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Abstract

Tax competition between two governments who choose nonlinear income tax schedules to maximize the average utility of its residents when skills are unobservable and labor is perfectly mobile is examined. We show that there are no Nash equilibria in which there is a skill type that pays positive taxes to one country and whose utility is larger than the average utility in the other country or in which the lowest skilled are subsidized. We also show that it is possible for the most highly skilled to receive a net transfer funded by taxes on lower skilled individuals in equilibrium. These findings confirm the race-to-the-bottom thesis in this setting.

JEL-Code: D820, H210, H870.

Keywords: income tax competition, labor mobility, optimal income taxation, race to the bottom.

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1. Introduction

When nonlinear income taxes are set optimally for a closed economy, the high skilled pay taxes in order to subsidize the low skilled. However, in the presence of interjurisdictional labor mobility, high taxes might act as a catalyst for the high skilled to exit from a region, thereby thwarting the redistributive goals of its government. In this article, we investigate income tax competition between two national governments who set nonlinear income tax schedules when individuals of all skills are perfectly mobile between the two countries. We consider a discrete-type version of the model used in the seminal paper of Mirrlees (1971) to analyze optimal nonlinear income taxation for a closed economy. In order to isolate the impact of adding interjurisdictional tax competition to the Mirrlees model without at the same time introducing other factors that might affect an individual's choice of country of residence, we employ the same assumptions as are typically adopted in the discrete version of the Mirrlees model and assume that there are no impediments to moving between countries.

Specifically, we suppose that each government designs a tax schedule to maximize an average utilitarian social welfare function defined over the utilities of its residents, given the tax schedule in the other country, but taking full account of any mobility that might result from its choice. With fixed, immobile populations, this welfare criterion provides a motivation for transferring income from higher to lower skilled individuals. We also suppose that there is perfect labor mobility. That is, no resources need to be expended in order to move and there are no frictions to mobility based on residential attachments. We assume that it is only possible for a government to tax the income at source of its residents. It is not possible to tax the earnings of citizens who reside in the other country, nor do such individuals make remittances to their relatives in the country of origin. We further assume that the labor productivity of an individual does not depend on the country of residence. When combined with our assumption that labor is perfectly mobile, this assumption implies that locational decisions only depend on the tax schedules offered by the two countries.

Free labor mobility leads to aggressive competition. We show that there are no equilibria in our tax-setting game in which there is a skill type that pays positive taxes to one country and whose utility is larger than the average utility in the other country. As a corollary to this result, it follows that there are no equilibria in which individuals with the highest skill make positive tax payments to either country. We also show that it is impossible for the lowest-skilled residents in either country to receive a subsidy in equilibrium. An example is also provided that demonstrates that competition for the most highly skilled is so intense that it is even possible for them to receive a net transfer funded by taxes on lower skilled individuals in equilibrium.

Individuals make choices on two margins. The labor-leisure decision operates on the intensive margin, whereas the locational decision operates at the extensive margin.[1]

[1] We are ruling out the possibility that individuals may work in more than one jurisdiction. This possibility is modeled by Osmundsen (1999). The related problems of taxing multinational firms is discussed

The labor-leisure decision is largely driven by marginal tax rates, whereas the locational decision is more sensitive to average rates of taxation. Because the adjustments at the extensive margin, not the adjustments at the intensive margin, are the driving force behind our results, we are confined to making statements about total tax liabilities.

There is widespread concern that tax competition among jurisdictions for mobile capital and labor places severe constraints on the ability of these jurisdictions to engage in substantial redistributive taxation, to provide welfare programs for the needy, to maintain high safety and environmental standards, and to regulate labor practices, among other desirable social objectives. In other words, tax competition results in a "race to the bottom." The importance of this phenomenon in the context of interjurisdictional competition among communities in the same country was recognized by Stigler (1957), but it is also a familiar feature of competition between national governments. With an ever expanding European Union, barriers to mobility across national boundaries within Europe are eroding.[2] Similarly, restrictions on the free flow of capital around the world are diminishing. These developments have been accompanied by increased worries that the welfare state will wither and die due to the inability of national governments to maintain their social policies as the impediments to capital and labor mobility are relaxed. It is therefore important to determine the extent to which these concerns are justified.

There is a substantial literature that investigates the constraints that the competition between governments for mobile capital and labor places on the ability of governments to raise tax revenue and redistribute income. What distinguishes our analysis from the existing literature is that we assume that *all* governments act strategically, that governments operate in a second-best environment in which they are restricted to using nonlinear income taxes, and that there is perfect labor mobility of all skill types. The existing literature has made one or more of the following assumptions: only one country acts strategically, first-best tax policies are possible, only linear income taxation is possible, or there is less than perfect labor mobility.

For the most part, the research on interjurisdictional tax competition assumes that there is full information about the relevant characteristics of the agents in the economy. For an overview of the main issues considered and a review of what has been learned in this literature, see Cremer and Pestieau (2004) and Wildasin (2006a). However, in the context of redistributive income taxation, it is not only the tax policies of foreign governments that constrain the design of a country's income tax schedule by creating incentives for the wealthy to emigrate or the foreign poor to immigrate. As emphasized by Mirrlees (1971), the inability of a government to distinguish individuals with different skills also limits the amount of redistribution that is possible. Surprisingly, very little attention has been directed to investigating the validity of the race-to-the-bottom thesis in the presence of such information asymmetries when labor is internationally mobile.

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by Gresik (2001) and Olsen and Osmundsen (2001). Calzolari (2004) studies regulating multinationals.

[2]The importance of international migration for OECD countries is apparent from the data presented in Wildasin (2006b). For example, he documents that gross migration flows for most European Union countries exceeded 0.5 percent of total population in 2000, and in some cases exceeded 1 percent.

Our findings demonstrate that concerns about a race to the bottom are well founded when there is perfect labor mobility.

Our results stand in sharp contrast to the findings of Hamilton, Lozachmeur, and Pestieau (2002). They show that there is no race to the bottom when the two countries choose linear income tax schedules to maximize a Rawlsian objective function when only the low skilled are mobile and these individuals do not work. However, the Rawlsian objective adopted is nonstandard. Governments are assumed to maximize the poll subsidy available to the low skilled, not to maximize the utility of the worst-off residents. With the latter objective, no equilibrium with a positive poll subsidy exists because either government would want to lower its poll subsidy in order to induce the low skilled to move.

There are relatively few articles that consider strategic interaction among governments who can employ fully nonlinear income taxes when some individuals are free to choose both their country of residence and how much to work.[3]

Hamilton and Pestieau (2005) consider a political economy model of competition between a large number of small countries when there are two skill types and only one of them is mobile. The objective function of a government is determined by majority rule, with the consequence that it wants to maximize the utility of the type of individual who is in the majority. Hamilton and Pestieau focus on identifying what distributions of skill types communities can achieve in equilibrium.

Like us, Piaser (2007) studies nonlinear income tax competition by two benevolent governments in the presence of labor mobility, but in a model with only two skills. However, there are important differences between our work and his that lead to more intense competition for the highly skilled in the problem that we consider. Piaser assumes that both countries are identical, both in the objective that they pursue and in their initial skill distribution. He restricts attention to symmetric equilibria and, hence, while the potential for free movement of labor constrains what tax schedules are sustainable, nobody actually moves. In contrast to our assumption that labor is perfectly mobile, Piaser assumes that only one type of individual is mobile, but even for the mobile type, it is costly to move. Naturally, the introduction of moving costs softens tax competition, but it also provides a greater role for adjustments at the intensive margin to shape tax policy, which permits Piaser to make statements about equilibrium marginal tax rates, which we do not. Piaser considers two kinds of social welfare functions, one Rawlsian and one that evaluates outcomes by taking a weighted sum of the utilities of representatives of each type of individual, with the relative weight chosen so as to favor redistribution towards the low skilled. Both objectives have the feature that they are entirely indifferent to the number of individuals of a given type who reside in a country; all that matters is the utility achieved by each type. In contrast, with the average utilitarian criterion employed here, every resident counts positively. As a consequence, there is an incentive to compete for skilled individuals because, due to the incentive constraints that operate

[3] Huber (1999) considers how competition for mobile capital affects the choice of an optimal nonlinear income tax schedule when labor is immobile.

within a country, they have the highest utilities, and this would be the case even when increasing the number of skilled individuals residing in a country does not increase that country's tax revenue.

Lipatov and Weichenrieder (2010) also consider nonlinear income tax competition between two governments when there are two types of individuals. They assume that only the high-skilled are mobile and that these individuals differ in their costs of moving. Lipatov and Wehenrieder consider three social objective functions: maximizing tax revenue (subject to a minimum individual utility constraint), Rawlsian maximin uility, and total utilitarianism with no weight given to immigrants. Their main findings are that the tax competition lowers the taxes imposed on the high skilled (compared to the no-mobility benchmark) which, in turn, relaxes their downward self-selection constraint, thereby reducing the need to distort the low-skilled, whose labor supply is therefore increased.

The only other article that considers nonlinear income tax competition with mobility of all skill types is Morelli, Yang, and Ye (2010). Individuals in their model differ not only in their skills, but also in their costs of moving. Morelli, Yang, and Ye are primarily interested in whether having unified taxation for two jurisdictions or having tax competition between them would be chosen by majority rule in a constitutional stage that determines this institutional design question. They assume that each government wants to maximize the total utility of those individuals that fall within its tax jurisdiction. Analytical results are obtained when there are three skill levels. In this case, the lowest skilled prefer the unitary government so as to receive a higher subsidy, whereas the highest skilled prefer tax competition so as to lower their tax payments. The middle class faces countervailing incentives. They want to receive transfers from the highest skilled, but do not want to subsidize the lowest skilled. As a consequence, which regime is chosen depends on the preferences of the middle class.

Considerably more attention has been devoted to the analysis of optimal income taxation with mobile labor in the absence of strategic competition between governments. Early contributions to this literature include Bhagwati and Hamada (1982) and Wilson (1980) for the case of linear taxation and Mirrlees (1982) for the case of nonlinear taxation. Wilson (2006) provides an insightful interpretive survey of this literature. In much of this literature, potential emigrants choose between the best labor-consumption bundle available at home and some predetermined bundle or utility abroad. There is a potential conflict between a government's desire to tax on the basis of ability to pay and the possibility that more able individuals might emigrate to avoid high tax burdens. As shown by Wilson (1980) for linear taxation and by Wilson (1992) for nonlinear taxation, individual decisions made at the intensive and extensive margins can interact in complex ways, with the possibility of migration by skilled individuals lowering their average taxes while at the same time raising their marginal tax rates compared to what would be the case in a closed economy.[4] With strategic interaction, as in our model, decisions

[4]See also the recent articles by Krause (2009) and Simula and Trannoy (2010), both of which deal with nonlinear taxation.

made at the extensive margin play an even more central role, further constraining the amount of redistribution that is possible compared to what would be the case when other jurisdictions act passively.

We present our model in Section 2. In Sections 3 and 4, we respectively consider taxation of the high skilled and subsidization of the low skilled. Examples of equilibria are provided in Section 5. In Section 6, we offer some concluding remarks.

2. The Model

There are two countries, A and B, who have access to the same constant-returns-to-scale production technology that can be used to transform a single input, effective labor, into a single output. Units of these goods are normalized so that one unit of effective labor produces one unit of output. There are $n \geq 2$ types of individuals who differ in their labor productivities. An individual's labor productivity is the same in both countries. A type i individual has productivity w_i; that is, a unit of labor time supplied by such an individual is equivalent to w_i units of effective labor. Thus, by supplying l_i units of labor, an individual of type i produces $y_i = w_i l_i$ units of output. The production sector is perfectly competitive and, hence, the wage rate of an individual of type i equals his productivity w_i and his pretax income equals the amount of output y_i that he produces. Types are ordered so that $w_1 < w_2 < \cdots < w_n$. The total number of individuals of type i is $N_i > 0$. The distribution of these types is common knowledge.

Every individual has the same preferences over consumption c and hours of work l. These preferences are represented by the cardinally significant utility function $u(c, l)$. The function $u(\cdot)$ is strictly increasing in c, strictly decreasing in l, strictly concave, and twice continuously differentiable. For an individual with labor productivity w, his utility expressed in terms of observable variables is given by

$$v(c, y; w) = u\left(c, \frac{y}{w}\right). \tag{2.1}$$

These preferences satisfy the usual single-crossing property. In (y, c)-space, each person's indifference curves are upward sloping, with the slope increasing along an indifference curve as y increases. Furthermore, the slope of the indifference curve through (y, c) decreases with an increase in w.

Utility is not directly affected by location; that is, holding consumption and labor supply fixed, individuals are indifferent between country of residence. Thus, countries possess no country-specific amenities that any individual might find attractive and that would therefore introduce frictions to labor mobility based on residential attachments. We also assume that no expenditure of resources is required to change country of residence. Together, these features of our model imply that labor is perfectly mobile. As a consequence, it does not matter whether we regard individuals as having an initial country of residence or if we treat them as being initially stateless. For concreteness, we make the latter assumption, so an individual acquires his nationality by his choice of residence.

The government in each country has the authority to levy taxes on its residents for the purpose of redistributing income among them. Neither government can observe the labor productivity or the hours worked of any individual, but each of them can observe who resides within its borders and what each resident's pretax income is. Accordingly, taxes are based on labor income. The governments simultaneously and independently announce type-independent income tax schedules, $\tau^A(\cdot)$ and $\tau^B(\cdot)$, where $\tau^j(y)$ is the tax paid by a resident of country j, $j = A, B$, whose income is y. Each individual then chooses where to reside and how much labor to supply taking these tax schedules as given. These choices determine the amount of tax he pays, the government to which he remits these payments, and his after-tax income. Because there are no resource costs associated with the choice of residence, his consumption of the single output good equals his after-tax income. The number of individuals of type i choosing to reside in country j is N_i^j, where $N_i^A + N_i^B = N_i$ for all i.

For expositional purposes, we mostly think of countries as offering income-consumption *bundles* to different skill types. Strictly speaking, countries offer a tax function. The bundle that is offered to type i by country j is type i's most preferred bundle on the tax function offered by country j. We refer to $a^j = (c_1^j, y_1^j, \ldots, c_n^j, y_n^j)$ as the *allocation* offered by country j.

Because labor is freely mobile and the tax schedules are anonymous, with the amount of tax paid only a function of an individual's own income, the consumption c_i^j and income y_i^j of a type i individual residing in country j must satisfy a set of self-selection constraints.[5] See, for example, Guesnerie and Seade (1982). These self-selection constraints only apply to income-consumption bundles that are actually chosen by some individual. The allocations offered by the two countries must satisfy

$$v(c_i^j, y_i^j; w_i) \geq v(c_h^k, y_h^k; w_i) \text{ for all } j, k = A, B \text{ and all } h, i = 1, 2, \ldots, n \text{ such that } N_i^j > 0. \tag{2.2}$$

We assume that if $v(c_i^j, y_i^j; w_i) = v(c_h^k, y_h^k; w_i)$ with $h \neq i$, then an individual of type i chooses the bundle intended for his type; that is, he chooses (c_i^j, y_i^j). This assumption is standard in finite population optimal nonlinear income tax models when there is no mobility.

An immediate implication of the self-selection constraints is that if individuals of the same type reside in both countries, then they must receive the same utility regardless of where they live. Furthermore, within a country, (i) consumption and income are nondecreasing in type and two types have the same income if and only if they have the same consumption, in which case they are said to be bunched, and (ii) utility is increasing in type.

A tax schedule $\tau^j(\cdot)$ supports the allocation a^j offered by country j if (c_i^j, y_i^j) maximizes a type i individual's utility subject to $c_i^j = y_i^j - \tau^j(y_i^j)$ for all i. In a closed

[5]Our restrictions on the tax schedules conform to the practice in the optimal tax literature, rather than in the mechanism design literature, which typically places no *a priori* restrictions on the admissible mechanisms. Note that we are not appealing to the revelation principle here, which, as discussed in Martimort and Stole (2002), is problematic in multi-principal settings.

economy, the taxation principle says that any allocation that satisfies the corresponding self-selection constraints can be supported by a tax schedule. See, for example, Guesnerie and Seade (1982). In our two-country model, the taxation principle needs slight qualification because not all pairs of allocations that satisfy (2.2) can be supported by a pair of tax schedules. If $N_i^j = 0$ and individuals of type i prefer a different bundle offered by country j to the one offered them in a^j, then it is possible to satisfy (2.2) without having a^j supportable by a tax schedule. However, for any such allocation a^j, there exists another allocation for country j that only differs in bundles for which $N_i^j = 0$ that is supportable by a tax schedule and for which any such type still chooses to reside in the other country. For example, for such i, they can be offered what this type is offered in the other country provided that they choose to live in the other country even though they are indifferent about where they reside. Thus, restricting attention to allocations that satisfy (2.2) does not rule out any tax schedules of interest.

An individual of type i who resides in country j pays

$$T_i^j = y_i^j - c_i^j, \quad j = A, B, \ i = 1, 2, \ldots, n, \tag{2.3}$$

in taxes to government j. If T_i^j is negative, then this individual receives a transfer of $|T_i^j|$.

Each government wants to maximize the average utility of its residents.[6] Formally, social welfare in country j is given by

$$W(u_1^j, u_2^j, \ldots, u_n^j; N_1^j, N_2^j, \ldots, N_n^j) = \begin{cases} \frac{\sum_{i=1}^n N_i^j u_i^j}{\sum_{i=1}^n N_i^j} & \text{if } \sum_{i=1}^n N_i^j > 0 \\ -\infty & \text{if } \sum_{i=1}^n N_i^j = 0 \end{cases}, \quad j = A, B, \tag{2.4}$$

where u_i^j is the utility of a type i individual residing in country j.[7] With this welfare criterion, the individuals whom a government cares for are exactly those individuals whom it taxes. Moreover, individuals are sufficiently footloose that there is no meaningful distinction between residents and citizens that might justify differential tax treatment of them.[8]

[6] Unlike Epple and Romer (1991) and Morelli, Yang, and Ye (2010), there is no voting in our model.

[7] If nobody of type i resides in country j, then the value of u_i^j can be chosen arbitrarily. Because all individuals of the same type obtain the same utility when the self-selection constraints are satisfied, it is only necessary to consider the number of individuals who reside in a country and their common level of utility.

[8] The benefits provided by citizenship introduce frictions to mobility that have been ruled out by our assumption that labor is freely mobile. In a model with mobility costs, it would be natural to consider the possibility of subjecting residents and citizens to different tax schedules. Actual practice in this regard varies from country to country. For example, a citizen of the United States is subject to U.S. tax on his worldwide income regardless of his country of residence, whereas a Canadian citizen is not liable for tax in Canada if he resides elsewhere. For discussions of the relative merits of different proposals about who should count in a country's social welfare function, who it should tax, and how it should compute its income tax base, see Mirrlees (1982), Cremer and Pestieau (2004), and Wilson (2006).

The average utilitarian form of each government's objective gives rise to an important trade-off. On the one hand, if individuals were not mobile, an average utilitarian government would want to depart from the laissez-faire outcome by redistributing income from more highly-skilled to less highly-skilled individuals, provided that the natural conditions on individual utility functions described by Dixit and Seade (1979) are satisfied. On the other hand, because the self-selection constraints imply that utilities are increasing in type, governments have an incentive to attract the most highly-skilled individuals and to encourage the lowest-skilled individuals to locate in the other country so as to increase the average utility of their residents. Thus, each government faces a tension between the desire to redistribute resources from higher skill types to lower skill types and the possibility that, if taken too far, this redistribution might lead to exit by higher skill types and entry of lower skill types with a concomitant decrease in average utility. As emphasized by Cremer and Pestieau (2004) and Wilson (2006), this tension plays a fundamental role in the literature on redistributive taxation in the presence of mobile labor.

In addition to taking account of the self-selection constraints, each government must ensure that it has sufficient resources to carry out the redistribution that is required to implement the labor-consumption choices of those individuals who choose to reside in its county. Formally, the allocation of consumption and pretax income (effective labor) offered by country j must satisfy the following materials balance constraint:

$$\sum_{i=1}^{n} N_i^j c_i^j \leq \sum_{i=1}^{n} N_i^j y_i^j, \quad j = A, B. \tag{2.5}$$

By Walras' Law, this feasibility constraint is equivalent to requiring that the net tax revenue is nonnegative:

$$\sum_{i=1}^{n} N_i^j T_i^j \geq 0. \tag{2.6}$$

The two governments choose their tax schedules simultaneously with a view to maximizing the welfare in their respective states. A pair of tax schedules $\tau^A(\cdot)$ and $\tau^B(\cdot)$ is a (Nash) *equilibrium* if:

1. they induce a distribution of allocations that satisfies (2.2);
2. the allocations chosen from the tax schedules satisfy the materials balance constraints (2.5).
3. neither government has an incentive to change its tax function given the tax function used in the other country. Formally, for $i = A, B$: given $\tau^j(\cdot)$, no $\tilde{\tau}^i(\cdot) \neq \tau^i(\cdot)$ induces a set of allocations and residential decisions that both increases the value of the objective (2.4) for country i and satisfies the materials balance constraint (2.5) for country i.

When deciding on the potential benefits of revising the allocation it offers, a government anticipates the income-consumption and locational responses of individuals to these changes. It is possible that the allocation announced by one country, perhaps one that is obtained by deviating from a candidate equilibrium, results in a set of individual residential decisions that leave the other government with a budget deficit. Indeed, whenever a government, say in country A, considers modifying the allocation it offers for the express purpose of attracting individuals who pay positive taxes in the other country without otherwise altering the distribution of the other types of individuals across countries, the policy change in question results in a loss of revenue in the other country. Imposing the restriction that country B's revenue constraint be satisfied after A changes its offer is inconsistent with the notion of Nash equilibrium, for such a restriction requires, contrary to the usual Nash conjectures, that country A foresees a budget-balancing response by country B.[9] Note, however, that the revenue constraints of both governments are satisfied in equilibrium.

3. The Impossibility of Taxing the High Skilled

A government engaging in redistributive taxation has an incentive to attract highly-skilled taxpayers from the other country. In single-country formulations of the optimal nonlinear income tax problem, the tax paid is nondecreasing in the skill level if, as is typically the case, the government wants to redistribute resources towards the lower skilled individuals.[10] The more skilled individuals pay taxes that help finance transfers to individuals with low skills. This observation suggests that a country might wish to attract highly-skilled individuals in order to make it easier to finance its redistributive goals. Moreover, because utility is increasing in the skill level, a government with an average utilitarian objective function would want to attract these individuals for the direct contribution they make to social welfare. The ability to announce a fully nonlinear tax schedule provides governments with a powerful tool to compete for the most highly-skilled individuals. As we now show in Proposition 1, this tool is flexible enough to allow a country to attract some individuals that live in the other country and whose presence would increase its average utility provided that the immigrants pay positive taxes.

Proposition 1. *There does not exist an equilibrium in which there is a type i and a country j for which $T_i^j > 0$, $N_i^j > 0$, and u_i is larger than the average utility in country $k \neq j$.*

Proof. Suppose, by way of contradiction, that such an equilibrium exists. Without loss of generality, suppose that $T_i^B > 0$, $N_i^B > 0$, and u_i is larger than the average utility in country A. For all i and j, let $x_i^j = (y_i^j, c_i^j)$ be the bundle offered by country j to a type i individual. Starting with the allocations on offer in the candidate equilibrium, we show how at least one country can modify its proposed allocation so as to increase its average

[9] Piaser (2007), Lipatov and Weichenrieder (2010), and Morelli, Yang, and Ye (2010) also employ this equilibrium concept in their studies of income tax competition.

[10] This result is a direct consequence of combining Proposition 6 in Guesnerie and Seade (1982) with Proposition 1 in Brito, Hamilton, Slutsky, and Stiglitz (1990).

utility while at the same time satisfying all of the constraints that it faces. We note that when considering self-selection constraints, the single-crossing property implies that it is sufficient to consider the self-selection constraints between adjacent types.[11]

We first suppose that $N_i^A > 0$. It then follows from the self-selection constraints (2.2) that x_i^A and x_i^B are on the same indifference curve.

We begin by showing that $x_i^A = x_i^B$. On the contrary, suppose that $x_i^A \neq x_i^B$. Without loss of generality, we may assume that $x_i^A \gg x_i^B$. Let country A modify its offer by replacing x_i^A with an $\tilde{x}_i^A$ lying above the type i indifference curve through x_i^A and x_i^B, below the type $i-1$ indifference curve through x_i^B, and below the type $i+1$ indifference curve through x_i^A, as illustrated in Figure 1.[12] By self-selection and single-crossing, all individuals of a type other than i strictly prefer the bundle designed for them in the country they are currently residing to $\tilde{x}_i^A$. Thus, the only effect of the change is to induce all type i individuals to now choose $\tilde{x}_i^A$. In particular, type i individuals initially residing in country B move to country A. Choose $k \in \{A, B\}$ such that $T_i^k = \max\{T_i^A, T_i^B\}$. (If $T_i^A = T_i^B$, k can be chosen arbitrarily.) Because $T_i^B > 0$, it follows that $T_i^k > 0$ as well. By choosing $\tilde{x}_i^A$ sufficiently close to x_i^k, the change in country A's tax revenue is arbitrarily close to $N_i^B T_i^k + N_i^A (T_i^k - T_i^A)$. For either choice of k, the first term in this expression is positive. The second term is arbitrarily close to 0 if $k = A$ and it is nonnegative if $k = B$. Hence, the change in country A's tax revenue is positive, and so replacing x_i^A with $\tilde{x}_i^A$ is feasible for country A. (The case in which $k = B$ is illustrated in Figure 1.) Because, by hypothesis, the utility of type i individuals initially living in country B is larger than the average utility in country A and because all type i individuals now live in country A and have a larger utility than they did before country A's offer is modified, the average utility in country A increases. Therefore, the initial allocation offered by country A is not a best response to the one offered by country B. We have thus shown that $x_i^A = x_i^B$, which implies that $T_i^A = T_i^B$.

There are three cases to consider. In the first case, either $i = n$ or there exists a $k > i$ such that all individuals of type k strictly prefer their bundle(s) to x_i^B with all types between i and k bunched with type i.[13] In this case, country A can offer an allocation $\tilde{x}_i^A$ like the one shown in Figure 1, where now if $i \neq n$ the indifference curve for type $i+1$ is replaced with the indifference curve of type k individuals through their initial bundle(s) (and x_i^A coincides with x_i^B). This offer induces all individuals of type $i, \ldots, k-1$ to choose $\tilde{x}_i^A$ regardless of their initial country of residence. If $\tilde{x}_i^A$ is chosen to be sufficiently close to x_i^B, reasoning as above, the new offer is feasible for country A because it attacts all individuals of these types from country B and the tax revenue raised from them more than compensates for any loss in revenue from the initial residents of these types in country A. Moreover, by incentive-compatibility, any individual who is more skilled than type i has an initial utility that exceeds u_i. Because u_i is larger than the initial

[11] See Matthews and Moore (1987) for a justification of this claim and related discussion.

[12] If $i = 1$ (resp. $i = n$), then we only need to consider the self-selection constraints between types $i+1$ and i (resp. types $i-1$ and i). Henceforth, we do not consider special cases like these explicitly.

[13] It is possible that $k = i + 1$, in which case nobody is bunched with type i.

average utility in country A, the immigration raises this country's average utility. The increase in the utilities of the initial residents of country A of types $i, \ldots, k-1$ raises it even more. Hence, the change in country A's offer results in it having a larger average utility.

In the second case, everybody more highly skilled than type i is bunched with type i. Formally, for all $j \in \{A, B\}$ and all $k > i$ for which $N_k^j > 0$, $x_k^j = x_i^B$. Without loss of generality, suppose that $N_n^B > 0$. By offering an allocation $\tilde{x}_n^A$ sufficiently close to x_i^B that lies below the type $n-1$ indifference curve through x_i^B but above the type n indifference curve through x_i^B, as illustrated in Figure 2, only the type n individuals change their choice; they now all choose $\tilde{x}_n^A$ and pay a positive tax. Such a change increases the utility of any type n individuals already resident in country A and attracts all of this type of individual living in country B. Because the type n individuals have the highest utility of all types, country A's average utility increases. As above, if there is any loss in tax revenue from the initial type n residents of country A, it is more than compensated for by the tax revenue received from the new residents.

In the third case, there exists a $j \in \{A, B\}$ and a $k > i$ for which $N_k^j > 0$ such that $x_k^j \neq x_i^B$, both x_k^j and x_i^B are on the same type k indifference curve, and all types between i and k (if any) are bunched with type i. There are two subcases.

In the first subcase, $N_k^A T_k^A \leq 0$. Then, country A can offer a bundle $\tilde{x}_k^A$ lying above the type k indifference curve though x_i^B, below the type $k-1$ indifference curve through x_i^B, and below the type $k+1$ indifference curve through x_k^j, but sufficiently close to x_i^B so that positive taxes are raised at $\tilde{x}_k^A$, as illustrated in Figure 3. Now, all individuals of type k choose to reside in country A and pay positive taxes there, whereas tax revenue in country A from this type was non-positive before the new offer. Moreover, by incentive-compatibility, the utility of type k individuals is larger than that of type i individuals. Because only the type k individuals change the bundle chosen, reasoning as above, this change is feasible for country A and increases its average utility.

In the second subcase, $N_k^A T_k^A > 0$. The bundle $\tilde{x}_k^A$ used in the previous subcase might result in a loss in revenue for country A, which may not be feasible. Suppose that country A instead makes simultaneous offers $\tilde{x}_i^A$ and $\tilde{x}_k^A$ chosen so that (i) $\tilde{x}_i^A$ lies below the type $i-1$ and above the type i indifference curve through x_i^B, but below the type $k+1$ indifference curve through x_k^A and (ii) $\tilde{x}_k^A$ lies above the type k indifference curve though x_i^B, below the type $k-1$ indifference curve through x_i^B, and below the type $k+1$ indifference curve through x_k^A. Everybody of type $i, \ldots, k-1$ chooses $\tilde{x}_i^A$, everybody of type k chooses $\tilde{x}_k^A$, and the rest of the population keeps their original bundles. This pair of offers is illustrated in Figure 4. For $\tilde{x}_i^A$ sufficiently close to x_i^B and $\tilde{x}_k^A$ sufficiently close to x_k^A, any potential small losses in tax revenue from individuals of types $i, \ldots, k$ initially residing in country A are outweighed by the inflow of taxpayers of these types from country B. There must be such an inflow because $N_i^B > 0$. As above, the inflow of these individuals and the increase in the utilities of these types initially resident in country A also increases in the average utility in country A.

If $N_i^A = 0$, it need not be the case that $x_i^A = x_i^B$. Nevertheless, in each of the three cases considered above, the same argument applies when all statements about bunching with type i are interpeted as being about bunching with type i residents of country B. The bundle x_i^A may or may not lie on the type i indifference curve though x_i^B and it may well be the case that some higher types may be indifferent between their bundles and x_i^A even though nobody chooses x_i^A. Of course, if $N_i^A = 0$, country A does not have to worry about losing tax revenue from type i residents when it makes them a new offer.

Having shown that the allocation offered by one of the countries in the candidate equilibrium is not a best reply to the one offered by the other country, we conclude that no such equilibrium exists. □

To gain some intuition for Proposition 1, suppose that individuals of skill type i live in country B, pay positive taxes, and have a utility that is larger than the average utility in country A (i.e., they are relatively high skilled). The simplest case to consider is the one in which (a) no downward self-selection constraint binds with what is offered to type i individuals by either country and (b) there are individuals with this skill initially resident in both countries. Given the assumptions of Proposition 1, when (b) holds, individuals with this skill must be offered the same income-consumption bundle and pay the same taxes in both countries in equilibrium. If (a) also holds, because preferences satisfy the single-crossing property, it is possible to attract this skill type from country B by making them a slightly better offer without affecting where an individual of any other skill type locates or what bundle he chooses. Because these immigrants raise average utility and because the utilities of the initial residents of country A of type i are increased with the new offer, it is worthwhile to induce this skill type to immigrate from country B. It is also also possible to do so without violating country A's budget constraint because the new offer can be made arbitrarily close to the initial offer for this type, which, by assumption, raises positive taxes. Even though country A might collect less tax revenue per person from some of its existing residents of this type, the additional tax revenue it receives from the immigrants more than compensates for any loss of pre-existing revenues. The small losses at the intensive margin are of marginal significance compared to the revenue gains at the extensive margin. If either there are no initial residents of type i in country A or individuals of this type face a binding downward self-selection contraint, the arguments are somewhat more complex and it may be that it is an even more highly-skilled type of individual that is enticed to move, but the basic intuition remains the same: nonlinear income taxes are flexible enough to feasibly attract some highly-skilled individuals who pay positive taxes even though there may be a small loss in tax revenue from some existing residents. Hence, either country can feasibly raid the other country for its highly-skilled workers. Because these are the individuals with above average utilities, governments that want to maximize the average utility of their residents have an incentive to engage in such self-defeating competition.

The most highly skilled have the highest utility in the population. As a consequence, Proposition 1 has immediate implications for the taxation of these individuals.

Corollary 1. *There does not exist an equilibrium in which either* (a) $T_n^A > 0$ *and* $N_n^A > 0$ *or* (b) $T_n^B > 0$ *and* $N_n^B > 0$.

Corollary 1 demonstrates that the intense competition for the most highly skilled makes it impossible for either government to raise any tax revenue from these individuals. Each government can design its tax schedule in such a way as to offer income-consumption bundles targeted directly at individuals of type n. Because these individuals are completely mobile, either government can attract all individuals of type n by offering them a utility level slightly above that offered by its competitor. Therefore, if the highest-skilled individuals pay positive taxes, the governments engage in Bertrand-type competition by undercutting the taxes levied on these individuals in the other country.

4. The Impossibility of Subsidizing the Low Skilled

In the absence of labor mobility between countries, a country with redistributive goals transfers resources to its lowest-skilled residents, who pay a negative tax. However, when the self-selection constraints are satisfied, these individuals are the residents with the lowest utilities post transfer. As a consequence, if labor is mobile, there are strong incentives to induce the lowest-skilled residents to emigrate. If a country succeeds in inducing its poorest residents to move without otherwise affecting who resides within its borders and without changing what it offers to the other types of individuals, then it will have increased the average utility of its remaining inhabitants. Furthermore, it will run a budget surplus, and it may be possible to use this surplus to further increase the utilities of these residents. However, simply reducing the subsidy offered to the lowest-skilled residents by reducing the consumption offered to them may not lead to any mobility because what is offered to these individuals in the other country may be so unattractive that none of these individuals want to relocate there. Nevertheless, in Proposition 2 below, we show that the competition between the two countries is sufficiently intense that no country will subsidize its least-skilled residents in equilibrium. Thus, the tax competition thwarts the redistributive goals of both governments.

To formally state Proposition 2, we need to introduce some further notation. Consider an equilibrium pair of allocations (a^A, a^B). Let $\underline{i}^j$, $j = A, B$, denote the smallest type that chooses to live in country j in this equilibrium. That is, $\underline{i}^j$ is the smallest type i for which $N_i^j > 0$.

Proposition 2. *There does not exist an equilibrium in which there is a country* j *for which* $T_{\underline{i}^j}^j < 0$.

An immediate implication of Proposition 2 is that there is no equilibrium in which the lowest skilled receive a subsidy in any country in which they reside.

Corollary 2. *There does not exist an equilibrium in which either* (a) $T_1^A < 0$ *and* $N_1^A > 0$ *or* (b) $T_1^B < 0$ *and* $N_1^B > 0$.

The proof of Proposition 2 is rather lengthy. It may be found in the Appendix. Here, we provide an overview of the proof strategy and offer some intuition for the result.

The proof is by contradiction. For concreteness, suppose that country A subsidizes its lowest-skilled residents, who are of type k^1, and that its next lowest-skilled residents are of type k^2. The proof proceeds by showing that it is then possible for one of the countries to offer a new allocation that increases its average utility without violating either its budget constraint or the self-selection constraints once the individuals have reoptimized. We first consider the case in which the upward self-selection constraint in country A between type k^1 and type k^2 does not bind.

If anybody of type k^1 also lives in country B, the self-selection constraints imply that individuals of this skill type are indifferent about where they live. Hence, if country A simply reduces the consumption offered to this type, its lowest-skilled residents emigrate. Because they are also the residents with the smallest utility, their out migration increases country A's average utility. Furthermore, because country A now runs a budget surplus, the new allocation offered by it is feasible.

If nobody of type k^1 lives in country B, then if country A reduces the consumption offered to type k^1 individuals, they may not move; they might instead either accept the new offer or choose one of the other income-consumption bundles offered by this country. When $N^B_{k^1} = 0$, there are two cases. (i) If the average utility of all but type k^1 is no smaller in country A than in country B, we show that it is then possible for country B to increase its average utility by inducing these types to immigrate without also attacting country A's type k^1 residents. (ii) If the average utility of all but type k^1 is smaller in country A than in country B, we show that it is then possible for country A to increase its average utility either by inducing its lowest-skilled residents to emigrate or by inducing all of the residents of country B to immigrate.

In case (i), the residents of country A with skill type at least k^2 pay positive taxes in the aggregate and they have an average utility no smaller than the residents of country B, which makes them a desirable target for the latter country. Country B can induce just these types of individuals to immigrate by changing what it offers to them. These offers are determined by first showing that there are income-consumption bundles $\bar{x}^B_i$ for $i \geq k^2$ with $\bar{x}^B_i$ on the initial indifference curve for this type that (a) would not violate any self-selection constraint and (b) would give country B a budget surplus if it attracts all of these types of individuals from country A. If everybody whose type is at least k^2 accepts the new income-consumption bundle offered to them, not only is country B'a allocation feasible, it also does not reduce its average utility. However, because the residents of country A with these skill types are not made strictly better off with what is now offered to them by country B, they may not choose to move. Nevertheless, their positive tax revenue can be used to induce them to move. By offering each of these types an appropriately chosen income-consumption bundle $\hat{x}^B_i$ arbitrarily close to $\bar{x}^B_i$, each of these types can be made better off, thereby increasing country B's average utility, without violating its budget constraint. Again, the tax revenue gained from the in migration more than compensates for any marginal loss of tax revenue from the existing residents of country B. The $\hat{x}_i^B$ bundles need to be chosen so that no self-selection constraint is violated. Because preferences satisfy the single-crossing property and the self-selection constraints are satisfied with the $\bar{x}_i^B$ bundles, this can be done by first increasing both the consumption and income of type n individuals in such a way that they are made better off and such that their new bundle, $\hat{x}_n^B$, lies below the type $n-1$ indifference curve through $\bar{x}_{n-1}^B$. Type $n-1$ is now not subject to any binding downward incentive constraint, so similar adjustments can be made to its bundle. Iterating this argument one type at a time, all individuals with skill types at least k^2 can be made better off. Hence, it is feasible for country B to increase its average utility by inducing everybody except the lowest-skilled residents of country A to move.

In case (ii), country A first reduces the consumption offered to type k^1 by an amount that keeps the upward self-selection constraint with type k^2 slack and then offers the resulting income-consumption bundle to everybody with a type less than k^2.[14] If this induces the lowest-skilled residents to emigrate, then, as above, country A has successfully raised its average utility. If the new bundle does not induce out migration, country A is nevertheless left with a budget surplus that it can use to attract all of the residents of country B, thereby raising its average utility. By making the reduction in consumption offered to the low skilled described above sufficiently small, country A can ensure that type k^1 individuals do not face any binding downward self-selection constraints. The offers country A uses to induce the residents of country B to relocate are constructed in the same way as in case (i). For those less skilled than k^1, this can be done without violating any self-selection constraint because the type k^1 individuals do not face any binding downward self-selection constraint.[15] Hence, in this case, country A can feasibly increase its average utility.

It remains to deal with the case in which the upward self-selection constraint in country A between its lowest-skilled residents (type k^1) and its second lowest-skilled residents (type k^2) binds. If the type k^2 individuals in country A do not receive at least as large as subsidy as the type k^1 individuals, then the latter individuals can be made a new offer arbitrarily close to what is offered to type k^2 that makes them better off and that only they will choose. This leaves country A with a budget surplus and raises its average utility. Hence, if the initial allocations are an equilibrium, then $0 > T_{k^1}^A \geq T_{k^2}^A$. Letting k^3 denote the next lowest-skilled type of resident in country A, the same basic proof strategy as described above with some modifications can be used to show (a) that if the upward self-selection constraint in country A between type k^2 and type k^3 does not bind, then there is a contradiction with the assumption that the initial situation is an equilibrium and (b) that otherwise, $0 > T_{k^1}^A \geq T_{k^2}^A \geq T_{k^3}^A$. Iterating this argument, we either obtain a contradiction as above in the case of a non-binding upward incentive

[14]Recall that of these types, only those of type k^1 live in country A.

[15]For those lower skilled than type k^1, the natural analogue of the procedure used to generate new offers for the high skilled would instead use reductions in consumption and income to induce the low skilled in country B (who may be paying taxes that are needed to satisfy country A's budget constraint) to relocate. However, this is not possible if the lowest skilled face a binding nonnegativity constraint.

constraint or we find that all of the residents of country A are being subsidized, which is not feasible. Thus, we conclude that the lowest-skilled residents of either country cannot be subsidized in equilibrium.

While a country that subsidizes its lowest-skilled residents would like them to emigrate so as to increase the average utilty of the remaining residents, it is not always possible to induce only them to move. Nevertheless, as our summary of the proof strategy of Proposition 2 makes clear, there is always some way for one of the countries to take advantage of any subsidy paid to these individuals. Hence, they cannot be subsidized in equilibrium. As in Proposition 1, the intense competition between the two countries reduces their ability to pursue their redistributive objectives.

5. Examples of Equilibria

While equilibria with positive taxes for the most highly-skilled individuals and subsidies for the lowest skilled do not exist, there are situations in which equilibria exist in the tax competition problem being considered here. In this section, we provide examples of two kinds of equilibria for the two-type case. In the first example, each country offers the laissez-faire allocation. With the laissez-faire allocation, an individual of type i receives the unique income-consumption bundle that maximizes his utility subject to the constraint that his consumption equals his pretax income. In the second example, the low skilled subsidize the high skilled.

Example 1. If $n = 2$, it is an equilibrium for each country to offer the laissez-faire allocation for its residents when one country has all of the type 1 individuals and all the type 2 individuals live in the other country. To show why this is an equilibrium, without loss of generality, suppose that all the type 1 (resp. type 2) individuals live in country A (resp. country B).

Country A has no incentive to tax the type 1 individuals so as to induce them to move because this would leave it without any inhabitants, resulting in social welfare of $-\infty$. It cannot induce the type 2 individuals to leave country B without offering a subsidy. If it tried to finance this subsidy by taxing individuals of type 1, they would leave and take up the laissez-faire offer of country B. Thus, no such subsidy is feasible. Country B has no incentive to attract the type 1 individuals because to do so would decrease average utility in country B. Thus, neither country can find a better reply to the laissez-faire tax schedule when the population is segregated.

When both economies are initially tax free, the only way for a country to increase the utility of a group of individuals—either by benefiting some of its existing residents or by attracting individuals from the other country—is to offer them a transfer. However, this transfer must be financed from taxes levied on another group. Because the other country has no taxes, these would-be taxpayers emigrate.

Laissez-faire equilibria are supported by only very special distributions of the population in the two-type case. There are no laissez-faire equilibria when there are more than two types. In a laissez-faire equilibrium, no self-selection constraints bind between different types of individuals. If there are three or more types, at least one country must have residents of more than one type. If it offers less consumption to all of these types except for the highest-skilled of them, the individuals with the new offers all emigrate, thereby increasing the average utility of the remaining residents.

The economic forces sustaining the laissez-faire equilibrium can also sustain equilibria in which some low-skilled individuals pay positive taxes in both countries in order to make transfers to the most highly-skilled individuals in the population. This point can be most simply illustrated when there are two types.

Example 2. Suppose that $n = 2$, $N_1 = N_2 = 2$, $w_1 = 1$, $w_2 = 2$, and

$$u(c, y) = \ln(c) - \frac{y}{w}. \tag{5.1}$$

Each country offers a common tax schedule that induces individuals to choose the income-consumption pairs illustrated in Figure 5, with one person of each skill type residing in each country. In this allocation, the marginal rate of substitution between income and consumption equals 1 for each type of individual; that is, everyone faces a zero implicit marginal tax rate on income. We suppose that $T_1 = 1$ and, hence, that $T_2 = -1$. Thus, the individuals with skill w_1 (type 1) are taxed so as to make transfers to the individuals with skill w_2 (type 2).

The utility level of individuals of type i can be found by solving

$$\max_{c_i, y_i} \ln(c_i) - \frac{y_i}{w_i} \quad \text{subject to} \quad c_i = y_i - T_i. \tag{5.2}$$

The optimized value of problem (5.2) is

$$u_i^* = \ln(w_i) - \frac{T_i}{w_i} - 1. \tag{5.3}$$

Setting $w_1 = 1$, we obtain

$$u_1^* = -T_1 - 1 \quad \text{and} \quad u_2^* = \ln(w_2) + \frac{T_1}{w_2} - 1. \tag{5.4}$$

The average utility in each country with this allocation is given by

$$W^* = \frac{u_1^* + u_2^*}{2} = \frac{1}{2}\left[T_1\left(\frac{1 - w_2}{w_2}\right) + \ln(w_2)\right] - 1. \tag{5.5}$$

We show that country A (and, hence, also country B) cannot increase its average utility. Because everybody has a marginal rate of substitution equal to 1, the only way to increase the utility of one type of individual is to lower the tax paid by any individual of that type. If country A could costlessly induce the individual of type 2 residing in country B to move without modifying what anybody else chooses, it would do so because this move would increase its average utility by changing the composition of its residents.

However, this type 2 individual will move only if given an incentive to do so in the form of an increased transfer. Country A would have to raise additional tax revenue from type 1 individuals in order to cover the combined cost of a larger transfer for its existing type 2 resident and the additional transfer it must now pay to its new type 2 resident. However, any attempt to increase the tax paid by the type 1 individual residing in country A induces him to move to country B. Thus, country A cannot feasibly attract the other type 2 individual in this way.

Moreover, country A has no incentive to attract the type 1 individual living in country B because this would decrease its average utility. To see why, first note that the most beneficial way for country A to attract the type 1 resident of country B and still raise enough tax revenue to pay for the transfer to its type 2 resident is to offer the income-consumption bundle that a type 1 individual would choose if it faced a lump-sum tax of $\widehat{T}_1 = T_1/2$. This new offer induces the type 1 individual residing in country B to relocate. The two type 1 individuals generate sufficient tax revenue to keep the transfer that country A provides to its type 2 resident constant. The utility levels associated with this modified tax system are

$$\hat{u}_1 = -\frac{T_1}{2} - 1 \quad \text{and} \quad \hat{u}_2 = u_2^* = \ln(w_2) + \frac{T_1}{w_2} - 1. \tag{5.6}$$

The average utility generated in country A is now

$$\widehat{W} = \frac{2\hat{u}_1 + \hat{u}_2}{3} = \frac{1}{3}\left[T_1\left(\frac{1-w_2}{w_2}\right) + \ln(w_2)\right] - 1. \tag{5.7}$$

The new proposal reduces the average utility in country A if $W^* - \widehat{W}$ is negative, which is equivalent to the term in square brackets on the right-hand sides of both (5.5) and (5.7) being positive. This term is positive if

$$\ln(w_2) > T_1\left(\frac{w_2 - 1}{w_2}\right). \tag{5.8}$$

For our assumed values of $w_2 = 2$ and $T_1 = 1$, the left-hand side of (5.8) is approximately 0.69, whereas the right-hand side is 0.5, so this inequality is satisfied.

Country A could also contemplate making offers that are only attractive to the type 1 individuals. The best such offer is the laissez-faire allocation. In this case, country A's average utility is

$$\widetilde{W} = 2[\ln(w_1) - 1], \tag{5.9}$$

which is equal to -2 for our assumed value of $w_1 = 1$. Because $u_1^* = -2$ and $u_2^* > u_1^*$, $W^* > \widetilde{W}$, so it is not in country A's interest to change its offer in this way.

Hence, for the parameter values used in this example, neither country can feasibly modify the allocation illustrated in Figure 5 in a welfare-improving way.[16]

[16]It is clear from the preceding discussion that this conclusion is quite robust with respect to the choice of parameter values.

Our analysis of tax competition has a number of features in common with the competition in insurance markets studied by Rothschild and Stiglitz (1976). However, there are also important differences. In the Rothschild-Stiglitz model, there are two types of individuals (high risk and low risk) and the firms that provide insurance only care about expected profits, with the free entry of firms driving profits for each risk class to zero. In contrast, the countries in our model want to maximize the average utility of its residents and there are are fixed number of countries. There is a dual motive for attracting the most highly skilled when they pay postive taxes—they bring tax revenue and they also contribute directly to the objective function of the government. Similarly, there is a dual motive for encouraging the lowest skilled to emigrate if they are subidized—doing so reduces the transfers being paid and increases the average utility of those who remain if this strategy is succssful. Thus, tax competition generates pressure for a race to the bottom, with the competition possibly being so fierce that it can be optimal to subsidize the highly skilled.

In the two-type insurance model considered by Rothschild and Stiglitz, whether an equilibrium exists depends on (i) the degree of risk aversion (preferences), (ii) the relative proportions of the two types (the type distribution), and (iii) the gap between the high-risk and low-risk probabilities of an accident (the support of the distribution). With more than two types, the existence of an equilibrium also depends on these three factors: preferences, the type distribution, and the support of the distribution. The same is true in our model.[17] However, we expect the existence problem to be less severe in our tax competition problem than in the insurance problem because the countries do not face the threat of entry by new jurisdictions or exit by existing jurisdictions.

6. Concluding Remarks

The threat posed to redistributive taxation by subnational governments led Stigler (1957) to advocate centralizing the redistributive role of government. The same reasoning could be applied to argue that national governments should transfer the responsibility for redistributive taxation to a supranational government, such as the European Union.

The economy considered in this article has two features that make it very difficult to sustain redistribution from the rich to the poor. First, the average utilitarian social welfare function used to evaluate tax policies, when combined with the requirements of self-selection, creates an incentive for countries to attract highly-skilled taxpayers.[18] Many studies of income taxation in the presence of migration cite the revenue loss associated with emigration as a reason to provide incentives for the highly skilled not to move.

[17] Existence becomes more problematic as the gap between adjacent types (the values of $w_{i+1} - w_i$ in our model) goes to 0. With a continuum of types, this gap is 0. See Riley (2001, Section 3.1) for a discussion of this issue when there is a zero-profit constraint and a continuum of types.

[18] In the words of Johnson (1965, p. 300), our model generates the fundamental motive for worrying about a brain drain: the highly skilled "make a contribution to national welfare that goes beyond the money value of the services they perform and that this contribution can only be enjoyed if these people are resident within the nation."

See, for example, the survey by Cremer and Pestieau (2004). But, as we have emphasized, this is not the only reason why the high skilled are desirable residents; they are also desirable for the direct contribution they make to social welfare. Assuming that the governments pursue an average utilitarian objective is just one way to capture this concern. Any social welfare function for which the more highly skilled make a greater contribution than do the more lowly skilled, *ceteris paribus*, generates such a concern. More generally, a desire to have a population that is, on average, more skilled might originate from the general equilibrium effects of population composition on wages or the popular view that skilled workers are particularly valuable to a country in a knowledge-based economy.

Second, the perfect mobility of labor results in the competition for skilled individuals being particularly intense. Costless mobility is an extreme assumption. When mobility is costless, marginal changes in government policy can induce discrete changes in population composition. Moreover, when a marginal tax change leads to non-marginal migration across borders, the effects of this migration on either welfare or budget balance outweigh whatever within-country effects the change might have. Thus, in our model, migration ultimately trumps other considerations. However, contrary to the view expressed by Piaser (2007, p. 75), free mobility does not necessarily lead to laissez-faire. Given a strong enough desire to attract the most highly skilled, governments might actually settle on an equilibrium in which the most highly skilled receive a net transfer.

In order to make clear-cut statements about optimal marginal tax rates, our model needs to be modified so that marginal changes in taxes induce marginal changes in population composition. Models with individual-specific mobility costs, like the ones presented by Piaser (2007), Lipatov and Weichenrieder (2010), and Morelli, Yang, and Ye (2010), or some form of residential attachment, as in Blackorby, Brett, and Cebreiro (2007), provide settings in which to examine how the interaction of labor mobility and strategic tax competition affects marginal, not just average, tax rates. Alternatively, allowing a greater degree of diversity between countries by, for example, allowing for governments to provide differentiated public services as in Zissimos and Wooders (2008) might serve to soften competition. The extent to which the race-to-the-bottom thesis remains valid for personal income taxation in these settings is not yet fully explored.

Appendix

Proof of Proposition 2. The proof is by contradiction. Without loss of generality, suppose that $T^A_{\underline{i}^A} < 0$. Let $k^1 = \underline{i}^A$. Note that $T^A_{k^1} < 0$ implies that $c^A_{k^1} > 0$. Furthermore, in order for country A's revenue constraint to be satisfied, it must be raising a positive tax revenue from types $i^A_{k^1} + 1$ through n.

For all i and j, let $x^j_i = (y^j_i, c^j_i)$ be the bundle offered to a type i individual by country j in the candidate equilibrium. If type i individuals live in both countries, by the self-selection constraints, they must receive the same utility. Thus, we can let u_i denote the utility a type i individual obtains when he chooses from this menu of options, regardless of whether individuals of this type reside in both countries.

We first show that there must be at least one individual living in country B. Suppose that this is not the case and, thus, that the level of social welfare in country B is $-\infty$. In order for country A to satisfy its revenue constraint, there must be some type k for which $N_k^A > 0$ and $T_k^A > 0$. Let $\hat{x}_k = (\hat{c}_k, \hat{y}_k)$ be the bundle that maximizes type k utility subject to the constraint that $c_k = y_k$. If country B offers the allocation in which $\hat{x}_k^B = \hat{x}_k$ and $\hat{x}_j^B = (0, 0)$ for all $j \neq k$, then all individuals of type k relocate to country B, as does any other individual whose initial bundle is dominated by one of the two bundles offered by country B. In this allocation, the residents of country B simply consume what they produce, so country B's revenue constraint holds with equality. Furthermore, all of the self-selection constraints are satisfied. The average utility in country B is no longer $-\infty$ and, hence, it is not an equilibrium to have nobody live in country B.

Let k^2 be the smallest type larger than k^1 for which $N_{k^2}^A > 0$. We consider two cases.

Case 1. The upward self-selection constraint in country A between type k^1 and type k^2 does not bind; that is, $v(x_{k^1}^A; w_{k^1}) > v(x_{k^2}^A; w_{k^1})$. There are three subcases.

Subcase 1. In this subcase, $N_{k^1}^B > 0$. By the self-selection constraints (2.2), $x_{k^1}^A$ and $x_{k^1}^B$ are on the same indifference curve. Choose $\varepsilon > 0$ so that $c_{k^1}^A - \varepsilon \geq 0$. Suppose that country A changes its offer to $\tilde{a}^A$ by letting $\tilde{c}_i^A = c_{k^1}^A - \varepsilon$ for all $i = 1, \ldots, k^2 - 1$, with all of the other components of a^A unchanged. The individuals of type k^1 all strictly prefer $x_{k^1}^B$ to any bundle offered by country A and so locate in country B. No other type of individual is affected by the change in the allocation offered by country A. Because $T_{k^1}^A < 0$ and $N_{k^1}^A > 0$, the exit of individuals of type k^1 from country A leave it with a budget surplus. By the self-selection constraints and the definition of k^1, these individuals have the lowest utility in country A. Therefore, when they move, country A's average utility increases. Hence, the initial allocations offered by the two countries are not an equilibrium.

Subcase 2. Now suppose that $N_{k^1}^B = 0$ and that the average utility of all the types residing in country A excluding type k^1 is no smaller than the average utility in country B. That is,

$$\frac{\sum_{i \neq k^1} N_i^A u_i}{\sum_{i \neq k^1} N_i^A} = \frac{\sum_{i=k^2}^n N_i^A u_i}{\sum_{i=k^2}^n N_i^A} \geq \frac{\sum_{i=1}^n N_i^B u_i}{\sum_{i=1}^n N_i^B} = \frac{\sum_{i \neq k^1} N_i^B u_i}{\sum_{i \neq k^1} N_i^B}. \tag{A.1}$$

We show that it is possible for country B to increase its average utility by attracting all of the residents of country A except for those of type k^1.

Country B constructs a new offer in a series of steps as follows. Let $\tilde{x}_{k^1}^B = x_{k^1}^B$ and

$$\tilde{x}_i^B = \frac{N_i^A x_i^A + N_i^B x_i^B}{N_i^A + N_i^B}, \quad i \neq k^1. \tag{A.2}$$

In (A.2), if $N_i^A = 0$ (resp. $N_i^B = 0$), then $\tilde{x}_i^B = x_i^B$ (resp. $\tilde{x}_i^B = x_i^A$).[19] Otherwise, $\tilde{x}_i^B$ is a weighted sum of the initial bundles chosen by the type i individuals, with the weights

[19] Note that $\tilde{x}_i^B = x_i^B$ for all $i < k^2$.

given by the initial population shares of these individuals in the two countries. By the strict concavity of the utility function v,

$$v(\tilde{x}_i^B; w_i) \geq u_i, \quad i \neq k^1, \tag{A.3}$$

with a strict equality for any i for which $N_i^A > 0$, $N_i^B > 0$, and $x_i^A \neq x_i^B$. Now for $i \neq k^1$, choose $\bar{c}_i$ so that

$$v(\bar{c}_i^B, \tilde{y}_i^B; w_i) = u_i, \quad i \neq k^1. \tag{A.4}$$

Note that $\bar{c}_i \leq \tilde{c}_i^B$ for all $i \neq k^1$. Let $\bar{x}_i^B = (\bar{c}_i^B, \tilde{y}_i^B)$ for $i \neq k^1$ and $\bar{x}_i^B = x_i^B$ for $i = k^1$.[20]

For any initial resident of country A of type $i \neq k^1$ (if any), it is a matter of indifference whether to choose x_i^A or $\bar{x}_i^B$. Later, we shall provide a positive incentive for these individuals to move, but for now we simply show that if they all relocate, then country B runs a budget surplus and all of the self-selection constraints are satisfied. More concretely, suppose that all individuals of type $i \neq k^1$ choose $\bar{x}_i^B$ and that the individuals of type k^1 (who all live in country A) keep their initial bundles and do not relocate.

In order to show that country B runs a budget surplus when it offers the allocation $\bar{a}^B = (\bar{x}_1^B, \ldots, \bar{x}_n^B)$, we first determine the revenue implications of having all individuals of type $i \neq k^1$ choose $\tilde{x}_i^B$. In this case, country B's tax revenue from all types is the sum of its initial tax revenue and the tax revenue raised from types $i \neq k^1$ in country A in the candidate equilibrium. The former is nonnegative and the latter is positive. Because $\bar{c}_i \leq \tilde{c}_i^B$ for all $i \neq k^1$, it follows that country B runs a budget surplus if the individuals of type $i \neq k^1$ instead choose $\bar{x}_i^B$.[21]

By construction, all individuals are indifferent between the bundles they receive in the candidate equilibrium and in the alternative situation that we are considering in which country B offers the allocation $\bar{a}^B$. Furthermore, the only individuals who receive a new bundle are individuals of type $i \neq k^1$ for whom $N_i^A > 0$, $N_i^B > 0$, and $x_i^A \neq x_i^B$. For such an individual, $\bar{x}_i^B$ lies between x_i^A and x_i^B on type i's initial indifference curve. Because all of the self-selection constraints are satisfied in the candidate equilibrium, by single-crossing, they are also satisfied when country B offers the allocation $\bar{a}^B$.[22]

We now modify the $\bar{x}_i^B$ bundles so that all individuals of types $i \geq k^2$ strictly prefer to locate in country B and so that no self-selection constraint that applies to individuals of these types binds. For $i < k^2$, let $\hat{x}_i^B = x_i^B$. For $i \geq k^2$, let $\hat{x}_i^B = \bar{x}_i^B + (\varepsilon_i^1, \varepsilon_i^2)$ with $(\varepsilon_i^1, \varepsilon_i^2) \gg (0,0)$ chosen so that $v(\hat{x}_i^B; w_i) > v(\bar{x}_i^B; w_i)$ and $v(\hat{x}_{i-1}^B; w_{i-1}) > v(\hat{x}_i^B; w_{i-1})$ for all $i \geq k^2$ and $v(\hat{x}_{i+1}^B; w_{i+1}) > v(\hat{x}_i^B; w_{i+1})$ for all $i = k^2 - 1, \ldots, n-1$. Because the self-selection constraints are satisfied before this modification, by single-crossing, this construction is possible with arbitrarily small values for ε_i^1 and ε_i^2. If country B offers the $\hat{x}_i^B$ bundles holding country A's offered allocation fixed, then all individuals of type $i < k^2$ retain their initial bundles and do not move, whereas everyone else locates

[20] In fact, by construction, $\bar{x}_i^B = x_i^B$ for all $i < k^2$.

[21] A more formal argument for this conclusion using Jensen's inequality can be provided by adapting the proof of Lemma 4 in Blackorby, Brett, and Cebreiro (2007).

[22] We do not claim that the self-selection constraints are satisfied when country B offers the $\tilde{x}_i^B$ bundles.

in country B. Furthermore, all of the self-selection constraints are satisfied. Because country B has a budget surplus if all individuals of type $i \neq k^1$ reside there and receive the bundle $\bar{x}_i^B$, by choosing the values of ε_i^1 and ε_i^2 sufficiently small, its revenue constraint will be satisfied.

It remains to show that the average utility in country B has increased. By the continuity and monotonicity of the social welfare function, there exist $\bar{u}^A$ and $\bar{u}^B$ such that

$$\bar{u}^j = \frac{\sum_{i \neq k^1} N_i^j \bar{u}^j}{\sum_{i \neq k^1} N_i^j} = \frac{\sum_{i \neq k^1} N_i^j u_i}{\sum_{i \neq k^1} N_i^j}, \quad j = A, B. \tag{A.5}$$

By (A.1), $\bar{u}^A \geq \bar{u}^B$. Let $\hat{u}_i$ be the utility a type i individual obtains when country B offers the $\hat{x}_i^B$ bundles and country A offers the x_i^A bundles. Because nobody of type k^1 lives in country B, the average utility in country B after it modifies its offered allocation is $(\sum_{i \neq k^1} N_i^A \hat{u}_i + \sum_{i \neq k^1} N_i^B \hat{u}_i)/(\sum_{i \neq k^1}(N_i^A + N_i^B))$. We have

$$\begin{aligned}
\frac{\sum_{i \neq k^1} N_i^A \hat{u}_i + \sum_{i \neq k^1} N_i^B \hat{u}_i}{\sum_{i \neq k^1}(N_i^A + N_i^B)} &> \frac{\sum_{i \neq k^1} N_i^A u_i + \sum_{i \neq k^1} N_i^B u_i}{\sum_{i \neq k^1}(N_i^A + N_i^B)} \\
&= \frac{\sum_{i \neq k^1} N_i^A \bar{u}^A + \sum_{i \neq k^1} N_i^B \bar{u}^B}{\sum_{i \neq k^1}(N_i^A + N_i^B)} \\
&\geq \frac{\sum_{i \neq k^1} N_i^A \bar{u}^B + \sum_{i \neq k^1} N_i^B \bar{u}^B}{\sum_{i \neq k^1}(N_i^A + N_i^B)} \\
&= \bar{u}^B,
\end{aligned} \tag{A.6}$$

where the first inequality in (A.6) follows from the fact that $\hat{u}_i > u_i$ for all $i \geq k^2$ and $\hat{u}_i = u_i$ for all $i < k^2$, the first equality follows from (A.5), and the second inequality follows from the fact that $\bar{u}^A \geq \bar{u}^B$. Thus, the average utility in country B increases when it offers the $\hat{x}_i^B$ bundles instead of the x_i^B bundles, which contradicts the assumption that the initial offers are in equilibrium.

Subcase 3. Now suppose that $N_{k^1}^B = 0$ and that the average utility of all the types residing in country A excluding type k^1 is smaller than the average utility in country B. That is,

$$\frac{\sum_{i \neq k^1} N_i^A u_i}{\sum_{i \neq k^1} N_i^A} = \frac{\sum_{i=k^2}^n N_i^A u_i}{\sum_{i=k^2}^n N_i^A} < \frac{\sum_{i=1}^n N_i^B u_i}{\sum_{i=1}^n N_i^B} = \frac{\sum_{i \neq k^1} N_i^B u_i}{\sum_{i \neq k^1} N_i^B}. \tag{A.7}$$

Suppose that for all $i < k^2$, country A replaces x_i^A with $(c_{k^1}^A - \varepsilon, y_{k^1}^A)$, where $\varepsilon > 0$ has been chosen sufficiently small so that $c_{k^1}^A - \varepsilon \geq 0$ and $v(c_{k^1}^A - \varepsilon, y_{k^1}^A; w_{k^1}) > v(x_{k^2}^A; w_{k^1})$. Nobody except for possibly the individuals of type k^1 will choose this new bundle. If this new offer induces the type k^1 individuals to move to country B, then reasoning as in Subcase 1, this change is feasible for country A and it increases its average utility, contradicting the assumption that the initial offers were an equilibrium. Therefore, after any such offer, we can suppose that all of the type k^1 individuals remain in country A. Consider such an offer with ε chosen sufficiently small so that $v(c_{k^1}^A - \varepsilon, y_{k^1}^A; w_{k^1}) > v(x_{k^1-1}^B; w_{k^1})$. Note that the reduction in country A's average utility in this step can be made arbitrarily small by choosing ε sufficiently close to 0. Single-crossing and self-selection imply that $(c^A_{k^1} - \varepsilon, y^A_{k^1}) \gg x^B_{k^1-1}$.

To facilitate the use of the arguments from Subcase 2, we use the same notation for country A's modified offers as for its original offer. Note that the downward incentive constraints for $x^A_{k^1}$ are slack and that country A has a budget surplus. We show that it is possible for country A to increase its average utility by attracting all of the residents of country B.

Let the bundles $\bar{x}^A_i$ (in obvious notation) be defined as in Subcase 2 but with the roles of the two countries reversed. For any initial resident of country B of type $i \neq k^1$, it is a matter of indifference whether to choose x^A_i or $\bar{x}^B_i$. If they all relocate to country A, then the latter's tax revenue from all types is the total tax revenue initially raised by both countries, which is positive because country A runs a surplus and country B has no deficit.

Let $\hat{x}^A_{k^1} = \bar{x}^A_{k^1} = x^A_{k^1}$. For all other i, let $\hat{x}^A_i = \bar{x}^A_i + (\varepsilon^1_i, \varepsilon^2_i)$ with $(\varepsilon^1_i, \varepsilon^2_i) \gg (0, 0)$ chosen so that $v(\hat{x}^A_i; w_i) > v(\bar{x}^A_i; w_i)$ for all $i \neq k^1$ and so that no self-selection constraint binds with any of the bundles offered by country A. This is possible because of the single-crossing property and the fact that the downward incentive constraints between $x^A_{k^1}$ and both $x^A_{k^1-1}$ and $x^B_{k^1-1}$ are slack. Everybody now locates in country$A$. Reasoning as in Subcase 2 (with the roles of the two countries reversed), this change is feasible for country A and it increases its average utility. By choosing ϵ sufficiently small, this increase in country A's average utility more than compensates for the loss in its average utility in the first step of this argument, a contradiction to the assumption that the initial offers are in equilibrium.

Case 2. The upward self-selection constraint in country A between type k^1 and type k^2 binds; that is, $v(x^A_{k^1}; w_{k^1}) = v(x^A_{k^2}; w_{k^1})$.

Step 1. We first show that

$$0 > T^A_{k^1} \geq T^A_{k^2}. \tag{A.8}$$

Hence, the second-lowest-skilled resident in country A is also subsidized.

If $x^A_{k^1} = x^A_{k^2}$, then (A.8) obviously holds because then $T^A_{k^1} = T^A_{k^2}$. If $x^A_{k^1} \neq x^A_{k^2}$, in order to satisfy type k^2's self-selection constraint with $x^A_{k^1}$, we must have $x^A_{k^2} \gg x^A_{k^1}$. Suppose by way of contradiction that $T^A_{k^1} < T^A_{k^2}$. Because $x^A_{k^1}$ and $x^A_{k^2}$ are on the same indifference curve of a type k^1 individual, if there exists a k with $k^1 < k < k^2$, then by single-crossing and self-selection, $x^B_k = x^A_{k^2}$ for any such k (recall that $N^A_k = 0$ for such k). Furthermore, if $x^B_{k^2}$ lies on the type k^2 indifference through $x^A_{k^2}$, then $x^B_{k^2} \geq x^A_{k^2}$. Suppose that country A replaces $x^A_{k^1}$ with $\hat{x}^A_{k^1} = x^A_{k^2} - (\varepsilon^1, \varepsilon^2)$, where $(\varepsilon^1, \varepsilon^2) \gg (0, 0)$ is chosen so that $v(\hat{x}^A_{k^1}; w_{k^1}) > v(x^A_{k^1}; w_{k^1})$, $v(\hat{x}^A_{k^1}; w_{k^1+1}) < v(x^A_{k^2}; w_{k^1+1})$, and $v(\hat{x}^A_{k^1}; w_{k^1-1}) < v(x^B_{k^1-1}; w_{k^1-1})$ (recall that $N^A_{k^1-1} = 0$). Figure 6 illustrates this construction for the case in which $k^2 = k^1 + 2$. It follows from the preceding discussion that only individuals of type k^1 will choose the new bundle, all of whom already live in country A. Thus, country A's average utility increases. By choosing ε^1 and ε^2 sufficiently close to 0, because $T^A_{k^1} < T^A_{k^2}$, country A now has a budget surplus, so the new allocation is feasible for it. Hence, the supposition that $T^A_{k^1} < T^A_{k^2}$ is false and, therefore, (A.8) also holds if $x^A_{k^1} \neq x^A_{k^2}$.

Step 2. It follows from (A.8) that in order to satisfy country A's budget constraint, it must have residents of some type $k > k^2$ who pay positive taxes. Let k^3 be the smallest type larger than k^2 for which $N^A_{k^3} > 0$.

If the upward self-selection constraint in country A between type k^2 and type k^3 does not bind, then to conclude that the initial allocations are not an equilibrium, we can apply the argument from Case 1 with k^2 in place of k^1 and k^3 in place of k^2 with the following modifications.

(a) In Subcase 1, all individuals of types k^1 and k^2 relocate from country A to country B, thereby raising the average utility in country A and leaving it with a budget surplus.

(b) In Subcase 2, when the $\hat{x}^B_i$ bundles are defined as above with k^3 substituting for k^2, the initial residents of types k^1 and k^2 in country A remain there, whereas everybody else lives in country B.[23] Because both the type k^1 and k^2 residents are subsidized in country A, country B has a budget surplus when it offers the $\hat{x}^B_i$ bundles. Furthermore, (A.6) now holds with k^2 instead of k^1 and the left-hand side of this modified version of (A.6) is smaller than the average utility in country B because the type k^1 (not just the type k^2) residents of country A do not move and they have the smallest utility in their country.

(c) In Subcase 3, we begin by having country A replace x^A_i with $\hat{x}^A_{k^1} = (c^A_{k^1} - \varepsilon, y^A_{k^1})$ for $i < k^2$ and by $\hat{x}^A_{k^2} = (c^A_{k^2} - \nu, y^A_{k^2})$ for $i = k^2, \ldots, k^3 - 1$, where $\varepsilon > 0$ and $\nu > 0$ have been chosen sufficiently small so that $c^A_{k^1} - \varepsilon \geq 0$, $c^A_{k^2} - \nu \geq 0$, $v(\hat{x}^A_{k^1}; w_{k^1}) = v(\hat{x}^A_{k^2}; w_{k^1})$, and $v(\hat{x}^A_{k^2}; w_{k^2}) > v(x^A_{k^3}; w_{k^2})$. We have a contradiction unless all individuals of types k^1 and k^2 remain in country A after any such modification to country A's offered allocation, in which case individuals of type k^1 choose $\hat{x}^A_{k^1}$ and individuals of type k^2 choose $\hat{x}^A_{k^2}$. In this case, we choose ε and ν sufficiently small so that $v(\hat{x}^A_{k^1}; w_{k^1}) > v(x^B_{k^1-1}; w_{k^1})$ and so that the loss in country A's average utility is arbitrarily small. It must then be the case that $k^2 = k^1 + 1$ and that $N^B_{k^1} = 0$ (not just, as has been assumed, that $N^B_{k^2} = 0$), for otherwise, by single-crossing and self-selection, for $i = k^1, \ldots, k^2 - 1$, x^B_i would lie on the type k^1 indifference curve through $x^A_{k^1}$ and, hence, the type k^1 individuals would not remain in country A after it modifies its offer. By letting $\hat{x}^A_i$ for $i = k^1, k^2$ be defined as above and $\hat{x}^A_i$ be defined for all other i as in Subcase 3, we proceed as in that subcase.

If the upward self-selection constraint in country A between type k^2 and type k^3 binds, then we can use the argument in Step 1 to conclude that

$$0 > T^A_{k^1} \geq T^A_{k^2} \geq T^A_{k^3}. \tag{A.9}$$

Step 3. By repeating the preceding argument a finite number of times (with appropriate modifications), we either eventually arrive at a contradiction (as above) or we find that

$$0 > T^A_{k^1} \geq T^A_{k^2} \geq T^A_{k^3} \geq \cdots \geq T^A_{k^m}, \tag{A.10}$$

where $k^1 < k^2 < k^3 < \cdots < k^m$ and $k^1, k^2, k^3, \ldots, k^m$ are the types of individuals resident in country A in the candidate equilibrium. But then every resident of country

[23] Note that it is now possible for $N^B_{k^1}$ to be positive.

A is subsidized, which violates its budget constraint. Hence, the initial supposition that $T_{k^1}^A < 0$ is false. □

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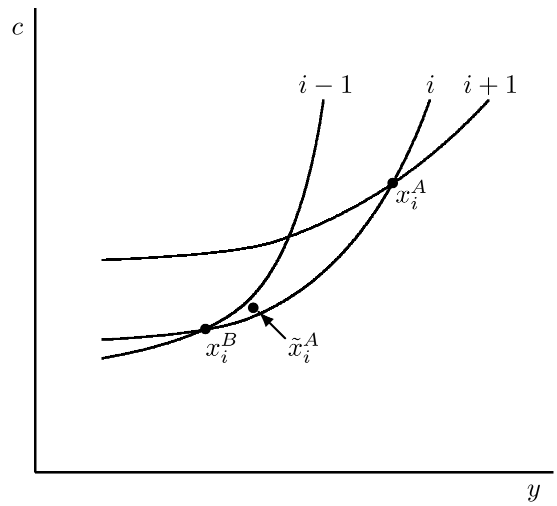

Figure 1: Reforming country A's tax schedule to attract type i individuals

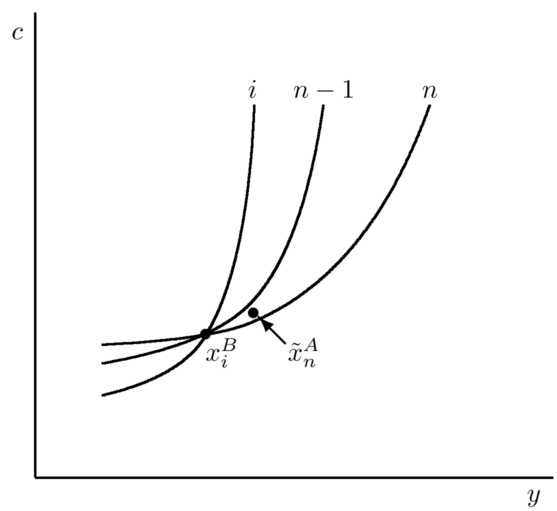

Figure 2: Reforming country A's tax schedule to attract type n individuals

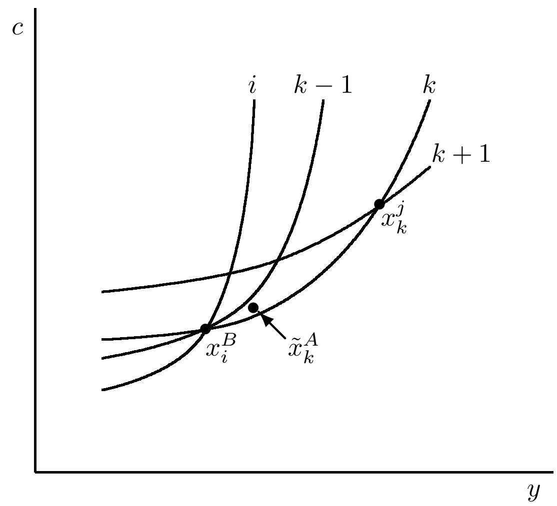

Figure 3: Reforming country A's tax schedule to attract type k individuals

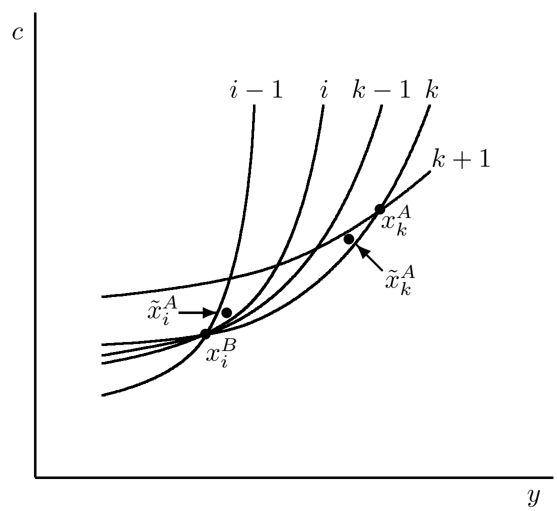

Figure 4: Reforming country A's tax schedule to attract individuals of types i and k

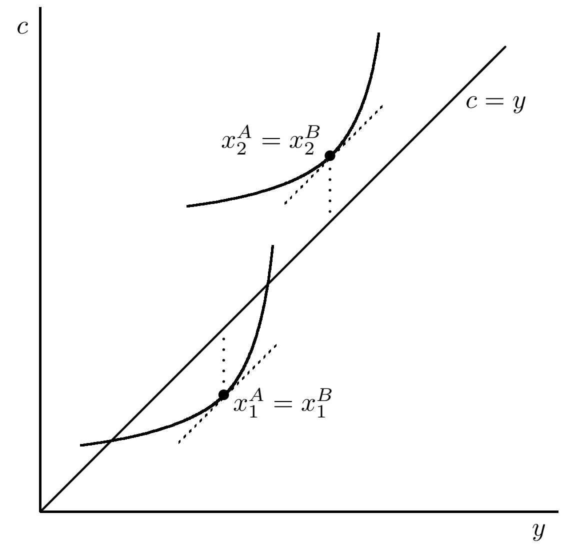

Figure 5: An equilibrium with transfers to the high skilled

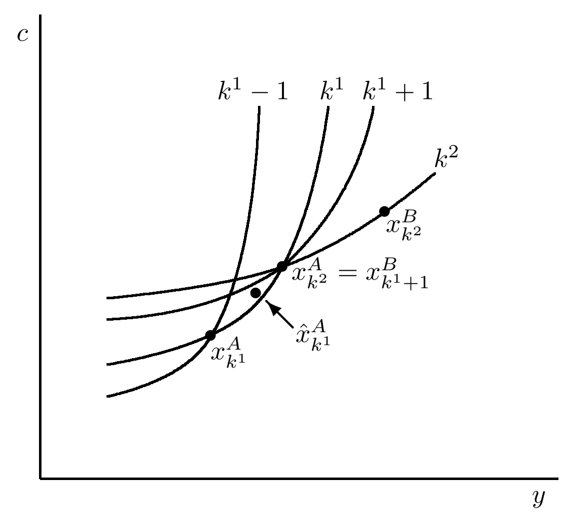

Figure 6: Reforming country A's tax schedule to increase utility for type k^1 individuals