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Abstract

In a partial equilibrium setting without price uncertainty, the balanced-budget substitution of an ad valorem tax on output for a specific (unit) tax can enhance welfare in imperfectly competitive markets and is without impact in a competitive world. This paper demonstrates that a substitution of this kind can also increase expected output and welfare in a competitive market characterised by uncertainty about the commodity price, if firms can respond to the revelation of demand conditions by altering output.

JEL-Code: H210, H250.

Keywords: ad valorem tax, commodity taxation, perfect competition, uncertainty, unit tax.

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> "We conclude .. that the choice between the two taxes (i.e. an ad valorem and a specific tax) is a matter of indifference under competition …" (Musgrave 1959, p. 305f)

> "In a world of perfect competition, and in which the nature of the product being sold is immutable, the balance between ad valorem and specific taxation is a matter of no significance. For the essence of perfect competition is that firms … take the price at which they can sell their product as given, … ." (Keen 1998, p. 4)

1. Introduction

The relative merits of an ad valorem tax and a specific (i.e. unit) tax on output are a longstanding issue. As the above quotations indicate, in a perfectly competitive market the two taxes are regarded as equivalent. Cournot (1838) – at least implicitly – and Wicksell (1896) clarify that this equivalence does not hold in a monopoly. Suits and Musgrave (1953) show that tax revenues resulting from a unit tax are lower than from an ad valorem tax, assuming tax rates which induce a monopolist to produce the same output. Skeath and Trandel (1994a), furthermore, make clear that the ad valorem tax Pareto-dominates a specific tax of equal yield. The analyses have been extended to monopolistic competition (Cheung 1998; Schröder 2004), oligopolies (Delipalla and Keen 1992; Denicolò and Matteuzzi 2000), and to frameworks in which firms can become informal and will then not pay taxes (Delipalla 2009a, b). In general, an ad valorem tax welfare-dominates a specific tax of equal yield, welfare being the sum of consumer and producer surplus.[1] However, this ranking is reversed for a monopsonist (Hamilton 1999) and may not hold in general equilibrium settings (Grazzini 2006; Blackorby and Murty 2007), in differentiated or multiproduct oligopolies (Anderson et al. 2001; Hamilton 2009; Wang and Zhao 2009), in the presence of externalities (Pirttilä 2002), or in two-sided markets (Kind et al. 2009).

Virtually all contributions to the debate assume payoffs to be certain. Such simplification will generally not affect the evaluation of a tax reform if risk-neutral firms are unable to react to the realisation of, for example, uncertain demand conditions; that is, if there is – what we call – *ex-post* uncertainty. In this paper, we also assume risk neutrality but *ex-ante* uncertainty instead of ex-post uncertainty, implying that the position of the demand curve becomes known *before* output decisions are made. We investigate how a balanced-budget substitution of an ad valorem tax for a specific tax in a perfectly competitive world of ex-ante uncertainty about demand conditions affects expected output and welfare. Accordingly, the analysis

[1] See also Bishop (1968). There is a closely related literature on the use of ad valorem and specific tariffs in trade policy in models of imperfect competition. See, for example, Helpman and Krugman (1989, Chap. 4), Skeath and Trandel (1994b), Jørgensen and Schröder (2005), Collie (2006), and Shea and Shea (2006).

pertains to products, such as regular consumption goods, for which demand does not vary too frequently, relative to the duration of the production process. Furthermore, we assume that tax rates are set prior to the revelation of the state of demand. This is a particularly relevant setting since consumption tax rates and the tax structure are altered only sporadically (cf. OECD 2008).

The basic analysis takes the number of firms as given. In this short-run perspective, firms make profits because they are assumed to produce under conditions of decreasing returns to scale. The substitution of an ad valorem tax for a specific tax of equal expected yield can result in higher expected output and a Pareto-improvement from an ex-ante perspective. This finding contrasts with Musgrave's and Keen's assertions for a world of certainty in which firms are price-takers, as quoted above. The intuition for the superiority of the ad valorem tax is as follows: since the ad valorem tax constitutes a fraction of the demand price, substituting it for a specific tax lowers the after-tax price variability, holding constant the expected tax payment per unit of output. This reduction induces the firm to raise expected output since the cost function is strictly convex. Nonetheless, the fall in the variability of after-tax prices lowers expected net profits because of the (strict) convexity of the profit function. If the government is to hold constant expected tax revenues, the quantity expansion requires a fall in the expected tax burden per unit of output. As a consequence, expected net profits go up. If expected output and profits rise, the shift towards ad valorem taxation can be a Pareto-improvement. In the long run, entry and exit of firms can be argued to ensure constant expected net profits and it is shown that in this particular case the tax structure becomes irrelevant again.

An income tax which allows for the deduction of losses can induce risk-averse agents to perform more of a taxable activity with uncertain outcomes than in the absence of taxation. The findings outlined above may, at first sight, therefore be based on this so-called Domar-Musgrave effect (1944). However, our analysis presumes risk neutrality. Moreover, firms benefit from uncertainty because of their assumed ability to respond to price variations and the convexity of the profit function. Accordingly, it is not the role of the government as an implicit insurer against risk which induces the increase in output but the interaction of lower expected costs, the ensuing rise in production, and the fall in expected taxes per unit of output.

A comparison between ad valorem and specific taxes in a perfectly competitive market of ex-ante uncertainty has not yet been undertaken. Assuming ex-post uncertainty, Fraser (1985) obtains ambiguous results for strictly risk-averse firms, whereas risk-neutral firms are unaffected by a substitution of an ad valorem tax for a specific tax, as mentioned above. Dickie and Trandel (1996) investigate a setting with ex-ante price uncertainty and a negative (production) externality. Dickie and Trandel compute the specific and ad valorem tax rates and the output quota which minimise the expected welfare loss due to the externality. In the absence of the externality – the case considered here – optimal Pigouvian taxes are zero because the government does not face a revenue constraint. Accordingly, Dickie and Trandel (1996) do not investigate the relative welfare effects of equal yield ad valorem and specific taxes. In a further contribution related to our investigation, Kotsogiannis and Serfes (2010) show that the relative superiority of ad valorem taxes in a Cournot oligopoly may no longer hold if marginal costs are uncertain. Finally, analyses of settings with endogenous quality choices are relevant to our study. This is the case because the equivalence of ad valorem and specific taxes in competitive markets vanishes (Liu 2003; Delipalla and Keen 2006) since the two types of taxes affect the firms' incentives to alter quality and output levels differently.

The remainder of the paper is structured as follows. In Section 2, the model is set up, assuming a given number of firms. Section 3 investigates the output effects of raising the ad valorem tax rate and lowering the specific tax, holding constant expected tax revenues. For expositional reasons and to convey the intuition for the result as succinctly as possible, a horizontal inverse demand curve is assumed. Subsequently, in Section 4, a downward-sloping demand curve is derived explicitly from the household's optimisation behaviour. It is shown that under mild additional conditions the findings obtained for the horizontal inverse demand curve continue to hold. Furthermore, Section 4 elaborates on the long run in which the entry and exit of firms ties down the level of profits. Section 5 analyses welfare effects, while Section 6 summarises the findings. Some calculations are relegated to an Appendix.

2. A Simple Model

The analysis takes a partial equilibrium perspective and focuses on one market. A given number of risk-neutral, identical firms produce a homogeneous commodity. With probability z^H, $0 < z^H < 1$, the pre-tax output price p^i is high, $p^i = p^H$, whereas with probability z^L the output price is low, $p^i = p^L$, where $z^L + z^H = 1$ and $0 < p^L < p^H$, i = L, H. Firms are price takers and use production technology exhibiting decreasing returns to scale. Accordingly, the cost function $C(x^i)$ is strictly convex and marginal costs $C'(x^i)$ are positive and increasing with the quantity x^i produced by the (representative) firm in state i, $0 = C(0) < C'(x^i), C''(x^i)$.

The inverse demand curve is horizontal (initially). Firms learn about the price before deciding about output (*ex-ante* uncertainty). They have to pay a specific tax at rate τ, $0 \leq \tau$, and an ad valorem tax t, $0 \leq t < 1$, defined as a fraction of the demand price p^i. We assume that tax rates are set prior to the revelation of the price and are determined in such a manner that the after-tax price will always be positive ($p^i(1 - t) - \tau > 0$). A representative firm's net profits in state i can, hence, be expressed as:

$$\pi(x^i) = (p^i(1-t)-\tau)x^i - C(x^i) \tag{1}$$

The firm maximises expected net profits $\Pi(x^H, x^L) = z^H\pi(x^H) + z^L\pi(x^L)$. Since the firm knows about the state of the world when making its decision, it chooses output in each state optimally, yielding $p^i(1 - t) - \tau = C'(x^i)$. This implies:

$$p^i \frac{\partial x^i}{\partial \tau} = \frac{-p^i}{C''(x^i)} = \frac{\partial x^i}{\partial t} < 0 \tag{2}$$

For simplicity, the number of firms is normalised to unity. In the presence of a horizontal inverse demand curve, equation (2) then also describes the change in the equilibrium quantity owing to a variation in tax rates.

The government incurs a fixed expenditure and – as mentioned above – has to set tax rates prior to the revelation of the state of demand. This implies that actual tax revenues are likely to vary with the price, for a given tax structure, because output levels adjust. Accordingly, the question arises what the appropriate definition of a balanced budget is, which then determines the extent to which one tax rate can be substituted for another. One possibility would be a restriction of constant revenues, irrespective of the state of the world. Keen (1998), however, shows that this requirement uniquely determines the tax structure as a function of the price elasticity of demand. A further option would be constant revenues for any tax structure in a given state of demand. A third possibility would be constant expected tax revenues. This is the definition of a balanced budget employed in the earlier contributions on the optimal commodity structure under uncertainty by Fraser (1985) and Kotsogiannis and Serfes (2010), and it will also be applied here.

For a given expenditure, the government will incur a budget deficit (surplus) if tax revenues are low (high), However, if the government could borrow and lend at the same interest rate, constant expected tax revenues would ensure a balanced budget in the long-run. An alternative justification for using the concept of constant expected revenues as restriction for tax policy could be the assumption that there are many markets, such as the one considered here, and that shocks are idiosyncratic. In consequence, z^i can be viewed as the probability that a particular market is in state i, and aggregate tax revenues will, hence, be constant if the number of markets is sufficiently large. Furthermore, the requirement of constant expected tax revenues can be the appropriate one if the government cannot insure against variations in tax revenues, either indirectly or directly, but the share of risk born by each individual tax payer is sufficiently small (cf. Arrow and Lind 1970). Assuming, therefore, that any tax reform must leave expected tax revenues unaffected, implies dB = 0, where B is given by:

$$B = B^H + B^L = z^H x^H (tp^H + \tau) + z^L x^L (tp^L + \tau) \tag{3}$$

Note, finally, that if the sequence of decisions were different and the government decided on tax rates after the state of the world had been revealed, the tax structure would not affect output. Put differently, as long as the government cannot perfectly condition tax rates on the output price, the subsequent findings continue to hold.

3. Output Effects of Commodity Tax Reform: Simple Model

Assume that the government marginally lowers the specific tax τ and raises the ad valorem tax rate t, holding constant expected tax revenues B. To calculate the feasible decline in the specific tax rate τ, we totally differentiate equation (3):

$$\frac{d\tau}{dt}\bigg|_{dB=0} = -\frac{B_t}{B_\tau} = -\frac{B_\tau^H p^H + B_\tau^L p^L}{B_\tau^H + B_\tau^L}, \text{ where} \tag{4}$$

$$B_\tau^i := \frac{\partial B^i}{\partial \tau} = \frac{1}{p^i}\frac{\partial B^i}{\partial t} := \frac{B_t^i}{p^i} = z^i\left(x^i + \left[p^i t + \tau \right] \frac{\partial x^i}{\partial \tau} \right), \text{ for i = L, H.} \tag{5}$$

We assume a positive budgetary effect of a rise in either tax rate. Clearly, B_τ^i, $B_t^i > 0$, B_t, $B_\tau > 0$, and B = 0 will hold if tax rates are zero. Therefore, starting from a tax rate level of zero, a rise in t or τ will ensure an increase in expected revenues. A further rise in tax rates will have a less pronounced positive budgetary effect because the tax base, ceteris paribus, shrinks with the decline in output. While, therefore, theoretically further increases in tax rates may eventually reduce tax revenues in the spirit of a Laffer-curve-type relationship, a government facing a budget constraint will never choose tax rates in such a manner that

$B_t < 0$ or $B_\tau < 0$ hold. If the government had (accidentally) done so, it could lower the respective tax rate, thereby increasing expected revenues, and also raising output in both states of the world because $\partial x^i/\partial\tau$, $\partial x^i/\partial t < 0$ (cf. equation (2)). As a consequence, the question of whether the government should substitute one tax rate for the other to increase output and welfare will only make sense if $B_t, B_\tau > 0$ holds.[2]

The change in expected output X, $X := z^H x^H + z^L x^L$, owing to the (marginal) substitution of the ad valorem tax for the specific tax is determined by:

$$\frac{dX}{dt}\bigg|_{dB=0} = z^H\left[\frac{\partial x^H}{\partial t} + \frac{\partial x^H}{\partial \tau}\frac{d\tau}{dt}\bigg|_{dB=0}\right] + z^L\left[\frac{\partial x^L}{\partial t} + \frac{\partial x^L}{\partial \tau}\frac{d\tau}{dt}\bigg|_{dB=0}\right] \tag{6}$$

Replacing the quantity adjustments and the required tax rate variation in accordance with equations (2), (4), and (5) and simplifying, we obtain:

$$\begin{aligned}\frac{dX}{dt}\bigg|_{dB=0} &= \frac{z^H}{C''(x^H)}\left[\frac{B_\tau^H p^H + B_\tau^L p^L}{B_\tau^H + B_\tau^L} - p^H\right] + \frac{z^L}{C''(x^L)}\left[\frac{B_\tau^H p^H + B_\tau^L p^L}{B_\tau^H + B_\tau^L} - p^L\right] \\ &= \frac{(p^H - p^L) z^H z^L}{B_\tau C''(x^L) C''(x^H)}\left[C''(x^H)x^H - C''(x^L)x^L - t(p^H - p^L)\right]\end{aligned} \tag{7}$$

The first term in equation (7) is positive since $p^H > p^L$, $B_\tau > 0$, and $C''(x^i) > 0$. The expression in square brackets will unambiguously be positive if *marginal* costs $C'(x^i)$ are not too concave and the ad valorem tax rate t, weighted by the price difference $p^H - p^L > 0$, is not too high initially. In general, the term in square brackets will be non-zero, irrespective of the extent of uncertainty, that is, the difference between p^H and p^L. Therefore, we obtain:

Proposition 1:

Assume a given number of risk-neutral, profitable firms, (some) ex-ante uncertainty about the demand price, a perfectly competitive output market, and a positive initial specific tax rate τ.

a) The substitution of an ad valorem tax t for a specific tax τ, holding constant expected tax revenues, alters expected output.

[2] Other contributions have approached this issue, following the approach pioneered by Suits and Musgrave (1953), by looking at what Delipalla and Keen (1992) call a P-shift, namely a substitution of one tax for the other, holding constant tax revenues per unit at a given price; that is, ignoring the budgetary repercussions of quantity and price adjustments.

b) If marginal costs are (weakly) convex, the introduction of an ad valorem tax t and a reduction of the specific tax τ such that expected tax revenues remain constant will raise expected output.

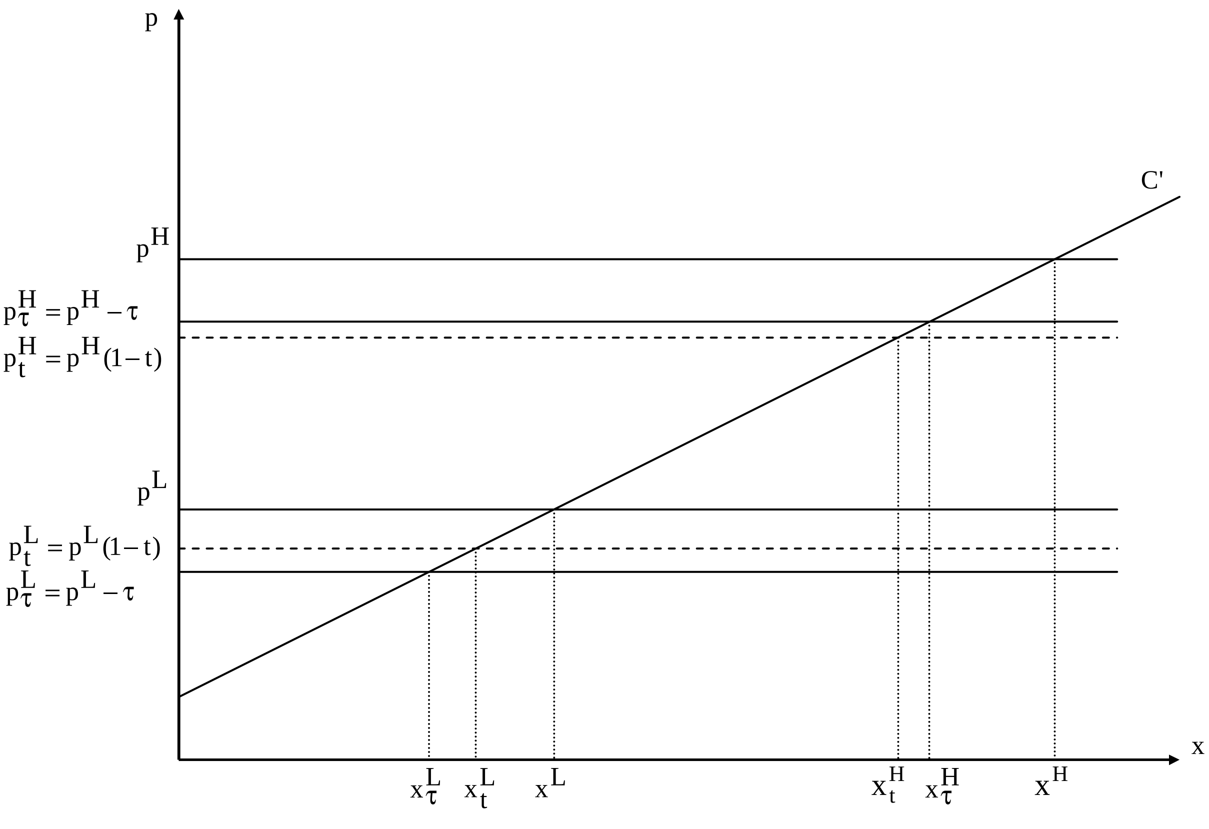
Assume, initially, linearly increasing marginal costs ($C''(x^H) = C''(x^L) > 0 = C'''(x^i)$). One rationalisation for part b) of Proposition 1 focuses on a setting in which the tax reform does not affect the expected output level X. Another feasible explanation takes constant expected tax payments per unit of output as its starting point. A tax reform not affecting expected output X will raise expected tax revenues since the increase in revenues from the ad valorem tax more than compensates for the loss resulting from the fall in the specific tax (as shown in Appendix 7.1). This is the case because the rise in tax receipts from the ad valorem tax t in the high-price state is greater than in the low-price state, not only because of the output difference but also because of the price differential. The fall in tax receipts owing to the decline in the specific tax rate τ, however, is unaffected by the price differential. If the firm produces the same expected output while tax revenues go up, a balanced-budget tax reform requires a reduction in at least one tax rate, inducing an increase in output.

Turning to the second explanation, assume that the specific tax is lowered while the ad valorem tax is raised to such an extent that expected tax payments per unit of output $(z^H p^H + z^L p^L)t + \tau$ are constant. The fall in the specific tax τ raises the after-tax price in both states of the world by the same amount. The ad valorem tax t, however, lowers the after-tax price in the high-price state by a larger absolute amount than in the low-price state. Accordingly, the firm reduces output in the high-price state and increases it in the low-price state. Given a strictly convex cost function, expected output rises. As the profit function, too, is strictly convex, the reduced difference in after-tax prices lowers expected net profits. However, the increase in output causes a budget surplus and the tax payment per unit can be reduced. This induces the firm to expand expected output further and guarantees an increase in expected net profits.[3] This reasoning will apply without limitations if the product of the initial tax rate t and the price difference $(p^H - p^L)$ is not too large. The greater the price difference, the larger the fall in expected output resulting from a given increase in the ad valorem tax rate. The higher the initial ad valorem tax rate, the more pronounced the decline in tax revenues will be due to a given reduction in expected output. An initial ad valorem tax rate of zero rules out such a tax base effect.

[3] See equation (11) below. Clearly, the increase in (expected) output will also arise in the present setting if the output market is not competitive but characterised, for example, by a monopoly. This will be the case since the uncertainty and market power effect reinforce each other.

In Figure 1, quadratic costs ($C''(x^H) = C''(x^L) > 0 = C'''(x^i)$), a horizontal inverse demand curve, and $z^H = z^L = 0.5$ are presumed. In the absence of taxes, output will be x^H (x^L) if the price is high (low) and equals p^H (p^L). A specific tax causes a downward shift of both inverse demand curves by an (identical) amount τ, resulting in after-tax prices p^H_τ and p^L_τ, where $p^H_\tau / p^L_\tau > p^H/p^L$. The output level in both states of the world falls by the same amount to x^H_τ and x^L_τ, implying $x^H - x^H_\tau = x^L - x^L_\tau > 0$. The ad valorem tax t shifts the inverse demand curves downward by the same fraction of the initial price ($p^H_t / p^L_t = p^H/p^L$). The new equilibrium quantities are x^H_t and x^L_t, where $x^H - x^H_t > x^L - x^L_t > 0$. Since the tax reform reduces output by less in the high-price state than it raises output in the low-price state ($(x^H - x^H_t) - (x^H - x^H_\tau) = x^H_\tau - x^H_t < x^L_t - x^L_\tau$), expected output increases. In addition, the ad valorem tax yields higher expected tax revenues than the specific tax.[4]

Figure 1: An Illustration



If marginal costs are strictly convex ($C''(x^H) > C''(x^L) > 0 < C'''(x^i)$), at least in the neighbourhood of the initial output levels, the positive output effect of the balanced-budget tax reform will be strengthened. As pointed out above, a high initial ad valorem tax rate t *and*

[4] As an example, suppose $p^H = 16 = 2p^L$, $C'(x) = 2 + 0.5x$, $z^H = z^L = 0.5$, $\tau = 2$ and $t = 0.15625$. We then have $x^H = 28$, $x^L = 12$, $x^H_\tau = 24 = 3x^L_\tau$, $X(\tau = 2) = 16$, $B(\tau = 2) = 32$, $p^H_t = 13.5 = 2p^L_t$, $x^H_t = 23$, $x^L_t = 9.5$, $X(t = 0.15625) = 16.25 > X(\tau = 2)$, and $B(t = 0.15625) = 0.5(23 \times 2.5 + 9.5 \times 1.25) = 34.6875 > B(\tau = 2)$.

a high price difference $p^H - p^L$ make it less likely that positive output effects occur. However, it is unlikely that both effects exactly cancel out. As long as there is some uncertainty of the ex-ante type, the tax reform alters expected output, as Part a) of Proposition 1 indicates.

4. Output Effects of Commodity Tax Reform: Partial Equilibrium Setting

The analysis has thus far been based on the simplifying assumption of a horizontal inverse demand curve to provide a clear intuition for the non-equivalence of equal expected yield ad valorem and specific taxes in a competitive output market with ex-ante price uncertainty. In this section we show that the same finding can be obtained in a setting in which demand is explicitly derived from the households' optimisation decisions, while price variations result from shocks to preferences.[5] Accordingly, the demand function is downward sloping and the prices p^H and p^L are no longer exogenous but determined endogenously as equilibrium outcomes.

Suppose, therefore, that the representative household's utility function is quasi-linear. Overall utility is increasing and strictly concave in the utility $\alpha^i u(x^i)$ from consuming the good under consideration, $\alpha^i u'(x^i) > 0 > \alpha^i u''(x^i)$, and linear in a second commodity, the price of which is normalised to unity. The parameter α^i, i = L, H, $\alpha^H > \alpha^L > 0$, captures shocks to preferences which induce ex-ante uncertainty about the position of the demand curve. Given an exogenous income and assuming an interior solution, the demand for the good is implicitly determined by $\alpha^i u'(x^i) - p^i = 0$. Therefore, the inverse demand function is downward sloping, its slope being given by $\partial p^i(x^i)/\partial x^i = \alpha^i u''(x^i) < 0$. Furthermore, the inverse demand function $p^H(x^H)$ is located above $p^L(x^L)$ for any given quantity. Normalising the number of price-taking households and firms to unity and incorporating the firm's first-order condition $p^i(1 - t) - \tau = C'(x^i)$, the market equilibrium can be defined by $C'(x^i) + \tau - (1 - t)\alpha^i u'(x^i) = 0$. For $\Omega(x^i) := C''(x^i) - (1 - t)\alpha^i u''(x^i) > 0$, the impact of a shock to preferences on the equilibrium quantity x^i is found to be positive:

$$\frac{\partial x^i}{\partial \alpha^i} = \frac{(1-t)u'(x^i)}{\Omega(x^i)} > 0 \qquad (8)$$

Accordingly, the equilibrium price p^i also rises with the parameter α^i:

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[5] We are grateful to an anonymous referee for suggesting this explicit derivation of the downward-sloping demand curve in the context of a proper and self-contained partial equilibrium model.

$$\frac{dp^i(x^i)}{d\alpha^i} = \frac{d(\alpha^i u'(x^i))}{d\alpha^i} = u'(x^i) + \alpha^i u''(x^i)\frac{\partial x^i}{\partial \alpha^i} = \frac{u'(x^i)C''(x^i)}{\Omega(x^i)} > 0 \tag{9}$$

Equations (8) and (9) clarify that the inequalities $x^H > x^L$ and $p^H > p^L$ also hold in a proper partial equilibrium setting, implying that the simplifications underlying the model of Section 2 are consistent with a more elaborate analytical specification.

The change in equilibrium output x^i owing to a rise in either of the tax rates is determined by:

$$p^i(x^i)\frac{\partial x^i}{\partial \tau} = -\frac{p^i(x^i)}{\Omega(x^i)} = \frac{\partial x^i}{\partial t} < 0 \tag{2'}$$

The decline in the specific tax rate τ, required to balance the budgetary impact resulting from a marginal increase in the ad valorem tax rate t, is determined by:

$$\left.\frac{d\tau}{dt}\right|_{d\tilde{B}=0} = -\frac{\tilde{B}_t}{\tilde{B}_\tau} = -\frac{\tilde{B}^H_\tau p^H + \tilde{B}^L_\tau p^L}{\tilde{B}^H_\tau + \tilde{B}^L_\tau}, \text{ where} \tag{4'}$$

$\tilde{B}^i_\tau$ and $\tilde{B}^i_t$, i = L, H, are given by:

$$\tilde{B}^i_\tau = \frac{\tilde{B}^i_t}{p^i(x^i)} = z^i\left(x^i + \left[p^i(x^i)t + \tau + \frac{dp^i}{dx^i}tx^i\right]\frac{\partial x^i}{\partial \tau}\right) \tag{5'}$$

In equation (5'), dp^i/dx^i describes the change in the equilibrium price p^i in state i due to a tax-induced variation in the (equilibrium) output level x^i. To distinguish the effects arising in the partial equilibrium setting characterised by the downward-sloping demand curve from those occurring in a world with a horizontal inverse demand curve (cf. Section 3), we use a tilda (~) for the relevant variables. $\tilde{B}^i_\tau$ and B^i_τ, for example, differ because, first, the output changes ($\partial x^i/\partial\tau$) of a given tax rate variation are not the same and, second, the tax rate change will induce price effects only if the demand curve is downward sloping, as the comparison of equations (5) and (5') clarifies.

Once again, we assume that $\tilde{B}_\tau$ and $\tilde{B}_t$ are positive. If this were not the case, the government could lower the respective tax rate to increase expected revenues. Since, moreover, equilibrium output levels decline with the tax rates (cf. equation (2')), substituting one tax rate for another will only represent a viable tax policy option if lowering a tax rate is costly to the government. Following the same procedure as in the derivation of dX/dt, i.e. substituting $\tilde{B}_\tau$ for B_τ and $\Omega(x^i)$ for $C''(x^i)$ in (7), the variation in expected aggregate output $\tilde{X}$ due to the introduction of an ad valorem tax (so that the initial ad valorem tax rate t is zero, $t = t^I = 0$) can be calculated as:

$$\left.\frac{d\tilde{X}}{dt}\right|_{d\tilde{B}=t^I=0} = \frac{(p^H(x^H)-p^L(x^L))z^H z^L}{\tilde{B}_\tau \Omega(x^L)\Omega(x^H)}\left[\Omega(x^H)x^H - \Omega(x^L)x^L\right] \tag{10}$$

If the marginal cost curve is (weakly) convex ($C''(x^H) \geq C''(x^L) > 0$), while the inverse demand curve $p^i(x^i)$ is (weakly) concave ($u'''(x^i) \leq 0$), $\Omega(x^H) \geq \Omega(x^L) > 0$ will apply. Under this restriction, expected output will continue to rise with the balanced-budget introduction of an ad valorem tax for a specific tax because $x^H > x^L$ holds (cf. equation (8)). In consequence, the result summarised in Proposition 1 continues to apply. The rationale for this is as follows: If the inverse demand curve is downward sloping and strictly concave, the reduction in the output differential in the two states will raise the average price received. Therefore, the effects of a given tax rate change on output are strengthened. In terms of Figure 1, a downward shift of a negatively-sloped inverse demand curve reduces the equilibrium quantity in the high-price state – characterised by a greater absolute slope of $p^H(x^H)$ – by less than in the low-price state. As a result, the fall in the quantity x^H_τ - x^H_t becomes less in relative terms and the positive expected output effect of the tax reform is enhanced.

The assumption of a strictly convex cost function is certainly plausible in the short run. In the long run, however, entry and exit of firms is likely to take place so that firms are producing at minimum expected average costs. The question then arises as to whether the non-neutrality of the proposed tax reform also applies in a competitive (very) long-run setting. To answer this question, we continue to assume a downward-sloping inverse demand curve and the following sequence of decisions: initially, the government sets the tax rates; subsequently, firms enter the market and, given the entry decision, the output price will be revealed; finally, firms select output. In such a setting, expected output will be unaffected by the balanced-budget introduction of an ad valorem tax for a specific tax if entry (and exit) of firms allows for no variation in expected profits (see Appendix 7.2 for the proof). The intuition is as follows: If expected net profits and tax revenues are to be unaffected by the tax reform, expected gross profits must remain constant. This will only be feasible if output in both states of the world is left unchanged by the tax reform. Effectively, the assumption of constant expected net profits rules out the possibility that tax reforms alter expected costs and, therefore, induce firms to change the expected output level.

5. Welfare Effects of Commodity Tax Reform

To analyse the welfare consequences of the substitution of an ad valorem tax for a specific tax, suppose that expected output rises, implying that entry and exit of firms is not feasible at a sufficient rate to rule out a change in expected profits. Initially, a horizontal inverse demand curve is considered. Since quantities are chosen optimally, following the same procedure as in the calculation of the output effect, will yield the subsequent expression for the variation in expected net profits Π if marginal costs are assumed to be linear and increasing ($C''(x^L) = C''(x^H) > 0$):

$$\frac{d\Pi}{dt}\bigg|_{dB=0} = -z^H x^H \left(p^H + \frac{d\tau}{dt}\bigg|_{dB=0} \right) - z^L x^L \left(p^L + \frac{d\tau}{dt}\bigg|_{dB=0} \right)$$

$$= \frac{(p^H - p^L) z^H z^L}{B_\tau C''(x^H)} \left[x^H (p^L t + \tau) - x^L (p^H t + \tau) \right] \qquad (11)$$

If the ad valorem tax is introduced, that is, if expression (11) is evaluated at an initial tax rate $t = t^I = 0$, expected net profits will unambiguously rise. Assuming identical households, their utility can be argued not to change with an output expansion if the inverse demand curve is horizontal. Since the budget is balanced in expected terms, the tax reform represents a Pareto-improvement from an ex-ante perspective.

However, the picture becomes more opaque in the presence of a downward-sloping inverse demand curve as derived in Section 4. Since expected aggregate output $\tilde{X}$ increases by assumption, while x^L rises and x^H declines due to the tax reform, $-z^L/z^H < \partial x^H/\partial x^L = -z^L/z^H + \varepsilon < 0$ results for $\varepsilon > 0$. For $dx^L > 0$, the change in expected consumer surplus $\tilde{S}$, $\tilde{S} := z^H \alpha^H u(x^H) + z^L \alpha^L u(x^L) + y - z^L p^L x^L - z^H p^H x^H$, is, using $\alpha^i u'(x^i) = p^i$, found to be:

$$\frac{d\tilde{S}}{dt}\bigg|_{d\tilde{B}=0} = -\left(z^L x^L \frac{dp^L}{dx^L} + z^H x^H \frac{dp^H}{dx^H} \frac{\partial x^H}{\partial x^L} \right) \underbrace{\left(\frac{\partial x^L}{\partial t} + \frac{\partial x^L}{\partial \tau} \frac{d\tau}{dt}\bigg|_{d\tilde{B}=0} \right)}_{:= dX^L / dt}$$

$$= z^L\left(\frac{dp^H}{dx^H}x^H - \frac{dp^L}{dx^L}x^L\right)\frac{dX^L}{dt} - z^H x^H \varepsilon \frac{dp^H}{dx^H}\frac{dX^L}{dt} \tag{12}$$

Based on the convention used in equation (5'), dp^i/dx^i in equation (12) captures the adjustment in the price p^i resulting from a reduction in the equilibrium quantity x^i due to a rise in a tax rate. Furthermore, we can write the variation in expected net profits $\tilde{\Pi} = z^H\tilde{\pi}(x^H) + z^L\tilde{\pi}(x^L)$, where $\tilde{\pi}(x^i) = ((1 - t)p^i(x^i) + \tau)x^i - C(x^i)$, as the sum of the variation in profits Π resulting in the setting with a horizontal inverse demand curve, $\Pi = z^H\pi(x^H) + z^L\pi(x^L)$, and an additional term which turns out to be a fraction of the variation in expected consumer surplus $\tilde{S}$. To do so, we use, first, the definition of dX^L/dt, as employed in equation (12) and, second, $\partial x^H/\partial t = (\partial x^H/\partial x^L)(\partial x^L/\partial t)$, where $\partial x^H/\partial x^L = -z^L/z^H + \varepsilon < 0$ for $\varepsilon > 0$, as defined above.

$$\left.\frac{d\tilde{\Pi}}{dt}\right|_{d\tilde{B}=0} = -z^H x^H\left(p^H + \left.\frac{d\tau}{dt}\right|_{d\tilde{B}=0}\right) - z^L x^L\left(p^L + \left.\frac{d\tau}{dt}\right|_{d\tilde{B}=0}\right)$$

$$+ (1-t)\left[z^H x^H \frac{dp^H}{dx^H}\frac{\partial x^H}{\partial x^L} + z^L x^L \frac{dp^L}{dx^L}\right]\left(\frac{\partial x^L}{\partial t} + \frac{\partial x^L}{\partial \tau}\left.\frac{d\tau}{dt}\right|_{d\tilde{B}=0}\right)$$

$$= \left.\frac{d\Pi}{dt}\right|_{d\tilde{B}=0} + \underbrace{(1-t)\left(z^H x^H \frac{dp^H}{dx^H}\frac{\partial x^H}{\partial x^L} + z^L x^L \frac{dp^L}{dx^L}\right)\frac{dX^L}{dt}}_{= -\left.\frac{d\tilde{S}}{dt}\right|_{d\tilde{B}=0}} \tag{13}$$

To determine the sign of $d\tilde{\Pi}/dt$, it is helpful to note that if the initial ad valorem tax rate t is zero ($t = t^I = 0$), the first term in the last line of equation (13) will be given by:

$$\left.\frac{d\Pi}{dt}\right|_{d\tilde{B}=t^I=0} = -z^H x^H\left(p^H + \left.\frac{d\tau}{dt}\right|_{d\tilde{B}=0}\right) - z^L x^L\left(p^L + \left.\frac{d\tau}{dt}\right|_{d\tilde{B}=0}\right)$$

$$= \frac{(p^H - p^L)z^H z^L \tau}{\tilde{B}_\tau}\left[\frac{x^H}{\Omega(x^L)} - \frac{x^L}{\Omega(x^H)}\right] \tag{14}$$

A sufficient condition for the term in square brackets in equation (14) to be positive, given $p^H > p^L$, $\tilde{B}_\tau > 0$, and $x^H > x^L$, is $\Omega(x^H) \geq \Omega(x^L)$. This (weak) inequality will hold, as outlined above, if the marginal costs curve is linear (implying $C'''(x^i) = 0$) and marginal utility is weakly concave (such that $u''(x^i) < 0 \geq u'''(x^i)$). Putting equations (13) and (14) together, observe, finally, that $d\tilde{\Pi}/dt + d\tilde{S}/dt = d\Pi/dt > 0$ for an initial ad valorem tax rate t of zero ($t = t^I = 0$) and $d\tilde{B} = 0$. Therefore, the welfare impact of the balanced-budget tax reform can be summarised as follows:

Proposition 2

Assume a given number of risk-neutral firms, linear marginal costs, (some) ex-ante uncertainty about the demand price, a perfectly competitive output market, and the balanced-budget introduction of an ad valorem tax t for a specific tax τ which is positive initially.

a) If the inverse demand curve is horizontal, the tax reform will be a Pareto-improvement.

b) In the partial equilibrium setting with an inverse demand curve explicitly derived from the households' optimisation behaviour, the sum of expected consumer and producer surplus will increase if expected output rises and marginal utility is weakly concave.

6. Conclusions

This paper has shown that the substitution of an ad valorem tax on output for a specific (unit) tax of the same expected yield will raise expected output in a perfectly competitive market in the presence of some ex-ante uncertainty if firms are risk-neutral and can respond to price variations by output adjustments, marginal production costs are increasing and weakly convex, the inverse demand curve is weakly concave and the initial ad valorem tax rate is sufficiently low. This increase in expected output implies higher welfare and may also represent a Pareto-improvement from an ex-ante view. Accordingly, an ad valorem tax can not only be superior to a specific tax of equal yield in imperfectly competitive markets but will be so – based on plausible assumptions – in a competitive setting. This effect is due to the strict convexity of the cost function, ensuring that the fall in after-tax price variability raises (expected) output. The positive output effect of a shift towards ad valorem taxation will no longer occur if aggregate output is determined by a zero expected profit condition or if marginal production costs are constant. However, a model with constant expected profits can be viewed as a limiting case of a set-up relevant for policy advice, because such a long-run equilibrium will – in the spirit of Keynes' (1923, p. 65) famous obiter dictum that "… this long run is a misleading guide to current affairs. In the long run we are all dead" – never actually be attained. Given this interpretation and following Keen's (1998, p. 4) assertion quoted at the beginning that " the essence of perfect competition is that firms … take the price at which they can sell their product as given", the findings summarised in Propositions 1 and 2 constitute valuable policy information because they suggest that even in the absence of market imperfections an ad valorem tax may be preferable to a specific tax.

7. Appendix

7.1 Change in Expected Revenues in Simple Model

The change in expected output $X = z^H x^H + z^L x^L$ owing to a rise in the ad valorem tax rate t and a decline in the specific tax rate τ (cf. equations (2) and (6)) will be zero if:

$$\frac{dX}{dt} = z^H\left[\frac{-p^H}{C''(x^H)} - \frac{1}{C''(x^H)}\frac{d\tau}{dt}\right] + z^L\left[\frac{-p^L}{C''(x^L)} - \frac{1}{C''(x^L)}\frac{d\tau}{dt}\right] = 0 \tag{A.1}$$

Assuming linear marginal costs ($C''(x^H) = C''(x^L) > 0$), the required decline in τ for $dX/dt = 0$ to hold equals $d\tau/dt = -(z^H p^H + z^L p^L) < 0$, since $z^H + z^L = 1$. Using the fact that $p^i B^i_\tau = B^i_t$ from equation (5), the change in expected revenues B, due to a marginal rise in t and a fall in τ, so that expected output X is unaffected, is given by:

$$\left.\frac{dB}{dt}\right|_{\frac{d\tau}{dt}=-(z^H p^H + z^L p^L)} = B^H_\tau \frac{d\tau}{dt} + B^L_\tau \frac{d\tau}{dt} + B^H_t + B^L_t$$

$$= -(B^H_\tau + B^L_\tau)(z^H p^H + z^L p^L) + (B^H_\tau p^H + B^L_\tau p^L)$$

$$= (p^H - p^L) z^H z^L \left[x^H + (p^H t + \tau)\frac{\partial x^H}{\partial \tau} - x^L + (p^L t + \tau)\frac{\partial x^L}{\partial \tau}\right]$$

$$= (p^H - p^L)\frac{z^H z^L}{C''(x^H)}\left[C''(x^H)(x^H - x^L) - t(p^H - p^L)\right] \tag{A.2}$$

In the derivation of (A.2) we have made use of equations (2) and (5). Evaluating the last line of (A.2) at an initial tax rate t of $t = t^I = 0$ clarifies that the budget experiences a surplus.

7.2 Long-run Model

With the exception of the modifications mentioned in the main text (cf. the last paragraph of Section 4), the analysis is based on the framework outlined in Section 2. In addition, we assume the inverse demand function to be downward sloping, as derived in Section 4. To simplify the exposition, we normalise the number of firms to unity and do not explicitly model the entry decision. Given that entry takes place until the net expected output price equals the minimum of expected average costs, the effects of entry and exit can be captured by calculating the variation in the expected output level of the representative firm, which takes the output price as given. Expected net profits equal $\tilde{\Pi}(x^H, x^L) = z^H \tilde{\pi}(x^H) + z^L \tilde{\pi}(x^L)$. The change in output x^i owing to a marginal rise in the specific tax τ can be calculated by differentiating expected net profits $\tilde{\Pi}(x^H, x^L)$ with respect to x^i and τ.

$$d\tilde{\Pi} = z^H \frac{dp^H}{dx^H} x^H dx^H + z^L \frac{dp^L}{dx^L} x^L dx^L - (z^H x^H + z^L x^L) d\tau \tag{A.3}$$

Setting $d\tilde{\Pi} = 0$ because expected profits $\tilde{\Pi}(x^H, x^L)$ are constant due to entry and exit, and solving for the change in output in state i due to a rise of the specific tax rate τ yields:

$$\left.\frac{dx^i}{d\tau}\right|_{d\tilde{\Pi}=0} = \frac{z^H x^H + z^L x^L}{z^i x^i \dfrac{dp^i}{dx^i}} \tag{A.4}$$

In equation (A.4), $dp^i(x^i)/dx^i < 0$ depicts the change in the equilibrium price owing to an increase in output in state i, i = L, H (see equation (5') as well). To simplify the subsequent exposition, we define the following variables:

$$x_\tau^{ii} := \frac{1}{\dfrac{dp^i}{dx^i}} < 0 \quad \text{(A.5a)} \qquad x_\tau^{HL} := \frac{z^L x^L}{z^H x^H \dfrac{dp^H}{dx^H}} < 0 \quad \text{(A.5b)} \qquad x_\tau^{LH} := \frac{z^H x^H}{z^L x^L \dfrac{dp^L}{dx^L}} < 0 \quad \text{(A.5c)}$$

Substituting these definitions into (A.4), we obtain:

$$\left.\frac{dx^i}{d\tau}\right|_{d\tilde{\Pi}=0} = \frac{z^H x^H}{z^i x^i \dfrac{dp^i}{dx^i}} + \frac{z^L x^L}{z^i x^i \dfrac{dp^i}{dx^i}} = x_\tau^{iH} + x_\tau^{iL} < 0 \tag{A.6}$$

The change in x^i due to an increase in the ad valorem tax rate t can be calculated in an analogous manner, using the variables defined in equations (A.5), and is given by:

$$\left.\frac{dx^i}{dt}\right|_{d\tilde{\Pi}=0} = p^H x_\tau^{iH} + p^L x_\tau^{iL} < 0 \tag{A.7}$$

The effects of a rise in the tax rates τ and t on the budget, again evaluating the respective terms at an initial tax rate $t = t^I = 0$, are – making use of equations (4'), (5'), (A.6) and (A.7) – given by $\hat{B}_\tau$ and $\hat{B}_t$, where the (^) indicates that the respective derivatives are calculated for the restriction $d\tilde{\Pi} = 0$:

$$\hat{B}_\tau := \hat{B}^H_\tau + \hat{B}^L_\tau = z^H\left(x^H + \tau \frac{dx^H}{d\tau}\bigg|_{d\tilde{\Pi}=0}\right) + z^L\left(x^L + \tau \frac{dx^L}{d\tau}\bigg|_{d\tilde{\Pi}=0}\right)$$

$$= z^H\left(x^H + \tau x^{HH}_\tau + \tau x^{HL}_\tau \right) + z^L\left(x^L + \tau x^{LH}_\tau + \tau x^{LL}_\tau \right) \qquad \text{(A.8a)}$$

$$\hat{B}_t = z^H\left(p^H x^H + \tau \frac{dx^H}{dt}\bigg|_{d\tilde{\Pi}=0}\right) + z^L\left(p^L x^L + \tau \frac{dx^L}{dt}\bigg|_{d\tilde{\Pi}=0}\right)$$

$$= z^H\left(p^H x^H + \tau p^H x^{HH}_\tau + \tau p^L x^{HL}_\tau \right) + z^L\left(p^L x^L + \tau p^H x^{LH}_\tau + \tau p^L x^{LL}_\tau \right)$$

$$= z^H\left(p^H x^H + \tau p^H (x^{HH}_\tau + x^{HL}_\tau) - \tau x^{HL}_\tau (p^H - p^L) \right)$$

$$+ z^L\left(p^L x^L + \tau p^L (x^{LH}_\tau + x^{LL}_\tau) + \tau x^{LH}_\tau (p^H - p^L) \right)$$

$$= p^H \hat{B}^H_\tau + p^L \hat{B}^L_\tau - \underbrace{\tau(p^H - p^L)\left[z^H x^{HL}_\tau - z^L x^{LH}_\tau \right]}_{:=T} \qquad \text{(A.8b)}$$

Following the argument made in the main text, we assume the derivatives $\hat{B}_t$ and $\hat{B}_\tau$ to be positive. The change in expected output $\hat{X}$ is determined by a modified equation (6). Substituting in accordance with (A.6) and (A.7) and taking into account $d\tau/dt = -\hat{B}_t / \hat{B}_\tau$ from equations (A.8) together with the assumption that the initial ad valorem tax rate t is zero ($t = t^I = 0$), yields:

$$\frac{d\hat{X}}{dt}\bigg|_{d\hat{B}=t^I=0} = z^H\left[\frac{dx^H}{dt} - \frac{dx^H}{d\tau}\frac{\hat{B}_t}{\hat{B}_\tau} \right] + z^L\left[\frac{dx^L}{dt} - \frac{dx^L}{d\tau}\frac{\hat{B}_t}{\hat{B}_\tau} \right]$$

$$= z^H\left[p^H x^{HH}_\tau + p^L x^{HL}_\tau - x^{HH}_\tau \frac{\hat{B}_t}{\hat{B}_\tau} - x^{HL}_\tau \frac{\hat{B}_t}{\hat{B}_\tau} \right]$$

$$+ z^L\left[p^H x^{LH}_\tau + p^L x^{LL}_\tau - x^{LH}_\tau \frac{\hat{B}_t}{\hat{B}_\tau} - x^{LL}_\tau \frac{\hat{B}_t}{\hat{B}_\tau} \right] \qquad \text{(A.9)}$$

Extracting $\hat{B}_\tau$ in the denominator of (A.9) and substituting $\hat{B}_\tau^H + \hat{B}_\tau^L$ for $\hat{B}_\tau$ and $p^H \hat{B}_\tau^H$ + $p^L \hat{B}_\tau^L$ - T for $\hat{B}_t$ in the numerator (cf. equations (A.8)) gives:

$$\frac{d\hat{X}}{dt}\bigg|_{d\hat{B}=t^I=0} = \frac{T}{\hat{B}_\tau}\left[z^H\left(x_\tau^{HH} + x_\tau^{HL}\right) + z^L\left(x_\tau^{LH} + x_\tau^{LL}\right)\right]$$

$$+\frac{p^H - p^L}{\hat{B}_\tau}\left[\hat{B}_\tau^L\left(z^H x_\tau^{HH} + z^L x_\tau^{LH}\right) - \hat{B}_\tau^H\left(z^H x_\tau^{HL} + z^L x_\tau^{LL}\right)\right] \quad \text{(A.10)}$$

Substituting for $\hat{B}_\tau^i$, i = L, H, (cf. (A.8a)), noting that $z^L x^L x_\tau^{HH} = z^H x^H x_\tau^{HL}$ and $z^L x^L x_\tau^{LH} = z^H x^H x_\tau^{LL}$ (from the definitions in (A.5)), and collecting terms, we obtain $d\hat{X}/dt = 0$.

8. References

Anderson, Simon P., de Palma, André and Brent Kreider (2001), The Efficiency of Indirect Taxes under Imperfect Competition, Journal of Public Economics, Vol. 81, 231-251.

Arrow, Kenneth J. and Robert C. Lind (1970), Uncertainty and the Evaluation of Public Investment Decisions, American Economic Review, Vol. 60(3), 364-378.

Bishop, Robert L. (1968), The Effects of Specific and Ad Valorem Taxation, The Quarterly Journal of Economics, Vol. 82(2), 198-218.

Blackorby, Charles and Sushama Murty (2007), Unit versus Ad Valorem Taxes: Monopoly in General Equilibrium, Journal of Public Economics, Vol. 91, 817-822.

Cheung, Francis K. (1998), Excise Taxes on a Non-Uniform Pricing Monopoly: Ad Valorem and Unit Taxes Compared, Canadian Journal of Economics, Vol. 31(5), 1192-1203.

Collie, David R. (2006), Tariffs and Subsidies under Asymmetric Oligopoly: Ad Valorem versus Specific Instruments, The Manchester School, Vol. 74(3), 314-333.

Cournot, Augustin (1838), Recherches sur les Principes Mathématiques de la Théorie des Richesses, Hachette: Paris.

Delipalla, Sofia (2009a), Tobacco Tax Structure and Smuggling, FinanzArchiv/ Public Finance Analysis, Vol. 65(1), 93-104.

Delipalla, Sofia (2009b), Commodity Tax Structure and Informal Activity, Bulletin of Economic Research, Vol. 61(3), 283-294.

Delipalla, Sophia and Michael Keen (1992), The Comparison Between Ad Valorem and Specific Taxation Under Imperfect Competition, Journal of Public Economics, Vol. 49, 351-367.

Delipalla, Sophia and Michael Keen (2006), Product Quality and the Optimal Structure of Commodity Taxes, Journal of Public Economic Theory, Vol. 8(4), 547-554.

Denicolò, Vincenzo and Massimo Matteuzzi (2000), Specific and Ad Valorem Taxation in Asymmetric Cournot Oligopolies, International Tax and Public Finance, Vol. 7, 335-342.

Dickie, Mark and Gregory A. Trandel (1996), Comparing Specific and Ad Valorem Pigouvian Taxes and Output Quotas, Southern Economic Journal, Vol. 63, 388-405.

Domar, Evsey D. and Richard A. Musgrave (1944), Proportional Income Taxation and Risk-Taking, The Quarterly Journal of Economics, Vol. 58(3), 388-422.

Fraser, R. W. (1985), Commodity Taxes under Uncertainty, Journal of Public Economics, Vol. 28, 127-134.

Grazzini, Lisa (2006), A Note on Ad Valorem and Per Unit Taxation in an Oligopoly Model, Journal of Economics, Vol. 89(1), 59-74.

Hamilton, Stephen F. (1999), The Comparative Efficiency of Ad Valorem and Specific Taxes Under Monopoly and Monopsony, Economics Letters, Vol. 63, 235-238.

Hamilton, Stephen F. (2009), Excise Taxes with Multiproduct Transactions, American Economic Review, Vol. 99(1), 458-471.

Helpman, Elhanan and Paul R. Krugman (1989), Trade Policy and Market Structure, The MIT Press: Cambridge (Mass.) and London.

Jørgensen, Jan G. and Philipp J. H. Schröder (2005), Welfare-ranking Ad Valorem and Specific Tariffs Under Monopolistic Competition, Canadian Journal of Economics, Vol. 38 (1), 228-241.

Keen, Michael J. (1998), The Balance between Specific and Ad Valorem Taxation, Fiscal Studies, Vol. 19(1), 1-37.

Keynes, John Maynard (1971), A Tract on Monetary Reform (1923), The Collected Writings of John Maynard Keynes, Vol. IV, Macmillan, Cambridge University Press: London and Basingstoke.

Kind, Hans Jarle, Koethenbuerger, Marko and Guttorm Schjelderup (2009), On Revenue and Welfare Dominance of Ad Valorem Taxes in Two-sided Markets, Economics Letters, Vol. 104, 86-88.

Kotsogiannis, Christos and Konstantinos Serfes (2010), The Comparison of Ad Valorem and Specific Taxation Under Uncertainty, Mimeo.

Liu, Liqun (2003), Relative Efficiency of Ad Valorem and Unit Taxes: the Case of Endogenous Quality, Public Finance Review, Vol. 31(3), 291-301.

Musgrave, Richard A. (1959), The Theory of Public Finance, McGraw-Hill Book Company Inc.: New York et al.

OECD (2008), Consumption Tax Trends: VAT/GST and Excise Rates, Trends and Administration Issues, Paris.

Pirttilä, Jukka (2002), Specific versus Ad Valorem Taxation and Externalities, Journal of Economics, Vol. 76(2), 177-187.

Schröder, Philipp J. H. (2004), The Comparison Between Ad Valorem and Unit Taxes Under Monopolistic Competition, Journal of Economics, Vol. 83(3), 281-292.

Shea, Koon Lam and Ester Y. P. Shea (2006), On the Equivalence of Ad Valorem Tariffs and Specific Tariffs under Duopoly, Review of International Economics, Vol. 14(3), 445-451.

Skeath, Susan E. and Gregory A. Trandel (1994a), A Pareto Comparison of Ad Valorem and Unit Taxes in Noncompetitive Environments, Journal of Public Economics, Vol. 55, 53-71.

Skeath, Susan E. and Gregory A. Trandel (1994b), Pareto-superior Trade Policy, Journal of International Trade and Economic Development, Vol. 3, 277-288.

Suits, D. B. and Richard A. Musgrave (1953), Ad Valorem and Unit Taxes Compared, The Quarterly Journal of Economics, Vol. 67(4), 598-604.

Wang, X. Henry and Jingang Zhao (2009), On the Efficiency of Indirect Taxes in Differentiated Oligopolies with Asymmetric Costs, Journal of Economics, Vol. 96, 223-239.

Wicksell, Knut (1896), Finanztheoretische Untersuchungen: nebst Darstellung und Kritik des Steuerwesens Schwedens, Verlag von Gustav Fischer: Jena.