Inefficient Group Organization as Optimal Adaption to Dominant Environments

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Abstract

Contests between groups are plagued by intra-group externalities (freeriding). Yet, costless incentive schemes that entirely avoid free-riding within a group might not be desirable, neither individually nor socially. In contests among two groups, a relatively weak (i.e., small or unproductive) group will optimally not implement them because they compound strength differences between groups. If both groups rein in their intra-group externalities, they are both worse off, compared to a situation with free-riding, if they are relatively similar. If they are sufficiently heterogenous, the weak group loses at the expense of the relatively strong group.

JEL-Code: Z13, D72, N40, D74.

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1 Introduction

"There are only 2 qualities in the world: efficiency and inefficiency, and only 2 sorts of people: the efficient and the inefficient."

George Bernard Shaw

"The average man has a carefully cultivated ignorance [...] – a sort of cheerful inefficiency which protects him."

Crystal Eastman

If groups rather than individuals compete against each other in a contest, a free-rider problem among members of each group arises: when contributing to its group's effort in the contest, every individual bears the full marginal costs while the marginal benefits partly spill over to the rest of the group (e.g., Konrad 2009, Chs. 5.5 and 7). The attending positive externalities arise both if the contested rent is a group-specific public good (Katz, Nitzan, and Rosenberg 1990, Esteban and Rey 2001, Epstein and Mealem 2009, Nitzan and Ueda 2009) or if the rent is a private good (Nitzan 1991a,b; Esteban and Rey 2001, Nitzan and Ueda 2009).

The intensity of free-riding or, conversely, the motivation of individuals to exert effort is determined by the intra-group incentive scheme. In standard contest games, these incentives are unseparably linked to the sharing rule, i.e. to the (usually exogenous) procedure by which acquired rents are distributed within the group.[1] In a number of economic applications, however, incentives to exert effort appear to be separate from the sharing rule. Think, e.g., of phenomena such as team spirit, identification with the group, or norms to contribute to the social good. In such scenarios, individual members behave co-operatively (i.e., in the interest of the entire group) – even if the distribution of rents follows a standard sharing rule. Alternatively, the group could apply an incentive scheme that marginally equates individual and group incentives. Such schemes were discussed by Nitzan and Ueda (2009) as a means to motivate group members. We call an incentive scheme that aligns individual behaviour with group interest an *intra-group efficient incentive scheme* (IGEIS).

Nitzan and Ueda (2009) study a contest where all competing groups jointly, but exogenously implement an (costless) IGEIS. They show that larger groups beenfit from such schemes at the expense of smaller ones. They do not, however, allow

[1]See, e.g., Nitzan (1991a,b), Baik and Lee (2001), Noh (2002) or Bloch et al. (2006).

for the possibility that groups voluntarily and unilaterally implement an IGEIS. More technically, they do not determine the Nash equilibrium for the adoption of incentive schemes. At first glance, this restriction seems innocuous. Why would a group that has free access to an IGEIS not utilize it? After all, an IGEIS internalizes all within-group externalities. We show that this intuition does not hold for contests. If the competing groups have sufficiently unequal strengths (due to unequal sizes or different comparative advantages), the weaker group may wish not to implement an IGEIS even if that was costless. The rationale is to avoid the contest from heating up: if group A is relatively weak, group B's effort is a strategic complement to group A's effort, whereas group A's effort is a strategic substitute to group B's effort. Thus, if an IGEIS makes group A more efficient (and, thus, more aggressive) in the contest, group B will show more aggression too. This compounds differences in contest strengths and the share of the rent that the weaker group can acquire shrinks. This effect may outweigh the benefits from a better intra-group organization.

This observation has an analogy in rent seeking contests with an endogenous order of moves (Baik and Shogren 1992, Leininger 1993, Nitzan 1994). These papers show that in a two-player Tullock contest where both players can choose the levels as well as the timing of effort, the weaker player, or – using the terminology of Dixit (1987) – the "underdog" always moves first. From the underdog's point of view, the favorite's effort is a strategic complement to its own effort, whereas the opposite is true from the point of view of the favorite. This strategic complementarity can be used to reduce effort if the underdog moves first. A similar mechanism is at work here: If a group is sufficiently weak, an increase in its efforts would be disproportionately retaliated by the other, stronger group. Sticking with inefficient group incentives helps to keep the contest temperate.

In group contests, a greater efficiency within groups may also be undesirable from the perspective of social welfare: it makes groups more aggressive, the costs of the contests increase, rent dissipation rises, and social welfare is reduced.

This finding adds a new element to the discussion about the group-size paradox (Olson 1965, Esteban und Rey 2001, Pecorino and Temimi 2008, Nitzan and Ueda 2009). This paradox posits that group size and effectiveness in collective action are inversely related for the case of collectively provided private goods, but positively associated when non-rival goods are collectively provided. Esteban and Rey (2001) have shown that this – in their terms – "general wisdom" need not be correct if the costs of effort are sufficiently convex.[2] Our result pushes this finding one step further: Independently of the nature of the rent earned from collective action, (relatively) larger group have a stronger incentive than smaller ones to increase their effectiveness in collective action (by introducing an IGEIS). Relatively small groups cannot only afford it more easily to remain inefficient (as their free-rider problem is less severe), any attempt of this group to solve its internal organization problem will be retaliated by the other group, diluting the potential benefits of efficient within-group incentives. The inefficient organization of a relatively small/weak group may therefore be simply an expression of its relative smallness and/or weakness, an optimal adoption to its dominant environment. Hence, relatively large groups may end up with a comparative advantage for collective action precisely because of their large free-rider problems, and seemingly inefficient organizational structures for collective action in smaller groups may be deliberately chosen to limit the adverse consequences of one's weakness.

Section 2 introduces a model that captures these effects. Section 3 analyzes the attending game before Section 4 discusses welfare implications. Section 5 concludes.

2 A model of group incentives

We model a contest between two groups, A and D. Group $k = A, D$ consists of $N_k \geq 2$ identical members. Without loss of generality, we set $N_D = N$ and write $N_A = \alpha N_D = \alpha N$ for some $\alpha \geq 2/N_D$.[3] The two groups compete against each other for a given rent. As in Esteban and Rey (2001), this rent has a rival component, R, as well as a public component, P.[4] Using $z \in [0, 1]$ to measure the degree of publicness of the rent and assuming that an equal-sharing rule is applied to distribute the rival part of the rent within a group, a generic member of group D [of group A] has a

[2] Pecorino and Temini (2008) at least partly re-established the "general wisdom" by allowing for small fixed costs. In the same spirit, Nitzan and Ueda (2009) explicitly model the within-group utilization of the good and show that this generalization strenghtens the possibility of the group-size paradox.

[3] At some places in this paper, we treat α as if it could vary continuously. This is for expositional simplicity only; restrictions on α to ensure that N_A is an integer would not affect any of our observations.

[4] See also Nitzan (1991a, b) who has first analyzed group contests for private and public goods.

benefit of

$$(1-z)\frac{R}{N} + zP \quad \text{[respectively, of } (1-z)\frac{R}{\alpha N} + zP\text{]} \tag{1}$$

in case his group wins the rent.

The members of groups A and D voluntarily invest efforts $a = \{a_1, \ldots, a_{N_A}\}$ and $d = \{d_1, \ldots, d_{N_D}\}$ in the contest. The probability p_k that the contest is won by group $k = A, D$ is determined by a generalized Tullock contest-success function[5]

$$p_A(a,d) = \frac{\theta \cdot \sum_{i=1}^{\alpha N} a_i}{\theta \cdot \sum_{i=1}^{\alpha N} a_i + \sum_{i=1}^{N} d_i} \quad \text{and} \quad p_D(a,d) = \frac{\sum_{i=1}^{N} d_i}{\theta \cdot \sum_{i=1}^{\alpha N} a_i + \sum_{i=1}^{N} d_i}. \tag{2}$$

The parameter $\theta > 0$ measures the relative effectiveness of group A; if $\theta > 1$ group A is c.p. more efficient than group B. Hence, we allow for two sources of asymmetry between the groups, relative group size α and an innate comparative (dis-)advantage θ. Individuals are risk neutral, and the opportunity costs of investments and the rent are perfect substitutes.

We analyze two different incentive schemes, one with non-internalized intra-group externalities and an intra-group efficient incentive scheme (IGEIS). With the former, group members choose their effort levels in view of individuals benefits, as given in (1). This coincides with the standard approach to contests. If an IGEIS is applied, every group member behaves *as if* she maximized *aggregate* group welfare, viewing the value of the rent as $(1-z)R + zN_kP$. This is tantamount to assuming that individuals act *as if* maximizing a utilitarian welfare maximization which, given our assumptions on the utility functions, is equivalent to aiming at Pareto efficiency. An IGEIS is merely an incentive device, which does not alter the total quantity of resources or their actual distribution: if a group wins the contest, the rent is still shared equally and every group member "only" consumes $(1-z)R/N_k + zP$.

In standard contests (i.e., without an IGEIS), individual efforts involve a positive externality: they increase the group's probability to win the rent, a benefit that spills over to all group members.[6] With an IGEIS, individual and social (marginal) benefits

[5]For expositional simplicity we suppress parameters θ, α, and N as arguments of functions when not needed.

[6]A small increase in effort by a member of group k increases the winning probablity for that group by $\mathrm{d}p_k$. Excluding effort costs, this increases group welfare by $[(1-z)R + zN_kP] \cdot \mathrm{d}p_k$ and individual utility by $[(1-z)R/N_k + zP] \cdot \mathrm{d}p_k$. The difference between these values is positive. Hence, a positive marginal externality.

from effort choice are equated; individuals behave as if intra-group externalities were internalized. To illustrate, an IGEIS could be a Clark-Groves-Vickrey type of incentive scheme or result from individuals adopting a team spirit or a group identity in the sense of Akerlof and Kranton (2003).

We assume that a perfect and costless IGEIS is available. We do, of course, not claim that this is the case in reality. Yet, the assumption of a costless IGEIS makes our point as strong as possible – as we plan to derive conditions under which a group will voluntary *abstain* from using such a seemingly ideal tool.

We code the adoption of an IGEIS by group k by $s_k = 1$ and non-adoption by $s_k = 0$. Formally, the two cases differ by what group members perceive as the behaviourally relevant rent (per group member) from the contest. Denoting these rents for groups A and D by, respectively, $W_A(s_A)$ and $W_D(s_D)$, we obtain:

$$W_A(s_A) = \begin{cases} (1-z)R + z\alpha NP & \text{if } s_A = 1 \\ (1-z)R/(\alpha N) + zP & \text{if } s_A = 0 \end{cases}$$

and

$$W_D(s_D) = \begin{cases} (1-z)R + zNP & \text{if } s_D = 1 \\ (1-z)R/N + zP & \text{if } s_D = 0 \end{cases}$$

(see Esteban and Rey 2001). Observe that $W_k(0)$ coincides with the actual rent (1), while $W_k(1) = N_k W_k(0)$ equals the total rent for the entire group.

We study a two-stage game where groups first and simultaneously decide whether to use the IGEIS or not and then, given these choices, play a simultaneous contest game at the second stage. The game is solved by backwards induction.

3 The game

3.1 The contest at stage 2

We determine the Nash equilibria of the contest subgame for all combinations of (s_A, s_D), i.e., for $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$. Members of groups A and D behave as if they maximized utility functions

$$u_k^i(a,d) = p_A(a,d) \cdot W_A - a_i \quad \text{and} \quad u_D^i(a,d) = p_D(a,d) \cdot W_D - d_i, \tag{3}$$

depending on (s_A, s_D) as specified above. Restricting attention to symmetric equilibria where all members of the same group make equal investments,[7] we obtain effort levels as

$$a_i^*(s_A, s_D) = \frac{W_A(s_A)}{\alpha N} \cdot \frac{\theta W_A(s_A) W_D(s_D)}{(W_D(s_D) + \theta W_A(s_A))^2}, \quad (4)$$

$$d_i^*(s_A, s_D) = \frac{W_D(s_A)}{N} \cdot \frac{\theta W_A(s_A) W_D(s_D)}{(W_D(s_D) + \theta W_A(s_A))^2}. \quad (5)$$

Since $W_k(1) > W_k(0)$ we obtain that, *given* the other group's choice of s, an individual exerts greater efforts in the contest when his group has adopted an IGEIS: $a_i^*(1, s_D) > a_i^*(0, s_D)$ and $d_i^*(s_A, 1) > d_i^*(s_A, 0)$. This simply reflects the IGEIS' potential to rein in within-group externalities.

We denote the associated utility levels of the contest subgame by $V_A(s_A, s_D)$ and $V_D(s_A, s_D)$, respectively. These values are calculated using actual benefits, i.e., using in (3) the rents from (1) rather than W_k (for $s_k = 1$).

3.2 The choice of an incentive scheme at stage 1

We assume that an IGEIS is implemented by a group if it increases the per-capita (indirect) utility of its group members, i.e., if $V_A(1, s_D) > V_A(0, s_D)$, $V_D(s_A, 1) > V_D(s_A, 0)$. Obviously, in a non-strategic environment it is always optimal to implement an IGEIS: it internalizes the otherwise persistent intra-group externalities. For a contest environment the case is less obvious. The adoption of an IGEIS by, say, group A not only influences the behavior of the members of this group, but has spillover effects on the behavior of group D. This repercussion is helpful from the point of view of group D if a more aggressive behavior of group A makes group D less aggressive. In the opposite case, however, the net effect is not clear. The optimal decision for or against an IGEIS depends on whether group A's and group D's investments are strategic substitutes or complements as well as on the absolute strength of the effect (because the choice problem is discrete). The following proposition characterizes the equilibrium choice of incentive schemes.

Proposition: There exist threshold values $\underline{\theta}(\alpha)$ and $\bar{\theta}(\alpha)$ with $0 < \underline{\theta}(\alpha) < \bar{\theta}(\alpha)$ such that:

[7] Baik (2008) has shown that the effort (sub-)game has multiple equilibria. All equilibria share, however, that the sum (within each group) of efforts is identical, which implies that our restriction on symetric equilibria has no influence on the aggregate winning probabilities of the two groups.

1. If $\theta < \underline{\theta}(\alpha)$ the Nash equilibrium is $s_A = 0, s_D = 1$.
2. If $\underline{\theta}(\alpha) < \theta < \bar{\theta}(\alpha)$ the Nash equilibrium is $s_A = s_D = 1$.
3. If $\theta > \bar{\theta}(\alpha)$ the Nash equilibrium is $s_A = 1, s_D = 0$.
4. If $\theta = \underline{\theta}(\alpha)$, there are two Nash equilibria: $s_A = 0, s_D = 1$ and $s_A = s_D = 1$. If $\theta = \bar{\theta}(\alpha)$, there are two Nash equilibria: $s_A = 1, s_D = 0$ and $s_A = s_D = 1$.

Figure 1 gives a graphical representation of Proposition 1.[8] The figure reveals that

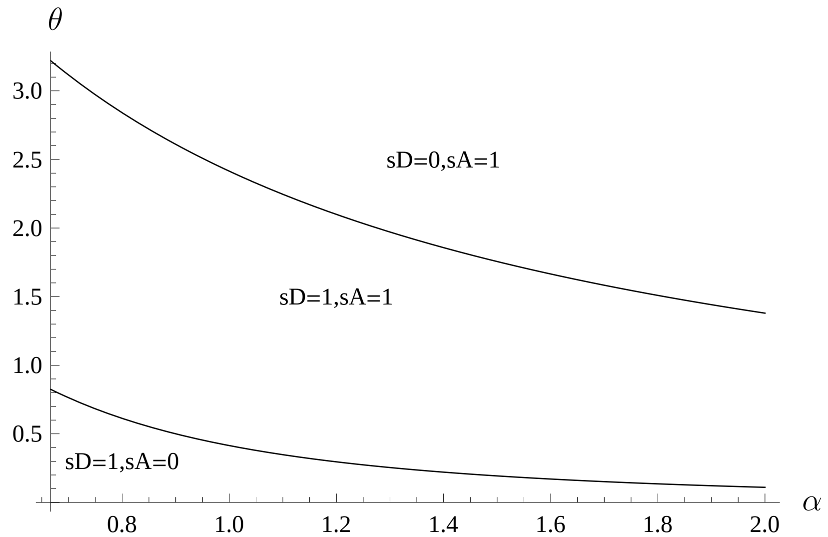

Figure 1: Equilibria for different α-θ-combinations ($N = 3$, $R = 1$, $P = 1$, $z = 0.5$).

relatively "weak" groups may abstain from introducing an IGEIS, where from the point of view of group A (the opposite is true for group D) weakness refers to a combination of low θ and low α. It is clear that group A is favored by large values of θ, however, in the light of the group-size paradox (Olson 1965, Esteban and Rey 2001) it is not clear that large values of α favor group A. A relatively large group size only creates the *potential* for strength that can only be exploited if the within-group incentive problem is solved. In that case, however, potential is transformed into actual strength. As a consequence, asymmetric equilibria are possible if differences in relative size and/or contest productivity are large.

[8]In Appendix A.1, we derive that $\underline{\theta}(\alpha) = 0.5\Phi(\alpha)(\sqrt{(\alpha N - 2)\alpha N + 5} - (\alpha N - 1))$, $\bar{\theta}(\alpha) = 0.5\Phi(\alpha)(\sqrt{(N-2)N+5} + (N-1))$, and $\Phi(\alpha) = \frac{(1-z)R+zNP}{(1-z)R+z\alpha NP}$. These are all decreasing functions of α, and it is straightforward to check that $\bar{\theta}(\alpha) > \Phi(\alpha) > \underline{\theta}(\alpha)$ for all α.

To get a better intuition for this result it is useful to extend the concepts of "favorite" and "underdog" from Dixit (1987) to group contests. Group A is called the underdog [the favorite] if the cross-partial derivative of the contest success function, evaluated at the Nash equilibrium of the contest subgame,

$$p_A^{ad} := \frac{\partial^2 p_A(a^*, d^*)}{\partial a_i \partial d_j} \tag{6}$$

(for generic group members i, j) is negative [positive]. Since $p_D^{da} = -p_A^{ad}$ by construction, group D is a favorite [underdog] whenever group A is an underdog [favorite]. If group A is a favorite, then its effort in the contest is a strategic complement to the underdog D's effort and, conversely, the underdog D's effort is a strategic substitute to group A's effort (and *mutatis mutandis* when A is the underdog). Favorites become more aggressive when the contestant increases its contest effort. Underdogs, however, duck out.

In the Appendix we show that, if an asymmetric equilibrium emerges, the group that implements an IGEIS (i.e., the relatively large or effective group) must be a favorite while the other group is an underdog. An increase in the effort of the favorite would reduce effort by the other group (whose investment is a strategic substitute to the favorite's investment), whereas an increase in the underdog's would encourage the other group to also increase effort (the favorite's investment being a strategic complement to the underdog's investment by j). In the intermediate cases (where a symmetric equilibrium occurs), the cross effects are relatively weak.

This finding adds an interesting new aspect to the discussion about the validity of the group-size paradox: relatively large groups may end up with a comparative advantage exactly because their large free-rider problem puts pressure on the establishment of efficient incentive schemes.

The second important aspect of this finding is that an apparently inefficient internal organizational structure may be optimal after all: it is rationally selected by underdogs to keep the contest more temperate. As a general lesson, incentive schemes can only be assessed properly in knowledge on the economic environment in which the organization exist that choose them.

4 Welfare

The strategic interdependence of the groups' organizational choices may lead to a social dilemma: even if it is optimal for both groups to introduce an IGEIS, the consequence may be that both groups are worse-off in equilibrium. Analogously, it is possible in an asymmetric equilibrium that a group that introduces an IGEIS profits at the expense of the other group. To analyze this question, we compare the utility levels in equilibria with and without IGEIS.

We start with the case of equally-sized groups:

Proposition 2: Assume that $\alpha = 1$. Compared to a situation $s_D = s_A = 0$,

1. group D is better off and group A is worse off if $\theta < \underline{\theta}$;
2. both groups are worse off if $\underline{\theta} \leq \theta \leq \bar{\theta}$; and
3. group A is better off and group D is worse off if $\bar{\theta} < \theta$.

This finding reveals two properties of the "incentive game". First, if neither group has a sufficiently large comparative advantage in the contest, the choice of an IGEIS has a typical prisoners' dilemma structure (item 2). Second, if one of the groups has a sufficiently large comparative disadvantage in the contest that an asymmetric equilibrium results, this group profits at the expense of the other group by the introduction of an IGEIS (items 1 and 3).

For groups of unequal sizes the following results can be established:

Proposition 3a: Assume that $\underline{\theta} \leq \theta \leq \bar{\theta}$ (i.e., both groups choose an IGEIS). Compared to a situation where both groups have a simple sharing rule,

1. both groups are worse off if they have similar sizes;
2. group A is better off [worse off] if its group size αN is sufficiently larger [smaller] than the group size of D, N.

The opposite result to 2. holds for group D.

Part 1 of Proposition 3a follows the same logic as Proposition 2. The introduction of an IGEIS makes both groups more aggressive. If both groups are relatively similar, the resulting effect on the equilibrium probability is relatively small such that the effect on welfare is negative. For unequal group sizes, part 2 shows that a large group can be better off in an equilibrium with IGEIS. Following the above line of argumentation, the effect on equilibrium probabilities is sufficiently biased in favor of the larger group in this case. This guarantees for this group an increase in group welfare.

We now show that this intuition carries over to the case of asymmetric equilibria:

> **Proposition 3b:** Assume that $\theta < \underline{\theta}$ $[\theta > \bar{\theta}]$. Group D $[A]$ benefits at the expense of group A $[D]$ compared to a situation where both groups apply a simple sharing rule.

Figure 2 summarizes our findings. The figure is a identical to Figure 1 with the

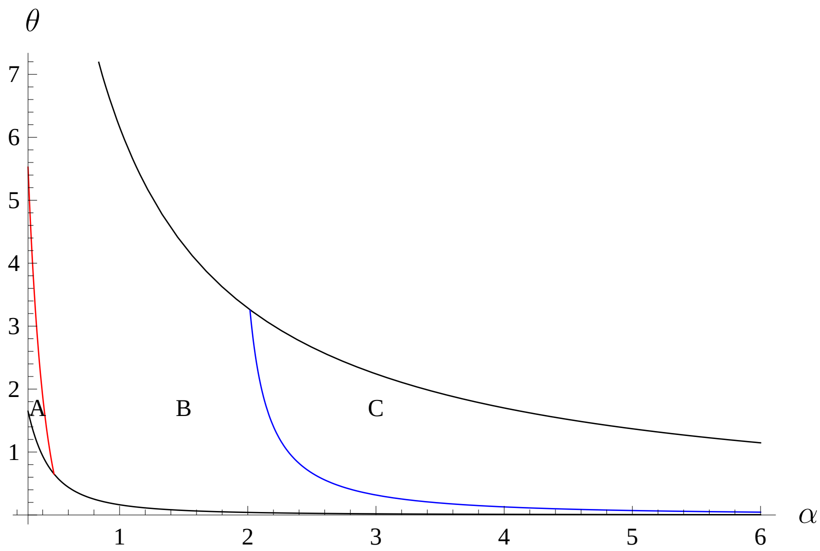

Figure 2: Welfare effects for different α-θ-combinations ($N = 7$, $R = 1$, $P = 1$, $z = 0.5$).

exception that we divide the range of (α, θ)-combinations that lead to an equilibrium where both groups choose an IGEIS is further divided into three subregions. Both groups are wore-off compared to the equilibrium with an equal-sharing rule in area B. Group D profits at the expense of group A in area A as well as at all points below the lower black graph $\underline{\theta}(\alpha)$. Group A profits at the expense of group D in area C as well as at all points above the upper black graph $\bar{\theta}(\alpha)$.

5 Conclusions

Incentive schemes are of crucial importance for the behavior of individuals in group contests. They help to rein in free-riding among group members and thus *ceteris paribus* help to increase the share of the rent that can be appropriated in the contest. Yet, this does not imply that the internalization of intra-group externalities is a dominant strategy or even welfare-improving. If a group is relatively weak, it may fare better when leaving its members with only weak incentives to expend (fruitless) effort in a contest with a stronger rival. Moreover, the additional aggression induced by high-powered incentives may be socially wasteful (increase in rent dissipation).

Our findings shed new light on the persistence of apparently inefficient organizational structures. Incentive schemes applied within organizations can in general not be evaluated without taking into account the competitive environment of the organization. What appears to be inefficient from an isolated, single-group perspective may be an optimal adaptation to a specific contest environment.

In addition our results in a sense turns the group-size paradox upside down. Esteban and Rey (2001) have shown that larger groups need not be less effective than smaller groups. On the contrary, if the costs of effort are sufficiently convex, larger groups may end up in a better position. Our paper shows that even in a situation with constant marginal costs, larger groups may end up in a better position because they have a more urgent need to introduce efficient incentive schemes. Of course this result recommits the discussion to a meta level: the implementation of an efficient incentive scheme has the character of a group-specific public good. If it comes along with some fixed costs of implementation (from which we abstract), it may create a group-size problem of its own.

Appendix A: Proofs for Section 3

A.1: Proof of Proposition 1

Group D: (i.) Suppose group A chooses $s_A = 0$. Then for a member of group D, the utility differential between $s_D = 1$ and $s_D = 0$, $\Delta_D(1, 0 \mid s_A = 0) = V_D(0, 1, \theta, \alpha, N) - V_D(0, 0, \theta, \alpha, N)$ is larger than zero if and only if

$$\frac{\alpha N^3((1-z)R + zNP)}{(1-z)R\theta + \alpha N((1-z)R + P(N+\theta)z))^2} - \frac{(N-1)R\theta(1-z) + \alpha N((1-z)R + zP(N + (N-1)\theta))}{(R(\alpha + (1-z)\theta) + \alpha(NzP(1+\theta) - R))^2} > 0.$$

This is non-negative iff

$$\theta \leq \theta_D^1 := \frac{\alpha N}{2}\Phi\left(\sqrt{N(N-2)+5} + (N-1)\right) > 0,$$

and $\Phi := ((1-z)R + zNP)/((1-z)R + z\alpha NP) > 0$ is the welfare-ratio between group D and A. The condition holds if the relative effectiveness of group D is not too strong.

(ii.) For $s_A = 1$, the utility differential $\Delta_D(1, 0 \mid s_A = 1) = V_D(1, 1, \theta, \alpha, N) - V_D(1, 0, \theta, \alpha, N)$ is larger than zero if and only if

$$\frac{(1-z)R + zNP}{((1-z)R(1+\theta) + z(1+\alpha\theta)NP))^2} - \frac{(1-z)R(1 + (N-1)\theta) + zNP(1 - \alpha(1-N)\theta}{((1-z)R(1 + N\theta + zNP(1 + \alpha N\theta))^2} > 0,$$

which is non-negative if and only if

$$\theta \leq \theta_D^2 := \frac{1}{2}\Phi(\sqrt{(N-2)N+5} + (N-1)) > 0.$$

Again, this condition holds if the relative effectiveness of group D is not too strong. Note that $\theta_D^1 > \theta_D^2$ because $\alpha \geq 2/N$.

Group A: (iii.) If $s_D = 0$, the utility differential between $s_A = 1$ and $s_A = 0$, $\Delta_A(1, 0 \mid s_D = 0) = V_A(1, 0, \theta, \alpha, N) - V_D(0, 0, \theta, \alpha, N)$ is larger than zero if and only if

$$\frac{\theta N^3((1-z)R + z\alpha NP)}{((1-z)R(1+\theta N) + zNP(1+\alpha\theta N))^2} - \frac{(1-z)R(N(\alpha+\theta) - 1) + zNP(\alpha N(1+\theta) - 1))}{((1-z)R(\theta - \alpha) + zNP\alpha(1+\theta))^2},$$

which is non-negative if and only if

$$\theta \geq \theta_A^1 := \frac{1}{2N}\Phi\left(\sqrt{(\alpha N-2)\alpha N+5}-(\alpha N-1)\right) > 0.$$

This finding and its interpretation are symmetric to case (i.).

(iv.) If $s_D = 1$, the utility differential between $s_A = 1$ and $s_A = 0$, $\Delta_A(1, 0 \mid s_D = 1) = V_A(1, 1, \theta, \alpha, N) - V_D(0, 1, \theta, \alpha, N)$ is larger than zero if and only if

$$\frac{\theta((1-z)R+z\alpha NP)}{((1-z)R(1+\theta)+zNP(1+\alpha\theta))^2} - \frac{(1-z)R(N\alpha+\theta)-1)+zNP(\alpha(N+\theta)-1)}{((1-z)R(\theta+\alpha N)+zNP(N+\theta))^2}$$

which is non-negative if and only if

$$\theta \geq \theta_A^2 := \frac{1}{2}\Phi\left(\sqrt{(\alpha N-2)\alpha N+5}-(\alpha N-1)\right).$$

This finding and its interpretation are symmetric to case (ii.). Note that $\theta_A^2 > \theta_A^1$.

To summarize, there are four equilibrium configurations:

$$\begin{aligned}
s_D = 0, s_A = 0 \quad &\text{if} \quad \theta_D^1 \leq \theta \leq \theta_A^1 \\
s_D = 1, s_A = 0 \quad &\text{if} \quad \theta \leq \min\{\theta_D^1, \theta_A^2\} \\
s_D = 0, s_A = 1 \quad &\text{if} \quad \theta \geq \max\{\theta_D^2, \theta_A^1\} \\
s_D = 1, s_A = 1 \quad &\text{if} \quad \theta_A^2 \leq \theta \leq \theta_D^2.
\end{aligned}$$

We have already established that $\theta_D^2 < \theta_D^1$ and $\theta_A^1 < \theta_A^2$. In addition it is straightforward to show that $\theta_A^2 < \theta_D^2$: The difference $\theta_D^2 - \theta_A^2 = (1+a)N - 2 + \sqrt{(N-2)N+5} - \sqrt{(\alpha N-2)\alpha N+5}$ has to be positive for all $\alpha \geq 2/N, N \geq 2$, or alternatively $N_D + NA - 2 + \sqrt{(N_D-2)N_D+5} \geq \sqrt{(N_A-2)N_A+5} \geq 0$. Since the lhs of this inequality is increasing in N_D, it is sufficient to show that $N_A + \sqrt{5} = 2 + N_A - 2 + \sqrt{(2-2)2+5} \geq \sqrt{(N_A-2)N_A+5}$ is always fulfilled. The rhs of this inequality is always smaller than $\sqrt{N_A^2+5}$. Squaring both sides and simplifying yields $2N_A\sqrt{5} > 0$. Putting $\underline{\theta} = \theta_A^2$ and $\bar{\theta} = \theta_D^2$, the claim follows. *q.e.d.*

A.2: Underdogs and favorites

For the Tullock contest success function (2) it is straightforward to check from (6) that group A is the underdog [favorite] if and only if the equilibrium winning probability p_A^* is smaller [larger] than 1/2. It is also straightforward to check that p_A^{ad} is equal in sign to $\theta W_A(s_A) - W_D(s_D)$. We now calculate the values of p_{ad}^A.

- Assume that $s_D = 0, s_A = 1$: This is an equilibrium if and only if $\theta \geq \bar{\theta}(\alpha)$. Then $p_A^{ad} \geq 0$ if and only if $\theta \geq \Phi(\alpha)/N$ (insert the appropriate rents and solve for θ). This function lies below $\Phi(\alpha)$, which in turn lies below $\bar{\theta}(\alpha)$ (cf. Figure 1). This property implies that $p_A^{ad}(1,0) > 0$, effort by group A is a strategic complement to effort by group D, and effort by D is a strategic substitute to effort by A.

- Assume that $s_D = 1$ and $s_A = 0$ which is an equilibrium if and only if $\theta \leq \underline{\theta}(\alpha)$. In that case, $p_A^{ad}(0,1) \geq 0$ if and only if $\theta \geq \tilde{\theta}(\alpha) := \alpha N \Phi(\alpha)$. Inspection of $\tilde{\theta}(\alpha)$ shows that this is an increasing function. Comparing $\tilde{\theta}(\alpha)$ and $\underline{\theta}(\alpha)$ at $\alpha = 2/N$ (the smallest value of α to make the problem well defined) reveals that $\tilde{\theta}(2/N) > \underline{\theta}(2/N)$. Given that $\underline{\theta}(\alpha)$ is decreasing, the (α, θ)-combinations that are consistent with $s_D = 1, s_A = 0$ are always in an area where $p_A^{ad}(0,1) < 0$: effort by group A is a strategic substitute to effort by group D, and effort by D is a strategic complement to effort by A.

- Assume that $s_D = s_A = 1$, which is an equilibrium if and only if $\underline{\theta}(\alpha) \leq \theta \leq \bar{\theta}(\alpha)$. Then, $p_A^{ad}(1,1) \geq 0$ iff $\theta \geq \Phi(\alpha)$. This type of equilibrium does dot depend on the sign of the cross derivative. However, it has to be relatively small in absolute terms. •

Appendix B: Proofs for Section 4

B.2: Proof of Proposition 2

1. If $\theta \leq \underline{\theta}$ the equilibrium is $\{0,1\}$, and we compare utility levels $V_D(0,0)$, $V_D(0,1)$ and $V_A(0,0)$, $V_A(0,1)$. Members of group A are better off by revealed preference. Individuals of group D are worse off, $V_D(0,1) - V_D(0,0) < 0$, iff

$$-\frac{(N-1)\theta(1+N+N(1+N)\theta(N-1)N\theta^2)}{(1+\theta)^2(1+N\theta)^2((1-z)R+zNP)} < 0, \qquad \text{(B.1)}$$

which is always fulfilled.

2. If $\theta \in [\underline{\theta}, \bar{\theta}]$ the equilibrium is $\{1,1\}$, and we compare utility levels $V_D(0,0)$, $V_D(1,1)$ and $V_A(0,0)$, $V_A(1,1)$. Members of group A are worse off, $V_A(1,1) < V_A(0,0)$, iff

$$-\frac{(N-1)}{(1+\theta)^2((1-z)R+zNP)} < 0, \qquad \text{(B.2)}$$

and members of group D are worse off, $V_D(1,1) < V_D(0,0)$, iff

$$-\frac{(N-1)\theta}{(1+\theta)^2((1-z)R+zNP)} < 0. \tag{B.3}$$

Both conditions are always fulfilled.

3. The proof of this case is symmetric to the proof of case 1. *q.e.d.*

B.2: Proof of Propositions 3

Proposition 3.a: As the equilibrium is $(s_A, s_D) = (1,1)$, we compare utilities $V_A(1,1)$, $V_A(0,0)$, and $V_D(1,1)$, $V_D(0,0)$, respectively. Our aim is to arrive at conditions on the model parameters (θ, α, N) such that members in both groups are better off with an IGEIS rather than without it.[9]

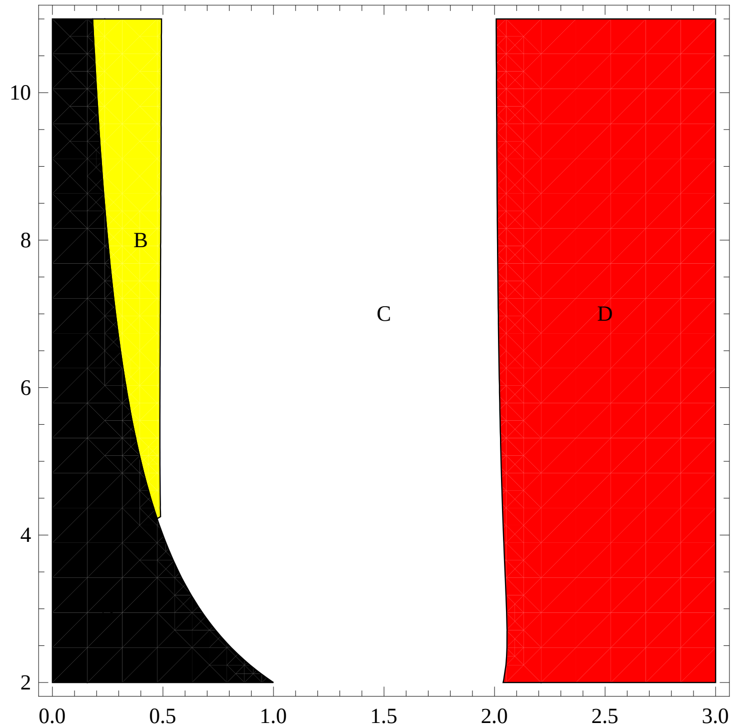

Figure 3: α-N-combinations for which $V_A(1,1) - V_A(0,0)$ or $V_D(1,1) - V_D(0,0)$ are positive. $\alpha \in [0,3]$ is on the abscissa and $N \in [2,11]$ is on the ordinate.

[9]The proof of the proposition has been carried out by the help of Mathematica 7.0. A file with the detailed programming code can be received from the authors upon request.

As a first set of conditions, we must ensure that both groups have at least two members and that $(s_A, s_D) = (1,1)$ is in fact a Nash equilibrium. This requires that:

- $\alpha \geq 2/N$,
- $\underline{\theta} \leq \theta \leq \bar{\theta}$.

Next, group members in A must better off with $(s_A, s_D) = (1,1)$. Checking $V_A(1,1) > V_A(0,0)$ yields that

- $\theta > \theta_A := \frac{\sqrt{(\alpha-1)^2 N\left((\alpha-1)^2 N + 4\right)} - (2 + ((\alpha-2)\alpha - 1)N)}{1 + (\alpha-2)N}$,
- $(1 + \alpha N) > 2N$.

Analogously, individuals of group D are better off iff $V_D(1,1) > V_D(0,0)$ which requires that

- $\theta < \theta_D := \frac{\sqrt{(\alpha-1)^2 N(4\alpha + (\alpha-1)^2 N)} - ((\alpha(2+\alpha) - 1)N - 2\alpha)}{\alpha(N-1)}$,
- $(1 + N) > 2\alpha N$.

While the set of (α, N)-combinations consistent with all six inequalities just presented is not empty, it cannot be characterized analytically in an explicit form. Yet, a graphical illustration is feasible. Figure 3 provides one. Area A violates $\alpha \geq 2/N$. Area B is the set of all pairs (α, N) such that $V_D(1,1) - V_D(0,0) > 0$. In all other cases (areas C and D), $V_D(1,1) - V_D(0,0) < 0$. Area D is the range where $V_A(1,1) - V_A(0,0) > 0$. In all other cases (areas B and C), $V_A(1,1) - V_A(0,0) < 0$. *q.e.d.*

Proposition 3.b: Suppose that $\theta < \underline{\theta}$ (the case $\theta > \bar{\theta}$ follows by a similar token). We compare $s_D = s_A = 0$ and $s_D = 1, s_A = 0$. By a revealed-preference argument, group D must be better off because group A's choice is the same in both situations. Denote by $R_A = (1-z)R + z\alpha NP$ and $R_D = (1-z)R + zNP$ the aggregate group rents. Group A is better off, $V_A(1,0) > V_A(0,0)$, iff

$$\frac{\Psi_1}{\Psi_2}\left(\frac{1}{\alpha}R_A\left((N-1)N\theta^2 R_A + N\theta(\alpha N + 1)R_D\right) + (N+1)R_D\right) < 0,$$

where $\Psi_1 = \theta(\alpha N - 1)R_D^2 R_A$ and $\Psi_2 = N^2(1+\theta)^2 R_A^2(R_D + N\theta R_A)$. This condition, however, can never be fulfilled given the assumptions of the model. *q.e.d.*

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