

# Technological Determinants of the Group-Size Paradox

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# Technological Determinants of the Group-Size Paradox

## Abstract

The present paper analyzes situations in which groups compete for rents. A major result in the literature has been that there are both cases where larger groups have advantages and cases where they have disadvantages. The paper provides two intuitive criteria which for groups with homogenous valuations of the rent determine whether there are advantages or disadvantages for larger groups. For groups with heterogenous valuations the complementarity of group members' efforts is shown to play a role as a further factor.

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# 1 Introduction

In many economic situations like R&D races, military conflicts, lobbying, or sports, groups compete for economic rents. In most cases, these groups will differ in many ways, including size, heterogeneity of the members' valuations of the rent, or productivity. Naturally, all these factors should have an effect on the relative success of each group to acquire the rent. The influence of these factors on the relative success of each group however depends on the way in which groups compete. Focussing on military conflicts, Carl von Clausewitz held the position that “...*in modern war one will search in vain for a battle in which the winning side triumphed over an army twice its size.*” (Clausewitz, 1943, 1834). He acknowledged that there are other factors like technological superiority (temporarily) influencing the outcome of a conflict, but if all these variables were stripped away, numbers would determine victory. Further research has shown that the advantages from having a larger army have varied greatly over time (Hirshleifer, 1995). In lobbying contests, there may even be disadvantages from larger group sizes, as Olson (1965) argued. His arguments gave rise to a debate about the so-called group-size paradox, which Esteban and Ray (2001) define as: “*larger groups may be less successful than smaller groups in furthering their interests*” (p.663).

Informal observation shows that individuals pervasively organize in groups in conflicts. In hunter-gatherer societies, individuals were apparently better equipped to be successful in predator-prey conflicts (either as predator or as prey) as well as in conflicts with rivals if they organized in groups (Tainter, 1990).<sup>1</sup> Clausewitz's observation shows that similar forces exist in modern warfare. But also individuals engaging in lobbying and rent-seeking activities often organize in groups. Esteban and Ray (2001) develop one explanation for these facts: larger groups may profit from cost advantages. If the costs of effort are sufficiently convex, *ceteris paribus*, members of larger groups face sufficiently lower marginal costs that reverse the group-size paradox. This is a very important insight that helps to explain the prevalence of groups in conflicts.<sup>2</sup>

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<sup>1</sup>The same is true for animals who organize in flocks, swarms, packs, etc., see the discussion at the end of this paper.

<sup>2</sup>See also Marwell and Oliver (1993); Pecorino and Temini (2008); Nitzan and Ueda (2009,

The group-size paradox has been extensively discussed in the theoretical and empirical literature. Agrawal and Goyal (2001) point to the inconclusive evidence: “[...] scholars writing on the subject have remarked on the ambiguities in Olsons argument and suggested that the relationship between group size and collective action is not very straightforward.” However, up to now it is unclear whether the specific cause for the possible reversal of the group-size paradox on which Esteban and Ray (2001) focus, namely convexities in costs, is exhaustive. The starting point of our paper is to ask if there are additional properties of the conflict environment that add to the explanation of the relative advantage or disadvantage of larger compared to smaller groups by focussing on the properties of the “production” of group impact. We believe that those properties are an important factor for the explanation of the effect of group size in conflicts.

We show that two intuitive properties of the ways in which the groups aggregate their efforts are responsible for its occurrence. The first property is an inherent advantage that may be given by the contest structure to larger groups: If there are two groups with the same total amount of effort but different numbers of individuals, one cannot in general expect them to have the same lobbying impact. For example, there may be a difference in impact whether 10 000 people demonstrate for 10 hours or 100 000 people demonstrate for 1 hour. The different demonstrations may receive very different media attention which in turn may lead to very different impacts on policymaking. If the 10 000 people would have a higher impact, we would expect the group size paradox to appear more likely. This property of the contest structure will be called *group-size biasedness*.

The second important property is *returns to scale*. Suppose group members increase their efforts by some factor and their relative strength increases by less than this factor (decreasing returns to scale). This may cause the group-size paradox to occur even if large groups are advantaged by group size biasedness. Larger groups tend to be disadvantaged by this because they are facing a larger problem of free riding. If returns to scale are decreasing, the marginal return of investing more effort is not large enough to make up for the better possibilities for free riding.

It turns out that if group members have homogenous valuations of winning the

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2010).

contest (which may differ *between* groups), these two properties completely determine whether the group-size paradox occurs or not.<sup>3</sup> Homogenous valuations are, however, rarely the case in reality. In general, we would expect to encounter groups where group members differ in many features: Not only the valuations of winning may be different, but also abilities, qualifications, or affections. Empirical research emphasizes that the level of heterogeneity in the group is an important mediator for the impact of group size (Hardin, 1982; Ostrom, 1997). Once we introduce heterogeneous group members, other factors may start to play a role, such as complementarity between group-members' efforts: Groups where agents are heterogeneous will often have the feature that group members have specialized according to their comparative advantage. Alchian and Demsetz (1972) see such non-additivity as constitutive for group or team production (pp. 777): *“Resource owners increase productivity through cooperative specialization. [...] With team production it is difficult, solely by observing total output, to either define or determine each individual’s contribution to this output of the cooperating inputs. The output is yielded by a team, by definition, and it is not a sum of separable outputs of each of its members.”* Despite the fact that there is a growing interest on the influence of heterogeneity within and between groups,<sup>4</sup> with only a few exceptions the literature on group contests<sup>5</sup> has focused attention on situations where the effort levels of group members are perfect substitutes, i.e. are aggregated by summation. In order to analyze the group-size paradox for heterogeneous valuations, this paper will introduce different degrees of complementarity for the case of a CES-type impact function. If we hold the other properties – group-size biasedness and returns to scale – at a neutral level, the complementarity between the efforts of group members determines a minimum valuation a new group member must have in order for the group-size paradox not to occur.

The paper is organized as follows. We introduce the model in Section 2 and cover the case of homogenous group members in Section 3. In Section 4 we allow

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<sup>3</sup>This claim may appear to be at odds with Esteban and Ray (2001) who focus on convexities in the cost-of effort functions. We will show that their model is isomorphic to a model with linear costs and nonlinear impact functions that is a special case of our model.

<sup>4</sup>See Esteban and Ray (2008, 2010).

<sup>5</sup>The literature on contests between groups has recently been surveyed by Corchón, 2007, Section 4.2, Garfinkel & Skaperdas, 2007, Section 7, and Konrad, 2009, Chapters 5.5 and 7.

for heterogeneity of agents and use a CES type impact function to aggregate group members' efforts. We characterize the simultaneous Nash equilibrium of the CES model in Subsection 4.1. In Subsection 4.2 we will show the effect of complementarity on the group-size paradox for heterogeneous agents. There will be an extended discussion of findings from related strands of the literature in Section 5. Section 6 concludes.

## 2 The model

Assume that  $n$  groups compete for a given rent  $R$ .  $m_i$  is the number of individuals in group  $i$  and  $k$  is the index of a generic member of this group. The rent is a group-specific public good that has a value  $v_i^k > 0$  to individual  $k$  of group  $i$ .  $p_i$  represents the probability of group  $i = 1, \dots, n$  to win the contest. Individuals can influence the winning probability by contributing effort  $x_i^k$ . The members' efforts of a group are then aggregated by a function  $q_i(x_i^1, \dots, x_i^{m_i})$ .  $p_i$  is then a function of these aggregated efforts. Following the literature, we will call  $q_i$  *impact function* and  $p_i$  *contest-success function*. We focus on Tullock-form contest-success functions where the winning probability of a group  $i$  is defined as:<sup>6</sup>

**Assumption 1.**  $p_i(q_1, \dots, q_n) = \frac{q_i(x_i^1, \dots, x_i^{m_i})}{\sum_{j=1}^n q_j(x_j^1, \dots, x_j^{m_j})}$ ,  $i = 1, \dots, n$ .

Further, we impose the following assumptions on the individuals:

**Assumption 2.** Individuals are risk neutral, face linear costs, and maximize their net rent.

Assumptions 1 and 2 imply that we can write expected utility as:

$$\pi_i^k = \frac{q_i(x_i^1, \dots, x_i^{m_i})}{\sum_{j=1}^n q_j(x_j^1, \dots, x_j^{m_j})} v_i^k - x_i^k \quad (1)$$

We are looking for a Nash equilibrium of this game where individuals choose their effort  $x_i^k$  simultaneously to maximize their expected utility,

$$x_i^{k*} \in \arg \max_{x_i^k} \pi_i^k(x_i^k, x_{-x_i^k}^*) \quad \forall i, k. \quad (2)$$

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<sup>6</sup>An axiomatic foundation for the Tullock function for group contests can be found in Münster (2009). An interpretation of the Tullock contest as a perfectly discriminatory noisy ranking contest can be found in Fu and Lu (2008).

where “\*” refers to equilibrium values and  $x_{-x_i^k}^*$  to the vector of efforts by all individuals except  $k$  in group  $i$ . In order to facilitate the analysis, we will focus on situations where a unique Nash equilibrium exists with respect to the total effort produced of each group. Formally,

**Assumption 3.** We assume that  $q_i(\cdot)$  is at least twice continuously differentiable and (weakly) monotonic in  $x_i^k \forall i, k$ . Further,  $q_i(\cdot)$  is weakly quasiconcave.

This still allows for multiple equilibria within groups as they may arise when effort levels are for example perfect substitutes.

### 3 Homogenous valuations within groups

The group size paradox was first discussed by Olson (1965), who stated that “*the larger the group, the farther it will fall short of providing an optimal amount of a collective good*” (p. 35). One particular interpretation of the statement has been given by Esteban and Ray (2001): In a contest environment in which different groups compete for a rent, larger groups should win with lower probability if the group size paradox was true. One could however also take a comparative-static perspective on the group size paradox, which seems to underscore its relevance even better:

**Definition 1. (Group-size paradox)** Suppose in a contest there are  $n$  groups  $i$  competing for a prize and have  $m_i$  individuals with equal valuations  $v_i$ . Then, the group-size paradox holds strictly (weakly) if and only if adding an individual with valuation  $v_i$  to group  $i$  will decrease (decrease or leaves constant) the probability of the group to win the prize.

Next we formulate two intuitive criteria that will turn out to be able to explain the occurrence of the group-size paradox if individuals of a group have identical valuations of the rent.

**Definition 2. (Group-size bias)** A class of impact functions  $q_m(x^1, \dots, x^m)$  with  $m$  being the number of group members is said to be *group-size unbiased* if for all numbers  $\xi > 1$  such that  $\xi \cdot m$  is a natural number, it holds that  $q_m(x, \dots, x) = q_{m \cdot \xi}(\frac{x}{\xi} \dots \frac{x}{\xi})$ . The impact function is said to be *positively group-size biased* or *negatively group-size biased* if  $q_m(x, \dots, x) < q_{m \cdot \xi}(\frac{x}{\xi} \dots \frac{x}{\xi})$  or  $q_m(x, \dots, x) > q_{m \cdot \xi}(\frac{x}{\xi} \dots \frac{x}{\xi})$ .

The reason why this property is interesting is that it reflects whether a redistribution of the same total amount of effort to more (heterogenous) group members will lead to an increase, a decrease, or no effect on the impact of the group. For example, the simple sum of efforts of all group members,  $\sum_{k=0}^m x^k$ , is group-size unbiased: If all group members exert the same effort  $x$ , then  $\sum_{k=0}^m x^k = m \cdot x$ . Multiplying  $m$  by  $\xi$  and dividing  $x$  by  $\xi$  will then naturally lead to the same result.

Another property of an impact function is whether it has increasing or decreasing returns to scale:

**Definition 3. (Returns to scale)** A class of impact functions  $q_m(\vec{x})$  is said to have *constant returns to scale* if  $\forall m : q_m(\xi \vec{x}) = \xi \cdot q_m(\vec{x})$ , *decreasing returns to scale* if  $\forall m : q_m(\xi \vec{x}) < \xi \cdot q_m(\vec{x})$ , *increasing returns to scale* if  $\forall m : q_m(\xi \cdot \vec{x}) > \xi \cdot q_m(\vec{x})$ .

Assume that the impact functions have the generalized CES-form

$$q_{m_i}(\vec{x}) = m_i^\delta \left( \sum (x_i^k)^\gamma / m_i \right)^{\beta/\gamma}. \quad (3)$$

In this case,  $q_{\xi m_i}(\frac{x}{\xi}, \dots, \frac{x}{\xi}) = \xi^{\delta-\beta} q_{m_i}(x, \dots, x)$  and  $q_{m_i}(\lambda x, \dots, \lambda x) = \lambda^\beta q_{m_i}(x, \dots, x)$ , which shows that returns to scale and group-size bias can be independently chosen.

To show the relationship between our approach and the one employed by Esteban and Ray (2001)<sup>7</sup>, note that their model would translate into

$$\pi_i^k = v_i^k \frac{\sum_l y_i^l}{\sum_j \sum_l y_j^l} - c(y_i^k),$$

with  $c(\dots)$  being an increasing, strictly convex function. The central result is that if the elasticity of the marginal rate of substitution between effort  $y_i^k$  and  $v_i^k \frac{\sum_l y_i^l}{\sum_j \sum_l y_j^l}$  is sufficiently high, the winning probability will strictly increase with group size. Note that we can write  $y_i^k = c^{-1}(x_i^k)$  where  $c^{-1}$  is the inverse of  $c$ . Since by their assumptions the cost function is a bijection, the maximization problem

$$\pi_i^k = v_i^k \frac{\sum_l c^{-1}(x_i^l)}{\sum_j \sum_l c^{-1}(x_j^l)} - x_i^k$$

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<sup>7</sup>Their model allows for rival as well as nonrival elements of the rent.



yields the same solutions as the original problem. We can now focus again on the case with  $c(y) = y^\alpha$ , from which we obtain  $q(x_i) = \sum_l (x_i^l)^{1/\alpha}$  as the input to the Tullock impact function.

In the special case that the costs of effort function is equal to  $(x_i^l)^\alpha$ , the group-size paradox does (not) occur for  $\alpha < (>)2$ . Since  $q_{m_i}(x_i) = \sum_l (x_i^l)^{1/\alpha}$  yields an isomorphic optimization problem, we can look for group-size bias and returns to scale of  $q_{m_i}(x_i)$ . It is easy to check that positive group-size bias as well as decreasing returns to scale exist if and only if  $\alpha > 1$ . In addition, a decrease in the returns to scale (as measured by an increase in  $\alpha$ ) reduces any positive group-size bias: changing the convexity of costs is equivalent to a simultaneous change in returns to scale and groups size bias in the isomorphic problem. The result by Esteban and Ray (2001) imply that if  $\alpha > 2$  the group-size-bias/returns to scale combinations work in favor of large groups, whereas the opposite is true for  $\alpha < 2$ . Given that this result has been derived in a situation where group-size bias and returns to scale cannot be disentangled, the question arises as to whether further insights can be gained if both effects are treated separately.

The following propositions hold (all proofs can be found in the appendix).

**Proposition 1.** *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions are group-size unbiased. Then for constant or decreasing returns to scale the group-size paradox holds weakly if all groups' members have equal valuations.*

**Proposition 2.** *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions have constant returns to scale. Then for negative (positive) group-size biased impact functions the group-size paradox holds (does not hold) weakly if all groups' members have equal valuations.*

These results provide a very intuitive explanation when the group-size paradox arises if group members have the same valuation of the rent. Returns to scale and group-size biasedness are indeed the main driving forces behind the different results on the group-size paradox by Olson (1965) and Esteban and Ray (2001). If one is willing to accept the assumption that the impact functions are homogenous, the above results can be generalized in the following way.

**Proposition 3.** *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions are homogenous as well as either negatively group-size biased or group-size unbiased, and have constant or decreasing returns to scale. Then, the group-size paradox holds weakly if all groups' members have equal valuations.*

The following corollary of the above propositions establishes an interesting special case.

**Corollary 1.** *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions have constant returns to scale and are group-size unbiased. Then, the group-size paradox holds weakly but not strictly if all groups' members have equal valuations.*

Corollary 1 establishes a link to the case of additively linear impact functions, which is a special case of an group-size unbiased impact function with constant returns to scale and that have been standard in the literature so far (see, for example, Baik (2008) and Konrad, 2009, Chapters 5.5 and 7). In this case, equilibrium group impact and therefore winning probability is independent of group size as the maximum valuation remains unchanged. Corollary 1 shows that this finding carries over to a larger class of impact functions.

The next result covers the case of positive group-size bias combined with decreasing returns to scale. Propositions 1 and 2 which cover the boundary cases as well as Esteban and Ray (2001) suggest that we cannot expect clear-cut results for this area which in fact turns out to be true. Nevertheless, we get a nice monotonicity property for the case of homogenous impact functions and equilibria where members of the same group behave identically.

**Definition 4.** For a class of homogenous impact functions,  $q_m(\vec{x})$ , the returns to scale are measured by the degree of homogeneity  $r$ , such that for all  $m$ :  $q_m(\lambda\vec{x}) = \lambda^r q_m(\vec{x})$

Note that this immediately implies that if we speak of a class of impact functions having certain returns to scale, this means that each of the impact functions of this class has the same returns to scale. Definition 5 can be used to define a measure of group-size bias:

**Definition 5.** For a class of impact functions,  $q_m(\vec{x})$ , the group size bias is measured by  $b(\xi, x, m) = q_{m\xi}(x/\xi, \dots, x/\xi)/q_m(x, \dots, x)$ .

Note that group size bias may change for different values of  $x$ ,  $m$ , and  $\xi$ . However, it is immediately clear that for homogenous classes of impact functions, the measure is independent of  $x$ .

**Proposition 4.** *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions are homogenous, positively group-size biased and have decreasing returns to scale. Consider a change in group  $i$  from  $m_i$  to  $\xi m_i$  group members. If a class of impact functions  $q_{m_i}$  has returns to scale  $r$  and group size bias  $b(\xi, x, m_i)$ , then:*

- a) *If the group size paradox holds for  $q_{m_i}$ , it also holds for all  $\hat{q}_{m_i}$  for which  $\hat{r} \leq r$  and  $\hat{b}(\xi, x, m_i) \leq b(\xi, x, m_i)$ .*
- b) *If the group size paradox does not hold for  $q_{m_i}$ , it also does not hold for all  $\hat{q}_{m_i}$  for which  $\hat{r} \geq r$  and  $\hat{b}(\xi, x, m_i) \geq b(\xi, x, m_i)$ .*

In other words, if for some class of impact functions the group size paradox holds, then decreasing the returns to scale further or decreasing the group size bias further, will imply that the group size paradox still holds. The reverse holds for classes of impact functions for which the group size paradox does not hold: Increasing the returns to scale or the group size bias will imply that for the new class of impact functions, the group size paradox also does not hold.

Returning to the special case of a CES-impact function given in (3) and restricting attention to the special case of two groups 1 and 2 with equal valuations of the rent,  $v_i^k = v$ , Propositions 1 - 4 can be illustrated as follows. The impact functions have decreasing, constant, and increasing returns to scale if  $\beta <, =, > 1$ . Group-size bias is positive (negative) if  $\delta > (<) \beta$ . It is straightforward to calculate the Nash equilibrium  $x_i^k = (\beta m_j^{-\beta+\delta} m_i^{-1+\beta+\delta} v) / (m_j^{-\beta+\delta} m_i^\beta + m_i^\delta)^2$  as well as the winning probabilities  $p_i^*$  for this example. The partial derivative of group  $i$ 's equilibrium winning probability with respect to  $m_i$ , evaluated at  $m_1 = m_2$  is then  $\partial p_i^* / \partial m_i = (\delta - \beta) / (4m_i)$ , which is positive (negative) if and only if the impact function has positive (negative) group-size bias.

## 4 Heterogenous valuations within groups

We now turn to the analysis of heterogenous groups and how the complementarity of the impact functions affect the group-size paradox. Notice first that Definition 1 cannot be used in a framework where group members have different valuations of the rent. For groups that consist of members with different valuations, it is not at all clear what valuation a new member should have. In this case, it is more interesting to see what the minimum valuation of a new group member has to be in order to increase the winning probability of the group. Second, we may want to remove the effects of changing returns to scale and group-size biasedness and introduce a parameter ( $\gamma$ ) to account for different degrees of complementarity of the group members' efforts. These properties are fulfilled by the following CES-type impact function:

**Assumption 4.**  $q_i(x_i^1, \dots, x_i^{m_i}) = m_i \left( \sum_{l=1}^{m_i} \frac{1}{m_i} (x_i^l)^\gamma \right)^{1/\gamma}$ ,  $\gamma \in (-\infty, 1]$ ,  $i = 1, \dots, n$ .

It is easy to check that this function has constant returns to scale and is group-size unbiased. Note that we obtain a closed-form solution only if  $\gamma \neq 0$ . The Cobb-Douglas case  $\gamma \rightarrow 0$  will be covered by a limit result. It follows from Assumptions 1, 2, and 4 that the individual expected utility functions are as follows:

$$\pi_i^k(x_1^1, \dots, x_n^{m_n}) := \pi_i^k(x_i^k, x_{/x_i^k}) = v_i^k \frac{m_i (\sum_l \frac{1}{m_i} (x_i^l)^\gamma)^{1/\gamma}}{\sum_j m_j (\frac{1}{m_j} \sum_l (x_j^l)^\gamma)^{1/\gamma}} - x_i^k, \quad (4)$$

where  $x_{/x_i^k}$  refers to the vector  $x_1^1, \dots, x_n^{m_n}$  without  $x_i^k$ . In order to have a lean notation, let  $y_i^k = (x_i^k)^\gamma$  and  $Y_i = (\sum_l y_i^l)$ . Further,  $Q = \sum_j Q_j = \sum_j Y_j^{\frac{1}{\gamma}} = Y_i^{\frac{1}{\gamma}} + \sum_{j \neq i} Y_j^{\frac{1}{\gamma}} = Q_i + Q_{/i}$  in the following. While deriving the equilibrium strategies, we will omit the parameters of these functions for better readability (e.g  $y_i^k$  instead of  $y_i^k(\gamma, x_i^k)$ ).

### 4.1 Nash equilibrium

We will now determine the Nash equilibrium of the given model. Hillman and Riley (1987) and Stein (2002) have shown that groups/individuals may prefer to stay inactive if the size of all groups is equal to 1. Baik (2008) has shown that only group members with maximum valuation participate in a contest for the special case  $\gamma = 1$ .

Hence, it is possible that some individuals and/or groups will stay inactive in our setup. We therefore start with an analysis of active individuals and groups and restrict attention to  $\gamma < 1$ .

**Definition 6.** An individual  $k$  of group  $i$  is said to participate if  $x_i^k > 0$ . A group  $i$  is said to *participate* if there exists some  $k$  such that  $x_i^k > 0$ . A group is said to *fully participate* if  $\forall k : x_i^k > 0$ .

**Lemma 1.** *In a Nash equilibrium of a contest fulfilling Assumptions 1, 2, and 4 if a group participates, it fully participates.*

Lemma 1 implies that in order to determine whether an individual participates, it is sufficient to determine whether its group participates and vice versa. Let  $V_i(\gamma) \equiv \left(1/m_i \sum_l v_i^l \frac{\gamma}{1-\gamma}\right)^{\frac{1-\gamma}{\gamma}}$ . Without loss of generality, suppose the groups are ordered such that  $V_i(\gamma) \geq V_{i+1}(\gamma)$  for a given  $\gamma$ .  $Q_i^*(\gamma)$  and  $Q^*(\gamma)$  shall denote  $Q_i$  and  $Q$  in equilibrium. The following Lemma determines the groups that participate in equilibrium.

**Lemma 2.** *a) There exist best-response strategies of the members of a group, if and only if the following group best-response function is fulfilled:*

$$\hat{Q}_i(\gamma, Q_{/i}) = \max \left( 0, \sqrt{Q_{/i} V_i(\gamma)} - Q_{/i} \right). \quad (5)$$

*b) Groups  $1 \dots n^*(\gamma)$  participate, where  $n^*(\gamma) \equiv \arg \max_i i$  such that  $V_i(\gamma) > Q^*(\gamma)$ .*  
*c) If the Nash equilibrium is unique,  $Q_i^*(\gamma)$  and  $Q^*(\gamma)$  are continuous functions for  $\gamma \neq 0$ .*

Lemma 2.c is useful for the comparative-static analysis. Given that the number and identity of active groups depends on  $\gamma$ , it is *a priori* not clear that aggregate effort and indirect utilities are continuous in  $\gamma$ . The Lemma reveals that continuity is in fact guaranteed except at  $\gamma = 0$ . The economic intuition is as follows: Assume that  $\hat{\gamma}$  is a point where a formerly active group becomes inactive or a formerly inactive group becomes active. The aggregate group effort of the active group is continuously reduced to zero as  $\gamma$  approaches  $\hat{\gamma}$ , and the formerly inactive group continuously increases its effort from 0 as  $\gamma$  increases from  $\hat{\gamma}$ . Hence, there is a “smooth” fade out or fade in of groups at those points.

The following proposition characterizes the unique Nash equilibrium of the game. For readability, the strategies  $x_i^k$  are defined as functions of  $Q^*(\gamma)$  and  $V_i(\gamma)$ .

**Proposition 5.** *The unique Nash equilibrium of the game characterized by Assumptions 1, 2, and 4 is given by strategies  $x_i^{k*}(\gamma)$  that fulfill*

$$x_i^{k*}(\gamma) = \begin{cases} Q^*(\gamma) \left(1 - \frac{Q^*(\gamma)}{V_i(\gamma)}\right) \frac{1}{m_i} \frac{(v_i^k)^{\frac{1}{1-\gamma}}}{V_i(\gamma)^{\frac{1}{1-\gamma}}}, & V_i(\gamma) > Q^*(\gamma) \\ 0, & V_i(\gamma) \leq Q^*(\gamma) \end{cases}, \quad (6)$$

where  $Q^*(\gamma) = \frac{n^*(\gamma)-1}{\sum_{i=1}^{n^*(\gamma)} V_i(\gamma)^{-1}}$  and  $n^*(\gamma)$  is defined in Lemma 2.a and groups are ordered such that  $V_i(\gamma) \geq V_{i+1}(\gamma)$ .

*Proof.* To obtain  $Q^*(\gamma)$  we sum (5) over all  $i \leq n^*(\gamma)$ :

$$Q^*(\gamma) = \frac{n^*(\gamma) - 1}{\sum_{i=1}^{n^*(\gamma)} V_i(\gamma)^{-1}}. \quad (7)$$

With an explicit solution for  $Q^*(\gamma)$ , we can now determine individual expenditures  $x_i^{k*}(\gamma)$  by solving equation (5) using (7). The participation condition of a group is given by Lemma 2, while Lemma 1 ensures that there does not exist an incentive for any group member to deviate to  $x_i^k = 0$ . It was further shown that the first-order conditions return local maxima. Since the system of equations given by the first-order conditions of the participating groups has a unique solution this is indeed the unique Nash equilibrium.  $\square$

It is of course interesting to see whether different degrees of complementarity have an effect on the equilibrium if all individuals have the same valuations, i.e.  $v_i^k = v_i \forall k \forall i$ . The following corollary of Proposition 5 can then be established.

**Corollary 2.** *If  $v_i^k = v_i \forall k \forall i$  the equilibrium efforts of all groups are independent of  $\gamma$ .*

*Proof.* The corollary directly follows from inserting  $v_i^k = v_i$  into the above definitions, since then  $V_i = v_i$ .  $\square$

This finding implies that an increase in complementarity between group members' effort *per se* has no effect on the within-group free-rider problem, as could

have been conjectured from Hirshleifer (1983). A further implication of the result is that the results on group contests that have been derived in the literature for the case of perfect substitutes or perfect complements carry over to arbitrary elasticities of substitution if groups differ only in their valuations of the rent. In particular, different elasticities of substitution will not affect the occurrence of the group size paradox for homogenous groups.

## 4.2 Group-size paradox for heterogenous valuations within groups

It is already clear that the group-size paradox will hold weakly but not strictly in this model if agents within groups have identical valuations. For heterogenous agents we can however no longer rely on Definition 1, since agents with different valuations can be added to the group. It therefore makes more sense to look at how high the valuation of a new member of the group needs to be to increase the winning probability of the group.

**Proposition 6.** *For groups with heterogenous valuations, there exists a minimum valuation a new group member must have in order to raise the winning probability of its group. This minimum valuation is increasing in the elasticity of substitution among the efforts of the group members.*

This result shows that for heterogenous valuations, a third property of a contest plays an important role with respect to the effect of the group size on the winning probabilities: The more complementary the efforts of the group members are, the lower the necessary valuation of a new member has to be in order to raise the winning probability. Since heterogeneity in valuations is a rather common feature of interest groups, the returns to scale, the group size biasedness and the effort complementarity of the impact function should play a role when analyzing the relative strength of interest groups.

## 5 Relationship of our results with other fields of the literature

The studies mentioned in the introduction – like Olson’s original argument and contrary to the specific interpretation given to it by Esteban and Ray (2001) – do not specifically focus attention to situations of conflict between groups, and there are only few strands of the literature that deal with contest-like situations.

The first one studies the effect of group size for non-human species<sup>8</sup>, especially bird groups and predation. The empirical evidence points in the direction that – for several reasons and up to a certain limit – larger bird groups are more successful than smaller ones, for example because larger groups may detect predators sooner than solitary individuals. Bertram (1980) found that the percentage of time a single ostrich has its head up is decreasing in group size, but the percentage of time one or more heads are up is increasing. For the case of wood-pigeons, Kenward (1978) found that the percentage of successful hawk attacks is decreasing in group size. An explanation for this positive effect of a larger group-size is the so-called ‘many eyes hypothesis:’ All members of a group are alerted if at least one member detects a potential predator<sup>9</sup> or the fact that it is harder for the predator to focus on a specific prey if the group is large. Cresswell and Quinn (2004) and Kenward (1978) analyzed Sparrowhawk attack success when hunting Redshanks and found that the probability of capture of Sparrowhawks increased when the group size of their prey decreased. This finding suggests that there are positive effects of group size for the prey in predator-prey contests. Translated into the formal language of our model and with the necessary prudence, the group advantage can be interpreted as a specific property of the contest that cannot be reduced to some form of strictly convex

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<sup>8</sup>Which of course implies that one has to be sufficiently cautious with respect to the implications for models based on the utility maximization paradigm. The seminal contribution to game theoretic application to animal behavior is Smith and Price (1972). For a critical comparison of models of utility maximization and Nash equilibrium as solution concept with models of fitness maximization and evolutionary stable strategies as solution concept in contests see Leininger (2003).

<sup>9</sup>Pulliam, Pyke, and Caraco (1982) found that individuals in groups use a ‘conditional vigilance strategy’ where individuals are cooperatively vigilant as long as all other group members remain so too (tit-for-tat); this implies that individuals monitor the behavior of others.



costs-of effort function but can better be explained by impact functions leading to some form of group-size bias and/or economies of scale. The main driving force for these positive effects, however, does not necessarily stem from the contest situation; the decrease in successful attacks may be a useful byproduct of some other positive effects. In order to test this, Beauchamp (2004) examined flock sizes of species living on islands where predation risk is either absent or negligible with flock sizes of the same species on the mainland (with higher predation risk). Controlled for other potential explanatory factors like population density, habitat type, food type, etc., mean and maximum flock size were smaller on islands than on the mainland. The results suggest that predation is a significant factor in the evolution of flocking in birds. The ‘many eyes hypothesis’ and the increasing difficulty of the predator to focus on prey in larger groups closely resemble a type of non-additive impact function with positive group-size bias.

There is also a lively discussion about the prevalence of a group-size paradox in contest environments in the sociological as well as the strategic-management literature with (as can be expected from Esteban and Ray (2001) as well as this paper) mixed empirical evidence. Siegel (2009) argues that in large groups, the large number of ties between group members can hamper collective action. Larger groups require more specialization to effectively manage the increasing complexity (McCarthy & Zald, 1977) and to allow for effective decision-making procedures (Benbasat and Lim (1993). The in general smaller diversity of members of smaller groups makes it easier to coordinate on shared goals and collective action (e.g. Gamson, 1995, Klandermans & deWeerd, 2000, Monge et al., 1998). It is also easier to speak with one voice if the group is smaller, avoiding inconsistent messages which are counterproductive for the success in any lobbying process (Dominelli, 1996), which implies that there must be a certain degree of complementarity between the group members’ efforts. Using data from Swedish firms, Wincent, Örtqvist, Eriksson, and Autio (2010) found evidence for the predicted adverse effect of group size on the amount of fundraising. Economies of scale and group-size bias therefore tend to favor smaller groups in these situations. However, there are also opposite findings. For example, the mere number of group members may give the group more media coverage and/or political power (McAdam, 1882), pointing to a positive group-size bias. Larger groups may also be able to provide more funds (Oliver, 1993, Zald

& Ash, 1966). A study by McCarthy and Wolfson (1996) showed that in fact the size of task committees had a positive effect on the amount of funding obtained in campaigns for local governments. Finally, Dejean, Penard, and Suire (2009) find a positive relation between the size of a community and the amount of collective good provided (with decreasing individual propensity to cooperate) in an empirical analysis of P2P file-sharing communities, a result that is strikingly similar to the findings about individual and group vigilance of birds mentioned above.

## 6 Concluding remarks

According to our model one can expect that four crucial “technological” factors determine the role of group size on the outcome of a group contest, group-size biasedness, returns to scale, and complementarity between group members’ efforts as well as the composition of their valuations in case of heterogenous valuations within groups. These findings complement and extend the results by Esteban and Ray (2008, 2010) who have shown that the convexity of individual cost functions may explain the reversal of the group-size paradox, and it turns out that convexity in costs is a special case of positive group-size bias.

Empirical findings support the existence of a group-size paradox, but as noted by Marwell and Oliver (1993), it also stands in contrast to a significant body of empirical findings pointing to a positive relationship between group size and group performance in conflicts. Our analysis reveals that this diverse empirical pattern may not be reduced to only one explanatory variable, namely the degree of convexity in costs. To continue the example from the introduction, the success of political demonstrations may depend on media coverage which in turn may depend on the number of demonstrators. This is an example of group-size biasedness that cannot be reduced to convexities in costs.

Our analysis of within-group heterogeneity, a constellation that should empirically rather be the rule than the exception, shows first that the composition of individual valuations is in fact important for contest success. Second, the degree of complementarity in reaching impact becomes important. The higher the degree of complementarity, the lower the threshold an individual valuation has to reach in

order to have a positive impact on group success. This finding sheds light on the empirical findings stressing that heterogeneity is likely to have adverse effects on the contribution to the group-specific public good (Bandiera, Barankay, & Rasul, 2005).

## Appendix A: Proof of Proposition 1

*Proof.* Suppose we have for all groups a group-size unbiased impact function  $q$ . Denote by  $Q^*$  and  $Q_{/i}^*$  the total impact of all groups and of all groups except group  $i$  in equilibrium. By Assumption 3 there exists a Nash equilibrium that is unique up to redistributions among the group members and that is characterized by the solution to the first-order conditions (FOCs):

$$\frac{\partial q(x_i^*)}{\partial x_i^k} \frac{Q_{/i}^*}{Q^{*2}} v_i - 1 = 0 \quad \forall i, k \quad (\text{A.1})$$

It is evident from the FOC that the equilibrium is symmetric among members of a group and we thus only need to focus on fulfilling the FOC of the first member of each group. By assumption, the impact function is group-size unbiased. For equal inputs we can then write  $q(x_i) = g(x_i^1 \cdot m_i)$ , since otherwise the functional equation induced by group-size unbiasedness cannot be fulfilled Aczél and Dhombres (1989). Note that  $\frac{\partial q(x_i)}{\partial x_i^k} = g'(x_i^1 \cdot m_i) \cdot m_i$  is not the correct partial derivative. However, we can employ the total differential:

$$\Delta q(x_i) = \sum_k \Delta x_i^k \frac{\partial q(x_i)}{\partial x_i^k} \quad (\text{A.2})$$

which becomes for symmetric agents:

$$\Delta g(x_i^1 m_i) = m_i \Delta x_i^1 \frac{\partial q(x_i)}{\partial x_i^k}, \quad (\text{A.3})$$

and thus

$$\frac{\partial q(x_i)}{\partial x_i^k} = \frac{\Delta g(x_i^1 \cdot m_i)}{m_i \cdot \Delta x_i^1} = g'(x_i^1 \cdot m_i) \quad (\text{A.4})$$

for  $\Delta \rightarrow 0$ . We can reinsert this expression into the FOC and replace  $Q_{/i}^*/Q^*$  by  $(1 - p_i^*)$  as this is the probability with which the other groups win,

$$g'(x_i^{1*} \cdot m_i) = \frac{Q^*}{v_i(1 - \frac{Q_i^*}{Q^*})} = \frac{Q^*}{v_i(1 - p_i^*)} \quad \forall k. \quad (\text{A.5})$$

We can now prove by contradiction that  $m_i$  and  $p_i^*$  cannot rise at the same time. Suppose this would be the case. For the behavior of  $Q^*$ , we can now distinguish three cases,  $Q^*$  increases, remains constant, or decreases.

Suppose  $Q^*$  increases. This implies by the definition of  $p_i^* = Q_i^*/Q^*$  that  $Q_i^*$  increases, as otherwise  $p_i^*$  would not increase. Since  $Q_i^*$  increases if and only if  $x_i^{k^*} \cdot m_i$  increases, the left-hand side (LHS) of (A.5) must decrease, as  $g'(\dots)$  is a decreasing function. But the fact that  $Q^*$  and  $p_i^*$  increase, implies that the right-hand side (RHS) increases, which means the FOCs cannot be fulfilled.

Second, suppose  $Q^*$  remains unchanged. This implies that the RHS of (A.5) increases, which in turn implies that  $x_i^{k^*} \cdot m_i$  falls. But if  $Q_i^*$  decreases,  $Q^*$  must decrease as well, since  $p_i^*$  increases.  $Q^*$  can thus not remain constant.

Third, consider  $Q^*$  decreasing. Since  $p_i^*$  is increasing, there must exist a group  $j$  where  $p_j^*$  is decreasing. Take the FOC of this group  $j$ : Since  $Q^*$  is decreasing and  $p_j^*$  is decreasing, the RHS is decreasing. This implies that  $x_j^{k^*} \cdot m_j$  is increasing which means  $Q_j^*$  is increasing and thus  $p_j^*$  as well. This contradicts the assumption that there exists a group where  $p_j^*$  is decreasing for decreasing  $Q^*$ . If there is no group with decreasing probability,  $p_i^*$  cannot be increasing.

Since for all possible cases of behavior for  $Q^*$ , there arises a contradiction from the assumption that  $p_i^*$  increases when  $m_i$  increases, it is established that  $p_i^*$  weakly decreases in  $m_i$  under the given assumptions.  $\square$

## Appendix B: Proof of Proposition 2

*Proof.* It is convenient to summarize the following properties in a Lemma.

**Lemma B.1.** *If  $q_{m_i}(\cdot), q_{m_i+1}(\cdot)$  have constant returns to scale and negative (positive) group-size bias for all  $m_i$ , it follows for a symmetric equilibrium  $x_i^k = x_i^l \forall k, l \forall i$  that*

$$\frac{\partial q_{m_i}(\hat{x}_i)}{\partial x_i^k} > (<) \frac{\partial q_{m_i+1}(\hat{x}_i)}{\partial x_i^k}.$$

and that  $\frac{\partial q_{m_i}(\hat{x}_i)}{\partial x_i^k}$  is invariant in the level of effort of a group.

*Proof of Lemma.* Due to constant returns to scale, we have  $q(a \cdot x_i) = a \cdot q(x_i)$ , and it follows that  $\frac{\partial q(x_i)}{\partial x_i^k}$  is homogenous of degree zero in  $x_i$ . Using the symmetry

of an equilibrium,  $q(\cdot)$  can be expressed by some function  $h(\cdot)$  such that  $q_{m_i}(x_i) = h(x_i^1, m_i)$ , and  $\frac{\partial q(x_i)}{\partial x_i^k}$  can only be a function of  $m_i$ . By Euler's homogenous function theorem,

$$\sum_k \frac{\partial q(x_i)}{\partial x_i^k} x_i^k = q(x_i),$$

we have in a symmetric equilibrium

$$\frac{\partial q(x_i)}{\partial x_i^k} = \frac{q(x_i)}{x_i^1 \cdot m_i} \equiv \frac{h(x_i^1, m_i)}{x_i^1 \cdot m_i}.$$

By homogeneity of degree one of  $h$  in its first argument, we know that  $\frac{\partial q_{m_i}(\hat{x}_i)}{\partial x_i^k}$  is indeed invariant in the level of effort as expected for constant returns to scale in a symmetric equilibrium. We further want to know whether

$$\frac{h(\hat{x}_i^1, m_i)}{\hat{x}_i^1 \cdot m_i} \geq \frac{h(\hat{x}_i^1, m_i + 1)}{\hat{x}_i^1 \cdot (m_i + 1)}$$

holds. Suppose that  $\beta = \frac{m_i + 1}{m_i}$ . Then, the RHS of the above inequality can be written as:

$$\frac{g(\hat{x}_i^1, m_i + 1)}{\hat{x}_i^1 \cdot (m_i + 1)} = \frac{h(\hat{x}_i^1, \beta \cdot m_i)}{\hat{x}_i^1 \cdot \beta \cdot m_i}. \quad (\text{B.1})$$

By constant returns to scale, (B.1) can be written as:

$$\frac{h(\hat{x}_i^1, \beta \cdot m_i)}{\hat{x}_i^1 \cdot \beta \cdot m_i} = \frac{h(\frac{\hat{x}_i^1}{\beta}, \beta \cdot m_i)}{\hat{x}_i^1 \cdot m_i},$$

which by  $q(\cdot)$  being negatively (positively) group size biased is strictly smaller (larger) than  $\frac{h(\hat{x}_i^1, m_i)}{\hat{x}_i^1 \cdot m_i}$ . This implies that  $\partial q(\hat{x}_i)/\partial x_i^k$  is indeed decreasing (increasing) in  $m_i$  for a symmetric equilibrium.  $\square$

We now turn to the proof for negative group-size bias. The FOCs are given by:

$$\frac{\partial q(x_i^*)}{\partial x_i^k} \frac{Q_{/i}^*}{Q^{*2}} v_i - 1 = 0 \quad \forall i, k. \quad (\text{B.2})$$

We want to establish that a joint increase in  $m_i$  and  $p_i^*$  in equilibrium leads to a contradiction. Examining the FOC of group member 1 of group  $i$ :

$$\frac{\partial q(x_i^*)}{\partial x_i^1} = \frac{Q^*}{(1 - p_i^*) v_i} \quad \forall i, k, \quad (\text{B.3})$$

we know that  $Q^*$  must decrease if  $m_i$  and  $p_i^*$  increase, since by Lemma B.1. the LHS is decreasing in  $m_i$  and invariant in  $x_i^{1*}$  for every given symmetric equilibrium. The

RHS is increasing in  $p_i^*$ . For the FOC to hold,  $Q^*$  must then decrease. Note that an increase in  $p_i^*$  implies a decrease in  $p_j^*$  for at least one group  $j$ . Looking at the FOC of group member 1 of this group  $j$ , we can derive the contradiction:

$$\frac{\partial q(x_j^*)}{\partial x_j^1} = \frac{Q^*}{(1 - p_j^*)v_j}. \quad (\text{B.4})$$

As we know, the LHS is constant for given group size  $m_j$ . The RHS however is both increasing in  $p_j^*$  and  $Q^*$ . Since both are decreasing, the FOC can no longer be fulfilled, which yields the contradiction.

The proof for positive group-size bias is similar. We again start with FOCs of an arbitrary member of an arbitrary group:

$$\frac{\partial q(x_i^*)}{\partial x_i^k} \frac{Q_{/i}^*}{Q^{*2}} v_i - 1 = 0 \quad \forall i, k. \quad (\text{B.5})$$

We want to establish that an increase in  $m_i$  and a decrease in  $p_i^*$  in equilibrium leads to a contradiction. Examining the FOC of group member 1 of group  $i$ :

$$\frac{\partial q(x_i^*)}{\partial x_i^1} = \frac{Q^*}{(1 - p_i^*)v_i} \quad \forall i, k, \quad (\text{B.6})$$

we know that  $Q^*$  must increase if  $m_i$  increases and  $p_i^*$  decreases, since by Lemma B.1. the LHS is decreasing in  $m_i$  and invariant in  $x_i^{1*}$  for every given symmetric equilibrium. The RHS is increasing in  $p_i^*$ . For the FOC to hold,  $Q^*$  must then increase. Note that a decrease in  $p_i^*$  implies an increase in  $p_j^*$  for at least one group  $j$ . Looking at the FOC of group member 1 of this group  $j$ , we can, as before, derive the contradiction:

$$\frac{\partial q(x_j^*)}{\partial x_j^1} = \frac{Q^*}{(1 - p_j^*)v_j}. \quad (\text{B.7})$$

As we know, the LHS is constant for given group size  $m_j$ . The RHS however is both increasing in  $p_j^*$  and  $Q^*$ . Since both are increasing, the FOC can no longer be fulfilled, which yields the contradiction.

□

## Appendix C: Proof of Proposition 3

*Proof.* Let  $\rho$  be the degree of homogeneity. Euler's Theorem for homogenous functions implies that  $q_{m_i}(x_i) = \frac{1}{\rho} \cdot \sum_k x_i^k \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i^k}$ , which boils down to

$$q_{m_i}(x_i) = \frac{m_i \cdot x_1^k}{\rho} \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i^k} \quad (\text{C.1})$$

for a symmetric equilibrium, which can (again for a symmetric equilibrium) alternatively be expressed as

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i^k} = \frac{\rho \cdot q_{m_i}(x_i)}{m_i \cdot x_1^k}. \quad (\text{C.2})$$

(Weakly) negative group-size biasedness implies  $q_{m_i}(x_i) \geq q_{\xi \cdot m_i}(x_i/\xi)$ . Using (C.2) this inequality can be expressed as

$$\begin{aligned} q_{m_i}(x_i) &= \frac{m_i \cdot x_1^k}{\rho} \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i^k} \geq \frac{m_i \cdot x_1^k}{\rho} \cdot \frac{\partial q_{\xi \cdot m_i}(x_i/\xi)}{\partial x_i^k} \\ &= \frac{\xi \cdot m_i}{\rho} \cdot \frac{x_1^k}{\xi} \cdot \frac{\partial q_{\xi \cdot m_i}(x_i/\xi)}{\partial x_i^k} = q_{\xi \cdot m_i}(x_i/\xi). \end{aligned}$$

This inequality reduces to

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i^k} \geq \frac{\partial q_{\xi \cdot m_i}(x_i/\xi)}{\partial x_i^k}. \quad (\text{C.3})$$

Assume that  $\xi = (m_i + 1)/m_i$ . The first-order condition of a representative member of group  $i$  is equal to

$$\frac{\partial q(x_i^*)}{\partial x_i^k} = \frac{Q^*}{v_i(1 - \frac{Q_i^*}{Q^*})} = \frac{Q^*}{v_i(1 - p_i^*)} \quad \forall k \quad (\text{C.4})$$

in equilibrium. We can now prove by contradiction that  $m_i$  and  $p_i^*$  cannot rise at the same time. Suppose this would be the case. For the behavior of  $Q^*$ , we can now distinguish three cases,  $Q^*$  increases, remains constant, or decreases.

Suppose  $Q^*$  increases. The increase in  $p_i^*$  also implies that  $Q_i^*$  has to increase. For fixed  $X_i = \sum x_i^k$ , the definition of group-size biasedness implies that  $q(x_i)$  goes down. In order to be consistent with the increase in  $Q_i^*$  it follows that  $X_i = \sum x_i^k$  has to go up.

The above assumptions imply that the RHS of (C.4) increases. For this to be an equilibrium, the LHS of (C.4) has to increase as well. For fixed  $X_i = \sum x_i^k$ , however,

(C.3) implies that  $\partial q(x_i)/\partial x_i^k$  goes down. (Weakly) decreasing returns to scale imply that  $\partial q(x_i)/\partial x_i^k$  is (weakly) decreasing along the symmetric array through the origin. Hence,  $X_i = \sum x_i^k$  has to go down to reestablish the equality, a contradiction.

Second, suppose  $Q^*$  remains unchanged. The contradiction follows along the same lines as before: An increase from  $m_i$  to  $m_i + 1$ , (C.3) implies that  $\partial q(x_i)/\partial x_i^k$  is reduced for constant  $X_i = \sum x_i^k$ . If  $p_i^*$  goes up, the LHS of (C.4) goes up. To reestablish the equality it follows that  $X_i = \sum x_i^k$  has to go down, which c.p. reduces  $Q_i^*$ , and for  $Q^*$  being constant, has to increase  $Q_j^*$  for some  $j$ . This is, however, inconsistent with the conjecture that  $p_i^*$  increases.

Third, consider  $Q^*$  decreasing. Since  $p_i^*$  is increasing, there must exist a group  $j$  where  $p_j^*$  is decreasing. Take the FOC of this group  $j$ : Since  $Q^*$  is decreasing and  $p_j^*$  is decreasing, the RHS is decreasing, which implies that the LHS has to decrease. In addition,  $Q_j^*$  has to go down as well. For given  $m_j$ , the LHS can only decrease if  $X_j = \sum x_j^k$  increases, which is inconsistent with the requirement that  $Q_j^*$  has to go down.

□

## Appendix D: Proof of Corollary 1

*Proof.* From group size unbiasedness we know that (A.4) holds. Due to constant returns to scale,  $g'(x_i^1 m_i)$  is a constant. The LHS of the first order condition (A.5) is thus constant. Suppose now  $p_i$  rises (falls) with a change in  $m_i$ . Then for the first order condition to still hold,  $Q^*$  needs to fall (rise). Also, the winning probability of at least one other group  $p_j$  needs to fall (rise). A fall (rise) in both  $Q^*$  and  $p_j$  is however incompatible with the first order condition of group  $j$ . Therefore, both  $p_i$  and  $Q^*$  need to remain constant. □



## Appendix E: Proof of Proposition 4

*Proof.* We assume throughout that we are in a symmetric, interior equilibrium. By homogeneity of degree  $r_i$ , we have from Euler's theorem

$$r_i \cdot q_{m_i}(x_i) = m_i \cdot x_i \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i}, \quad (\text{E.1})$$

and further

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i} = \frac{r_i \cdot q_{m_i}(x_i)}{m_i \cdot x_i} = \frac{r_i \cdot q_{m_i}(1)}{m_i \cdot (x_i)^{1-r_i}}. \quad (\text{E.2})$$

By the definition of group size biasedness we have that a function is positively group size biased, if  $q_{m_i}(x_i) < q_{m_i\xi}(x_i/\xi)$ . In the homogenous case, this is equivalent to  $q_{m_i}(1) < q_{m_i\xi}(1)\xi^{-r_i}$ . Using the above equation, we can reformulate this as:

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i} < \frac{\partial q_{m_i\xi}(x_i)}{\partial x_i} \xi^{-r_i}. \quad (\text{E.3})$$

By these conditions, a natural measure for group size bias emerges that does not depend on  $x_i$ :

$$b_i(\xi) = \frac{q_{m_i\xi}(1)}{q_{m_i}(1)\xi^{r_i}} = \frac{\frac{\partial q_{m_i\xi}(1)}{\partial x_i}}{\frac{\partial q_{m_i}(1)}{\partial x_i} \xi^{r_i}}. \quad (\text{E.4})$$

Clearly, if  $b_i(\xi) > 1$ ,  $q$  is positively group size biased.<sup>10</sup> For homogenous functions,  $q_{m_i\xi}$  can be uniquely determined by  $b_i(\xi)$ ,  $q_{m_i}$ , and  $r_i$ . We can derive two helpful expressions for the derivatives of  $q$  with respect to  $x_i$ :

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i} = \frac{r_i q_{m_i}(1)}{m_i x_i^{1-r_i}} = \frac{\partial q_{m_i\xi}(x_i)}{\partial x_i} \cdot \frac{\xi^{1-r_i}}{b_i(\xi)} \quad (\text{E.5})$$

$$= \frac{\partial q_{m_i}(1)}{\partial x_i} \cdot \frac{1}{x_i^{1-r_i}}. \quad (\text{E.6})$$

We now compare two classes of impact functions,  $q$  and  $\hat{q}$  for which  $b_i(\xi) > \hat{b}_i(\xi)$  for some<sup>11</sup>  $\xi$  and  $r_i = \hat{r}_i$ .

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<sup>10</sup>Note that this measure is a local measure which depends on  $\xi$ . A function that is positively group size biased for some increase in members may be negatively group size biased for others.

<sup>11</sup>Alternatively we could assume that this condition holds for all  $\xi$ , from which we could derive the result that if the group size paradox holds for all  $\xi$  under  $q$ , then it must hold for all  $\xi$  under  $\hat{q}$  as well.

Suppose that the first-order condition holds for the impact function  $q_{m_i}$  at  $x_i^*$ . After adding  $m_i\xi - m_i$  group members and employing the impact function  $q_{m_i\xi}$ , let the equilibrium contribution be  $x_{i,\xi}^*$ . Similarly, denote by  $\hat{x}_i^*$ ,  $\hat{x}_{i,\xi}^*$  the equilibrium efforts when using impact functions  $\hat{q}_{m_i}$  and  $\hat{q}_{m_i\xi}$ , respectively.

We can now employ the first-order condition to obtain that if the group size paradox holds, then  $\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} > \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i}$ . The first-order condition of an interior solution becomes after rearranging terms:

$$\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} = \frac{Q^*}{v_i^k(1-p_i^*)}. \quad (\text{E.7})$$

By (E.6) we can reformulate the LHS:

$$\frac{\partial q_{m_i}(1)}{\partial x_i} \cdot (x_i^*)^{r_i-1} = \frac{Q^*}{v_i^k(1-p_i^*)}. \quad (\text{E.8})$$

Suppose the group-size paradox holds. Then we can see that for an increase in  $m_i$ , the LHS of the equation has to decrease. This follows since an increase in the LHS would imply that on the RHS  $Q^*$  must rise, because by assumption  $p_i^*$  decreases. Since  $p_i^*$  decreases and probabilities must sum to one, there must exist another group  $j$ , for which  $p_j$  increases. But then the RHS of the first order condition of group  $j$  must increase and since  $Q^*$  increases,  $q_j^*$  and thus  $x_j^*$  must increase as well. However, the LHS is decreasing in  $x_j$ , implying that there does not exist  $x_j$  such that the first-order condition is satisfied anymore. Concluding, if the group size paradox holds and  $m_i$  increases, then

$$\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} > \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i}, \quad (\text{E.9})$$

i.e. the LHS of the first-order condition of group  $i$  has to be lower in the new equilibrium. Similarly, it can be derived that if the winning probability does not change for a change in  $m_i$ , then  $\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} = \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i}$ , and lastly, if the winning probability increases,  $\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} < \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i}$ .

We will now assume that the group-size paradox holds for  $q$ , but not for  $\hat{q}$  and show that this yields a contradiction. Combining the above properties of the partial derivatives of  $q_{m_i}$ ,  $q_{m_i\xi}$  with equation (E.9) and remembering that the derivative of a function that is homogenous of degree  $r$  is homogenous of degree  $r - 1$ , we obtain:

$$\frac{\partial q_{m_i}(x_i^*)}{\partial x_i^*} = \frac{\partial q_{m_i\xi}(1)}{\partial x_i} \cdot \frac{\xi^{1-r_i}}{b_i(\xi)(x_i^*)^{1-r_i}} > \frac{\partial q_{m_i\xi}(1)}{\partial x_i} \cdot \frac{1}{(x_{i,\xi}^*)^{1-r_i}} = \frac{\partial q_{m_i\xi}(x_{i,m_i}^*)}{\partial x_i} \quad (\text{E.10})$$

which implies:

$$b_i(\xi) < \left( \frac{\xi x_{i,\xi}^*}{x_i^*} \right)^{1-r_i} \quad (\text{E.11})$$

Similarly, with the inequality reversed (since we assume that the group size paradox does not hold for  $\hat{q}$ ):

$$\hat{b}_i(\xi) > \left( \frac{\xi \hat{x}_{i,\xi}^*}{\hat{x}_i^*} \right)^{1-r_i} \quad (\text{E.12})$$

Combining these two inequalities via the assumption that  $q$  has a greater group size bias than  $\hat{q}$ :

$$\left( \frac{\xi \hat{x}_{i,\xi}^*}{\hat{x}_i^*} \right)^{1-r_i} < \hat{b}_i(\xi) < b_i(\xi) < \left( \frac{\xi x_{i,\xi}^*}{x_i^*} \right)^{1-r_i}. \quad (\text{E.13})$$

Exponentiating by  $r_i/(1-r_i)$  and using homogeneity of degree  $r_i$  and (E.4), we have:

$$\frac{\hat{q}_{m_i\xi}(\hat{x}_{i,\xi}^*)}{\hat{q}_{m_i}(\hat{x}_i^*)\hat{b}_i(\xi)} < \frac{q_{m_i\xi}(x_{i,\xi}^*)}{q_{m_i}(x_i^*)b_i(\xi)}. \quad (\text{E.14})$$

Finally, we know that if the group-size paradox holds,  $q_{m_i}(x_i^*) > q_{m_i\xi}(x_{i,\xi}^*)$  must hold since otherwise  $x_{i,\xi}^*$  could not be a best response<sup>12</sup>. Together with  $b_i(\xi) > \hat{b}_i(\xi)$ , we conclude that  $\hat{q}_{m_i\xi}(\hat{x}_{i,\xi}^*) < \hat{q}_{m_i}(\hat{x}_i^*)$ . This contradicts the assumption that the group-size paradox does not hold for  $\hat{q}$ , since then a lower  $\hat{q}$  would have to yield a higher winning probability, in which case  $\hat{x}_i^*$  could not be a best response.

The reverse argument, stating that if we increase group size bias, the group size paradox cannot hold if it did not held before can be derived by reversing inequalities in E.13 above. The proof for a change in  $r_i > \hat{r}_i$  for  $b_i(\xi) = \hat{b}_i(\xi)$  now follows the same lines. Assuming the group size paradox holds under  $q$  but not under  $\hat{q}$ , we have:

$$\left( \frac{\xi \hat{x}_{i,\xi}^*}{\hat{x}_i^*} \right)^{1-\hat{r}_i} < \hat{b}_i(\xi) = b_i(\xi) < \left( \frac{\xi x_{i,\xi}^*}{x_i^*} \right)^{1-r_i} \quad (\text{E.15})$$

This inequality, similarly to (E.13), implies (note that the term  $\frac{r_i}{1-r_i}/\frac{\hat{r}_i}{1-\hat{r}_i}$  is strictly greater than 1 and since the RHS is strictly greater than 1 we can exponentiate it with the term):

$$\frac{\hat{q}_{m_i\xi}(\hat{x}_{i,\xi}^*)}{\hat{q}_{m_i}(\hat{x}_i^*)} < \frac{q_{m_i\xi}(x_{i,\xi}^*)}{q_{m_i}(x_i^*)}. \quad (\text{E.16})$$

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<sup>12</sup>To see this, note that the winning probability is higher for a group impact  $q_{m_i}(x_i^*)$ . Since group impact is increasing in  $x_i$ , it would be profitable for some group member to choose  $x_i^k < x_{i,\xi}^*$

From here, the same argument as above applies and therefore if the group size paradox holds for  $q$  with returns to scale  $r_i$ , then it must hold as well for  $\hat{q}$  with returns to scale  $\hat{r}_i < r_i$ . The reverse argument again follows from reversing the inequalities in (E.15) and assuming  $\hat{r}_i > r_i$ .

□

## Appendix F: Proof of Lemma 1

*Proof.* We first check that the interior solution is a local maximum. The FOC of the maximization problem (2) can be written as

$$\frac{Q_{/i}Q_i}{Y_iQ^2} = \frac{(y_i^k)^{\frac{1}{\gamma}}}{y_i^k v_i^k}. \quad (\text{F.1})$$

The second-order condition is satisfied if

$$\frac{v_i^k Q_{/i}Q_i}{\gamma Q^2 Y_i^2} \left( \frac{1 - 2\frac{Q_i}{Q}}{\gamma} - 1 \right) - \frac{\frac{1}{\gamma} - 1}{\gamma} (y_i^k)^{\frac{1}{\gamma} - 2} < 0. \quad (\text{F.2})$$

Solving the FOC for  $v_i^k$  and inserting the expression into the second-order condition we obtain, upon rearranging:

$$\frac{1 - \frac{1}{\gamma}}{\gamma} \left( 1 - \frac{y_i^k}{Y_i} \right) - 2\frac{1}{\gamma^2} \frac{Q_i y_i^k}{Q Y_i} < 0, \quad (\text{F.3})$$

which holds for all  $\gamma \in (-\infty, 1)$ . Therefore, all solutions of the FOC are local maxima taking the other players' strategies as given. The best responses are either given by the solution to the FOC, or by a corner solution. From equation (4) it is clear that the only possible corner solutions are non-participation with  $x_i^k = 0$ . We thus need to verify that whenever the best response of one member of the group is given by the solution to the FOC, it is not possible for any member of the group to have the best response  $x_i^k = 0$ . First, we will show that whenever there exists a solution of the FOC for one individual of a group, it exists for all individuals: From the FOCs of two representative group members  $l, k$  we obtain the within-group equilibrium condition:

$$\forall l, k < m_i : \frac{(y_i^k)^{\frac{1}{\gamma} - 1}}{v_i^k} = \frac{(y_i^l)^{\frac{1}{\gamma} - 1}}{v_i^l} \quad (\text{F.4})$$

for all members  $k, l$  of group  $i$ . Both, the LHS and RHS of (F.4) are strictly increasing in  $y_i^k, y_i^l$  if  $\gamma \in (0, 1)$ . For  $\gamma \in (-\infty, 0)$  both LHS and RHS of (F.4) are strictly decreasing in  $y_i^k, y_i^l$ . Thus, for each  $y_i^k$  there exists a  $y_i^l$  such that the within-group equilibrium condition holds. Since for all group members the LHS of (F.1) is equal, there exists a positive solution to the FOC for either all group members or none.

Second, we need to show that  $x_i^k = 0$  is not a best response if it is a best response for another individual  $l$  in the group to play  $x_i^l > 0$ . We do so by contradiction: Obviously, for a corner solution with  $x_i^k = 0$  and  $x_i^l > 0$  the following condition needs to hold:

$$\frac{\partial \pi_i^k}{\partial x_i^k} = \frac{Q_{/i} Q_i}{Q^2 Y_i} (x_i^k)^{\gamma-1} v_i^k - 1 \Big|_{x_i^k = 0, x_i^l > 0} \leq 0. \quad (\text{F.5})$$

$(x_i^k)^{\gamma-1}$  converges to  $\infty$  if  $x_i^k \rightarrow 0$ , which contradicts the above inequality if  $Q_{/i} Q_i / Q^2 Y_i > 0$ . Given that  $x_i^l > 0$  and the structure of the CES-impact function,  $Q_i > 0, Y_i > 0$ , and it cannot be an equilibrium that no group participates. To complete the proof, note that from the fact that there is an individual  $l$  in the group which participates with strictly positive effort, we know that

$$\frac{\partial \pi_i^l}{\partial x_i^l} = \frac{Q_{/i} Q_i}{Q^2 Y_i} (x_i^l)^{\gamma-1} v_i^l - 1 \Big|_{x_i^k = 0, x_i^l > 0} = 0. \quad (\text{F.6})$$

Inserting (F.6) into (F.5) yields:

$$\frac{(x_i^l)^{1-\gamma}}{v_i^l} - \frac{(x_i^k)^{1-\gamma}}{v_i^k} \Big|_{x_i^k = 0, x_i^l > 0} \leq 0 \quad (\text{F.7})$$

from which we obtain by inserting  $x_i^k = 0$ :

$$(x_i^l)^{1-\gamma} \Big|_{x_i^l > 0} \leq 0, \quad (\text{F.8})$$

which is a contradiction for all  $\gamma < 1$ . Thus there does not exist an equilibrium in which for one player in the group a corner solution at zero effort investments is obtained while for another an interior solution holds.  $\square$

## Appendix G: Proof of Lemma 2

*Proof.* If there exists a solution to the FOC, it is characterized by the following equation, obtained by solving (F.4) for  $y_i^l$  and summing over all  $l$ ,

$$Y_i = y_i^k \sum_l \left( \frac{v_i^l}{v_i^k} \right)^{\frac{\gamma}{1-\gamma}}. \quad (\text{G.1})$$

We can now solve equation (F.1) for  $Y_i$  explicitly:

$$Y_i = \left( \sqrt{Q_{/i} V_i(\gamma)} - Q_{/i} m_i^{\frac{1-\gamma}{\gamma}} \right)^\gamma. \quad (\text{G.2})$$

Thus, the condition for a strictly interior solution is  $(1/m_i \sum_l v_i^l)^{\frac{1-\gamma}{\gamma}} > Q_{/i}$ . Note that this condition is the same for all members of a group. In all other cases, we get  $y_i^k = 0$  for  $\gamma \in (0, 1)$  and  $y_i^k = \infty$  for  $\gamma \in (-\infty, 0)$  as was to be expected since both cases correspond to  $x_i^k = 0$ . In these cases we have  $\forall l : y_i^k = y_i^l$  by equation (F.4) and by the definition of  $Q_i$ , we have:  $Q_i = Y_i^{\frac{1}{\gamma}} / m_i^{(1-\gamma)/\gamma} = 0$ . We can write a group best-response function as

$$\hat{Q}_i(\gamma, Q_{/i}) = \max \left( 0, \sqrt{Q_{/i} V_i(\gamma)} - Q_{/i} \right). \quad (\text{G.3})$$

establishing part a), since by Lemma 1 either for all group members we obtain an interior solution or for none. Since the best-response function is continuous in  $\gamma \neq 0$  and in the strategies of the other groups  $Q_{/i}$ , if a unique Nash equilibrium exists, the equilibrium strategies must also be continuous in  $\gamma$ . This establishes part c) of Lemma 2. What remains to be shown is which groups participate in equilibrium. Suppose a group  $\zeta$  participates in equilibrium with strictly positive effort, while a group  $\zeta + 1$  does not participate. Let  $Q_i^*(\gamma)$  be  $Q_i$  in equilibrium (we ignore here that these are best responses and should thus be functions of  $Q_{/i}^*$ ) and let the other variables introduced above be defined correspondingly in equilibrium. Then by the above condition in equilibrium we have for any given  $\gamma$ :

$$\begin{aligned} V_\zeta(\gamma) &> Q_{/\zeta}^*(\gamma) \\ V_{\zeta+1}(\gamma) &\leq Q_{/(\zeta+1)}^*(\gamma) \end{aligned} \quad (\text{G.4})$$

Since by assumption  $Q_{\zeta+1}^*(\gamma) = 0$ , we have  $Q_{/(\zeta+1)}^*(\gamma) = Q^*(\gamma)$ . Solving (5) for  $Q_{/i}$  tells us that in an equilibrium where group  $\zeta$  participates, the following needs to be

true:

$$Q_{/\zeta}^*(\gamma) = \frac{Q^*(\gamma)^2}{V_\zeta(\gamma)}. \quad (\text{G.5})$$

We now insert (G.5) into the first equation of (G.4) and the condition  $\hat{Q}_{/\zeta+1} = \hat{Q}$  into the second equation. Thus the condition (G.4) becomes

$$\begin{aligned} V_p(\gamma) &> Q^*(\gamma) \\ V_{\zeta+1}(\gamma) &\leq Q^*(\gamma) \end{aligned} \quad (\text{G.6})$$

in equilibrium. It follows that  $V_\zeta(\gamma) > V_{\zeta+1}(\gamma)$ . We can thus order the groups such that  $V_i(\gamma) \geq V_{i+1}(\gamma)$  and define  $n^*(\gamma)$  as the group with the highest index number that still participates with strictly positive effort. By (G.6), all groups  $i \leq n^*(\gamma)$  participate. This establishes part b) of Lemma 2.  $\square$

## Appendix H: Proof of Proposition 6

*Proof.* The winning probability of each group is given by:

$$\frac{Q_i^*(\gamma)}{Q^*(\gamma)} = \left( 1 - \frac{n^*(\gamma) - 1}{\sum_{j=1}^{n^*(\gamma)} V_j(\gamma)^{-1}} \frac{1}{V_i(\gamma)} \right), \quad (\text{H.1})$$

where  $V_i(\gamma) = \left( \frac{1}{m_i} \cdot \sum_l (v_i^l)^{\frac{\gamma}{1-\gamma}} \right)^{\frac{1-\gamma}{\gamma}}$ . Since the winning probability increases in  $V_i$ , the winning probability is increased by an additional member  $x$  of the group if the following holds:

$$\left( \frac{\sum (v_i^l)^{\frac{\gamma}{1-\gamma}}}{m_i} \right)^{\frac{1-\gamma}{\gamma}} \leq \left( \frac{\sum (v_i^l)^{\frac{\gamma}{1-\gamma}} + (v_i^x)^{\frac{\gamma}{1-\gamma}}}{m_i + 1} \right)^{\frac{1-\gamma}{\gamma}}. \quad (\text{H.2})$$

This condition yields after solving:

$$v_i^x \geq V_i. \quad (\text{H.3})$$

Notice, if we substitute  $\theta = \frac{\gamma}{1-\gamma}$ ,  $V_i(\theta)$  is a power mean with the exponent ranging over  $\theta \in (-1, \infty)$  where the greatest lower bound is given by  $\gamma \rightarrow -\infty$  and the least upper bound given by  $\gamma \rightarrow 1$ . It follows from the power-mean inequality that the power mean is weakly increasing in its exponent  $\theta$  and strictly increasing in  $\theta$

if there are two distinct  $v_i^k \neq v_i^l$  (Bullen (2003), chapter 3). Since  $\theta$  is an increasing function of  $\gamma$ ,  $V_i$  is strictly increasing in  $\gamma$ . This in turn implies that the minimum valuation a new group member must have in order to raise the winning probability of this group is also increasing in  $\gamma$ .  $\square$



## References

- Aczél, J., & Dhombres, J. (1989). *Functional equations in several variables*. Cambridge, UK: Cambridge University Press.
- Agrawal, A., & Goyal, S. (2001). Group size and collective action, third-party monitoring in common-pool resources. *Comparative Political Studies*, *34*, 63-93.
- Alchian, A. A., & Demsetz, H. (1972). Information costs, and economic organization. *The American Economic Review*, *62*(5), 777-795.
- Baik, K. H. (2008). Contests with group-specific public goods prizes. *Social Choice and Welfare*, *30*(1), 103-117.
- Bandiera, O., Barankay, I., & Rasul, I. (2005). Cooperation in collective action. *Economics of Transition*, *13*, 473-498.
- Beauchamp, G. (2004). Reduced flocking by birds on islands with relaxed predation. *The Royal Society, Proceedings: Biological Sciences*, *271*, 1039-1042.
- Benbasat, I., & Lim, L.-H. (1993). The effects of group, task, context, and technology variables on the usefulness of group support systems: A meta-analysis of experimental studies. *Small Group Research*, *24*, 430-62.
- Bertram, B. (1980). Vigilance and group size in ostriches. *Animal Behavior*, *28*, 278-286.
- Bullen, P. (2003). *Handbook of means and their inequalities*. Dordrecht: Kluwer.
- Clausewitz, C. (1943). *On war*. New York: Random House.
- Corchón, L. C. (2007). The theory of contests: A survey. *Review of Economic Design*, *11*(2), 69-100.
- Cresswell, W., & Quinn, J. L. (2004). Faced with a choice, sparrowhawks more often attack the more vulnerable prey group. *Oikos*, *104*, 71-76.
- Dejean, S., Penard, T., & Suire, R. (2009). *Olson's paradox revisited: An empirical analysis of incentives to contribute in p2p file-sharing communities*. (Mimeo)
- Dominelli, L. (1996). Deprofessionalizing social work: Anti-oppressive practice, competencies and postmodernism. *British Journal of Social Work*, *26*, 15375.
- Esteban, J., & Ray, D. (2001). Collective action and the group paradox. *American Political Science Review*, *95*(3), 663-672.
- Esteban, J., & Ray, D. (2008). On the salience of ethnic conflict. *The American*

- Economic Review*, 98(5), 2185-2202.
- Esteban, J., & Ray, D. (2010). *A model of ethnic conflict*. (forthcoming in: The Journal of the European Economic Association.)
- Fu, Q., & Lu, J. (2008). *A micro foundation of generalized multi-prize lottery contests: A noisy ranking perspective*. (Unpublished working paper)
- Gamson, W. A. (1995). Hiroshima, the holocaust, and the politics of exclusion: 1994 presidential address to the american sociological association. *American Sociological Review*, 60, 1-20.
- Garfinkel, M. R., & Skaperdas, S. (2007). Chapter 22 economics of conflict: An overview. In T. Sandler & K. Hartley (Eds.), *Handbook of defense economics - defense in a globalized world* (Vol. 2, p. 649 - 709). Elsevier.
- Hardin, R. (1982). *Collective action*. Baltimore: John Hopkins University Press.
- Hillman, A. L., & Riley, J. G. (1987). Politically contestable rents and transfers. *Economics and Politics*, 1(1), 17-39.
- Hirshleifer, J. (1983). From weakest-link to best-shot: The voluntary provision of public goods. *Public Choice*, 41(3), 371-386.
- Hirshleifer, J. (1995). Anarchy and its breakdown. *Journal of Political Economy*, 103, 26-52.
- Kenward, R. (1978). Hawks and doves: factors affecting success and selection in goshawk attacks on wood-pigeons. *Journal of Animal Ecology*, 47, 449-460.
- Klandermans, B., & deWeerd, M. (2000). Group identification and political protest. In S. Stryker, T. J. Owens, & R. W. White (Eds.), *Self, identity and social movements* (p. 68-92). Minneapolis: University of Minnesota Press.
- Konrad, K. (2009). *Strategy and dynamics in contests*. Oxford, UK: Oxford University Press.
- Leininger, W. (2003). On evolutionarily stable behavior in contests. *Economics of Governance*, 4, 177-186.
- Marwell, G., & Oliver, P. (1993). *The critical mass in collective action*. New York: Cambridge University Press.
- McAdam, D. (1982). *Political process and the development of black insurgency 1930 - 1970*. Chicago, IL: University of Chicago Press.
- McCarthy, J., & Wolfson, M. (1996). Resource mobilization by local social movement organizations: Agency, strategy and organization in the movement against

- drinking and driving. *American Sociological Review*, 61, 107088.
- McCarthy, J., & Zald, M. (1977). Resource mobilization and social movements: A partial theory. *American Journal of Sociology*, 82, 1212-41.
- Monge, P. R., Fulk, J., Kalman, M. E., Flanagin, A. J., Parnassa, C., & Rumsey, S. (1998). Production of collective action in alliance-based interorganizational communication and information systems. *Organization Science*, 9, 411-33.
- Münster, J. (2009). Group contest success functions. *Economic Theory*, 41(2), 345-357.
- Nitzan, S., & Ueda, K. (2009). Collective contests for commons and club goods. *Journal of Public Economics*, 93(1), 48-55.
- Nitzan, S., & Ueda, K. (2010). *Prize sharing in collective contests*. (forthcoming in: The European Economic Review)
- Oliver, P. E. (1993). Formal models of collective action. *Annual Review of Sociology*, 19, 271-300.
- Olson, M. (1965). *The logic of collective action*. Cambridge, MA: Harvard University Press.
- Ostrom, E. (1997). *Self-governance of common-pool resources (w97 workshop in political theory and policy analysis)*. Bloomington: Indiana University.
- Pecorino, P., & Temini, A. (2008). The group size paradox revisited. *Journal of Public Economic Theory*, 10, 785-799.
- Pulliam, H., Pyke, G., & Caraco, T. (1982). The scanning behavior of juncos: a game-theoretical approach. *Journal of theoretical Biology Economics*, 95, 89-103.
- Siegel, D. A. (2009). Social networks and collective action. *American Journal of Political Science*, 53, 122-38.
- Smith, J. M., & Price, G. (1972). The logic of animal conflict. *Nature*, 246, 15-18.
- Stein, W. E. (2002). Asymmetric rent-seeking with more than two contestants. *Public Choice*, 113(3), 325-336.
- Tainter, J. A. (1990). *The collapse of complex societies*. Cambridge: Cambridge University Press.
- Wincent, J., Örtqvist, D., Eriksson, J., & Autio, E. (2010). The more the merrier? the effect of group size on effectiveness in sme funding campaigns. *Strategic Organization*, 8, 43-68.

Zald, M. N., & Ash, R. (1966). Social movement organizations: Growth, decay and change. *Social Forces*, 44, 327-41.