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Abstract

We review the labor market implications of recent real-business-cycle models that successfully replicate the empirical equity premium. We document the fact that all models considered in this survey with the exception of Boldrin, Christiano, and Fisher (2001) imply a negative correlation of working hours and output that is not observed empirically, while in their model, the equity premium does not result from variation in the firm value, but from changes in the relative price of two goods. In addition, we calibrate the models with regard to characteristics from the German economy and show that the equity premium is very sensitive with regard to the utility parameters.

JEL-Code: G120, C630, E220, E320.

Keywords: equity premium, production CAPM, real-business cycle, labor market statistics.

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1 Introduction

Mehra and Prescott (1985) estimate an equity premium for the US during 1889 and 1978 of approximately 6 percent per annum.[1] As shown by Jerman (1988), the equity premium puzzle of Mehra and Prescott (1985) is solved in the standard real-business-cycle model with inelastic labor once both habit in consumption and capital adjustment costs are introduced. The basic intuition for this observation is given by the observation that, in such a model, household utility is more sensitive to consumption volatility due to habits and the household can only smooth his intertemporal consumption with the help of his wealth at high costs due to the capital adjustment costs. If labor is elastic, instead, the household can reduce the volatility of his utility with the help of its labor supply and the equity premium disappears. In fact, we demonstrate in this paper that in the case of the Jerman (1988) model with elastic labor, the equity premium falls close to zero.

One possible way to introduce such rigidity is by means of habit in leisure which serves as a short-cut to the modeling of either adjustment costs of labor or search frictions in the labor market. Bouakez and Kano (2006) argue that habit formation in leisure fits the US data better with regard to the persistence and propagation of shocks than other standard real-business-cycle models, in particular those allowing for learning-by-doing such as Chang, Gomes, and Schorfheide (2002). Lettau and Uhlig (2000), however, argue that, with habit formation in leisure, labor input is too smooth over the cycle and output and hours are negativey correlated, which is clearly at odds with the stylized facts of the business cycle.[2] The two sector model of Boldrin, Christiano, and Fisher (2001) (BCF for short) does not share this property. In this model, it is not possible to reallocate labor from the consumption goods sector to the investment goods sector after the observation of the shock. Accordingly, the equity premium results from variations in the relative price of the two goods rather than from variations in the firm's value.

Most studies of the equity premium and asset prices are constrained to the analysis of the real economy that is subject to a technology shock. As one of the very few

[1]In Germany, the equity premium has been lower according to a study by Kyriacou, Madsen, and Mase (2004). During 1900-2002, the equity premium in Germany amounted to 5.18 percent compared to 6.88 percent in the US if the risk-free rate is measured by the short-term government bill rate. In their study, the years 1922-23 of the German hyperinflation were excluded from the data.

[2]See, among others, Ambler, Clarida, and Zimmermann (2004), Basu and Taylor (1994), and Maußner (1994), for a survey of these facts.

exceptions, De Paoli, Scott and Weeken (2010) examine the behavior of asset prices in a New-Keynesian model with sticky prices. They find that the effect of nominal rigidities on the risk premium depends on the nature of the shock. While the risk premium is reduced if cycles are driven by technology shocks, it increases in the case of monetary shocks.

In the following we review the models in the present literature on the equity premium in the production economy. We analyze their ability to explain the behavior of labor market variables. In addition to the existing literature, we formulate and analyze a model of the equity premium that features sticky wages.

Our results are summarized in Table 1.1. In the first column, you find the names of the models that we consider in the following sections. The first row presents the empirical values in Germany that we aim to match. Evidently, some of the models are able to generate an annual equity premium that is close to the empirical value (5.18) or at least of sizeable amount. However, only one of the models, the two-sector model of BCF, is able to replicate a sizeable equity premium and labor market correlations of hours with output and real wages that are observed empirically (0.40 and 0.27, respectively). In the last column, we present a statistics that measures the squared deviations of the model's second moments from their empirical values. Along this measure, the two-sector model and the model with predetermined hours by the household perform best.

The paper is organized as follows. In Section 2, we first present the Jermann (1988) model as a benchmark case to which we add one model element after the other. In Section 3, we show that the equity premium disappears once labor is supplied elastically. In Section 4, habit in leisure is demonstrated to fail to reestablish the equity premium to its full extent for the German calibration. Moreover, in this case, hours are strongly negatively correlated with both output and wages. Section 5 analyzes frictions in the labor market in the form of predetermined working hours. Again, output and hours are negatively correlated. We review the two-sector model of Boldrin, Christiano and Fisher (2001) in its various variants in Section 6. With labor being immobile between the consumption goods and the investment goods production sectors, the labor market statistics improve considerably. Again, we find that the equity premium is very sensitive with regard to the parameterization of the model. In Section 7, we study whether the time-to-plan model of Christiano and Todd (1996) rather than the capital-adjustment-cost mechanism is able to generate both a sizeable equity premium and a realistic labor market behavior. However, the equity premium falls far short of observed values.

Table 1.1
Summary of Results

	Equity premium	s_Y	s_I/s_Y	s_N/s_Y	s_w/s_Y	r_{YN}	r_{wN}	Score
Data	5.18	1.14	2.28	0.69	1.03	0.40	0.27	
Models								
Benchmark (Jerman)	5.18	0.90	2.28					
Benchmark with endogenous labor	0.55	0.51	1.47	1.27	2.08	−0.68	−0.94	26.16
Habit formation in leisure	1.84	0.57	2.07	0.83	1.78	−0.89	−0.97	14.98
Predetermined hours								
Firms	5.83	0.76	2.87	0.44	15.65	−0.47	0.15	215.35
Housholds	6.10	0.76	2.87	0.44	1.27	−0.47	−0.71	3.03
Two sector model								
Stationary growth	5.18	0.96	2.36	0.14	2.64	0.72	0.0	3.08
Integrated growth	3.73	0.95	1.63	0.08	2.46	0.73	0.03	5.11
Adjustment costs	4.19	0.84	0.79	0.13	2.07	−0.62	−0.19	5.85
Time to plan								
Utility (7.5a)	0.02	1.89	4.86	1.05	0.45	0.90	−0.31	34.33
Utility (7.5b)	0.04	1.57	4.82	1.01	0.56	0.85	−0.29	33.71
Sticky price model	0.41	0.61	1.04	1.75	34.85	0.19	0.60	1169.36
Sticky wage model	1.37	0.58	1.84	1.66	1.93	0.02	−0.85	17.86

Notes: s_x:=Standard deviation of time series x, where $x \in \{Y, I, N, w\}$ and Y, I, and N denote output, investment, hours, and the wage, respectively. Empirical as well as model generated time series were HP-filtered with weight 1600. The empirical moments relate to per capita magnitudes, except for the real wage which was measured as hourly worker compensation. s_x/s_y:=standard deviation of variable x relative to standard deviation of output y. r_{NY}:=Cross-correlation of variable hours with output, r_{wN}:=Cross-correlation of the real wage with hours. The column Score presents the sum of squared differences between the moments from simulations of the model and the moments from the data. The moments considered are those in columns 2, 4, 5, 6, and 7.

Section 8 analyzes the New-Keynesian model of de Paoli, Scott, and Weeken (2010) and shows that the replication of labor market statistics is also difficult in a model with monetary shocks and nominal rigidities. In Section 9, we demonstrate that our model with rigid wages displays similar deficiencies to generate empirical labor market statistics. In this case, hours and wages are almost perfectly negatively correlated. All equilibrium conditions and derivations of the individual models are presented in the Appendix.

2 The Jermann Model

2.1 The Model

The first model that we consider is the asset pricing model of Jermann (1998). We follow the description of this model in Herr and Maußner (2009). Time is discrete and denoted by t.

Households. A representative household supplies labor in a fixed amount of $N_t \equiv N$ at the real wage w_t. Besides labor income he receives dividends d_t per unit of share S_t he holds of the representative firm. The current price of shares in units of the consumption good is v_t. His current period utility function u depends on current and past consumption, C_t and C_{t-1}, respectively. Given his initial stock of shares S_t the households maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\eta} - 1}{1-\eta} \right\}, \quad \eta \geq 0,\ \chi^N \in [0,1),\ \beta \in (0,1)$$

subject to the sequence of budget constraints

$$v_t(S_{t+1} - S_t) \leq w_t N_t + d_t S_t - C_t. \tag{2.1}$$

The operator $\mathbb{E}_t$ denotes mathematical expectations with respect to information as of period t. The first-order conditions of this problem are:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{2.2a}$$

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} Q_{t+1}, \tag{2.2b}$$

$$R_t := \frac{d_t + v_t}{v_{t-1}}, \tag{2.2c}$$

where Λ_t is the Lagrange multiplier of the budget constraint.

Firms. The representative firm uses labor N_t and capital K_t to produce output Y_t according to the production function

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \quad \alpha \in (0,1). \tag{2.3}$$

The level of total factor productivity Z_t is governed by the AR(1)-Process

$$\ln Z_t = \rho^Z \ln Z_{t-1} + \epsilon_t^Z, \quad \epsilon_t^Z \sim N\left(0, (\sigma^Z)^2\right). \tag{2.4}$$

The firm finances part of its investment I_t from retained earnings RE_t and issues new shares to cover the remaining part:

$$I_t = v_t(S_{t+1} - S_t) + RE_t. \tag{2.5}$$

It distributes the excess of its profits over retained earnings to the household sector:

$$d_t S_t = Y_t - w_t N_t - RE_t. \tag{2.6}$$

Investment increases the firm's future stock of capital according to:

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \quad \delta \in [0,1], \tag{2.7}$$

where we parameterize the function Φ as

$$\Phi(I_t/K_t) := \frac{a_1}{1-\zeta}\left(\frac{I_t}{K_t}\right)^{1-\zeta} + a_2, \quad \zeta > 0. \tag{2.8}$$

The firm's ex-dividend value at the end of the current period t, V_t, equals the number of outstanding stocks S_{t+1} times the current stock price v_t. This definition implies:

$$\begin{aligned} V_t = v_t S_{t+1} &\overset{(2.5)}{=} I_t + v_t S_t - RE_t \overset{(2.6)}{=} I_t + w_t N_t - Y_t + (v_t + d_t)S_t, \\ &\overset{(2.2c)}{=} I_t + w_t N_t - Y_t + R_t V_{t-1}. \end{aligned}$$

Rearranging and taking expectations as of period t, yields

$$V_t = \mathbb{E}_t\left\{\frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + V_{t+1}}{R_{t+1}}\right\}$$

Iterating on this equation using the law of iterated expectations and assuming

$$\lim_{s\to\infty} \mathbb{E}_t\left\{\frac{V_{t+s}}{R_{t+1}R_{t+2}\dots R_{t+s}}\right\} = 0$$

establishes that the end-of-period value of the firm equals the discounted sum of its future cash flows $CF_{t+s} = Y_{t+s} - w_{t+s}N_{t+s} - I_{t+s}$:

$$V_t = \mathbb{E}_t \sum_{s=1}^{\infty} \varrho_{t+s} CF_{t+s}, \quad \varrho_{t+s} = \frac{1}{R_{t+1}R_{t+2}\dots R_{t+s}} \tag{2.9}$$

The firm's objective is to maximize its beginning-of-period value, which equals $V_t^{bop} = V_t + CF_t$. Defining $\varrho_t = 1$ allows us to write

$$V_t^{bop} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} CF_{t+s}. \tag{2.10}$$

The first-order conditions for maximizing (2.10) subject to (2.7) are:

$$w_t = (1-\alpha)Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{2.11a}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{2.11b}$$

$$q_t = \mathbb{E}_t \varrho_{t+1}\Big\{\alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta\big]\Big\}. \tag{2.11c}$$

In addition, the transversality condition

$$\lim_{s\to\infty} \mathbb{E}_t \varrho_{t+s} q_{t+s} K_{t+s+1} = 0 \tag{2.11d}$$

must hold.

Market Equilibrium. Using equations (2.5) and (2.6), the household's budget constraint implies the economy's resource restriction:

$$Y_t = C_t + I_t. \tag{2.12}$$

In equilibrium, the labor market clears at the wage w_t so that $N_t = 1$ for all t. Furthermore, using (2.2b), ϱ_{t+1} can be replaced by $\beta\Lambda_{t+1}/\Lambda_t$ so that at any date t the set of equations

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{2.13a}$$

$$Y_t = Z_t K_t^{\alpha}, \tag{2.13b}$$

$$Y_t = C_t + I_t, \tag{2.13c}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta b \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{2.13d}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda t}\Big\{\alpha Z_{t+1} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta\big]\Big\} \tag{2.13e}$$

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \tag{2.13f}$$

determines $(Y_t, C_t, I_t, K_{t+1}, \Lambda_{t+1}, q_{t+1})$ given (K_t, Λ_t, q_t).

Deterministic Stationary Equilibrium. Since our solution strategy rests on a second order approximation of the model we must consider the stationary equilibrium of the deterministic counterpart of our model that we get if we put $\sigma^Z = 0$ so that Z_t equals its unconditional expectation $Z = 1$ for all t. In this case we can ignore the expectations operator $\mathbb{E}_t$. Stationarity implies $x_{t+1} = x_t = x$ for any variable in our model. As usual, we specify Φ so that adjustment costs play no role in the stationary equilibrium, i.e., $\Phi(I/K)K = \delta K$ and $q = \Phi'(\delta) = 1$. This requires that we choose

$$\begin{aligned} a_1 &= \delta^{\zeta}, \\ a_2 &= \frac{-\zeta\delta}{1-\zeta}. \end{aligned}$$

These assumptions imply via equation (2.13e) the stationary solution for the stock of capital:

$$K = \left(\frac{1-\beta(1-\delta)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}}. \tag{2.14a}$$

Output, investment, consumption, and the stationary solution for Λ are then given by

$$Y = K^{\alpha}, \tag{2.14b}$$

$$I = \delta K, \tag{2.14c}$$

$$C = Y - I, \tag{2.14d}$$

$$\Lambda = C^{-\eta}(1-\chi^C)^{-\eta}(1-\chi^C\beta). \tag{2.14e}$$

2.2 Calibration and the Equity Premium

Calibration. We calibrate the model using seasonally adjusted quarterly data for the West German economy over the period 1975.i through 1989.iv. The parameter settings are taken from Heer and Maußner (2009), Section 6.3.4. Table 2.1 displays the respective values. Notice that the wage share in the German data, $1-\alpha = 0.73$, is larger than the value of 0.64 that is often found in comparable studies relying upon US data,[3] while the depreciation rate, $\delta = 0.011$, is much smaller and amounts to approximately half the US value. In addition, $N = 0.13$ is chosen to match the average quarterly fraction of hours spent on work by the typical German household. Notice that many studies set $N = 1/3$ arguing that the typical worker spends 8 hours per day on the job (see, for example, Hansen (1985)). We consider the typical household to be an average over the total population including children and retired persons rather than consisting of a single worker who is also working on the weekend and does not take any vacation. The discount factor $\beta = 0.994$ yields an annual risk free rate in the simulation of the model of about 1 percent. We choose the unobserved parameters χ^C and ζ to match two statistics: the relative volatility of investment expenditures and the equity

[3] See, for example, King, Plosser, and Rebelo (1988) and Plosser (1989).

premium. The former, measured as the standard deviation of the cyclical component of investment expenditures relative to the standard deviation of the cyclical component of GDP, is 2.28 in our data set. The latter equals 5.18 according to a recent study by Kyriacou, Madsen, and Mase (2004) covering the period 1900-2002 (see footnote 1). Table 2.1 summarizes our choice of parameter values.[4]

Table 2.1

Benchmark calibration

Preferences	β=0.994	χ^C=0.793	η=2	N=0.13
	ν_1=5.0			
Production	α=0.27	δ=0.011	ρ^Z=0.90	σ^Z=0.0072
	ζ=5.53			

Computation of the Equity Premium. The solution of the model are functions g^i, $i \in \{K, Y, C, I, \Lambda, q\}$, that determine K_{t+1}, Y_t, C_t, I_t, Λ_t, and q_t given the current period state variables K_t, C_{t-1}, and the log of the productivity shock $\ln Z_t$.

In our model the risk free rate of return r_t is given by

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}} - 1. \tag{2.15}$$

Since

$$\begin{aligned}
\Lambda_{t+1} &= g^\Lambda(K_{t+1}, C_t, \ln Z_{t+1}) \\
&= g^\Lambda(g^K(K_t, C_{t-1}, \ln Z_t), g^C(K_t, C_{t-1}, \ln Z_t), \rho^Z \ln Z_t + \epsilon^Z_{t+1}) \\
&=: \tilde{g}^\Lambda(K_t, C_{t-1}, \rho^Z \ln Z_t + \epsilon^Z_{t+1},)
\end{aligned}$$

and ϵ^Z_{t+1} is normally distributed, the expected value of the Lagrange multiplier equals

$$\mathbb{E}_t \Lambda_{t+1} = \int_{-\infty}^{\infty} \tilde{g}^\Lambda(K_t, C_{t-1}, \rho^Z \ln Z_t + \epsilon^Z_{t+1},) \frac{1}{\sigma^Z \sqrt{2\pi}} e^{\frac{-(\epsilon^Z_{t+1})^2}{(\sigma^Z)^2}} d\epsilon^Z_{t+1}.$$

We use the quadratic approximation of g^Λ at the stationary equilibrium and the Gauss-Hermite 6-point quadrature formula to approximate the integral on the right-hand-side of this equation.

[4]For future reference it also presents parameters that will be introduced below.

The labor market equilibrium condition (2.11a) and equation (2.7) imply that the right-hand-side of (2.11c) can be written as

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \\
&= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{d_{t+1} + v_{t+1}}{v_t} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1}
\end{aligned},
$$

where the second equality follows from equations (2.5) and (2.6) and the observation that $q_t K_{t+1} = v_t S_{t+1}$ (see Heer and Maußner (2009), p. 317). Therefore, the gross rate of return on the shares of the representative firm equals[5]

$$
R_{t+1} = \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}. \tag{2.16}
$$

We use the quadratic approximations of g^i and a random number generator to compute a long artificial time series for $R_{t+1} - r_t$. The average of this time series is our measure of the ex-post equity premium implied by the model.

We compute the equity premium from a time series of 1,000,000 observations and the second moments of simulated time series from averages over 300 simulations with 80 observations. As our empirical data we pass the artificial time series through the Hodrick-Prescott filter with weight 1600. As noted above, using the parameters in Table 2.1 and a pseudo random number generator, this yields an equity premium of 5.18 and a relative standard deviation of investment of 2.28.[6]

3 Endogenous Labor Supply

In this section, we introduce flexible labor in the model of Jermann (1988). As a consequence, the equity premium drops from 5.18 to 0.55 percent (see Table 1.1).

The Model. Let

$$
\begin{aligned}
U_t &\equiv \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} N_{t+s}^{1+\nu_1} \right\}, \\
&\beta \in (0,1),\ \chi^C \in [0,1),\ \eta, \nu_0, \nu_1 \geq 0
\end{aligned} \tag{3.1}
$$

[5]Note, $\alpha Y_{t+1} = Y_{t+1} - w_{t+1}N_{t+1}$.

[6]The Fortran computer programs are available from Alfred Maußner on request. The solution algorithm is the same as in Heer and Maußner (2009), Chapter 2. The respective code is available from `http://www.wiwi.uni-augsburg.de/vwl/maussner/dgebook/download3.html`.

denote the household's expected life-time utility. Maximizing this expression subject to the budget constraint (2.1) implies the first-order condition:

$$\nu_0 N_t^{\nu_1} = \Lambda_t w_t \tag{3.2}$$

in addition to equations (2.2). The model's dynamics consists of equations (3.2), (2.11a), (2.11b), (2.3), the resource constraint, (2.2a), (2.11c), and (2.7). The equilibrium conditions for this and the following models are summarized in the Appendix.

We follow Heer and Maußner (2008) and choose $\nu_1 = 5$ implying a Frisch elasticity of labor supply with respect to the real wage of 0.2.

Equity Premium. In this model the ex post gross return on the firm's shares equals

$$R_{t+1} = \frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \tag{3.3}$$

since

$$Y_{t+1} - w_{t+1}N_{t+1} = \alpha Z_{t+1}(K_{t+1}/N_{t+1})^{\alpha}$$

due to the labor market clearing condition (2.11a).

Using the same sequence of random numbers as in Section 2, we find an average risk free rate of return of 2.2 percent p.a. and an equity premium of 0.55 percent p.a. Evidently, the size of the equity premium depends critically on the variability of working hours over the business cycle. Besides the small premium the model has two other deficiencies: hours and output as well as hours and the real wage are negatively correlated (see Table 1.1), which is clearly as odds with the empirical evidence provided in the first row of entries in Table 1.1.

4 Habit Formation in Leisure

Lettau and Uhlig (2000) introduce habit formation in both consumption and leisure in the standard real business cycle model in order to study the implications for the optimal responses of output, consumption, labor input, and investment to exogenous shocks. Different from our model, they do not allow for capital adjustment costs. Consequently, the equity premium falls close to zero in their model. In the following, we introduce habit in leisure in the above model explicitly allowing for capital adjustment costs. We show that even in this case, the equity premium is almost zero and well below the value of 5.18 percent found in the Jermann (1988) model.

The Model. With habit in leisure, the household expected life-time utility is given by[7]

$$U_t \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \nu_0 \frac{(N_{t+s} - \chi^N N_{t+s-1})^{1+\nu_1}}{1+\nu_1} \right\}, \tag{4.1}$$
$$\eta,\, \nu_0, \nu_1 \geq 0,\, \chi^C, \chi^N \in [0,1).$$

Maximizing (4.1) subject to (2.1) implies the first-order condition

$$\nu_0 (N_t - \chi^N N_{t-1})^{\nu_1} - \beta \nu_0 \chi^N \mathbb{E}_t (N_{t+1} - \chi^N N_t)^{\nu_1} = \Lambda_t w_t \tag{4.2}$$

in addition to equations (2.2). The model's dynamics consists of equations (4.2), (2.11a), (2.11b), (2.3), the resource constraint, (2.2a), (2.11c), and (2.7). We test for different values of $\chi^N \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. In addition to $\nu_1 = 5$ we also consider the case where hours are almost perfectly elastic with respect to the real wage. The equity premium is computed from (3.3).

Results. Table 4.1 summarizes our results. The moments were computed from simulated logged and HP filtered model data. The filter weight was 1,600. As compared to the model of the previous section – which is the current model with $\chi^N = 0$ – note that the equity premium is increasing with the size of the habit parameter χ^N, but still diverges significantly from our benchmark value of 5.18. For $\chi^N = 0.50$ both the relative standard deviation of investment and of hours come close the their empirical counterparts (2.28 and 0.69, respectively). Yet, also the size of the counterfactual negative correlation between output and hours and hours and the real wage increases. As a minor result, we observe that in the case of $\nu_1 = 0.01$ the second moments displayed in the table are insensitive to the degree of habit in leisure d and the values implied for both the equity premium and the labor market statistics are far away from their empirical counterparts.

5 Predetermined Working Hours

In this section, we follow Boldrin, Christiano, and Fisher (2001) and consider frictions in the allocation of labor. In particular, we assume that firms must hire worker before

[7] The exact utility function used by Lettau and Uhlig (2000) differs from ours. They specify the utility as a function of leisure, $1 - N_t$. Bouakez and Kano (2006) use the fraction of labor and the habit stock rather than the first difference.

Table 4.1
Second Moments from the Model with Habit in Leisure

χ^N	Equity premium	s_I/s_Y	s_H/s_Y	$r_{N,Y}$	$r_{N,w}$
		$\nu_1 = 5$			
0.10	0.65	1.53	1.22	−0.71	−0.94
0.20	0.81	1.62	1.17	−0.74	−0.94
0.30	1.03	1.75	1.08	−0.73	−0.98
0.40	1.37	1.90	0.98	−0.84	−0.96
0.50	1.84	2.07	0.83	−0.89	−0.97
		$\nu_1 = 0.01$			
0.10	0.03	1.04	5.14	−0.66	−0.99
0.20	0.03	1.04	5.14	−0.66	−0.99
0.30	0.03	1.04	5.14	−0.66	−0.99
0.40	0.03	1.04	5.13	−0.67	−0.99
0.50	0.03	1.04	5.13	−0.67	−0.99

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, $x \in \{Y, I, N, w\}$. Y, I, N, and w denote output, investment, hours, and the real wage, respectively. r_{xY}:=Cross-correlation of variable x with output Y.

the productivity shock is revealed. First, we consider predetermined working hours in the standard one-sector model before we examine the two-sector model of BCF in the next section. We also study the question if it makes a difference whether the household's labor supply or the firm's labor demand is predetermined.[8] We show that the distinction mainly concerns the business cycle properties of the real wage, and to a much lesser extend the equity premium, whereas it has no discernable impact on other variables. Both model variants fail to replicate the labor market statistics observed empirically.

The Model. We study the two variants that hours are predetermined 1) by the firms and 2) by the households. In the first case, maximizing (2.10) with respect to N_{t+1} yields the first-order condition

$$0 = \mathbb{E}_t \Lambda_{t+1} \left((1-\alpha) Z_{t+1} N_{t+1}^{-\alpha} K_{t+1}^{\alpha} - w_{t+1} \right), \tag{5.1}$$

[8]BCF assume that firms must determine labor demand prior to the technology shock.

which replaces (2.11a). Note, however, that equation (5.1) no longer implies

$$R_{t+1} = \frac{d_{t+1} + v_{t+1}}{v_t} = \frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}},$$

since $\alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} \neq Y_{t+1} - w_{t+1}K_{t+1}$. Therefore, we assume that the firm uses internal funds only to finance investment. This allows us to employ (2.2c) to compute the return on equity from

$$R_{t+1} = (v_{t+1} + d_{t+1})/v_t. \tag{5.2}$$

In the second version, we assume that the household rather than the firm must determine her labor supply before the productivity shock is revealed. Maximizing (3.1) subject to (2.1) with respect to N_{t+1} yields the first-order condition

$$0 = \mathbb{E}_t \left\{ \nu_0 N_{t+1}^{\nu_1} - \Lambda_{t+1} w_{t+1} \right\} \tag{5.3}$$

that replaces (3.2), whereas (2.11a) reflects the firm's labor demand schedule. Besides, the model is the same as in Section 3.

Table 5.1

Second Moments from the Model with Endogenous Hours

Variable	s_x	s_x/s_Y	r_{xY}	r_{xN}	r_x
		Hours Predetermined by Firms			
Output	0.76	1.00	1.00	−0.47	0.46
Consumption	0.61	0.81	0.89	−0.72	0.79
Investment	2.18	2.87	0.81	0.01	0.04
Real Wage	11.87	15.65	0.72	0.15	−0.05
Hours	0.33	0.44	−0.47	1.00	0.51
		Hours Predetermined by Households			
Output	0.76	1.00	1.00	−0.47	0.46
Consumption	0.61	0.81	0.89	−0.72	0.79
Investment	2.18	2.87	0.81	0.01	0.04
Real Wage	0.96	1.27	0.95	−0.71	0.70
Hours	0.33	0.44	−0.47	1.00	0.51

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, where x stands for any of the variables in column 1. s_x/s_Y:=standard deviation of variable x relative to standard deviation of output Y. r_{xY}:=Cross-correlation of variable x with output Y. r_{xN}:=Cross-correlation of variable x with hours N, r_x:=First order autocorrelation of variable x.

Results. Table 5.1 presents second moments from simulations of both models. They are averages over 300 simulations of sample size 80. Except for the time series properties of the real wage both models have virtually the same implications for the second moments of the variables in the model. The real wage is much more volatile if hours are determined by firms. If households choose their labor supply before they know the real wage, hours and the real wage are negatively correlated. Also note that in both models hours and output are negatively correlated.

The annual equity premium is 5.83 percent in the first version of the model and 6.10 percent in the second version.

As a measure of fit, consider the sum of squared deviations of the model implied moments in Table 1.1 from those empirically observed. Since wages are extremely volatile if firms determine employment, this sum is 215 as opposed to 3 for the second model. For this reason we will only consider the case with hours predetermined by the household sector in the next section.

6 A Two-Sector Model

In this section, we consider the two sector model of Boldrin, Christiano, and Fisher (2001). As a distinctive feature of their model, investment goods are produced in a separate production sector and the mobility of labor between this sector and the sector producing the consumption good is limited. Therefore, the price of the investment good is volatile and generates a sizeable equity premium. We study the sensitivity of their model with respect to the assumption on the technology process. In the following, we first consider the case that the (natural) logarithm of total factor productivity $\ln Z_t$ follows the AR(1) given in equation (2.4). Subsequently, we compare our results to the case studied in BCF (2001) where labor augmenting technical progress is driven by a random walk with drift.

6.1 Stationary Technology Shocks

The Model. Consumption goods C_t are produced according to the technology

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \quad \alpha \in (0,1) \tag{6.1a}$$

where N_{Ct} and K_{Ct} denote labor and capital employed in this sector. The investment goods sector (subscript I) uses the same technology so that

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha} \tag{6.1b}$$

is the amount of investment goods I_t which sell at the relative price p_t. Total labor and capital in the economy equal

$$N_t = N_{Ct} + N_{It}, \tag{6.2a}$$
$$K_t = K_{Ct} + K_{It}. \tag{6.2b}$$

The first-oder conditions with respect to labor demand of both sectors are:

$$w_t = (1-\alpha) Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \tag{6.3a}$$
$$w_t = p_t (1-\alpha) Z_t N_{It}^{-\alpha} K_{It}^{\alpha}. \tag{6.3b}$$

Both sectors rent capital services from the household at the rates r_{Ct} and r_{It}, respectively, so that equilibrium in the respective markets implies:

$$r_{Ct} = \alpha Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha-1}, \tag{6.4a}$$
$$r_{It} = p_t \alpha Z_t N_{It}^{1-\alpha} K_{It}^{\alpha-1}. \tag{6.4b}$$

The representative household maximizes the same intertemporal utility function (3.1) as in the previous section. Since ex ante the wages in both sectors may differ from each other as do the rental rates of capital, his budget constraint is

$$0 \leq w_{Ct} N_{Ct} + w_{It} N_{It} + r_{Ct} K_{Ct} + r_{It} K_{It} + \Pi_{Ct} + \Pi_{It} - C_t - p_t I_t, \tag{6.5}$$

where w_{Ct} and w_{It} denote the real wage paid in the consumption and the investment goods sector, respectively. Maximizing (3.1) subject to (6.5) and the law of motion for the aggregate capital stock

$$K_{t+1} = I_t + (1-\delta) K_t, \tag{6.6}$$

implies

$$0 = E_t \left\{ \nu_0 N_{t+1}^{\nu_1} - \Lambda_{t+1} w_{Ct+1} \right\}, \tag{6.7a}$$
$$0 = E_t \left\{ \nu_0 N_{t+1}^{\nu_1} - \Lambda_{t+1} w_{It+1} \right\}, \tag{6.7b}$$
$$p_t \Lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(p_{t+1}(1-\delta) + r_{Ct+1} \right), \tag{6.7c}$$
$$p_t \Lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(p_{t+1}(1-\delta) + r_{It+1} \right) \tag{6.7d}$$

in addition to (2.2a) and (3.2).

In equilibrium the budget constraint implies the resource restriction $Y_t = C_t + p_t I_t$. BCF argue that the measure of real output in the national income and product accounts is output at constant prices. They choose the base period price $p = 1$, the relative price of investment goods in the stationary equilibrium of the deterministic version of the model, and compute output as $Y_t = C_t + I_t$. The dynamics of the model is, thus, determined by (6.1)-(6.4), (6.6), (6.7) as well as (2.2a) and (3.2).

Equity Premium. The household's first-order conditions (6.7) imply that the gross rate of return on investment in sector C or I are given by:

$$R_{Ct+1} = \frac{p_{t+1}(1-\delta) + r_{Ct+1}}{p_t} = \frac{p_{t+1}(1-\delta) + \alpha Z_{t+1} N_{Ct+1}^{1-\alpha} K_{Ct+1}^{\alpha}}{p_t}, \tag{6.8a}$$

$$R_{It+1} = \frac{p_{t+1}(1-\delta) + r_{It+1}}{p_t} = \frac{p_{t+1}(1-\delta) + \alpha p_{t+1} Z_{t+1} N_{It+1}^{1-\alpha} K_{It+1}^{\alpha}}{p_t}. \tag{6.8b}$$

BCF compute the average gross rate of return on equity from

$$R_{t+1} = \frac{K_{Ct+1}}{K_{t+1}} R_{Ct+1} + \frac{K_{It+1}}{K_{t+1}} R_{It+1}. \tag{6.8c}$$

The risk free rate of return is the same expression as in the previous models, namely

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}}.$$

We compute the ex-post average equity premium from the time series average of $R_{t+1} - r_t$.

Results. Table 6.1 displays second moments from this model for two different parameter settings. The numbers in the first panel are from a simulation that used the parameter settings of BCF. The corresponding equity premium is 3.34 percent p.a. When we used the parameters from Table 2.1 we found an equity premium of almost 37 percent p.a. As it turned out, two parameters are responsible for this result: the intertemporal elasticity of substitution in consumption $1/\eta$ and the habit parameter χ^C. Assuming a logarithmic utility function in consumption (i.e., reducing η from 2 to 1) lowered the equity premium to about 10 percent p.a.. Setting $\chi^C = 0.756$ (instead of $\chi^C = 0.793$) brought the equity premia to our benchmark value of 5.18 percent p.a.

With respect to the labor market statistics the two sector model predicts a positive correlation between working hours and no correlation (for the second parameter set)

Table 6.1
Second Moments from the BCF Model

Variable	s_x	s_x/s_Y	r_{xY}	r_{xN}	r_x
$\beta = 0.99999$, $b = 0.73$, $\nu_1 = 0$, $\eta = 1.0$, $\alpha = 0.36$, $\delta = 0.021$.					
Output	1.84	1.00	1.00	0.94	0.80
Consumption	0.78	0.42	0.79	0.54	0.53
Investment	4.10	2.23	0.98	0.98	0.76
Total Hours	1.76	0.96	0.94	1.00	0.64
Total Capital	0.30	0.16	−0.13	−0.10	0.95
Real Wage	3.82	2.07	0.11	−0.24	−0.09
Relative Price	8.95	4.86	0.07	−0.27	−0.07
Rental Rate of Capital C	1.90	1.03	0.99	0.92	0.80
Rental Rate of Capital I	9.39	5.10	0.26	−0.07	0.02
$\beta = 0.994$, $b = 0.756$, $\nu_1 = 5.0$, $\eta = 1.0$, $\alpha = 0.27$, $\delta = 0.011$.					
Output	0.96	1.00	1.00	0.72	0.70
Consumption	0.81	0.84	0.95	0.51	0.55
Investment	2.25	2.36	0.83	0.90	0.88
Total Hours	0.14	0.14	0.72	1.00	0.64
Total Capital	0.10	0.11	−0.34	−0.22	0.96
Real Wage	2.52	2.64	0.68	0.00	0.05
Relative Price	11.21	11.72	0.44	−0.28	−0.07
Rental Rate of Capital C	1.00	1.04	1.00	0.72	0.71
Rental Rate of Capital I	11.71	12.24	0.51	−0.21	−0.05

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, where x stands for any of the variables from column 1. s_x/s_Y:=Standard deviation of variable x relative to standard deviation of output Y. r_{xY}:=Cross-correlation of variable x with output y, r_{xN}:=cross-correlation of variable x with hours N. r_x:=First order autocorrelation of variable x.

between hours and the real wage. In terms of our distance measure the model fares slightly worse than the one sector model with predetermined hours by the household (3.08 versus 3.04).

6.2 Integrated Technology Shocks

The Model. In the following, we consider the model of the previous paragraph for the case that the technical progress is a difference stationary stochastic process. This is the assumption of the original BCF model. We reformulate the production functions of both sectors accordingly:

$$C_t = (Z_t N_{Ct})^{1-\alpha} K_{Ct}^{\alpha}, \quad \alpha \in (0,1), \tag{6.9a}$$

$$I_t = (Z_t N_{It})^{1-\alpha} K_{It}^{\alpha}. \tag{6.9b}$$

The growth factor of Z_t is governed by:

$$\begin{aligned} z_t &:= \frac{Z_t}{Z_{t-1}}, \\ \ln z_t &= a + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2). \end{aligned} \tag{6.10}$$

Calibration. BCF employ the parameter values for $\bar{z}$ and σ from Christiano and Eichenbaum (1992). These authors equate $\bar{z}$ with the average growth rate of GDP and compute Z_t from actual data on output, hours, and the capital stock. Their measure of σ is the standard deviation of the growth rate of Z_t. We apply their method to our German data set on per capita GDP, hours, and the capital stock. Our estimate of $\bar{z}$ is 0.6 percent per quarter and $\sigma = 0.0101$ as compared to $\bar{z} = 0.44$ percent and $\sigma = 0.018$. The remaining parameters are set to the values present in the second panel of Table 6.1.

Results. The summary statistics from simulations of the model are presented in Table 1.1. As compared to the model of the previous subsection, the equity premium drops from 5.18 to 3.73 percent p.a. Investment, hours, and the real wage are less volatile, whereas the cross-correlations are almost unchanged as compared to the model of the previous section. With a score of 5.11 the model fits the data less accurately than the model with a stationary technology shock.

6.3 A Two Sector Adjustment Cost Model

The equity premium in the model of the previous two subsections results from variations of the relative price of two goods. In order to study the equity premium that results from variations in the firm value we introduce adjustment costs in the BCF model.

The Model. The representative household holds stocks S_{Xt} of both industries, where, as before, the index $X = C$ denotes consumption goods and $X = I$ refers to the investment goods sector. He chooses his labor supply before the period t shock is realized.

The budget constraint is:

$$v_{Ct}(S_{Ct+1} - S_{Ct}) + v_{It}(S_{It+1} - S_{It}) \leq w_{Ct}N_{Ct} + w_{It}N_{It} + d_{Ct}S_{Ct} + d_{It}S_{It} - C_t. \quad (6.11)$$

Maximizing (3.1) subject to (6.11) with respect to consumption C_t, labor supply N_{Ct+1}, N_{It+1}, S_{Ct+1}, and S_{It+1} yields the first-order conditions (2.2a), (6.7a), (6.7b), and

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{d_{Ct+1} + v_{Ct+1}}{v_{Ct}}, \quad (6.12a)$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{d_{It+1} + v_{It+1}}{v_{It}}, \quad (6.12b)$$

which determine his portfolio allocation.

Let

$$\varrho_{t+s} = \prod_{i=0}^{s} R_{ti}^{-1} = \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}, \quad R_{ti} = \begin{cases} 1 & \text{for } i = 0, \\ \frac{\Lambda_{t+i-1}}{\beta \Lambda_{t+i}} & \text{for } i = 1, 2, \dots. \end{cases} \quad (6.13)$$

denote the stochastic discount factor. The representative firm in the consumption goods sector maximizes

$$V_{Ct} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} \left[C_{t+s} - w_{Ct+s}N_{Ct+s} - p_{t+s}I_{t+s} \right] \quad (6.14)$$

subject to

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \quad \alpha \in (0, 1), \quad (6.15a)$$

$$K_{Ct+1} = \Phi(I_{Ct}/K_{Ct})K_{Ct} + (1 - \delta)K_{Ct}, \quad \delta \in (0, 1]. \quad (6.15b)$$

The first-order conditions for the optimal choice of N_{Ct}, I_{It}, and K_{Ct+1} are:

$$w_{Ct} = (1 - \alpha)Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \quad (6.16a)$$

$$q_{Ct} = \frac{p_t}{\Phi'(I_{Ct}/K_{Ct})}, \quad (6.16b)$$

$$q_{Ct} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \alpha Z_{t+1} N_{Ct+1}^{1-\alpha} K_{Ct+1}^{\alpha-1} - \frac{p_{t+1}I_{Ct+1}}{K_{Ct+1}} + q_{Ct+1} \left(\Phi(I_{Ct+1}/K_{Ct+1}) + 1 - \delta \right) \right\}, \quad (6.16c)$$

where q_{Ct} (Tobin's q) is the Lagrange multiplier on the equation governing capital accumulation. In addition, the transversality condition

$$\lim_{s \to \infty} \mathbb{E}_t \varrho_{t+s} q_{Ct+s} K_{Ct+s+1} = 0$$

must hold. As in the one-sector model of Section 3, it can be shown that $V_{Ct} = q_{Ct}K_{Ct+1}$.

Analogously, the representative firm in the investment goods sector maximizes

$$V_{It} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} \left[p_{t+s}I_{t+s} - w_{It+s}N_{It+s} - p_{t+s}I_{It+s}\right] \tag{6.17}$$

subject to

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha}, \quad \alpha \in (0,1), \tag{6.18a}$$

$$K_{It+1} = \Phi(I_{It}/K_{It})K_{It} + (1-\delta)K_{It}, \quad \delta \in (0,1]. \tag{6.18b}$$

The respective first-order conditions are:

$$w_{It} = (1-\alpha)Z_t N_{It}^{-\alpha} K_{It}^{\alpha}, \tag{6.19a}$$

$$q_{It} = \frac{p_t}{\Phi'(I_{It}/K_{It})}, \tag{6.19b}$$

$$q_{It} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg\{ p_{t+1}\alpha Z_{t+1} N_{It+1}^{1-\alpha} K_{It+1}^{\alpha-1} - \frac{p_{t+1}I_{It+1}}{K_{It+1}} + q_{It+1}\left(\Phi(I_{It+1}/K_{It+1}) + 1 - \delta\right) \Bigg\}. \tag{6.19c}$$

Firms from both sectors transfer their profits less retained earnings as dividends to the household sector

$$d_{Ct}S_{Ct} = C_t - w_{Ct}N_{Ct} - RE_{Ct},$$

$$d_{It}S_{It} = p_tI_t - w_{It}N_{It} - RE_{It},$$

and finance the remaining investment expenditures by issuing new equity $v_{Xt}(S_{Xt+1} - S_{Xt}) = p_tI_{Xt} - RE_{Xt}$. Thus, in equilibrium, the budget constraint of the household implies the definition of GDP, $Y_t = C_t + p_tI_t$.

Results. We compute the equity premium of each sector in the same way as in the one sector model of Section 5, i.e.,

$$R_{Ct+1} = \frac{C_{t+1} - w_{Ct+1}N_{Ct+1} - p_{t+1}I_{Ct+1} + q_{Ct+1}K_{Ct+2}}{q_{Ct}K_{Ct+1}}, \tag{6.20a}$$

$$R_{It+1} = \frac{p_{t+1}I_{t+1} - w_{It+1}N_{It+1} - p_{t+1}I_{It+1} + q_{It+1}K_{It+2}}{q_{It}K_{It+1}}, \tag{6.20b}$$

Table 6.2

Calibration of the Two Sector Adjustment Cost Model

Preferences	β=0.994	χ^N=0.756	η=1	N=0.13
	ν_1=5.0			
Production	α=0.27	δ=0.011	ρ^Z=0.90	σ^Z=0.0072
	ζ=5.53			

are the gross rates of return on equity in the consumption goods and the investment goods sector, respectively. The average gross rate of return is the weighted average of these rates with the respective shares of capital employed in each sector as weights.

Except for the values of η and χ^C, which we set at the values used in the previous subsection, we use the parameter settings presented in Table 2.1. For convenience, we summarize our choice of parameter values in Table 6.2.

Table 1.1 presents second moments from simulations of the model. Note the following:

1) Different from the one sector model (see Table 5.1), the model is not able to generate the well-documented fact that investment is about 2 to 3 times more volatile than output.

2) As the one sector adjustment cost model, the model predicts that output and hours are negatively correlated.

3) The average equity premium implied by the model is about one percentage point below our benchmark value predicted by the BCF model.

Among the three different two sector models considered in this section, the model has the worst test score.

7 Time to Plan

In this section we consider yet another way to explain the equity premium. We embed a consumption habit in the model of Heer and Maußner (2009), Section 2.6.2. This model is a stripped down version of the Kydland and Prescott (1982) model of economic fluctuations. The parameterization of the investment equation follows Beaubrun-Diant (2005), who employs the time-to-plan model of Christano and Todd (1996) to investigate the equity premium puzzle.

Households. At time t the representative household maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, C_{t+s-1}, N_{t+s}) \tag{7.1}$$

subject to the budget constraint

$$v_t A_t (S_{t+1} - S_t) \leq w_t A_t N_t + A_t d_t S_t - C_t. \tag{7.2}$$

v_t, w_t, and d_t are the share price, the real wage, and dividends per share, all measured per unit of A_t, the level of labor augmenting technical progress, which evolves according to

$$A_{t+s} = a^s \tag{7.3}$$

This yields the first-order conditions:

$$\Lambda_t = u_1(C_t, C_{t-1}, N_t) + \beta \mathbb{E}_t u_2(C_{t+1}, C_t, N_{t+1}), \tag{7.4a}$$

$$\Lambda_t w_t A_t = -u_3(C_t, C_{t-1}, N_t), \tag{7.4b}$$

$$\Lambda_t = a\beta \mathbb{E}_t \Lambda_{t+1} \frac{d_{t+1} + v_{t+1}}{v_t}, \tag{7.4c}$$

where u_i denotes the partial derivative of u with respect to its i-th argument and where Λ_t is the Lagrange multiplier of the budget constraint (7.2).

We will explore two different current-period utility functions. The function that we used in the previous subsections with $\eta = 1$ to allow for a balanced growth path

$$u(C_t, C_{t-1}, N_t) = \ln(C_t - \chi^C C_{t-1}) - \nu_0 N_t^{1+\nu_1}, \quad \chi^C \in [0, 1),\ \nu_0, \nu_1 \geq 0, \tag{7.5a}$$

and

$$u(C_t, C_{t-1}, N_t) = \frac{(C_t - \chi^C C_{t-1})^{1-\eta}(1 - N_t)^{\theta(1-\eta)} - 1}{1 - \eta}, \quad \chi^C \in [0, 1),\ \eta, \theta \geq 0. \tag{7.5b}$$

Firms. The firm maximizes its current value V_t. By using (6.13), this can be written as:

$$V_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[Z_{t+s}(A_{t+s}N_{t+s})^{1-\alpha} K_{t+s}^{\alpha} - w_{t+s}A_{t+s}N_{t+s} - I_{t+s} \right] \tag{7.6}$$

subject to

$$I_t = \sum_{i=1}^{4} \omega_i X_{it}, \quad \sum_{i=1}^{4} \omega_i = 1, \tag{7.7a}$$

$$K_{t+4} = (1-\delta)K_{t+3} + X_{4t}, \tag{7.7b}$$
$$X_{1t+1} = X_{2t}, \tag{7.7c}$$
$$X_{2t+1} = X_{3t}, \tag{7.7d}$$
$$X_{3t+1} = X_{4t}. \tag{7.7e}$$

The time-to-build model assumes that the resource costs are equally spread over the construction period so that $\omega_i = 0.25\ \forall i = 1, 2, 3, 4$. The time-to-plan model instead assumes that in the start-up phase little resources are required. Thus, $\omega_4 = 0.01$ and $\omega_i = 0.33\ \forall i = 2, 3, 4$. This is the parameterization which we will employ here.

Results. The household's Euler equation (7.4c) implies that the return on equity equals

$$R_{t+1} = a\frac{d_{t+1} + v_{t+1}}{v_t}$$

and that the risk free rate is

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}} = \frac{\lambda_t}{\beta a^{-\eta} \mathbb{E}_t \lambda_{t+1}}$$

with $\lambda_t := \Lambda_t A_t^\eta$.

We can use the stationary version of (7.4c) to determine v_t. With $y_t \equiv (Y_t/A_t)$ and $i_t \equiv (I_t/A_t)$ dividends are given by $d_t = y_t - w_t N_t - i_t$ if the firm entirely finances its investment from retained earnings.

Calibration. Besides the weights ω_i, which implement the time to plan assumption, the model has just on extra parameter, the growth factor of labor augmenting technical progress. Heer and Maußner (2009) estimate $a = 1.005$ from quarterly German data between 1975-1989. As before, ν_0 and θ are set in order to imply $N = 0.13$ in the stationary equilibrium. All other parameters are set as in Table 2.1.

Results. Table 7.1 displays selected second moments from simulations of both versions of the model. The more curved utility function (7.5b) implies that output and hours are less volatile than in the case of utility function (7.5a) with $\eta = 1$. As compared to the adjustment cost model of Section 5 investment is more volatile. Contrary to the former model, however, output and hours are positively correlated, as they are in the data. Both utility functions imply that hours and the real wage are negatively correlated, and both generate an equity premium close to zero: 0.02 for (7.5a) and 0.04 for (7.5b).

Table 7.1
Second Moments from the Time to Plan Model

Variable	s_x	s_x/s_Y	r_{xY}	r_{xN}	r_x
			Utility function (7.5a)		
Output	1.89	1.00	1.00	0.90	0.69
Consumption	0.24	0.13	0.61	0.44	0.91
Investment	9.21	4.86	1.00	0.91	0.67
Hours	1.98	1.05	0.90	1.00	0.34
Real Wage	0.86	0.45	0.12	−0.31	0.09
			Utility function (7.5b), $\eta = 2$		
Output	1.57	1.00	1.00	0.85	0.70
Consumption	0.22	0.14	0.64	0.40	0.89
Investment	7.59	4.82	1.00	0.86	0.67
Hours	1.58	1.01	0.85	1.00	0.24
Real Wage	0.88	0.56	0.26	−0.29	0.20

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, where x stands for any of the variables from column 1. s_x/s_Y:=Standard deviation of variable x relative to standard deviation of output Y. r_{xY}:=Cross-correlation of variable x with output Y. r_{xN}:=Cross-correlation of variable x with hours N. r_x:=First order autocorrelation of variable x.

8 A Neo-Keynesian Model with Sticky Prices

In the following two sections, we will study two monetary models with nominal rigidities. First, we introduce frictions in the form of price staggering, before we analyze a model of sticky wages in the next section.

In this section, we consider a slightly simplified version of the model of De Paoli, Scott, and Weeken (2010). They build on the model described in Section 4 and introduce money via the household's utility function. Money prices do not adjust perfectly due to convex costs of price adjustment. However, these costs are modeled as intangible, i.e., they appear in the firms objective function but do not reduce the firm's output.

Households. Households enter the current period t with a given amount of firm shares S_t and given stocks of nominal Bonds B_t.[9] The current price level is P_t. Bonds

[9]The original model also considers the stock of money. However, since monetary policy is modeled via a Taylor rule and since real money balances enter the current period utility function additively, the time path of money holdings does not interfere with the rest of the model. Therefore, we strip

pay a predetermined nominal rate of interest $Q_t - 1$. The real share price is v_t and real dividend payments per share are d_t. Firms pay the real wage w_t per unit of working hours N_t. Thus,

$$v_t(S_{t+1} - S_t) + \frac{B_{t+1} - B_t}{P_t} \leq w_t N_t + (Q_t - 1)\frac{B_t}{P_t} + d_t S_t - C_t \tag{8.1}$$

is the household's budget constraint. Households maximize (3.1) subject to (8.1) and given initial values of S_t and B_t.

De Paoli, Scott, and Weeken (2010) assume that the household treats previous consumption C_{t-1} and previous working hours N_{t-1} as given, when he decides on current consumption and working hours. Thus, different from equations (2.2a) and (4.2), the first order conditions are:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \tag{8.2a}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1}, \tag{8.2b}$$

$$v_t = \beta \mathbb{E}_t \Lambda_{t+1}(v_{t+1} + d_{t+1}), \tag{8.2c}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}, \tag{8.2d}$$

where Λ_t is the Lagrange multiplier of the time t budget constraint.

Firms. Final output Y_t is produced from differentiated inputs $Y_t(j)$ distributed on the unit interval according to the function

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon - 1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon - 1}}, \quad \epsilon > 1. \tag{8.3}$$

The zero-profit condition

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$$

implies the usual demand function for the intermediate product $Y_t(j)$:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t, \tag{8.4}$$

and the price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{8.5}$$

down the presentation of the model. The full version is considered in the Appendix.

Consider an arbitrary producer of intermediate product $j \in [0,1]$. His production function is

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} K_t(j)^{\alpha}, \quad \alpha \in (0,1), \tag{8.6}$$

where total factor productivity Z_t is common to all producers and evolves as stated in equation (2.4). The producer finances investment $I_t(j)$ out of retained earnings and distributes the remaining surplus as dividends:

$$D_t(j) = Y_t j - w_t N_t(j) - I_t(j). \tag{8.7}$$

Capital accumulation is subject to adjustment costs so that

$$K_{t+1}(j) = (1-\delta)K_t(j) + \Phi\left(\frac{I_t(j)}{K_t(j)}\right) K_t(j), \tag{8.8}$$

with $\Phi(\cdot)$ specified in equation (2.8). Producer j determines his nominal price $P_t(j)$, demand for labor $N_t(j)$, and investment expenditures $I_t(j)$ to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[D_{t+s}(j) - \frac{\psi}{2}\left(\frac{P_{t+s}(j)}{\pi P_{t+s-1}(j)} - 1\right)^2 Y_{t+s} \right]$$

subject to (8.4), (8.6)-(8.8), and a given initial stock of capital $K_t(j)$. In this expression π denotes the inflation factor in a stationary environment without exogenous shocks. Also note, that the convex cost function in this expression indicates intangible costs, since it appears in the objective function of the producer but does not reduce his profits.

Let Γ_t denote the Lagrange multiplier in minimizing production costs subject to the production function.[10] The first-order conditions are given by:

$$w_t = (1-\alpha)\Gamma_t Z_t N_t(j)^{-\alpha} K_t(j)^{\alpha}, \tag{8.9a}$$

$$q_t = \frac{1}{\Phi'(I_t(j)/K_t(j))}, \tag{8.9b}$$

$$\begin{aligned} q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg[& \alpha \Gamma_{t+1} Z_{t+1} N_{t+1}(j)^{1-\alpha} K_{t+1}(j)^{\alpha-1} - \frac{I_{t+1}(j)}{K_{t+1}(j)} \\ & + q_{t+1}\left(1-\delta+\Phi(I_{t+s}(j)/K_{t+1}(j))\right) \Bigg], \end{aligned} \tag{8.9c}$$

$$\begin{aligned} 0 = {} & \epsilon \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} - \psi \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right) \frac{Y_t}{\pi P_{t-1}(j)} \\ & + \epsilon \Gamma_t \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon-1} \frac{Y_t}{P_t} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right) \frac{P_{t+1}(j) Y_{t+1}}{\pi P_t(j)^2}. \end{aligned} \tag{8.9d}$$

[10] This multiplier is independent of the firm index j, since all firms face the same wages and rental prices for capital and since the production function is linear homogenous.

Monetary Policy. The central bank sets the nominal interest rate Q_{t+1} according to the Taylor-rule

$$Q_{t+1} = Q_t^{\delta_1} \left(\frac{\pi}{\beta}\right)^{1-\delta_1} \left(\frac{\pi_t}{\pi}\right)^{\delta_2} e^{\epsilon_t^Q}, \quad \delta_1 \in [0,1), \quad \epsilon_t^Q \sim N(0, \sigma^Q). \tag{8.10}$$

The elasticity of Q_{t+1} with respect to the deviation of the inflation factor π_t from its steady state value π will be chosen so that the equilibrium is determinate. Usually, this requires $\delta_2 > 1$.

Calibration. The model has several additional parameters. We assume an inflation target of zero and set χ^N equal to our benchmark value of $\chi^C = 0.793$. Linnemann (1999) presents estimates of markups for Germany, which imply a price elasticity of $\epsilon = 6.0$. We compare the flexible price version of the model obtained from $\psi = 0$ with the sticky price version for $\psi = 77$, the value used by De Paoli, Scott, and Weeken (2010). This implies that the intangible cost of a one percent increase of the price amounts to less than 0.4 percent of the firm's value added. We consider two different interest rate rules: without persistence, $\delta_1 = 0$, and with $\delta_1 = 0.75$, the value used by De Paoli, Scott, and Weeken (2010). In addition, we simulate the model for three different values of the standard deviation of the innovation in (8.10).

Results. The equity premium in the model with flexible prices amounts to 1.06 percentage points.[11] It does not dependent on the parameters of the Taylor rule. When we use the parameter values employed by De Paoli, Scott, and Weeken (2010) in the flexible price model the premium increases to 2.34 percent p.a.[12] Sticky prices reduce the volatility of output, investment, and hours (see Table 8.1), and, thus, asset returns become less risky. As a consequence, the equity premium declines. As can bee seen from Table 8.1, the premium increases with the relative importance of monetary policy shocks, as measured by the ratio of the standard deviations of the innovations in (8.10) and (2.4).

An interest rate shock reduces output and increases profits whereas a technology shock increases both output and profits. Depending on the relative strength of both shocks,

[11] Monopolistic price setting introduces a wedge between the marginal product of labor and the real wage (see equations (8.9a) and (8.9c)). As a consequence, $\alpha\Gamma_{t+1}Z_{t+1}N_{t+1}^{1-\alpha}K_{t+1}^{\alpha} \neq Y_{t+1} - w_{t+1}N_{t+1}$ and $v_t \neq q_t K_{t+1}$ so that equation (3.3) cannot be used to compute the equity premium. Instead, $R_{t+1} = (v_{t+1} + d_{t+1})/v_t$ has to be used.

[12] The respective parameter values are $\beta = 0.99$, $\eta = 5.0$, $\chi^C = 0.8$, $\chi^N = 0.8$, $\nu_1 = 2.5$, $\alpha = 0.36$, $\zeta = 3.33$, $\delta = 0.025$, $\rho^Z = 0.95$, $\sigma^Z = 0.01$, $\epsilon = 6$.

Table 8.1

Second Moments from the Sticky Price Model

σ_Q/σ_Z	δ_1	Equity premium	s_Y	s_I/s_Y	s_w/s_Y	r_{NY}	r_{Nw}
		Flexible Prices					
		1.06	0.73	1.18	1.32	−0.98	−0.99
		Sticky Prices $\psi = 77$					
0.5	0.75	0.14	0.39	0.9	42.11	−0.57	0.59
1.0	0.75	0.20	0.44	0.96	39.49	−0.31	0.59
2.0	0.75	0.41	0.61	1.04	34.85	0.19	0.60
0.5	0.0	0.26	0.37	1.01	34.06	−0.85	0.58
1.0	0.0	0.29	0.38	1.02	34.20	−0.82	0.58
2.0	0.0	0.37	0.39	1.06	34.62	−0.72	0.58

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, where $x \in \{Y, I, w\}$, and Y, I, and w denote output, investment, and the real wage respectively. r_{NY}:=Cross-correlation of variable hour N with output Y. r_{Nw}:=Cross-correlation of variable of hours N with the real wage w.

profits can be counter-cyclical, which is at odds with empirical observations (see, for example, Christiano, Eichenbaum, and Evans (1999)). In the simulations reported in Table 8.1 profits are acyclical with correlation between 0 and -0.05.

In the sticky price version of the model, hours and the real wage are positively correlated as in the data. Furthermore, if monetary policy shocks are relatively more important than technology shocks and are sufficiently persistent, also the negative correlation between output and hours disappears. Yet, the relative volatility of the real wage is far beyond any reasonable empirical bound.

9 A Neo-Keynesian Model with Sticky Wages

As our second model with nominal frictions, we set up a model with wage staggering as introduced by Erceg, Henderson, and Levin (2000). From the previous section we borrow the modeling of the government sector. The production sector is the same as in Section (2) with one exception to which we turn next.

Labor Demand. Labor input N_t in production $Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}$ is an index of the different types of labor $N_t(h)$ supplied by the members $h \in [0, 1]$ of the representative household:

$$N_t = \left[\int_0^1 N_t(h)^{\frac{\epsilon_w - 1}{\epsilon_w}} dh \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \epsilon_w > 1. \tag{9.1}$$

Let W_t denote the nominal wage rate at date t and $W_t(h)$ the wage paid to labor of type h. Minimizing the wage bill

$$W_t N_t = \int_0^1 W_t(h) N_t(h) dh$$

subject to (9.1) yields the demand function for labor and the wage index:

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\epsilon_w} N_t, \tag{9.2}$$

$$W_t = \left[\int_0^1 W_t(h)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}}. \tag{9.3}$$

Since everything else is unchanged, conditions (2.11) continue to describe the firm's optimal decisions with respect to capital accumulation and aggregate labor demand N_t, where $w_t = W_t/P_t$ on the left hand side of (2.11a). As in the previous section P_t denotes the money price of output Y_t.

Wage Setting. The preferences of household member $h \in [0, 1]$ are:[13]

$$u(C_t(h), C_{t-1}(h), N_t(h)) \equiv \frac{(C_t(h) - \chi^C C_{t-1}(h))^{1-\eta} - 1}{1 - \eta} - \frac{\nu_0}{1 + \nu_1} N_t(h)^{1+\nu_1}, \tag{9.4}$$
$$\eta, \nu_0, \nu_1 > 0, \chi^C \in [0, 1),$$

where $C_t(h)$ denote consumption of household member h.

In each period a fraction φ_w of households updates their wage rate according to the steady state inflation factor π:

$$W_t(h) = \pi W_{t-1}(h). \tag{9.5}$$

The fraction $1 - \varphi_w$ of the households can choose their wage rate $W_t(h)$ optimally. These households maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \varphi_w)^s u(C_{t+s}(h), C_{t+s-1}(h), N_{t+s}(h)) \tag{9.6}$$

[13] As in the previous section we do not model the demand for money.

subject to the series of budget constraints

$$\begin{aligned}&\frac{W_{t+s}(h)}{P_{t+s}}N_{t+s}(h)+S_{t+s}d_{t+s}(h)+(Q_{t+s}-1)\frac{B_{t+s}(h)}{P_{t+s}}-C_{t+s}(h)\\&\geq\frac{B_{t+s+1}(h)-B_{t+s}(h)}{P_{t+s}}+v_{t+s}(S_{t+s+1}(h)-S_{t+s}(h)),\end{aligned}\tag{9.7}$$

and the demand function (9.2). As before, d_t are dividends per share S_t with price v_t and B_t are bonds in money units that earn the nominal interest rate Q_t-1. The maximand (9.6) is the expected life time utility assuming that the household were never able to readjust its wage after period t. We assume that there is a sufficiently rich set of contingent security markets so that a representative agent exists. Therefore, all wage setters will opt for the same relative wage $w_{At}\equiv W_t(h)/W_t$. In the Appendix we show that this wage is determined by the set of equations:

$$w_{At}=\frac{\epsilon_w}{\epsilon_w-1}\frac{\Gamma_{1t}}{\Gamma_{2t}},\tag{9.8a}$$

$$\Gamma_{1t}=\nu_0 w_{At}^{-\epsilon_w(1+\nu_1)}N_t^{1+\nu_1}+\beta\varphi_w\mathbb{E}_t\left(\frac{\pi w_{At}}{\omega_{t+1}w_{At+1}}\right)^{-\epsilon_w}\Gamma_{1t+1},\tag{9.8b}$$

$$\Gamma_{2t}=\Lambda_t w_t w_{At}^{-\epsilon_w}N_t+\beta\varphi_w\left(\frac{\pi}{\omega_{t+1}}\right)^{1-\epsilon_w}\left(\frac{w_{At}}{w_{At+1}}\right)^{-\epsilon_w}\Gamma_{2t+1},\tag{9.8c}$$

$$w_t=\frac{W_t}{P_t}\equiv\frac{\omega_t}{\pi_t}w_{t-1},\tag{9.8d}$$

$$1=(1-\varphi_w)w_{At}^{1-\epsilon_w}+\varphi_w(\pi/\omega_t)^{1-\epsilon_w},\tag{9.8e}$$

$$\omega_t=\frac{W_t}{W_{t+1}}.\tag{9.8f}$$

Consumption and Portfolio Choice. The pooling assumption allows us to derive the demand for consumption, bonds, and stocks from maximizing

$$\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s u(C_{t+s},C_{t+s-1},N_{t+s})$$

subject to the sequence of budget constraints

$$w_{t+s}N_{t+s}+S_t d_{t+s}+(Q_{t+s}-1)\frac{B_{t+s}}{P_{t+s}}-C_{t+s}\geq\frac{B_{t+s+1}-B_{t+s}}{P_{t+s}}+v_{t+s}(S_{t+s+1}-S_{t+s}).$$

The respective first-order conditions coincide with (2.2a), (8.2c), (8.2d).[14]

[14]Different from de Paoli, Scott, and Weeken (2010) we return to our assumption from Section 2 that the consumption habit is endogenous in the households decision on consumption.

Dynamics. The equilibrium conditions of the model consist of the firm's optimality conditions stated in (2.11), the production function (2.3), the capital accumulation equation (2.7), the economy's resource constraint implied by the household's budget constraint, the wage setting equations (9.8a)-(9.8d), and the household's optimality conditions (2.2a), (8.2c),(8.2d), and the Taylor rule (8.10). These conditions determine the time path of Y_t, C_t, I_t, N_t, K_t, w_t, w_{At}, ω_t, Q_t, π_t, q_t, Λ_t, Γ_{1t}, and Γ_{2t}.

Results. The equity premium implied by this model can be computed as in Section (3) from equation (3.3). The model has two new parameters, the wage markup implied by ϵ_w and the degree of wage stickiness determined by ϕ_w. We set ϵ_w equal to 6 so that the wage markup is 20 percent. The value of $\phi_w = 0.75$ implies that wage adjustment requires about one year, which is the usual length of wage contracts signed by German trade unions and employer's federations. As in the previous section we simulate the model for different values of the parameters in the Taylor rule (8.10). All other parameters are chosen as in Table 2.1. Table 9.1 summarizes our results.

Table 9.1

Second Moments from the Sticky Wage Model

σ_Q/σ_Z	δ_1	Equity premium	s_Y	s_I/s_Y	s_w/s_Y	r_{NY}	r_{Nw}
		Flexible Wages					
		0.58	0.51	1.46	2.07	−0.68	−0.94
		Sticky Wages					
0.5	0.75	0.67	0.40	1.94	1.91	−0.76	−0.97
1.0	0.75	0.83	0.44	1.91	1.83	−0.51	−0.94
2.0	0.75	1.37	0.58	1.84	1.66	0.02	−0.85
0.5	0.0	0.10	0.08	3.28	15.49	−0.20	−1.00
1.0	0.0	0.13	0.08	3.33	14.60	−0.18	−1.00
2.0	0.0	0.23	0.10	3.43	12.09	−0.11	−1.00

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, where $x \in \{Y, I, w\}$, and Y, I, and w denote output, investment, and the real wage respectively. r_{NY}:=Cross-correlation of variable hour N with output Y. r_{Nw}:=Cross-correlation of variable of hours N with the real wage w.

The flexible wage version of the model differers from the model of section 3 only in one respect: the inefficient allocation of labor introduced by monopolistic wage setting. Thus, it is not surprising that the time series properties of both models are very similar.

Different from the sticky price model, an expansionary, i.e., a negative interest rate shock increases hours, output, and profits. Output and profits also move together in the wake of a positive technology shock. Thus, asset returns are relatively more negatively correlated with the stochastic discount factor and investors demand a higher equity premium.

Note also that the volatility of output and thus the size of the equity premium strongly depend on the persistence of the monetary policy and increase with relative importance of monetary policy shocks σ^Q/σ^Z.

Since hours and output move in the same direction after a monetary policy shock and move in opposite directions after a technology shock, the negative correlation between hours and output vanishes if interest rate shocks are relatively more important than technology shocks. Different from the sticky price model (where an interest rate shock increases the real wage and working hours) and contrary to the sign of the correlation found in the data, the real wage and average working hours are negatively correlated

With respect to the sum of squared deviations from the empirical moments presented in Table 1.1 the sticky wage model with $\delta_1 = 0.75$ and $\sigma^Q/\sigma^Z = 2.0$ slightly outperforms the model from Section 5 (see the rightmost column of Table 1.1).

10 Conclusion

We have evaluated the current-state of the art business-cycle models that try to replicate the empirically observed equity premium with regard to their labor market behavior. In addition to the current studies, we also analyzed a model of the equity premium with sticky wages.

As our main result, we find that, except for the two-sector model with growth and the sticky price model, none of the models is able to account for both the positive correlation between hours and output and the slightly positive correlation between hours and the real wage. Yet the sticky price model is unable to account for the relative volatility of hours and wages (compare Table 1.1). The model predicts that hours and real wages are about twice and 35 times more volatile than found in the data (expressed relative to the output volatility). All models with adjustment costs of capital predict a negative correlation between hours and output. The time to plan and the two-sector model of Boldrin, Christiano and Fisher (2001) do not share this property. Yet, the time to plan model is unable to generate a non-negligible equity premium, and the equity premium in the BCF model results from changes in the relative price of the consumption and the investment good rather than from changes in the firm value. Furthermore, the BCF model is sensitive to the calibration. We had to adjust our benchmark parameter values used throughout the paper to get a reasonable equity premium from this model. For instance, for $\eta = 2$ and $\chi^C = 0.793$ (the values from Table 2.1) the models predicts an equity premium of almost 37 percent p.a.

Further results are:

- The model with habit in consumption and leisure is not able to generate the observed equity premium even in the presence of capital adjustment costs.
- The same applies to the New-Keynesian models even though the economy is subject to both a technology shock and a monetary shock.

In future work, we are planning to improve the behavior of the above models with respect to the labor market. This feature can become crucial, for example, if one is considering the behavior of monetary policy in a New-Keynsian model where the monetary authority is also considering asset prices. As an example, Lim and McNelis (2008) introduce q-targeting into the Taylor rule for the monetary authority. Possible candidates for modifications of the existing models include the consideration of labor market search.

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Appendix

In the following, we summarize the equilibrium conditions for the various model types that are not reported in the main part of the paper.

A.1 Endogenous Labor Supply

The model with endogenous labor supply in Section 3 is described by the following equilibrium conditions:

$$\nu_0 N_t^{\nu_1} = \Lambda_t w_t, \tag{A.1.1a}$$
$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.1.1b}$$
$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.1.1c}$$
$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.1.1d}$$
$$Y_t = C_t + I_t, \tag{A.1.1e}$$
$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta b \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.1.1f}$$
$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda t} \Big\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta \big] \Big\}, \tag{A.1.1g}$$
$$K_{t+1} = \Phi(I_t/K_t) K_t + (1-\delta) K_t. \tag{A.1.1h}$$

In the stationary equilibrium, equation (A.1.1f) reduces to

$$\frac{K}{N} = \left(\frac{1 - \beta(1-\delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}}. \tag{A.1.2}$$

For $N = 0.13$, equations (A.1.2) allows us to infer K, and we can compute the stationary values of the remaining variables in the same way is in the model of the previous section. Finally, equation (A.1.1a) allows us to fix the value of ν_0.

A.2 Habit in Leisure

The model with habit in leisure in Section 4 is described by the following equilibrium conditions:

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.2.1a}$$
$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.2.1b}$$
$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.2.1c}$$
$$Y_t = C_t + I_t, \tag{A.2.1d}$$

$$\Lambda_t w_t = \nu_0(N_t - \chi^N N_{t-1})^{\nu_1} - \beta\nu_0\chi^N(N_{t+1} - \chi^N N_t)^{\nu_1}, \tag{A.2.1e}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta\chi^C \mathbb{E}_t(C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.2.1f}$$

$$q_t = \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda t}\Big\{\alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta\big]\Big\}, \tag{A.2.1g}$$

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t. \tag{A.2.1h}$$

In the stationary equilibrium, equations (A.2.1c) and (A.2.1g) imply

$$\frac{K}{N} = \left(\frac{1-\beta(1-\delta)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}}. \tag{A.2.2}$$

Equation (A.2.2) allows us to infer K with the help of $N = 0.13$, and we can compute the stationary values of the remaining variables in the same way as in the model of the previous section. Finally, equation (A.2.1e) allows us to fix the value of ν_0 for given value of d.

A.3 Two-Sector Model with Predetermined Working Hours by the Households

The entire two-sector model where households decide upon their labor supply prior to the observation of the technology shock consists of the equations:

$$w_{Ct} = (1-\alpha)Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \tag{A.3.1a}$$

$$w_{It} = p_t(1-\alpha)Z_t N_{It}^{-\alpha} K_{It}^{\alpha}, \tag{A.3.1b}$$

$$w_t = \frac{N_{Ct}}{N_t} w_{Ct} + \frac{N_{It}}{N_t} w_{It}, \tag{A.3.1c}$$

$$r_{Ct} = \alpha Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha-1}, \tag{A.3.1d}$$

$$r_{It} = p_t \alpha Z_t N_{It}^{1-\alpha} K_{It}^{\alpha-1}, \tag{A.3.1e}$$

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \tag{A.3.1f}$$

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha}, \tag{A.3.1g}$$

$$Y_t = C_t + I_t, \tag{A.3.1h}$$

$$N_t = N_{Ct} + N_{It}, \tag{A.3.1i}$$

$$K_t = K_{Ct} + K_{It}, \tag{A.3.1j}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{Ct+1}, \tag{A.3.1k}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{It+1}, \tag{A.3.1l}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta\chi^C \mathbb{E}_t(C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.3.1m}$$

$$p_t\Lambda_t = \beta\mathbb{E}_t\lambda_{t+1}\left(p_{t+1}(1-\delta) + r_{Ct+1}\right), \tag{A.3.1n}$$

$$p_t\Lambda_t = \beta\mathbb{E}_t\lambda_{t+1}\left(p_{t+1}(1-\delta) + r_{It+1}\right), \tag{A.3.1o}$$

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{A.3.1p}$$

A.4 Two-Sector Model with Stochastic Trend

In order to compute linear or quadratic approximate solutions of the model, we must transform it into a model in stationary variables. However, this requires $\eta = 1$, the assumption used by BCF. It will be convenient to put

$$\begin{aligned} x_t &:= \frac{X_t}{Z_{t-1}}, \quad X_t \in \{K_t, K_{Ct}, K_{It}, Y_t, C_t, I_t, w_{Ct}, w_{It}\}, \\ \lambda_t &:= \Lambda_t Z_{t-1}. \end{aligned} \tag{A.4.1}$$

This allows us to transform equations (A.3.1) in into the following system:[15]

$$w_{Ct} = (1-\alpha) z_t^{1-\alpha} N_{Ct}^{-\alpha} k_{Ct}^{\alpha}, \tag{A.4.2a}$$
$$w_{It} = p_t (1-\alpha) z_t^{1-\alpha} N_{It}^{-\alpha} k_{It}^{\alpha}, \tag{A.4.2b}$$
$$w_t = \frac{N_{Ct}}{N_t} w_{Ct} + \frac{N_{It}}{N_t} w_{It}, \tag{A.4.2c}$$
$$r_{Ct} = \alpha z_t^{1-\alpha} N_{Ct}^{1-\alpha} K_{Ct}^{\alpha-1}, \tag{A.4.2d}$$
$$r_{It} = p_t \alpha z_t^{1-\alpha} N_{It}^{1-\alpha} K_{It}^{\alpha-1}, \tag{A.4.2e}$$
$$c_t = z_t^{1-\alpha} N_{Ct}^{1-\alpha} k_{Ct}^{\alpha}, \tag{A.4.2f}$$
$$i_t = z_t^{1-\alpha} N_{It}^{1-\alpha} k_{It}^{\alpha}, \tag{A.4.2g}$$
$$y_t = c_t + i_t, \tag{A.4.2h}$$
$$N_t = N_{Ct} + N_{It}, \tag{A.4.2i}$$
$$k_t = k_{Ct} + k_{It}, \tag{A.4.2j}$$
$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \lambda_{t+1} w_{Ct+1}, \tag{A.4.2k}$$
$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \lambda_{t+1} w_{It+1}, \tag{A.4.2l}$$
$$\lambda_t = (c_t - \chi^C (c_{t-1}/z_{t-1}))^{-\eta} - \beta \chi^C \mathbb{E}_t (z_t c_{t+1} - \chi^C c_t)^{-\eta}, \tag{A.4.2m}$$
$$p_t \lambda_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{z_t} \left(p_{t+1}(1-\delta) + r_{Ct+1} \right), \tag{A.4.2n}$$
$$p_t \lambda_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{z_t} \left(p_{t+1}(1-\delta) + r_{It+1} \right), \tag{A.4.2o}$$
$$z_t k_{t+1} = i_t + (1-\delta) k_t. \tag{A.4.2p}$$

In a stationary environment (i.e., when $z_t \equiv z$ and all scaled variables are constant) equations (A.4.2n) and (A.4.2o) imply $r = r_C = r_I$. Together with the stationary versions of (A.4.2a) and (A.4.2b) this implies $k_C/N_c = k_I/N_I = k/N$ and $p = 1$. This allows one to compute k/N from (A.4.2d) as:

$$\frac{k}{N} = \left(\frac{z - \beta(1-\delta)}{\alpha \beta z^{1-\alpha}} \right)^{1/(\alpha-1)}. \tag{A.4.3a}$$

[15] Note, that (A.4.1) implies that we redefine wages without using new symbols.

Given the stationary value of average hours N, this equation also implies k. We can then use (A.4.2p) to find

$$i = (z - 1 + \delta)k. \tag{A.4.3b}$$

From $i = z^{1-\alpha}N_I(k/N)^{\alpha}$ and $i = z^{1-\alpha}k_I(k/N)^{\alpha-1}$, we get the stationary values of N_I and k_I, and, therefore $N_C = N - N_I$ and $k_C = k - k_I$. Given these solutions, $c = z^{1-\alpha}N_C^{1-\alpha}k_C^{\alpha}$ so that (A.4.2m) yields

$$\lambda = \frac{z - \beta b}{c(z - b)}. \tag{A.4.3c}$$

As before, we compute second moments from logged and HP-filtered levels of the variables. Note, our solution procedure yields linear or quadratic policy functions for the stationary variables x_t. Therefore,

$$X_t = Z_{t-1}x_t.$$

We assume $Z_0 \equiv 1$. Given a random sequence $\{\hat{z}_i\}_{i=1}^{t}$, $\hat{z}_i = \ln(z_i/z)$,

$$Z_{t-1} = \prod_{i=1}^{t-1} e^{\hat{z}_i}z.$$

The risk free rate of return is given by

$$r_t = \frac{\Lambda_t}{\beta\mathbb{E}_t\Lambda_{t+1}} = \frac{\lambda_t z_t}{\beta\mathbb{E}_t\lambda_{t+1}}$$

and can be computed via Gauss-Hermite quadrature from the policy function for λ_t as explained in Section 3. The rates of return on equity from both sectors as defined in (6.8a) involve stationary values only so that no change in the program is required.

A.5 Two-Sector Model with Capital Adjustment Costs

The two-sector model with capital adjustment costs is described by the following 18 equations:

$$w_{Ct} = (1 - \alpha)Z_tN_{Ct}^{-\alpha}K_{Ct}^{\alpha}, \tag{A.5.1a}$$
$$w_{It} = (1 - \alpha)Z_tN_{It}^{-\alpha}K_{It}^{\alpha}, \tag{A.5.1b}$$
$$q_{Ct} = \frac{p_t}{\Phi'(I_{Ct}/K_{Ct})}, \tag{A.5.1c}$$
$$q_{It} = \frac{p_t}{\Phi'(I_{It}/K_{It})}, \tag{A.5.1d}$$
$$N_t = N_{Ct} + N_{It}, \tag{A.5.1e}$$
$$K_t = K_{Ct} + K_{It}, \tag{A.5.1f}$$

$$w_t = \frac{N_{Ct}}{N_t} w_{Ct} + \frac{N_{It}}{N_t} w_{It}, \tag{A.5.1g}$$

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \tag{A.5.1h}$$

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha}, \tag{A.5.1i}$$

$$Y_t = C_t + I_t, \tag{A.5.1j}$$

$$I_t = I_{Ct} + I_{It}, \tag{A.5.1k}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.5.1l}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{Ct+1}, \tag{A.5.1m}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{It+1}, \tag{A.5.1n}$$

$$q_{Ct} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg\{ \alpha Z_{t+1} N_{Ct+1}^{1-\alpha} K_{Ct+1}^{\alpha-1} - \frac{p_{t+1} I_{Ct+1}}{K_{Ct+1}} \tag{A.5.1o}$$

$$+ q_{Ct+1} \left(\Phi(I_{Ct+1}/K_{Ct+1}) + 1 - \delta \right) \Bigg\},$$

$$q_{It} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg\{ p_{t+1} \alpha Z_{t+1} N_{It+1}^{1-\alpha} K_{It+1}^{1-\alpha} - \frac{p_{t+1} I_{It+1}}{K_{It+1}} \tag{A.5.1p}$$

$$+ q_{It+1} \left(\Phi(I_{It+1}/K_{It+1}) + 1 - \delta \right) \Bigg\},$$

$$K_{Ct+1} = \Phi(I_{Ct}/K_{Ct}) K_{Ct} + (1-\delta) K_{Ct}, \tag{A.5.1q}$$

$$K_{It+1} = \Phi(I_{It}/K_{It}) K_{It} + (1-\delta) K_{It}. \tag{A.5.1r}$$

We employ the assumptions about the function Φ from Section 3. Therefore, the model has the same stationary solution as the two sector model in the previous subsection.

A.6 Time-to-Plan Model

In the time-to-plan model in Section 7, the first-order conditions of the firm's problem are:

$$w_t A_t = (1-\alpha) Z_t A_t^{1-\alpha} N_t^{-\alpha} K_t^{\alpha}, \tag{A.6.1a}$$

$$q_t = \omega_4 + \beta \omega_3 \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} + \beta^2 \omega_2 \mathbb{E}_t \frac{\Lambda_{t+2}}{\Lambda_t} + \beta^3 \omega_1 \mathbb{E}_t \frac{\Lambda_{t+3}}{\Lambda_t}, \tag{A.6.1b}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1} (1-\delta) + \beta^4 \mathbb{E}_t \frac{\Lambda_{t+4}}{\Lambda_t} \alpha Z_{t+4} (A_{t+4} N_{t+4})^{1-\alpha} K_{t+4}^{\alpha-1}, \tag{A.6.1c}$$

where q_t denotes the Lagrange multiplier of the constraint (7.7b). Equations (A.6.1b) and (A.6.1c) can be condensed to

$$0 = \mathbb{E}_t \Bigg\{ \omega_4 \left[\beta(1-\delta) \Lambda_{t+1} - \Lambda_t \right] + \omega_3 \beta \left[\beta(1-\delta) \Lambda_{t+2} - \Lambda_{t+1} \right] \tag{A.6.1d}$$

$$+ \omega_2 \beta^2 \left[\beta(1-\delta) \Lambda_{t+3} - \Lambda_{t+2} \right] + \omega_1 \beta^3 \left[\beta(1-\delta) \Lambda_{t+4} - \Lambda_{t+3} \right],$$

$$+ \alpha\beta^4\Lambda_{t+4}Z_{t+4}(A_{t+4}N_{t+4})^{1-\alpha}K_{t+4}^{\alpha-1}\Bigg\}.$$

Temporary Equilibrium in Stationary Variables. We use lower case letters to denote variables per unit of A_t, except for $\lambda_t := \Lambda_t A_t^\eta$. A temporary equilibrium is defined by equations (7.4a), (7.4b), (7.7), (A.6.1a), (A.6.1d), the production function $Y_t = Z_t(A_tN_t)^{1-\alpha}K_t^\alpha$, and the economy's resource constraint $Y_t = C_t + I_t$. Where necessary, we transform these equations so that they only consist of variables without trend. To put the ensuing system into the canonical form of equations (2.51) of Heer and Maußner (2009), we include a set of auxiliary variables v_{it}, $i = 1, 2, \dots, 10$. In the case of utility function (7.5a), the model's equations in terms of stationary variables are given by:

$$\nu_0 N_{t1}^{\nu_1} = \lambda_t w_t, \tag{A.6.2a}$$
$$w_t = (1-\alpha)Z_t N_t^{-\alpha}k_t^\alpha, \tag{A.6.2b}$$
$$y_t = Z_t N_t^{1-\alpha}k_t^\alpha, \tag{A.6.2c}$$
$$y_t = c_t + i_t, \tag{A.6.2d}$$
$$i_t = \sum_{i=1}^4 \omega_i x_{it}, \tag{A.6.2e}$$
$$a x_{1t+1} = x_{2t}, \tag{A.6.2f}$$
$$a x_{2t+1} = x_{3t}, \tag{A.6.2g}$$
$$a x_{3t+1} = x_{4t}, \tag{A.6.2h}$$
$$a k_{t+1} = (1-\delta)k_t + x_{1t}, \tag{A.6.2i}$$
$$\lambda_t = (c_t - (b/a)v_{1t})^{-1} - b\beta(ac_{t+1} - bc_t)^{-1}, \tag{A.6.2j}$$
$$\begin{aligned} 0 = \mathbb{E}_t\Bigg\{ & \omega_4\left[\left(\beta a^{-\eta}\right)(1-\delta)\lambda_{t+1} - \lambda_t\right] \\ & + \left(\beta a^{-\eta}\right)\omega_3\left[\left(\beta a^{-\eta}\right)(1-\delta)v_{2t+1} - v_{2t}\right] \\ & + \left(\beta a^{-\eta}\right)^2\omega_2\left[\left(\beta a^{-\eta}\right)(1-\delta)v_{3t+1} - v_{3t}\right] \\ & + \left(\beta a^{-\eta}\right)^3\omega_1\left[\left(\beta a^{-\eta}\right)(1-\delta)v_{4t+1} - v_{4t}\right] \\ & + \alpha\left(\beta a^{-\eta}\right)^4 v_{4t+1}(Z_{t+1})^{\left(\rho^Z\right)^3} v_{7t+1}^{1-\alpha}v_{10t+1}^{\alpha-1}\Bigg\}, \end{aligned} \tag{A.6.2k}$$
$$v_{1t} = c_{t-1}, \tag{A.6.2l}$$
$$v_{2t} = \lambda_{t+1}, \tag{A.6.2m}$$
$$v_{3t} = v_{2t+1} = \lambda_{t+2}, \tag{A.6.2n}$$
$$v_{4t} = v_{2t+1} = \lambda_{t+3}, \tag{A.6.2o}$$
$$v_{5t} = N_{t+1}, \tag{A.6.2p}$$
$$v_{6t} = v_{5t+1} = N_{t+2}, \tag{A.6.2q}$$
$$v_{7t} = v_{6t+1} = N_{t+3}, \tag{A.6.2r}$$

$$v_{8t} = k_{t+1}, \tag{A.6.2s}$$
$$v_{9t} = v_{8t+1} = k_{t+2}, \tag{A.6.2t}$$
$$v_{10t} = v_{9t+1} = k_{t+3}. \tag{A.6.2u}$$

In the case of utility function (7.5b) equations (A.6.2a) and (A.6.2j) are replaced by

$$w_t\lambda_t = \theta(c_t - (\chi^C/a)c_{t-1})^{1-\eta}(1-N_t)^{\theta(1-\eta)-1}, \tag{A.6.3a}$$
$$\lambda_t = (c_t - (\chi^C/a)c_{t-1})^{-\eta}(1-N_t)^{\theta(1-\eta)} - \chi^C\beta\mathbb{E}_t(ac_{t+1} - \chi^C c_t)^{-\eta}(1-N_{t+1})^{\theta(1-\eta)}. \tag{A.6.3b}$$

On the balanced growth path (i.e., when $Z_t \equiv 1\ \forall t$, and all variables remain unchanged), equation (A.6.2k) reduces to

$$\frac{y}{k} = \frac{a^\eta - \beta(1-\delta)}{\alpha\beta}\left[\omega_1 + \left(\beta a^{-\eta}\right)^{-1}\omega_2 + \left(\beta a^{-\eta}\right)^{-2}\omega_3 + \left(\beta a^{-\eta}\right)^{-3}\omega_4\right].$$

From this equation we can compute k/N, and given N, the stationary stock of capital. k and N allow us to determine y, and, since $i = x = (a-1+\delta)k$ from (A.6.2e)-(A.6.2i), $c = y - i$. w follows from $w = (1-\alpha)(y/N)$ and λ is implied by (A.6.2j). Finally, we use equation (A.6.2a) to determine ν_0 in the model with utility function (7.5a). In the case of utility function (7.5b), equations (A.6.3a) and (A.6.3b) imply θ.

A.7 New-Keynesian Model with Sticky Prices

Households. Households enter the current period t with a given amount of firm shares S_t and given stocks of nominal money balances M_t and nominal Bonds B_t. The current price level is P_t. Bonds pay a predetermined nominal rate of interest $Q_t - 1$. The real share price is v_t and real dividend payments per share are d_t.[16] Firms pay the real wage w_t per unit of working hours N_t. Government transfers to the households are T_t in units of money. Thus,

$$v_t(S_{t+1} - S_t) + \frac{M_{t+1} - M_t}{P_t} + \frac{B_{t+1} - B_t}{P_t} \le w_tN_t + (Q_t - 1)\frac{B_t}{P_t} + d_tS_t + \frac{T_t}{P_t} - C_t \tag{A.7.1}$$

is the household's budget constraint.

[16] De Paoli, Scott, and Weeken (2010) distinguish between real and nominal bonds and consider different maturities of nominal bonds. They also assume that share prices and dividends are denoted in units of money and not in units of goods. However, since the equilibrium conditions of the model boil down to conditions in real share prices and real dividends, we can assume this right away. Furthermore, since our focus is on the cross-correlations of output, hours, and the real wage, we restrict the spectrum of financial assets to a one-period nominal bond, money, and stocks.

The current period utility function is

$$\begin{aligned} u(C_t, C_{t-1}, N_t, N_{t-1}, M_{t+1}/P_t) = {} & \frac{(C_t - \chi^C C_{t-1})^{1-\eta} - 1}{1-\eta} \\ & - \nu_0 \frac{(N_t - \chi^N N_{t-1})^{1+\nu_1} - 1}{1+\nu_1} \\ & + \theta^M \frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\gamma} - 1}{1-\gamma}. \end{aligned} \tag{A.7.2}$$

Households maximize

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, C_{t+s-1}, N_{t+s}, N_{t-1+s}, M_{t+1+s}/P_{t+s})$$

subject to (A.7.1) and given initial values of S_t, M_t, and B_t. De Paoli, Scott, and Weeken (2010) assume that the household treats previous consumption C_{t-1} and previous working hours N_{t-1} as given, when he decides on current consumption and working hours. Thus, different from equations (2.2a) and (4.2), the first order conditions are:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \tag{A.7.3a}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1}, \tag{A.7.3b}$$

$$v_t = \beta \mathbb{E}_t \Lambda_{t+1}(v_{t+1} + d_{t+1}), \tag{A.7.3c}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}, \tag{A.7.3d}$$

$$\Lambda_t = \beta \mathbb{E}_t \left(\theta^M m_{t+1}^{-\gamma} + \frac{\Lambda_{t+1}}{\pi_{t+1}} \right), \quad m_{t+1} = \frac{M_{t+1}}{P_t}, \tag{A.7.3e}$$

where Λ_t is the Lagrange multiplier of the time t budget constraint.

Firms. The Lagrange function of the firm's maximization problem can be written as:

$$\begin{aligned} \mathscr{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Bigg\{ & \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} - w_{t+s} N_{t+s}(j) - I_{t+s}(j) \\ & - \frac{\psi}{2} \left(\frac{P_{t+s}(j)}{\pi P_{t+s-1}(j)} - 1 \right)^2 Y_{t+s} \\ & + q_{t+s} \left[(1-\delta) K_{t+s}(j) + \Phi \left(\frac{I_{t+s}(j)}{K_{t+s}(j)} \right) K_{t+s}(j) - K_{t+s+1}(j) \right] \\ & + \Gamma_{t+s} \left[Z_{t+s} N_{t+s}(j)^{1-\alpha} K_{t+s}(j)^{\alpha} - \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right] \Bigg\}. \end{aligned}$$

Differentiating this expression with respect to $N_t(j)$, $I_t(j)$, $K_{t+1}(j)$, and $P_t(j)$ and setting the ensuing results equal to zero yields the first-order conditions stated in (8.9).

Money Supply. The central bank satisfies the money demand that originates from the Taylor rule (8.10). This implies the money growth factor μ_t:

$$\mu_t = \frac{M_{t+1}}{M_t}. \tag{A.7.4}$$

The government transfers the seignorage lump sum to the households so that

$$\frac{T_t}{P_t} = \frac{M_{t+1} - M_t}{P_t}. \tag{A.7.5}$$

Temporary Equilibrium. In equilibrium the supply of bonds is zero, $B_t = 0$, the supply of shares is constant, and all intermediate producers choose the same nominal price $P_t(j)$ so that – via the definition of the price index (8.5) – the relative price of each producer equals unity, and individual prices $P_t(j)$, output $Y_t(j)$, working hours $N_t(j)$, capital services $K_t(j)$, investment expenditures $I_t(j)$, and dividend payments $D_t(j)$ equal the respective aggregate quantities. Therefore, the budget constraint (A.7.1) simplifies to the economy's resource constraint $Y_t = C_t + I_t$, and (8.6) implies the aggregate production function $Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}$. The dynamics of the model is governed by equations (A.7.3), the simplified equations (8.7), (8.8), (8.9), the resource constraint, the production function, the Taylor rule (8.10), and (A.7.4). For convenience, we repeat this set of equations, yet in a different ordering with the static equations appearing first.

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \tag{A.7.6a}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1}, \tag{A.7.6b}$$

$$w_t = (1-\alpha)\Gamma_t Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.7.6c}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.7.6d}$$

$$Y_t = C_t + I_t, \tag{A.7.6e}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.7.6f}$$

$$d_t = Y_t - w_t N_t - I_t, \tag{A.7.6g}$$

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t/K_t)K_t, \tag{A.7.6h}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg[\alpha \Gamma_{t+1} Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \left(1 - \delta + \Phi(I_{t+1}/K_{t+1})\right) \Bigg], \tag{A.7.6i}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \tag{A.7.6j}$$

$$\Lambda_t = \mathbb{E}_t \left(\theta^M m_{t+1}^{-\gamma} + \beta \frac{\Lambda_{t+1}}{\pi_{t+1}} \right), \tag{A.7.6k}$$

$$v_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (v_{t+1} + d_{t+1}), \tag{A.7.6l}$$

$$m_{t+1} = \frac{\mu_t m_t}{\pi_t}, \tag{A.7.6m}$$

$$Q_{t+1} = Q_t^{\rho^Q} \left(\frac{\pi}{\beta}\right)^{1-\rho^Q} \left(\frac{\pi_t}{\pi}\right)^{\varphi(1-\rho^Q)} e^{\epsilon_t^Q}, \tag{A.7.6n}$$

$$\begin{aligned} 0 = {} & \epsilon Y_t + \epsilon \Gamma_t Y_t - \psi \left(\frac{\pi_t}{\pi} - 1\right) \frac{\pi_t}{\pi} Y_t \\ & + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{\pi_{t+1}}{\pi} - 1\right) \frac{\pi_{t+1}}{\pi} Y_{t+1}. \end{aligned} \tag{A.7.6o}$$

Note that equation (A.7.6g) derives from equation (8.7) if we normalize the outstanding shares to unity. Equation (A.7.6m) is just another way to write the definition of end-of-period real money balances $m_t = M_{t+1}/P_t$ given the definition of the money growth factor μ_t and the inflation factor π_t. Since the nominal interest rate Q_{t+1} is determined in period t, it is non-stochastic with respect to the conditional expectations operator $\mathbb{E}_t$. Thus, condition (8.2d) can be written as

$$\frac{\Lambda_t}{Q_{t+1}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\pi_{t+1}},$$

which allows one to reduce the first-order condition (A.7.6k) to a static equation by using the definition (A.7.6m) and the Taylor rule (A.7.6n).[17] Considering the maximization problem of the firm, equation (A.7.6l) recursively defines the end-of-period value of the firm, if investment is entirely financed from internal funds.

Stationary Equilibrium. As usual, the stationary equilibrium is defined by setting the shocks equal to their unconditional means and by assuming $x_{t+1} = x_t = x$ for all variables x of the model. In this case, equation (A.7.6o) simplifies to

$$\Gamma = \frac{\epsilon - 1}{\epsilon}, \tag{A.7.7a}$$

and equation (A.7.6i) reduces to

$$\frac{Y}{K} = \frac{1 - \beta(1-\delta)}{\alpha \beta \Gamma}, \tag{A.7.7b}$$

so that for given N the stationary stock of capital equals

$$K = N(Y/K)^{\frac{1}{\alpha - 1}}. \tag{A.7.7c}$$

and output Y is determined by (A.7.6d). Given the properties of the adjustment cost function Φ (see Section 3), equation (A.7.6h) implies

$$I = \delta K, \tag{A.7.7d}$$

and we get the stationary value of consumption from the resource constraint (A.7.6e). Given the solution for C we can compute the solution for Λ from (A.7.6a). The

[17] Otherwise, the model must be solved by using the generalized Schur factorization.

stationary real wage follows from equation (A.7.6c). This allows us to determine the parameter ν_0:

$$\nu_0 = \Lambda w \left(N - \chi^N N\right)^{-\nu_1}. \tag{A.7.7e}$$

Dividends d follow from equation (A.7.6g). The stationary share price derives from (A.7.6l):

$$v = \frac{\beta}{1-\beta} d. \tag{A.7.7f}$$

In the stationary equilibrium, the Taylor rule (A.7.6n) fixes the nominal interest rate factor Q for a given inflation target π:

$$Q = \frac{\pi}{\beta}, \tag{A.7.7g}$$

and (A.7.6m) implies $\mu = \pi$. Finally, given θ^M, equation (A.7.6k) can be used to determine the stationary end-of-period level of real money balances m:

$$m = \left(\frac{\Lambda(1-(\beta/\pi))}{\theta^M}\right)^{-1/\gamma}. \tag{A.7.7h}$$

A.8 New Keynesian Model with Sticky Wages

The Optimal Relative Wage. Substituting from (9.2) in (9.6) and (A.7.1) yields the Lagrangian for choosing the optimal wage:

$$\begin{aligned}
\mathscr{L} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Bigg\{ \Bigg[& \frac{(C_{t+s}(h) - \chi^C C_{t+s-1}(h))^{1-\eta} - 1}{1-\eta} \\
& - \frac{\nu_0}{1+\nu_1} \left(\frac{\pi^s W_t(h)}{W_{t+s}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+s}^{1+\nu_1} + \theta \frac{\left(\frac{M_{t+s+1}(h)}{P_{t+s}}\right)^{1-\gamma} - 1}{1-\gamma} \Bigg] \\
& + \Lambda_{t+s}(h) \Bigg[\frac{\pi^s W_t(h)}{P_{t+s}} \left(\frac{\pi^s W_t(h)}{W_{t+s}}\right)^{-\epsilon_w} N_{t+s} + S_{t+s}(h) d_{t+s} \\
& + (Q_{t+s} - 1) \frac{B_{t+s}(h)}{P_{t+s}} - \frac{T_t(h)}{P_{t+s}} - C_{t+s}(h) \\
& - \frac{M_{t+s+1}(h) - M_{t+s}(h) + B_{t+s+1}(h) - B_{t+s}(h)}{P_{t+s}} \\
& - v_{t+s}(S_{t+s+1}(h) - S_{t+s}(h)) \Bigg] \Bigg\}.
\end{aligned}$$

Differentiating with respect to $W_t(h)$ and setting the ensuing expression equal to zero delivers

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \left\{ N_{t+s}(h) \left[\nu_0 N_{t+s}(h)^{\nu_1} - \frac{\epsilon_w - 1}{\epsilon_w} \Lambda_{t+s}(h) \frac{\pi^s W_t(h)}{P_{t+s}} \right] \right\}.$$

We assume that there is a sufficiently rich set of contingent security markets so that a representative agent exists. Thus, $\Lambda_{t+s}(h) = \Lambda_{t+s}$ and all wage setters will opt for the same relative wage $w_{At} \equiv \frac{W_t(h)}{W_t}$. Therefore, the preceding condition can be stated as:

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\Gamma_{1t}}{\Gamma_{2t}}, \tag{A.8.1a}$$

$$\Gamma_{1t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \nu_0 \left(\frac{\pi^s W_t(h)}{W_{t+s}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+s}^{1+\nu_1}, \tag{A.8.1b}$$

$$\Gamma_{2t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Lambda_{t+s} \frac{\pi^s W_t}{P_{t+s}} \left(\frac{\pi^s W_t(h)^{-\epsilon_w}}{W_{t+s}}\right) N_{t+s}. \tag{A.8.1c}$$

The auxiliary variables Γ_{1t} and Γ_{2t} have a recursive definition. Consider (9.8b):

$$\begin{aligned}\Gamma_{1t} = \mathbb{E}_t \Bigg\{ & \nu_0 \left(\frac{W_t(h)}{W_t}\right)^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + (\beta\phi_w)\nu_0 \left(\frac{\pi W_t(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+1}^{1+\nu_1} \\ & + (\beta\phi_w)^2 \nu_0 \left(\frac{\pi^2 W_t(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+2}^{1+\nu_1} + \dots \Bigg\}\end{aligned} \tag{A.8.2}$$

Therefore,

$$\begin{aligned}\Gamma_{1t+1} = \mathbb{E}_{t+1} \Bigg\{ & \nu_0 \left(\frac{W_{t+1}(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+1}^{1+\nu_1} + (\beta\phi_w)\nu_0 \left(\frac{\pi W_{t+1}(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+2}^{1+\nu_1} \\ & + (\beta\phi_w)^2 \nu_0 \left(\frac{\pi^2 W_{t+1}(h)}{W_{t+3}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+3}^{1+\nu_1} + \dots \Bigg\}\end{aligned}$$

From the perspective of period $t+1$ the variables $W_t(h)$, $W_{t+1}(h)$, and W_{t+1} are non-random. Thus, multiplying the previous equation on both sides by

$$(\beta\phi_w)\left(\pi \frac{(W_t(h)/W_t)}{W_{t+1}(h)/W_{t+1}} \frac{W_t}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} \equiv (\beta\phi_w)\left(\frac{\pi w_{At}}{w_{At+1}} \frac{1}{\omega_{t+1}}\right)^{-\epsilon_w(1+\nu_1)}$$

and taking expectations as of period t yields (since $\mathbb{E}_t\mathbb{E}_{t+1}\{\cdot\} = \mathbb{E}_t\{\cdot\}$ by the law of iterated expecations)

$$\begin{aligned}&(\beta\phi_w)\mathbb{E}_t\left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}}\right)^{-\epsilon_w(1+\nu_1)} \Gamma_{1t+1} \\ &= \mathbb{E}_t \Bigg\{ (\beta\phi_w)\nu_0 \left(\frac{\pi W_t(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+1}^{1+\nu_1} \\ &\qquad + (\beta\phi_w)^2 \nu_0 \left(\frac{\pi^2 W_t(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+2}^{1+\nu_1} + \dots \Bigg\}.\end{aligned}$$

Together with (A.8.2) this establishes:

$$\Gamma_{1t} = \nu_0 w_{At}^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + \beta\varphi_w \mathbb{E}_t \left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}}\right)^{-\epsilon_w} \Gamma_{1t+1}, \tag{A.8.3a}$$

Analogously, the recursive definition of the auxiliary variable Γ_{2t},

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta\varphi_w \left(\frac{\pi}{\omega_{t+1}}\right)^{1-\epsilon_w} \left(\frac{w_{At}}{w_{At+1}}\right)^{-\epsilon_w} \Gamma_{2t+1}, \tag{A.8.3b}$$

can be derived, where

$$w_t = \frac{W_t}{P_t}, \tag{A.8.3c}$$

$$\omega_t = \frac{W_t}{W_{t-1}}. \tag{A.8.3d}$$

Finally, note that $W_{t-1}(h) = W_{t-1}$ for those that cannot adjust their wage optimally. Thus, equation (9.3) implies:

$$W_t^{1-\epsilon_w} = (1-\varphi_w) W_{At}^{1-\epsilon_w} + \varphi_w (\pi W_{t-1})^{1-\epsilon_w}$$

or

$$1 = (1-\varphi_w) w_{At}^{1-\epsilon_w} + \varphi_w (\pi/\omega_t)^{1-\epsilon_w}. \tag{A.8.4}$$

Equilibrium Conditions. The equilibrium conditions of the model consist of the firm's optimality conditions stated in (2.11), the production function (2.3), the capital accumulation equation (2.7), the economy's resource constraint implied by the household's budget constraint, the wage setting equations (9.8a)-(9.8d), and the household's optimality conditions (2.2a), (A.7.3c)-(A.7.3e), and the Taylor rule (8.10). We disregard the solution for the stock of real balances so that the following 14 equations determine the time path of Y_t, C_t, I_t, N_t, K_t, w_t, w_{At}, ω_t, Q_t, π_t, q_t, Λ_t, Γ_{1t}, and Γ_{2t}.

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.8.5a}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.8.5b}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.8.5c}$$

$$Y_t = C_t + I_t, \tag{A.8.5d}$$

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\Gamma_{1t}}{\Gamma_2 t}, \tag{A.8.5e}$$

$$1 = (1-\varphi_w) w_{At}^{1-\epsilon_w} + \varphi_w (\pi/\omega_t)^{1-\epsilon_w}, \tag{A.8.5f}$$

$$w_t = \frac{\omega_t}{\pi_t} w_{t-1}, \tag{A.8.5g}$$

$$K_{t+1} = (1-\delta) K_t + \Phi(I_t/K_t) K_t, \tag{A.8.5h}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Big\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta\big] \Big\} \tag{A.8.5i}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.8.5j}$$

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} \frac{Q_{t+1}}{\pi_{t+1}}, \tag{A.8.5k}$$

$$\Gamma_{1t} = \nu_0 w_{At}^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + \beta \varphi_w \mathbb{E}_t \left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}} \right)^{-\epsilon_w} \Gamma_{1t+1}, \tag{A.8.5l}$$

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta \varphi_w \left(\frac{\pi}{\omega_{t+1}} \right)^{1-\epsilon_w} \left(\frac{w_{At}}{w_{At+1}} \right)^{-\epsilon_w} \Gamma_{2t+1}, \tag{A.8.5m}$$

$$Q_{t+1} = Q_t^{\rho^Q} \left(\frac{\pi}{\beta} \right)^{1-\rho^Q} \left(\frac{\pi_t}{\pi} \right)^{\varphi(1-\rho^Q)} e^{\epsilon_t^Q}. \tag{A.8.5n}$$

Stationary Solution. In the stationary equilibrium of the deterministic counterpart of the model, equation (A.8.5i) implies

$$\frac{Y}{K} = \frac{1-\beta(1-\delta)}{\alpha\beta}.$$

Given the stationary value of hours N, (A.8.5e) yields the stationary stock of capital

$$K = N \left(\frac{1-\beta(1-\delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}}.$$

Given the assumptions with respect to $\Phi(I/K)$ investment equals $I = \delta K$ so that consumption follows from (A.8.5d). Given C the stationary version of (A.8.5j) yields Λ.

In equilibrium, wage inflation ω must equal price inflation π – the target of the monetary authority. Equation (A.8.5f), thus, implies $w_A = 1$. Therefore, equations (A.8.5e), (A.8.5l), and (A.8.5m) reduce to

$$1 = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\nu_0 N^{1+\nu_1}}{\Lambda w N}.$$

We use this equation to fix the unknown parameter ν_0 yielding:

$$\nu_0 = (1-\alpha) \frac{\epsilon_w - 1}{\epsilon_w} \Lambda K^\alpha N^{-(\alpha+\nu_1)}.$$