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Competition between State Universities

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Abstract

We analyse how state university competition to collect resources may affect both research and the quality of teaching. By considering a set-up where two state universities behave strategically, we model their interaction with potential students as a sequential noncooperative game. We show that different types of equilibrium may arise, depending on the mix of research and teaching supplied by each university, and the mix of low- and high-ability students attending each university. The most efficient equilibrium results in the creation of an élite institution attended only by high-ability students who enjoy a higher teaching quality but pay higher tuition fees.

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1 Introduction

Notwithstanding its importance for researchers, the economic literature on education has traditionally ignored the competition for students and public funding among public universities (Boroah (1994), De Fraja and Iossa (2002), Johnes (2007), Gautier and Wauthy (2007)). Instead, there exist several theoretical and empirical papers on competition between private and public schools and universities (Epple and Romano (1998, 2008), Bailey et al. (2004), Bertola and Checchi (2003), Oliveira (2006)).

This paper aims to analyse how state university competition to collect resources may affect the quality of teaching and the level of research. In this respect, two main remarks are in order. First, as suggested by Rothschild and White (1995), universities compete for students because universities adopt a customer-input technology, i.e. students are at once inputs and customers of the educational process. More precisely, students are inputs needed to produce education, but they also provide funds to universities both by paying tuition fees and by allowing universities to receive transfers from the government. In fact, most public funding mechanisms, such as the European ones for example, have a per-student transfer component in addition to a lump-sum component. Second, Cohn and Cooper (2004) stress the fact that universities are multi-product institutions that supply three types of output: teaching, research, and public services. Teaching aims to deliver knowledge both at undergraduate and postgraduate level. Research, instead, aims to create knowledge with externalities for all society. Research may be considered as complementary to teaching in the case of postgraduate courses, while it is probably a substitute in the case of undergraduate courses. Finally, universities produce a third output which can be thought of as a public service: university diplomas certify that students have acquired specific competencies. In many countries university diplomas have a legally recognized value.

We consider a set-up where two state universities behave strategically in the same jurisdiction.[1] Their interaction with potential students is thus modelled as a sequential noncooperative game. Given a public funding mechanism, at the first stage, the universities choose their tuition fees and investments in teaching and research; at the second stage, students choose which university to attend depending on a cost-benefit comparison. Under the assumption of perfect mobility of students, the cost of attending one university only depends on tuition fees (for simplicity, other costs are assumed to be equal). The benefit derived from attending one university or the other, instead, depends on each student's own ability and on the quality of teaching which includes a peer group effect. Consequently also the average ability of students attending each university is relevant from an individual point of view (Epple and Romano (1998)).

By solving the model, we show that different types of equilibrium may arise, depending on the levels of the public transfers. Each equilibrium is characterized from two points of view: the mix of research and teaching quality supplied by each university, and the mix of low- and high-ability students attending each university. On the one side, universities may choose to specialize only in

[1] See Aghion et al. (2010) for an empirical analysis of the link between university autonomy, competition, and research performance. See also Veugelers and Van Der Ploeg (2008).

research or teaching, or instead to supply both. On the other side, students with different ability allocate between universities in different ways. We show that there does not exist an equilibrium where both high- and low-ability students attend both universities. Thus, possible equilibria are the following: 1) an equilibrium where there is complete segregation and an élite institution is created, i.e. all high-ability students attend one university, and all low-ability students attend the other university; 2) a mixed equilibrium where all students of one type and part of the students of the other type attend one university, and the rest attend the other university; 3) a specialized equilibrium where all students attend one university, and the other institution only produces research. From a social point of view, we show that the first equilibrium is the most efficient. When compared to the second equilibrium, the first one allows the attainment of higher teaching quality at the same public extra-research cost. Also research is higher, reaching its technically efficient level. When compared to the third equilibrium, the first one allows the same teaching quality and research level at a lower public cost.

Our paper is related to two strands of economic literature which we try to combine in order to gather some new hints on university incentives. More specifically, we refer both to the literature on public university competition, and to the literature on capital tax competition with household mobility. As we stressed above, competition between public universities has received limited attention, even if some recent papers have tried to shed some light on the issue. Del Rey (2001) uses a spatial competition model to analyse a game between two universities which provide both research and teaching, and use admission standards to control the average ability of enrolled students. Depending on preferences and technologies different types of symmetric equilibrium may arise: both universities admit only some of the applicants and provide research; both universities satisfy all students' demand and provide research; both are 'teaching only' universities; both are 'research only' universities. In a related paper, De Fraja and Iossa (2002) focus attention on how students' mobility costs may affect the equilibrium configuration. In particular, if mobility costs are high, as in Del Rey (2001), the equilibrium is symmetric: both universities admit the same number of students, and research investments are the same. If mobility costs are sufficiently low, instead, the resulting equilibrium (provided it exists) is asymmetric, i.e. one university (the 'élite institution') admits the best students, and provides more research than the other.[2] More recently, Kemnitz (2007) examines how different public funding schemes may affect competition between universities, and thus the quality of their teaching and research. Hubner (2009) extends the previous analyses by showing that the introduction of tuition fees can raise the quality of education and the number of students when both central and local governments lack sufficient instruments to tax the high-skilled population.

Contrary to what happens with university competition, the literature on capital tax competition is quite large (for surveys see Wellish (2000), Hindriks and Myles (2006)). In this respect, a familiar result is that tax competition for perfectly mobile capital results in underprovision of

[2] Optimal research and teaching decisions are also analysed by De Fraja and Valbonesi (2009) who, however, assume that in each local education market there is a single university that acts as a monopolist because no mobility of students is allowed.

local public goods when households are perfectly immobile. Such a result, however, does not hold when households are allowed to be perfectly mobile. Fiscal externalities, which are at the basis of the result on local public good underprovision, disappear when households are mobile: each region/country internalizes the effects of its own policies on the welfare of non-residents by taking the migration equilibrium into account. Accordingly, introducing mobility of households in the standard capital tax competition model mitigates the downward pressure on local public goods provision (Wellish 2000, p.105).

In the present paper we use the methodological tools offered by the literature on capital tax competition in order to analyse how student mobility affects university competition on both tuition fees, and expenditure in research and teaching. To the best of our knowledge this represents a novelty with respect to the existing literature which uses spatial competition models to analyse state university competition, and does not allow universities to set tuition fees. The main contribution of this paper is to characterize different configurations of the university system (élite institution, mixed system and specialization in research) in a unified framework, where the differences depend on the public transfers chosen by the government. This allows us to select the élite system as the most efficient. On the contrary, existing literature on state university competition does not analyse the role of the government in shaping the university system. Further, in our paper, universities do not set admission standards, thus students are free to attend the university they prefer on the basis of a cost-benefit analysis. This scenario fits the European set-up better than the U.S. one, and is probably more suitable to describe undergraduate degrees.

The plan of the paper is as follows. Section 2 describes the model. Section 3 analyses students' university choice and characterizes three different type of stable equilibria that may arise. Section 4 examines how universities compete with respect to their choice of tuition fees and expenditure for research and teaching. Section 5 compares the outcomes of the three equilibria from a social point of view. Finally, section 6 contains some concluding remarks. All the proofs can be found in the Appendix.

2 The model

Consider two universities denoted by j, $j = A, B$, operating in the same jurisdiction, and (possibly) differing with respect to quality of teaching, q_j, and level of research, r_j. Students have to choose which university to attend. Students differ with respect to their ability, e^i, which can be high, e^h, or low, e^l, with $e^h > e^l$. The preferences of the students are represented by the following utility function

$$U^i(q_j) - b_j, \qquad i = h, l, \quad j = A, B, \tag{1}$$

where $b_j \geqslant 0$ denotes the per-student tuition fee paid to university j. We assume that high-ability students derive a higher level of utility from any given level of $q_j > 0$, i.e. $U^h(q_j) > U^l(q_j)$, and $U^i(0) = 0$, $i = h, l$. We also assume that university quality positively affects students' utility at a decreasing rate, $\frac{dU^i}{dq_j} > 0$, $\frac{d^2U^i}{d(q_j)^2} < 0$ with $\frac{dU^h}{dq_j} > \frac{dU^l}{dq_j}$. Further, the reservation level of utility of both types of students is normalized to zero. The exogenous total number of students is $N = \sum_{i=h,l} N^i$, where N^h is the total number of high-ability students, and N^l the total number of low-ability students with $N^l \geq N/2$. Thus, it is $N = n_A + n_B$, where n_j denotes the total number of students attending university j, $j = A, B$, i.e. all students attend one of the two universities.[3] Moreover, n_j^i, $i = h, l$, denotes the total number of students belonging to each type and attending each university so that $n_j = \sum_{i=h,l} n_j^i$, $j = A, B$, and $N^i = \sum_{j=A,B} n_j^i$, $i = h, l$. Let us denote with $\bar{e}_j$ the average ability of students attending university j. Accordingly, the average ability of students attending university j obtains as

$$\bar{e}_j = \frac{\sum_{i=h,l} n_j^i e^i}{n_j} = \frac{n_j^h}{n_j}\Delta + e^l, \qquad j = A, B, \tag{2}$$

with $\Delta \equiv e^h - e^l$.

Each university may receive two types of transfer from the government. Let $t_j \geq 0$ denote a per-student transfer to university j, and $\tau_j \geq 0$ denote a lump-sum transfer, $j = A, B$. Accordingly, the budget constraint of university j, $j = A, B$, obtains as

$$(t_j + b_j)n_j + \tau_j = T_j + R_j, \qquad j = A, B, \tag{3}$$

where $T_j \geq 0$ and $R_j \geq 0$ represent expenditure on teaching and research by university j, $j = A, B$, respectively. Notice that universities are not constrained in the destination of the transfers. The sums thus received can be used either to finance teaching or research.

Each university produces teaching according to the following production function[4]

$$q_j = \alpha\bar{e}_j + \beta\tfrac{T_j}{n_j}, \quad \text{when } n_j > 0, \quad \alpha, \beta > 0, \tag{4}$$

$$q_j = 0, \quad \text{when } n_j = 0, \quad j = A, B.$$

Teaching quality can be improved by augmenting the average quality of the students and/or teaching expenditure, for example by increasing the teacher/students ratio. The parameters α and β, measure how the peer group effect and per-student teaching expenditure, respectively, translate into teaching quality and are the same in both universities. The quality of teaching is assumed to be independent of research. This means that we mostly refer to undergraduate courses.

Further, each university produces research according to the following production function with decreasing returns[5]

[3] In other words, we consider only those young people who benefit from university education. We assume that secondary school performance is informative enough to divide school leavers between potential university students and workers.

[4] This is a common form for the teaching production function, see e.g. Del Rey (2001). Notice that this production function implies that $q_j > 0$ even if $T_j = 0$. This can be interpreted in two ways. We can assume that when $n_j > 0$, T_j is always higher than the minimum level needed to be active in teaching. Alternatively, even if universities devote no funds to teaching, they can be thought to operate as a screening device or as a network that makes attendance beneficial to students anyway, as in Del Rey (2001).

[5] See also Gautier and Wauthy (2007).

$$r_j = R_j^{\gamma_j}, \qquad j = A, B, \quad 0 < \gamma_j < 1, \tag{5}$$

where γ_j represents an index of efficiency of research activity specific to each university. Then, each university can improve the quality of its research by augmenting its expenditure on research activity, for example, by recruiting better researchers and by purchasing more sophisticated equipment.

Finally, each university cares about both teaching and research, and thus we assume the following objective functions

$$W_j = \sum_{i=h,l} n_j^i q_j + r_j, \qquad j = A, B, \tag{6}$$

according to which, in the intent of the universities, there is perfect substitutability between total quality of teaching and research.[6]

The game is solved by backward induction. We first examine the students' decisions on which university to attend and then the universities' decisions on tuition fees, research and teaching expenditure.

3 Students' university choice and characterization of stable equilibria

Consider the second stage of the game when students make their decisions. If both universities enrol students of a given type, at equilibrium, those students must be indifferent with respect to which university to attend. This implies that the following arbitrage condition has to hold[7]

$$U^i(q_A) - b_A = U^i(q_B) - b_B, \qquad i = h, l. \tag{7}$$

Recall that the quality of teaching depends on per-student expenditure and on average student ability. It is consequently affected both by the number of students and by the proportion of high-ability individuals. By substituting (4), and (3) into (1), the effect of the number of students on individual utility obtains as

$$\frac{\partial U^i}{\partial n_j^i} = \frac{dU^i}{dq_j}\frac{\partial q_j}{\partial n_j^i}, \qquad i = h, l; \quad j = A, B. \tag{8}$$

Accordingly, $sign\frac{\partial U^i}{\partial n_j^i} = sign\frac{\partial q_j}{\partial n_j^i}$, because $\frac{dU^i}{dq_j} > 0$ by assumption. By using (2) and (3) into (4), the effect of the number of students on teaching quality obtains as

$$\frac{\partial q_j}{\partial n_j^i} = \alpha\frac{\partial \bar{e}_j}{\partial n_j^i} + \beta\frac{\partial (T_j/n_j)}{\partial n_j^i}, \qquad i = h, l, \qquad j = A, B, \tag{9}$$

[6] In order to sum up the two components of the objective function, q_j and r_j indexes must be normalized. The same type of objective function is also used by Del Rey (2000) and a similar one by de Fraja and Iossa (2002). The latter assume that universities are interested in maximising their prestige which is formalized as a function of the number of students, the average ability of the student body, and research expenditure. More recently, De Fraja and Valbonesi (2008) suppose that universities are only interested in maximising their amount of research, so that teaching is not an end in itself, but a means to fund research.

[7] This condition is quite familiar in the literature on tax competition with household mobility. See for instance Wellish (2000, p.111).

when $n_j > 0$. More specifically, for high-ability students, $i = h$, equation (9) rewrites as

$$\frac{\partial q_j}{\partial n_j^h} = \frac{1}{n_j^2}\left[\alpha \Delta n_j^l + \beta\left(R_j - \tau_j\right)\right], \qquad j = A, B, \tag{10}$$

and for low-ability students, $i = l$, equation (9) rewrites as

$$\frac{\partial q_j}{\partial n_j^l} = \frac{1}{n_j^2}\left[-\alpha \Delta n_j^h + \beta\left(R_j - \tau_j\right)\right], \qquad j = A, B. \tag{11}$$

Notice that the effect of the number of students of type i, n^i, on teaching quality of university j depends on two terms. The first one represents the direct effect of an additional student on average ability and is positive (negative) for high (low) ability students. Notice that for each university, the effect of the number of high (low) ability students on the quality of teaching depends on the number of low (high) ability students. The second term represents the indirect effect of an additional student on per-student teaching expenditure and is positive (negative) if research expenditure is higher (lower) than the lump-sum transfer. The reason is that an excess of research expenditure over the lump-sum transfer has to be financed by the fees paid by students. When the lump-sum transfer exceeds research expenditure, instead, an additional student subtracts per-capita teaching resources.

Considering that $t_j + b_j + \tau_j - R_j = T_j \geq 0$, the sign of $\frac{\partial q_j}{\partial n_j^i}$, $i = h, l$, is determined in the following

Lemma 1 : *For* $n_j > 0$, $i)$ $\frac{\partial q_j}{\partial n_j^h} \gtreqless 0$ *iff* $R_j - \tau_j \gtreqless -\frac{\alpha}{\beta}\Delta n_j^l$, *with* $n_j^l \geq 0$,
$ii)$ $\frac{\partial q_j}{\partial n_j^l} \gtreqless 0$ *iff* $R_j - \tau_j \lesseqgtr \frac{\alpha}{\beta}\Delta n_j^h$, *with* $n_j^h \geq 0$.
For $n_j = 0$, $\left.\frac{\partial q_j}{\partial n_j^i}\right|_{n_j=0} = \alpha e_j^i + \beta\left(t_j + b_j + \tau_j - R_j\right) > 0,\ j = A, B,\ i = h, l.$

The sign of $\frac{\partial q_j}{\partial n_j^i}$, $i = h, l$, $j = A, B$, is crucial in determining the type of locally stable equilibrium which occurs at the students' subgame. In this respect, we can state the following

Proposition 1 *There does not exist an equilibrium where each university is attended by both types of students.*

The reason why there cannot exist an equilibrium where both h and l students are found in both universities is that such undifferentiated structure contradicts the arbitrage condition, i.e. the requirement that the utility levels must be the same in both universities for each type of students. Given the difference in marginal utilities, if the utility achievable in the two universities is equalized for one type, it cannot be equalized for the other type. We are then left with the following three kinds of equilibria:[8]

Equilibrium E (élite university system): all h students attend university A and all l students attend university B.

[8] More precisely, for each type, there actually exist two symmetric equilibria. The second one can be obtained by simply exchanging the subscript A for B and viceversa.

Equilibrium M (mixed university system): all students of one type and part of the students of the other type attend university A and the rest attend university B.

Equilibrium S (specialized university system): all students attend university A. University B only produces research.

In the following we focus on locally stable equilibria, and derive the conditions on public transfers which characterize each kind of equilibrium.

3.1 Equilibrium E: An élite university system

In this equilibrium a process of perfect segregation takes place. Formally, for all h students to choose university A and all l students to choose university B, the following conditions must be satisfied[9]

$$U^h\left(\alpha e^h + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N^h}\right)\right) - b_A \geq U^h\left(\alpha e^l + \beta\left(t_B + b_B + \frac{\tau_B - R_B}{N^l}\right)\right) - b_B, \quad (12)$$

and

$$U^l\left(\alpha e^l + \beta\left(t_B + b_B + \frac{\tau_B - R_B}{N^l}\right)\right) - b_B \geq U^l\left(\alpha e^h + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N^h}\right)\right) - b_A. \quad (13)$$

From the above conditions we can derive the following

Proposition 2 *In equilibrium E, either 1)* $q_A^{E_1} = q_B^{E_1} = q^{E_1}$ *and* $b_A^{E_1} = b_B^{E_1} = b^{E_1}$ *or 2)* $q_A^{E_2} > q_B^{E_2}$ *and* $b_A^{E_2} > b_B^{E_2}$.

Proposition 2 identifies two specifications of equilibrium E. In equilibrium E_1, teaching quality and tuition fee reach the same level in both universities. In equilibrium E_2, both the teaching quality and the tuition fee are higher in university A, where all h students are enrolled, than in university B, which is attended only by l students.

Notice that in specification E_1, conditions (12) and (13) hold as equalities. Then, local stability implies $\frac{\partial q_A}{\partial n_A^l} < 0$ and $\frac{\partial q_B}{\partial n_B^h} < 0$ which in turn implies $\frac{\partial q_B}{\partial n_B^l} < 0$. By Lemma 1, this equilibrium (with $n_A^l = n_B^h = 0$) arises only if

$$\begin{aligned} \tau_A - R_A &> -\frac{\alpha}{\beta}\Delta N^h, \\ \tau_B - R_B &> \frac{\alpha}{\beta}\Delta N^l. \end{aligned} \quad (14)$$

In university B, the lump-sum transfer must exceed research expenditure by an amount representing the compensation for the lower quality of its students while in university A the lump-sum transfer can fall short of research expenditure by an amount proportional to the higher quality of its students. In both universities, an increase in the number of students lowers the teaching quality. In section 4.1.2, we will show that also the stability condition for equilibrium E_2 implies $\frac{\partial q_B}{\partial n_B^h} < 0$.

[9] We assume that universities fix tuition fees without taking into account the marginal effect of a student movement on teaching quality. Given that N is large, such effect is negligible.

3.2 Equilibrium M: A mixed university system

Recalling that in this equilibrium both types of students attend university A while university B is attended by students of the same type, we distinguish two specifications according to the type found in university B. In equilibrium M_1, university B is attended by low-ability students while in equilibrium M_2, university B is attended by high-ability students.

3.2.1 Equilibrium M_1

Formally, for all h students and part of l students to attend university A and the rest of l students to attend university B, the following conditions must be satisfied[10]

$$\begin{aligned} &U^h\left(\alpha\left(e^l+\tfrac{\Delta N^h}{N^h+n_A^l}\right)+\beta\left(t_A+b_A+\tfrac{\tau_A-R_A}{N^h+n_A^l}\right)\right)-b_A> \\ &U^h\left(\alpha e^l+\beta\left(t_B+b_B+\tfrac{\tau_B-R_B}{n_B^l}\right)\right)-b_B, \end{aligned} \tag{15}$$

and

$$\begin{aligned} &U^l\left(\alpha\left(e^l+\tfrac{\Delta N^h}{N^h+n_A^l}\right)+\beta\left(t_A+b_A+\tfrac{\tau_A-R_A}{N^h+n_A^l}\right)\right)-b_A= \\ &U^l\left(\alpha e^l+\beta\left(t_B+b_B+\tfrac{\tau_B-R_B}{n_B^l}\right)\right)-b_B. \end{aligned} \tag{16}$$

In order for this equilibrium to be stable it must be the case that $\frac{\partial q_j}{\partial n_j^l}<0$, $j=A,B$. This means that, at equilibrium, quality decreases with low-ability students for both universities. By Lemma 1, this implies

$$\tau_A-R_A>-\frac{\alpha}{\beta}\Delta N^h, \tag{17}$$

$$\tau_B-R_B>0. \tag{18}$$

For university B, the lump-sum transfer τ_B must exceed research expenditure. Funds in excess can be used to improve teaching quality. As a consequence of the high lump-sum transfer, university B has no need to attract too many (l) students. For university A, τ_A may exceed or be lower than R_A. In university A, there may be an incentive to attract students in order to finance teaching and possibly research.

Further, we derive the impact of universities' decisions on the location of low-ability students, by stating the following

Lemma 2. *At equilibrium M_1, for low-ability students it is* $\frac{dn_j^l}{db_j}=\frac{1-\beta\frac{\partial U_j^l}{\partial q_j}}{J^l}$, *and* $\frac{dn_j^l}{dR_j}=\frac{\frac{\beta}{n_j}\frac{\partial U_j^l}{\partial q_j}}{J^l}<0$, *where* $J^l\equiv\sum_{j=A,B}\frac{\partial U_j^l}{\partial q_j}\frac{\partial q_j}{\partial n_j^l}<0$, $j=A,B$.

Lemma 2 shows that the number of low-ability students attending one university depends negatively on R_j. The higher the value of β, the greater the effect because R_j represents resources

[10] Notice that condition (15) cannot hold as an equality by the same argument used in the proof of Proposition 1 to exclude that both types are shared between the universities.

that are subtracted from teaching expenditure. In Section 4.2, we show that also $\frac{dn_j^l}{db_j} < 0$. Notice that such a negative effect is greater the lower the value of β, i.e. the lower the impact of per-student teaching expenditure on quality. With a low β, a large number of low-ability students decide to move away from the university, which raises the tuition fee. The location choice of high-ability students is not affected by marginal changes in b_j and R_j, because the corresponding solution is a corner one.

As to the relation between tuition fees and teaching quality in university A and B, we can state the following

Proposition 3 *In equilibrium M_1, $b_A^{M_1} > b_B^{M_1}$ and $q_A^{M_1} > q_B^{M_1}$.*

Notice that teaching quality and tuition fees are higher in the university where high-ability students are enrolled and average ability is higher.

3.2.2 Equilibrium M_2

In this specification of equilibrium M, university A is attended by both types of students, while university B is attended only by high-ability students. Formally, for all l students and part of h students to attend university A and the rest of h students to attend university B, the following conditions must be satisfied[11]

$$\begin{gathered} U^l\left(\alpha\left(e^l + \frac{\Delta n^h}{N^l + n_A^h}\right) + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N^l + n_A^h}\right)\right) - b_A > \\ U^l\left(\alpha\left(e^l + \Delta\right) + \beta\left(t_B + b_B + \frac{\tau_B - R_B}{n_B^h}\right)\right) - b_B, \end{gathered} \tag{19}$$

and

$$\begin{gathered} U^h\left(\alpha\left(e^l + \frac{\Delta n^h}{N^l + n_A^h}\right) + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N^l + n_A^h}\right)\right) - b_A = \\ U^h\left(\alpha\left(e^l + \Delta\right) + \beta\left(t_B + b_B + \frac{\tau_B - R_B}{n_B^h}\right)\right) - b_B. \end{gathered} \tag{20}$$

In order for this equilibrium to be stable, it must be the case that $\frac{\partial q_j}{\partial n_j^h} < 0$, $j = A, B$, which implies $\frac{\partial q_j}{\partial n_j^l} < 0$. This means that, at equilibrium, quality decreases with the number of students for both universities. By Lemma 1, this implies

$$\tau_A - R_A > \frac{\alpha}{\beta}\Delta N^l, \tag{21}$$

$$\tau_B - R_B > 0. \tag{22}$$

For university A, the lump-sum transfer τ_A must exceed research expenditure so as to compensate for the lower ability of part of its students. Funds in excess can thus be used to improve teaching quality. Also for university B, τ_B must exceed R_B so that per capita teaching resources diminish

[11] Again condition (19) cannot hold as an equality by the same argument of the proof of Proposition 1.

with the enrolment of students. If this were not the case, all h ability students would migrate to university B.

Again, we derive the impact of universities' decisions on the location of low-ability students, by stating the following that can be interpreted along the same lines as Lemma 2.

Lemma 3. *At equilibrium M_2, for high-ability students it is* $\frac{dn_j^h}{db_j} = \frac{1-\beta\frac{\partial U_j^h}{\partial q_j}}{J^h}$, *and* $\frac{dn_j^h}{dR_j} = \frac{\frac{\beta}{n_j}\frac{\partial U_j^h}{\partial q_j}}{J^h} < 0$, *where* $J^h \equiv \sum_{j=A,B} \frac{\partial U_j^h}{\partial q_j}\frac{\partial q_j}{\partial n_j^h} < 0$, $j = A, B$.

Finally, as to the relation between tuition fees and teaching quality in university A and B, we can state the following

Proposition 4 *In equilibrium M_2, $b_A^{M_2} < b_B^{M_2}$ and $q_A^{M_2} < q_B^{M_2}$.*

Notice that teaching quality and tuition fees are higher in the university where only high-ability students are enrolled and average ability is higher.

3.3 Equilibrium S: A specialized university system

In this equilibrium, university B is fully specialized, i.e. there are no students and only research is carried on. University A, on the contrary, produces both teaching and research. Formally, for all students to choose university A, so that university B only produces research, the following conditions must be satisfied:

$$U^i\left(\alpha\left(e^l + \frac{\Delta N^h}{N}\right) + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N}\right)\right) - b_A \geq \widehat{U}^i > 0, \qquad i = h, l,$$

where $b_B = 0$, and $\widehat{U}^i$ represents the level of utility that a student of ability i could obtain if teaching activity were started in university B, based only on his tuition payment. In order for this equilibrium to be stable, it must be the case that $\frac{\partial q_A}{\partial n_A^i} > 0$, $i = h, l$. By Lemma 1, this implies that

$$R_A - \tau_A > \frac{\alpha}{\beta}\Delta N^h.$$

Notice that for $n_B = 0$, the condition $R_B = \tau_B$ must be satisfied. Moreover, it is $\frac{\partial q_B}{\partial n_B^i} > 0$, $i = h, l$, by Lemma 1. In other words, this means that equilibrium S may arise only if i) university A's investment in research, R_A, is greater than the lump-sum transfer, τ_A, so that part of the fees are used to finance research, and ii) the effect of an increase in the number of low-ability students on university A's investment in teaching is greater than the effect on university A's average ability of students. University B only produces research, and thus the government only provides a lump-sum transfer which is entirely spent on research.

Further, at equilibrium S, the location choices of both high and low-ability students are not affected by marginal changes in universities' decisions.

4 University competition: Research expenditure and tuition fees

At the first stage of the game, each university solves its maximization problem in accordance with the type of equilibrium arising at the second stage. In particular, each university behaves à la Nash with respect to its competitor but is a Stackelberg leader with respect to students. This means that each university decides tuition fees b_j, and research expenditure R_j, taking into account the reaction of students, i.e. their subsequent location decisions. Starting from each equilibrium of the second stage, we then solve the first stage considering that the objective function (6) must incorporate the corresponding equilibrium.

4.1 Equilibrium E: An élite university system

Considering that, at equilibrium E of the second stage, the students' location decisions are such that $n_A = N^h$, $n_B = N^l$, the universities' objective functions (6) take the following form

$$W_A = N^h \alpha e^h + \beta[(t_A + b_A)N^h + (\tau_A - R_A)] + R_A^{\gamma_A},$$

and

$$W_B = N^l \alpha e^l + \beta[(t_B + b_B)N^l + (\tau_B - R_B)] + R_B^{\gamma_B}.$$

Accordingly, the first-order conditions w.r.t. R_j, $j = A, B$, are

$$\partial W_j / \partial R_j = \gamma_j R_j^{\gamma_j - 1} - \beta = 0, \qquad j = A, B. \tag{23}$$

As far as the tuition fees are concerned, we have that both universities payoffs are monotonic increasing functions of b_j, $j = A, B$. Tuition fees are then bound by the characteristics of the equilibrium itself (see section 4.1.2).

4.1.1 Optimal research expenditure

From (23), the optimal level of research expenditure, R_j^E, obtains as

$$R_j^E = \left(\frac{\beta}{\gamma_j}\right)^{\frac{1}{\gamma_j - 1}}, \qquad j = A, B, \tag{24}$$

and thus, the optimal level of research is

$$r_j^E = \left(\frac{\beta}{\gamma_j}\right)^{\frac{\gamma_j}{\gamma_j - 1}}, \qquad j = A, B.$$

The optimal level of research is given by two technological parameters β and γ_j. The first represents the impact of per-student teaching expenditure on the quality of teaching (efficacy of teaching expenditure), and the second is the coefficient transforming expenditure in effective research activity (efficacy of research expenditure). Given that $\gamma_j < 1$, r_j^E is increasing in γ_j and decreasing in β. The greater the efficacy of research expenditure, the higher the optimal level of research. The greater the efficacy of teaching, on the contrary, the lower the amount of research expenditure and consequently the level of research because the higher is its opportunity cost. Recall that it is $\tau_B > R_B$ while in university A it can be $\tau_A < R_A$, in which case tuition fees are used to finance teaching. Notice that $\frac{\partial R_j^E}{\partial \tau_j} = 0$, and $\frac{\partial R_j^E}{\partial t_j} = 0$, i.e. expenditure on research is independent of the lump-sum transfer by the central government as well as of the per-student transfer. Marginal changes in τ_j and t_j only affect the quality of teaching.

4.1.2 Optimal per-student tuition fee

On the basis of Proposition 2, we distinguish equilibrium E_1, where $b_A^{E_1} = b_B^{E_1}$, and equilibrium E_2, where $b_A^{E_2} > b_B^{E_2}$.

Equilibrium E_1. Given that the payoff of the universities is monotonically increasing in b_j, each university chooses the highest possible value of $b_j^{E_1}$, $j = A, B$, compatible with this equilibrium. Such values result from the solution to the system of equations (12) and (13), when they hold as equalities. Proposition 2 shows that the quality of teaching is the same in both universities and that the same tuition fee is charged. Low-ability students, even if segregated, are not penalized in terms of quality of teaching and pay exactly the same as high-ability ones.

The next Corollary to Proposition 2 shows that the government must give relatively higher per-capita transfers to university B, where the difference in the transfers between B and A is given by the amount compensating for the lower ability of university B's students.

Corollary 1: *For equilibrium E_1 to exist, $t_A^{E_1} + \frac{\tau_A^{E_1} - R_A^{E_1}}{N^h} = t_B^{E_1} + \varepsilon^{E_1}$, where $\varepsilon^{E_1} \equiv \frac{\tau_B^{E_1} - R_B^{E_1}}{N^l} - \frac{\alpha}{\beta}\Delta >$ 0.*

Corollary 1 implies that the transfer to university B, net of the compensation for the lower ability of its students, must be the same as the transfer to university A. If $t_B^{E_1} = t_A^{E_1}$, the per-capita lump-sum transfer net of research cost to university A must be equal to the excess of the net per-capita lump-sum transfer over ability compensation to university B.

In equilibrium E_1, where the values of teaching quality are the same, Proposition 2 implies that $b_B^{E_1} = b_A^{E_1} = b^{E_1}$, but it does not impose any constraint on the level of the fee. As a consequence, considering that the payoff of each university is monotonically increasing in b_j and considering that for any q_j, $U^h(q_j) > U^l(q_j)$ by assumption, the value of b^{E_1} is found from the solution to

$$U^l\left(\alpha e^l + \beta\left(t_B^{E_1} + b^{E_1} + \frac{\tau_B^{E_1} - R_B^{E_1}}{N^l}\right)\right) - b^{E_1} = 0. \tag{25}$$

Thus, low-ability students are kept at their reservation level of utility while high-ability students enjoy higher utility because $U^h(q_j) > U^l(q_j)$. For university B, equation (25) shows that $t_B^{E_1}$ and b^{E_1} are complements. A higher level of $t_B^{E_1}$ (and the consequent increase in $t_A^{E_1}$ implied by Corollary 1) in fact enables the universities to raise b^{E_1} and, consequently, to further raise teaching quality.

Notice that for equilibrium E_1 to exist, public transfers must be sufficiently high. If this is not the case, the level of q^{E_1} that solves (25) would be such that $\left.\frac{\partial U_j^h}{\partial q_j}\right|_{q_A} > \frac{1}{\beta}$, i.e. for high-ability students the marginal utility from an increase in b_A would be higher than the marginal cost. As a consequence, university A could raise the tuition fee and hence its teaching quality.

Equilibrium E_2. For this equilibrium, Proposition 2 shows that the quality of teaching and the tuition fee in university A are higher than in university B. The level of $b_B^{E_2}$ is found from

$$U^l\left(\alpha e^l+\beta\left(t_B^{E_2}+b_B^{E_2}+\frac{\tau_B^{E_2}-R_B^{E_2}}{N^l}\right)\right)-b_B^{E_2}=0, \tag{26}$$

while the level of $b_A^{E_2}$ is found from

$$U^h\left(q_A^{E_2}\right)-b_A^{E_2}=U^h\left(q_B^{E_2}\right)-b_B^{E_2}. \tag{27}$$

Given (27), in order to be stable, equilibrium E_2 must satisfy $\frac{\partial q_B}{\partial n_B^h}<0$ which implies $\frac{\partial q_B}{\partial n_B^l}<0$. By Lemma 1, we then have the same stability condition as in equilibrium E_1 (see (14)). Again, university B is compensated for the lower quality of its students. Notice that, contrary to what happens in equilibrium E_1, now local stability does not impose any restriction on $\frac{\partial q_B}{\partial n_A^l}$.

Considering (27), the following Corollary to Proposition 2 holds

Corollary 2: *For equilibrium* E_2 *to exist,* $t_A^{E_2}+b_A^{E_2}+\frac{\tau_A^{E_2}-R_A^{E_2}}{N^h}>t_B^{E_2}+b_B^{E_2}+\varepsilon^{E_2}$, *where* $\varepsilon^{E_2}\equiv\frac{\tau_B^{E_2}-R_B^{E_2}}{N^l}-\frac{\alpha}{\beta}\Delta>0$.

4.2 Equilibrium M: A mixed university system

In this equilibrium university A is attended by both types of students, while university B is attended only by low-ability students in equilibrium M_1, and only by high-ability students in equilibrium M_2. Denoting by i the type of students that attend both universities, so that $i=l$ in equilibrium M_1 and $i=h$ in equilibrium M_2, we then have that $n_A=n_A^i+N^{-i}$ and $n_B=n_B^i$, and we can write university j's maximization problem as follows

$$\begin{aligned}
\max_{b_j,R_j}\quad & W_j=n_jq_j+r_j\\
s.t.\quad & q_j=\alpha\bar{e}_j+\beta\frac{T_j}{n_j},\\
& r_j=R_j^{\gamma_j},\\
& (t_j+b_j)n_j+\tau_j=T_j+R_j,\\
& b_j\geqslant 0,\qquad j=A,B.
\end{aligned}$$

Considering that $\frac{\partial n_j^h}{\partial R_j}=\frac{\partial n_j^h}{\partial b_j}=0$ in equilibrium M_1 and $\frac{\partial n_j^l}{\partial R_j}=\frac{\partial n_j^l}{\partial b_j}=0$ in equilibrium M_2, the first-order conditions[12] for an interior solution are

$$R_j:\quad \alpha\frac{\partial n_j^i}{\partial R_j}e^i+\beta\left[(t_j+b_j)\frac{\partial n_j^i}{\partial R_j}-1\right]+\gamma_jR_j^{\gamma_j-1}=0, \tag{28}$$

[12] We consider parameter values such that these conditions are also sufficient for a maximum. Notice that Lemma 2 and 3 guarantee that students' reactions go in the right direction.

and

$$b_j: \quad \alpha \frac{\partial n_j^i}{\partial b_j} e^i + \beta \left[(t_j + b_j) \frac{\partial n_j^i}{\partial b_j} + n_j \right] = 0. \tag{29}$$

Notice that (29) implies that $\frac{\partial n_j^i}{\partial b_j} < 0$ (see Lemma 2 and 3).

4.2.1 Optimal research expenditure

Substituting (29), the solution of (28) gives the optimal level of research expenditure $R_j^{M_k}$, $j = A, B$, $k = 1, 2$

$$R_j^{M_k} = \left[\frac{\beta}{\gamma_j} \left(1 + \Omega_j^{M_k} \right) \right]^{\frac{1}{\gamma_j - 1}}, \tag{30}$$

and thus the optimal level of research obtains as

$$r_j^{M_k} = \left[\frac{\beta}{\gamma_j} \left(1 + \Omega_j^{M_k} \right) \right]^{\frac{\gamma_j}{\gamma_j - 1}},$$

where

$$\Omega_j^{M_k} \equiv - \frac{\frac{\partial n_j^i}{\partial R_j}}{D_j^{M_k}} > 0, \;\; i = l \;\; when \;\; k = 1 \; ; \; i = h \;\; when \;\; k = 2, \tag{31}$$

and

$$D_j^{M_k} \equiv - \frac{\frac{\partial n_j^i}{\partial b_j}}{n_j} > 0, \;\; i = l \;\; when \;\; k = 1 \; ; \; i = h \;\; when \;\; k = 2.$$

$D_j^{M_k}$ is positive because $\frac{dn_j^l}{db_j} < 0$. Thus, considering that $\frac{\partial n_j^i}{\partial R_j} < 0$ by Lemma 2 and 3, it follows that $\Omega_j^{M_k}$ is positive too. Notice that $D_j^{M_k}$ is an index of tuition fee competition, because it measures the semi-elasticity of students with respect to the fee, i.e. the percentage of student outflight due to an increase in the fee. Further, $\Omega_j^{M_k}$ is an index of the student outflight due to an increase in expenditure on research, relatively to the index of tuition fee competition $D_j^{M_k}$. If university j increases its expenditure in research, students tend to leave because, everything else being equal, expenditure in teaching is reduced.

While in equilibrium E_1 and E_2, r_j^E was determined by technological parameters, now $r_j^{M_k}$ results from the product of a 'technological factor' $\left(\frac{\beta}{\gamma_j}\right)^{\frac{\gamma_j}{\gamma_j - 1}}$ and a 'students' response factor' $\left(1 + \Omega_j^{M_k}\right)^{\frac{\gamma_j}{\gamma_j - 1}}$. When $\Omega_j^{M_k}$ is low, $r_j^{M_k}$ tends to be determined only by technological parameters as in equilibrium E_1 and E_2. When $\Omega_j^{M_k}$ increases, $r_j^{M_k}$ decreases. Observe that, given $\left(1 + \Omega_j^{M_k}\right)^{\frac{\gamma_j}{\gamma_j - 1}} < 1$, research is lower in equilibrium M than in equilibrium E_1 and E_2, i.e. $r_j^E > r_j^{M_k}$, $k = 1, 2$.

As far as the relation between $R_A^{M_k}$ and $R_B^{M_k}$ is concerned, notice that the relation between $\Omega_A^{M_k}$and $\Omega_B^{M_k}$ in (30) depends on the relative quality of teaching. Consider equilibrium M_1. By substituting $\frac{dn_j^l}{db_j}$ and $\frac{dn_j^l}{dR_j}$ from Lemma 2 into (31), $\Omega_j^{M_1}$ can be re-written as

$$\Omega_j^{M_1} = \frac{\frac{dU_j^l}{dq_j}}{1/\beta - \frac{dU_j^l}{dq_j}}, \tag{32}$$

with $\frac{dU_j^l}{dq_j} < 1/\beta$ because we know that $\frac{dn_j^l}{db_j} < 0$. $U(.)$ is concave, if $q_A^{M_1} \gtreqless q_B^{M_1}$, then $\Omega_A^{M_1} \lesseqgtr \Omega_B^{M_1}$, and thus $R_A^{M_1} \gtreqless R_B^{M_1}$, unless γ_A is much lower than γ_B. Exactly the same argument can be applied to equilibrium M_2 by substituting h for l.

4.2.2 Optimal per-student tuition fee

The optimal level of the tuition fee is obtained by solving (29), for $i = l$ when $k = 1$; $i = h$ when $k = 2$,

$$b_A^{M_k} = -\frac{\alpha}{\beta}e^i - \frac{N^{-i} + n_A^i}{\frac{\partial n_A^i}{\partial b_A}} - t_A = -\frac{\alpha}{\beta}e^i + \frac{1}{D_A^{M_k}} - t_A, \tag{33}$$

where $D_A^{M_k} = -\frac{\frac{\partial n_A^i}{\partial b_A}}{N^{-i}+n_A^i}$, and

$$b_B^{M_k} = -\frac{\alpha}{\beta}e^i - \frac{n_B^i}{\frac{\partial n_B^i}{\partial b_B}} - t_B = -\frac{\alpha}{\beta}e^i + \frac{1}{D_B^{M_k}} - t_B, \tag{34}$$

where $D_B^{M_k} = -\frac{\frac{\partial n_B^i}{\partial b_B}}{n_B^i}$. Therefore $b_j^{M_k}$, $j = A, B$, $k = 1, 2$, decreases with α/β, e^i, and $D_j^{M_k}$.

Consider equilibrium M_1. In university A, the level of the tuition fee is not so high as to discourage too many low-ability students from enrolling; in university B, it is high enough to avoid being attended by all low-ability students (which would be the case covered by equilibrium E). Recall that in this equilibrium $\tau_B^{M_1} > R_B^{M_1}$. Thus, in university B, part of the lump-sum transfer is devoted to finance teaching and this helps raise teaching quality. Given that university B has no high-ability students, its quality would otherwise be too low. Such a positive effect on quality of the sum $\tau_B^{M_1} - R_B^{M_1}$, however, increases as the number of students diminishes. For university A, instead, $\tau_A^{M_1}$ can be either lower or higher than $R_A^{M_1}$. If it is lower, students contribute to financing both teaching and research.

A similar comment applies to equilibrium M_2. Now the level of $b_A^{M_2}$ must not be so high as to discourage any low-ability student to enrol in university A, and $b_B^{M_2}$ must be high enough so as not to attract too many students. In university A, $\tau_A^{M_2} > R_A^{M_2} + \frac{\alpha}{\beta}\Delta N^l$ so as to compensate for the presence of all low-ability students, and in university B the lump-sum transfer must exceed research expenditure. The relation between the public transfers to the two universities is determined in the following corollary to Proposition 4.

Corollary 3: *For equilibrium M_2 to exist,* $\dfrac{\tau_B^{M_2} - R_B^{M_2}}{n_B^h} > t_A^{M_2} - t_B^{M_2} + \varepsilon^{M_2}$*, where* $t_A^{M_2} - t_B^{M_2} > 0$ *and* $\varepsilon^{M_2} \equiv \frac{\tau_A^{M_2} - R_A^{M_2} - \frac{\alpha}{\beta}\Delta N^l}{N^l + n_A^h} > 0$.

Finally, notice that in equilibrium M, given (33) and (34), the tuition fee and the per-student transfer can be substitute,[13] contrary to what happens in equilibrium E. Now the tuition fee has an opportunity cost for university j, because of students' response. In equilibrium E, such opportunity cost does not exist as university j does not gain anything by marginally reducing $b_j^{E_k}$ (the derivative of the university objective function w.r.t. $b_j^{E_k}$ is always positive). In equilibrium M, instead, university j directly gains by marginally reducing $b_j^{M_k}$ because it can attract students.

4.3 Equilibrium S: A specialized university system

Considering that, at equilibrium S of the second stage, $n_A = N$ and $n_B = 0$, the universities' objective functions are

$$W_A = N\alpha(\frac{N^h}{N}\Delta + e^l) + \beta[(t_A + b_A)N + (\tau_A - R_A)] + R_A^{\gamma_A},$$

and

$$W_B = R_B^{\gamma_B}.$$

Accordingly for university A, the f.o.c. w.r.t research expenditure is

$$\partial W_A/\partial R_A = \gamma_A R_A^{\gamma_A - 1} - \beta = 0, \tag{35}$$

while w.r.t the tuition fee b_A, the pay-off is monotonically increasing. For university B, we obviously have that the pay-off is increasing in research expenditure.

4.3.1 Optimal research expenditure

From (35), the optimal level of research expenditure for university A, R_A^S, obtains as

$$R_A^S = \left(\frac{\beta}{\gamma_A}\right)^{\frac{1}{\gamma_A - 1}}, \tag{36}$$

and the optimal level of research obtains as

$$r_A^S = \left(\frac{\beta}{\gamma_A}\right)^{\frac{\gamma_A}{\gamma_A - 1}}.$$

For university B, the optimal level of research expenditure, R_B^S, is simply

$$R_B^S = \tau_B \leq \left(\frac{\beta}{\gamma_B}\right)^{\frac{1}{\gamma_B - 1}}, \tag{37}$$

and thus the optimal level of research obtains as

$$r_B^S = \tau_B^{\gamma_B}.$$

[13] They are substitute unless the semi-elasticity of low ability students w.r.t. the fee decreases so much with the per-student transfer as to counterbalance the direct effect of t_j.

Notice that the level of the lump-sum transfer must not exceed the efficient level $\left(\frac{\beta}{\gamma_B}\right)^{\frac{1}{\gamma_B-1}}$ otherwise university B would have an incentive to use part of the lump-sum transfer to start teaching activity.

In university A, where all the students are, the level of expenditure in research is the same as that in both specifications of equilibrium E. Again, R_A^S depends only on technological parameters, and thus it is independent of the public lump-sum transfer, i.e. $\frac{\partial R_A^S}{\partial \tau_A} = 0$. Now however $\tau_A < R_A$ and students fees are partly used to finance research. In university B, only research is carried on, and expenditure just equals the public lump-sum transfer, i.e. $\frac{\partial R_B^S}{\partial \tau_B} = 1$.

4.3.2 Optimal per-student tuition fee

At equilibrium S, the government does not finance teaching at university B, and consequently $t_B = 0$. In order that university B does not find it profitable to start teaching activity financed only by tuition fees, the tuition fee of university A must not be too high

$$b_A^S \leq \min[\widehat{b}_{A,}^l \widehat{b}_A^h],$$

where $\widehat{b}_A^i$, $i = h, l$, is the solution to

$$U^i\left(q_A^i\right) - b_A^i = V^i, \tag{38}$$

and

$$V^i \equiv \max_{b_B^i}[U^i\left(q_B^i\right) - b_B^i],$$

with $q_B^i = \alpha e^i + \beta b_B^i$.

Notice that for (38) to have a solution, public transfers to university A must be sufficiently high, i.e.

$$t_A^S + \frac{\tau_A^S - R_A^S}{N} > \frac{\alpha}{\beta}\Delta\frac{N^l}{N},$$

otherwise the RHS of (38) is always higher than the LHS.

Given that $V^i > 0$, in this equilibrium both types of students obtain a positive level of utility. Now

$$U^h\left(q_A\right) - b_A^S > U^l\left(q_A\right) - b_A^S > 0, \tag{39}$$

i.e. high-ability students enjoy a higher level of utility than low-ability ones.

As in equilibrium E, t_A and b_A^S are complements, being the tuition fee with no opportunity cost. A higher level of t_A in fact enables university A to raise b_A^S and, consequently, to raise teaching quality.

5 A social comparison among equilibria

In order to compare the three equilibria from a social point of view, we suppose that the government aims to obtain a high level of both total research and teaching quality, subject to an efficient use of financial resources. As we adopt a partial equilibrium approach taking into account only students' utility, we do not consider a welfarist objective function for the government. In other words, we consider research and teaching quality as objectives *per se*, although their provision is constrained by budget concerns. Both research and teaching quality could in fact be considered as instruments for human capital accumulation and then for growth. In the following, we make pairwise comparisons between equilibria, and then we show that equilibrium E_2 is the most efficient.

Equilibrium E_2 vs. equilibrium E_1. Recall that in equilibrium E, university A is an élite institution attended only by high-ability students, and university B is only attended by low-ability students. Notice that the level of research is the same in both specifications of this equilibrium which then differs only as to teaching quality. The level of the public transfers is crucial in determining the specification that is achieved. In order to have the same teaching quality, in equilibrium E_1 the government must compensate university B for the lower quality of its students and then give the same amount of resources to both universities. In equilibrium E_2 a higher teaching quality in university A can be obtained by transferring more funds to it (after compensating university B for the lower quality of its students through the lump-sum transfer). Recall that for equilibrium E_1 to exist, public transfers must be high enough, otherwise only equilibrium E_2 may obtain. If this is the case, we can prove the following

Proposition 5 *For a given level of public expenditure, equilibrium E_2 allows a higher average teaching quality than equilibrium E_1.*

Thus we may say that equilibrium E_2 is more efficient than equilibrium E_1.

Equilibrium E_2 vs. equilibrium M_2. Let us now compare equilibrium E_2 to equilibrium M_2, recalling that in the latter university A is attended by both types of students while university B is only attended by high-ability students. We know that in equilibrium E research is at its technically efficient level and is higher than in equilibrium M. Given that also research expenditure is then higher in equilibrium E, we compare the two equilibria in terms of equal levels of extra research resources, i.e. total public transfers net of expenditure devoted to research activity. In this respect, we state the following

Proposition 6 *For equal levels of extra-research resources, average teaching quality is higher in equilibrium E_2 than in equilibrium M_2.*

Proposition 6 means that at the same teaching cost, teaching quality is higher in the segregated state university system of equilibrium E_2 than in the mixed state university system of equilibrium M_2.

Equilibrium E_1 vs. equilibrium M_1. We may then compare equilibrium E_1 to equilibrium M_1, recalling that in the latter university A is attended by both types of students while university B is only attended by low-ability students.

Proposition 7 *For equal levels of extra-research resources, average teaching quality is higher in equilibrium E_1 than in equilibrium M_1.*

Proposition 7 means that, at the same teaching cost, teaching quality is higher in the élite university system of equilibrium E_1 than in the mixed university system of equilibrium M_1. Considering that equilibrium E_1 is dominated by equilibrium E_2, we may say that equilibrium E_2 dominates equilibrium M_1.

Equilibrium E_1 vs. equilibrium S. Let us finally compare equilibrium E_1 and S. In the latter, university A supplies both research and teaching, and is attended by all students, while university B is only a research institution.

Proposition 8 *For any given level of teaching quality and research, public expenditure is lower in equilibrium E_1 than in equilibrium S.*

In terms of resource allocation, this proposition implies that equilibrium E_1 is more efficient than equilibrium S. Since equilibrium E_1 is in turn dominated by equilibrium E_2, the government should choose the structure of grants corresponding to equilibrium E_2.

According to our propositions, we may conclude that equilibrium E_2 is more efficient than all the other equilibria. The question arises whether the government can effectively implement equilibrium E_2 by choosing appropriate public transfers. The following proposition provides sufficient conditions on the lump-sum and the per-student transfers that guarantee that E_2 and not another equilibrium will be selected. Let us define $R_{j\min}^{M_k}$ as the minimum value that can be taken by $R_j^{M_k}$ in equilibrium M_k, $j = A, B$, $k = 1, 2$. We then show

Proposition 9 *Sufficient conditions on the public transfers for equilibrium E_2 to be selected by the universities are:*

$$\tau_B > R_B^E + \frac{\alpha}{\beta}\Delta N^l, \tag{40}$$

$$\tau_A < \min\left[R_{A\min}^{M_1} - \frac{\alpha}{\beta}\Delta N^h, R_{B\min}^{M_2}\right], \tag{41}$$

$$t_A + b_A^{E_2}(t_A) + \frac{\tau_A - R_A^E}{N^h} > t_B + b_B^{E_2}(t_B) + \varepsilon, \textit{ where } \varepsilon \equiv \frac{\tau_B - R_B^E}{N^l} - \frac{\alpha}{\beta}\Delta > 0. \tag{42}$$

Conditions (40)-(42) are not more restrictive than the necessary conditions for equilibrium E_2 to exist and be stable (see Corollary 2 and subsequent discussion). In other words Proposition 9 does not impose additional conditions on the total amount of public transfers but simply points out how to shape them in order to avoid multiple equilibria.

Finally notice that, if the government has an equity concern about the average teaching quality of the university system, equilibrium E_1 could be preferred.[14] Moreover, if the government should

[14] To implement equilibrium E_1 instead of equilibrium E_2 the government should give per student transfer that are sufficiently high and satisfy the condition in Corollary 1. Moreover for E_1 to be stable the lump sum transfers must satisfy (14).

decide to devote very low resources to the university sector, there could be not enough funds to finance the solution designed by equilibria E, and equilibrium M_1 might be preferred.[15]

6 Concluding remarks

We have analysed the impact of student mobility on the characteristics of two competing state universities. Assuming there are two types of students ('high-ability' and 'low-ability'), the composition of the population of students impacts on the quality of teaching ('peer effect'). The latter is an argument of the individual utility function as well as of the universities' objective functions. The level of research (which is linked to research expenditure by efficiency parameters) is the other argument of the universities' objective functions. Each university decides the level of its tuition fees and of its research expenditure. The government contributes to financing the universities with a lump-sum transfer and a matching grant per-student. The aim of the government is to promote a high level of research and teaching quality by making an efficient use of financial resources.

By selecting locally stable equilibria, the analysis has ruled out some institutional settings in favour of some others. One of the main results is that there cannot exist a stable equilibrium where both high- and low-ability students divide between different universities. We have then three types of equilibria. In equilibrium E, an élite institution is created with only high-ability students while low-ability students are segregated in a different institution. In equilibrium M, all students of one type and part of the students of the other type attend one university while the rest attend the other university. In equilibrium S, all students are concentrated in one university, while the other institution becomes a research center.

Equilibrium E stands out as the most efficient. When compared to equilibrium M, equilibrium E allows the attainment of a higher teaching quality at the same public extra research cost. In equilibrium E, the level of research expenditure is at its efficient level being entirely explained by the technological parameters of the research production function. Thus, research productivity is crucial in defining the level of public expenditure. When compared to equilibrium S, equilibrium E allows the same teaching quality and research level at a lower public cost.

Concerning the debate on the appropriate form for the universities' objective function, one could think that universities maximize average teaching quality instead of total teaching quality in addition to research as in the present paper. We have checked that equilibrium E is robust to such a change and therefore our results still hold.

[15] Equilibrium M_1 requires lower transfers than equilibrium E_1 and E_2. Recall in fact that local stability conditions impose $\tau_B > R_B^E + \frac{\alpha}{\beta}\Delta N^l$ for both specifications of equilibrium E and $\tau_A > R_A^E - \frac{\alpha}{\beta}\Delta N^h$ for equilibrium E_1. For equilibrium M_1 instead, $\tau_B > R_B^{M_1}$, and $\tau_A > R_A^{M_1} - \frac{\alpha}{\beta}\Delta N^h$ where $R_j^E > R_j^{M_1}$, $j = A, B$. Moreover in equilibrium E_1 and E_2 the per-student transfers are constrained by the conditions in Corollary 1 and 2, respectively, while in equilibrium M_1 there are no constraints and thus could be equal to zero. Notice that, as regards equilibrium E_2, the condition in Corollary 2 counterbalances the fact that $\tau_A^{E_2}$ can be very low.

7 Appendix

Proof of Lemma 1. It follows directly by signing (10) and (11).

Proof of Proposition 1. Suppose, contrary to proposition 1, that there exists an equilibrium where students of both types l and h attend both universities A and B. The following arbitrage condition should then be satisfied for $i = h, l$:

$$U^i\left(\alpha\left(e^l + \tfrac{\Delta n_A^h}{n_A^h + n_A^l}\right) + \beta\left(t_A + b_A + \tfrac{\tau_A - R_A}{n_A^h + n_A^l}\right)\right) - b_A =$$

$$= U^i\left(\alpha\left(e^l + \tfrac{\Delta n_B^h}{n_B^h + n_B^l}\right) + \beta\left(t_B + b_B + \tfrac{\tau_B - R_B}{n_B^h + n_B^l}\right)\right) - b_B.$$

But these equations cannot be simultaneously satisfied for $i = h, l$ because of the assumption that $\frac{\partial U^h}{\partial q_j} > \frac{\partial U^l}{\partial q_j}$.□

Proof of Propositon 2. Let us rewrite conditions (12) and (13) as follows

$$U^h\left(\alpha e^h + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N^h}\right)\right) - U^h\left(\alpha e^l + \beta\left(t_B + b_B + \frac{\tau_B - R_B}{N^l}\right)\right) \geq b_A^E - b_B^E,$$

$$U^l\left(\alpha e^h + \beta\left(t_A + b_A + \frac{\tau_A - R_A}{N^h}\right)\right) - U^l\left(\alpha e^l + \beta\left(t_B + b_B + \frac{\tau_B - R_B}{N^l}\right)\right) \leq b_A^E - b_B^E.$$

Considering that $\frac{dU^h}{dq_j} > \frac{dU^l}{dq_j}$ the proposition is immediately proved.□

Proof of Lemma 2. By totally differentiating (7), the following equation obtains

$$\begin{aligned} &\tfrac{dU_A^i}{dq_A}\textstyle\sum_{i=h,l}\tfrac{\partial q_A}{\partial n_A^i}dn_A^i + \tfrac{dU_A^i}{dq_A}\tfrac{\partial q_A}{\partial R_A}dR_A + \tfrac{dU_A^i}{dq_A}\tfrac{\partial q_A}{\partial b_A}db_A - db_A + \\ &-\tfrac{dU_B^i}{dq_B}\textstyle\sum_{i=h,l}\tfrac{\partial q_B}{\partial n_B^i}dn_B^i - \tfrac{dU_B^i}{dq_B}\tfrac{\partial q_B}{\partial R_B}dR_B - \tfrac{dU_B^i}{dq_B}\tfrac{\partial q_B}{\partial b_B}db_B + db_B = 0. \end{aligned} \tag{43}$$

By using the market clearing condition for low-ability students, $dn_B^l = -dn_A^l$ and $dn_B^h = dn_A^h = 0$ into (43), it follows that

$$\frac{dn_j^l}{db_j} = \frac{1 - \beta\frac{\partial U_j^l}{\partial q_j}}{J^l}, \qquad j = A, B, \tag{44}$$

$$\frac{dn_j^l}{dR_j} = \frac{\frac{\beta}{n_j}\frac{\partial U_j^l}{\partial q_j}}{J^l}, \qquad j = A, B, \tag{45}$$

where

$$J^l = \sum_{j=A,B}\frac{\partial U_j^l}{\partial q_j}\frac{\partial q_j}{\partial n_j^l}, \qquad j = A, B. \tag{46}$$

Given that $\frac{\partial q_j}{\partial n_j^l} < 0$ for equilibrium M_1 to be stable, $J^l < 0$ in (46) because $\frac{\partial U_j^l}{\partial q_j} > 0$ by assumption. Then $\frac{dn_j^l}{dR_j} < 0$ follows immediately from (45). Moreover, from (44), it follows that $\frac{\partial U_j^l}{\partial q_j} \gtreqless \frac{1}{\beta} \iff \frac{dn_j^l}{db_j} \gtreqless 0$.□

Proof of Proposition 3. (i) It cannot be $b_A^{M_1} = b_B^{M_1}$, because this implies $q_A^{M_1} > q_B^{M_1}$ from (15), but $q_A^{M_1} = q_B^{M_1}$ from (16). (ii) It cannot be $b_A^{M_1} < b_B^{M_1}$, because this implies that

$$b_B^{M_1} - b_A^{M_1} =$$

$$= U^l \left(\alpha e^l + \beta \left(t_B^{M_1} + b_B^{M_1} + \frac{\tau_B^{M_1} - R_B^{M_1}}{n_B^l} \right) \right) - U^l \left(\alpha \left(e^l + \frac{\Delta N^h}{N^h + n_A^l} \right) + \beta \left(t_A^{M_1} + b_A^{M_1} + \frac{\tau_A^{M_1} - R_A^{M_1}}{N^h + n_A^l} \right) \right) >$$

$$U^h \left(\alpha e^l + \beta \left(t_B^{M_1} + b_B^{M_1} + \frac{\tau_B^{M_1} - R_B^{M_1}}{n_B^l} \right) \right) - U^h \left(\alpha \left(e^l + \frac{\Delta N^h}{N^h + n_A^l} \right) + \beta \left(t_A^{M_1} + b_A^{M_1} + \frac{\tau_A^{M_1} - R_A^{M_1}}{N^h + n_A^l} \right) \right) > 0$$

from (15) and (16). But the inequalities cannot be satisfied because of the assumption $\frac{dU^h}{dq_j} > \frac{dU^l}{dq_j}$.□

Proof of Lemma 3. The proof exactly follows that of Lemma 2 with superscripts h in place of l.□

Proof of Proposition 4. The proof follows the same argument of Proposition 3.□

Proof of Corollary 1. $\varepsilon^{E_1} \equiv \frac{\tau_B^{E_1} - R_B^{E_1}}{N^l} - \frac{\alpha}{\beta}\Delta$ is strictly positive because of stability condition (14). The rest of the Corollary follows immediately from $q_A^{E_1} = q_B^{E_1}$, i.e. $\alpha \left(e^l + \Delta \right) + \beta \left(t_A^{E_1} + b_A^{E_1} + \frac{\tau_A^{E_1} - R_A^{E_1}}{N^h} \right) = \alpha e^l + \beta \left(t_B^{E_1} + b_B^{E_1} + \frac{\tau_B^{E_1} - R_B^{E_1}}{N^l} \right)$ considering that in this equilibrium $b_A^{E_1} = b_B^{E_1}$.□

Proof of Corollary 2. $\varepsilon^{E_2} \equiv \frac{\tau_B^{E_2} - R_B^{E_2}}{N^l} - \frac{\alpha}{\beta}\Delta$ is strictly positive because of stability condition (14). The rest of the Corollary follows immediately from $q_A^{E_2} > q_B^{E_2}$, i.e. $\alpha \left(e^l + \Delta \right) + \beta \left(t_A^{E_2} + b_A^{E_2} + \frac{\tau_A^{E_2} - R_A^{E_2}}{N^h} \right) > \alpha e^l + \beta \left(t_B^{E_2} + b_B^{E_2} + \frac{\tau_B^{E_2} - R_B^{E_2}}{N^l} \right)$.□

Proof of Corollary 3. $\varepsilon^{M_2} \equiv \frac{\tau_A^{M_2} - R_A^{M_2} - \frac{\alpha}{\beta}\Delta N^l}{N^l + n_A^h}$ is strictly positive because of stability condition (21). To prove that $t_A^{M_2} > t_B^{M_2}$ consider that the f.o.c. (29) implies

$$t_A^{M_2} + b_A^{M_2} - \frac{n_A^h + N^l}{\left| \frac{\partial n_A^h}{\partial b_A} \right|} = t_B^{M_2} + b_B^{M_2} - \frac{n_B^h}{\left| \frac{\partial n_B^h}{\partial b_B} \right|}.$$

Notice that $\left| \frac{\partial n_A^h}{\partial b_A} \right| = \left| \frac{1 - \beta \frac{\partial U_A^h}{\partial q_A}}{J^h} \right|$ is lower than $\left| \frac{\partial n_B^h}{\partial b_B} \right| = \left| \frac{1 - \beta \frac{\partial U_B^h}{\partial q_B}}{J^h} \right|$ because $q_B^{M_2} > q_A^{M_2}$ (see Proposition 4) and $\frac{\partial U_j^h}{\partial q_j} < 1/\beta$ is decreasing. Since $b_B^{M_2} > b_A^{M_2}$ (see Proposition 4) and $n_A^h + N^l > n_B^h$, then $t_A^{M_2} > t_B^{M_2}$, and the Corollary follows from $q_B^{M_2} > q_A^{M_2}$, i.e.

$$\alpha \left(e^l + \Delta \right) + \beta \left(t_B^{M_2} + b_B^{M_2} + \frac{\tau_B^{M_2} - R_B^{M_2}}{n_B^h} \right) >$$

$$\alpha \left(e^l + \Delta \frac{n_A^h}{N^l + n_A^h} \right) + \beta \left(t_A^{M_2} + b_A^{M_2} + \frac{\tau_A^{M_2} - R_A^{M_2}}{N^l + n_A^h} \right) =$$

$$\alpha \left(e^l + \Delta \right) + \beta \left(t_A^{M_2} + b_A^{M_2} + \varepsilon^{M_2} \right).□$$

Proof of Proposition 5.

Starting from a given equilibrium E_1, we want to prove that, by appropriately redistributing public resources, an equilibrium E_2 can be generated that has a higher average teaching quality than that of the equilibrium E_1. Notice that in both equilibria E_2 and E_1, it must be $\tau_B - R_B > \frac{\alpha}{\beta}\Delta N^l$ while in equilibrium E_2 there are no restrictions on $\tau_A - R_A$. Given q^{E_1}, the government can then decrease t_B by an amount $\sigma > 0$, and correspondingly increase t_A by $\sigma\frac{N^l}{N^h}$, where $\frac{N^l}{N^h} \geq 1$. If the level of the tuition fee did not change, i.e. for $b_A^{E_2} = b_B^{E_2} = b^{E_1}$, total quality would not change, then $q^{E_1}N = q_A^{E_2}N^h + q_B^{E_2}N^l$. However, the decrease in t_B implies a decrease in b_B, as (26) must be satisfied and $\frac{dU^l}{dq}|_{q=q^{E_1}} < 1/\beta$. Notice that (26) also implies that the decrease in b_B, Δb_B, must satisfy

$$\frac{dU^l}{dq}|_{q=q^{E_1}}\beta(\sigma + \Delta b_B) = \Delta b_B. \tag{47}$$

Correspondingly we have that b_A increases so as to satisfy

$$\frac{dU^h}{dq}|_{q=q^{E_1}}\beta(\sigma\frac{N^l}{N^h} + \Delta b_A) = \Delta b_A + \mu(\sigma), \tag{48}$$

where the term $\mu(\sigma)$ is positive and increasing because in equilibrium E_2, from (27), b_A and q_A must satisfy

$$U^h\left(q_A^{E_2}\right) - b_A^{E_2} = U^h\left(q_B^{E_2}\right) - b_B^{E_2} > U^h\left(q^{E_1}\right) - b^{E_1},$$

and in equilibrium E_1 $\frac{dU^h}{dq}|_{q=q^{E_1}} < 1/\beta$ must be satisfied. Moreover, $\mu(\sigma) \to 0$ as $\sigma \to 0$, i.e. as $q_B^{E_2} \to q^{E_1}$. Rewrite (47) and (48) as

$$\begin{aligned}
\Delta b_B &= \frac{\frac{dU^l}{dq}|_{q=q^{E_1}}\beta\sigma}{1 - \frac{dU^l}{dq}|_{q=q^{E_1}}\beta}, \\
\Delta b_A &= \frac{\frac{dUh}{dq}|_{q=q^{E_1}}\beta\sigma\frac{N^l}{N^h} - \mu(\sigma)}{1 - \frac{dU^h}{dq}|_{q=q^{E_1}}\beta}.
\end{aligned}$$

Considering that $\frac{dU^h}{dq_j} > \frac{dU^l}{dq_j}$, $\frac{N^l}{N^h} \geq 1$ and $\mu(\sigma)$ increasing, the government can find a level of σ such that $\Delta b_A N^h > \Delta b_B N^l$ implying that the increase in q_A more than compensates for the decrease in q_B making the average teaching quality increase. $\square$

Proof of Proposition 6.

Let us consider an equilibrium M_2. From stability condition (21) and Corollary 3, public transfers are

$$\begin{aligned}
\tau_A^{M_2} - R_A^{M_2} &= \frac{\alpha}{\beta}\Delta N^l + \varepsilon^{M_2}(N^l + n_A^h), \\
\tau_B^{M_2} - R_B^{M_2} &= \left(t_A^{M_2} - t_B^{M_2} + \varepsilon^{M_2} + \delta^{M_2}\right) n_B^h,
\end{aligned} \tag{49}$$

where $t_A^{M_2} - t_B^{M_2} > 0$, ε^{M_2}, $\delta^{M_2} > 0$. If the government induces an equilibrium E_2 where the transfers to university B are

$$\begin{aligned}
\tau_B^{E_2} - R_B^{E_2} &= \frac{\alpha}{\beta}\Delta N^l + \varepsilon^{M_2}N^l, \\
t_B^{E_2} &= t_A^{M_2},
\end{aligned}$$

the teaching quality for the low-ability students is (weakly) higher in equilibrium E_2 than in equilibrium M_2, i.e.

$$q_B^{E_2} = \alpha(e^l + \Delta) + \beta(t_A^{M_2} + b_B^{E_2} + \varepsilon^{M_2}) \geq \alpha(e^l + \Delta) + \beta(t_A^{M_2} + b_A^{M_2} + \varepsilon^{M_2}) = q_A^{M_2}, \qquad (50)$$

because $b_B^{E_2} \geq b_A^{M_2}$ follows from (26) and from the fact that in M_2 it is $U^l\left(q_A^{M_2}\right) - b_A^{M_2} \geq 0$.

Let us now define

$$z^{M_2} \equiv U^h\left(q_A^{M_2}\right) - b_A^{M_2} = U^h\left(q_B^{M_2}\right) - b_B^{M_2}.$$

Given that $\frac{dU^h}{dq}|_{q=q_A^{M_2}} < 1/\beta$ at equilibrium M_2, and (50) holds, using condition (27) it follows that in equilibrium E_2 it must be

$$U^h\left(q_A^{E_2}\right) - b_A^{E_2} = U^h\left(q_B^{E_2}\right) - b_B^{E_2} \leq z^{M_2}. \qquad (51)$$

In order to prove that the teaching quality for the high-ability students is higher in equilibrium E_2 than in equilibrium M_2 for equal extra-research costs, let the government give the following transfers to university A

$$\begin{aligned} \tau_A^{E_2} &= R_A^{E_2} + \varepsilon^{M_2} N^h + \delta^{M_2} n_B^h, \\ t_A^{E_2} &= t_A^{M_2}, \end{aligned}$$

so that the extra-research cost is the same in equilibrium E_2 as in equilibrium M_2. Notice that such values of $\tau_A^{E_2}$ and $t_A^{E_2}$ also imply that the extra-research per-capita transfer for high ability students in equilibrium E_2 is the same as the average extra-research per-capita transfer for high ability students in equilibrium M_2, i.e.

$$\begin{aligned} t_A^{E_2} + \frac{\tau_A^{E_2} - R_A^{E_2}}{N^h} &= t_A^{M_2} + \varepsilon^{M_2} + \delta^{M_2} \frac{n_B^h}{N^h} = \\ & \frac{n_A^h}{N^h}\left[t_A^{M_2} + \varepsilon^{M_2}\right] + \frac{n_B^h}{N^h}\left[t_B^{M_2} + \frac{\tau_B^{M_2} - R_B^{M_2}}{n_B^h}\right], \end{aligned}$$

where in equilibrium M_2 we do not consider the amount $\frac{\alpha}{\beta}\Delta N^l$ for university A because it is attributed to low-ability students in order to reach quality level $q_A^{M_2}$ as in (50). It then follows that

$$b_A^{E_2} > \frac{n_A^h}{N^h} b_A^{M_2} + \frac{n_B^h}{N^h} b_B^{M_2},$$

and

$$q_A^{E_2} > \frac{n_A^h}{N^h} q_A^{M_2} + \frac{n_B^h}{N^h} q_B^{M_2}.$$

Suppose in fact that it were $b_A^{E_2} = \frac{n_A^h}{N^h} b_A^{M_2} + \frac{n_B^h}{N^h} b_B^{M_2}$ implying $q_A^{E_2} = \frac{n_A^h}{N^h} q_A^{M_2} + \frac{n_B^h}{N^h} q_B^{M_2}$, then by the concavity of U^h we would have that

$$U^h\left(q_A^{E_2}\right)-b_A^{E_2}=$$
$$U^h\left(\alpha\left(e^l+\Delta\right)+\beta\left(t_A^{M_2}+\varepsilon^{M_2}+\delta^{M_2}\frac{n_B^h}{N^h}+\frac{n_A^h}{N^h}b_A^{M_2}+\frac{n_B^h}{N^h}b_B^{M_2}\right)\right)-\left(\frac{n_A^h}{N^h}b_A^{M_2}+\frac{n_B^h}{N^h}b_B^{M_2}\right)>$$
$$\frac{n_A^h}{N^h}\left[U^h\left(\alpha\left(e^l+\Delta\right)+\beta(t_A^{M_2}+\varepsilon^{M_2}+b_A^{M_2})\right)-b_A^{M_2}\right]+$$
$$+\frac{n_B^h}{N^h}\left[U^h\left(\alpha\left(e^l+\Delta\right)+\beta(t_A^{M_2}+\varepsilon^{M_2}+\delta^{M_2}+b_B^{M_2})\right)-b_B^{M_2}\right]=$$
$$\frac{n_A^h}{N^h}\left[U^h(q_A^{M_2})-b_A^{M_2}\right]+\frac{n_B^h}{N^h}\left[U^h(q_B^{M_2})-b_B^{M_2}\right]=z^{M_2},$$

where we have used (49) in $q_B^{M_2}$. But if this were the case, (51) would be contradicted. Then, considering that $\frac{dU^h}{dq}|_{q=q_j^{M_2}}<1/\beta$, $j=A,B$ implies $\frac{dU^h}{dq}|_{q=q_A^{E_2}}<1/\beta$, it must be $b_A^{E_2}>\frac{n_A^h}{N^h}b_A^{M_2}+\frac{n_B^h}{N^h}b_B^{M_2}$ because if it were $b_A^{E_2}<\frac{n_A^h}{N^h}b_A^{M_2}+\frac{n_B^h}{N^h}b_B^{M_2}$ the difference between the RHS and the LHS of the above inequality would be even higher. Consequently it is $q_A^{E_2}>\frac{n_A^h}{N^h}q_A^{M_2}+\frac{n_B^h}{N^h}q_B^{M_2}$.□

Proof of Proposition 7.

Let us consider the extra research cost in equilibrium M_1

$$\widetilde{C}^{M_1}=t_A^{M_1}\left(N^h+n_A^l\right)+t_B^{M_1}n_B^l+\tau_B^{M_1}-R_B^{M_1}+\tau_A^{M_1}-R_A^{M_1},$$

and in equilibrium E_1

$$\widetilde{C}^{E_1}=t_A^{E_1}N^h+t_B^{E_1}N^l+\tau_B^{E_1}-R_B^{E_1}+\tau_A^{E_1}-R_A^{E_1}.$$

Using Corollary 1, $\widetilde{C}^{E_1}$ can be rewritten as

$$\widetilde{C}^{E_1}=t_A^{E_1}N+(\tau_A^{E_1}-R_A^{E_1})\frac{N}{N^h}+\frac{\alpha}{\beta}\Delta N^l.$$

Considering stability condition (17) for equilibrium M_1, let the government fix

$$\tau_A^{M_1}-R_A^{M_1}=-\frac{\alpha}{\beta}\Delta N^h+\delta^{M_1}, \tag{52}$$

with $\delta^{M_1}>0$, and set

$$(\tau_A^{E_1}-R_A^{E_1})\frac{N}{N^h}=\tau_A^{M_1}-R_A^{M_1}. \tag{53}$$

Using (52) and (53) we obtain that $\widetilde{C}^{E_1}=\widetilde{C}^{M_1}$ implies

$$t_A^{E_1}N+\frac{\alpha}{\beta}\Delta N^l=t_A^{M_1}\left(N^h+n_A^l\right)+t_B^{M_1}n_B^l+\tau_B^{M_1}-R_B^{M_1}. \tag{54}$$

Recall that teaching quality in equilibrium E_1 can be written as

$$q^{E_1}=\alpha(e^l+\Delta)+\beta\left(t_A^{E_1}+b^{E_1}+\frac{\tau_A^{E_1}-R_A^{E_1}}{N^h}\right).$$

By substituting from (53) and (52), the above expression can be re-written as

$$q^{E_1} = \alpha(e^l+\Delta)+\beta\left(t_A^{E_1}+b^{E_1}+\frac{-\frac{\alpha}{\beta}\Delta N^h+\delta^{M_1}}{N}\right)= \tag{55}$$

$$\alpha e^l+\beta\left(t_A^{E_1}+b^{E_1}+\frac{\alpha}{\beta}\Delta\frac{N^l}{N}+\frac{\delta^{M_1}}{N}\right). \tag{56}$$

Let us denote $\widehat{q}^{M_1}$ the average teaching quality in equilibrium M_1, which obtains as

$$\begin{aligned}\widehat{q}^{M_1} &= q_A^{M_1}\frac{N^h+n_A^l}{N}+q_B^{M_1}\frac{n_B^l}{N}= \\ &\left[\alpha\left(e^l+\Delta\frac{N^h}{N^h+n_A^l}\right)+\beta\left(t_A^{M_1}+b_A^{M_1}+\frac{\tau_A^{M_1}-R_A^{M_1}}{N^h+n_A^l}\right)\right]\frac{N^h+n_A^l}{N}+ \\ &\left[\alpha e^l+\beta\left(t_B^{M_1}+b_B^{M_1}+\frac{\tau_B^{M_1}-R_B^{M_1}}{n_B^l}\right)\right]\frac{n_B^l}{N}.\end{aligned}$$

Substituting $t_A^{M_1}\left(N^h+n_A^l\right)+t_B^{M_1}n_B^l$ from (54) and using (52), $\widehat{q}^{M_1}$ can be re-written as

$$\widehat{q}^{M_1}=\alpha e^l+\beta\left(t_A^{E_1}+\widehat{b}+\frac{\alpha}{\beta}\Delta\frac{N^l}{N}+\frac{\delta^{M_1}}{N}\right), \tag{57}$$

where $\widehat{b}=b_A^{M_1}\frac{N^h+n_A^l}{N}+b_B^{M_1}\frac{n_B^l}{N}$.

In what follows we want to exclude that $\widehat{q}^{M_1}\geq q^{E_1}$ when $\widetilde{C}^{E_1}=\widetilde{C}^{M_1}$.

In order for $\widehat{q}^{M_1}=q^{E_1}$ it should be $\widehat{b}=b^{E_1}$ where b^{E_1} is determined by (25). By concavity of $U(.)$ this would in turn imply

$$0=U^l(\widehat{q}^{M_1})-\widehat{b}>\left[U^l\left(q_A^{M_1}\right)-b_A^{M_1}\right]\frac{N^h+n_A^l}{N}+\left[U^l\left(q_B^{M_1}\right)-b_B^{M_1}\right]\frac{n_B^l}{N},$$

which is inconsistent with students' enrolment in both universities. In order to exclude $\widehat{q}^{M_1}>q^{E_1}$, notice that all terms in $\widehat{q}^{M_1}$ and q^{E_1} are equal with the exception of $\widehat{b}$. Then, in order to have $\widehat{q}^{M_1}>q^{E_1}$, we should raise $\widehat{b}$ above b^{E_1}. But, from (25), b^{E_1} is the highest possible value of the tuition fee that can be charged given these values of the non-tuition-fee terms, in the sense that $\widehat{b}>b^{E_1}$ would imply

$$0>U^l(\widehat{q}^{M_1})-\widehat{b}>\left[U^l\left(q_A^{M_1}\right)-b_A^{M_1}\right]\frac{N^h+n_A^l}{N}+\left[U^l\left(q_B^{M_1}\right)-b_B^{M_1}\right]\frac{n_B^l}{N},$$

which is inconsistent with student enrolment in both universities. Then it must be the case that $\widehat{q}^{M_1}<q^{E_1}$.□

Proof of Proposition 8.

Consider that $r_A^S=r_A^E=\left(\frac{\beta}{\gamma_A}\right)^{\frac{\gamma_A}{\gamma_A-1}}$. Let us fix $r_B^S=r_B^E=\left(\frac{\beta}{\gamma_B}\right)^{\frac{\gamma_B}{\gamma_B-1}}$, which results in research expenditure $R_B^S=\left(\frac{\beta}{\gamma_B}\right)^{\frac{1}{\gamma_B-1}}=R_B^E$. We show that any level of $q^{E_1}=q_A^S$ can be obtained with a lower public expenditure in equilibrium E_1 than in equilibrium S.

Considering (14), the government can fix the lump-sum transfers as follows:

$$\begin{aligned}\tau_A^S &= R_A^S - \frac{\alpha}{\beta}\Delta N^h - \epsilon^S, \quad \tau_B^S = R_B^S, \\ \tau_B^{E_1} &= R_B^{E_1} + \frac{\alpha}{\beta}\Delta N^l + \epsilon^{E_1},\end{aligned} \tag{58}$$

and can fix the per-student transfers as follows

$$t_A^S = t_B^{E_1} + \frac{\alpha}{\beta}\Delta + \frac{\epsilon^{E_1}}{N^l} + \frac{\epsilon^S}{N}, \tag{59}$$

in order to guarantee equal public costs. It is in fact

$$\begin{aligned}C^S &= \tau_A^S + \tau_B^S + t_A^S N = \\ &R_A^S + R_B^S - \frac{\alpha}{\beta}\Delta N^h - \epsilon^S + t_A^S N.\end{aligned}$$

Substituting t_A^S from (59), C^S becomes

$$C^S = R_A^S + R_B^S + \frac{\alpha}{\beta}\Delta N^l + \epsilon^{E_1}\frac{N}{N^l} + t_B^{E_1} N,$$

which is equal to

$$\begin{aligned}C^E &= \tau_A^{E_1} + \tau_B^{E_1} + t_B^{E_1} N^l + t_A^{E_1} N^h = \\ &R_A^{E_1} + R_B^{E_1} + \frac{\alpha}{\beta}\Delta N^l + \epsilon^{E_1}\frac{N}{N^l} + t_B^{E_1} N,\end{aligned}$$

since $R_A^S + R_B^S = R_A^{E_1} + R_B^{E_1}$.

Let us now compare the teaching quality in the two equilibria. At equilibrium S, teaching quality at university A obtains as

$$q_A^S = \alpha\left(e^l + \frac{\Delta N^h}{N}\right) + \beta\left(t_A^S + b_A^S + \frac{\tau_A^S - R_A^S}{N}\right).$$

Substituting from (58) and (59), q_A^S can be written as

$$q_A^S = \alpha e^l + \beta\left(b_A^S + t_B^{E_1} + \frac{\alpha}{\beta}\Delta + \frac{\epsilon^{E_1}}{N^l}\right),$$

which is lower than

$$q^{E_1} = \alpha e^l + \beta\left(b^{E_1} + t_B^{E_1} + \frac{\alpha}{\beta}\Delta + \frac{\epsilon^{E_1}}{N^l}\right),$$

because $b^{E_1} > b_A^S$ as b^{E_1} is the solution to

$$U^l\left(q^{E_1}\right) - b^{E_1} = 0,$$

while b_A^S is such that $U^l\left(q_A^S\right) - b_A^S > 0$.□

Proof of Proposition 9.

Condition (40) is the stability condition for equilibrium E_2 which contradicts condition (37) for the existence of equilibrium S. Condition (41) contradicts the stability conditions (17) and (22) for equilibria M_k, $k = 1, 2$. Condition (42) from Corollary 2 guarantees the existence of equilibrium E_2.

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