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Abstract

Matching mechanisms are regarded as an important instrument to bring about Pareto optimal allocations in a public good economy and to cure the underprovision problem associated with private provision of public goods. The desired Pareto optimal interior matching equilibrium, however, emerges only under very special conditions. But we show in this note that corner solutions, in which some agents choose zero flat contributions, normally avoid underprovision and illustrate and interpret our results by a simple numerical example.

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1. Introduction

It is well-known that public good provision is inefficiently low in the voluntary contribution model (see, e.g., Cornes and Sandler 1996). The use of "matching mechanisms", under which the contributions of independently acting agents are subsidized by others, has been suggested as a way to achieve efficiency.[1] Ideally, such mechanisms implement a Pareto optimal solution as a Nash equilibrium.

A matching mechanism works as intended if all agents equate their marginal rates of substitution between the public and the private good with their individual price ratios as modified by matching. Unfortunately, under any given matching scheme, such interior matching equilibria only emerge for specific initial income distributions. Interiority of Nash equilibria is even much harder to get with matching than without (see Buchholz, Cornes and Rübbelke 2011). Rather, corner solutions involving zero flat contributions by some players are a likely outcome of voluntary public good provision with matching.[2] This is bad news concerning the usefulness of matching mechanisms. Better news, however, is that for matching schemes that would lead to Pareto optimality in an interior equilibrium underprovision of the public good will be avoided even if a corner solution occurs in which all agents still have strictly positive private consumption and thus is non-degenerate. To demonstrate this striking result we use the Aggregative Game Approach (see Cornes and Hartley 2007).

In addition to our formal analysis, we construct a simple numerical 2-agent example, in which we use the Kolm triangle (see Kolm 1969, Chapter 9), to illustrate our claims. In addition to the "Pareto optimal" matching mechanism, which is treated in our general analysis, the example also considers the standard voluntary contribution and a "partial" matching mechanism. The comparison between the outcomes under these mechanisms further elucidates the source of our overprovision result.

[1] The long list of papers dealing with matching in a public good economy includes Guttman (1978), Boadway, Pestieau and Wildasin (1989), Danziger and Schnytzer (1991), Althammer and Buchholz (1993), Varian (1994), Andreoni and Bergstrom (1996), Falkinger (1996), Kirchsteiger and Puppe (1997), Falkinger and Brunner (1999), and Boadway, Song and Tremblay (2007). Several authors – e.g. Falkinger, Hackl and Pruckner (1996) – have proposed matching in the context of global climate policy.

[2] On the importance of corner solutions in voluntary public good provision games without matching see Bergstrom, Blume and Varian (1986), Itaya, de Meza and Myles (1997) and Cornes and Sandler (2000).

2. The Framework

There are n agents $i = 1,...,n$ with utility functions $u_i(x_i, G)$ where x_i is private consumption of agent i and G is public good supply. All utility functions are twice partially differentiable, strictly monotone increasing in both variables and strictly quasi-concave. Indifference curves are assumed to asymptote to the two axes. Both goods are strictly normal for every agent $i = 1,...,n.$ Agent i´s initial private good endowment ("income") is w_i . Total income is $W = \sum_{i=1}^{n} w_i$. We assume that $G = \sum_{i=1}^{n} z_i$, where $z_i = w_i - x_i$ is agent i´s total contribution to the public good which, under a given matching scheme, consists of a direct *flat contribution* y_i, chosen independently of the actions of the other agents, and of an *indirect contribution* that agent *i* makes by matching the flat contributions y_j of all other agents. Neither component of an agent's total contribution can be negative. For a linear matching scheme as considered in this paper we have

(1) $$z_i = \sum_{j=1}^{n} \mu_{ij} y_j .$$

Here for $j \neq i$ the exogenously given and constant *matching rates* $\mu_{ij} \geq 0$ express how much agent i adds to the flat contributions of agent *j* and where $\mu_{ii} = 1$. Given a matching scheme agent i´s marginal rate of transformation between the private and the public good is

(2) $$\pi_i = \sum_{j=1}^{n} \mu_{ji} .$$

The reciprocal, $1/\pi_i$ thus is the subsidized personal public good price agent *i* has to pay for an additional unit of the public good.

Given any linear matching scheme, preferences and income, a Nash equilibrium is defined in the usual way.

Definition: An n-tuple $(y_1^M,...,y_n^M)$ is a *matching equilibrium* in flat contributions if for any agent $i = 1,...,n$ the flat contribution y_i^M maximizes

(3) $$u_i(w_i - y_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \mu_{ij} y_j^M , G_{-i}^M + (\sum_{j=1}^{n} \mu_{ji}) y_i)$$

where G_{-i}^M denotes aggregate public good supply of all agents $j \neq i$.

A matching equilibrium is called *interior* if all flat contributions are strictly positive, i.e. $y_i^M > 0$ for all $i = 1,...,n$. Furthermore, let public good supply in a matching equilibrium be G^M and private consumption of agent i be x_i^M.

To apply the Aggregative Game Approach, let $e_i^{\pi_i}(.)$ denote agent *i´s* income expansion path, described as a function of public-good supply G, on which agent *i´s* marginal rate of substitution between the private and the public good equals π_i. Given strict normality for both the private and the public good, the function $e_i^{\pi_i}(.)$ is defined for all $G > 0$ and is strictly increasing in G with $e_i^{\pi_i}(0) = 0$ for all agents $i = 1,...,n$. We now compare the levels of public good supply that result in different matching equilibria**.**

3. Public Good Supply in Interior and in Corner Solutions

Let $(\hat{x}_1,...,\hat{x}_n,\hat{G})$ denote the levels of private consumption and the level of public good supply in an *interior matching equilibrium* given a certain matching mechanism, individual preferences and some distribution of total income W. Then we have

(4) $$\hat{x}_i = e_i^{\pi_i}(\hat{G}) \qquad \text{for all } i = 1,...,n \qquad \text{and}$$

(5) $$\hat{G} + \sum_{i=1}^{n} e_i^{\pi_i}(\hat{G}) = W .$$

Condition (4) holds since any agent that chooses a strictly positive flat contribution is in a position in which her marginal rate of substitution equals π_i, so that her choice is on the income expansion path $e_i^{\pi_i}(.)$. Condition (5), the aggregate budget constraint, holds since each agent spends her income either for private consumption or her contribution to the public good. A given matching mechanism, given preferences and any given level of aggregate wealth $W > 0$, generate unique values $(\hat{x}_1,...,\hat{x}_n,\hat{G})$ that fulfil conditions (4) and (5). This follows from strict monotonicity of all income expansion paths $e_i^{\pi_i}(.)$, and $e_i^{\pi_i}(0) = 0$ for all agents i.

If we apply some matching mechanism and start from some arbitrary distribution of income it is, however, an unlikely eventuality that an interior matching equilibrium really results. Rather, without adjusting the matching mechanism to the income distribution which is informationally quite demanding, we have to face corner matching solutions, in which the flat contribution of at least one agent is zero (see Buchholz, Cornes and Rübbelke 2011). For corner solutions that in addition are non-degenerate in the sense that $x_i^M > 0$ for $i = 1,...,n$ it is now possible to compare public good supply G^M with public good supply $\hat{G}$ in an interior solution.

Proposition 1: If $(x_1^M,...,x_n^M,G^M)$ is a non-degenerate corner matching equilibrium, then $G^M \geq \hat{G}$ holds.

Proof: At a non-degenerate matching equilibrium, $x_i^M = e_i^{\pi_i}(G^M)$ for all agents i that make strictly positive flat contributions to the public good. But, for any agent whose flat contribution is zero, $x_i^M \leq e_i^{\pi_i}(G^M)$. Otherwise the marginal rate of substitution between the private and the public good would be smaller than π_i and there would be an incentive for her to make a positive contribution to the public good. Now assume that $G^M < \hat{G}$. From strict monotonicity of all income expansion paths it then follows that

(6) $$G^M + \sum_{i=1}^{n} x_i^M \leq G^M + \sum_{i=1}^{n} e_i^{\pi_i}(G^M) < \hat{G} + \sum_{i=1}^{n} e_i^{\pi_i}(\hat{G}) = W .$$

Hence the aggregate resource constraint would not hold, which gives a contradiction. QED

Proposition 1 implies that public good supply in a matching equilibrium is smallest when the number of active contributors is highest, i.e. if all agents in the economy make a strictly positive contribution to the public good.[3] The argument given in the proof also implies that if some agent j is not at the verge of contributing in $(x_1^M,...,x_n^M,G^M)$, i.e. $x_j^M < e_i^{\pi_i}(G^M)$ holds, we even get $G^M > \hat{G}$.

Turning to matching schemes which seek to implement a Pareto optimal allocation it directly follows from the Samuelson rule that an interior matching equilibrium is Pareto optimal if and only if

(7) $$\sum_{i=1}^{n} \frac{1}{\pi_i} = 1 .$$

[3] Letting $\mu_{ij} = 0$ for all $i \neq j$, the result in Bergstrom, Blume and Varian (1986), that compares the levels of public good supply in interior and in corner solutions of the standard voluntary contribution game without matching, is obtained as a special case of Proposition 1.

A matching mechanism that fulfils condition (7) is called a *Pareto matching scheme*. We have the following result.

Proposition 2: If $(x_1^M,...,x_n^M,G^M)$ is a non-degenerate matching equilibrium given some Pareto matching scheme, then there is a Pareto optimal allocation with public good supply $G^* \leq G^M$.

Proof: Choose $G^* := \hat{G}$ when $\hat{G}$ again is the public good supply in the interior matching equilibrium of the given scheme. Proposition 2 then is an immediate implication of Proposition 1.
QED

Proposition 2 says that applying a matching mechanism helps in any case to avoid underprovision of the public good. If the desired Pareto optimal interior matching equilibrium is missed and a corner solution is attained, then the public good will, in a sense, be overprovided as long as the outcome is non-degenerate. Our overprovision result is particularly striking when optimal public-good provision is independent of income distribution, as is the case under Bergstrom-Cornes preferences (see Bergstrom and Cornes 1983).

Note, however, that Proposition 2 does not hold for degenerate matching equilibria. For example, suppose that three agents $i = 1,2,3$ all have the same Cobb-Douglas utility function $u_i(x_i,G) = x_i G$. The matching scheme is given by $\mu_{12} = \mu_{21} = 0$, $\mu_{13} = \mu_{23} = 1$ and $\mu_{31} = \mu_{32} = 2$. Obviously, if $\pi_1 = \pi_2 = \pi_3 = 3$, then condition (7) is fulfilled and the interior matching equilibrium is Pareto optimal. Let $w_1 = w_2 = 10$ and $w_3 = 4$. Then the Pareto optimal public good provision is $G^* = 12$. It can readily be confirmed that the matching equilibrium is given by $x_1^M = x_2^M = G^M = 8$ and $x_3^M = 0$, i.e. the matching equilibrium is degenerate and in contrast to the result in Proposition 2, public good underprovision arises.

4. A Numerical Example

In this section, we exploit the Kolm triangle to work through three different matching rules within the context of a simple numerical 2-person example. This diagram, although unable to cope with more than two agents, has the advantage that, within this limitation, it can show every magnitude of interest. We refer the reader to the paper by Thomson (1999) for an excellent explanation of the diagram, and for demonstrating a number of its applications. An earlier paper by Schlesinger (1989), unfortunately not widely available, gives a clear account of the voluntary contribution mechanism in such an economy.[4]

Consider an economy with 2 agents, one pure public good and one private good - in short, the standard 2-agent public good economy. Our example supposes that the two agents have identical Cobb-Douglas preferences:

$$u_i(x_i, G) = x_i G, \qquad i = 1, 2.$$

Budget constraints are

$$x_i + z_i = w_i, \qquad i = 1, 2.$$

Finally, we set total income at 12 units:

$$w_1 + w_2 = 12.$$

We do not at this stage pin down the precise distribution of initial income. We will see as we go along how different income distributions give rise to different outcomes. The reader should think of the large equilateral triangle in Figures 1-3 as having height 12, reflecting the total endowment of the economy. Each of the smaller triangles that make up the grid has height 1 unit.

4.1 Preparation: The Kolm Triangle Method

The key idea underlying the Kolm diagram is the following. Any point in the equilateral triangle represents the trio of values (x_1, x_2, G). Given the point E, say, in Figure 1, the value of x_1 is measured by the length of the perpendicular from E to AB, that of x_2 by the perpendicular from E to AC, and that of G by the perpendicular from E to BC. Thus, given that the height of each small triangle is taken to be one, the point E represents the allocation $(x_1, x_2, G) = (4,4,4)$. The point I represents the allocation $(x_1, x_2, G) = (5,7,0)$ - and so on. The fact that the sum of these three perpendiculars is a constant reflects the fact that the sum of the three economic quantities equals the given overall resource endowment: $x_1 + x_2 + G = w_1 + w_2 = 12$.

Our example is a member of the class, identified by Bergstrom and Cornes (1983), with a unique optimal level of public good provision, regardless of the distribution of private good consumption. Straightforward calculation of the Samuelson rule for optimal provision identifies the optimal level as $G^{PO} = 6$. In each figure the horizontal dashed line along which $G = 6$ represents the set of Pareto optimal allocations.

[4] In view of the difficulty of obtaining Schlesinger's contribution, we explain the depiction of the voluntary mechanism by the Kolm triangle in some detail, before going on to the partial and "Pareto optimal" matching mechanisms.

4.2 Equilibrium of the Voluntary Contribution Mechanism [$\mu_{12} = \mu_{21} = 0$]

At an interior equilibrium - that is, an equilibrium at which every agent chooses an interior allocation - each agent equates her marginal rate of substitution between private and public good with the relative price that she faces. In the present example this implies that:

$$\frac{\partial u_i(.)/\partial x_i}{\partial u_i(.)/\partial G} = \frac{G}{x_i} = 1 \ \ \forall i = 1, \ldots, n.$$

Before we identify an equilibrium, consider first agent 1's behavior. Whatever her endowment, the sacrifice of one unit of private good increases the total quantity available of the public good. For example, if $w_1 = 5$, and the other agent's contribution to the public good is zero, agent 1 can transform private into public good at the rate of one-for-one along the ray ID in Figure 1.

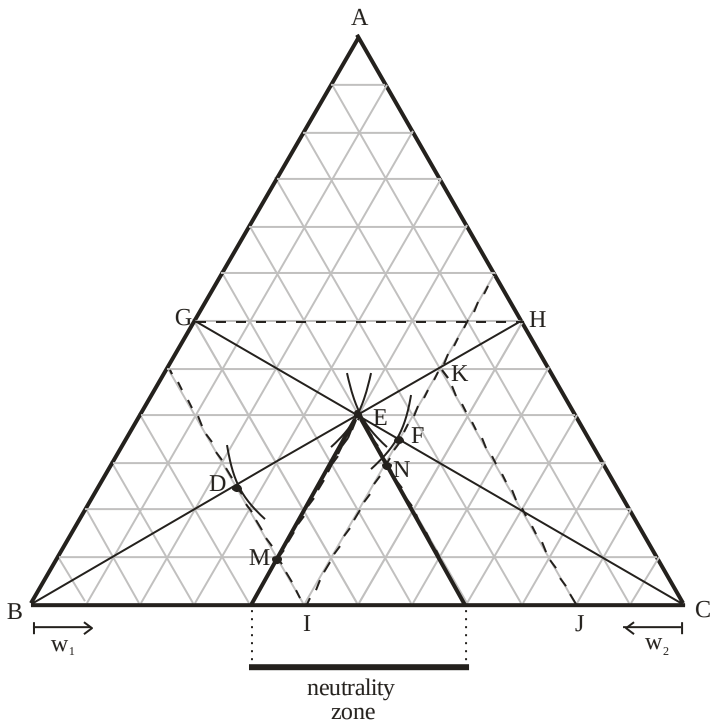

Figure 1: The voluntary contribution mechanism [$\mu_{12} = \mu_{21} = 0$]

Her most preferred choice is the point of tangency with an indifference curve, D, where $(x_1, z_1) = \left(\frac{5}{2}, \frac{5}{2}\right)$. Now vary agent 2's contribution parametrically. As agent 2's contribution increases, agent 1's endowment point shifts upward along IF. Thus her implied transformation curve shifts upwards to the right. The implied locus of tangencies between the transformation curves and her indifference map, her 'income expansion path', is the ray BH. [5]

If both agents are to be choosing an interior solution, then the allocation must be at the sole intersection of the income expansion paths at E, where G = 4. This will, indeed, be the equilibrium under the present mechanism arising from any initial income distribution within the indicated neutrality zone in Figure 1 in which

$$4 \leq w_1 = 12 - w_2 \leq 8.$$

For example, suppose that $(w_1, w_2) = (5, 7)$. This is the point I. The points E and I can be seen as opposite corners of a parallelogram, ENIM, whose sides have the slopes of the agents' transformation curves. At E, the agents' contributions are $(z_1, z_2) = (1, 3)$. Agent 2's contribution implies that agent 1 has a full income represented by the point N - she has 5 units of money income plus 3 units of public good. Given her implied full income endowment point, N, her utility-maximizing choice is clearly to give up one unit of the private good and augment the public good total by one unit, taking her to E. Similarly, agent 1's contribution of one unit implies that agent 2 enjoys a full income represented by the point M. Again, it is clear that her best choice is to move along her transformation curve to her point of tangency at E. Consider the thick continuous lines through E, the slopes of which equal those of the two agents' transformation curves. Their points of intersection with the base of the triangle identify the limits of the neutrality zone, within which income distributions generate the equilibrium at E.

Now suppose that the initial distribution of income is outside the neutrality zone, say at the point J where $(w_1, w_2) = (10, 2)$. Starting from this point, agent 1 will sacrifice 5 units of private good consumption to contribute 5 units of the public good, taking her to the point K. Agent 2 will contribute zero. She would like to be able to undo a part of agent 1's contribution, but the nonnegativity constraint on contributions rules this out. Thus the equilibrium is at the point K, where $(x_1, x_2, G) = (5,2,5)$.

In short, the locus of equilibrium outcomes is the piecewise linear curve GEH. All initial distributions within the neutrality zone map into the equilibrium E. As agent 1's initial income

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[5] If agent 1's income, w_1, is fixed at 5, not all points on this expansion path can be attained. The constrained income expansion path starts at the point D - points on the segment BD will never be observed under the present mechanism. Furthermore, if agent 2 contributes 5 or more units to the public good, agent 1 will not be on her expansion path, since this would involve her making a negative contribution to the public good - something we do not allow. Hence, in this case, points on the segment KH will not be observed. These observations simply reflect the fact that the tangency condition only characterizes interior solutions for agent 1.

increases further beyond the point where $w_1 = 8$, the equilibrium level of public good provision rises. Indeed, as $w_1 \rightarrow 12$, the equilibrium provision gets closer to the unique Pareto optimal level that we identified at the outset. The same observations apply if the income distribution is sufficiently skewed in favour of agent 2 to take us outside the neutrality zone to the left in Figure 1.

4.3 Equilibrium with [Weak] Matching: $[\mu_{12} = \mu_{21} = 0.5]$

We retain all features of the numerical example, except that we now suppose there is strictly positive matching. Specifically, suppose that

$$\begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}.$$

In this case, agent *i*'s utility is

$$u_i(w_i - y_i - \frac{y_j}{2}, G_{-i} + y_i + \frac{y_i}{2}),\ i, j = 1,2,\ j \neq i.$$

Inspection of the arguments of this function shows the implied trade-off between private good consumption and the total level of public good as agent *i* varies her choice of y_i. By giving up a unit of private good consumption, agent *i* augments public good provision by $\frac{3}{2}$ units. Of this, she herself contributes one unit. In addition, the other agent is bound by the matching rule to contribute half a unit.

At any internal allocation, agent *i* equates her private marginal rate of transformation to her marginal rate of substitution. Figure 2 depicts the expansion paths for the two agents.

Whatever her initial endowment point, agent 1's transformation curve is steeper than under zero matching. As a consequence, the locus of tangencies between transformation and indifference curves, which is agent 1's income expansion path, is the ray BH′ . This ray is steeper than its counterpart in Figure 1. Since the same observations apply also to agent 2, the common intersection between the two expansion paths, BH′ and CG′ - which is the Nash equilibrium under this mechanism - is at E′ in Figure 2, where = $(x_1, x_2, G) = \left(\frac{24}{7}, \frac{24}{7}, \frac{36}{7}\right)$.

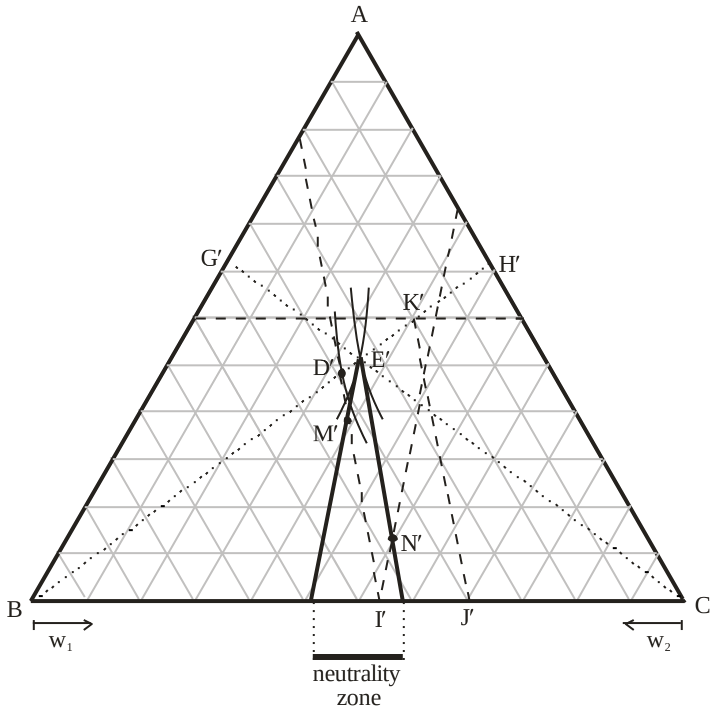

Figure 2: The weak matching mechanism [$\mu_{12} = \mu_{21} = 0.5$]

Again, the allocation E′ will only be achieved if the initial income distribution is such that both agents are at an interior solution. Since each agent's transformation curve is steeper than in Figure 1 - reflecting the more favorable rate at which each can transform her private good into public good through the matching rule - the set of distributions consistent with interior solutions is smaller than under the standard zero matching mechanism. In Figure 2, the end-points of this set of distributions are where the rays E′ N′ and E′ M′ intersect the base of the triangle.

Outside the neutrality zone, the larger the positive contributor's income, the larger is the equilibrium public good level. For example, if $(w_1, w_2) = (8,4)$ - the point J′ in Figure 2 - agent 1 will choose a flat contribution rate of $y_1 = 4$. Under the matching rule, this requires agent 2 to make a matching contribution of 2 units, even if the latter chooses a flat rate of $y_2 = 0$ - which she will do at equilibrium. The equilibrium quantities are $(x_1, x_2, G) = (4,2,6)$. Note that this particular equilibrium implies precisely the unique Pareto optimal quantity of the public good. If the income distribution were even more unequal, the resulting equilibrium would imply an even higher level of public good provision, as agent 1 chooses higher points on her expansion path, forcing matching contributions from agent 2.

4.4 Equilibrium with "Pareto" Matching $[\mu_{12} = \mu_{21} = 1]$To give an illustration of our genera analysis in Section 3 we now consider, as a final step, a specific Pareto matching scheme which This matching scheme is given by

$$\begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

In this case, agent *i*'s utility at any allocation is

$$u_i(w_i - y_i - y_j, G_{-i} + 2y_i), \qquad i, j = 1, 2, \qquad j \neq i.$$

This is the matching scheme which, under ideal circumstances, produces an efficient Lindahl equilibrium. For each agent, the sacrifice of one unit of private consumption generates 2 units of additional public good. However, the set of income distributions that lead to an equilibrium in which both agents are at an interior solution has shrunk yet further by comparison with the partial matching mechanism. Indeed, the alert reader can perhaps anticipate the outcome under this matching rule.

In Figure 3, which depicts this case, the parallelogram to which we drew attention in our discussion of the zero and partial matching rules has degenerated to a line, so that the neutrality zone has shrunk to a singleton. Each agent, in return for sacrificing one unit of the private good, gains 2 units of public good. Of this, one unit is the matching contribution by the other agent. The equilibrium reached from the single income distribution consistent with interior outcomes, E'', is the point of common tangency between the transformation curves and the agents' indifference curves.

It is therefore clearly Pareto optimal. All other possible equilibria, reached from other income distributions, involve levels of G that are 'too high'. For example, starting from a distribution such as J'', where $(w_1, w_2) = (7{,}5)$, the resulting equilibrium is at $(x_1, x_2, G) = \left(\frac{7}{2}, \frac{3}{2}, 7\right)$. The implied value of *G* exceeds the unique Pareto optimal level of 7. By choosing $y_1 = \frac{7}{2}$, agent 1 forces a matching contribution of $\frac{7}{2}$ from agent 2, who chooses $y_2 = 0$, in equilibrium.

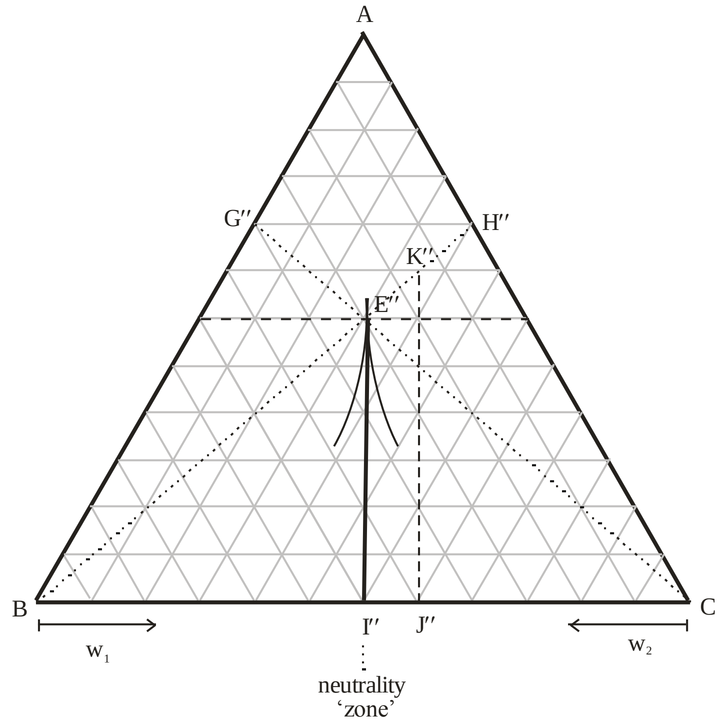

Figure 3: The Pareto matching mechanism [$\mu_{12} = \mu_{21} = 1$]

Since the equilibrium attained when $(w_1, w_2) = (6{,}6)$ implies the unique Pareto optimal level of public good provision, it clearly follows that the further the income distribution departs from equality in either direction, the more will the resulting equilibrium level of the public good exceed its unique optimal level, as the higher income agent forces matching contributions out of the other. The locus of equilibria under this matching rule is the piecewise linear curve G″E″H″.

4.5 Lessons of the Numerical Example

Our example is, of course, very simple and special since, e.g., preferences are of the Bergstrom-Cornes type which gives independence of the Pareto optimal level of public good from the distribution of private consumption. But some properties of the example are robust and significant.

First, under each of the three mechanisms, it is the normality assumption that is responsible for the fact that, as the income of the set of positive contributors increases, so too does the equilibrium level of public good provision. Second, any increase in the extent to which players' flat contributions are matched by others increases the absolute value of the slope of the agent's transformation curves in the Kolm triangle. This feature by itself shrinks the set of initial income distributions that constitute what we have called the neutrality zone – so much so that, under full matching, this set becomes a singleton. These two observations show that it is not so surprising that the Pareto matching mechanism never generates a level of public good provision that is below the level in a Pareto optimal solution.

In particular, income distributions that imply a nonempty set of noncontributors yield a higher level of public good provision than that achieved at the interior equilibrium, i.e. over-provision of the public good results. At such corner equilibria, those who choose positive flat contributions are forcing others, even those who choose zero flat contributions, to sacrifice private consumption by making matching contributions. In a certain sense thus negative externalities are being generated by those who choose positive flat contributions.

We have noted, without further exploring, the possibility of degenerate equilibria at which the private consumption of some agents is driven to zero by the matching requirement. It is easily confirmed that, in our example of the Pareto optimal matching mechanism depicted in Figure 3, if $w_1 > 8$, the equilibrium level of public good provision falls as income distribution becomes more unequal. Moreover, if $w_1 > 9$, it will fall below its Pareto optimal level. Such allocations would not satisfy participation constraints, since agent 2 would be better off. In our example, this requires very unequal income distributions. It may be interesting to explore modified matching rules that respect voluntary participation requirements, and also to analyze matching rules that permit individual agents to opt out.

5. Conclusion

In a public good economy the use of matching mechanisms is attractive because in principle they may generate Pareto optimal allocations as the outcome of a voluntary provision game. However, there is the significant problem that, without exact knowledge of individual preferences, there is much risk that a corner matching equilibrium instead of the desired interior solution is attained. But the deviation from Pareto optimal public good supply as associated with non-degenerate corner solutions entails throughout an overprovision of the public good. This result, which is the main message of this paper and has been illustrated and interpreted through a simple numerical example, may sound reassuring to all those who perceive under-provision of essential public goods (as in the sphere of global public goods, e.g., greenhouse gas abatement) as the most important danger.

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