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Limited Liability, Asymmetric Taxation, and Risk Taking – Why Partial Tax Neutralities can be Harmful

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Abstract

We examine the combined effects of asymmetric taxation and limited liability on optimal risk taking of investors. Given an optimal risk level in the pre-tax case under full liability, loss-offset restrictions reduce, and limited liability enhances the incentives for taking risk. For every degree of limited liability we can find corresponding loss-offset limitations inducing the same optimal risk level as in the reference case. Thereby we get tax neutrality with respect to risk taking. We show that tax neutrality with respect to risk taking is incompatible with tax neutrality with respect to the choice of the legal form. In our model, full liability requires symmetric taxation and limited liability requires asymmetric taxation of profits and losses.

JEL-Code: H25, M41.

Keywords: limited liability, loss-offset, tax neutrality, risk taking.

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– Why Partial Tax Neutralities Can Be Harmful –

1. Introduction

Liability-related issues are often regarded as one of the main reasons for the current financial crises. As an example, mortgage debtors in the U.S. who give real estate as a security for a loan can get rid of the mortgage if they hand over the real estate to the lender, even if the current value falls short of the nominal debt. As a result, mortgage debtors can be held liable only to a limited extent. Banks which “securitize” loans and do not retain a fraction of these securities for their own account are another example of limited liability. In the case of default, these banks do not suffer losses although they impose substantial risks on their trading partners. The trading behaviour of investment banks − which are typically subject to limited liability − has been described by the metaphor “picking up nickels in front of a steamroller”. Obviously, limited liability provides additional incentives for risk taking by creating a convex cash flow structure.

The treatment of losses is closely related to liability issues. If investors take risks and conditions turn sufficiently unfavourable, losses may occur. Then the question arises to what extent these losses can be used for tax purposes. Current tax systems are characterized by loss-offset limitations, creating concavities in the after-tax cash flow structures.

Rational investors can anticipate limited liability as well as loss-offset restrictions when making their decision on the amount of risk they want to bear. Thus, investors are facing a combination of tax- and non-tax asymmetries of the cash flows. On an ex ante basis, it is not clear which combined incentives are induced by these multiple asymmetries on risk taking.

In Business Taxation, it is often argued that taxation should be neutral with respect to a firm’s legal form. Proponents of legal-form-tax-neutrality point out that every taxpayer should be taxed considering the financial results of its economic activity[1]. This requires a uniform taxation regardless of a firm’s legal form. Since different legal forms face different liability rules, it remains an open question which risk taking incentives are induced by an equal tax treatment of losses incurred by firms of different legal forms. Moreover, we ask whether legal-form-tax-neutrality is compatible with tax neutrality regarding risk taking. More generally, we analyze whether different partial tax neutralities are congruent or conflicting tax policy objectives.

We consider a risky investment with a risk level to be chosen by the investor. We assume that there is an individually optimal risk level in a world without taxation under full liability. This

[1] See, for instance, Siegel (2004). Unlike the principles-based proponents of legal-form-tax-neutrality, business economists have recently been questioning the economic meaningfulness of legal-form-tax-neutrality. See Wagner (2006) and the references cited there.

symmetric case serves as the reference case for the effects of tax- and non-tax asymmetries. We show that proportional income taxation with loss-offset limitations reduces the optimal risk level. By contrast, limited liability – as a parameter reflecting a firm's legal form – enhances the incentives for taking risk. The final decision about the risk level is affected by a combination of loss-offset restrictions, the degree of limited liability, and the tax rate.

Our results indicate that risk-taking-tax-neutrality is not compatible with legal-form-tax-neutrality. Risk-taking-tax-neutrality requires asymmetric taxation under limited liability and symmetric taxation under full liability. Thus, we show that partial tax neutralities do not necessarily provide guidance to overall tax neutrality.

This paper is organized as follows: Section 2 gives a review of the tax literature on risk taking and limited liability. Section 3 illustrates the structure of our model. Furthermore, we examine the incentive effects induced by asymmetric taxation and limited liability. Section 4 analyzes conditions for risk-taking-tax-neutrality. The results are illustrated with some numerical examples in section 5. Section 6 summarizes the results and provides a critical discussion of the model assumptions. We conclude with a perspective on further research questions.

2. Literature review

The tax impact on risk taking has been a focal issue of public finance and business tax research for decades. In their seminal paper, Domar / Musgrave (1944) prove that risk taking strongly depends on loss-offset rules. Improving loss-offset opportunities results in higher risk taking. Mossin (1968) generalizes the Domar / Musgrave results by means of expected utility theory. Näslund (1968) uses mathematical optimisation and Russell / Smith (1970) use criteria of stochastic dominance in their analyses of taxes and risk taking. Richter (1960), Stiglitz (1969), Allingham (1972) and Sandmo (1989) also use the risk-utility-theory to analyze the impact of taxation with and without loss-offset on the demand for risky investments[2]. These articles can be attributed to the public finance literature. Research in business taxation gained similar insights[3].

In the following decades, asymmetric taxation was examined intensively. Important contributions to the public finance literature in this area were Barlev / Levy (1975), Auerbach (1986), Auerbach / Poterba (1987), MacKie-Mason (1990), Eeckhoudt / Gollier / Schlesinger (1997), van Wijnbergen / Estache (1999) and Panteghini (2001a, 2001b, 2005). The empirical relevance of loss-offset regulations is clarified by Altshuler / Auerbach (1990), Mintz (1988) as well as by Shevlin (1990). Their results are examined using effective tax rates.

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[2] For a comprehensive literature review on risk taking and taxation see Niemann / Sureth (2008). The relevance of taxation in the current financial crisis is explained by Shaviro (2009) and Hemmelgarn / Nicodème (2010).

[3] See Schneider (1980). Eeckhoudt / Hansen (1982) derive conditions which increase demand for risky assets under more restrictive loss-offset rules.

Real-world loss-offset regulations often induce a path dependence of investment decisions. Thus, in many cases analytical results for investment models cannot be obtained. Therefore, numerical methods are necessary for the assessment of investment projects. Numerical simulations were realized by Majd / Myers (1986) and Majd / Myers (1987). Since the 1970s Monte-Carlo-simulations have been used for the evaluation of loss-offset limitations in the German literature[4].

The analysis of loss-offset regulations is not restricted to public finance research. The regulations are examined in the finance literature as well. For example, Ball / Bowers (1982), Cooper / Franks (1983), Majd / Myers (1986), Majd / Myers (1987), Schnabel / Roumi (1990), and Lund (2000) emphasize parallels between a call option and the treasury's tax claim.

Since the 1940s neutral tax systems under certainty have been derived by Brown (1948), Preinreich (1951), Samuelson (1964) and Johansson (1969), for instance. Neutral tax systems under uncertainty were proved later by Hartman (1978), Fane (1987), Buchholz (1988), Bond / Devereux (1995). Under risk neutrality, such neutral tax systems have already been derived in a real option context by Sureth (2002), and Niemann / Sureth (2005) as well as under risk aversion by Niemann / Sureth (2004).

Neutral tax systems mainly require a proportional tax rate and symmetric taxation of profits and losses. Loss-offset limitations induce violations of tax neutrality. Moreover, most models of capital budgeting with taxes implicitly assume that investments are carried out by individuals rather than corporations, because these models consider only one taxpayer rather than a combination of individual and corporate taxation. The taxation of partnerships in many countries corresponds to this assumption. By contrast, a comprehensive analysis of tax effects in corporations requires the integration of individual and corporate taxation as well as assumptions about the profit situation and corporate dividend policy.

Different liability laws restrict comparability of partnerships and corporations[5]. Partners are directly liable for debts of partnerships whereas shareholders cannot be held liable for debts of their corporation. Thus, limited liability generates asymmetric cash flow structures: Corporate cash flows can be transferred to shareholders approximately proportionally. By contrast, corporate losses do not generate equivalent payment obligations for shareholders.

Because of its practical relevance limited liability has been examined in law and economics for a long time. These analyses especially focus on the relationship between shareholders and creditors of corporations. Obviously, risk neutral decision makers with limited liability prefer risky investments. Sinn (1980) and Golbe (1988) show that these results are particularly relevant for companies close to default ("gamble for resurrection"). Gollier / Koehl / Rochet

[4] See Haegert / Kramm (1977), Niemann (2004), and Dahle / Sureth (2008).

[5] For a historical overview of the international development of corporate law see Guinnane / Harris / Lamoreaux / Rosenthal (2007).

(1997) prove that limited liability increases the level of individually optimal risk taking. However, risk-averse investors with limited liability generally do not choose the highest possible risk level.

Jensen / Meckling (1976) examine the effects of limited liability on agency costs and capital structure[6]. Rose-Ackerman (1991) argues that corporate managers prefer very secure investments in order to minimize the probability of insolvency. By contrast, there also exist incentives for excessive risk taking. Risky investments offer the opportunity of receiving positive results without being held responsible for negative results.

Limited liability and its effects on the banking sector were already examined before the current financial crisis[7]. In connection with the Asian banking crisis, Sinn (2003) illustrates effects of limited liability and asymmetric information. He shows that investment banks have an incentive to invest excessively in risky investments and not to generate enough equity.

Tax payments have not been analyzed in these papers. However, Meade (1978) discusses the correlation between corporate taxation and limited liability. He argues that limited liability is a special benefit of corporations, which should be taxed. By contrast, Musgrave / Musgrave (1973) state that limited liability does not imply substantial social costs. Therefore, it should not be taxed. John / Nair / Senbet (2005) show that limited liability can lead to overinvestment. In their model, a corporate tax can contribute to reducing conflicts of interest between shareholders and other stakeholders. As a result, a corporation tax must be regarded as the price of limited liability.

Becker / Fuest (2007) illustrate in a two-state-model that a corporate tax can be justified by the existence of limited liability[8]. In contrast to John / Nair / Senbet (2005), they identify the corporate tax as a corrective to capital market failure under asymmetric information. In their study, they derive a socially optimal extra burden on corporations. Another rarely examined aspect is the interaction between corporate law and / or contractual limited liability and tax loss-offset. In their model, Becker / Fuest (2007) distinguish between companies with full loss-offset and companies with restricted loss-offset.

The impact of taxation on the decision between insolvency and merger with solvent companies is analyzed by Bulow / Shoven (1978). They show that asymmetric taxation favors mergers. The reason is that loss carry-forwards are lost in the case of insolvency or liquidation. By contrast, taxation of highly positive profits induces the opposite effect and tends to favor insolvency or liquidation.

[6] For incentive problems under limited liability see for instance Palomino / Prat (2003), Budde / Kräkel (2008), Malcomson (2009). For incentive problems under limited liability and taxation see Banerjee / Besley (1990).

[7] For limited liability and risk taking in banking see, for example, Esty (1998), Grossman (2001).

[8] See Miglo (2007) for a slightly modified version of the model.

In this paper we examine the simultaneous effects of tax- and non-tax asymmetries in the investment's cash flow structure on the individually optimal risk level. We derive a substitutional relationship of limited liability and loss offset restrictions.

3. Model design

3.1 Benchmark situation: Tax-free case with full liability

We consider a one-period model with a risk-neutral investor. The investor has an initial wealth denoted by *I*. He puts his entire initial wealth in an investment project with a risk level to be chosen. The project's expected rate of return *v* depends on the chosen risk level *r*: $v \equiv v(r)$. In addition, the realized rate of return is a function of the stochastic variable $\tilde{\varepsilon}$.

Assuming a sufficiently well-behaved risk-return function *v*, there is an individually optimal risk level r^* in the tax-free case with full liability:

$$\tilde{\mu}(r) = v(r) + r\tilde{\varepsilon} \quad \text{with} \quad v(0) = z, v'\left(r < r^*\right) > 0, v'\left(r^*\right) = 0, v'\left(r > r^*\right) < 0, v''(r) < 0 .$$

If $r = 0$, the rate of return $v(r)$ corresponds to the risk-free interest rate *z*. Increasing *r* initially leads to an increased expected rate of return. The expected rate of return reaches its unique maximum at *r**. Subsequently, the expected rate of return decreases[9]. The risk level *r* can be varied continuously. The stochastic variable $\tilde{\varepsilon}$ is distributed over the interval $\left[\underline{\varepsilon}, \overline{\varepsilon}\right]$ with $\underline{\varepsilon} < 0, \overline{\varepsilon} > 0$. Its probability density function is denoted by $f(\varepsilon)$. The expected value of the stochastic variable is zero: $\mathrm{E}\left[\tilde{\varepsilon}\right] = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon f(\varepsilon)\, d\varepsilon = 0$.

We assume that the investor is fully liable for payment obligations resulting from the investment. This means that he faces a reserve liability if the rate of return falls short of –100%, i.e, if $\varepsilon < -\frac{1+v(r)}{r}$. Examples for this setting are investments in sole proprietorships or in partnerships. Partners have full liability for the partnership's payment obligations.

Using the above definitions, the investor's expected future value *W* in our reference case can be easily computed as:

$$W = I\left(1 + \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(v(r) + r\varepsilon\right) f(\varepsilon)\, d\varepsilon\right) = I\left(1 + v(r)\right)$$

Obviously, the expected future value is maximized if $v(r)$ is maximized. Thus, r^* represents the investor's optimal risk level in the symmetric benchmark situation.

[9] Assuming that increasing risk requires additional effort by the investor, this effect can be interpreted as a result of the investor's increasing and convex disutility associated with higher effort induced by higher risk.

3.2 Symmetric taxation with full liability

We use a uniform tax rate s for all kinds of capital income. The rate of return $\tilde{\mu}$ is subject to the same tax rate, regardless of the chosen risk level. Tax-exempt income and non-deductible expenses do not exist. Therefore, tax base and tax rate effects cannot occur under symmetric taxation.

Symmetric taxation does not alter the individually optimal risk taking r^*, because a multiplication by the factor $(1-s)$ does not change the optimality properties of the expected rate of return. Hence, symmetric taxation does not distort risk taking if the investor is risk neutral[10]. This effect can be easily verified by computing the expected future value after symmetric taxation:

$$W^s = I\left(1+\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}(1-s)\left(v(r)+r\varepsilon\right)f(\varepsilon)d\varepsilon\right) = I\left(1+(1-s)v(r)\right).$$

3.3 Asymmetric taxation with full liability

We implement a tax-induced asymmetry in the form of a loss-offset limitation. This is the first step in a gradual introduction of several asymmetries. Positive profits are subject to the full tax rate s. For losses, i.e., for $\tilde{\mu}(r)<0 \Leftrightarrow \varepsilon < -\frac{v(r)}{r}$ only a limited or no tax refund applies. This is formally displayed by a loss-offset parameter $\gamma \le 1$. This parameter represents the proportion of deductible losses. As a result, γ is regarded as a tax policy variable. By adjusting γ, the tax legislator can change investors' willingness to realize risky investments. Under asymmetric taxation, the tax base TB is defined as:

$$TB = \begin{cases} I\cdot\gamma\cdot(v(r)+r\varepsilon) & \text{for} \quad \varepsilon < -\dfrac{v(r)}{r} \\ I\cdot(v(r)+r\varepsilon) & \text{for} \quad \varepsilon \ge -\dfrac{v(r)}{r} \end{cases}$$

For $\gamma = 1$, the special case of symmetric taxation emerges. A case of no loss-offset is given by $\gamma = 0$. Existing tax systems with limited loss carry-backs, non-interest-bearing loss carry-forwards or different types of minimum taxation are characterized by $0<\gamma<1$. Stricter loss-offset rules (in terms of decreasing the investor's wealth) correspond with a reduction of γ[11].

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[10] This result is distorted by risk aversion. See Domar / Musgrave (1944).

[11] In most jurisdictions the use of losses for tax purposes depends on the amount of losses. Losses that cannot be offset against current profits must be carried forward to subsequent periods, which induces a negative time effect. The higher a loss, the later it can be offset against future profits. In our notation, this effect would imply that the loss offset parameter γ would be a function of ε: $\gamma \equiv \gamma(\varepsilon)$. Such a model specification would

The relationship of the realisation of the random variable ε and the investor's future value $W(\varepsilon)$ can be easily illustrated graphically:

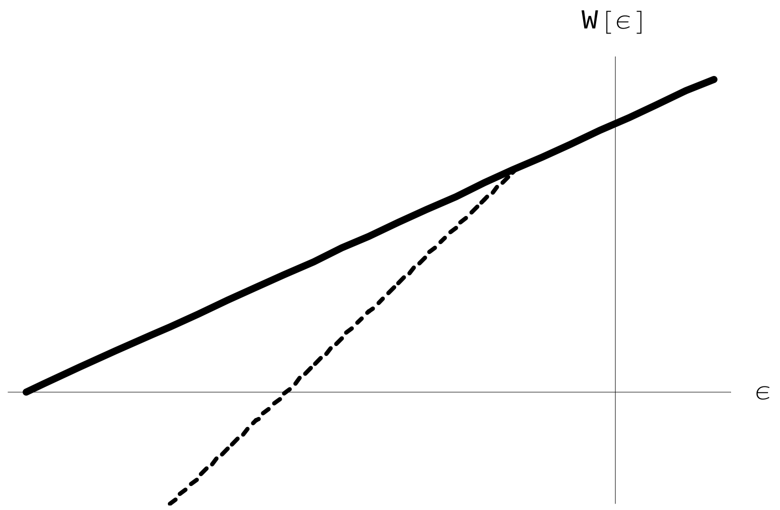

Figure 1: Future value under symmetric and asymmetric taxation as functions of the random variable ε

The solid line illustrates the future value under symmetric taxation. In the case of loss-offset limitations, the future value is described by the dashed line. The two lines coincide for $\varepsilon \geq -\frac{v(r)}{r} \Leftrightarrow \mu \geq 0$. Asymmetric taxation concavifies the future value function. Therefore, the decision on risk taking is probably influenced by similar effects as the concave utility function of a risk-averse decision-maker.

The expected future value considering asymmetric taxation is given by:

$$W^s = I\left(1 + \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} \big(v(r)+r\varepsilon\big)\big(1-\gamma s\big) f(\varepsilon)d\varepsilon + \int_{-\frac{v(r)}{r}}^{\overline{\varepsilon}} \big(v(r)+r\varepsilon\big)\big(1-s\big) f(\varepsilon)d\varepsilon\right)$$

$$= I\left(1 + \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} \big(v(r)+r\varepsilon\big)\big(1-s+s-\gamma s\big) f(\varepsilon)d\varepsilon + \int_{-\frac{v(r)}{r}}^{\overline{\varepsilon}} \big(v(r)+r\varepsilon\big)\big(1-s\big) f(\varepsilon)d\varepsilon\right)$$

$$= I\left(1 + \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} \big(v(r)+r\varepsilon\big)\big(1-s\big) f(\varepsilon)d\varepsilon + \int_{-\frac{v(r)}{r}}^{\overline{\varepsilon}} \big(v(r)+r\varepsilon\big)\big(1-s\big) f(\varepsilon)d\varepsilon + (1-\gamma)s \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} \big(v(r)+r\varepsilon\big) f(\varepsilon)d\varepsilon\right)$$

require extensive assumptions regarding future profits, which are difficult to explain in a one-period model. Hence, we assume γ to be constant. A one-period model does not permit time effects of taxation. As a consequence, we have to approximate time effects by tax base effects.

$$= I\left(1 + v(r)(1-s) + (1-\gamma)s \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} \left(v(r) + r\varepsilon\right) f(\varepsilon)\, d\varepsilon\right)$$

$$\leq I\left(1 + v(r)(1-s)\right)$$

The inequality arises because of $M \equiv \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} \left(v(r) + r\varepsilon\right) f(\varepsilon)\, d\varepsilon \leq 0$.

To determine the optimal risk level under asymmetric taxation $\hat{r}$, the following first-order condition must be met:

$$\frac{\partial W^s(\hat{r})}{\partial r} = I\left(v'(\hat{r})(1-s) + (1-\gamma)s \int_{\underline{\varepsilon}}^{-\frac{v(\hat{r})}{\hat{r}}} \left(v'(\hat{r}) + \varepsilon\right) f(\varepsilon)\, d\varepsilon\right) = 0.$$

It can be shown that the relation $\hat{r} \leq r^*$ applies. Because of $v'(r^*) = 0$ the following inequality holds at r^*:

$$\left.\frac{\partial W^s}{\partial r}\right|_{r=r^*} = I\left((1-\gamma)s \int_{\underline{\varepsilon}}^{-\frac{v(r^*)}{r^*}} \varepsilon f(\varepsilon)\, d\varepsilon\right) \leq 0.$$

Thus, r^* cannot be an optimum. To show that $\hat{r} < r^*$, we use the second partial derivative of W_r^s with respect to r, which is negative:

$$\frac{\partial^2 W^s}{\partial r^2} = I\left(v''(r)(1-s) + (1-\gamma)s \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} v''(\hat{r}) f(\varepsilon)\, d\varepsilon\right.$$

$$\left. +(1-\gamma)s\left(\frac{v(r) - rv'(r)}{r^2}\right) f\left(-\frac{v(r)}{r}\right)\left(v'(r) - \frac{v(r)}{r}\right)\right).$$

Due to our concavity assumption $v''(r) < 0$, the first two terms (upper line) are negative. The last term (lower line) is negative as well because of:

$$(1-\gamma)s\left(\frac{v(r) - rv'(r)}{r^2}\right) f\left(-\frac{v(r)}{r}\right)\left(v'(r) - \frac{v(r)}{r}\right) = -\frac{1}{r}\left(\frac{v(r) - rv'(r)}{r}\right)^2 (1-\gamma)sf\left(-\frac{v(r)}{r}\right) < 0.$$

As a result, asymmetric taxation under full liability decreases the willingness for risk taking. It should be noted that this result is still based on the assumption of risk-neutral behaviour.

3.4 Pre-tax case with limited liability

If the investment project's rate of return falls short of –100%, i.e. if $\tilde{\varepsilon} < -\frac{1+v(r)}{r}$, a liability exemption implies that a reserve liability as under unlimited liability cannot occur. This means that zero is a lower boundary for the investor's future value. Thus, negative future values are not possible. With regard to a meaningful analysis of limited liability, we assume that this case has a strictly positive probability: $\underline{\varepsilon} < -\frac{1+v(r)}{r}$.

In the case of partially limited liability, the reserve liability will not be completely removed. This variation can be interpreted as an insurance with proportional retention. The compensation by the insurance is a fraction of $0 \le \beta \le 1$ of the “damage”. Equivalently, the investor has a proportional retention of $(1-\beta)$. A rate of return before insurance benefits of $\tilde{\mu}(r) < -1$ is regarded as the “insured event”. This corresponds to a total loss of the investor's initial wealth. For $\beta = 1$, there is a liability exemption for the investor. This is typical for shareholders of corporations who cannot incur negative future values from their shareholdings. For $\beta = 0$, full liability applies. Practically, this can be found by sole proprietors or partners in partnerships.

Within the scope of trade and company law, regulations on liability are at the legislator's discretion. Hence, β can be regarded as the legislator's action variable. In addition, β can be interpreted as a tax policy action variable.

The conditional future value for a risky investment amounts to:

$$W = \begin{cases} I(1-\beta)(1+(v(r)+r\tilde{\varepsilon})) & \text{for} \quad \tilde{\varepsilon} < -\dfrac{1+v(r)}{r} \\ I(1+v(r)+r\tilde{\varepsilon}) & \text{for} \quad \tilde{\varepsilon} \ge -\dfrac{1+v(r)}{r}. \end{cases}$$

We can graphically illustrate the connection of the random variable ε and the investor’s future value with partially limited liability and unlimited liability:

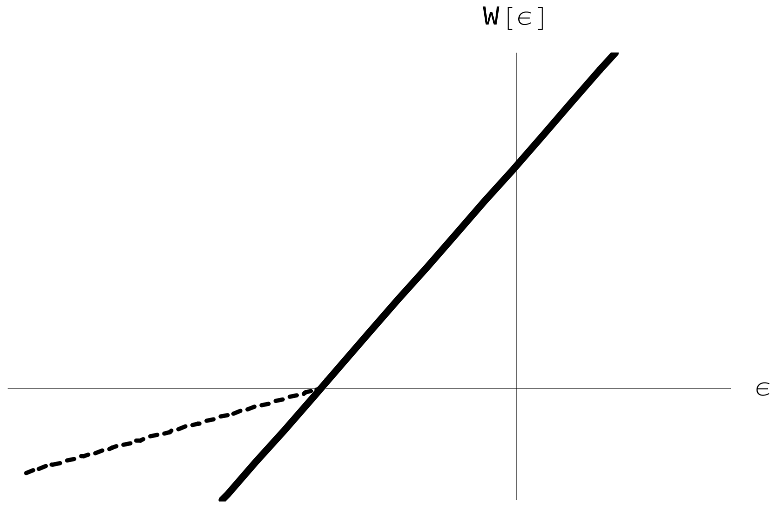

Figure 2: Future value under partially limited and unlimited liability as a function of the random variable ε

The solid line displays the future value with unlimited liability. The case of partially limited liability (with $\beta < 1$) is described by the dashed line. The two lines coincide for $\varepsilon \geq -\frac{1+v(r)}{r}$. Obviously, limited liability generates a convexity of the future value function. Therefore, an incentive for higher risk taking can be expected.

The explicit formal analysis confirms this conjecture. Now, the expected future value is:

$$\begin{aligned} W &= \int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}} I(1-\beta)\left(1+v(r)+r\varepsilon\right) f\left(\varepsilon\right) d\varepsilon + \int_{-\frac{1+v(r)}{r}}^{\bar{\varepsilon}} I\left(1+v(r)+r\varepsilon\right) f\left(\varepsilon\right) d\varepsilon \\ &= I\left(1+v(r)-\beta \int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}} \left(1+v(r)+r\varepsilon\right) f\left(\varepsilon\right) d\varepsilon\right) \\ &= I\left(1+v(r)-\beta N\left(r\right)\right) \\ &\geq I\left(1+v(r)\right). \end{aligned}$$

The inequality holds because $N \equiv N(r) = \int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}} \underbrace{\left(1+v(r)+r\varepsilon\right)}_{<0} \underbrace{f\left(\varepsilon\right)}_{>0} d\varepsilon < 0$.

The first-order condition for the optimal solution $\hat{r}$ is:

$$\frac{\partial W\left(\hat{r}\right)}{\partial r} = I\left(v'\left(\hat{r}\right)-\beta \int_{\underline{\varepsilon}}^{-\frac{1+v(\hat{r})}{\hat{r}}} \left(v'\left(\hat{r}\right)+\varepsilon\right) f\left(\varepsilon\right) d\varepsilon\right) = 0.$$

Because of $v'(r^*) = 0$, at the point r^* the inequality $\left.\frac{\partial EW}{\partial r}\right|_{r=r^*} = -I\beta \int_{\underline{\varepsilon}}^{-\frac{1+v(r^*)}{r^*}} \varepsilon f(\varepsilon) d\varepsilon \geq 0$ is satisfied. As a result, the individually optimal risk level is higher than under full liability: $\hat{r} \geq r^*$.

3.5 Symmetric taxation with limited liability

In the case of limited liability ($\beta > 0$) the calculation of the expected future value may differ from the pre-tax case. In principle, the following interpretations are possible:

1. The tax base is defined as the net change in value $I \cdot \mu(r)$ taking into account limited liability. In the case of a total loss of the initial wealth the treasury would participate proportionally under symmetric taxation. This would result in a net future value of $sI > 0$, which consists of the loss-induced tax reimbursement only. In practice, this is the case if investors do not need to provide their loss-induced tax reimbursement to cover the liabilities of the corporation. This corresponds to actual tax law for shareholders of corporations in most jurisdictions.
2. The loss-induced tax reimbursement must be paid to cover losses until the net future value equals zero. This is the case for sole proprietors or for partners of partnerships who are legally facing an unlimited liability, but whose net wealth might not be sufficient to cover all obligations.

That is the reason why there are two possibilities even under full exemption from liability excluding negative future values. There are similar differentiations in the case of partial liability $0 < \beta < 1$. Case 1 occurs if the partially limited liability is valid for rates of return below $\mu \leq -1$. In this case, a positive future value is possible only because of a tax shield. Case 2 is relevant for rates of return below $\mu \leq -\frac{1}{1-s}$.

Case 1 corresponds to the situation of corporation shareholders and is independent of the initial wealth. Case 2 just matches with the situation of partners of partnerships if this investment is the investor's only asset. Otherwise, the other components of the investor's wealth would have to be considered in the calculation, too. However, in this case, the character of a partial analysis would be lost. Since we focus on the typical liability rules for the different legal forms, we will analyze case 1 only.

In this case, the future value as a function of μ is given by[12]:

[12] In order to avoid a multitude of subscripts and superscripts, the expected future value under symmetric (asymmetric) taxation with limited liability is denoted by W_1 (W_2).

$$W_1(\mu) = \begin{cases} I\left[(1-(1-s)\beta)+(1-s)(1-\beta)\mu\right] & \text{for} \quad \mu < -1 \\ I\left[1+(1-s)\mu\right] & \text{for} \quad \mu \geq -1 \end{cases}$$

As in the pre-tax case, limited liability increases the incentives for risky investments under symmetric taxation. This can be easily verified for $\mu = v(r) + r\varepsilon$:

$$\begin{aligned} W_1 = & \int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}} I\left[1-(1-s)\beta+(1-s)(1-\beta)\left(v(r)+r\varepsilon\right)\right] f\left(\varepsilon\right) d\varepsilon \\ & + \int_{-\frac{1+v(r)}{r}}^{\overline{\varepsilon}} I\left[1+(1-s)(v(r)+r\varepsilon)\right] f\left(\varepsilon\right) d\varepsilon \\ = & \, I\left[1+(1-s)\left(v(r)-\beta N\left(r\right)\right)\right]. \end{aligned}$$

The first-order condition for the optimal risk level $\hat{r}$ is:

$$\frac{\partial W_1\left(\hat{r}\right)}{\partial r} = I(1-s)\left(v'\left(\hat{r}\right) - \beta \int_{\underline{\varepsilon}}^{-\frac{1+v(\hat{r})}{\hat{r}}} \left(v'\left(\hat{r}\right)+\varepsilon\right) f\left(\varepsilon\right) d\varepsilon \right) = 0 .$$

At the point $r = r^*$ the inequality $\dfrac{\partial W_1\left(r^*\right)}{\partial r} \geq 0$ holds. Consequently, the optimal risk level exceeds the one from the symmetric case again: $\hat{r} \geq r^*$.

The impact of the tax rate on the optimal risk level is negative:

$$\begin{aligned} \frac{dW_1\left(\hat{r}\right)}{ds} & = \underbrace{\frac{\partial W_1\left(\hat{r}\right)}{\partial r}}_{=0} \frac{d\hat{r}}{ds} + \frac{\partial W_1\left(\hat{r}\right)}{\partial s} \\ & = \frac{\partial W_1\left(\hat{r}\right)}{\partial s} = I\left(\beta \int_{\underline{\varepsilon}}^{-\frac{1+v(\hat{r})}{\hat{r}}} \left(1+v\left(\hat{r}\right)+\hat{r}\varepsilon\right) f\left(\varepsilon\right) d\varepsilon - v(\hat{r}) \right) < 0. \end{aligned}$$

Hence, the higher the tax rate, the lower the optimal risk level.

3.6 Asymmetric taxation with limited liability

In this section we combine tax- and non-tax asymmetries. Therefore, the future value is a piecewise linear function of the pre-tax return μ (or the random variable ε, respectively) and is kinked twice.

Figure 3: Future value under asymmetric taxation and partial limited liability $0<\beta<1$ as function of the random variable ε

Restricting our analysis to case 1 (limited liability is effective for $\mu \leq -1$), the conditional future values as functions of μ are:

$$W_2(\mu)=\begin{cases} I\left[(1-(1-\gamma s)\beta)+(1-\gamma s)(1-\beta)\mu\right] & \text{for } \mu<1 \\ I\left[1+(1-\gamma s)\mu\right] & \text{for } -1\leq\mu<0 \\ I\left[1+(1-s)\mu\right] & \text{for } \mu\geq 0 \end{cases}$$

The future value will be exactly γsI if the rate of return equals –100%. This future value corresponds to the tax shield given a total loss of the initial wealth. For losses exceeding this value, the future value depends on the liability parameter β. Furthermore, γsI represents a lower boundary for the case of liability exemption ($\beta=1$). Under full liability ($\beta=0$), the results correspond to those in section 3.3.

Using $\tilde{\mu}(r)=v(r)+r\tilde{\varepsilon}$, the expected future value W_2 can be computed as:

$$\begin{aligned} W_2 = & \int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}} I\left[1-(1-\gamma s)\beta+(1-\gamma s)(1-\beta)\big(v(r)+r\varepsilon\big)\right] f(\varepsilon)\,d\varepsilon \\ & + \int_{-\frac{1+v(r)}{r}}^{-\frac{v(r)}{r}} I\left[1+(1-\gamma s)(v(r)+r\varepsilon)\right] f(\varepsilon)\,d\varepsilon \\ & + \int_{-\frac{v(r)}{r}}^{\bar{\varepsilon}} I\left[1+(1-s)(v(r)+r\varepsilon)\right] f(\varepsilon)\,d\varepsilon \end{aligned}$$

$$= I\left[1+(1-s)v(r)+(1-\gamma)s\int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}}\left(v(r)+r\varepsilon\right)f(\varepsilon)d\varepsilon-(1-\gamma s)\beta\int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}}\left(1+v(r)+r\varepsilon\right)f(\varepsilon)d\varepsilon\right].$$

Using $M(r)=\int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}}\left(v(r)+r\varepsilon\right)f(\varepsilon)d\varepsilon<0$ and $N(r)=\int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}}\left(1+v(r)+r\varepsilon\right)f(\varepsilon)d\varepsilon<0$ as defined above, the expected future value can be simplified to: $W_2(r)=I\left[1+(1-s)v(r)+(1-\gamma)sM(r)-(1-\gamma s)\beta N(r)\right]$.

The main trade-offs are obvious and result from the isolated effects of asymmetric taxation and limited liability. It depends on $M(r)$ and $N(r)$ and hence on the risk-return function $v(r)$, the probability density function $f(\varepsilon)$, the tax rate s, the loss-offset parameter γ, and the liability parameter β, whether the combined asymmetries lead to increased or decreased incentives for risk taking.

4. Neutral tax systems

The trade-off between loss-offset limitations and limited liability implies that there should be a loss-offset-liability combination that leaves the investor's risk taking decision unaffected. Such a combination is called "neutral with respect to risk taking". Under a neutral tax system, the investor's individually optimal risk level would coincide with the one in the symmetric case. Thus, the neutrality condition is defined by: $\hat{r}(\beta,\gamma)=r^*$.

To meet this neutrality condition, the first-order condition $\partial W_2(r^*)/\partial r=0$ must be fulfilled in the presence of asymmetric taxation and limited liability at $r=r^*$. Using $W_2(r)=I\left[1+(1-s)v(r)+(1-\gamma)sM(r)-(1-\gamma s)\beta N(r)\right]$, the neutrality condition can be written as:

$$\frac{\partial W_2(r^*)}{\partial r}=0$$

$$\Rightarrow (1-s)\underbrace{v'(r^*)}_{=0}+(1-\gamma)s\frac{\partial M(r^*)}{\partial r}-(1-\gamma s)\beta\frac{\partial N(r^*)}{\partial r}=0$$

$$\Rightarrow \frac{(1-\gamma)s}{(1-\gamma s)\beta}=\frac{\dfrac{\partial N(r^*)}{\partial r}}{\dfrac{\partial M(r^*)}{\partial r}} \quad \text{for } \beta>0.$$

Using the expressions for $M(r)$ and $N(r)$ from section 3.6 yields:

$$\frac{\partial M\left(r^*\right)}{\partial r} = \int_{\underline{\varepsilon}}^{-\frac{v\left(r^*\right)}{r^*}} \left(v'\left(r^*\right)+\varepsilon\right) f\left(\varepsilon\right) d\varepsilon$$

$$\frac{\partial N\left(r^*\right)}{\partial r} = \int_{\underline{\varepsilon}}^{-\frac{1+v\left(r^*\right)}{r^*}} \left(v'\left(r^*\right)+\varepsilon\right) f\left(\varepsilon\right) d\varepsilon = \frac{\partial M\left(r^*\right)}{\partial r} - \int_{-\frac{1+v\left(r^*\right)}{r^*}}^{-\frac{v\left(r^*\right)}{r^*}} \left(v'\left(r^*\right)+\varepsilon\right) f\left(\varepsilon\right) d\varepsilon .$$

Because of $v'(r^*) = 0$, these partial derivatives are related as follows:

$$0 > \frac{\partial N\left(r^*\right)}{\partial r} = \frac{\partial M\left(r^*\right)}{\partial r} \underbrace{- \int_{-\frac{1+v\left(r^*\right)}{r^*}}^{-\frac{v\left(r^*\right)}{r^*}} \varepsilon f\left(\varepsilon\right) d\varepsilon}_{<0} > \frac{\partial M\left(r^*\right)}{\partial r} = \int_{\underline{\varepsilon}}^{-\frac{v\left(r^*\right)}{r^*}} \varepsilon f\left(\varepsilon\right) d\varepsilon.$$

Finally, we arrive at the following condition to ensure risk-taking-tax-neutrality:

$$\frac{(1-\gamma)s}{(1-\gamma s)\beta} = \frac{\frac{\partial N\left(r^*\right)}{\partial r}}{\frac{\partial M\left(r^*\right)}{\partial r}} = \frac{\frac{\partial M\left(r^*\right)}{\partial r} - \int_{-\frac{1+v\left(r^*\right)}{r^*}}^{-\frac{v\left(r^*\right)}{r^*}} \varepsilon f\left(\varepsilon\right) d\varepsilon}{\frac{\partial M\left(r^*\right)}{\partial r}} \equiv Q^* < 1.$$

Thus, we can show for an arbitrary tax rate $0<s<1$ and a given value Q^*:

(a) Under unlimited liability taxation has to be symmetric, et vice versa, to ensure risk-taking-tax-neutrality.

This can be derived for $\beta = 0$ (unlimited liability) from the initial neutrality condition:

$$\left(1-\gamma\right)s\frac{\partial M\left(r^*\right)}{\partial r} - \left(1-\gamma s\right)\beta\frac{\partial N\left(r^*\right)}{\partial r} = 0$$

From this condition immediately follows: $\gamma = 1 \Leftrightarrow \beta = 0$.

(b) If liability is at least partially limited, taxation has to be asymmetric to ensure risk-taking-tax-neutrality.

For $0 \leq s < 1$ and $\beta > 0$ the condition for risk-taking-tax-neutrality denotes:

$$\frac{\left(1-\gamma\right)s}{\left(1-\gamma s\right)\beta} \equiv Q\left(\gamma, s\right) = Q^* < 1$$

With respect to the ratio $Q(\gamma,s)$ the following relations hold:

$$Q(0,s)=\frac{s}{\beta}$$
$$Q(1,s)=0$$
$$\frac{\partial Q}{\partial \gamma}=-\frac{s(1-s)}{\beta(1-\gamma s)^2}<0.$$

So it follows $\beta>0 \Rightarrow \gamma<1$, because only for $\gamma<1$ can Q take strictly positive values. Since Q is strictly decreasing in γ, the neutrality condition can be met for any given value $0 < \beta \leq 1$. If the loss-offset parameter is required to take only non-negative values $\gamma \geq 0$, however, this is only true if the tax rate s exceeds the liability parameter β:

$$s>\beta>0 \Rightarrow \frac{s}{\beta}>1 \Rightarrow \exists\, \gamma \in [0,1) \left| \frac{(1-\gamma)s}{(1-\gamma s)\beta}=Q^*<1 \right. .$$

For $0<s\leq\beta$, it is not guaranteed that the neutrality condition can be met within the range of existing loss-offset limitations ($\gamma \geq 0$). Especially for low tax rates ($s \to 0$) or for extensive liability limitations ($\beta \to 1$), the condition for risk-taking-tax-neutrality requires negative values $\gamma<0$. This parameter setting is incompatible with existing loss-offset rules, because it demands not only neglecting losses for tax purposes, but even a taxation of losses!

Our results indicate that the request for legal-form-tax-neutrality may be misleading as far as risk allocation is concerned. Under full liability, symmetric taxation is a necessary condition for tax neutrality with respect to risk taking. This result is intuitive, because under full liability, the investor participates equally in positive and negative returns. Only symmetric taxation keeps the resulting risk-return incentives intact.

For limited liability firms, in contrast, asymmetric taxation is necessary to maintain risk choice undistorted. The economic reason is given by the opposing effects of limited liability and loss-offset restrictions. Whereas limited liability increases the demand for risky investment, loss-offset limitations decrease investors' willingness to invest in risky projects. For any given liability parameter β, there exists a unique loss-offset parameter γ which leaves risk taking undistorted compared to the symmetric case. As a consequence, if both policy action variables β and γ are variable, there is an infinite number of neutral $\beta-\gamma$ combinations for each given set of tax rates s and ratios Q^*.

From a tax policy perspective, it is problematic that every neutral $\beta-\gamma$ combination depends on the ratio Q^*, which must be determined individually for each risk-return function $v(r)$ and each distribution of returns $f(\varepsilon)$. Although a neutral loss-offset parameter γ does exist for any given tax rate s and any given liability parameter β, it ensures neutrality only for an individual case rather than for a general setting. For other risk-return functions and other distributions of returns, the supposedly neutral $\beta-\gamma$ combination will distort risk taking.

In any case, our results refute the proponents of legal-form-tax-neutrality. Our model clearly proves that firms with different liability parameters, i.e., incorporated and non-incorporated firms, must be subject to differential taxation if risk-taking-tax-neutrality is used as a tax policy objective. More generally, our model reveals an example of opposing partial tax neutralities.

5. Numerical examples

5.1. Tax effects

This section explains the effects of tax- and non-tax asymmetries using a simple distribution of the rates of return. Since the uniform distribution is probably the simplest continuous distribution function, it is adequate to illustrate the emerging effects. The probability density function is given by:

$$f(\varepsilon)=\begin{cases}\dfrac{1}{\overline{\varepsilon}-\underline{\varepsilon}} & \text{for} \quad \underline{\varepsilon}\le\varepsilon\le\overline{\varepsilon}\\ 0 & \text{otherwise.}\end{cases}$$

We need a symmetric distribution, because the expected value of the stochastic term should equal zero: $E[\varepsilon]=0$. Hence, $\overline{\varepsilon}=-\underline{\varepsilon}$, and

$$f(\varepsilon)=\begin{cases}-\dfrac{1}{2\underline{\varepsilon}} & \text{for} \quad \underline{\varepsilon}\le\varepsilon\le\overline{\varepsilon}\\ 0 & \text{otherwise.}\end{cases}$$

For the following numerical examples we use a quadratic risk-return function: $v(r)=z+ar+br^2$ with $z\ge 0$ as the risk-free interest rate. In order to meet the properties $v'\left(r<r^*\right)>0,\ v'\left(r^*\right)=0,\ \ v'\left(r>r^*\right)<0,\ v''\left(r\right)<0$, we need coefficients $a>0,\ b<0$. The individually optimal risk level in the symmetric case is given by the unique root of $v(r)$, which maximizes the expected future value $W=I\cdot\left[1+v(r)\right]$:

$$v'(r)=a+2br\overset{!}{=}0 \quad\Rightarrow\quad r^*=-\frac{a}{2b}>0 \quad\Rightarrow\quad v(r^*)=z-\frac{a^2}{4b}>0.$$

Thus, the expected rate of return in the optimum is strictly positive.

Assuming a uniform distribution yields the following auxiliary variables *M* and *N* and their partial derivatives with respect to *r*:

$$M = \int_{\underline{\varepsilon}}^{-\frac{v(r)}{r}} [v(r) + r\varepsilon] f(\varepsilon) d\varepsilon = \frac{\left[z + r(a+\underline{\varepsilon}) + br^2\right]^2}{4r\underline{\varepsilon}},$$

$$\frac{dM}{dr} = \frac{\left[z + r(a+\underline{\varepsilon}) + br^2\right]\left[-z + r(a+\underline{\varepsilon}) + 3br^2\right]}{4r^2\underline{\varepsilon}},$$

$$N = \int_{\underline{\varepsilon}}^{-\frac{1+v(r)}{r}} [1 + v(r) + r\varepsilon] f(\varepsilon) d\varepsilon = \frac{\left[1 + z + r(a+\underline{\varepsilon}) + br^2\right]^2}{4r\underline{\varepsilon}},$$

$$\frac{dN}{dr} = \frac{\left[1 + z + r(a+\underline{\varepsilon}) + br^2\right]\left[-1 - z + r(a+\underline{\varepsilon}) + 3br^2\right]}{4r^2\underline{\varepsilon}},$$

so that the expected future value can be written as

$$W_2 = I\left[1 + (1-s)z + (1-\gamma)sM - (1-\gamma s)\beta N\right]$$

$$= I \cdot \{1 + (1-s)z + (1-\gamma)s\frac{\left[z + r(a+\underline{\varepsilon}) + br^2\right]^2}{4r\underline{\varepsilon}} - (1-\gamma s)\beta\frac{\left[1 + z + r(a+\underline{\varepsilon}) + br^2\right]^2}{4r\underline{\varepsilon}}\}.$$

Basically, this expression seems to have a rather simple polynomial form, which should be easily optimized. However, although the roots of $\frac{dW_2}{dr} = I\left[(1-\gamma)s\frac{dM}{dr} - (1-\gamma s)\beta\frac{dN}{dr}\right]$ can be determined analytically, they are very complicated and cannot be interpreted in an economically meaningful way.

Numerical examples illustrate the substitution effect of limited liability and loss-offset restrictions derived above. We use the following assumptions:

- Tax rate: s=25%
- Risk-return function: $v(r) = z + ar + br^2 = 0.1 + 0.5r - 0.3r^2$, i.e., the risk-free interest is 10%
- Uniform distribution of the rate of return: $f(\varepsilon) = \begin{cases} -\frac{1}{2\underline{\varepsilon}} & \text{for} \quad \underline{\varepsilon} \le \varepsilon \le \overline{\varepsilon} \\ 0 & \text{otherwise} \end{cases}$,

  with $\underline{\varepsilon} = -10$

A moderate loss-offset restriction with γ=0.8 induces the following relationship between the investor's optimal risk level *r* and the expected future value. Our reference case is defined by full liability under symmetric taxation ($\beta = 0, \gamma = 1$; solid line). In addition, three versions with partially limited liability ($0 < \beta < 1$) are shown as well (thick dashed line: β= 0.1; medium dashed line: β=0.2; thin dashed line: β=0.3):

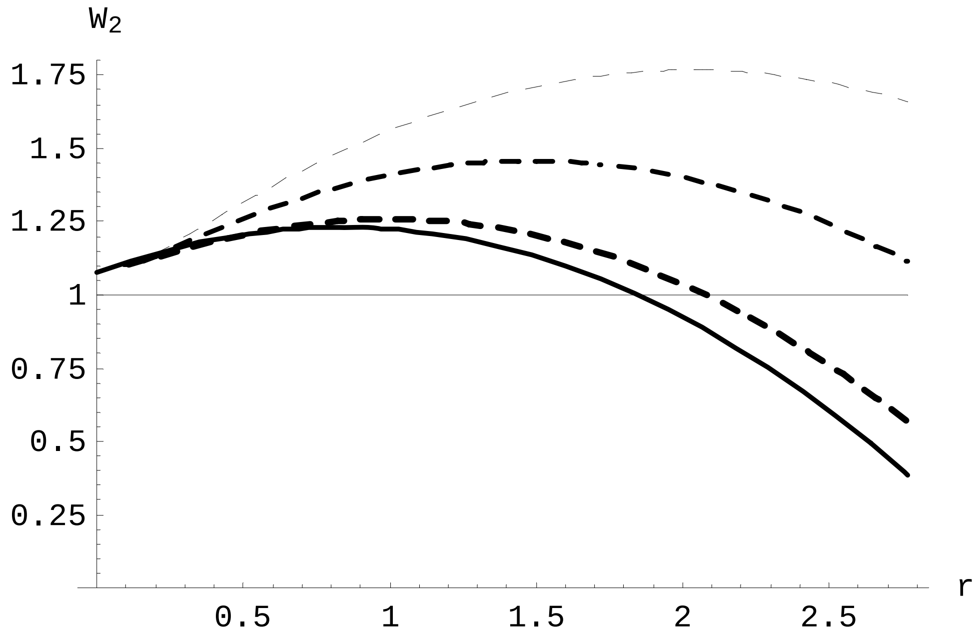

Figure 4: Expected future values as functions of the chosen risk level *r*

As has been formally shown above, the individually optimal risk level $\hat{r}$ increases with increasing liability limit β.

Even moderate liability limits (for example, β=0.3) induce rather high optimal risk levels which would cause negative returns if the investor was facing full liability (for example, *r* > 1.8471). The optimal risk levels associated with figure 4 are:

Liability- and loss-offset parameters	Optimal risk level $\hat{r}$
$\beta = 0 \quad \gamma = 1$	0.8333 (reference case)
$\beta = 0.1 \quad \gamma = 0.8$	0.9949
$\beta = 0.2 \quad \gamma = 0.8$	1.4811
$\beta = 0.3 \quad \gamma = 0.8$	2.0294

Table 1: Optimal risk levels depending on liability- and loss-offset parameters

Figure 4 clarifies that limited liability may generate substantial increases in expected future value compared to full liability. By contrast, the impact of tighter loss-offset limitations is rather small. As shown above, for lower values of γ the investor chooses a lower risk level. However, the general effects are confirmed for other parameter settings (for example, $\gamma = 0.6$). Figure 5 displays the expected future value for $\beta = 0, \gamma = 1$ (solid line) as well as for $\gamma = 0.6; \beta = 0.1/0.2/0.3$ (thick, medium and thin dashed line):

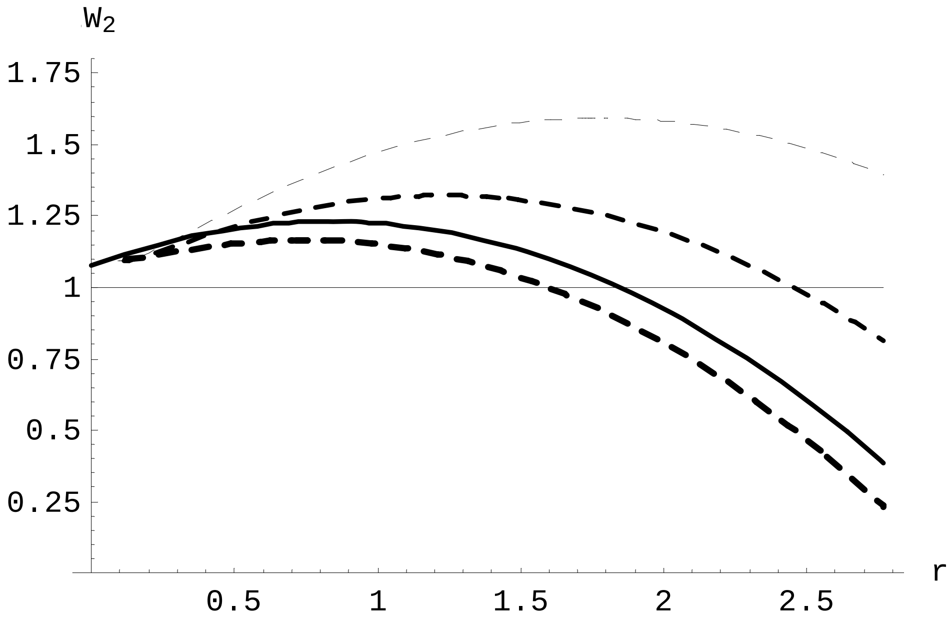

Figure 5: Expected future values as functions of the chosen risk level *r*

Now, the investor's optimal risk levels are:

Liability- and loss-offset parameters	Optimal risk level $\hat{r}$
$\beta = 0 \quad \gamma = 1$	0.8333 (reference case)
$\beta = 0.1 \quad \gamma = 0.6$	0.7378
$\beta = 0.120455 \quad \gamma = 0.6$	0.8333
$\beta = 0.2 \quad \gamma = 0.6$	1.2268
$\beta = 0.3 \quad \gamma = 0.6$	1.7780

Table 2: Optimal risk levels depending on liability- and loss-offset parameters

Again, this example shows that combinations of β and γ could generate lower, higher, or constant optimal risk levels compared to the symmetric case.

For quadratic risk-return functions $v(r) = z + ar + br^2$, the expected future value is a fourth-degree polynomial in *r*. Thus, taking infinite risks could generate economically useless infinite values: $\lim_{r \to \infty} W_2(r) = \pm\infty$. For this reason, we have to put an exogenously-given upper boundary $r^{\max}$ on the selectable risk level *r*. We think that this assumption reflects reality quite well, because the risk level of real investment cannot be chosen completely arbitrarily. Rather, the risk will be within reasonable limits.

5.2 Neutral Tax systems

Assuming a quadratic risk-return function $v(r)=z+ar+br^2$ the neutrality condition $\frac{\partial W_2(r^*)}{\partial r}=0$ for $r^*=-\frac{a}{2b}$ is written as:

$$\frac{\partial EW_2(r^*)}{\partial r}=I\left[(1-\gamma)s\frac{\partial M(r^*)}{\partial r}-(1-\gamma s)\beta\frac{\partial N(r^*)}{\partial r}\right]\overset{!}{=}0$$

$$\Rightarrow \frac{\frac{\partial M(r^*)}{\partial r}}{\frac{\partial N(r^*)}{\partial r}}=\frac{-\frac{(a^2-2a\underline{\varepsilon}-4bz)(a^2+2a\underline{\varepsilon}-4bz)}{16a^2\underline{\varepsilon}}}{-\frac{(a^2-2a\underline{\varepsilon}-4b(1+z))(a^2+2a\underline{\varepsilon}-4b(1+z))}{16a^2\underline{\varepsilon}}}$$

$$=\frac{(a^2-2a\underline{\varepsilon}-4bz)(a^2+2a\underline{\varepsilon}-4bz)}{(a^2-2a\underline{\varepsilon}-4b(1+z))(a^2+2a\underline{\varepsilon}-4b(1+z))}\overset{!}{=}\frac{(1-\gamma s)\beta}{(1-\gamma)s}.$$

There is no analytical solution in this special case. Rather, the neutrality condition is just given in implicit form. For this reason, the interaction between limited liability and loss-offset restrictions is illustrated by numerical examples.

The optimal risk level in the symmetric case given the parameters of section 5.1 ($s=0.25$; $v(r)=z+ar+br^2=0.1+0.5r-0.3r^2$; $f(\varepsilon)=-1/(2\underline{\varepsilon})$; $\underline{\varepsilon}=-10$) is $r^*=5/6$. In this setting the β^*-γ-combinations, which are neutral with respect to risk taking can be displayed graphically:

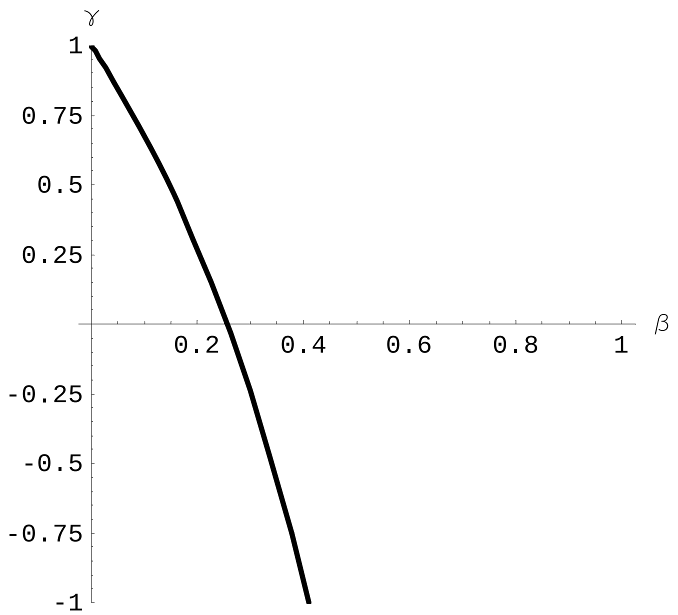

Figure 6: Neutral β-γ-combinations

The substitutional relation between loss-offset restrictions and limited liability can be confirmed for this functional specification. Moreover, for a sufficiently tight liability limit (for example, $\beta \geq 0.25$), risk-taking-tax-neutrality not only requires neglecting losses for tax purposes ($\gamma = 0$), rather, it would be necessary to tax losses as income ($\gamma < 0$). This apparently absurd result can be interpreted as follows. The (involuntary) loss incurred by the contractual partners of a limited liability investor would be allocated to the investor as taxable income. In this sense, negative values $\gamma < 0$ could be interpreted as a fiscal measure against moral hazard of limited liability investors. The liability limit prescribed by corporate law would be eliminated by tax law. Of course, tax debts would have to be excluded from limited liability.

The broader the range of the possible rates of return (i.e., the lower the values of $\underline{\varepsilon} < 0$) the larger are the potential benefits of limited liability. Thus, loss-offset rules have to be more restrictive for a higher variability of ε in order to keep risk-taking-tax-neutrality. This effect is clarified under the assumption of $\underline{\varepsilon} \in \{-10; -8; -6; -4; -2; -1.57\}$ in the following figure:

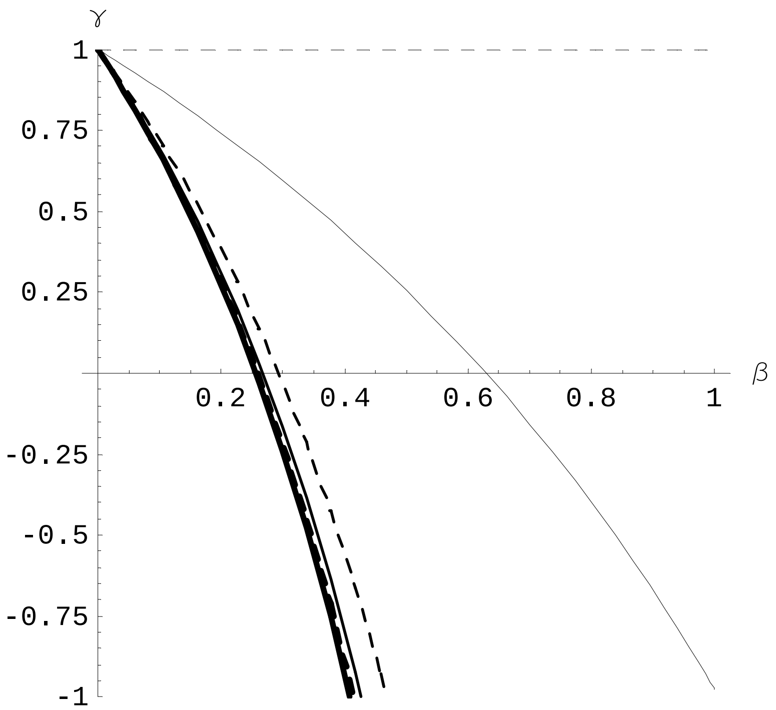

Figure 7: Neutral β-γ-combinations for different $\underline{\varepsilon}$-values

The neutral β-γ-combinations are relatively close together for sufficiently high probabilities of default ($\underline{\varepsilon} \in \{-10; -8; -6; -4\}$, solid and dashed bottom lines). For lower default probabilities and hence low probabilities of a total loss (e.g., $\underline{\varepsilon} = -2$, thin solid line) the loss-offset restrictions have to be relaxed. For $0 > \underline{\varepsilon} \geq -1,57$ (horizontal dashed line) rates of return below -100% cannot occur and limited liability cannot be binding. Hence, the neutrality condition is independent of β and a complete loss-offset must be granted ($\gamma = 1$).

The tax rate also has a substantial impact on the neutral β-γ-combinations. Low tax rates result in low loss-induced tax reimbursements, whereas high tax rates aggravate the relevance of loss-offset restrictions. To compensate for the benefits of limited liability, low tax rates require a much more severe tax "punishment" of losses than high tax rates. Thus, for low tax rates, the risk-taking neutral γ can be dramatically lower than for high tax rates. This is illustrated in figure 8, which shows neutral β-γ-combinations for different tax rates $s \in \{10\%, 25\%, 40\%, 55\%, 70\%, 99\%\}$. The left, thick solid line is valid for $s = 10\%$, the thin, dashed line top right for $s = 99\%$, respectively. Since the other parameters are held constant ($v(r) = z + ar + br^2 = 0{,}1 + 0{,}5r - 0{,}3r^2;\ \ f(\varepsilon) = -1/(2\underline{\varepsilon}); \underline{\varepsilon} = -10$), our reference case still is $r^* = 5/6$.

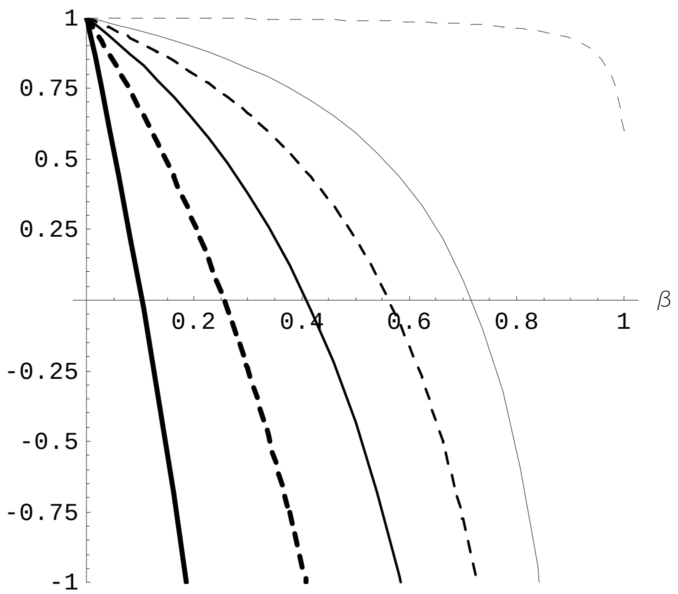

Figure 8: Neutral β-γ-combinations for different tax rates

As before, the negative slope of the functions indicates the substitutional relationship of limited liability and loss-offset restrictions. Furthermore, figure 8 illustrates that neutral β-γ-combinations may not be available within traditional loss-offset methods characterized by $0 \leq \gamma \leq 1$. A higher risk level r increases the probability for default and enhances the relevance of limited liability. For a compensation of the resulting benefit to the investor, a denial of loss-offset or even a taxation of losses could be necessary. The higher the tax rate, the more likely it is that this compensation effect can be achieved within the range of traditional loss-offset methods.

6. Summary and conclusion

This paper analyzes the simultaneous incentives arising from asymmetric taxation and limited liability on optimal risk taking. We show that loss-offset restrictions reduce incentives for risk taking by inducing concavities in the investor's value function. By contrast, limited liability convexifies the investor's value function and encourages risk taking. The combined effects of asymmetric taxation and limited liability depend on the intensity of both components. An important result is the fact that risk-taking-tax-neutrality and legal-form-tax-neutrality are incompatible: Under full liability, risk-taking-tax-neutrality requires symmetric taxation of profits and losses, whereas asymmetric taxation is a necessary condition for risk-taking-tax-neutrality under limited liability.

Again, these results reveal that legal-form-tax-neutrality – if defined simply as equal tax treatment of different legal forms – is at best irrelevant, but typically distorts risk taking decisions.

Although our model considers a risk-neutral investor only, we expect the results to remain qualitatively unchanged if risk-averse investors are taken into account. Compared to symmetric taxation, the basic incentives for risk taking induced by asymmetric taxation and limited liability are similar. Of course, the quantitative effects are different for risk-averse investors. However, it should be noted that symmetric taxation can increase risk taking compared to the pre-tax case under risk aversion[13].

Our model is based on some *ceteris paribus* assumptions. For example, we assume the distribution of the rates of return to be independent of the liability parameter. Implicitly, this means that contractual partners of the investor are indifferent with respect to (or unaware of) whether the investor is subject to full liability. Alternatively, we could assume that the contractual partners adjust the terms to which they are willing to deal with the investor in accordance with the liability conditions. I.e., market participants would demand a risk premium in the case of limited liability. Of course, a risk premium would imply different market prices for investors with limited and full liability.

The existence of a risk premium means that identical risk levels taken by different investors are economically different if the investors are subject to different liability parameters. This does not speak against our type of analysis. Rather, this insight points to difficulties in defining tax neutrality if more and more changes of market processes are considered. As a result, the problem of risk transfer to contractual partners as well as the question of tax shifting must currently be regarded as unsolved. In principle, risk shifting and tax shifting could be modelled within a general equilibrium model. However, this approach requires assumptions about the risk preferences of all market participants. For reasons of feasibility, we use a partial model which excludes these problems.

[13] See Domar / Musgrave (1944).

In any case, accepting the risk premium argument would rather support our results concerning legal-form-tax-neutrality. If risk taking decisions under full and limited liability were economically different, differential taxation of different legal forms would not violate the principle of equitable taxation.

As a further caveat, our model requires detailed information about the distributions of investments' returns. Even among professional investors these distributions are known only in exceptional cases. Furthermore, the loss-offset- and liability parameters are investor-specific as well as investment-specific and cannot be directly observed from tax law or corporate law. Liability-induced risk shifting between market participants is unlikely to be observable, too. Hence, risk-taking-tax-neutrality should not be regarded as a realistic tax policy objective. At best, it can serve as a benchmark for revealing the allocative effects of fiscal policy. Therefore, our model serves as a starting point to analyze the interdependencies between limited liability and asymmetric taxation from a tax policy perspective.

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