $A = 1$ and preferences as in (50). Optimal behavior as from (47) demands

$$1 - H(W_t) = \frac{\psi}{1-\alpha}\frac{C(W_t)H(W_t)^\alpha}{W_t^\alpha}, \quad 1 - H(e^\nu W_t) = \frac{\psi}{1-\alpha}\frac{C(e^\nu W_t)H(e^\nu W_t)^\alpha}{(e^\nu W_t)^\alpha}. \tag{52}$$

This pins down the optimal jump terms as

$$\tilde{C}(W_t) = \frac{1 - \tilde{H}(W_t)H(W_t)}{1 - H(W_t)}\tilde{H}(W_t)^{-\alpha}e^{\alpha\nu}, \tag{53}$$

where $\tilde{C}(W_t) \equiv C(e^\nu W_t)/C(W_t)$, and $\tilde{H}(W_t) \equiv H(e^\nu W_t)/H(W_t)$, such that e.g. $1 - \tilde{C}(W_t)$ denotes the percentage drop of optimal consumption after a disaster. As a result, we can neglect the Euler equation for consumption since technically (52) and (53) give optimal consumption as functions of optimal hours and financial wealth, $C(W_t) = C(H(W_t), W_t)$ and $\tilde{C}(W_t) = \tilde{C}(H(W_t))$.[12] Economically, optimal behavior of consumption is described completely by optimal hours and financial wealth through the algebraic condition (47).

As shown in the appendix, for $0 < H_t < 1$ the reduced form can thus be summarized as

$$\begin{aligned} dH_t &= \frac{\rho - (1-\theta)r_t + (1-\alpha\theta)\delta + \lambda - \alpha\theta C_t/W_t}{\alpha\theta H_t^{-1} - (\psi - \theta\psi - \theta)(1-H_t)^{-1}}dt \\ &\quad - \frac{\tilde{C}(W_t)^{-\theta+(1-\theta)\psi}\tilde{H}(W_t)^{(1-\theta)\psi\alpha}e^{\nu-(1-\theta)\psi\alpha\nu}\lambda}{\alpha\theta H_t^{-1} - (\psi - \theta\psi - \theta)(1-H_t)^{-1}}dt + (H(e^\nu W_{t-}) - H(W_{t-}))dN_t, \\ dW_t &= ((r_t - \delta)W_t + H_t w_t^H - C_t)dt - (1-e^\nu)W_{t-}dN_t, \end{aligned}$$

[12] This approach requires an interior solution for optimal hours which is assumed throughout the analysis.

which we use for our numerical solutions. Finally, the risk premium (49) is obtained from

$$RP_t = \tilde{C}(W_t)^{(1-\theta)\psi-\theta}\tilde{H}(W_t)^{(1-\theta)\psi\alpha}e^{-(1-\theta)\psi\alpha\nu}(1-e^{\nu})\lambda. \tag{54}$$

As a result, the implicit risk premium depends on the optimal jumps in consumption and hours immediately after a disaster. Accordingly, we obtain the jump terms and thus the risk premium from (54) by evaluating the optimal policy functions at points in the state space.

3.2.5 Results

In what follows, we restrict our discussion to the empirically most relevant case where $\theta \geq 1$. The key result is that effective risk aversion, except for the singular case $\rho = \bar{\rho}$, is still a function of financial wealth. This in turn implies a time-varying risk premium since wealth is changing stochastically over time. As shown in the appendix, elastic labor supply, $\psi \neq 0$, primarily has an effect on the optimal hours supplied, but does not substantially affect the shape and properties of the risk premium (cf. Figures A.1 and A.2).

The knife-edge value $\rho = \bar{\rho}$ ensures that the individual's optimal choice of leisure is constant (cf. Bodie et al., 1992). In this particular case, the expected proportional decline of marginal utility with respect to consumption matches the expected rate of return apart from a constant. Moreover, we obtain that the marginal propensity to save (to consume), $s(W_t) = s$, the supplied hours, $H(W_t) = H$, and the risk premium are all constant measures over time (consumption becomes a homogeneous function of degree α). For $\rho < \bar{\rho}$ the individual prefers a higher saving rate, $s(W_t) > s$, and supplies more hours, $H(W_t) > H$. Because both optimal policy functions for consumption and hours are concave, the effective risk aversion of the value function is lower than for $\psi = 0$. The risk premium is convex in financial wealth and has the upper bound $e^{-\alpha\theta\nu}(1-e^{\nu})\lambda$ for wealth approaching zero. For $\rho > \bar{\rho}$ the marginal propensity to save is smaller, $s(W_t) < s$, and the individual supplies less hours, $H(W_t) < H$, which in fact raises the effective risk aversion. Since the saving rate is decreasing in wealth, the risk premium is concave with lower bound $e^{-\theta\alpha\nu}(1-e^{\nu})\lambda$.

An empirically testable implication is the correlation between hours and consumption. In the data, hours and consumption are positively correlated which in turn implies a negative correlation between consumption and leisure (cf. Lettau and Uhlig, 2000). We may infer this property directly from the policy functions. For $\rho = \bar{\rho}$ there is zero correlation, while for $\rho < \bar{\rho}$ consumption and hours are concave functions of financial wealth (which has the usual interpretation of the capital stock per effective worker), and we obtain a positive correlation. It is only for $\rho > \bar{\rho}$ that the optimal policy function for hours is convex. In turn this implies a counterfactual negative correlation as long as the consumption function is concave. Thus, the empirically most plausible case $\rho < \bar{\rho}$ implies strictly concave policy functions for both consumption and hours, as well as time-varying and asymmetric risk premia similar to the benchmark case of constant labor supply $\psi = 0$.

Summarizing, the extension to endogenous labor supply is able to generate empirically plausible correlations for consumption and leisure. Though our main results on the shape and the time-varying property of the risk premium are not affected, the ability to buffer risk trough the labor-leisure choice makes it even more challenging to generate sizable risk premia in production economies. One interesting extension could therefore examine the role of different types of non-linearities such as capital adjustment cost and/or habit formation which would affect effective risk aversion and thus the risk premium (cf. Jermann, 2010).

4 Conclusion

In this paper we study how non-linearities affect asset pricing implications in a production economy. We derive closed-form solutions of the Lucas' fruit-tree model and compare the implicit risk premium to those obtained from models which account for non-linearities in the form of a neoclassical production function. For this purpose, we formulate our DSGE models in continuous time which gives analytical benchmark solutions for numerical analysis. Our key result is that these non-linearities can generate time-varying and asymmetric risk premia over the business cycle. The economic intuition is that individual's effective risk aversion, except for singular cases, is not constant in a neoclassical production economy. We show that non-normalities in the form of rare disasters substantially increase the economic relevance of these (empirical) key features.

From a methodological point of view, this paper shows that formulating the endowment economy or non-trivial production models in continuous time gives analytical solutions for reasonable parametric restrictions or functional forms. Analytical solutions are useful for macro-finance models for at least two reasons. First, they are points of reference from which numerical methods can be used to explore a broader class of models. Second, they shed light on asset market implications without relying purely on numerical methods. This circumvents problems induced by approximation schemes which could be detrimental when studying the effects of uncertainty. Along these lines, we propose the continuous-time formulation of DSGE models as a workable paradigm in macro-finance.

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A Appendix

A.1 Lucas fruit-tree model in continuous-time

A.1.1 Deriving the budget constraint

Consider a portfolio strategy which holds n_t units of the risky asset and $n_0(t)$ units of the riskless asset with default risk, such that

$$W_t = n_0(t)p_0(t) + p_t n_t$$

denotes the portfolio value. Using Itô's formula, it follows that

$$\begin{aligned} dW_t &= p_0(t)dn_0(t) + n_0(t)p_0(t)rdt + p_t dn_t + w_t\mu W_t dt + w_t\sigma W dB_t \\ &\quad + (w_{t-}J_t + (1-w_{t-})D_t)\, W_{t-}dN_t, \end{aligned}$$

where $w_t W_t \equiv n_t p_t$ denotes the amount invested in the risky asset. Since investors use their savings to accumulate assets, assuming no dividend payments, $p_0(t)dn_0(t) + p_t dn_t = -C_t dt$,

$$\begin{aligned} dW_t &= ((\mu - r)w_t W_t + rW_t - C_t)\, dt + \sigma w_t W_t dB_t \\ &\quad + ((J_t - D_t)w_{t-} + D_t)\, W_{t-}dN_t. \end{aligned}$$

Finally, we obtain the budget constraint (6) by defining

$$\mu_M \equiv (\mu - r)w_t + r, \quad \sigma_M \equiv w_t\sigma, \quad \zeta_M(t) \equiv (D_t - J_t)w_t - D_t.$$

A.1.2 The short-cut approach

As a necessary condition for optimality the Bellman's principle gives at time s

$$\rho V(W_s) = \max_{C_s}\left\{u(C_s) + \frac{1}{dt}E_s dV(W_s)\right\}. \tag{55}$$

Using Itô's formula (see e.g. Sennewald, 2007),

$$\begin{aligned} dV(W_s) &= \left((\mu_M W_s - C_s)\, V_W + \tfrac{1}{2}\sigma_M^2 W_s^2 V_{WW}\right) dt + \sigma_M W_s V_W dB_t + (V(W_s) - V(W_{s-}))dN_t \\ &= \left((\mu_M W_s - C_s)\, V_W + \tfrac{1}{2}\sigma_M^2 W_s^2 V_{WW}\right) dt + \sigma_M W_s V_W dB_t \\ &\quad + (V((1-\zeta_M(t_-))W_{s-}) - V(W_{s-}))dN_t, \end{aligned}$$

where σ_M^2 is the instantaneous variance of the risky asset's return from the Brownian motion increments. If we take the expectation of the integral form, and use the property of stochastic integrals, we may write using $\zeta_M \equiv E(\zeta_M(t)|D_t = \exp(\kappa) - 1) = 1 - e^{\kappa} - (e^{\nu_1} - e^{\kappa})w$ and $\zeta_M^0 \equiv E(\zeta_M(t)|D_t = 0) = (1 - e^{\nu_2})w$,

$$\begin{aligned} E_s dV(W_s) &= \big((\mu_M W_s - C_s) V_W + \tfrac{1}{2}\sigma_M^2 W_s^2 V_{WW} \\ &\quad + (V((1-\zeta_M)W_s)q + V((1-\zeta_M^0)W_s)(1-q) - V(W_s))\lambda \big) dt. \end{aligned}$$

Inserting into (55) gives the Bellman equation

$$\begin{aligned} \rho V(W_s) &= \max_{C_s} \big\{ u(C_s) + (\mu_M W_s - C_s) V_W + \tfrac{1}{2}\sigma_M^2 W_s^2 V_{WW} \\ &\quad + (V((1-\zeta_M)W_s)q + V((1-\zeta_M^0)W_s)(1-q) - V(W_s))\lambda \big\}. \end{aligned}$$

The first-order condition (9) makes consumption a function of the state variable. Using the maximized Bellman equation for all $s = t \in [0, \infty)$,

$$\begin{aligned} \rho V(W_t) &= u(C(W_t)) + (\mu_M W_t - C(W_t))V_W + \tfrac{1}{2}\sigma_M^2 W_t^2 V_{WW} \\ &\quad + (V((1-\zeta_M)W_t)q + V((1-\zeta_M^0)W_s)(1-q) - V(W_t))\lambda. \end{aligned}$$

Use the envelope theorem to compute the costate

$$\begin{aligned} \rho V_W &= (\mu_M V_W + (\mu_M W_t - C(W_t))V_{WW} + \sigma_M^2 W_t V_{WW} + \tfrac{1}{2}\sigma_M^2 W_t^2 V_{WWW} \\ &\quad + (V_W((1-\zeta_M)W_t)(1-\zeta_M)q + V_W((1-\zeta_M^0)W_s)(1-\zeta_M^0)(1-q) - V_W(W_t))\lambda. \end{aligned}$$

Collecting terms, we obtain

$$\begin{aligned} (\rho - \mu_M + \lambda)V_W &= (\mu_M W_t - C(W_t))V_{WW} + \sigma_M^2 W_t V_{WW} + \tfrac{1}{2}\sigma_M^2 W_t^2 V_{WWW} \\ &\quad + (V_W((1-\zeta_M)W_t)(1-\zeta_M)q + V_W((1-\zeta_M^0)W_s)(1-\zeta_M^0)(1-q))\lambda. \end{aligned} \tag{56}$$

Using Itô's formula, the costate obeys

$$\begin{aligned} dV_W(W_t) &= (\mu_M W_t - C_t) V_{WW} dt + \tfrac{1}{2}\sigma_M^2 W_t^2 V_{WWW} dt + \sigma_M W_t V_{WW} dB_t \\ &\quad + (V_W((1-\zeta_M(t_-))W_{t-}) - V_W(W_{t-}))dN_t \\ &= \big((\rho - \mu_M + \lambda)V_W - \sigma_M^2 W_t V_{WW} - V_W((1-\zeta_M)W_t)(1-\zeta_M)q\lambda \\ &\quad - V_W((1-\zeta_M^0)W_s)(1-\zeta_M^0)(1-q)\lambda \big) dt \\ &\quad + \sigma_M W_t V_{WW} dB_t + (V_W((1-\zeta_M(t_-))W_{t-}) - V_W(W_{t-}))dN_t, \end{aligned}$$

where we inserted the costate from (56). As a final step we insert the first-order condition and obtain the Euler equation (10).

A.1.3 Proof of Proposition 2.1

The idea of this proof is to show that using an educated guess of the value function, the maximized Bellman equation and the first-order condition (9) are both fulfilled. For constant relative risk aversion, θ, the utility function has the form

$$u(C_t) = \mathbb{C}_1 \frac{C_t^{1-\theta}}{1-\theta} + \mathbb{C}_2, \quad \theta > 0,\ (\mathbb{C}_1, \mathbb{C}_2) \in \mathbb{R}_+ \times \mathbb{R}. \tag{57}$$

From (8), we obtain the maximized Bellman equation using the functional equation for consumption from the condition (9), i.e., $C(W_t) = \mathbb{C}_1^{1/\theta} V_W^{-1/\theta}$. We use the *educated guess*

$$\bar{V} = \mathbb{C}_0 \mathbb{C}_1 \frac{W_t^{1-\theta}}{1-\theta} + \mathbb{C}_2/\rho, \tag{58}$$

where $\bar{V}_W = \mathbb{C}_0 \mathbb{C}_1 W_t^{-\theta}$ and $\bar{V}_{WW} = -\theta \mathbb{C}_0 \mathbb{C}_1 W_t^{-\theta-1}$, to solve the resulting equation. Note that optimal consumption is linear in wealth, $C(W_t) = \mathbb{C}_0^{-1/\theta} W_t$, and we arrive at

$$\begin{aligned} \rho \mathbb{C}_0 \mathbb{C}_1 \frac{W_t^{1-\theta}}{1-\theta} + \mathbb{C}_2 &= \mathbb{C}_1 \frac{\mathbb{C}_0^{-\frac{1-\theta}{\theta}} W_t^{1-\theta}}{1-\theta} + \mathbb{C}_2 + \left(\mu_M W_t - \mathbb{C}_0^{-1/\theta} W_t\right) V_W + \tfrac{1}{2}\sigma_M^2 W_t^2 V_{WW} \\ &\quad + \left((1-\zeta_M)^{1-\theta} q + (1-\zeta_M^0)^{1-\theta}(1-q) - 1\right) \mathbb{C}_0 \mathbb{C}_1 \frac{W_t^{1-\theta}}{1-\theta} \lambda. \end{aligned}$$

Collecting terms gives

$$\begin{aligned} \rho &= \mathbb{C}_0^{-1/\theta} + (1-\theta)\left(\mu_M - \mathbb{C}_0^{-1/\theta}\right) - (1-\theta)\tfrac{1}{2}\sigma_M^2 \theta \\ &\quad + \left((1-\zeta_M)^{1-\theta} q + (1-\zeta_M^0)^{1-\theta}(1-q) - 1\right) \lambda \\ \Rightarrow \quad \mathbb{C}_0^{-1/\theta} &= \frac{\rho - (1-\theta)\mu_M + \lambda - (1-\zeta_M)^{1-\theta} q \lambda - (1-\zeta_M^0)^{1-\theta}(1-q)\lambda}{\theta} + (1-\theta)\tfrac{1}{2}\sigma_M^2 \\ &= \frac{\rho - (1-\theta)\mu_M + \lambda - (1-\zeta_M)^{1-\theta}\lambda}{\theta} + (1-\theta)\tfrac{1}{2}\sigma_M^2, \end{aligned}$$

where the last equality used that in general equilibrium asset prices in (16) imply $\zeta_M = \zeta_M^0$. This proofs that the guess (58) indeed is a solution, and by inserting the guess together with the constant, we obtain the policy function for consumption.

Table A.1: Calibrated model and the risk premium (endowment economy)

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
					Parameters			
		No disasters	Baseline	Low θ	High λ	Low q	Low $\bar{\mu}$	Low ρ
θ	(coef. of relative risk aversion)	4	**4**	**3**	4	4	4	4
$\bar{\sigma}$	(s.d. of growth rate, no disasters)	0.02	**0.02**	0.02	0.02	0.02	0.02	0.02
ρ	(rate of time preference)	0.03	**0.03**	0.03	0.03	0.03	0.03	**0.02**
$\bar{\mu}$	(growth rate, deterministic part)	0.025	**0.025**	0.025	0.025	0.025	**0.020**	0.025
λ	(disaster probability)	0	**0.017**	0.017	**0.025**	0.017	0.017	0.017
q	(default probability in disaster)	0	**0.4**	0.4	0.4	**0.3**	0.4	0.4
$1 - e^{\bar{\nu}}$	(size of disaster)	0	**0.4**	0.4	0.4	0.4	0.4	0.4
$1 - e^{\kappa}$	(size of default)	0	**0.4**	0.4	0.4	0.4	0.4	0.4
						Variables		
Default risk		0	**0.021**	0.012	0.03	0.016	0.021	0.021
Disaster risk		0	**0.031**	0.019	0.046	0.036	0.031	0.031
Residual risk		0.002	**0.002**	0.001	0.002	0.002	0.002	0.002
Implicit risk premium		0.002	**0.054**	0.032	0.078	0.054	0.054	0.054
Expected market rate		0.128	**0.06**	0.067	0.028	0.06	0.04	0.05
Expected bill rate		0.126	**0.031**	0.051	-0.013	0.026	0.011	0.021
Market premium		0.002	**0.029**	0.016	0.041	0.033	0.029	0.029
Expected market rate, conditional		0.128	**0.066**	0.074	0.038	0.066	0.046	0.056
Face bill rate		0.126	**0.034**	0.054	-0.009	0.028	0.014	0.024
Market premium, conditional		0.002	**0.033**	0.02	0.047	0.038	0.033	0.033
Sharpe ratio, conditional		0.08	**1.641**	0.996	2.366	1.901	1.641	1.641
Expected growth rate		0.025	**0.016**	0.016	0.012	0.016	0.011	0.016
Expected growth rate, conditional		0.025	**0.025**	0.025	0.025	0.025	0.02	0.025

Table A.2: Calibrated model and the risk premium (endowment economy)

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
						Parameters		
		No default	Baseline	High $\bar{\sigma}$	High λ	High q	Low $1-e^{\bar{\nu}}$	Low $1-e^{\kappa}$
θ	(coef. of relative risk aversion)	4	**4**	4	4	4	4	4
$\bar{\sigma}$	(s.d. of growth rate, no disasters)	0.02	**0.02**	**0.05**	0.02	0.02	0.02	0.02
ρ	(rate of time preference)	0.03	**0.03**	0.03	0.03	0.03	0.03	0.03
$\bar{\mu}$	(growth rate, deterministic part)	0.025	**0.025**	0.025	0.025	0.025	0.025	0.025
λ	(disaster probability)	0.017	**0.017**	0.017	**0.2**	0.017	0.017	0.017
q	(default probability in disaster)	**0**	**0.4**	0.4	0.4	**1**	0.4	0.4
$1-e^{\bar{\nu}}$	(size of disaster)	0.4	**0.4**	0.4	**0.034**	0.4	**0.2**	0.4
$1-e^{\kappa}$	(size of default)	0.4	**0.4**	0.4	**0.034**	0.4	0.4	**0.2**
						Variables		
Default risk		0	**0.021**	0.021	0.003	0.052	0.007	0.01
Disaster risk		0.052	**0.031**	0.031	0.004	0	0.002	0.042
Residual risk		0.002	**0.002**	0.01	0.002	0.002	0.002	0.002
Implicit risk premium		0.054	**0.054**	0.062	0.009	0.054	0.01	0.054
Expected market rate		0.06	**0.06**	0.047	0.102	0.06	0.108	0.06
Expected bill rate		0.013	**0.031**	0.01	0.099	0.058	0.106	0.022
Market premium		0.047	**0.029**	0.037	0.002	0.002	0.003	0.038
Expected market rate, conditional		0.066	**0.066**	0.054	0.108	0.066	0.112	0.066
Face bill rate		0.013	**0.034**	0.013	0.102	0.065	0.108	0.023
Market premium, conditional		0.054	**0.033**	0.041	0.006	0.002	0.003	0.043
Sharpe ratio, conditional		2.681	**1.641**	0.824	0.292	0.08	0.162	2.161
Expected growth rate		0.016	**0.016**	0.015	0.019	0.016	0.021	0.016
Expected growth rate, conditional		0.025	**0.025**	0.024	0.025	0.025	0.025	0.025

Table A.3: Calibrated model and the risk premium (production economy)

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
					Parameters			
		No disasters	Baseline	High θ	Low α	Low δ	High λ	High $\lvert\nu\rvert$
θ	(coef. of relative risk aversion)	4	**4**	**6**	4	4	4	4
α	(output elasticity of capital)	0.75	**0.75**	0.75	**0.33**	0.75	0.75	0.75
δ	(capital depreciation, deterministic part)	0.1	**0.1**	0.1	0.1	**0.05**	0.1	0.1
ρ	(rate of time preference)	0.05	**0.05**	0.05	0.05	0.05	0.05	0.05
σ	(s.d. of stochastic depreciation, no disasters)	0	**0**	0	0	0	0	0
$\bar{\sigma}$	(s.d. of TFP growth)	0	**0**	0	0	0	0	0
$\bar{\mu}$	(growth rate TFP, deterministic part)	0	**0**	0	0	0	0	0
λ	(disaster probability)	**0**	**0.017**	0.017	0.017	0.017	**0.02**	0.017
$1 - e^{\nu}$	(size of disaster)	0	**0.4**	0.4	0.4	0.4	0.4	**0.5**
					Variables			
Implied knife-edge value $\bar{\rho}$		0.200	**0.230**	0.435	0.035	0.130	0.236	0.251
Implicit risk premium								
steady state, conditional		0	**0.024**	0.034	0.014	0.027	0.028	0.045
zero wealth (left limit)		0	**0.032**	0.068	0.013	0.032	0.037	0.068
Market rate, steady state (gross)		0.150	**0.131**	0.116	0.147	0.077	0.128	0.122
Bill rate, steady state (gross)		0.150	**0.107**	0.081	0.133	0.051	0.101	0.078

A.2 A model of growth under uncertainty

A.2.1 The Bellman equation and the Euler equation

As a necessary condition for optimality the Bellman's principle gives at time s

$$\rho V(W_s, A_s) = \max_{C_s} \left\{ u(C_s) + \frac{1}{dt} E_s dV(W_s, A_s) \right\}.$$

Using Itô's formula yields

$$\begin{aligned} dV &= V_W(dW_s - J_s W_{s-} dN_t) + V_A dA_s + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2 A_s^2 + V_{WW}\sigma^2 W_s^2\right) dt \\ &\quad + [V(W_s, A_s) - V(W_{s-}, A_{s-})] dN_t \\ &= ((r_s - \delta)W_s + w_s^L - C_s)V_W dt + V_W \sigma W_s dZ_s + V_A \bar{\mu} A_s dt + V_A \bar{\sigma} A_s dB_s \\ &\quad + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2 A_s^2 + V_{WW}\sigma^2 W_s^2\right) dt + [V(e^{\nu} W_{s-}, A_{s-}) - V(W_{s-}, A_{s-})] dN_t. \end{aligned}$$

Using the property of stochastic integrals, we may write

$$\begin{aligned} \rho V(W_s, A_s) = \max_{C_s} \big\{ & u(c_s) + ((r_s - \delta)W_s + w_s^L - C_s)V_W + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2 A_s^2 + V_{WW}\sigma^2 W_s^2\right) \\ & + V_A \bar{\mu} A_s + [V(e^{\nu} W_s, A_s) - V(W_s, A_s)]\lambda \big\} \end{aligned}$$

for any $s \in [0, \infty)$. Because it is a necessary condition for optimality, we obtain the first-order condition (29) which makes optimal consumption a function of the state variables.

For the *evolution of the costate* we use the maximized Bellman equation

$$\begin{aligned} \rho V(W_t, A_t) &= u(C(W_t, A_t)) + ((r_t - \delta)W_t + w_t^L - C(W_t, A_t))V_W + V_A \bar{\mu} A_t \\ &\quad + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2 A_t^2 + V_{WW}\sigma^2 W_t^2\right) + [V(e^{\nu} W_t, A_t) - V(W_t, A_t)]\lambda, \end{aligned} \tag{59}$$

where $r_t = r(W_t, A_t)$ and $w_t^L = w(W_t, A_t)$ follow from the firm's optimization problem, and the envelope theorem (also for the factor rewards) to compute the costate,

$$\begin{aligned} \rho V_W &= \bar{\mu} A_t V_{AW} + ((r_t - \delta)W_t + w_t^L - C_t)V_{WW} + (r_t - \delta)V_W + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2 A_t^2 + V_{WWW}\sigma^2 W_t^2\right) \\ &\quad + V_{WW}\sigma^2 W_t + [V_W(e^{\nu} W_t, A_t)e^{\nu} - V_W(W_t, A_t)]\lambda. \end{aligned}$$

Collecting terms we obtain

$$\begin{aligned} (\rho - (r_t - \delta) + \lambda)V_W &= V_{AW}\bar{\mu} A_t + ((r_t - \delta)W_t + w_t^L - C_t)V_{WW} + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2 A_t^2 + V_{WWW}\sigma^2 W_t^2\right) \\ &\quad + \sigma^2 V_{WW} W_t + V_W(e^{\nu} W_t, A_t)e^{\nu}\lambda. \end{aligned}$$

Using Itô's formula, the costate obeys

$$\begin{aligned} dV_W &= V_{AW}\bar{\mu} A_t dt + V_{AW}\bar{\sigma} A_t dB_t + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2 A_t^2 + V_{WWW}\sigma^2 W_t^2\right) dt \\ &\quad + ((r_t - \delta)W_t + w_t^L - C_t)V_{WW} dt + V_{WW}\sigma W_t dZ_t + [V_W(W_t, A_t) - V_W(W_{t-}, A_{t-})] dN_t, \end{aligned}$$

where inserting yields

$$
\begin{aligned}
dV_W &= (\rho - (r_t - \delta) + \lambda)V_W dt - V_W(e^\nu W_t, A_t)e^\nu \lambda - \sigma^2 V_{WW} W_t dt + V_{AW} A_t \bar{\sigma} dB_t \\
&\quad + V_{WW} W_t \sigma dZ_t + [V_W(e^\nu W_{t-}, A_{t-}) - V_W(W_{t-}, A_{t-})]dN_t,
\end{aligned}
$$

which describes the evolution of the costate variable. As a final step, we insert the first-order condition (29) to obtain the Euler equation (30).

A.2.2 Proof of Proposition 3.1

The idea of this proof is to show that using an educated guess of the value function, the maximized Bellman equation (59) and the first-order condition (29) are both fulfilled. We guess that the value function reads

$$
V(W_t, A_t) = \frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta} + f(A_t). \tag{60}
$$

From (29), optimal consumption is a constant fraction of wealth,

$$
C_t^{-\theta} = \mathbb{C}_1 W_t^{-\theta} \quad \Leftrightarrow \quad C_t = \mathbb{C}_1^{-1/\theta} W_t.
$$

Now use the maximized Bellman equation (59), the property of the Cobb-Douglas technology, $F_K = \alpha A_t K_t^{\alpha-1} L^{1-\alpha}$ and $F_L = (1-\alpha) A_t K_t^\alpha L_t^{-\alpha}$, together with the transformation $K_t \equiv L W_t$, and insert the solution candidate,

$$
\begin{aligned}
\rho \frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta} &= \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{1-\theta}}{1-\theta} + (\alpha A_t W_t^{\alpha-1} W_t - \delta W_t + (1-\alpha) A_t W_t^\alpha - \mathbb{C}_1^{-1/\theta} W_t)\mathbb{C}_1 W_t^{-\theta} \\
&\quad - \tfrac{1}{2}\theta \mathbb{C}_1 W_t^{1-\theta}\sigma^2 - g(A_t) + (e^{(1-\theta)\nu} - 1)\frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta}\lambda,
\end{aligned}
$$

where we defined $g(A_t) \equiv \rho f(A_t) - f_A \bar{\mu} A_t - \frac{1}{2} f_{AA} \bar{\sigma}^2 A_t^2$. When imposing the condition $\alpha = \theta$ and $g(A_t) = \mathbb{C}_1 A_t$ it can be simplified to

$$
\begin{aligned}
(\rho - (e^{(1-\theta)\nu} - 1)\lambda)\frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta} + g(A_t) &= \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{1-\theta}}{1-\theta} + (A_t W_t^{\alpha-\theta} - \delta W_t^{1-\theta} - \mathbb{C}_1^{-1/\theta} W_t^{1-\theta})\mathbb{C}_1 \\
&\quad - \tfrac{1}{2}\theta \mathbb{C}_1 W_t^{1-\theta}\sigma^2 \\
\Leftrightarrow \; (\rho - (e^{(1-\theta)\nu} - 1)\lambda) W_t^{1-\theta} &= \theta \mathbb{C}_1^{-1/\theta} W_t^{1-\theta} - (1-\theta)\delta W_t^{1-\theta} - \tfrac{1}{2}\theta(1-\theta) W_t^{1-\theta}\sigma^2,
\end{aligned}
$$

which implies that

$$
\mathbb{C}_1^{-1/\theta} = \frac{\rho - (e^{(1-\theta)\nu} - 1)\lambda + (1-\theta)\delta + \frac{1}{2}\theta(1-\theta)\sigma^2}{\theta}.
$$

This proofs that the guess (60) indeed is a solution, and by inserting the guess together with the constant, we obtain the optimal policy function for consumption.

A.2.3 Proof of Proposition 3.5

The idea of this proof follows Section A.2.2. An educated guess of the value function is

$$V(W_t, A_t) = \frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1-\alpha\theta} A_t^{-\theta}. \tag{61}$$

From (29), optimal consumption is a constant fraction of income,

$$C_t^{-\theta} = \mathbb{C}_1 W_t^{-\alpha\theta} A_t^{-\theta} \quad \Leftrightarrow \quad C_t = \mathbb{C}_1^{-1/\theta} W_t^{\alpha} A_t.$$

Now use the maximized Bellman equation (59), the property of the Cobb-Douglas technology, $F_K = \alpha A_t K_t^{\alpha-1} L^{1-\alpha}$ and $F_L = (1-\alpha) A_t K_t^{\alpha} L^{-\alpha}$, together with the transformation $K_t \equiv L W_t$, and insert the solution candidate,

$$\begin{aligned} \rho V(W_t, A_t) \quad = \quad & \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{\alpha-\alpha\theta} A_t^{1-\theta}}{1-\theta} + ((r_t - \delta) W_t + w_t^L - C(W_t, A_t)) V_W + V_A \bar{\mu} A_t \\ & + \tfrac{1}{2} \left(V_{AA} \bar{\sigma}^2 A_t^2 + V_{WW} \sigma^2 W_t^2 \right) + [V(e^{\nu} W_t, A_t) - V(W_t, A_t)] \lambda, \end{aligned}$$

Inserting the guess and collecting terms gives

$$\rho + \theta\bar{\mu} - \tfrac{1}{2} \left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\theta)\sigma^2 \right) + (1-\alpha\theta)\delta = \left(\frac{\theta}{1-\theta} \mathbb{C}_1^{-1/\theta} + 1 \right) (1-\alpha\theta) A_t W_t^{\alpha-1},$$

which has a solution for $\mathbb{C}_1^{-1/\theta} = (\theta - 1)/\theta$ and

$$\rho = (e^{(1-\alpha\theta)\nu} - 1)\lambda - \theta\bar{\mu} + \tfrac{1}{2} \left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\theta)\sigma^2 \right) - (1-\alpha\theta)\delta.$$

This proofs that the guess (61) indeed is a solution, and by inserting the guess together with the constant, we obtain the optimal policy function for consumption.

A.3 A model of growth under uncertainty with leisure

A.3.1 The Bellman equation and the Euler equation

As a necessary condition for optimality the Bellman's principle gives at time s

$$\rho V(W_s, A_s) = \max_{C_s, H_s} \left\{ u(C_s, H_s) + \frac{1}{dt} E_s dV(W_s, A_s) \right\}.$$

Using Itô's formula yields

$$\begin{aligned} dV \quad = \quad & V_W (dW_s - (e^{\nu} - 1) W_{s-} dN_t) + V_A dA_s + \tfrac{1}{2} \left(V_{AA} \bar{\sigma}^2 A_s^2 + V_{WW} \sigma^2 W_s^2 \right) dt \\ & + [V(W_s, A_s) - V(W_{s-}, A_{s-})] dN_t \\ = \quad & ((r_s - \delta) W_s + H_s w_s^H - C_s) V_W dt + V_W \sigma W_s dZ_s + V_A \bar{\mu} A_s dt + V_A \bar{\sigma} A_s dB_s \\ & + \tfrac{1}{2} \left(V_{AA} \bar{\sigma}^2 A_s^2 + V_{WW} \sigma^2 W_s^2 \right) dt + [V(e^{\nu} W_{s-}, A_{s-}) - V(W_{s-}, A_{s-})] dN_t. \end{aligned}$$

Using the property of stochastic integrals, we may write

$$\rho V(W_s, A_s) = \max_{C_s, H_s} \{u(C_s, H_s) + ((r_s - \delta)W_s + H_s w_s^H - C_s)V_W \\ + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2 A_s^2 + V_{WW}\sigma^2 W_s^2\right) + V_A \bar{\mu} A_s + [V(e^\nu W_s, A_s) - V(W_s, A_s)]\lambda\}$$

for any $s \in [0, \infty)$. Because it is a necessary condition for optimality, we obtain the first-order conditions (45) and (46) which make optimal consumption and hours functions of the state variables, $C_t = C(W_t, A_t)$ and $H_t = H(W_t, A_t)$, respectively.

For the *evolution of the costate* we use the maximized Bellman equation

$$\begin{aligned} \rho V(W_t, A_t) &= u(C(W_t, A_t), H(W_t, A_t)) + ((r_t - \delta)W_t + H(W_t, A_t)w_t^H - C(W_t, A_t))V_W \\ &\quad + V_A \bar{\mu} A_t + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2 A_t^2 + V_{WW}\sigma^2 W_t^2\right) + [V(e^\nu W_t, A_t) - V(W_t, A_t)]\lambda, \end{aligned} \quad (62)$$

where $r_t = r(W_t, A_t)$ and $w_t^L = w(W_t, A_t)$ follow from the firm's optimization problem, and the envelope theorem (also for the factor rewards) to compute the costate,

$$\begin{aligned} \rho V_W &= \bar{\mu} A_t V_{AW} + ((r_t - \delta)W_t + H_t w_t^H - C_t)V_{WW} + (r_t - \delta)V_W \\ &\quad + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2 A_t^2 + V_{WWW}\sigma^2 W_t^2\right) + V_{WW}\sigma^2 W_t + [V_W(e^\nu W_t, A_t)e^\nu - V_W(W_t, A_t)]\lambda. \end{aligned}$$

Collecting terms we obtain

$$\begin{aligned} (\rho - (r_t - \delta) + \lambda)V_W &= V_{AW}\bar{\mu} A_t + ((r_t - \delta)W_t + H_t w_t^H - C_t)V_{WW} \\ &\quad + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2 A_t^2 + V_{WWW}\sigma^2 W_t^2\right) + \sigma^2 V_{WW} W_t + V_W(e^\nu W_t, A_t)e^\nu \lambda. \end{aligned}$$

Using Itô's formula, the costate obeys

$$\begin{aligned} dV_W &= V_{AW}\bar{\mu} A_t dt + V_{AW}\bar{\sigma} A_t dB_t + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2 A_t^2 + V_{WWW}\sigma^2 W_t^2\right)dt + V_{WW}\sigma W_t dZ_t \\ &\quad + ((r_t - \delta)W_t + H_t w_t^H - C_t)V_{WW} dt + [V_W(W_t, A_t) - V_W(W_{t-}, A_{t-})]dN_t, \end{aligned}$$

where inserting yields

$$\begin{aligned} dV_W &= (\rho - (r_t - \delta) + \lambda)V_W dt - V_W(e^\nu W_t, A_t)e^\nu \lambda - \sigma^2 V_{WW} W_t dt + V_{AW} A_t \bar{\sigma} dB_t \\ &\quad + V_{WW} W_t \sigma dZ_t + [V_W(e^\nu W_{t-}, A_{t-}) - V_W(W_{t-}, A_{t-})]dN_t, \end{aligned}$$

which describes the evolution of the costate variable. As a final step, we insert the first-order condition (45) to obtain the Euler equation (48).

A.3.2 Proof of Proposition 3.5

The idea of this proof follows Section A.2.2. An educated guess of the value function is

$$V(W_t, A_t) = \frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1 - \alpha\theta} A_t^{-\theta}. \quad (63)$$

From the first-order conditions (45) and (46), we obtain

$$\begin{aligned} C_t^{-\theta}(1-H_t)^{(1-\theta)\psi} &= \mathbb{C}_1 W_t^{-\alpha\theta} A_t^{-\theta}, \\ \psi C_t^{1-\theta}(1-H_t)^{(1-\theta)\psi-1} &= w_t^H \mathbb{C}_1 W_t^{-\alpha\theta} A_t^{-\theta} \quad \Rightarrow \quad \psi C_t/(1-H_t) = (1-\alpha)A_t W_t^{\alpha} H_t^{-\alpha}. \end{aligned}$$

Suppose that optimal hours are constant, $H_t = H$, then optimal consumption becomes a constant fraction of income,

$$C_t = (1-s)A_t W_t^{\alpha} H^{1-\alpha}, \quad 1-s \equiv (1-\alpha)\frac{1-H}{\psi H}, \quad \psi \neq 0.$$

Inserting everything into (62) and collecting terms gives

$$\begin{aligned} &\left(\rho + (1-\alpha\theta)\delta + \left(\theta\bar{\mu} - \tfrac{1}{2}\left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\sigma)\sigma^2\right)\right) - (e^{\nu(1-\alpha\theta)}-1)\lambda\right)\frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1-\alpha\theta} A_t^{-\theta} = \\ &\quad \left((1-s)^{1-\theta} H^{(1-\theta)(1-\alpha)}(1-H)^{(1-\theta)\psi} + \left(H^{1-\alpha} - (1-s)H^{1-\alpha}\right)(1-\theta)\mathbb{C}_1\right)\frac{A_t^{1-\theta} W_t^{\alpha-\alpha\theta}}{1-\theta}. \end{aligned}$$

Hence, for $\rho = \bar{\rho}$ and

$$\mathbb{C}_1 = -\frac{(1-s)^{1-\theta} H^{(1-\alpha)(1-\theta)}(1-H)^{(1-\theta)\psi}}{(1-\theta)H^{1-\alpha} - (1-\theta)(1-s)H^{1-\alpha}},$$

the constant saving rate is indeed the optimal solution. The optimal hours can be obtained from the first-order condition for consumption

$$\begin{aligned} C_t(1-H)^{-\frac{1-\theta}{\theta}\psi} &= \mathbb{C}_1^{-1/\theta} W_t^{\alpha} A_t \\ \Leftrightarrow \quad \frac{1-\alpha}{\psi} H^{-\alpha}(1-H)^{1-\frac{1-\theta}{\theta}\psi} &= \mathbb{C}_1^{-1/\theta}. \end{aligned}$$

Inserting the condition for $\mathbb{C}_1$, we obtain

$$\begin{aligned} \left(\frac{1-\alpha}{\psi}\right)^{-\theta} H^{\alpha\theta}(1-H)^{-\theta+(1-\theta)\psi} &= -\frac{(1-s)^{1-\theta} H^{(1-\alpha)(1-\theta)}(1-H)^{(1-\theta)\psi}}{(1-\theta)H^{1-\alpha} - (1-\theta)(1-s)H^{1-\alpha}} \\ \Leftrightarrow \quad \frac{\psi}{1-\alpha} &= -\frac{1-H}{(1-\theta)H - (1-\theta)(1-\alpha)(1-H)/\psi}. \end{aligned}$$

Collecting terms yields

$$\begin{aligned} \psi &= -\frac{(1-\alpha)(1-H)}{(1-\theta)H - (1-\theta)(1-\alpha)(1-H)/\psi} \\ \Leftrightarrow \quad -\psi(1-\theta)H &= \theta(1-\alpha)(1-H) \\ \Leftrightarrow \quad H &= \frac{\theta(1-\alpha)}{\theta(1-\alpha) - \psi(1-\theta)} \end{aligned}$$

which are admissible solutions if and only if $0 < H < 1$, which holds for $\theta > 1$.

A.3.3 Obtaining the reduced form

In order to keep notation simple, this section provides the full derivation for a deterministic system ($\lambda = 0$). The complete derivation for the stochastic system is available on request from the author. Observe that the dynamic system can be summarized as

$$\begin{aligned}
dC_t &= -\frac{u_C}{u_{CC}}(r_t - \rho - \delta)\,dt - \frac{u_{CH}}{u_{CC}} dH_t, \\
dH_t &= \frac{u_{HC}u_C - u_{CC}u_H}{Y_{HH}/Y_H u_H u_{CC} + \bar{u}}(\rho - (r_t - \delta))dt \\
&\quad - \frac{u_{CC}u_H}{Y_{HH}/Y_H u_H u_{CC} + \bar{u}} \frac{Y_{HK}}{Y_H}((r_t - \delta)W_t + H_t w_t^H - C_t)dt, \\
dW_t &= ((r_t - \delta)W_t + H_t w_t^H - C_t)dt.
\end{aligned}$$

We can neglect the first equation because in equilibrium $C_t = C(H(W_t))$. We find that

$$\begin{aligned}
dH_t &= \frac{-(1-\theta)\psi C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-1} - \theta\psi C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-1}}{Y_{HH}/Y_H \theta\psi C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-1} + \bar{u}}(\rho - (r_t - \delta))dt \\
&\quad - \frac{\theta\psi C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-1}}{Y_{HH}/Y_H \theta\psi C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-1} + \bar{u}} \frac{Y_{HK}}{Y_H}((r_t - \delta)W_t + H_t w_t^H - C_t)dt,
\end{aligned}$$

where

$$\begin{aligned}
\bar{u} &= (1-\theta)^2\psi^2 C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-2} + (\theta\psi^2 - \theta^2\psi^2 - \psi\theta)C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-2} \\
&= \left((1-\theta)^2\psi^2 + \theta\psi^2 - \theta^2\psi^2 - \psi\theta\right) C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-2} \\
&= \psi\left(\psi - \theta\psi - \theta\right) C_t^{-2\theta}(1-H_t)^{2(1-\theta)\psi-2}.
\end{aligned}$$

Hence, inserting $\bar{u}$ and collecting terms yields

$$\begin{aligned}
dH_t &= \frac{-1}{Y_{HH}/Y_H\theta + ((1-\theta)\psi - \theta)(1-H_t)^{-1}}(\rho - (r_t - \delta))dt \\
&\quad - \frac{\theta}{Y_{HH}/Y_H\theta + ((1-\theta)\psi - \theta)(1-H_t)^{-1}} \frac{Y_{HK}}{Y_H}((r_t - \delta)W_t + H_t w_t^H - C_t)dt.
\end{aligned}$$

Inserting remaining partial derivatives yields,

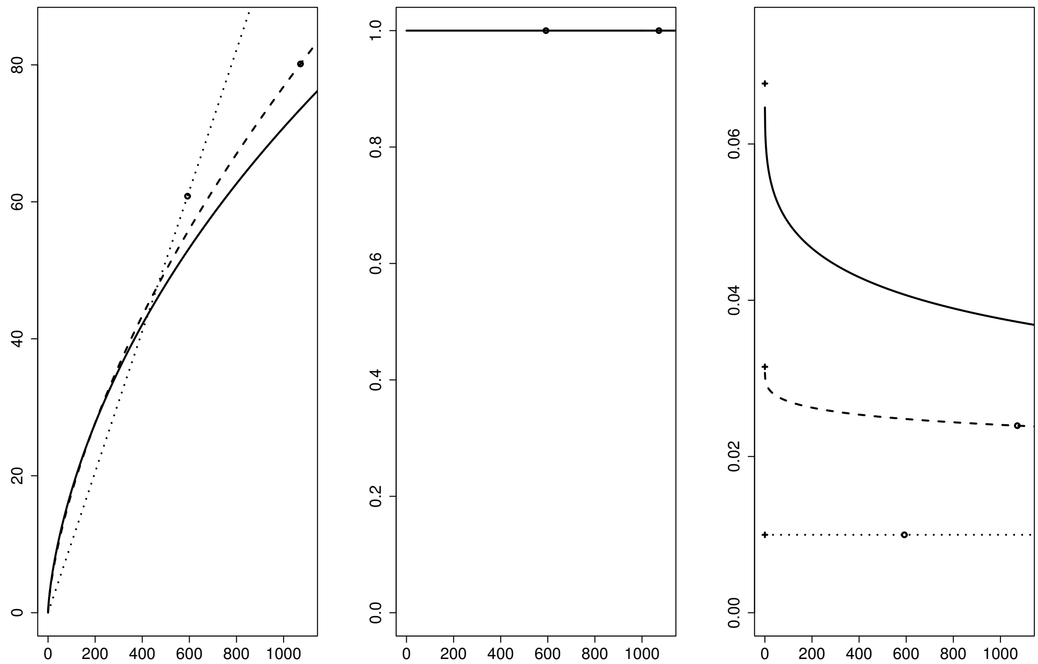
$$\begin{aligned}
dH_t &= \frac{-\rho + r_t - \delta}{-\alpha\theta H_t^{-1} + ((1-\theta)\psi - \theta)(1-H_t)^{-1}}dt + \frac{-\theta(r_t - \alpha\delta - \alpha C_t/W_t)}{-\alpha\theta H_t^{-1} + ((1-\theta)\psi - \theta)(1-H_t)^{-1}}dt \\
&= \frac{\rho - r_t + \delta + \theta(r_t - \alpha\delta - \alpha C_t/W_t)}{\alpha\theta H_t^{-1} + (\theta - (1-\theta)\psi)(1-H_t)^{-1}}dt.
\end{aligned}$$

To summarize, the reduced form description of the deterministic model can be written as

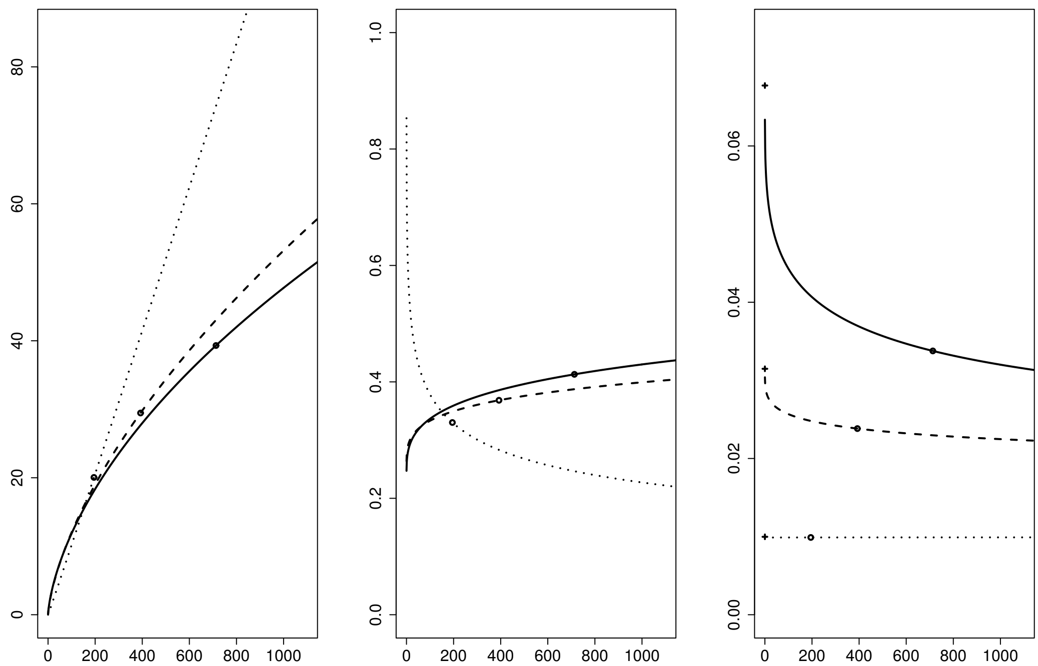
$$\begin{aligned}
dH_t &= \frac{\rho + (1-\alpha\theta)\delta - (1-\theta)r_t - \alpha\theta C_t/W_t}{\alpha\theta H_t^{-1} + (\theta - (1-\theta)\psi)(1-H_t)^{-1}}dt \\
dW_t &= ((r_t - \delta)W_t + H_t w_t^H - C_t)dt,
\end{aligned}$$

where $C_t = C(H(W_t), W_t)$.

Figure A.1: Risk premia in a production economy

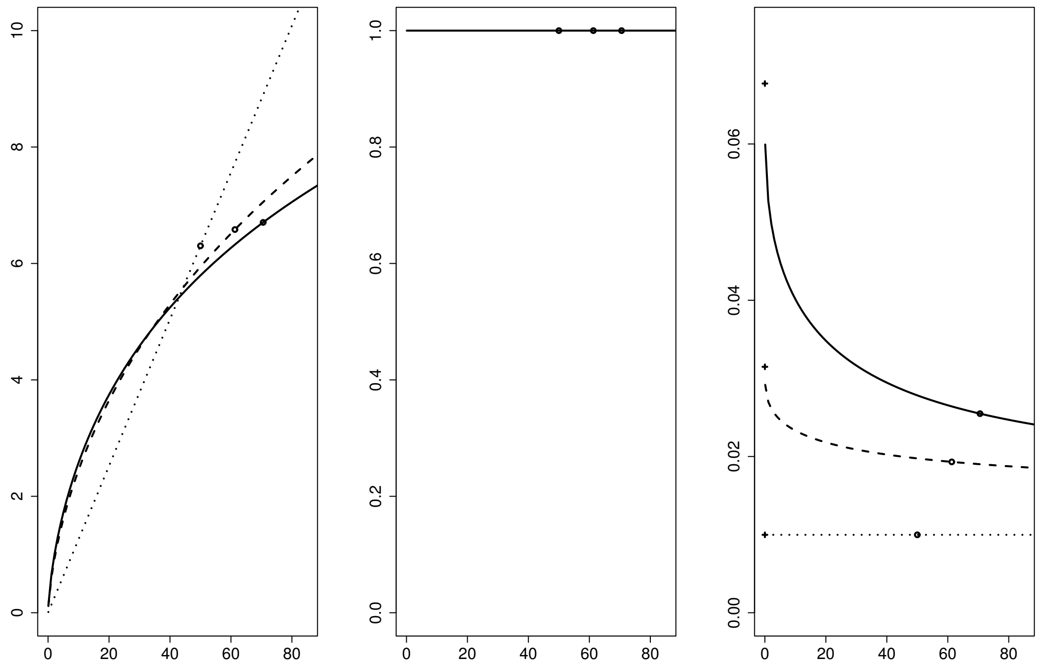


Notes: These figures illustrate the optimal policy functions for consumption (left panel), for hours (middle panel) and the risk premium (right panel) as functions of individual wealth for different levels of relative risk aversion for the case of $\sigma = \bar{\sigma} = \bar{\mu} = 0$, for calibrations $(\rho, \alpha, \theta, \delta, \lambda, 1 - e^{\nu}, \psi) = (.05, .75, \cdot, .1, .017, .4, 0)$ where $\theta = .75$ (dotted), $\theta = 4$ (dashed), and $\theta = 6$ (solid).

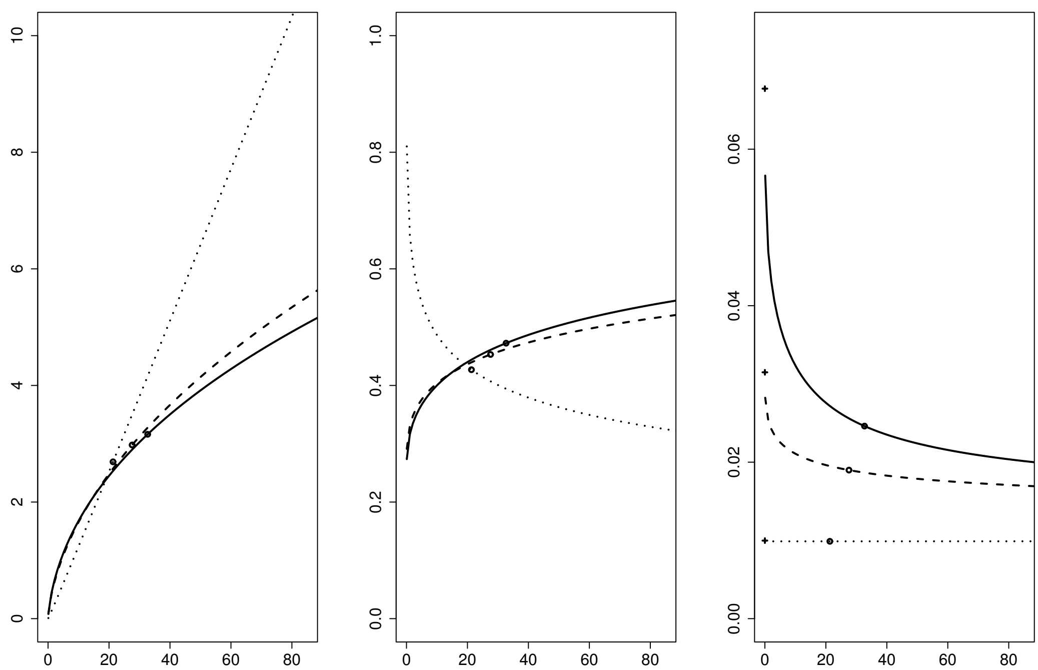


Notes: These figures illustrate the optimal policy functions for consumption (left panel), for hours (middle panel) and the risk premium (right panel) as functions of individual wealth for different levels of relative risk aversion for the case of $\sigma = \bar{\sigma} = \bar{\mu} = 0$, for calibrations $(\rho, \alpha, \theta, \delta, \lambda, 1 - e^{\nu}, \psi) = (.05, .75, \cdot, .1, .017, .4, 1)$ where $\theta = .75$ (dotted), $\theta = 4$ (dashed), and $\theta = 6$ (solid).

Figure A.2: Risk premia in a production economy



Notes: These figures illustrate the optimal policy functions for consumption (left panel), for hours (middle panel) and the risk premium (right panel) as functions of individual wealth for different levels of relative risk aversion for the case of $\sigma = \bar{\sigma} = \bar{\mu} = 0$, for calibrations $(\rho, \alpha, \theta, \delta, \lambda, 1 - e^{\nu}, \psi) = (.03, .75, \cdot, .25, .017, .4, 0)$ where $\theta = .75$ (dotted), $\theta = 4$ (dashed), and $\theta = 6$ (solid).



Notes: These figures illustrate the optimal policy functions for consumption (left panel), for hours (middle panel) and the risk premium (right panel) as functions of individual wealth for different levels of relative risk aversion for the case of $\sigma = \bar{\sigma} = \bar{\mu} = 0$, for calibrations $(\rho, \alpha, \theta, \delta, \lambda, 1 - e^{\nu}, \psi) = (.03, .75, \cdot, .25, .017, .4, 1)$ where $\theta = .75$ (dotted), $\theta = 4$ (dashed), and $\theta = 6$ (solid).

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